

Menu Cost Model

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Timing

Within a period t

1. Idiosyncratic and aggregate shocks realize
2. Firms make pricing decisions
3. Aggregate prices realize

Final Good Aggregators

There's a final goods producer who is just a CES aggregator. They simply exist to give us three things: - The demand for each intermediate good - The Aggregate price index - Aggregate output

Aggregate output is

$$Y_t = \left(\int y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

Demand for a variety i is given by

$$y_{it} = p_{it}^{-\epsilon} Y_t$$

Where p_{it} is the relative price of variety i . (i.e. their nominal price divided by the aggregate price index)

The aggregate price index written in terms of relative prices implies

$$\left(\int p_{it}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} = 1$$

Intermediate Goods

The production function is given by $y_{it} = e^{Z_t a_{it}} L_{it}$

Real operating profit is given by:

$$\Pi_{it} = p_{it} y_{it} - w_t L_{it}$$

Where w_t is the real wage.

Substituting in the demand for good i from the final goods aggregator we get

$$\Pi(p_{it}, a_{it}, Z_t) = \left(p_{it}^{1-\epsilon} - p_{it}^{-\epsilon} \frac{w_t}{e^{Z_t a_{it}}} \right) Y_t$$

The recursive representation of the intermediate goods producer is given by:

$$V_t^{NA}(p_i, a_i) = \Pi\left(\frac{p_i}{1 + \pi_t}, a_i\right) + \mathbb{E}_t \Theta_{t,t+1} V_{t+1}\left(\frac{p_i}{1 + \pi_t}, a'_i\right)$$

$$V_t^A(a_i) = \max_p \Pi(p, a_i) - \kappa + \mathbb{E}_t \Theta_{t,t+1} V_{t+1}(p, a'_i)$$

$$V_t(p_i, a_i) = \max\{V_t^{NA}(p_i, a_i), V_t^A(a_i)\}$$

Where $\pi_t = P_t/P_{t-1}$ is the inflation rate. If a firm doesn't adjust its prices its real price is deflated by the period's inflation rate.

Let $\chi(p_{i,t-1}, a_{it}) = \mathbf{1}\{V^A(a_{it}) \geq V^{NA}(p_{i,t-1}, a_{it})\}$ be the discrete decision on whether to change prices or not.

Conditional on changing prices let the optimal pricing policy be given by $p'(a_{it})$. It only depends on a_{it} .

We can derive the aggregate Labour demand:

$$L_t^d = \int \frac{y_{it}}{e^{Z_t a_{it}}} di \tag{1}$$

$$= \left(\int \frac{p_{it}^{-\epsilon}}{e^{Z_t a_{it}}} di \right) Y_t \tag{2}$$

Step 2 substitutes in the final demand for y_{it} from the final goods aggregator.

Cross Sectional Distribution

Let $\hat{\Omega}_t(p, a)$ be the cross sectional distribution at the beginning of t after shocks have been realized but before any pricing decisions have been made. Let $\Omega_t(p, a)$ be the joint distribution of firms at the end of period t i.e. after pricing decisions have been made.

Consumers

Consumers are standard infinitely lived consumers who give us an euler equation and a static labour equation. We assume their utility functions are $u(C, L) = \log(C) - \zeta \frac{L_t^{1+1/\eta}}{1+1/\eta}$

The Euler equation is:

$$\frac{1}{C_t} = \beta \frac{1+r_t}{1+\pi_{t+1}} \frac{1}{C_{t+1}}$$

The static labour optimality condition is

$$L_t^{1/\eta} \zeta = \frac{w_t}{C_t}$$

Central bank

The central bank runs a taylor rule

$$r_t = r_{t-1} + \phi_\pi(\pi_t - \pi^*) + \phi_Y(Y_t - Y^*)$$

Equilibrium

The recursive equilibrium is a set of functions $\{V^A, V^{NA}, p', \chi, C, Y, w, r, L, \pi, \Omega, \hat{\Omega}, Z\}$ satisfying the equations

$$V_{t-1}^{NA}(p_i, a_i) = \Pi\left(\frac{p_i}{1 + \pi_{t-1}}, a_i\right) + \mathbb{E}_{t-1}\Theta_{t-1,t}V_t\left(\frac{p_i}{1 + \pi_{t-1}}, a'_i\right) \quad (3)$$

$$V_{t-1}^A(a_i) = \Pi(p'_{t-1}(a_i), a_i) - \kappa + \mathbb{E}_{t-1}\Theta_{t-1,t}V_t(p'_{t-1}(a_i), a'_i) \quad (4)$$

$$p'_{t-1}(a_i) = \arg \max_p \Pi(p, a_i) + \mathbb{E}_{t-1}\Theta_{t-1,t}V_t(p, a'_i) \quad (5)$$

$$\chi_{t-1}(p_i, a_i) = \mathbf{1}\{V_{t-1}^A(a_i) > V_{t-1}^{NA}(p_i, a_i)\} \quad (6)$$

$$\hat{\Omega}_t(p_i, a'_i) = \int \Omega_{t-1}(p_i, a_i) \Gamma(a'_i | a_i) da_i \quad (7)$$

$$\begin{aligned} \Omega_t(p'_i, a_i) &= \int \chi(p_i, a_i) \mathbf{1}\{p'(p_i, a_i) = p'_i\} \\ &\quad + (1 - \chi(p_i, a_i)) \mathbf{1}\left\{\frac{p_i}{1 + \pi_t} = p'_i\right\} \hat{\Omega}_t(p_i, a_i) dp_i \end{aligned} \quad (8)$$

$$\frac{1}{C_t} = \beta \frac{1 + r_t}{1 + \pi_{t+1}} \frac{1}{C_{t+1}} \quad (9)$$

$$L_t^{1/\eta} \zeta = \frac{w_t}{C_t} \quad (10)$$

$$r_t = r_{t-1} + \phi_\pi(\pi_t - \pi^*) + \phi_Y(Y_t - Y^*) + \epsilon_t^r \quad (11)$$

$$1 = \left(\int \int p_i^{1-\epsilon} \Omega_t(p_i, a_i) dp_i da_i \right)^{\frac{1}{1-\epsilon}} \quad (12)$$

$$Y_t = C_t + \int \int \kappa \mathbf{1}\{\chi_t(p_i, a_i) = 1\} \hat{\Omega}_t(p_i, a_i) dp_i da_i \quad (13)$$

$$L_t = \int \int p_i^{-\epsilon} e^{-a_i} Y_t \Omega_t(p_i, a_i) dp_i da_i \quad (14)$$

$$Z_t = \rho Z_{t-1} + \epsilon_t^z \quad (15)$$