Menu Cost Model

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Timing

Within a period t

1. Idiosyncratic and aggregate shocks realize
2. Firms make pricing decisions
3. Aggregate prices realize

Final Good Aggregators

There's a final goods producer who is just a CES aggregator. They simply exist to give us three things: - The demand for each intermediate good - The Aggregate price index - Aggregate output

Aggregate output is

$$Y_t = \left(\int y_{it}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} di$$

Demand for a variety i is given by

$$y_{it} = p_{it}^{-\epsilon} Y_t$$

Where p_{it} is the relative price of variety i. (i.e. their nominal price divided by the aggregate price index)

The aggregate price index written in terms of relative prices implies

$$\left(\int p_{it}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} = 1$$

Intermediate Goods

The production function is given by $y_{it} = e^{Z_t a_{it}} L_{it}$

Real operating profit is given by:

$$\Pi_{it} = p_{it} y_{it} - w_t L_t$$

Where w_t is the real wage.

Substituting in the demand for good i from the final goods aggregor we get

$$\Pi(p_{it}, a_{it}, Z_t) = \left(p_{it}^{1-\epsilon} - p_{it}^{-\epsilon} \frac{w_t}{e^{Z_t a_{it}}} \right) Y_t$$

The recursive representation of the intermediate goods producer is given by:

$$V_t^{NA}(p_i, a_i) = \Pi(\frac{p_i}{1+\pi_t}, a_i) + \mathbb{E}_t \Theta_{t,t+1} V_{t+1}(\frac{p_i}{1+\pi_t}, a_i')$$

$$V_t^{A}(a_i) = \max_p \Pi(p, a_i) - \kappa + \mathbb{E}_t \Theta_{t,t+1} V_{t+1}(p, a_i')$$

$$V_t(p_i, a_i) = \max\{V_t^{NA}(p_i, a_i), V_t^{A}(a_i)\}$$

Where $pi_t = P_t / P_{t-1}$ is the inflation rate. If a firm doesnt adjust its prices its real price is deflated by the period's inflation rate.

Let $\chi(p_{i,t-1}, a_{it}) = \mathbf{1}\{V^A(a_{it}) \geq V^{NA}(p_{i,t-1}, a_{it})\}$ be the discrete decision on whether to change prices or not.

Conditional on changing prices let the optimal pricing policy be given by $p'(a_{it})$. It only depends on a_{it}.

We can derive the aggregate Labour demand:

$$L_t^d = \int \frac{y_{it}}{e^{Z_t a_{it}}} di \tag{1}$$

$$= \left(\int \frac{p_{it}^{-\epsilon}}{e^{Z_t a_{it}}} di \right) Y_t \tag{2}$$

Step 2 substitutes in the final demand for y_{it} from the final goods aggregator.

Cross Sectional Distribution

Let $\hat{\Omega}_t(p, a)$ be the cross sectional distribution at the beginning of t after shocks have been realized but before any pricing decisions have been made. Let $\Omega_t(p, a)$ be the joint distribution of firms at the end of period t i.e. after pricing decisions have been made.

Consumers

Consumers are standard infinitely lived consumers who give us an euler equation and a static labour equation. We assume their utility functions are $u(C, L) = \log(C) - \zeta \frac{L_t^{1+1/\eta}}{1+1/\eta}$

The Euler equation is:

$$\frac{1}{C_t} = \beta \frac{1 + r_t}{1 + \pi_{t+1}} \frac{1}{C_{t+1}}$$

The static labour optimality condition is

$$L_t^{1/\eta} \zeta = \frac{w_t}{C_t}$$

Central bank

The central bank runs a taylor rule

$$r_t = r_{t-1} + \phi_\pi(\pi_t - \pi^*) + \phi_Y(Y_t - Y^*)$$

Equilibrium

I denote the aggregate state $S_t = (A_t, Y_t, r_t)$ as a tuple for convenience where required to be explicit.

The recursive equilibrium is a set of functions $\{V^A, V^{NA}, p', \chi, C, Y, w, r, L, \pi, \Omega, \hat{\Omega}, Z\}$ satisfying the equations

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\begin{aligned}
V^{NA}(p_i, a_i; S_{t-1}) &= \Pi(\frac{p_i}{1+\pi_{t-1}}, a_i) + \mathbb{E}_{t-1}\Theta_{t-1,t}V(\frac{p_i}{1+\pi_{t-1}}, a_i'; S_t) && (3)\\
V^{A}(a_i; S_{t-1}) &= \Pi(p_{t-1}'(a_i), a_i) - \kappa + \mathbb{E}_{t-1}\Theta_{t-1,t}V(p_{t-1}'(a_i), a_i'; S_t) && (4)\\
p'(a_i; S_{t-1}) &= \arg\max_p \Pi(p, a_i) + \mathbb{E}_{t-1}\Theta_{t-1,t}V(p, a_i'; S_t) && (5)\\
\chi(p_i, a_i; S_t) &= \mathbf{1}\{V^A(a_i; S_t) > V^{NA}(p_i, a_i; S_t)\} && (6)\\
\hat{\Omega}_t(p_i, a_i') &= \int \Omega_{t-1}(p_i, a_i)\Gamma(a_i'|a_i)da_i && (7)\\
\Omega_t(p_i', a_i) &= \int \chi(p_i, a_i)\mathbf{1}\{p'(p_i, a_i) = p_i'\} \\
&\quad + (1-\chi(p_i, a_i))\mathbf{1}\{\frac{p_i}{1+\pi_t} = p_i'\}\hat{\Omega}_t(p_i, a_i)dp_i && (8)\\
\frac{1}{C_t} &= \beta\frac{1+r_t}{1+\pi_{t+1}}\frac{1}{C_{t+1}} && (9)\\
L_t^{1/\eta}\zeta &= \frac{w_t}{C_t} && (10)\\
r_t &= r_{t-1} + \phi_\pi(\pi_t - \pi^*) + \phi_Y(Y_t - Y^*) + \epsilon_t^r && (11)\\
1 &= \left(\int\int p_i^{1-\epsilon}\Omega_t(p_i, a_i)dp_i da_i\right)^{\frac{1}{1-\epsilon}} && (12)\\
Y_t &= C_t + \int\int \kappa\mathbf{1}\{\chi_t(p_i, a_i) = 1\}\hat{\Omega}_t(p_i, a_i)dp_i da_i && (13)\\
L_t &= \int\int p_i^{-\epsilon}e^{-a_i}Y_t\Omega_t(p_i, a_i)dp_i da_i && (14)\\
Z_t &= \rho Z_{t-1} + \epsilon_t^z && (15)
\end{aligned}
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