

Linear combination of vectors and Computation of determinant, rank of a matrix

1. **Vector Addition:** Write a Python program to add two vectors of the same dimension using lists.
2. **Scalar Multiplication:** Implement a function that multiplies a vector by a scalar value.
3. **Dot Product of Vectors:** Create a function that computes the dot product of two vectors using basic loops.
4. **Matrix Creation:** Write a function to create a 2x2 matrix using nested lists.
5. **Matrix Addition:** Implement a function that adds two matrices of the same size element-wise.
6. **Linear Combination of Two Vectors:** Write a function to compute a linear combination of two vectors, given the vectors and scalar coefficients.
7. **Transpose of a Matrix:** Write a program that transposes a 2x3 matrix using nested loops.
8. **Check Linear Dependence:** Implement a function that checks if two vectors are linearly dependent.
9. **2x2 Determinant Calculation:** Create a function that calculates the determinant of a 2x2 matrix using a simple formula.
10. **Matrix Rank Using Row Echelon Form:** Write a program to find the rank of a 2x2 matrix using basic row reduction techniques.
11. **Cross Product of Vectors:** Write a function that calculates the cross product of two 3D vectors.
12. **Matrix Multiplication:** Implement matrix multiplication for two 3x3 matrices using nested loops.
13. **Determinant of a 3x3 Matrix:** Write a function that computes the determinant of a 3x3 matrix using the cofactor expansion method.
14. **Matrix Rank Using NumPy:** Use NumPy to compute the rank of a given matrix and compare it with manual calculations.
15. **Vector Space Basis Identification:** Write a program to identify if a set of vectors forms a basis of a vector space using the rank of the matrix formed by the vectors.
16. **Cramer's Rule for Solving Linear Equations:** Implement a function using the determinant of matrices to solve a system of linear equations with 2 variables using Cramer's rule.
17. **LU Decomposition and Determinant:** Use SciPy to perform LU decomposition on a matrix and compute its determinant.

18. **Eigenvalues and Determinants:** Write a program to find the eigenvalues of a matrix and verify that the product of the eigenvalues equals the determinant of the matrix.
19. **Singular Value Decomposition (SVD):** Use SciPy to perform SVD on a matrix and determine its rank using the singular values.
20. **Principal Component Analysis (PCA):** Implement PCA on a set of data points using NumPy to reduce dimensionality and understand the rank implications.