Group 8 - GLM Exercises

November 10, 2015

1. A density that belongs in the one-parameter exponential family has the following (canonical) representation:

$p(z) = ...$

where θ is called the natural parameter, and c and h are functions whose exact form depends on the particular density. For many members of this family, q > 0, is a further unknown parameter, often called a precision parameter.

1. The following densities belong to the exponential family. Identify θ, q and an appropriate $c(\theta)$, for each of them (take into account that for some z might be a transformation of t):

Normal

[ADD ME]

Bernoulli

$t\tilde{\ }Bern(n,p) = p^z(1-p)^{1-z}$

$exp\left\{log(p^z) + log((1-p)^{1-z})\right\}$

$exp\left\{zlog(p) + (1-z)log(1-p)\right\}$

$exp\left\{zlog(p) + log(1-p) - zlog(1-p)\right\}$

$exp\left\{zlog(\frac{p}{1-p}) + log(1-p)\right\}$

Result:

$\theta = log(\frac{p}{1-p})$

$q = 1$

$c(\theta) = -log(1-p)$

Binomial

$t\tilde{\ }Bin(n,p) = \binom{n}{k}p^k(1-p)^{n-k}$

$exp\left\{log(\binom{n}{k}) + log(p^k) + log((1-p)^{n-k})\right\}$

$$exp\left\{log(\binom{n}{k}) + klog(p) + (n-k)log(1-p)\right\}$$

$$exp\left\{log(\binom{n}{k}) + klog(\frac{p}{1-p}) + nlog(1-p)\right\}$$

$$exp\left\{n\left[\frac{k}{n}log(\frac{p}{1-p}) + log(1-p)\right] + log(\binom{n}{k})\right\}$$

Result:

$\theta = log(\frac{p}{1-p})$

$q = n$

$c(\theta) = log(\binom{n}{k})$

Poisson

[ADD ME]

2. Identify the canonical link function for each of the models given above.

I JUST LOOKED THIS UP I'M NOT SURE IT'S CORRECT.

Normal

μ

Bernoulli

$logit(\mu)$

Binomial

$logit(\mu)$

Poisson

$log(\mu)$

3. Consider now the generalised linear model:

$t_n|x_n -> NdEF(\theta(x_n, w), q\gamma_n)$ with

$\theta(x_n, w) = (c')^{-1}(g^{-1}(\phi(x_n)^T w)) =: f(\phi(x_n)^T w)$

[ADD ME]