Multiple Testing Project

2.2 P-value

1. Distribution of the p-value under the null

1.1 Show that for any α, $c_\alpha = F_H^{-1}(1-\alpha)$

We know that $\alpha = 1 - F_H(c_\alpha)$ from the definition of c_α.

$\alpha = 1 - F_H(c_\alpha)$

$\alpha - 1 = -F_H(c_\alpha)$

$-(\alpha - 1) = F_H(c_\alpha)$

$1 - \alpha = F_H(c_\alpha)$

$F_H^{-1}(1-\alpha) = c_\alpha$

$c_\alpha = F_H^{-1}(1-\alpha)$

Q.E.D.

1.2 Show that the p-value of the test, as a function of the data X used, is given by $p(\mathbf{X}) = 1 - F_H(T(\mathbf{X}))$.

The p-value is defined as $p - value = inf\{\alpha : T(X) \in R_\alpha\}$

Which is to say that the p-value is the *smallest* α for which $T(X)$ is in the region R_α of the probability distribution P_H

So the p-value is an instance of α, which is defined as $\alpha = 1 - F_H(c_\alpha)$ where c_α is chosen so that the equation is true. Therefore, if we replace c_α with our test statistic $T(\mathbf{X})$, we get a $p(\mathbf{X}) = 1 - F_H(\mathbf{X})$.

This matters because it highlights the choice of α as evaluating the minimum value of the p-value $p(\mathbf{X})$ such that we reject the hypothesis that X is from the same distribution as Y.