

Dynamics

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9. a. $a = \frac{dv}{dt}$

c. $v = \frac{ds}{dt}$

$$\int_0^t a dt = v - u$$

$$\frac{ds}{dt} = u + at$$

$$\therefore v = u + at$$

$$\Rightarrow \Delta s = ut + \frac{1}{2}at^2$$

b. $v^2 = u^2 + 2uat + a^2t^2$

$$= u^2 + 2a(ut + \frac{1}{2}at^2)$$

$$= u^2 + 2as$$

d. $s = \frac{1}{2}gt^2 = 80m$

$$v = gt = 40ms^{-1}$$

If it took 50 seconds, air resistance would not be negligible so suvat can't be used.

e. a. $v = u + \bar{a}t$ where \bar{a} is ^{time} averaged acceleration

b, c. can't be easily fixed but do hold for short t but a can't be considered constant

10. $P = Fv \Rightarrow \frac{dv}{dt} \cdot v = \frac{P}{m}$

$$\Rightarrow \frac{1}{2} v^2 = \frac{P}{m} t$$

$$\Rightarrow v = \sqrt{\frac{2P}{m} t}$$

$$a = \frac{P}{mv} = \frac{P}{\sqrt{2Pt_m}} = \sqrt{\frac{P}{2tm}}$$

a. $v(7) = 18.7 \text{ ms}^{-1}$
 $\approx 19 \text{ ms}^{-1} \text{ (2sf)}$

b. $a(7) = 1.3 \text{ ms}^{-2} \text{ (2sf)}$

This is fairly reasonable. A gearbox means as ω (angular speed of wheel) increases, r decreases so $P = T\omega$ is roughly constant.

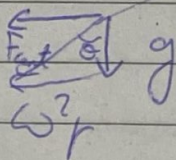
11. a. $v = \omega r$

b. $T = \frac{2\pi r}{\omega}$

c. $a = \omega^2 r$

d. $E_k = \frac{1}{2} m r^2 \omega^2$

e.



$$\theta = \arctan\left(\frac{\omega^2 r}{g}\right)$$

$$T = m \sqrt{g^2 + (\omega^2 r)^2}$$

12. a. $E_{k,1} = \frac{1}{2} m r^2 \omega^2$ fixed axis

$$E_{k,2} = \frac{1}{2} m r^2 \omega^2 + \frac{1}{2} m (r\omega)^2 = m r^2 \omega^2 \quad \text{rolling w/o slipping}$$

\uparrow rotational \uparrow translational

$$b. i. dm = \frac{2\pi r dr}{\pi R^2} m = \frac{2mr}{R^2} dr$$

$$ii. v = \omega r$$

$$iii. dE_k = \frac{mr}{R^2} dr \cdot \omega^2 r^2 = \frac{m\omega^2 r^3 dr}{R^2}$$

$$iv. \Rightarrow E_k = \int_0^R dE_k = \frac{1}{4} m \omega^2 \frac{R^4}{R^2} \\ = \frac{1}{4} m R^2 \omega^2$$