

Oxford Engineering Pre-course Revision Sheets

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1 Mathematics

1.1 Differentiation

1. $\frac{d}{dx} 5x^2 = 10x$
2. $\frac{d}{dx} 4e^x = 4e^x$
3. $\frac{d}{dx} 4 \tan x = 4 \sec^2 x$
4. $\frac{d}{dx} \sqrt{1+x} = \frac{1}{2\sqrt{1+x}}$
5. $\frac{d}{dx} 6 \cos(x^2) = -12x \sin(x^2)$
6. $\frac{d}{dx} e^{3x^4} = 12x^3 e^{3x^4}$
7. $\frac{d}{dx} x^2 \sin x = 2x \sin x + x^2 \cos x$
8. $\frac{d}{dx} \frac{\tan x}{x} = \frac{x \sec^2 x - \tan x}{x^2}$
9. $a(t) = \dot{v}(t) = 40t + 400e^{-t} \implies a(2) = 80 + 400e^{-2} = 134 \text{ ms}^{-2}$. Answer given to 3sf and SI units everywhere are assumed for the units.
10. $\frac{dy}{dx} = 2xe^{-x} - x^2e^{-x} = 0 \implies x(x-2) = 0 \implies x = 0 \text{ or } x = 2$. Consider that the function is everywhere non-negative, and at $x = 0$, it takes the value of 0, so this is a minima and the other stationary point is a maxima. Therefore, minima at $(0, 0)$ and maxima at $(2, 4e^{-2})$.

1.2 Integration

11. $\int_a^b 3x^2 dx = [x^3]_a^b = b^3 - a^3$
12. $\int \sin x \cos^5 x dx = -\frac{1}{6} \cos^6 x + c$
13. $\int x^4 + x^3 dx = \frac{1}{5} x^5 + \frac{1}{4} x^4 + c$
14. $\int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} + c$
15. $\int_0^{2\pi} \sin^2 x dx = \int_0^{2\pi} \frac{1}{2} - \frac{1}{2} \cos 2x dx = [\frac{1}{4}(2x - \sin 2x)]_0^{2\pi} = \pi$
16. $\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\ln |\cos x| + c = \ln |\sec x| + c$

17. Using $x = \sin \theta$, $dx = \cos \theta$, so $\int \frac{1}{\sqrt{1-x^2}} dx = \int \frac{\cos \theta}{\sqrt{1-\sin^2 \theta}} \theta = \theta + c = \arcsin x + c$
18. Using $x = a \sin \theta$, $dx = a \cos \theta$, so $\int \frac{1}{\sqrt{a^2-x^2}} dx = \int \frac{a \cos \theta}{a \sqrt{1-\sin^2 \theta}} \theta = \theta + c = \arcsin \frac{x}{a} + c$
19. $\int x \sin x = -x \cos x + \int \cos x = -x \cos x + \sin x + c$
20. $y = 0 \implies x(8-x^3) = 0 \implies x = 0$ or 2 so $A = \int_0^2 8x - x^4 dx = [4x^2 - \frac{1}{5}x^5]_0^2 = 16 - \frac{32}{5} = \frac{48}{5}$ so the area is $\frac{48}{5}$ units squared.
21. $x(2) = x(0) + \int_0^2 v(t) dt = \int_0^2 (20t^2 - 400e^{-t}) dt = [\frac{20}{3}t^3 + 400e^{-t}]_0^2 = \frac{160}{3} - 400(1 - e^{-2}) = -293$ m. So, the particle is 293 metres from the origin (3sf, assuming SI everywhere).

1.3 Series

22. $10.0 + 11.1 + 12.2 \dots + 19.9 = 10 \times \frac{10+19.9}{2} = 149.5$
23. $S_{10} = \frac{x(1-(2x)^{10})}{1-2x}$
24. $(a+2x)^n = a^n + 2na^{n-1}x + 2n(n-1)a^{n-2}x^2 + \frac{4}{3}n(n-1)(n-2)a^{n-3}x^3$

1.4 Functions

25. The function is undefined when the denominator is 0, which means $x = \pm 1$. When approaching $x = -1$ from below, the function goes to negative infinity. When approaching from above, it goes to positive infinity. When approaching $x = 1$ from below, the limit is negative infinity and from above the limit is positive infinity.

1.5 Complex Algebra

26. (i) $(1+2i) + (2+3i) = 3+5i$ (ii) $(1+2i)(2+3i) = -4+7i$ (iii) $(1+2i)^3 = 1+6i-12-8i = -11-2i$, (iv) $\arg(1+2i) = \arctan 2$ and $\arg(1+2i)^3 = 3\arctan 2 - 2\pi$ where the last correction is because by convention, $\theta \in [-\pi, \pi]$.
27. $z\bar{z} = (x+iy)(x-iy) = x^2 + y^2 + xyi - xyi = x^2 + y^2$.
28. $\frac{1+2i}{3+4i} = \frac{(1+2i)(3-4i)}{(3+4i)(3-4i)} = \frac{1}{25}(11+2i)$
29. $z = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$. When the coefficients of a quadratic are real, the complex solutions are always conjugates
30. $(\cos \theta + i \sin \theta)^2 = (\cos^2 \theta - \sin^2 \theta) + (2 \sin \theta \cos \theta)i = \cos 2\theta + i \sin 2\theta$

1.6 Vectors

31. This can be done by just dividing by the norm so $\hat{\mathbf{v}} = \frac{1}{\sqrt{6}}(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$
32. This can be done by dividing by the norm and then multiplying by $|\vec{OP}| = 3$. (i) $\vec{OP} = \sqrt{3}(\mathbf{i} + \mathbf{j} + \mathbf{k})$. (ii) $\vec{OP} = \frac{3}{\sqrt{14}}(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$.
33. (i) $\mathbf{r} = \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$. (ii) $\mathbf{r} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$.
34. Suppose the relevant point is $\mathbf{r} = \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$. The vector from this point to the reference point is $\mathbf{r}_1 = (\lambda - 3)\mathbf{i} + (\lambda - 4)\mathbf{j} + (\lambda - 5)\mathbf{k}$. Because the distance is minimal, the dot product with the direction vector of the line is 0. So, $3\lambda - 12 = 0$. So, the point is $(4, 4, 4)$.
35. $\cos \theta = \frac{10}{\sqrt{14} \times \sqrt{14}} = \frac{5}{7}$ so the angle between them is $\theta = \arccos \frac{5}{7} = 44.4^\circ$.
36. This is given by $\mathbf{r}_1 + \frac{1}{3}(\mathbf{r}_2 - \mathbf{r}_1) = \frac{2}{3}\mathbf{r}_1 + \frac{1}{3}\mathbf{r}_2$. So the point required is $(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}, \frac{2z_1+z_2}{3})$.
37. The resultant force is given by $\mathbf{f} = 3\mathbf{i} - \mathbf{j}$ and the acceleration takes the same value (adjusting for units) because the body is unit mass. So, $\mathbf{r}(t) = (\frac{3}{2}t^2 + 1)\mathbf{i} + (-\frac{1}{2}t^2 + 2)\mathbf{j}$. For $t > 2$, matching the initial velocities: $\mathbf{r}(t) = (6(t-2) + 1)\mathbf{i} + (-\frac{1}{2}t^2 + 2)\mathbf{j} = (6t - 11)\mathbf{i} + (-\frac{1}{2}t^2 + 2)\mathbf{j}$

2 Electricity

2.1 Current as a flow of charge

$$I = Anev = \frac{\pi}{4}d^2nev, \text{ and } t = \frac{l}{v} = \frac{\pi d^2 nel}{4I} = 1800 \text{ s}$$

2.2 Resistance and Resistivity

Total length is given by $L = N \times D_{\text{avg}} \times \pi$, so the resistance is $R = \frac{4\rho N D_{\text{avg}}}{d^2} = 3.4 \Omega$

Power is given by $P = I^2 R = 120 \text{ W}$

2.3 More resistivity

Suppose you have ρ for Silver and $x\rho$ for Tin where $x = 7.2$. Now consider the perpendicular resistivity. Suppose the specific dimensions are so that if the whole block was Silver, the Resistance would be R . Similarly, it would be xR if the block were Tin. Since $2/3$ of the block is Tin, the resistance due to Tin is $\frac{2xR}{3}$ and for Silver is $\frac{R}{3}$. Therefore, $\rho_{\perp} = (\frac{2}{3}x + \frac{1}{3})\rho$. Now for parallel, using r instead for the same concept as R before, the resistance of the Tin is $\frac{3}{2}xr$ and silver is $3r$ so the effective resistance is $\frac{1}{\frac{1}{3} + \frac{2}{3x}}r$ so $\rho_{\parallel} = \frac{1}{\frac{1}{3} + \frac{2}{3x}}\rho$ and so the final result is that $\frac{\rho_{\perp}}{\rho_{\parallel}} = (\frac{2}{3}x + \frac{1}{3})(\frac{1}{3} + \frac{2}{3x}) = \frac{1771}{810} = 2.19$

2.4 Simplified models

- Ideal conductors are shown as wires and they don't have any resistance in theory but in reality they do have some small non-zero resistance.
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