Oxford Engineering Pre-course Revision Sheets

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1 Mathematics

1.1 Differentiation

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}5x^2 = 10x$

2. $\dfrac{\mathrm{d}}{\mathrm{d}x}4e^x = 4e^x$

3. $\dfrac{\mathrm{d}}{\mathrm{d}x}4\tan x = 4\sec^2 x$

4. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{1+x} = \dfrac{1}{2\sqrt{1+x}}$

5. $\dfrac{\mathrm{d}}{\mathrm{d}x}6\cos\left(x^2\right) = -12x\sin\left(x^2\right)$

6. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x^4} = 12x^3e^{3x^4}$

7. $\dfrac{\mathrm{d}}{\mathrm{d}x}x^2\sin x = 2x\sin x + x^2\cos x$

8. $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\tan x}{x} = \dfrac{x\sec^2 x - \tan x}{x^2}$

9. $a(t) = \dot{v}(t) = 40t + 400e^{-t} \implies a(2) = 80 + 400e^{-2} = 134\ \mathrm{ms}^{-2}$. Answer given to 3sf and SI units everywhere are assumed for the units.

10. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2xe^{-x} - x^2e^{-x} = 0 \implies x(x-2) = 0 \implies x = 0$ or $x = 2$. Consider that the function is everywhere non-negative, and at $x = 0$, it takes the value of 0, so this is a minima and the other stationary point is a maxima. Therefore, minima at $(0,0)$ and maxima at $(2, 4e^{-2})$.

1.2 Integration

11. $\int_a^b 3x^2\mathrm{d}x = [x^3]_a^b = b^3 - a^3$

12. $\int \sin x\cos^5 x\mathrm{d}x = -\frac{1}{6}\cos^6 x + c$

13. $\int x^4 + x^3\mathrm{d}x = \frac{1}{5}x^5 + \frac{1}{4}x^4 + c$

14. $\int \frac{x}{\sqrt{1-x^2}}\mathrm{d}x = -\sqrt{1-x^2} + c$

15. $\int_0^{2\pi} \sin^2 x\mathrm{d}x = \int_0^{2\pi} \frac{1}{2} - \frac{1}{2}\cos 2x\mathrm{d}x = [\frac{1}{4}(2x - \sin 2x)]_0^{2\pi} = \pi$

16. $\int \tan x\mathrm{d}x = \int \frac{\sin x}{\cos x}\mathrm{d}x = -\ln|\cos x| + c = \ln|\sec x| + c$

17. Using $x = \sin\theta$, $\mathrm{d}x = \cos\theta$, so $\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \int \frac{\cos\theta}{\sqrt{1-\sin^2\theta}}\theta = \theta + c = \arcsin x + c$

18. Using $x = a\sin\theta$, $\mathrm{d}x = a\cos\theta$, so $\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \int \frac{a\cos\theta}{a\sqrt{1-\sin^2\theta}}\theta = \theta + c = \arcsin \frac{x}{a} + c$

19. $\int x\sin x = -x\cos x + \int \cos x = -x\cos x + \sin x + c$

20. $y = 0 \implies x(8 - x^3) = 0 \implies x = 0$ or 2 so $A = \int_0^2 8x - x^4\mathrm{d}x = [4x^2 - \frac{1}{5}x^5]_0^2 = 16 - \frac{32}{5} = \frac{48}{5}$ so the area is $\frac{48}{5}$ units squared.

21. $x(2) = x(0) + \int_0^2 v(t)\mathrm{d}t = \int_0^2 (20t^2 - 400e^{-t})\mathrm{d}t = [\frac{20}{3}t^3 + 400e^{-t}]_0^2 = \frac{160}{3} - 400(1 - e^{-2}) = -293$ m. So, the particle is 293 metres from the origin (3sf, assuming SI everywhere).

1.3 Series

22. $10.0 + 11.1 + 12.2... + 19.9 = 10 \times \frac{10+19.9}{2} = 149.5$

23. $S_10 = \frac{x(1-(2x)^10)}{1-2x}$

24. $(a + 2x)^n = a^n + 2na^{n-1}x + 2n(n-1)a^{n-2}x^2 + \frac{4}{3}n(n-1)(n-2)a^{n-3}x^3$

1.4 Functions

25. The function is undefined when the denominator is 0, which means $x = \pm 1$. When approaching $x = -1$ from below, the function goes to to negative infinity. When approaching from above, it goes to positive infinity. When approaching $x = 1$ from below, the limit is negative infinity and from above the limit is positive infinity.

1.5 Complex Algebra

26. (i) $(1 + 2i) + (2 + 3i) = 3 + 5i$ (ii) $(1 + 2i)(2 + 3i) = -4 + 7i$ (iii) $(1 + 2i)^3 = 1 + 6i - 12 - 8i = -11 - 2i$, (iv) $\arg(1 + 2i) = \operatorname{atan} 2$ and $\arg(1 + 2i)^3 = 3\operatorname{atan} 2 - 2\pi$ where the last correction is because by convention, $\theta \in [-\pi, \pi]$.

27. $z\bar{z} = (x + iy)(x - iy) = x^2 + y^2 + xyi - xyi = x^2 + y^2$.

28. $\frac{1+2i}{3+4i} = \frac{(1+2i)(3-4i)}{(3+4i)(3-4i)} = \frac{1}{25}(11 + 2i)$

29. $z = \frac{-2\pm\sqrt{4-8}}{2} = -1 \pm i$. When the coefficients of a quadratic are real, the complex solutions are always conjugates

30. $(\cos\theta + i\sin\theta)^2 = (\cos^2\theta - \sin^2\theta) + (2\sin\theta\cos\theta)i = \cos 2\theta + i\sin 2\theta$

1.6 Vectors

31. This can be done by just dividing by the norm so $\hat{\boldsymbol{v}} = \frac{1}{\sqrt{6}}(\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$

32. This can be done by dividing by the norm and then multiplying by $|\vec{OP}| = 3$. (i) $\vec{OP} = \sqrt{3}(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$. (ii) $\vec{OP} = \frac{3}{\sqrt{14}}(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$.

33. (i) $\boldsymbol{r} = \lambda(\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$. (ii) $\boldsymbol{r} = (\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}) + \lambda(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$.

34. Suppose the relevant point is $\boldsymbol{r} = \lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$. The vector from this point to the reference point is $\boldsymbol{r_1} = (\lambda - 3)\boldsymbol{i} + (\lambda - 4)\boldsymbol{j} + (\lambda - 5)\boldsymbol{k}$. Because the distance is minimal, the dot product with the direction vector of the line is 0. So, $3\lambda - 12 = 0$. So, the point is $(4, 4, 4)$.

35. $\cos\theta = \frac{10}{\sqrt{14}\times\sqrt{14}} = \frac{5}{7}$ so the angle between them is $\theta = \arccos\frac{5}{7} = 44.4°$.

36. This is given by $\boldsymbol{r_1} + \frac{1}{3}(\boldsymbol{r_2} - \boldsymbol{r_1}) = \frac{2}{3}\boldsymbol{r_1} + \frac{1}{3}\boldsymbol{r_2}$. So the point required is $(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}, \frac{2z_1+z_2}{3})$.

37. The resultant force is given by $\boldsymbol{f} = 3\boldsymbol{i} - \boldsymbol{j}$ and the acceleration takes the same value (adjusting for units) because thebody is unit mass. So, $\boldsymbol{r}(t) = (\frac{3}{2}t^2 + 1)\boldsymbol{i} + (-\frac{1}{2}t^2 + 2)\boldsymbol{j}$. For $t > 2$, matching the initial velocities: $\boldsymbol{r}(t) = (6(t-2) + 1)\boldsymbol{i} + (-\frac{1}{2}t^2 + 2)\boldsymbol{j} = (6t - 11)\boldsymbol{i} + (-\frac{1}{2}t^2 + 2)\boldsymbol{j}$