Oxford Engineering Pre-course Revision Sheets

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1 Mathematics

1.1 Differentiation

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}5x^2 = 10x$
2. $\dfrac{\mathrm{d}}{\mathrm{d}x}4e^x = 4e^x$
3. $\dfrac{\mathrm{d}}{\mathrm{d}x}4\tan x = 4\sec^2 x$
4. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{1+x} = \dfrac{1}{2\sqrt{1+x}}$
5. $\dfrac{\mathrm{d}}{\mathrm{d}x}6\cos\left(x^2\right) = -12x\sin\left(x^2\right)$
6. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x^4} = 12x^3e^{3x^4}$
7. $\dfrac{\mathrm{d}}{\mathrm{d}x}x^2\sin x = 2x\sin x + x^2\cos x$
8. $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\tan x}{x} = \dfrac{x\sec^2 x - \tan x}{x^2}$
9. $a(t) = \dot{v}(t) = 40t + 400e^{-t} \implies a(2) = 80 + 400e^{-2} = 134\ \mathrm{ms}^{-2}$. Answer given to 3sf and SI units everywhere are assumed for the units.
10. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2xe^{-x} - x^2e^{-x} = 0 \implies x(x-2) = 0 \implies x = 0$ or $x = 2$. Consider that the function is everywhere non-negative, and at $x = 0$, it takes the value of 0, so this is a minima and the other stationary point is a maxima. Therefore, minima at $(0, 0)$ and maxima at $(2, 4e^{-2})$.

1.2 Integration

11. $\int_a^b 3x^2\mathrm{d}x = [x^3]_a^b = b^3 - a^3$
12. $\int \sin x\cos^5 x\mathrm{d}x = -\frac{1}{6}\cos^6 x + c$
13. $\int x^4 + x^3\mathrm{d}x = \frac{1}{5}x^5 + \frac{1}{4}x^4 + c$
14. $\int \frac{x}{\sqrt{1-x^2}}\mathrm{d}x = -\sqrt{1-x^2} + c$
15. $\int_0^{2\pi}\sin^2 x\mathrm{d}x = \int_0^{2\pi}\frac{1}{2} - \frac{1}{2}\cos 2x\mathrm{d}x = [\frac{1}{4}(2x - \sin 2x)]_0^{2\pi} = \pi$
16. $\int \tan x\mathrm{d}x = \int \frac{\sin x}{\cos x}\mathrm{d}x = -\ln|\cos x| + c = \ln|\sec x| + c$

17. Using $x = \sin\theta$, $\mathrm{d}x = \cos\theta$, so $\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \int \frac{\cos\theta}{\sqrt{1-\sin^2\theta}}\theta = \theta + c = \arcsin x + c$

18. Using $x = a\sin\theta$, $\mathrm{d}x = a\cos\theta$, so $\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \int \frac{a\cos\theta}{a\sqrt{1-\sin^2\theta}}\theta = \theta + c = \arcsin\frac{x}{a} + c$

19. $\int x\sin x = -x\cos x + \int \cos x = -x\cos x + \sin x + c$

20. $y = 0 \implies x(8-x^3) = 0 \implies x = 0$ or 2 so $A = \int_0^2 8x - x^4\mathrm{d}x = [4x^2 - \frac{1}{5}x^5]_0^2 = 16 - \frac{32}{5} = \frac{48}{5}$ so the area is $\frac{48}{5}$ units squared.

21. $x(2) = x(0) + \int_0^2 v(t)\mathrm{d}t = \int_0^2 (20t^2 - 400e^{-t})\mathrm{d}t = [\frac{20}{3}t^3 + 400e^{-t}]_0^2 = \frac{160}{3} - 400(1-e^{-2}) = -293$ m. So, the particle is 293 metres from the origin (3sf, assuming SI everywhere).

1.3 Series

22. $10.0 + 11.1 + 12.2... + 19.9 = 10 \times \frac{10+19.9}{2} = 149.5$

23. $S_10 = \frac{x(1-(2x)^10)}{1-2x}$

24. $(a+2x)^n = a^n + 2na^{n-1}x + 2n(n-1)a^{n-2}x^2 + \frac{4}{3}n(n-1)(n-2)a^{n-3}x^3$

1.4 Functions

25. The function is undefined when the denominator is 0, which means $x = \pm 1$. When approaching $x = -1$ from below,