25. ii. as $|x| \to \infty$, denominator $\sim x^2$
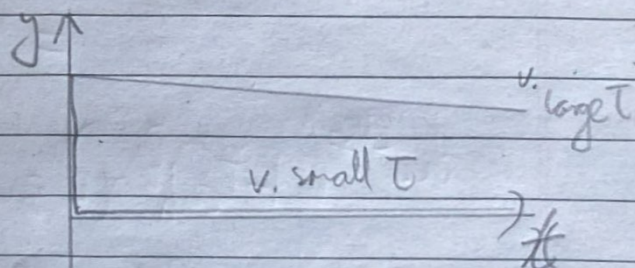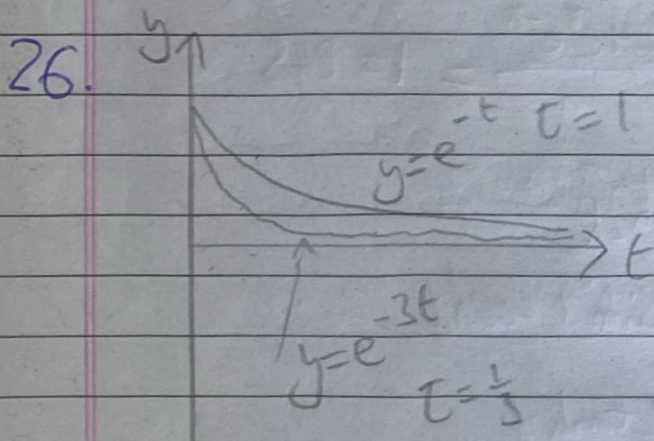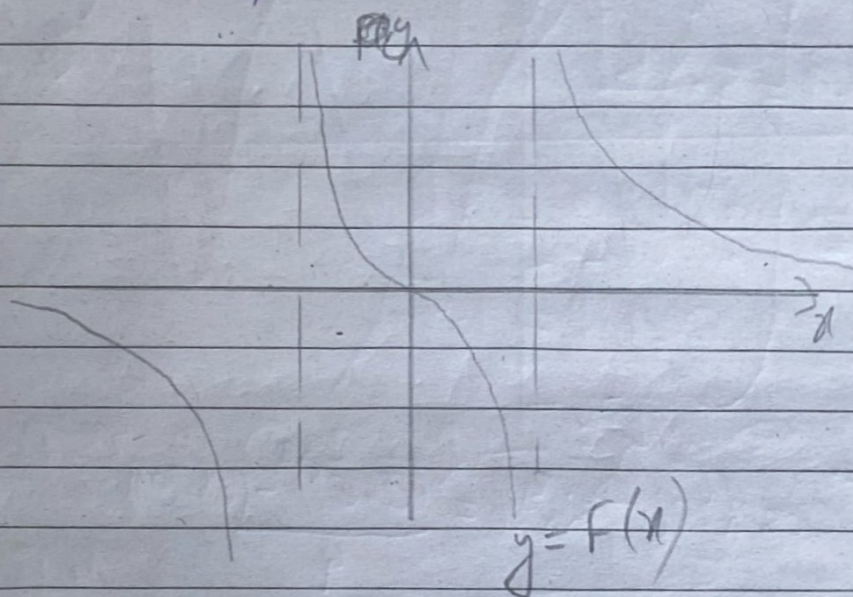
so $f \sim \dfrac{x}{x^2} = \dfrac{1}{x}$

So, $f \to 0$ and $f' \to 0$ as $x \to \pm \infty$

iii. $f'(x) = \dfrac{x^2 - 1 - 2x^2}{(x^2-1)^2} = \dfrac{-(x^2+1)}{(x^2-1)^2} \neq 0$ for any $x \in \mathbb{R}$

iv.

$y = f(x)$

26.

$y = e^{-t}$ $\tau = 1$

$y = e^{-3t}$ $\tau = \frac{1}{3}$

v. small τ

v. large τ

27. $e^{-\frac{t}{10}}$ long timescales

$1000 e^{-10t}$ short timescales

28. $y_1 = 2\cosh t \qquad y_2 = \cos\left(2\omega t + \frac{\pi}{4}\right)$

amplitude 2   amplitude 1

frequency $\dfrac{\omega}{2\pi}$ frequency $\dfrac{\omega}{\pi}$
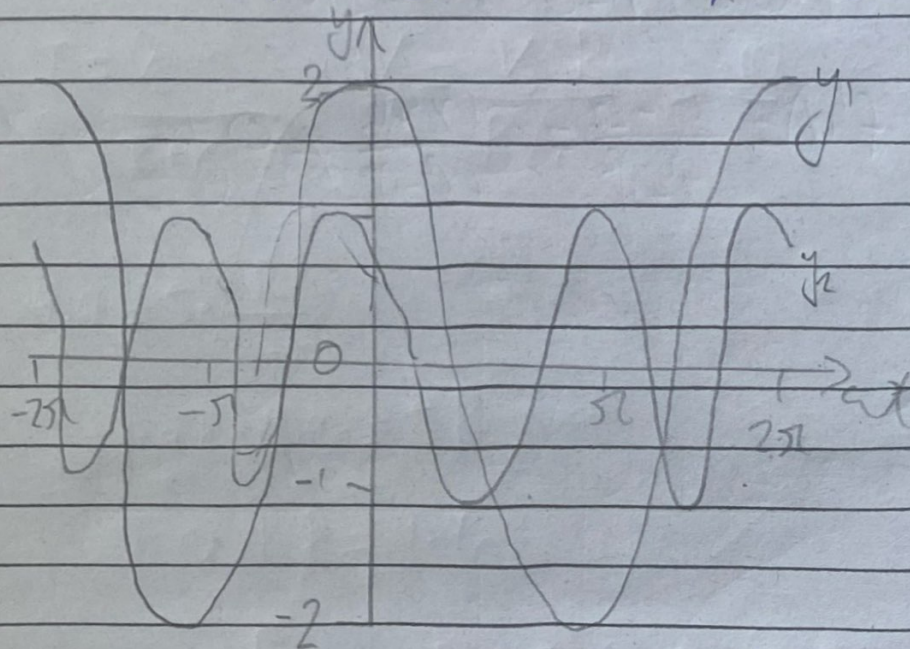
29. i. LHS= $\cosh^2 x - \sinh^2 x = \dfrac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{4}$

$$= \frac{4e^x e^{-x}}{4} = 1 = RHS$$

ii. LHS= $\left(1 - \tanh^2 x\right)\sinh 2x$

$$= \text{sech}^2 x \cdot 2\sinh x \cosh x$$

$$= 2\frac{\sinh x}{\cosh x} = 2\tanh x$$

30. a. $\dfrac{d}{dx}(\cosh x) = \dfrac{e^x - e^{-x}}{2} = \sinh x$,   b. $\dfrac{d}{dx}(\sinh x) = \dfrac{e^x + e^{-x}}{2} = \cosh x$