Oxford Engineering Pre-course Revision Sheets

Harik Sodhi

1 Mathematics

1.1 Differentiation

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}5x^2 = 10x$
2. $\dfrac{\mathrm{d}}{\mathrm{d}x}4e^x = 4e^x$
3. $\dfrac{\mathrm{d}}{\mathrm{d}x}4\tan x = 4\sec^2 x$
4. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{1+x} = \dfrac{1}{2\sqrt{1+x}}$
5. $\dfrac{\mathrm{d}}{\mathrm{d}x}6\cos\left(x^2\right) = -12x\sin\left(x^2\right)$
6. $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x^4} = 12x^3e^{3x^4}$
7. $\dfrac{\mathrm{d}}{\mathrm{d}x}x^2\sin x = 2x\sin x + x^2\cos x$
8. $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{\tan x}{x} = \dfrac{x\sec^2 x - \tan x}{x^2}$
9. $a(t) = \dot{v}(t) = 40t + 400e^{-t} \implies a(2) = 80 + 400e^{-2} = 134\ \mathrm{ms}^{-2}$. Answer given to 3sf and SI units everywhere are assumed for the units.
10. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2xe^{-x} - x^2e^{-x} = 0 \implies x(x-2) = 0 \implies x = 0$ or $x = 2$. Consider that the function is everywhere non-negative, and at $x = 0$, it takes the value of 0, so this is a minima and the other stationary point is a maxima. Therefore, minima at $(0,0)$ and maxima at $(2, 4e^{-2})$.

1.2 Integration

11. $\int_a^b 3x^2\mathrm{d}x = [x^3]_a^b = b^3 - a^3$
12. $\int \sin x\cos^5 x\mathrm{d}x = -\frac{1}{6}\cos^6 x + c$
13. $\int x^4 + x^3\mathrm{d}x = \frac{1}{5}x^5 + \frac{1}{4}x^4 + c$
14. $\int \frac{x}{\sqrt{1-x^2}}\mathrm{d}x = -\sqrt{1-x^2} + c$
15. $\int_0^{2\pi} \sin^2 x\mathrm{d}x = \int_0^{2\pi} \frac{1}{2} - \frac{1}{2}\cos 2x\mathrm{d}x = [\frac{1}{4}(2x - \sin 2x)]_0^{2\pi} = \pi$
16. $\int \tan x\mathrm{d}x = \int \frac{\sin x}{\cos x}\mathrm{d}x = -\ln|\cos x| + c = \ln|\sec x| + c$

17. Using $x = \sin\theta$, $\mathrm{d}x = \cos\theta$, so $\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \int \frac{\cos\theta}{\sqrt{1-\sin^2\theta}}\theta = \theta + c = \arcsin x + c$

18. Using $x = a\sin\theta$, $\mathrm{d}x = a\cos\theta$, so $\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \int \frac{a\cos\theta}{a\sqrt{1-\sin^2\theta}}\theta = \theta + c = \arcsin\frac{x}{a} + c$

19. $\int x\sin x = -x\cos x + \int \cos x = -x\cos x + \sin x + c$

20. $y = 0 \implies x(8 - x^3) = 0 \implies x = 0$ or 2 so $A = \int_0^2 8x - x^4\mathrm{d}x = [4x^2 - \frac{1}{5}x^5]_0^2 = 16 - \frac{32}{5} = \frac{48}{5}$ so the area is $\frac{48}{5}$ units squared.

21. $x(2) = x(0) + \int_0^2 v(t)\mathrm{d}t = \int_0^2 (20t^2 - 400e^{-t})\mathrm{d}t = [\frac{20}{3}t^3 + 400e^{-t}]_0^2 = \frac{160}{3} - 400(1 - e^{-2}) = -293$ m. So, the particle is 293 metres from the origin (3sf, assuming SI everywhere).

1.3 Series

22. $10.0 + 11.1 + 12.2... + 19.9 = 10 \times \frac{10+19.9}{2} = 149.5$

23. $S_10 = \frac{x(1-(2x)^10)}{1-2x}$

24. $(a + 2x)^n = a^n + 2na^{n-1}x + 2n(n-1)a^{n-2}x^2 + \frac{4}{3}n(n-1)(n-2)a^{n-3}x^3$

1.4 Functions

25. The function is undefined when the denominator is 0, which means $x = \pm 1$. When approaching $x = -1$ from below, the function goes to to negative infinity. When approaching from above, it goes to positive infinity. When approaching $x = 1$ from below, the limit is negative infinity and from above the limit is positive infinity.

1.5 Complex Algebra

26. (i) $(1+2i)+(2+3i) = 3+5i$ (ii) $(1+2i)(2+3i) = -4+7i$ (iii) $(1+2i)^3 = 1+6i-12-8i = -11-2i$, (iv) $\arg(1+2i) = \operatorname{atan} 2$ and $\arg(1+2i)^3 = 3\operatorname{atan} 2 - 2\pi$ where the last correction is because by convention, $\theta \in [-\pi, \pi]$.

27. $z\bar{z} = (x+iy)(x-iy) = x^2+y^2+xyi-xyi = x^2+y^2$.

28. $\frac{1+2i}{3+4i} = \frac{(1+2i)(3-4i)}{(3+4i)(3-4i)} = \frac{1}{25}(11+2i)$

29. $z = \frac{-2\pm\sqrt{4-8}}{2} = -1 \pm i$. When the coefficients of a quadratic are real, the complex solutions are always conjugates

30. $(\cos\theta + i\sin\theta)^2 = (\cos^2\theta - \sin^2\theta) + (2\sin\theta\cos\theta)i = \cos 2\theta + i\sin 2\theta$

1.6 Vectors

31. This can be done by just dividing by the norm so $\hat{\boldsymbol{v}} = \frac{1}{\sqrt{6}}(\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$

32. This can be done by dividing by the norm and then multiplying by $|\vec{OP}| = 3$. (i) $\vec{OP} = \sqrt{3}(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$. (ii) $\vec{OP} = \frac{3}{\sqrt{14}}(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$.

33. (i) $\boldsymbol{r} = \lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$. (ii) $\boldsymbol{r} = (\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}) + \lambda(\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k})$.

34. Suppose the relevant point is $\boldsymbol{r} = \lambda(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$. The vector from this point to the reference point is $\boldsymbol{r_1} = (\lambda-3)\boldsymbol{i} + (\lambda-4)\boldsymbol{j} + (\lambda-5)\boldsymbol{k}$. Because the distance is minimal, the dot product with the direction vector of the line is 0. So, $3\lambda - 12 = 0$. So, the point is $(4,4,4)$.

35. $\cos\theta = \frac{10}{\sqrt{14}\times\sqrt{14}} = \frac{5}{7}$ so the angle between them is $\theta = \arccos\frac{5}{7} = 44.4°$.

36. This is given by $\boldsymbol{r_1} + \frac{1}{3}(\boldsymbol{r_2} - \boldsymbol{r_1}) = \frac{2}{3}\boldsymbol{r_1} + \frac{1}{3}\boldsymbol{r_2}$. So the point required is $(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}, \frac{2z_1+z_2}{3})$.

37. The resultant force is given by $\boldsymbol{f} = 3\boldsymbol{i} - \boldsymbol{j}$ and the acceleration takes the same value (adjusting for units) because thebody is unit mass. So, $\boldsymbol{r}(t) = (\frac{3}{2}t^2+1)\boldsymbol{i} + (-\frac{1}{2}t^2+2)\boldsymbol{j}$. For $t > 2$, matching the initial velocities: $\boldsymbol{r}(t) = (6(t-2)+1)\boldsymbol{i} + (-\frac{1}{2}t^2+2)\boldsymbol{j} = (6t-11)\boldsymbol{i} + (-\frac{1}{2}t^2+2)\boldsymbol{j}$

2 Electricity

2.1 Current as a flow of charge

$I = Anev = \frac{\pi}{4}d^2nev$, and $t = \frac{l}{v} = \frac{\pi d^2 nel}{4I} = 1800$ s

2.2 Resistance and Resistivity

Total length is given by $L = N \times D_{\text{avg}} \times \pi$, so the resistance is $R = \frac{4\rho N D_{\text{avg}}}{d^2} = 3.4\ \Omega$
Power is given by $P = I^2R = 120$ W

2.3 More resistivity

Suppose you have ρ for Silver and $x\rho$ for Tin where $x = 7.2$. Now consider the perpendicular resistivity. Suppose the specific dimensions are so that if the whole block was Silver, the Resistance would be R. Similarly, it would be xR if the block were Tin. Since 2/3 of the block is Tin, the resistance due to Tin is $\frac{2xR}{3}$ and for Silver is $\frac{R}{3}$. Therefore, $\rho_\perp = \left(\frac{2}{3}x + \frac{1}{3}\right)\rho$. Now for parallel, using r instead for the same concept as R before, the resistance of the Tin is $\frac{3}{2}xr$ and silver is $3r$ so the effective resistance is $\frac{1}{\frac{1}{3}+\frac{2}{3x}}r$ so $\rho_\parallel = \frac{1}{\frac{1}{3}+\frac{2}{3x}}\rho$ and so the final result is that $\frac{\rho_\perp}{\rho_\parallel} = (\frac{2}{3}x + \frac{1}{3})(\frac{1}{3} + \frac{2}{3x}) = \frac{1771}{810} = 2.19$

2.4 Simplified models

a. Ideal conductors are shown as wires and they don't have any resistance in theory but in reality they do have some small non-zero resistance.