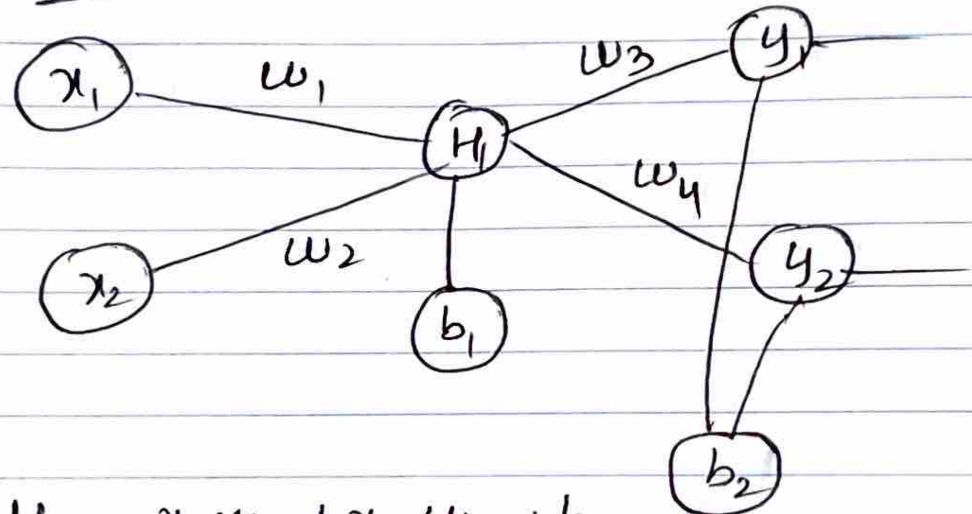
Step 8:-



$H_1 = x_1 w_1 + x_2 w_2 + b_1$

Activation function used is sigmoid $= \dfrac{1}{1+e^{-x}}$

$\boxed{\text{Out } H_1 = \dfrac{1}{1+e^{-H_1}}}$ → activation function used at H_1

Initialising x_1, x_2 as 0.05, 0.10 & w_1, w_2, w_3, w_4 as 0.15, 0.20, 0.25, 0.30 respectively.

Assuming target values $T_1 = 0.01$ and $T_2 = 0.99$ respectively and bias values b_1, b_2 as 0.35 & 0.60

Forward pass:-

$H_1 = x_1 \times w_1 + x_2 \times w_2 + b_1$

$= 0.05 \times 0.15 + 0.10 \times 0.20 + 0.35$

$= 0.3775$

Applying activation function on H_1

$$out\ H_1 = \frac{1}{1+e^{-H_1}}$$

$$= \frac{1}{1+e^{-0.3775}} = 0.59326$$

Calculating the value of y_1 and y_2

$$y_1 = out\ H_1 \times w_3 + b_2$$

$$= 0.59326 \times 0.25 + 0.60$$

$$= 0.748315$$

Now applying activation function on y_1

$$out\ y_1 = \frac{1}{1+e^{-y_1}} = \frac{1}{1+e^{-0.748315}} = \frac{1}{1+0.47316}$$

$$\boxed{out\ y_1 = 0.678812}$$

$$y_2 = out\ H_1 \times w_4 + b_2$$

$$= 0.59326 \times 0.3 + 0.6$$

$$= 0.777978$$

Now applying activation function on y_2

$$out\ y_2 = \frac{1}{1+e^{-y_2}} = \frac{1}{1+e^{-0.777978}} = \frac{1}{1+0.459333}$$

$$\boxed{out\ y_2 = 0.685244}$$

Calculating total error, Here using M.S.E loss function

$$E_{Total} = \sum \frac{1}{2}(\text{target} - \text{output})^2$$

$$= \frac{1}{2}(T_1 - out y_1)^2 + \frac{1}{2}(T_2 - out y_2)^2$$

$$= \frac{1}{2}(0.01 - 0.678812)^2 + \frac{1}{2}(0.99 - 0.685244)^2$$

$$\Rightarrow \frac{1}{2}(-0.668812)^2 + \frac{1}{2}(0.304756)^2$$

$$\Rightarrow \frac{1}{2} \times 0.447309 + \frac{1}{2} \times 0.09287621$$

$$\Rightarrow 0.2236545 + 0.046438105$$

$$= 0.270092605$$

For calculating backward pass using gradient descent optimizer and to update weights of all the w_1, w_2, w_3, w_4.

First, calculating error at $w_3 = \frac{\partial E_{total}}{\partial w_3}$

$$\frac{\partial E_{total}}{\partial w_3} = \frac{\partial E_{total}}{\partial out\, y_1} \times \frac{\partial out\, y_1}{\partial y_1} \times \frac{\partial y_1}{\partial w_3}$$

$$\frac{\partial E_{total}}{\partial out y_1} = 2 \times \frac{1}{2}(T_1 - out y_1) - 1$$

$$\Rightarrow -(T_1 - out y_1)$$

$\Rightarrow -(0.01 - 0.678812)$

$\Rightarrow \cancel{-0.01+}\ 0.668812$

$$\frac{\partial out_{y_1}}{\partial y_1} = out_{y_1}(1 - out_{y_1})$$

$= 0.678812(1 - 0.678812)$

$= 0.678812 \times 0.321188$

$= 0.218026$

$$\frac{\partial y_1}{\partial w_3} = 1 \times out_{H_1} + 0$$

$= out_{H_1} = 0.59326.$

$$\frac{\partial E_{total}}{\partial w_3} = \frac{\partial E_{total}}{\partial out_{y_1}} \times \frac{\partial out_{y_1}}{\partial y_1} \times \frac{\partial y_1}{\partial w_3}$$

$0.0865082 = 0.668812 \times 0.218026 \times 0.59326$

error partial differentiation with w_3 is 0.08650

New weight of w_3 = original $w_3 - \eta \times \frac{\partial E_{total}}{\partial w_3}$

$\Rightarrow$ original $w_3 - \eta \times \frac{\partial E_{total}}{\partial w_3}$

Assume $\eta = 0.1$ learning rate

$\Rightarrow 0.25 - 0.1 \times 0.0865082$

$\Rightarrow 0.25 - 0.00865082$

$= 0.24134$

In the similar way, calculating error at 'w_4'.

$$\frac{\delta E_{total}}{\delta w_4} = \frac{\delta E_{total}}{\delta out y_2} \times \frac{\delta out y_2}{\delta y_2} \times \frac{\delta y_2}{\delta w_4}$$

$$\frac{\delta E_{total}}{\delta out y_2} \Rightarrow 2 \times \frac{1}{2}(T_2 - out y_2) - 1$$

$$= -(T_2 - out y_2)$$

$$= -(0.99 - 0.685244)$$

$$= -0.304756$$

$$\frac{\delta out y_2}{\delta y_2} = out y_2 (1 - out y_2)$$

$$= 0.685244(1 - 0.685244)$$

$$= 0.685244 \times 0.314756$$

$$= 0.21568466$$

$$\frac{\delta y_2}{\delta w_4} = out H_1 \times 1 + 0$$

$$= 0.59326$$

$$\frac{\delta E_{total}}{\delta w_4} = -0.304756 \times 0.21568466 \times 0.59326$$

$$= -0.038995688$$

error partial differentiation with w_4 is

-0.038995688

New weight of w_4 = Original $w_4 - \eta \times \frac{\delta E_{total}}{\delta w_4}$

$= 0.30 - 0.1(-0.038995688)$ Assuming learning rate $\eta = 0.1$

$\Rightarrow 0.30 + 0.0038995688$

$= 0.303899568$

New weight of w_4 is 0.303899568

Calculating error at $w_1 = \frac{\delta E_{total}}{\delta w_1}$

$$\frac{\delta E_{total}}{\delta w_1} = \frac{\delta E_{total}}{\delta out y_1} \times \frac{\delta out y_1}{\delta y_1} \times \frac{\delta y_1}{\delta out H_1} \times \frac{\delta out H_1}{\delta H_1} \times \frac{\delta H_1}{\delta w_1}$$

From the previous calculations, we have values for $\frac{\delta E_{total}}{\delta out y_1}$, $\frac{\delta out y_1}{\delta y_1}$

$\frac{\delta E_{total}}{\delta out y_1} = 0.668812$, $\frac{\delta out y_1}{\delta y_1} = 0.218026$

$\frac{\delta y_1}{\delta out H_1} = 1 \times w_3 + 0 = w_3 \Rightarrow \cancel{0.24134}\ 0.25$

$\frac{\delta out H_1}{\delta H_1} = out H_1 (1 - out H_1)$

$= 0.59326(1 - 0.59326)$

$= 0.59326(0.40674) = 0.2413025$

$\frac{\delta H_1}{\delta w_1} = x_1 \times 1 + 0 \times x_2 + 0 = x_1 = 0.05$

$$\frac{\delta E_{total}}{\delta w_1} = \frac{\delta E_{total}}{\delta outy_1} \times \frac{\delta outy_1}{\delta y_1} \times \frac{\delta y_1}{\delta outH_1} \times \frac{\delta outH_1}{\delta H_1} \times \frac{\delta H_1}{\delta w_1}$$

$$\frac{\delta E_{total}}{\delta w_1} = 0.668812 \times 0.218026 \times 0.25 \times 0.2413025 \times 0.05$$

$$= 0.000439829$$

New weight of w_1 = Original $w_1 - \eta \times \frac{\delta E_{total}}{\delta w_1}$

$$= 0.15 - 0.1 \times 0.000439829$$

$$= 0.15 - 0.0000439829$$

$$= 0.149956017$$

η = learning rate = 0.1

Calculating the error at $w_2 = \frac{\delta E_{total}}{\delta w_2}$

$$\frac{\delta E_{total}}{\delta w_2} = \frac{\delta E_{total}}{\delta outy_1} \times \frac{\delta outy_1}{\delta y_1} \times \frac{\delta y_1}{\delta outH_1} \times \frac{\delta outH_1}{\delta H_1} \times \frac{\delta H_1}{\delta w_2}$$

From the previous calculations, we have the values for $\frac{\delta E_{total}}{\delta outy_1}$, $\frac{\delta outy_1}{\delta y_1}$, $\frac{\delta y_1}{\delta outH_1}$, $\frac{\delta outH_1}{\delta H_1}$.

$\frac{\delta E_{total}}{\delta outy_1} = 0.668812$, $\frac{\delta outy_1}{\delta y_1} = 0.218026$,

$\frac{\delta y_1}{\delta outH_1} = 0.25$, $\frac{\delta outH_1}{\delta H_1} = 0.2413025$

$$\frac{\delta H_1}{\delta w_2} = 0 \times 0 + x_2 \times 1 + 0$$

$$= x_2 = 0.10$$

$$\frac{\delta E_{total}}{\delta w_1} = \frac{\delta E_{total}}{\delta out y_1} \times \frac{\delta out y_1}{\delta y_1} \times \frac{\delta y_1}{\delta out H_1} \times \frac{\delta out H_1}{\delta H_1} \times \frac{\delta H_1}{\delta w_2}$$

$$= 0.668812 \times 0.218026 \times 0.25 \times 0.2413025 \times 0.10$$

$$\Rightarrow 0.0008796586$$

New weight of w_2 = original $w_2 - \eta \times \frac{\delta E_{total}}{\delta w_2}$

$$= 0.20 - 0.1 \times 0.0008796586$$

$$= 0.20 - 0.00008796586$$

$$= 0.199912034$$

Assume $\eta = 0.1$

New weight of w_2 is 0.199912034

New weight of w_1 is 0.149956017
New weight of w_2 is 0.199912034
New weight of w_3 is 0.24134
New weight of w_4 is 0.303899568