

### ASSIGNMENT-5

- 1) To implement the Median of Medians algorithm ensures that you handle the worst-case time complexity efficiently while finding the  $k$ th smallest element in an unsorted array.

$$\text{arr} = [12, 3, 5, 7, 19] \quad k=2$$

$$\text{arr} = [12, 3, 5, 7, 4, 19, 26] \quad k=3$$

$$\text{arr} = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] \quad k=6$$

i)  $\text{arr} = [12, 3, 5, 7, 19] = [3, 5, \textcircled{7}, 12, 19]$

$$\text{Median} = 7$$

Arrange the values less than 7 in leftside of '7' and greater than 7 in rightside.

$$[12, 3, 5, 7, 19] = [3, 5, \textcircled{7}, 12, 19]$$

2nd Smallest element = 5.

ii)  $\text{arr} = [12, 3, 5, 7, 4, 19, 26] \quad k=3$   
Divide into sub arrays:  $[3, 4, 5, \textcircled{7}, 12, 19, 26]$

$$A_1 = [12, 3, 5, 7, 4] \quad \text{and} \quad A_2 = [19, 26]$$

From  $A_1$  Median is = 5

From  $A_2$  Median is = 19

$$\therefore \text{Medians } [5, 19] = 5$$

Partition around 5, Arrange the values less than 5 in leftside of '5' and greater than

$$5 \text{ in rightside, } [3, 4, \underline{5}, 7, 12, 19, 26]$$

= 5 is the 3rd smallest element

arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]  
Divide into subarrays

A<sub>1</sub> = [1, 2, 3, 4, 5] ; A<sub>2</sub> = [6, 7, 8, 9, 10]

Median = 3 ; Median = 8

Median of Medians = [3, 8]

= 3

∴ Partition around 3, Arrange the elements  
than 3 are in left side and greater elements  
right side.

= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

∴ 6<sup>th</sup> smallest ele = 6

2)

To implement a function median-of-medians(arr, k)  
that takes an unsorted array arr and an  
integer k and returns the k<sup>th</sup> smallest element  
in an array.

arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] ; k = 6

arr = [23, 17, 31, 44, 55, 21, 20, 18, 19, 27] ; k = 5

1) arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

sorted arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Median = 6

k = 6 (6<sup>th</sup> smallest element).

2) arr = [23, 17, 31, 44, 55, 21, 20, 18, 19, 27] ; k = 5

Mid =  $\frac{0+9}{2} = 4.5 \approx 5$

∴ Median = 21

sorted  
 $\{17, 18, 19, 20, 21, 23, 27, 31, 40, 55\}$   
0 1 2 3 4 5 6 7 8 9

$\therefore 5 = \text{Median} = 23$

$\therefore k^{\text{th}}$  (smallest element is  $= 21$ ,) ( $6^{\text{th}} = k$ )

Closest pair of points:

- 3) Given an array of points where  $\text{points}[i] = [x_i, y_i]$  represents a point on the X-Y plane and an integer  $k$ , return the  $k$ -closest pair to the origin.

(1) points =  $[[1, 3], [-2, 2], [5, 8], [0, 1]]$ ,  $k=2$ ,

Given

points =  $[[1, 3], [-2, 2], [5, 8], [0, 1]]$

Distance =  $x^2 + y^2$

$$[1, 3] = 1^2 + 3^2 = 10$$

$$[-2, 2] = (-2)^2 + 2^2 = 8$$

$$[5, 8] = 5^2 + 8^2 =$$

$$= 25 + 64 = 89$$

$$[0, 1] = 0^2 + 1^2$$

$$= 1$$

Distance =  $[8, 89, 10, 1]$ .

Now, arrange the points in that order, close to origin.

=  $[[0, 1], [-2, 2], [1, 3], [5, 8]]$

At the value,  $k=2$

consider first 2 points, so the closest pair

=  $[[0, 1], [-2, 2]]$

2) points =  $[[1, 3], [-2, 2]]$ ,  $k=1$

Distance =  $x^2 + y^2$

$$[1, 3] = 1^2 + 3^2 = 10$$

$$\text{Distance} = [10, 8]$$

$$[-2, 2] = 2^2 + 2^2 = 8$$

Arrange the points in such order that are close to the origin by considering distance  
 $= \{[2, 2], [1, 3]\}$

$\rightarrow k=1$

$[-2, 2],$

(iii) points =  $\{[3, 3], [5, -1], [-3, 4]\}$ ,  $k=2$

Distance =  $x^2 + y^2$

$[3, 3] = [9 + 9] = 18$

$[5, -1] = [25 + 1] = 26$

$[-3, 4] = [9 + 16] = 25$

As the arrangement of points should be done in a such a way that are close to origin.  
 $\therefore k=2$ .

$[3, 3], [-3, 4]$

4) Given four lists A, B, C, D of integer values, write a program to compute how many tuples  $(i, j, k, l)$  there are such that  $A[i] + B[j] + C[k] + D[l]$  is zero.

$A = [1, 2]; B = [-2, -1]; C = [-1, 2]; D = [0, 2]$

from collections import defaultdict

def fourlists (A, B, C, D):

AB = defaultdict(int)

for a in A:

for b in B:

AB[a+b] += 1

count = 0

for c in c:

for d in D:

complement = -(c+d)

if complement in AB-sum-counts:

count += AB-sum-counts[complement]

return count.

A = [1, 2]

B = [-3, -1]

C = [-1, 2]

D = [0, 2]

print (four\_sum\_count(A, B, C, D))

(ii) A = [0], B = [0], C = [0], D = [0].

for collections import defaultdict

def four\_sum\_count(A, B, C, D):

AB-sum-counts = defaultdict(int)

for a in A:

for b in B:

AB-sum-counts[a+b] += 1

count = 0

for c in C:

for d in D:

complement = -(c+d)

if complement in AB-sum-counts:

count += AB-sum-counts[complement]

return count.

= 5 is the 3<sup>rd</sup> smallest element.