22/6/24

Y. Harika
192325023
CSA0669

ANALYTICAL ASSIGNMENT - 2

1) Calculate the number of ways to acheive a sum of 15 when rolling four six-sided dice. provide a detailed step-by-step solution

let us assume

$$x_1 + x_2 + x_3 + x_4 = 15 \quad (1 < x_i \leq 6)$$

consider $x_i = y+1$

$$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) = 15$$

$$y_1 + y_2 + y_3 + y_4 = 11$$

By using stars and Bars theorem

$$\binom{n+k-1}{n-1} = \binom{11+4-1}{4-1} = \binom{14}{3} = \frac{14 \times 13 \times 12}{1 \times 2 \times 3} = 364$$

Assume $y_1 = 6 \ (i \leq 6)$

$$y_1' + y_2 + y_3 + y_4 = 5$$

$$\binom{5+4-1}{4-1} = \binom{5+3}{3} = \binom{8}{3} = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 56.$$

Since any of four variables $= 4 \times 56 = 224$

$\Rightarrow y_2 = 6. \ (i \leq 6)$

$$y_1' + y_2' + y_3 + y_4 = -1$$

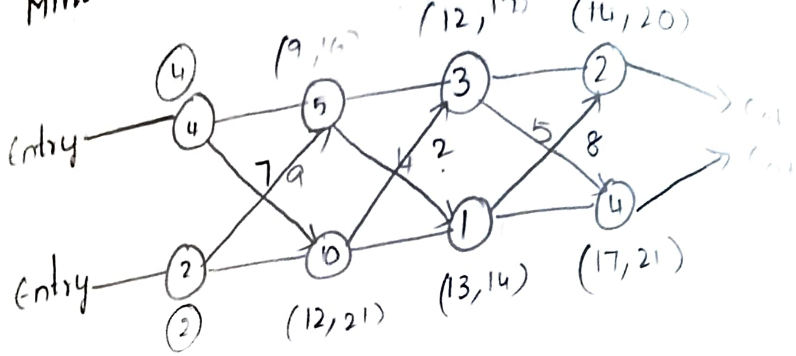
As only non negative values are considered.

$\therefore$ The Number of ways

$$= 364 - 224$$

$$= 140$$

2) Two assembly lines have station times ...
line 1: [4, 5, 3, 2], line 2: [2, 10, 1, 4] ...
times between lines are: from line 1 to line 2: [7, 4, 5], from line 2 to line 1: [9, 2, 8] ...
the Minimum time to assemble a p...



$F_1[j]$	4	9	12	14
$F_2[j]$	2	12	13	17

$l_1[j]$	1	1	1	1
$l_2[j]$	2	2	2	2

3) Given keys {10, 20, 30, 40} with access probabilities {0.1, 0.2, 0.4, 0.3}. Construct the optimal binary search tree. Calculate total cost of tree.

Given

$k = \{10, 20, 30, 40\}$

$P = \{0.1, 0.2, 0.4, 0.3\}$

	0	1	2	3	4
0	0	0.1	[illegible]	[illegible]	[illegible]
1		0	0.2	[illegible]	[illegible]
2			0	0.4	[illegible]
3				0	0.3
4					0

1) $j-i=0$

0-0 = 0 [0,0]

1-1 = 0 [1,1]

2-2 = 0 [2,2]

3-3 = 0 [3,3]

4-4 = 0 [4,4]

2) $j-i=1$

1-0 = [0,1]

2-1 = [1,2]

3-2 = [2,3]

4-3 = [3,4]

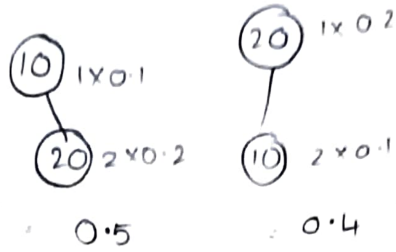
3) $j-i=2$

$2-0=[0,2]$

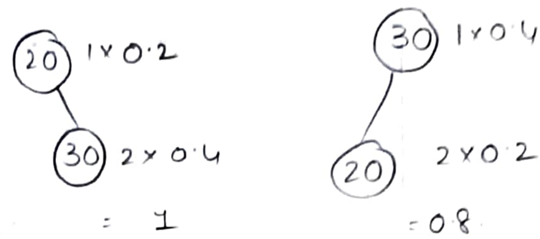
$3-1=[1,3]$

$4-2=[2,4]$

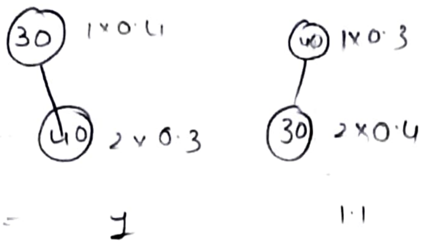
1)



2)



3)



4) $j-i=3$

$3-0=[0,3]$

$4-1=[1,4]$.

$$\mathrm{cost}\overset{i,j}{(0,3)} = \mathrm{Min}\left\{\begin{array}{l}\mathrm{cost}(0,1-1)+\mathrm{cost}(1,3)\\ \mathrm{cost}(0,2-1)+\mathrm{cost}(2,3)\\ \mathrm{cost}(0,3-1)+\mathrm{cost}(3,3)\end{array}\right\} + 0.7$$

$$= \mathrm{Min}\left\{\begin{array}{l}0+0.8\\ 0.1+0.4\\ 0.4+0\end{array}\right\} + 0.7$$

$$= \mathrm{Min}\left\{\begin{array}{l}1.5\\ 1.2\\ 1.1\end{array}\right\} [3].$$

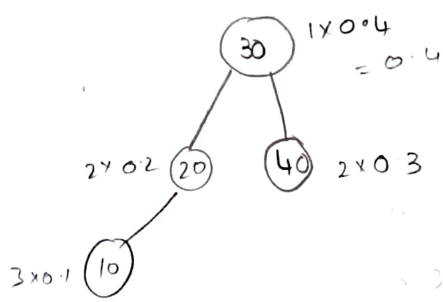
$$\underset{K=2,3,4}{\mathrm{cost}\overset{i,j}{(1,4)}} = \mathrm{Min}\left\{\begin{array}{l}1.0\\ 1.4\\ 1.7\end{array}\right\}$$

$$= 1.0$$

5) $j-i=4$

$4-0=[0,4]$

$$= \mathrm{Min}\left\{\begin{array}{l}2.0\\ 2.1\\ 1.7\\ 2.1\end{array}\right\} = 1.7$$



$= \underline{\underline{1.7}}$

4) Solve the TSP for the following 5-city distance Matrix using dynamic programming.

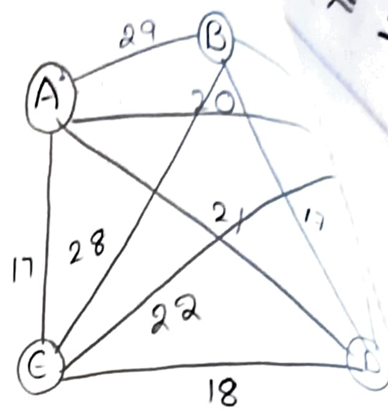
A: [0, 29, 20, 21, 17]

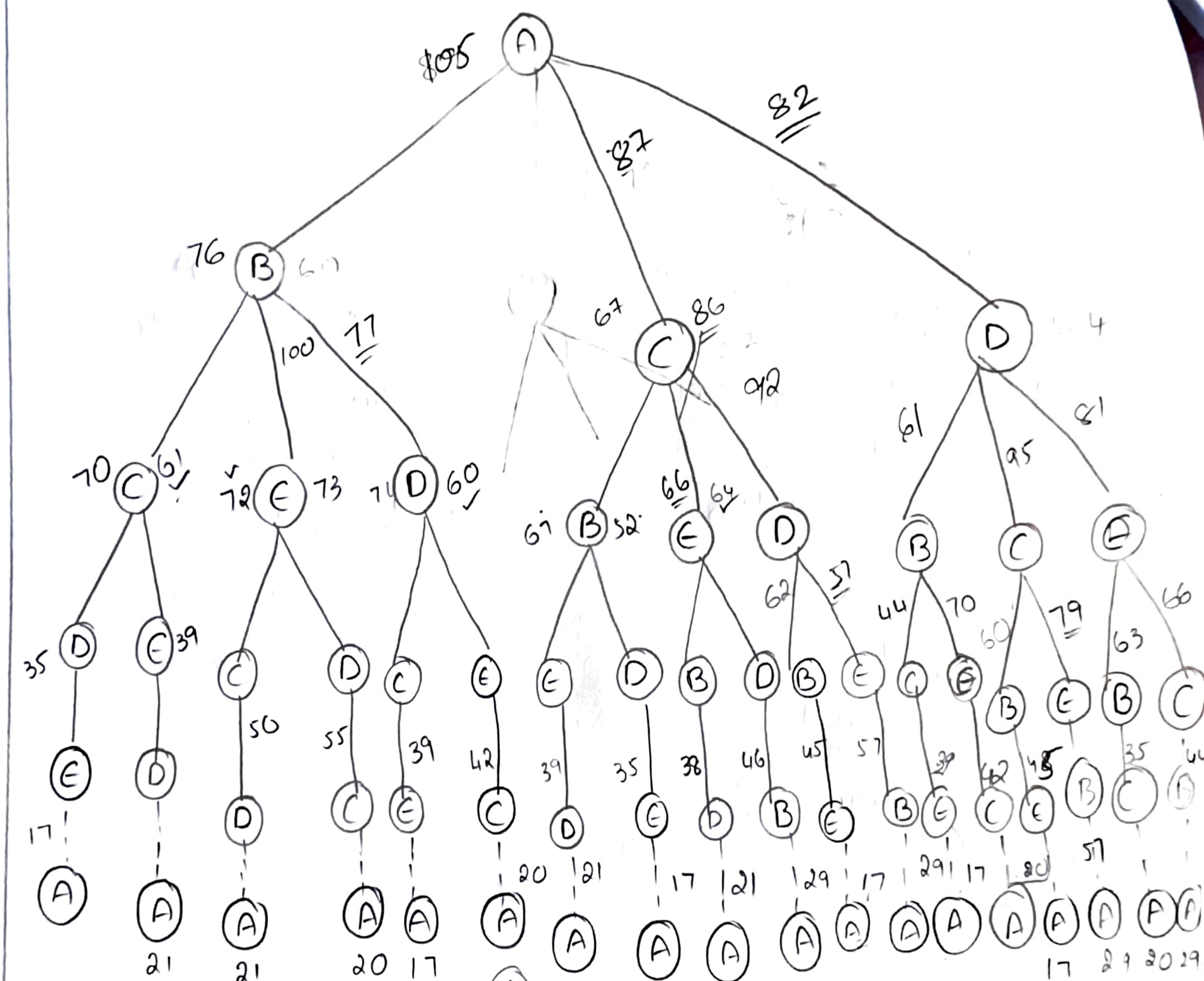
B: [29, 0, 15, 17, 28]

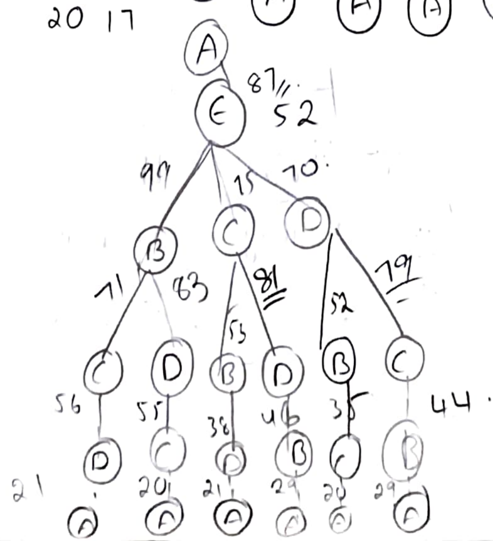
C: [21, 17, 35, 0, 18]

E: [17, 28, 22, 18, 0]

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 29 & 20 & 21 & 17 \\ B & 29 & 0 & 15 & 17 & 28 \\ C & 20 & 15 & 0 & 35 & 22 \\ D & 21 & 17 & 35 & 0 & 18 \\ E & 17 & 28 & 22 & 18 & 0 \end{array}$$







The Min cost

A - E ~~D~~B - ~~C~~ - A

A - E - D - B - C - A

17 + 21 + 15 + 17 + 21

= 82

3 You have a knapsack with a capacity of 50 units. There are 4 items with following weights and values:

I_1 : $W_1 = 10$; $V = 60$

I_2 : $W_2 = 20$; $V = 100$

I_3 : $W_3 = 30$, $V = 120$ $\}$ 220

I_4 : $W_4 = 40$, $V = 200$. Determine using 0/1 knapsack.

W/V	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	220
4	0	60	100	120	200	$\boxed{260}$

I	W	V
1	10	60
2	20	100
3	30	120
4	40	200

6) Given the following directed graph with vertices A, B, C, D and edges with weights.

A → BA \ rightarrow BA → B with wt 1

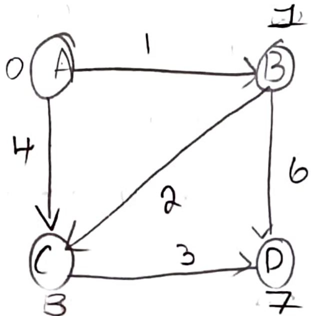
A → CA \ " CA → C " wt 4

B → CB \ " CB → C " wt 2

B → DB \ " DB → D " wt 6

C → DC \ " DC → D " wt 3.

Use the BellmanFord algorithm to find the shortest path from vertex AAA to all other vertices . ST steps.



Initialize:

v	A	B	C	D
d	0	∞	∞	∞
P	-	-	-	-

A - B	1
A - C	4
B - C	2
B - D	6
C - D	3

1)

v	A	B	C	D
d	0	1	4	∞
P	-	A	A	B

2)

v	A	B	C
d	0	1	3
P	-	A	B

3)

v	A	B	C	D
d	0	1	3	6
P	-	A	B	C

path	Shortest distance	shortest path
A - B	1	A - B
A - C	3	A - B - C
A - D	6	A - B - C - D

7. Determine the probability of rolling five dice such that sum is exactly 20. Include a combinatorial approach to arrive at solution.

sum = 20

dice = 5 = $6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

By stars and bars

$\binom{15+5-1}{5-1} = \binom{19}{4} = 3876$

$y_1' + y_2 + y_3 + y_4 + y_5 = 9.$

$$\binom{9+5-1}{5-1} = \binom{13}{4} = 715$$

$$= 5 \times 715 = 3575$$

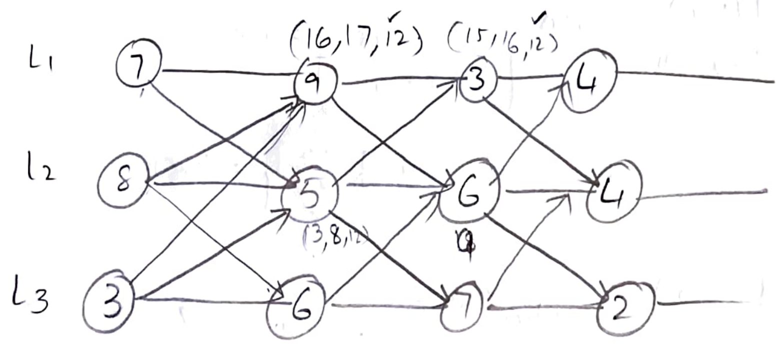
$$y_1' + y_2' + y_3 + y_4 + y_5 = 3$$

$$\binom{3+5-1}{5-1} = \binom{7}{4} = 35$$

$$= 10 \times 35 = 350$$

$$\Rightarrow 3876 - 3575 + 350 = 651 = \frac{651}{7776} \approx 0.0837$$

8). For three assembly lines with station times: Line 1: [7, 9, 3, 4], L2: [8, 5, 6, 4], L3: [3, 6, 7, 2] and transfer times b/w lines given, determine the optimal scheduling and total Min assembly time.



	1	2	3	4
$F_1[j]$	7	13	12	16
$F_2[j]$	8	9	15	17
$F_3[j]$	3	9	16	15

$L_1[j]$	1	3	1	1
$L_2[j]$	2	3	2	1
$L_3[j]$	3	3	3	1

9) Consider keys {15, 25, 35, 45, 55} with acces
{0.05, 0.15, 0.4, 0.25, 0.15}. Determine the struc
of optimal binary search tree and compute

	0	1	2	3	4	
0	0	0.05	0.25 [2]	1.35 [3]	[illegible]	
1		0	0.15	0.70 [3]	[illegible]	
2			0	0.4	0.90	
3				0	0.25	[illegible]
4					0	0
5						0

k = {15, 25, 35, 45, 55}

v = {0.05, 0.15, 0.4, 0.25, 0.15}

j - i = 1

1 - 0 = 1(0,1)
2 - 1 = 1(1,2)
3 - 2 = 1(2,3)
4 - 3 = (3,4)
5 - 4 = (4,5)

j - i = 2

2 - 0 = (0,2)
3 - 1 = (1,3)
4 - 2 = (2,4)
5 - 3 = (3,5)

j - i = 3

3 - 0 = 3(0,3)
4 - 1 = 3 (1,4)
5 - 2 = 3 (2,5)

$$\underset{k=1,2,3}{cost(0,3)} = \min \begin{Bmatrix} 0 + 0.70 \\ 0.05 + 0.4 \\ 0.25 + 0 \end{Bmatrix} + 0.6$$

$$= \min \begin{Bmatrix} 1.30 \\ 1.05 \\ 0.85 \end{Bmatrix} = 0.85$$

$$\underset{k=2,3,4}{cost(1,4)} = \min \begin{Bmatrix} 0 + 0.90 \\ 0.15 + 0.25 \\ 0.70 + 0 \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 1.70 \\ 1.20 \\ 1.50 \end{Bmatrix} = 1.20$$

$$cost(2,5) = \min \begin{Bmatrix} 0 + 0.55 \\ 0.4 + 0.15 \\ 0.90 + 0 \end{Bmatrix} + 0.8 = \min \begin{Bmatrix} 1.35 \\ 1.35 \\ 1.70 \end{Bmatrix} = 1.35$$

j - i = 4

4 - 0 = 4 [0,4]
5 - 1 = 4 [1,5]

$$\underset{k=1,2,3,4}{cost(0,4)} = \min \begin{Bmatrix} 0 + 1.20 \\ 0.05 + 0.90 \\ 0.25 + 0.25 \\ 0.85 + 0 \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.71 \end{Bmatrix} = 1.35$$

$$cost(1,5) = \min_{K=2,3,4,5} \begin{Bmatrix} 0+1.35 \\ 0.15+1.35 \\ 0.70+0.15 \\ 1.20+0 \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 2.30 \\ 2.45 \\ 1.80 \\ 2.15 \end{Bmatrix} = 1.80$$

$j-i=5$

$$c0 = (0,5) = \min_{k=1,2,3,4,5} \begin{Bmatrix} 0+1.80 \\ 0.05+1.35 \\ 0.25+0.55 \\ 0.85+0.15 \\ 1.35+0 \end{Bmatrix} + 1$$

$$= \min \begin{Bmatrix} 2.80 \\ 2.65 \\ 1.80 \\ 2.00 \\ 2.35 \end{Bmatrix} = 1.80.$$

10th person

10) Solve the TSP for 4 cities using simulated annealing with the following distance Matrix

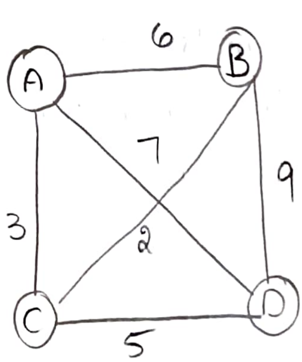
A: [0, 6, 3, 7]

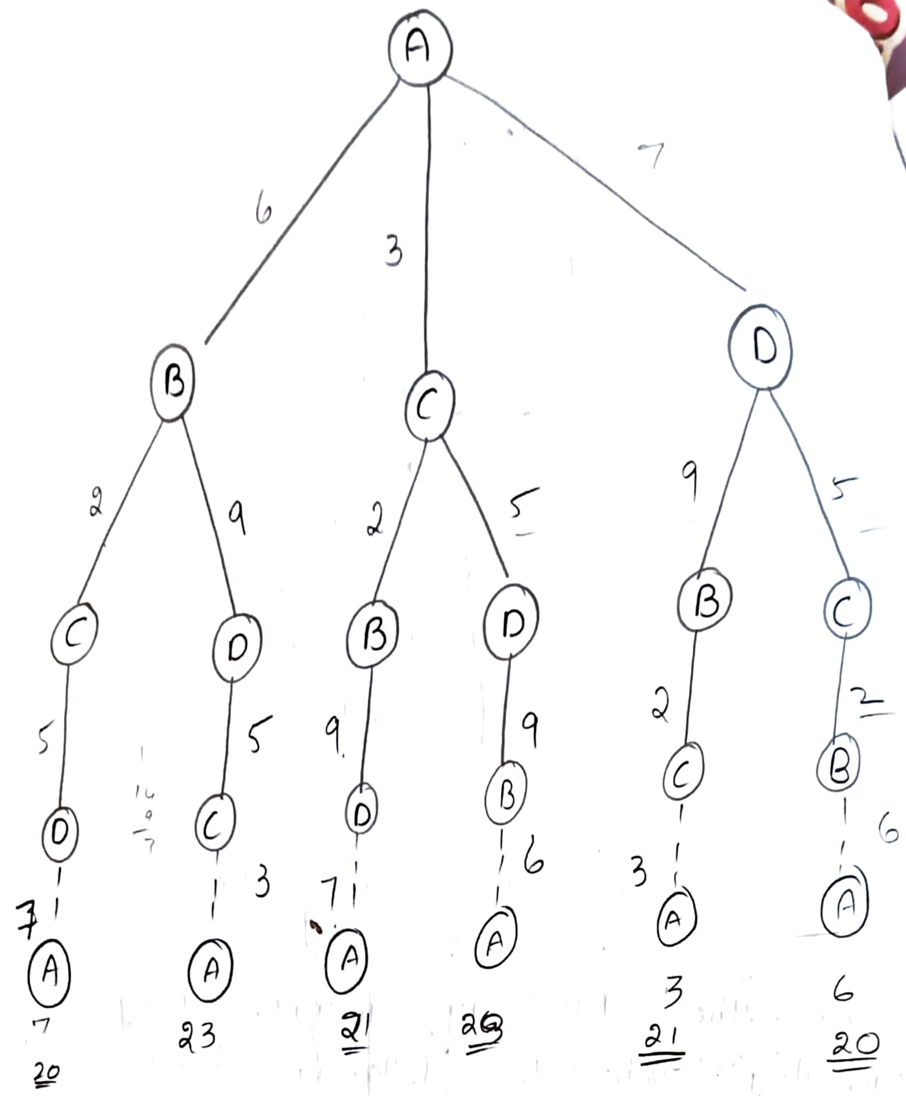
B: [6, 0, 2, 9]

C: [3, 2, 0, 5]

D: [7, 9, 5, 0]

$$\begin{array}{c|c} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 6 & 3 & 7 \\ 6 & 0 & 2 & 9 \\ 3 & 2 & 0 & 5 \\ 7 & 9 & 5 & 0 \end{bmatrix} \end{array}$$





A - B - C - D - A - 20 } Min optimal path
A - D - C - B - A - 20

11) You have a knapsack with capacity of 50 units. There are 4 items with following weights and values.

I_1 W = 10 V = 60

I_2 W = 20 V = 100

I_3 W = 30 V = 120

I_4 W = 40 V = 200

Determine the Min. value by using 0/1 knapsack.

W|V	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	$\boxed{260}$

12). For a graph with vertices A, B, C, D, E, F and the following edges and weights.

A → BA | rightarrow	BA → B - wt 6
A → DA | "	DA → D - wt 7
B → CB | "	CB → C - wt 5
B → EB | "	EB → E - wt -8
B → DB | "	DB → D - wt 8
C → BC | "	BC → B - wt -2
D → CD | "	CD → C - wt -3
D → ED | "	ED → E - wt -9
E → FE | "	FE → F - wt 7
F → CF | "	CF → C - wt 2

Use the Bellman Ford algorithm starting from vertex AAA.

v	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
p	-	-	-	-	-	-

1)

v	A	B	C	D	E	F
d	0	6	~~6~~4	7	2	9
p	-	A	D	A	B	E

2)

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	-	C	D	A	B	E

3)

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	-	C	D	A	B	E.

4)

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	-	C	D	A	B	E.

5)

v	A	B	C	D	E	F
d	0	2	4	7	2	9
p	-	C	D	A	B	E

Vertex	Distance	path
A	0.	A
B	2	A – D – C – B
C	4	A – D – C
D	7.	A – D
E	2	A – D – C – B – E
F	9	A – D – C – B – E – F

13) Find the expected value of sum of the outcomes when rolling there. four sided dice.

sum – 3 (1+1+1)

Sum

$4 = \frac{3}{64}$ (1+1+2, 1+2+1, 2+1+1)

$5 = \frac{6}{64}$ (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

$6 = \frac{10}{64}$ (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2,
3+2+1, 1+4+1, 2+2+2, 2+3+1)

$\frac{12}{64} = 7$ (1+3+3, 2+2+3, 2+3+2, 3+1+3, 3+2+2, 3+3+1,
1+4+2, 2+3+2, 2+4+1, 3+2+2, 3+3+1, 4+1+2))

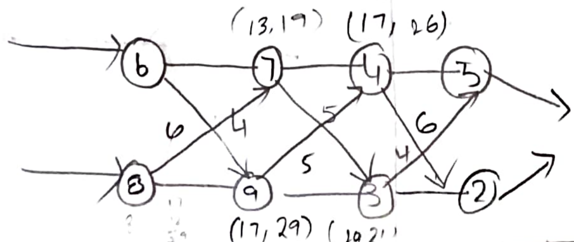
8 - 12/64
9 = 10/64
10 = 6/64
11 = 3/64
12 = 1/64

Σ(sum * probability)

$$= \left(3\times\frac{1}{64}\right) + \left(4\times\frac{3}{64}\right) + \left(5\times\frac{6}{64}\right) + \left(6\times\frac{10}{64}\right) + \left(7\times\frac{12}{64}\right) + 8\left(\times\frac{12}{64}\right)$$
$$\left(9\times\frac{10}{64}\right) + 10\left(\times\frac{6}{64}\right) + \left(11\times\frac{3}{64}\right) + \left(12\times\frac{1}{64}\right)$$

$$= \frac{480}{64} = 7.5.$$

14) Calculate the Min time for product assembly for two assembly lines where line 1 = [6,7,4,5] & l_2 [8,9,3,2] with transfer times b/w lines [4,5,6] from l_1 to l_2 [6,5,4] from [l_2 to l_1].



	1	2	3	4
F1[j]	6	13	17	22
F2[j]	8	17	20	22

	1	2	3	4
l_1	1	1	1	1
l_2	2	1		2

15) Keys $\{10, 20, 30\}$ have probabilites $\{0.2, 0.5, 0.3\}$. the optimal binary search tree & calculate total search cost.

$K = \{10, 20, 30\}$

$V = \{0.2, 0.5, 0.3\}$.

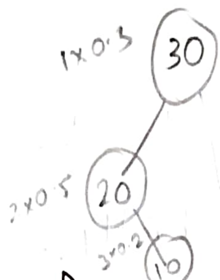
$j - i = 3$

$3 - 0 = (0, 3)$

	0	1	2	
0	0	0.2	[0.7]	[1.1]
1		0	0.5	[1.1]
2			0	0.3
3				0

$$\underset{K=1,2,3}{cost(0,3)} = Min \begin{Bmatrix} 0 + 1.1 \\ 0.2 + 0.3 \\ 0.1 + 0 \end{Bmatrix} + 1.0$$

$$= \begin{Bmatrix} 2.1 \\ 1.5 \\ 1.1 \end{Bmatrix} [3]$$



16) Given a distance Matrix for 6 cities, find the shortest path using the nearest Neighbor heuristic

A : [0, 10, 8, 9, 7, 5]

B : [10, 0, 10, 5, 6, 9]

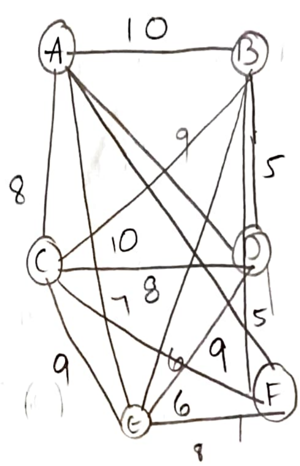
C : [8, 10, 0, 8, 9, 7]

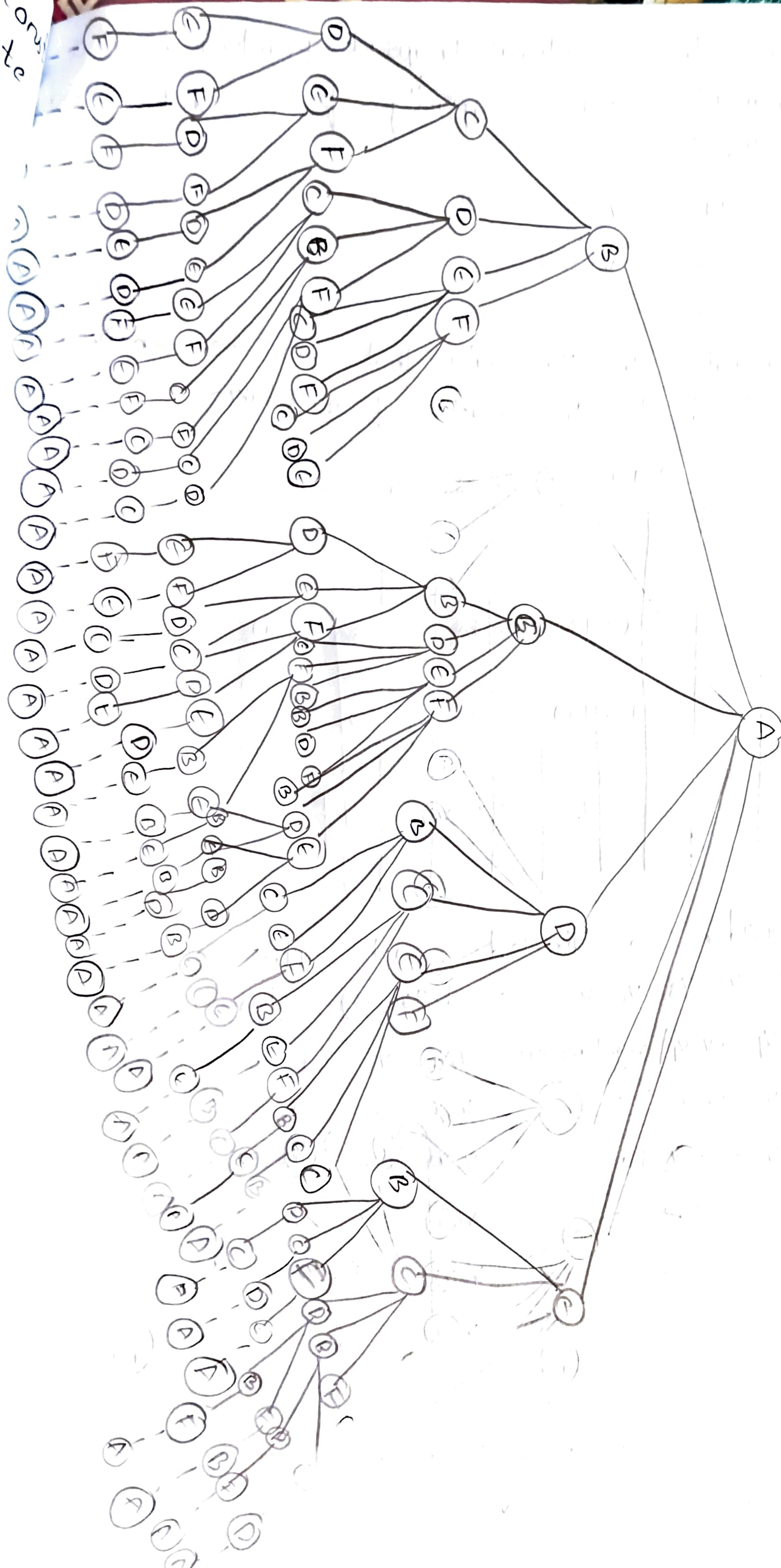
D : [9, 5, 8, 0, 6, 5]

E : [7, 6, 9, 6, 0, 8]

F : [5, 9, 7, 5, 8, 0].

$$\begin{array}{c|cccccc} & A & B & C & D & E & F \\ \hline A & 0 & 10 & 8 & 9 & 7 & 5 \\ B & 10 & 0 & 10 & 5 & 6 & 9 \\ C & 8 & 10 & 0 & 8 & 9 & 7 \\ D & 9 & 5 & 8 & 0 & 6 & 5 \\ E & 7 & 6 & 9 & 6 & 0 & 8 \\ F & 5 & 9 & 7 & 5 & 8 & 0 \end{array}$$





17) Solve the fractional knapsack problem for knapsack with capacity of 60 units follo items

I_1 : W=20; V=100

I_2 : W=30; V=120

I_3 : W=10; V=60

Calculate the Max value that can be acheived and describe the fractions of. items taken

	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	100	100	120	120	120	120
3	0	60	100	120	180	220	$\boxed{280}$

1) 20 100

2) 30 120

4) 10 60

18) Consider a directed graph with 5 vertices V_1, V_2, V_3, V_4, V_5, & following edges and weights

$V_1 \rightarrow V_2V_1$ | rightarrow — $V_2V_1 \rightarrow V_2$ - wt 3

$V_1 \rightarrow V_3V_1$ | '' — $V_3V_1 \rightarrow V_3$ - wt 8

$V_2 \rightarrow V_3V_2$ | '' — $V_3V_2 \rightarrow V_3$ wt 2

$V_2 \rightarrow V_4V_2$ | '' — $V_4V_2 \rightarrow V_4$ wt 5

$V_3 \rightarrow V_4V_3$ | '' — $V_4V_3 \rightarrow V_4$ wt 1

$V_4 \rightarrow V_5V_4$ | '' — $V_5V_4 \rightarrow V_5$ wt 7

$V_5 \rightarrow V_3V_5$ | '' — $V_3V_5 \rightarrow V_3$ wt -4

Apply Bellman Ford algorithm from vertex V_1.

$V_1 - V_2$ - 3

$V_1 - V_3$ - 8

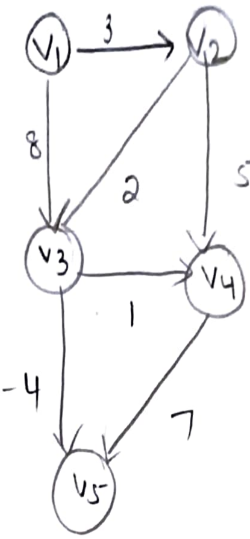
$V_2 \to V_4$ - 5

$V_3 \to V_4$ - 1

$V_3 \to V_5 = -4$

$V_4 \to V_5 = 7$

$V_2 \to V_3 = 2.$



v	V_1	V_2	V_3	V_4	V_5
d	0	∞	∞	∞	∞
p	-	-	-	-	-

1)

v	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	V_4
p	-	V_1	V_2	V_3	13

2)

v	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
p	-	V_1	V_2	V_3	V_4

3)

v	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
p	-	V_1	V_2	V_3	V_4

4)

v	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
p	-	V_1	V_2	V_3	V_4

Vertex	Distance	Shortest path
V_1	0	-
V_2	3	$V_1 \to V_2$
V_3	5	$V_1 \to V_2 \to V_3$
V_4	6	$V_1 \to V_2 \to V_3 \to V_4$
V_5	13	$V_1 \to V_2 \to V_3 \to V_4 \to V_5$

(5)

19) Given two eight-sided dice, compute the NO of ways to acheive a sum of 10. Then extend this to three dice and find the new NO of ways to get sum.

Given

$i = 8(2)$

$S = 10$

possible combination

(2,8) (3,7) (4,6) (5,5) (6,4) (7,3) (8,2)

∴ There are 7 combinations.

~~20~~) Extending to 3 dice

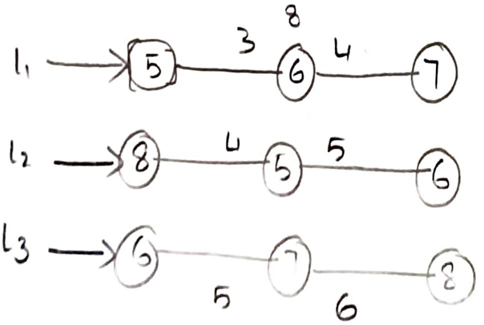
(1,3,6) (1,4,5) (1,5,4) (1,6,3) (2,2,6) (2,3,5) (2,4,4) (2

(3,1,6) (3,2,5) (3,3,4) (3,4,3) (3,5,2). (4,1,5) (4,2,4)

(4,3,3) (4,4,2) (5,1,4) (5,2,3) (5,3,2) (6,1,3) (6,2,2)

∴ There are 21 combinations.

20) Given station times for Line 1: [5, 6, 7], l_2: [8, 5, 6] and l_3: [6, 7, 8] and transfer times b/w lines: [3, 4], [4, 5] and [5, 6], Calculate the Min time req to complete product assembly.



	1	2	3
$F_1[j]$	5	11	18
$F_2[j]$	8	10	16
$F_3[j]$	6	12	21

l_1	1	1	1
l_2	2	2	1
l_3	1	2	1

Given keys {5, 15, 25, 35, 45, 55} with access possibilities
{0.1, 0.05, 0.2, 0.25, 0.30, 0.1) use dp to find obst.

K = {5, 15, 25, 35, 45, 55}

V = {0.1, 0.05, 0.02, 0.25, 0.3, 0.1}.

j - i = 2

2 - 0 = (0,2) (0,1)

3 - 1 = (1,3) (1,2)

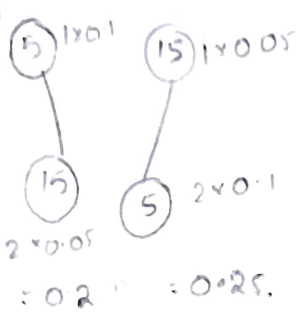
4 - 2 = (2,4) (2,3)

5 - 3 = (3,5) (3,4)

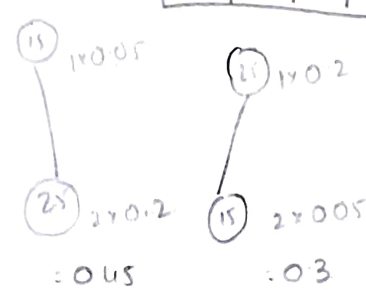
6 - 4 = (4,6) (4,5)

	0	1	2	3	4	5	6
0	0	0.1	$(0.2)^{1}$	$(0.55)^{3}$	$(1.05)^{3}$	[illegible] ^{1,4}	[1.45]
1		0	0.05	$(0.3)^{3}$	$(0.8)^{4}$	$(0.85)^{4}$	$[1.7]^{4}$
2			0	0.2	$(0.65)^{4}$	$[1.25]^{4}$	$[1.6]^{4}$
3				0	0.25	$(0.8)^{4}$	$[1]^{5}$
4					0	0.3	$(0.5)^{4}$
5						0	0.1
6							0

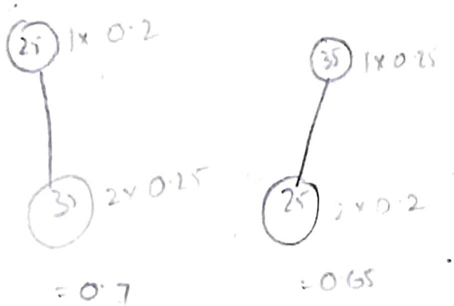
1)



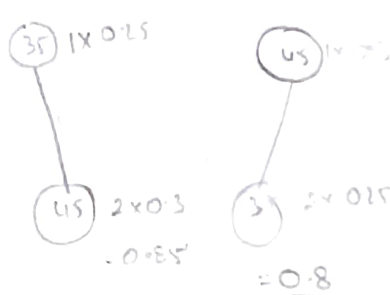
2)



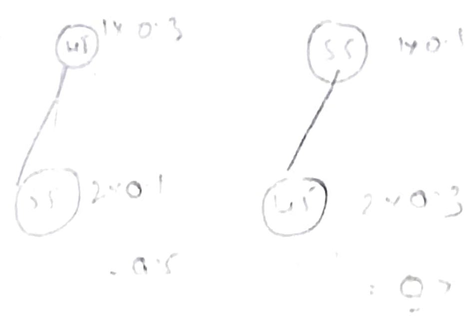
3)



4)



5)



$j-i=3$

$3-0=(0,3)$

$4-1=(1,4)$

$5-2=(2,5)$

$6-3=(3,6).$

$$\underset{k=(1,2,3)}{cost(\overset{i,j}{0,3})} = Min\left\{ cost(i,k-1)+cost(k,j) \right\} + \ldots$$

$$= Min\left\{\begin{array}{l} cost(0,1-1)+cost(1,3) \\ cost(0,2-1)+cost(2,3) \\ cost(0,3-1)+cost(3,3) \end{array}\right\} + 0$$

$$= Min\left\{\begin{array}{l} 0+0.3 \\ 0.1+0.2 \\ 0.2+0 \end{array}\right\} + 0.35 = Min\left\{\begin{array}{l} 0.65 \\ 0.65 \\ \underline{\underline{0.55}} \end{array}\right\}$$

$$\underset{k=2,3,4}{cost(\overset{i,j}{1,4})} = Min\left\{\begin{array}{l} cost(1,2-1)+cost(2,4) \\ cost(1,3-1)+cost(3,4) \\ cost(1,4-1)+cost(4,4) \end{array}\right\} + 0.5$$

$$= Min\left\{\begin{array}{l} 0+0.75 \\ 0.2+0.25 \\ 0.3+0 \end{array}\right\} + 0.5 = Min\left\{\begin{array}{l} 1.25 \\ 0.95 \\ 0.8 \end{array}\right\}$$

$$\underset{k=3,4,5}{cost(\overset{i,j}{2,5})} = Min\left\{\begin{array}{l} cost(2,3-1)+cost(3,5) \\ cost(2,4-1)+cost(4,5) \\ cost(2,5-1)+cost(5,5) \end{array}\right\} + 0.75$$

$$= Min\left\{\begin{array}{l} 0+0.8 \\ 0.2+0.3 \\ 0.65+0 \end{array}\right\} + 0.75 = Min\left\{\begin{array}{l} 1.55 \\ 1.25 \\ 1.4 \end{array}\right\}$$

$$\underset{k=4,5,6}{cost(\overset{i,j}{3,6})} = Min\left\{\begin{array}{l} cost(3,4-1)+cost(4,6) \\ cost(3,5-1)+cost(5,6) \\ cost(3,6-1)+cost(6,6) \end{array}\right\} + 0.65 = Min\left\{\begin{array}{l} 0+0.5 \\ 0.25+0.1 \\ 0.8+0 \end{array}\right\} + 0.65$$

$$= \left\{\begin{array}{l} 1.15 \\ 1 \\ 1.45 \end{array}\right\}$$

$j-i=4$

$4-0=(0,4)$

$6-2=(2,6)$

$5-1=(1,5)$

$$\text{cost}\overset{i,j}{(0,4)} = \begin{Bmatrix} \text{cost}(0,1-1) + \text{cost}(1,4) \\ \text{cost}(0,2-1) + \text{cost}(2,4) \\ \text{cost}(0,3-1) + \text{cost}(3,4) \\ \text{cost}(0,4-1) + \text{cost}(4,4) \end{Bmatrix} + 0.6$$

$k=1,2,3,4$

$$= \text{Min}\begin{Bmatrix} 0+0.8 \\ 0.1+0.65 \\ 0.2+0.25 \\ 0.55+0 \end{Bmatrix} + 0.6 \quad = \text{Min}\begin{Bmatrix} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{Bmatrix} ✓$$

$$\text{cost}\overset{i,j}{(2,6)} = \text{Min}\begin{Bmatrix} \text{cost}(2,3-1) + \text{cost}(3,6) \\ \text{cost}(2,4-1) + \text{cost}(4,6) \\ \text{cost}(2,5-1) + \text{cost}(5,6) \\ \text{cost}(2,6-1) + \text{cost}(6,6) \end{Bmatrix} + 0.85$$

$k=3,4,5,6$

$$= \text{Min}\begin{Bmatrix} 0+1 \\ 0.2+0.5 \\ 0.65+0.1 \\ 1.25+0 \end{Bmatrix} + 0.85 \quad = \text{Min}\begin{Bmatrix} 1.85 \\ 1.55 \\ 1.6 \\ 2.1 \end{Bmatrix} = \underline{\underline{1.55}}$$

$$\text{cost}\overset{i,j}{(1,5)} = \text{Min}\begin{Bmatrix} \text{cost}(1,2-1) + \text{cost}(2,5) \\ \text{cost}(1,3-1) + \text{cost}(3,5) \\ \text{cost}(1,4-1) + \text{cost}(4,5) \\ \text{cost}(1,5-1) + \text{cost}(5,5) \end{Bmatrix} + \underline{\underline{0.8}}$$

$k=2,3,4,5$

$$= \text{Min}\begin{Bmatrix} 0+1.25 \\ 0.05+0.8 \\ 0.55+0.3 \\ 0.8+0 \end{Bmatrix} + 0.8 = \begin{Bmatrix} 2.05 \\ 1.65 \\ 0.85 \\ 1.6 \end{Bmatrix} ✓$$

$j-i=5$

$5-0=[0,5]$

$6-1=[1,6]$

$$\text{cost}(\overset{i,j}{0,5}) = \text{Min}\begin{cases} \text{cost}(0,1-1)+\text{cost}(1,5) \\ \text{cost}(0,2-1)+\text{cost}(2,5) \\ \text{cost}(0,3-1)+\text{cost}(3,5) \\ \text{cost}(0,4-1)+\text{cost}(4,5) \\ \text{cost}(0,5-1)+\text{cost}(5,5) \end{cases} + 0.9$$

$k=1,2,3,4,5$

$$= \text{Min}\begin{Bmatrix} 0+0.85 \\ 0.1+1.25 \\ 0.2+0.8 \\ 0.55+0.3 \\ 1.05+0 \end{Bmatrix}+0.9 = \text{Min}\begin{Bmatrix} 1.75 \\ 2.25 \\ 1.9 \\ 1.75 \\ 1.95 \end{Bmatrix}$$

$$\text{cost}(\overset{i,j}{1,6}) = \text{Min}\begin{cases} \text{cost}(1,2-1)+\text{cost}(2,6) \\ \text{cost}(1,3-1)+\text{cost}(3,6) \\ \text{cost}(1,4-1)+\text{cost}(4,6) \\ \text{cost}(1,5-1)+\text{cost}(5,6) \\ \text{cost}(1,6-1)+\text{cost}(6,6) \end{cases} + 0.9.$$

$k=2,3,4,5,6$

$$= \text{Min}\begin{Bmatrix} 0+1.55 \\ 0.05+1 \\ 0.3+0.5 \\ 0.8+0.1 \\ 0.85+0 \end{Bmatrix}+0.9$$

$$= \text{Min}\begin{Bmatrix} 2.45 \\ 1.95 \\ 1.7 \\ 1.8 \\ 1.75 \end{Bmatrix}$$

Given

I_1 :

I_2 :

I

Given a knapsack capacity of 70 units.

I_1 : $W_1 = 25$; $V = 80$

I_2 : $W_2 = 35$ $V = 90$

I_3 : $W_3 = 45$ $V = 120$

I_4 : $W_4 = 30$ $V = 70$. Use DP to solve 0/1 knapsack.

W/V	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	30	120	$\boxed{150}$

24) For a graph with vertices A, B, C, D, E & the follo weighted edges.

A → BA | rightarrow BA → B - wt -1

A → CA | rightarrow CA → C - wt 4

B → CB | " CB → C - wt 3

B → DB | " DB → D - wt 2

B → EB | " EB → E - wt 2

D → BD | " BD → B - wt 1

D → CD | " CD → C - wt 5

E → DE | " DE → D - wt -3.

Use the Bellman - Ford algorithm from vertex A to shortest path to all other vertices.

$A \to B = -1$

$A \to C = 4$

$B \to C = 3$

$B \to D = 2$

$B \to E = 2$

$D \to B = 1$

$D \to C = 5$

$E \to D = -3$

v	A	B	C	D	E
d	0	∞	∞	∞	∞
p	-		-	-	

1)

v	A	B	C	D	E
d	0	-1	4	∞	∞
p	-	A	A	-	-

2)

v	A	B	C	D	E
d	0	-1	4	∞	1
p	-	A	A	-	B

3)

v	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

4)

v	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

vertex	distance	shortest path
A	0	A
B	-1	$A \to B$
C	4	$A \to C$
D	3	$A \to E \to D$
E	1	$A \to B \to C$.

[8]

25) Using the given distance Matrix for 5 cities determine the shortest route using genetic algorithms.

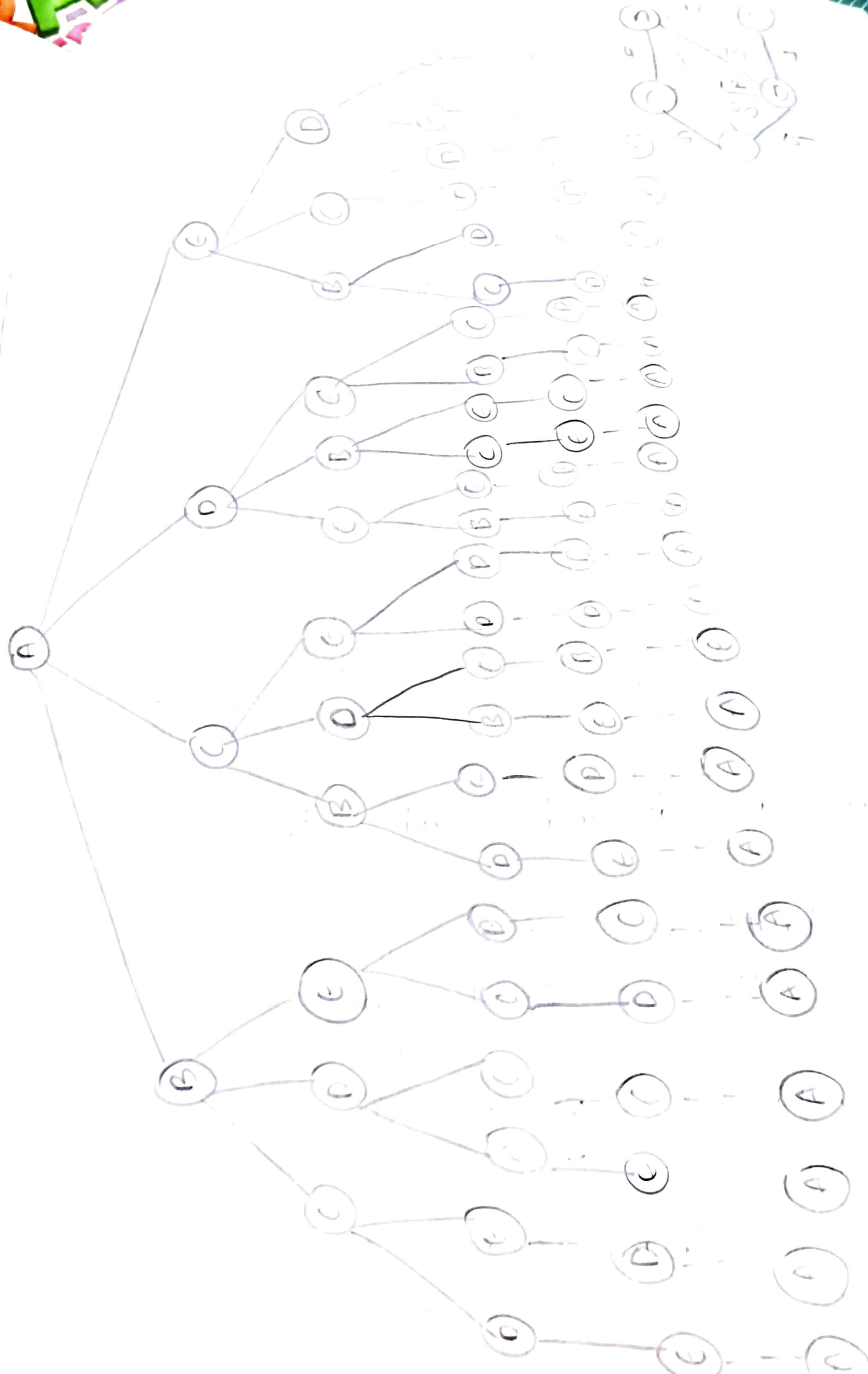
A : [0, 14, 4, 10, 20]

B : [14, 0, 7, 8, 7]

C : [4, 7, 0, 12, 6]

D : [10, 8, 12, 0, 15]

E : [20, 7, 6, 15, 0]



26) For a knapsack with a capacity of 50 u and items with weights and values.

I_1 : W=10 V=50

I_2 : W=20 V=70

I_3 : W=30 V=90

I_4 : W=25 V=60

I_5 : W=15 V=40

Determine the optimal sol using 0/1 knapsack method

	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

27) Given a directed graph with vertices 1,2,3,4 & edges:

1→21| right arrow .21→2 wt - 4

1→31| 〃 31→1 wt - 5

2→32| 〃 32→2 wt - 2

3→43| 〃 43→4 wt 3

4→24| 〃 24→2 wt -10.

Apply Bellman Ford Algorithm from vertex 1|1.
to determine shortest paths.

$1 \to 2 \quad - 4$

$1 \to 3 \quad - 5$

$2 \to 3 \quad - (-2)$

$3 \to 4 \quad - 3$

$4 \to 2 \quad = -10$

v	1	2	3	4
d	0	∞	∞	∞
p	-	-	-	-

1)

v	1	2	3	4
d	0	4	5	∞
p	-	1	1	-

2)

v	1	2	3	4
d	0	4	2	∞
p	-	1	2	-

3)

v	1	2	3	4
d	0	4	2	5
p	-	1	2	3.

vertex	Distance	path
1	0	1 –
2	4	$1 \to 2$
3	2	$1 \to 2 \to 3$
4	5	$1 \to 2 \to 3 \to 4$

3). Roll - six - six - sided dice. Determine the no of ways to get a sum of 18. ensuring that at least one die shows a 6,,.

$x + x^2 + x^3 + x^4 + x^5 + x^6$

$\Rightarrow x(1+x+x^2+x^3+x^4+x^5)$

$= \dfrac{x(1-x^6)}{1-x}$

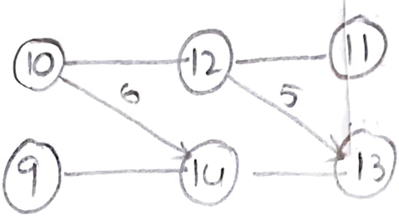
For six dice

$$\left(\frac{x(1-x^6)}{1-x} \right)^6$$

$$\left(x \left(\frac{1-x^6}{1-x} \right) \right)^6 = x^6 (1-x^6)^6 (1-x)^{-6} = (x \wedge 18)$$

$= 340$

29) Given line 1: [10, 12, 11], L2: [9, 14, 13], and transfer times b/w lines: [6, 5]. Calculate Min assembly line, considering a reduction in one of the transfer times by 2 units.



Before Reduction

L1	6	5
L2	30	30

After Red (2)

L1	4	~~2~~ 5
L2	28	27

30) For keys (8, 12, 16, 20, 24) with access possibilities {0.2, 0.05, 0.4, 0.25, 0.1}. determine the OBST using dp approach.

K = (8, 12, 16, 20, 24)

V = (0.2, 0.05, 0.4, 0.25, 0.1)

j - i = 0

j - i = 1

	0	1	2	3	4	5
0	0	0.2^{1}	$[0.3]^{1}$	$[0.95]^{3}$	$[1.45]^{3}$	$[1.65]^{3}$
1		0	[illegible]	$[0.5]^{3}$	$[1]^{3}$	$[1.3]^{3}$
2			0	[illegible]	$[0.9]^{3}$	$[1.2]^{3}$
3				0	[illegible]	0.45
4					0	0.1
5						0

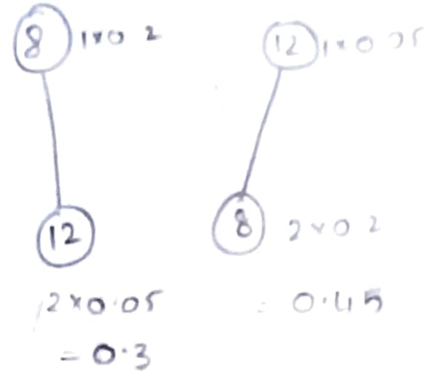
$j-i=2$

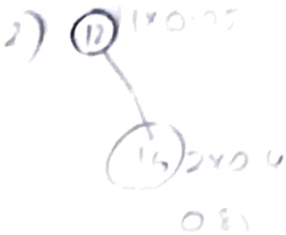
$2-0=[0,2]$

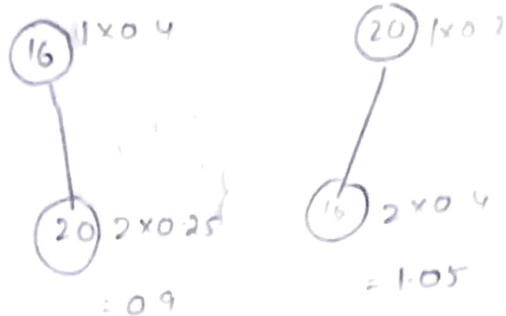
$3-1=[1,3]$

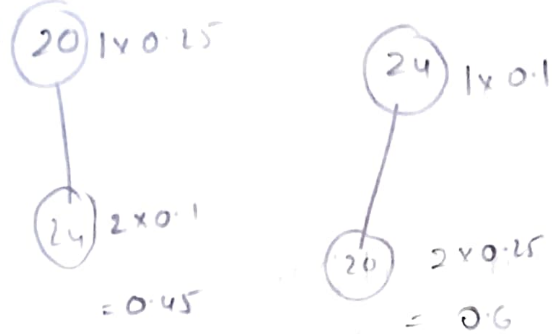
$4-2=[2,4]$

$5-3=[3,5]$

1) 

2) 

3) 

4) 

$j-i=3$

$3-0=(0,3)$

$4-1=(1,4)$

$5-2=(2,5)$.

$$\underset{k=1,2,3}{cost\overset{i,j}{(0,3)}} = Min\begin{Bmatrix} cost(0,1-1)+cost(1,3) \\ cost(0,2-1)+cost(2,3) \\ cost(0,3-1)+cost(3,3) \end{Bmatrix} + 0.65$$

$$= Min\begin{Bmatrix} 0+0.5 \\ 0.2+0.4 \\ 0.3+0 \end{Bmatrix} + 0.65$$

$$= Min\begin{Bmatrix} 1.15 \\ 1.25 \\ 0.95 \end{Bmatrix}.$$

$$\underset{k=2,3,4}{cost\overset{i,j}{(1,4)}} = Min\begin{Bmatrix} cost(1,2-1)+cost(2,4) \\ cost(1,3-1)+cost(3,4) \\ cost(1,4-1)+cost(4,4) \end{Bmatrix} + 0.7 = Min\begin{Bmatrix} 0+0.9 \\ 0.05+0.25 \\ 0.5+0 \end{Bmatrix} + 0.7$$

$$= \begin{Bmatrix} 1.6 \\ 1.0 \\ 1.2 \end{Bmatrix}$$

$$cost(\overset{i,j}{2,5}) = Min \begin{cases} cost(2,3-1) + cost(3,5) \\ cost(2,4-1) + cost(4,5) \\ cost(2,5-1) + cost(5,5) \end{cases} + 0.75$$

$K = 3,4,5.$

$$= Min \begin{cases} 0 + 0.45 \\ 0.4 + 0.1 \\ 0.9 + 0 \end{cases} + 0.75$$

$$= Min \begin{cases} 1.2 \\ 1.25 \\ 1.65 \end{cases}$$

$j - i = 4.$

$4 - 0 = [0,4]$

$5 - 1 = [1,5]$

$$cost(\overset{i,j}{0,4}) = Min \begin{cases} cost(0,1-1) + cost(1,4) \\ cost(0,2-1) + cost(2,4) \\ cost(0,3-1) + cost(3,4) \\ cost(0,4-1) + cost(4,4) \end{cases} + 0.9.$$

$K = 1,2,3,4$

$$= Min \begin{cases} \cancel{cost}\, 0 + 1 \\ 0.2 + 0.9 \\ 0.3 + 0.25 \\ 0.95 + 0 \end{cases} + 0.9 = Min \begin{cases} 1.9 \\ 2 \\ 1.45 \\ 1.85 \end{cases}$$

$$cost(\overset{i,j}{1,5}) = Min \begin{cases} cost(1,2-1) + cost(2,5) \\ cost(1,3-1) + cost(3,5) \\ cost(1,4-1) + cost(4,5) \\ cost(1,5-1) + cost(5,5) \end{cases} + 0.8$$

$K = 2,3,4,5$

$$= Min \begin{cases} 0 + 1.2 \\ 0.05 + 0.45 \\ 0.5 + 0.1 \\ 1 + 0 \end{cases} + 0.8 = Min \begin{cases} 2 \\ 1.3 \\ 1.4 \\ 1.8 \end{cases}$$