Supplementary: Emergence of robust oscillatory dynamics in the interlocked negative and feedback-feed-forward loops

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S1 DYNAMICS OF A COHERENT FEED-FORWARD LOOP

The coherent feed-forward loop (CFFL) is a three-tiered cascade where we can expect the presence of the threshold for the appropriate choice of parameters. In Fig-S1, we show the simulation of the four types of coherent feed-forward loops in both AND (Figs-S1a-d) and OR (Figs-S1e-h) logic. We take x as an input variable, a step function that steps up at Time = 15 units and switches off at Time = 35 units. With the input signal correspondingly, the variables y and z also kick up and shut down.

For the chosen set of parameters, in the case of AND logic, when the input signal is ON, there is no delay in the y and z variables to kick start in C1-FFL and C3-FFL. While in C2-FFL and C4-FFL there is a delay in variable z. When the system is OFF, there is no delay in shutting down the variables in the case of C1-FFL , C2-FFL and C4-FFL, while C3-FFL shows a delay in shutting down the variable z.

In the case of OR logic, when the input signal is ON, there is no delay in the y and z variables to kick start in C1-FFL and C3-FFL. While in C2-FFL and C4-FFL there is a delay in variable z. When the system is OFF, there is no delay in shutting down the variables in the case of C2-FFL and C4-FFL, while C1-FFL and C3-FFL show a delay in shutting down the variable z.

S2 INCOHERENT FEED FORWARD LOOP

In Fig-S1, we show the simulation of the four types of incoherent feed-forward loops in both AND (Figs-S1a-d) and OR (Figs-S1e-h) logic. We take x as an input variable, a step function that steps up at Time = 15 units and switches off at Time = 35 units. With the input signal correspondingly, the variables y and z also kick up and shut down. While the variable y shows immediate response for both ON and OFF of the input signal, the variable z shows the properties of (i) pulse response, (ii) accelerated response, (iii) biphasic response, and (iv) initial rapid rise. The pulse response is seen in the AND logic of I4-FFL and I1-FFL. The accelerated and initial rapid rise is seen in the AND logic of I1-FFL and the OR logic of I3-FFL. The biphasic response is seen in the AND logic of I3-FFL and the OR logic of all the IFFLs.

S3 NEGATIVE FEEDBACK LOOPS: GOODWIN AND REPRESSILATOR

S3.1 GW1 - Goodwin 1

Fig-2 in the main article show the Goodwin 1 (Fig-2A in the main article) model with coherent (Figs-(2B, 2C) in the main article) and incoherent (Figs-(2D, 2E) in the main article) feed-forward loops combined with both AND and OR logic. The Goodwin 1 model and the Goodwin 1 with C1-FFL have only a negative feedback loop in the circuit. In these cases only Hopf bifurcation is seen. When an Incoherent feedforward loop is added to the Goodwin 1 model, the outer loop becomes an interlocked positive feedback loop, making the circuit a combination of both positive and negative feedback loops. For the chosen parameters, in the case of AND logic, the bifurcation diagram shows bistability and supercritical Hopf bifurcation however, both are separated, and the amplitude-frequency plot shows amplitude fine-tuned with constant frequency. But in case of OR logic, the bifurcation diagram shows the Saddle node infinite period (SNIPER), and the amplitude-frequency plot shows a broad range of tunable frequency.

S3.2 GW2 - Goodwin 2

Figs-S3-6 and Fig-3B in the main article, show the Goodwin 2 (Fig-S3) model with coherent (Figs-S4 and S5) and incoherent (Fig-S6 and Fig-3B in the main article) feed-forward loops combined with both AND and OR logic. The Goodwin 2 model and the Goodwin 2 with C2-FFL have only a negative feedback loop in the circuit. In these cases, only Hopf bifurcation is seen. When an Incoherent feedforward loop is added to the Goodwin 2 model, the outer loop becomes an interlocked positive feedback loop, making the circuit a combination of both positive and negative feedback loops. In the case of AND logic, the model is capable of showing bistability. However, with the given set of parameters, the bifurcation diagram shows only Hopf bifurcation and the amplitude-frequency plot shows amplitude fine-tuned with constant frequency. But in the case of OR logic, the bifurcation diagram shows the Saddle node infinite period (SNIPER), and the amplitude-frequency plot shows a broad range of tunable frequency.

S3.3 GW3 - Goodwin 3

Figs-S7-10 and Fig-3C in the main article show the Goodwin 3 (Fig-S7) model with coherent (Figs-S8 and S9) and incoherent (Fig-S10 and Fig-3C in the main article) feed-forward loops combined with both AND and OR logic. The Goodwin 3 model and the Goodwin 3 with C3-FFL, has only negative feedback loop in the circuit, and the frequency is not tunable. In the case of only Goodwin model 3 and its combination with C3-FFL using AND logic, only supercritical hopf bifurcation is seen. With its combination with C3-FFL using OR logic, the bifurcation diagram shows Supercritical flip bifurcation. When an Incoherent feedforward loop is added to the Goodwin 3 model, the outer loop becomes an interlocked positive feedback loop, making the circuit a combination of both positive and negative feedback loops. For the chosen parameters, in the case of AND logic, the bifurcation diagram shows bistability and Subcritical Hopf bifurcation; however, both are separated, and the amplitude-frequency plot shows amplitude fine-tuned with constant frequency. But in case of OR logic, the bifurcation diagram shows The bifurcation diagram shows Saddle node infinite period (SNIPER) and the amplitude-frequency plot shows a broad range of tunable frequency.

S3.4 REP - Repressilator

Fig-S11-14 and Fig-3D in the main article show the Repressilator (Fig-S11) model with coherent (Fig-S12 and S13) and incoherent (Fig-3D in the main article and Fig-S14) feed-forward loops combined with both AND and OR logic. The Repressilator model and the Repressilator with C4-FFL have only a negative feedback loop in the circuit. In these cases only supercritical hopf bifurcation is seen. When an Incoherent feedforward loop is added to the Repressilator model, the outer loop becomes an interlocked positive feedback loop, making the circuit a combination of both positive and negative feedback loops. For the chosen parameters, in case of OR logic, the bifurcation diagram shows bistability and supercritical Hopf bifurcation however, both are separated and the amplitude-frequency plot shows amplitude fine-tuned with constant frequency. But in case of AND logic, the bifurcation diagram shows Saddle node infinite period (SNIPER), and the amplitude-frequency plot shows a broad range of tunable frequency.

We have performed the bifurcation analysis and compared the amplitude vs frequency plots for eight combinations of negative feedback and feed-forward loop with both AND and OR logic for each. The comparison is summarised in the Table-1 in the main article. Among all the combinations, four combinations, which include the Goodwin models with an incoherent feedback loop using OR logic and the repressilator with an incoherent feedback loop with AND logic, show saddle-node infinite period bifurcation. The SNIPER bifurcation occurs when the sable node and an unstable saddle annihilate each other. This gives rise to a large amplitude cycle. At this point, the period of oscillation is infinite, which then quickly drops (Hong et al. (2007)). This explains the large range of frequency in the Amplitude vs Frequency plot.

S3.5 ODEs of GW and REP models

GW-1

$$\frac{d[x]}{dt} = \frac{\beta 1 K_{mx}^{n}}{K_{mx}^{n} + [z]^{n}} - kd1[x] \tag{1}$$

$$\frac{d[y]}{dt} = \frac{\beta [x]^{n}}{K_{m1}^{n} + [x]^{n}} - kd2[y] \tag{2}$$

$$\frac{d[z]}{dt} = \frac{\gamma [y]^{n}}{K_{m2}^{n} + [y]^{n}} - kd3[z] \tag{3}$$

GW-1 + C1-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma [y]^{n}}{K_{m2}^{n} + [y]^{n}}\right)\left(\frac{[x]^{n}}{K_{m3}^{n} + [x]^{n}}\right) - kd3[z] \tag{4}$$

GW-1 + C1-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma [y]^{n}}{K_{m2}^{n} + [y]^{n}}\right) + \left(\frac{[x]^{n}}{K_{m3}^{n} + [x]^{n}}\right) - kd3[z] \tag{5}$$

GW-1 + I3-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma [y]^{n}}{K_{m2}^{n} + [y]^{n}}\right)\left(\frac{K_{m3}^{n}}{K_{m3}^{n} + [x]^{n}}\right) - kd3[z] \tag{6}$$

GW-1 + I3-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma[y]^n}{K_{m2}^n + [y]^n}\right) + \left(\frac{K_{m3}^n}{K_{m2}^n + [x]^n}\right) - kd3[z] \tag{7}$$

GW-2

$$\frac{d[x]}{dt} = \frac{\beta 1[z]^n}{K_{mx}^n + [z]^n} - kd1[x] \tag{8}$$

$$\frac{d[y]}{dt} = \frac{\beta K_{m1}^n}{K_{m1}^n + [x]^n} - kd2[y] \tag{9}$$

$$\frac{d[z]}{dt} = \frac{\gamma[y]^n}{K_{m2}^n + [y]^n} - kd3[z] \tag{10}$$

GW-2 + C2-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma[y]^n}{K_{m2}^n + [y]^n}\right)\left(\frac{K_{m3}^n}{K_{m3}^n + [x]^n}\right) - kd3[z] \tag{11}$$

GW-2 + C2-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma[y]^n}{K_{m2}^n + [y]^n}\right) + \left(\frac{K_{m3}^n}{K_{m3}^n + [x]^n}\right) - kd3[z] \tag{12}$$

GW-2 + I4-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma[y]^n}{K_{m2}^n + [y]^n}\right)\left(\frac{[x]^n}{K_{m3}^n + [x]^n}\right) - kd3[z] \tag{13}$$

GW-2 + I4-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma[y]^n}{K_{m2}^n + [y]^n}\right) + \left(\frac{[x]^n}{K_{m2}^n + [x]^n}\right) - kd3[z] \tag{14}$$

GW-3

$$\frac{d[x]}{dt} = \frac{\beta 1[z]^n}{K_{mx}^n + [z]^n} - kd1[x] \tag{15}$$

$$\frac{d[y]}{dt} = \frac{\beta [x]^n}{K_{m1}^n + [x]^n} - kd2[y] \tag{16}$$

$$\frac{d[z]}{dt} = \frac{\gamma K_{m2}^n}{K_{m2}^n + [y]^n} - kd3[z] \tag{17}$$

GW-3 + C3-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^n}{K_{m2}^n + [y]^n}\right)\left(\frac{K_{m3}^n}{K_{m3}^n + [x]^n}\right) - kd3[z] \tag{18}$$

GW-3 + C3-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^n}{K_{m2}^n + [y]^n}\right) + \left(\frac{K_{m3}^n}{K_{m3}^n + [x]^n}\right) - kd3[z] \tag{19}$$

GW-3 + I1-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^n}{K_{m2}^n + [y]^n}\right)\left(\frac{[x]^n}{K_{m3}^n + [x]^n}\right) - kd3[z] \tag{20}$$

GW-3 + I1-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^n}{K_{m2}^n + [y]^n}\right) + \left(\frac{[x]^n}{K_{m2}^n + [x]^n}\right) - kd3[z] \tag{21}$$

REP

$$\frac{d[x]}{dt} = \frac{\beta 1 K_{mx}^n}{K_{mx}^n + [z]^n} - kd1[x] \tag{22}$$

$$\frac{d[y]}{dt} = \frac{\beta K_{m1}^n}{K_{m1}^n + [x]^n} - kd2[y] \tag{23}$$

$$\frac{d[z]}{dt} = \frac{\gamma K_{m2}^n}{K_{m2}^n + [y]^n} - kd3[z] \tag{24}$$

REP + C4-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^{n}}{K_{m2}^{n} + [y]^{n}}\right)\left(\frac{[x]^{n}}{K_{m3}^{n} + [x]^{n}}\right) - kd3[z] \tag{25}$$

REP + C4-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^{n}}{K_{m2}^{n} + [y]^{n}}\right) + \left(\frac{[x]^{n}}{K_{m3}^{n} + [x]^{n}}\right) - kd3[z] \tag{26}$$

REP + I2-FFL (AND)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^{n}}{K_{m2}^{n} + [y]^{n}}\right)\left(\frac{K_{m3}^{n}}{K_{m3}^{n} + [x]^{n}}\right) - kd3[z] \tag{27}$$

REP + I2-FFL (OR)

$$\frac{d[z]}{dt} = \left(\frac{\gamma K_{m2}^{n}}{K_{m2}^{n} + [y]^{n}}\right) + \left(\frac{K_{m3}^{n}}{K_{m2}^{n} + [x]^{n}}\right) - kd3[z] \tag{28}$$

S4 PC: PLANT CIRCADIAN CLOCK MODEL

The circuit for the plant circadian model proposed by De-caluwe (De Caluwé et al. (2017)) is taken as an example. The plant circadian circuit has a combination of negative feedback loops and feed-forward loops, giving rise to an outer positive feedback loop. In the case of the light phase, with the given parameters, the period range is from 23.25 hours to 32.26 hours with a variable amplitude with a maximum value of 1.14. With the changed parameters (Table-2 in the main article), the amplitude vs frequency plot shows a wide range of frequencies ranging from 28 hours to 3.3e+6 hours with a constant amplitude of 2.93. The model parameters are given in Table-S1

S4.1 ODEs

PC: Full

$$\frac{d[CLm]}{dt} = (v1 + (v1L * L * [P])) * \left(\frac{1}{1 + \left(\frac{P97p}{K1}\right)^{2} + \left(\frac{P51p}{K2}\right)^{2}}\right) - (k1L * L + k1D * D) * [CLm] \tag{29}$$

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{30}$$

$$\frac{d[P97m]}{dt} = \left((v2L * L * [P]) + v2A + \left(v2B * \frac{[CLp]^2}{K3^2 + [CLp]^2}\right)\right) * \left(\frac{1}{1 + \left(\frac{P51p}{K4}\right)^2 + \left(\frac{ELp}{K5}\right)^2}\right) - k2 * [P97m] \tag{31}$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \tag{32}$$

$$\frac{d[P51m]}{dt} = v3 * \left(\frac{1}{1 + \left(\frac{CLp}{K6}\right)^2 + \left(\frac{P51p}{K7}\right)^2}\right) - k3 * [P51m] \tag{33}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{34}$$

$$\frac{d[ELm]}{dt} = L * v4 * \left(\frac{1}{1 + \left(\frac{CLp}{K8}\right)^2 + \left(\frac{P51p}{K9}\right)^2 + \left(\frac{ELp}{K10}\right)^2}\right) - k4 * [ELm] \tag{35}$$

$$\frac{d[ELp]}{dt} = p4 * [ELm] - (d4D * D + d4L * L) * [ELp] \tag{36}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{37}$$

PC: Represssilator 1

$$\frac{d[CLm]}{dt} = (v1 + (v1L * L * [P])) * \left(\frac{1}{1 + \left(\frac{P97p}{K1}\right)^2}\right) - (k1L * L + k1D * D) * [CLm] \tag{38}$$

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{39}$$

$$\frac{d[P97m]}{dt} = ((v2L * L * [P]) + v2A)) * \left(\frac{1}{1 + \left(\frac{ELp}{K5}\right)^2}\right) - k2 * [P97m] \tag{40}$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \tag{41}$$

$$\frac{d[ELm]}{dt} = L * v4 * \left(\frac{1}{1 + \left(\frac{CLp}{K8}\right)^2}\right) - k4 * [ELm] \tag{42}$$

$$\frac{d[ELp]}{dt} = p4 * [ELm] - (d4D * D + d4L * L) * [ELp] \tag{43}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{44}$$

110 **PC: Repressilator 2**

$$\frac{d[CLm]}{dt} = (v1 + (v1L * L * [P])) * \left(\frac{1}{1 + \left(\frac{P97p}{K1}\right)^2}\right) - (k1L * L + k1D * D) * [CLm] \tag{45}$$

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{46}$$

$$\frac{d[P97m]}{dt} = ((v2L * L * [P]) + v2A) * \left(\frac{1}{1 + \left(\frac{P51p}{K4}\right)^2}\right) - k2 * [P97m] \tag{47}$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \tag{48}$$

$$\frac{d[P51m]}{dt} = v3 * \left(\frac{1}{1 + \left(\frac{CLp}{K6}\right)^2}\right) - k3 * [P51m] \tag{49}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{50}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{51}$$

111 **PC: Positive feedback**

$$\frac{d[CLm]}{dt} = (v1 + (v1L * L * [P])) * \left(\frac{1}{1 + \left(\frac{P51p}{K2}\right)^2}\right) - (k1L * L + k1D * D) * [CLm] \tag{52}$$

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{53}$$

$$\frac{d[P51m]}{dt} = v3 * \left(\frac{1}{1 + \left(\frac{CLp}{K6}\right)^2}\right) - k3 * [P51m] \tag{54}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{55}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{56}$$

112 **PC: Repressilator 2 + Positive feedback**

$$\frac{d[CLm]}{dt} = (v1 + (v1L * L * [P])) * \left(\frac{1}{1 + \left(\frac{P97p}{K1}\right)^2 + \left(\frac{P51p}{K2}\right)^2} \right) - (k1L * L + k1D * D) * [CLm] \quad (57)$$

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \quad (58)$$

$$\frac{d[P97m]}{dt} = ((v2L * L * [P]) + v2A+) * \left(\frac{1}{1 + \left(\frac{P51p}{K4}\right)^2} \right) \quad (59)$$

$$- k2 * [P97m]$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \quad (60)$$

$$\frac{d[P51m]}{dt} = v3 * \left(\frac{1}{1 + \left(\frac{CLp}{K6}\right)^2} \right) - k3 * [P51m] \quad (61)$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \quad (62)$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \quad (63)$$

PC: C4-FFL 1

[CLm] is taken as an input square wave function which steps up and down every 12hrs.

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \quad (64)$$

$$\frac{d[P97m]}{dt} = \left((v2L * L * [P]) + v2A + \left(v2B * \frac{[CLp]^2}{K3^2 + [CLp]^2} \right) \right) * \left(\frac{1}{1 + \left(\frac{ELp}{K5}\right)^2} \right) - k2 * [P97m] \quad (65)$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \quad (66)$$

$$\frac{d[ELm]}{dt} = L * v4 * \left(\frac{1}{1 + \left(\frac{CLp}{K8}\right)^2} \right) - k4 * [ELm] \quad (67)$$

$$\frac{d[ELp]}{dt} = p4 * [ELm] - (d4D * D + d4L * L) * [ELp] \quad (68)$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \quad (69)$$

PC: C4-FFL 2

[CLm] is taken as an input square wave function which steps up and down every 12hrs.

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{70}$$

$$\frac{d[P97m]}{dt} = \left((v2L * L * [P]) + v2A + \left(v2B * \frac{[CLp]^2}{K3^2 + [CLp]^2} \right) \right) * \left(\frac{1}{1 + \left(\frac{P51p}{K4}\right)^2} \right) - k2 * [P97m] \tag{71}$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \tag{72}$$

$$\frac{d[P51m]}{dt} = v3 * \left(\frac{1}{1 + \left(\frac{CLp}{K6}\right)^2} \right) - k3 * [P51m] \tag{73}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{74}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{75}$$

PC: I2-FFL 1

[P51] is taken as an input square wave function which steps up and down every 12hrs.

$$\frac{d[CLm]}{dt} = (v1 + (v1L * L * [P])) * \left(\frac{1}{1 + \left(\frac{P97p}{K1}\right)^2 + \left(\frac{P51p}{K2}\right)^2} \right) - (k1L * L + k1D * D) * [CLm] \tag{76}$$

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{77}$$

$$\frac{d[P97m]}{dt} = ((v2L * L * [P]) + v2A) * \left(\frac{1}{1 + \left(\frac{P51p}{K4}\right)^2} \right) - k2 * [P97m] \tag{78}$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \tag{79}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{80}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{81}$$

PC: I2-FFL 2

[P51] is taken as an input square wave function which steps up and down every 12hrs.

$$\frac{d[P97m]}{dt} = ((v2L * L * [P]) + v2A) * \left(\frac{1}{1 + \left(\frac{P51p}{K4}\right)^2 + \left(\frac{ELp}{K5}\right)^2} \right) - k2 * [P97m] \tag{82}$$

$$\frac{d[P97p]}{dt} = p2 * [P97m] - (d2D * D + d2L * L) * [P97p] \tag{83}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{84}$$

$$\frac{d[ELm]}{dt} = L * v4 * \left(\frac{1}{1 + \left(\frac{P51p}{K9}\right)^2} \right) - k4 * [ELm] \tag{85}$$

$$\frac{d[ELp]}{dt} = p4 * [ELm] - (d4D * D + d4L * L) * [ELp] \tag{86}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{87}$$

PC: I2-FFL 3

[CLm] is taken as an input square wave function which steps up and down every 12hrs.

$$\frac{d[CLp]}{dt} = (p1 + p1L * L) * [CLm] - d1 * [CLp] \tag{88}$$

$$\frac{d[P51m]}{dt} = v3 * \left(\frac{1}{1 + \left(\frac{CLp}{K6}\right)^2} \right) - k3 * [P51m] \tag{89}$$

$$\frac{d[P51p]}{dt} = p3 * [P51m] - (d3D * D + d3L * L) * [P51p] \tag{90}$$

$$\frac{d[ELm]}{dt} = L * v4 * \left(\frac{1}{1 + \left(\frac{CLp}{K8}\right)^2 + \left(\frac{P51p}{K9}\right)^2} \right) - k4 * [ELm] \tag{91}$$

$$\frac{d[ELp]}{dt} = p4 * [ELm] - (d4D * D + d4L * L) * [ELp] \tag{92}$$

$$\frac{d[P]}{dt} = 0.3 * (1 - P) * D - [P] * L \tag{93}$$

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Table S1. Plant Circadian clock model parameters

Parameter description	Parameter	Value	Units
CL synthesis	v1	4.6	nM.h^{-1}
CL light-induced synthesis	v1L	3	nM.h^{-1}
P97 synthesis	v2A	1.3	nM.h^{-1}
P97 CL-induced synthesis	v2B	1.5	nM.h^{-1}
P97 light-induced synthesis	v2L	5	nM.h^{-1}
P51 synthesis	v3	1	nM.h^{-1}
EL synthesis	v4	1.5	nM.h^{-1}
CL mRNA degradation (light)	k1L	0.53	h^{-1}
CL mRNA degradation (dark)	k1D	0.21	h^{-1}
P97 mRNA degradation	k2	0.35	h^{-1}
P51 mRNA degradation	k3	0.56	h^{-1}
EL mRNA degradation	k4	0.57	h^{-1}
CL translation	p1	0.76	h^{-1}
CL light-induced translation	p1L	0.42	h^{-1}
P97 translation	p2	1	h^{-1}
P51 translation	p3	0.64	h^{-1}
EL translation	p4	1	h^{-1}
CL degradation	d1	0.68	h^{-1}
P97 degradation (dark)	d2D	0.5	h^{-1}
P97 degradation (light)	d2L	0.3	h^{-1}
P51 degradation (dark)	d3D	0.48	h^{-1}
P51 degradation (light)	d3L	0.78	h^{-1}
EL degradation (dark)	d4D	1.2	h^{-1}
EL degradation (light)	d4L	0.38	h^{-1}
Inhibition: CL by P97	K1	0.16	nM
Inhibition: CL by P51	K2	1.2	nM
Activation: P97 by CL	K3	0.24	nM
Inhibition: P97 by P51	K4	0.23	nM
Inhibition: P97 by EL	K5	0.3	nM
Inhibition: P51 by CL	K6	0.46	nM
Inhibition: P51 by P51	K7	2	nM
Inhibition: EL by CL	K8	0.36	nM
Inhibition: EL by P51	K9	1.9	nM
Inhibition: EL by EL	K10	1.9	nM

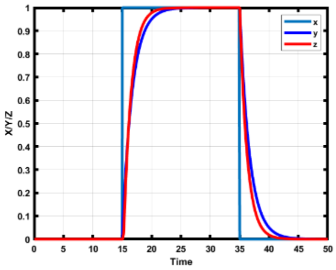


Fig S1a. C1-FFL AND

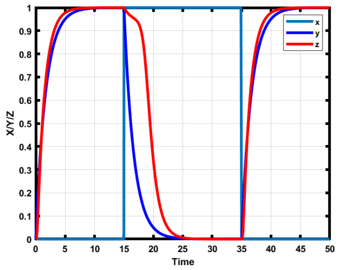


Fig S1b. C2-FFL AND

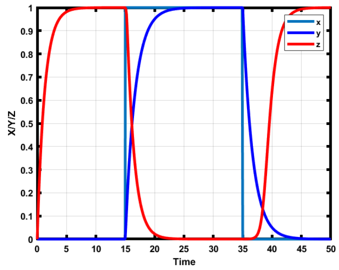


Fig S1c. C3-FFL AND

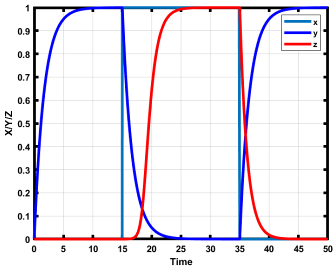


Fig S1b. C4-FFL AND

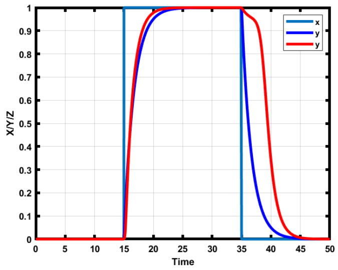


Fig S1e. C1-FFL OR

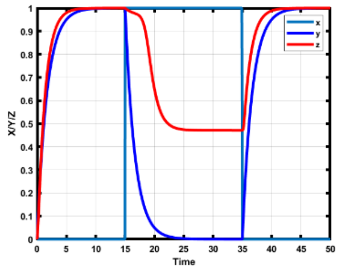


Fig S1f. C2-FFL OR

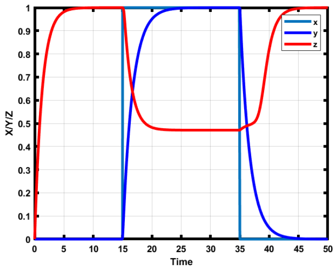


Fig S1g. C3-FFL OR

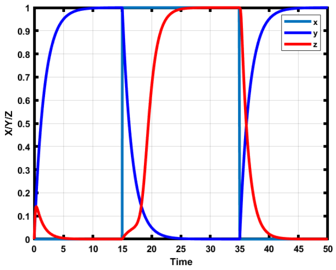


Fig S1h. C4-FFL OR

Fig S1. Simulation for coherent feed-forward loops type 1 to 4 in both AND (a - d) and OR (e - h) logic. The variable x is considered to be the input variable as a step function and the properties of the response of y and z upon ON and OFF were observed.

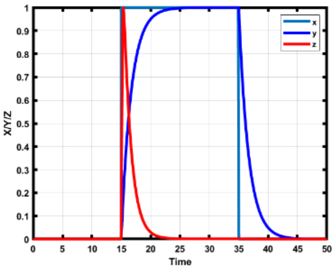


Fig S2a. I1-FFL AND

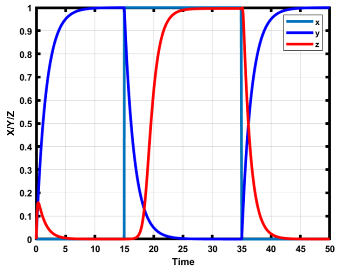


Fig S2b. I2-FFL AND

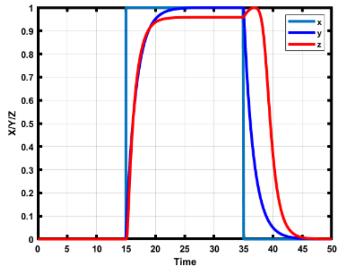


Fig S2c. I3-FFL AND

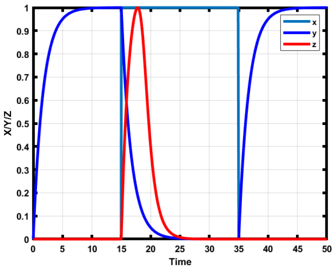


Fig S2d. I4-FFL AND

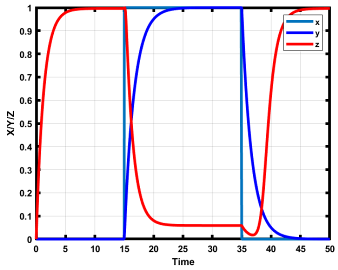


Fig S2e. I1-FFL OR

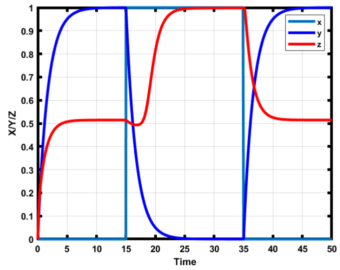


Fig S2f. I2-FFL OR

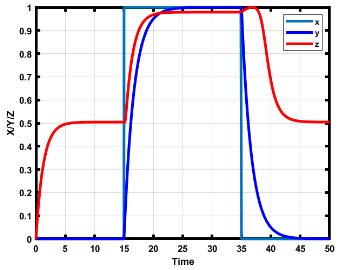


Fig S2g. I3-FFL OR

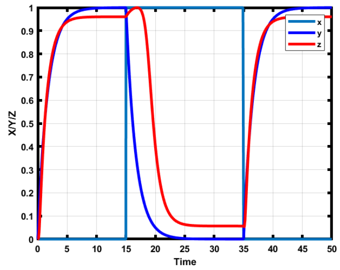


Fig S2h. I4-FFL OR

Fig S2. Simulation for incoherent feed-forward loops type 1 to 4 in both AND (a - d) and OR (e - h) logic. The variable x is considered to be the input variable as a step function and the the properties of the response of y and z upon ON and OFF were observed.

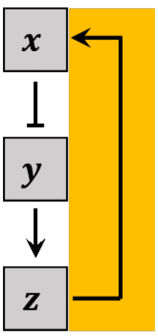


Fig S3a.

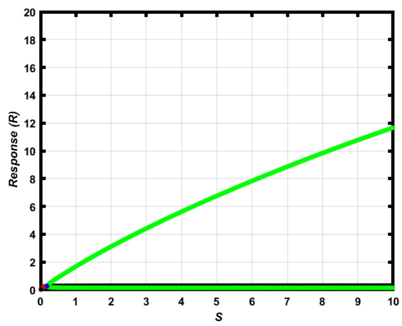


Fig S3b. Bifurcation

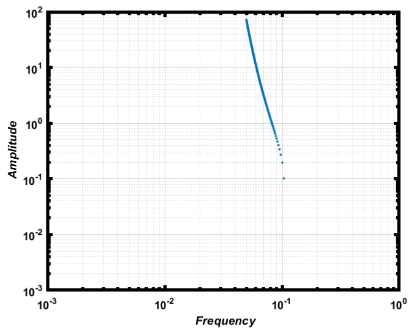
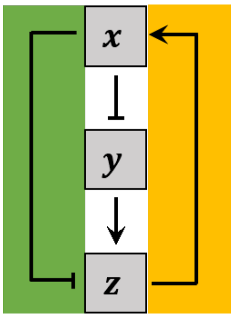
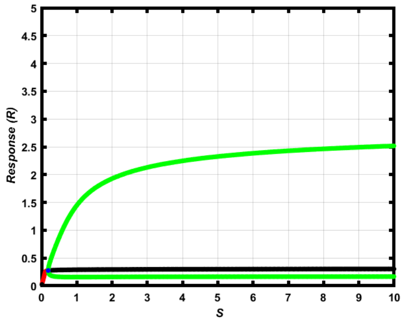


Fig S3c. Amp vs Fre

Fig S3. GW-2: The simulation of the bifurcation diagram for the negative feedback (Goodwin 2) shows Hopf bifurcation. The Amplitude vs Frequency plot shows amplitude fine-tuned with constant frequency.





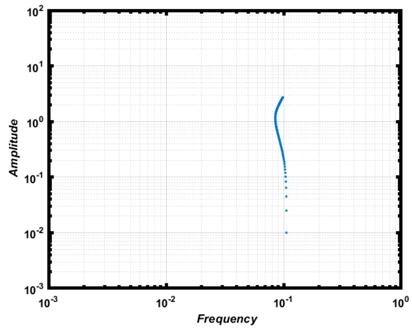
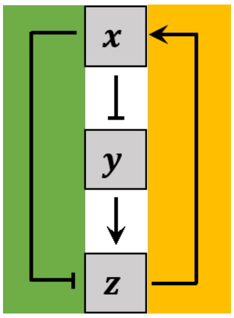
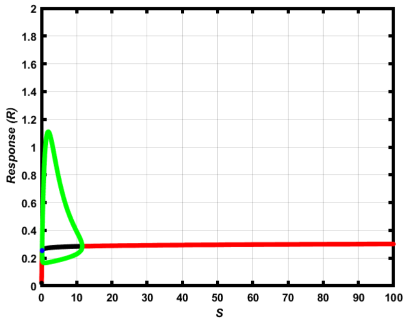


Fig S4a. **Fig S4b.** Bifurcation **Fig S4c.** Amp vs Fre

Fig S4. GW-2 + C2-FFL (AND): To the negative feedback (Goodwin 2), we added a coherent feed-forward loop (shaded in green) using AND logic. The bifurcation diagram shows Hopf bifurcation. Upon adding the coherent feed-forward loop using AND logic, there is no change in the range of frequency in the Amplitude vs frequency plot. It shows amplitude fine-tuned with constant frequency.





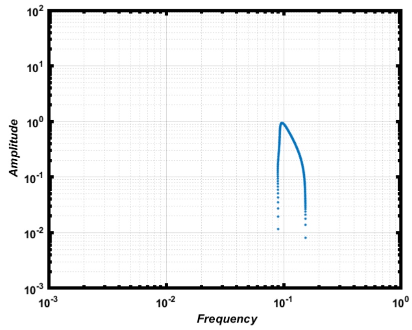
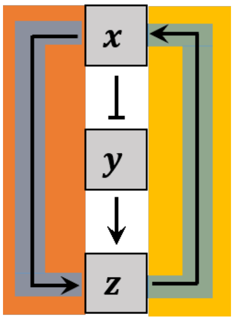
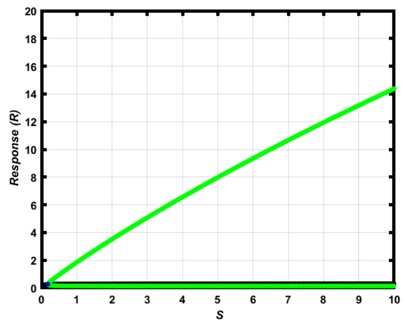


Fig S5a. **Fig S5b.** Bifurcation **Fig S5c.** Amp vs Fre

Fig S5. GW-2 + C2-FFL (OR): To the negative feedback (Goodwin 2), we added a coherent feed-forward loop (shaded in green) using OR logic. The bifurcation diagram shows Hopf bifurcation. Upon adding the coherent feed-forward loop using OR logic the range of frequency is narrow and the fine-tuning is not robust.





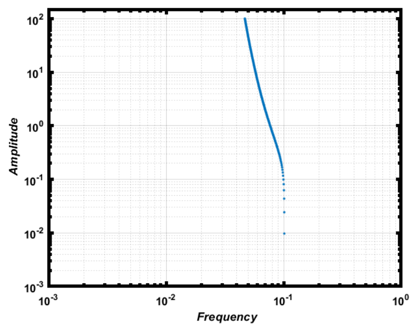


Fig S6a. **Fig S6b.** Bifurcation **Fig S6c.** Amp vs Fre

Fig S6. GW-2 + I4-FFL (AND): To the negative feedback (Goodwin 2), we added an incoherent feed-forward loop (shaded in red) using AND logic. The addition of the incoherent feed-forward loop, makes the outer loop (shaded in blue) positive feedback. The model is capable of showing bistability. However, with the given set of parameters, the bifurcation diagram shows only Hopf bifurcation. Upon adding the incoherent feed-forward loop, there is no change in the range of frequency in the Amplitude vs frequency plot. It shows amplitude fine-tuned with constant frequency.

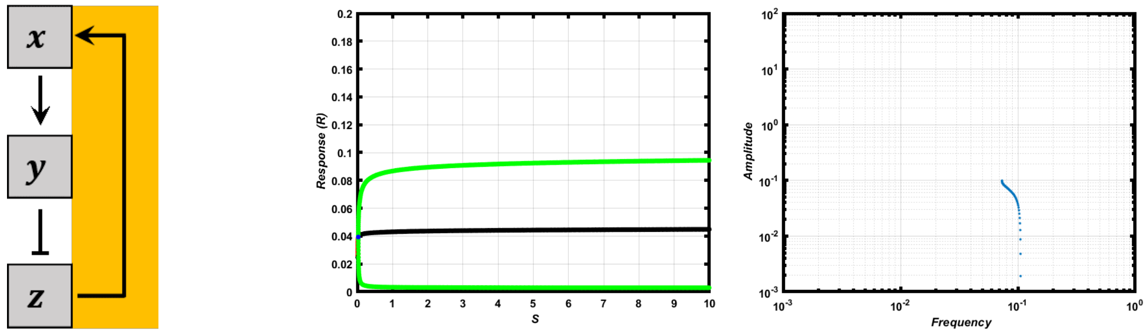


Fig S7a. **Fig S7b.** Bifurcation **Fig S7c.** Amp vs Fre

Fig S7. GW3: The simulation of the bifurcation diagram for the negative feedback (Goodwin 3) shows Hopf bifurcation. The Amplitude vs Frequency plot shows amplitude fine-tuned with constant frequency.

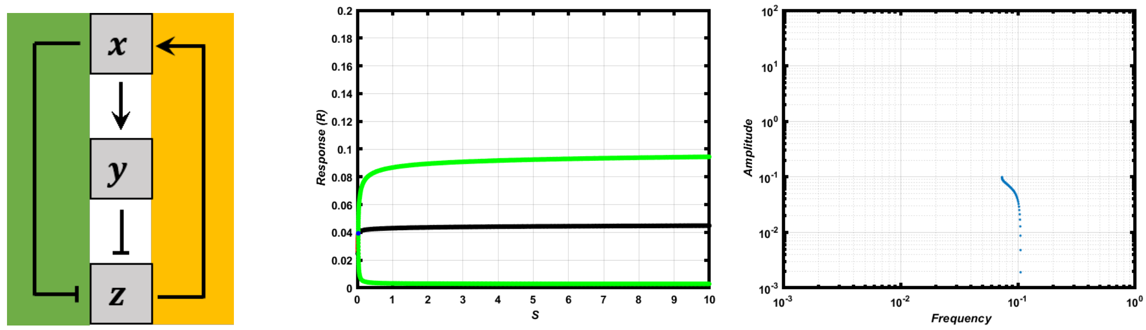


Fig S8a. **Fig S8b.** Bifurcation **Fig S8c.** Amp vs Fre

Fig S8. GW-3 + C3-FFL (AND): To the negative feedback (Goodwin 3), we added a coherent feed-forward loop (shaded in green) using AND logic. The bifurcation diagram shows Hopf bifurcation. Upon adding the coherent feed-forward loop using AND logic, there is no change in the range of frequency in the Amplitude vs frequency plot. It shows amplitude fine-tuned with constant frequency.

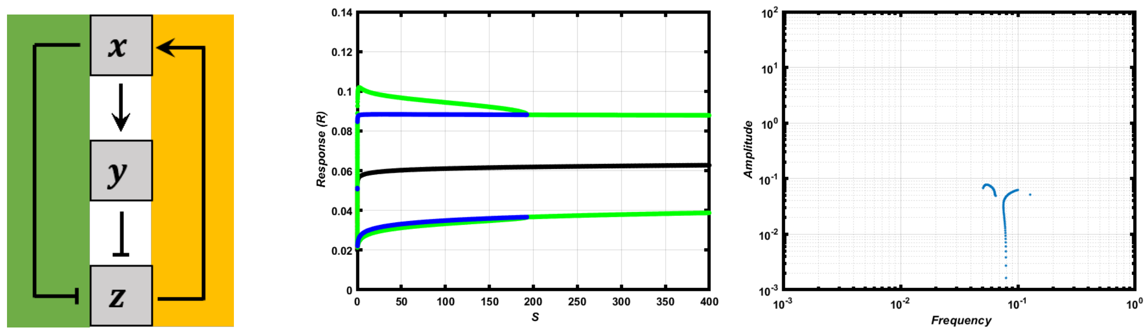


Fig S9a. **Fig S9b.** Bifurcation **Fig S9c.** Amp vs Fre

Fig S9. GW-3 + C3-FFL (OR): To the negative feedback (Goodwin 3), we added a coherent feed-forward loop (shaded in green) using OR logic. The bifurcation diagram shows Supercritical flip bifurcation. Upon adding the coherent feed-forward loop using OR logic the range of frequency is narrow and the fine-tuning is not robust. It shows amplitude fine-tuned with constant frequency.

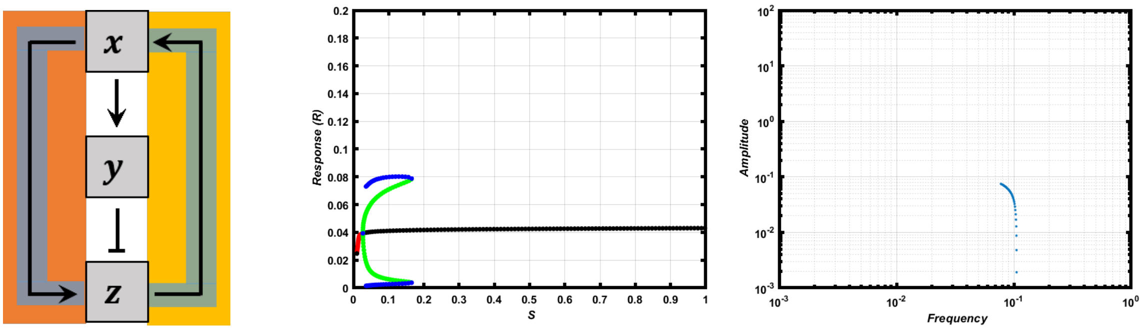


Fig S10a. **Fig S10b.** Bifurcation **Fig S10c.** Amp vs Fre

Fig S10. GW-3 + I1-FFL (AND): To the negative feedback (Goodwin 3), we added an incoherent feed-forward loop (shaded in red) using AND logic. The addition of the incoherent feed-forward loop makes the outer loop (shaded in blue) positive feedback. The bifurcation diagram shows bistability and Subcritical Hopf bifurcation. Upon adding the incoherent feed-forward loop, the frequency range is narrow in the Amplitude vs frequency plot and the fine-tuning is not robust. It shows amplitude fine-tuned with constant frequency.

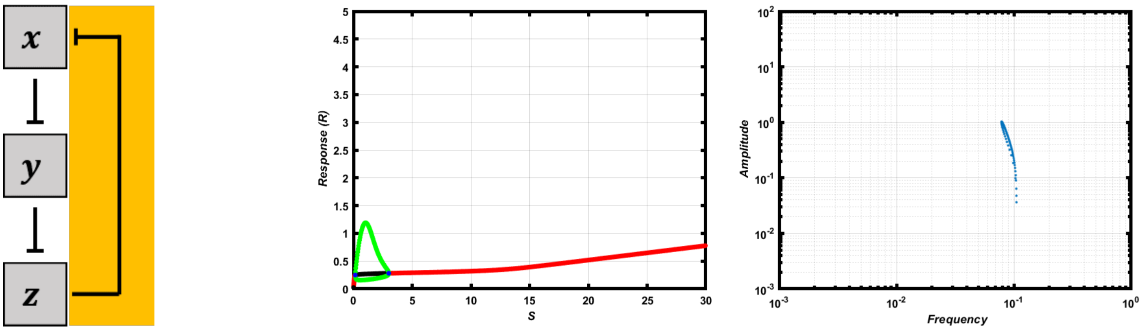


Fig S11a. **Fig S11b.** Bifurcation **Fig S11c.** Amp vs Fre

Fig S11. REP: The simulation of the bifurcation diagram for the negative feedback (Repressilator) shows Hopf bifurcation. The Amplitude vs Frequency plot shows that the frequency range is narrow and the fine-tuning of frequency is not robust. Showing, amplitude fine-tuned with constant frequency.

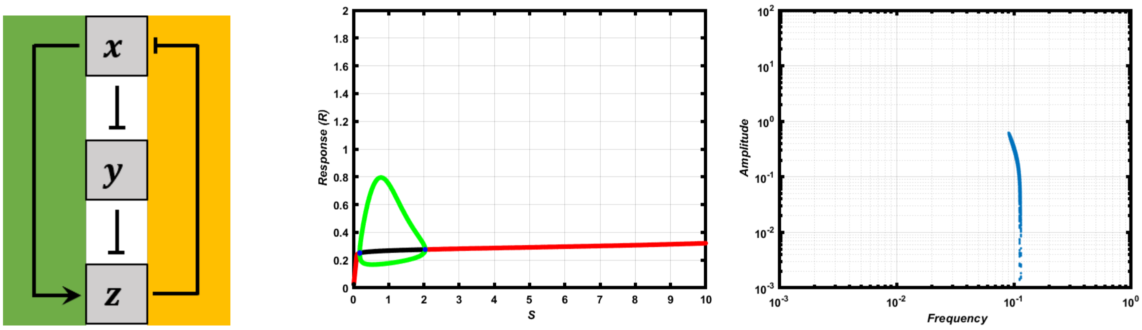


Fig S12a. **Fig S12b.** Bifurcation **Fig S12c.** Amp vs Fre

Fig S12. REP + C4-FFL (AND): To the negative feedback (Repressilator), we added a coherent feed-forward loop (shaded in green) using AND logic. The bifurcation diagram shows Supercritical Hopf bifurcation. Upon adding the coherent feed-forward loop using AND logic, the frequency range is narrow in the Amplitude vs frequency plot. The range of frequency is narrow and the fine-tuning is not robust. Showing amplitude fine-tuned with constant frequency.

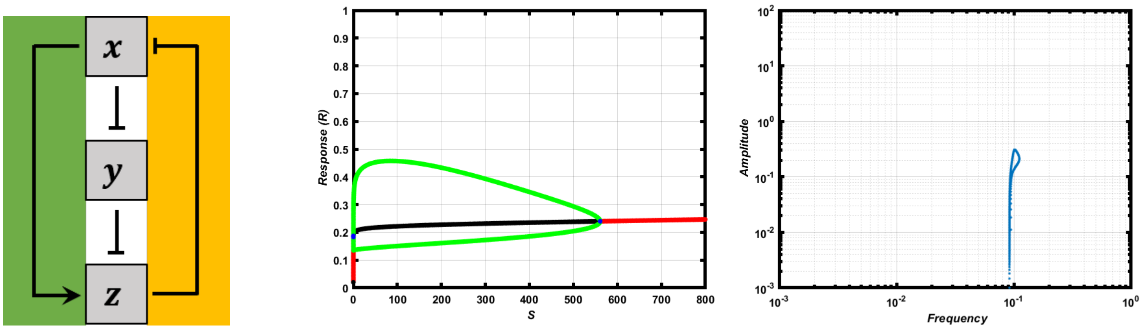


Fig S13a. **Fig S13b.** Bifurcation **Fig S13c.** Amp vs Fre

Fig S13. REP + C4-FFL (OR): To the negative feedback (Repressilator), we added a coherent feed-forward loop (shaded in green) using OR logic. The bifurcation diagram shows Supercritical Hopf bifurcation. Upon adding the coherent feed-forward loop using OR logic the range of frequency is narrow and the fine-tuning is not robust. Showing amplitude fine-tuned with constant frequency.

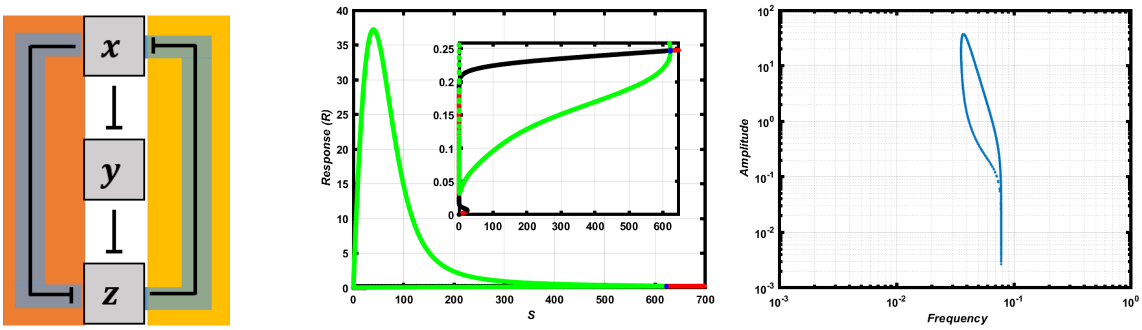


Fig S14a. **Fig S14b.** Bifurcation **Fig S14c.** Amp vs Fre

Fig S14. REP + I2-FFL (OR): To the negative feedback (Repressilator), we added an incoherent feed-forward loop (shaded in red) using AND logic. The addition of the incoherent feed-forward loop makes the outer loop (shaded in blue) positive feedback. The bifurcation diagram shows bistability and Supercritical Hopf bifurcation. Upon adding the incoherent feed-forward loop, the frequency range is narrow in the Amplitude vs frequency plot and the fine-tuning is not robust.It shows amplitude fine-tuned with constant frequency.