ASSIGNMENT – 2

Solution for 1(a):

Given data

Computer center open from 8 am to midnight.

Below is the shift available and minimum number of consultants required for each shift.

Time of day	Minimum number of consultants required to be on duty
8 am–noon	4
Noon–4 pm	8
4 pm–8 pm	10
8 pm–midnight	6

Two types of employees

1: Full-time: works for 8 hours and paid $14/hour and shifts available are

morning (8 am – 4 pm)

afternoon (noon – 8 pm)

evening (4 pm – midnight)

2: Part-time: works for 4 hours and paid $12/hour and shifts available are as above table.

Requirement is for every part-time there should be at least one full time.

Decision Variables:

$X_1 =$ Number of part-time consultants required for 8am to noon (shift 1).

$X_2 =$ Number of part-time consultants required for noon to 4 pm (shift 2).

$X_3 =$ Number of part-time consultants required for 4pm to 8pm (shift 3).

$X_4 =$ Number of part-time consultants required for 8 pm to midnight (shift 4).

$X_5 =$ Number of full-time consultants required for morning shift.

$X_6 =$ Number of full-time consultants required for afternoon shift.

$X_7 =$ Number of full-time consultants required for evenings shift.

Objective Function

$Z_{min} =$ (14*8) $(X_5 + X_6 + X_7)$ + (12*4) $(X_1 + X_2 + X_3 + X_4)$

Constraints

a. Minimum number of consultants required on each shift:

$X_1 + X_5 \geq 4$

$X_2 + X_6 + X_5 \geq 8$

$X_3 + X_6 + X_7 \geq 10$

$X_4 + X_7 \geq 6$

b. Number of hours of full-time consultants and part-time consultants:

$X_5 \geq 2X_1$

$X_5 + X_6 \geq 2X_2$

$X_6 + X_7 \geq 2X_3$

$X_7 \geq 2X_4$

c. Non-negativity condition:

$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 0$

Mathematical Linear Programming Formulation

Standard Form:

$Z_{min} = (14*8)(X_5 + X_6 + X_7) + (12*4)(X_1 + X_2 + X_3 + X_4)$

Subject to restrictions:

$X_1 + X_5 \geq 4$

$X_5 + X_6 + X_2 \geq 8$

$X_3 + X_6 + X_7 \geq 10$

$X_4 + X_7 \geq 6$

$X_5 \geq 2X_1$

$X_5 + X_6 \geq 2X_2$

$X_6 + X_7 \geq 2X_3$

$X_7 \geq 2X_4$

$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 0$

Now by excel tabular form explanation

$X_1 = 2$

$X_2 = 4$

$X_3 = 5$

$X_4 = 3$

$X_5 = 2$

$X_6 = 2$

$X_7 = 3$

Number of full-time consultants required is 7

Number of part-time consultants required is 14

Minimum cost Z = $1456.00

Full Time	$ 14.00
Part Time	$ 12.00

	Shift 1				Shift 2				Shift 3				Shift 4						
	8:00- 9:00	9:00 - 10:00	10:00 - 11:00	11:00 - 12:00	12:00 - 1:00	1:00 - 2:00	2:00 - 3:00	3:00 - 4:00	4:00 - 5:00	5:00 - 6:00	6:00 - 7:00	7:00 - 8:00	8:00 - 9:00	9:00 - 10:00	10:00 - 11:00	11:00-12:00			
Consultant 1	1	1	1	1	1	1	1	1									8	Full Time	$ 112.00
Consultant 2	1	1	1	1	1	1	1	1									8	Full Time	$ 112.00
Consultant 3	1	1	1	1													4	Part Time	$ 48.00
Consultant 4	1	1	1	1													4	Part Time	$ 48.00
Consultant 5									1	1	1	1	1	1	1	1	8	Full Time	$ 112.00
Consultant 6									1	1	1	1	1	1	1	1	8	Full Time	$ 112.00
Consultant 7									1	1	1	1	1	1	1	1	8	Full Time	$ 112.00
Consultant 8					1	1	1	1									4	Part Time	$ 48.00
Consultant 9					1	1	1	1	1	1	1	1					8	Full Time	$ 112.00
Consultant 10					1	1	1	1	1	1	1	1					8	Full Time	$ 112.00
Consultant 11					1	1	1	1									4	Part Time	$ 48.00
Consultant 12									1	1	1	1					4	Part Time	$ 48.00
Consultant 13									1	1	1	1					4	Part Time	$ 48.00
Consultant 14									1	1	1	1					4	Part Time	$ 48.00
Consultant 15									1	1	1	1					4	Part Time	$ 48.00
Consultant 16									1	1	1	1					4	Part Time	$ 48.00
Consultant 17													1	1	1	1	4	Part Time	$ 48.00
Consultant 18													1	1	1	1	4	Part Time	$ 48.00
Consultant 19					1	1	1	1									4	Part Time	$ 48.00
Consultant 20					1	1	1	1									4	Part Time	$ 48.00
Consultant 21													1	1	1	1	4	Part Time	$ 48.00
minimum consultant required in particular shift	4	4	4	4	8	8	8	8	10	10	10	10	6	6	6	6			$ 1,456.00

<u>Solution for 1(b):</u>

In this problem break time of one-hour is included which can be taken after 3 hours of works or 4 hours of works for full-time consultants only.

Decision Variables:

X_1 = Number of part-time consultants required for 8am to noon (shift 1).

X_2 = Number of part-time consultants required for noon to 4 pm (shift 2).

X_3 = Number of part-time consultants required for 4pm to 8pm (shift 3).

X_4 = Number of part-time consultants required for 8 pm to midnight (shift 4).

X_5 = Number of full-time consultants required for morning shift.

X_6 = Number of full-time consultants required for afternoon shift.

X_7 = Number of full-time consultants required for evenings shift.

Objective Function

Z_{min} = (14*8) $(X_5 + X_6 + X_7)$ + (12*4) $(X_1 + X_2 + X_3 + X_4)$

Assuming that full time consultant's break time is paid.

Subject to restrictions:

$X_1 + X_5 \geq 4$

$X_5 + X_6 + X_2 \geq 8$

$X_3 + X_6 + X_7 \geq 10$

$X_4 + X_7 \geq 6$

$X_5 \geq 2X_1$

$X_5 + X_6 \geq 2X_2$

$X_6 + X_7 \geq 2X_3$

$X_7 \geq 2X_4$

$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 0$

Full Time	$ 14.00
Part Time	$ 12.00

	Shift 1				Shift 2				Shift 3				Shift 4						
	8:00- 9:00	9:00 - 10:00	10:00 - 11:00	11:00 - 12:00	12:00 - 1:00	1:00 - 2:00	2:00 - 3:00	3:00 - 4:00	4:00 - 5:00	5:00 - 6:00	6:00 - 7:00	7:00 - 8:00	8:00 - 9:00	9:00 - 10:00	10:00 - 11:00	11:00-12:00			
Consultant 1	1	1	1		1	1	1	1									7	Full Time	$ 112.00
Consultant 2	1	1	1	1		1	1	1									7	Full Time	$ 112.00
Consultant 3	1	1	1	1		1	1	1									7	Full Time	$ 112.00
Consultant 4	1	1	1	1													4	Part Time	$ 48.00
Consultant 5	1	1	1	1													4	Part Time	$ 48.00
Consultant 6									1	1	1		1	1	1	1	7	Full Time	$ 112.00
Consultant 7									1	1	1		1	1	1	1	7	Full Time	$ 112.00
Consultant 8									1	1	1		1	1	1	1	7	Full Time	$ 112.00
Consultant 9									1	1	1		1	1	1	1	7	Full Time	$ 112.00
Consultant 10									1	1	1		1	1	1	1	7	Full Time	$ 112.00
Consultant 11					1	1	1	1		1	1	1					7	Full Time	$ 112.00
Consultant 12					1	1	1	1		1	1	1					7	Full Time	$ 112.00
Consultant 13					1	1	1	1		1	1	1					7	Full Time	$ 112.00
Consultant 14					1	1	1	1									4	Part Time	$ 48.00
Consultant 15					1	1	1	1		1	1	1					7	Full Time	$ 112.00
Consultant 16					1	1	1	1		1	1	1					7	Full Time	$ 112.00
Consultant 17					1	1	1	1									4	Part Time	$ 48.00
Consultant 18									1	1	1	1					4	Part Time	$ 48.00
Consultant 19									1	1	1	1					4	Part Time	$ 48.00
Consultant 20									1	1	1	1					4	Part Time	$ 48.00
Consultant 21									1	1	1	1					4	Part Time	$ 48.00
Consultant 22									1	1	1	1					4	Part Time	$ 48.00
Consultant 23													1	1	1	1	4	Part Time	$ 48.00
minimum consultant required in particular shift	5	5	5	4	8	10	10	10	10	15	15	10	6	6	6	6			$ 1,936.00

So

Now by excel tabular form explanation

$$X_1 = 2$$

$$X_2 = 2$$

$$X_3 = 5$$

$$X_4 = 1$$

$$X_5 = 3$$

$$X_6 = 5$$

$$X_7 = 5$$

In this consideration full-time consultant required is 13

Part-time consultant required is 10.

$Z_{min} = 14 * 8 * 13 + 12 * 4 * 10$ = \$ 1936.00.

Solution 2:

According to assignment-1 solution that is as follows:

Data given

- Two models produced by back savers are Collegiate and Mini.
- Material required to produce each Collegiate is 3 square feet and each Mini is 2 square feet.
- Shipment of material per week is 5000 square feet.
- 1000 Collegiate and 1200 Mini sold per week.
- Time required to produce each Collegiate and Mini is 45 **minutes** and 40 **minutes** each respectively.
- Profit produced by each Collegiate and Mini is $32 and $24 each.
- Total **laborers** available for the company is **35** and each provided with **40 hours** per week.

Model	Material required for each model	Sold per week	Time required for each labor to produce	profit
Collegiate	3 sq. Ft	1000	45 minutes	$32
Mini	2 sq. Ft	1200	40 minutes	$24

Decision Variables

X- Number of Collegiate model quantity required to produce per week.

Y-Number of Mini model required quantity to produce per week.

Z_{max} -Maximize Profit per week.

Objective Function

Z_{max} = 32X+24Y

Constraints

1.Sales forecast mentioned is X model is 1000 and Y model is 1200 per week so it’s a restriction.

X ≤ 1000

Y ≤ 1200

2.Total shipment they can receive per week is 5000 square feet.

3X+2Y ≤ 5000

3.Labor time required to produce each model in minutes.

45X+40Y ≤ 84000

(Explanation for 84000 minutes

According to given data 40 hours per week which should be converted into minutes is 40*60=2400 minutes. Number of laborers mentioned is 35 so 35*2400=84000 total minutes available.)

4.Non-negativity restrictions.

$X \geq 0$

$Y \geq 0$.

Mathematical Linear Programming Formulation

Standard form:

Maximize Z = 32X+24Y

Subject to the restrictions:

Constraint1: $X \leq 1000$

Constraint2: $Y \leq 1200$

Constraint3: $3X+2Y \leq 5000$

Let X=0

3X+2Y = 5000

Y= 2500

Substitute Y in above equation

X= 1666.6

Constraint4: $45X+40Y \leq 84000$

Let X=0

445X + 40Y =84000

Y=2100

Substitute Y in above equation

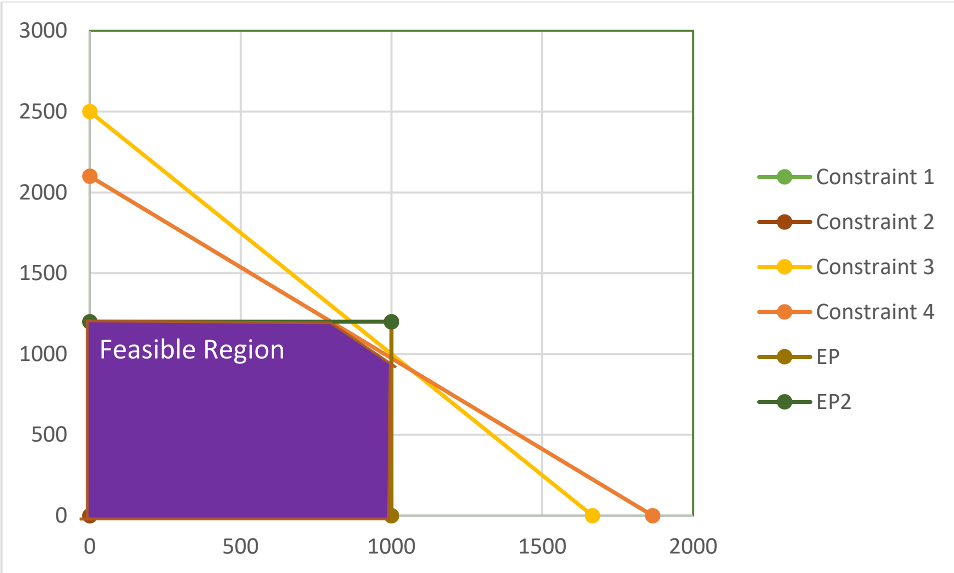
X=1866

and

$X \geq 0$

$Y \geq 0$.

By using all above constraints value plotted graph as below



EP points is extra points for finding optimal solution.

Maximize Z = 32X+24Y

By EP point Y=1200

Then substitute in constraint 4 that is

9X + 8Y =16800

9X=16800-9600

X=800

(X, Y) = (800,1200)

SUBSTITUTE above points in Maximize Z

Then Z=54400.

By EP2 point X=1000

Then substitute in constraint 4 that is

9X + 8Y =16800

8Y=16800-900

Y=975

(X, Y) = (1000,975)

SUBSTITUTE above points in Maximize Z

Then Z=55400.

So, the optimal solution for maximize Z= 55400 for points (X, Y) = (1000,975).

Solution 3:

Given data

Decision Variables:

X_{ij} = Number of each size of products need to be produced by each plant .

Whereas i = 1 =Plant1

i = 2 =Plant2

i = 3 =Plant3

j = 1 =Large

j = 2 =Medium

j = 3 =Small

Objective Function:

Maximize Z = $420(X_{11} + X_{21} + X_{31}) + 360(X_{12} + X_{22} + X_{32}) + 300(X_{13} + X_{23} + X_{33})$.

Constraints:

a. Excess capacity for each plant

$X_{11} + X_{12} + X_{13} \leq 750$

$X_{21} + X_{22} + X_{23} \leq 900$

$X_{31} + X_{32} + X_{33} \leq 450$

b. In-process storage space

$12X_{11} + 15X_{12} + 20X_{13} \leq 13000$

$12X_{21} + 15X_{22} + 20X_{23} \leq 12000$

$12X_{31} + 15X_{32} + 20X_{33} \leq 5000$

c. Sales forecast

$X_{11} + X_{21} + X_{31} \leq 900$

$X_{12} + X_{22} + 20X_{32} \leq 1200$

$X_{13} + X_{23} + X_{33} \leq 750$

d. To avoid layoff that the plants should use the same percentage of their excess capacity to produce the new product.

1/750($X_{11} + X_{12} + X_{13}$)-1/900($X_{21} + X_{22} + X_{23}$)=0

1/900($X_{21} + X_{22} + X_{23}$)-1/450($X_{31} + X_{32} + X_{33}$) =0

1/450($X_{31} + X_{32} + X_{33}$)- 1/750($X_{11} + X_{12} + X_{13}$)=0

e. Non-negativity constraints

X_{ij} = Number of each size of products need to be produced by each plant ≥ 0.

Whereas i = 1 =Plant1

i = 2 =Plant2

i = 3 =Plant3

j = 1 =Large

j = 2 =Medium

j = 3 =Small.

Mathematical Linear Programming Formulation

Standard form:

Maximize Z = $420(X_{11} + X_{21} + X_{31}) + 360(X_{12} + X_{22} + X_{32}) + 420(X_{13} + X_{23} + X_{33})$.

Subject to restrictions:

$X_{1j} \leq 750$

$X_{2j} \leq 900$

$X_{3j} \leq 450$

$12X_{11} + 15X_{12} + 20X_{13} \leq 13000$

$12X_{21} + 15X_{22} + 20X_{23} \leq 12000$

$12X_{31} + 15X_{32} + 20X_{33} \leq 5000$

$X_{11} + X_{21} + X_{31} \leq 900$

$X_{12} + X_{22} + 20X_{32} \leq 1200$

$X_{13} + X_{23} + X_{33} \leq 750$

1/750($X_{11} + X_{12} + X_{13}$)-1/900($X_{21} + X_{22} + X_{23}$)=0

1/900($X_{21} + X_{22} + X_{23}$)-1/450($X_{31} + X_{32} + X_{33}$) =0

1/450($X_{31} + X_{32} + X_{33}$)- 1/750($X_{11} + X_{12} + X_{13}$)=0

X_{ij} = Number of each size of products need to be produced by each plant ≥ 0.

Whereas i = 1 =Plant1

i = 2 =Plant2

i = 3 =Plant3

j = 1 =Large

j = 2 =Medium

j = 3 =Small.