ASSIGNMENT-3

Solution 1:

Decision Variables:

$L1$ = number of large units produced per day at Plant 1,
$M1$ = number of medium units produced per day at Plant 1,
$S1$ = number of small units produced per day at Plant 1,
$L2$ = number of large units produced per day at Plant 2,
$M2$ = number of medium units produced per day at Plant 2,
$S2$ = number of small units produced per day at Plant 2,
$L3$ = number of large units produced per day at Plant 3,
$M3$ = number of medium units produced per day at Plant 3,
$S3$ = number of small units produced per day at Plant 3.

Objective function:

Maximize $Z = 420\ L1 + 360\ M1 + 300\ S1 + 420\ L2 + 360\ M2 + 300\ S2 + 420\ L3 + 360\ M3 + 300\ S3$

Constraints:

a. Excess capacity for each plant

$$L1 + M1 + S1 \leq 750$$
$$L2 + M2 + S2 \leq 900$$
$$L3 + M3 + S3 \leq 450$$

b. In-process storage space

$$20\ L1 + 15\ M1 + 12\ S1 \leq 13000$$
$$20\ L2 + 15\ M2 + 12\ S2 \leq 12000$$
$$20\ L3 + 15\ M3 + 12\ S3 \leq 5000$$

c. Sales forcast

$$L1 + L2 + L3 \leq 900$$
$$M1 + M2 + M3 \leq 1200$$
$$S1 + S2 + S3 \leq 750$$

d. To avoid layoff that the plants should use the same percentage of their excess capacity to produce the new product. Using only two constraints from this particular case.

$$\frac{1}{750}(L1 + M1 + S1) - \frac{1}{900}(L2 + M2 + S2) = 0$$
$$\frac{1}{750}(L1 + M1 + S1) - \frac{1}{450}(L3 + M3 + S3) = 0$$

e. Non-negativity

$$L1 \geq 0,\ M1 \geq 0,\ S1 \geq 0,\ L2 \geq 0,\ M2 \geq 0,\ S2 \geq 0, L3 \geq 0,\ M3 \geq 0,\ S3 \geq 0.$$

Mathematical Linear Programming Formulation

Standard form:

Maximize Z: $420\ L1 + 360\ M1 + 300\ S1 + 420\ L2 + 360\ M2 + 300\ S2 + 420\ L3 + 360\ M3 + 300\ S3$;

Subject to restrictions:

L1 + M1 + S1 <= 750
L2 + M2 + S2 <= 900
L3 + M3 + S3 <= 450
20 L1 + 15 M1 + 12 S1 <= 13000
20 L2 + 15 M2 + 12 S2 <= 12000
20 L3 + 15 M3 + 12 S3 <= 5000
L1 + L2 + L3 <= 900
M1 + M2 + M3 <= 1200
S1 + S2 + S3 <= 750
900 L1 + 900 M1 + 900 S1 - 750 L2 - 750 M2 - 750 S2 = 0
450 L1 + 450 M1 + 450 S1 - 750 L3 - 750 M3 - 750 S3 = 0

Solved using lpsolve in R is in separate file.

Solution 2:

To find shadow price, reduced cost, dual solution using "get.sensitivity.rhs(x)".

Shadow Price:
0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

Reduced Cost:
0.00 0.00 -24.00 -40.00 0.00 0.00 -360.00 -120.00 0.00

Dual Solution:
0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

Solution 3:

The range of price and cost

$duals
[1] 0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56 0.00 0.00 -24.00 -40.00 0.00
[17] 0.00 -360.00 -120.00 0.00

$dualsfrom is lower limit
[1] -1.000000e+30 -1.000000e+30 -1.000000e+30 1.122222e+04 1.150000e+04 4.800000e+03 -1.000000e+30 -1.000000e+30 -1.000000e+30
[10] -2.500000e+04 -1.250000e+04 -1.000000e+30 -1.000000e+30 -2.222222e+02 -1.000000e+02 -1.000000e+30 -1.000000e+30 -2.000000e+01
[19] -4.444444e+01 -1.000000e+30

$dualstill is upper limit
[1] 1.000000e+30 1.000000e+30 1.000000e+30 1.388889e+04 1.250000e+04 5.181818e+03 1.000000e+30 1.000000e+30 1.000000e+30 2.500000e+04
[11] 1.250000e+04 1.000000e+30 1.000000e+30 1.111111e+02 1.000000e+02 1.000000e+30 1.000000e+30 2.500000e+01 6.666667e+01 1.000000e+30

The range for objective function variables

$objfrom is lower limit
[1] 3.60e+02 3.45e+02 -1.00e+30 -1.00e+30 3.45e+02 2.52e+02 -1.00e+30 -1.00e+30 2.04e+02

$objtill is upper limit
[1] 4.60e+02 4.20e+02 3.24e+02 4.60e+02 4.20e+02 3.24e+02 7.80e+02 4.80e+02 1.00e+30

Solution 4:

Dual Solution

Objective Function:

Minimize $750y1 + 900y2 + 450y3 + 13000y4 + 12000y5 + 5000y6 + 900y7 + 1200\ y8 + 750y9 + 0y10 + 0y11$

Constraints:

$y1+20y4+y8+900y10+450y11 \geq 420$
$y1+15y4+y8+900y10+450y11 \geq 360$
$y1+12y4+y9+900y10+450y11 \geq 300$
$y2 + 20\ y5 + y7 - 750y10 >= 420;$
$y2 + 15\ y5 + y8 - 750y10 >= 360;$
$y2 + 12\ y5 + y9 - 750y10 >= 300;$
$y3 + 20\ y6 + y7 - 750y11 >= 420;$
$y3 + 15\ y6 + y8 - 750y11 >= 360;$
$y3 + 12\ y6 + y9 - 750y11 >= 300;$

$y1,y2,y3,y4,y5,y6,y7,y8,y9 \geq 0$
$y10$, $y11$ are unrestricted.

Problem is solved using lpsolve in R observed that solution of dual problem is same as shadow price of primal problem and also shadow price of dual problem is same as solution of primal problem.