

Boosting — Sequential  
— Additive Model  
— Boosting reduces Bias

Stage 0 — Predicts  $\hat{y}$

$$D^{100} \rightarrow \{(x_i, y_i)\}_{i=1}^n$$

$$\hat{y} = f_0(x) \quad \boxed{\hat{y} = f(x)}$$

Base Models

high Bias + low Variance

Weak Learners

Stage 1 — Predicts  $E_0$

$$\{(x_i, E_{0i})\}_{i=1}^n$$

$$\hat{E}_0 = f_1(x)$$

$$\begin{aligned} \hat{E}_1 &= \hat{E}_0 - E_0 \\ &= \hat{y} - f_0(x) - f_1(x) \end{aligned}$$

Stage 2 — Predicts  $E_1$

$$\{(x_i, E_{1i})\}_{i=1}^n \quad \hat{E}_1 = f_2(x)$$

$$\begin{aligned} E_2 &= E_1 - \hat{E}_1 \\ &= y - f_0(x) - f_1(x) - f_2(x) \end{aligned}$$

$$\hat{y} = f_0(x) + f_1(x) + f_2(x) +$$

Stage 0

	$x_1$	$x_2$	$y$	$\hat{y}$	$f_0(x)$	$E_0$
Area	Bedrooms	Pence				
1500	2	50	32	18		
1800	2	70	58	12		
2200	3	83	80	7		
2800	3	98	89	9		

Stage 2

	$x_1$	$x_2$	$y$	$\hat{y}$	$f_2(x)$
Area	Bedroom				
1500	2	9	3	6	
1800	2	6	2	4	
2200	3	5	2	3	
2800	3	6	3	3	

Stage 1

 $f_1(x)$  $\uparrow$  $\wedge$ 

	$x_1$	$x_2$	$E_0$	$E_0$	$E_1$
Area	Bedrooms				
1500	2	18	9	9	
1800	2	12	6	6	
2200	3	7	2	5	
2800	3	9	3	6	

$$y = 32 + 9 + 3 \\ = 44$$

$$\rightarrow \frac{53}{50} \downarrow \rightarrow 49 \cdot 9$$

$$\rightarrow 43$$

$$\rightarrow 44 \cdot 9$$

$$\rightarrow 42 \cdot 5$$

$$\rightarrow 43 \uparrow$$

$$\rightarrow 41$$

$$\uparrow$$

$\rightarrow 32$  Rough prediction of  $y$

## Classification

Partition feature space  
into pure regions assigned  
to each class

$PL < 3.5$   
Setosa

Else

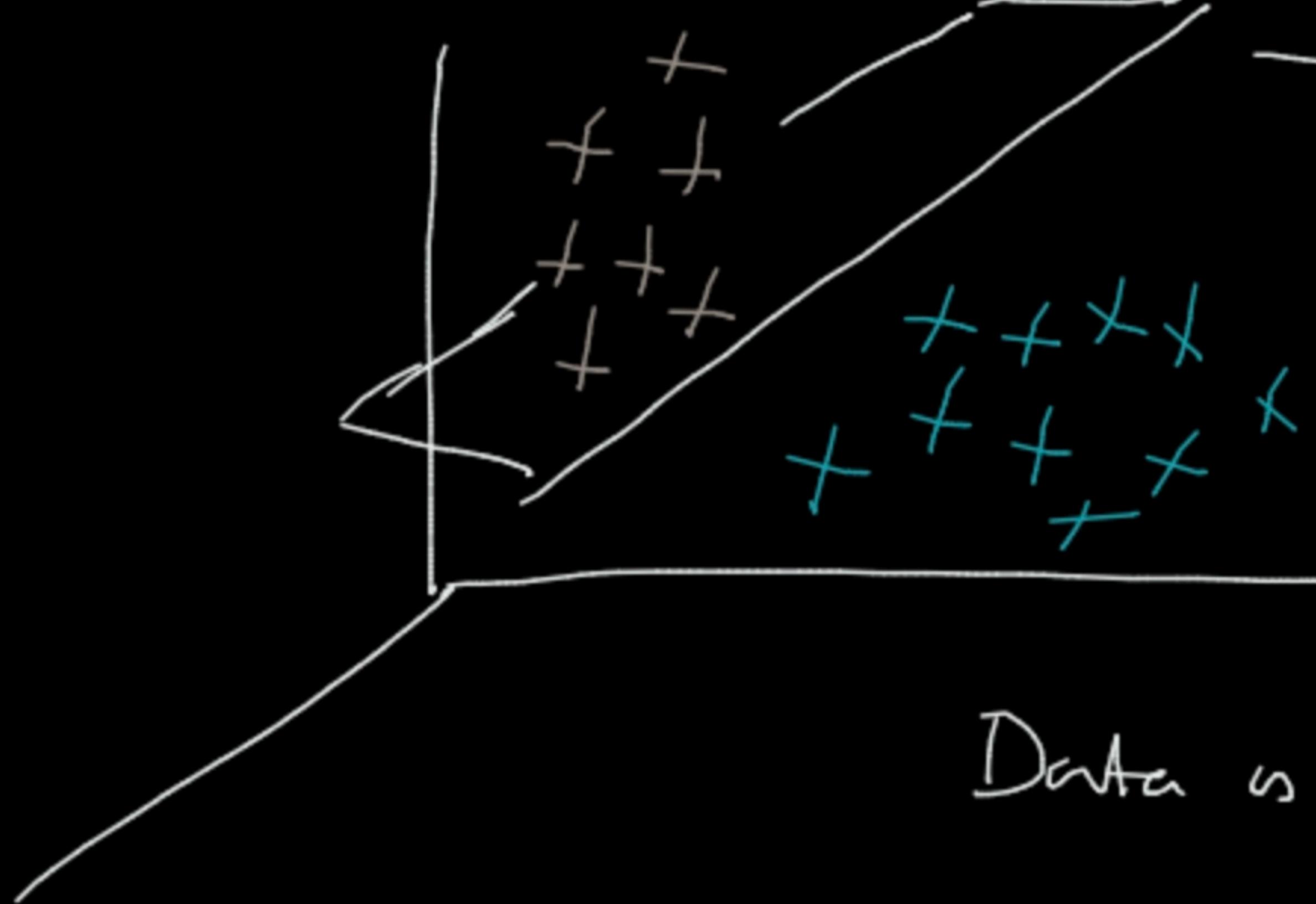
$PL > 4.2$

Versicolor

else

Virginica

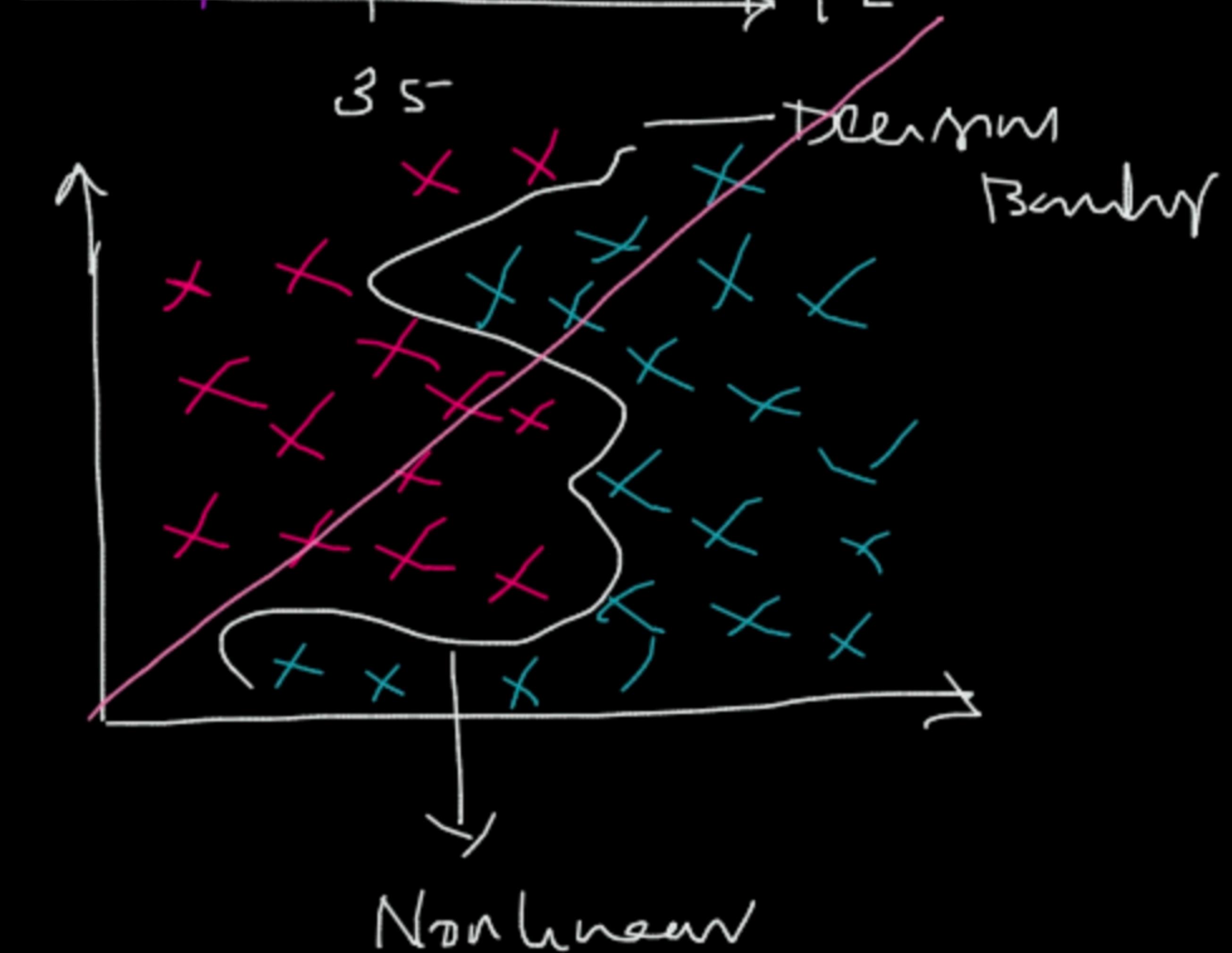
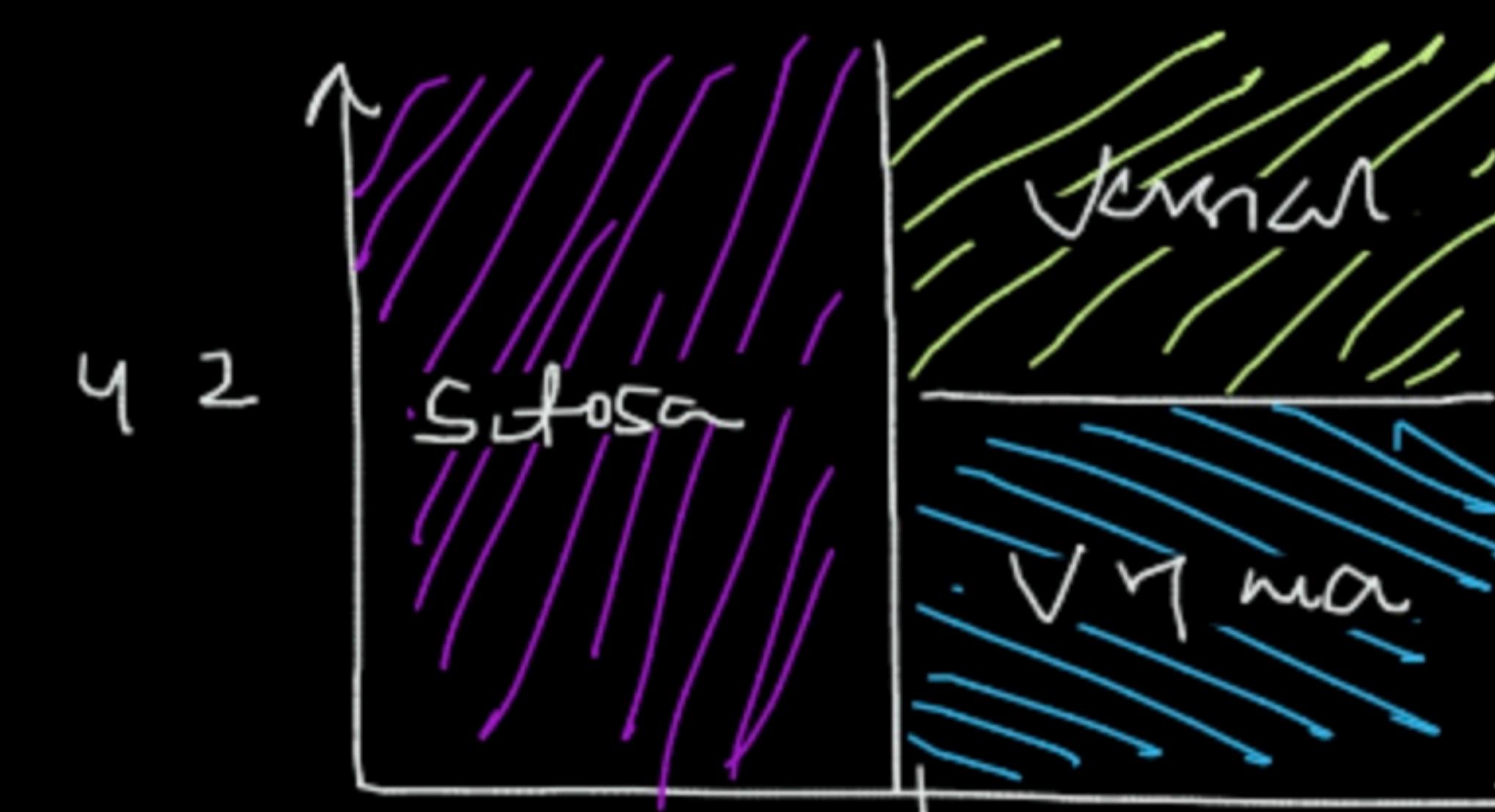
→ Decision Boundary



Data is linearly separable

DTree → Axes parallel

SL



Nonlinear

$\begin{matrix} 0 & 0 \\ \uparrow \\ M_1 & M_2 & M_3 & M_4 \\ \downarrow \\ 0 & 0 \end{matrix}$

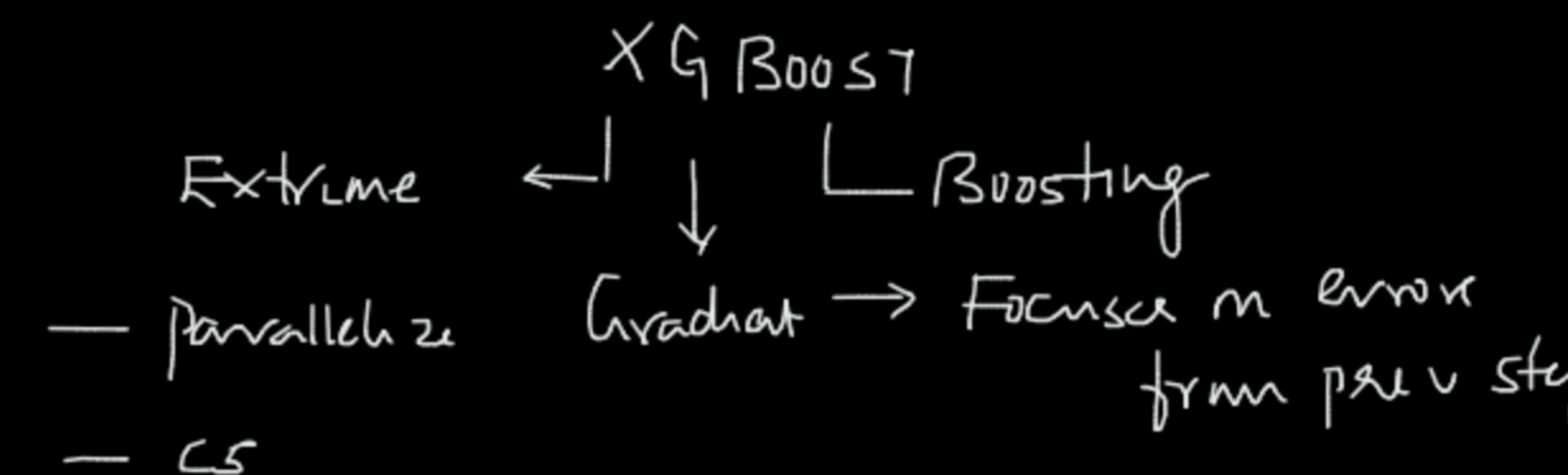
1 Assigns weightage to models  
based on their performance

2 Every Data point is given a  
Weightage

	$x_1$	$x_2$	$y$	
$0.13 \downarrow 0, 2$	-	-	0	0
$0.3 \uparrow 0, 2$	-	-	1	0
$0.13 \downarrow 0, 2$	-	-	1	1
$0.13 \downarrow 0, 2$	-	-	0	0
$0.3 \uparrow 0, 2$	-	-	1	0
$\overline{1.0}$	$\overline{1.0}$			

$y \rightarrow f_1(x) + f_2(x) -$

Pip install xgboost



$$\frac{dy}{dx} \rightarrow \frac{\Delta y}{\Delta x}$$

Loss Fn  $\rightarrow$  Fn of error  
 $\rightarrow$  Squared Error

$$\text{Loss Fn(L)} = \sum_{n=1}^N (y_i - \hat{y}_i)^2$$

$$y - \hat{y}$$

$$\begin{aligned} y &= x \\ &= 2x^2 \end{aligned}$$

"We can customize

$$\text{Loss}(L) = (y - \hat{y})^2$$

(The loss fn provided  
 If n differentiable)

↑

$$\frac{\partial L}{\partial \hat{y}} = \text{Partial differential}$$

$$\frac{\partial L}{\partial \hat{y}} = -\alpha(y - \hat{y})$$

Pseudo residual  $\rightarrow -\frac{1}{2} \frac{\partial L}{\partial \hat{y}} = (y - \hat{y})$

$\hookrightarrow$  residual

$x$	$y$	$\hat{y}$	$E_0$	$\hat{y}$	$E_1$	$\hat{y}$	$E_2$
Dosage	Efficiency						
15	12	-10	0,5	-10,5	-2,65	-7,35	-2,88
18	7	0,5	6,5	6,5	+2,6	4,4	
27	8	0,5	7,5	7,5	+2,6	5,4	
33	-7	0,5	-7,5	-7,5	-1,75	-5,25	

<sup>↑ posw</sup>

$$f_0(x) = 0,5$$

$$f_1(x) \rightarrow \hat{E}_0$$

$$\text{SIM} = \frac{(\sum \text{residual})^2}{n \text{ obs} + \lambda} = 0$$

$$\boxed{-10,5 \ 6,5 \ 7,5 \ -7,5}$$

$$\text{SIM} = \frac{(-4)^2}{4} = 4,0$$

$$\boxed{-10,5}$$

$$\text{SIM} = (-10,5)^2$$

$$= 110,25$$

$$0,3 \times (-10,5)$$

$$-3,15$$

$$\boxed{6,5 \ 7,5 \ -7,5} \quad \text{SIM} = \frac{(6,5)^2}{3} = 14,08$$

$$\boxed{6,5 \ 7,5} \quad +7,0$$

$$\hat{y} = f_0(x) + f_1(x)$$

$$= 0,5 + 0,3(7,0)$$

$$\begin{aligned}\hat{y} &= 0,5 + 0,3(7,0) \\ &= +2,6\end{aligned}$$

$$\text{Gain} = \text{Left SIM} + \text{Right SIM} - \text{Parent SIM}$$

$$\begin{aligned}&= 110,25 + 14,08 - 4 \\ &= 120,33\end{aligned}$$

$$\alpha_2 = 0,5$$

$$\boxed{-7,35 \ 4,4 \ 5,4 \ -5,25}$$

$$\downarrow \boxed{\text{Dosage} < 23}$$

$$\boxed{-7,35, 4,4}$$

$$\boxed{5,4, -5,25}$$

$$\begin{aligned}\hat{y} &= 0,5 + (-2,65) + 0,5(-1,47) \\ &= 0,5 + (-2,65) + 0,5(-1,47) \\ &= -2,88\end{aligned}$$

$$\begin{aligned}\hat{y} &= 0,5 + 0,3(-7,5) \\ &= 0,5 - 2,25\end{aligned}$$

$$\hat{y} = f_0(x) + \alpha_1 f_1(x) + \alpha_2 f_2(x) + \dots$$
$$y - \hat{y} \rightarrow \text{pseudo residual}$$

