Time Series

	X_1	X_2	X_3	y
1	–	–	–	1
2	–	–	–	0
n	–	–	–	1

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

Jan 2001	1	–
Feb 2001	1	–
	3	–
Dec 2025	300	–
Jan 2026	301	y_t

Future ← Past Values

(y) (x)

	Time	y	
Jan	1	100	y_{t-11}
Feb	2	98	
Mar	3	110	
Oct			y_{t-2}
Nov	n	123	y_{t-1}
Dec		y_t	

Stocks

Time	y
1	–
2	–
3	–
$n = 12$	–
	y_t

$$\hat{y}_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \quad + \beta_{11} y_{t-11}$$

Regression

x_1	x_2	x_3	y
Age			

← Auto Regression →

Time	y
2000 →	–
2004	–
	–
2012 Jan	–
2012 Feb	NA
2012 Mar	–
	–
	–
	–
Dec 2025	y_t

500

past Values 13K

Frequency →

Era →

Yearly

Monthly →

Weekly

Hourly →

ms →

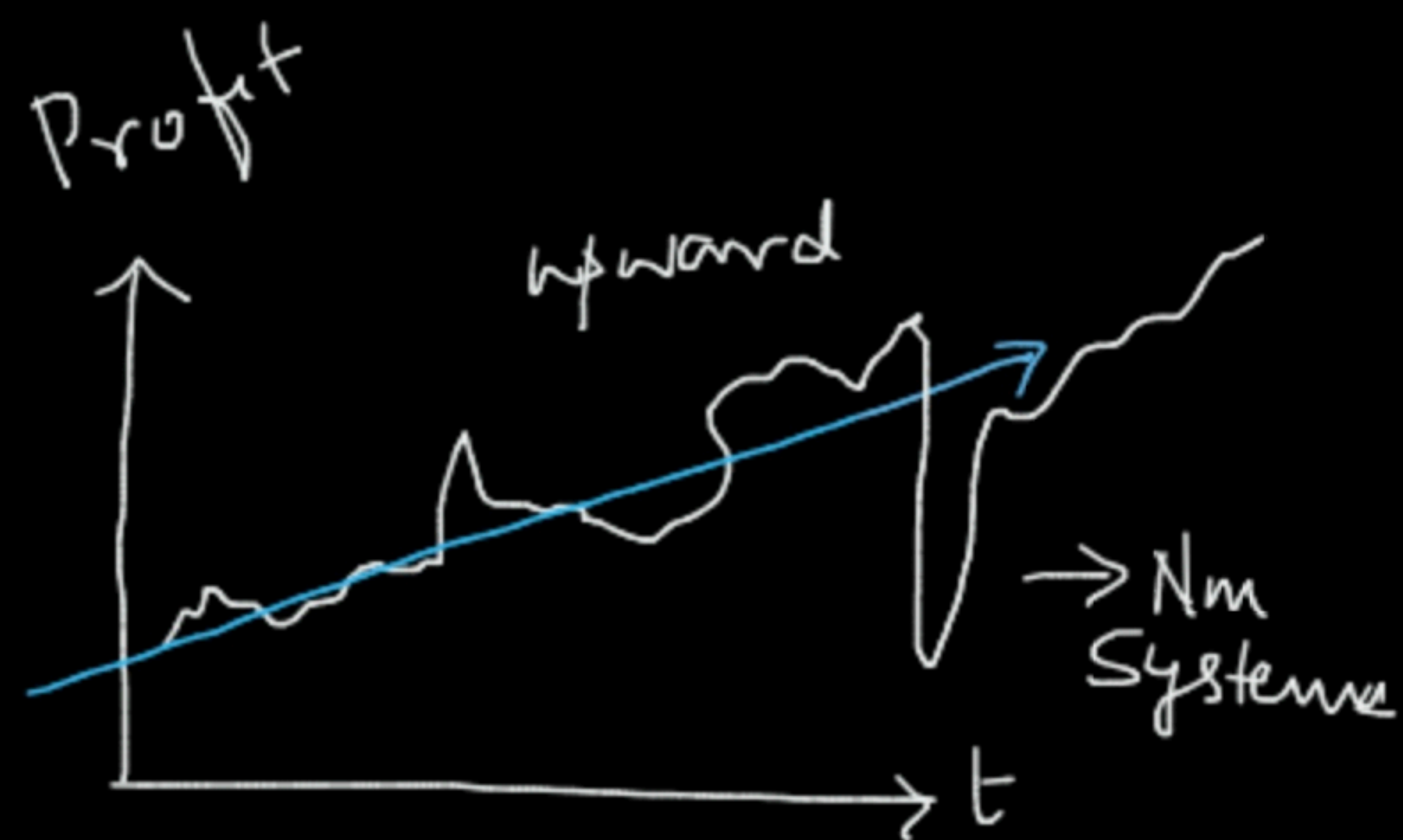
Forward / Back Fill

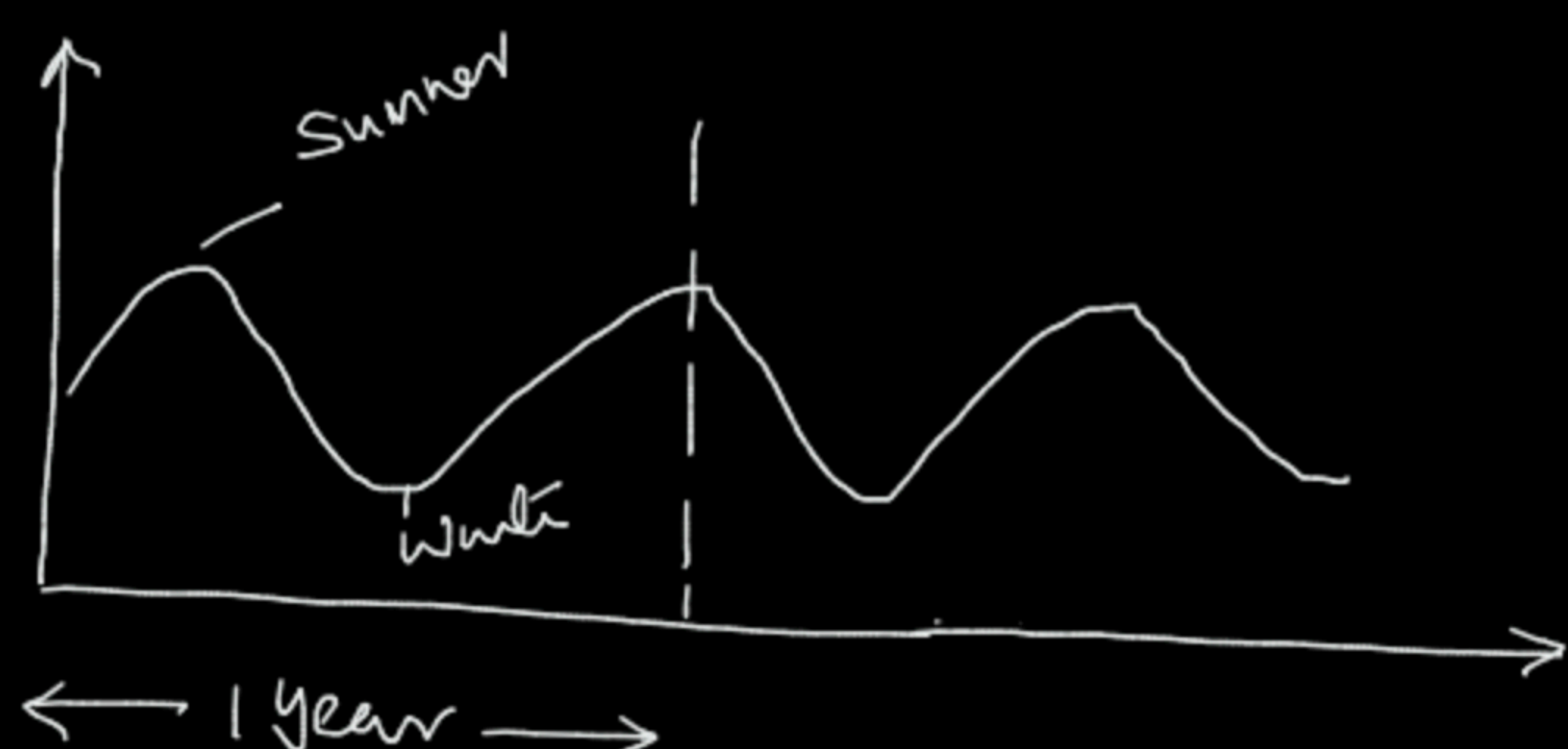
Components of time Series

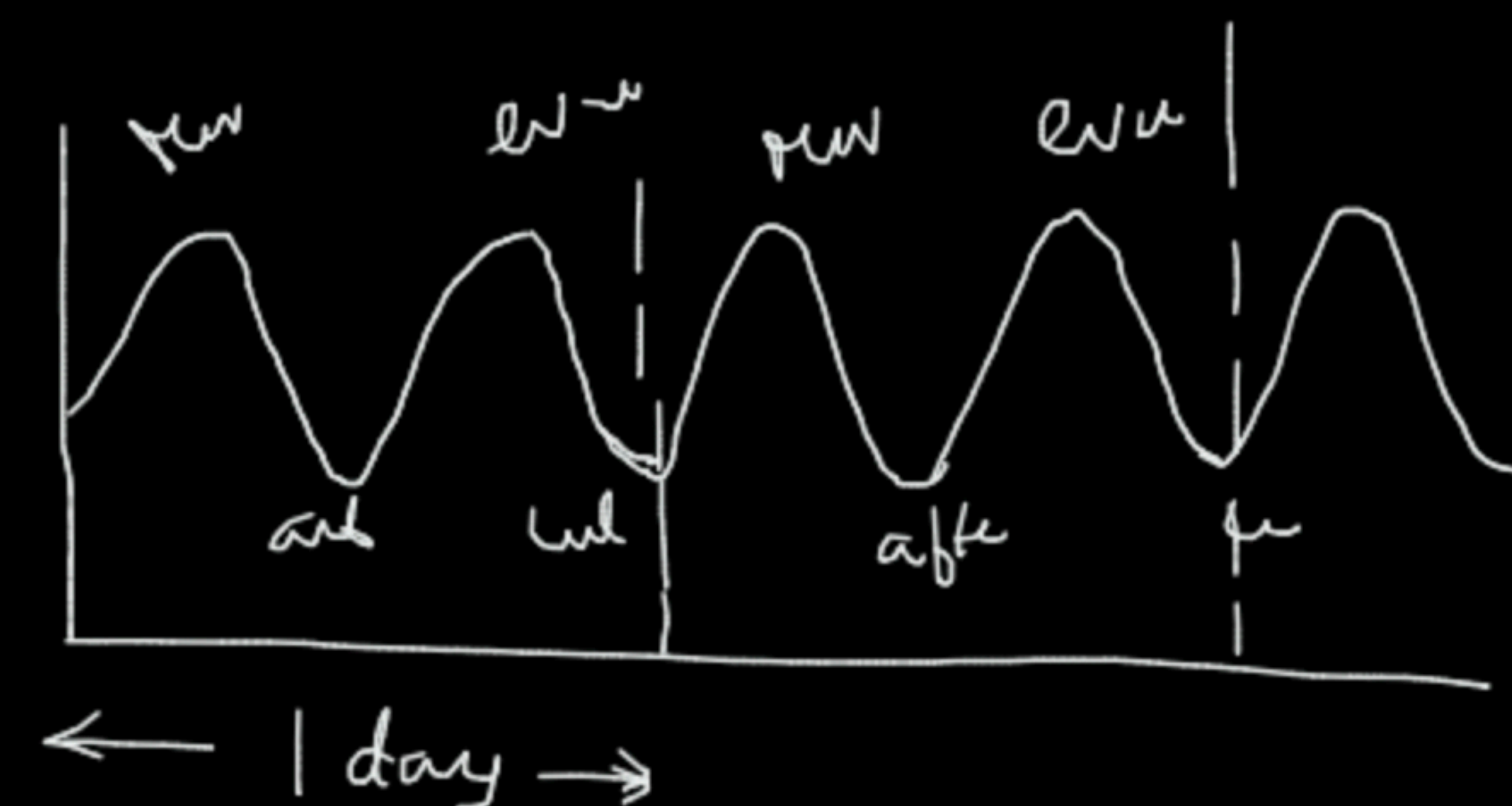
Systematic
- Level
- trend
- Seasonality

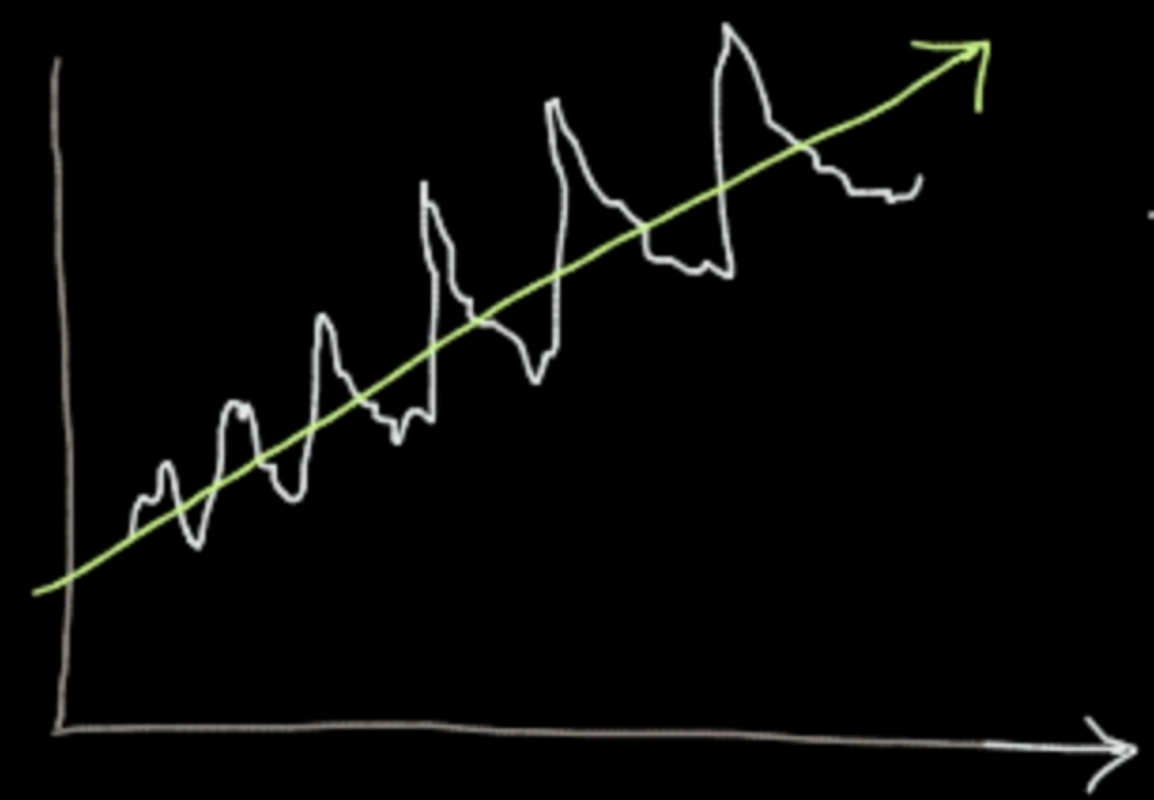
Non Systematic
- Error
- unpredictable

Cyclicity → freq > year









Trend + seasonality

ETS – Decomposition

Error

Trend

Seasonality

Line Plot → time

Chronology should be maintained

← Retrain →

188 36 36

← Training Data → ← Test → ← Forecast →

Jan 2001 Dec 2009 Dec 2012

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Time	y	$\hat{y}$	$(y-\hat{y})$	
1	—	100	—	
2	—	110	—	—
3	—	108	—	—
	—	120	—	—
	—	114	—	—
n				

MAPE →

AR → Auto Regression

MA → Moving Average

ARMA → Auto Regressive Moving Average

ARIMA → Integrated " " "

Time	Prices
2000	50
2001	63
2002	68
	—
	—
2025	99

$\boxed{1000 \quad 3 \quad B1} \rightarrow \boxed{53}$

1200 3 B2 68

5 / 5

Linear Regression

x_1	x_2	x_3	y

(a) Scatter plot

(b) Correlation coeff (r) → ACF

— strength of the linear rel betn x & y

Auto Regression

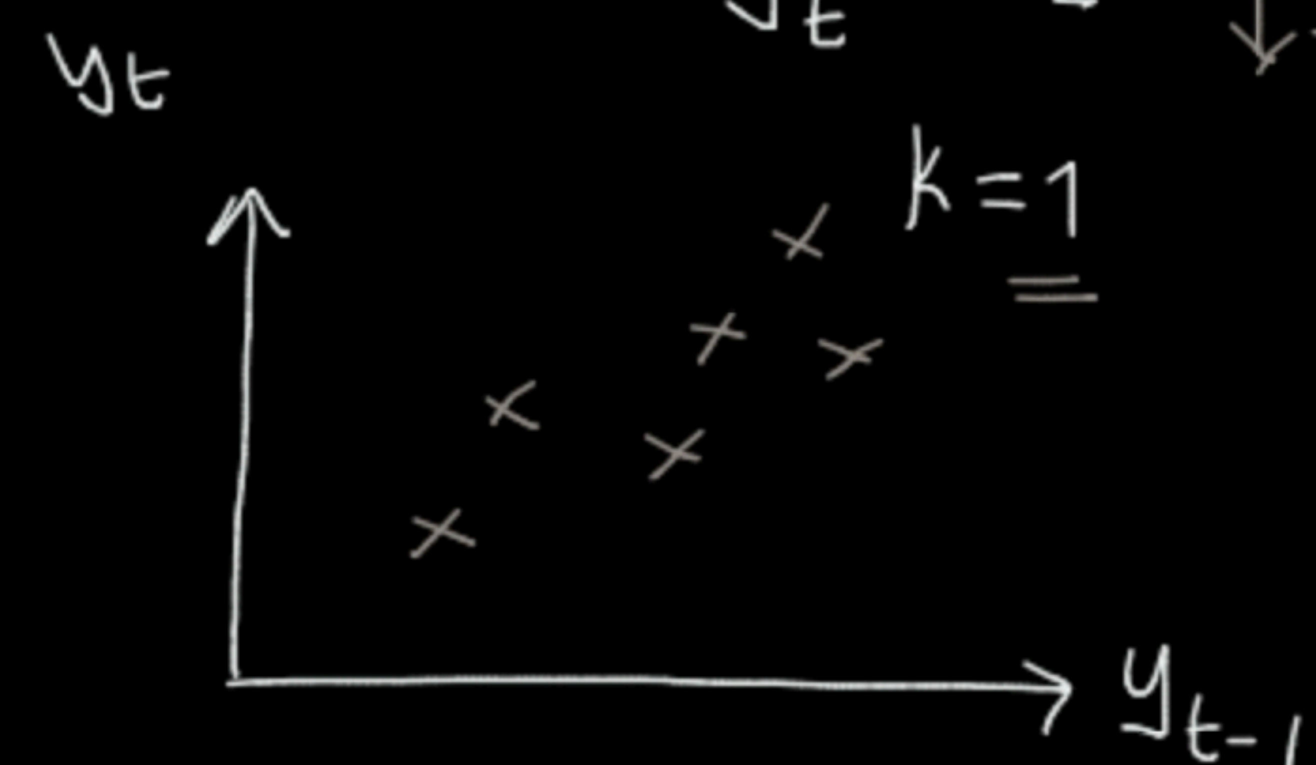
Time	y
1	y_{t-300}
2	
3	
4	
	y_{t-3}
	y_{t-2}
n	y_{t-1}

y_t

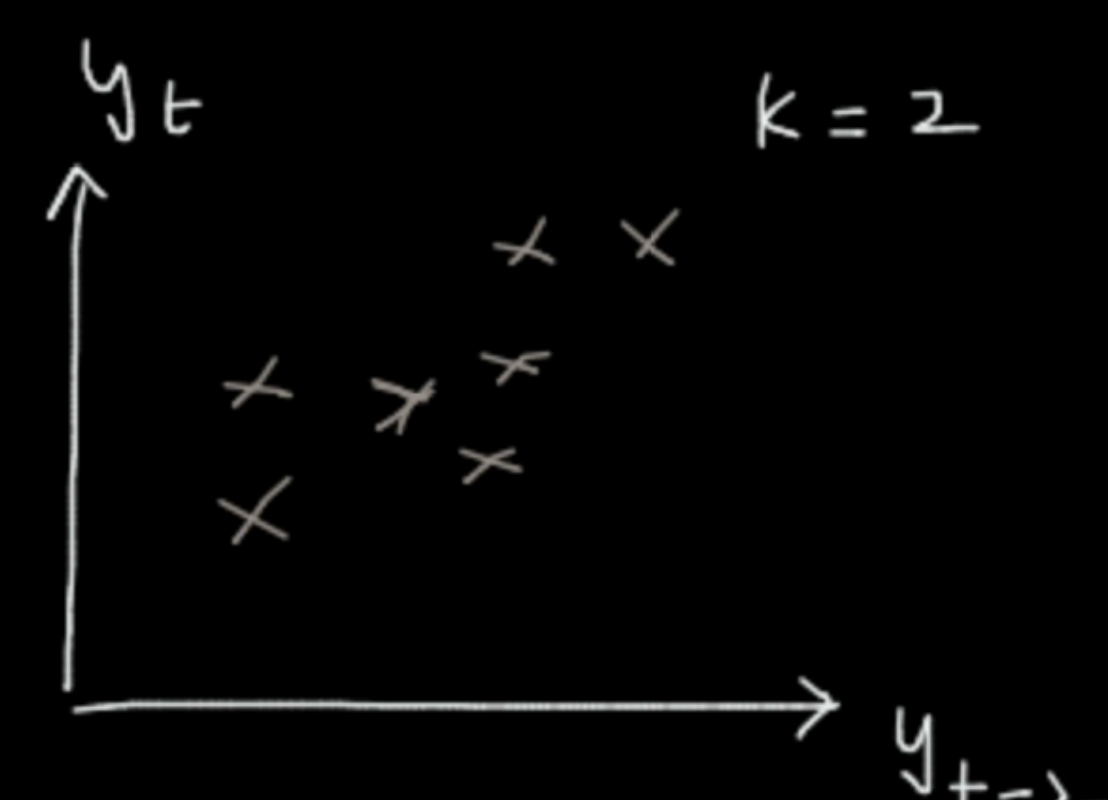
1, 2, 3 → lags (k)

x

y



Lag plot

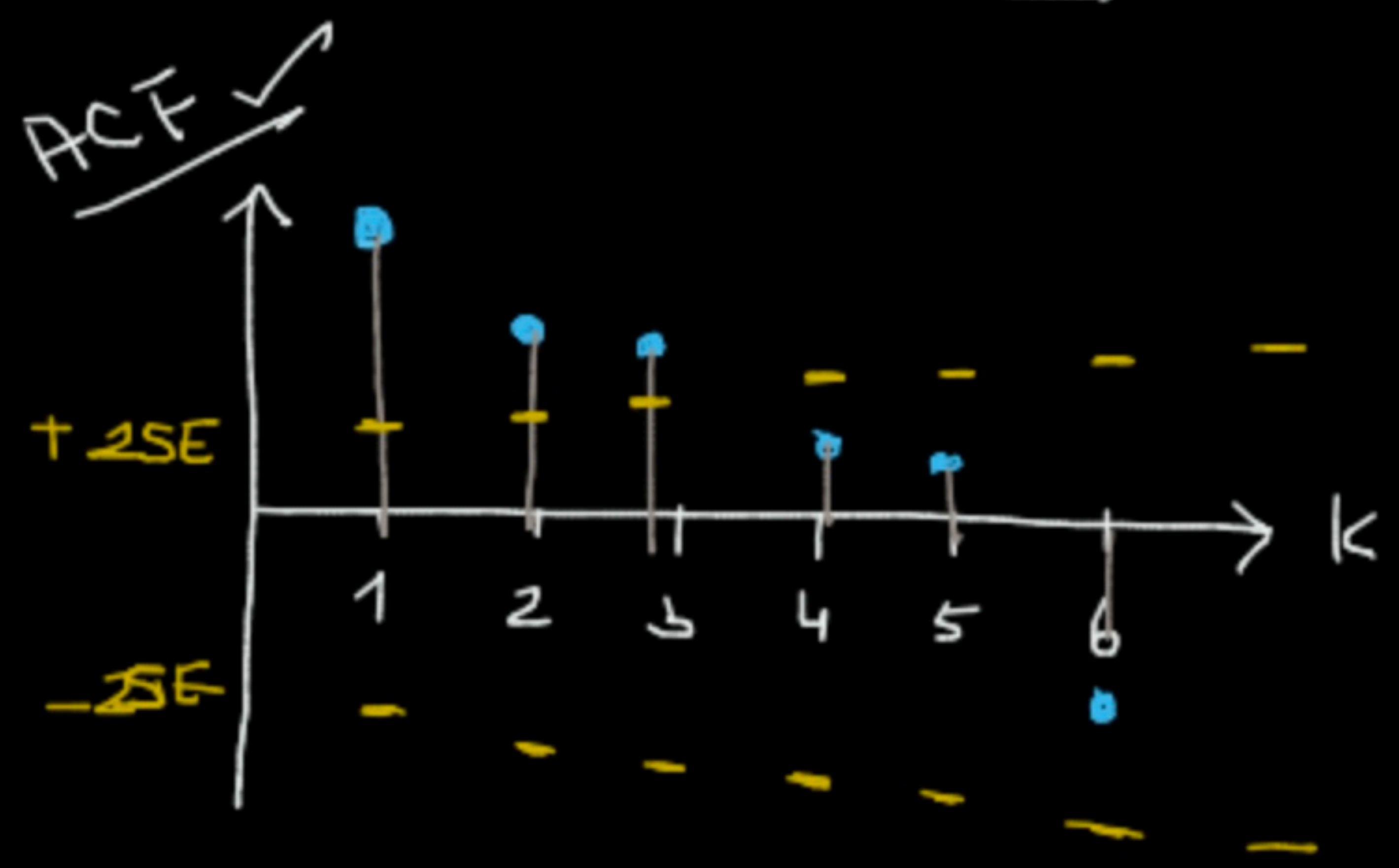


lag plots

ACF Plot — MA

SE → ACF

→ Standard Error

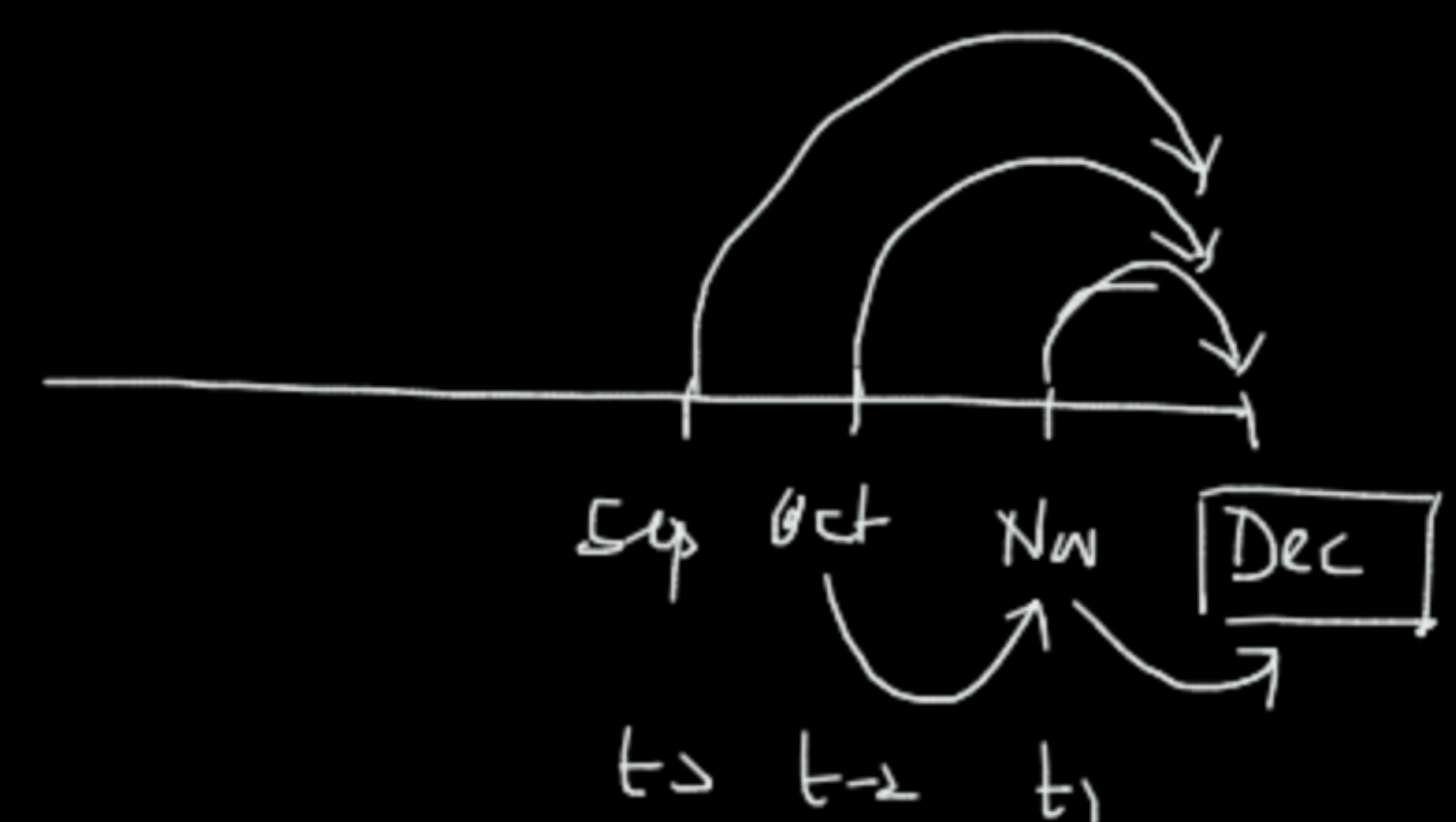


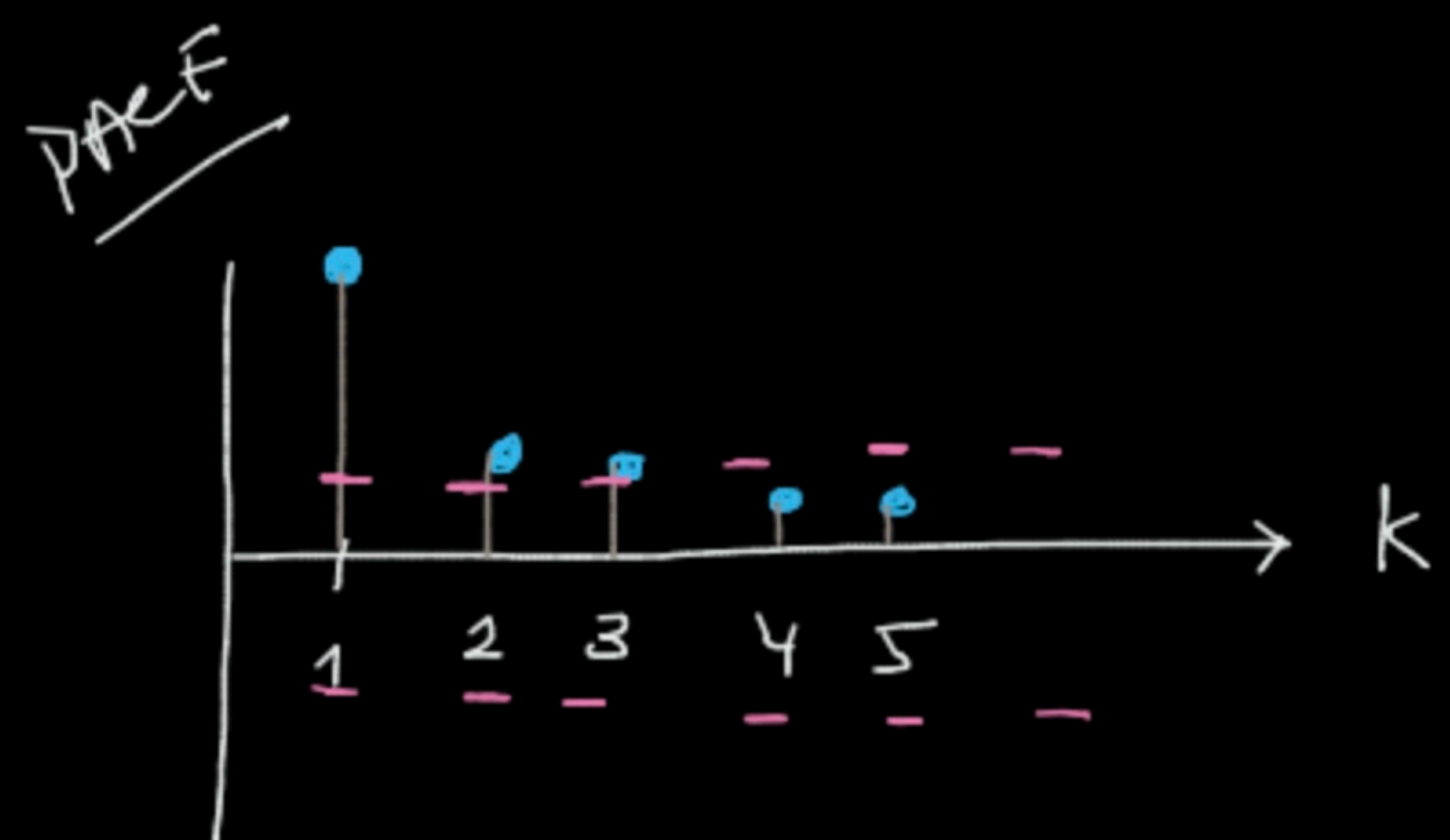
ACF ⇒ Direct + Indirect

PACF → Direct

MA(3)

$\varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}$





AR → $y_{t-1}, y_{t-2}, y_{t-3}$

$$\hat{y} = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3}$$

AR(3)

Moving Average (MA)

— Errors

	y	$\hat{y}$	Error
Week 1	12	10	2
Week 2	9	11	−2
Week 3	9	9	0
Week 4		10	

$$\hat{y} = \underbrace{\phi_0}_{10} + \underbrace{\phi_1}_{0.5} \varepsilon_{t-1} + \underbrace{\phi_2}_{0.2} \varepsilon_{t-2}$$

$\Rightarrow 10 + 1$

$\Rightarrow 10 + 0.5(-2)$

ARMA

$$\hat{y} = \beta_0 + \beta y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \phi_0 + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2}$$

→ PACF

Order of AR Model (p)

→ ACF

Order of MA Model (q)

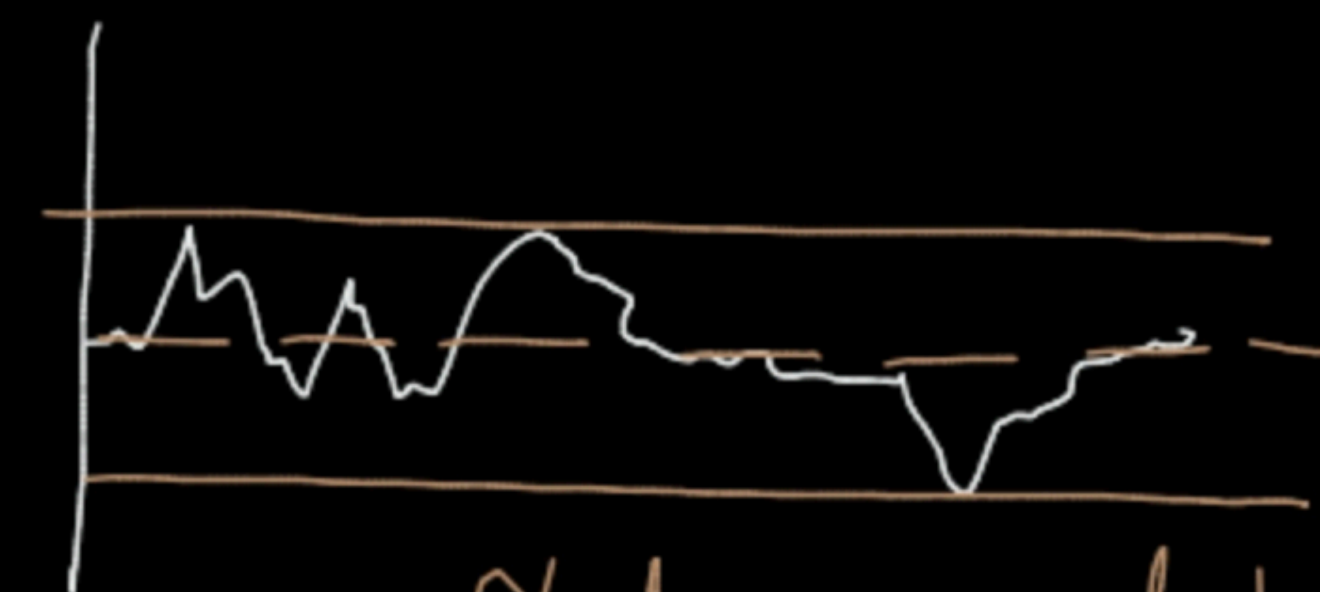
AR(p) MA(q) →

AR
MA
ARMA

→ Stationary Data

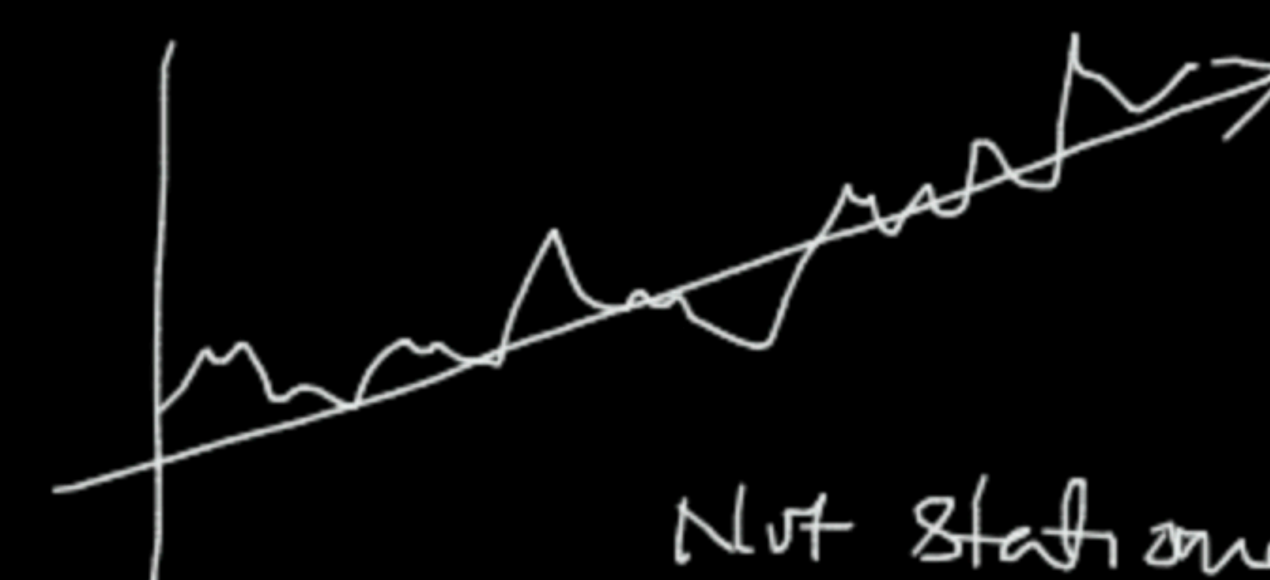
└ Constant Variance

→ No trend, No seasonality



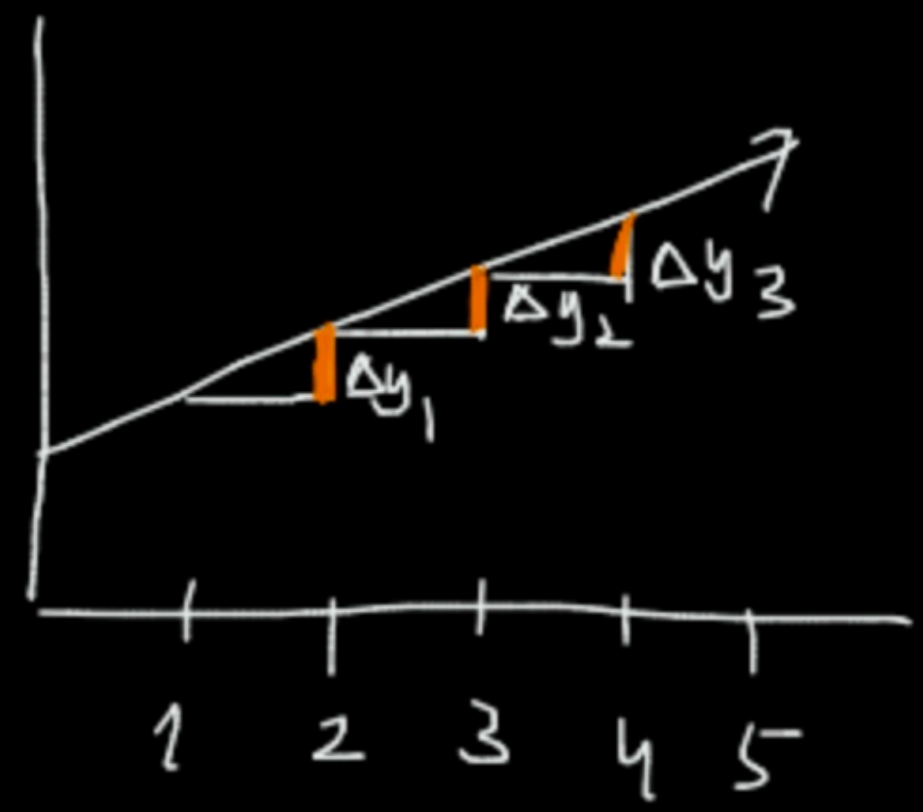
Stationary data

Augmented Dickey Fuller test (ADF)



Not Stationary

ARIMA → Trend



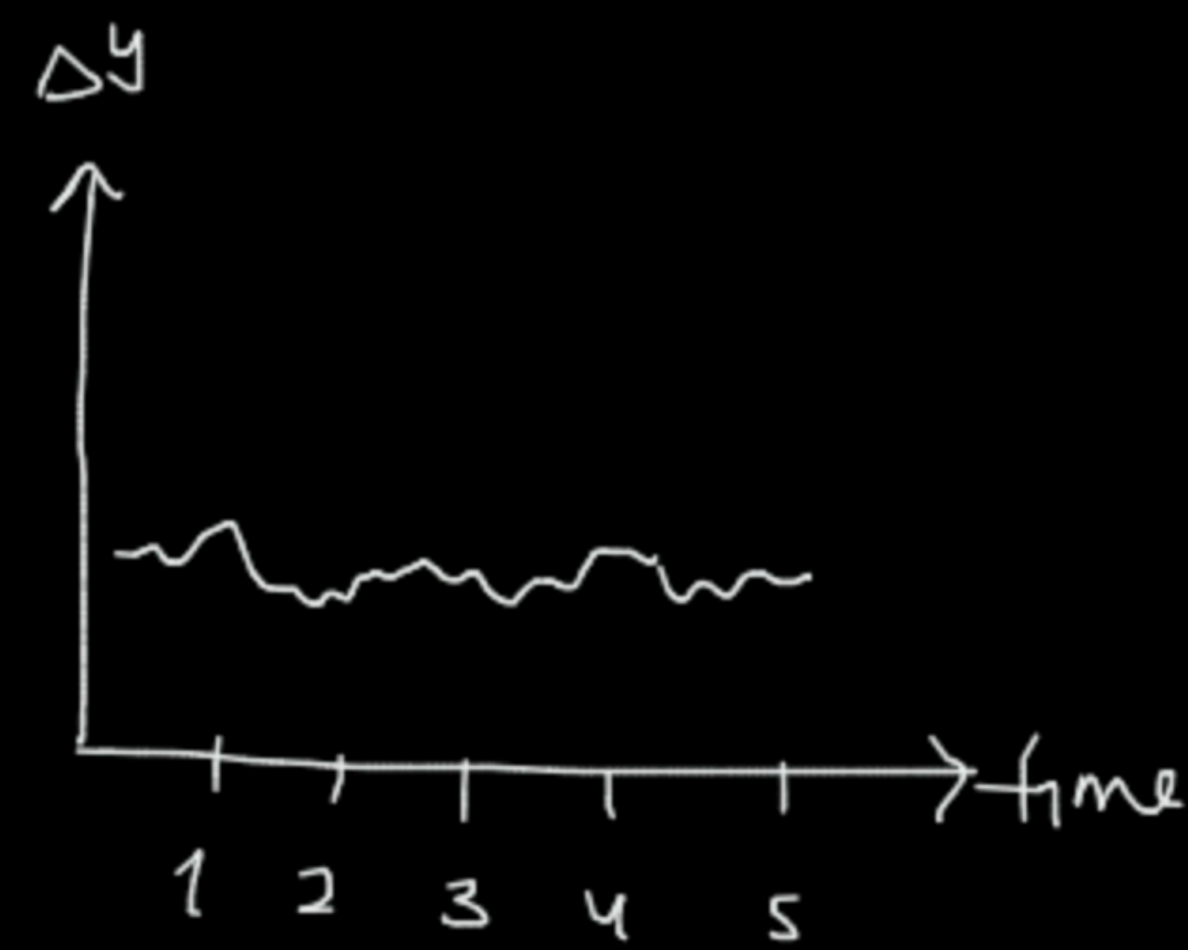
Time	y	Δy	$\Delta' y$
1	y_1		
2	y_2	Δy_1	
3	y_3	Δy_2	$\Delta' y_1$
4	y_4	Δy_3	$\Delta' y_2$
5	y_5	Δy_4	$\Delta' y_3$
6			

$d = 2$

→ Second order difference

→ First order differencing

$d = 1$



$$AR(\overset{3}{p})\ I(\overset{1}{d})\ MA(\overset{2}{q}) \longrightarrow ARIMA$$

$$AR(p)\ I(0)\ MA(0) \rightarrow AR(4)$$

$$AR(0)\ I(0)\ MA(q) \rightarrow MA(3)$$

$$AR(p)\ I(0)\ MA(q) \rightarrow \underset{4\ \ 3}{ARMA}$$

ARIMA (order = (3, 1, 2)

order = (4, 0, 0)

order = (0, 0, 3)

order = (4, 0, 3)