Unit -2 (Combinatorics)
Mathematical Induction

Non negative $n \geq 0$

Mathematic Induction:

Procedure:

Let (P(n)) be a statement or proposition involving the natural numbers n

Step1: Prove P(1) is true

Step2: Assume P(k) is true

} weak Induction

Step 3: Prove that P(k+1) is true

Now we can say P(n) is true

Strong Induction

Step1: Prove P(1) is true

Step2: Assume P(i) is true for i = 1, 2, 3 ... k Eg $2 \leq n \leq k$

Step3: Prove that P(k+1) is true

WEAK INDUCTION

P1) use Mathematical Induction to prove that

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

when n is positive integer $n \geq 1$ positive integer

Soln:

Step1: TP P(1) True

$LHS = 1^2 = 1$

$RHS = \frac{1(2-1)(2+1)}{3} = 1$

P(1) is True

Step 2: Assume P(k) is true

$$1^2 + 2^2 \dots \boxed{(2k-1)^2} = \frac{k(2k-1)(2k+1)}{3} \quad ①$$

$2k+2-1$

$(2k+1)^2$

Step: TP P(k+1) is true

$$1^2+2^2+3^2 \dots (2(k+1)-1)^2 = \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

$$\text{TP} \quad 1^2+2^2+3^2 \dots + (2k-1)^2 + (2(k+1)-1)^2 = \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

$$= 1^2+2^2 \dots (2k-1)^2 + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$$

LHS

$$1^2+2^2+3^2 + \boxed{(2k-1)^2} + (2k+1)^2$$

$$\frac{k(2k-1)(2k+1)}{3} + (2k+1)^2 \quad \text{by } ①$$

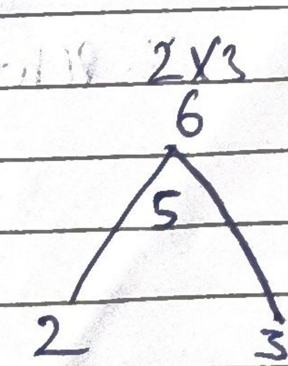
Put in same fraction

$$= \frac{k(2k-1)(2k+1)+3(2k+1)^2}{3}$$

$$= \frac{(2k+1)[k(2k-1)+3(2k+1)]}{3}$$

$$= 2k+1\,[2k^2 - k + 6k + 3] \times \frac{1}{3}$$

$$= 2k+1\,[2k^2 - 5k + 3] \times \frac{1}{3}$$



$$= \frac{(2k+1)(k+1)(2k+3)}{3} = \frac{(2k+1)(k+1)(2k+3)}{3}$$

LHS = RHS ∴ proved

M	T	W	T	F	S	S

Page No.: YOUVA

Date:

P2) use Mathematical Induction and prove

$2^0 + 2^1 + 2^2 \ldots \ldots 2^n = 2^{(n+1)} - 1$

$2^0 + 2^1 + 2^2 \ldots \ldots 2^n = 2^{n-1} - 1$

Soln

Step 1: prove P(1) is true which is P(0)

$LHS = 2^0 = 1$

$RHS = 2^{0+1} - 1$

$= 1$

$LHS = RHS$

P(0) is true

Step 2 : P(k) is true

$2^0 + 2^1 \ldots 2^k = 2^{(k+1)} - 1$ — (1)

Step 3 : P(k+1) is true

$2^0 + 2^1 \ldots \ldots 2^k + 2^{k+1} = 2^{(k+1)+1} - 1$

$LHS = 2^{k+1} - 1 + 2^{k+1}$ By (1)

$= 2^1 \cdot 2^{k+1} - 1$

$= 2^{(k+1+1)} - 1 = RHS$

$\therefore$ P(k+1) is true

P3) use mathematical induction to show that

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 \dots n^2 = \frac{n(n+1)(2n+1)}{6}$$

Step 1: assum P(1) is true

$$1^2 + 2^2 \dots 1^2 = \frac{1(1+1)(2(1)+1)}{6}$$

$$1 = 1$$

$\therefore$ P(1) is true

Step 2: assume P(K) is true

$$1^2 + 2^2 \dots k^2 = \frac{k(k+1)(2k+1)}{6} \quad \leftarrow (1)$$

Step 3: Assume P(K+1) is true

$$1^2 + 2^2 \dots k^2 + (k+1)^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

simplified

$$\text{LHS} = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

from (1) Eq

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \boxed{\frac{(k+1)(k+2)(2k+3)}{6}}$$

$$\text{LHS} = \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{k+1[k(2k+1) + 6(k+1)}{6} = \frac{(k+1)(2k^2 + k + 6k + 6)}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

12
7
4 3

$$= \frac{(k+1)[2k^2+4k+3k+6]}{6}$$

$$= \frac{(k+1)(2k(k+2)+3(k+2))}{6}$$

$$= \frac{(k+1)(2k+3)(2k+2)}{6} = RHS$$

$\therefore$ prove

P4) use Mathematical Induction

$$3^0 + 3^1 + \dots 3^n = \frac{3^{n+1}-1}{2}$$

Step 1: TP $P(0)$ is True

$3^0 = 1$ LHS

$$RHS = \frac{3^{0+1}-1}{2}$$

$$= \frac{3-1}{2}$$

$$= 1$$

LHS = RHS

$P(0)$ is true

Step 2: Assume P(k) is true

$$3^0 + 3^1 \dots 3^k = \frac{3^{k+1}-1}{2} \quad \leftarrow (i)$$

Step 3: Assum P(k+1) is true

$$3^0 + 3^1 \dots 3^k + 3^{k+1} = \frac{3^{(k+1+1)}-1}{2}$$

from (i) we get

$$LHS = \frac{3^{k}-1}{2} + 3^{k+1}$$

$$= \frac{3^{k+1}-1+2\cdot 3^{k+1}}{2}$$

$$= \frac{3^{(1)}\cdot 3^{k+1}-1}{2}$$

$$= \frac{3^{(k+1+1)}-1}{2} = RHS$$

∴ proved by mathematical Induction

P5) Prove that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}\ldots\frac{1}{n(n+1)}=\frac{n}{n+1}$

$\odot$ is not mult in Multiplication

Sol :

Step 1 : TP ~~4~~ P(1) is True

$$\frac{1}{1(1+1)}=\frac{1}{1+1}$$

$$\frac{1}{2}=\frac{1}{2}$$

LHS = RHS
P(1) is true

Step 2 : TP P(K) is true

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}\ldots\frac{1}{k(k+1)}=\frac{k}{(k+1)} \leftarrow (1)$$

Step 3 : TP P(K+1) is true

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}\ldots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+1+1)}=\frac{k+1}{(k+1+1)}$$

$$\frac{k}{k+1}+\frac{1}{(k+1)(k+1+1)}=\frac{k(k+1+1)+1}{(k+1)(k+1+1)}$$

$$=\frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+1+1)}$$

$$=\frac{k+1}{(k+1+1)}=\text{RHS}$$

∴ Proved

Page No.:
Date:
YOUVA

P6) By mathematical induction prove n^3-n is divisible by 3 where n is a positive integer.

Soln:

i) To P(P(1)) is true

$n^3-n = 1^3-1 = 0$

$= 3\times 0$ (multiple of 3)

$\therefore$ True

ii) Assume (P(K)) is true

$k^3-k = 3\times M$ — ① M = Integer

iii) Assume P(K+1) is true

i.e $(k^3-k)+(k+1)^3-(k+1) = 3\times M$

$3M + (k+1)^3 - (k+1) = 3\times M$

$= (k+1)^3-(k+1)$ Simplify to get

$\Rightarrow 3[M+k^2+k]$

$\therefore$ Multiple of ~~k~~ 3 then it is true

wrong method

P7) Using Induction principle, prove that n^3+2n is divisible by 3

Soln Given $n^3+2n = 3\times M$

i) P(1) is true

$(1)^3+2n = 3$ ①

$\therefore$ True

ii) P(k) is true

$\boxed{k^3+2k = 3\times M}$ ①

iii) P(k+1) is true

$(k^3+2k)+(k+1)^3+2(k+1)=3\times M$

$3M+(k+1)^3+2k+2)=3\times M$

$= k^3+1+2k+2$

$= k^3+2k+3$

$= \cancel{3(k+1)}$

$= k(k^2+2k+3)$

$= k(k(1+2k+3))$

$= 3\times M$

Since the multiple of 3

Wrong Method

correct method diff sum $\Rightarrow$

7) Use Mathematical Induction to show that n^2-1 is divisible by 8 n is odd integer

Soln Given n^2-1

i) TP P(1) is true

$1-1=0\times 8$

multiple of 8 Thus True

ii) TP P(k) is true

$\boxed{k^2-1=8M}$ — (1)

The reason for using k+2 is because n is odd

iii) TP P(k+2) is true

$(k+2)^2-1 = k^2+4+4k-1$

$= k^2-1+4+4k$

$= (k^2-1)+4(k+1)$

$= 8m+4(2l)$

$= 8M+8l \Rightarrow 8(M+l)$

P(k+2) is true $\therefore$

M T W T F S S
Page No.:
Date:
YOUVA

10) Prove by Mathematical Induction $2^n > n$ for all all positive Integers

sol

Eqr $2^n > n$ or $2^n > n$ $n < 2^n$

i) $P(1)$

$1 < 2^1$

$1 < 2$

$\therefore$ True

ii) $P(k)$

$\boxed{k < 2^k}$ $\to$ ①

iii) $P(k+1)$

$k+1 < 2^{k+1}$

From ① :

$k < 2^k$

add one on bs

$k+1 < 2^k + 1$

$k+1 < 2^k + 2^k$ Since $1 < 2^1$

$\boxed{k+1 < 2^{k+1}}$

$\therefore$ $P(k+1)$ is true

11) Using Mathematical Induction, Prove that if n is a positive Integer, then 133 divides $11^{(n+1)} + 12^{(2n-1)}$

Soln: $11^{n+1} + 12^{2n-1} = 133 \times$ integer

i) $P(1)$

$11^{1+1} + 12^{2 \cdot 1-1} = 121 + 12 = 133 \times 1$

is True

ii) Assume P(K) true

$$11^{k+1} + 12^{2k-1} = 133M$$

$\to$ (1)

iii) Assume P(k+1) is true

$$\cancel{11^{k+1+1} + 12^{2k+1-1} = 133M}$$

$$11^{k+2} + 12^{2k+1} = \cancel{133M} \quad \cancel{11^{k+1} \cdot 11^1 + 12^{2k+1}}$$

LHS

$$11^{k+2} + 12^{2k+1} = 11^{(k+1)} \cdot 11^1 + 12^{2k+1}$$

$$= (133M - 12^{2k-1}) \, 11 + 12^{2k-1}$$

$$= 133 \times 11M - 11 \cdot 12^{2k-1} + 12^{2k-1} \cdot 12^2$$

$$= 133 \times 11M + 133 \cdot 12^{2k-1}$$

$$= 133 \left[11M + 12^{2k-1} \right]$$

$\boxed{\text{True}}$

Strong Induction / **Second Principle of Mathematical Induction**

1) Any positive integer $n \geq 2$ is either a prime or a product of primes by using strong mathematical induction

Sol:

i) P(2) true:

2 is a prime

P(2) is true

Page No.:	YOUVA
Date:	

ii) Assume given statement is true $\boxed{n=2,3 \ldots k}$
it can either be prime or product of prime

iii) TP $P(k+1)$ is true

Case i) $(k+1)$ prime is true
Case ii) $(k+1)$ is not a prime number

$\therefore k+1 = p \cdot q$ $\boxed{\begin{matrix} 2 \le p \le k \\ 2 \le q \le k \end{matrix}}$

p, q are prime or product of prime
It is true

Pigeon Hole Principle

If $(n+1)$ pigeon occupies 'n' holes then atleast one hole has more than 1 pigeon

Generalized pigeon hole principle

If 'm' pigeon occupies 'n' holes $(m>n)$, then one of the holes must contain atleast $\left[\frac{m-1}{n}\right]+1$ pigeons.

m = ~~pigeon holes~~ number of pigeons
n = number of pigeons holes

1) Show that in any group of 8 people at least two have birthdays which falls on same day of the week in any year

Sol

$m = 8$

$n = 7$

$\left[\frac{m-1}{n}\right]+1 = \left[\frac{8-1}{7}\right]+1 = 2$ people have same birthdays

2) Prove that in a group of six people, there must be atleast 3 mutual friends or atleast 3 mutual enemies

Sol name : A B C, D, E, F

i) Friends of A

$m = 5$

$n = 2$

$\left[\frac{5-1}{2}+1\right]$

$= 2$

ii) ~~Friends~~ of enemies of A

$m = 5$

$n = 2$

$\left[\frac{5-1}{2}\right]+1$

$= 2$

3) What is the maximum number of students required in a mathematics class to be sure that atleast 6 will receive the same grade, if there are 5 possible grad A, B, C, D, F?

Soh

$m =$ No of students

$n = 5$

$\left[\frac{M-1}{5}\right]+1 = 6$

$\frac{M-1}{5} = 5$

$M-1 = 25$

$\boxed{M = 26}$

DM Sem Preparation

Unit-2 part II

Recurrence Relations (or) Difference Equation

$\boxed{a_n = a_{n-1} + 3 \quad \text{where } 3 \text{ is } n}$

$\boxed{a_{n+1} = a_n - 2a_{n-1}}$

Ex : Fibbonacs serie 0, 1, 1, 2, 3, 5

$\boxed{a_n = a_{n-1} + a_{n-2}}$

Degree :

$a_{n-1} + 3a_{n-2} - a_{n-3} + a_{n-4}$

Thus the above sequences degree is 4.

Homogeneous Recurrence Relation

no terms occur that are not multiples of a_i's

es: 1) $a_n = 3a_{n-1} + a_{n-2}$ Homogenous

$a_n = a_{n-1} + 3a_{n-2} + 2 \to$ not homogenous

Linear : If all sequence have power 1, they are linear
if not they are not linear

If they are both homogeneous and linear they are called homogenous linear difference equation

Page No.:
Date:

Problems

1) Find the recurrence relation for the sequence

$\boxed{a_n = 2n+4 \ , n \geq 1}$

Sol: Given $a_n = 2n+4$

$$a_{n-1} = 2(n-1)+4$$
$$= (2n+4)-2$$
$$a_{n-1} = a_n - 2$$

$a_{n-1} + 2 = a_n$ i.e $\boxed{a_n = a_{n-1} + 2}$

2) Find the recurrence relation

$\boxed{y_n = A3^n + B(-4)^n}$ — (1)

It has two unknowns then
do not use y_{n-1} and y_{n-2}
rather y_{n+1}, y_{n+2}

$$y_{n+1} = A3^{n+1} + B(-4)^{n+1}$$
$$= A3^n \cdot 3^1 + B(-4)^n \cdot (-4)^1$$
$$= 3A3^n - 4B(-4)^n \quad \text{— (2)}$$

$$y_{n+2} = A3^{n+2} + B(-4)^{n+2}$$
$$= A^n \cdot 3^2 + B(-4)^n \cdot (-4)^2$$

$\boxed{y_{n+2} = 9A^n + 16B(-4)^n}$ — (3)

Add (1) (2) (3) |A| |B| to get recurrence solution

3

4

M T W T F S S	
Page No.:	YOUVA
Date:	

Solution of Recurrence Relation

2)

It has three methods
(1) Iteration
(2) characteristic root method (use this if no method is specified)
(3) Generating function method

Ex for characteristic method

Step 1

Let $a_n + \alpha_1 a_{n-1} + \alpha_2 a_{n-2} + \dots \alpha_k a_{n-k} = 0$
be a homogeneous linear difference equation of degree k and α_i's are Constant

Step 1: write down the characteristic equation
Ex $\boxed{r^3 + 3r^2 + 2r^1 - 1}$ degree

Step 2: find all the roots of characteristic equations
Step 3: write solution
Case (i) If roots are real & distinct
$$a_n = c_1 r_1^n + c_2 r_2^n + c_3 r_3^n \dots c_k r_k^n$$
case (ii) real and equal
$$a_n = (c_1 + n c_2 + n^2 c_3 + \dots n^{k-1} c_k) r^n$$
Case (iii) Imaginary

$$a_n = r^n (c_1 \cos n\theta + c_2 \sin n\theta)$$
$$r = |x + iy| = \sqrt{x^2 + y^2} \quad \& \quad \theta = \tan^{-1}\left[\frac{y}{x}\right]$$
$$r = \sqrt{4 + 9} = \sqrt{13}$$

Step 4: Apply Initial conditions and find out the arbitrary constants $c_1, c_2, \dots$

1) Solve the recurrence equation $a_n = 8a_{n-1} - 16a_{n-2}$ for $n \geq 2$

$a_0 = 16$, $a_1 = 80$

$a_n - 8a_{n-1} - 16a_{n-2}$

deg = 2

$r^2 - 8r + 16 = 0$

16
8
-4 -4

$(r-4)(r-4) = 0$

$r - 4 = 0$ & $r - 4 = 0$

$r = 4$ & $r = 4$

$r = 4, 4$ equal roots

soln $\boxed{a_n = (c_1 + c_2 n) 4^n}$

$a_0 = 16$ $a_1 = 80$

$a_n = (c_1 + 0) 4^0$

$16 = (c_1) 1$

$\boxed{c_1 = 16}$

$a_1 = 80$

$a_1 = (c_1 + c_2) 4$

$a_1 = (16 + c_2) 4$

$\frac{80}{4} = (16 + c_2)$

$20 = 16 + c_2$

$\boxed{c_2 = 4}$

$\boxed{a_n = (16 + 4n) 4^n}$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

use this formula if you don't have a calc calculator

2) $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$

$a_0 = 5, \quad a_1 = 9, \quad a_2 = 15$

$a_n + 3a_{n-1} + 3a_{n-2} + a_{n-3} = 0$

$\boxed{r^3 + 3r^2 + 3r + 1} = 0$

Now lets simplify the eq

$$
\begin{array}{r|rrrr}
-1 & 1 & 3 & 3 & 1 \\
 & 0 & -1 & -2 & -1 \\
\hline
 & 1 & 2 & 1 & \boxed{0} \\
-1 & & & & \\
\hline
-1 & & & &
\end{array}
$$

$r = -1, -1, -1$

soln

$\boxed{a_n = (c_1 + c_2 n + c_3 n^2)(-1)^n + (4}$

Apply a_0, a_1, a_2 and find c_1, c_2, c_3 and write

There are too many models so just reffering them, its not covered deep in this note

Page No.:
Date:

YOUVA

1, Generating function method

for both homo and non-Homo

1) Rewrite RHS = 0
2) Multiply by x^n
3) Draw summation for all
4) put $a_0 + a_1 x$

$$\boxed{\sum_{n=0}^{\infty} a_n x^n = G(x)}$$

1) Solve relation $S_n + 3S_{n-1} - 4S_{n-2} = 0$, $n \geq 2$. given $S_0 = 3$, $S_1 = -2$

It's hard and long so I am avoiding Recurrence relation except for Important

So Just follow these rules to at least put some steps

If there is no $(k-1)$ but it has $(k-2)(k-3)$
then

1) $r^3 + 0r^2 - 7r + 6$
2) By long division always look at the near value to get 0
3) If you get a non homogeneous.

Ex: $S(k) - 7S(k-1) + 10S(k-2) = \boxed{8k + 6}$

Non homo
Just omit
this and
continue

4) If $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Consider this

$\frac{2 \pm \sqrt{-4}}{2} \Rightarrow \frac{2 \pm 2i}{2}$

$= \frac{2[1 \pm i]}{2} = 1 \pm i$

imaginary so
use that for

Permutation and Combinations

Combinations : Selection, choosing certain from all $nCr = \frac{n!}{(n-r)!\,r!}$

permutation : arrange , choose order

$nPr = \frac{n!}{(n-r)!}$

1) A team of 11 players is to be choosen from 15 members. In how many ways can this be done if
(i) one particular player is always included
(ii) Two such players have always to be included

Soln: i) If team has 11 and one is always included
Remain 10 and 14
$14C_{10} = 1001$

ii) Two such players are always included

$13C_9 =$
$13C_9 = 715$ ways

2) In how many was 5 men and 5 women sit around the table of 10 (arranging)

$10!$

11 people

1) There are 6 men and 5 women in a room. Find the number of ways 4 persons can be drawn from the room if

i) They can be male or female

ii) 2 men & 2 women

iii) All are same sex

i) $11C_4 = (\ )$

ii) $6C_2 \times 5C_2 = 150$

iii) The both are sam

$6C_4 + 5C_4 = 20$

2) From a committee consisting of $\boxed{6 \text{ men and } 7 \text{ women}}$ in how many ways can be select a committee of (a) 3 men and 4 women $\boxed{\text{max}}$

(b) 4 members which has $\boxed{\text{atleast}}$ one woman

(c) 4 persons that has both sex

~~a) 8 6C~~

a) $6C_3 \times 4C_7$

~~b) possibilities: (1w3m) or (2w2m) or (3w ...w)~~

b) possibilities : (1F & 3M) or (2F & 2M) or (3F & 1M) or (4F & 0M)

$\Rightarrow (7C_1 \times 6C_3) + (7C_2 \times 6C_2)$ or $(7C_3$ or $6C_4)$ or $(7C_4)$

c) (1M or 3F) (0M or 4F) ~~3&4~~ ~~we can't get~~

d) all are same

(1M & 3W) or (2M & 2W) or (3M & 1W)

11

3) A box contains 6 white balls and 5 red balls
4 balls are drawn
i) These can be any colour
ii) 2 must be white and 2 red
iii) All are same

i) $^{11}C_4$ ii) $^6C_2$ and $^5C_2$
iii) $^6C_4 + {^5C_4}$
Put them in calculator to find your solution

4) How many permutations can be made out of BASIC
i) Begin with B
ii) End with C
iii) B and C occupy the end place
iv)

i) B _ _ _ _ $^4P_4 = 24$
ii) _ _ _ _ C $^4P_4 = 24$
iii) B _ _ _ C $^3P_3 = 6$

5) How many permutations of {a, b, c, d, e, f, g}
i) End with a
ii) begin with c
iii) c and a
iv) c and a

i) _ _ _ _ _ _ a $^6P_6 = 6!$
ii) c _ _ _ _ _ _ $^6P_6 = 6!$
iii) c _ _ _ _ _ a $^5P_5 = 5!$
iv) a _ _ _ _ _ c ~~$^6P_6 = 6!$~~ $^2P_2$ and $^5P_5$

M	T	W	T	F	S	S

Page No.:
Date:

YOUVA

6) How many ways are there to assign 5 different Job to 4 different Employee if every employee is assigned one job

Soln It can be Assi-

	e1	e2	e3	e4
	1	1	2	1
(or)	2	1	1	1
(or)	1	2	1	1
(or)	1	1	~~2~~ 1	2

7) How many bit strings of length 10 contain (0 1 1)

Max

a) almost four 1's
b) atleast four 1's
c) equal of 0's and 1's
d) either begin with 1 or end with 0

Sol

a) Max of 4(1) => 4 & 6(0) → $\frac{10!}{4!\,6!}$

or

$\frac{10!}{3!\,7!}$ — => 3 & 7(0)

or

=> 2 & 8(0) — $\frac{10!}{2!\,8!}$

or

$\frac{10}{1!\,9!}$ — => 1 & 9(0)

=> 0 & 10(0) → $\frac{10!}{0!\,10!}$

$= \frac{10!}{4!\,6!} + \frac{10!}{3!\,7!} + \frac{10!}{2!\,8!} + \frac{10!}{1!\,9!} + 1$

b) Atleast four

4(1) and 6(0) or
5(1) and 5(0) or
6(1) and 4(0) or
7(1) and 3(0) or
8(1) and 2(0) or
9(1) and 1(0)

or
10(1) and 0(0)

c) equal 0's and 1's

$S(0) \Delta S(1)$

d) either begin with 1 or end with 0

A → bit starting with 1
B → Start with 0

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

so for $n(A) = 2^4$ or
$n(B) = 2^4$ or
$n(A \cap B) = 2^3$

$\boxed{\text{Inclusion and Exclusion}}$

$|A \cup B| = |A| + |B| - |A \cap B|$
$|A \cup B \cup C| = (|A| + |B| + |C|) - (|A \cap B| + |B \cap C| + |A \cap C|)$
$\boxed{1\ \Sigma} \quad + |A \cap B \cap C| \quad \boxed{\Sigma}$
$\boxed{\cap\ \Sigma}$

So for 2nd mid I just covered the basic so just use them to revise.

M T W T F S S	
Page No.:	YOUVA
Date:	

Sum

1) State the Inc and Ext

how many faculty members can speak either French or Russian
if 200 faculty can speak French & 50 can speak Russian
20 both

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 200 + 50 - 20$
$= 250 - 20$
$\boxed{= 230}$

2) In a class of 50, 20 play football, 16 play Hockey. 10 both

$|U| = 50$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 20 + 16 - 10$
$= 36 - 10$
$= 26$
$|\overline{A \cup B}| = |U| - |A \cup B|$
$= 50 - 26$
$\boxed{= 24}$

3) 1232 Spanish = A

879 French = B

114 Russian = C

$103 = A \cap B$

$23 = A \cap C$

$2092 = |A \cup B \cup C|$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$2092 = 1232 + 879 + 114 - 103 - 14 - 23 + |A \cap B \cap C|$$

$$= 7$$

And Just check Type-2