A Gentle Introduction to Chaos

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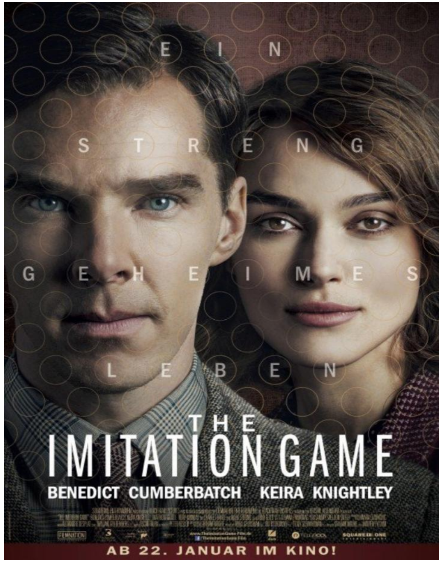
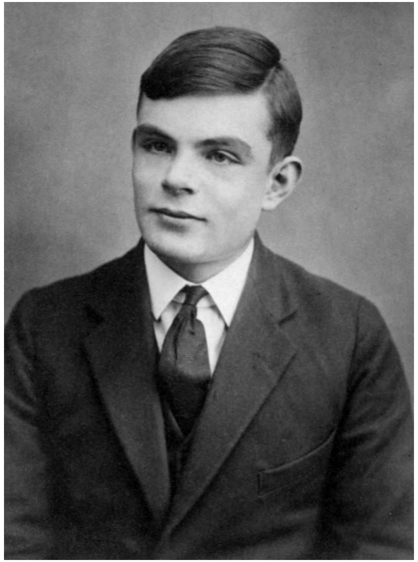
Turing’s Dream/ Artificial Intelligence/ Chaos

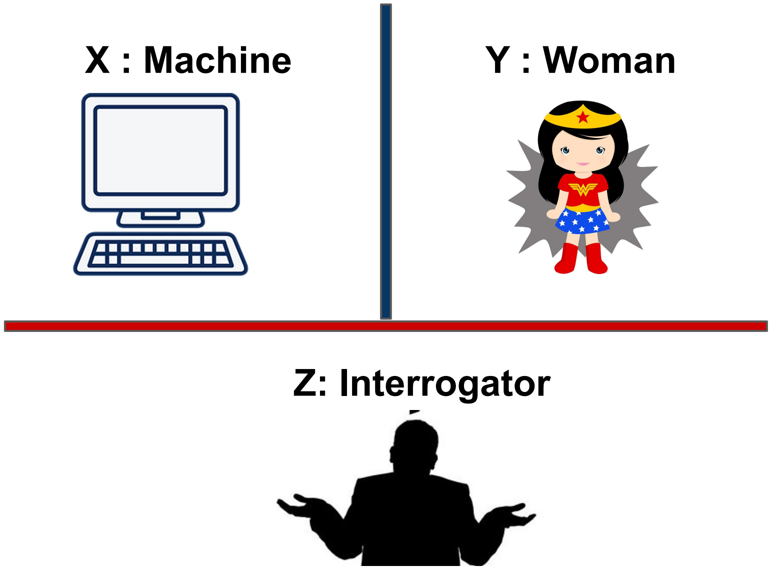


- Mathematician
- Computer Theorist
- **World War II Code Breaker**

- Father of Modern Computer Science
- Founding Fathers of AI (along with along with John McCarthy, Marvin Minsky, Allen Newell and Herbert A Simon)



Can Machines Think? -->The Imitation Game*



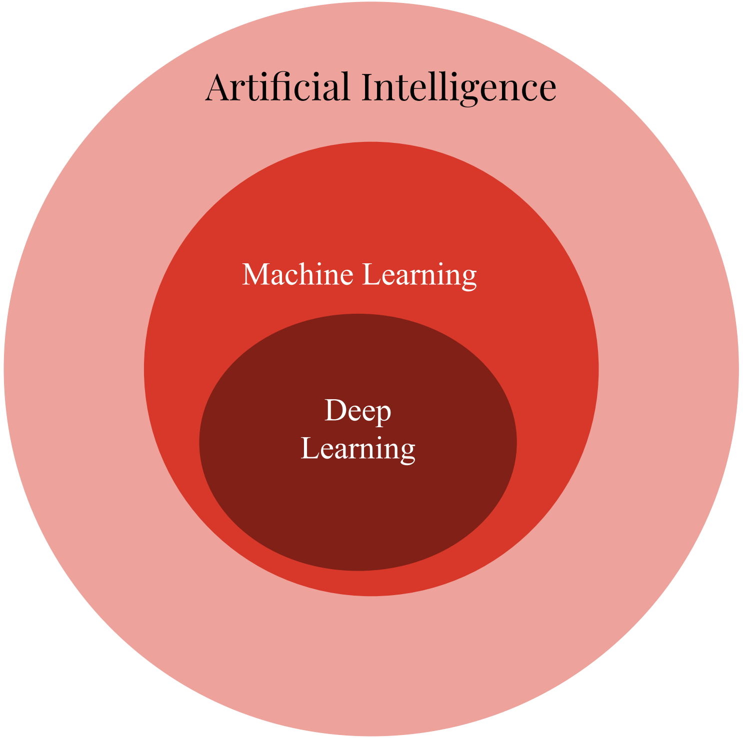
- In **50 years** time it will be possible to programme computers, with a storage capacity of 10^9 , to make them play the imitation game so well that an average interrogator will not have more than **70** % chance of making the right identification after **5 minutes of questioning**.

* Machinery, C. (1950). Computing machinery and intelligence-AM Turing. *Mind*, *59*(236), 433.

Where are we today?

01	Artificial Narrow Intelligence	• Ability to accomplish narrow set of tasks.
02	Artificial General Intelligence 	• Ability to accomplish any cognitive tasks (including learning). • Understand people's emotion, thoughts, expectations and to be able to interact socially. • Self Aware.
03	Artificial Super Intelligence	• Greatly exceeds the cognitive performance of humans in virtually all domains of interest.

Artificial Narrow Intelligence

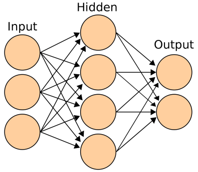


Why we need to investigate the current learning algorithms?

- Brain science is said to be in *Faraday Stage.* [1]
- ~86 billion neurons [2]
- complex network of neurons
- neurons are inherently **non-linear** and found to exhibit **chaos.**
- **Artificial Intelligence: interdisciplinary field - neuroscience, computer science, cognitive science, non-linear dynamics.**

1. Ramachandran, Vilayanur S., Sandra Blakeslee, and Neil Shah. *Phantoms in the brain: Probing the mysteries of the human mind*. New York: William Morrow, 1998.

2. Azevedo, Frederico AC, et al. "Equal numbers of neuronal and nonneuronal cells make the human brain an isometrically scaled-up primate brain." *Journal of Comparative Neurology* 513.5 (2009): 532-541.



Artificial Neural Networks and Neurons in Brain

<< Research Gap >>

Artificial Neural Networks (ANN)	*Biological Neural Networks*
Linearity + Non-linear activation.	**Non-linearity at the neuronal level.** [3, 4]
Current deep learning architectures does not exhibit chaotic behaviour at the neuronal level for classification.	**Exhibits different behaviours - from periodic to chaotic at different spatiotemporal scales.**
Not robust to noise.	Robust to noise and interference.
Need huge amount of training data.	**Learning from limited samples.**

3. Faure, Philippe, and Henri Korn. "Is there chaos in the brain? I. Concepts of nonlinear dynamics and methods of investigation." *Comptes Rendus de l'Académie des Sciences-Series III-Sciences de la Vie* 324.9 (2001): 773-793.

4. Korn, Henri, and Philippe Faure. "Is there chaos in the brain? II. Experimental evidence and related models. " *Comptes rendus biologies* 326.9 (2003): 787-840.

Brain: The Ultimate Machine

- High complexity (non-linear)
- Very high neural noise and interference
- Very low SNR: -29 dB to -20 dB*
- Neural signal multiplexing¶
- Low power "neural computation" (~12.6 Watts**)
- How does it work? No idea!

***Ref:** G. Czanner et al., Measuring the signal-to-noise ratio of a neuron, PNAS 112 (23), 2015
¶Ref: M L R Meister et al., Signal multiplexing and single-neuron computations in Lateral Intraparietal Area
during decision-making, J. Neurosci. 33 (6), 2013
****Ref:** Scientific American, 18 July 2012

My view is to throw it all away and start again

Geoffrey E Hinton
Turing Award winner 2018

https://cacm.acm.org/news/221108-artificial-intelligence-pioneer-says-we-need-to-start-over/fulltext

NATURE VOL. 323 9 OCTOBER 1986 — LETTERS TO NATURE — 533

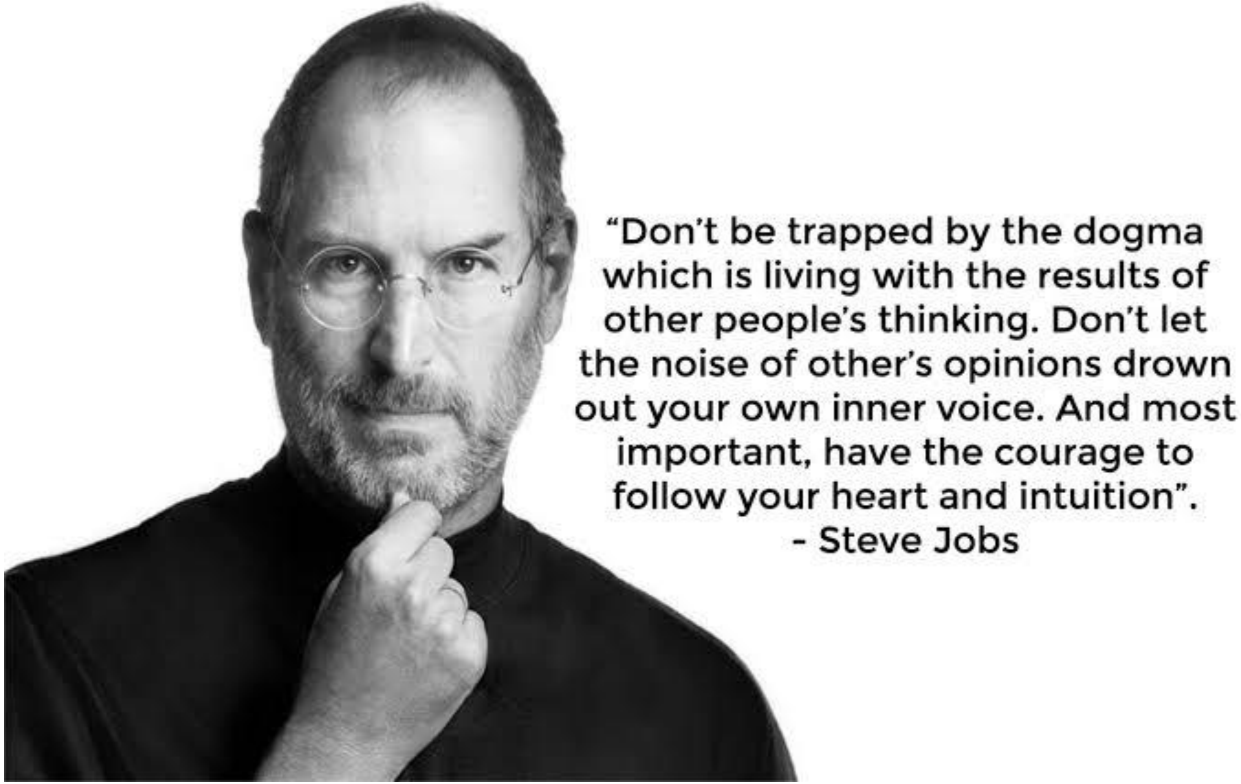
Learning representations by back-propagating errors

David E. Rumelhart*, Geoffrey E. Hinton† & Ronald J. Williams*

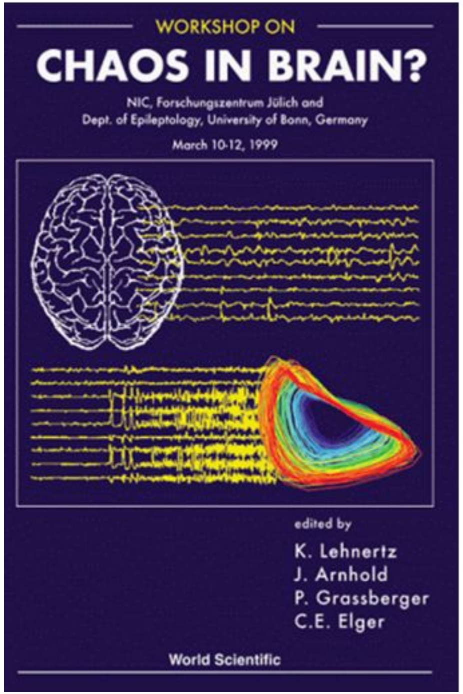
* Institute for Cognitive Science, C-015, University of California, San Diego, La Jolla, California 92093, USA
† Department of Computer Science, Carnegie-Mellon University, Pittsburgh, Philadelphia 15213, USA

We describe a new learning procedure, back-propagation, for networks of neurone-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure[1].

Let us think differently than following the ML dogma



Why should I learn Chaos Theory?



ELSEVIER
Comptes Rendus de l'Académie des Sciences - Series III - Sciences de la Vie
Volume 324, Issue 9, September 2001, Pages 773-793
Comptes Rendus de l'...

Is there chaos in the brain? I. Concepts of nonlinear dynamics and methods of investigation

Pierre Buser
Philippe Faure, Henri Korn

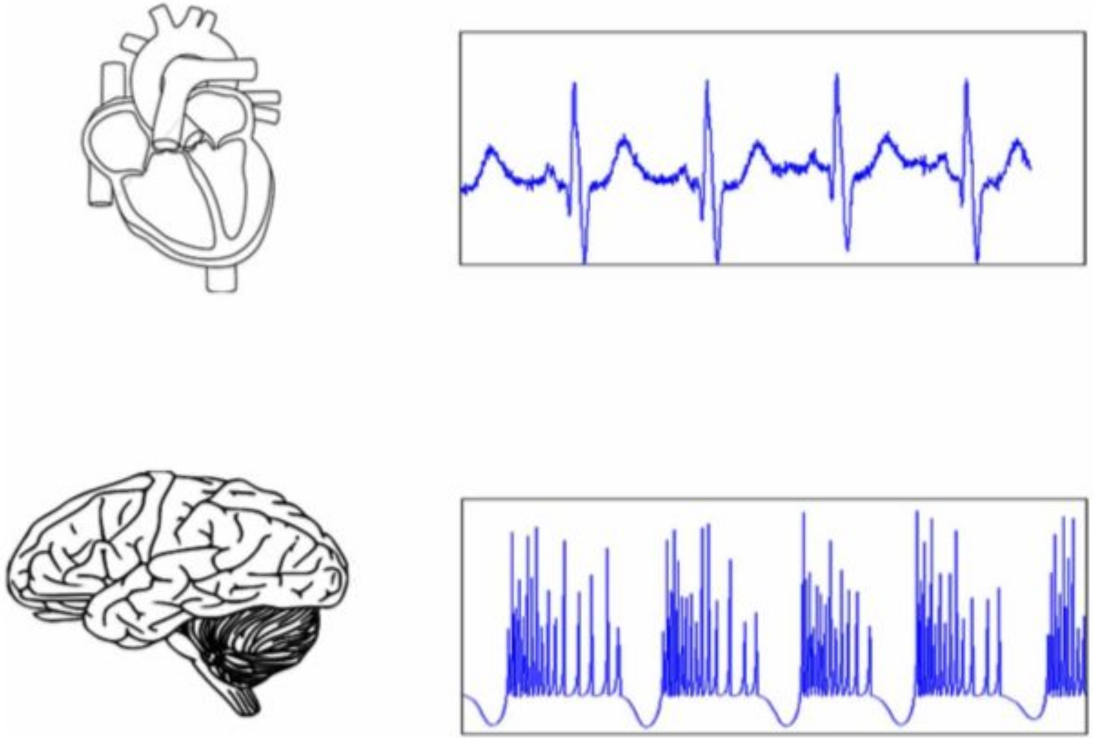
ELSEVIER
Comptes Rendus Biologies
Volume 326, Issue 9, September 2003, Pages 787-840

Neurosciences

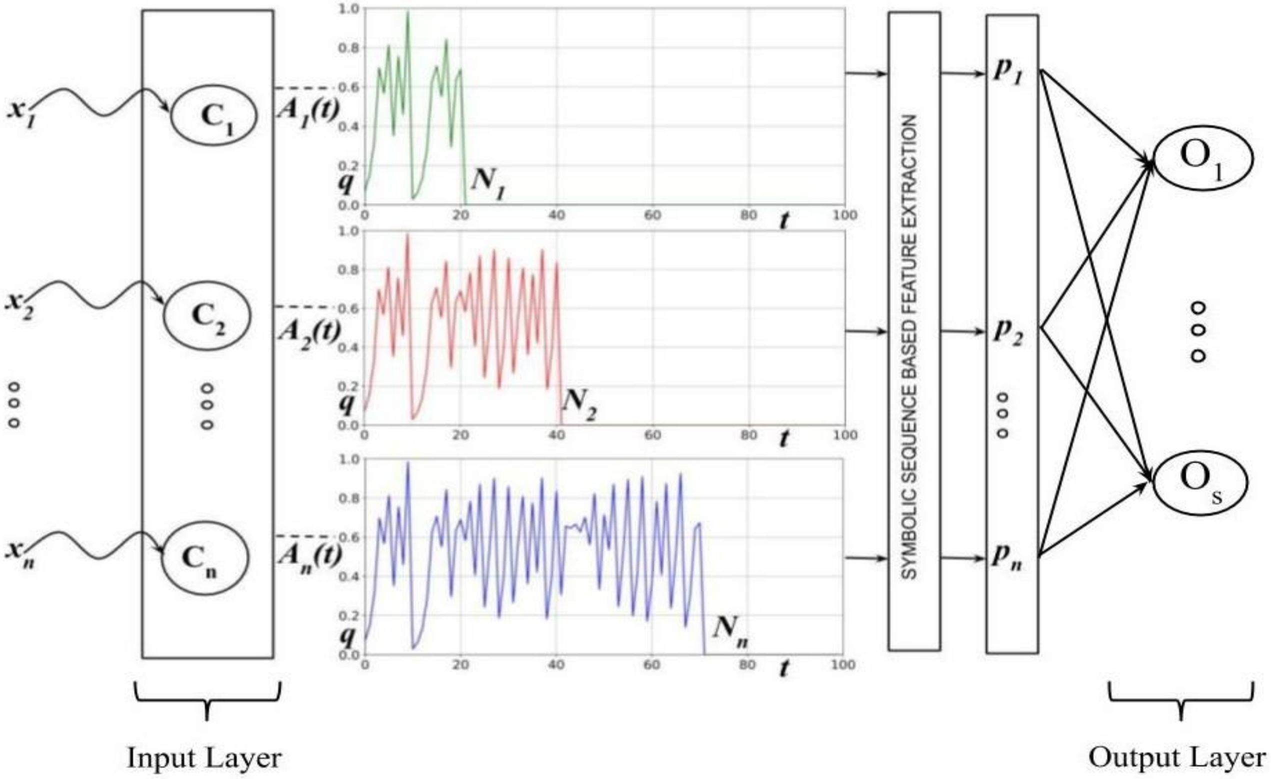
Is there chaos in the brain? II. Experimental evidence and related models

Presented by Pierre Buser
Henri Korn, Philippe Faure

Chaos in Heart and Brain



ChaosNet: used in ML



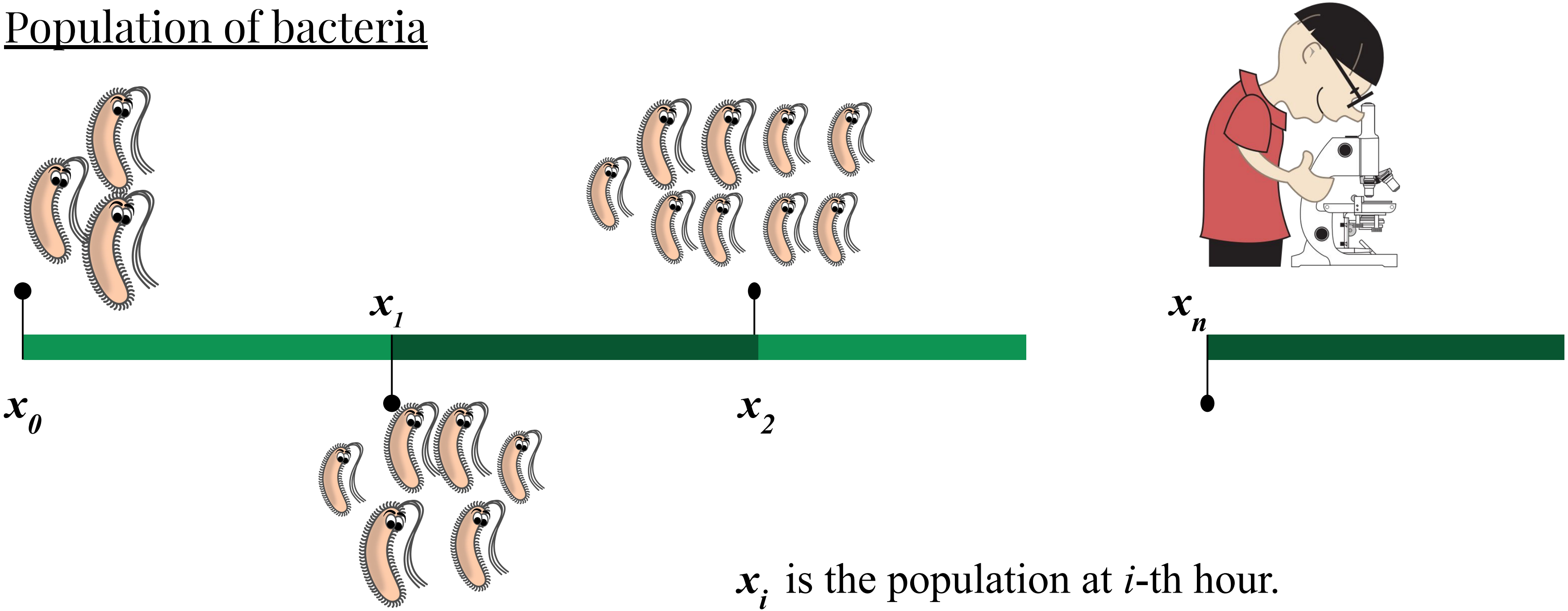
Applications of Chaos Theory

- Cosmology – universe underwent chaotic phase after Big Bang
- Weather and Climate, Turbulence, Convection
- Double pendulum, Atomic and Molecular dynamics
- Quantum chaos, nonlinear optics
- Chemical Reactions
- Chaos Cryptography, Chaos Computing, Nonlinear circuits, Fractal Image Compression, Nonlinear signal processing
- Hydrology, Economics, Sociology, Social networks, Epidemiology, Population Dynamics
- Ergodic theory of Numbers
- **Recurrent Neural Networks show Chaos**

Introduction to Chaos Theory

Let us learn:)

<u>Population of bacteria</u>

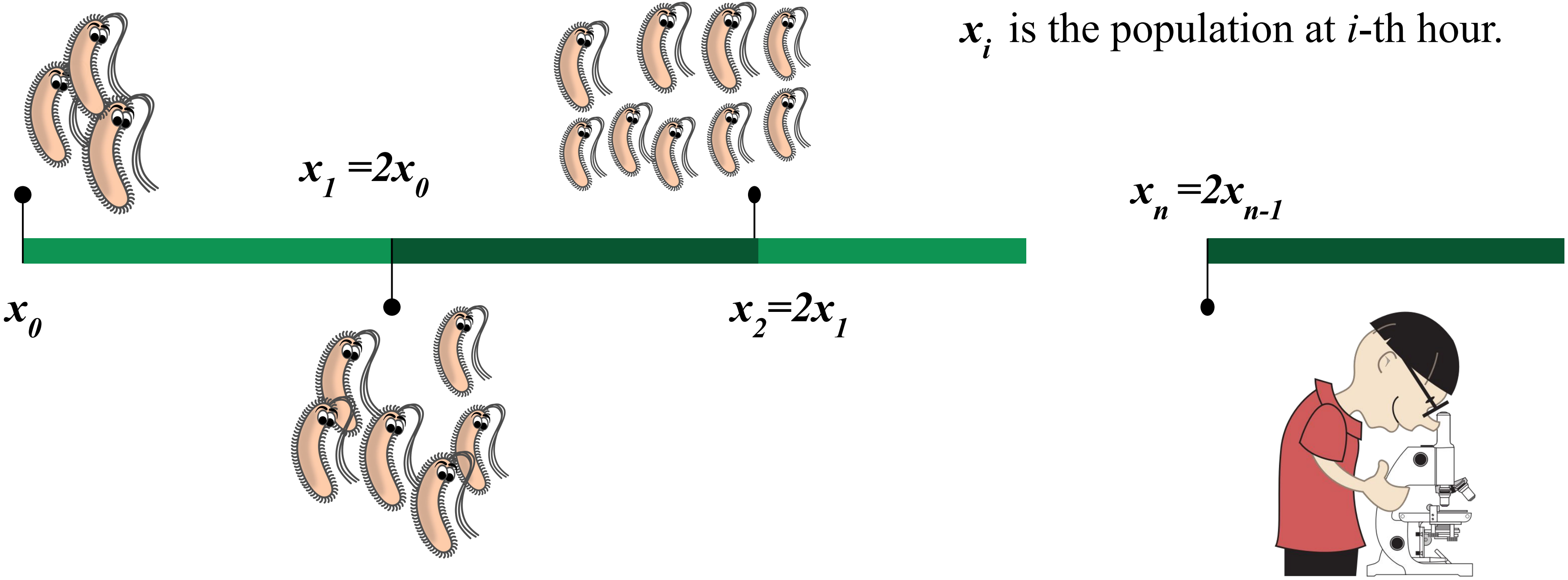


x_i is the population at *i*-th hour.

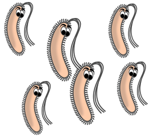
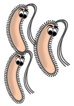
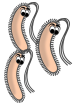
Model - A

<u>Population of bacteria</u>

x_i is the population at i-th hour.



Model - A

 $= f($  $) = 2($  $)$

$$x_n = f(x_{n-1}) = 2x_{n-1}$$

$(x_0, x_1, \ldots, x_n)$: are the states of this model, where $\boldsymbol{x_i}$ corresponds to the population at the $\boldsymbol{i}$-th hour.

- **Deterministic, iterative, discrete time**

$$x_0 \to x_1 \to x_2 \ldots \to x_n$$

$$x_0 \to f(x_0) \to f(x_1) \ldots \to f(x_{n-1})$$

$$x_0 \to f(x_0) \to f^2(x_0) \ldots \to f^n(x_0)$$

Discrete time deterministic dynamical systems

$$\boxed{x_n = f(x_{n-1}) = 2x_{n-1}}$$

Deterministic

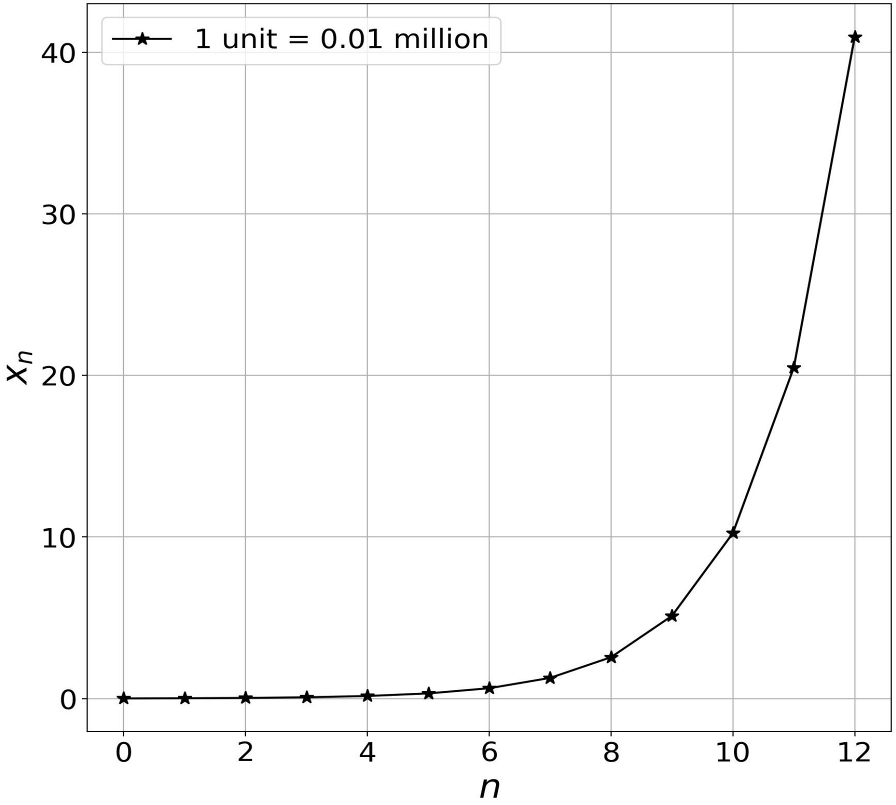
$$(x_0, x_1, \ldots, x_n) \longrightarrow$$

are the states of this model, where $\boldsymbol{x_i}$ corresponds to the population at the $\boldsymbol{i}$-th hour.

$$\{0, 1, 2, \ldots n\} \longrightarrow$$

Discrete time in hours.

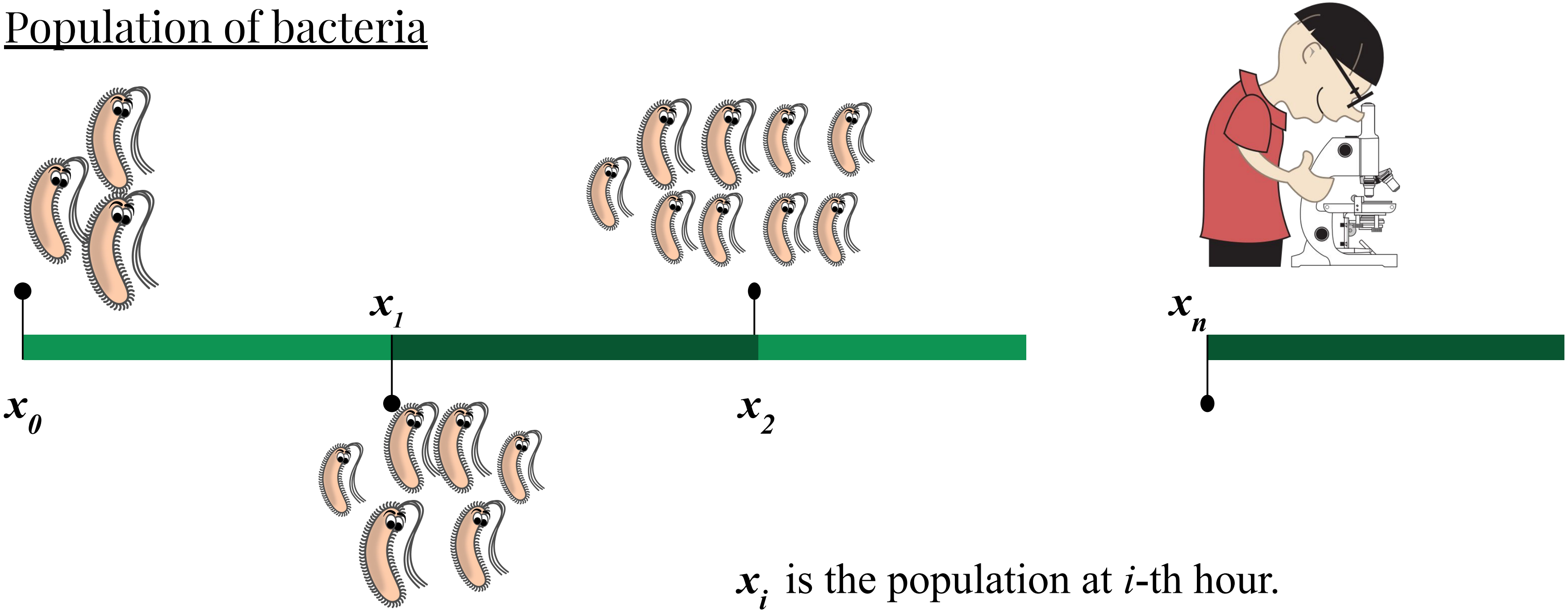
Model - A



- Exponential growth
- Assumptions - Infinite resources

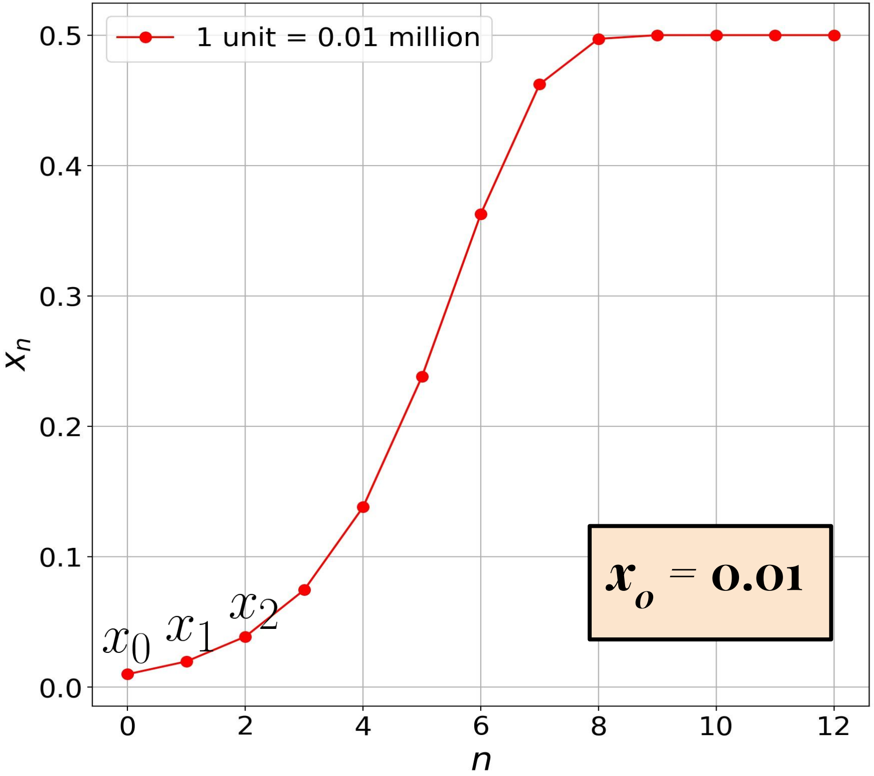
Model - B $x_n = 2x_{n-1}(1 - x_{n-1})$

<u>Population of bacteria</u>



x_i is the population at *i*-th hour.

Model - B $x_n = 2x_{n-1}(1 - x_{n-1})$



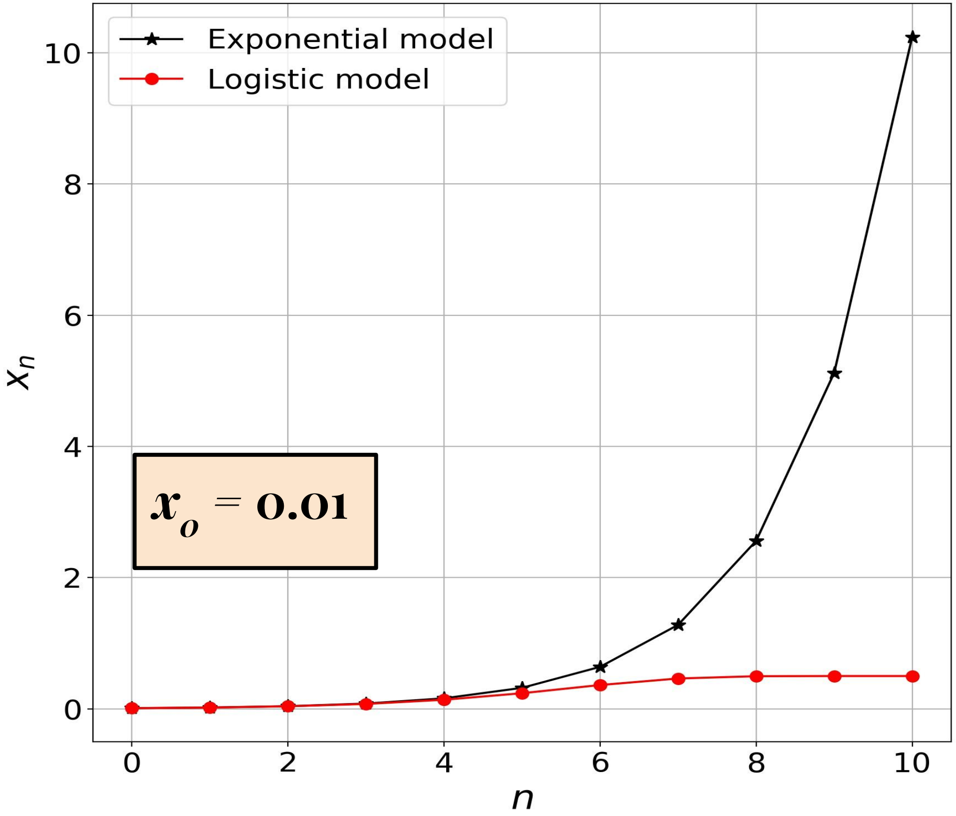
- Built in Carrying capacity.

$$x_1 = f(x_0)$$
$$x_1 = 2x_0(1 - x_0)$$

Orbit or **trajectory** of x_o under the map f is represented as

$$x_0 \to x_1 \to x_2 \ldots \to x_n$$

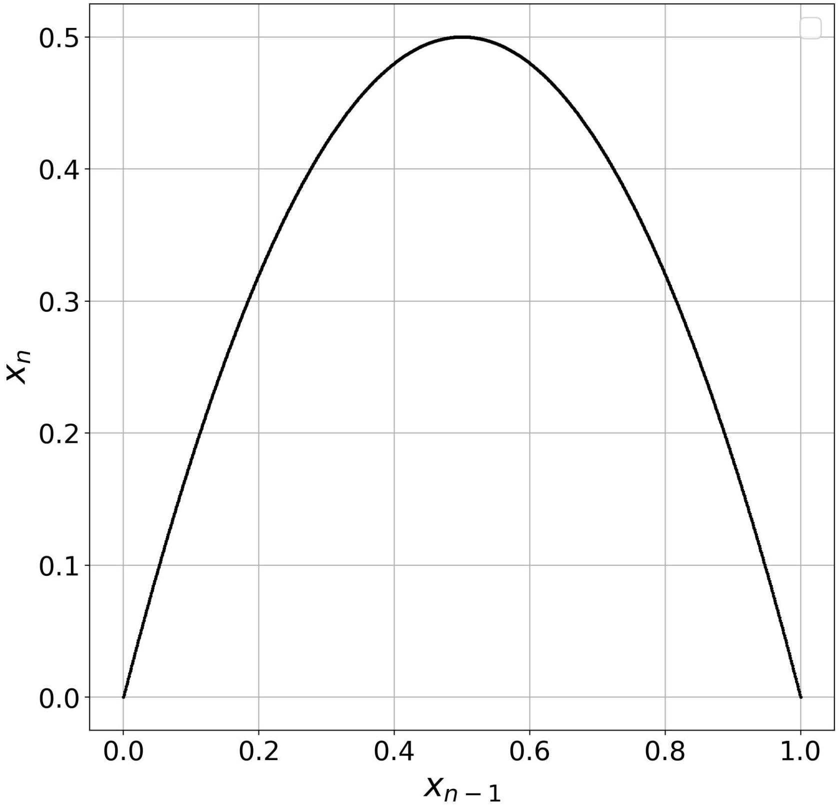
Model A vs Model B

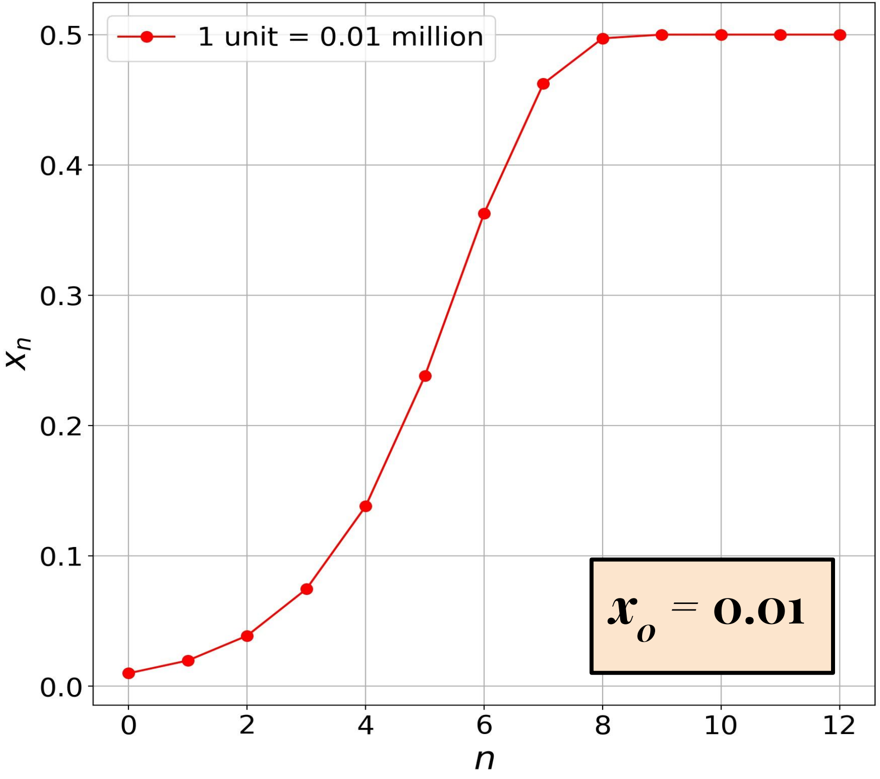


$$x_n = f(x_{n-1}) = 2x_{n-1}$$

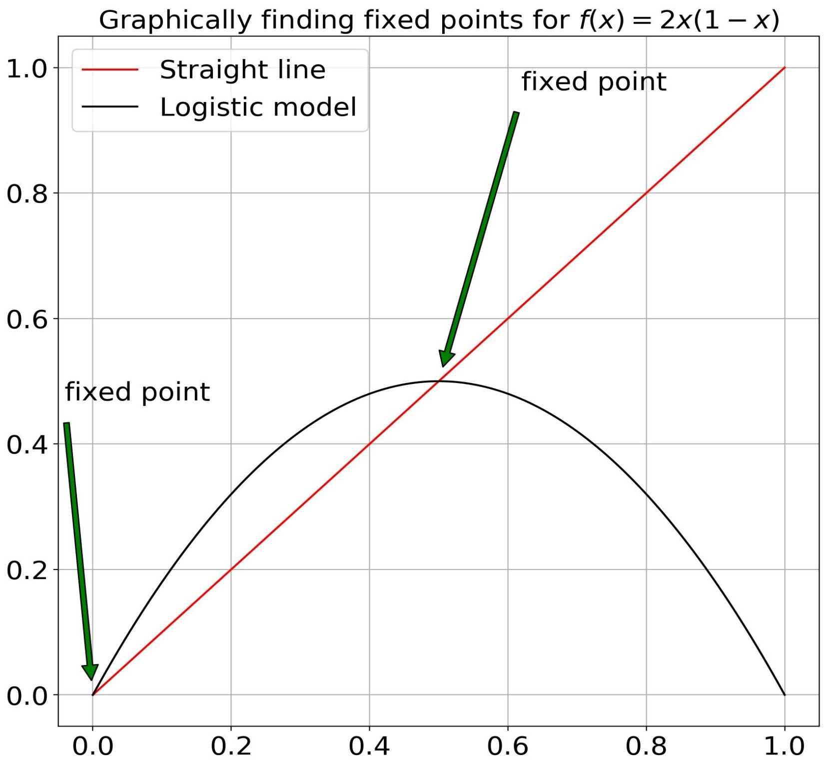
$$x_n = 2x_{n-1}(1 - x_{n-1})$$

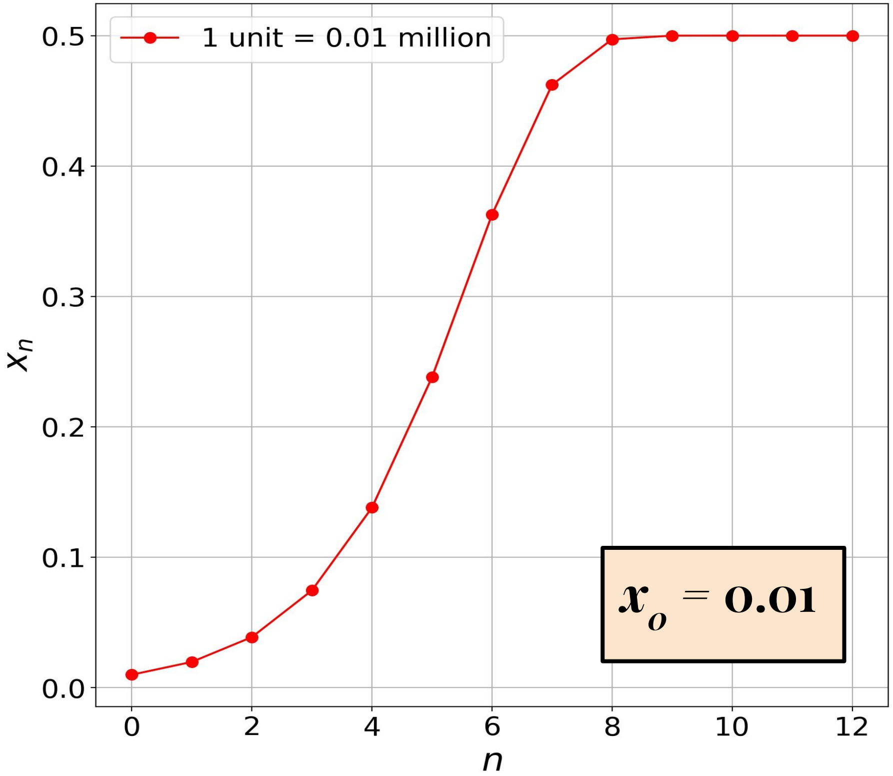
Model B: x_n vs x_{n-1} and x_n vs n



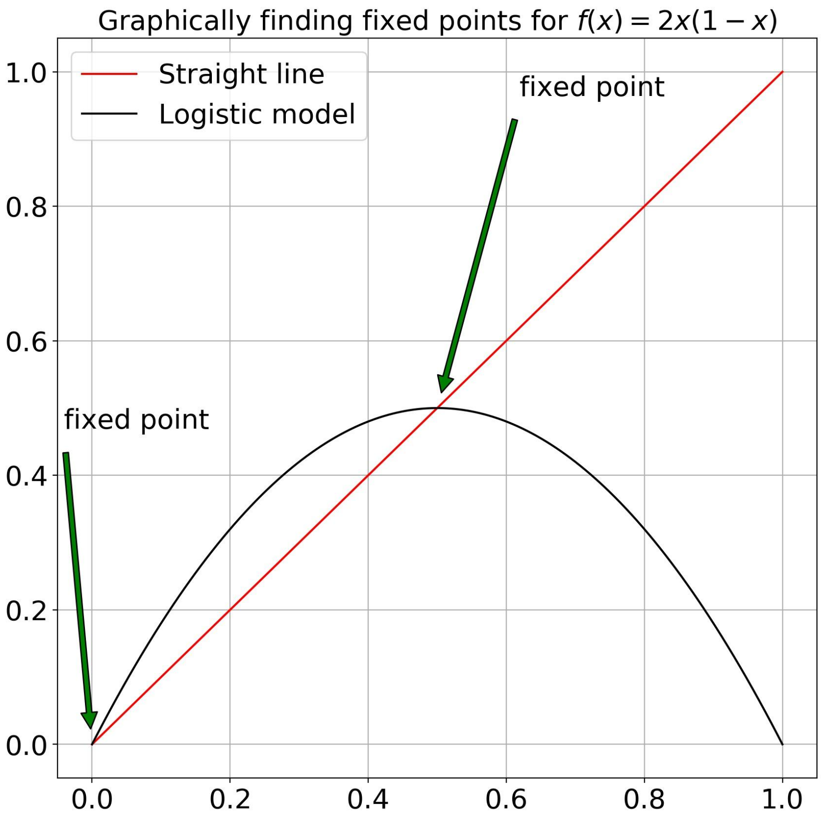


Model B: x_n vs x_{n-1} and x_n vs n





Fixed points

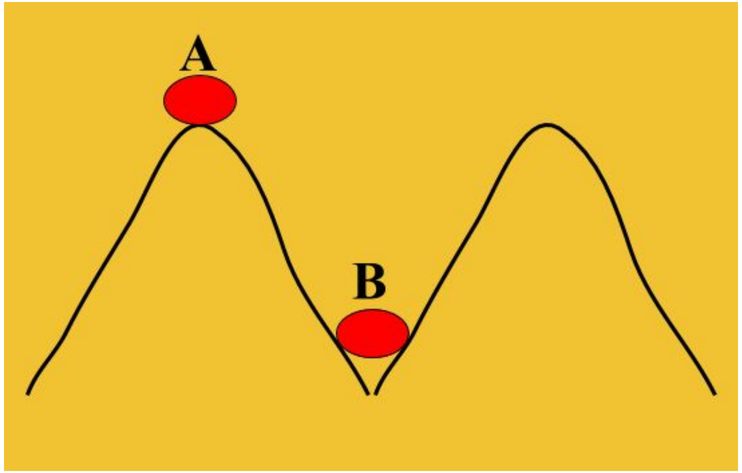


A point ***p*** is a fixed point of a **map *f***, if ***f(p)*** = ***p.***

Orbits starting with Initial conditions as ***p*** (fixed point) are orbits with period **1.**

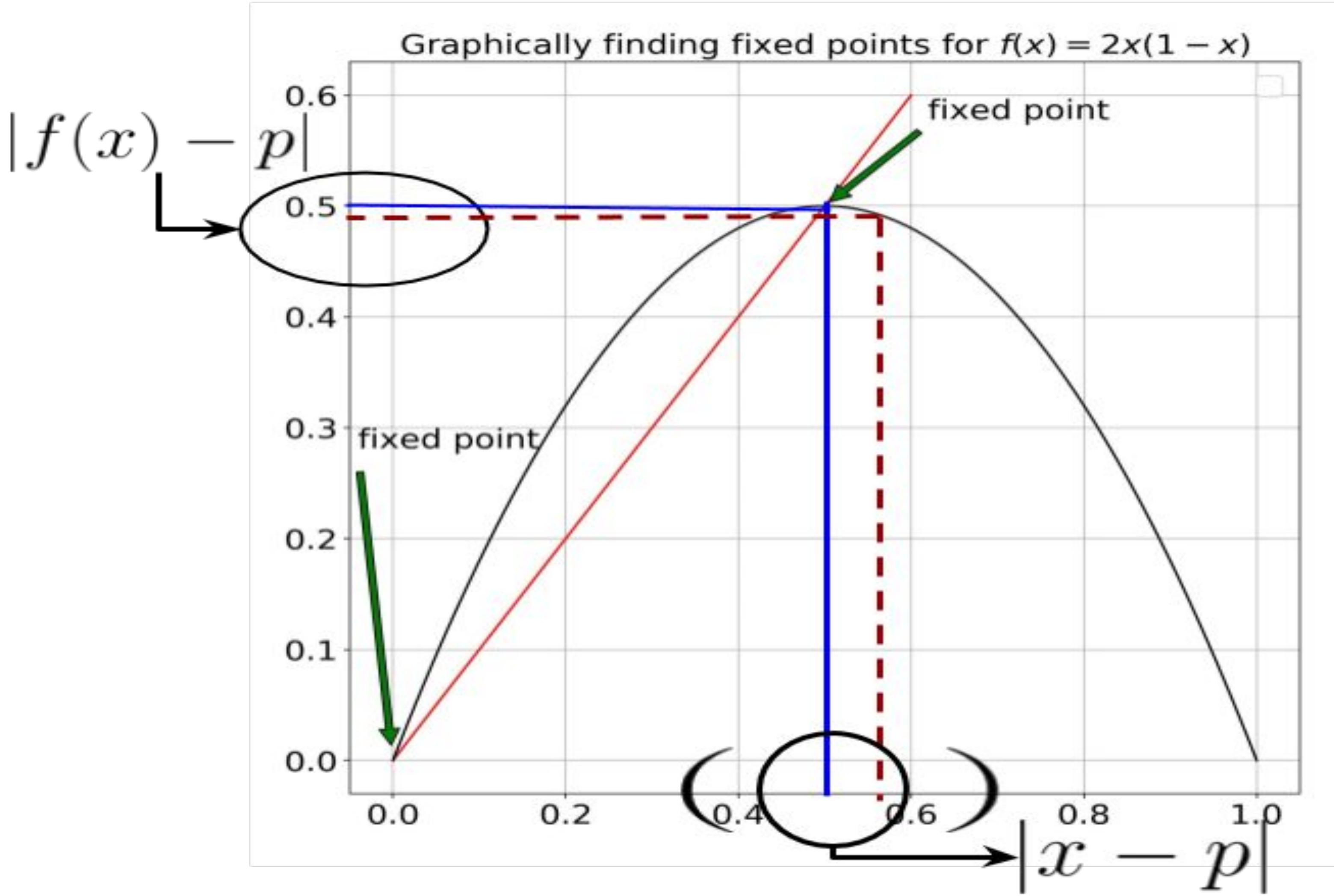
$$p \to f(p) \to f^2(p) \to \ldots . f^n(p)$$

Stability of fixed points



1. Is the ball at location A stable?

2. What will happen if you slightly perturb the position of the ball at location B?



Graphically finding fixed points for $f(x) = 2x(1-x)$
fixed point
fixed point
$|f(x) - p|$
$|x - p|$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.0
0.2
0.4
0.6
0.8
1.0

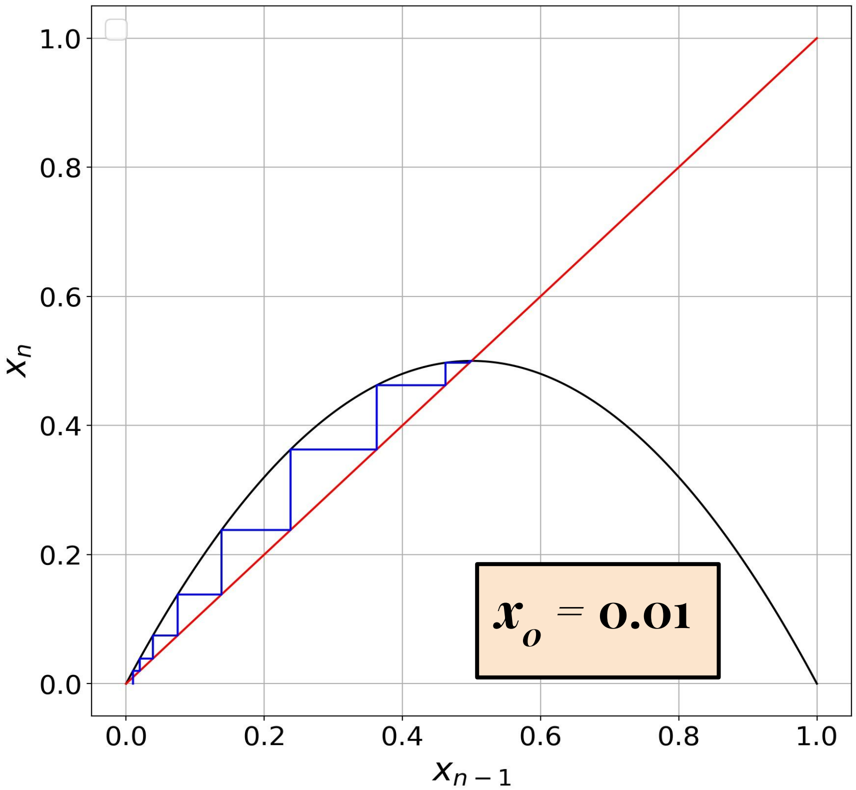
Conditions for Stability

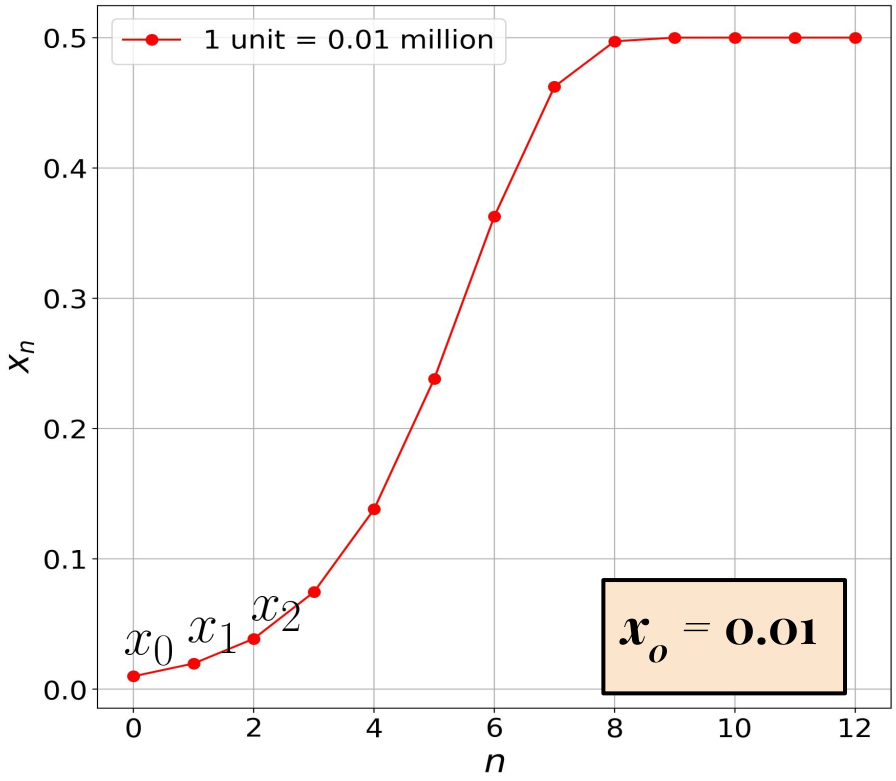
Theorem : Let ***f*** be a (smooth) map on R, and assume that p is a fixed point of ***f***.

$$\text{If } |f^{'}(p)| < 1, \text{ then } p \text{ is a sink}$$

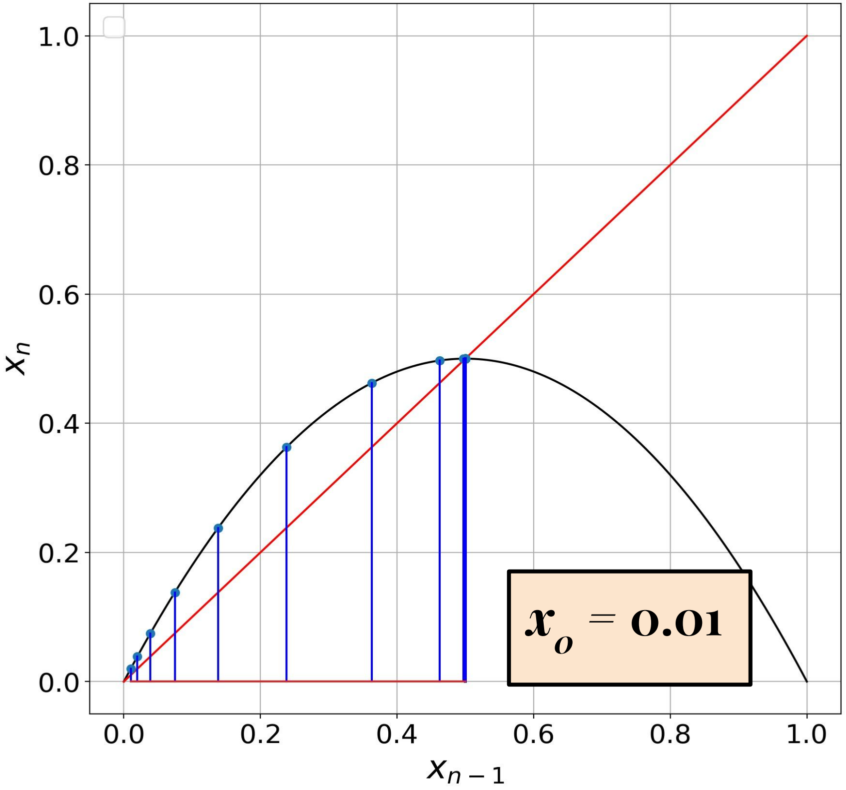
$$\text{If } |f^{'}(p)| > 1, \text{ then } p \text{ is a source}$$

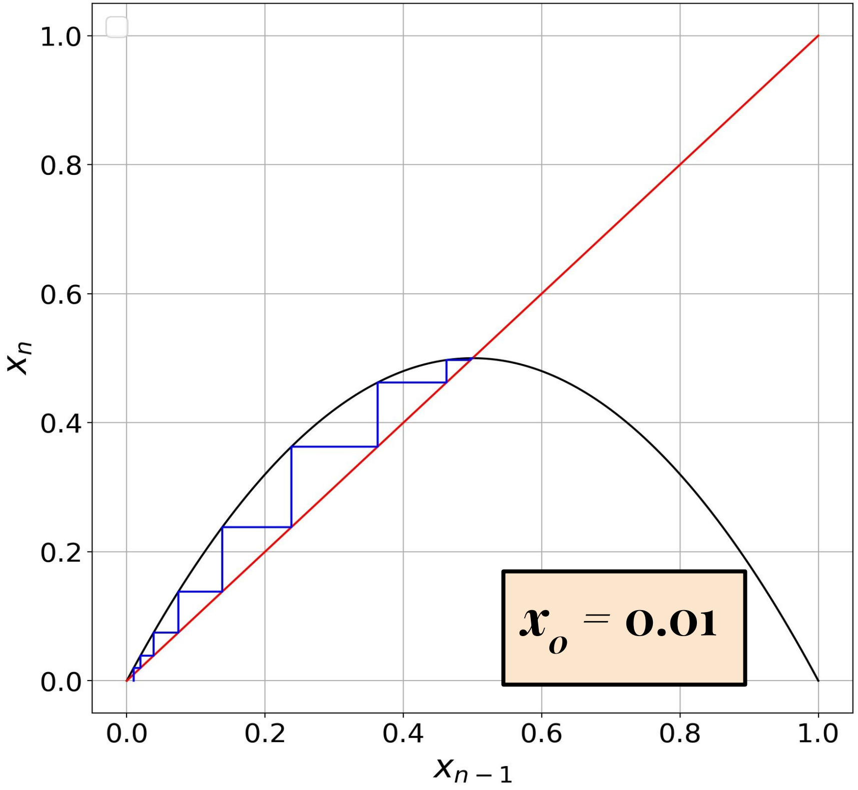
Nature of Fixed points and trajectory (x_0= 0.01)



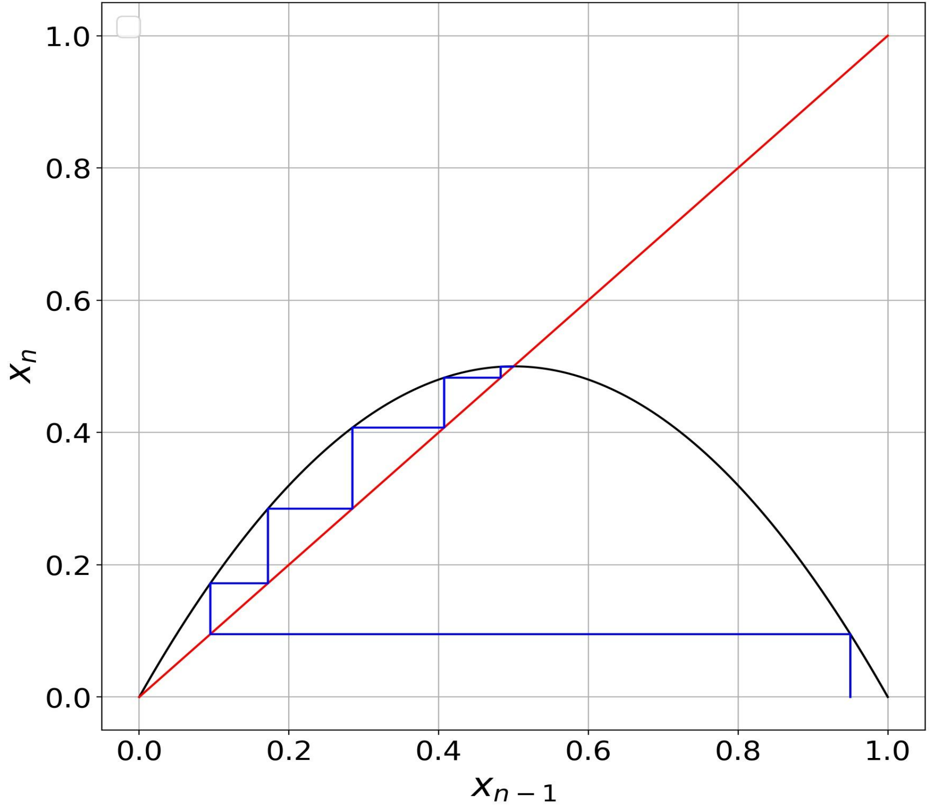


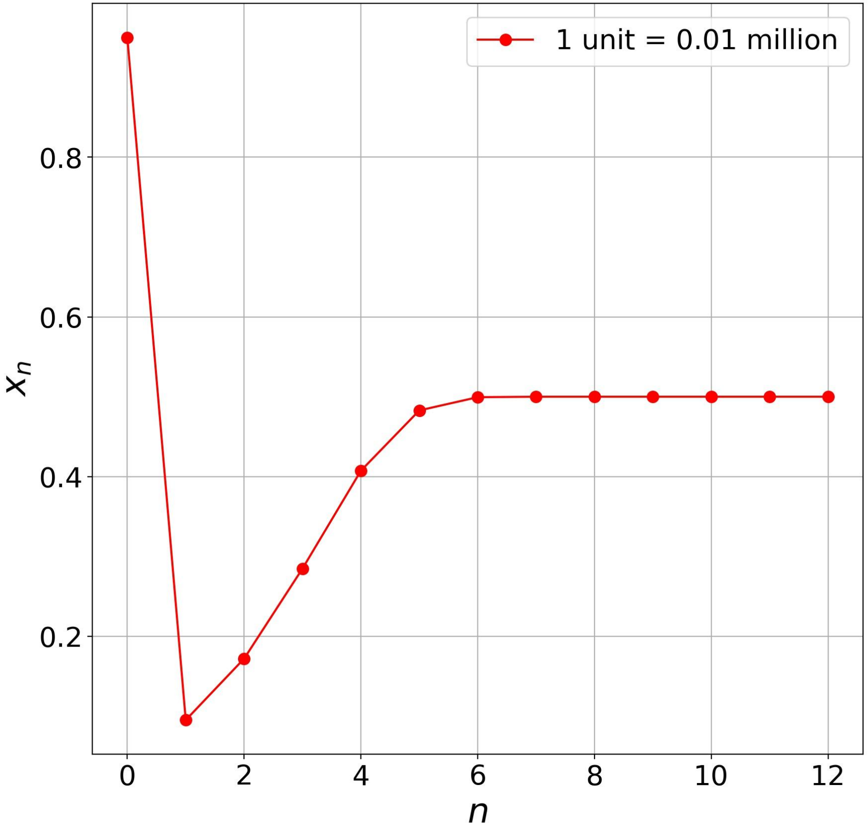
Understanding Cobweb Plot





Cobweb plots: (x_0= 0.95)





Second Half :)

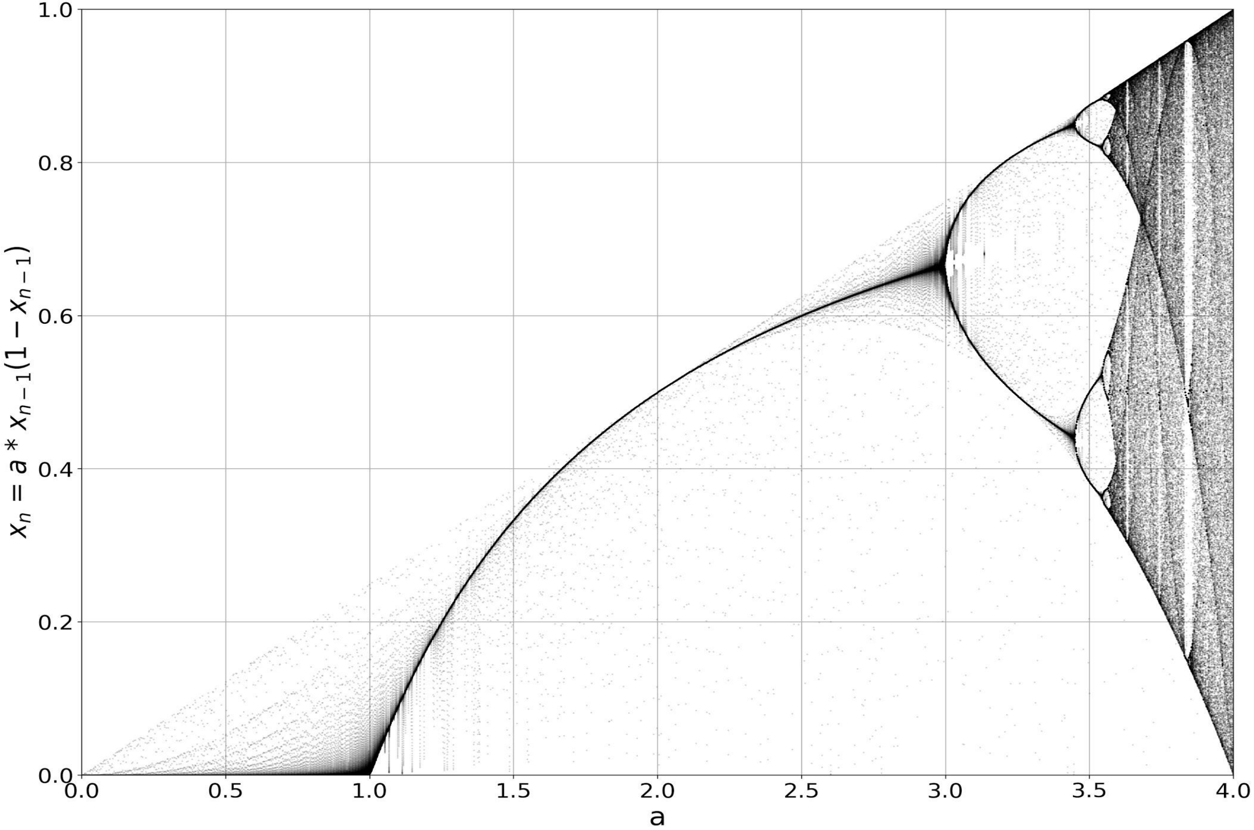
$$x_n = a x_{n-1}(1 - x_{n-1})$$

Challenge: Find the fixed points ?

$$x_n = a x_{n-1}(1 - x_{n-1})$$

$$0 \leq a \leq 4$$

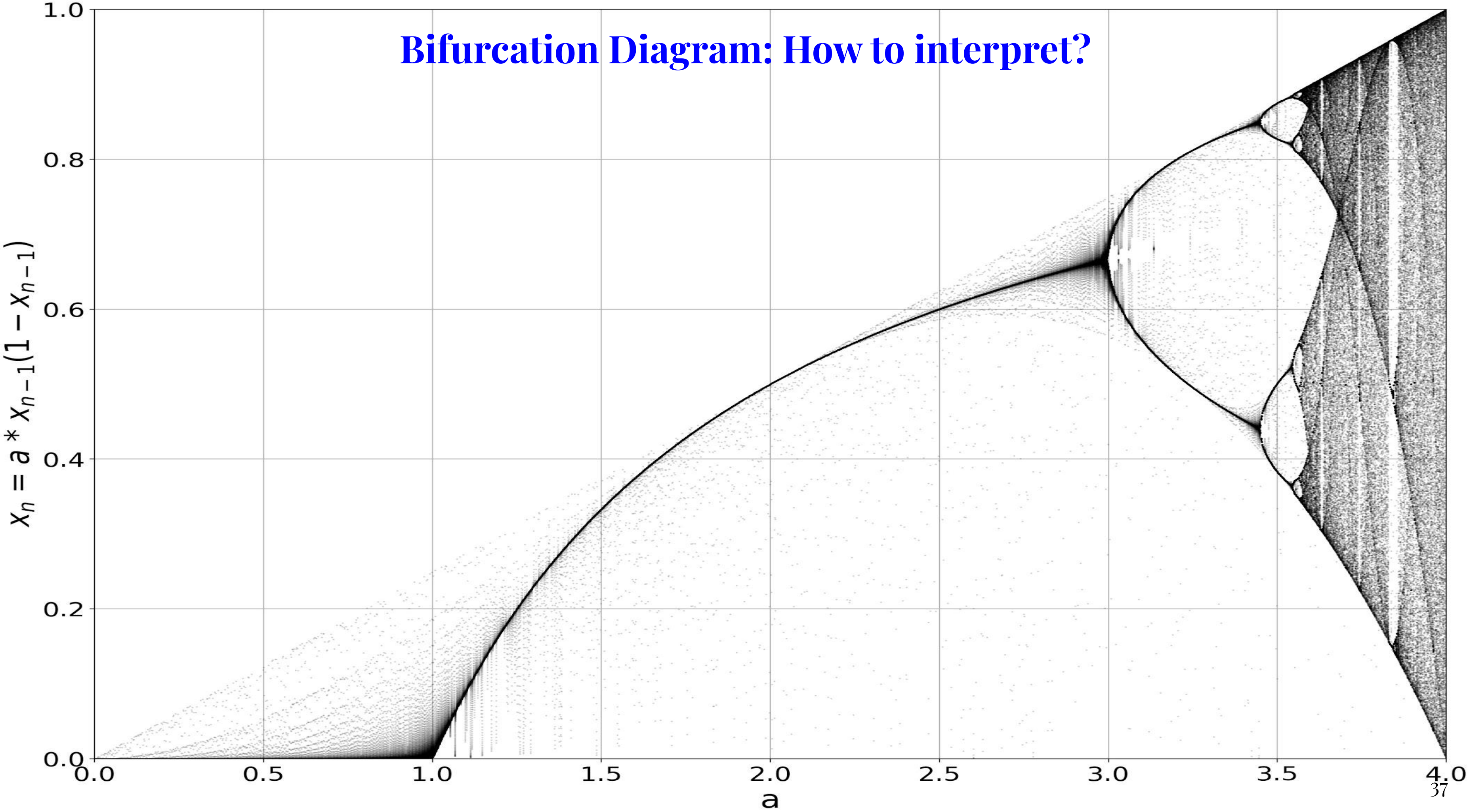
Behaviour of logistic map for different a value

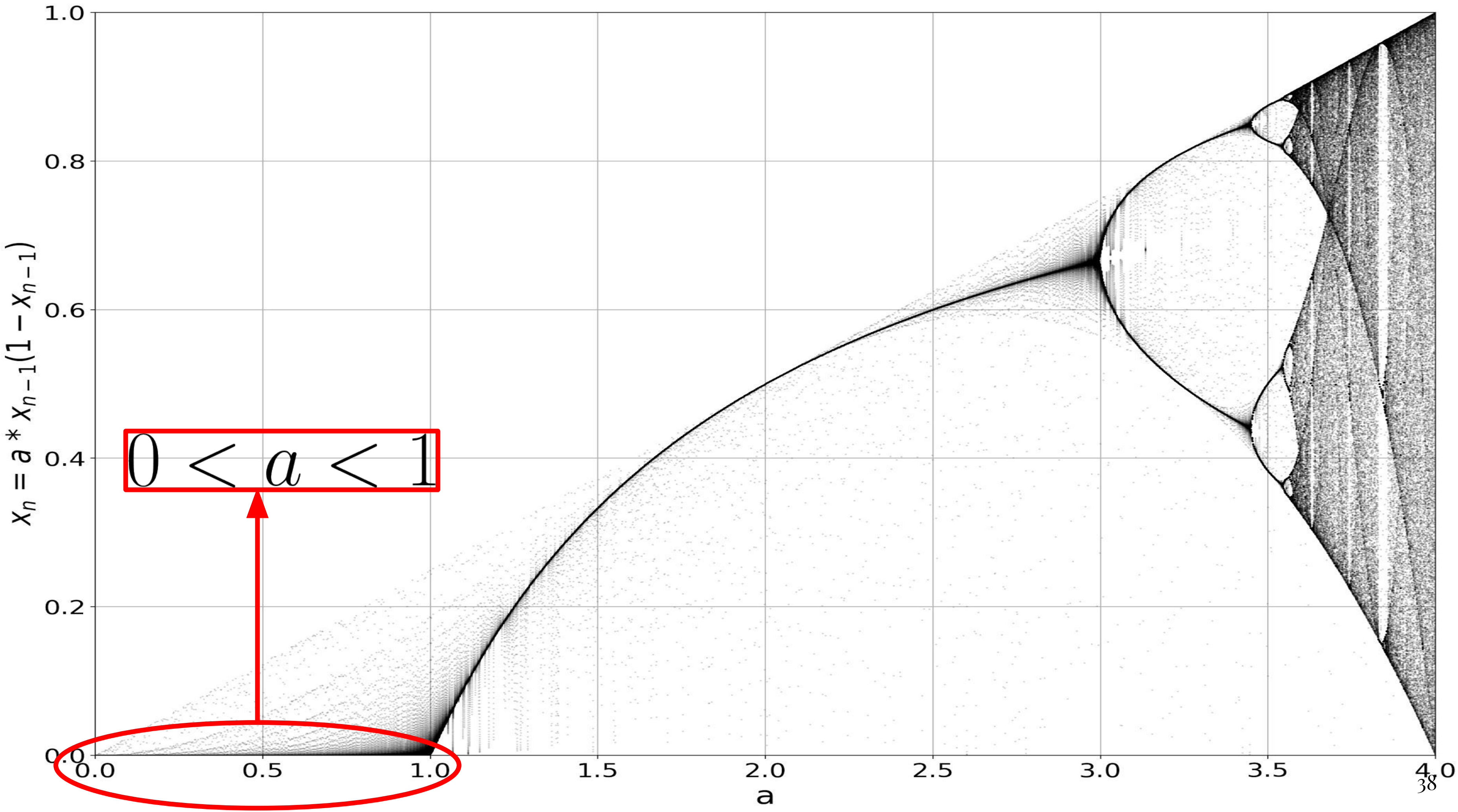


$$x_n = ax_{n-1}(1 - x_{n-1})$$

$$0 \leq a \leq 4$$

Bifurcation Diagram: How to interpret?

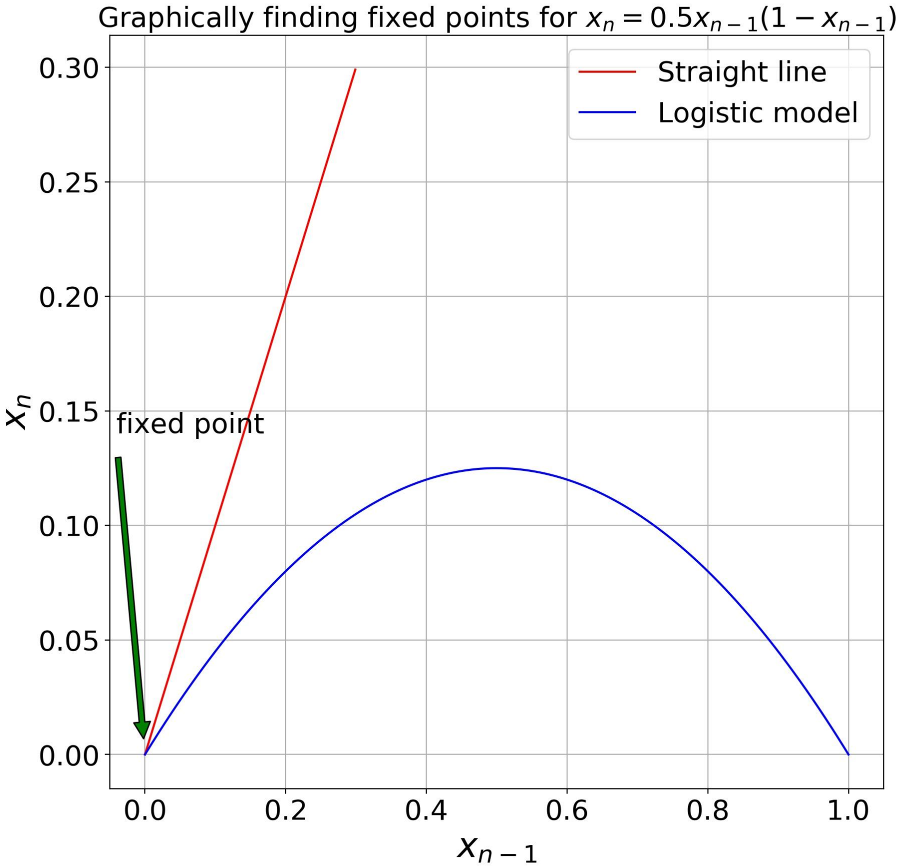


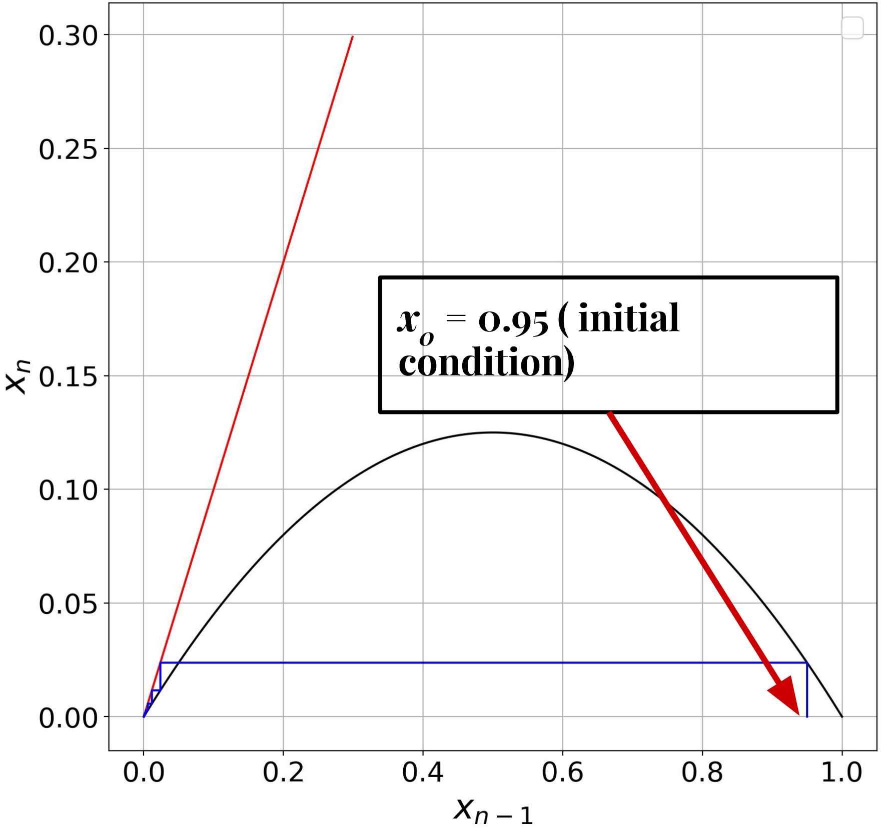


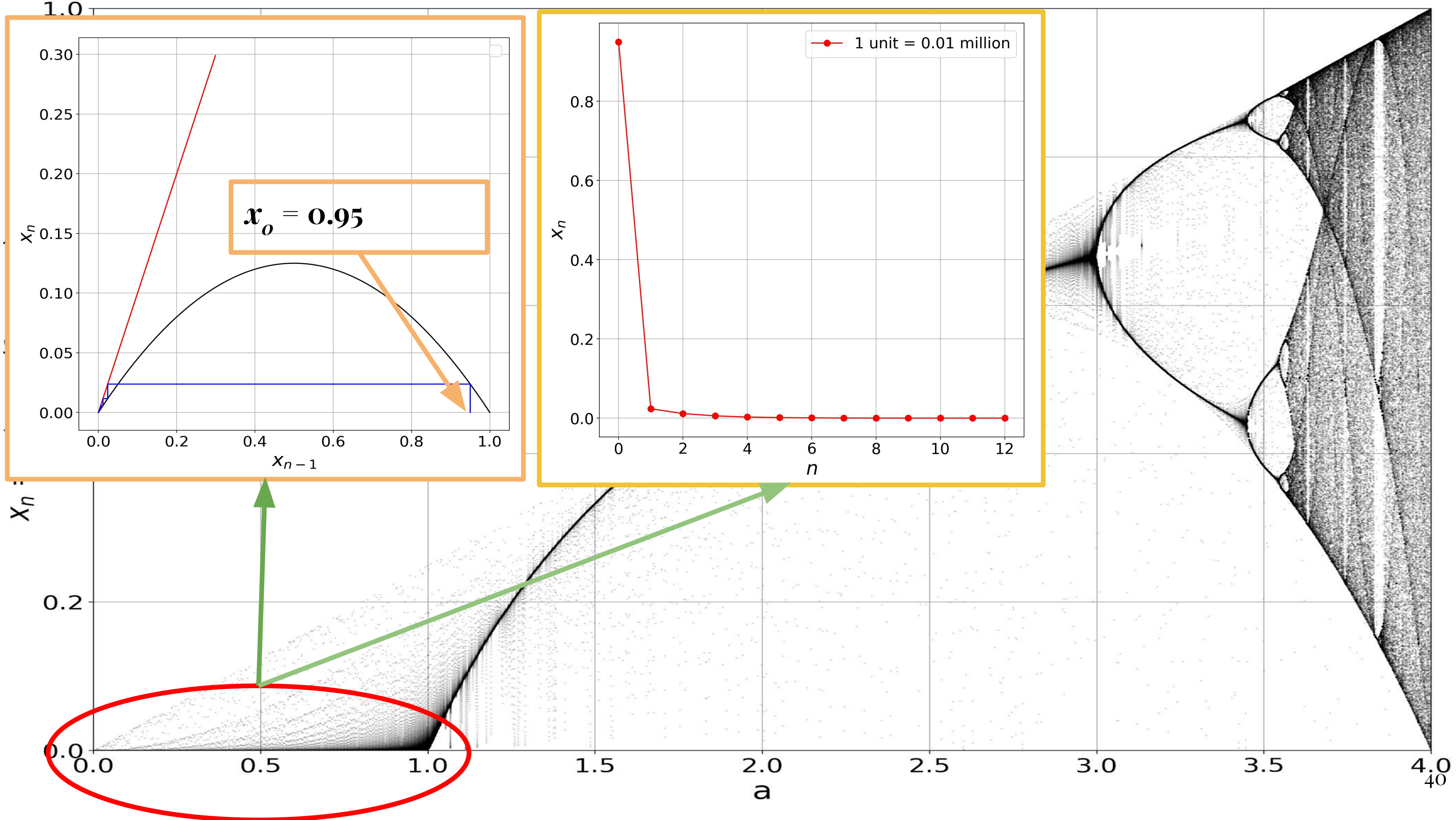
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
a
$x_n = a * x_{n-1}(1 - x_{n-1})$
$0 < a < 1$

0 < a < 1 (Let a = 0.5)

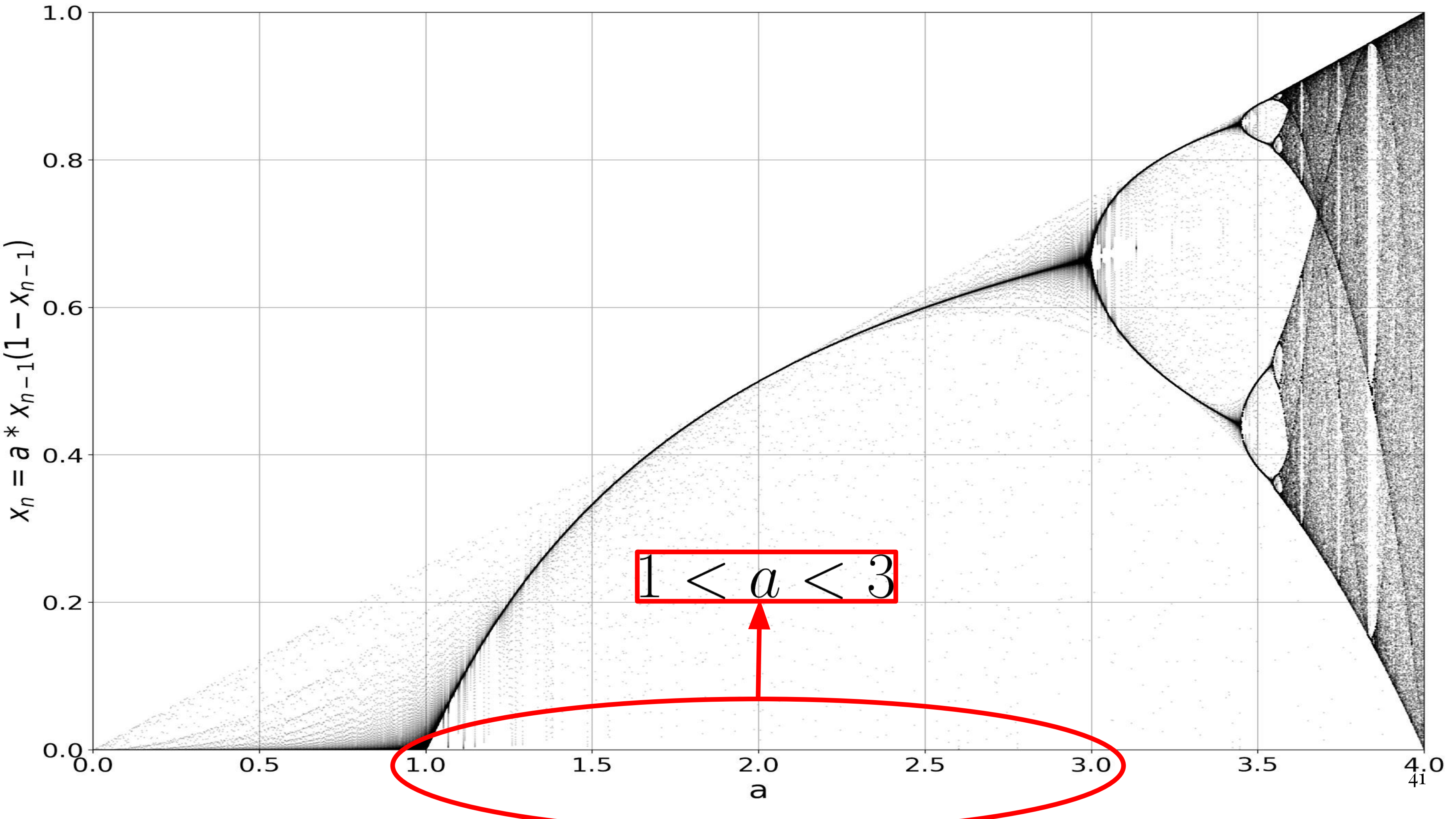
$$x_n = 0.5x_{n-1}(1 - x_n)$$





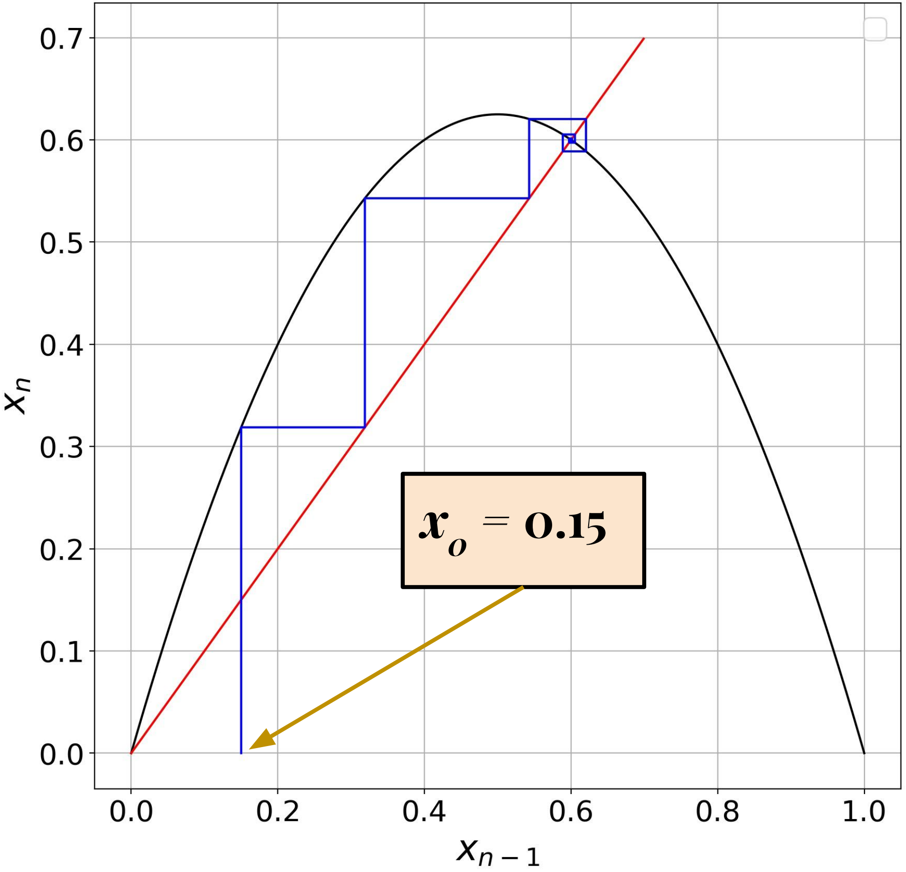


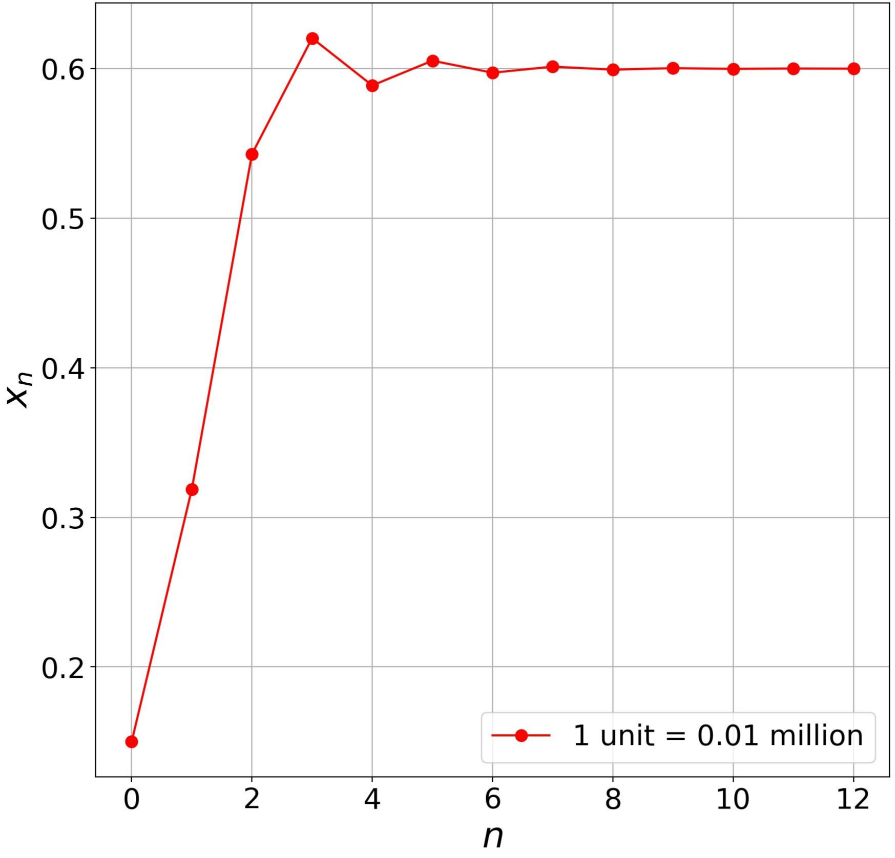
1.0
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
a
x_n
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0
x_{n-1}
x_n
$x_o = 0.95$
1 unit = 0.01 million
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
n
x_n

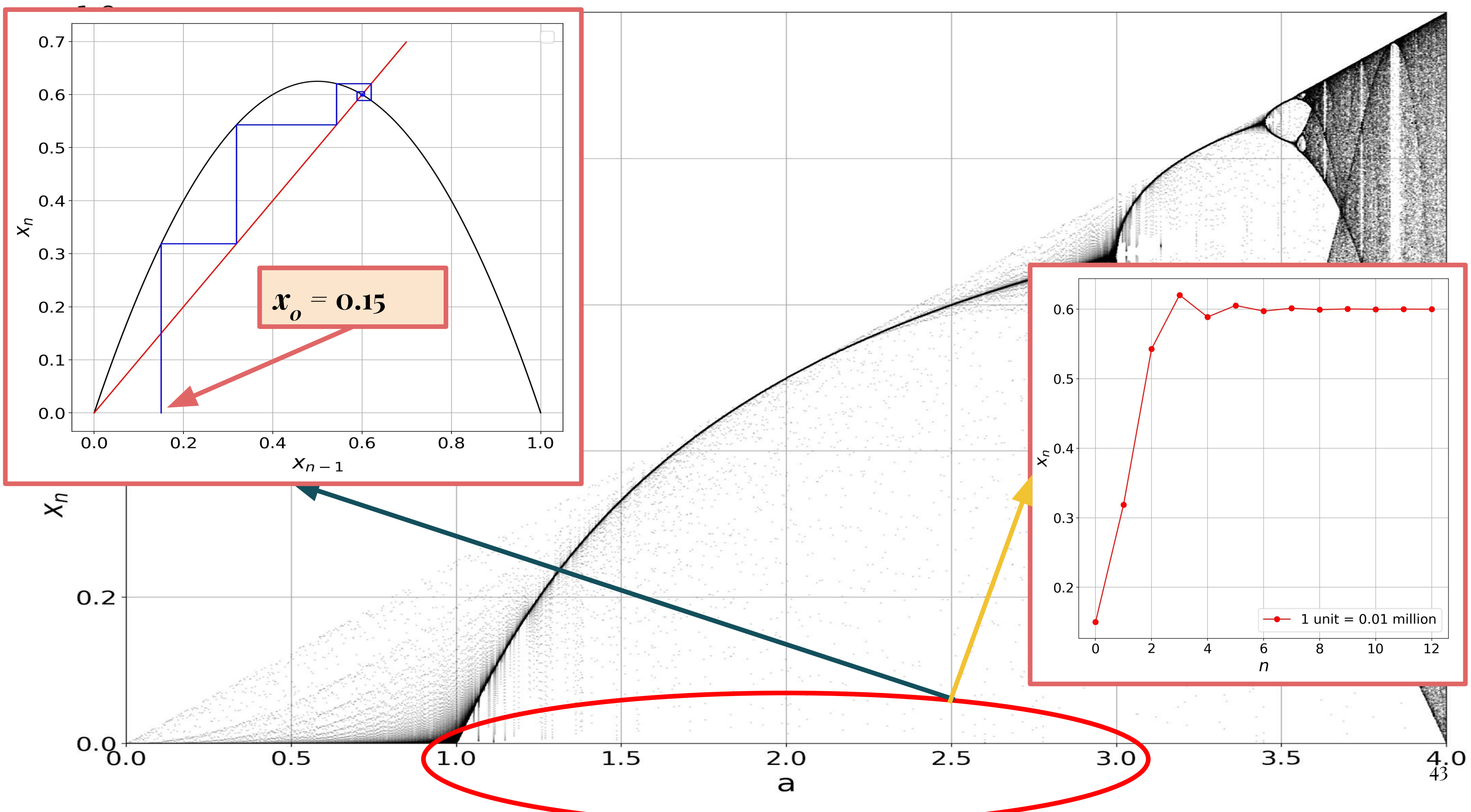

1.0
0.8
0.6
0.4
0.2
0.0
$x_n = a * x_{n-1}(1 - x_{n-1})$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
a
$1 < a < 3$

1 < a < 3 (Let a = 2.5)

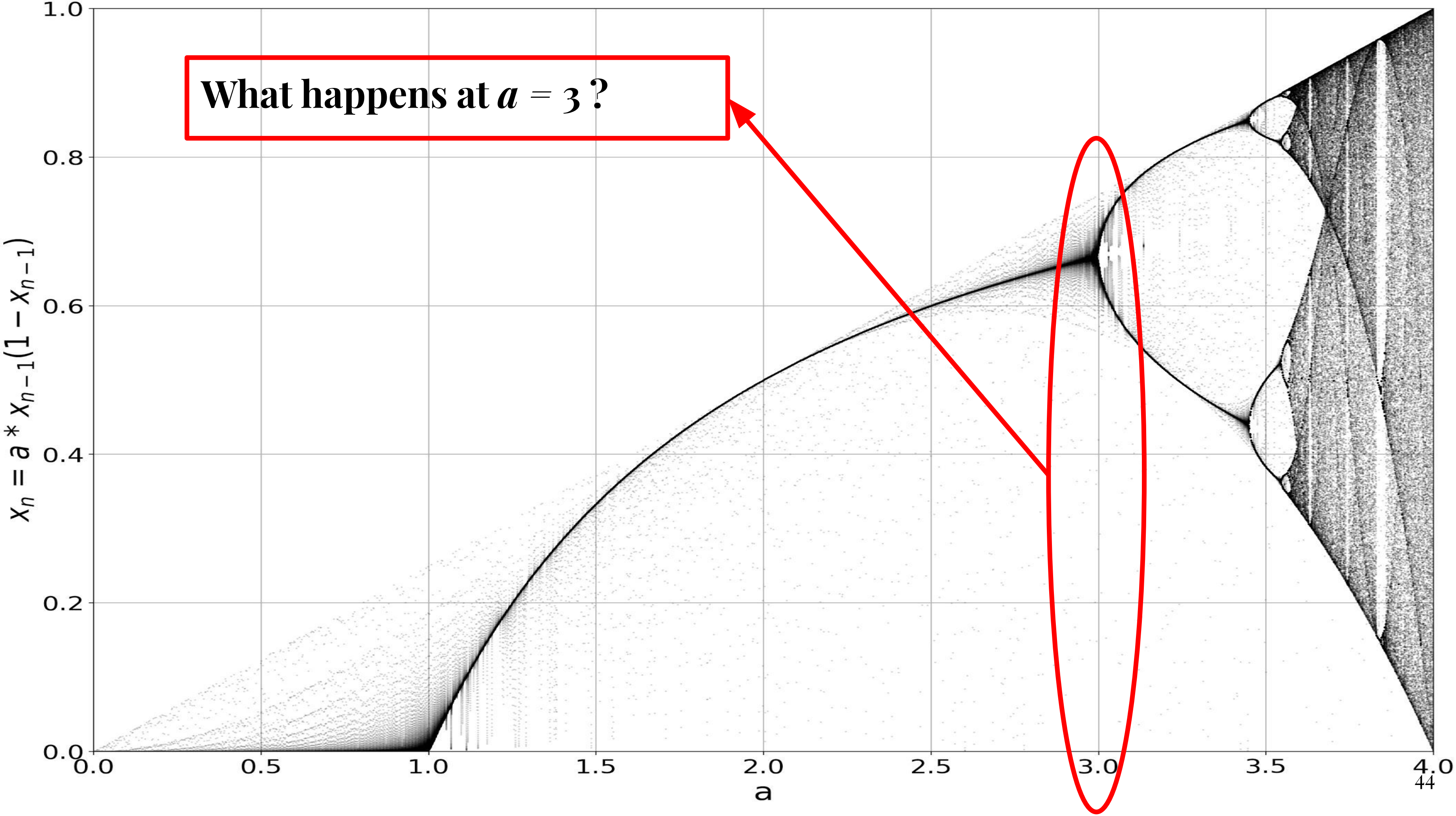
$$x_n = 2.5x_{n-1}(1 - x_n)$$





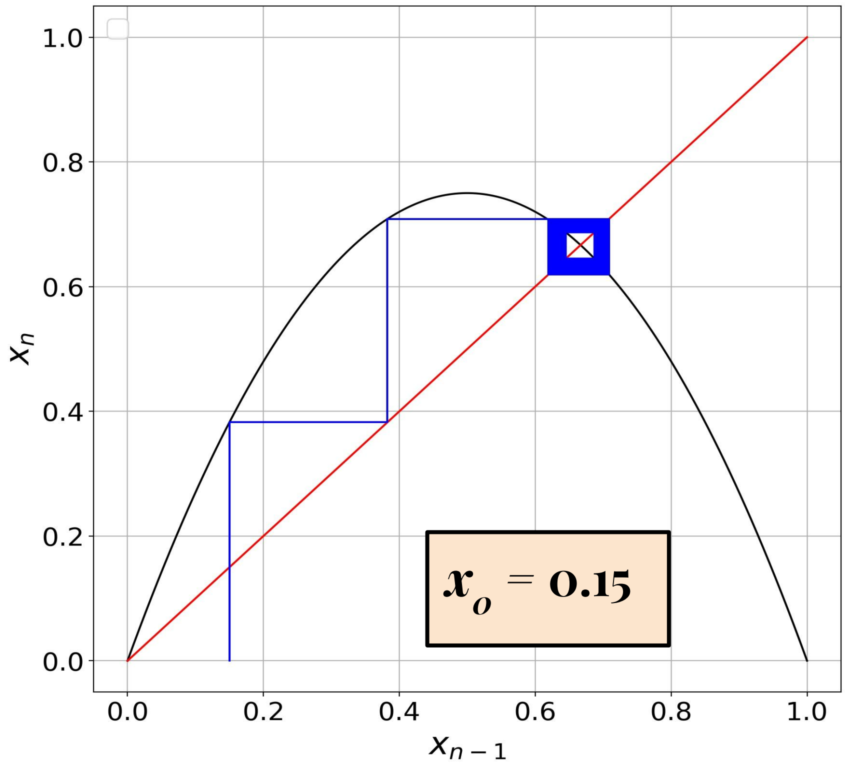


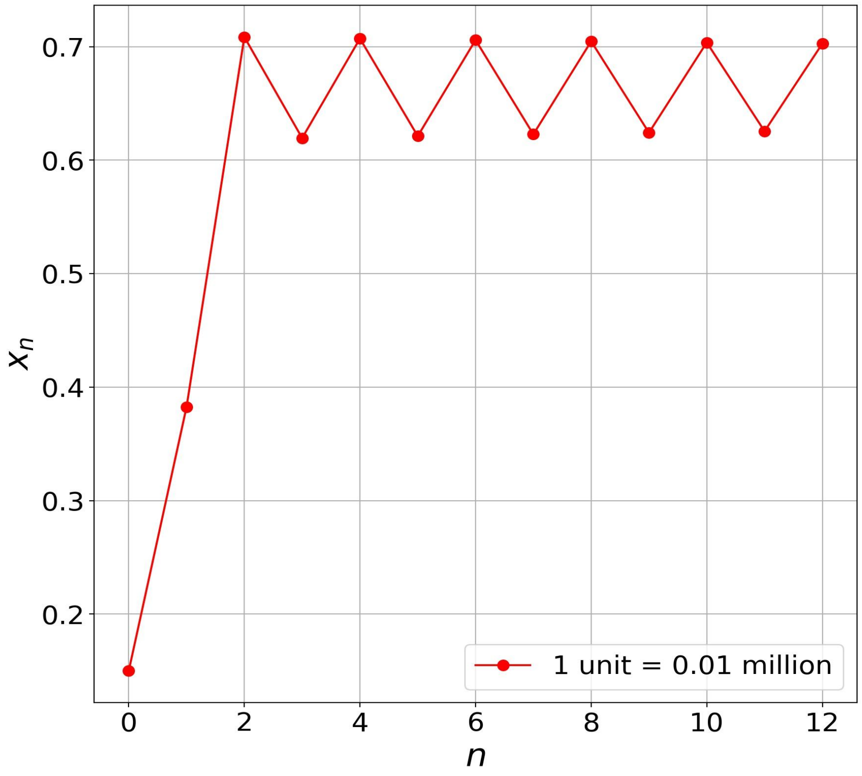
x_o = 0.15
x_n
x_{n-1}
1 unit = 0.01 million
n
a


What happens at $a = 3$?
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
a
$x_n = a * x_{n-1}(1 - x_{n-1})$

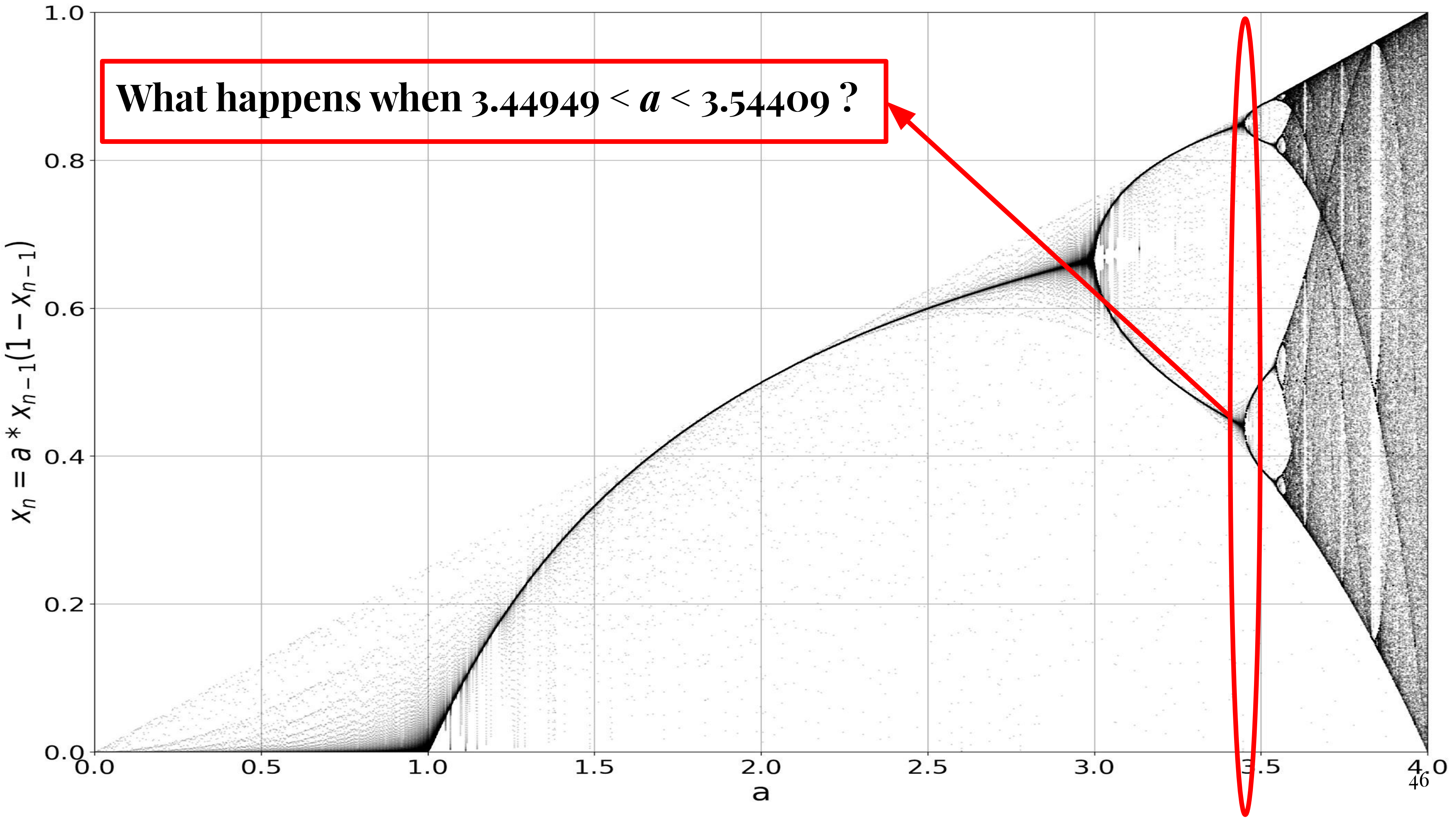
Birth of period 2

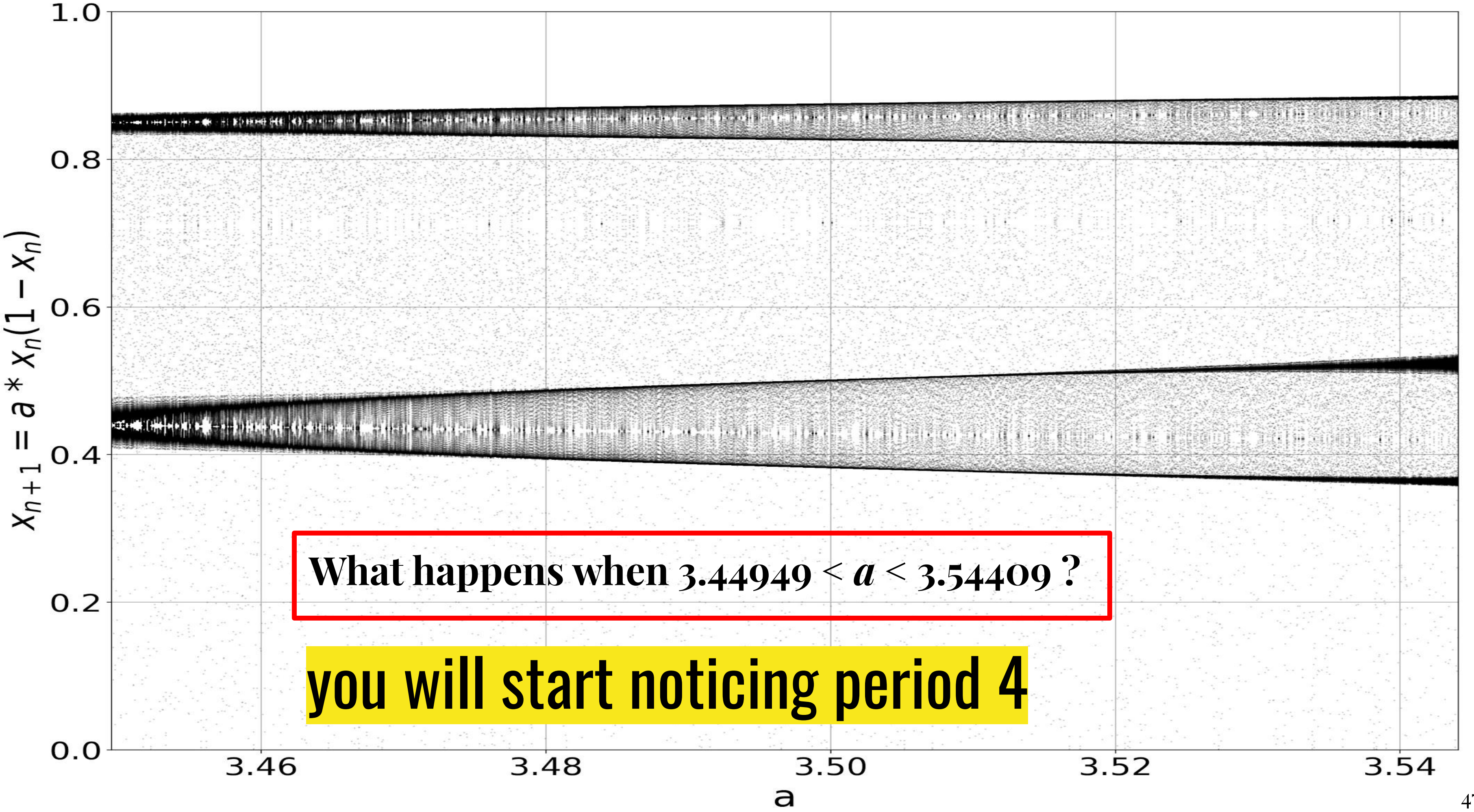
$$x_n = 3x_{n-1}(1 - x_n)$$





What happens when $3.44949 < a < 3.54409$?



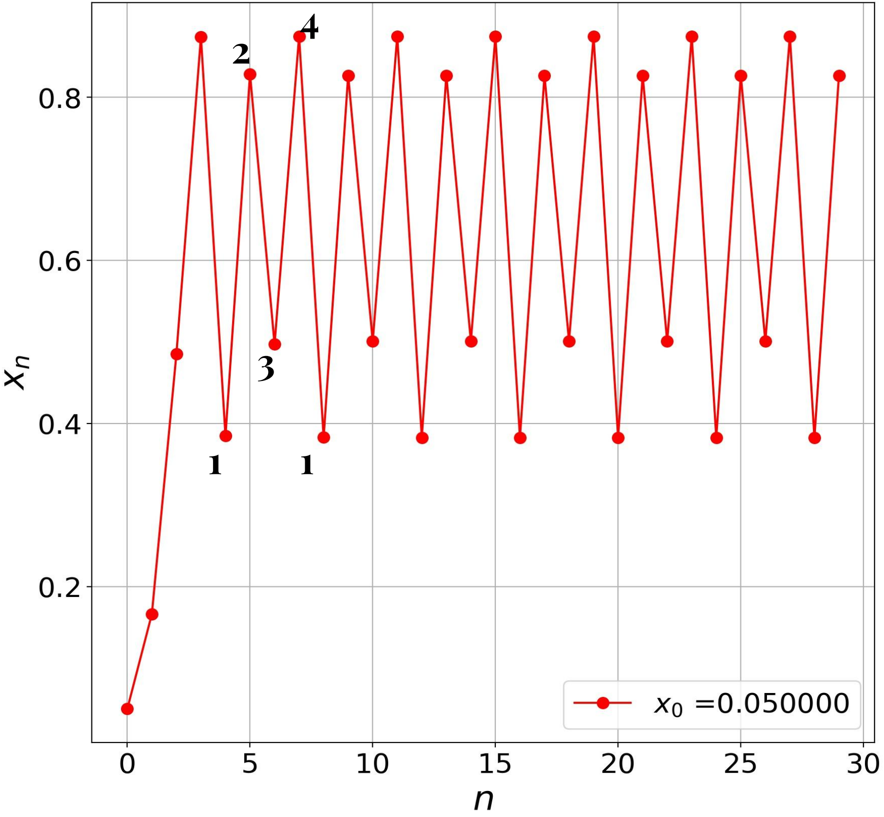


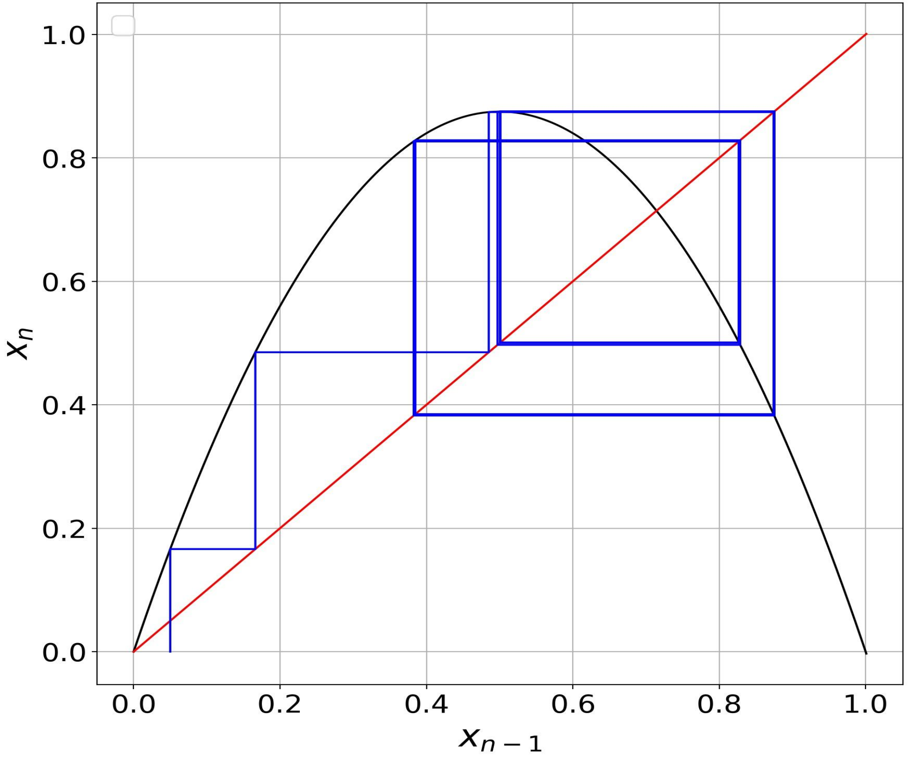
What happens when $3.44949 < a < 3.54409$?

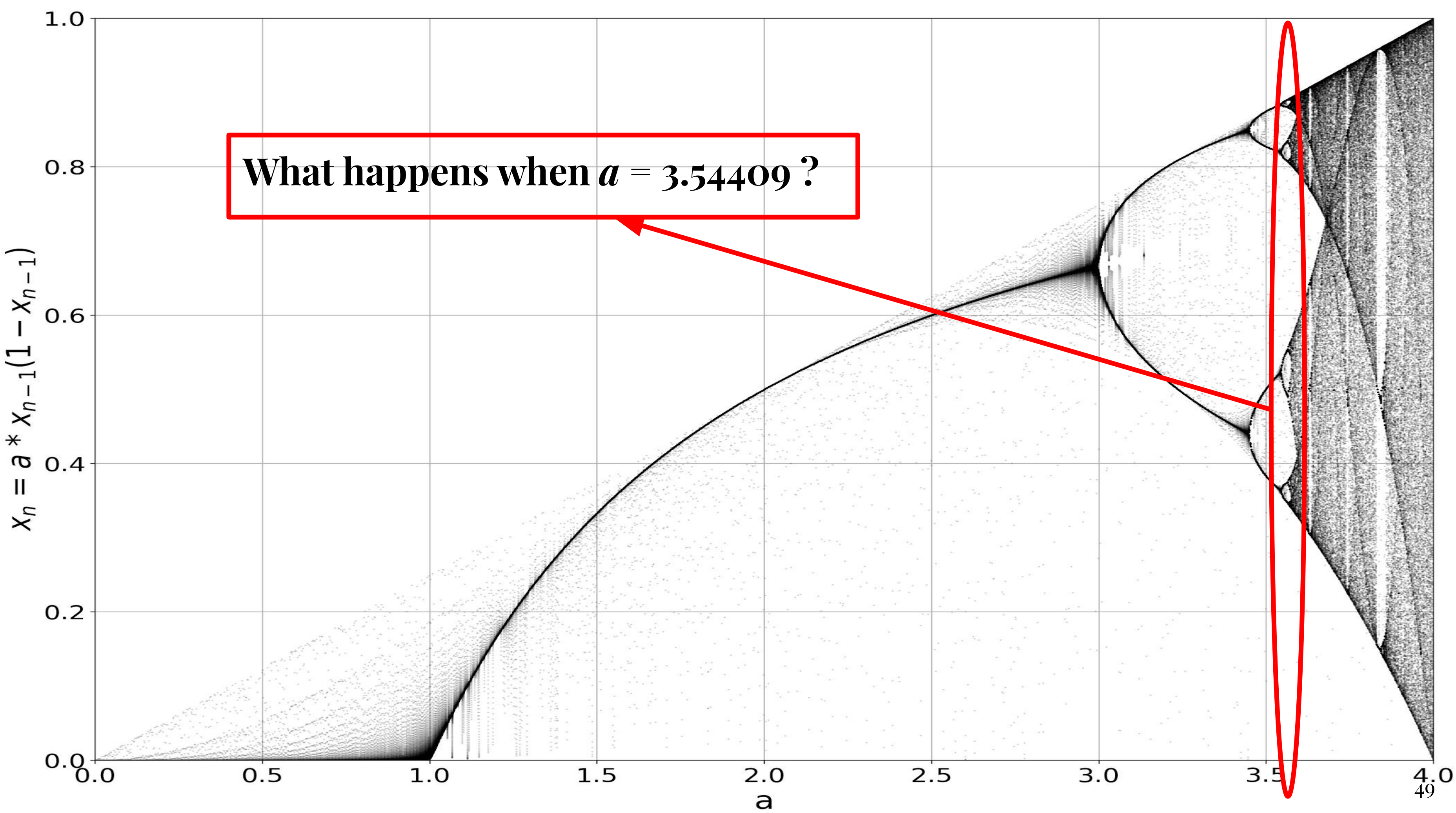
you will start noticing period 4

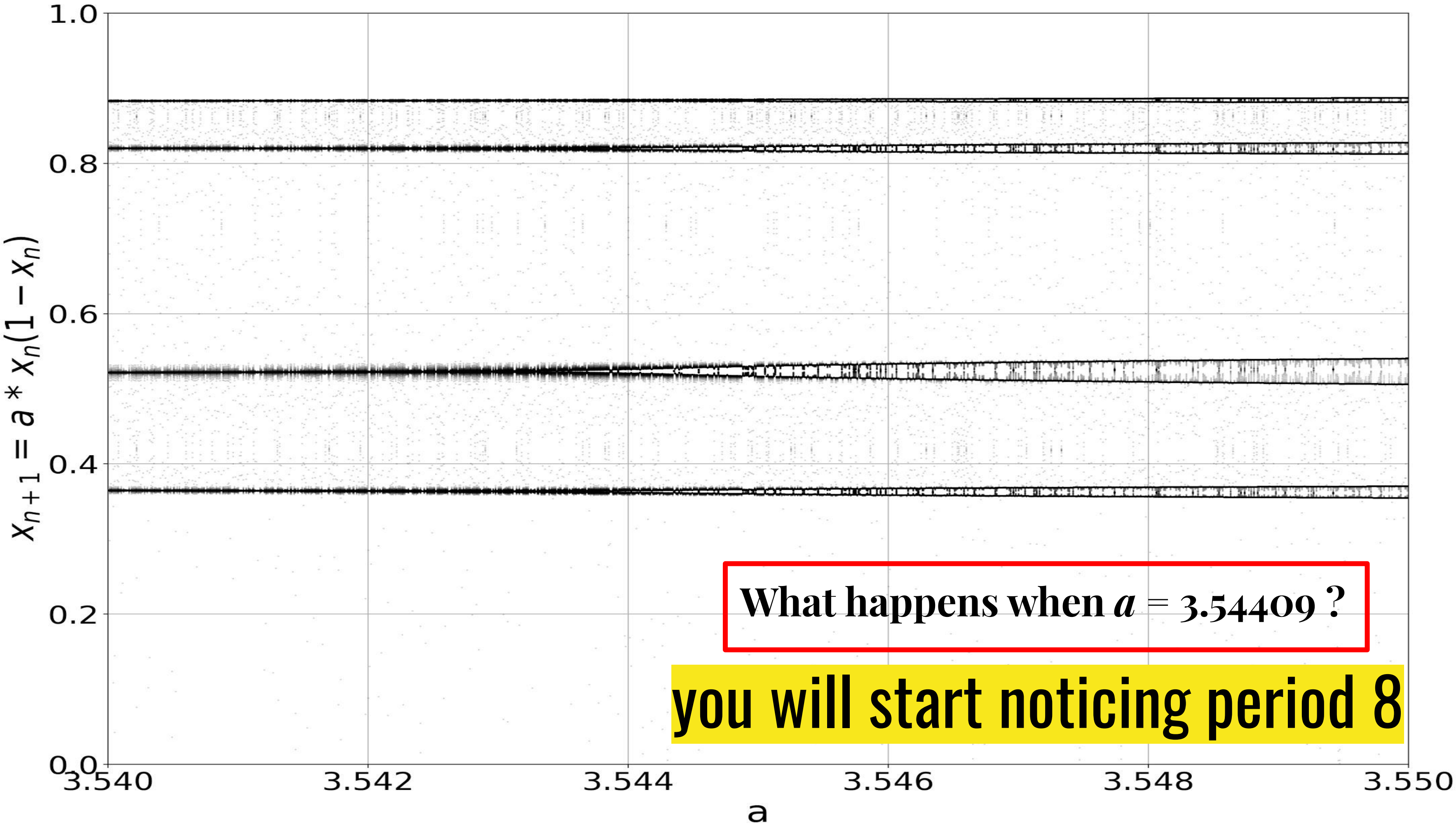
At a =3.5, we observe a period 4

$$x_n = 3.5x_{n-1}(1 - x_n)$$





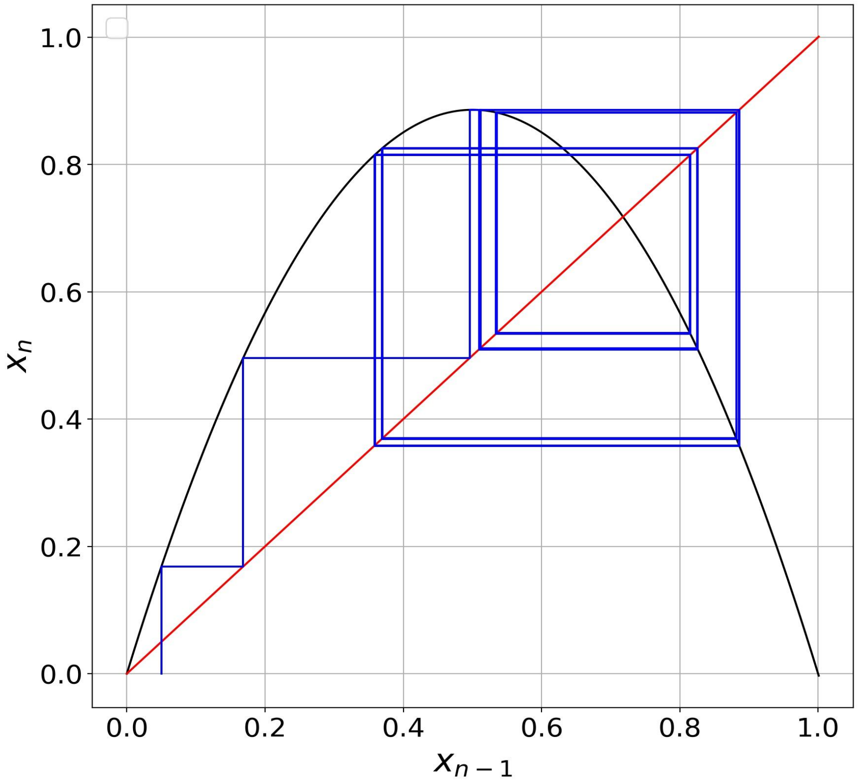

What happens when a = 3.54409 ?
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
a
$x_n = a * x_{n-1}(1 - x_{n-1})$

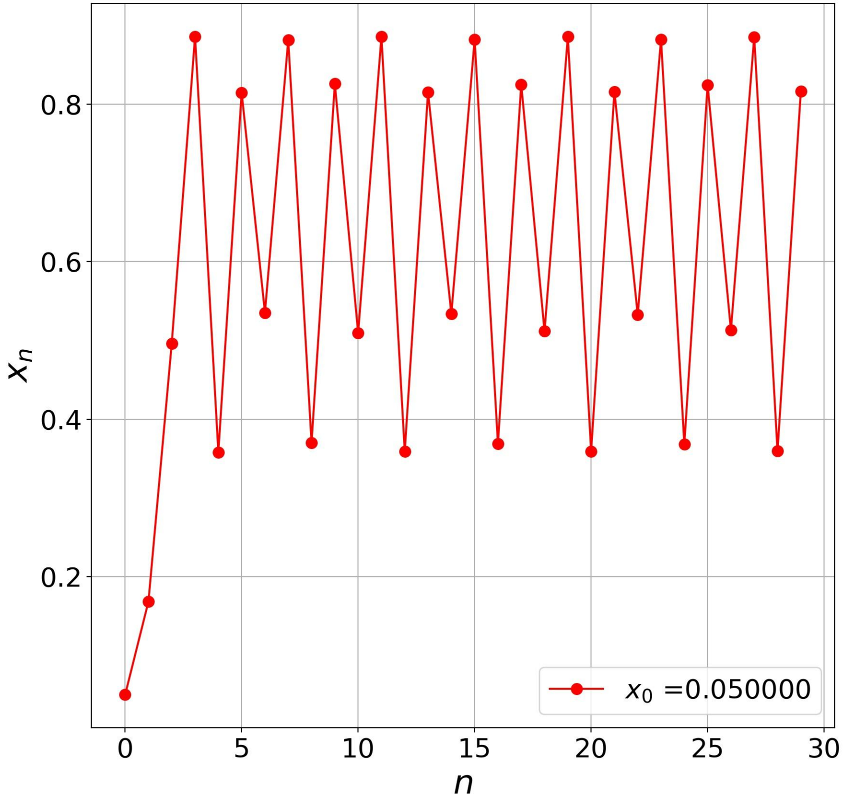


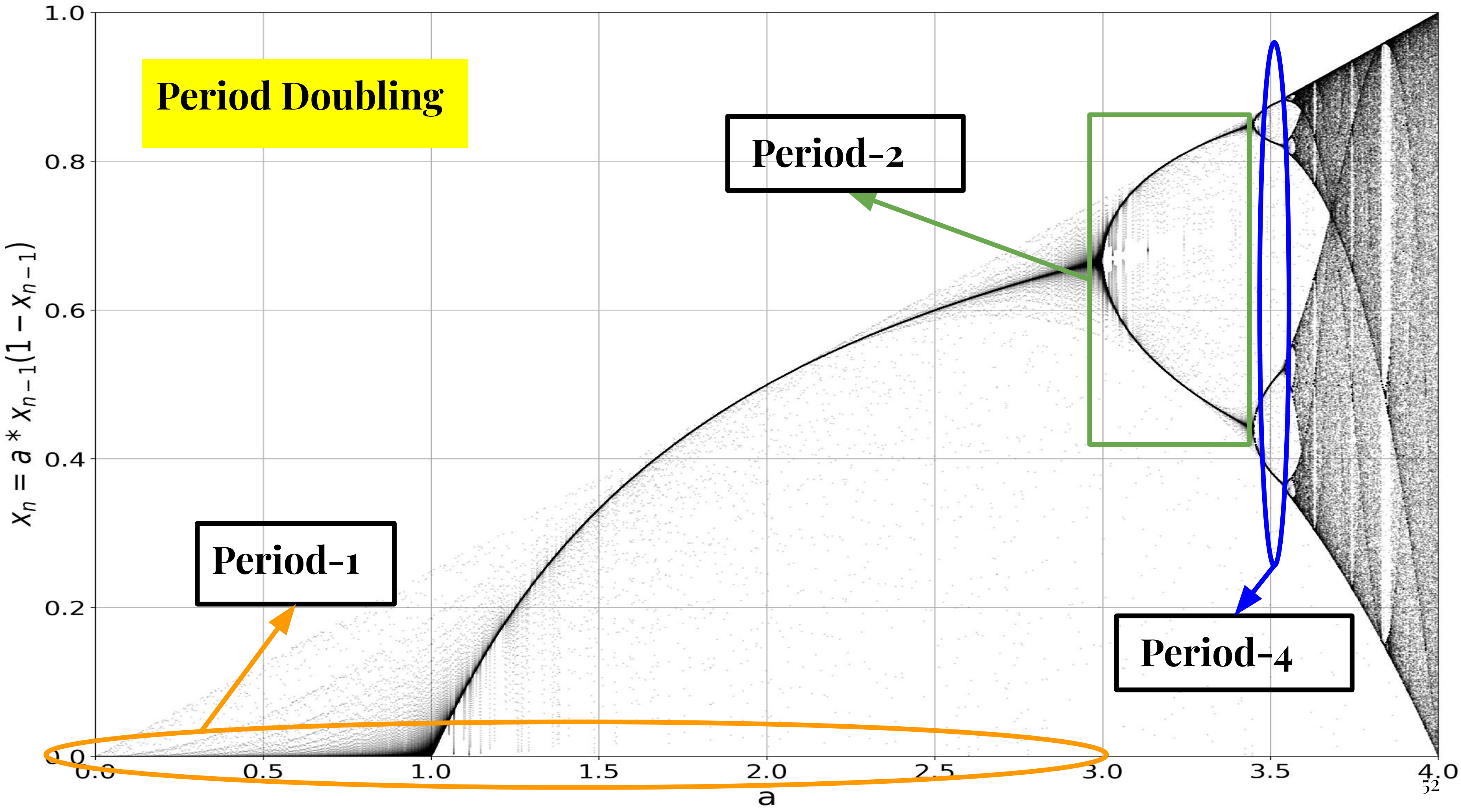
What happens when a = 3.54409 ?

you will start noticing period 8

a = 3.54409, start noticing period 8



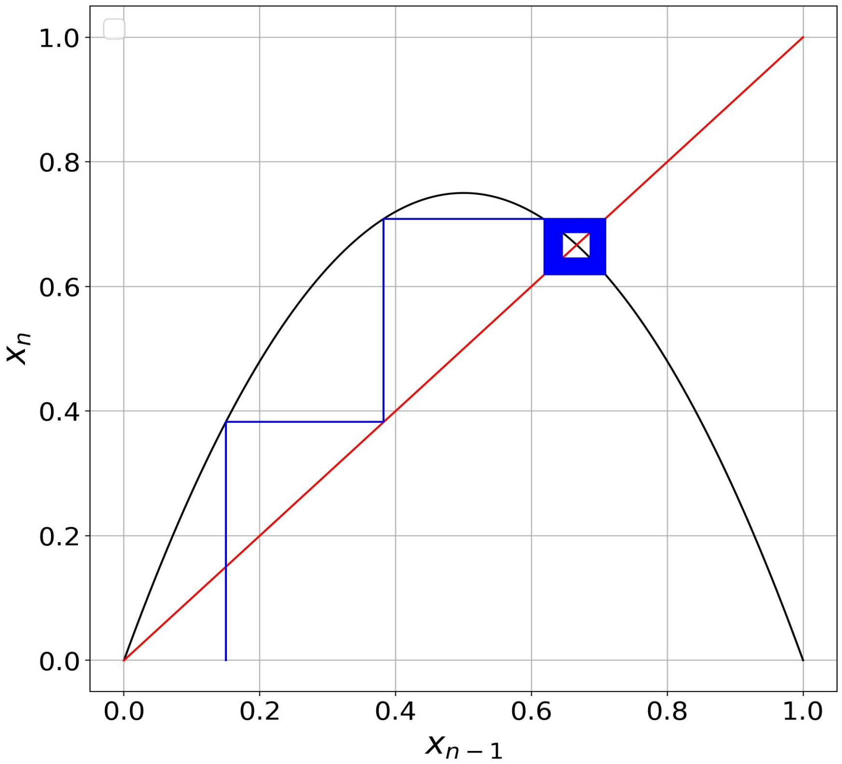


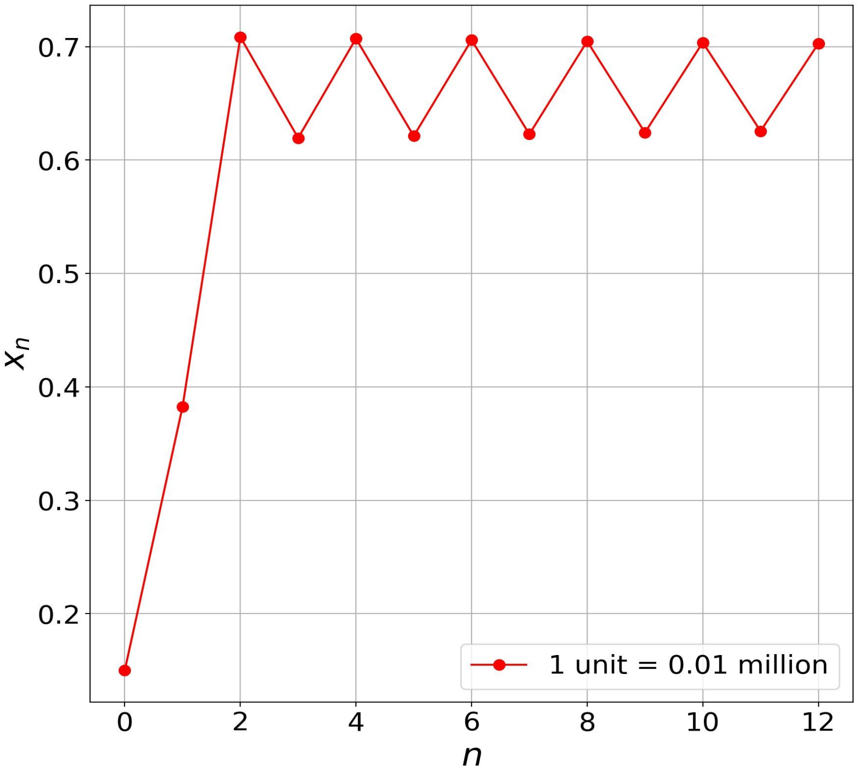

Period Doubling
Period-1
Period-2
Period-4
$x_n = a * x_{n-1}(1 - x_{n-1})$
a
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Condition for period doubling

Birth of period 2

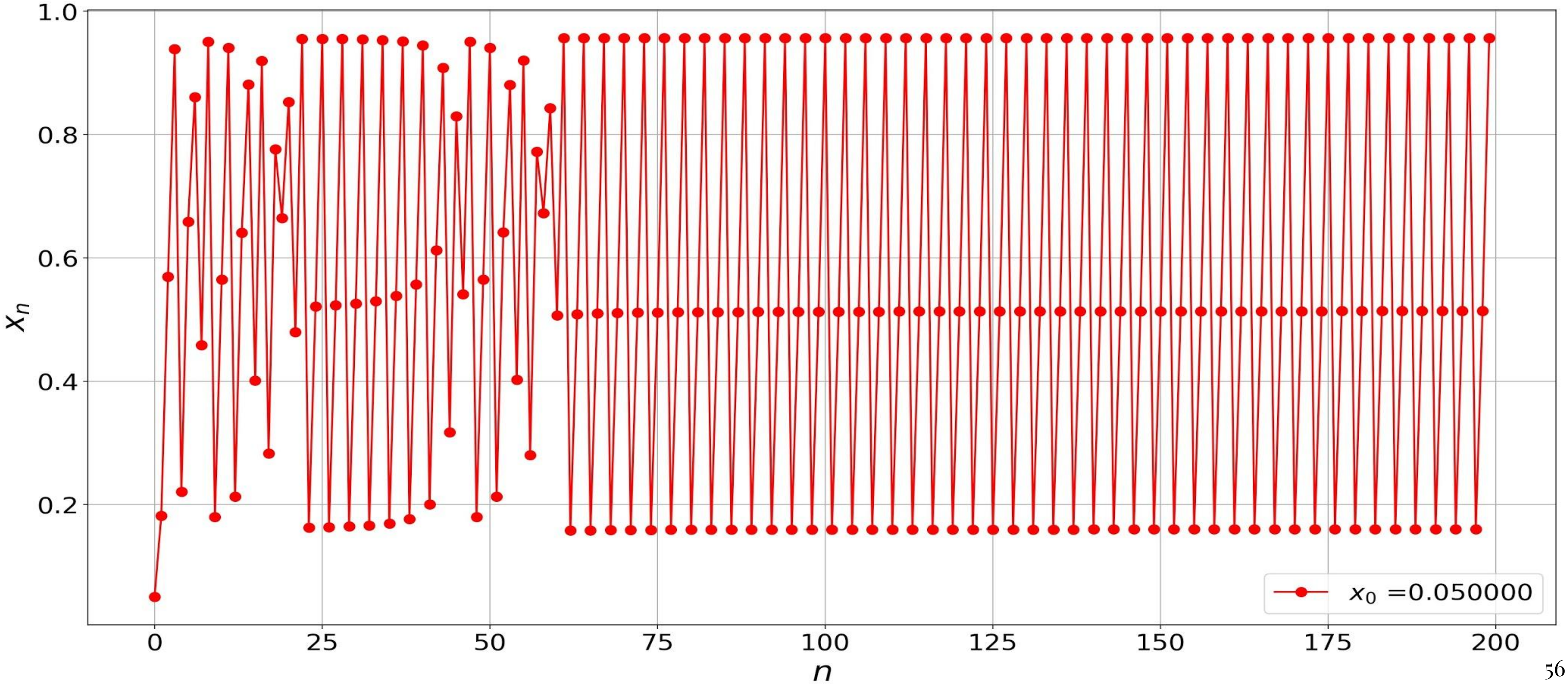
$$x_n = 3x_{n-1}(1 - x_n)$$





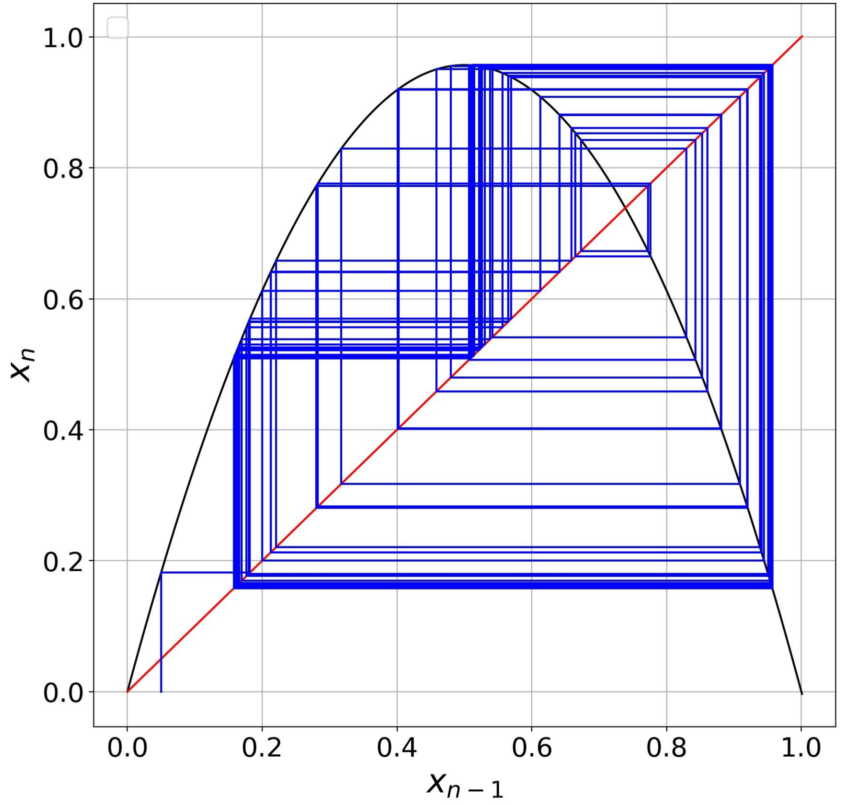
When is the birth of period 3?

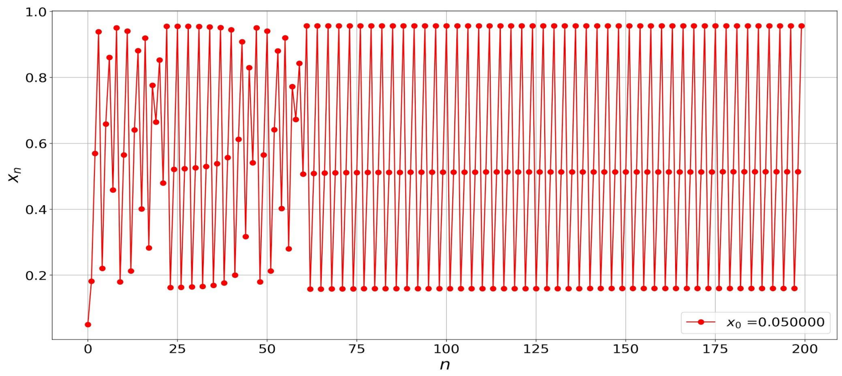
a =3.82842712475 $a = 1 + 2\sqrt{2}$

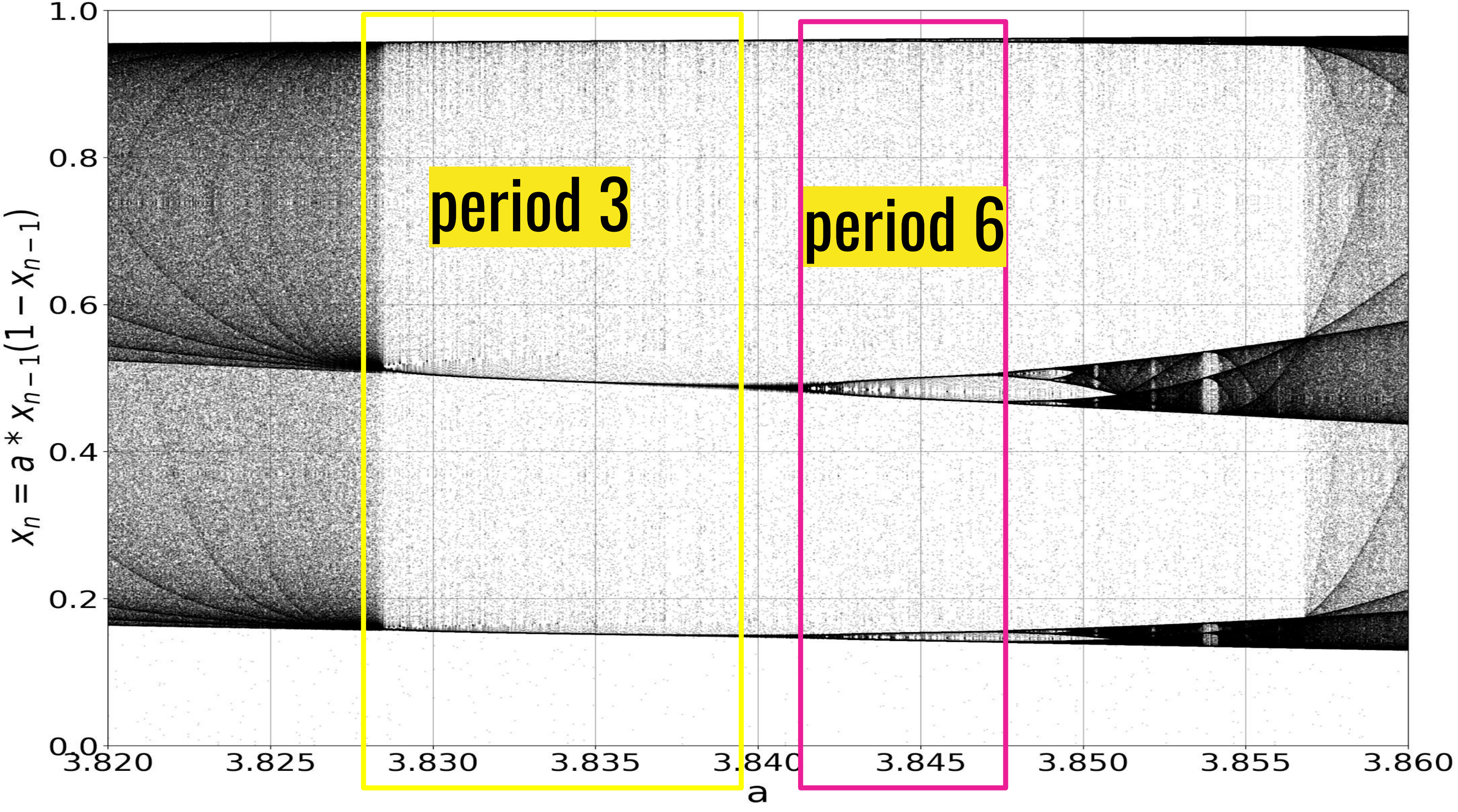


Cobweb plot (x_0 =0.05)

$$a = 1 + 2\sqrt{2}$$

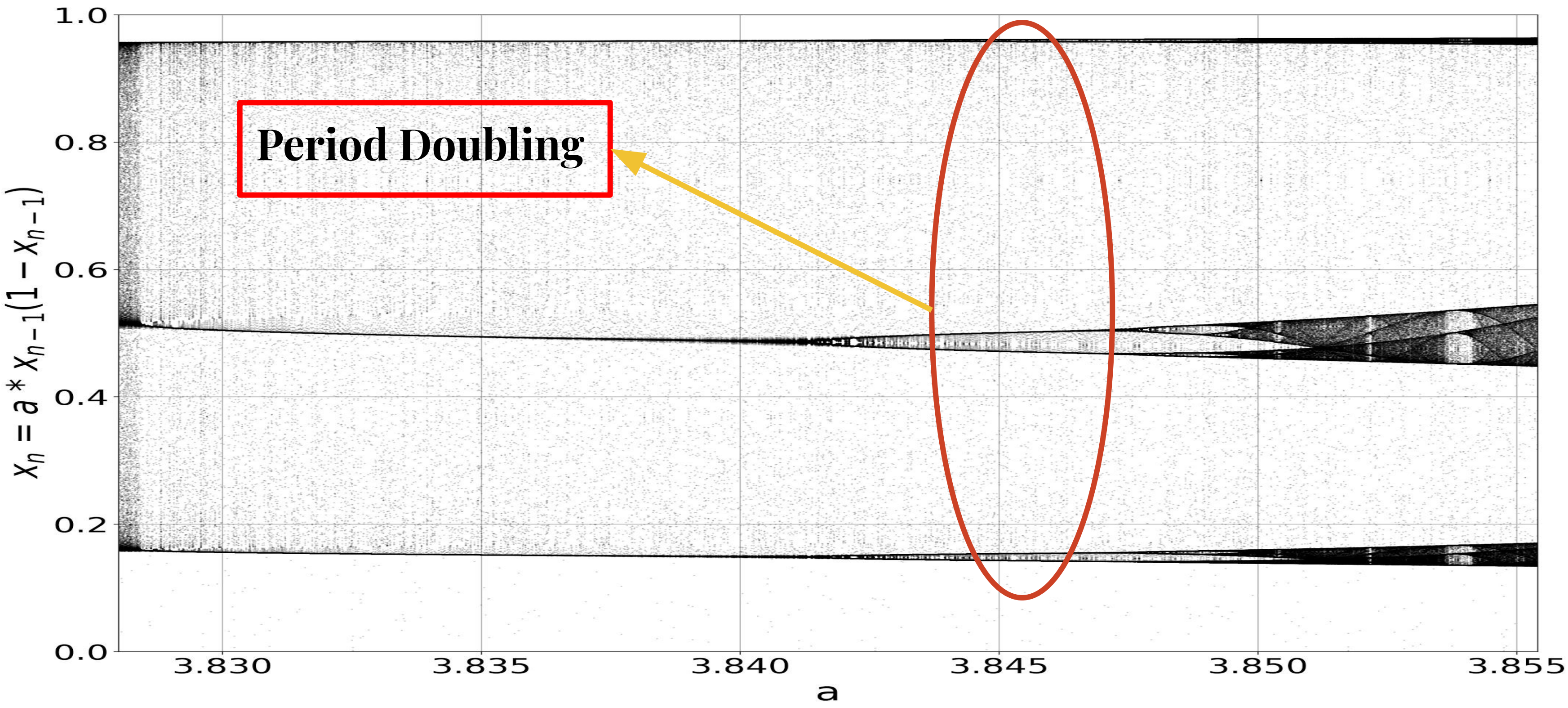




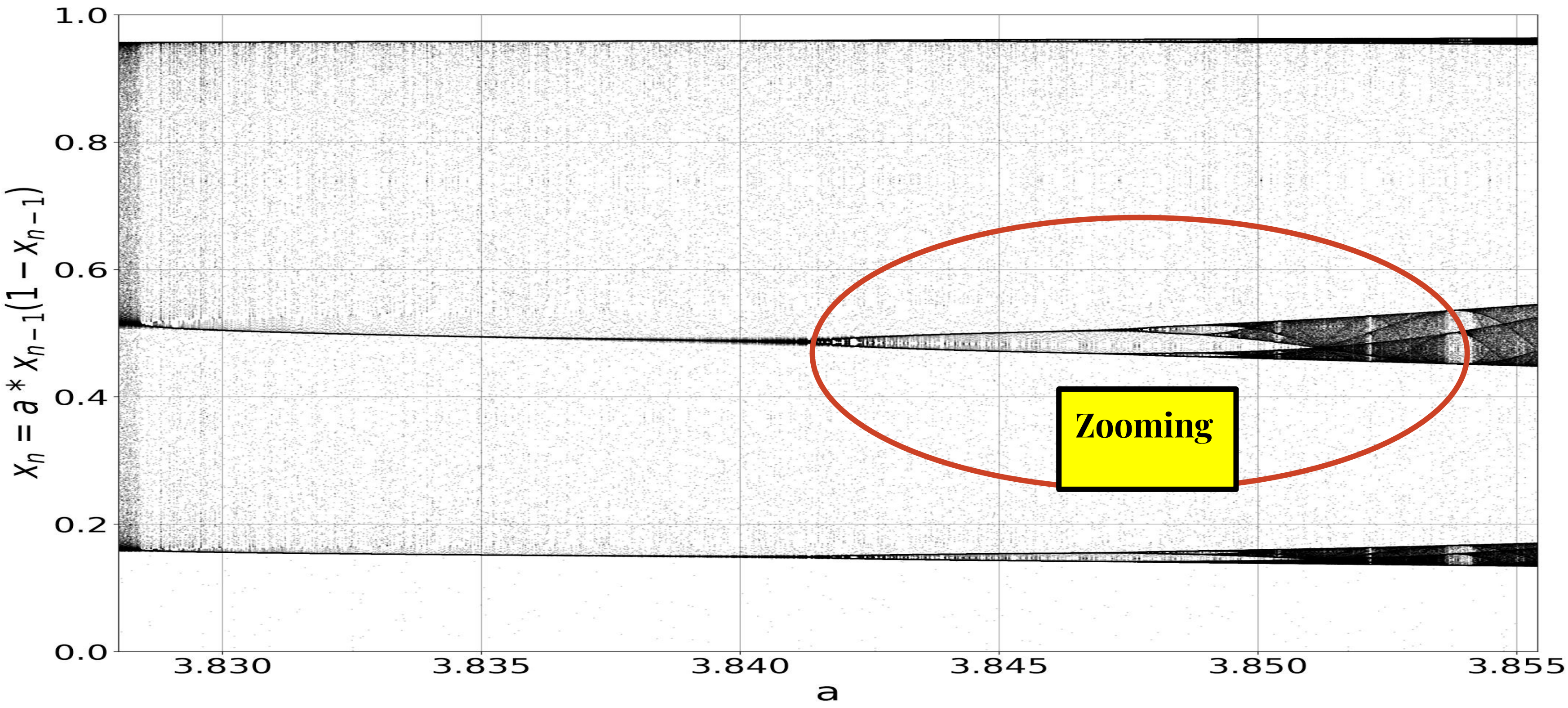


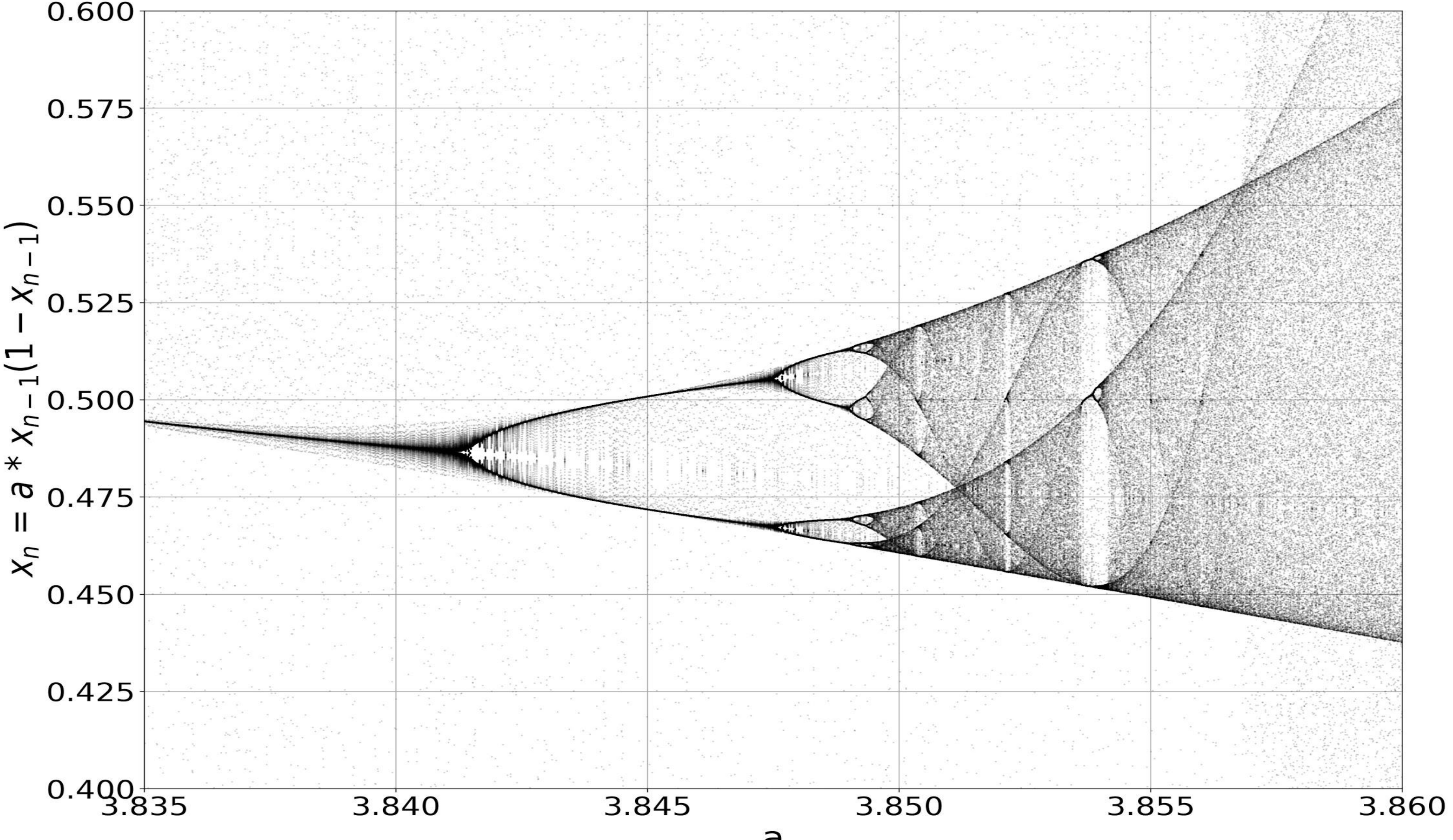
period 3
period 6
$x_n = a * x_{n-1}(1 - x_{n-1})$
1.0
0.8
0.6
0.4
0.2
0.0
3.820
3.825
3.830
3.835
3.840
3.845
3.850
3.855
3.860
a

Interesting Observation- Zooming period 3 window

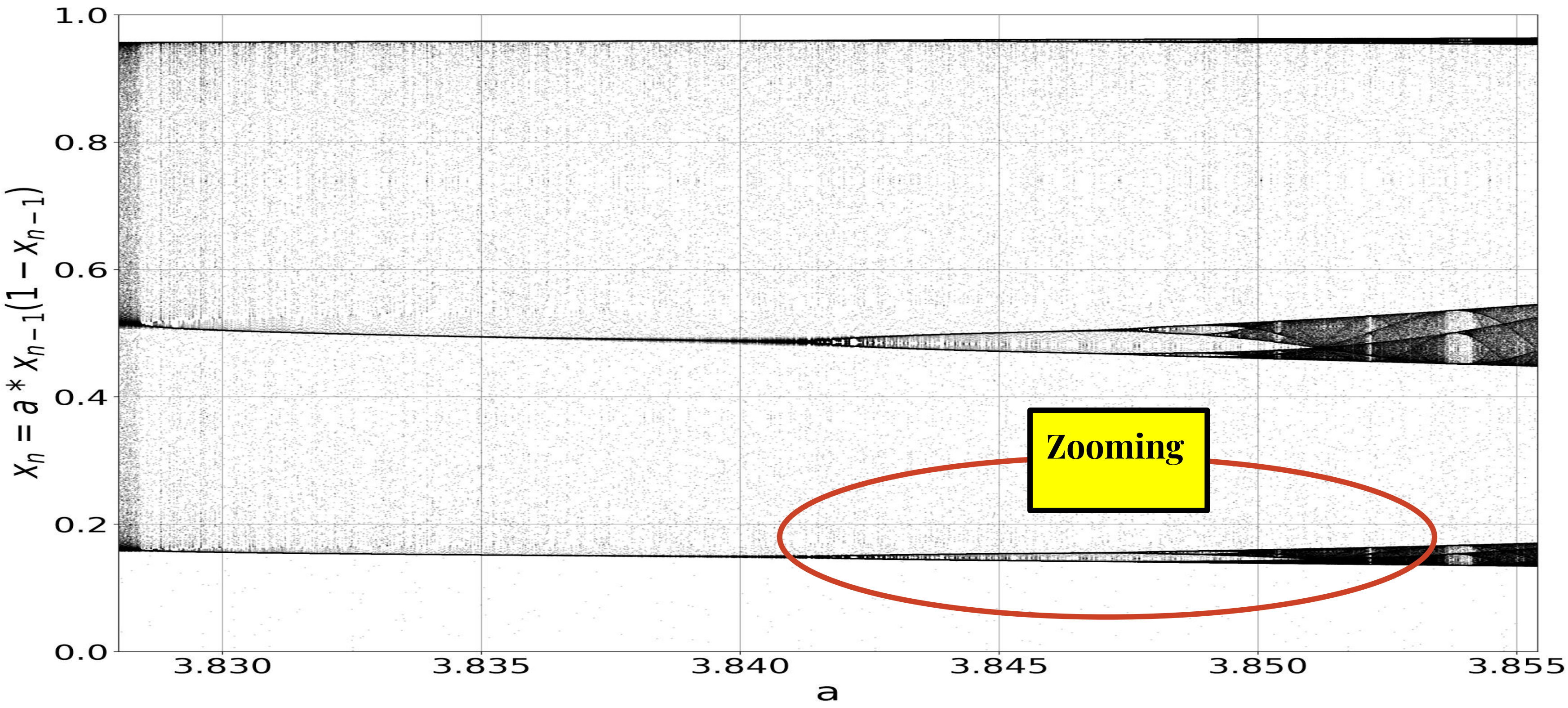


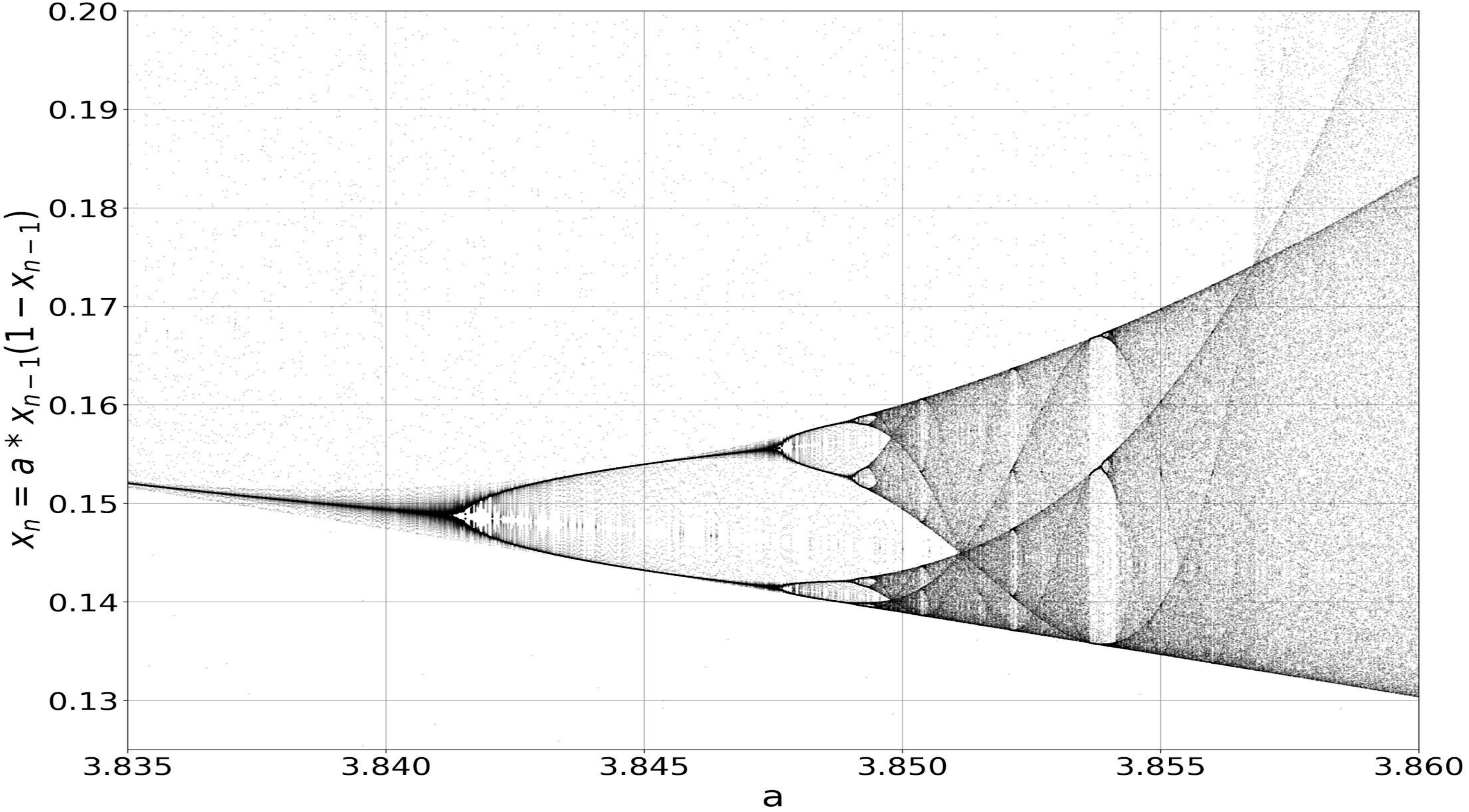
Zooming : to see the copy of the bifurcation diagram

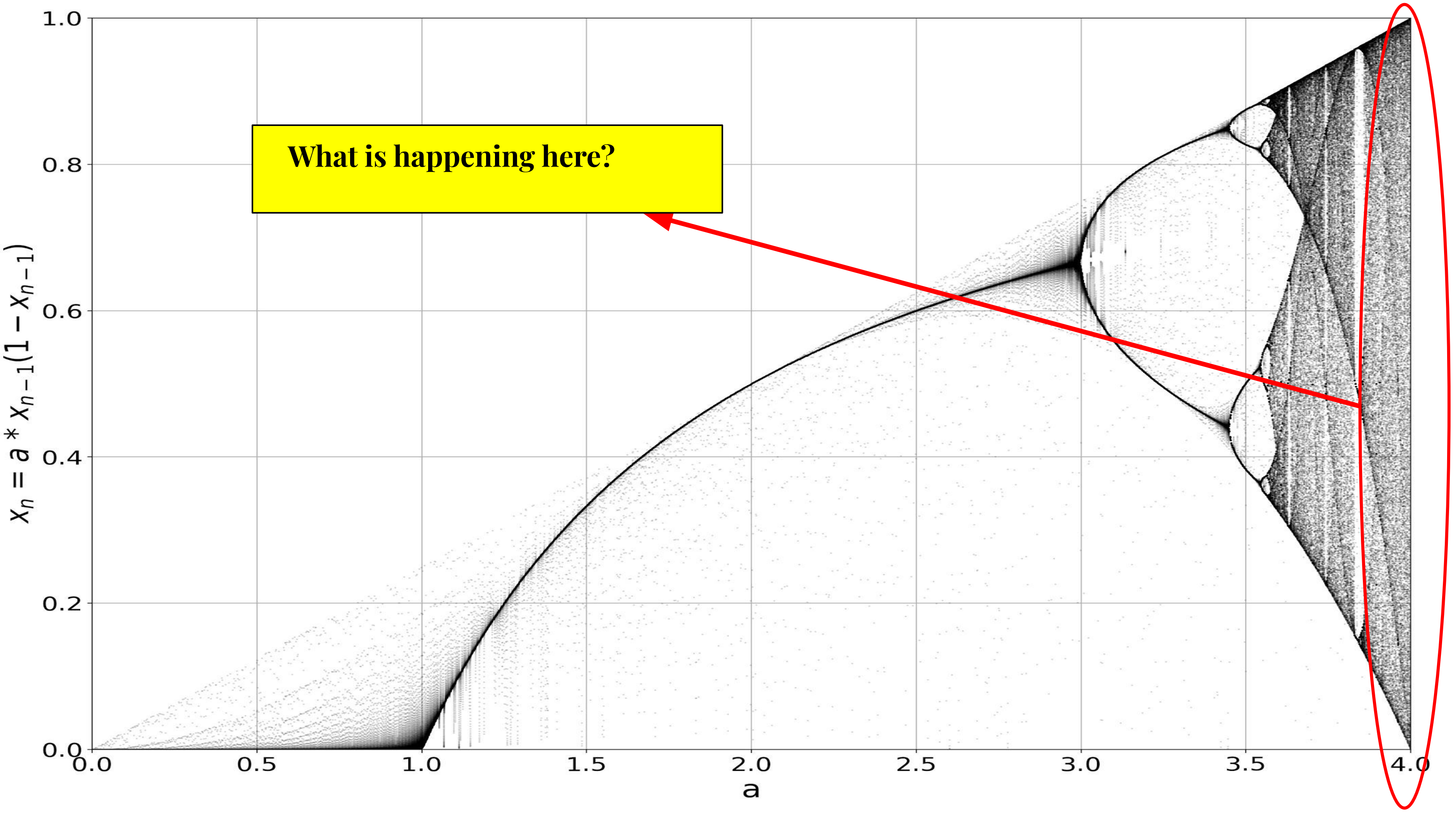



0.600
0.575
0.550
0.525
0.500
0.475
0.450
0.425
0.400
$x_n = a * x_{n-1}(1 - x_{n-1})$
3.835
3.840
3.845
3.850
3.855
3.860
a

What about the other two? They also have a copy of bigurcation diagram?




0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
$x_n = a * x_{n-1}(1 - x_{n-1})$
3.835
3.840
3.845
3.850
3.855
3.860
a

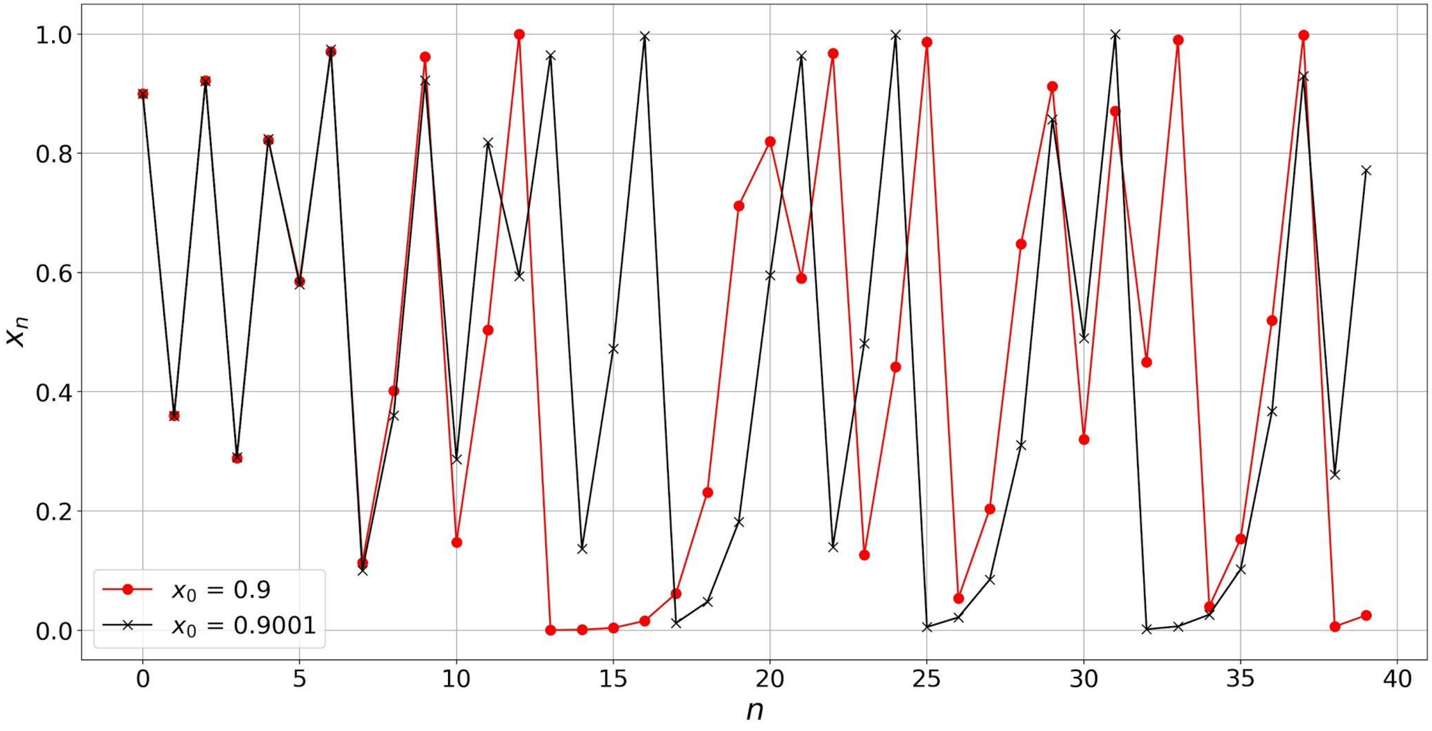


What is happening here?
$x_n = a * x_{n-1}(1 - x_{n-1})$
a
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

$$x_n = 4x_{n-1}(1 - x_{n-1})$$

Butterfly Effect: Sensitivity to initial condition

$$x_n = 4x_{n-1}(1 - x_{n-1})$$



Chaos

- Deterministic, yet unpredictable
- Bounded, non-linear, iterations
- Looks/feels like Randomness, but has rich structure and order
- Sensitive dependence on initial value (Butterfly Effect)
- Periodic, quasi-periodic and non-periodic solutions/trajectories
- Topological Transitivity

ChaosNet

A Novel Chaos Theory Inspired Neuronal Architecture

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Consciousness Studies Programme
National Institute of Advanced Studies
Indian Institute of Science Campus, Bengaluru, India
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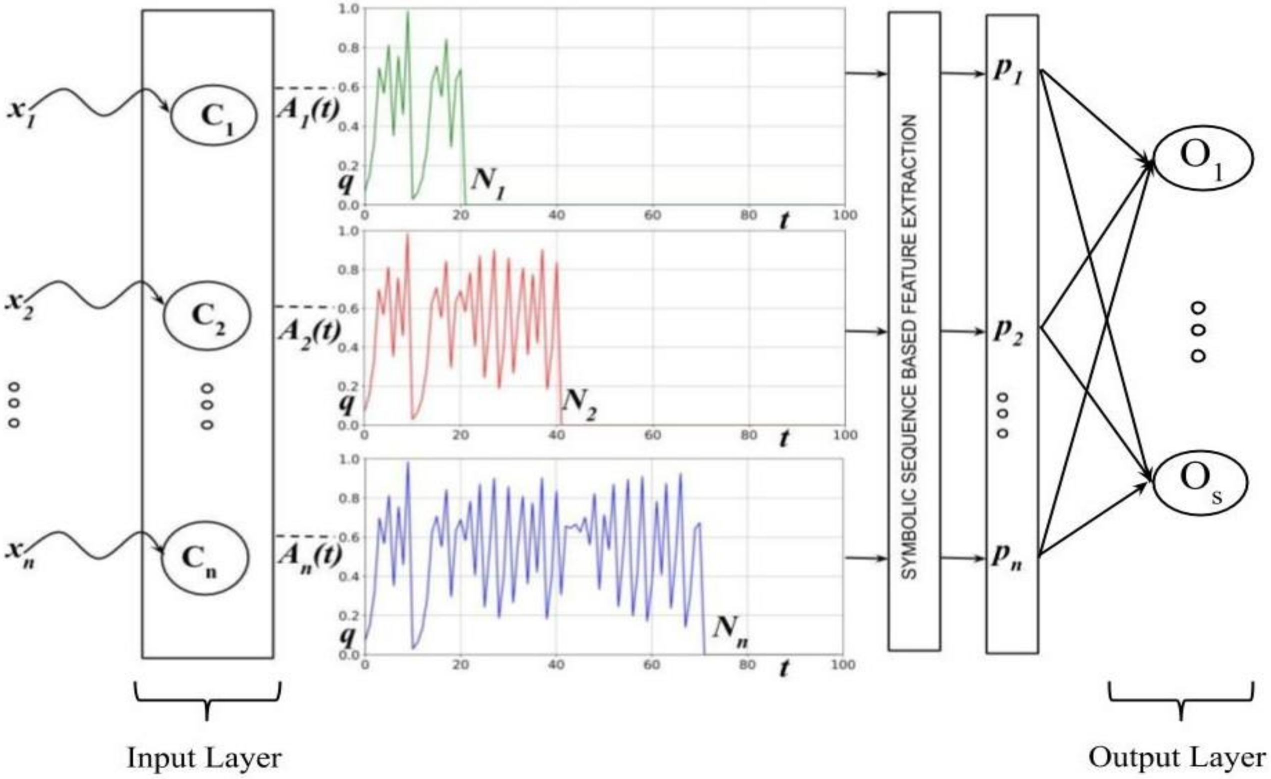
AIP Chaos: An Interdisciplinary Journal of Nonlinear Science

Published Online: 20 November 2019 Accepted: November 2019

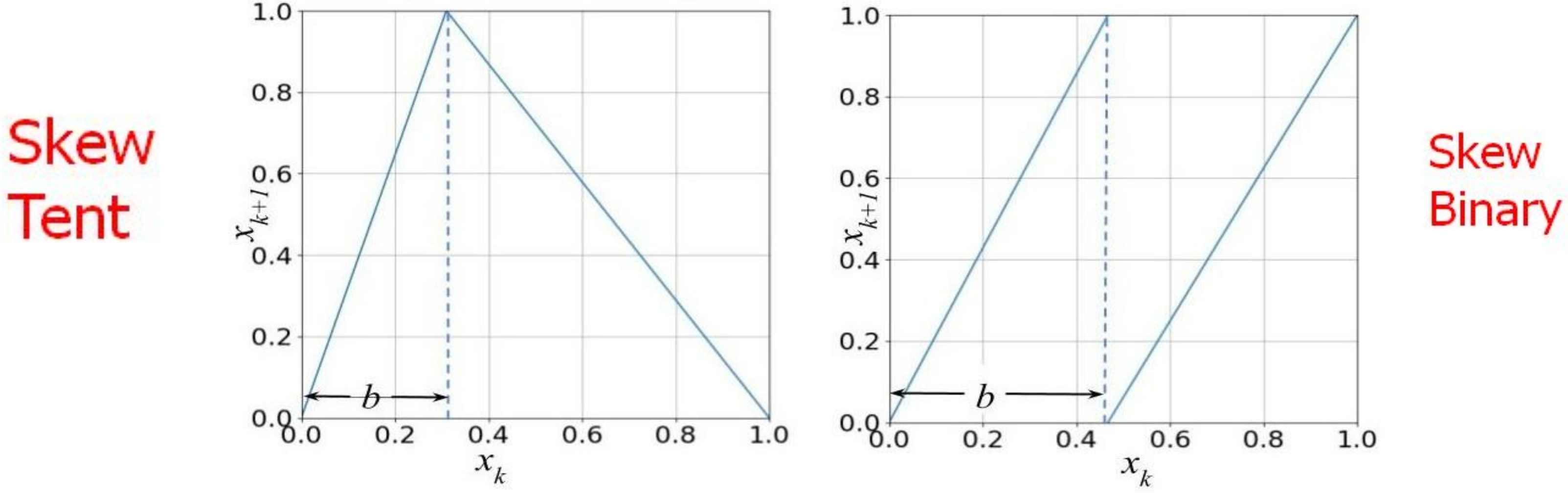
ChaosNet: A chaos based artificial neural network architecture for classification

Chaos **29**, 113125 (2019); https://doi.org/10.1063/1.5120831

ChaosNet: used in ML



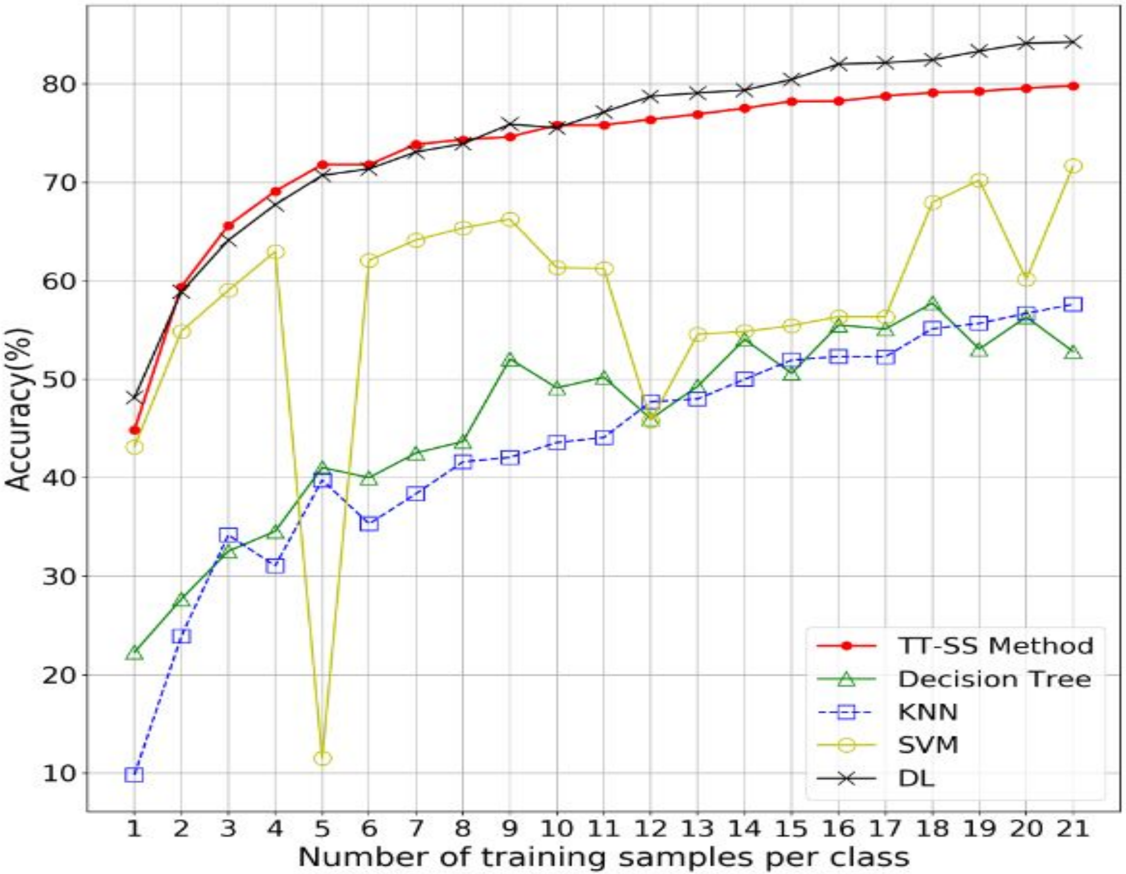
GLS maps and why?



- Simplest Chaotic map (piecewise linear)
- Universal Approximation theorem

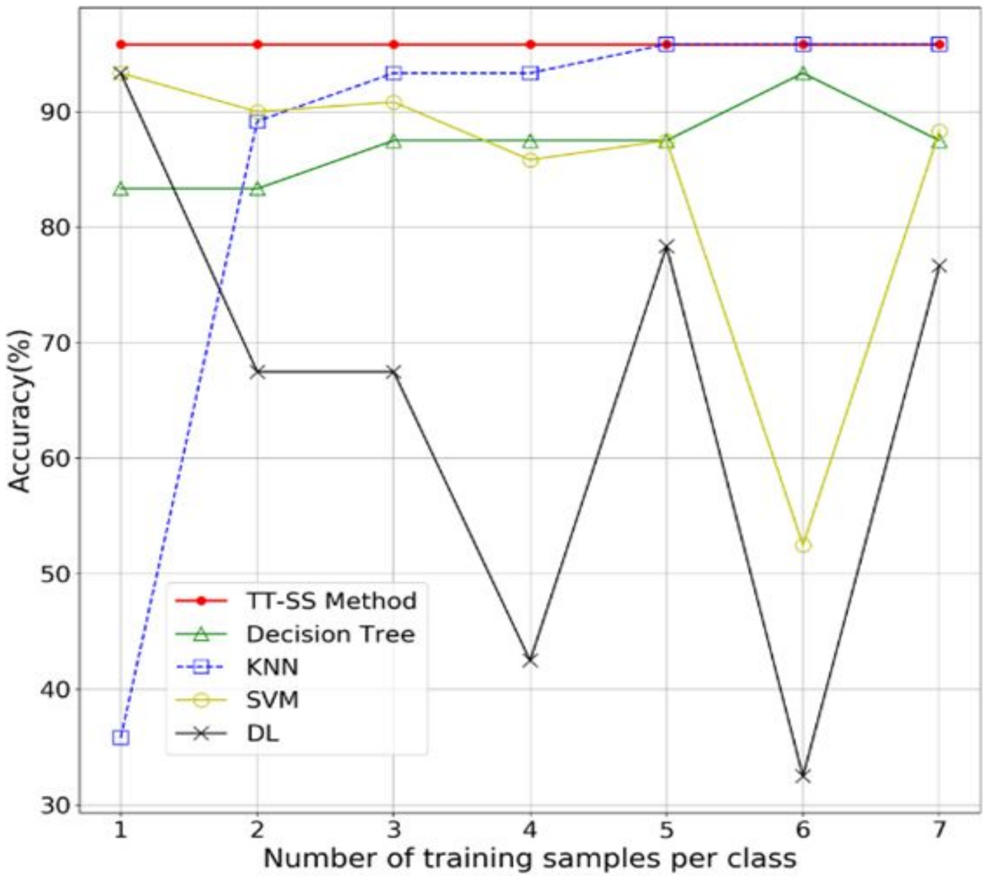
Topological Transitivity

MNIST



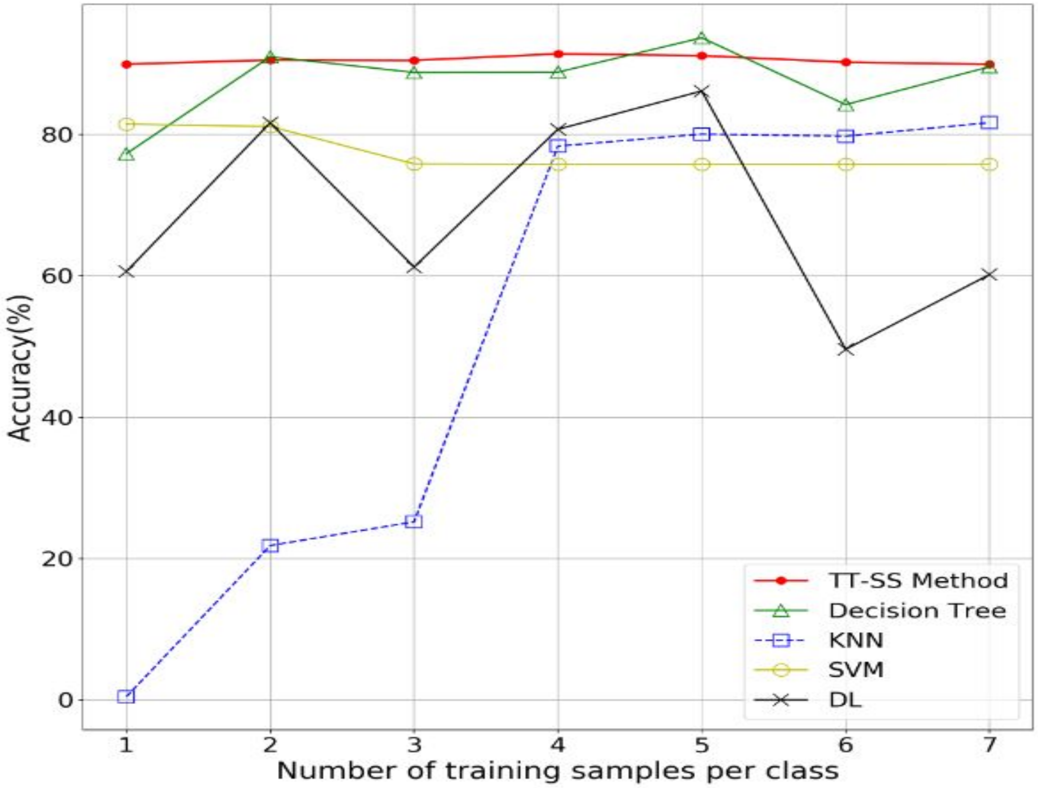
Accuracy(%)
Number of training samples per class
TT-SS Method
Decision Tree
KNN
SVM
DL
80
70
60
50
40
30
20
10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Iris



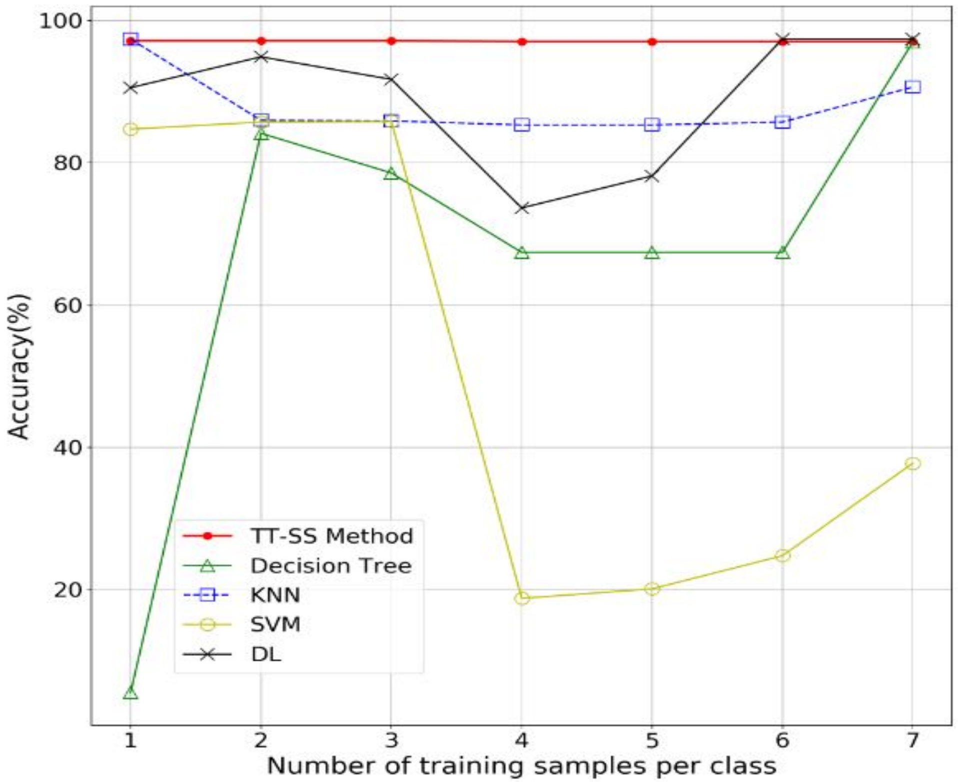
TT-SS Method
Decision Tree
KNN
SVM
DL
Accuracy(%)
Number of training samples per class
30
40
50
60
70
80
90
1
2
3
4
5
6
7

KDD Cup'99



Accuracy(%)
0
20
40
60
80
1
2
3
4
5
6
7
Number of training samples per class
TT-SS Method
Decision Tree
KNN
SVM
DL

Exoplanet



Accuracy(%)
100
80
60
40
20
1
2
3
4
5
6
7
Number of training samples per class
TT-SS Method
Decision Tree
KNN
SVM
DL

To Conclude

- **ChaosNet** – a neural net with chaotic neurons (GLS)
- GLS-neuron has excellent properties
- Accuracy: 73.89% to 98.33% with <0.05% of data for training
- No backprop., very few hyperparameters (x_o, b)

Going forward:

- How to handle noise? Low SNR processing (use Fractals)
- Further improve and test **ChaosNet**
- Use the rich properties of Neuro-Chaos for ML

Enrich AI with Non-linear Physics & Neuroscience

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(contd.)

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Thank You

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