DSP Assignment

R161598
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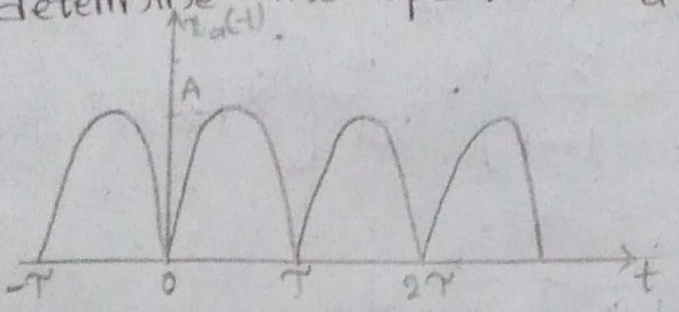
1 Consider the full wave rectified Sinusoid

a) determine the Spectrum $X_a(F)$.



$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi t/T} \qquad T = \tau$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi t/\tau}$$

$$c_k = \frac{1}{\tau}\int_0^{\tau} A\sin\left(\frac{\pi t}{\tau}\right) e^{-j2\pi kt/\tau}\, dt$$

$$= \frac{1}{\tau}\int_0^{\tau} A\cdot\left[\frac{e^{j\pi t/\tau} - e^{-\frac{j\pi t}{\tau}}}{2j}\right] e^{-j2\pi kt/\tau}\, dt$$

$$= \frac{A}{2j\tau}\int_0^{\tau}\left[\left(e^{j\pi t/\tau}\cdot e^{-j2\pi kt/\tau}\right) - \left(e^{-\frac{j\pi t}{\tau}}\cdot e^{-j2\pi kt/\tau}\right)\right] dt$$

$$= \frac{A}{2j\tau}\int_0^{\tau}\left(e^{j\pi(1-2k)t/\tau} - e^{-j\pi(1+2k)t/\tau}\right) dt$$

$$= \frac{A}{2j\tau}\int_0^{\tau} e^{j\pi(1-2k)t/\tau}\, dt - \frac{A}{2j\tau}\int_0^{\tau} e^{-j\pi(1+2k)t/\tau}\, dt$$

$$= \frac{A}{2j\tau}\left[\frac{e^{j\pi(1-2k)t/\tau}}{j\pi(1-2k)t/\tau}\right]_0^{\tau} - \frac{A}{2j\tau}\left[\frac{e^{-j\pi(1+2k)t/\tau}}{-j\pi(1+2k)t/\tau}\right]_0^{\tau}$$

$$= \frac{A}{2j\tau}\left[\frac{e^{j\pi(1-2k)} - 1}{j\pi(1-2k)t/\tau}\right] - \frac{A}{2j\tau}\left[\frac{e^{-j\pi(1+2k)} - 1}{+j\pi(1+2k)t/\tau}\right]$$

$$= \frac{A}{2jT} \cdot \frac{T}{j\pi}\left[\frac{e^{j\pi(1-2k)}-1}{(1-2k)} + \frac{e^{-j\pi(1+2k)}-1}{(1+2k)}\right]$$

$$= \frac{-A}{2\pi}\left[\frac{-1-1}{(1-2k)} + \frac{-1-1}{(1+2k)}\right]$$

$$= \frac{-A}{2\pi}\left[\frac{-2}{1-2k} - \frac{2}{1+2k}\right] = \frac{-A}{2\pi}\left[-2\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]\right]$$

$$= \frac{A}{\pi}\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]$$

$$= \frac{A}{\pi}\left[\frac{1+2k+1-2k}{(1-2k)(1+2k)}\right] = \frac{A}{\pi}\left[\frac{2}{1+2k-2k-4k^2}\right]$$

$$= \frac{2A}{\pi(1-4k^2)}$$

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi Ft}\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_k e^{j2\pi kF_0 t} \cdot e^{-j2\pi Ft}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} C_k e^{-j2\pi(Ft - kF_0 t)}\, dt \quad \because F_0 = \frac{1}{T} = \frac{1}{T}$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} C_k e^{-j2\pi\left(F-\frac{k}{T}\right) \cdot t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} C_k \int_{-\infty}^{\infty} e^{-j2\pi\left(F-\frac{k}{T}\right)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} C_k\, \delta\left(F - \frac{k}{T}\right)$$

b) Compute the Power of the Signal

$$P = \frac{1}{T}\int_0^T |x(t)|^2\,dt \qquad T = \tau$$

$$= \frac{1}{\tau}\int_0^\tau \left(A\sin\frac{\pi t}{\tau}\right)^2 dt$$

$$= \frac{1}{\tau}\int_0^\tau A^2 \sin^2\frac{\pi t}{\tau}\,dt$$

$$= \frac{1}{\tau}\int_0^\tau A^2\left[\frac{1-\cos\frac{2\pi t}{\tau}}{2}\right] dt$$

$$= \frac{A^2}{T}\int_0^\tau \left[\frac{1}{2} - \frac{\cos 2\left(\frac{\pi}{\tau}\right)t}{2}\right] dt$$

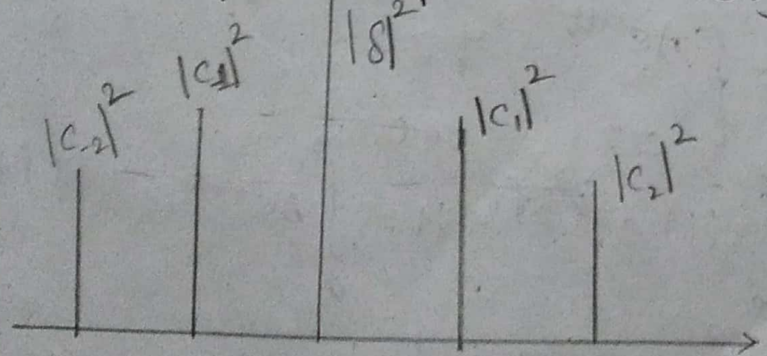
$$= \frac{A^2}{\tau}\int_0^\tau \frac{1}{2}\,dt - \frac{A^2}{\tau}\int_0^\tau \frac{\cos 2(\pi/\tau)t}{2}\,dt$$

$$= \frac{A^2}{\tau}\left[\frac{\tau}{2}\right] - \frac{A^2}{\tau}\left(\frac{\sin 2\pi}{4}\right)$$

$$= \frac{A^2}{\cancel{\tau}}\cdot\frac{\cancel{\tau}}{2} - 0$$

$$= \frac{A^2}{2}.$$

c) Plot the power spectral density.



d) Check the validity of Parseval's relation for the given signal.

$$P_x = \sum_{k=-\infty}^{\infty} |C_k|^2$$

$$C_k = \frac{2A}{\pi(1-4k^2)}$$

$$P_x = \sum_{k=-\infty}^{\infty} \left|\frac{2A}{\pi(1-4k^2)}\right|^2$$

$$= \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[\frac{1}{(4k^2-1)^2}\right] + 2\sum_{k=1}^{\infty}\left(\frac{1}{(4k^2-1)^2}\right)$$

$$= \frac{4A^2}{\pi^2}\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \frac{2}{35^2} + \cdots\right]$$

$$= \frac{4A^2}{\pi^2}\left[1.041\right]$$

$$= A^2\left[\frac{1.041 \times 4}{\pi^2}\right]$$

$$= \frac{A^2\left[4.164\right]}{\pi^2} = 0.422A^2$$

2 Compute and Sketch the magnitude and phase spectra for the following Signals $(a>0)$

a) $x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0 & t<0 \end{cases}$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = \int_{0}^{\infty} Ae^{-at} \cdot e^{-j2\pi ft}\, dt$$

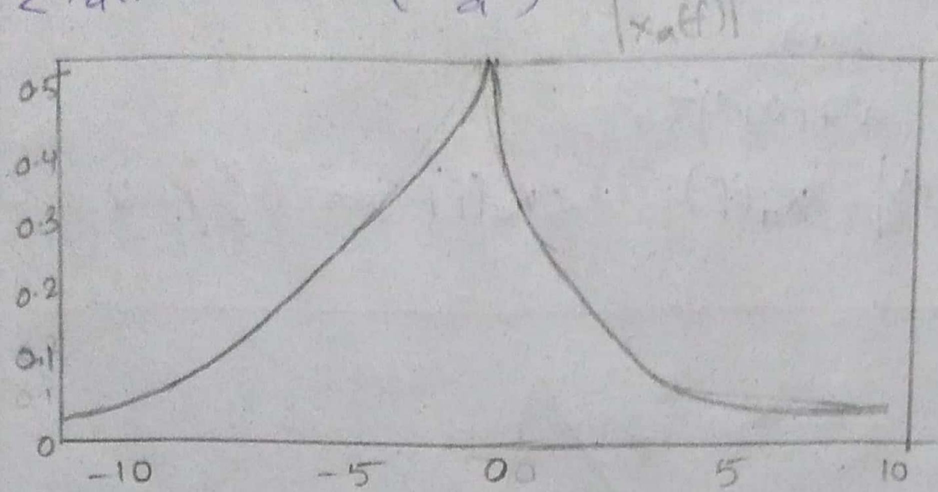
$$= A\int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt$$

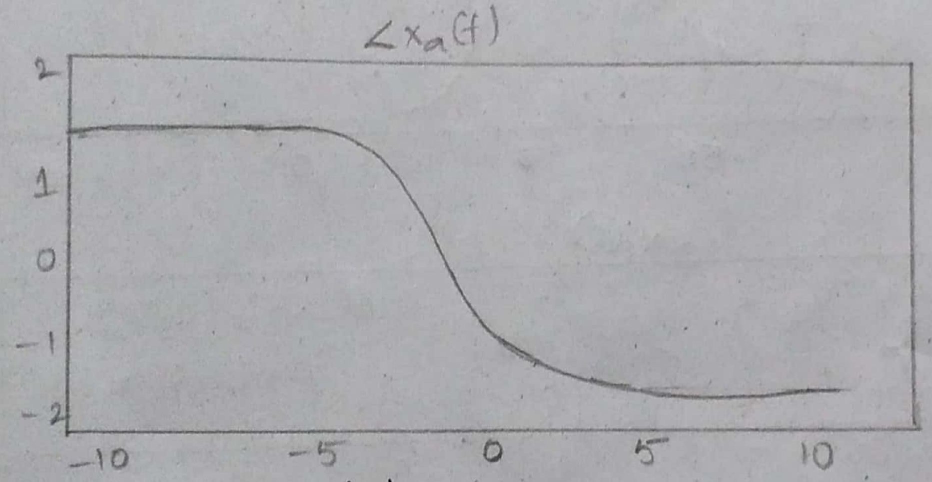
$$= A\left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)}\right]_0^{\infty}$$

$$= A \cdot \frac{1}{a + j2\pi f} = \frac{A}{a + j2\pi f}$$

$$|x_a(f)| = \frac{A}{\sqrt{a^2 + (2\pi f)^2}} = \frac{A}{\sqrt{a^2 + 4\pi^2 f^2}}$$

$$\angle x_a(f) = -\tan^{-1}\left(\frac{2\pi f}{a}\right)$$





b) $x_a(t) = Ae^{-a|t|}$

$$x(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} Ae^{at} \cdot e^{-j2\pi ft}\, dt + \int_{0}^{\infty} Ae^{-at} \cdot e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{0} Ae^{-(j2\pi f - a)t}\, dt + \int_{0}^{\infty} Ae^{-(a+j2\pi f)t}\, dt$$

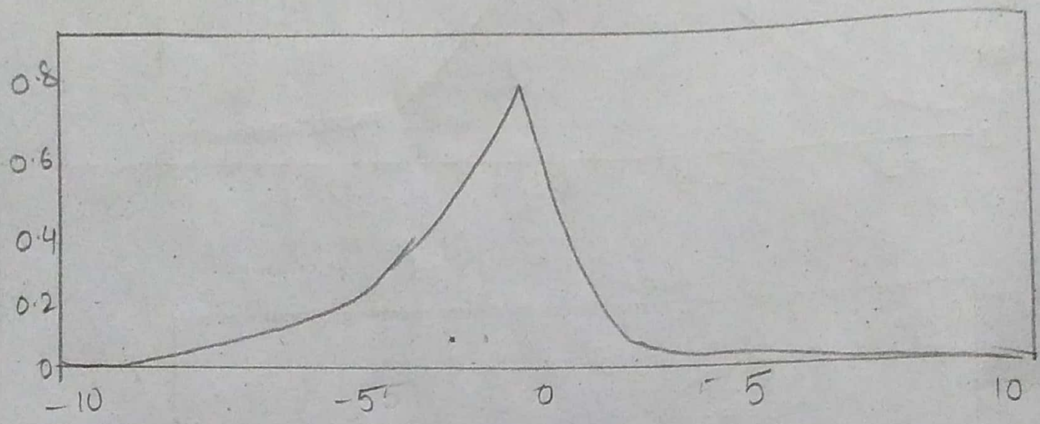
$$= A\left[\frac{e^{-(j2\pi f - a)t}}{-(-a + j2\pi f)}\right]_{-\infty}^{0} + A\left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)}\right]_{0}^{\infty}$$

$$= A\left[\frac{1}{a-j2\pi f}\right] + A\left[\frac{1}{a+j2\pi f}\right]$$

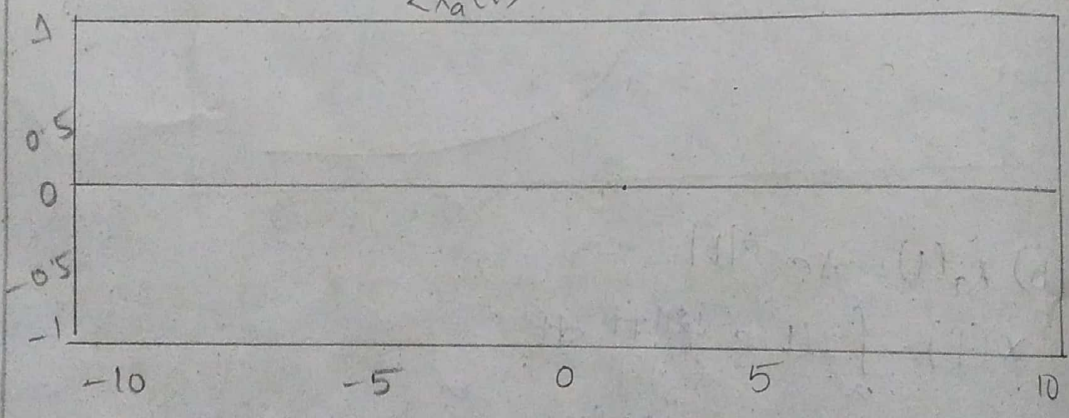
$$= \frac{Aa + Aj2\pi f + Aa - Aj2\pi f}{a^2 - (j2\pi f)^2}$$

$$= \frac{2aA}{a^2+(2\pi f)^2}$$

$|x_a(f)| = x_a(f)$ $\quad \angle x_a(f) = \tan^{-1}\left(\frac{0}{2}\right) = 0$



$\angle x_a(t)$



3 Consider the Signal $x(t) = \begin{cases} 1 - \frac{|t|}{\tau} & , |t| \leq \tau \\ 0 & , \text{elsewhere} \end{cases}$

a) Determine and sketch the Magnitude and Phase Spectra, $|x_a(f)|$ and $\angle x_a(f)$

$$X(f) = \int_{-\tau}^{0} x(t)e^{-j2\pi ft}\,dt + \int_{0}^{\tau} x(t)e^{-j2\pi ft}\,dt$$

$$= \int_{-\tau}^{0}\left(1+\frac{t}{\tau}\right)e^{-j2\pi ft}\,dt + \int_{\tau}^{0}\left(1-\frac{t}{\tau}\right)e^{-j2\pi ft}\,dt$$

Fourier transform $y(t) = x'(t) = \begin{cases} \frac{1}{\tau} & , -T < t \leq 0 \\ \frac{-1}{\tau} & , 0 < t \leq T \end{cases}$

$$Y(f) = \int_{-\tau}^{0}\frac{1}{\tau}e^{-j2\pi ft}\,dt + \int_{0}^{\tau}\frac{-1}{\tau}e^{-j2\pi ft}\,dt$$

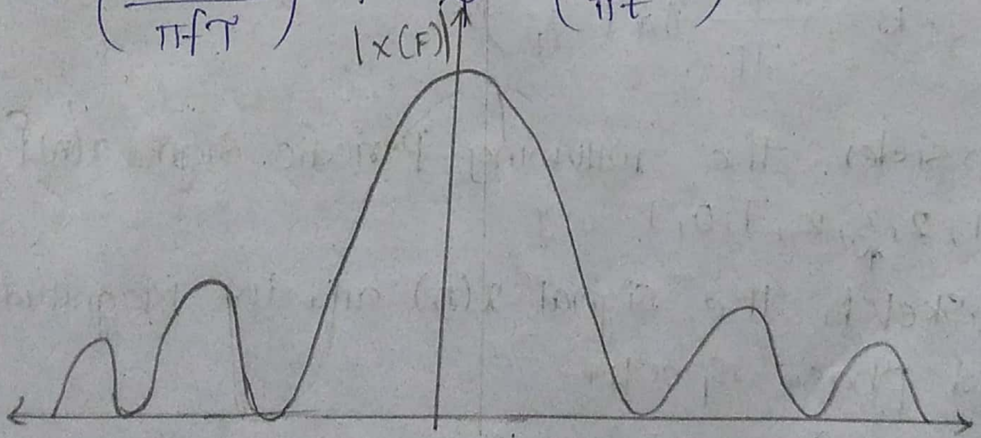
$$= -2\,\frac{\sin^2 \pi ft}{j\pi ft}$$

$$X(F) = \frac{1}{j2\pi f}Y(f) = \tau\left(\frac{\sin \pi f\tau}{\pi f\tau}\right)^2$$

$$|X(F)| = \tau\left(\frac{\sin \pi f\tau}{\pi f\tau}\right)^2$$

$$\angle X(F) = 0$$

$$\tau\left(\frac{\sin \pi f\tau}{\pi f\tau}\right)^2 \quad f = \frac{1}{\tau} \Rightarrow \left(\frac{\sin \pi t}{\pi t}\right)^2 = \text{sinc}^2(t)$$



b) Create a Periodic Signal $x_p(t)$ with fundamental Period $T_P \geq 2T$, So that $x(t) = x_p(t)$ for $|t| < T_P/2$. what are the fourier coefficients C_k for the Signal $x_p(t)$?

$$C_k = \frac{1}{TP}\int_{-TP/2}^{TP/2} x(t)e^{-j2\pi ft/TP}.dt$$

$$= \frac{1}{TP}\int_{-\tau}^{0}\left(1+\frac{t}{\tau}\right)e^{-j2\pi kt/TP}dt + \int_{0}^{\tau}\left(1-\frac{t}{\tau}\right)e^{-j2\pi kt/TP}dt$$

$$= \frac{\tau}{TP}\left[\frac{\sin\pi k\tau/TP}{\pi k\tau/TP}\right]^2$$

c) using the results in Part a & b show that

$$C_k = \left(\frac{1}{TP}\right)x_a\left(\frac{k}{TP}\right)$$

$$= \frac{1}{TP}x_a\left(\frac{k}{TP}\right)$$

$$= \frac{1}{TP}.\tau\left(\frac{\sin\pi\frac{k}{TP}\tau}{\pi\frac{k}{TP}\tau}\right)^2$$

$$= \frac{\tau}{TP}\left(\frac{\sin\pi k\tau/TP}{\pi k\tau/TP}\right)^2 \Rightarrow C_k$$

$$C_k = \frac{1}{TP}x_a\left(\frac{k}{TP}\right)$$

4 Consider the following Periodic Signal $x(n)\{\ldots 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1 \ldots\}$

a) sketch the signal $x(n)$ and its Magnitude and phase Spectra.

$$x(n) = \{\ldots 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots\}$$

$$\underbrace{1\ 0\ 1\ 2\ \underset{\uparrow}{3}\ 2\ 1\ 0\ 1}$$

$N=6$

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{6}}$$

for $n=0 \rightarrow x(0) \quad e^{-\frac{j2\pi k(0)}{6}} = 3\times1 = 3$

$n=1 \rightarrow x(1) \quad e^{-\frac{j2\pi k(1)}{6}} = 2e^{-\frac{j2\pi k(1)}{6}} \quad 2e^{-\frac{jk2\pi}{6}}$

$n=2 \rightarrow x(2) \quad e^{-\frac{j2\pi k(2)}{6}} = e^{-\frac{j2\pi k}{3}}$

$n=3 \rightarrow x(3)\, e^{-\frac{j2\pi k(3)}{6}} \quad x(3)\, e^{-\frac{j2\pi k}{2}} = x(3) e^{-j\pi k} = 0$

$n=4 \rightarrow x(4)\, e^{-\frac{j2\pi k(4)}{6}} = 1\cdot e^{-\frac{j2\pi k}{3}}$

$n=5 \rightarrow x(5)\, e^{-\frac{j2\pi k5}{6}} = 2\cdot e^{-\frac{j10\pi k}{6}}$

$$= \frac{1}{6}\left[3+2e^{-\frac{j\pi 2k}{6}} + e^{-\frac{j2\pi k}{3}} + 0 + e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j10\pi k}{6}}\right]$$

for $k=0$:

$$= \frac{1}{6}\left[3+2+1+0+1+2\right]$$

$$= \frac{1}{6}[9] = 9/6$$

$$= \frac{3}{2}$$

for $k=1 \Rightarrow \frac{1}{6}\left[3+2e^{-j\pi/3} + e^{-j2\pi/3} + 0 + e^{-j4\pi/3} + 2e^{-j10\pi/6}\right]$

$$= \frac{1}{6}\left[3+2e^{-j\pi/3} + e^{-j2\pi/3} + 0 + e^{-j4\pi/3} + 2e^{-j5\pi/3}\right]$$

$$= \frac{1}{6}\left\{3+2\left(\cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right) + \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right) + \cos\left(\frac{4\pi}{3}\right)\right.$$

$$\left. - j\sin\left(\frac{4\pi}{3}\right) + 2\cos\left(\frac{5\pi}{3}\right) - 2j\cdot\sin\left(\frac{5\pi}{3}\right)\right.$$

$$= \frac{1}{6}\left[3+2\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \left(\frac{-1}{2}\right) - j\frac{\sqrt{3}}{2} + \left(\frac{-1}{2}\right) - j\left(\frac{-\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right) - 2j\left(\frac{-\sqrt{3}}{2}\right)\right]$$

$$= \frac{1}{6}\left[3+1-j\cancel{\sqrt{3}} - \frac{1}{2} - \frac{\cancel{\sqrt{3}j}}{\cancel{2}} - \frac{1}{2} + \frac{\cancel{\sqrt{3}j}}{\cancel{2}} + 1 + \cancel{\sqrt{3}j}\right]$$

$$= \frac{1}{6}[5-1] = \frac{4}{6}$$

$$= 2/3$$

for $k=2$; $C_2=0$

$k=3$; $C_3 = 1/6$

$k=4$; $C_4=0$

$k=5$; $C_5 = 4/6$

b) Using the results in part(a) verify parseval's relation by computing the power in the time and frequency domains.

$$P_t = \frac{1}{6}\sum_{n=0}^{6} |x(n)|^2$$

$$= \frac{1}{6}\left[1^2+0+1+2^2+3^2+4\right] = \frac{1}{6}\left[1+1+4+9+4\right]$$

$$= \frac{19}{6}$$

$$P_f = \sum_{n=0}^{5} |C_n|^2 = \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{4}{6}\right)^2 = \frac{114}{36}$$

$$= \frac{19}{6}$$

5 Consider the signal $x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$

a) Determine and sketch its PDs

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

$$= 2+2\left[\frac{e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}}{2}\right] + \left[\frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2}\right] + \frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}}{2}\right]$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{-\frac{j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{-\frac{j3\pi n}{4}}$$

$N=8$

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n) e^{-\frac{j\pi k n}{4}}$$

$$x(n) = \left\{ \frac{11}{2}, 2+\frac{3}{4}\sqrt{2}, 1, 2-\frac{3}{4}\sqrt{2}, \frac{1}{2}, 2-\frac{3}{4}\sqrt{2}, 1, 2+\frac{3}{4}\sqrt{2} \right\}$$

$C_0 = 2,\ C_1 = C_7 = 1,\ C_2 = C_6 = \frac{1}{2},\ C_3 = C_5 = \frac{1}{4},\ C_4 = 0$

b) Evaluate the power of the Signal.

$$\sum_{n=0}^{7} |C_k|^2 \Rightarrow \left[2^2 + 1^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \right]$$

$$\Rightarrow \left[4 + 2 + \frac{1}{2} + \frac{1}{8} \right]$$

$$\Rightarrow \left(\frac{32+16+4+1}{8} \right)$$

$$= \frac{53}{8}$$

6 Determine & Sketch the magnitude and Phase Spectra of the following Periodic signals.

a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

$$= 4\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{-\frac{j\pi(n-2)}{3}}}{2j} \right]$$

$$= 4\left[e^{\frac{3j\pi(n-2)}{5}} - e^{-3j\pi(n-2)} \right]$$

$N = 6,\ C_k = \frac{1}{6}\sum_{n=0} x(n) e^{-\frac{j2\pi k n}{6}}$

$$= \frac{4}{6}\sum_{n=0}^{5} \left(\frac{e^{\frac{j\pi(n-2)}{3}} - e^{-\frac{j\pi(n-2)}{3}}}{2j} \right) e^{-\frac{j2\pi k n}{6}}$$

$$= \frac{1}{\sqrt{3}} \left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{j\pi k/3} + e^{-j2\pi k/3} \right]$$

$$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\frac{2\pi k}{6} + \sin\frac{\pi k}{3} \right] e^{-j2\pi k/3}$$

$C_0 = 0,\ C_1 = -j2e^{-j2\pi/3},\ C_2 = C_3 = C_4 = 0,\ C_5 = C_1$

$\angle C_1 = \frac{5\pi}{6} \quad \angle C_5 = \frac{-5\pi}{6} \quad \angle C_0 = \angle C_2 = \angle C_3 = \angle C_4 = 0$

b) $x(n) = \cos\frac{2\pi}{3}n + \sin\frac{2\pi}{5}n$

$N = 15$

$\cos\frac{2\pi}{3}n$

$\frac{1}{2}\left[e^{\frac{j2\pi}{3}n} + e^{-\frac{j2\pi}{3}n}\right]$

$e^{-\frac{j2\pi}{3}n} = e^{-\frac{j2\pi kn}{N}}$

$\frac{k}{N} = \frac{1}{3}$

$k = \frac{N}{3} = 5$

$15 - 5 = 10$

$C_{1k} \begin{cases} 1/2 \ ; k = 5, 10 \\ 0 \ ; \text{otherwise} \end{cases}$

$\sin\frac{2\pi}{5}n$

$\frac{1}{2j}\left[e^{\frac{j2\pi n}{5}} - e^{-\frac{j2\pi}{5}n}\right]$

$e^{-\frac{j2\pi}{5}n} = e^{-\frac{2j\pi kn}{N}}$

$\frac{k}{N} = \frac{1}{5}$

$5k = N \Rightarrow k = 3$

$15 - 3 = 12$

$C_{2k} \begin{cases} 1/2j \ ; k = 3 \\ -1/2j \ ; k = 12 \\ 0 \ ; \text{otherwise} \end{cases}$

$C_k = C_{1k} + C_{2k}$

$$C_k = \begin{cases} \frac{1}{2j}, & k = 3 \\ \frac{1}{2}, & k = 5 \\ \frac{1}{2}, & k = 10 \\ \frac{-1}{2j}, & k = 12 \\ 0, & \text{otherwise} \end{cases}$$

c) $x(n) = \cos\frac{2\pi}{3}n \cdot \sin\frac{2\pi}{5}n$

$\cos a \sin b = \frac{\sin a+b}{2} - \frac{\sin(a-b)}{2}$

$= \frac{1}{2}\left[\sin\frac{(10\pi n + 6\pi n)}{15} - \sin\frac{(10\pi n - 6\pi n)}{15}\right]$

$= \frac{1}{2}\left[\sin\frac{16\pi n}{15} - \sin\frac{4\pi n}{15}\right]$

$= \frac{1}{2}\left(\frac{e^{\frac{j16\pi n}{15}} - e^{-\frac{j16\pi n}{15}}}{2j}\right) - \frac{1}{2}\left(\frac{e^{\frac{4\pi n j}{15}} - e^{-\frac{j4\pi n}{15}}}{2j}\right)$

$= \frac{1}{4j}\left[e^{\frac{j16\pi n}{15}} - e^{-\frac{j16\pi n}{15}}\right] - \frac{1}{4j}\left[e^{\frac{4\pi n j}{15}} - e^{-\frac{j4\pi n}{15}}\right]$

$e^{\frac{j16\pi n}{15}} \Rightarrow e^{j2\pi k/N}$ $\qquad$ $e^{\frac{j4\pi n}{15}} \Rightarrow e^{\frac{j2\pi k}{N}}$

$\frac{k}{N} = \frac{8}{15}$ $\qquad$ $\frac{k}{N} = \frac{2}{15}$

$k = 8 \Rightarrow \frac{1}{4j}$ $\qquad$ $k = 2$

$\frac{-1}{4j} \Rightarrow k = 2$

$15 - 8 = 7 \Rightarrow \frac{-1}{4j}$ $\qquad$ $15 - 2 = 13 \Rightarrow \frac{1}{4j}$

$$C_k = \begin{cases} \frac{1}{4j} & ; 8, 13 \\ \frac{-1}{4j} & ; 7, 2 \\ 0 & , \text{otherwise} \end{cases}$$

d) $x(n) = \{ \cdots -2, -1, 0, 1, 2, -2, -1, 0, 1, 2, \cdots \}$

$x(n) = \{ \cdots \underbrace{-2, -1, \underset{\uparrow}{0}, 1, 2}, -2, -1, 0, 1, 2, \cdots \}$

$N = 5$

$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-\frac{j2\pi kn}{N}}$

$= \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-\frac{j2\pi kn}{5}}$

$= \frac{1}{5}\left[0 + e^{-\frac{j2\pi k}{5}} + 2e^{-\frac{j2\pi k(2)}{5}} + -2e^{-\frac{j6\pi k}{5}} - e^{-\frac{j8\pi k}{5}}\right]$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$$

If $k=0;\ C_0=0$

$k=1;\ C_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) - 2\sin\left(\frac{4\pi}{5}\right)\right]$

$k=2;\ C_2 = \frac{2j}{5}\left[-\sin\left(\frac{4\pi}{5}\right) - 2\sin\left(\frac{8\pi}{5}\right)\right]$

$C_3 = -C_2\ ;\quad C_4 = -C_1$

e) $x(n) = \{-1, 2, 1, 2, -1, 0, -1, 2, 1, 2 \ldots\}$

$$C_k = \frac{1}{N}\sum_{n=0}^{5} x(n)\, e^{-\frac{j2\pi kn}{N}}$$

from 0 to 5 in above equation we get

$$= \frac{1}{6}\left[1 + 2e^{-\frac{j\pi k}{3}} - e^{-\frac{j2\pi k}{3}} - e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3}\right]$$

$C_0 = \frac{1}{2};\ C_1 = \frac{2}{3};\ C_2 = 0;\ C_3 = \frac{-5}{6};\ C_4 = 0;\ C_5 = \frac{2}{3}$

f) $x(n) = \{\ldots 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0 \ldots\}$

$N=5$

$$C_k = \frac{1}{N}\sum_{n=0}^{4} x(n)\, e^{-\frac{j2\pi kn}{5}}$$

$$= \frac{1}{5}\left[1 + e^{-\frac{j2\pi k}{5}}\right]$$

$$= \frac{2}{5}\cos\left(\frac{\pi k}{5}\right) e^{-\frac{j\pi k}{5}}$$

$\therefore C_0 = \frac{2}{5};\ C_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right)e^{-j\pi/5};\ C_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right)e^{-j2\pi/5}$

$C_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right)e^{-\frac{j3\pi}{5}};\ C_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right)e^{-j4\pi/5}$

g) $x(n) = 1 \qquad -\infty < n < \infty$

$N = 1$

$C_0 = 1$

$C_k = x(0) = 1$

h) $x(n) = (-1)^n, \quad -\infty < n < \infty$

$N = 2$

$$C_k = \frac{1}{2}\sum_{n=0}^{1} x(n)\, e^{-j\pi nk}$$

$$= \frac{1}{2}\left(1 - e^{-j\pi k}\right)$$

$\therefore C_0 = 0;\ C_1 = 1$

7 Determine the Periodic Signals $x(n)$ with fundamental Period $N=8$ if their fourier Coefficients are given by a) $C_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

$$x(n) = \sum_{n=0}^{7} C_k\, e^{\frac{j2\pi nk}{N}}$$

Let $C_k = e^{\frac{j2\pi pk}{N}}$

$$\sum_{n=0}^{7} e^{\frac{j2\pi pk}{N}} \cdot e^{\frac{j2\pi nk}{N}}$$

$$\sum_{n=0}^{7} e^{\frac{j2\pi (p+n)k}{N}}$$

it gives 8; when $p = -n$

0; when $p \neq n$

$$\therefore C_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}}\right] - \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{\frac{-j6\pi k}{8}}\right]$$

$$x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n-1) - 4j\delta(n+3) + 4j\delta(n-3)$$

$j;\ -3 \le n \le 5$

b) $C_k = \begin{cases} \sin\frac{k\pi}{3} & ; 0 \le k \le 16 \\ 0 & ; k = 7 \end{cases}$

$C_0 = 0;\ C_1 = \frac{\sqrt{3}}{2};\ C_3 = 0;\ C_4 = \frac{-\sqrt{3}}{2};\ C_5 = \frac{-\sqrt{3}}{2};\ C_6 = C_7 = 0$

$$x(n) = \sum_{k=0}^{7} C_k\, e^{\frac{j2\pi nk}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n 2}{4}} - e^{\frac{-j4\pi n}{4}} - e^{\frac{-j5\pi n}{4}}\right]$$

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$$= \sqrt{3}\left[\frac{\sin \pi n}{2} + \sin\frac{\pi n}{4}\right] e^{\frac{j\pi(3n-2)}{4}}$$

c) $C_k = \{\ldots 0, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}, \frac{1}{4}, 0 \ldots\}$

$$x(n) = \sum_{k=-3}^{4} C_k e^{j2\pi nk/8}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{-\frac{j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{-\frac{j3\pi n}{4}}$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{2}.$$

8 Two DT signals $s_k(n)$ and $s_1(n)$ are said to be orthogonal over an integral $[N_1\ N_2]$ if $\sum_{n=N_1}^{N_2} s_k(n)\, s_1^*(n) =$
$\begin{cases} A_k & k=1 \\ 0 & k \neq 1 \end{cases}$ If $A_k = 1$ the signals are orthogonal.

a) Prove the relation $\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \begin{cases} N & k = 0, \pm N, \pm 2N \\ 0 & \text{otherwise} \end{cases}$

$k = 0 \pm N, \pm 2N$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \text{sum}_{n=0}^{N-1}\ 1 = N$$

If $k \neq 0, \pm N_1, \pm 2N$ $\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \frac{1 - e^{j2\pi k}}{1 - e^{j2\pi k/N}}$

9 Compute the fourier transform of the following signals.

a) $x(n) = \mu(n) - \mu(n-6)$

$x(n) = \mu(n) - \mu(n-6)$

$$x(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{5} e^{-j\omega n}$$

$$= \frac{1 - e^{-j6\omega}}{1 - e^{-j\omega}}$$

b) $x(n) = 2^n \mu(-n)$

$x(n) = 2^n \mu(-n)$

$$x(\omega) = \sum_{n=-\infty}^{\infty} 2^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n$$

$$= \frac{2}{2 - e^{j\omega}}$$

c) $x(n) = \left(\frac{1}{4}\right)^n \mu(n+4)$

$$x(n) = \left(\frac{1}{4}\right)^n \mu(n+4)$$

$$x(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty}\left(\left(\frac{1}{4}\right)^m e^{-j\omega m}\right) 4^4 e^{j4\omega}$$

$$= \frac{4^4 e^{j4\omega}}{1 - \frac{1}{4}e^{-j\omega}}$$

d) $x(n) = (a^n \sin\omega_0 n)\mu(n) \quad |a|<1$

$$x(n) = (a^n \sin\omega_0 n)\mu(n) \quad |a|<1$$

$$x(\omega) = \sum_{n=0}^{\infty} a^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j}\sum_{n=0}^{\infty}\left[a e^{-j(\omega-\omega_0)}\right]^n - \frac{1}{2j}\sum_{n=0}^{\infty}\left[a e^{-j(\omega+\omega_0)}\right]^n$$

$$= \frac{1}{2j}\left[\frac{1}{1 - a e^{-j(\omega-\omega_0)}} - \frac{1}{1 - a e^{-j(\omega_0+\omega_0)}}\right]$$

$$= \frac{a \sin\omega_0 e^{-j\omega}}{1 - 2a\cos\omega_0 e^{-j\omega} + a^2 e^{-j2\omega}}$$

e) $x(n) = |a|^n \sin\omega_0 n \quad |a|<1$

$$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |a|^n |\sin\omega_0 n|$$

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$\omega_0 = \frac{\pi}{2} \qquad |\sin\omega_0 n| = 1$

$$\sum_{n=-\infty}^{\infty} |a|^n = \sum_{n=-\infty}^{\infty} |x(n)| \to \infty$$

Fourier Transform does not exist

f) $x(n) = \begin{cases} 2-\left(\frac{1}{2}\right)n & |n| \le 4 \\ 0 & \text{, elsewhere} \end{cases}$

$$x(n) = \begin{cases} 2-\left(\frac{1}{2}\right)n & |n| \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

$$X(\omega) = \sum_{n=-4}^{4} x(n) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left[2-\left(\frac{1}{2}\right)n\right] e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}}$$

$$= \frac{1}{2}\left[-4e^{j4\omega} + 4e^{-j4\omega} - 3e^{j3\omega} + e^{-j3\omega} - 2e^{j2\omega} + 2e^{-j2\omega} - e^{j\omega} + e^{-j\omega}\right]$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\left[4\sin4\omega + 3\sin3\omega + 2\sin2\omega + \sin\omega\right]$$

g) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= -2e^{-j2\omega} - e^{j\omega} + e^{j\omega} + 2e^{-j2\omega}$$

$$= -2j\left[2\sin2\omega + \sin\omega\right]$$

h) $x(n) = \begin{cases} A(2M+1-|n|) & |n| \le M \\ 0 & |n| > M \end{cases}$

$$X(\omega) = \sum_{n=-M}^{M} x(n) e^{-j\omega n}$$

$$= A \sum_{n=-M}^{M} (2M+1-|n|) e^{-j\omega n}$$

$$= (2M+1)A + A \sum_{k=1}^{M} (2M+1-k)(e^{-j\omega k} + e^{j\omega k})$$

$$= (2M+1)A + 2A \sum_{k=1}^{M} (2M+1-k) \cos \omega k$$

10 Determine the signals having the following fourier transforms.

a) $X(\omega) = \begin{cases} 0 & 0 < |\omega| \le \omega_0 \\ 1 & \omega_0 < |\omega| \le \pi \end{cases}$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\omega_0} X(\omega) e^{j\omega n} + \int_{\omega_0}^{\pi} X(\omega) e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \left\{ \left[\frac{e^{-j\omega n}}{jn} \right]_{-\pi}^{-\omega_0} + \left[\frac{e^{j\omega n}}{-jn} \right]_{\omega_0}^{\pi} \right\}$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} + \frac{e^{j\pi n} - e^{j\omega_0 n}}{jn} \right]$$

$$= \frac{1}{2\pi} \left[2 \cdot \frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} + 2 \cdot \frac{e^{j\pi n} - e^{-j\pi n}}{2jn} \right]^{0}$$

$$= \frac{1}{2\pi} \left[\frac{2}{n} - \sin \omega_0 n + 2 \cdot \frac{\sin \pi n}{2jn} \right] \quad \text{since } \sin \pi = 0$$

$$= -\frac{\sin \omega_0 n}{n\pi} \; ; \; n \neq 0$$

for $n=0$ from eq ①

$$= \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0)$$

$$= \frac{\pi - \omega_0 + (\pi - \omega_0)}{2\pi} = \frac{2(\pi - \omega_0)}{2\pi}$$

$$= \pi - \omega_0 \text{ ; when } n=0$$

b) $x(\omega) = \cos^2\omega$

$$= \left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^{j\omega})^2 + (e^{-j\omega})^2 + 2e^{j\omega}e^{-j\omega}\right]$$

$$= \frac{1}{4}\left[e^{j2\omega} + e^{-j2\omega} + 2\right]$$

$$= \frac{1}{4}e^{j2\omega} + \frac{1}{2} + \frac{1}{4}e^{-j2\omega}$$

$\downarrow$ IFT

$$= \frac{1}{4}\delta(n+2) + \delta(n)\frac{1}{2} + \frac{1}{4}\delta(n-2)$$

c) $x(\omega) = \begin{cases} 1 \text{ ; } \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2} \\ 0 \text{ ; elsewhere} \end{cases}$

$$\omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2}$$

$$\omega_0 - \frac{\delta\omega}{2} \le -\omega \le \omega_0 + \frac{\delta\omega}{2}$$

$$-\omega_0 + \frac{\delta\omega}{2} \le \omega \le -\omega_0 - \frac{\delta\omega}{2}$$

Consider limits $\omega_0 - \frac{\delta\omega}{2} \le \omega \le \omega_0 + \frac{\delta\omega}{2}$

$$= \frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} 1 \cdot e^{j\omega n}\, d\omega$$

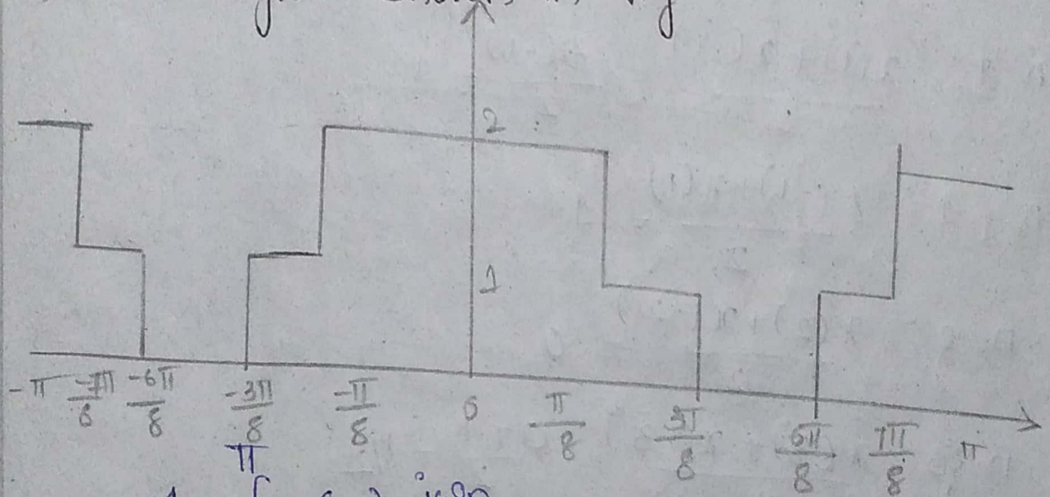
$$\frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}}$$

$$\frac{2}{\pi}\ \frac{e^{j(\omega_0+\frac{\delta\omega}{2})n}-e^{j(\omega_0-\frac{\delta\omega}{2})n}}{2jn}$$

$$\delta\omega\,\frac{2}{\pi}\left[\frac{\sin\left(\frac{\delta\omega}{2}n\right)}{\frac{n\delta\omega}{2}}\right]e^{jn\omega_0}$$

$$\delta\omega.\frac{2}{\pi}\,Sa\left(\frac{\delta\omega}{2}n\right)e^{jn\omega_0}$$

d) The signal shown in fig



$$\frac{1}{2\pi}\int_{-\pi}^{\pi} x(\omega)e^{j\omega n}\,d\omega$$

Let consider limits 0 to π

$$2.\frac{1}{2\pi}\left[\int_0^{\pi/8}2e^{j\omega n}d\omega+\int_{\pi/8}^{3\pi/8}e^{j\omega n}d\omega+\int_{6\pi/8}^{7\pi/8}e^{j\omega n}d\omega+\int_{7\pi/8}^{\pi}2e^{j\omega n}d\omega\right]$$

$$=\frac{1}{\pi}\left[\int_0^{\pi/8}2\cos\omega n\,d\omega+\int_{\pi/8}^{3\pi/8}\cos\omega n\,d\omega+\int_{6\pi/8}^{7\pi/8}\cos\omega n\,d\omega+\int_{7\pi/8}^{\pi}2\cos\omega n\,d\omega\right]$$

$$\because j\sin\omega_n=\sin(\pi n\pi)=0$$

$$=\frac{1}{\pi}\left[-2\sin\omega n\right]_0^{\pi/8}+\left[-\sin\omega n\right]_{\pi/8}^{3\pi/8}+\left[-\sin\omega n\right]_{6\pi/8}^{7\pi/8}+\left[2\sin\omega n\right]_{-7\pi/8}^{\pi}$$

$$=\frac{1}{\pi}\left[-2\sin\frac{\pi n}{8}-\sin\frac{3\pi}{8}n+\sin\frac{\pi}{8}n-\sin\frac{7\pi}{8}n+\sin\frac{6\pi n}{8}-2\sin\pi n+2\sin\frac{7\pi}{8}n\right]$$

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$= \frac{1}{\pi}\left[\sin\frac{\pi \pi n}{8} n - \frac{\sin 7\pi n}{8} + \sin\frac{6\pi}{8} n - \sin\frac{3\pi}{8} n\right]$

11 Consider the signal $x(n) = \{-1, 0, -1, \underset{\uparrow}{2}, 3\}$ with Fourie transform $X(\omega) = X_R(\omega) + jX_I(\omega)$. Determine and sketch the signal $y(n)$ with fourier transform $Y(\omega) = X_I(\omega) + X_R(\omega)\, e^{j2\omega}$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$n = 0;\ x_e(n) = \frac{x(0) + x(-0)}{2} = \frac{2+2}{2} = 2$

$n = 1;\ \frac{x(1) + x(-1)}{2} = \frac{2(-1)}{2} = 1$

$n = -1;\ \frac{x(-1) + x(1)}{2} = 1$

$n = 2;\ \frac{x(2) + x(-2)}{2} = 0$

$n = 3;\ \frac{x(3) + x(-3)}{2} = \frac{0+1}{2} = 1/2$

$n = -2;\ \frac{x(-2) + x(2)}{2} = 0$

$n = -3;\ \frac{x(-3) + x(3)}{2} = 1/2$

$x_e(n) = \{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2}\}$.

$x_o(n) = \left(\frac{x(n) - x(-n)}{2}\right)$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-jn\omega}$$

$$jX_I(\omega) = \sum_{n=-3}^{3} x_o(n) e^{-jn\omega}$$

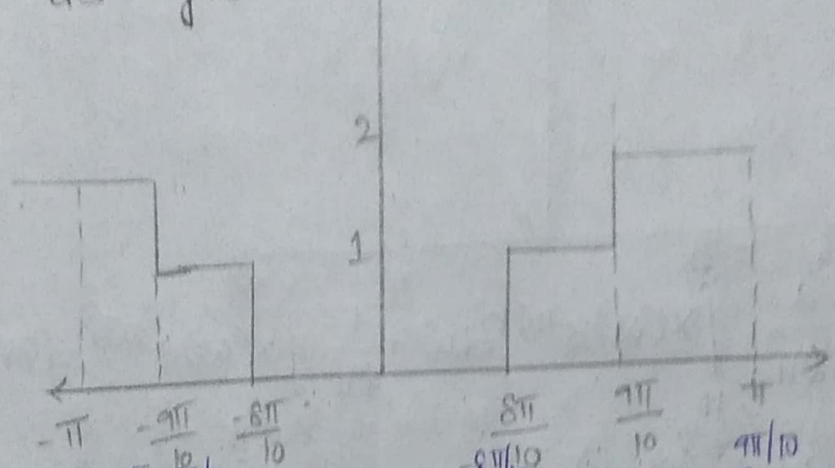
$$Y(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}$$

$= \frac{x_o(n)}{j} + x_e(n+2) \to$ IFT of $Y(\omega)$

$= -jx_o(n) + x_e(n+2)$

$= \{\frac{1}{2}, 0, 1, -\frac{j}{2}, 2, 1+\frac{j}{2}, 0, \frac{1}{2} - j2, 0, \frac{j}{2}\}$

12 Determine the signal $x(n)$ if its fourier transform is as given as



$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-9\pi/10} 2 \cdot e^{jn\omega} d\omega + \int_{-9\pi/10}^{-8\pi/10} 1 \cdot e^{jn\omega} d\omega + \int_{8\pi/10}^{9\pi/10} e^{jn\omega} d\omega + \frac{1}{2}\int_{9\pi/10}^{\pi} e^{jn\omega} d\omega\right]$$

$$= \frac{1}{2\pi}\left[2\left(\frac{e^{jn\omega}}{jn}\right)_{-\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{8\pi/10}^{\frac{9\pi}{10}} + 2\left[\frac{e^{jn\omega}}{jn}\right]_{9\pi/10}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[2\left[e^{-jn\frac{9\pi}{10}} - e^{-jn\pi}\right] + e^{jn-\frac{8\pi}{10}} - e^{-\frac{jn9\pi}{10}} + e^{\frac{jn9\pi}{10}} - e^{\frac{jn8\pi}{10}} + 2e^{jn\pi} - 2e^{jn\frac{9\pi}{10}}\right.$$

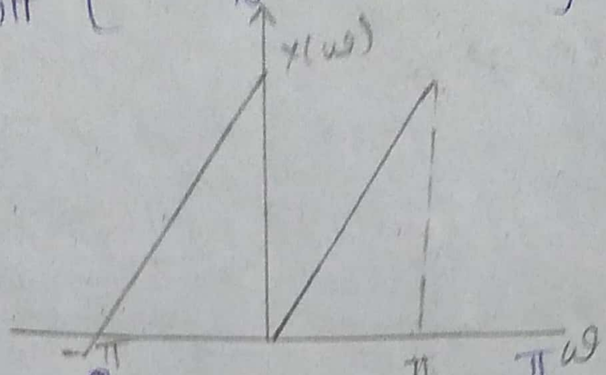
$$= \frac{1}{2\pi jn}\left[2e^{-jn\frac{9\pi}{10}} - 2e^{-jn\pi} + e^{-\frac{jn8\pi}{10}} - e^{\frac{jn9\pi}{10}} - e^{\frac{jn8\pi}{10}} - e^{-\frac{jn8\pi}{10}} + 2e^{jn\pi} - e^{\frac{j9\pi}{10}}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-jn\frac{9\pi}{10}} - 2e^{-jn\pi} + 2e^{jn\pi} + e^{-\frac{jn8\pi}{10}} - e^{\frac{jn8\pi}{10}} + e^{\frac{jn9\pi}{10}} - 2e^{jn\frac{9\pi}{10}}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-jn\frac{9\pi}{10}} - e^{jn\frac{9\pi}{10}} - 2e^{-jn\pi} + 2e^{jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{jn\frac{8\pi}{10}}\right]$$

$$= \frac{+1}{n\pi}\left[-\sin\left(\frac{9\pi n}{10}\right) - \sin\frac{8n\pi}{10} + \sin n\pi\right]$$

$$= \frac{-1}{n\pi}\left[\sin\frac{9\pi n}{10} + \sin\frac{4n\pi}{5}\right]$$

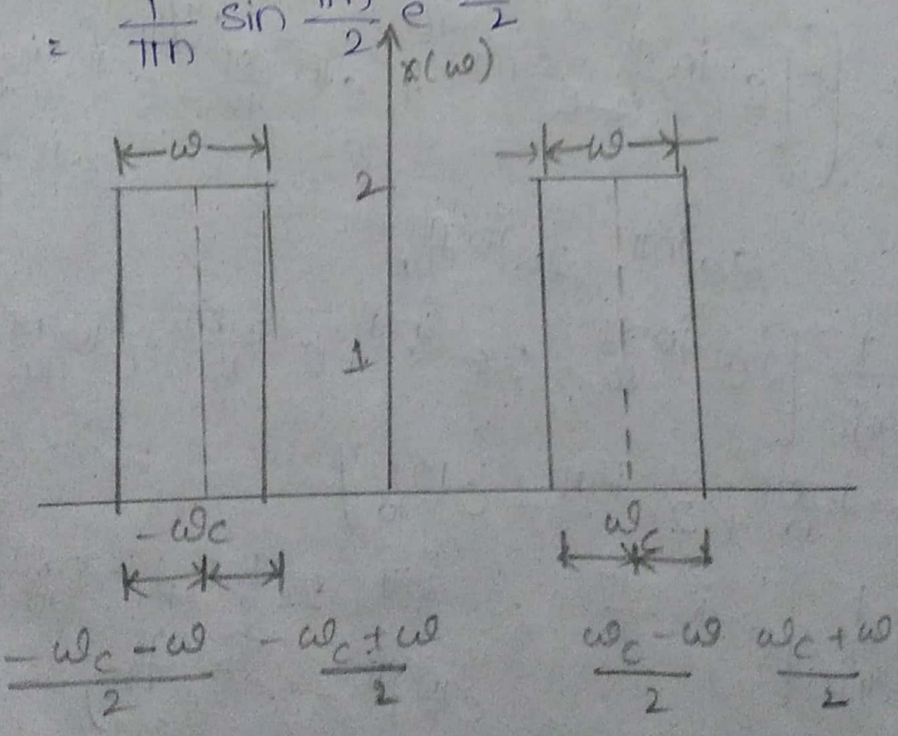


$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega)e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega)e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right)e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi}e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\left[\left[\frac{\omega}{jn\pi}e^{j\omega n}\right]_{-\pi}^{\pi} + \left(\frac{e^{j\omega n}}{jn}\right)\Bigg|_{-\pi}^{0}\right]$$

$$= \frac{1}{\pi n}\sin\frac{\pi n}{2}e^{\frac{-jn\pi}{2}}$$



$$x(n) = \frac{1}{2\pi}\left[\int_{\frac{-\omega_c-\omega}{2}}^{\frac{-\omega_c+\omega}{2}} 2e^{jn\omega}\, d\omega + \int_{\frac{\omega_c-\omega}{2}}^{\frac{\omega_c+\omega}{2}} 2\, e^{jn\omega} d\omega\right]$$

$$= \frac{1}{\pi}\left(\left[\frac{e^{jn\omega}}{jn}\right]_{\frac{-\omega_c+\omega}{2}}^{\frac{-\omega_c+\omega}{2}} + \left[\frac{e^{jn\omega}}{jn}\right]_{\frac{\omega_c-\omega}{2}}^{\frac{\omega_c+\omega}{2}}\right)$$

$$= \frac{1}{jn\pi}\left[e^{jn(-\omega_c+\frac{\omega}{2})} - e^{jn(\omega_c-\frac{\omega}{2})} + e^{jn(\omega_c+\frac{\omega}{2})} - e^{jn(\omega_c-\frac{\omega}{2})}\right]$$

$$= \frac{1}{jn\pi}\left[e^{jn(-\omega_c+\frac{\omega}{2})} - e^{jn(-\omega_c-\frac{\omega}{2})} + e^{-jn(-\omega_c-\frac{\omega}{2})} - e^{-jn(\frac{\omega}{2}-\omega_c)}\right]$$

$$= \frac{2}{n\pi}\left[\sin\left(\frac{\omega}{2}-\omega_c\right)n - \sin\left(-\omega_c-\frac{\omega}{2}\right)n\right]$$

$$= \frac{2}{n\pi}\left[\sin\left(\frac{\omega}{2}-\omega_c\right)n - \sin\left(-\omega_c-\frac{\omega}{2}\right)n\right]$$

$$= \frac{2}{n\pi}\left[-\sin\left(\omega_c-\frac{\omega}{2}\right) + \sin\left(\omega_c+\frac{\omega}{2}\right)n\right]$$

$$= \frac{2}{n\pi}\left[\sin\left(\omega_c+\frac{\omega}{2}\right)n - \sin\left(\omega_c-\frac{\omega}{2}\right)n\right]$$

3 Given the fourier transform of the signal $x(n) = \begin{cases} 1; & -m \le n \le m \\ 0; & \text{otherwise} \end{cases}$ was shown to be $x(\omega) = 1 + 2\sum_{n=1}^{M} \cos\omega n$ then show that the fourier transform of $x_1(n) = \begin{cases} 1; & 0 \le n \le m \\ 0; & \text{otherwise} \end{cases}$ is $X_1(\omega) = \frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}}$

$x_2(n) = \begin{cases} 1; & -m \le n \le -1 \\ 0; & \text{otherwise} \end{cases}$ is $X_2(\omega) = \frac{e^{j\omega} - e^{j\omega(m+1)}}{1-e^{j\omega}}$

$$X_1(\omega) = \sum_{n=0}^{m} 1 \cdot e^{-j\omega n}$$

$$1 + e^{-j\omega} + e^{-j\omega 2} + e^{-j\omega 3} + \cdots - \left[e^{-j\omega(m+1)} + e^{-j\omega(m+2)}\right.$$

$$\frac{1}{1-e^{-j\omega}} - \frac{e^{-j\omega(m+1)}}{1-e^{-j\omega}} \qquad \frac{e^{j\omega(m+1)}}{1-e^{-j\omega}}$$

$$X_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega n} = \sum_{n=1}^{m} e^{j\omega n} = \frac{1-e^{j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1 - e^{-j\omega(m+1)}}{1-e^{-j\omega}} + \frac{1-e^{-j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$= \frac{1 - e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(m+1)} - e^{j\omega(m+1)} + e^{j\omega n} + e^{-j\omega n}}{2 - e^{-j\omega} - e^{j\omega}}$$

$$= \frac{2\cos\omega m - 2\cos\omega(m+1)}{2 - 2\cos\omega}$$

$$= \frac{2\sin\left(\omega m + \frac{\omega}{2}\right)\cos\frac{\omega}{2}}{2\sin^2\frac{\omega}{2}}$$

$$= \frac{\sin\left(m + \frac{1}{2}\right)\omega}{\sin\left(\frac{\omega}{2}\right)}$$

$$\therefore 1 + 2\sum_{n=1}^{M} \cos\omega n = \frac{\sin(m+\frac{1}{2})\omega}{\sin\left(\frac{\omega}{2}\right)}$$

14 Consider the signal $x(n) = \{-1, 2, -3, 2, -1\}$ with fourier transform $X(\omega)$. compute the following quantities without explicitly computing $X(\omega)$:

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a) $x(0)$

$$x(\omega)\Big|_{\omega=0} = \sum x(n) e^{-j\omega n}\Big|_{\omega=0}$$

$$\Rightarrow x(0) = -3e^0$$

$$= -3$$

b) $\angle x(\omega) = \pi$ for all 'ω'

c) $\int_{-\pi}^{\pi} x(\omega)\, d\omega$ $\quad x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(\omega)\, d\omega$

$$\int_{-\pi}^{\pi} x(\omega)\, d\omega = 2\pi x(\omega)\Big|_{\omega=0}$$

$$= 2\pi(-3) \Rightarrow -6\pi$$

d) $x(\pi)$

$$x(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$$

$$= \sum_n e^{-jn\pi} \cdot x(n)$$

$$= \sum_n \left[\cos(n\pi) - j\sin(n\pi)\right] x(n)$$

$$= \sum_n (-1)^n x(n)$$

for $n=0$ $\quad (-1)^0 x(0) \Rightarrow 1\cdot(-3) = -3$

$n=1$ $\quad (-1)^1 x(1) \Rightarrow -1\cdot 2 = -2$

$n=2$ $\quad (-1)^2 x(2) \Rightarrow -1 = -1$

$n=-1$ $\quad (-1)^{-1} x(-1) = -2$

$n=-2$ $\quad (-1)^{-2} x(-2) = -1$

$\Rightarrow -3-2-1-2-1 \Rightarrow -3-4-2 \Rightarrow -9$

e) $\int_{-\pi}^{\pi} |x(\omega)|^2 d\omega$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |x(\omega)|^2 d\omega = \sum_n |x(n)|^2$$

$$= (-1)^2 + (2)^2 + (-3)^2 + (2)^2 + (-1)^2$$

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$= 1+4+9+4+1$

$= 19$

$$\int_{-\pi}^{\pi} |X(\omega)|^2 = 19 \times 2\pi = 38\pi$$

15 The Center of gravity of a signal $x(n)$ is defined as $c = \dfrac{\sum_{n=-\infty}^{\infty} n x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$ and provides a measure of the "time delay" of the signal.

a) Express c in terms of $X(\omega)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n) e^{0}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n)$$

$$n x(n) \xleftrightarrow{F.T} j \frac{d}{d\omega} X(\omega)$$

$$-jn\, x(n) \xleftrightarrow{F.T} \frac{d}{d\omega} X(\omega).$$

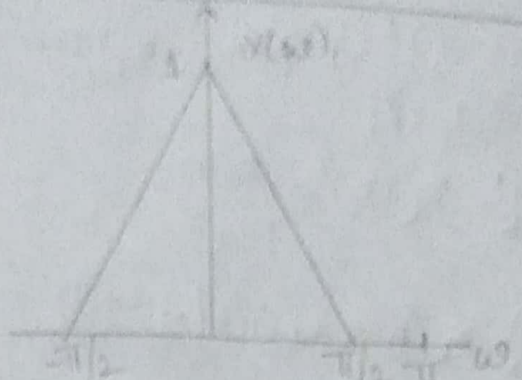
$$\frac{d}{d\omega} X(\omega) = \sum_{n=-\infty}^{\infty} -jn x(n) e^{-jn\omega} d\omega$$

$$= -j \sum_{n=-\infty}^{\infty} n x(n) e^{-jn\omega} d\omega$$

$$j \frac{d}{d\omega} X(\omega) = \sum_{n=-\infty}^{\infty} n x(n) e^{-jn\omega} d\omega$$

$$c = \left. \frac{j \frac{d}{d\omega} X(\omega)}{X(\omega)} \right|_{\omega=0}$$

c) Compute c for the signal $x(n)$ whose fourier transform is shown.

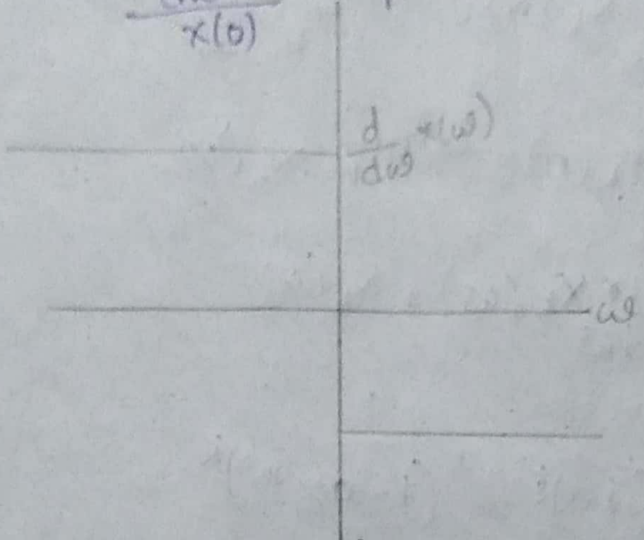


$x(0) = 1$

$\Lambda\left(1 - \frac{|t|}{1}\right)$

$1\left(1 - \frac{|t|}{\pi/2}\right)$

$$C = \frac{j\frac{d}{d\omega}x(0)}{x(0)} = \frac{0}{1} = 0$$



16 Consider the fourier transform pair $a^n\mu(n) \overset{FT}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}}$ $|a|<1$ use the differentiation in frequency theorem and introduction shown that $x(n) = \frac{(n+l-1)!}{n!(l-1)!}a^n\mu(n) \overset{F}{\longleftrightarrow} x(\omega)$

$$= \frac{1}{(1-ae^{-j\omega})^l}$$

Let $l = k+1$

$$x(n) = \frac{(n+k+1-1)!}{n!(k+1-1)!}a^n\mu(n)$$

$$= \frac{(n+k)!}{n!\,k!}a^n\mu(n)$$

$$= \frac{(n+k)(n+k-1)!}{kn!(k-1)!}a^n\mu(n)$$

Let $x_k(n) = \frac{(n+k-1)!}{n!(k-1)!} a^n \mu(n)$ $x_{k+1}(n) = \frac{n+k}{k} x_k(n)$

$$X_{k+1}(\omega) = \sum_{n=-\infty}^{\infty} \frac{n+k}{k} x_k(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{n}{k} x_k(n) + x_k(n)\right] e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + X_k(\omega)$$

$$= \frac{1}{k} \cdot j \frac{d X_k(\omega)}{d\omega} + X_k(\omega)$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^k}$$

17 Let $x(n)$ be a arbitary signal, not necessarily real valued with F.T $X(\omega)$ Express the fourier transform of the following signals in terms of $X(\omega)$

a) $x^*(n)$

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[x(n) e^{-j\omega n}\right]^*$$

b) $x^*(-n)$

$$\sum_{n=-\infty}^{\infty} x^*(-n) e^{-j\omega n}$$

replace $-n$ with n

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n}$$

$$\sum_{n=-\infty}^{\infty} \left(x(n) e^{-j\omega n}\right)^*$$

c) $y(n) = x(n) - x(n-1)$

$$\sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

$$X(\omega) - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

$l = n-1$

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega(l+1)}$$

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega l} \cdot e^{-j\omega}$$

$$= X(\omega) - e^{-j\omega} \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega l}$$

Replace l by n

$$= X(\omega) - e^{-j\omega} \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= X(\omega) - e^{-j\omega} X(\omega)$$

$$= X(\omega)\left[1 - e^{-j\omega}\right]$$

d) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$$= y(n) - y(n-1)$$

$$= x(n)$$

$$X(\omega) = Y(\omega)\left[1 - e^{-j\omega}\right]$$

$$Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}$$

e) $y(n) = x(2n)$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} x(2n) e^{-jn\omega}$$

Let $l = 2n$

$$= \sum_{l=-\infty}^{\infty} x(l) e^{\frac{-jl}{2}\omega}$$

$$= \sum_{l=-\infty}^{\infty} x(l) e^{-jl\frac{\omega}{2}}$$

$$= X\left(\frac{\omega}{2}\right)$$

f) $y(n) = \begin{cases} x\left(\frac{n}{2}\right) & \text{'n' even} \\ 0 & \text{'n' odd} \end{cases}$

$$Y(\omega) = \sum_{n} x\left(\frac{n}{2}\right) e^{jn\omega}$$

Let $n = 2l$

$$= \sum_{n} x\left(\frac{2l}{2}\right) e^{jn\omega}$$

$$= \sum_{n} x(2) e^{jn\omega}$$

$$= X(2\omega)$$

18 Determine & sketch the fourier transforms $X_1(\omega), X_2(\omega)$ and $X_3(\omega)$ of the following Signals

a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$$X_1(f) = \sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n} = \sum_{n=-2}^{2} x_1(n) e^{-j\omega n}$$

$n = -2$; $(1)e^{j2\omega} = e^{j2\omega}$

$n = -1$; $(1)e^{j\omega} = e^{j\omega}$

$n = 0$; $(1)e^{0} = 1$

$n = 1$; $(1)e^{-j\omega} = e^{-j\omega}$

$n = 2$; $(1)e^{-2j\omega} = e^{-2j\omega}$

$$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$$

$$= 2\cos(2\omega) + 2\cos(\omega) + 1$$

b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

$$X_2(f) = \sum_{n=-\infty}^{\infty} x_2(n) e^{-j\omega n} = \sum_{n=-4}^{4} x_2(n) e^{-j\omega n}$$

$n=-4$; $(1)e^{j4\omega} = e^{j4\omega}$

$n=-3$; $(0)e^{j3\omega} = 0$

$n=-2$; $(1)e^{j2\omega} = e^{j2\omega}$

$n=-1$; $(0)e^{j\omega} = e^{j\omega}(0) = 0$

$n=0$; $(1)e^{0} = 1$

$n=1$; $(0)e^{-j\omega} = 0$

$n=2$; $(1)e^{-j2\omega} = e^{-j2\omega}$.

$n=3$; $(0)e^{-j3\omega} = 0$

$n=4$; $(1)e^{-j4\omega} = e^{-j4\omega}$

$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-4j\omega}$

$= 2\cos(2\omega) + 2\cos(4\omega) + 1$

c) $x_3(n) = \{1,0,0,1,0,0,\underset{\uparrow}{1},0,0,1,0,0,1\}$

$$X_3(n) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = \sum_{n=-6}^{6} x(n)e^{-j\omega n}$$

$n=-6$; $1\times e^{j6\omega} = e^{j6\omega}$

$n=-3$; $1\times e^{j3\omega} = e^{j3\omega}$

$n=0$; $1\times e^{j0} = 1$

$n=3$; $1\times e^{-j3\omega} = e^{-j3\omega}$

$n=6$; $1\times e^{-j6\omega} = e^{-j6\omega}$

$= e^{j6\omega} + e^{-j6\omega} + e^{j3\omega} + e^{-j3\omega} + 1$

$= 2\cos(6\omega) + 2\cos(3\omega) + 1$

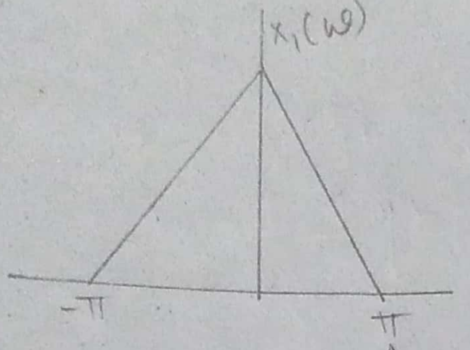
d) Find the relationship between $X_1(\omega), X_2(\omega)$ and $X_3(\omega)$? what is its physical meaning

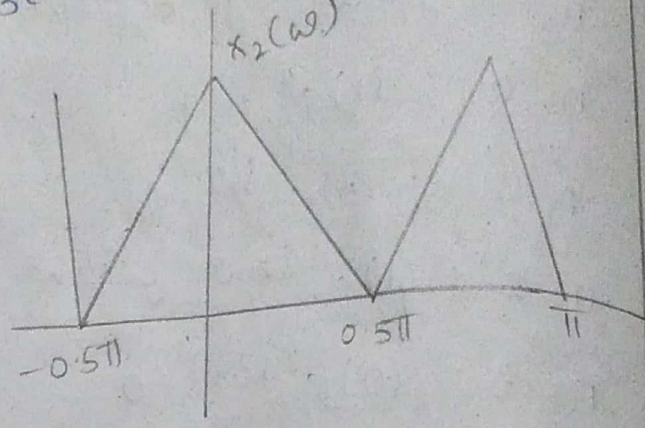
$X_1(\omega) = 2\cos(2\omega) + 2\cos(\omega) + 1$

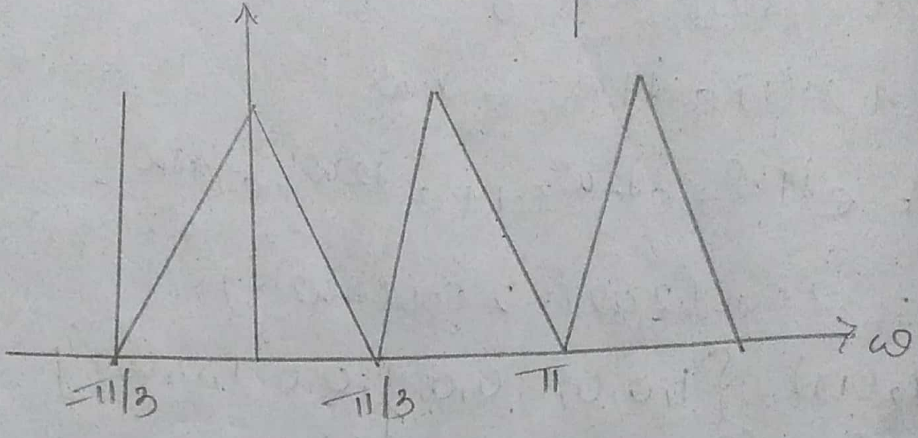
$X_2(\omega) = 2\cos(2\omega) + 2\cos(4\omega) + 1$

$X_3(\omega) = 2\cos(6\omega) + 2\cos(3\omega) + 1$

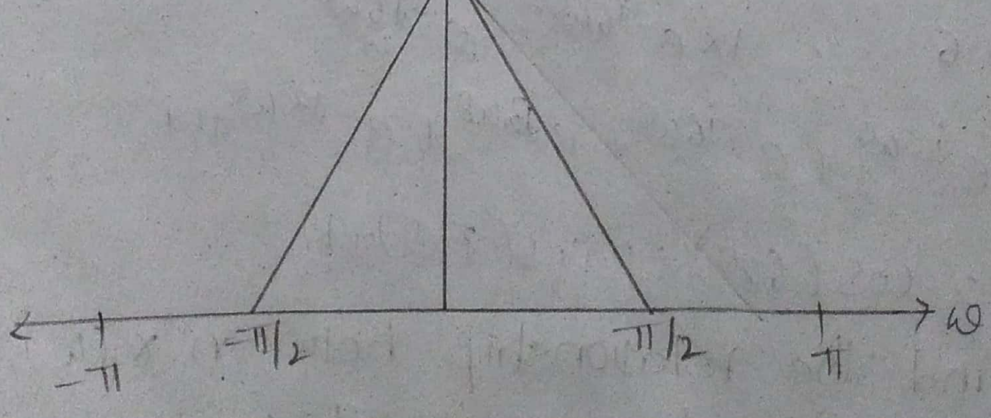
$X_2(\omega) = X_1(2\omega)$; $X_3(\omega) = X_1(3\omega)$







19 Let $x(n)$ be a signal with fourier transform as shown. Determine & sketch the fourier transforms of the following signals. Note that these signal sequence are obtained by amplitude modulation of a carrier $\cos\omega_c n$ and $\sin\omega_c n$ by the sequence $x(n)$)



a) $x_1(n) = x(n) \cdot \cos(\frac{\pi n}{4})$

$x(n)\cos(\omega_0 n) \xleftrightarrow{F.T} \frac{1}{2}\left[x(\omega-\omega_0) + x(\omega+\omega_0)\right]$

$x_1(\omega) = \frac{1}{2}\left[x(\omega+\frac{\pi}{4}) + x(\omega-\frac{\pi}{4})\right]$

limits: $-\pi + \frac{\pi}{4} = \frac{-3\pi}{4}$

$\frac{-\pi}{2} + \frac{\pi}{4} = \frac{-\pi}{4}$

$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$0 + \frac{\pi}{4} = \frac{\pi}{4}$

$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

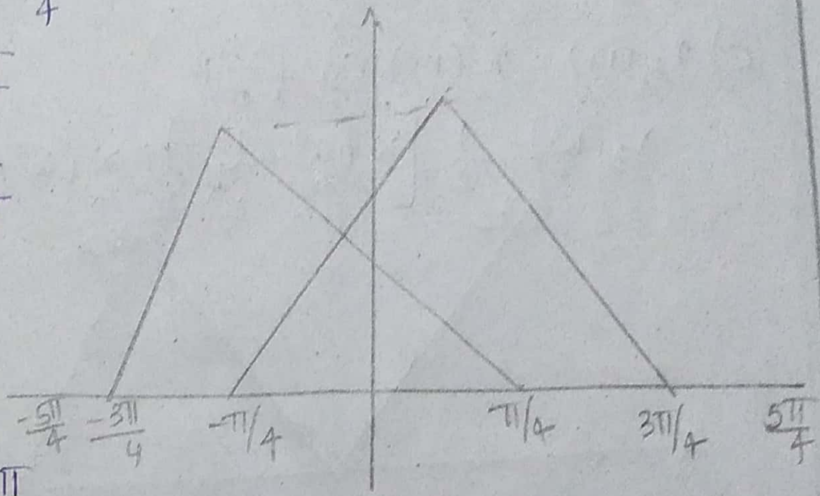
$-\pi - \frac{\pi}{4} = \frac{-5\pi}{4}$

$\frac{-\pi}{2} - \frac{\pi}{4} = \frac{-3\pi}{4}$

$0 - \frac{\pi}{4} = -\pi/4$

$\pi/2 - \frac{\pi}{4} = \frac{\pi}{4}$

$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$



b) $x_2(n) = x(n)\sin(\pi n/2)$

$x(n)\sin(n\pi/2) = \frac{1}{2j}\left[x(\omega-\omega_0) + x(\omega+\omega_0)\right]$

$x_2(\omega) = \frac{1}{2j}\left[x(\omega-\frac{\pi}{2}) + x(\omega+\frac{\pi}{2})\right]$

limits:–

$-\pi - \frac{\pi}{2} \Rightarrow \frac{-3\pi}{2}$

$\frac{-\pi}{2} - \frac{\pi}{2} \Rightarrow -\pi$

$\frac{-\pi}{2} + 0 = \frac{-\pi}{2}$

$\frac{\pi}{2} - \frac{\pi}{2} = 0$

$\pi - \frac{\pi}{2} = \frac{\pi}{2}$

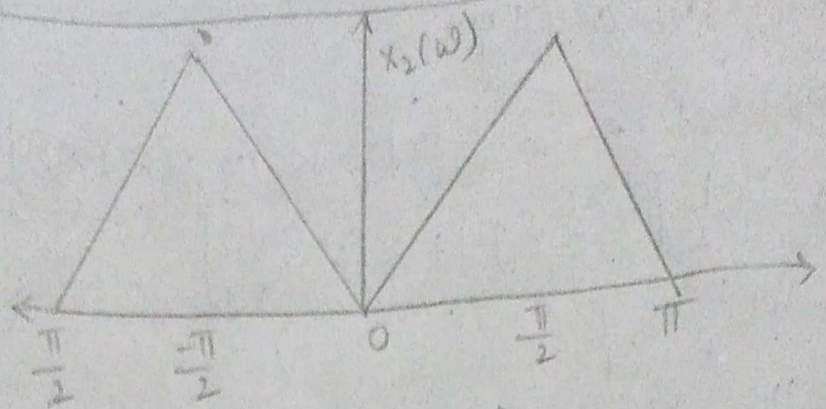
$-\pi + \frac{\pi}{2} = \frac{-\pi}{2}$

$\frac{-\pi}{2} + \frac{\pi}{2} = 0$

$0 + \frac{\pi}{2} = \frac{\pi}{2}$

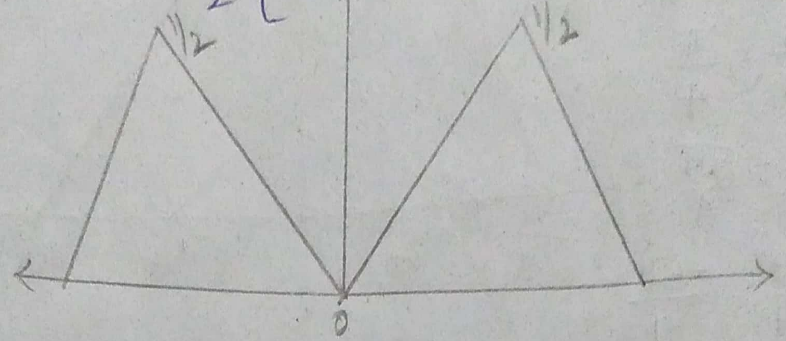
$\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$\pi + \frac{\pi}{2} = \frac{3\pi}{2}$



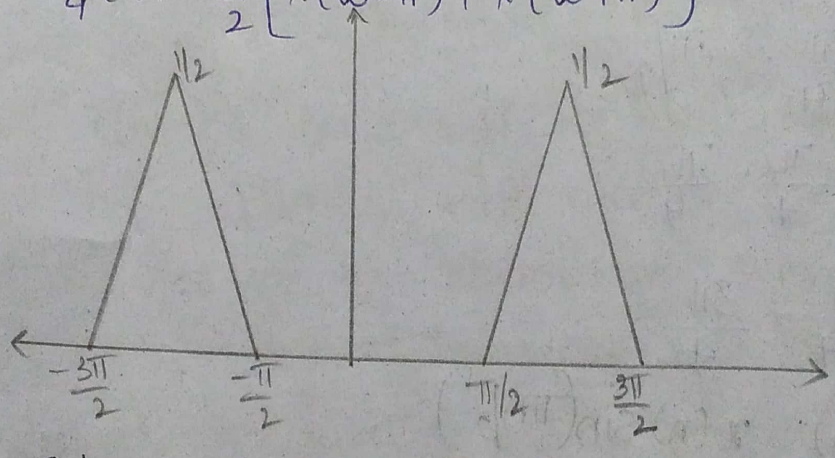
c) $x_3(n) = x(n)\cos\left(\frac{n}{2}\pi\right)$

$$X_3(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega + \frac{\pi}{2}\right)\right]$$



d) $x_4(n) = x(n)\cos\pi n$

$$X_4(n) = \frac{1}{2}\left[X(\omega - \pi) + X(\omega + \pi)\right]$$



20 Consider an Periodic signal $x(n)$ with FT $X(\omega)$. Show that the fourier series coefficients C_k^y of the periodic signal $y(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$ are given by $C_k^y = \frac{1}{N} X\left(\frac{2\pi}{N}k\right)$ $k = 0, 1, \ldots N-1$

$$C_k^y = \frac{1}{N}\sum_{n=0}^{N-1} y(n)\, e^{-j2\pi kn/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{k=-\infty}^{\infty} x(n - lN)\, e^{\frac{-j2\pi kn}{N}}\right.$$

$$\frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)e^{-j2\pi k(m+lN)/N}$$

But $\sum_{l=-\infty}^{\infty}\sum_{m=-ln}^{N-1-lN} x(m)e^{-j\omega(m+N)} = X(\omega)$

$$\therefore C_k^y = \frac{1}{N}X\left(\frac{2\pi k}{N}\right)$$

21 Prove that $X_N(\omega) = \sum_{n=-N}^{N}\frac{\sin\omega_c n}{\pi n}e^{-j\omega n}$ may be expressed as $X_N(\omega) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\frac{\sin[(2N+1)(\omega-\frac{\theta}{2})]}{\sin[(\omega-\theta)/2]}\,d\theta$

Let $x_N(n) = \frac{\sin\omega_c n}{\pi n}$ $\quad -N \le n \le N$

$$= x(n)\,w(n)$$

$x(n) = \frac{\sin\omega_c n}{\pi n}$ $\quad -\infty < n \le \infty$

$w(n) = 1$; $-N \le n \le N$

$= 0$; otherwise

$\frac{\sin\omega_c n}{\pi n} \xleftrightarrow{F.T} X(\omega)$

$= 1$; $|\omega| \le \omega_c$

0; otherwise

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\theta)W(\omega-\theta)\,d\theta$$

$$= \int_{-\omega_c}^{\omega_c}\frac{\sin(2N+1)(\omega-\theta)/2}{\sin(\omega-\theta)/2}\,d\theta$$

22 A signal $x(n)$ has the following fourier transform $X(\omega) = \frac{1}{1-ae^{-j\omega}}$ Determine the fourier transform of the following signals.

a) $x(2n+1)$

$\sum_{n=-\infty}^{\infty} x(2n+1)e^{-j\omega n}$ let $2n+1=l$

$\sum_{l=-\infty}^{\infty} x(l)e^{-j\omega\left[\frac{l-1}{2}\right]}$

$\sum_{l=-\infty}^{\infty} x(l)e^{-\frac{j\omega l}{2}} \cdot e^{j\omega/2}$

$e^{\frac{j\omega}{2}} \sum_{l=-\infty}^{\infty} x(l)e^{-j\omega l/2}$

$e^{j\omega/2} \sum_{l=-\infty}^{\infty} x(l)e^{-\frac{j\omega l}{2}}$

$e^{\frac{j\omega}{2}} \sum_{l=-\infty}^{\infty} x(n)e^{-j\frac{\omega}{2}n}$ l by 'n'

$= e^{j\omega/2}\, X\left(\frac{\omega}{2}\right)$

$= e^{j\omega/2} \frac{1}{1-ae^{-j(\omega/2)}}$

$= \frac{e^{j\omega/2}}{1-ae^{-j(\omega/2)}}$

b) $e^{\pi j/2}\, x(n+2)$

$e^{j2\omega}\, X\left(\omega-\frac{\pi j}{2}\right)$

$x(n) \longleftrightarrow X(\omega)$

$x(n+2) \longleftrightarrow e^{j2\omega}X(\omega)$

$e^{\pi j/2}\, x(n+2) \longleftrightarrow e^{j2\omega}X\left(\omega-\frac{\pi j}{2}\right)$

$e^{j2\omega} \cdot X\left(\omega-\frac{\pi j}{2}\right)$

c) $x(-2n)$

$x(n) \longleftrightarrow X(\omega)$

$x(2n) = X(\omega/2)$

$x(-2n) = X(-\omega/2)$

d) $x(n)\cos(0.3\pi n)$

$x(n)\cos(0.3\pi n) \longleftrightarrow \frac{1}{2}\left[X(\omega+0.3\pi)+X(\omega-0.3\pi)\right]$

$x(n)\cos\omega_0 n \longleftrightarrow \frac{1}{2}\left[X(\omega+\omega_0)+X(\omega-\omega_0)\right]$

e) $x(n) * x(n-1)$

$X(\omega)\cdot X^*(\omega)$

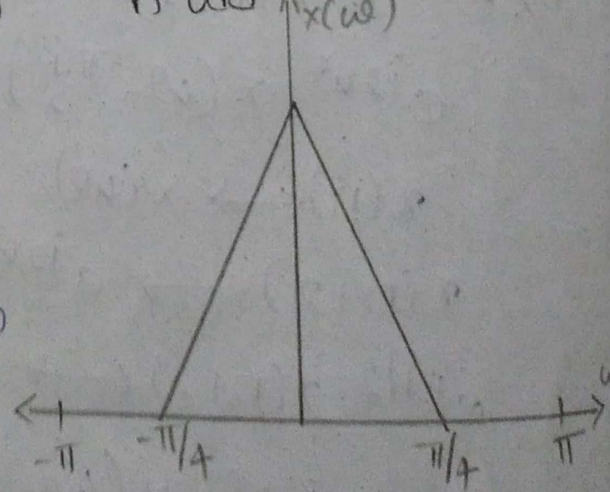
$\frac{1}{1-ae^{-j\omega}} \cdot \frac{1}{1-ae^{j\omega}}$

$\frac{1}{1-ae^{j\omega}-ae^{-j\omega}+a^2} \Rightarrow \frac{1}{1+a^2-2\cos\omega}$

23 From a discrete time Signal $x(n)$ with fourier transform $X(\omega)$ shown in figure. Determine and Sketch the fourier transform of the following signals.

Note that $y_1(n) = x(n)\,s(n)$ where $s(n) = \{\ldots 0,1,0,1,0,1,0,1\}$

a) $y_1(n) = \begin{cases} x(n) & \text{'n' even} \\ 0 & \text{'n' odd} \end{cases}$



b) $y_2(n) = x(2n)$

$y_2(n) = x(2n)$

$\frac{1}{2}(\omega) = \sum_n y_2(n)e^{-j\omega n}$

$= \sum_n x(2n)e^{-j\omega n}$

$= X\left(\frac{\omega}{2}\right)$

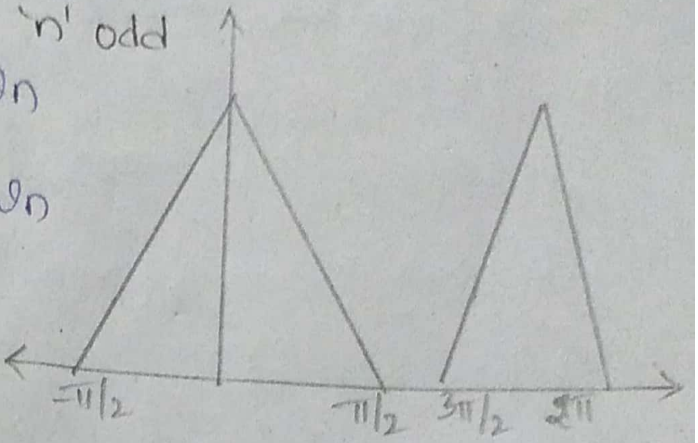
c) $y_3(n) = \begin{cases} x(n/2) & \text{'n' even} \\ 0 & \text{'n' odd} \end{cases}$

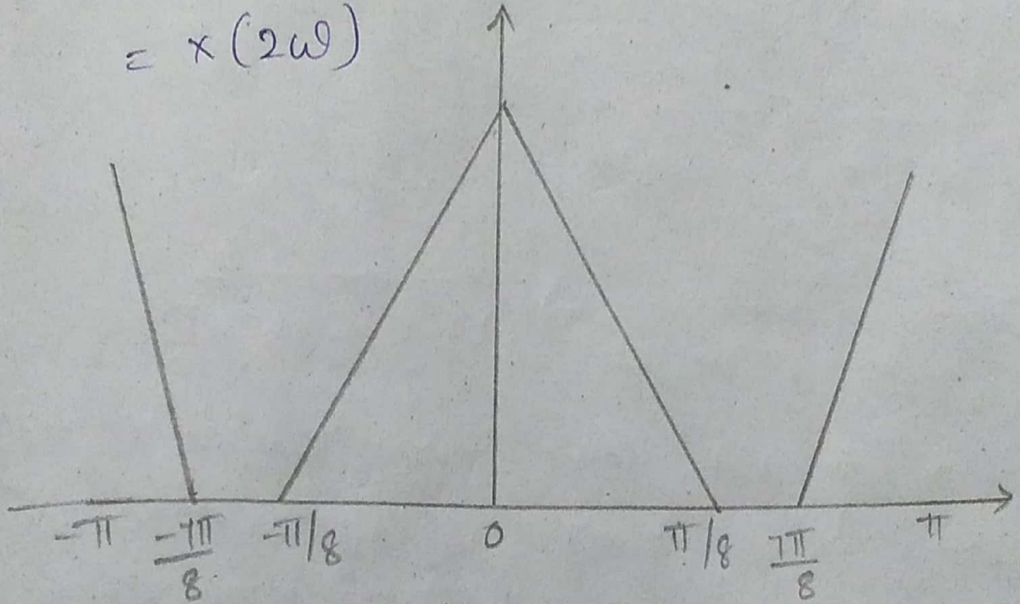
$Y_3(\omega) = \sum_n y_3(n) e^{-j\omega n}$

$= \sum_{n\,\text{even}} x\left(\frac{n}{2}\right) e^{-j\omega n}$

$= \sum_m x(m) e^{-j2\omega n}$

$= X(2\omega)$





$y_1(n) = \begin{cases} y_2(n/2) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$