Fidelity Analysis of Quantum Walk : A Monte Carlo Simulation using the Quantum Galton Board

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1. Introduction

In the current era of Noisy Intermediate-Scale Quantum (NISQ) computers, environmental noise and gate imperfections pose a significant challenge to achieving reliable computation. This project presents a detailed analysis of this challenge by simulating and evaluating the impact of noise on quantum circuits. This project uses a "Quantum Galton Board" as an intuitive analogy to model a one-dimensional quantum walk, enabling a clear and quantitative study of computational accuracy.

2. Methodology

The project utilized the PennyLane library to build and simulate three distinct quantum circuits, each designed to produce a different probability distribution.

2.1. Quantum Galton Board (Plinko) Circuit

This circuit models a quantum walk by simulating a particle's journey through a series of layers. The logic uses a for loop that iterates through each layer of the board. Inside the loop, the circuit performs two main actions:

1. **Prepare the Decision Qubit:** An `RY` gate is applied to a single decision qubit (the "coin"), determining the probability of a left or right turn.
2. **Record the Path:** A `CNOT` gate entangles the decision qubit with a dedicated output qubit for that layer, effectively recording the turn taken.

This circuit can be configured to produce different distributions by adjusting the `RY` gate's angle:

- A balanced RY gate (e.g., $\pi/2$) simulates an unbiased walk, leading to an approximation of a "Gaussian (Bell Curve)" distribution.
- A biased RY gate (e.g., $\pi/4$) skews the probabilities, resulting in a non-symmetric distribution.

2.2. Hadamard Quantum Walk Circuit

This circuit implements a standard one-dimensional quantum walk using a 'Hadamard' gate as the "coin flip." The output distribution of this circuit is distinct from the Galton Board model and is also explored in a noiseless environment.

2.3. Amplitude Encoding Circuit

To demonstrate the project's ability to handle other distributions, We used the 'qml.MottonenStatePreparation' function to directly encode a target "exponential distribution" into a quantum state. The circuit is verified to prepare this state accurately before being tested with noise.

3. Noise Model Analysis and Monte Carlo Simulation

To simulate the effects of a real-world quantum computer, We applied a "depolarizing channel" after each gate in all the three circuits. This model is more realistic because it adds errors after each gate, which accounts for gate-level imperfections rather than simply applying a single noise operation at the end. A **Monte Carlo simulation** technique is then used to gather a statistical sample of the circuit's output. The simulation runs the noisy quantum circuit thousands of times, and for each run, the final state of the position qubits is measured. The results of these measurements are then aggregated to form a final empirical probability distribution, which accounts for the stochastic uncertainty of the noisy system. Our Project also provides a framework for exploring the effects of various advanced and specific noise models on quantum circuits. Beyond the general `DepolarizingChannel`, the implementation explicitly includes `AmplitudeDamping` (simulating energy dissipation), `PhaseDamping` (modeling dephasing without energy loss), `BitFlip`(modeling a classical bit error where a qubit's state flips with a certain probability), and `PhaseFlip`(modeling a quantum error where the phase of a superposition state flips) channels.

4. Analysis & Findings

We compared the perfect (noiseless) results with the noisy results using a variety of key quantitative metrics to assess the degradation of the quantum state.

- **Fidelity:** Measures the similarity between the noisy and target states. A score of 1.0 indicates a perfect match.
- **Hellinger, KL, and Total Variation Distances:** These metrics quantify the statistical distance between the probability distributions. A score of 0.0 indicates a perfect match.

Our analysis gave a clear and direct relationship between noise and computational accuracy. As the `noise_prob_per_gate` was increased, the probability distribution generated by the noisy simulation increasingly deviated from the ideal target distribution. This degradation was quantitatively captured by our metrics, Fidelity systematically decreased from 1.0, while the distance metrics all increased from 0.0. The results visually demonstrated a flattening of the distribution peaks in the noisy simulation, indicating that the quantum state was losing its intended structure and becoming more randomized—a direct consequence of the depolarizing noise.

5. Conclusion & Future Work

This project successfully established a comprehensive framework for exploring quantum noise and its impact on circuit fidelity. Through an intuitive Quantum Galton Board analogy, We demonstrated the fragility of quantum computation in the NISQ era and provided a quantitative method for assessing this challenge. The Future work could include:

- **Error Mitigation:** Implementing and testing basic quantum error mitigation techniques to determine if they can successfully restore the fidelity of the noisy simulation.
- **Using More Complex Noise Models:** Investigate More Complex Noise Models like Thermal Relaxation (T1 and T2 Errors), Readout Error, and Stochastic Pauli Channels.