PERFORMANCE

Term	Remarks	Staff Member's Signature
I	Completed	
II	Completed	AV 01/02/2020

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Practical No. 1

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3ax} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \frac{\sqrt{3a+x} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\cancel{2}^{2}}{\cancel{2}} \frac{\sqrt{\cancel{a}}}{3\sqrt{\cancel{a}}}$

$= \frac{2}{3\sqrt{3}}$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$= \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$

$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{y}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$= \frac{1}{\sqrt{a+0}\,\sqrt{a+0} + \sqrt{a}}$

$= \frac{1}{\sqrt{a} + (\sqrt{a} + \sqrt{a})}$

$= \frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}$

3. $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By ~~con~~ substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \frac{\pi}{6})}{\pi - 6(h + \pi/6)}$

using
$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$
$\sin(A+B) = \sin A \cos B + \cos B \sin A$

$\lim_{h \to 0} \frac{\cosh \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6 - \sqrt{3}\,\sin h \cos \pi/6 + \cosh \sin \pi/6}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$

$\cos \pi/6 = \cos 30° = \sqrt{3}/2$
$\sin \pi/6 = \sin 30° = 1/\sqrt{2}$

$\lim_{h \to 0} \frac{\cosh \cdot \frac{\sqrt{3}}{2} - \sin h \, 1/\sqrt{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cosh \cdot 1/\sqrt{2}\right)}{\pi - 6h + \pi}$

$\lim_{h \to 0} \frac{\cos \frac{\sqrt{3}}{2} h - \sin \frac{h}{2} - \sin \frac{3h}{2} - \cos \frac{\sqrt{3}h}{2}}{-6h}$

$\lim_{h \to 0} \frac{+\sin \frac{4h}{2}}{+6h}$

$\lim_{h \to 0} \frac{\sin 4h}{{}_3 12h}$

$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = 1/3 \times 1 = 1/3$ //

4.) $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator and Denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^3+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^3+3)\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\,(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$$

After applying limit we get,
$= 4$

5. $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 \le x \le \pi/2$
$= \dfrac{\cos x}{\pi-2x}$, for $\frac{\pi}{2} < x < \pi$ } at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\quad \therefore f(\pi/2) = 0$

f at $x = \pi/2$ define

(i) $\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to\pi/2^+} \dfrac{\cos x}{\pi-2x}$

By substituting method
$x - \frac{\pi}{2} = h$
$x = h + \pi/2$
where $h \to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi-2(h+\frac{\pi}{2})}$$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi-2(\frac{2h+\pi}{2})}$$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{-2h}$$

using
$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$\lim_{h\to 0}\frac{\cos h\cdot\cos\pi/2-\sin h\cdot\sin\pi/2}{-2h}$$

$$\lim_{h\to 0}\frac{\cos h\cdot 0-\sin h}{-2h}$$

$$\lim_{h\to 0}\frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$\frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h} = \frac{1}{2}$$

b. $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2^-}\dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to\pi/2^-}\frac{2\sin x\cdot\cos x}{\sqrt{2\sin^2 x}}$$

using
$\sin 2x = 2\sin x\cos x$

$$\lim_{x\to\pi/2^-}\frac{2\sin x\cdot\cos x}{\sqrt{2}\,\sin x}$$

$$\lim_{x\to\pi/2^-}\frac{2\cos x}{\sqrt{2}}$$

$2/\sqrt{2}\ \lim_{x\to\pi/2^-}\cos x)$

$\therefore$ L.H.L $\ne$ R.H.L

f is not continuous at $x = \pi/2$

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5)
i) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$= x+3$ $\quad 3 \le x \le 6$
$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$
$\Big\}$ at $x=3$ & $x=6$

at $x=3$

(i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

(ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3)$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} \quad \frac{(\cancel{x-3})(x+3)}{(\cancel{x-3})}$

$\therefore$ LHL = RHL

f is continuous at $x = 3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

2. $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 6$

$\lim_{x\to 6^-} (x+3) = \cancel{3}+6 = 9$

$\therefore$ L.H.L $\neq$ R.H.L

function is not dei

6) i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x<0$
$= k$ $\quad x=0$
$\Big\}$ at $x=0$

soln:

f is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$2 \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$

$2 \lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$k = 8$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$= k$ $\quad x=0$
$\Big\}$ at $x=0$

soln

$f(x) = (\sec^2 x)^{\cot^2 x}$

using
$\tan^2 x - \sec^2 x = 1$
$\sec^2 x = 1+\tan^2 x$
$\cot^2 x = 1/\tan^2 x$

S8

$$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$$

we know that

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$

$= k$ $\quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

using

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1 - \tan \pi/3 \cdot \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

using $\tan \pi/3 = \tan 60 = \sqrt{3}$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$4/3 \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$$

$$\frac{\tan h}{h} = 1$$

$$= 4/3 \frac{1}{(1 - \sqrt{3}(0))}$$

$$= 4/3 (1/1)$$

$$= 4/3$$

6)
$f(x) = \frac{1-\cos 3x}{x \tan x}$ $x \neq 0$
$= 9$ $x = 0$ } at $x = 0$

$f(x) = \frac{1-\cos 3x}{x \tan x}$

$\lim_{x \to 0} \frac{2 \sin^2 \frac{3}{2} x}{x \tan x}$

$\lim_{x \to 0} \frac{\frac{2 \sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \frac{\tan x}{x^2} \times x^2}$

$= 2 \lim_{x \to 0} \frac{(3/2)^2}{1}$

$= 2 \times \frac{9}{4} = \frac{9}{2}$

$\lim_{x \to 0} f(x) = \frac{9}{2}$ $9 = f(0)$

$\therefore$ f is not continuous at $x = 0$

Redefine function

$f(x) = \begin{cases} \frac{1-\cos 3x}{x - \tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$

Now $\lim_{x \to 0} f(0) = f(x)$

f has removable discontinuity at $x = 0$

7) (i) $f(x) = \frac{(e^{3x}-1) \sin x^{0}}{x^2}$ $x \neq 0$
$= \pi/6$ $x = 0$ } at $x = 0$

$\lim_{x \to 0} \frac{(e^{3x}-1) \sin(\frac{\pi x}{180})}{x^2}$

$\lim_{x \to 0} \frac{e^{3x}-1}{x}$ $\lim_{x \to 0} \frac{\sin(\frac{\pi x}{180})}{x}$

$\lim_{x \to 0} 3 \cdot \frac{e^{3x}-1}{3x}$ $\lim_{x \to 0} \sin(\frac{\pi x}{180})$

$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x}$ $\lim_{x \to 0} \frac{\sin(\frac{\pi x}{180})}{x}$

$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is continuous at $x = 0$

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

$\therefore$ given

f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \lim_{x \to 0} \frac{(e^{x^2} - 1)(1 + \cos x)}{x^2}$

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$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$

multiply with 2 on Num and Donominator

$= 1 + 2 \times \frac{1}{4} 2 = 3/2 = f^0$

a) $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(x_0)$ is continous at $x = \pi/2$

$\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$\lim_{x\to\pi/2} \frac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$> \lim_{x\to\pi/2} \frac{(1+\sin x)}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1}{2\sqrt{2}+\sqrt{2}}$

$= \frac{1}{2(2\sqrt{2})} = 1/4\sqrt{2}$

$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$

Practical No. 2

TOPIC : Derivative

Q.1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan(a) - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \cancel{\times} \tan(a+h)\tan a}$$

formula : $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1 - \tan A \tan B$

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$$= \lim_{h\to 0} \frac{(a-a-h) - (1+\tan a \cdot \tan(a+h))}{h \times \tan(a+h)\ \tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\mathrm{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$

ii) $\mathrm{cosec}\, x$

$f(x) = \mathrm{cosec}\, x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\mathrm{cosec}\, x - \mathrm{cosec}\, a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula:

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos(a+a+h) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a\ \mathrm{cosec}\, a$$

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iii) sec x

$f(x) = \sec x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\sec x - \sec a}{\sec x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$

$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cos a \cos x}$

Put $x - a = h$

$x = a + h$

cos $x \to a$, $h \to 0$

$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

formula : $-2\sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$

$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \sin\left(\frac{-h}{2}\right)}{\cos a \cos(a+h) \times \frac{-h}{2}} \times -\frac{1}{2}$

$= -\frac{1}{2} \times -2 \frac{\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$

$= -\frac{1}{2} \times 2 \frac{\sin a}{\cos a \cdot \cos a}$

$= \tan a \sec a$

Q.2 If $f(x) = 4x + 1$, $x \le 2$

$= x^2 + 5$, $x > 0$ at $x = 2$,

then function is differentiable or not

solution :

LHD :

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$

$= \lim_{x \to 2^-} \frac{4x + 1 - 9}{x - 2}$

$= \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$

$= \lim_{x \to 2^-} \frac{4(\cancel{x-2})}{(x-2)} = 4$

$Df(2^-) = 4$

RHD :

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$

$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 2 + 2 = 4$

$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x = 2$

Q.3) If $f(x) = 4x+7$, $x < 3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$, then

find f is differentiable or not?

solution:

RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3+3+1)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6=9$$

$Df(3^+)=9$

LHD, $Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^+)=4$

RHD $\neq$ LHD

f is not differentiable at $x=3$

Q.4) If $f(x) = 8x-5$, $x \leq 2$
$= 3x^2-4x+7$, $x>2$ at $x=2$, Then

find f is differentiable or not

solution:

$f(2) = 8 \times 2 - 5 = 16-5 = 11$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2+4x+7-11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

CA

2HD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$$= 8$$

$Df(2^-) = 8$

$2HD = RHD$

f is differentiable at $x = 3$

PRACTICAL NO. 3

TOPIC: Application of Derivatives

1) Find the intervals in which function is increasing or decreasing

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards

(i) $y = 3x^2 - 2x^3$

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(iii) $y = x^3 - 27x + 5$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

(v) $y = 2x^3 + x^2 - 20x + 4$

Solution:

Q.1)

i) $f(x) = x^3 - 5x - 11$

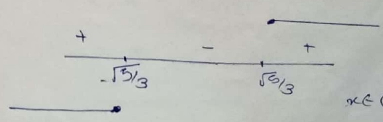
$\therefore f'(x) = 3x^2 - 5$

$\because$ f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x + \sqrt{5/3})(x - \sqrt{5/3}) > 0$



$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

+ − +

$-\sqrt{5/3}$ $\sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

(2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\because 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

+ − +

-2 $5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$x = -2, 5/3$

+ − +

-2 $5/3$

$x \in (-2, 5/3)$

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(4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$x = 3, -3$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-3, 3)$

(5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore (x-4)(x+1) > 0$

$x = 4, -1$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + 1x - 4 < 0$

$\therefore x(x-1) + 1(x-4) < 0$

$\therefore (x-4)(x+1)$

$\therefore x = -1, 4$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-1, 4)$

Q.2

i) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upwards if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12\left(\frac{6}{12} - x\right) > 0$

$x - \frac{1}{2} > 0$

$x > \frac{1}{2}$

$\because f''(x) > 0$

$x \in \left(\frac{1}{2}, \infty\right)$

(2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 + 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

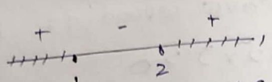
$\therefore x^2 - 3x + 2 > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

$x = 1, 2$



$x \in (-\infty, 1) \cup (2, \infty)$

(3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 64 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

5 $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

~~fi~~

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x > -1/6$

$\therefore f''(x) \neq 0 \quad (-1/6, \infty)$

$\therefore$ There is exist ~~no~~ in~~terva~~

21/12/19

Practical No. 4

Topic - Application of derivative & Newtons method

Q.1) Find minimum & maximum value of following function

(i) $f(x) = x^2 + \frac{16}{x^2}$

(ii) $f(x) = 3 - x 5x^3 + 3x^5$

(iii) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[2, 3]$

(iv) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

Q.2 Find the root of following derivation by Newton's method (Take 4 iteration only correct) upto decimal

(i) $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)

(ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$.

Q.1)

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6 = 8$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4 = 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

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(ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5 = 1$

$\therefore f''(-1) = 30(-1) + 60(-1)^3$

$= -30 - 60$

$= -30 < 0$ $\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12 = \underline{\underline{-3}}$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 36x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1 = -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 2(1)^3 - 3(-1)^2 + 12(+1)$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = 1$

and

f has minimum value -19 at $x = 2$

Q.2

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad \underline{x_0 = 0} \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newtons method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = \underline{0.1727}$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= \underline{-0.0829}$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= \underline{-55.9467}$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829 / 55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= \underline{0.0011}$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= \underline{-55.9393}$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.011 / 55.9393$

$= 0.1712$

$\therefore$ Root of the equation is $\underline{0.1712}$

(ii) $f(x) = x^3 - 4x - 9$ $\quad [2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.526 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$

$= \underline{2.7015}$

ॐ

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7168 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901 / 17.8943$

$= 2.7015 + 0.0050$

$= \underline{2.7065}$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.2x - 10$

$f(1) = (1)^3 - (1.8)(1)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = \underline{-2.2}$

Let $x_0 = 2$ be initial approximation By Newtons Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x - f(x_2) / f'(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.04230$

$= \underline{1.577}$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= \underline{0.6755}$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= \underline{-8.6162}$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= \underline{1.6592}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^3 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 1.6592 + 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(3) / f'(x_3)$

$= 1.8618 + \frac{0.0004}{7.6977}$

$= 1.6618$

12.

PRACTICAL NO. 5

TOPIC : INTEGRATION

Q.1 Solve the following integration

(i) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

(ii) $\int (4e^{3x}+1)\,dx$

(iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

(iv) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$

(v) $\int t^7 \times \sin(2t^4)\,dt$

(vi) $\int \sqrt{x}\,(x^2-1)\,dx$

(vii) $\int \dfrac{1}{x^3}\sin\left(\dfrac{1}{x^2}\right)dx$

(viii) $\int \dfrac{\cos x}{\sqrt[3]{\sin x}}\,dx$

(ix) $\int e^{\cos^2 x}\sin 2x\,dx$

(x) $\int \left(\dfrac{x^2-2x}{x^3-3x^2+1}\right)dx$

(i) $\int \dfrac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \dfrac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \dfrac{1}{\sqrt{(x+1)^2-4}}\,dx$

= Put $(x+1) = t$

$dx = \frac{1}{2} \times dt$

where $t = 1$

$t = x+1$

$\int \dfrac{1}{\sqrt{t^2-4}}\,dt$

using

$\int \dfrac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x+3}|)+C$

2) $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}dx + \int 1\,dx$

$= 4\int e^{3x}dx + \int 1\,dx$ # $\int e^{ax}dx = \frac{1}{a}e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + 3$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}dx$

$= \int 2x^2 dx - \int 3\sin(x)dx + \int 5x^{1/2}dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\,dx$

$= \int x^{5/2}dx + \int 3x^{1/2}dx + \int \frac{4}{x^{1/2}}dx$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}du = t^4 \frac{\times \sin(2t^4)}{8}\,du$

substituting t^4 with $u/2$

$= \int \frac{u/2 + \sin(u)}{8}\,du$

$= \int \frac{u \times \sin(u)}{2} / 8\,du$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= \frac{1}{16}\int u \times \sin(u)\,du$

$\int u\,dv = uv - \int v\,du$

where $u = v$

$dv = \sin(u) \times du \qquad v = -\cos(u)$

$du = 1\,du$

$= \frac{1}{16} \left(u \times (-\cos(u)) - \int -\cos(u)\,du\right)$

$= \frac{1}{16} \times \left(u \times (-\cos u)\right) + \int \cos(u)\,du$

$\int \cos x\,dx = \sin x$

$= \frac{1}{16} \times \left(u \times (-\cos(u)) + \sin(u)\right)$

Return the substitution $u = 2t^4$

$= \frac{1}{16} \times \left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4)\right)$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

(ii) $\int \sqrt{x}(x^2 - 1)\,dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}dx - \int x^{1/2}dx$

$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

(iii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cancel{\cos(x)}}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}dt$

$= \frac{1}{\sin x^{3/2}}\,dt$

$= \frac{1}{t^{2/3}}\,dt$

$$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3 - 1)t^{2/3 - 1}}$$

$$= \frac{-1}{-\frac{1}{3} t^{2/3 - 1}} = \frac{1}{\frac{1}{3} t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Return substitution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + C$$

(x) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$

put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\, dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3}\, dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)}\, dt = \int \frac{1}{3t}\, dt$$

$$= \frac{1}{3} \int \frac{1}{t}\, dt$$

$\# \int \frac{1}{x}\, dx = \ln|x|$

$$= \frac{1}{3} \times \ln|t| + C$$

$$= \frac{1}{3} \times \ln(|x^3 - 3x^2 + 1|) + C$$

7.) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$$

let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\frac{-2}{x^3}\, dx = dt$

$$I = -\frac{1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$$

$$= -\frac{1}{2} \int \sin t$$

$$= -\frac{1}{2}(-\cos t) + C$$

$$= \frac{1}{2} \cos t + C$$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$

9.) $\int e^{\cos^2 x} \cdot \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

Let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-\sin 2x \, dx = dt$

$I = \int - \sin 2x \, e^{\cos^2 x} \cdot dx$

$= - \int e^t \, dt$

$= e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

04/01/2020

PRACTICAL NO : 6

Topic : Application of Integration and numericals of Integration

Q.1) Find the length of following curve

1) $x = t \sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

$= \int_0^{2\pi} \sqrt{\quad}$

PRACTICAL-6

Aim: Application of Integration & Numerical Integrations

Q.1) Find the length of following curve

1.) $x = t\sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$

2.) $y = \sqrt{4-x^2}$ $x \in [-2, 2]$

3.) $y = x^{3/2}$ in $[0, 4]$

4.) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

5.) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2) Using Simpson's rule to solve the following

1) $\int_0^2 e^{x^2}\,dx$ with $x = 4$

2) $\int_0^4 x^2\,dx$ with $n = 4$

3) $\int_0^{\pi/2} \sqrt{\sin x}\,dx$ sin $n = 6$

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1.) $x = t\sin t \quad , \quad y = 1-\cos t \qquad t \in [0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1-\cos t^2)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt \quad - - - \sin t/_2 = \frac{1-\cos t}{2}$$

$$= \int_0^{2\pi} 2 \sin t/_2 \, dt$$

$$\left[\left(-4\cos\left(t/_2\right)\right)\right]_0^{2\pi} = (-4(\cos\pi) - (-4\cos 0))$$

$$= 4 + 4$$

$$= 8$$

2.) $y = \sqrt{4-x^2} \qquad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$$

$$= \frac{-2x}{\sqrt{4-x^2}}$$

$$L = \int_{-2}^{2} \sqrt{1 + (dy/dx)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x}}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{(2)^2 - (x)^2}}\, dx$$

$$= 2\left[\sin^{-1}\left(x/_2\right)\right]^2$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= 2\left[\left(\pi/_2\right) - \left(-\pi/_2\right)\right]$$

$$= 2\left[\pi/_2 + \pi/_2\right)$$

$$= 2\pi$$

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3] $y = x^{3/2}$ in $[0, 4]$

$$\frac{dy}{dx} = \frac{3}{2} \cdot x^{3/2 - 1}$$

$$L = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \left(\frac{3\sqrt{x}}{2}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{\left(1 + \frac{9x}{4}\right)}\, dx$$

$$= \int_0^4 \sqrt{\left(\frac{4+9x}{4}\right)}\, dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x}\, dx$$

$$= \frac{1}{2} \left[\frac{(4+9x)^{1/2+1}}{1/2 + 1} \right]_0^4 \cdot dx$$

$$= \frac{1}{2} \left[\frac{(4+9x)^{3/2}}{\cancel{3}/2} \times \frac{1}{9} \right]_0^4$$

$$= \frac{1}{27} \left[(9x+4)^{3/2} \right]_0^4$$

$$\frac{1}{27} \left[(4+0)^{3/2} - (4+36)^{3/2} \right]$$

$$= \frac{1}{27} \left[(4)^{3/2} - (40)^{3/2} \right]$$

$$= \frac{1}{27} \left[(4)^{3/2} - 8 \right]$$

(4) $x = 3\sin t$, $y = 3\cos t$, $t \in (0, 2\pi)$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t} \cdot dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\int_0^{2\pi} dt$$

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$= 3\,[x]_0^{2\pi}$

$= 3\,(2\pi - 0)$

$= L = 6\pi$ units

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in (1,2)$

Soln:

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{1 + \frac{(y^4-1)}{2y^2}}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} + \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{17}{3} \cdot \frac{7}{3} + \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$= \frac{17}{12}$ units

Solve the following using Simpson's rule:

1) $\int_0^2 e^{x^2} dx$ with $n = 4$

$\int_0^2 e^{x^2} dx = 16.4526$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

By Simpson's rule:

$\int_0^2 dx = \frac{1/2}{3}\left(y_0 + 4 \cdot 1/2 + 2\,1/2 + 4\,1/2 + 1/4\right]$

$= \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + (2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right)$

≈ 17.3536

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(2) $\int_0^4 x^2 \, dx$; $n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_a^b f(x)\,dx = \frac{\Delta x}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$

$= \frac{1}{3}[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)]$

$= \frac{1}{3}[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$

$= \frac{64}{3} \approx 21.333$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ $n = 6$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.875
	y_0	y_1	y_2	y_3	y_4	y_5

$\int_0^{\pi/3} \sqrt{\sin x}\,dx \approx \frac{\Delta x}{3}(y_0 + 4(y_1+y_3+y_5) + 2(y_2+y_4) + y_6)$

$= \frac{\pi/18}{3}(0 + 4(0.4167 + 0.707 + 0.8715) +$

$+2(0.584 + 0.801) + 0.930)$

≈ 0.681

Practical No. 7

Topic: Differential equation

Q.1 Solve the following differential equation

(1) $x\frac{dy}{dx} + y = e^x$

(2) $e^x\frac{dy}{dx} + 2e^x y = 1$

(3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

(4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

(5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

(6) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

(7) $\frac{dy}{dx} = \sin^2(x - y + 1)$

(8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

PRACTICAL NO. 7

Topic: Differential Equation

Q.1)

$x \frac{dy}{dx} + y = e^{x}$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^{x}}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^{x}}{x}$

$I.F = e^{\int P(x) dx}$

$= e^{\int 1/x \, dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x)(I.F) dx + C$

$= \int \frac{e^{x}}{x} \cdot x \cdot dx + C$

$= \int e^{x} dx + 1$

$x y = e^{x} + C$

2) $e^{x} \frac{dy}{dx} + 2e^{x} y = 1$

$\frac{dy}{dx} + \frac{2e^{x}}{e^{x}} y = 1/e^{x}$ $\quad$ ($\div$ by e^{x})

$\frac{dy}{dx} + 2y = 1/e^{x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $\quad Q(x) = e^{-x}$

$\int P(x) dx$

$I.F = e^{\int 2 dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F) dx + C$

$y \cdot e^{2x} \int e^{-x + 2x} dx + C$

$= \int e^{x} dx + C$

$y \cdot e^{2x} = e^{x} + C$

(iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \cos x / x^2$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{2/x\ x}$

$= \ln x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + C$

$= \int \frac{\cos x}{x^2} \cdot x^2 dx + C$

$= \int \cos x + C$

$x^2 y = \sin x + C$

(iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both sides)

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x)\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{3/x\ x}$

$= e^{\ln x^3}$

$I.F = x^3$

$Y.(I.F) = \int Q(x)(I.F)\,dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + C$

$= \int \sin x\,dx + C$

$x^3 y = -\cos x + C$

(v) $e^{2x}\frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$Y.(I.F) = \int Q(x)(I.F)\,dx + C$

$= \int 2x e^{-2x} e^{2x} dx + C$

$= \int 2x\,dx + C$

$y e^{2x} = x^2 + C$

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(vi) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$

$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + C$

$\log|\tan x - \tan y| = C$

$\tan x \cdot \tan y = e^C$

(vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

differentiating on both sides

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + C$

$\tan(x+y-1) = x + C$

(viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

05.

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3\,dx$$

$$= \int \frac{v+1}{v}\,dx + \int \frac{1}{\cancel{2}v+\cancel{x}1}\,dv = 3x$$

$$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$$

$$= v + \log|x| = 3x + C$$

$$= 2x + 3y + \log|2x+3y+1| = 3x + C$$

$$\therefore = 3y = x - \log|2x+3y+1| + C$$

AK 11/01/2020

PRACTICAL NO. 8

Aim: Eulers method

(1) using Eulers method find the following

i) $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$

ii) $\frac{dy}{dx} = 1 + y^2$ $y(0) = 0$ $h = 0.2$ find $y(1)$

iii) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$

iv) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$ for $h = 0.5$ & $h = 0.25$

v) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h = 0.2$

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7) $\frac{dy}{dx} = y + e^{x} - 2 \quad y(0) = 2 \quad h = 0.5$ find $y(2)$

Sol^n:

$f(x) = y + e^{x} - 2$

$y(0) = 2$

$y(x_0) = y_0$

$\therefore x_0 = 0 \quad y_0 = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

8) $\frac{dy}{dx} = 1 + y^2 \quad y(0) = 0 \quad h = 0.2$ find $y(1)$

Solution:

$x_0 = 0$

$y(0) = 0$

$y(x_0) = y_0$

$x_0 = 0 \quad y_0 = 0 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.412	0.9234
4	0.8	0.9234	1.8526	1.2939
5	0.1	1.2939		

$\therefore y(1) = 1.2439$

3.) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$

Solution:

$y(0) = 1$

$y(x_0) = y(0) y_0$

$\therefore x_0 = 0$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7699	1.5051
5	1	1.5051		

$y(1) = 1.5051$

4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ for $h = 0.5$ & $h = 0.25$

Solution:

$y(1) = 2$

$x_0 = 1$ $y_0 = 2$ $h = 0.75$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	1.875
2	2	7.875		

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9966
4	2	299.996		

$\therefore y(2) = 299.9966$

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5. $\frac{dy}{dx} = \sqrt{xy} + 2$ find 0.2

$y(1) = 1$

Solution

$y(1) = 1$

$y(x_0) = y_0$

$\therefore x_0 = 1 \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

11/01/22

Practical No. 9

Topic = limits & partial order derivative

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(1) Evaluate the following limits

i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$ (ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

2) Find f_x, f_y for each of the following f

i) $f(x,y) = xy\, e^{x^2+y^2}$ (ii) $f(x,y) = e^x \cos y$

iii) $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

3) Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

4) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2 - xy}{x^2}$ (ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

5) Find the linearization of $f(x,y)$ at given point

i) $f(x,y) = \sqrt{x^2 + y^2}$ at $(1,1)$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

Soln:

① $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

$\Rightarrow \lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-4)+(-4)^2-1}{-4(-1)+5}$

$= -61/9$

② $\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{(x+3y)}$

$\Rightarrow \lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\therefore$ At $(-2,0)$ Denominator $\neq 0$

$\therefore$ By applying circuit

$= \frac{(0+1)(4+0-8)}{2}$

$= -4/2$

$= -2$

③ $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$\Rightarrow$ At $(1,1,1)$ Denominator $= 0$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

on applying limit

$= \frac{1+1(1)}{1^2}$

$= 1$

Q2)

① $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \frac{\partial f}{\partial x}(x,y)$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= y\cancel{e}^{x^2+y^2}(2x)$

$\therefore f_x = 2xy^{e^{x^2+y^2}}$

$f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial}{dx}(xyx^{2+y^2})$

$= x e^{x^2+y^2} (2y)$

$\therefore \; f_y = 2y x e^{x^2+y^2}$

(2) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x} f(x,y)$

$= \frac{\partial}{\partial x} (e^x \cos y)$

$f_x = e^x \cos y$

$f_y = \frac{\partial f(x,y)}{\partial y}$

$= \frac{\partial (e^x \cos y)}{\partial y}$

$f_y = -e^x \sin y$

(3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial f}{\partial y} (x^2, y)$

$= \frac{\partial}{\partial x} f(x^3y^2 - 3x^2y + y^3 + 1)$

$= 3x^2y^2 - 6xy$

$f_y = \frac{\partial f}{\partial y} (x,y)$

$= \frac{\partial}{\partial y} (x^3y^2 - 3x^2y + y^3 + 1)$

$= 2x^3y - 3x^2 + 3y^2$

(3) $f(x,y) = \frac{2x}{1+y^2}$

$\Rightarrow f_x = \frac{\partial}{\partial x} f(x,y)$

$= \frac{\partial}{\partial x} f(2x/1+y^2)$

$= \frac{1+y^2 \frac{\partial (2x)}{\partial x} - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2}{(1+y^2)^2}$

$= 2/1+y^2$

At $f(0,0)$

$= \frac{2}{1+0} = 2$

$f_y = \frac{\partial}{\partial y} f(2x/1+y^2)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$

$= \frac{-4(0)(0)}{(1+0)^2} = 0$

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Q.4) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\Rightarrow f_x = \frac{x^2 \frac{d}{dx}(y^2-xy) - (y^2-xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$

$= \frac{-x^2y - 2x(y^2-xy)}{x^4}$

$f_y = 2y - \frac{x}{x^2}$

$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2-xy)}{x^4}\right)$

$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{\partial}{\partial x}(x^4)}{x^4}$

$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$ --- (i)

$f_{yy} = \frac{\partial}{\partial y}\left(2y - \frac{x}{x^2}\right)$

$= \frac{2-0}{x^2} = \frac{2}{x^2}$ --- (ii)

$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$

$= \frac{x^2 - 4xy + 2x^2}{x^4}$ --- (iii)

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$f_{yx} = \frac{\partial}{\partial x}\left(2y - \frac{x}{x^2}\right)$

$= \frac{x^2 \cdot \frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{x^2 - 4xy - 2x^2}{x^4}$ ----- (iv)

from (iii) & (iv)

$f_{xy} = f_{yx}$.

Q.2 $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x+1}$

$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 \frac{\cancel{(x^2+} \frac{\partial}{\partial x}(2x) - 2x\left(\frac{\partial}{\partial x}(x^2-1)\right)}{(x^2+1)^2}$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ ... (ii)

$f_{yy} = x^2 \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2$ ... (2)

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 \cdot \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$ ------ (3)

$f_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$= 12xy$ ------ (4)

from (3) & (2)

$f_{xy} = f_{yx}$

(3) $f(x,y) = \sin(xy) + e^{x+y}$

$\Rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)(y) + e^{x+y}(e)$

$= -y^2\sin(xy) + e^{x+y}$ --- (1)

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x).1\ e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ --- (2)

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) - \cos(xy) + e^{x+y}$ --- (3)

$f_{yx} = \frac{\partial}{\partial x}(x\cos(yx) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ --- (4)

from (3) & (4)

$f_{xy} \neq f_{yx}$

$f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = 1/\sqrt{2}$

f_y at $(1,1) = 1/\sqrt{2}$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}((x-1) + (y-1))$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

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(ii) $f(x,y) = 1 - x + y\sin x$ at $x = \pi/2$

$f(\pi/2, 0) = 1 - \pi/2 + 0 + \sin(\pi/2)$

$f(\pi/2, 0) = 2 - \pi/2$

$f_x = -1 + y\cos x$ $f_y = \sin x$

$f(x)(\pi/2, 0) = \sin(\pi/2)$

$L(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2)$
$+ f_y(\pi/2, 0)(y - 0)$

$= \frac{2-\pi}{x} + (-1)(x - \pi/2) + 1(y))$

$L(x,y) = 1 - x + y$

(iii) $f(x,y) = \log x + \log y$ at $x = (1,1)$

$f(1,1) = \log 1 + \log 1$
$= 0$

$f_x = 1/x$ $f_y = 1/y$

$f_x = (1,1) = 1$ $f_y\ (1,1) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1)$
$+ f_y(1,1)(y-1)$

$= 0 + 1(x-1) + (y-1)$

$L(x,y) = x + y - 2$

AV
25/01/2020

Topic :- Directional derivative, gradient vector & maxima, minima tangent & normal vector.

Q-1) Find the directional derivative of the following function of given points & in the direction of vector.

(i) $f(x,y) = x + 2y - 3$, $a = (1,-1)$, $u = 3i - j$

(ii) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

(iii) $f(x,y) = 2x + 3y$, $a(1,2)$, $u = 3i + 4j$

Q-2) Find gradient vector for the following func. at given pts.

(i) $f(x,y) = x^y + y^x$, $a = (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$ $a = (1,-1)$

(iii) $f(x,y) = xyz - e^{x+y+z}$, $a(1,-1,0)$

Q-3) Find the equation of tangent & normal to each of the following curves at given pts.

(1) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

(2) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

Q-4) Find the equation of tangent & normal line to each of following surfaces.

(1) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

(2) $3xyz - x - y + z = -4$ at $(1,-1,2)$

Q.5) Find the local minima & maxima for the following function

(1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

(2) $f(x,y) = 2x^4 + 3x^2y - y^2$

(3) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$.

Solutions

Q.1)

i) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$ $\quad u = 3i - j$

Here $u = 3i - j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{10}\right)$$

$$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{10}\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{10}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + h}{h}$$

$$D_u f(a) = 1/\sqrt{10}$$

ii) $f(x) = y^2 - 4x + 1$ $\quad a(3,4)$ $\quad u = i + 5j$

Here $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$$= \left(1/\sqrt{26}, 5/\sqrt{26}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25}{26}h^2 + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

(iii) $2x+3y$ $\quad a=(1,2)$, $u=(3i+4j)$

Here $u=3i+4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1)+3(2) = 8$

$f(a+hu) = f(1,2)+h\left(\frac{3}{5},\frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right)+3\left(2+\frac{4h}{5}\right)$

$= 2+\frac{6h}{5}+6+\frac{12h}{5}$

$= \frac{18h}{5}+8$

$D_u f(a) = \lim_{h\to 0} \frac{\frac{18\cancel{h}}{5}+\cancel{8}-\cancel{8}}{\cancel{h}}$

$= 18/5$

Q.2)

i) $f(x,y) = x^y + y^x$ $\quad a=(1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1}, y^x \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1}x)\cdot y^2$ $\quad a=(1,-1)$

$f_x = \frac{1}{1+x^2}\cdot y^2$

$f_y = 2y\tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\quad a=(1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0)-e^{1+(-1)+0}, (1)(0)-e^{1+(-1)+0}, (1)(-1)-e^{(1)+(-1)+0})$

$= (0-e^0, 0-e^0, -1-e^0)$

$= (-1,-1,-2)$

Q-3)

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \; 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1, y_0 = 0$

eqⁿ of tangent

$f(x)(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 \; 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^{0} \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0 \rightarrow$ eqⁿ of tangent

eqⁿ of Normal:

$ax + by + c = 0$

$bx + ay + d = 0$

$= 1(1) + 2(y) + d = 0$

$= y + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eqⁿ of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0 \rightarrow$ It is required eqⁿ of tangent

eqⁿ of Normal:

$= ax + by + c = 0$

$= bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$

at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q.4) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= -2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x (x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y (x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z (x_0, y_0, z_0) = -2(1) + 2 = 0$

eq^n of tangent

$f_x (x - x_0) + f_y (y - y_0) + f_z (z - z_0) = 0$

$= 4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$= 4x + 3y - 11 = 0$ → this is required eq^n of tangent

eq^n of normal at $(4, 3, -1)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{0}$$

(ii) $3xyz - x - y + z = -4$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, 1, -2)$ $\therefore x_0 = 1, y_0 = 1, z_0 = 2$

$f_x = (x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y \, f(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$

$f_z (x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x - 1) + 5(y + 1) - 2(z - 2) = 0$

$-7x + 7 + 5y - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → this is required equation of tangent

Equation of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$ //

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Q5)

1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \longrightarrow (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \longrightarrow (2)$

Multiply eq^n (1) with (2)

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substitute value of x in eq^n (1)

$2(0) - y = -2$

$-y = -2$ $\therefore y = 2$

$\therefore$ critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 - 0 - 8$

$= -4$

2) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 - 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \longrightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0 \longrightarrow (2)$

Multiply eq^n (1) with (3)

(2) with (4)

$12x^2 - 9y = 0$

$-12x^2 - 8y = 0$

$17y = 0$

$\therefore y = 0$

Substitute value of y in eq^n (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

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Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s^2)$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(Nothing to say)

$f(x,y)$ at $(0,0)$

$= (0)^4 + 3(0)^2 \cdot 0 - 0$

$0 + 0 0$

$= 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = -2/2 = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = -8/-2$

$\therefore y = 4$

$\therefore$ critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$

AR 01/02/2020