MOS DECEMBER 2017

PART B

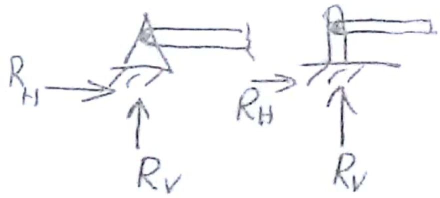
4 a) Name and explain the various types of beam supports indicating the reaction components diagrammatically.

Various types of beam supports are-

- Roller support.

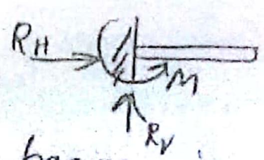
In a roller support, there is only one reaction from the support which is perpendicular to the plane on which roller is resting

- Hinged or pin jointed support



In this kind of support, there can be two types of support reactions One is perpendicular to the plane of support and the other reaction is parallel to the plane of support.

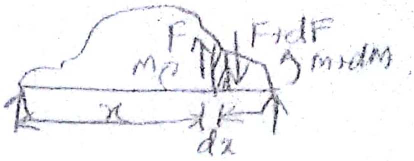
- Fixed Support | Built in Support



In this kind of support, as the beam is built into the support, there can be three support reactions formed One vertical reaction, one horizontal reaction and one ~~normal~~ moment reaction.

4 b Relationship between bending moment and shear force.

consider an element at a distance x from the support of the beam.



Let 'w' be intensity of load per unit length.

The equilibrium equation for the element can be written as

F - Shear Force
M - Bending moment

$\Sigma V = 0 \Rightarrow \quad F - w\,dx - F - dF = 0$

$$\therefore \frac{dF}{dx} = -w$$

ie Rate of change of shear force F is equal to load intensity.

$\Sigma M = 0 \Rightarrow \quad F \times dx + M - M - dM - w\frac{dx^2}{2} = 0$

dx being small, its square $\frac{dx^2}{2}$ will be too small and hence can be approximated to 0.

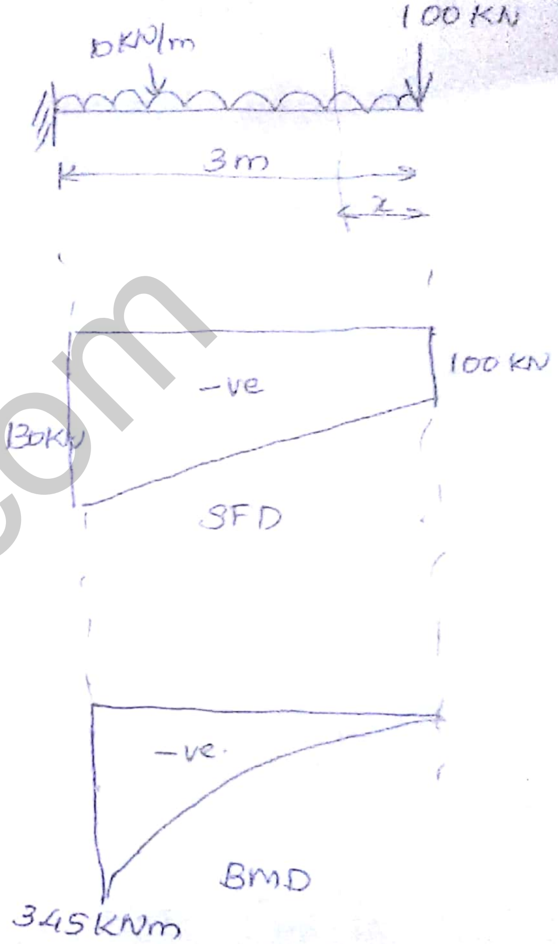
$$\therefore \quad F\,dx - dM = 0$$

$$\text{or} \quad \frac{dM}{dx} = F$$

ie Rate of change of bending moment is shear force.

4. c Draw SFD & BMD for a cantilever of span 3m with a UDL of 10 KN/m on the entire span and a point load of 100 KN at the free end

Sign Convention



Calculation of SF

$SF_x = -100 - 10x$ (st. line variation)

$SF_{x=0} = -100$ KN

$SF_{x=3} = -100 - 10 \times 3 = -130$ KN.

Calculation of BM

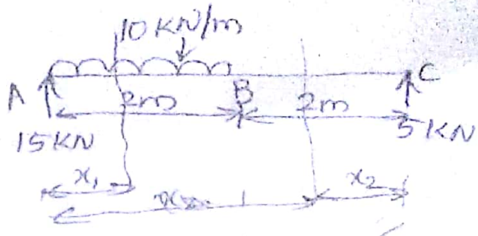
$BM_x = -100x - 10 \times \frac{x^2}{2}$ (Parabolic variation)

$BM_{x=0} = 0$

$BM_{x=3} = -100 \times 3 - \frac{10}{2} \times 3^2 = -345$ KNm

5a Draw SFD and BMD for a SSB of span 4m, with a udl of 10 KN/m on the left half of its span.

Calculation of support reaction

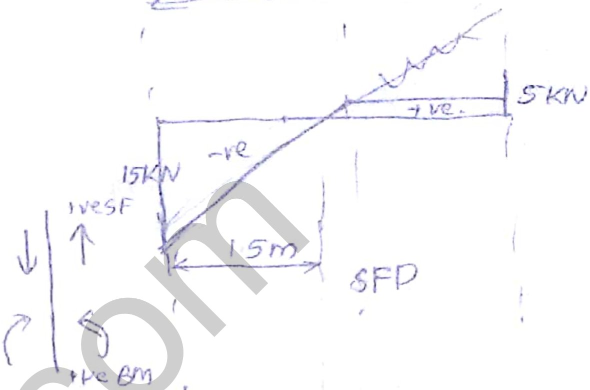


$R_A + R_C = 10 \times 2 = 20$ KN

$R_A \times 4 - 10 \times 2 \times 3 = 0$

$R_A = \frac{60}{4} = 15$ KN

$R_C = 20 - 15 = 5$ KN.



Calculation of Shear Force

Part AB

$SF_{x_1} = -15 + 10x_1$ (st. line)

$SF_{x_1=0} = -15$ KN

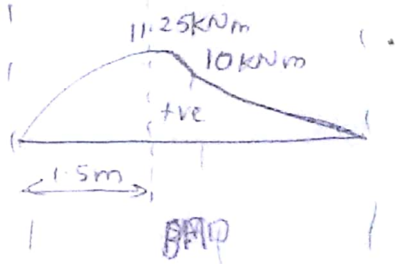
$SF_{x_1=2} = -15 + 10 \times 2 = 5$ KN

$SF_{x_1=4} =$

At one position, $SF_{x_1} = 0$

ie $-15 + 10x_1 = 0$

$x_1 = \frac{15}{10} = 1.5$ m.



Part BC

$SF_{x_2} = 5$ KN.

Calculation of Bending Moment

Part AB

$M_{x_1} = 15 \times x_1 - 10 \frac{x_1^2}{2}$ (Parabola)

$M_{x_1=0} = 0$

$M_{x_1=2m} = 15 \times 2 - \frac{10}{2} \times 2^2$

$= 30 - 20 = 10$ KNm

$M_{x_1=1.5m} = 15 \times 1.5 - \frac{10}{2} \times 1.5^2 = 11.25$ KNm

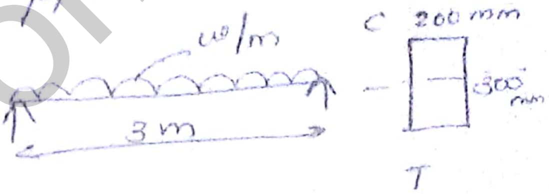
Part BC

$M_{x_2} = 5x_2$ (st. line)

$M_{x_2=0} = 0$

$M_{x_2=2} = 5 \times 2 = 10$ KNm

5.b A cantilever beam with span 3m and cross section 200 x 300 mm is to carry a udl on entire span. If tensile stress is limited to 3 MPa, what is the maximum UDL that can be applied on the beam?



$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_T = \frac{M\, y_T}{I}$$

$$M = \frac{w l^2}{8} = \frac{w \times 3000^2}{8}$$

$$y_T = \frac{300}{2} = 150$$

$$I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12}$$

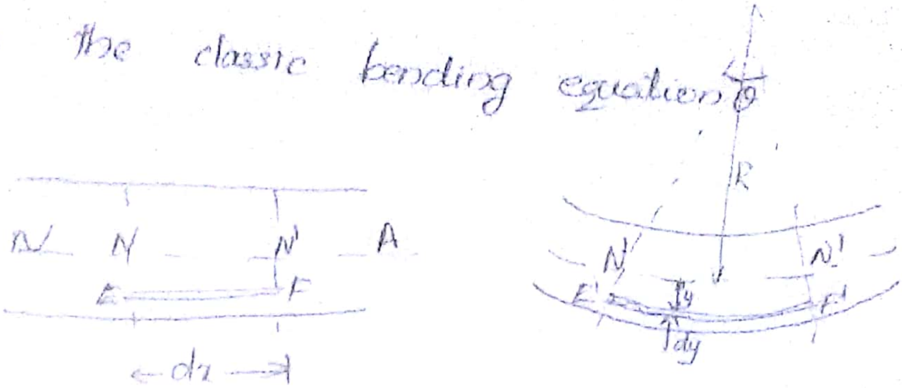
$$= 4.5 \times 10^7 \text{ mm}^4$$

$$3 = \frac{\frac{w \times 3000^2}{8} \times 150}{4.5 \times 10^7}$$

$$w = 8 \text{ N/mm}$$

$$= \underline{\underline{8 \text{ KN/m}}}.$$

2a Derive the classic bending equation



Original length $dx = NN$

After bending, NN will not change.

$NN = N'N' = dx = R\theta$

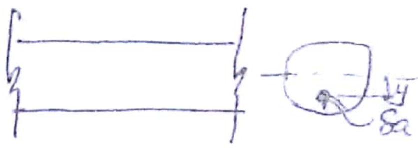
After bending $E'F' = (R+y)\theta$

Increase in length $= E'F' - EF = y\theta$

Strain in layer $EF = \frac{y\theta}{EF} = \frac{y\theta}{R\theta} = \frac{y}{R}$

stress in layer, $\sigma = E\frac{y}{R}$

$$\frac{\sigma}{y} = \frac{E}{R}$$



stress in the area $\sigma = \frac{Ey}{R}$

Force " $F = \sigma\, \delta a = \frac{Ey}{R}\delta a$

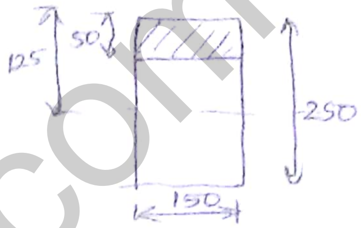
Moment of this resisting force about NA $= \frac{E}{R} y\, \delta a \cdot y = \frac{E}{R} y^2 \delta a$

ie $M' = \frac{E}{R} I$ where I is the centroid moment of inertia.

$$\therefore \frac{\sigma}{y} = \frac{E}{R} = \frac{M}{I}$$

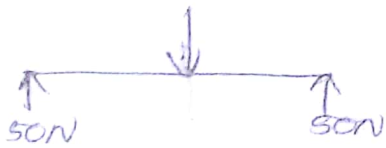
This is the classical bending formula.

6b A SSB of rectangular section of span 2.5m has cross section 150 mm x 250 mm and carries a central point load of 100 N. Find the shear stress at 50 mm below the top edge of the middle cross section.



$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12} =$

$= 195312500 \ mm^4$



SF at Middle = 50kN

$F = 50 \ kN$

$A\bar{y} = 50 \times 150 \times \left(\frac{50}{2} + 75\right)$

$= 750000$

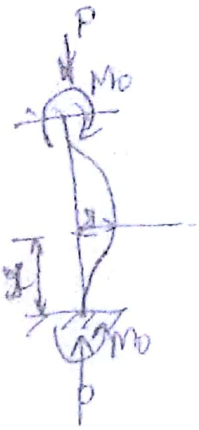
$b = 150 \ mm$

$\tau = \frac{FA\bar{y}}{Ib} = \frac{50 \times 10^3 \times 750000}{195312500 \times 150}$

$= \frac{1.28}{10^3} \ N/mm^2$

$= \underline{\underline{1.28 \ kN/m^2}}$

8b Derive an expression for Euler's buckling load for a column fixed at both ends



Let M_0 be the fixed end moment

$$M_Q = M_0 - Py = EI\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} = \frac{M_0}{EI} = \frac{M_0}{P}\cdot\frac{P}{EI}$$

Solution of the equation is

$$y = c_1 \cos x\sqrt{\frac{P}{EI}} + c_2 \sin x\sqrt{\frac{P}{EI}} + \frac{M_0}{P}$$

At $x=0$, $y=0$, $\frac{dy}{dx}=0$.

$$\therefore\ 0 = c_1 + \frac{M_0}{P} \Rightarrow c_1 = \frac{-M_0}{P}$$

$$\therefore\ y = \frac{-M_0}{P}\cos x\sqrt{\frac{P}{EI}} + c_2 \sin x\sqrt{\frac{P}{EI}} + \frac{M_0}{P}.$$

At $x=0$, $\frac{dy}{dx}=0 \Rightarrow$

$$0 = 0 + c_2\sqrt{\frac{P}{EI}} \Rightarrow c_2 = 0.$$

$$y = \frac{-M_0}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

At $x=l$, $y=0$.

$$0 = -\frac{M_0}{P}\cos l\sqrt{\frac{P}{EI}} + \frac{M_0}{P}$$

ie $$\frac{M_0}{P}\cos l\sqrt{P/EI} = \frac{M_0}{P}$$

$$\cos l\sqrt{P/EI} = 1 \Rightarrow$$

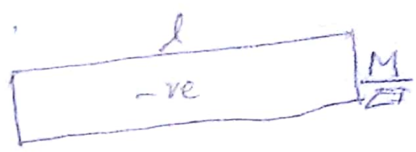
$$l\sqrt{\frac{P}{EI}} = 0, 2\pi, 4\pi, \ldots$$

ie $l\sqrt{P/EI} = 2\pi$ or $P = \frac{4\pi^2 EI}{l^2}$ is Euler's buckling load

9a Using moment area method, find the deflection and slope at the free end of a cantilever applied with a couple at the free end

Change in slope at B $\Big\}$ = Area of $\frac{M}{EI}$ diagram

$$= \frac{M}{EI} \times l$$



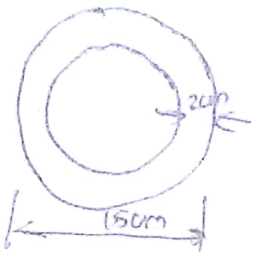
$\frac{BMD}{EI}$

Change in tangential deviation at B from A = Moment of Area of $\frac{M}{EI}$ diagram about A

$$= \frac{M}{EI} l \times \frac{l}{2} = \frac{Ml^2}{2EI}$$

9b Find the buckling load given by Rankine's formula for a tubular steel hinged at both ends, 6m long having outer dia 15cm and thickness 2cm. Given $E = 2 \times 10^5 \ N/mm^2$, $\sigma_c = 567 \ N/mm^2$ and Rankine's constant $a = \frac{1}{1600}$. For what length of column does the Euler's formula ceases to apply?

$l_e = 6000 \ mm \qquad A = 81.64 \ mm^2$



$$I = \frac{\pi(d_o^4 - d_i^4)}{64} = \frac{\pi}{64}\left[15^4 - 11^4\right]$$

$$= 1765.465 \ mm^4$$

$$\text{Rankine's load} = \frac{\sigma_c A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{567 \times \frac{\pi}{4}\left[15^2 - 11^2\right]}{1 + \frac{1}{1600} \times \left(\frac{6000}{\sqrt{\frac{1765.465}{\pi/4(15^2 - 11^2)}}}\right)^2}$$

$$= \frac{46289.88}{1041.46}$$

$$= 44.45 \ N$$

$$\text{Euler's buckling load} = \frac{\pi^2 EI}{l_e^2}$$

$$= 96.70 \ N$$

$$44.45 = \frac{\pi^2 EI}{l_e^2}$$

$$l_e = \underline{\underline{8.85 \ m}}$$