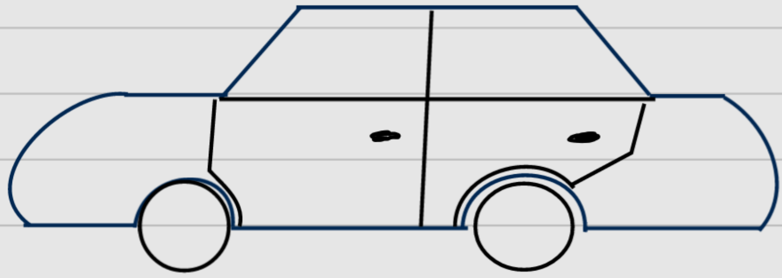
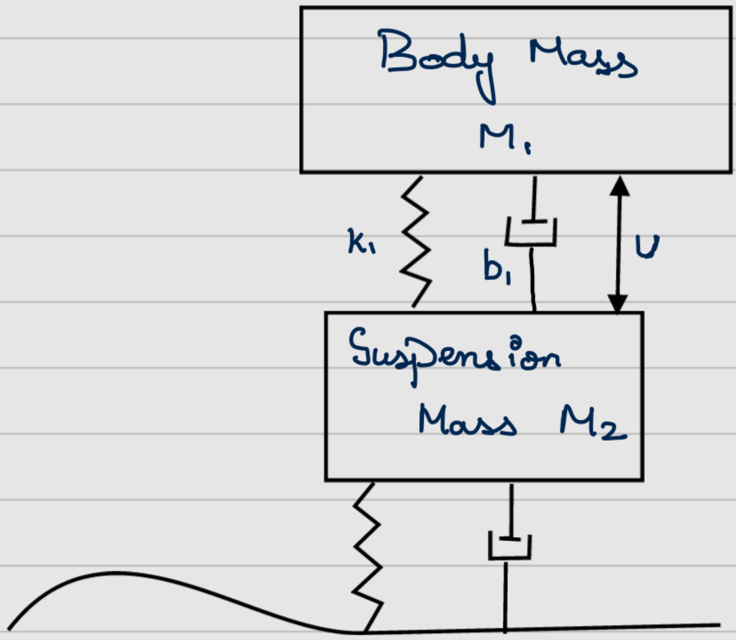


Model of Car Suspension System (1/4 Car)



$M_1 \rightarrow$ 1/2 car body mass - 500 kg

$M_2 \rightarrow$ Suspension mass - 120 kg

$K_1 \rightarrow$ Spring Constant of Suspension System - 50,000 N/m.

$K_2 \rightarrow$ Spring Constant of wheel and tire - 300,000 N/m.

$b_1 \rightarrow$ Damping Constant of Suspension System - 150 N·s/m

$b_2 \rightarrow$ Damping Constant of wheel and tire - 8,000 N·s/m

$U \rightarrow$ Control force.

Equations of Motion

$$M_1 \ddot{x}_1 = -b_1(\dot{x}_1 - \dot{x}_2) - K_1(x_1 - x_2) + u$$

$$M_2 \ddot{x}_2 = b_1(\dot{x}_1 - \dot{x}_2) + K_1(x_1 - x_2) + b_2(\dot{w} - \dot{x}_2) + K_2(w - x_2) - U$$

Transfer function model

Assuming that all of the initial conditions are 0

$$(M_1 s^2 + b_1 s + K_1)\, X_1(s) - (b_1 s + K_1)\, X_2(s) = U(s)$$

$$-(b_1 s + K_1)\, X_1(s) + (M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2))\, X_2(s) = (b_2 s + K_2) W(s) - U(s)$$

$$\begin{bmatrix} (M_1 s^2 + b_1 s + K_1) & -(b_1 s + K_1) \\ -(b_1 s + K_1) & (M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)) \end{bmatrix} \begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \begin{bmatrix} U(s) \\ (b_2 s + k_2) W(s) - U(s) \end{bmatrix}$$

$$A = \begin{bmatrix} (M_1 s^2 + b_1 s + K_1) & -(b_1 s + K_1) \\ -(b_1 s + K_1) & (M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)) \end{bmatrix}$$

$$\Delta = \det \begin{bmatrix} (M_1 s^2 + b_1 s + K_1) & -(b_1 s + K_1) \\ -(b_1 s + K_1) & (M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)) \end{bmatrix}$$

Find the inverse of matrix A and then multiply with inputs U(s) and W(s) on the right hand side as follow:

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)) & (b_1 s + K) \\ (b_1 s + K_1) & (M_1 s^2 + b_1 s + k_1) \end{bmatrix} \begin{bmatrix} U(s) \\ (b_2 s + k_2) W(s) - U(s) \end{bmatrix}$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (M_2 s^2 + b_2 s + k_2) & (b_1 b_2 s^2 + (b_1 k_2 + b_2 k_1) s + k_1 k_2 \\ -M_1 s^2 & (M_1 b_2 s^3 + (M_1 k_2 + b_1 b_2) s^2 + (b_1 k_2 + b_2 k_1) s + k_1 k_2 \end{bmatrix} \times \begin{bmatrix} U(s) \\ W(s) \end{bmatrix}$$

$$G_1(s) = \frac{X_1(s) - X_2(s)}{U(s)} = \frac{(M_1 + M_2) s^2 + b_2 s + k_2}{\Delta}$$

$$G_2(s) = \frac{X_1(s) - X_2(s)}{W(s)} = \frac{-M_1 b_2 s^3 - M_1 k_2 s^2}{\Delta}$$