07_homework_svm

February 2, 2018

1 Programming assignment 7: SVM

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline

        from sklearn.datasets import make_blobs

        from cvxopt import matrix, solvers
```

1.1 Your task

In this sheet we will implement a simple binary SVM classifier.

We will use CVXOPT http://cvxopt.org/ - a Python library for convex optimization. If you use Anaconda, you can install it using

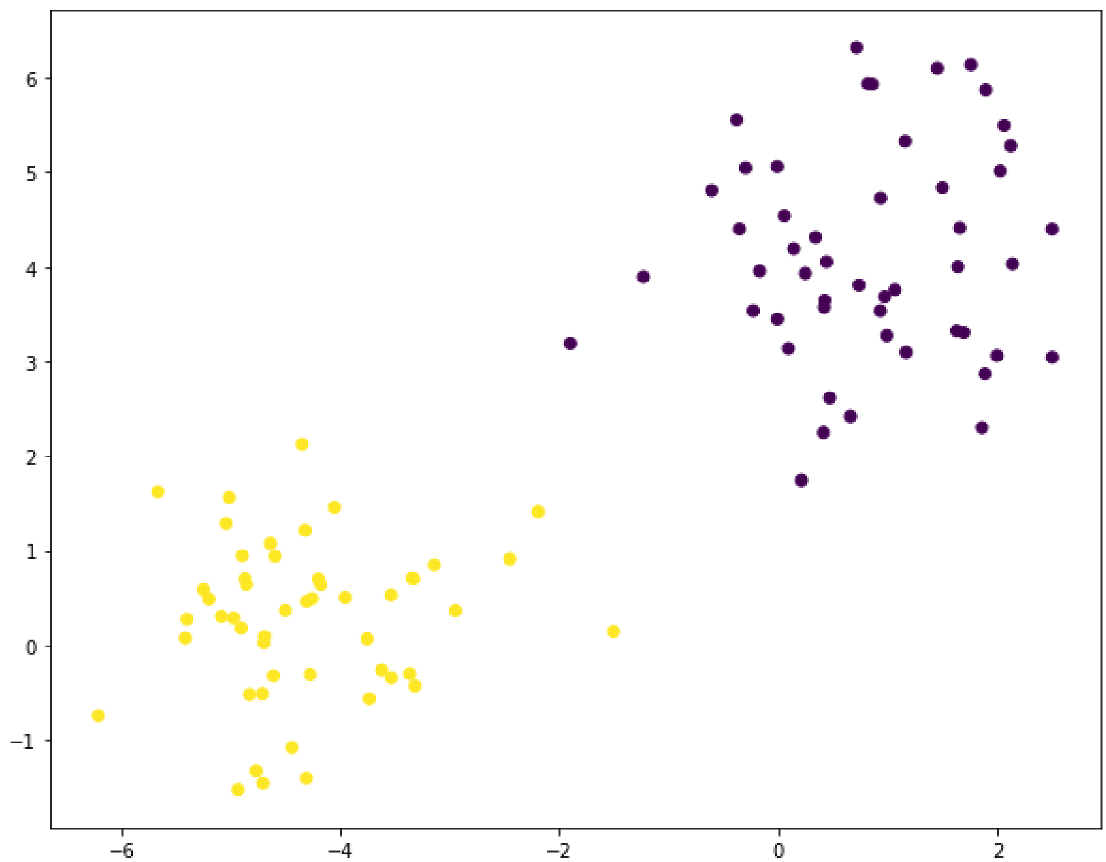
```
conda install cvxopt
```

As usual, your task is to fill out the missing code, run the notebook, convert it to PDF and attach it you your HW solution.

1.2 Generate and visualize the data

```
In [2]: N = 100  # number of samples
        D = 2  # number of dimensions
        C = 2  # number of classes
        seed = 3  # for reproducible experiments

        X, y = make_blobs(n_samples=N, n_features=D, centers=2, random_state=seed)
        y[y == 0] = -1  # it is more convenient to have {-1, 1} as class labels (instead of {0,
        y = y.astype(np.float)
        plt.figure(figsize=[10, 8])
        plt.scatter(X[:, 0], X[:, 1], c=y)
        plt.show()
```



1.3 Task 1: Solving the SVM dual problem

Remember, that the SVM dual problem can be formulated as a Quadratic programming (QP) problem. We will solve it using a QP solver from the `CVXOPT` library.

The general form of a QP is

$$\min_{\mathbf{x}} \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x}$$

$$\text{subject to} \quad \mathbf{G}\mathbf{x} \preceq \mathbf{h}$$

$$\text{and} \quad \mathbf{A}\mathbf{x} = \mathbf{b}$$

where $\preceq$ denotes "elementwise less than or equal to".

Your task is to formulate the SVM dual problems as a QP and solve it using `CVXOPT`, i.e. specify the matrices $\mathbf{P}, \mathbf{G}, \mathbf{A}$ and vectors $\mathbf{q}, \mathbf{h}, \mathbf{b}$.

```
In [3]: def solve_dual_svm(X, y):
            """Solve the dual formulation of the SVM problem.

            Parameters
```

```
    ----------
    X : array, shape [N, D]
        Input features.
    y : array, shape [N]
        Binary class labels (in {-1, 1} format).

    Returns
    -------
    alphas : array, shape [N]
        Solution of the dual problem.
    """
    # TODO
    # These variables have to be of type cvxopt.matrix
    P = matrix(np.multiply(np.dot(y[:,None],np.transpose(y[:,None])),
                           np.dot(X,np.transpose(X))))
    q = matrix(-1*np.ones((N,1)))
    G = matrix(np.diag(-1*np.ones(y.shape[0])))
    h = matrix(np.zeros(y.shape[0]))
    A = matrix(np.transpose(y[:,None]))
    b = matrix([0.0])
    solvers.options['show_progress'] = False
    solution = solvers.qp(P, q, G, h, A, b)
    alphas = np.array(solution['x'])
    return alphas
```

1.4 Task 2: Recovering the weights and the bias

```
In [4]: def compute_weights_and_bias(alpha, X, y):
            """Recover the weights w and the bias b using the dual solution alpha.

            Parameters
            ----------
            alpha : array, shape [N]
                Solution of the dual problem.
            X : array, shape [N, D]
                Input features.
            y : array, shape [N]
                Binary class labels (in {-1, 1} format).

            Returns
            -------
            w : array, shape [D]
                Weight vector.
            b : float
                Bias term.
            """
            w = np.dot(np.transpose(X),np.multiply(alpha,y[:,None]))
            d=np.nonzero(alpha > 1e-4)
```

```
    d=np.array((d[0],d[1]))
    d=np.transpose(d)
    b=np.sum(np.subtract(y[d[:,0],None],np.dot(X[d[:,0],:],w)))/d.shape[0]
    return w, b
```

1.5 Visualize the result (nothing to do here)

```
In [5]: def plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b):
            """Plot the data as a scatter plot together with the separating hyperplane.

            Parameters
            ----------
            X : array, shape [N, D]
                Input features.
            y : array, shape [N]
                Binary class labels (in {-1, 1} format).
            alpha : array, shape [N]
                Solution of the dual problem.
            w : array, shape [D]
                Weight vector.
            b : float
                Bias term.
            """
            plt.figure(figsize=[10, 8])
            # Plot the hyperplane
            slope = -w[0] / w[1]
            intercept = -b / w[1]
            x = np.linspace(X[:, 0].min(), X[:, 0].max())
            plt.plot(x, x * slope + intercept, 'k-', label='decision boundary')
            # Plot all the datapoints
            plt.scatter(X[:, 0], X[:, 1], c=y)
            # Mark the support vectors
            support_vecs = (alpha > 1e-4).reshape(-1)
            plt.scatter(X[support_vecs, 0], X[support_vecs, 1], c=y[support_vecs], s=250, marker
            plt.xlabel('$x_1$')
            plt.ylabel('$x_2$')
            plt.legend(loc='upper left')
```

The reference solution is

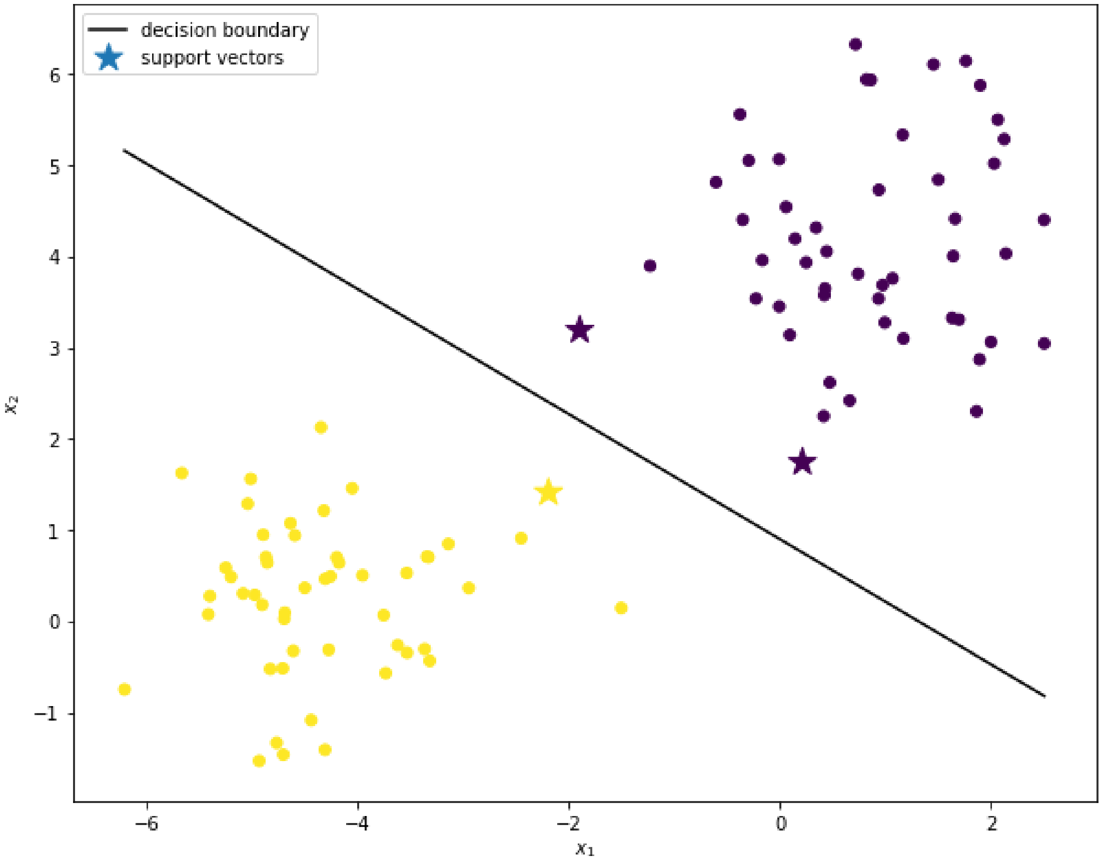
```
w = array([[-0.69192638],
           [-1.00973312]])

b = 0.907667782
```

Indices of the support vectors are

```
[38, 47, 92]
```

```
In [6]: alpha = solve_dual_svm(X, y)
        w, b = compute_weights_and_bias(alpha, X, y)
        plot_data_with_hyperplane_and_support_vectors(X, y, alpha, w, b)
        plt.show()
        d=np.nonzero(alpha > 1e-4)
        d=np.array((d[0],d[1]))
        d=np.transpose(d)
        print("My solution is\n w = array({})\n\n b = {}\n\n Indices of support vectors are:\n\n
```




```
My solution is
 w = array([[-0.69192638]
 [-1.00973312]])

 b = 0.9076677822380542

 Indices of support vectors are:

 [38 47 92]
```