04_homework_linear_regression

February 2, 2018

1 Programming assignment 4: Linear regression

```
In [1]: import numpy as np

        from sklearn.datasets import load_boston
        from sklearn.model_selection import train_test_split
```

1.1 Your task

In this notebook code skeleton for performing linear regression is given. Your task is to complete the functions where required. You are only allowed to use built-in Python functions, as well as any `numpy` functions. No other libraries / imports are allowed.

1.2 Load and preprocess the data

I this assignment we will work with the Boston Housing Dataset. The data consists of 506 samples. Each sample represents a district in the city of Boston and has 13 features, such as crime rate or taxation level. The regression target is the median house price in the given district (in $1000's).

More details can be found here: http://lib.stat.cmu.edu/datasets/boston

1.3 Task 1: Fit standard linear regression

```
In [2]: def fit_least_squares(X, y):
            """Fit ordinary least squares model to the data.

            Parameters
            ----------
            X : array, shape [N, D]
                (Augmented) feature matrix.
            y : array, shape [N]
                Regression targets.

            Returns
            -------
            w : array, shape [D]
                Optimal regression coefficients (w[0] is the bias term).
```

```
    """
    X_XTranspose_product=np.dot(np.transpose(X),X);
    X_XTranspose_product_as_matrix=np.asmatrix(X_XTranspose_product);
    X_XTranspose_product_inverse=X_XTranspose_product_as_matrix.getI();
    X_XTranspose_product_inverse_asndarray = np.squeeze(np.asarray(
                                                X_XTranspose_product_inverse));
    w=np.dot(X_XTranspose_product_inverse_asndarray,np.dot(np.transpose(X),y));
    return w
```

1.4 Task 2: Fit ridge regression

```
In [3]: def fit_ridge(X, y, reg_strength):
            """Fit ridge regression model to the data.

            Parameters
            ----------
            X : array, shape [N, D]
                (Augmented) feature matrix.
            y : array, shape [N]
                Regression targets.
            reg_strength : float
                L2 regularization strength (denoted by lambda in the lecture)

            Returns
            -------
            w : array, shape [D]
                Optimal regression coefficients (w[0] is the bias term).

            """
            X_size=np.shape(X);
            lambda_plus_X_XTranspose=np.add(np.multiply(
                                            reg_strength,np.identity(X_size[1])),
                                            np.dot(np.transpose(X),X))
            lambda_plus_X_XTranspose_as_matrix=np.asmatrix(lambda_plus_X_XTranspose);
            lambda_plus_X_XTranspose_inverse=lambda_plus_X_XTranspose_as_matrix.getI();
            lambda_plus_X_XTranspose_inverse_asndarray = np.squeeze(np.asarray(
                                                lambda_plus_X_XTranspose_inverse));
            w=np.dot(lambda_plus_X_XTranspose_inverse_asndarray,np.dot(np.transpose(X),y));
            return w;
```

1.5 Task 3: Generate predictions for new data

```
In [4]: def predict_linear_model(X, w):
            """Generate predictions for the given samples.

            Parameters
            ----------
            X : array, shape [N, D]
```

```
        (Augmented) feature matrix.
    w : array, shape [D]
        Regression coefficients.

    Returns
    -------
    y_pred : array, shape [N]
        Predicted regression targets for the input data.

    """
    y_pred=np.dot(X,w);
    return y_pred
```

1.6 Task 4: Mean squared error

```
In [5]: def mean_squared_error(y_true, y_pred):
            """Compute mean squared error between true and predicted regression targets.

            Reference: `https://en.wikipedia.org/wiki/Mean_squared_error`

            Parameters
            ----------
            y_true : array
                True regression targets.
            y_pred : array
                Predicted regression targets.

            Returns
            -------
            mse : float
                Mean squared error.

            """
            y_true_size=np.shape(y_true);
            mse=np.sum(np.power(np.subtract(y_pred,y_true),2))/y_true_size[0];
            return mse;
```

1.7 Compare the two models

The reference implementation produces * MSE for Least squares ≈ **23.98** * MSE for Ridge regression ≈ **21.05**

You results might be slightly (i.e. ±1%) different from the reference soultion due to numerical reasons.

```
In [6]: # Load the data
        np.random.seed(1234)
        X , y = load_boston(return_X_y=True)
        X = np.hstack([np.ones([X.shape[0], 1]), X])
```

```python
test_size = 0.2
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=test_size)

# Ordinary least squares regression
w_ls = fit_least_squares(X_train, y_train)
y_pred_ls = predict_linear_model(X_test, w_ls)
mse_ls = mean_squared_error(y_test, y_pred_ls)
print('MSE for Least squares = {0}'.format(mse_ls))

# Ridge regression
reg_strength = 1
w_ridge = fit_ridge(X_train, y_train, reg_strength)
y_pred_ridge = predict_linear_model(X_test, w_ridge)
mse_ridge = mean_squared_error(y_test, y_pred_ridge)
print('MSE for Ridge regression = {0}'.format(mse_ridge))
```

```
MSE for Least squares = 23.984307611777403
MSE for Ridge regression = 21.051487033772723
```