Sequences and Infinite Series

(Problem Set)

- One problem to always consider is the proof to theorems / mathematical facts. Such is the case with the very many convergence tests. Take for instance the limit comparison test. You might have a good intuitive sense as to what the test means. But just that isn't a proof. –

Q_1 : Prove that the Harmonic-Series is divergent.

Q_2 : Use limit comparison test for:

ⓐ $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n-3}$

ⓑ $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-n}}{n^2}$

ⓒ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n+1}$

ⓓ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4+e^n}$

ⓔ $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

ⓕ $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$

ⓖ $\displaystyle\sum_{n=1}^{\infty} \frac{n\sin^2 n}{n^3+1}$

ⓗ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+1}{n2^n-1}$

Q_3 : Ratio - Test

ⓐ $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

ⓑ $\sum_{n=1}^{\infty} \frac{1}{r^n n!}$, $r>0$

ⓒ $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

ⓓ $\sum_{n=0}^{\infty} \frac{2^n}{n^3+1}$

Q_4 : Alternating Series test

ⓐ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

ⓑ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

Q_5 : Test convergences:

ⓐ $\sum (-1)^n \left(1 + \frac{1}{n^2}\right)$

ⓑ $\sum \frac{(-1)^{n-1} \tan^{-1}(1/n)}{n^2}$

(c) $\sum_{n\geq 1}^{\infty} \frac{8^n}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{n(n+1)}{\sqrt{n^3+2n^2}}$

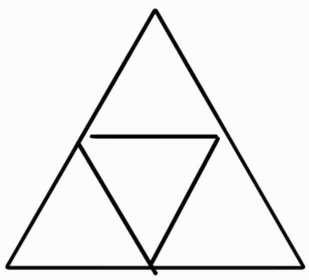
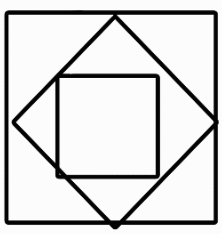
(d) $\sum_{n=1}^{\infty} \frac{n\,2^n}{3^n}$

(f) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(g) $\sum_{n} \frac{n^3}{e^{\sqrt{n}}}$

(h) $\sum_{k} \frac{(-1)^k}{\ln k}$

Q_6: Explore sequen. describing embedded shapes such as:

 , 

(Think fractals..)

Try finding the infinite sum.

Q_7: Tower of Hanoi;

Search the web for the problem of the tower of Hanoi.

Try working out the least amount of steps needed to solve the puzzle as a function of n (no. of discs).

- Note: Not explicitly linked to series but a good counting problem that requires intuition and counting, well. _

Q_8: Consider the itterative scheme;

$$F_n = F_{n-1} + F_{n-2}$$

with $F_0 = 0$; $F_1 = 1$

Compute $\lim_{n \to \infty} \frac{F_{n+1}}{F_n}$.