Calculus and Analytical Geometry 2

Integration Practice Test

18th April, 2025

Time allowed: 1 hour

Q1) Compute these indefinite integrals:

a) $\int x^2 \cos(1+x^3)\, dx$

b) $\int \frac{dx}{(1-x^2)^{3/2}}$

c) $\int \left(\varphi + \frac{2}{\sin(2\varphi)}\right) d\varphi$

d) $\int 3x\cos(2x)\, dx$

e) $\int \frac{x^2}{\sqrt{x-1}}\, dx$

f) $\int \frac{x+3}{(x-1)(x^2-4x+4)}\, dx$

g) $\int \frac{\sqrt{1-x^2}}{x^2}\, dx$

h) $\int \tan^3(x)\sec^3(x)\, dx$

i) $\int \frac{1}{\sin^{-1}(x)\sqrt{1-x^2}}\, dx$

j) $\int \frac{x+2}{x+1}\, dx$

Q2) Compute these definite integrals:

a) $\int_e^1 \frac{\sqrt{\ln x}}{x}\, dx$

b) $\int_0^3 \frac{x}{(x^2-1)^{2/3}}\, dx$

c) $\int_0^\infty x^2 e^{-2x}\, dx$

d) $\int_{-3}^0 \left(2+\sqrt{9-x^2}\right) dx$

e) $\int_{-1}^2 \left(\sqrt{2}+|x|\right) dx$

f)

g) $\int_1^\infty \frac{1}{x^{3/2}}\, dx$

h)

Q3)

The function $f(x) = \sqrt{1 - x^4}$ does not have a closed-form integral. Approximate:

$$\int_0^1 \sqrt{1 - x^4}\, dx$$

using Simpson's Rule and the Trapezoidal Rule with 10 intervals.

Q4)

Using the Midpoint Rule with three subdivisions ($n = 3$), estimate:

$$\int_0^{\pi/2} \sin^2 x\, dx$$

Make a table of midpoint values and compute the estimate.

Q5)

Find the area bounded by the curve $y = \frac{1}{x^2+9}$, the x-axis, and $x \geq 0$.

Q6) Find the area of each region.

(a) Between $y = -x^3 - 2x^2 + 7x - 2$ and $y = -x - 2$

(b) Between $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$

Additional Problems

(a) Two unit circles centered at $(0, 0)$ and $(1, 0)$. What is the area of their intersection?

(b) Between the two towers of a suspension bridge, each of the two main cables follows the parabola $y = x^2$ (units: km). The towers are 2 km apart, and 10 vertical cables are equally spaced between them.

   i. Set up a definite integral for the length of one main cable.

   ii. Find the average length (to the nearest meter) of the vertical cables.