Post-Infinite Series Problem Set

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Coordinate Geometry and Related Topics

1. Prove the Pythagorean Theorem from first principles.

2. Prove that the medians of a regular polygon are concurrent.

3. (a) The points A and B have coordinates $(1, 2, -3)$ and $(-2, 0, 5)$ respectively. Given that the point P divides the line segment AB in the ratio $1 : 2$, find the position vector of P.

   (b) Relative to an origin O, the position vectors of points A, B, and C are $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ respectively. Assume $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-parallel. Let M be the midpoint of AC, and let P be a point on the extension of AB such that the ratio $AB : BP = 2 : 3$. The line segment PM intersects line BC at point S. Show that the position vector of S is $\frac{1}{8}(5\mathbf{b} + 3\mathbf{c})$.

4. Find the general formula for the distance between a point and a line. (This distance will, of course, be the perpendicular distance, as it is both unique and shortest.)

   Additional: Prove that the shortest distance from a point to a line is along the perpendicular.

5. Find a parametrization for the circle $(x-2)^2 + y^2 = 1$, starting at the point $(1, 0)$ and moving clockwise once around the circle. Use the central angle θ (as shown in the accompanying figure) as the parameter.

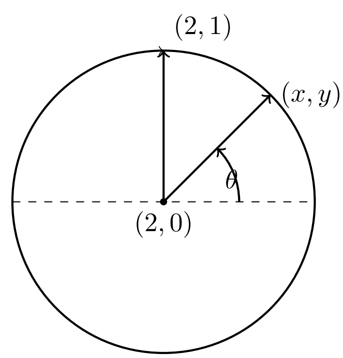

Figure 1: Parametrization of the circle centered at $(2, 0)$ with radius 1, showing angle θ and a point (x, y) on the circumference.

6. **Trochoids:** A wheel of radius a rolls along a horizontal straight line without slipping. Find parametric equations for the curve traced out by a point P on a spoke of the wheel, located b units from its center. Use the angle θ (through which the wheel turns) as the parameter. This curve is called a *trochoid*; it is a *cycloid* when $b = a$.

7. Plot the following polar functions:

$$\begin{aligned} r &= 1 \\ r\sin\theta &= \ln r + \ln\cos\theta \\ r^2 &= 4r\sin\theta \\ r &= 2\cos\theta - \sin\theta \end{aligned}$$

8. A limaçon is a polar curve of the general form $r = a + b\cos\theta$. Sketch the limaçon $r = 1.5 + \cos\theta$ on a clean grid.

9. Try plotting these polar equations to produce interesting graphs. Explore the symmetry embedded in polar notation, which may not be apparent in Cartesian (x-y) form:

$$
\begin{aligned}
r &= \sec(4\theta) \\
r &= \arctan(\theta) \\
r &= \theta + \sin\theta \\
r &= \tan(0.2\theta) \\
r &= 3\tan(\sin(1.5\theta) - 1) - 1 \\
r &= 7\sin(6\sin(5\sin(4\sin(3\sin(2\sin(\theta)))))) \\
r &= \csc(0.6\theta + 2\sin(2\theta)) \\
r &= 3\tan(\sin(\theta)) - \sin(5\theta) \\
r &= 3\tan(\sin(\theta)) - 3\sin(11\theta) \\
r &= 9\sin(0.4\theta) \\
r &= 4\tan(\sin(88\theta)) - 2\sin(4\theta) \\
9r &= \sec(9\theta) - 9\theta \\
r &= 6\sin(6.66\theta) - \cos(6\theta) \\
r &= \sec(99\theta)
\end{aligned}
$$

Note: These are just for fun. Plot them using a graphing calculator or software. Try modifying parameters to explore even more interesting shapes.

10. A limaçon is given by $r = 3 + 3\cos\theta$. Find the slope $\frac{dy}{dx}$ on this curve as a function of θ.