

# Practice Test 1

Semester 2, Department of Physics, QAU

16th March, 2025

## Integration

1. Evaluate the following indefinite integrals:

$$(a) \int \cos(3x) dx \quad (b) \int x e^{x^2} dx$$

2. Evaluate the following definite integrals:

$$(a) \int_0^2 \frac{dx}{1+x^2}$$

- (b) Decompose into partial fractions and evaluate:

$$\int_{-\pi/2}^{\pi/2} \frac{6x^2}{(x^2+1)(x+2)} dx$$

3. First make a substitution and then use integration by parts to evaluate:

$$\int \sin(\sqrt{x}) dx$$

4. Sketch the region enclosed by the parabolas  $y = 2x^2$  and  $y = 3 - x^2$ , and then find its area.
5. When a particle is located a distance  $x$  meters from the origin, a force  $f(x) = \cos\left(\frac{\pi x}{2}\right)$  newtons acts on it. How much work is done in moving the particle from  $x = 1$  m to  $x = 5$  m? (Express your answer in Joules.)
6. Find the area of the region bounded by the curves  $y = \sqrt{x}$  and  $y = x^3$  from  $x = 3$  to  $x = 9$ .
7. Evaluate the integral:

$$\int_0^3 \sqrt{x+1} dx$$

8. Between the two towers of a suspension bridge, each of the two main cables has the shape of the parabola  $y = \frac{1}{10}x^2$  (units are kilometers). The two towers are 2 km apart, and the 10 vertical cables from the main cable to the horizontal roadway are closely and equally spaced.

- (a) Set up a definite integral that gives the length of each main cable between the two towers.
- (b) What is the average length (to the nearest meter) of the vertical cables?

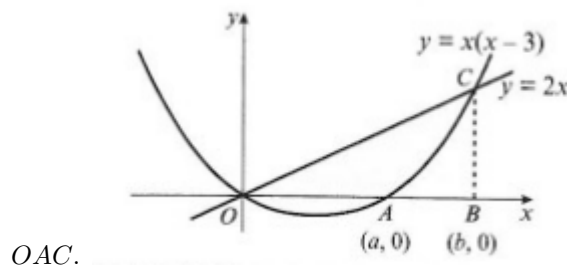
9. Evaluate:

$$\int_1^{\infty} x^{-3/2} dx$$

10. Integrate:

$$\int \frac{2e^{2x} - e^x}{\sqrt{6e^{2x} - 3e^x - 1}} dx$$

11. The diagram shows a sketch of the curve with the equation  $y = x(x - 3)$  and the line with the equation  $y = 2x$ . Find the area of the shaded region



12. Using substitution, show that:

$$\int_0^{\sqrt{2}} \sqrt{2 - x^2} dx = \frac{\pi}{2}$$

13. Sketch the region enclosed by the curves  $y = |x|$  and  $y = 6 - x^2$ , then find its area.

14. Evaluate:

$$\int \frac{\cos x + \tan x}{1 + \tan^2 x} dx$$