Cal - 2

Challanging Problems for Eid.

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Q:1 It is often claimed that integration is 'anti-derivative'. By first principle prove that the area under the curve $y = ax + b$ from c to d is given by $ax^2 + bx \Big|_c^d$.

Can you extend these ideas to a general polynomial?

Q:2 The fundamental theorem of calculus states

$$f(b) - f(a) = \int_a^b f'(x)\, dx$$

How does the area of the derivative function depend only on the end points of the original function. Convince yourself that it is the case.

Q:4: Compute the definite integrals by first integration technique and then by taking six intervals using trapezium and Simpson's rules. Compare the results

(a) $\int_0^4 \sqrt[3]{x^4 + 6} \, dx$

(d) $\int_0^5 x \, dx$

(b) $\int_3^5 \frac{1}{1 - \ln x} \, dx$

(e) $\int_0^5 2 \, dx$

(c) $\int_1^5 e^{\cos x} \, dx$

(f) $\int_0^5 x^2 \, dx$

Part (2) → Having noted the results and the discrepancies. Comment on which numerical integration technique is better and what quality of a function make the numerical results (even for smaller number of intervals) closer to the actual results.

Q:5. Determine the area of the region bounded by $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$

$A = \frac{343}{12}$

Q:6. Find the area bounded by the curves $y = 2 - e^{2-x}$, $y = x^2 - 4x$, $x = 3$ and the y-axis.

Note: These functions don't intersect.

Q:7: Determine the value of $\int_{-5}^{-1} f(x)\,dx$ given that $\int_{2}^{-5} f(x)\,dx = 56$,

$\int_{7}^{2} f(x)\,dx = -90$ and $\int_{-1}^{7} f(x)\,dx = 45$

Q:8: Evaluate, $\int 2z^5 e^{1-z}\,dz$

Q:9: $\int \frac{x^6 - 6x^5 + 3x^4 - 10x^3 - 9x^2 + 12x - 27}{x^4 + 3x^2} \, dx$

Q:9: A probability destribution function (PDF) is given by $f(x)$ s.t.

$$f(x) \geq 0 \quad , \quad \forall \; x$$

and $\int_{-\infty}^{\infty} f(x) \, dx = 1$

Mean of a variable (random) is given by;

$$\bar{x} = \int_{-\infty}^{\infty} x f(x) \, dx$$

(a) Show that $f(t) = \begin{cases} 0 & , \; t < 0 \\ 0.1 e^{-\frac{t}{10}} & , \; t \geq 0 \end{cases}$

fullfills the criteron for being a PDF.

Also, calculate $\bar{t}$

Q: 10 Integrals:

(a) $$\int_0^5 \frac{1}{4w-20}\, dw$$

(f) $$\int_0^\infty \frac{1}{x-1}\, dx$$

(b) $$\int_{-1}^{2} \frac{3}{\sqrt[6]{4-2z}}\, dz$$

(c) $$\int_0^\infty x\, e^{2+3x}\, dx$$

(d) $$\int_{-\infty}^{0} \cos w\, dw$$

(e) $$\int_0^\infty \frac{1}{z(\ln z)^2}\, dz$$

Note: All of these are improper integrals. So, keep an eye out for divergence in the integrand.