1)

(a) $\int \cos(3x)\,dx = \dfrac{\sin 3x}{3} + C$

(b) $u = x^2$; $du = 2x\,dx$

$$\int \frac{e^u\,du}{2} = \frac{e^u}{2} + C = \frac{e^{x^2}}{2} + C$$

2)

(a)

$$\begin{array}{r|l} & 1 \\ a^2+x^2 & \cancel{x^2} \\ & \underline{\pm \cancel{x^2} \pm a^2} \\ & -a^2 \end{array}$$

$$\int 1 - \frac{a^2}{a^2+x^2}$$

$$x\Big|_0^2 - a\tan^{-1}\left(\frac{x}{a}\right)\Big|_0^2$$

Known

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$2 - a\left(\tan^{-1}\left(\frac{2}{a}\right) - \tan^{-1}(0)\right)$$

$$2 - a\left(\tan^{-1}\left(\frac{2}{a}\right)\right)$$

(b)

Q:3

$u = \sqrt{x}$ $\quad\quad du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\sin u \;\; du$

$\int 2u\sin u\, du \rightarrow 2\left(-u\cos u + \int \cos u\, du\right)$

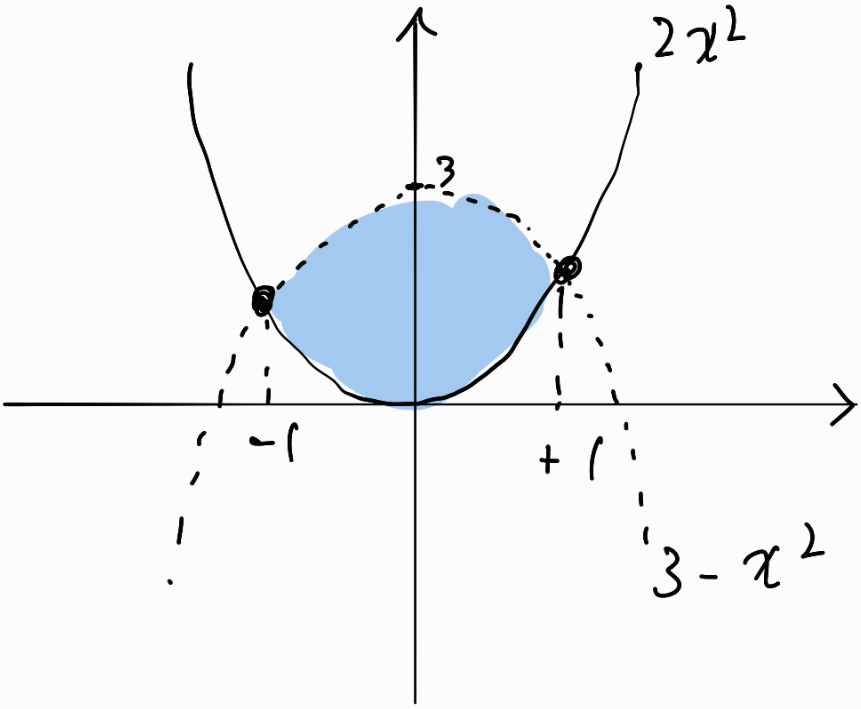
$2(-u\cos u + \sin u) + C$

$2(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x})$

Q : 4

$y = 2x^2$

$y = 3 - x^2$



$2x^2 = 3 - x^2$

$3x^2 = 3$

$x = \pm 1$

Shaded Area is required :

$$A = \int_{-1}^{1} \left((3 - x^2) - x^2 \right) dx$$

$$= \int_{-1}^{1} (3 - 2x^2)\, dx$$

$$= 3x \Big|_{-1}^{1} - \frac{2x^3}{3} \Big|_{-1}^{1}$$

$$= (3 + 3) - \left(\frac{2}{3} + \frac{2}{3} \right)$$

$$= 6 - \frac{4}{3} = \frac{14}{3}$$

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(5) As we know:

$$W = \int F(x)\, dx$$

$\longrightarrow$ For variable force

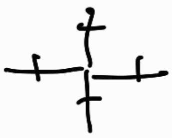
$$W = \int_{1}^{5} \cos\left(\frac{\pi x}{2}\right) dx$$

$$= \sin\left(\frac{\pi x}{2}\right) \frac{2}{\pi} \Big|_{1}^{5}$$

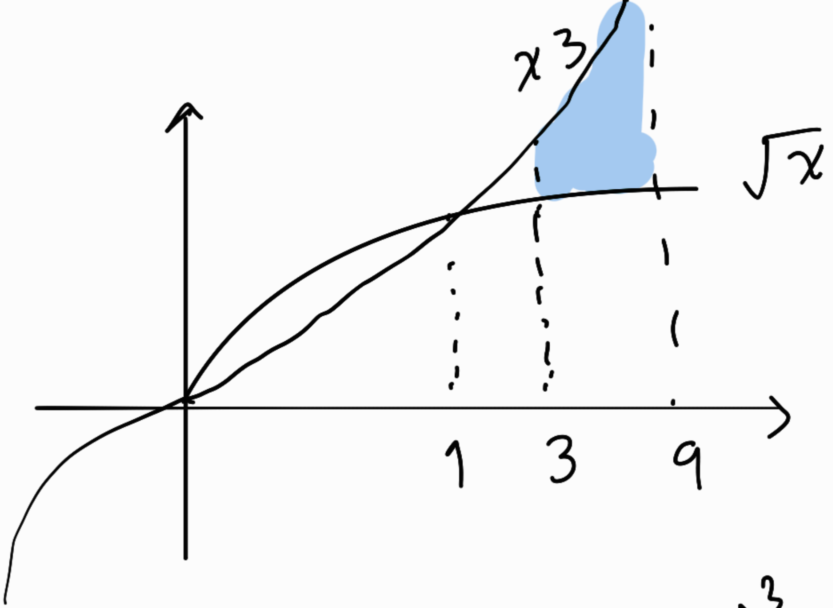
$$= \frac{2}{\pi}\left(\sin\left(\frac{5\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{2}{\pi}(1 - 1) = 0$$

5×90



6



$\sqrt{x} = x^3$

$x = x^6$

$x^6 - x = 0$

$x(x^5 - 1) = 0$

$x = 0 \; ; \; x^5 = 1$

$x = 1$

as $x^3 > \sqrt{x}$

$\forall \; x > 1$

$$A = \int_3^9 (x^3 - \sqrt{x})\, dx$$

7

$$\int_0^3 \sqrt{x+1}\, dx$$

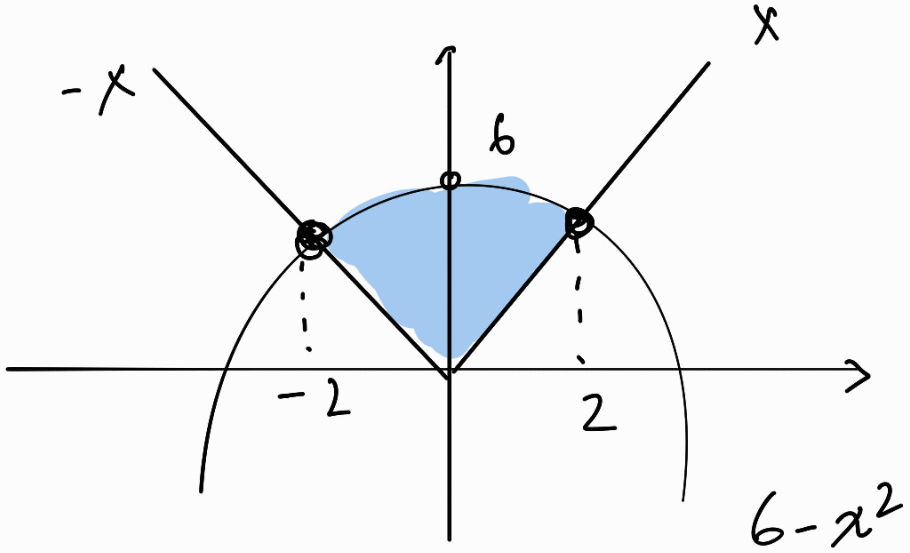
$u = x + 1$

x	u
0	1
3	4

$$\int_1^4 \sqrt{u}\, du \qquad \left.\frac{3u^{3/2}}{2}\right|_1^4 = 12 - \frac{3}{2}$$

$$= 21/2$$

13



$6-x^2 > |x|$
in Region

$x = 6 - x^2$

$x^2 + x - 6 = 0$

$x = \dfrac{-1 \pm \sqrt{25}}{2}$

$x = \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$

do with $-x$ too

$$A = \int_{-2}^{2} 6 - x^2 - |x| \; dx$$

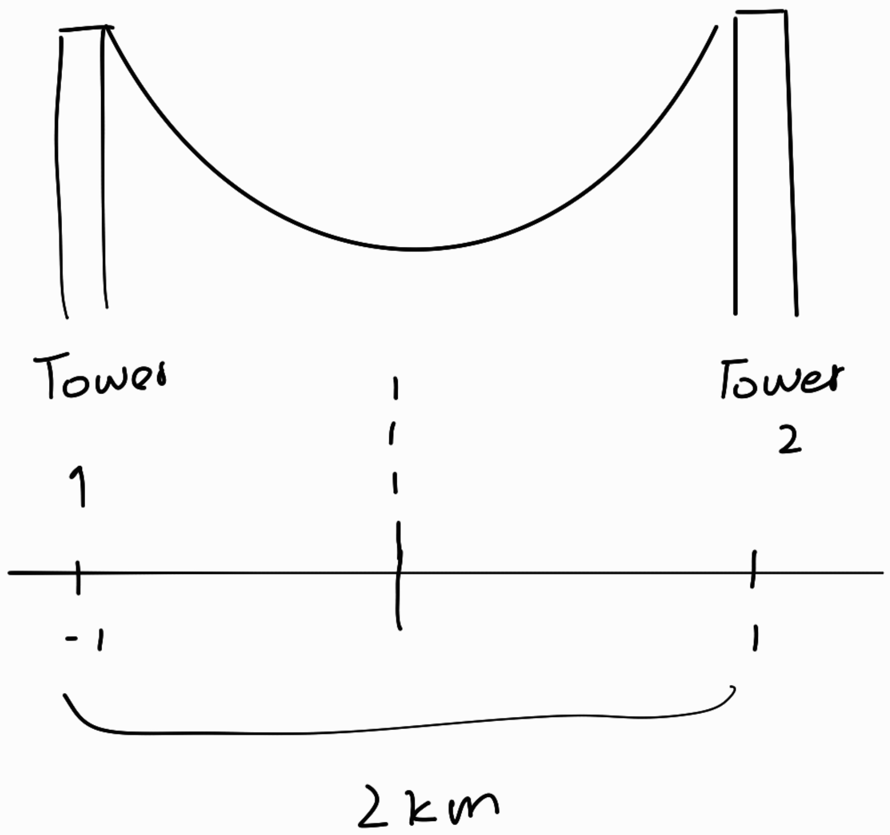
$$A = \int_{-2}^{0} (6 - x^2 + x)\, dx + \int_{0}^{2} (6 - x^2 - x)\, dx$$

$$A = 6x \Big|_{-2}^{0} - \frac{x^3}{3}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{-2}^{0} + 6x\Big|_{0}^{2} - \frac{x^3}{3}\Big|_{0}^{2} - \frac{x^2}{2}\Big|_{0}^{2}$$

$$= 12 - \frac{8}{3} - 2 + 12 - \frac{8}{3} - 2$$

$$= 24 - \frac{16}{3} - 4 = 28 - \frac{16}{3} =$$

Q : 8



ⓐ $\displaystyle\int_{-1}^{1} \sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)}\, dx$

L

$$\int_{-1}^{1} \sqrt{1 + \frac{x^2}{25}}\, dx$$

(b) avg. length

↳ It's the same as the notion of averages of non-constant functions

$$\text{avg} = \frac{\int f\,dx}{\text{length}} = \frac{1}{2}\int_{-1}^{1}\frac{x^2}{10}\,dx$$

$$= \frac{x^3}{30}\Big|_{-1}^{1}\left(\frac{1}{2}\right)$$

$$= \frac{2}{30}\left(\frac{1}{2}\right) = \frac{1}{30}\text{ km}$$

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9) $$\int_1^\infty \frac{dx}{x^{3/2}}$$

$$\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\Bigg|_1^\infty = \frac{x^{-1/2}}{\frac{-1}{2}} = \frac{-2}{\sqrt{x}}\Bigg|_1^\infty$$

$$-\frac{2}{\sqrt{\infty}} - \left(-\frac{2}{1}\right) = 0 + 2 = 2$$

$u = e^x$

$du = e^x dx$

<u>10)</u>

$u = e^x$

$$\frac{2u^2 - u}{\sqrt{6u^2 - 3u - 1}}$$

let $t = 6u^2 - 3u - 1$

$dt = (12u - 3)\, du$

$$; = \frac{4u^2 - u - u}{2\sqrt{t}} = \frac{u(4u-1)}{2\sqrt{t}}$$

$$-\frac{u}{2\sqrt{t}} = \frac{u}{6}\frac{dt}{\sqrt{t}} - \frac{12u-3u}{24\sqrt{t}}$$