Integration full test;

Q1)

a) $\int x^2 \cos(1+x^3)\,dx$

$u = x^3+1 \qquad du = 3x^2\,dx$

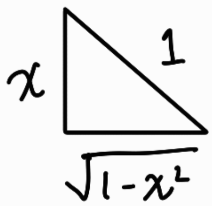
b) $\int \frac{dx}{(\sqrt{1-x^2})^3}$ $\qquad x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\frac{\cancel{\cos\theta}\, d\theta}{\cos^{\cancel{3}\,2}\theta} \qquad \frac{d\theta}{\cos^2\theta}$

$\sec^2\theta\, d\theta$

$\tan\theta + c$



$\frac{x}{\sqrt{1-x^2}}$

c) $\int \varphi\, d\varphi + \int {}^{2}/_{\sin 2\varphi}\, d\varphi$ $\qquad 2\varphi = \phi$

$\varphi^2/2 \qquad + \int \frac{d\phi}{\sin\phi}$ $\qquad d\varphi = \frac{d\phi}{2}$

$+ \int \frac{\sin\phi}{\sin^2\phi}\, d\phi$

$$+ \int \frac{\sin\phi}{1-\cos^2\phi}\, d\phi$$

$t = \cos\theta$

$dt = -\sin\theta\, d\theta$

$$+ \int \frac{-1}{1-t^2}\, dt \rightarrow \ln\left|\frac{t-1}{t+1}\right|$$

(d) $\int 3x \cos 2x \, dx$

$$3x \int \cos 2x \, dx - \int 3 \frac{\sin 2x}{2} \, dx$$

$$\frac{3x \sin 2x}{2} - \frac{3}{2} \frac{\cos 2x}{2} \quad //$$

(e) $\int \frac{x^2}{\sqrt{x-1}} \, dx$

$u = x-1 \quad ; \quad x = u+1$

$du = dx$

$$\sim \frac{(u+1)^2}{\sqrt{u}} = \frac{u^2+2u+1}{\sqrt{u}} \rightarrow \text{Term by Term}$$

(f) $\dfrac{x+3}{(x-1)(x^2-4x+4)} \sim$ Ans

$4\ln(|x-1|)$

$-4\ln|x-2|$

$-\dfrac{5}{x-2} + C$

(g) $\dfrac{\sqrt{1-x^2}}{x^2}$ $\qquad x = \sin t$

$\sim \dfrac{\cos t}{\sin^2 t} \cos t \, dt \sim \dfrac{\cos^2 t}{\sin^2 t} dt$

$\dfrac{1-\sin^2 t}{\sin^2 t} \sim \dfrac{1}{\sin^2 t} - 1$ ✓

$\downarrow$

$\operatorname{cosec}^2 t$

$\hookrightarrow \cot t$

$= \dfrac{-\sqrt{1-x^2}}{x} - \sin^{-1}(x) + C$

(h) $\tan^3 x \quad \sec^3 x$

$\sim \underbrace{\tan^2 x}\sec^2 x \quad \sec x \tan x \, dx$

$(\sec^2 x - 1)\sec^2 x \quad \sec x \tan x \quad dx$

$\sim (u^2-1)u^2 \quad du$

$\frac{u^5}{5} - \frac{u^3}{3} + C$

$u = \sec x$

$du = \sec x \tan x \, dx$

(i) $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

$\sim \boxed{\frac{1}{2}(\sin^{-1}(x))^2 + C}$

j) $\frac{x+2}{x+1} = \frac{x+1+1}{x+1} = 1 + \frac{1}{x+1}$

$\qquad\qquad \swarrow$

$\sim\ = x + \ln|x+1|$

Q_2

a) $\int_e^1 \frac{\sqrt{\ln x}}{x}\,dx$ $\qquad u = \ln x$ ✓

$du = \frac{1}{x}\,dx$ ✓

b) $\frac{x}{(x^2-1)^{2/3}}$ $\qquad x = \sec\theta$

$dx = \sec\theta \tan\theta \; d\theta$

$$\frac{\sec\theta}{(\sec^2\theta-1)^{2/3}} \sec\theta \tan\theta \, d\theta$$

$$\frac{\sec\theta}{(\tan\theta)^{4/3}} \sec\theta \cancel{\tan\theta} \, d\theta$$

$$\frac{\sec^2\theta}{(\tan\theta)^{4/3}}$$

$u = \tan\theta$

$du = \sec^2\theta \; d\theta$

c) $\sim x^2 e^{-2x}$

$$\frac{x^2 e^{-2x}}{-2} - \int \frac{\cancel{2}x e^{-2x}}{-\cancel{2}}$$

$$-\frac{x^2 e^{-2x}}{2} + \left(\frac{x e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2} \right)$$

$$-\frac{x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4}$$

$\sim -1/4$

d) $2 + \sqrt{9 - x^2}$

$\int 2 + \int \sqrt{9 - x^2}\, dx$ let

$\downarrow$ $\downarrow$

$x = 3 \sin u$

$$\int 2 + \left(\frac{9}{2} \sin^{-1}\left(\frac{x}{3}\right) + \frac{x}{2}\sqrt{9 - x^2} + c \right)$$

(e) $\int_{-1}^{2}$ → easy just break it on zero as $|x|$ is a piece-wise function.

(f) Trivial

Q:3

$n=10$ $\int_{0}^{1}$

Simpson's Rule → 0.8688149150 2

Trapezoidal Rule → 0.860993381576

Q:4 $n=3$

midpoint

$\longrightarrow 0.7853981633\text{97}$

Q:5 $$\int_0^{\infty} \frac{1}{x^2+9}\,dx = \frac{1}{3}\tan^{-1}(x)\Big|_0^{\infty}$$

$$\tan^{-1}(\infty) - \tan^{-1}(0)$$

$$\left(\frac{\pi}{2} - 0\right)\frac{1}{3} = \frac{\pi}{6} \,?$$

Q:6 (a)

$$-x^3 - 2x^2 + 7x - \cancel{2} = -x - \cancel{2}$$

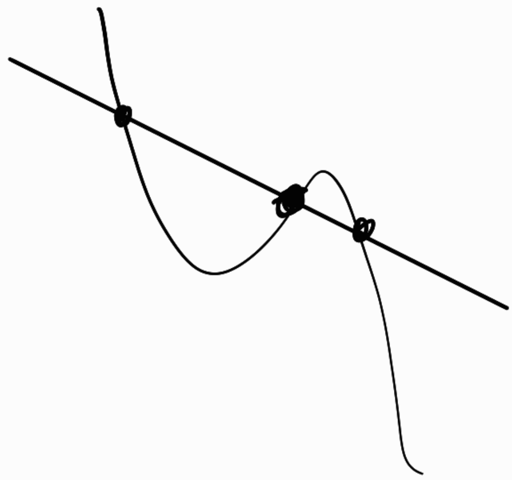
$$-x^3 - 2x^2 + 8x = 0$$

$$-x(x^2 + 2x - 8) = 0$$

$x = 0$; $x = \dfrac{-2 \pm \sqrt{4 + 4(8)}}{2}$

$$x = \frac{-2 \pm 6}{2} = \frac{6-2}{2} = 2$$

$$, \frac{-6-2}{2} = -4$$



(b)

$$x^3+2x^2 = x^2+2x$$

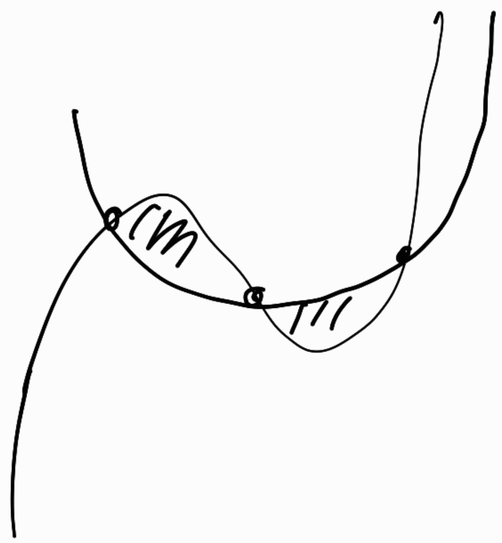
$$x^3+x^2-2x = 0$$

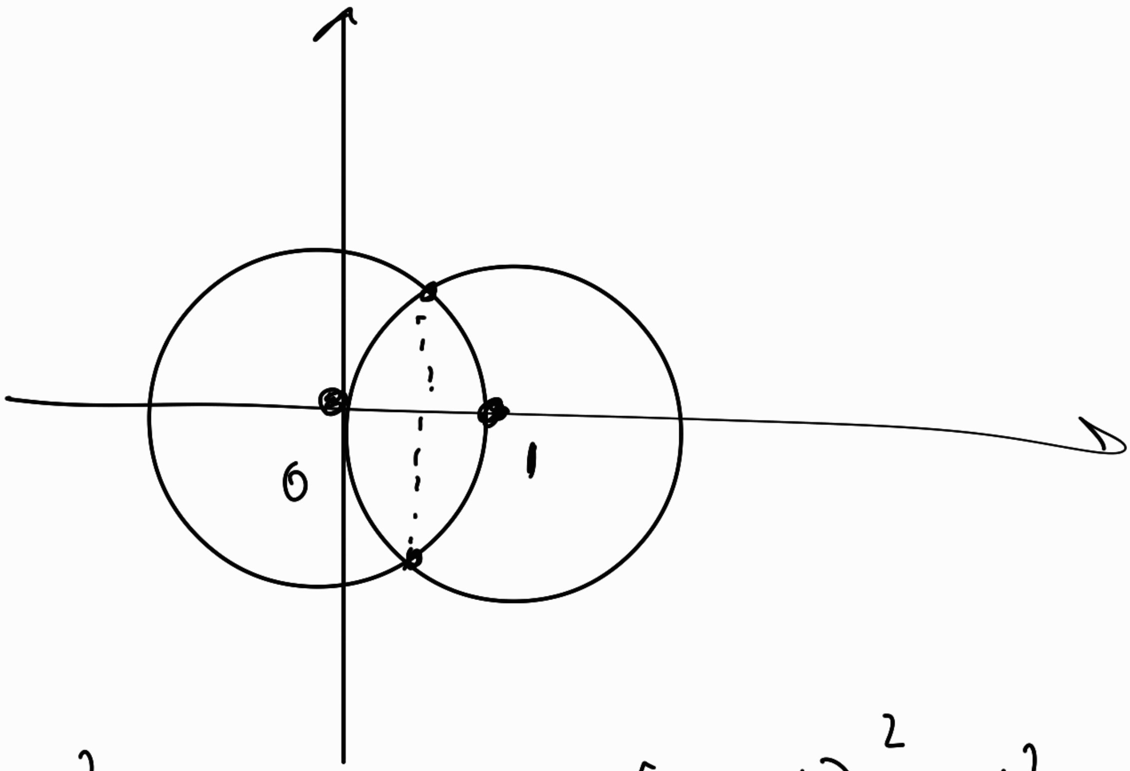
$x=0$; $x^2+x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$x=2$; $x=-1$





$x^2+y^2=1$ $\qquad$ $(x-1)^2+y^2=1$

$y=\sqrt{1-x^2}$