Recurrent Neural Networks

CEP Lecture

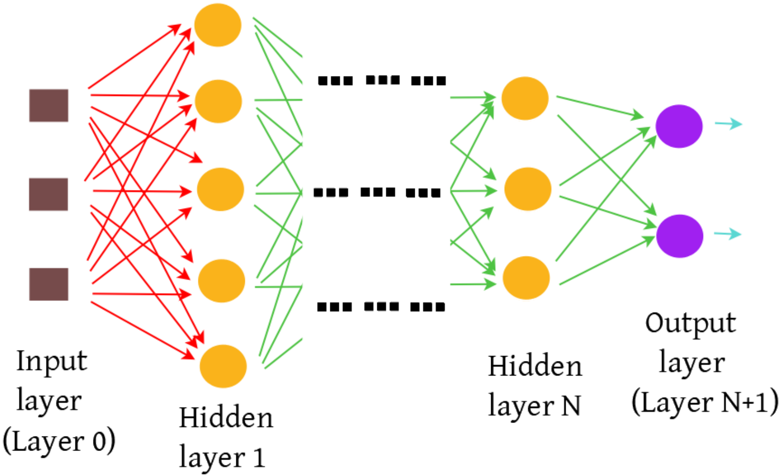
CSRE, IIT Bombay

March 6, 2024

1. Recap: Multi Layer Perceptron

2. Recurrent Neural Networks

Multi Layer Perceptron



Multi Layer Perceptron: Block structure

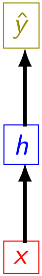


Figure: MLP

Recurrent Neural Networks

Sequential outputs - Motivating Applications

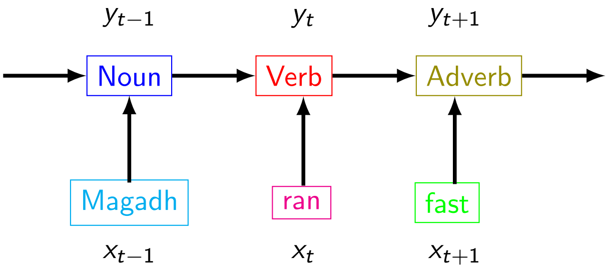


Figure: Part-of-Speech Tagging

Sequential outputs - Motivating Applications

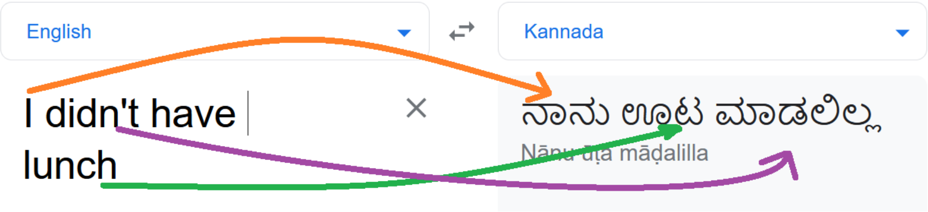


Figure: Language Translation

Sequential outputs - Motivating Applications

Figure: Protein Sequence Alignment

Earliest Recurrent Network Architectures

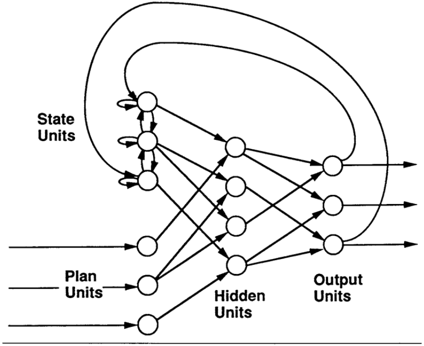


FIGURE 1. The processing units and basic interconnection scheme (not all connections are shown). The plan and state units together constitute the input units for the network.

Figure: Jordan Network†

† Michael Jordan. *Serial order: A parallel distributed processing approach (Tech. Rep. No. 8604).*, Institute for Cognitive Science, UCSD, San Diego, 1986

Earliest Recurrent Network Architectures

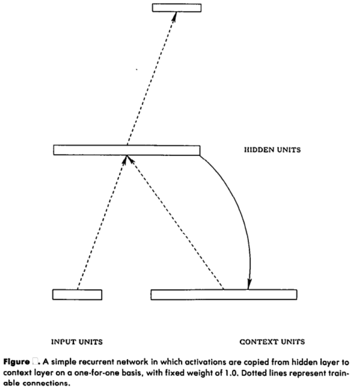


Figure: Elman Network[‡]

[‡] Jeffrey L. Elman, *Finding structure in time*, Cognitive Science, Vol 14(2), pp. 179-211, 1990.

Recurrent Neural Network

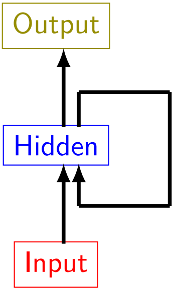


Figure: A simple recurrent network

Recurrent Neural Network

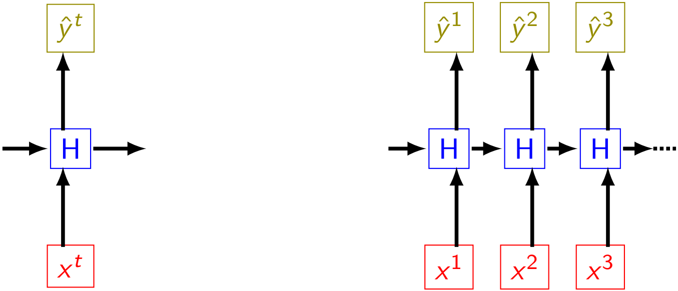


Figure: A simple recurrent network

Recurrent Layers

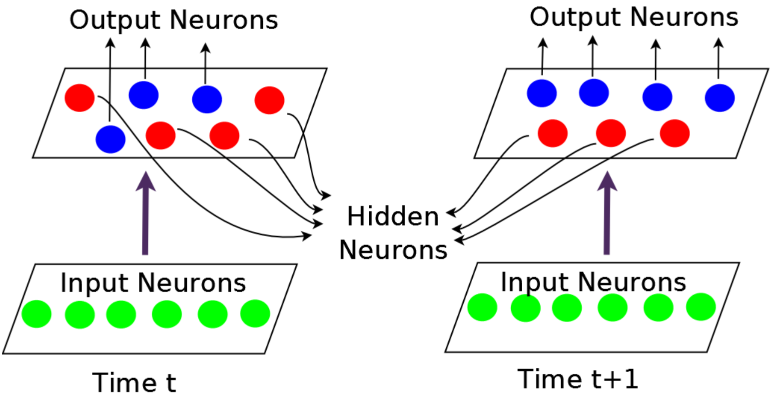


Figure: Recurrent layers

Recurrent Layers - Formalism

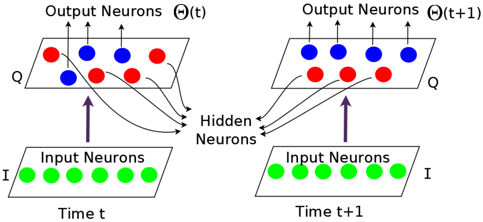


Figure: Recurrent layers

- S be the set of all neurons
- $S = \mathtt{I} \cup Q$
  - $\mathtt{I}$ is the set of input neurons
  - Q is the set of hidden neurons
- Some neurons in set Q can be used as output neurons at particular time steps (this set is denoted by $\Theta(t) \subseteq Q$).

Recurrent Layers

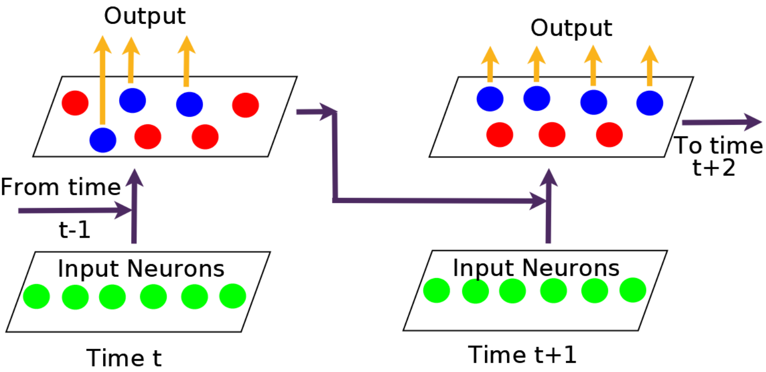


Figure: Recurrent layers

Recurrent Layers - Formalism

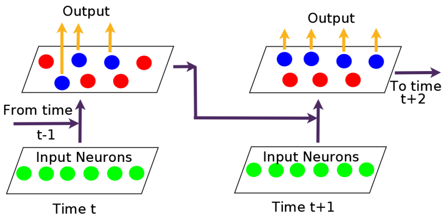


Figure: Recurrent layers

- At the time step t, we define:

$$\xi_k(t) = \begin{cases} a_k(t-1) & if \;\; k \in Q \\ x_k(t) & if \;\; k \in I \end{cases}$$

- $a_k(t-1)$ denotes activation of neuron k at time $t-1$.
- $x_k(t)$ denotes input at neuron k at time t.
- Note: $\hat{y}_k(t-1) = a_k(t-1)$.

Recurrent Layers - Formalism†

- At the time step t, we define:

$$\xi_k(t) = \begin{cases} a_k(t-1) & \textit{if } \; k \in Q \\ x_k(t) & \textit{if } \; k \in I \end{cases}$$

- For all neurons belonging to S we can now define:

$$z_k(t) = \sum_{\ell \in S} w_{k\ell} \xi_\ell(t) \;\; \forall k \in S$$
$$a_k(t) = \phi[z_k(t)]$$

- ϕ denotes activation function. (*e.g.* sigmoidal, ReLU)

† R. J. Williams and D. Zipser, *Gradient-based Learning Algorithms for Recurrent Networks and Their Computational Complexity*, Backpropagation, L. Erlbaum Associates Inc. Hillsdale, NJ, USA. 1995.

Recurrent Layers - Formalism

- For time step t, denote the set of output neurons as $\Theta(t) \subseteq Q$.
- Let $y_k(t)$ be the actual output associated with neuron k at time t.
- Error at time step t for a particular neuron k:

$$e_k(t) = \begin{cases} y_k(t) - a_k(t) & \text{when} \quad k \in \Theta(t), \\ 0 & \text{else.} \end{cases}$$

Recurrent Layers - Formalism

- For time step t, denote the set of output neurons as $\Theta(t) \subseteq Q$.
- Let $y_k(t)$ be the actual output associated with neuron k at time t.
- Error at time step t for a particular neuron k:

$$e_k(t) = \begin{cases} y_k(t) - a_k(t) & \text{when} \quad k \in \Theta(t), \\ 0 & \text{else.} \end{cases}$$

- Error at time step t can be defined as a squared error function:

$$E(t) = \sum_{k \in S} e_k^2(t)$$

Recurrent Layers - Formalism

- For time step t, denote the set of output neurons as $\Theta(t) \subseteq Q$.
- Let $y_k(t)$ be the actual output associated with neuron k at time t.
- Error at time step t for a particular neuron k:

$$e_k(t) = \begin{cases} y_k(t) - a_k(t) & \text{when} \quad k \in \Theta(t), \\ 0 & \text{else.} \end{cases}$$

- Error at time step t can be defined as a squared error function:

$$E(t) = \sum_{k \in S} e_k^2(t)$$

- Net error over all time steps (assuming that the network gets initialized at time t_0 and runs during time interval $(t_0, t_f])$ can be defined as:

$$E = \sum_{t=t_0+1}^{t_f} E(t) = \sum_{t=t_0+1}^{t_f} \sum_{k \in S} e_k^2(t)$$

Recurrent Neural Networks - Loss Minimization

Optimization problem

$$\min_{w} E = \sum_{t=t_0+1}^{t_f} E(t) = \sum_{t=t_0+1}^{t_f} \sum_{k \in S} e_k^2(t)$$

Recurrent Neural Networks - Loss Minimization

Optimization problem

$$\min_{w} E = \sum_{t=t_0+1}^{t_f} E(t) = \sum_{t=t_0+1}^{t_f} \sum_{k \in S} e_k^2(t)$$

Equivalent Optimization problem

$$\min_{w} E = \sum_{t=t_0+1}^{t_f} \sum_{k \in \Theta(t)} (y_k(t) - a_k(t))^2$$

Recall:

- $a_k(t) = \phi[z_k(t)]$
- $z_k(t) = \sum_{\ell \in S} w_{k\ell} \xi_\ell(t) \ \ \forall k \in S$

Recurrent Neural Networks - Loss Minimization

Equivalent Optimization problem

$$\min_{w} E = \sum_{t=t_0+1}^{t_f} \sum_{k\in\Theta(t)} \left(y_k(t) - \phi\left[\sum_{\ell\in S} w_{k\ell}\xi_\ell(t)\right]\right)^2$$

Recurrent Neural Networks - Parameter Update

Gradient Descent Type Update

$$w_{ij} = w_{ij} - \eta \frac{\partial E}{\partial w_{ij}}$$

Recurrent Neural Networks - Parameter Update

Gradient Descent Type Update

$$w_{ij} = w_{ij} - \eta \frac{\partial E}{\partial w_{ij}}$$

Homework: Write expressions for computing the error gradients with respect to the weights. Discuss the recurrence relations obtained as part of your derivations.