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1 The curvature of a circle of radius 'r' is $\boxed{\frac{1}{r}}$

2 The parametric equation of a parabola $y^2 = 4ax$ is $\boxed{x = at^2, \; y = 2at}$

3 The locus of $\boxed{\text{centre}}$ of curvature is called $\boxed{\text{Evolute}}$

4 The radius of curvature at (3, 4) on the curve $x^2 + y^2 = 25$ is $\boxed{5}$.

Answer

Given $x^2 + y^2 = 25$

$\Rightarrow (x-0)^2 + (y-0)^2 = 5^2$ which is a circle, with centre (0,0) and radius 5.

$\therefore$ Radius of curvature $\boxed{\rho = 5}$

5 The evolute of a curve is the locus of $\boxed{\text{centre of curvature}}$

6 The radius of curvature of a curve $y = 4\sin x$ at $x = \pi/2$ is $\boxed{\frac{1}{4}}$

Answer

$y = 4\sin x$

$\frac{dy}{dx} = 4\cos x$

$\frac{d^2y}{dx^2} = -4\sin x$

at $x = \pi/2$

$\left(\frac{dy}{dx}\right)_{x=\pi/2} = 4\cos \pi/2 = 0$

$\left(\frac{d^2y}{dx^2}\right)_{x=\pi/2} = -4\sin \pi/2 = -4(1) = -4$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)} = \frac{(1 + (0)^2)^{3/2}}{-4} = \frac{1^{3/2}}{-4} = -\frac{1}{4}$$

$|\rho| = \frac{1}{4}$.

7 The curvature of the straight line is $\boxed{0}$

8 The radius of curvature of a circle at any point is same as its $\boxed{\text{radius}}$.

9. The radius of curvature of the curve $y=e^x$ at $x=0$ is $\boxed{2\sqrt{2}}$

Answer:

$y=e^x$ | at $x=0$

$\frac{dy}{dx}=e^x$ | $\left(\frac{dy}{dx}\right)_{x=0} = e^0 = 1$

$\frac{d^2y}{dx^2}=e^x$ | $\left(\frac{d^2y}{dx^2}\right)_{x=0} = e^0 = 1$

$\frac{3}{2}=1+\frac{1}{2}$

$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left(\frac{d^2y}{dx^2}\right)} = \frac{(1+1^2)^{3/2}}{1} = 2^{3/2} = 2^1 \cdot 2^{1/2} = 2\sqrt{2}.$$

10. The envelope of the family of curves $A\alpha^2+B\alpha+C=0$ (α is parameter) is $\boxed{B^2-4AC=0}$

11. If the radius of curvature and curvature of a curve at any point are ρ and κ respectively, then $\boxed{\rho=\frac{1}{\kappa}}$

12. The curvature at any point of the circle is equal to $\boxed{\text{Reciprocal}}$ of its radius.

13. A curve which touches each member of the family of curves is called $\boxed{\text{Envelope}}$ of that family.

14. The equation of the circle of curvature, with centre of curvature at $(\bar{x},\bar{y})$ and radius of curvature ρ, is $(x-\bar{x})^2+(y-\bar{y})^2=\rho^2$.

15. The parametric form of $xy=c^2$ is $\boxed{x=ct,\ y=\frac{c}{t}}$

16. Evolute of a curve is $\boxed{\text{Envelope}}$ of the $\boxed{\text{normals}}$ of that curve.

17. The radius of curvature in polar coordinates is

$$\rho = \frac{(r^2+r'^2)^{3/2}}{r^2-rr''+2r'^2}$$

18. Evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is $\boxed{\text{cycloid}}$

19. Curvature of the circle $x^2+y^2=a^2$ is $\boxed{\frac{1}{a}}$

20. Envelope of the curve $y=mx+\frac{a}{m}$, 'm' being parameter is $\boxed{y^2-4ax=0}$

21. The radius of curvature of $r^n = a^n \cos n\theta$ at (r, θ) is $\boxed{\rho = \frac{a^n}{(n+1) r^{n-1}}}$

22. The radius of curvature of $r = a\cos\theta$ at (r, θ) is $\boxed{\rho = \frac{a}{2}}$

23. Write the equation of the circle of curvature
$(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$.

24. The envelope of the family of straight line is $y = mx + \frac{a}{m}$ is $\boxed{y^2 = 4ax}$

Answer

$$y = mx + \frac{a}{m}$$

$$\Rightarrow ym = m^2 x + a$$

$$m^2 x - ym + a = 0 \qquad Ax^2 + Bx + C = 0.$$

Envelope is $B^2 - 4AC = 0$, $A = x$, $B = -y$, $C = a$.

$$(-y)^2 - 4xa = 0$$

$$y^2 - 4ax = 0$$

$$y^2 = 4ax.$$

25. The radius of curvature at any point (r, θ) for the curve $r = a\cos\theta$ is $\boxed{\frac{a}{2}}$

26. The evolute of the parabola $y^2 = 4ax$ is
$\boxed{27ay^2 = 4(x-2a)^3}$

27. The parametric equation of a hyperbola is
$\boxed{x = a\sec\theta,\ y = b\tan\theta}$

28. The envelope of the normal to the curve is the $\boxed{\text{Evolute}}$ of a curve.

29. The reciprocal of the curvature of the curve at any point 'p' is called $\boxed{\text{Radius of curvature}}$

30. The envelope of the normal to the curve is the $\boxed{\text{Evolute}}$ of a curve.

31. $\boxed{\text{Evolute}}$ is defined as the locus of centre of curvature.

32. The envelope of $ty - x = at^2$, t is the parameter, is $\boxed{y^2 = 4ax}$