VOLUME OF A SPHERE

REMEMBER

(i) Volume of a sphere $= \frac{4}{3}\pi r^3$, where 'r' is the radius of the sphere.

(ii) Volume of a hemisphere $= \frac{2}{3}\pi r^3$, where 'r' is the radius of the hemisphere.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.8 (*Page 236*)

Note: Assume $\pi = \frac{22}{7}$, unless stated otherwise.

Question 1. *Find the volume of a sphere whose radius is*

(i) 7 cm *(ii) 0.63 m*

Solution: (i) Here, radius (r) = 7 cm

$$\therefore \quad \text{Volume of the sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (7)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3$$

$$= \frac{4 \times 22 \times 7 \times 7}{3} \text{ cm}^3$$

$$= \frac{4312}{3} \text{ cm}^3 = 1437\frac{1}{3} \text{ cm}^3$$

Thus, the required volume $= 1437\frac{1}{3}\text{ cm}^3$

(*ii*) Here, radius (r) = 0.63 m

$$\therefore \quad \text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{63}{100}\right)^3 \text{ m}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{63}{100} \times \frac{63}{100} \times \frac{63}{100} \text{ m}^3$$

$$= \frac{4 \times 22 \times 3 \times 63 \times 63}{100 \times 100 \times 100} \text{ m}^3$$

$$= \frac{1047816}{1000000} \text{ m}^3$$

$$= 1.047816 \text{ m}^3 = 1.05 \text{ m}^3 \text{ (approx)}$$

Thus, the required volume is 1.05 m^3 (approx).

Question 2. *Find the amount of water displaced by a solid spherical ball of diameter*

(i) 28 cm *(ii) 0.21 m*

Solution: (i) Diameter of the ball = 28 cm

$$\therefore \quad \text{Radius of the ball (r)} = \frac{28}{2} \text{ cm} = 14 \text{ cm}$$

$$\Rightarrow \text{Volume of the spherical ball} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 \text{ cm}^3$$

$$= \frac{34496}{3} \text{ cm}^3 = 11498\frac{2}{3} \text{ cm}^3$$

Thus, the amount of water displaced $= 11498\frac{2}{3}\text{ cm}^3$.

(*ii*) Diameter of the solid spherical ball = 0.21 m

$$\Rightarrow \text{Radius (r)} = \frac{0.21}{2} \text{ m} = \frac{21}{200} \text{ m}$$

$$\therefore \text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{200}\right)^3 \text{ m}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{200} \times \frac{21}{200} \times \frac{21}{200} \text{ m}^3$$

$$= \frac{11 \times 21 \times 21}{1000000} \text{ m}^3 = \frac{4851}{1000000} \text{ m}^3$$

$$= 0.004851 \text{ m}^3$$

Question 3. *The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³?*

Solution: Diameter of the metallic ball = 4.2 cm

$$\Rightarrow \quad \text{Radius (r)} = \frac{4.2}{2} \text{ cm} = 2.1 \text{ cm}$$

$$\therefore \quad \text{Volume of the metallic ball} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10} \text{ cm}^3$$

$$= \frac{4 \times 22 \times 21 \times 21}{10 \times 10 \times 10} \text{ cm}^3$$

$$\because \quad \text{Density} = 8.9 \text{ g per cm}^3$$

$$\therefore \quad \text{Mass of the ball} = 8.9 \times [\text{Volume of the ball}]$$

$$= \frac{89}{10} \times \frac{4 \times 22 \times 21 \times 21}{10 \times 10 \times 10} \text{ g}$$

$$= \frac{3453912}{10000} \text{ g} = 345.3912 \text{ g}$$

$$= 345.39 \text{ g (approx.)}$$

Thus, the mass of ball is 345.39 g (approx.).

Question 4. *The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?*

Solution: Let diameter of the earth be 2r.

$$\Rightarrow \quad \text{Radius of the earth} = \frac{2r}{2} = r$$

$$\because \quad \text{Diameter of the moon} = \frac{1}{4}(\text{Diameter of the earth})$$

$$\therefore \quad \text{Radius of the moon} = \frac{1}{4}(\text{Radius of the } \cancel{\text{moon}} \text{ earth})$$

$$\Rightarrow \quad \text{Radius of the moon} = \frac{1}{4}(r) = \frac{r}{4}$$

$$\therefore \quad \text{Volume of the earth} = \frac{4}{3}\pi r^3$$

$$\text{Volume of the moon} = \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = \frac{4}{3} \times \pi \times \frac{r \times r \times r}{4 \times 4 \times 4} = \frac{\pi r^3}{48}$$

$$\text{Now,} \quad \frac{[\text{Volume of the earth}]}{[\text{Volume of the moon}]} = \frac{\left[\frac{4}{3}\pi r^3\right]}{\left[\frac{\pi r^3}{48}\right]} = \frac{4}{3}\pi r^3 \times \frac{48}{\pi r^3} = \frac{64}{1}$$

$$\text{or} \quad \frac{[\text{Volume of the moon}]}{[\text{Volume of the earth}]} = \frac{1}{64}$$

$$\therefore \quad \text{The required fraction is } \frac{1}{64}.$$

Let ~~radius of~~ earth be r

~~radius~~ of moon $= \frac{r}{4}$

$$\frac{\text{volume of earth}}{\text{volume of moon}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi \left(\frac{r}{4}\right)^3}$$

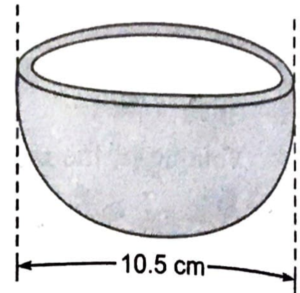
$$= \frac{1}{\left(\frac{1}{4}\right)^3} = \frac{4^3}{1} = \frac{64}{1}$$

$\therefore$ volume of moon $= \frac{1}{64}$ (volume of earth)

The fraction is $\boxed{\frac{1}{64}}$

Question 5. *How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?*

Solution: Diameter of the hemisphere = 10.5 cm



$\Rightarrow$ Radius of the hemisphere (r) $= \frac{10.5}{2}$ cm $= \frac{105}{20}$ cm

Volume of the hemispherical bowl $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{105}{20} \times \frac{105}{20} \times \frac{105}{20} \text{ cm}^3 = \frac{11 \times 105 \times 105}{20 \times 20} \text{ cm}^3$$

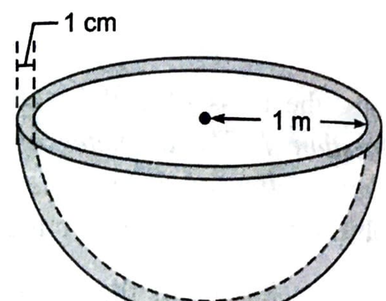
$$= \frac{121275}{400} = 303.1875 \text{ cm}^3$$

$\therefore$ Capacity of the hemispherical bowl = 303.1875 cm^3

$$= \frac{3031875}{10000 \times 1000} = 0.3031875\ l = 0.303\ l \text{ (approx.)} \qquad [\because 1000 \text{ cm}^3 = 1\ l]$$

Thus, the capacity of the bowl = 0.303 l (approx.)

Question 6. *A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.*



Solution: Inner radius (r) = 1m

$\because$ Thickness $= 1 \text{ cm} = \frac{1}{100} \text{ m} = 0.01 \text{ m}$

$\therefore$ Outer radius (R) = 1 m + 0.01 m = 1.01 m

Now, outer volume of the hemispherical bowl

$$= \frac{2}{3}\pi R^3 = \frac{2}{3} \times \frac{22}{7} \times (1.01)^3 \text{ m}^3$$

Inner volume of the hemispherical bowl

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} (1)^3 \text{ m}^3$$

$\therefore$ Volume of the iron used = [Outer volume] – [Inner volume]

$$= \frac{2}{3} \times \frac{22}{7} \times (1.01)^3 - \frac{2}{3} \times \frac{22}{7} \times (1)^3$$

$$= \frac{2}{3} \times \frac{22}{7} [(1.01)^3 - (1)^3] \text{ m}^3$$

$$= \frac{44}{21} [1.030301 - 1] \text{ m}^3$$

$$= \frac{44}{21} \times [0.030301] \text{ m}^3$$

$$= 0.063487776 \text{ m}^3$$

$$= 0.06348 \text{ m}^3 \text{ (approx.)}$$

Thus, the required volume of the iron = 0.06348 m^3

Question 7. *Find the volume of a sphere whose surface area is 154 cm^2*

Solution: Let 'r' be the radius of the sphere.

$\therefore$ Its surface area $= 4\pi r^2$

$\Rightarrow$ $4\pi r^2 = 154$

$\Rightarrow \quad r^2 = \frac{154}{4\pi} = \frac{154 \times 7}{4 \times 22} = \frac{7 \times 7}{4}$

$\Rightarrow \quad r^2 = \left(\frac{7}{2}\right)^2$

$\Rightarrow \quad r = \frac{7}{2}$ cm

Now, volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$ cm^3

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$ cm^3

$= \frac{11 \times 7 \times 7}{3}$ cm^3 $= \frac{539}{3}$ cm^3

$= 179\frac{2}{3}$ cm^3

Thus, the required volume of the sphere $= 179\frac{2}{3}$ cm^3

Question 8. *A dome of a building is in the form of a hemisphere. From inside, it was white washed at the cost of ₹ 498.96. If the cost of white washing is ₹ 2.00 per square metre, find the*

(i) inside surface area of the dome, *(ii) volume of the air inside the dome.*

Solution: Total cost of white washing = ₹ 498.96

Rate of white washing = ₹ 2 per m

$\therefore \quad$ Area $= \frac{\text{Total cost}}{\text{Rate}} = \frac{498.96}{2} = 249.48$ m^2

Let 'r' be the radius of the hemispherical dome.

$\therefore \quad$ Surface area $= 2\pi r^2$

$\therefore \quad 2\pi r^2 = 249.48$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = \frac{249.48}{100}$

$\Rightarrow \quad r^2 = \frac{24948}{100} \times \frac{7}{2 \times 22} = \frac{567 \times 7}{100} = \frac{3969}{100}$

$\Rightarrow \quad r^2 = \left(\frac{63}{10}\right)^2$ m

$\Rightarrow \quad r = \frac{63}{10} = 6.3$ m

(*i*) To find the inside surface area of the dome:

$\because \quad$ Radius of the hemisphere (r) = 6.3 m

Surface area of a hemisphere $= 2\pi r^2$ m^2

$\therefore \quad$ Surface area of the dome $= 2 \times \frac{22}{7} (6.3)^2$ m^2

$$= \frac{2 \times 22 \times 63 \times 63}{7 \times 10 \times 10} \text{ m}^2$$

$$= \frac{2 \times 22 \times 9 \times 63}{100} \text{ m}^2 = \frac{24948}{100} \text{ m}^2$$

$$= 249.48 \text{ m}^2$$

Thus, the required surface area of the dome = 249.48 m^2

(*ii*) To find the volume of air in the dome:

$$\text{Volume of a hemisphere} = \frac{2}{3}\pi r^3$$

$$\therefore \quad \text{Volume of the dome} = \frac{2}{3} \times \frac{22}{7} \times (6.3)^3 \text{ m}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{63}{10} \times \frac{63}{10} \times \frac{63}{10} \text{ m}^3$$

$$= \frac{2 \times 22 \times 3 \times 63 \times 63}{1000} \text{ m}^3$$

$$= \frac{523908}{1000} \text{ m}^3 = 523.9 \text{ m}^3 \text{ (approx.)}$$

Thus, the required volume of air inside the dome is 523.9 m^3 (approx.).

Question 9. *Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the (i) radius r′ of the sphere, (ii) ratio of S and S′.*

Solution: (*i*) To find r′:

$$\because \quad \text{Radius of a small sphere} = r$$

$$\therefore \quad \text{Its volume} = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \text{Volume of 27 small spheres} = 27 \times \left[\frac{4}{3}\pi r^3\right]$$

$$\because \quad \text{Radius of the new sphere} = r'$$

$$\therefore \quad \text{Volume of the new sphere} = \frac{4}{3}\pi (r')^3$$

$$\text{Since,} \quad \frac{4}{3}\pi (r')^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad (r')^3 = \frac{27 \times \frac{4}{3}\pi r^3}{\frac{4}{3}\pi} = 27 r^3$$

$$\Rightarrow \quad (r')^3 = (3r)^3$$

$$\Rightarrow \quad r' = 3r$$

(*ii*) To find ratio of S and S′:

$\because$ Surface area of a small sphere $= 4\pi r^2$

$$\therefore \quad S = 4\pi r^2$$

$$\text{and} \quad S' = 4\pi(3r)^2$$

Now, $$\frac{S}{S'} = \frac{4\pi r^2}{4\pi(3r)^2} = \frac{4\pi r^2}{4\pi(9r^2)} = \frac{1}{9}$$

Thus, $S : S' = 1 : 9$

Question 10. *A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?*

Solution: Diameter of the spherical capsule = 3.5 mm

$\Rightarrow$ Radius of the spherical capsule (r) = $\frac{3.5}{2}$ mm

$\therefore$ Volume of the spherical capsule $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{35}{20}\right)^3 \text{ mm}^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times \frac{35}{20} \text{ mm}^3$$

$$= \frac{22 \times 35 \times 35}{3 \times 20 \times 20} \text{ mm}^3$$

$$= \frac{26950}{1200} \text{ mm}^3 = 22.45833 \text{ mm}^3$$

$$= 22.46 \text{ mm}^3 \text{ (approx.)}$$

Thus, the required quantity of medicine = 22.46 mm^3 (approx).