Assignment - 1

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I affirm that I have neither given nor received help or used any means which would make this assignment unfair.
— M. Harish

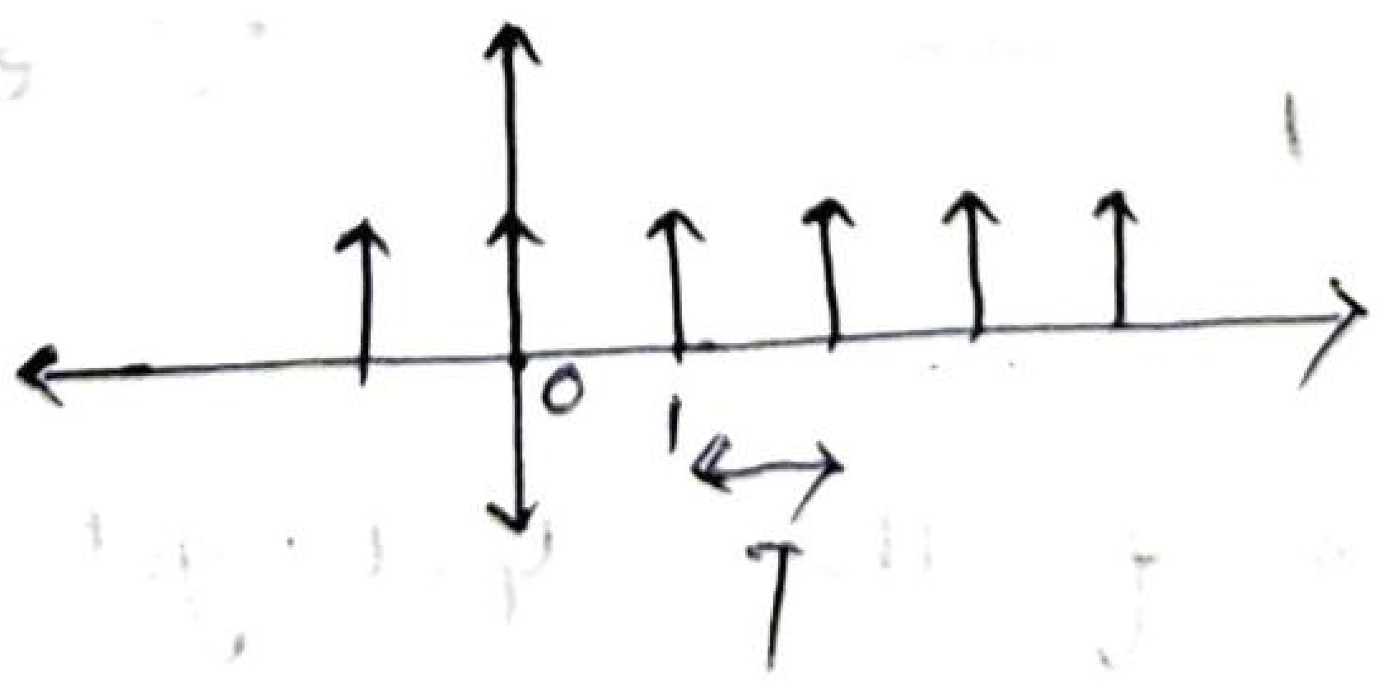
3.34) given

$$h(t) = e^{-4|t|}$$

we know that $e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$

$$\Rightarrow H(\omega) = \frac{8}{16+\omega^2}$$

(a) $$x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-n)$$



Time period $T = 1$

$$\Rightarrow \omega_0 = \frac{2\pi}{T} = 2\pi$$

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$$a_K = \frac{1}{T}\int_T x(t)\cdot e^{-jKw_0 t}\,dt$$

• for $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$

$$= \delta(t-(-\infty)) + \dots\ \delta(t-(n-1)) + \delta(t-n) + \delta(t-(n+1)) \dots$$

$$= \frac{1}{1}\int_{n-1/2}^{n+1/2} \sum_{n=-\infty}^{\infty}\delta(t-n)\, e^{-jKw_0 t}\,dt$$

⇒ By property we know that

$$\int \delta(t-t_0)\, x(t) = x(t_0).$$

$$= e^{-jK2\pi n}$$

$$= e^{-j2\pi nK}$$

$a_K = 1$ $\quad\because e^{i2\pi} = 1$

The coefficients of the output $y(t)$ b_K can write as

$$b_K = a_K H(Kw_0).$$

$$= a_K \cdot \frac{8}{16+w_0^2K^2} = \frac{8}{16+w_0^2K^2}$$

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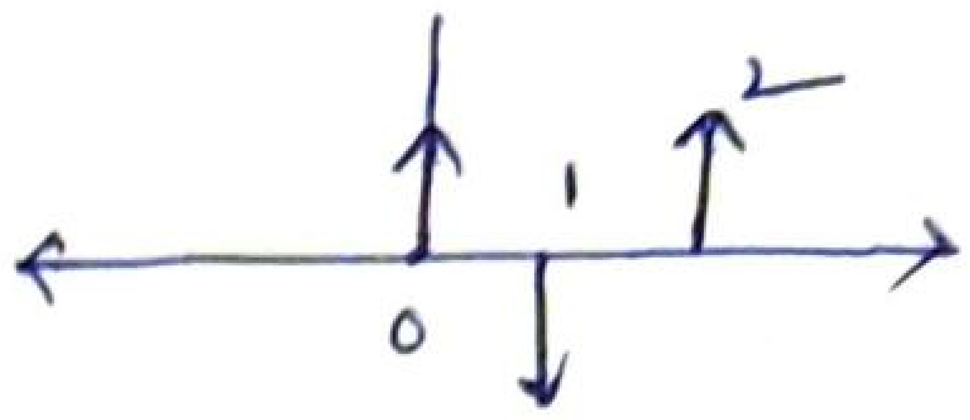
(b) given

$$x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n)$$



Time period $= 2$

$$\Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$$

$$a_K = \frac{1}{T} \int_T x(t)\, e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{2} \int_{-1}^{1} (-1)\Big(\delta(t+1) + \delta(t)\Big)\, e^{-jK\omega_0 t}\, dt$$

$$= \frac{1}{2} \left(1 - e^{jK\omega_0}\right)$$

$$= \frac{1}{2} \left(1 - e^{jK\pi}\right)$$

$$= \frac{1}{2} \Big(1 - \left(\cos(K\pi) + j\sin(K\pi)\right)\Big)$$

$$= \frac{1}{2} \Big(1 - \cos(K\pi) - j\sin(K\pi)\Big)$$

$$\Rightarrow \sin(K\pi) = 0 \quad \forall\ K \in Z$$

$$\cos(K\pi) = \begin{cases} 1 & \forall\ K = 0, 2, 4 \ldots \text{ even} \\ -1 & \forall\ K = \text{odd} \end{cases}$$

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Fourier coefficients of output signal

$$b_K = a_K \cdot H(K\omega_0).$$

$$= a_K \cdot H(K\pi)$$

$$= a_K \cdot \frac{8}{16 + K^2\pi^2}$$

$$\therefore \quad b_K = \begin{cases} 0, & \forall\ K = \text{even} \\ 8/16 + K^2\pi^2 & \forall\ K = \text{odd} \end{cases}$$

$$\Rightarrow \therefore \quad y(t) = \sum_{K=-\infty}^{\infty} b_K \cdot e^{-jK\omega_0 t}$$

$$y(t) = \sum_{K=-\infty}^{\infty} b_K\, e^{-jK\omega t}.$$

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3.44 Suppose we are given the following information about a signal $x(t)$:

1) $x(t)$ is a real signal.

2) $x(t)$ is periodic with period $T = 6$ and has fourier coefficients a_k.

3) $a_k = 0$ for $k=0$ and $k > 2$

4) $x(t) = -x(t-3)$

5) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = 1/2$

6) a_1 is a positive real number.

Sol

$\because$ $x(t)$ is a real signal

1 $\Rightarrow$

$\Rightarrow a_k = -a_k$

2 $\Rightarrow$ given time period $T = 6$

$$w_0 = \frac{2\pi}{6} = \pi/3.$$

3 $\Rightarrow$ & $a_k = 0$ for $k=0$ & $k > 2$

$\Rightarrow a_1, a_{-1}, a_{-2}, a_2 \neq 0.$

4 $\Rightarrow x(t) = -x(t-3)$

$$x(t) = a_1 e^{jw_0 t} + a_{-1} e^{-jw_0 t} + a_{-2} e^{-2jw_0 t} + a_2 e^{2jw_0 t}$$

$$= a_1 e^{jw_0 t} + a_1 e^{-jw_0 t} + a_2 e^{-2jw_0 t} + a_2 e^{2jw_0 t}$$

$$= a_1 \left[e^{jw_0 t} + e^{-jw_0 t} \right] + a_2 \left(e^{2jw_0 t} + e^{-2jw_0 t} \right)$$

$$= a_1 \, 2\cos(w_0 t) + a_2 \cdot 2\cos(2w_0 t)$$

$$= 2a_1 \cos(w_0 t) + 2a_2 \cos(2w_0 t).$$

$$= 2a_1 \cos\left(\frac{\pi}{3} t\right) + 2a_2 \cos\left(\frac{2\pi}{3} t\right).$$

$$= 2a_1 \cos\left(\frac{\pi}{3} t\right) + 2a_2 \cos\left(\pi - \frac{\pi}{3} t\right)$$

$$= 2a_1 \cos\left(\frac{\pi}{3} t\right) - 2a_2 \cos\left(\frac{\pi}{3} t\right).$$

$$= 2\cos\left(\frac{\pi}{3} t\right) \left[a_1 - a_2 \right] \longrightarrow (i)$$

$$\Rightarrow -x(t-3) = -\left[a_1 e^{jw_0(t-3)} + a_1 e^{-jw_0(t-3)} + a_2 e^{2jw_0(t-3)} + a_2 e^{-2jw_0(t-3)} \right]$$

$$= -\left[2a_1 \cos\left(\frac{\pi}{3} t - \pi\right) + 2a_2 \cos\left(\frac{2\pi}{3} t - 2\pi\right) \right].$$

$$= -\left[-2a_1 \cos\left(\frac{\pi}{3} t\right) + 2a_2 \cos\left(\frac{2\pi}{3} t\right) \right)$$

$$= -\left[-2a_1 \cos\left(\frac{\pi}{3} t\right) \mp 2a_2 \cos\left(\frac{\pi}{3} t\right) \right]$$

$$= 2\cos\left(\frac{\pi}{3} t\right) \left[a_1 + a_2 \right].$$

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$\because \; x(t) = -x(t-3)$.

$$2\cos\left(\cancel{\tfrac{\pi}{3}}t\right)[a_1 - a_2] = 2\cos\left(\cancel{\tfrac{\pi}{3}}t\right)[a_1 + a_2]$$

$$a_1 - a_2 = a_1 + a_2$$

$$2a_2 = 0$$

$$a_2 = 0$$

$\Rightarrow \; a_2 = 0 \;\; \because \; x(t)$ is real then

$a_{-2} = 0$.

⑤ $\Rightarrow \; \underbrace{\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt}_{} = 1/2$

↳ parseval's theorem.

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} |a_k|^2 = 1/2$$

$$|a_1|^2 + |a_{-1}|^2 = \frac{1}{2}$$

$$2|a_1|^2 = 1/2$$

$$|a_1|^2 = 1/4$$

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$a_1 = 1/2$ or $-1/2$

from (6) $\Rightarrow$ a_1 is positive

$\therefore \boxed{a_1 = 1/2}$

given
$$x(t) = A\cos(Bt + C)$$

$$x(t) = 2\cos\left(\frac{\pi}{3}t\right)(a_1 - a_2) \quad \text{from (i)}$$

$$= \cancel{2}\cos\left(\frac{\pi}{3}t\right)\left(1/\cancel{2}\right)$$

$$x(t) = \cos\left(\frac{\pi}{3}t\right)$$

By comparing

$$\boxed{A = 1,\ B = \frac{\pi}{3},\ C = 0}$$

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3.61

(a) what are the eigen-functions of the LTI system with unit Impulse response $h(t) = \delta(t)$? what are the associated eigen values?

Sol $h(t) = \delta(t)$.

All functions of a LTI system with unit impulse response all functions are eigen functions with a eigen value of one.

(b) Consider the LTI system with unit impulse response $h(t) = \delta(t-T)$. Find a signal that is not of the form e^{st}, but that is an eigenfunction of the system with eigen value 1. Similiarly, find the eigen functions with eigen values $\frac{1}{2}$ & 2 that are not complex exponentials?

Sol

LTI system with impulse response $h(t) = \delta(t-T)$.

generilazation :-

$$h(t) = \cancel{\alpha^k} \sum_{k=-\infty}^{\infty} (x)^k \delta(t-kT)$$

where x is a eigen value.

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$\Rightarrow$ eigen function with eigen value $1/2$

$$h(t) = \sum_{K=-\infty}^{\infty} \left(\frac{1}{2}\right)^K \delta(t-KT).$$

$\Rightarrow$ eigen function with eigen value 2

$$h(t) = \sum_{K=-\infty}^{\infty} (2)^K \delta(t-KT).$$

(c) Consider a stable LTI system with impulse response $h(t)$ that is real and even. Show that $\cos(\omega t)$ and $\sin(\omega t)$ are eigen functions of this system.

Sol Since $h(t)$ is real & even so,

$$H(-\omega) = H(\omega)$$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$e^{j\omega t} \longrightarrow \boxed{H(\omega)} \longrightarrow H(\omega)e^{j\omega t} \rightarrow (1)$$

$$e^{-j\omega t} \longrightarrow \boxed{H(\omega)} \longrightarrow H(\omega)e^{-j\omega t} \rightarrow (2)$$

from (1) & (2).

$$\frac{e^{j\omega t} + e^{-j\omega t}}{2} \longrightarrow \boxed{H(\omega)} \longrightarrow H(\omega)\cdot\frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$= H(\omega)\cdot\cos(\omega t)$$

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$\therefore$ $\cos(\omega t)$ is an eigen-function of given LTI system.

Similarly we can write

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2}$$

$$\frac{e^{j\omega t} - e^{-j\omega t}}{2} \longrightarrow \boxed{H(\omega)} \longrightarrow H(\omega)\,\frac{e^{j\omega t} - e^{-j\omega t}}{2}$$

$$= H(\omega)\,\sin(\omega t).$$

$\therefore$ $\sin(\omega t)$ is an eigen function of given LTI system

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4.13

Let $x(t)$ be a signal whose fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)$$

and let $h(t) = u(t) - u(t-2)$

Solution.

(a)

given $X(j\omega) = \delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)\right) e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}\, d\omega + \int_{-\infty}^{\infty} \delta(\omega-\pi)\, e^{j\omega t}\, d\omega + \int_{-\infty}^{\infty} \delta(\omega-5)\, e^{j\omega t}\, d\omega.$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t}\, d\omega = e^{j\omega_0 t}}$$

$$= \frac{1}{2\pi}\left[e^{j0.t} + e^{j\pi t} + e^{j5t} \right].$$

$$= \frac{1}{2\pi} + \frac{e^{j\pi t}}{2\pi} + \frac{e^{j5t}}{2\pi}$$

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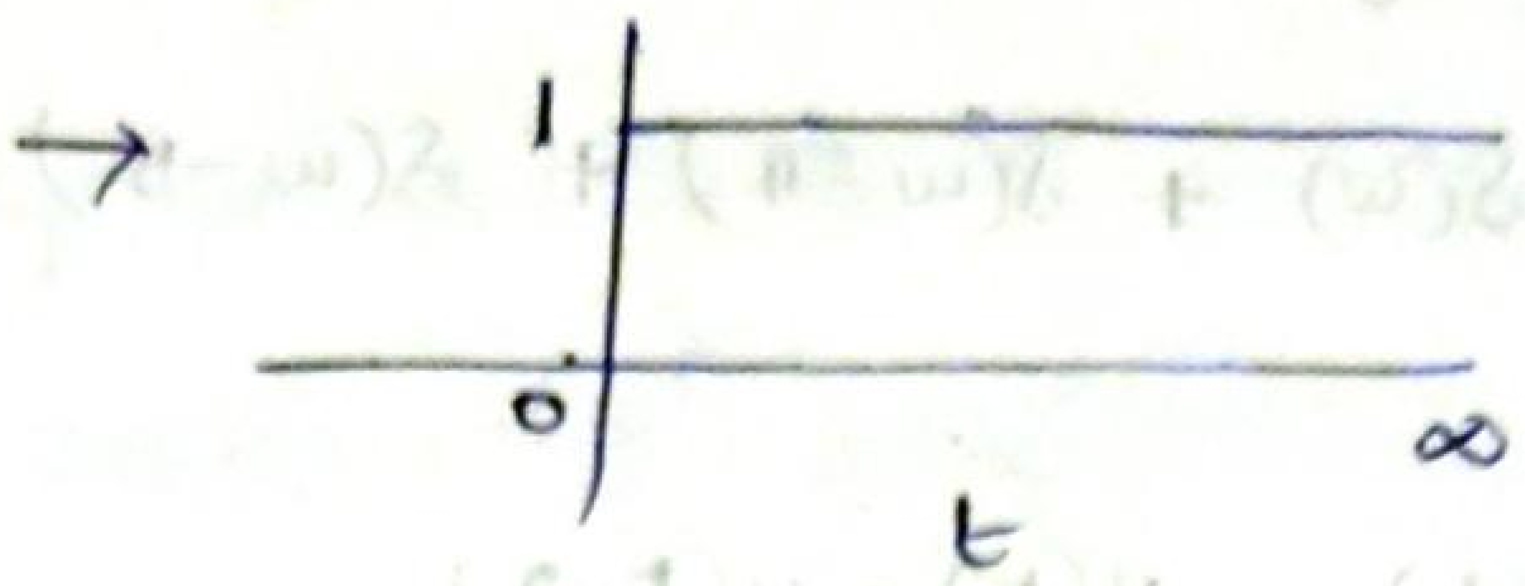
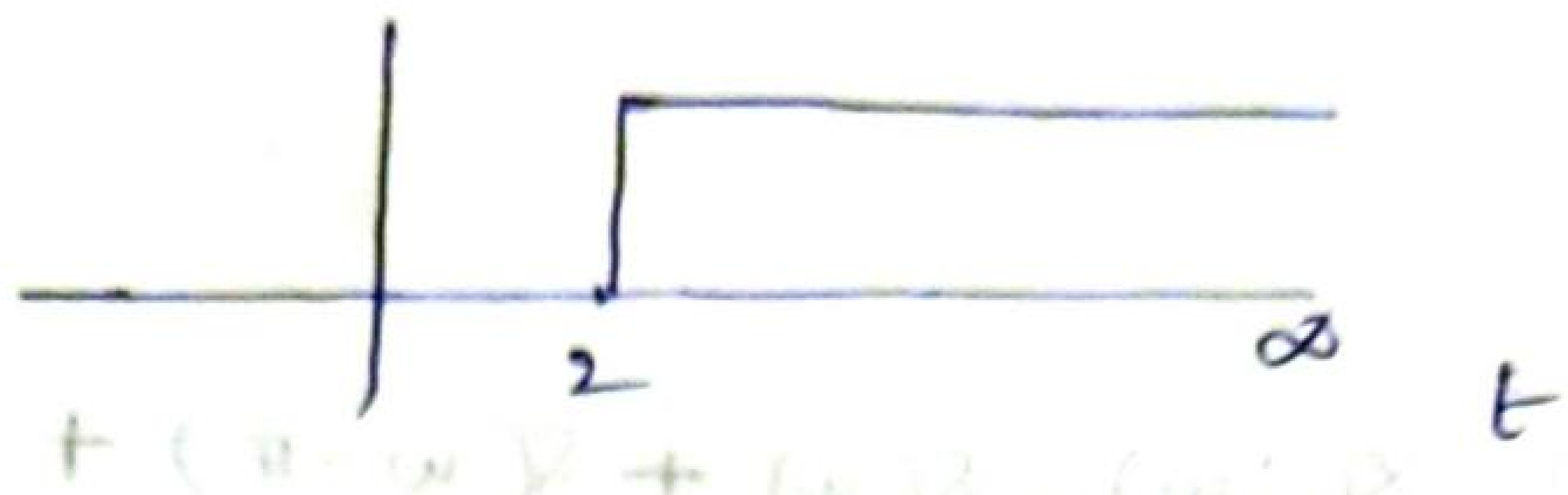
$h(t) = u(t) - u(t-2)$.

$u(t) \rightarrow$



$u(t-2)$



$$\Rightarrow u(t) - u(t-2) = \begin{cases} 1 & 0 < t < 2 \\ 0 & \text{~~at~~ for others.} \end{cases}$$

(a) $x(t)$ is periodic :

we got $x(t) = \frac{1}{2\pi} + \frac{e^{j\pi t}}{2\pi} + \frac{e^{j5t}}{2\pi}$.

Time period of second term $\Rightarrow T = \frac{2\pi}{\omega}$

$= 2$.

time period for third term $T = \frac{2\pi}{5}$

→ These time periods are not Harmonically Related. It cannot be integral multiples of a common fundamental frequency.

$\Rightarrow$ signal is not periodic.

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(b) Is $x(t) * h(t)$ periodic ?

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

$h(\tau) = 1 \quad 0<\tau<2$

$$= \int_{0}^{2} x(t-\tau)\, d\tau.$$

$$= \int_{0}^{2} \frac{1}{2\pi} + \frac{e^{j\pi(t-\tau)}}{2\pi} + \frac{e^{j5(t-\tau)}}{2\pi}\, d\tau.$$

$$= \int_{0}^{2} \frac{1}{2\pi}\, d\tau + \int_{0}^{2} \frac{e^{j\pi(t-\tau)}}{2\pi}\, d\tau + \int_{0}^{2} \frac{e^{j5(t-\tau)}}{2\pi}\, d\tau.$$

$$= \frac{1}{2\pi}(2) + \frac{1}{2\pi} e^{j\pi t} \int_{0}^{2} e^{-j\pi\tau}\, d\tau + \frac{e^{j5t}}{2\pi} \int_{0}^{2} e^{-j5\tau}\, d\tau$$

$$= \frac{1}{2\pi}(2) + \frac{e^{j\pi t}}{2\pi} \left. \frac{e^{-j\pi\tau}}{-j\pi} \right|_{0}^{2} + \frac{e^{j5t}}{2\pi} \left. \frac{e^{-j5\tau}}{-j5} \right|_{0}^{2}$$

$$= \frac{1}{2\pi}(2) + \frac{e^{j\pi t}}{2\pi(-j\pi)} \left(\cancel{e^{-j2\pi} - 1} \right) + \frac{e^{j5t}}{-2\pi \cdot j5} \left(e^{-j5(2)} - 1 \right)$$

0

$$= \frac{1}{2\pi}(2) + 0 - \frac{e^{j(5t-10)}}{2\pi\, 5j} + \frac{e^{j5t}}{5j\, 2\pi}$$

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$$= \frac{1}{2\pi}\left[2 - \frac{e^{j(5t-10)}}{5j} + \frac{e^{j5t}}{5j} \right].$$

Time period :- $T = \frac{2\pi}{5}$.

$\Rightarrow$ periodic signal.

(c) Can the convolution of two aperiodic signals be periodic.

$\therefore$ from a and b, we know

$x(t)$ and $h(t)$ are aperiodic signals but their convulution $y(t) = x(t) * h(t)$ is periodic.

$\therefore$ convulution of two aperiodic signals is periodic.

(4.21)

(e) $[t e^{-2t} \sin 4t] u(t)$.

let $x(t) = (t e^{-2t} \sin 4t) \cdot u(t)$.

$x(t) \rightleftharpoons X(w)$.

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$$x(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} t \cdot e^{-2t} \sin(4t)(u(t))\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} t \cdot \sin(4t)\, e^{-2t - j\omega t}\, dt$$

$$= \int_{0}^{\infty} t \cdot \left(\frac{e^{j4t} - e^{-j4t}}{2j}\right) \cdot e^{-t(+2+j\omega)}\, dt$$

$$= \frac{1}{2j}\left[\int_{0}^{\infty} t \cdot e^{j4t} \cdot e^{-t(2+j\omega)}\, dt - \int_{0}^{\infty} t \cdot e^{-j4t} \cdot e^{-t(2+j\omega)}\, dt\right.$$

$$= \frac{1}{2j}\left[\int_{0}^{\infty} t \cdot e^{-t(-4j+2+j\omega)}\, dt - \int_{0}^{\infty} t \cdot e^{-t(+4j+2+j\omega)}\, dt\right].$$

$$= \frac{1}{2j}\left[\frac{1}{(-4j+2+j\omega)^2} - \frac{1}{(+4j+2+j\omega)^2}\right].$$

$$\boxed{X(\omega) = \frac{1}{2j}\left[\frac{1}{(2+j(\omega-4))^2} - \frac{1}{(2+j(\omega+4))^2}\right]}$$

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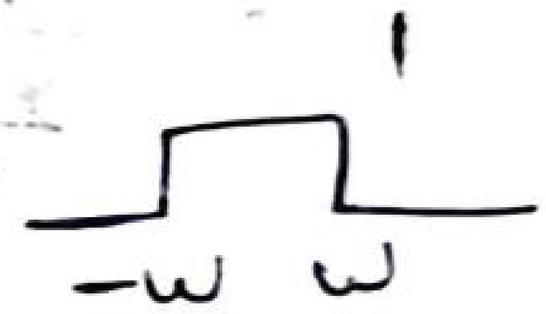
(f) $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$.

let $x_1(t) = \dfrac{\sin \pi t}{\pi t}$

~~As~~ we discussed in the class this looks likes sinc function which is a Inverse fourier transform of rectangular pulse

ie

$$\frac{\sin(wt)}{\pi t} \xrightarrow{FT} \text{rectangular pulse}$$



$$\Rightarrow \quad x_1(t) \xrightarrow{FT} X_1(w)$$

$$X_1(w) = \begin{cases} 1 & |w| < \pi \\ 0 & \text{otherwise} \end{cases}$$

let

$$x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)}$$

$$x_2(t) \xleftrightarrow{FT} X_2(w) \qquad \text{time shifting}$$

$$X_2(w) = \begin{cases} e^{-2w} & |w| < 2\pi \\ 0 & \text{other cases.} \end{cases}$$

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$x(t) = x_1(t) \cdot x_2(t)$.

multiplication in ~~time~~ domain is ~~freq~~ $\frac{1}{2}$ times Convulution in ~~frequency~~ domain

$$X(w) = \frac{1}{2\pi} \{ X_1(w) * X_2(w) \}.$$

we get

$$X(w) = \begin{cases} e^{-jw} & |w| < \pi \\ (1/2\pi)(3\pi + w) e^{-jw} & -3\pi < w < -\pi \\ (1/2\pi)(3\pi - w) e^{-jw} & \pi < w < 3\pi \\ 0 & \text{otherwise} \end{cases}$$

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4.29

given a real valued function $x(t)$ which has fourier tranform $X(j\omega)$.

Relation between $X_a(j\omega)$ and $X(j\omega)$

*
$$X_a(j\omega) = |X(j\omega)| e^{j\measuredangle X(j\omega) - ja\omega}$$
$$= X(j\omega) e^{-ja\omega}$$

using the time shift property

$$x(t-a) \xleftrightarrow{FT} X(j\omega) e^{-ja\omega}$$

So we can use time shifting property

$$\therefore \boxed{x_a(t) = x(t-a)}$$

*
$$X_b(j\omega) = |X(j\omega)| e^{j\measuredangle X(j\omega) + jb\omega}$$
$$= X(j\omega) e^{jb\omega}.$$

Similarly like above
from the time shifting property we write

$$\boxed{x_b(t) = x(t+b)}$$

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* given the relation b/w $X_c(j\omega)$ & $X(j\omega)$ it as follows.

$$X_c(j\omega) = |X(j\omega)|\, e^{-j\angle X(j\omega)} = X^*(j\omega)$$

from the properties conjugation & time reversal we know that

$$x^*(-t) \xleftrightarrow{FT} X^*(j\omega).$$

we can write

$$x_c(t) = x^*(-t) = x(-t).$$

$$\boxed{x_c(t) = x(-t)}$$

* given relation b/w $X_d(j\omega)$ & $X(j\omega)$

$$X_d(j\omega) = |X(j\omega)|\, e^{-j\angle X(j\omega) + jd\omega}.$$

$$= X^*(j\omega)\, e^{jd\omega}.$$

By using conjugation, time reversal & time shifting property.

$$x_d(t) = x^*(-t-d)$$

Since $x(t)$ is real we know $\boxed{x_d(t) = x(-t-d)}$

$\Downarrow$

$x^*(t) = x(t).$

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