classmate

Date
Page

EC-348 IMAGE & VIDEO PROCESSING

PRE - ASSIGNMENT

HARISH BACHU

181EC214

Q1 i) $x(t) = 3\cos(2000\pi t) + 5\sin(6000\pi t) + 10\cos(12000\pi t)$

$f_{max} = 6000 Hz$

$\therefore$ Nyquist Rate $= 2f_{max}$

$= \underline{\underline{12000 Hz}}$

Q1 ii) $f_s = 5000$ samples/sec

$\therefore x[n] = x(n/F_s)$

$= 3\cos\left(\frac{2\pi n}{5}\right) + 5\sin\left(\frac{6\pi n}{5}\right) + 10\cos\left(\frac{12\pi n}{5}\right)$

$= 3\cos\left(\frac{2\pi n}{5}\right) + 5\sin\left(\frac{6\pi n}{5}\right) + 10\cos\left(\frac{2\pi n}{5}\right)$

$= 3\cos(0.4\pi n) - 5\sin(0.8\pi n)$

Q2 $x(t) = 10 + 10\sin(500t)$

$f = \frac{500}{2\pi} = \frac{250}{\pi} Hz$ $(\approx 79.57 Hz)$

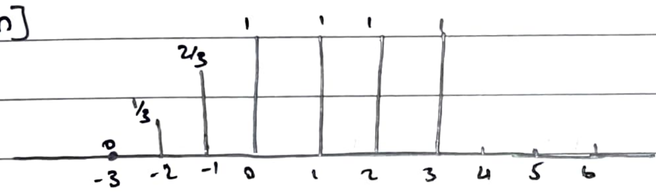
$\therefore$ Nyquist Rate $= \frac{500}{\pi} Hz$ $(\approx 159.15 Hz)$

Max allowable interval b/w samples $= \frac{\pi}{500} s$

No. of samples in 2 seconds $= 2f_{nyq} = \frac{1000}{\pi}$ (≈ 318.3)

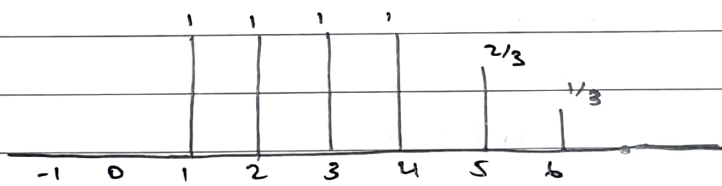
Q3 $$x(n) = \begin{cases} 1 + \frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

a) $x[n]$



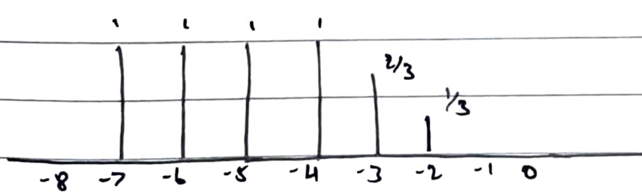
b) ii) Folded $\rightarrow$ $x[-n]$

Folded & ~~Delyo~~ Delayed $\Rightarrow$ $x[-n-4]$

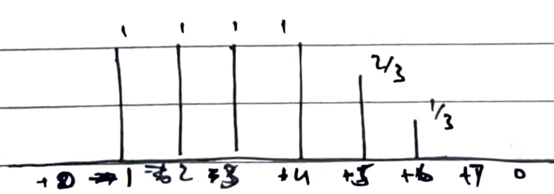


i) Delayed $x[n-4]$

Delayed & Folded $x[4-n]$



c) $x[-n+4]$



d) $x[n] = \frac{1}{3}\delta[n+2] + \frac{2}{3}\delta[n+1] + u[n] - u[n-4]$

classmate

Date

Page

Q4 a) $y[n] = \cos[x(n)]$

i) Static ∵ $y[n]$ depends only on input sample $\forall n$

ii) Non Linear :

$$T(a_1 x_1[n]) = \cos(a_1 x_1[n])$$
$$\neq a_1 \cos(x_1[n]) = a_1 y_1[n]$$

$$T(x_1[n] + x_2[n]) = \cos(x_1[n] + x_2[n])$$
$$\neq \cos(x_1[n]) + \cos(x_2[n]) = y_1[n] + y_2[n]$$

iii) Time Invariant

$$y[n] = \cos[x[n])$$
$$\Rightarrow y[n \pm k] = \cos(x[n \pm k])$$

iv) Causal ∵ depends only on present i/p samples

v) Stable : $|x[n]| \le M_x \le \infty$

$$\Rightarrow |y[n]| \le M_y \le \infty$$

b) $y[n] = \sum_{k=-\infty}^{n+1} x(k)$

i) Dynamic ∵ depends on previous samples

ii) Linear :

$$T(a_1 x[n]) = \sum_{k=-\infty}^{n+1} a_1 x(k) = a_1 \sum_{k=-\infty}^{n+1} x(k) = a_1 y[n]$$

$$T(x_1[n] + x_2[n]) = \sum_{k=-\infty}^{n+1} x_1(k) + \sum_{k=-\infty}^{n+1} x_2(k)$$

$$= y_1(n) + y_2(n)$$

iii.) Time Invariant :

$$y[n \pm m] = \sum_{k=-\infty}^{n \pm m+1} x[k]$$

iv.) Not Causal

v.) Not Stable : Let $x[k] = u[k]$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{n+1} u[k] = \begin{cases} 0 & n < -1 \\ n+2 & n \geq -1 \end{cases}$$

$n \to \infty \Rightarrow y(n) \to \infty$

$\Rightarrow$ Unstable

c.) $y(n) = x(n) \cos(\omega_0 n)$

i.) Static

ii.) Linear : $T(a_1 x_1[n]) = a_1 x_1[n] \cos(\omega_0 n)$
$= a_1 y_1[n]$

$$T(x_1[n] + x_2[n]) = (x_1[n] + x_2[n]) \cos(\omega_0 n)$$
$$= y_1[n] + y_2[n]$$

iii.) Time Variant

iv.) Causal

v.) Stable

d.) $y(n) = x(-n+4)$

i.) Dynamic ii.) Linear iii.) Time Variant
iv.) Non Causal v.) Stable

classmate

Date

Page

Q7 $x[n] = \{3, 5, -2, 4\}$

$h[n] = \{3, 1, 3\}$

$y[n] = x[n] * h[n]$

$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$y[0] = x[0]h[0] = 9$

$y[1] = x[0]h[1] + x[1]h[0] = 18$

$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0] = 8$

$y[3] = x[1]h[2] + x[2]h[1] + x[3]h[0] = 21$

$y[4] = x[2]h[2] + x[3]h[1] + x[4]h[0] = -2$

$y[5] = x[3]h[2] = 12$

$\Rightarrow y[n] = \{9, 18, 8, 21, -2, 12\}$

Q5 a) $\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

$X(k) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta_{T_0}(t)\, e^{-j2\pi kt/T_0}\, dt$

$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t)\, e^{-j2\pi kt/T_0}\, dt = \frac{1}{T_0}$

$\Rightarrow X(\omega) = \frac{2\pi}{T_0} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T_0}\right)$

b) $\text{IDTFT}\left(Xe^{j\omega} = \frac{1}{(1+ae^{-j\omega})^2}\right) = (n+1)(-a)^n u[n]$

classmate
Date
Page

Q6 a) $X_1(e^{j\omega}) = \sum_{m=-\infty}^{\infty} \left[2\pi\delta(\omega - 2\pi m) + \pi\delta\left(\omega - \frac{\pi}{2} - 2\pi m\right) + \pi\delta\left(\omega + \frac{\pi}{2} - 2\pi m\right) \right]$

$= \sum_{m=-\infty}^{\infty} 2\pi\delta(\omega - 2\pi m) + \sum_{m=-\infty}^{\infty} \pi\delta\left(\omega - \frac{\pi}{2} - 2\pi m\right)$

$+ \sum_{m=-\infty}^{\infty} \pi\delta\left(\omega + \frac{\pi}{2} - 2\pi m\right)$

$= \underbrace{\sum_{m=-\infty}^{\infty} 2\pi\delta(\omega - 2\pi m)}_{I} + \text{DTFT}(\cos\omega_0 n)\Big|_{\omega_0 = \pi/2}$

IDTFT (I) :> $x_I[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi m)$

$= 1$

$\therefore \quad x_1[n] = 1 + \cos\frac{\pi}{2}n$

b) $X_2(e^{j\omega}) = \begin{cases} 2j & 0 < \omega < \pi \\ -2j & -\pi < \omega \leq 0 \end{cases}$

$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{0} -2j\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} 2j\, e^{j\omega n}\, dn$

$= \frac{-j}{\pi}\left[\frac{e^{j\omega n}}{n}\Big|_{-\pi}^{0} \right] + \frac{j}{\pi}\left[\frac{e^{j\omega n}}{n}\Big|_{0}^{\pi} \right]$

$= \frac{-j}{\pi n}\left[1 - e^{-j\pi n} \right] + \frac{j}{\pi n}\left[e^{j\pi n} - 1 \right]$

$= \left(e^{j\pi n/2} - e^{-j\pi n/2} \right)^2 \frac{j}{\pi n}$

classmate
Date
Page

Q8 a) i) $x[n] = \{1,2,3,4\}$

Direct Method

$$X(k) = 1 + 2e^{-j2\pi k/4} + 3e^{-j2\pi \cdot 2k/4} + 4e^{-j2\pi \cdot 3k/4}$$

$k = 0 \Rightarrow X(k) = 10$
$k = 1 \Rightarrow X(k) = -2+2j$
$k = 2 \Rightarrow X(k) = -2$
$k = 3 \Rightarrow X(k) = -2-2j$

$\Rightarrow X(k) = \{10 \quad -2+2j \quad -2 \quad -2-2j\}$

L.T. method

$X(k) = W_4 x[n]$

$$W_4 x[n] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

ii) $x(n) = \{1,1,1,1,1,1,1,1\}$

Direct Method

$$X(k) = \sum_{n=0}^{7} e^{-j2\pi nk/8} = \sum_{n=0}^{7} W_8^{kn}$$

$W_8^0 = 1$ $\quad W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $\quad W_8^2 = \cancel{\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}}\ -j$

$W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $\quad W_8^4 = -1$ $\quad W_8^5 = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$W_8^6 = j$ $\quad W_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$\therefore$ $X[0] = 8$ $\quad X[4] = 0$
$X[1] = 0$ $\quad X[5] = 0$
$X[2] = 0$ $\quad X[6] = 0$
$X[3] = 0$ $\quad X[7] = 0$

$X[k] = \{8,0,0,0,0,0,0,0\}$

L.T. method

$$X(k) = W_8\, x[n] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ 1 & W_8^2 & W_8^4 & W_8^6 & 1 & W_8^2 & W_8^4 & W_8^6 \\ 1 & W_8^3 & W_8^6 & W_8^1 & W_8^4 & W_8^7 & W_8^2 & W_8^5 \\ 1 & W_8^4 & 1 & W_8^4 & 1 & W_8^4 & 1 & W_8^4 \\ 1 & W_8^5 & W_8^2 & W_8^7 & W_8^4 & W_8^1 & W_8^6 & W_8^3 \\ 1 & W_8^6 & W_8^4 & W_8^2 & 1 & W_8^6 & W_8^4 & W_8^2 \\ 1 & W_8^7 & W_8^6 & W_8^5 & W_8^4 & W_8^3 & W_8^2 & W_8^1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ //

classmate

Date
Page

b) $x_1[n] = \{1,2,3,1\}$ $x_2[n] = \{4,3,2,2\}$

a) Matrix Method

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

b) DFT, IDFT method

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$$

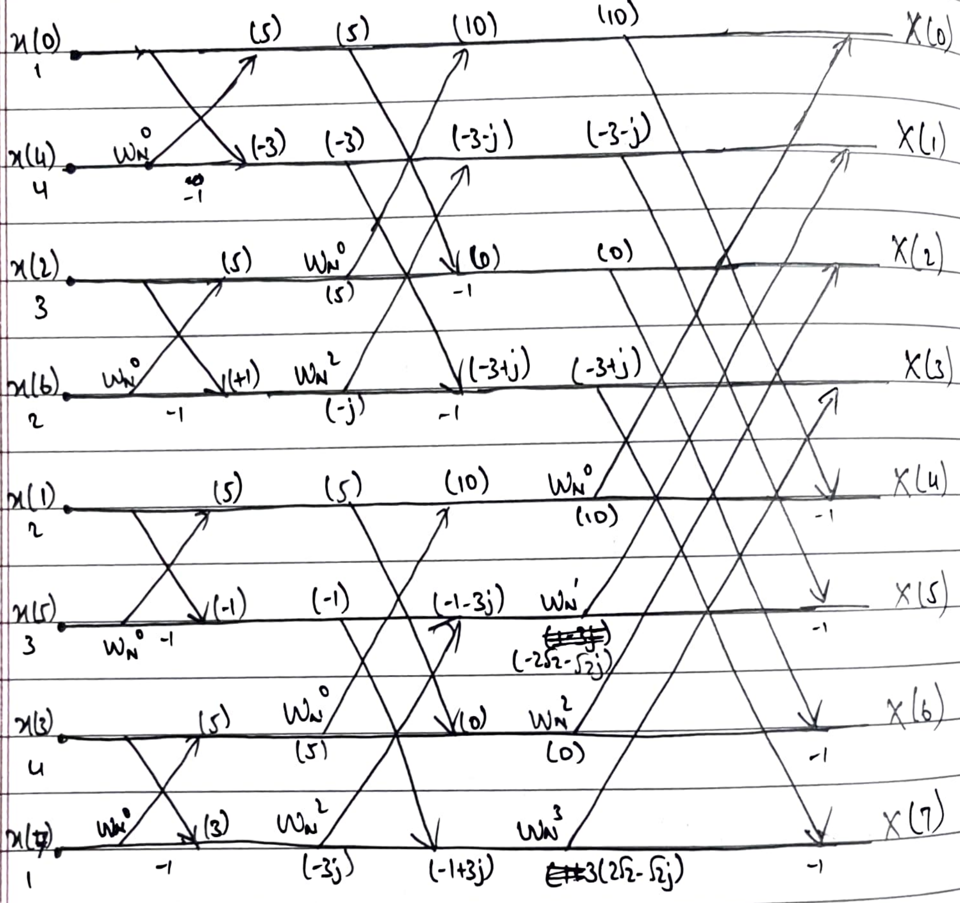
$$\Rightarrow X_1(k) = [3.5 \quad -1+0.5j \quad 0.5 \quad -1-0.5j]$$
$$X_2(k) = [5.5 \quad 1+0.5j \quad 0.5 \quad 1+0.5j]$$

$$Y(k) = X_1(k) \cdot X_2(k)$$
$$= [19.25 \quad -1.25 \quad 0.25 \quad -0.75]$$

$$y[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y[k] e^{-j2\pi nk/N}$$

$$= [17, 19, 22, 19]$$

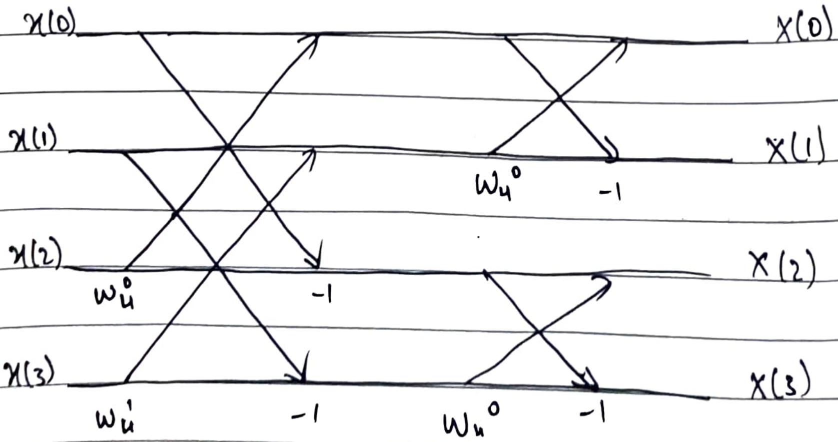
Q9



$x(n) = \{1, 2, 3, 4, 4, 3, 2, 0\}$ (Append 0 to make 8 point)

$\Rightarrow X(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}$

Q10



a) Cyclic

$W_4^0 = 1$ $\quad$ $W_4^4 = 1$

$W_4^1 = -j$

$W_4^2 = -1$ $\quad$ $W_4^{4k} = 1$

$W_4^3 = j$

$$W_N^{(k+N)} = W_N^k$$

b) Symmetry $\quad W_N^{(k+N/2)} = -W_N^k$

These properties reduce computation time

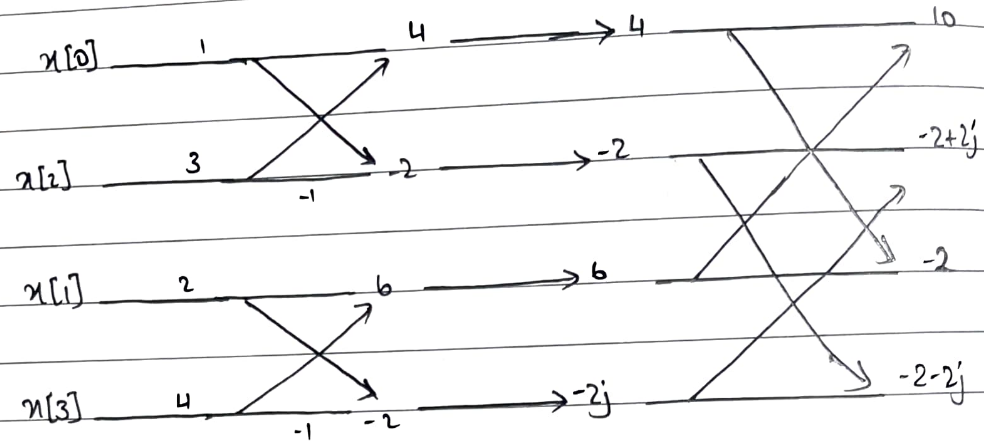
Q11 i) FFT is an improvement over DFT. It has a computational complexity of $O(n\log(n))$ while DFT has a complexity of $O(n^2)$

ii) Data ordering in FFT is required in bit reversed format. Due to this, MSB becomes LSB & vice versa

iii) Efficient use of memory is important for designing fast hardware to calculate the ~~FFT~~. FFT. "In place" computation is used to describe this memory.

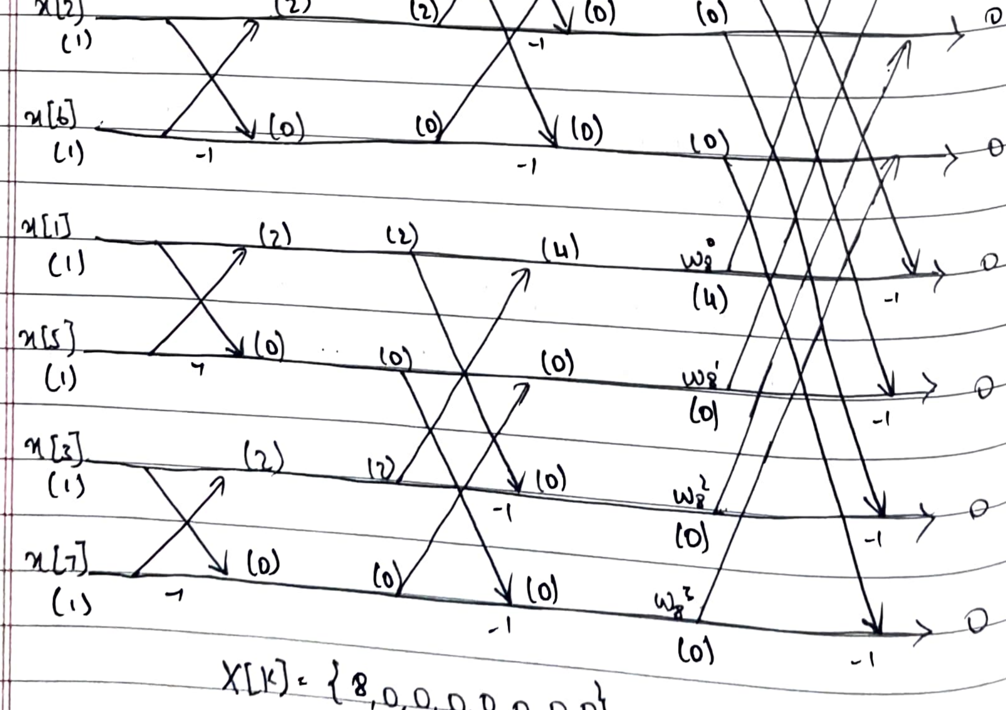
For a $2\times N$ real to complex FFT, the memory required is only $2\times N$ ~~are~~ complex memory locations broken into 4 parts.

Q12 a) $x(n) = \{1, 2, 3, 4\}$



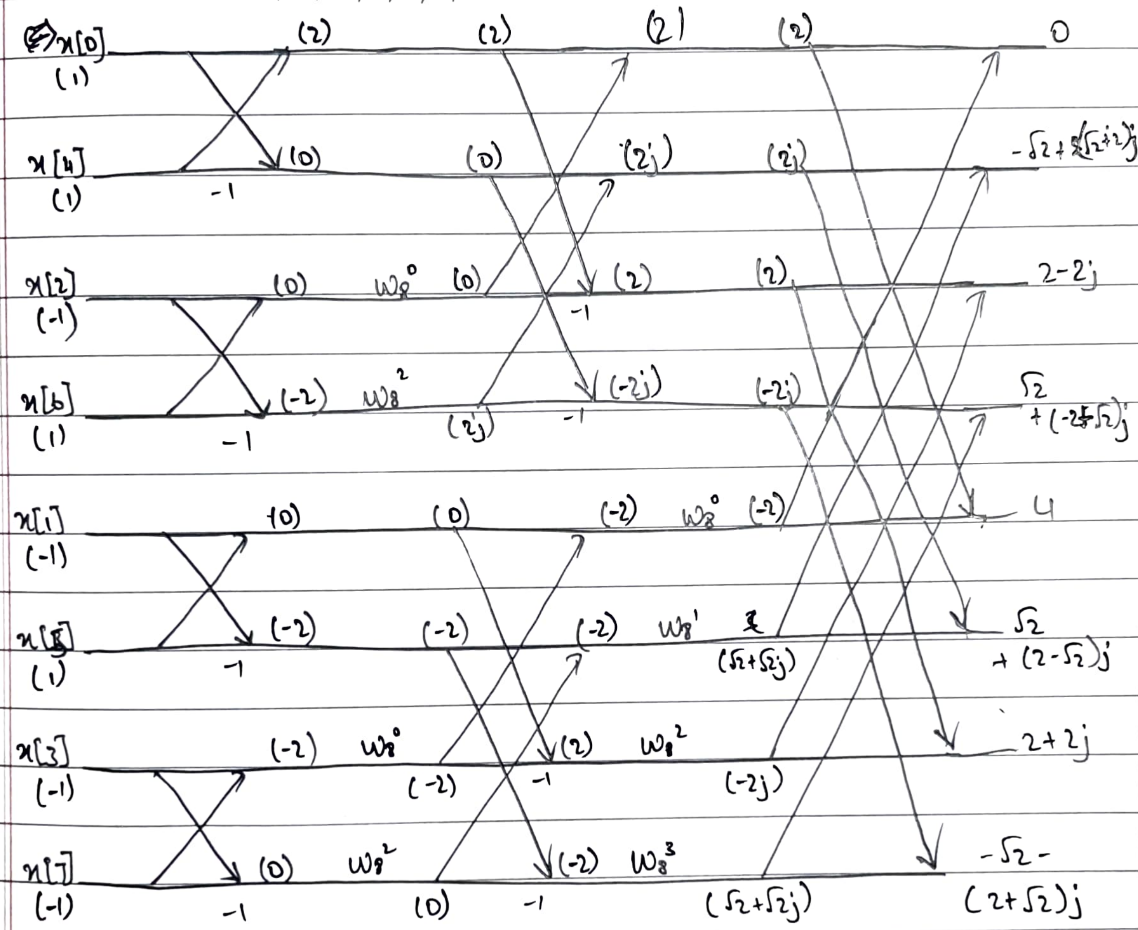
$X[k] = \{10, -2+2j, -2, -2-2j\}$

b) $x(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}$



$X[K] = \{8, 0, 0, 0, 0, 0, 0, 0\}$

(c) $x[n] = \{1, -1, -1, -1, 1, 1, 1, -1\}$



$\Rightarrow X(k) = \{0,\ -\sqrt{2}+(2+\sqrt{2})j,\ 2-2j,\ \sqrt{2}+(-2+\sqrt{2})j,$
$4,\ \sqrt{2}+(2-\sqrt{2})j,\ 2+2j,\ -\sqrt{2}-(2+\sqrt{2})j\}$