IVP MODULE 2 ASSIGNMENT 3

IMAGE TRANSFORMS

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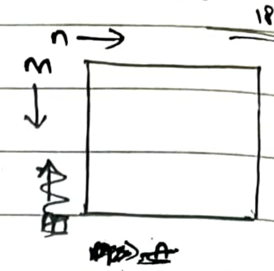

Q1 * Shifting Property :

$$f(m,n) \xrightarrow{\text{shift}} f(m-m_0, n-n_0)$$

DFT$(f(m-m_0, n-n_0))$

$$= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m-m_0, n-n_0)\, e^{-j\frac{2\pi m k}{N}}\, e^{-j\frac{2\pi n l}{N}}$$

$$\cancel{= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m-m_0, n-n_0)\, e^{\frac{-j2\pi mk - ej2\pi nl}{N}}}$$

$$= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m-m_0, n-n_0)\, e^{\frac{-j2\pi(m-m_0+m_0)k}{N}}\, e^{\frac{-j2\pi(n-n_0+n_0)l}{N}}$$

$$= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m-m_0, n-n_0)\, e^{\frac{-j2\pi(m-m_0)k}{N}}\, e^{\frac{-j2\pi(n-n_0)l}{N}}\, e^{\frac{-j2\pi m_0 k}{N}}\, e^{\frac{-j2\pi n_0 l}{N}}$$

$$= e^{\frac{-j2\pi m_0 k}{N}} \cdot e^{\frac{-j2\pi n_0 l}{N}} \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m-m_0, n-n_0)\, e^{\frac{-j2\pi(n-n_0)l}{N}}\, e^{\frac{-j2\pi(m-m_0)k}{N}}$$

$$= e^{\frac{-j2\pi(m_0 k + n_0 l)}{N}} \sum_{m=0}^{N-1} f(m-m_0, l)\, e^{\frac{-j2\pi(m-m_0)k}{N}}$$

$$= e^{\frac{-j2\pi(m_0 k + n_0 l)}{N}}\, f(k,l)$$

=) Spatial Shift :

$$\text{DFT}(f(m-m_0, n-n_0)) = e^{\frac{-j2\pi[m_0 k + n_0 l]}{N}}\, f(k,l)$$

* Separable Property: A 2D DFT can be computed as 2 1D DFTs.

$$F(k,l) \cancel{F(u,v)} = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(\overset{m,n}{\cancel{x,y}})\, e^{-j2\pi(mk+nl)/N}$$

$$= \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m,n)\, e^{-j2\pi nl/N}\, e^{-j2\pi mk/N}$$

$$= \sum_{m=0}^{N-1} f(m,l)\, e^{-j2\pi mk/N}$$

$$= F(k,l) \qquad [\text{LHS} = \text{RHS}]$$

⇒ Separable Property:

$$DFT(f(m,n)) = F(k,l)$$

$$= \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m,n)\, e^{-j\frac{2\pi mk}{N}}\, e^{-j\frac{2\pi nl}{N}}$$

* Rotation Property: $m \to r\cos\theta$ $\quad n \to r\sin\theta$

$k \to w\cos\phi$ $\quad l \to w\sin\phi$

$\Rightarrow f(m,n) \to f(r,\theta)$

$F(k,l) \to F(w,\phi)$

Then, by shifting property

$$f(r, \theta+\theta_0) = F(w, \phi+\phi_0)$$

Q2 b) i.) Matrix : $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = X$

Simulated Result : $\begin{bmatrix} 4 + 0j & 2 - 1.22468 \times 10^{-16} j \\ -2 - 3.67394044 \times 10^{-16} j & 0 + 0j \end{bmatrix}$

$$\approx \begin{bmatrix} 4 + 0j & 2 + 0j \\ -2 + 0j & 0 + 0j \end{bmatrix}$$

Calculated Result :

$DFT = W_2 X W_2 \qquad W_2 = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\cancel{\Rightarrow DFT = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$

$$DFT = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -2 & 0 \end{bmatrix}$$

$DFT = \begin{bmatrix} 4 + 0j & 2 + 0j \\ -2 + 0j & 0 + 0j \end{bmatrix}$ = Simulated ii.) Matrix: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = X$

Simulated Result: $\begin{bmatrix} 1.36\times10^{+2} + 0j & -8 + 8j & -8-0j & -8-8j \\ -32+32j & 0+0j & 0+0j & 0+0j \\ -32+0j & 0+0j & 0+0j & 0+0j \\ -32-32j & 0+0j & 0+0j & 0+0j \end{bmatrix}$

Calculated Result: $DFT = W_4 X W_4$

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\Rightarrow DFT = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & 1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 32 & 36 & 40 \\ -8+8j & -8+8j & -8+8j & -8+8j \\ 18 & 20 & 22 & 24 \\ -8-8j & -8-8j & -8-8j & -8-8j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\Rightarrow DFT = \begin{bmatrix} 136+0j & -8+8j & -8-0j & -8-8j \\ -32-32j & 0+0j & 0+0j & 0+0j \\ -32+0j & 0+0j & 0+0j & 0+0j \\ -32-32j & 0+0j & 0+0j & 0+0j \end{bmatrix} = \text{Simulated Result}$$

Q3 b) $I \cancel{g} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ filter $f = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

$\Rightarrow H_0 = \begin{bmatrix} -1 & -1 & -2 \\ -2 & -1 & -1 \\ -1 & -2 & -1 \end{bmatrix}$ $H_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$H_2 = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

$\Rightarrow$ Doubly Block Matrix & Circular Convolution:

$$g = \begin{bmatrix} -1 & -1 & -2 & 1 & 1 & 2 & 0 & 0 & 0 \\ -2 & -1 & -1 & 2 & 1 & 1 & 0 & 0 & 0 \\ -1 & -2 & -1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -2 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 & -1 & -1 & 2 & 1 & 1 \\ 0 & 0 & 0 & -1 & -2 & -1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 0 & 0 & 0 & -1 & -1 & -2 \\ 2 & 1 & 1 & 0 & 0 & 0 & -2 & -1 & -1 \\ 1 & 2 & 1 & 0 & 0 & 0 & -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 9 \\ \hline 4 \\ 5 \\ 6 \\ \hline 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -12 \\ -12 \\ -12 \\ \hline -12 \\ -12 \\ -12 \\ \hline 24 \\ 24 \\ 24 \end{bmatrix} = \begin{bmatrix} 24 & 24 & 24 \\ -12 & -12 & -12 \\ -12 & -12 & -12 \end{bmatrix} = \text{Simulated Result}$$