Chapter 21

Public-Key Cryptography and Message Authentication

RSA Public-Key Encryption

- By Rivest, Shamir & Adleman of MIT in 1977
- Best known and widely used public-key algorithm
- Uses exponentiation of integers modulo a prime
- Encrypt: $C = M^e \bmod n$
- *Decrypt:* $M = C^d \bmod n = (M^e)^d \bmod n = M$
- Both sender and receiver know values of n and e
- Only receiver knows value of d
- Public-key encryption algorithm with public key $PU = \{e, n\}$ and private key $PR = \{d, n\}$

Key Generation

Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p - 1)(q - 1)$	
Select integer e	$\gcd(\phi(n), e) = 1;\ 1 < e < \phi(n)$
Calculate d	$de \bmod \phi(n) = 1$
Public key	$KU = \{e, n\}$
Private key	$KR = \{d, n\}$

Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \pmod{n}$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \pmod{n}$

Figure 21.7 The RSA Algorithm

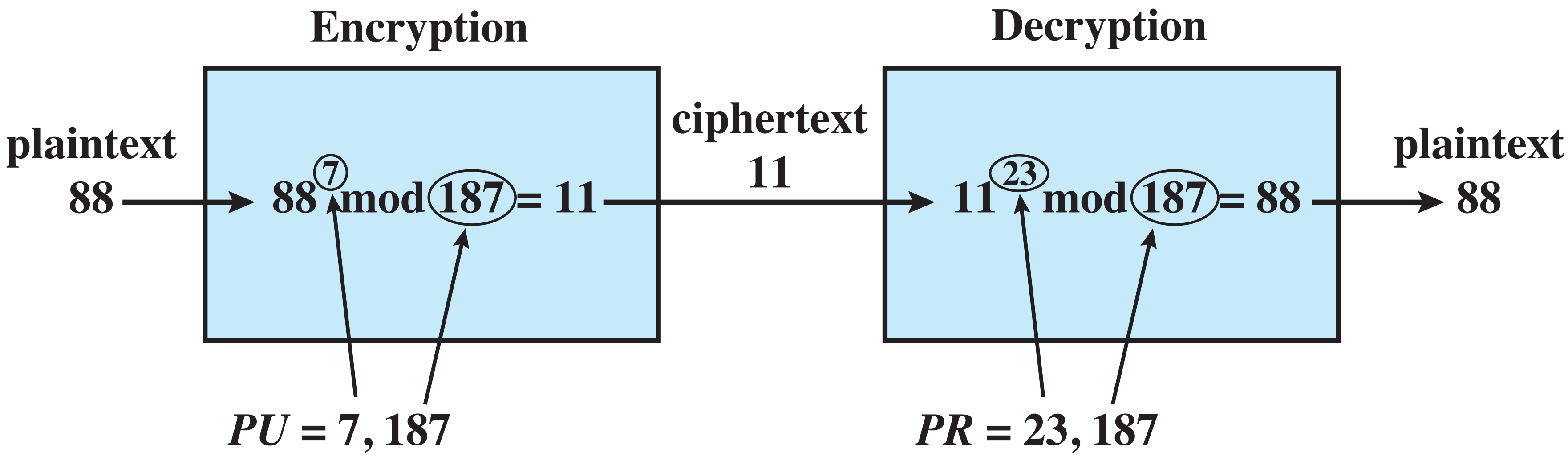


Figure 21.8 Example of RSA Algorithm

Security of RSA

Brute force

- Involves trying all possible private keys

Mathematical attacks

- There are several approaches, all equivalent in effort to factoring the product of two primes

Timing attacks

- These depend on the running time of the decryption algorithm

Chosen ciphertext attacks

- This type of attack exploits properties of the RSA algorithm

Table 21.2

Progress in Factorization

Number of Decimal Digits	Number of Bits	Date Achieved
100	332	April 1991
110	365	April 1992
120	398	June 1993
129	428	April 1994
130	431	April 1996
140	465	February 1999
155	512	August 1999
160	530	April 2003
174	576	December 2003
200	663	May 2005
193	640	November 2005
232	768	December 2009

Timing Attacks

- Paul Kocher, a cryptographic consultant, demonstrated that a snooper can determine a private key by keeping track of how long a computer takes to decipher messages
- Timing attacks are applicable not just to RSA, but also to other public-key cryptography systems
- This attack is alarming for two reasons:
  - It comes from a completely unexpected direction
  - It is a ciphertext-only attack

Timing Attack Countermeasures

Constant exponentiation time

- Ensure that all exponentiations take the same amount of time before returning a result
- This is a simple fix but does degrade performance

Random delay

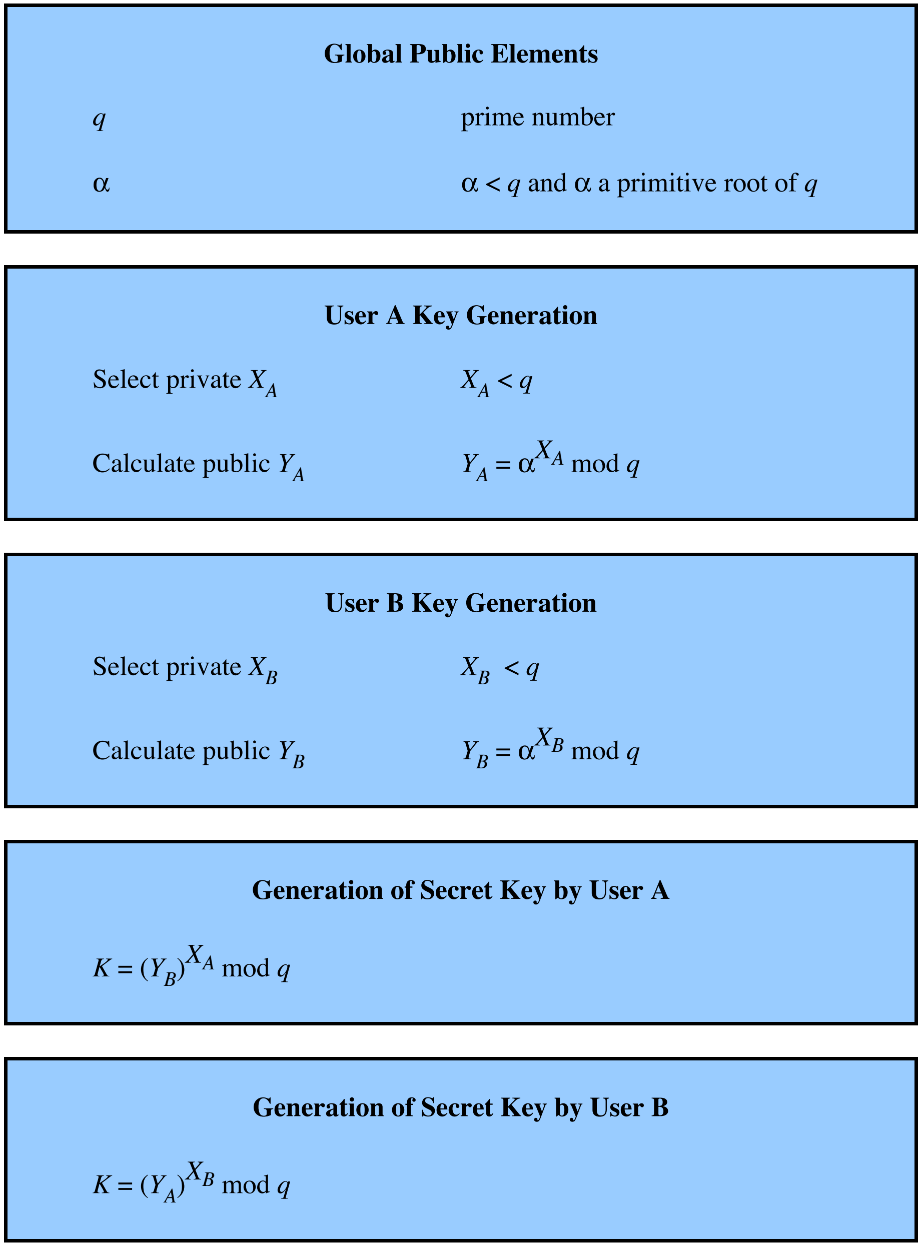
- Better performance could be achieved by adding a random delay to the exponentiation algorithm to confuse the timing attack
- If defenders do not add enough noise, attackers could still succeed by collecting additional measurements to compensate for the random delays

Blinding

- Multiply the ciphertext by a random number before performing exponentiation
- This process prevents the attacker from knowing what ciphertext bits are being processed inside the computer and therefore prevents the bit-by-bit analysis essential to the timing attack

Diffie-Hellman Key Exchange

- First published public-key algorithm
- By Diffie and Hellman in 1976 along with the exposition of public key concepts
- Used in a number of commercial products
- Practical method to exchange a secret key securely that can then be used for subsequent encryption of messages
- Security relies on difficulty of computing discrete logarithms



Global Public Elements

q prime number

α $\alpha < q$ and α a primitive root of q

User A Key Generation

Select private X_A $X_A < q$

Calculate public Y_A $Y_A = \alpha^{X_A} \bmod q$

User B Key Generation

Select private X_B $X_B < q$

Calculate public Y_B $Y_B = \alpha^{X_B} \bmod q$

Generation of Secret Key by User A

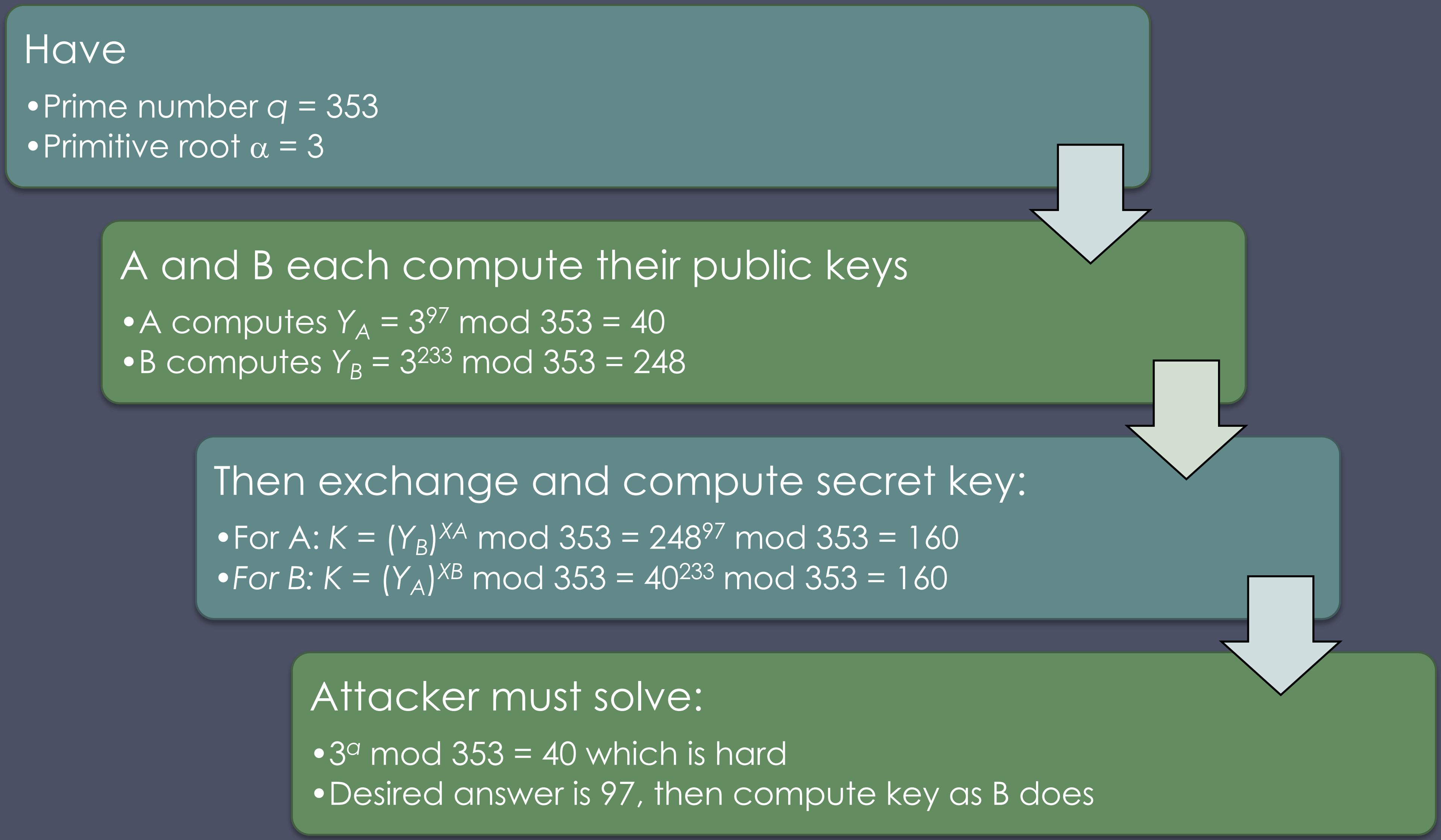
$K = (Y_B)^{X_A} \bmod q$

Generation of Secret Key by User B

$K = (Y_A)^{X_B} \bmod q$

Figure 21.9 The Diffie-Hellman Key Exchange Algorithm

Diffie-Hellman Example



Have

- Prime number q = 353
- Primitive root α = 3

A and B each compute their public keys

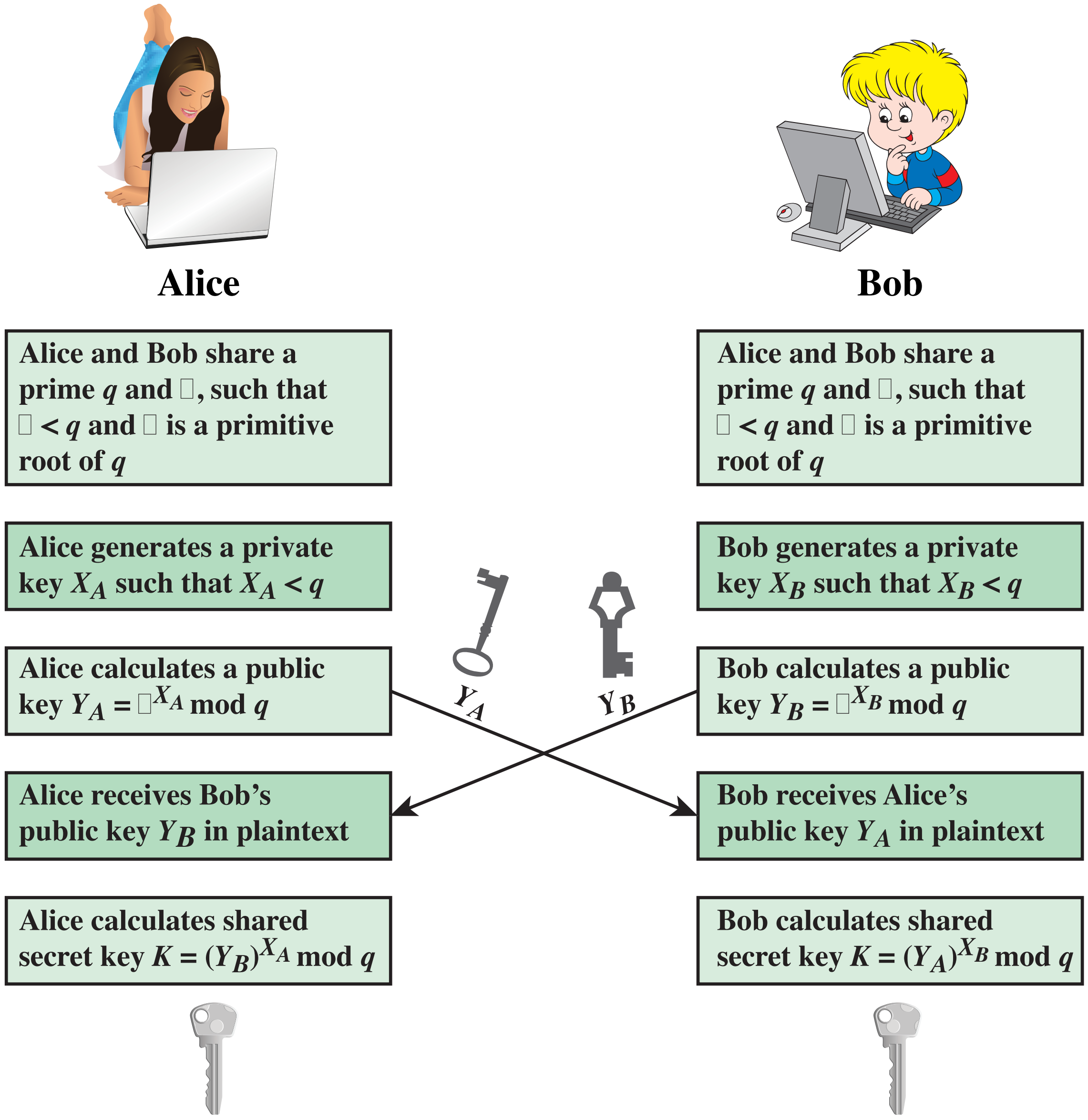
- A computes $Y_A = 3^{97} \bmod 353 = 40$
- B computes $Y_B = 3^{233} \bmod 353 = 248$

Then exchange and compute secret key:

- For A: $K = (Y_B)^{XA} \bmod 353 = 248^{97} \bmod 353 = 160$
- *For B:* $K = (Y_A)^{XB} \bmod 353 = 40^{233} \bmod 353 = 160$

Attacker must solve:

- $3^a \bmod 353 = 40$ which is hard
- Desired answer is 97, then compute key as B does



Alice

Bob

Alice and Bob share a prime q and α, such that $\alpha < q$ and α is a primitive root of q

Alice and Bob share a prime q and α, such that $\alpha < q$ and α is a primitive root of q

Alice generates a private key X_A such that $X_A < q$

Bob generates a private key X_B such that $X_B < q$

Alice calculates a public key $Y_A = \alpha^{X_A} \bmod q$

Bob calculates a public key $Y_B = \alpha^{X_B} \bmod q$

Alice receives Bob's public key Y_B in plaintext

Bob receives Alice's public key Y_A in plaintext

Alice calculates shared secret key $K = (Y_B)^{X_A} \bmod q$

Bob calculates shared secret key $K = (Y_A)^{X_B} \bmod q$

Figure 21.10 Diffie-Hellman Key Exchange

Man-in-the-Middle Attack

- Attack is:
  1. Darth generates private keys X_{D1} and X_{D2}, and their public keys Y_{D1} and Y_{D2}
  2. Alice transmits Y_A to Bob
  3. Darth intercepts Y_A and transmits Y_{D1} to Bob. Darth also calculates K2
  4. Bob receives Y_{D1} and calculates K1
  5. Bob transmits X_A to Alice
  6. Darth intercepts Y_B and transmits Y_{D2} to Alice. Darth calculates K1
  7. Alice receives Y_{D2} and calculates K2
- All subsequent communications compromised

Other Public-Key Algorithms

Digital Signature Standard (DSS)

- FIPS PUB 186
- Makes use of SHA-1 and the Digital Signature Algorithm (DSA)
- Originally proposed in 1991, revised in 1993 due to security concerns, and another minor revision in 1996
- Cannot be used for encryption or key exchange
- Uses an algorithm that is designed to provide only the digital signature function

Elliptic-Curve Cryptography (ECC)

- Equal security for smaller bit size than RSA
- Seen in standards such as IEEE P1363
- Confidence level in ECC is not yet as high as that in RSA
- Based on a mathematical construct known as the elliptic curve

Summary

- Secure hash functions
  - Simple hash functions
  - The SHA secure hash function
  - SHA-3
- Diffie-Hellman and other asymmetric algorithms
  - Diffie-Helman key exchange
  - Other public-key cryptography algorithms
- Authenticated encryption
- The RSA public-key encryption algorithm
  - Description of the algorithm
  - The security of RSA
- HMAC
  - HMAC design objectives
  - HMAC algorithm
  - Security of HMAC