University of Colorado **Boulder**

CSCI 5622: Machine Learning

Lecture 8

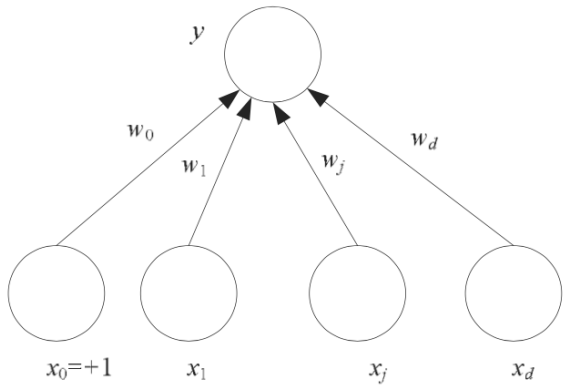
Overview

- Perceptron
  - Representation
  - Learning
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Overview

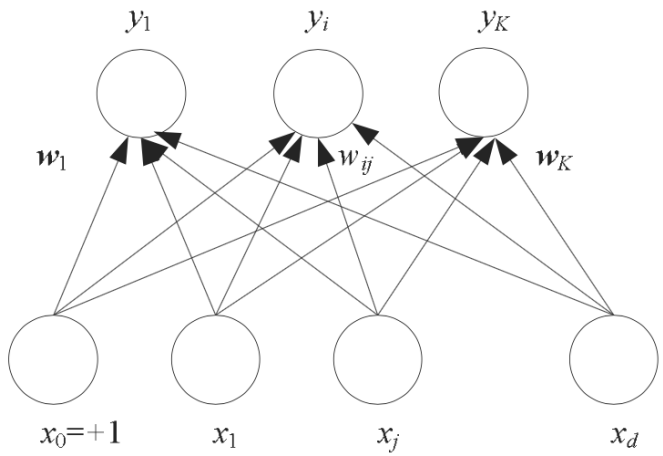
- Perceptron
  - Representation
  - Training
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Perceptron: Basic processing unit



- Inputs $x_d \in \mathbb{R}$, $d = 1, \ldots, D$
  - might come from the environment
  - might be the output of other perceptrons
- Associated with a connection weight $w_d \in \mathbb{R}$, $d = 1, \ldots, D$
- Output is some function of the linear combination of inputs
  - $y = s\left(\sum_{j=1}^{D} w_d x_d + w_0\right) = s(\mathbf{w}^T \mathbf{x})$
    where $s(\alpha) = 1$, if $\alpha > 0$, $s(\alpha) = 0$, otherwise
    e.g. sigmoid activation: $s(\mathbf{x}, \mathbf{w}) = \frac{1}{1+\exp(-\mathbf{w}^T \mathbf{x})}$
- can be used for classification, i.e. choose C_1, if $s(\alpha) > 0.5$

Perceptron: Basic processing unit



- Multiclass: $K > 2$ outputs
  - $y_k = s\left(\sum_{d=1}^{D} w_{kd} x_d + w_{k0}\right) = s(\mathbf{w_k}^T \mathbf{x})$
    where w_{kj} is the weight from input x_j to output y_k
    e.g. $s(\mathbf{x}, \mathbf{w_1}, \ldots, \mathbf{w_K}) = \frac{\exp(\mathbf{w_k}^T \mathbf{x})}{1+\sum_{k=1}^{K} \exp(\mathbf{w_k}^T \mathbf{x})}$
  - 0/1 encoding for output vector
    - e.g. in a 4-class problem: if class=3, then $y = [0, 0, 1, 0]$

Overview

- Perceptron
  - Representation
  - Training
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Perceptron: Training

Stochastic gradient descent

- Evaluation: cross-entropy function for 1 instance $(\mathbf{x_n}, y_n)$
  $\mathcal{E}(\mathbf{w}) = -y_n \log\left[\sigma(\mathbf{w}^T\mathbf{x_n})\right] - (1-y_n)\log\left[1-\sigma(\mathbf{w}^T\mathbf{x_n})\right]$
  $\mathcal{E}(\mathbf{w_1},\ldots,\mathbf{w_K}) = -\sum_{k=1}^{K} y_{nk}\log p(y_{nk}=1|\mathbf{w_1},\ldots\mathbf{w_K})$
- Optimization: gradient descent
  $\frac{\vartheta\mathcal{E}(\mathbf{w})}{\vartheta w_d} = \left(\sigma(\mathbf{w}^T\mathbf{x_n}) - y_n\right)x_{nd}$
  $\frac{\vartheta\mathcal{E}(\mathbf{w})}{\vartheta w_{kd}} = \left(\sigma(\mathbf{w}^T\mathbf{x_n}) - y_{nk}\right)x_{nd}$

We could have also performed batch or mini-batch gradient descent.

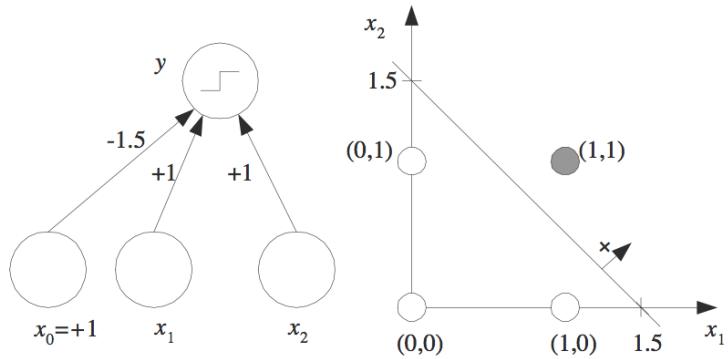
Overview

- Perceptron
  - Representation
  - Training
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Approximating linear functions

Example: Boolean AND

x_1	x_2	r
0	0	0
0	1	0
1	0	0
1	1	1



Example of a perceptron implementing AND

$y = s(x_1 + x_2 - 1.5)$, where $s(x) = 1$ if $x > 0$; and $s(x) = 0$ if $x \leq 0$

$\mathbf{w} = [-1.5\ 1\ 1]^T$

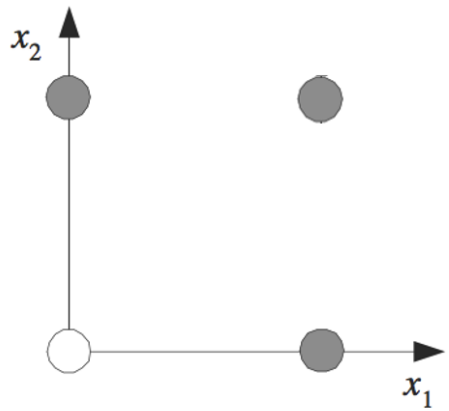
$\mathbf{x} = [1\ x_1\ x_2]^T$

The above weights were empirically selected, but we could have also learned them through gradient descent

Approximating linear functions

Example: Boolean OR

x_1	x_2	r
0	0	0
0	1	1
1	0	1
1	1	1



Example of a perceptron implementing OR

$y = s(x_1 + x_2 - 0.5)$, where $s(x) = 1$ if $x > 0$; and $s(x) = 0$ if $x \leq 0$

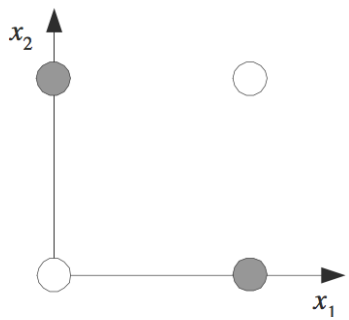
$\mathbf{w} = [-0.5\ 1\ 1]^T$

$\mathbf{x} = [1\ x_1\ x_2]^T$

Approximating linear functions

Example: Boolean XOR

x_1	x_2	r
0	0	0
0	1	1
1	0	1
1	1	0



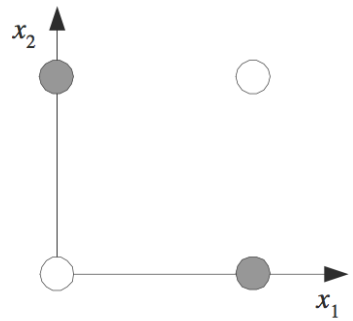
Not linearly separable

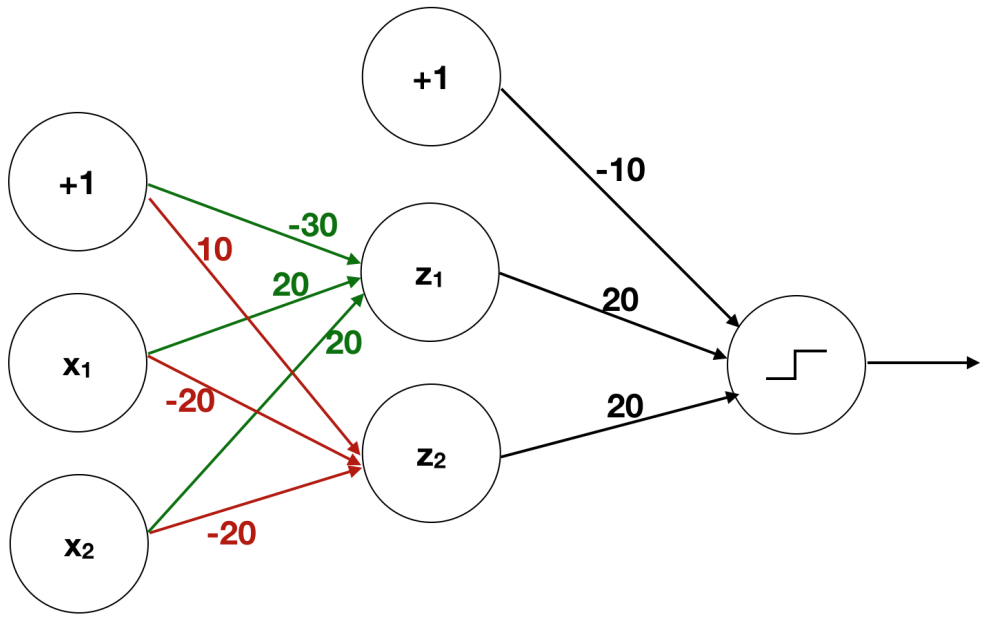
Need combination of more than one perceptrons $\rightarrow$ multilayer perceptrons

Multilayer Perceptron: Approximating non-linear functions

Example: Boolean XOR with multilayer perceptrons

x_1	x_2	z_1	z_2	r
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1



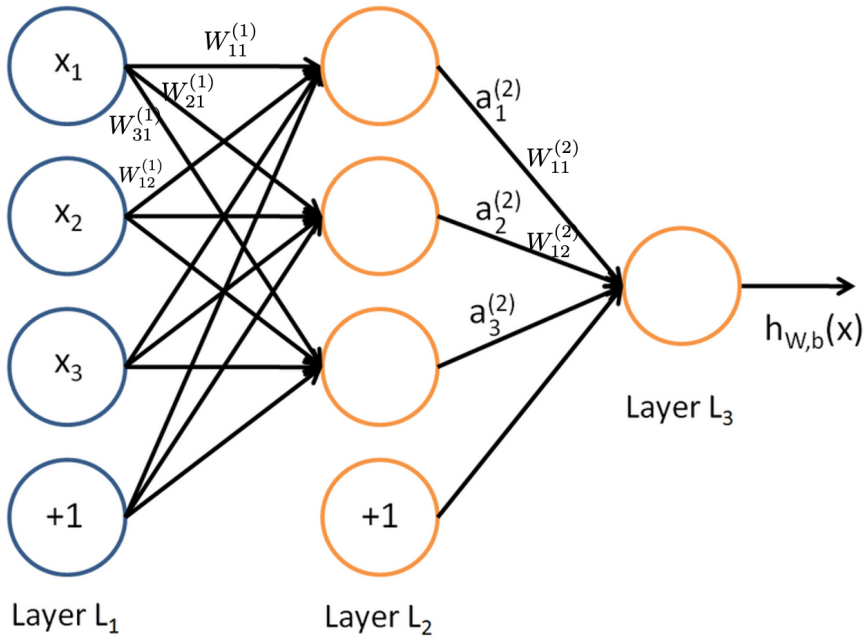


Overview

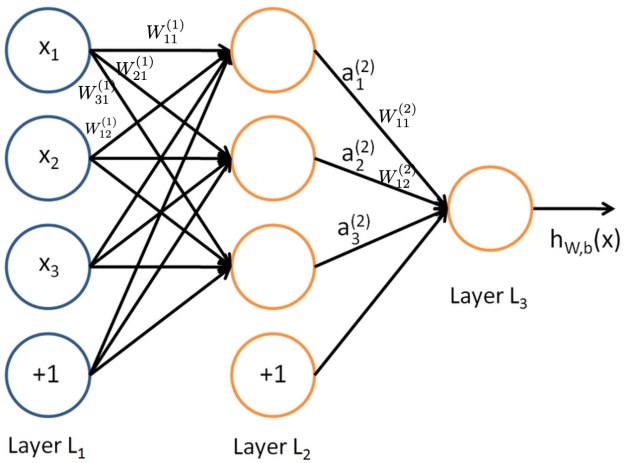
- Perceptron
  - Representation
  - Learning
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Multilayer Perceptron

- Type of feedforward neural network
- Can model non-linear associations
- "Multi-level combination" of many perceptrons



Multilayer Perceptron: Representation



$$\alpha_1^{(2)} = f(W_{11}^{(1)}x_1 + W_{12}^{(1)}x_2 + W_{13}^{(1)}x_3 + b_1^{(1)})$$

$$\alpha_2^{(2)} = f(W_{21}^{(1)}x_1 + W_{22}^{(1)}x_2 + W_{23}^{(1)}x_3 + b_2^{(1)})$$

$$\alpha_3^{(2)} = f(W_{31}^{(1)}x_1 + W_{32}^{(1)}x_2 + W_{33}^{(1)}x_3 + b_3^{(1)})$$

$$h_{\mathbf{W},\mathbf{b}}(\mathbf{x}) = \alpha_1^{(3)} = f(W_{11}^{(2)}\alpha_1^{(2)} + W_{12}^{(2)}\alpha_2^{(2)} + W_{13}^{(2)}\alpha_3^{(2)} + b_1^{(2)})$$

Terminology

$W_{ij}^{(l)}$: connection between unit j in layer l to unit i in layer $l+1$

$\alpha_i^{(l)}$: activation of unit i in layer l

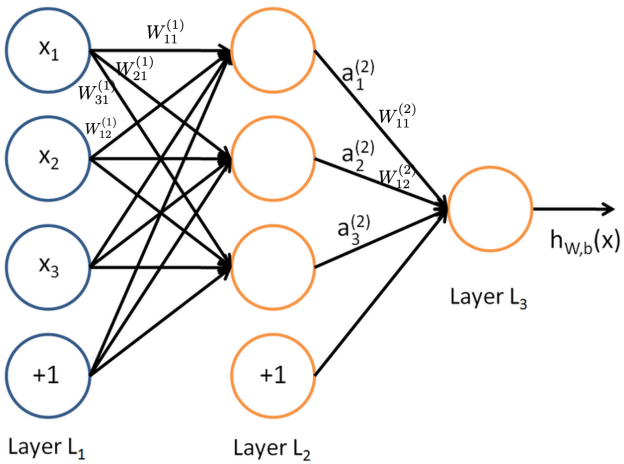
$b_i^{(l)}$: bias connected with unit i in layer $l+1$

f: activation function

Forward propagation: The process of propagating the input to the output through the activation of inputs and hidden units to each node

Multilayer Perceptron: Representation

Matrix notation



$$\alpha^{(2)} = f(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)})$$

$$h_{\mathbf{W},\mathbf{b}}(\mathbf{x}) = \alpha^{(3)} = f(\mathbf{W}^{(2)}\alpha^{(2)} + \mathbf{b}^{(2)})$$

$$\mathbf{W}^{(1)} = \begin{bmatrix} W_{11}^{(1)} & W_{12}^{(1)} & W_{13}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} & W_{23}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} & W_{33}^{(1)} \end{bmatrix}, \mathbf{b}^{(1)} = [b_1^{(1)}\ b_2^{(1)}\ b_3^{(1)}], \text{ etc.}$$

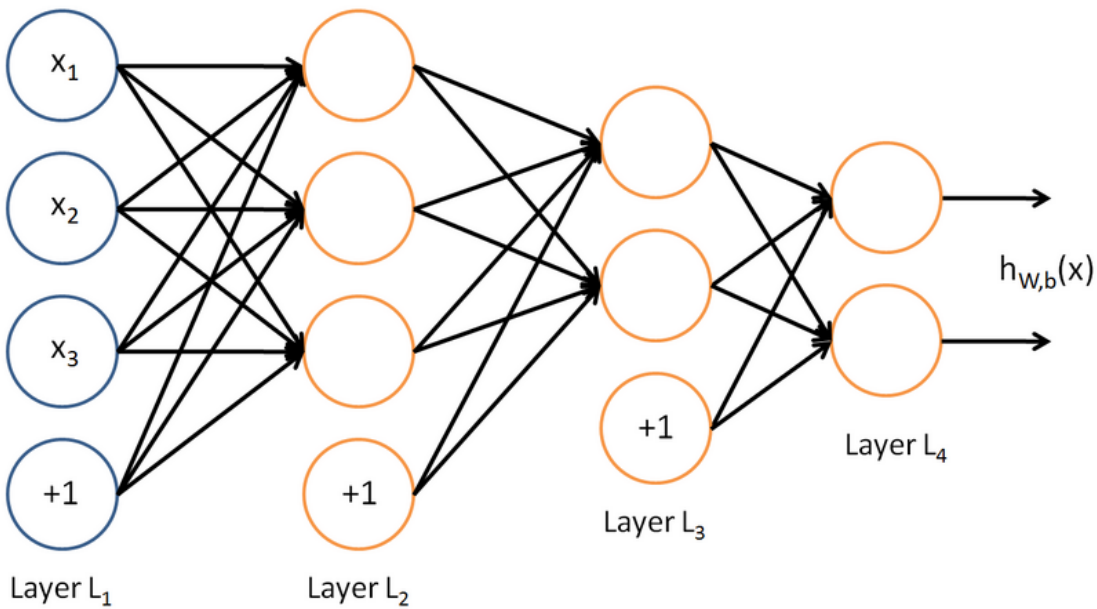
Multilayer Perceptron: Representation

Alternative architectures

2 hidden layers, multiple output units

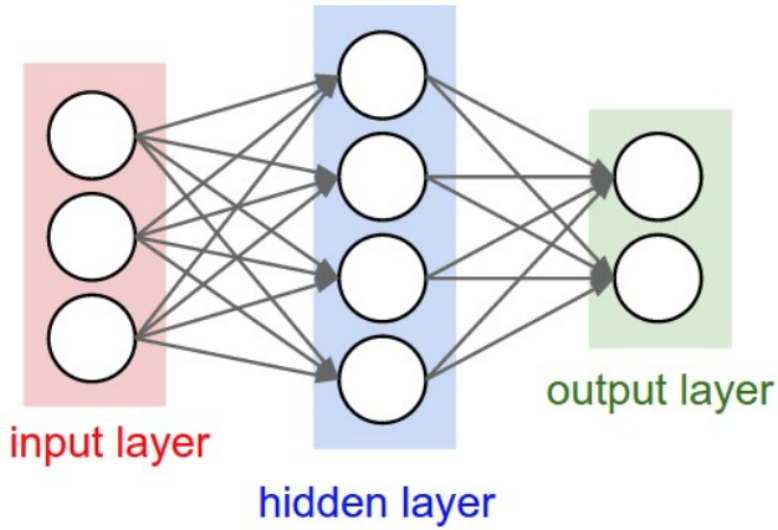
Layer L_1: input layer; Layers L_2, L_3: hidden layers, layer L_4: output layer

e.g. medical diagnosis: different outputs might indicate presence or absence of different diseases



Multilayer Perceptron

Question: How many parameters does this network have to learn?

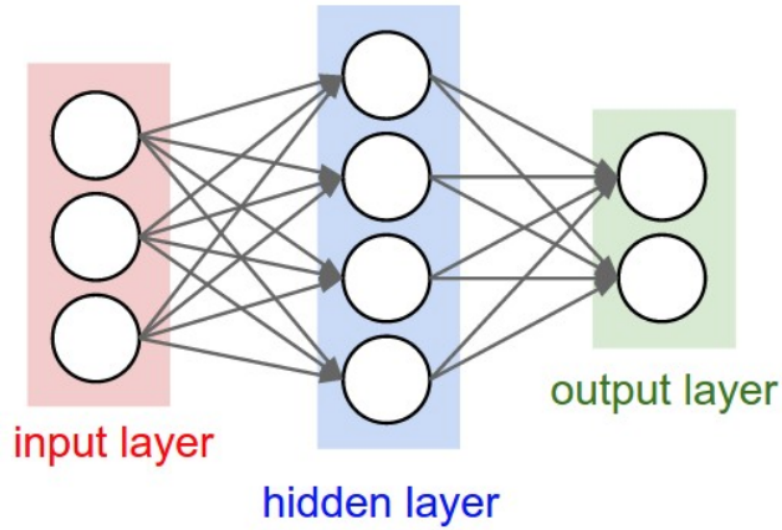


A) 20
B) 26
C) 6
D) 12

Multilayer Perceptron

Multilayer Perceptron

Question: How many parameters does this network have to learn?



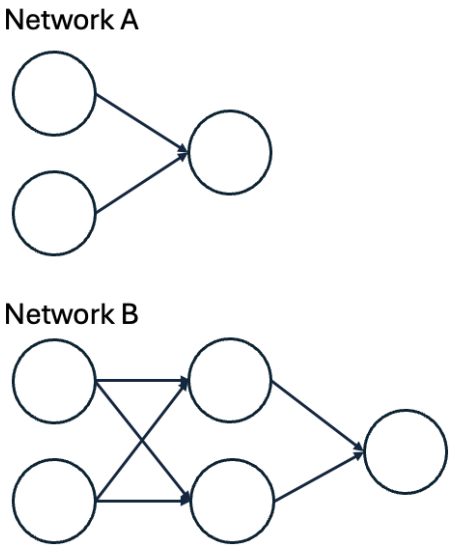
A) 20
B) 26
C) 6
D) 12

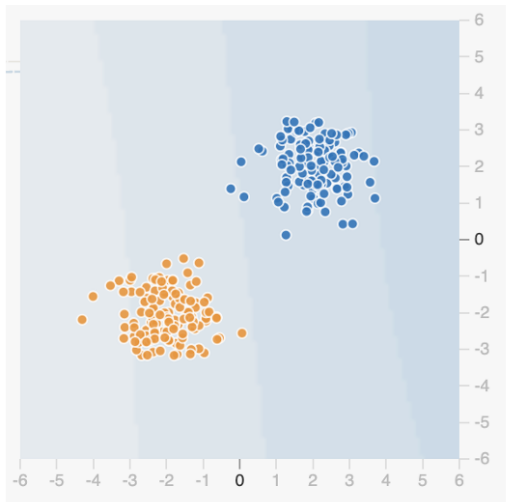
The correct answer is B

[3 x 4] + [4 x 2] = 20 weights, 4 + 2 = 6 biases

Multilayer Perceptron

Question: Which neural network(s) can correctly classify the following data?



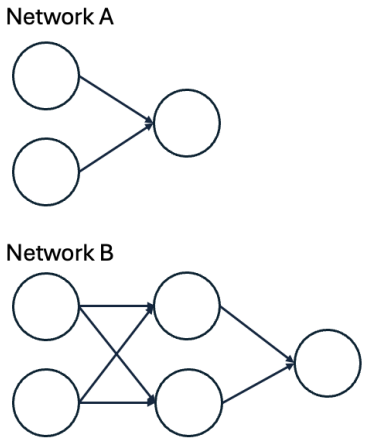


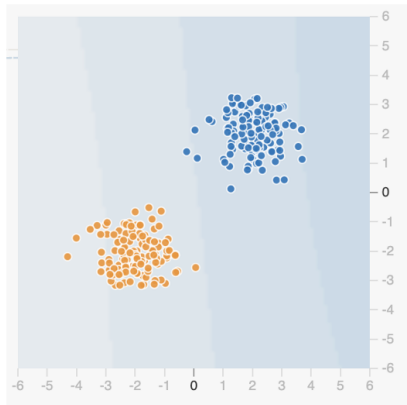
A) Network A
B) Network B
C) Both Network A and Network B
D) None

Multilayer Perceptron

Multilayer Perceptron

Question: Which neural network(s) can correctly classify the following data?



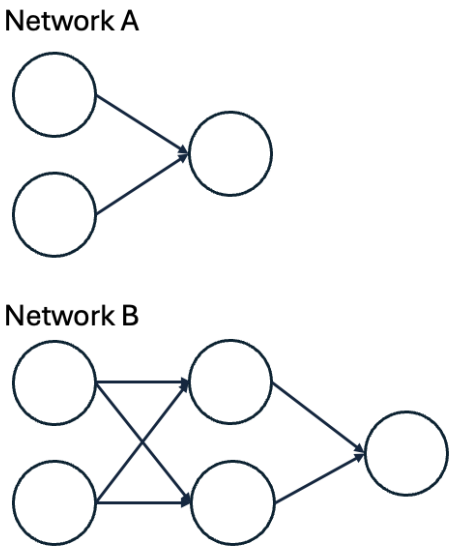


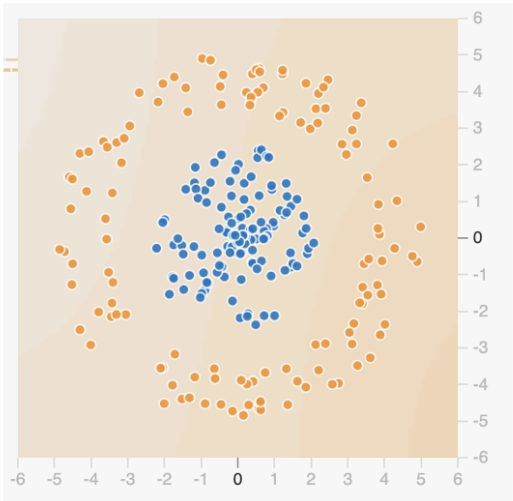
A) Network A
B) Network B
C) Both Network A and Network B
D) None

The correct answer is C. Network A can only classify correctly linearly separable data, since it does not have a hidden layer. Although Network B is a more complex network, it can also be used for the same data (given that we pay attention to overfitting).

Multilayer Perceptron

Question: Which neural network(s) can correctly classify the following data?



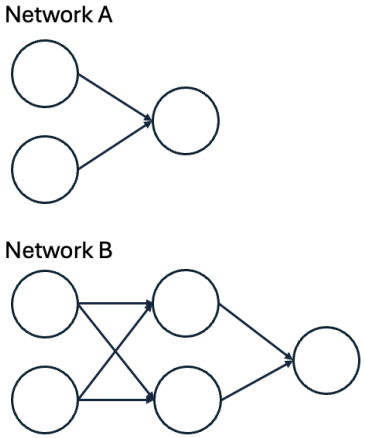


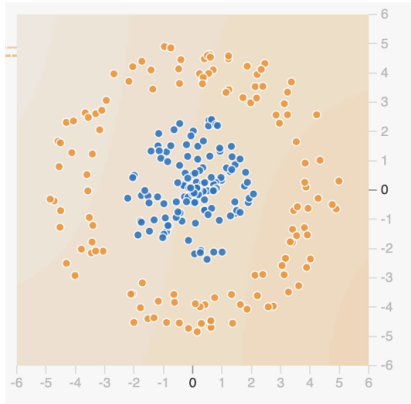
A) Network A
B) Network B
C) Both Network A and Network B
D) None

Multilayer Perceptron

Multilayer Perceptron

Question: Which neural network(s) can correctly classify the following data?



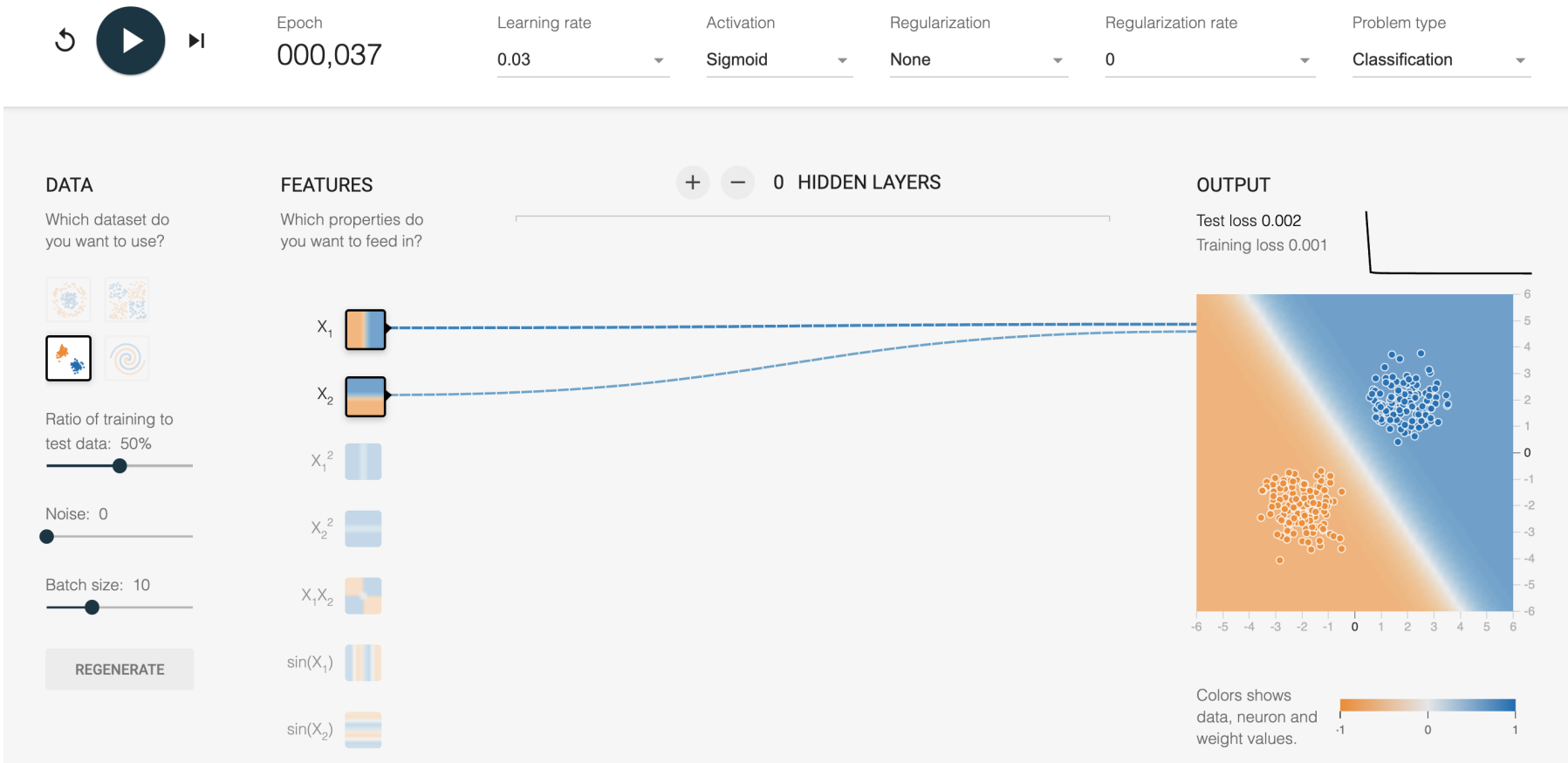


A) Network A
B) Network B
C) Both Network A and Network B
D) None

The correct answer is B. The data is not linearly separable, so Network A will not classify all the data points correctly. Network B has a hidden layer which can be used to create non-linear transformations of the input features, so it can be used for effectively classifying the data.

Multilayer Perceptron

Non-linear feature learning

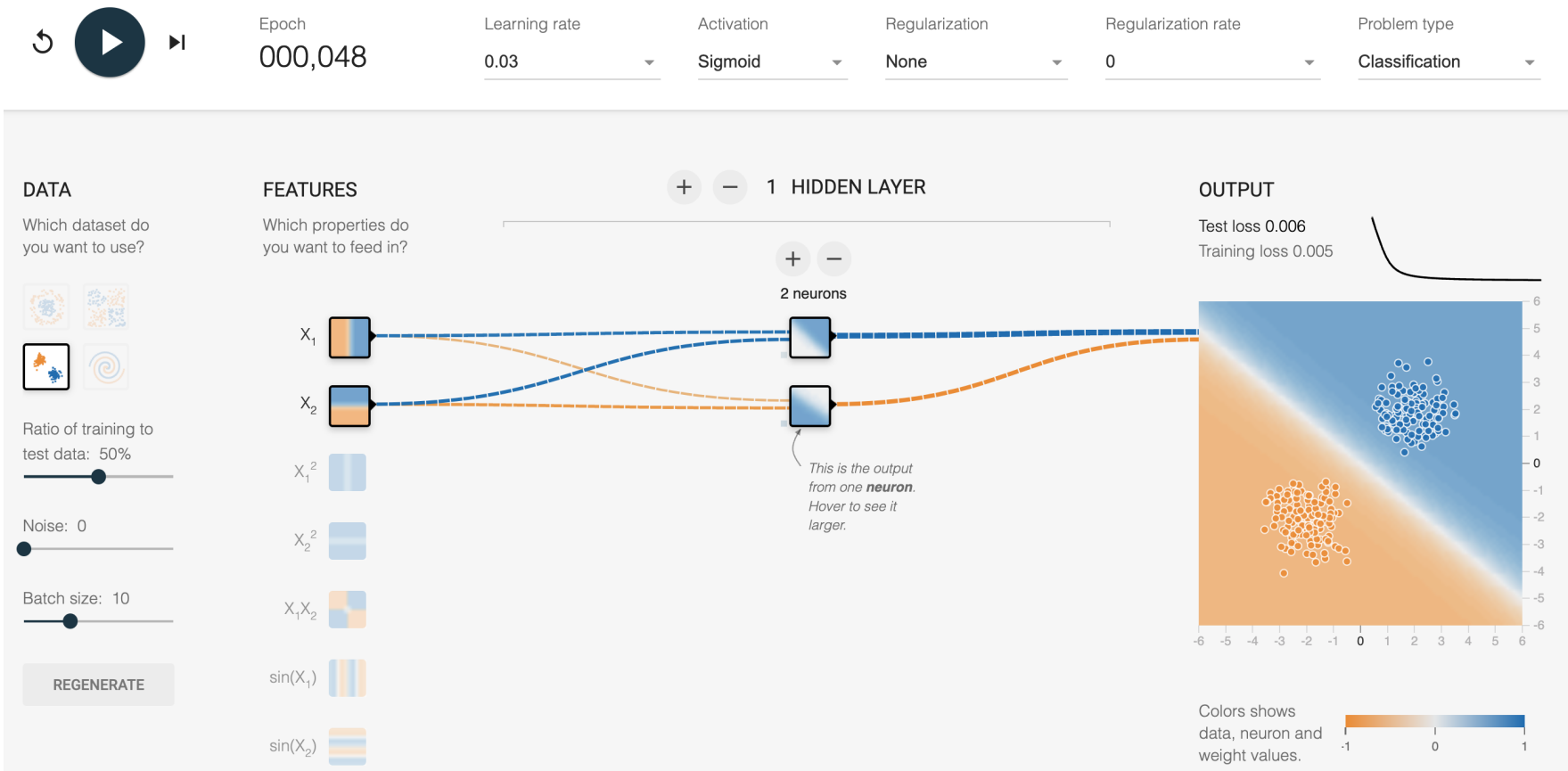


Equivalent to a logistic regression model.

Source: `https://playground.tensorflow.org/`

Multilayer Perceptron

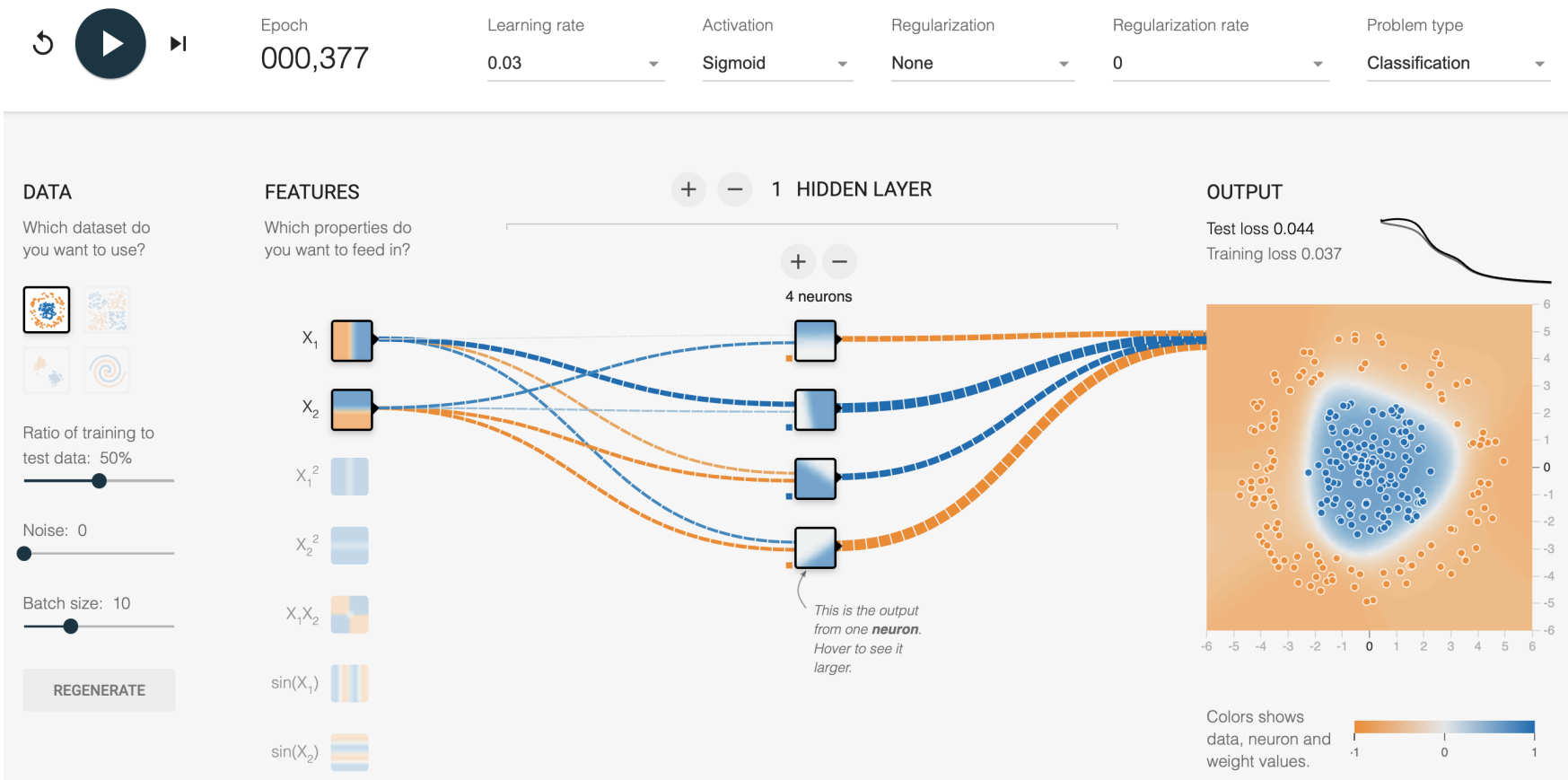
Non-linear feature learning



Source: `https://playground.tensorflow.org/`

Multilayer Perceptron

Non-linear feature learning



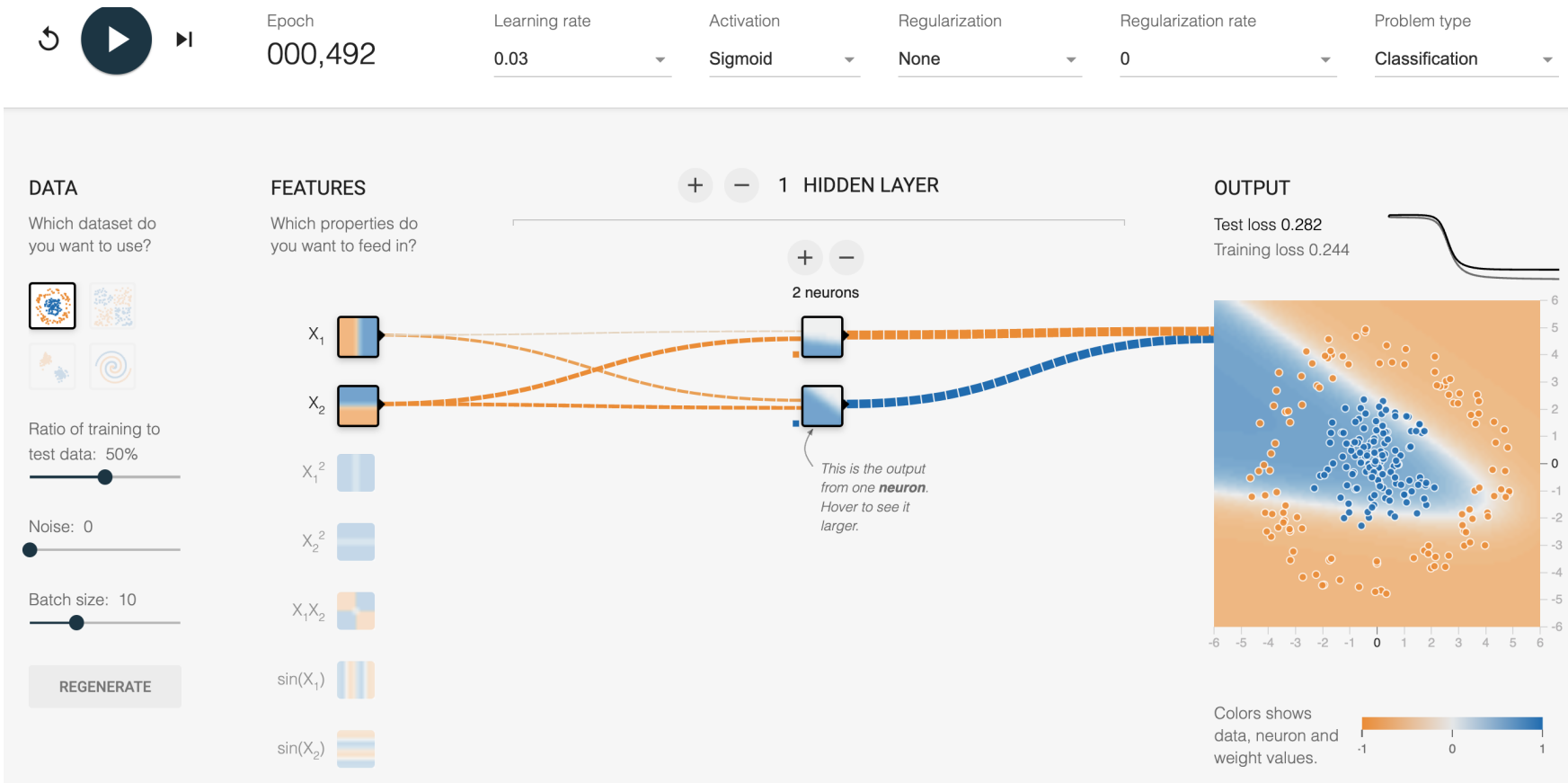
The neurons of the hidden layer learn different transformations of the input.

When combined, they learn non-linear transformations at the output.

Source: `https://playground.tensorflow.org/`

Multilayer Perceptron

Non-linear feature learning



The hidden layer does not have enough neurons to be able to adequately learn the structure of the data.

Source: `https://playground.tensorflow.org/`

Overview

- Perceptron
  - Representation
  - Learning
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Backpropagation

Multilayer Perceptron: Representation

- Input: $\mathbf{x} \in \mathbb{R}^D$
- Output:
  $y \in \{0, 1\}$ or $y \in \{1, \ldots, K\}$ (classification)
  $y \in \mathbb{R}$ or $y \in \mathbb{R}^K$ (regression)
- Training data: $\mathcal{D}^{train} = \{(\mathbf{x_1}, y_1), \ldots, (\mathbf{x_N}, y_N)\}$
- Model: $h_{\mathbf{W},\mathbf{b}}(\mathbf{x})$
  represented through forward propagation (see previous slides)
- Model parameters: weights $\mathbf{W}^{(1)}, \ldots, \mathbf{W}^{(\mathbf{L})}$ and biases $\mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(\mathbf{L})}$

Multilayer Perceptron: Evaluation criterion

$J(\mathbf{W}, \mathbf{b}, \mathcal{D}^{train}) = \frac{1}{2} \| h_{\mathbf{W},\mathbf{b}}(\mathbf{x}) - y \|_2^2$ (regression)

$J(\mathbf{W}, \mathbf{b}, \mathcal{D}^{train}) = y \log h_{\mathbf{W},\mathbf{b}}(\mathbf{x}) + (1 - y) \log(1 - h_{\mathbf{W},\mathbf{b}}(\mathbf{x}))$ (classification)

Backpropagation

Multilayer Perceptron: Evaluation criterion

Regression

$$J(\mathbf{W},\mathbf{b}) = \frac{1}{N}\sum_{n=1}^{M}\frac{1}{2}\|h_{\mathbf{W},\mathbf{b}}(\mathbf{x_n}) - y_n\|_2^2 + \frac{\lambda}{2}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(W_{ji}^{(l)})^2$$

s_l: # nodes in l^{th} layer

Classification

$$J(\mathbf{W},\mathbf{b}) = \frac{1}{N}\sum_{n=1}^{M}(y_n \log h_{\mathbf{W},\mathbf{b}}(\mathbf{x_n}) + (1-y_n)\log(1-h_{\mathbf{W},\mathbf{b}}(\mathbf{x_n})))$$
$$+ \frac{\lambda}{2}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(W_{ji}^{(l)})^2$$

We will perform gradient descent

Backpropagation

Gradient descent for regression

$$J(\mathbf{W},\mathbf{b}) = \frac{1}{N}\sum_{n=1}^{M}\frac{1}{2}\|h_{\mathbf{W},\mathbf{b}}(\mathbf{x_n}) - y_n\|_2^2 + \frac{\lambda}{2}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(W_{ji}^{(l)})^2$$

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta W_{ij}^{(l)}}$$
$$b_i^{(l)} := b_i^{(l)} - \alpha\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta b_i^{(l)}}$$

Note: Initialize the parameters randomly $\rightarrow$ symmetry breaking

Use backpropagation to compute partial derivatives $\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta W_{ij}^{(l)}}$ and $\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta b_i^{(l)}}$

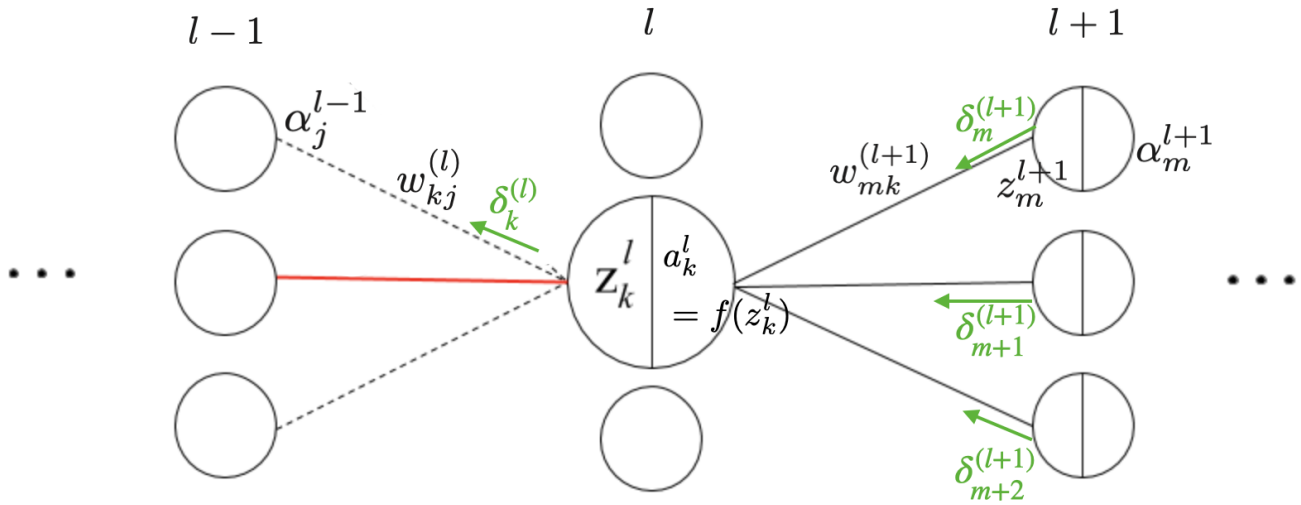
Backpropagation

Intuition

- Given a training example $(\mathbf{x_n}, y_n)$, we run a "forward pass" to compute all the activations
- For each node i in layer l, we compute an error term $\delta_i^{(l)}$ that measures how much that node was "responsible" for any errors in the output
  - Output node: difference between activation and target value
  - Hidden nodes: weighted average of the error terms of the nodes from the previous layer (i.e. $l + 1$)

Backpropagation

$J(\mathbf{W}, \mathbf{b})$ is the loss function, minimized with respect to $\mathbf{W}$ and $\mathbf{b}$.
$f(x)$, $x \in \mathbb{R}$ is the activation function (e.g., sigmoid), not being optimized.

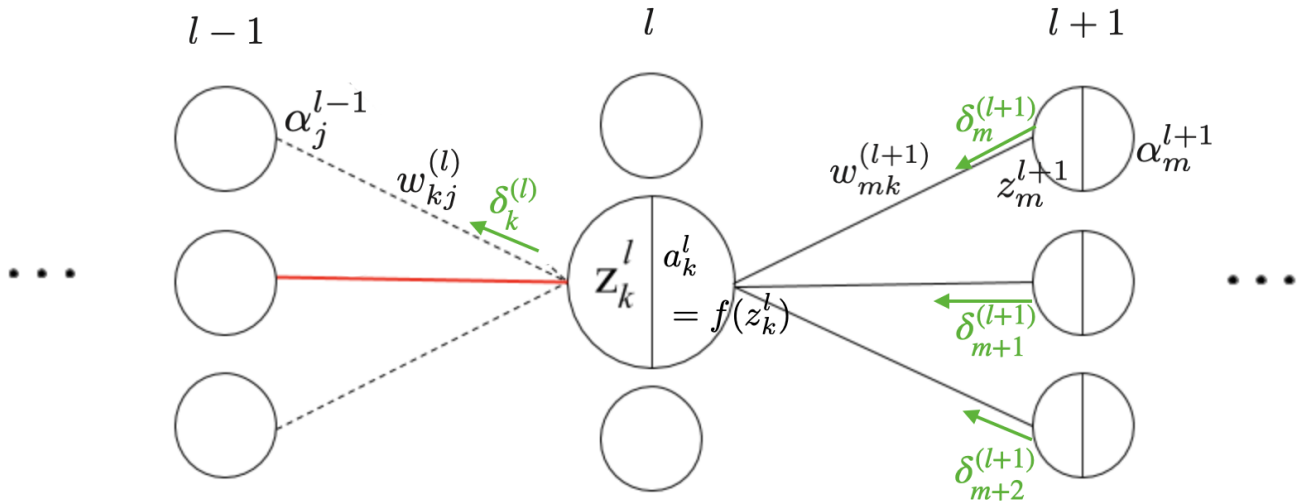


$$\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta w_{kj}^{(l)}} = \underbrace{\alpha_j^{(l-1)}}_{\text{output from layer } l-1} \cdot \underbrace{delta_k^{(l)}}_{\text{error from layer } l}, \text{ where } \alpha_j^{(l-1)} = f(z_j^{(l-1)})$$

$$w_{kj}^{(l)} := w_{kj}^{(l)} - a \cdot \frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta w_{kj}^{(l)}}, \; a\text{: learning rate}$$

Backpropagation

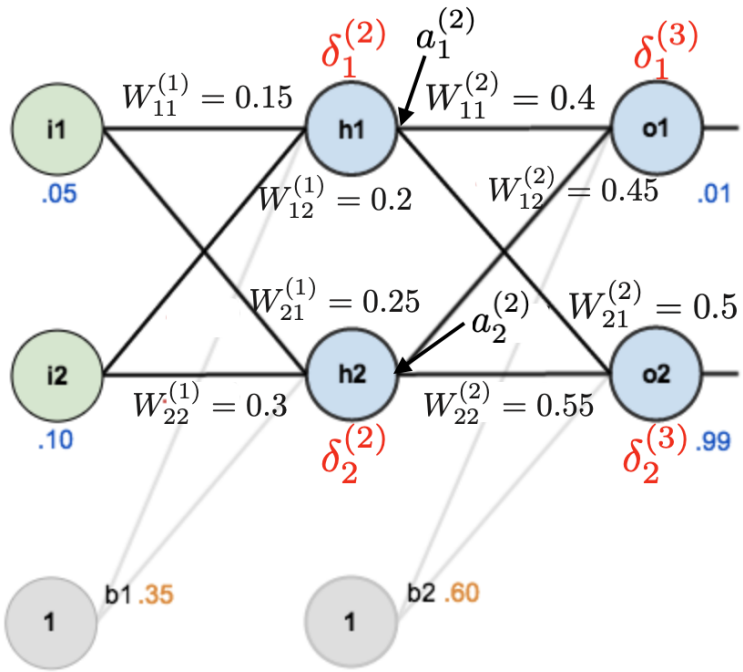
$J(\mathbf{W}, \mathbf{b})$ is the loss function, minimized with respect to $\mathbf{W}$ and $\mathbf{b}$.
$f(x)$, $x \in \mathbb{R}$ is the activation function (e.g., sigmoid), not being optimized.



$$\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta w_{kj}^{(l)}} = \underbrace{\alpha_j^{(l-1)}}_{\text{output from layer } l-1} \cdot \underbrace{delta_k^{(l)}}_{\text{error from layer } l}, \text{ where } \alpha_j^{(l-1)} = f(z_j^{(l-1)})$$

$$\delta_k^{(l)} = f'(z_k^{(l)}) \sum_m \delta_m^{(l+1)} w_{mk}^{(l+1)}$$

Backpropagation



Backpropagation Implementation

- For each node i in output layer L
  - $\delta_i^{(L)} = (\alpha_i^{(L)} - y_n) f'(z_i^{(L)})$
- For each node i in layer $l = L-1, L-2, \ldots, 2$
  - Hidden nodes: $\delta_i^{(l)} = \left(\sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$
- Compute the desired partial derivatives as:
  $\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta W_{ij}^{(l)}} = \alpha_j^{(l)} \delta_i^{(l+1)}$
  $\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta b_i^{(l)}} = \delta_i^{(l+1)}$
- Update the weights as:
  $W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta W_{ij}^{(l)}}$
  $b_i^{(l)} := b_i^{(l)} - \alpha \frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta b_i^{(l)}}$

[Detailed solution of example in Handouts]

Backpropagation

Implementation

- Given a training example $(\mathbf{x_n}, y_n)$, we run a "forward pass" to compute all the activations
- For each node i in output layer L
  - $\delta_i^{(L)} = (y_n - \alpha_i^{(L)}) f'(z_i^{(L)})$
- For each node i in layer $l = L-1, L-2, \ldots, 2$
  - Hidden nodes: $\delta_i^{(l)} = \left(\sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$
- Compute the desired partial derivatives as:
  $\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta W_{ij}^{(l)}} = \alpha_j^{(l)} \delta_i^{(l+1)}$
  $\frac{\vartheta J(\mathbf{W},\mathbf{b})}{\vartheta b_i^{(l)}} = \delta_i^{(l+1)}$

Backpropagation

Additional visualization tools

- https://xnought.github.io/backprop-explainer/
- https://hmkcode.com/ai/backpropagation-step-by-step/

Additional videos

- Andrej Karpathy: The spelled-out intro to neural networks and backpropagation: building micrograd
  https://www.youtube.com/watch?v=VMj-3S1tku0/

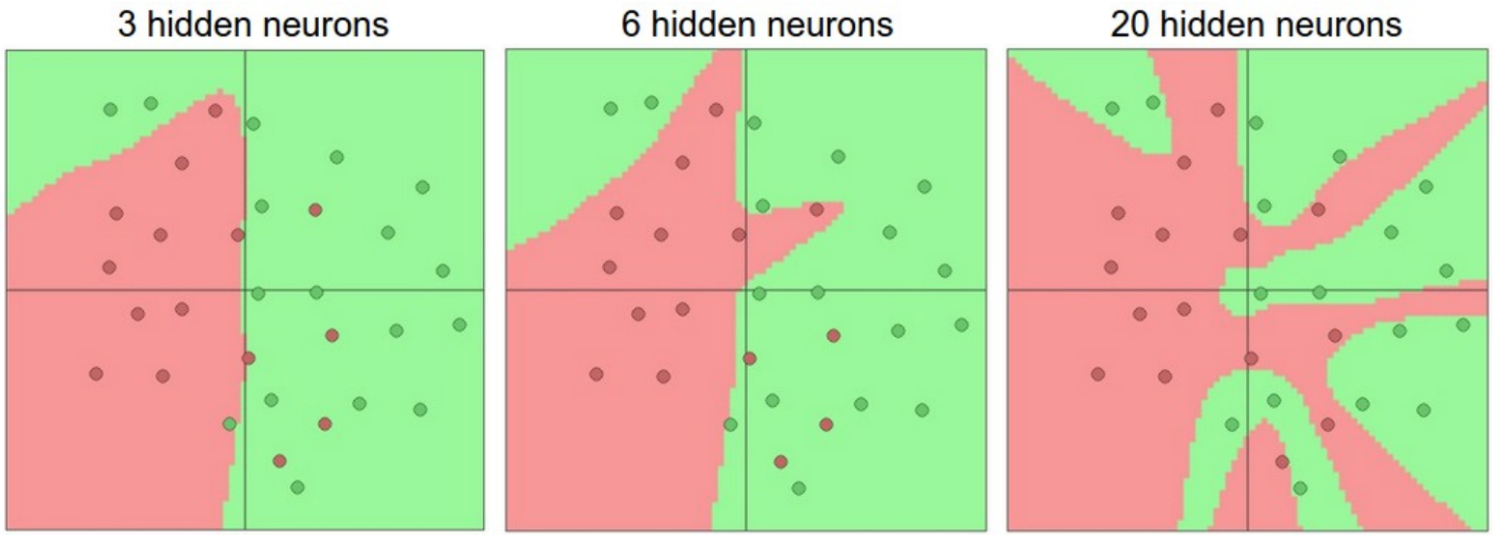
Overview

- Perceptron
  - Representation
  - Learning
  - Examples
- Multilayer Perceptron
  - Representation
  - Learning: Backpropagation
  - Practical issues

Determining number of layers and their sizes

Implementation

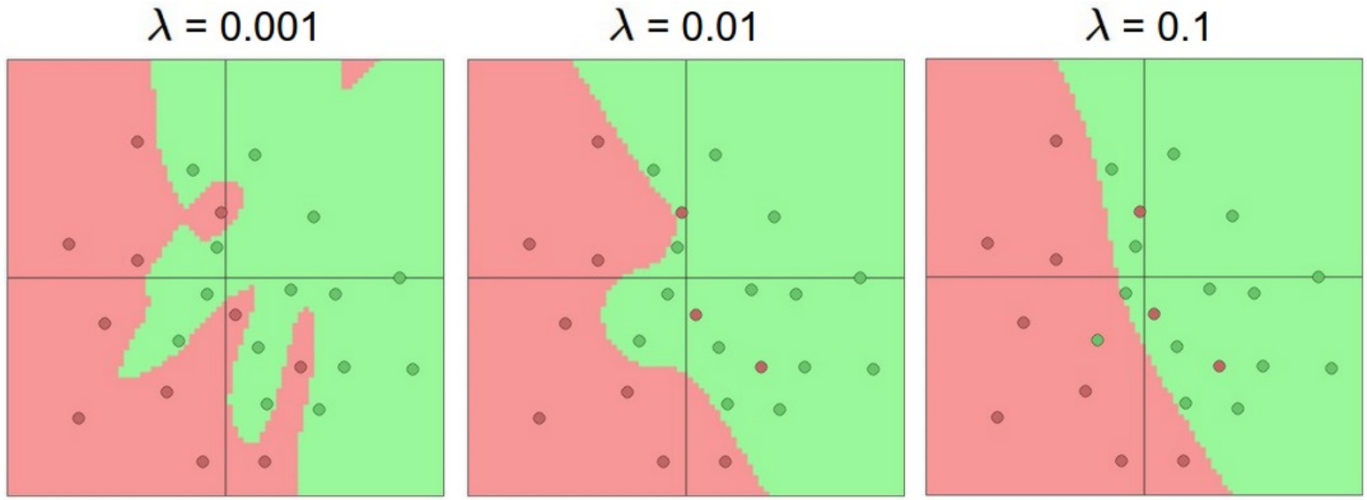
- The capacity of the network (i.e. the number of representable functions) increases as we increase the number of layers
- How to avoid overfittting?



Determining number of layers and their sizes

How to avoid overfitting

- Limit # layers and #hidden units per layers
- Early stopping: start with small weights and stop learning early
- Weight decay: penalize large weights (regularization)
- Noise: add noise to the weights



The effects of regularization strength: Each neural network above has 20 hidden neurons, but changing the regularization strength makes its final decision regions smoother with a higher regularization. You can play with these examples in this ConvNetsJS demo.

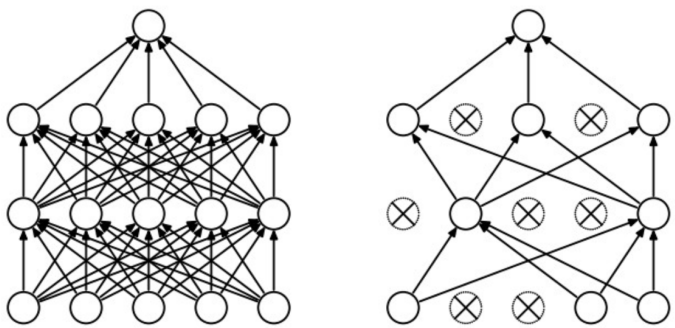
5

http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html

Determining number of layers and their sizes

How to avoid overfitting

- An alternative method that complements the above is dropout
- While training, dropout keeps a neuron active with some probability p (a hyperparameter), or sets it to zero otherwise



(a) Standard Neural Net (b) After applying dropout.

`https://machinelearningmastery.com/dropout-for-regularizing-deep-neural-networks/`

Determining number of layers and their sizes

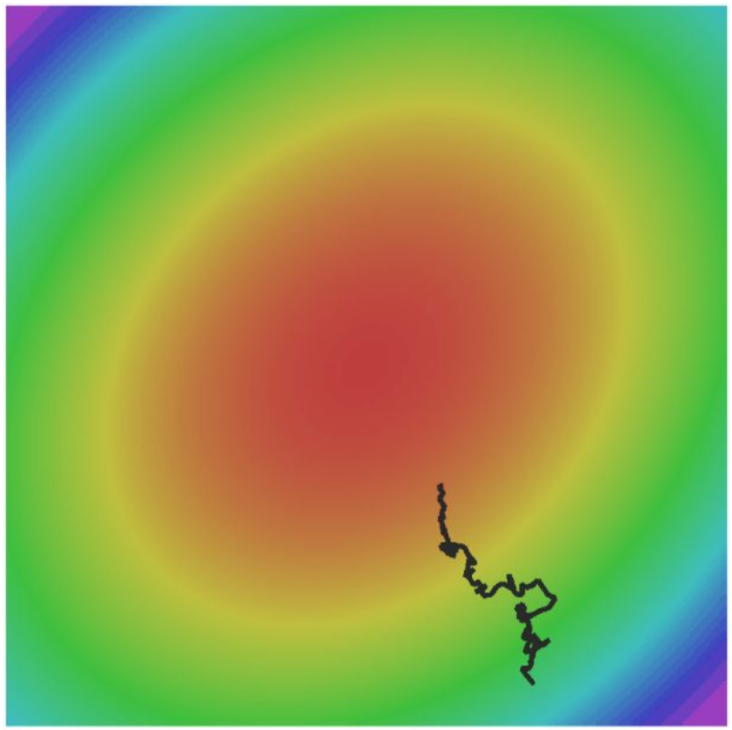
How to chose the number of layers and nodes

- No general rule of thumb, this depends on:
  - Amount of training data available
  - Complexity of the function that is trying to be learned
  - Number of input and output nodes
- If data is linearly separable, you don't need any hidden layers at all
- Start with one layer and hidden nodes proportional to input size
- Gradually increase

Alternative optimization methods

Problems with stochastic gradient descent

- Gradient becomes smaller as we increase the # layers
- Local optima and saddle points become more common in high dimensions



Alternative optimization methods

Stochastic gradient descent + Momentum

- Movement through the parameter space is averaged over multiple time steps
- Momentum speeds up movement along directions of strong improvement (loss decrease) and also helps the network avoid local minima

SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

```
while True:
  dx = compute_gradient(x)
  x += learning_rate * dx
```

SGD+Momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

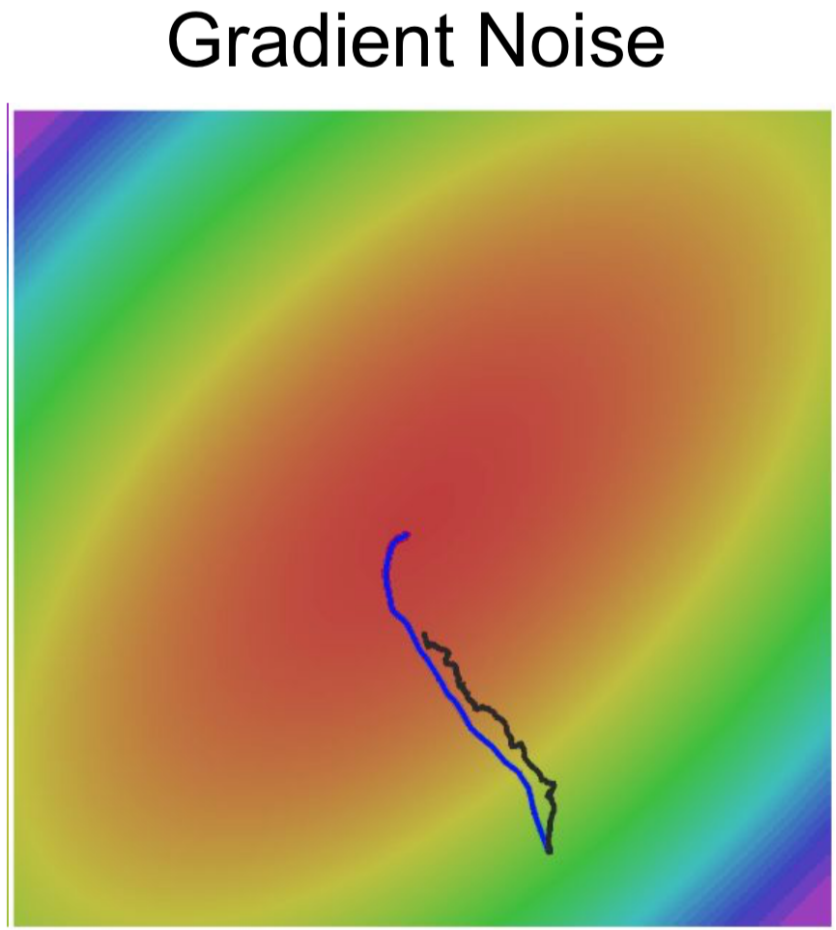
```
vx = 0
while True:
  dx = compute_gradient(x)
  vx = rho * vx + dx
  x += learning_rate * vx
```

- Build up “velocity” as a running mean of gradients
- Rho gives “friction”; typically rho=0.9 or 0.99

Alternative optimization methods

Stochastic gradient descent + Momentum

Issue with noisy trajectories that diverge from optima



https://towardsdatascience.com/
a-visual-explanation-of-gradient-descent-methods-momentum-adagrad-rmsprop-adam-f898b102325c

Alternative optimization methods

AdaGrad & RMSProp

Added element-wise scaling of the gradient based on the historical sum of squares in each dimension.

Able to escape saddle points much better than gradient descent.

Weights with larger gradients in the past will depict reduced learning rates. The opposite will occur for weights with with smaller gradients in the past.

AdaGrad

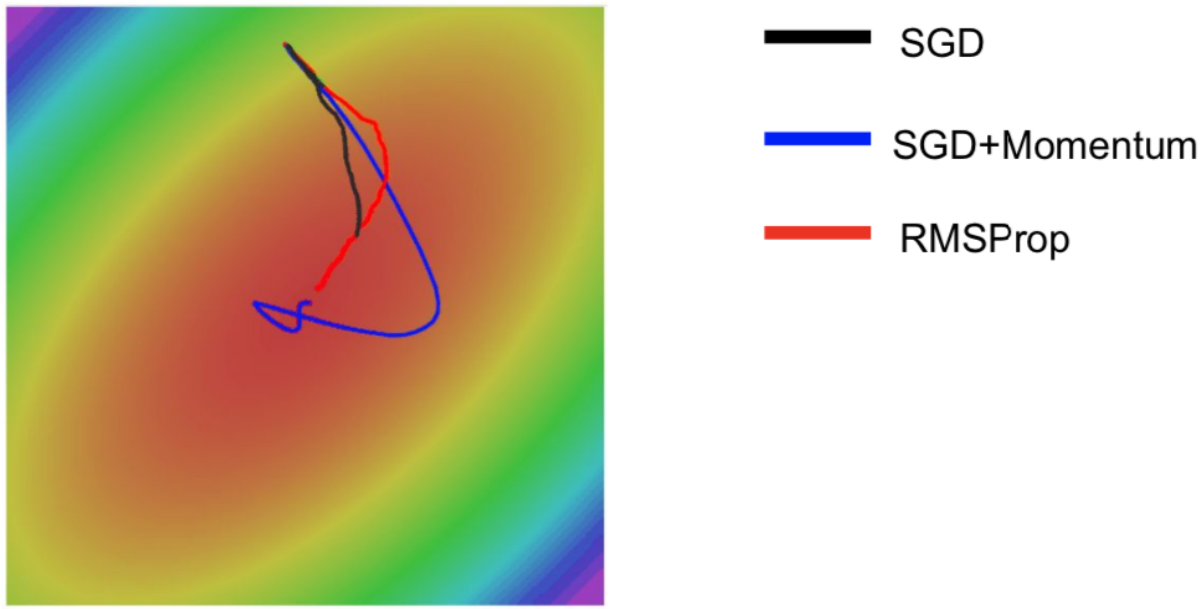
```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared += dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

RMSProp

```
grad_squared = 0
while True:
  dx = compute_gradient(x)
  grad_squared = decay_rate * grad_squared + (1 - decay_rate) * dx * dx
  x -= learning_rate * dx / (np.sqrt(grad_squared) + 1e-7)
```

Alternative optimization methods

RMSProp



Alternative optimization methods

Adam

Combination of RMSProp and Momentum

```
first_moment = 0
second_moment = 0
while True:
  dx = compute_gradient(x)
  first_moment = beta1 * first_moment  + (1 - beta1) * dx
  second_moment = beta2 * second_moment + (1 - beta2) * dx * dx
  x -= learning_rate * first_moment / (np.sqrt(second_moment) + 1e-7))
```

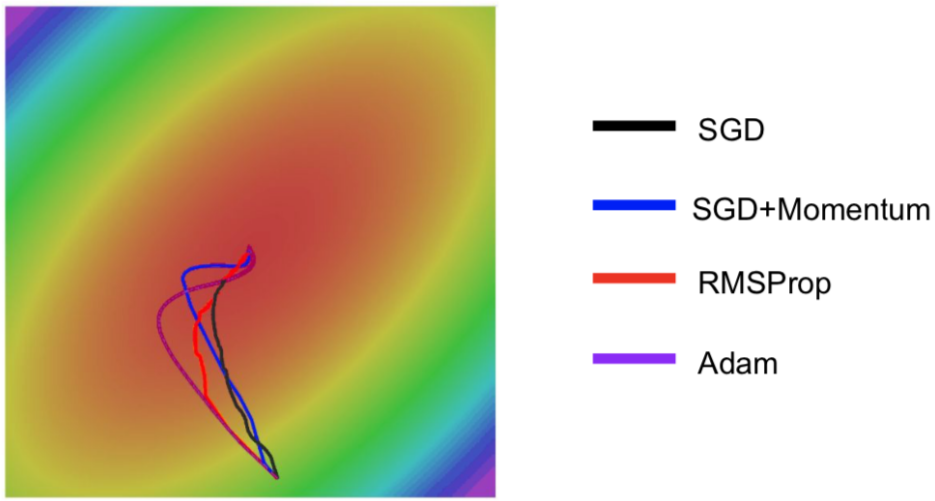
Momentum

AdaGrad / RMSProp

Alternative optimization methods

Adam

Issue with noisy trajectories that diverge from optima



Alternative optimization methods

- Adam is a good default choice
- A more informed selection of the optimization method can be done through hyper-parameter tuning
- Also see: `https://github.com/lilipads/gradient_descent_viz`
- Also see: Bengio, Yoshua. "Practical recommendations for gradient-based training of deep architectures." Neural networks: Tricks of the trade. Springer, Berlin, Heidelberg, 2012. 437-478.

Activation Function

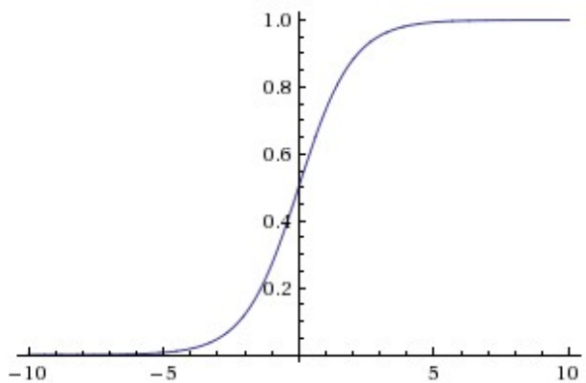
Transforms the activation level of a node (weighted sum of inputs) to an output signal

- Sigmoid: $\sigma(x) = \frac{1}{1+e^{-x}}$
- Hyperbolic tangent: $s(x) = \tanh(x) = 2\sigma(2x) - 1$
- Rectified Linear Unit (ReLU): $f(x) = \max(0, x)$
- Leaky ReLU: $f(x) = (ax) \cdot \mathbb{I}(x < 0) + (x) \cdot \mathbb{I}(x \geq 0)$ (e.g. $a = 0.01$)

Activation Function

Sigmoid: $\sigma(x) = \frac{1}{1+e^{-x}}$, $\frac{d\sigma(x)}{dx} = \sigma(x)(1-\sigma(x))$

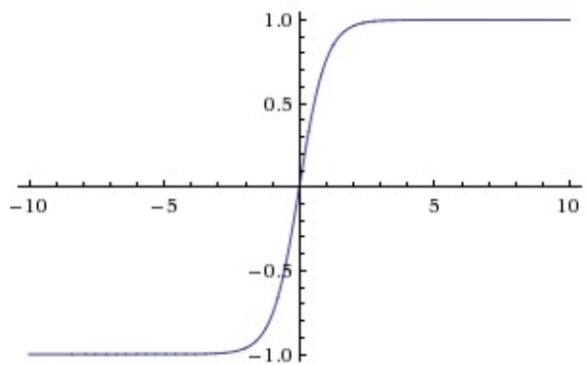
- Transforms a real-valued number between 0 and 1
- Large negative numbers become 0 (not firing at all)
- Large positive numbers become 1 (fully-saturated firing)
- Used historically because of its nice interpretation
- Saturates gradients: The gradient at either extremes (0 or 1) is almost zero, "killing" the signal will flow
- Non-zero centered output: Can be problematic during training, since it can bias outputs toward being always positive or always negative, causing unnecessary oscillations during the optimization



Activation Function

Hyperbolic tangent: $s(x) = \tanh(x) = 2\sigma(2x) - 1$

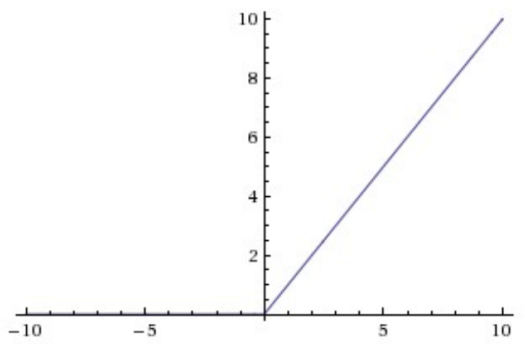
- Scaled version of sigmoid
- Transforms a real-valued number between -1 and 1
- Saturates gradients: Similar to sigmoid
- Output is zero-centered, avoiding some oscillation issues

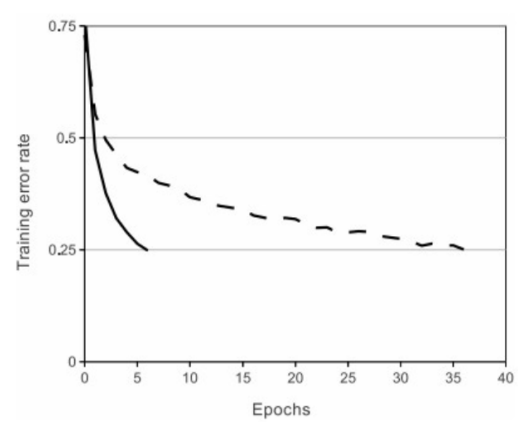


Activation Function

Rectified Linear Unit (ReLU): $f(x) = \max(0, x)$

- Activation simply thresholded at zero
- Very popular during the last years
- Accelerates convergence (e.g. a factor of 6, see bellow) compared to the sigmoid/tanh (due to its linear, non-saturating form)
- Cheap implementation by simply thresholding at zero
- Activation can "die": a large gradient flowing through a ReLU neuron could cause the weights to update in such a way that the neuron will never activate on any datapoint again, proper adjustment of learning rate can mitigate that

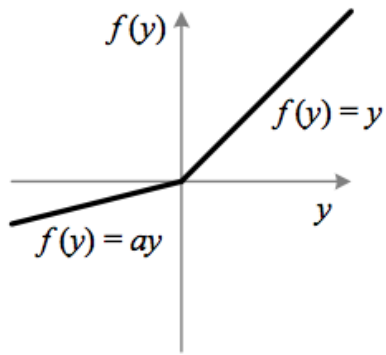




Activation Function

Leaky ReLU: $f(x) = (ax) \cdot \mathbb{I}(x < 0) + (x) \cdot \mathbb{I}(x \geq 0)$

- Instead of the function being zero when $x < 0$, leaky ReLU will have a small negative slope (e.g. $a = 0.01$)
- Some successful results, but not always consistent



Hyperparameter tuning

- Learning rate: how much to update the weight during optimization
- Number of epochs: number of times the entire training set pass through the neural network
- Batch size: the number of samples in the training set for weight update
- Activation function: the function that introduces non-linearity to the model (e.g. sigmoid, tanh, ReLU, etc.)
- Number of hidden layers and units
- Weight initialization: e.g., uniform distribution
- Dropout for regularization: probability of dropping a unit
- Optimization method: optimization method to learn the weights (e.g., Adam, RMSProp)

What have we learnt so far

- Perceptrons are the basic processing unit of neural networks
- Simulate the "neural connectivity"
- Implemented by the linear combination of input features followed by an activation function, e.g. sigmoid
- Online learning
  - updating weights based on one sample at a time
- Examples implementing boolean functions
  - XOR: non-linear $\rightarrow$ impossible to implement with single perceptron

What have we learnt so far

- Multilayer perceptron is the basic feedforward neural network
- Hidden nodes allow non-linear associations
- Backpropagation to find network weights
- Readings: Alpaydin 11.1-11.8.2
- Additional links
  `https://www.i-am.ai/index.html`