Model selection in R

In this lesson, we will implement various model selection methods on real data in R.

Let's use the `fabric` data to illustrate the implementation of model selection techniques. This dataset contains the acoustic absorption coefficients of 24 woven fabrics with different air gap distances ($d = 0, 1, 2, 3 cm$). There are four different possible response variables based on these air gap distances:

1. `acoustic0`: air gap distance $d = 0 cm$
2. **`acoustic1`:air gap distance** $d = 1 cm$
3. `acoustic2`: air gap distance $d = 2 cm$
4. `acoustic3`: air gap distance $d = 3 cm$

For simplicity, we'll focus on `acoustic1`. The following predictors are given:

1. `thickness`: the thickness of the woven fabric, in mm
2. `diameter`: the diameter of the woven fabric, in mm
3. `perforation`: perforation was measured as a percentage, and is relate to the pore width and yarn width of the woven fabric
4. `weight`: the weight of the woven fabric, measured in g/m^2
5. `stiffness`: the weight of the woven fabric, measured in mN x cm
6. `airPerm`: the air permeability of the woven fabric, measured in mm/s

According to Tang, Kong, and Yan, "It has been found that the acoustic absorption properties" of a woven fabric are "mainly determined by the perforation ratio and air permeability." The goal of this analysis is to see if other fabric properties can useful in predicting the absorption coefficient with $d = 1$, i.e., `acoustic1`. We'll use several different model selection techniques to show how they are implemented and how they might differ from one another.

Source: X. Tang, D. Kong, X. Yan (2018). "Multiple Regression Analysis of a Woven Fabric Sound Absorber," Textile Research Journal, https://doi.org/10.1177/0040517518758001.

```
#Another way to read the data:
#fabric =
read.table("http://users.stat.ufl.edu/~winner/data/fabricsoundabsorb.c
sv", header = TRUE, sep = ",")
library(ggplot2)

fabric =
read.csv(paste0("https://raw.githubusercontent.com/bzaharatos/",
```

```
                    "-Statistical-Modeling-for-Data-Science-
Applications/",
                    "master/Modern%20Regression%20Analysis%20/",
                    "Datasets/fabricsoundabsorb.csv"), sep = ",")

head(fabric)
summary(fabric)
```

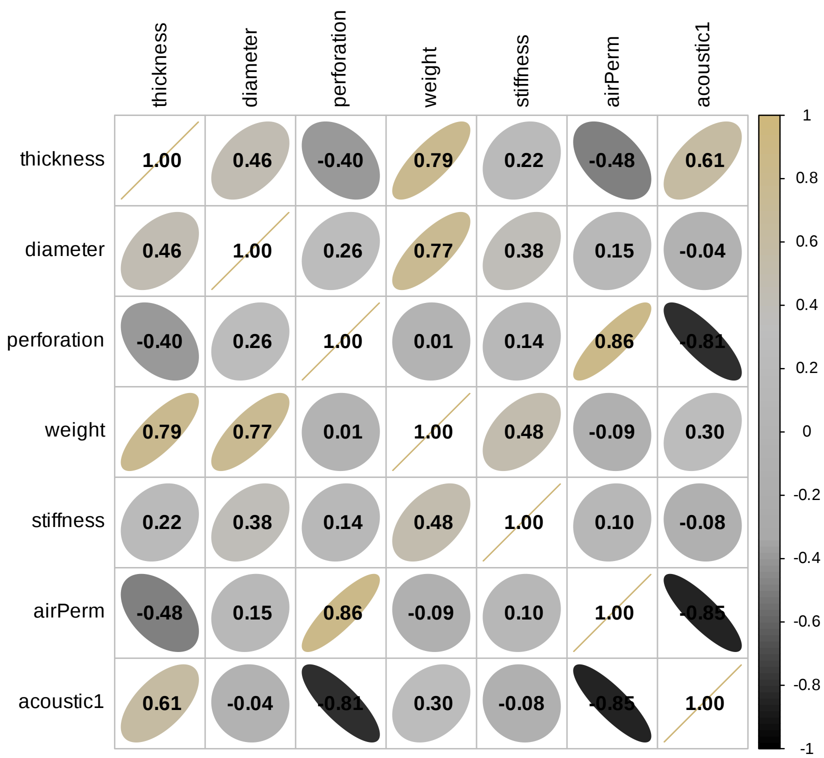
```
  sampleID thickness diameter perforation weight stiffness airPerm
acoustic0
1 1        0.547     0.269    6.14        247    181.90    805.4
0.076
2 2        0.541     0.378    6.08        253     74.40    887.8
0.092
3 3        0.875     0.584    4.97        366     71.41    598.4
0.129
4 4        0.640     0.527    4.87        319    161.50    799.2
0.115
5 5        0.750     0.534    4.51        292    156.20    753.2
0.095
6 6        0.539     0.368    4.39        232    171.50    704.0
0.073
  acoustic1 acoustic2 acoustic3
1 0.330     0.413     0.313
2 0.339     0.347     0.261
3 0.455     0.411     0.316
4 0.433     0.427     0.321
5 0.362     0.382     0.273
6 0.371     0.422     0.312
```

```
    sampleID       thickness         diameter        perforation
 Min.   : 1.00   Min.   :0.5260   Min.   :0.2200   Min.   :1.210
 1st Qu.: 6.75   1st Qu.:0.6438   1st Qu.:0.3395   1st Qu.:2.460
 Median :12.50   Median :0.7685   Median :0.4705   Median :3.110
 Mean   :12.50   Mean   :0.7302   Mean   :0.4320   Mean   :3.378
 3rd Qu.:18.25   3rd Qu.:0.8097   3rd Qu.:0.5340   3rd Qu.:4.195
 Max.   :24.00   Max.   :0.9310   Max.   :0.5840   Max.   :6.140
     weight        stiffness         airPerm         acoustic0
 Min.   :228.0   Min.   : 49.60   Min.   :401.5   Min.   :0.0730
 1st Qu.:251.8   1st Qu.: 81.91   1st Qu.:510.4   1st Qu.:0.0905
 Median :303.5   Median :139.41   Median :600.6   Median :0.1120
 Mean   :302.0   Mean   :134.81   Mean   :601.4   Mean   :0.1163
 3rd Qu.:347.8   3rd Qu.:171.40   3rd Qu.:685.7   3rd Qu.:0.1355
 Max.   :373.0   Max.   :275.56   Max.   :887.8   Max.   :0.1890
   acoustic1        acoustic2        acoustic3
 Min.   :0.3300   Min.   :0.3470   Min.   :0.2610
 1st Qu.:0.4412   1st Qu.:0.4203   1st Qu.:0.3175
 Median :0.4775   Median :0.4800   Median :0.3625
 Mean   :0.4793   Mean   :0.4834   Mean   :0.3699
```

```
 3rd Qu.:0.5420   3rd Qu.:0.5355   3rd Qu.:0.4148  
 Max.   :0.6170   Max.   :0.6260   Max.   :0.4830  
```

```
library(corrplot)
col4 = colorRampPalette(c("black", "darkgrey", "grey","#CFB87C"))
corrplot(cor(fabric[c(2:7,9)]), method = "ellipse", col = col4(100),
addCoef.col = "black", tl.col = "black")
```

```
corrplot 0.95 loaded
```



Let's start with the full model, which uses `acoustic1` as the response and `thickness`, `diameter`, `perforation`, `weight`, `stiffness`, and `airPerm` as predictors.

```
# 1) CODE HERE
lm_fabric_full = lm(acoustic1 ~
thickness+diameter+perforation+weight+stiffness+airPerm, data =
fabric)
summary(lm_fabric_full)
```

```
Call:
lm(formula = acoustic1 ~ thickness + diameter + perforation + 
    weight + stiffness + airPerm, data = fabric)
```

```
Residuals:
      Min        1Q    Median        3Q       Max 
-0.062713 -0.021894 -0.002938  0.020573  0.065304 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.5858162  0.0896400   6.535 5.09e-06 ***
thickness   -0.0288504  0.1606300  -0.180   0.8596    
diameter    -0.1340331  0.1192525  -1.124   0.2767    
perforation -0.0215229  0.0128301  -1.678   0.1117    
weight       0.0008418  0.0004612   1.825   0.0856 .  
stiffness   -0.0002042  0.0001622  -1.258   0.2253    
airPerm     -0.0003019  0.0001228  -2.458   0.0250 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03871 on 17 degrees of freedom
Multiple R-squared:  0.8393,	Adjusted R-squared:  0.7826 
F-statistic:  14.8 on 6 and 17 DF,  p-value: 6.511e-06
```

Here, we see that several of the predictors have t-tests with high p-values (higher than the standard $\alpha = 0.05$). Instead of just removing all of those, let's perform backward selection, removing the predictor with the largest p-value greater than $\alpha_0 = 0.15$, `thickness` in this case, and then refit the model. We can do this with the `update()` function. Remember that α_0, sometimes called the "p-to-remove", is used as a cuttoff in backward selection, and if the goal is prediction, then 0.15 or 0.2 is thought to work well.

```r
# 2) CODE HERE
lm_fabric_full = update(lm_fabric_full, .~.-thickness)
summary(lm_fabric_full)


Call:
lm(formula = acoustic1 ~ diameter + perforation + weight + stiffness + 

    airPerm, data = fabric)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.061055 -0.024693 -0.003761  0.020839  0.064457 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.5766761  0.0717833   8.034 2.31e-07 ***
diameter    -0.1299283  0.1138523  -1.141   0.2687    
perforation -0.0211186  0.0122868  -1.719   0.1028    
weight       0.0007765  0.0002758   2.815   0.0115 *  
stiffness   -0.0001961  0.0001516  -1.293   0.2122    
airPerm     -0.0002959  0.0001150  -2.573   0.0192 *  
```

```
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03765 on 18 degrees of freedom
Multiple R-squared:  0.839,Adjusted R-squared:  0.7943
F-statistic: 18.76 on 5 and 18 DF,  p-value: 1.416e-06
```

Notice that our p-values have changed! For example, before, `weight` was not sigificant at the $\alpha = 0.05$ level and now it is. And, importantly, we still have p-values above $\alpha_0 = 0.15$. So, let's remove the predictor with the largest p-value, namely, `diameter`.

```
# 3) CODE HERE
lm_fabric_full = update(lm_fabric_full, .~.-diameter)
summary(lm_fabric_full)


Call:
lm(formula = acoustic1 ~ perforation + weight + stiffness + airPerm,
    data = fabric)

Residuals:
      Min        1Q    Median        3Q       Max
-0.056134 -0.032975 -0.000806  0.025339  0.071500

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.6009065  0.0691148   8.694 4.76e-08 ***
perforation -0.0241498  0.0120914  -1.997  0.06032 .
weight       0.0005408  0.0001842   2.935  0.00849 **
stiffness   -0.0001887  0.0001526  -1.236  0.23144
airPerm     -0.0002958  0.0001159  -2.551  0.01950 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03795 on 19 degrees of freedom
Multiple R-squared:  0.8274,     Adjusted R-squared:  0.791
F-statistic: 22.77 on 4 and 19 DF,  p-value: 5.01e-07
```

It looks like we have at least one more iteration, since `stiffness` has a p-value greater than α_0.

```
# 4) CODE HERE
lm_fabric_full = update(lm_fabric_full, .~.-stiffness)
summary(lm_fabric_full)


Call:
lm(formula = acoustic1 ~ perforation + weight + airPerm, data =
fabric)
```

```
Residuals:
      Min        1Q    Median        3Q       Max
-0.074211 -0.025324  0.003821  0.020601  0.076329

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.6159414  0.0689289   8.936 2.03e-08 ***
perforation -0.0244787  0.0122470  -1.999   0.0594 .
weight       0.0004294  0.0001628   2.638   0.0158 *
airPerm     -0.0003053  0.0001172  -2.605   0.0169 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.03845 on 20 degrees of freedom
Multiple R-squared:  0.8135,	Adjusted R-squared:  0.7855
F-statistic: 29.08 on 3 and 20 DF,  p-value: 1.717e-07
```

Notice that each of our remaining predictors have a p-value less than $\alpha_0 = 0.15$. This is the "best" model, according to backward selection. Of course, there's nothing about backward selection that is consistent with statistical inference, so we should use this procedure with caution! The same is true for forward selection and other stepwise selection methods.

AIC, BIC, and adjusted R^2

Recall that AIC is defined as

$$AIC\left(g\left(x;\hat{\beta}\right)\right) = 2(p+1) + n\log(RSS/n).$$

We can use AIC (or BIC, or adjusted R^2) to help us choose a "best" model. One way to do this would be:

1. Fit *all* simple linear regression models, and report the one with the lowest RSS. This is allowed because when the number of predictors is the same, RSS (and equivalently, R^2) can be used to compare models.

2. Fit all two-predictor models, and report the one with the lowest RSS.

3. Continue this process: fit all k-predictor models, and report the one with the lowest RSS.

4. At this point, we should have the "best" models (in terms of RSS) for all models of size k, where k ranges from 1 to p. We can no longer use RSS to compare across *these* models, since each is of a difference size. So, we use a criterion that takes into account the tradeoff between fit (RSS) and model size/complexity (p).

The `regsubsets()` function in the `leaps` library can help us streamline this process.

```
install.packages("leaps")
library(leaps)
```

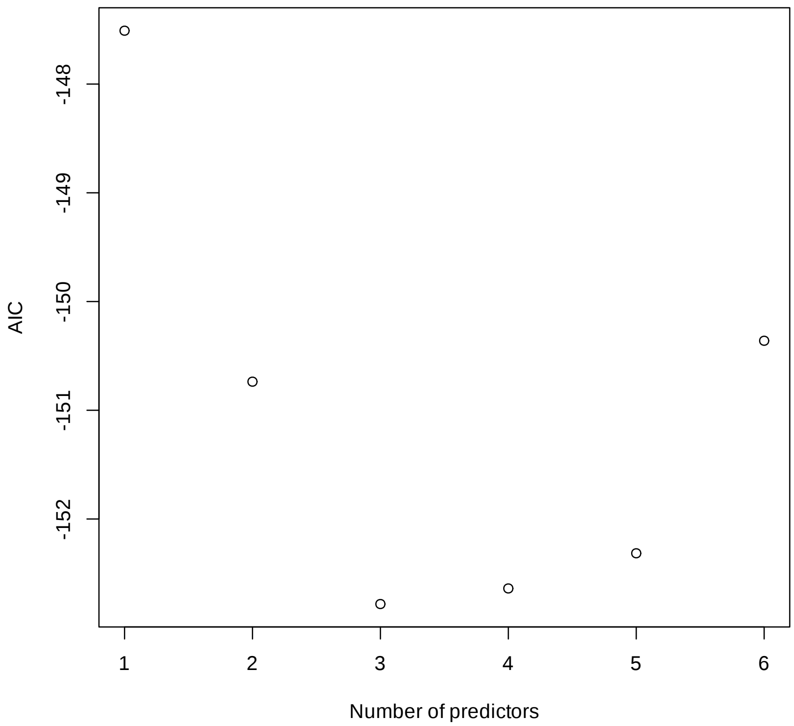
```r
library(MASS)

# 5) CODE HERE

n = dim(fabric)[1]
reg1 = regsubsets(acoustic1 ~
thickness+diameter+perforation+weight+stiffness+airPerm, data =
fabric)
rs = summary(reg1)
rs$which
```

```
Updating HTML index of packages in '.Library'

Making 'packages.html' ...
 done
```

	(Intercept)	thickness	diameter	perforation	weight	stiffness	airPerm
1	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE
2	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	TRUE
3	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE
4	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
5	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
6	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

The table above provides the best model (in terms of RSS) of size k, for $k=1,2,\ldots,6$. For example, the best simple linear regression model is the model `acoustic1` $= \hat{\beta}_0 + \hat{\beta}_1 \times$ `airPerm`. Now, to compare these models with each other, we calculate AIC, and plot the AIC values as a function of model size.

```r
# 6) CODE HERE

AIC = 2 *(2:7)+n*log(rs$rss/n)
plot(I(1:6),AIC, xlab = "Number of predictors", ylab="AIC")
```



In this plot, we see that the model of size $k = 3$ has the lowest AIC. That means that our model selection procedure has chosen:

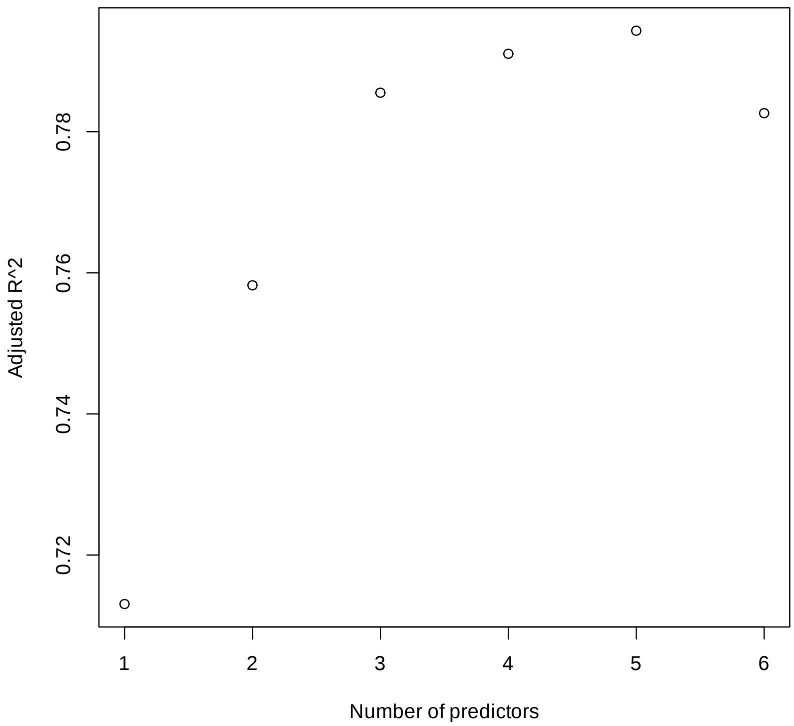
`acoustic1` = $\widehat{\beta}_0 + \widehat{\beta}_1 \times$ `perforation` $ + \widehat\beta_2 \times$ `weight` $ + \widehat\beta_3 \times$ `airPerm`.

Interestingly, using

$$R_a^2 = 1 - \frac{RSS/(n-(p+1))}{TSS/(n-1)},$$

we get a different model, namely the model with five predictors (`diameter`, `perforation`, `weight`, `stiffness`, `airPerm`):

```
# 7) CODE HERE
plot(I(1:6),rs$adjr2, xlab="Number of predictors", ylab="Adjusted
R^2")
```

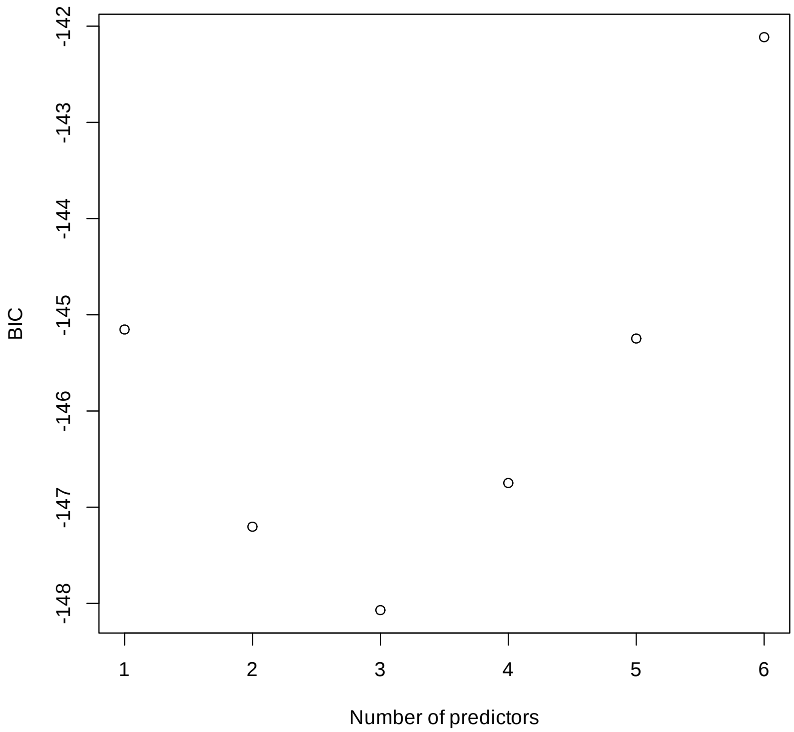


And BIC, which is given as

$$BIC\left(g\left(x;\hat{\beta}\right)\right)=\left(p+1\right)\log\left(n\right)-2\log L\left(\hat{\beta}\right),$$

chooses the same model as AIC.

```
# 8) CODE HERE
BIC = log(n)*(2:7)+n*log(rs$rss/n)
plot(I(1:6),BIC, xlab="Number of predictors", ylab="BIC")
```



Is there evidence of collinearity in the data using the full model? How about using the model chosen by AIC?

```
#install.packages("car")
#library(car)

source("vif_function.r")

# 9) CODE HERE
lm_fabric_full = lm(acoustic1 ~
thickness+diameter+perforation+stiffness+airPerm, data=fabric)
vif(lm_fabric_full)

  thickness    diameter perforation   stiffness     airPerm
   2.161076    1.902876    4.215372    1.189251    4.207542

lm_fabric_vif_thickness = lm(thickness ~ diameter + perforation +
weight + stiffness+airPerm,data=fabric)
vif = 1/(1-summary(lm_fabric_vif_thickness)$r.squared)
vif

[1] 5.717772
```

For the full model, we see:

1. The VIF for the estimators associated with `thickness` and `weight` are in the "some evidence of collinearity" range (i.e., $5 < VIF < 10$).

2. The condition number is very high (¿>30), suggesting collinearity is an issue.

3. The correlation matrix for the predictors shows high pairwise correlations.

```
# 10) CODE HERE
source("vif_function.r")
lm_fabric_aic = lm(acoustic1 ~ perforation + weight + airPerm, data =
fabric)
vif(lm_fabric_aic)
kappa(lm_fabric_aic)
cor(model.matrix(lm_fabric_aic)[,-1])
```

```
perforation      weight     airPerm
   3.927635    1.037730    3.957534
```

```
[1] 4531.029
```

```
            perforation weight      airPerm
perforation 1.00000000   0.01141017  0.85896316
weight      0.01141017   1.00000000 -0.08765898
airPerm     0.85896316  -0.08765898  1.00000000
```

For the full model, we see:

1. The VIF for the estimators are now below 5, which is not evidence of collinearity.

2. The condition number is still very high (¿>30), suggesting collinearity is an issue.

3. The correlation matrix for the predictors shows high pairwise correlations. Specifically, `airPerm` and `perforation` are highly correlated. We might remove one and refit the model.