Perfectly stirred reactor

A perfectly stirred reactor (PSR) is a way to perform a zero-dimensional approach of combustion occurring in a gas turbine combustion chamber using two characteristic timescales: a chemical time τ_{ch} and a residence time τ_s. A schematics of such a reactor is shown in Fig. 1. We will study its operation on the lean side of the equivalence ratio.

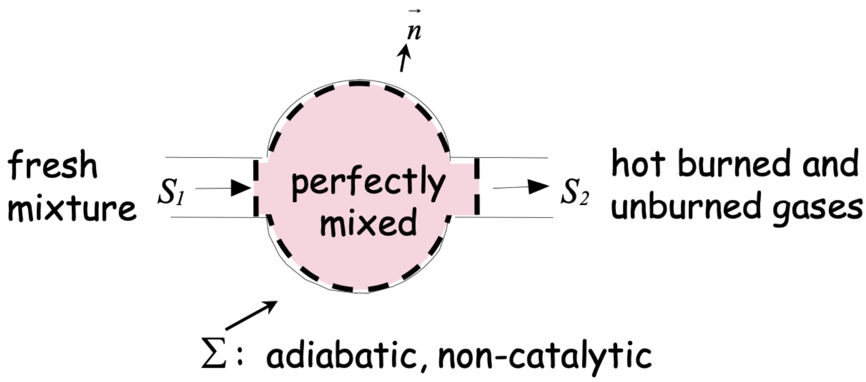

Figure 1: Schematics of a PSR

Assumptions

- Fresh mixture enters, burns and mixes instantaneously with the contents of the PSR
- Chemical reactions occur in the perfectly mixed, homogeneous mixture
- Steady state for the flow rate through the reactor
- One step chemistry following Arrhenius equation
- No conductive heat flux and no species diffusion at the boundaries
- Constant c_p

Modeling

- Total mass in the reactor:

$$M = \int_V \rho dV \tag{1}$$

- Mass balance in the reactor:

$$\frac{dM}{dt} = -\int_S \rho \mathbf{v}.\mathbf{n} dS = 0 \Rightarrow -\int_{S_1} \rho \mathbf{v}.\mathbf{n} dS = \int_{S_2} \rho \mathbf{v}.\mathbf{n} dS = \dot{m} \tag{2}$$

  The residence time τ_s can thus be expressed as $\frac{M}{\dot{m}}$.

- Rate of fuel consumption per cubic meter:

$$R_{Fu} = -k_0 e^{-\frac{T_{act}}{T}} \rho_{Fu} \tag{3}$$

  where $k_0 = \frac{1}{\tau_{ch}}$ and ρ_{Fu} is the fuel partial density in $kg.m^{-3}$.

- Simplified thermal balance:

$$\frac{d}{dt}\int_V \rho c_p T dV = \int_V -q_p R_{Fu} dV - \int_{S_1} \rho c_p T \mathbf{v}.\mathbf{n} dS - \int_{S_2} \rho c_p T \mathbf{v}.\mathbf{n} dS \tag{4}$$

  where q_p is the lower heating value of the reaction.

- Mean inlet and outlet temperatures:

$$T_{fr} = \frac{-\int_{S_1} \rho T \mathbf{v}.\mathbf{n} dS}{\dot{m}}; T_{br} = \frac{\int_{S_2} \rho T \mathbf{v}.\mathbf{n} dS}{\dot{m}} \tag{5}$$

- Simplified fuel mass balance:

$$\frac{d}{dt}\int_V \rho Y_{Fu} dV = \int_V R_{Fu} dV - \int_{S_1} \rho Y_{Fu} \mathbf{v}.\mathbf{n} dS - \int_{S_2} \rho Y_{Fu} \mathbf{v}.\mathbf{n} dS \tag{6}$$

- Mean inlet and outlet mass fractions of fuel:

$$Y_{Fu,fr} = \frac{-\int_{S_1} \rho Y_{Fu} \mathbf{v}.\mathbf{n} dS}{\dot{m}}; Y_{Fu,br} = \frac{\int_{S_2} \rho Y_{Fu} \mathbf{v}.\mathbf{n} dS}{\dot{m}} \tag{7}$$

Questions

1°) Show that one can write the following equation for the temperature in the PSR:

$$\frac{dT}{dt} = -\frac{q_p}{c_p}\tilde{R}_{Fu} - \frac{(T_{br} - T_{fr})}{\tau_s} \tag{8}$$

where $\tilde{R}_{Fu} = -k_0 Y_{Fu} e^{-\frac{T_{act}}{T}}$.

Discuss the behaviour of the PSR for different relative values of $\frac{q_p}{c_p}\tilde{R}_{Fu}$ and $\frac{(T_{br}-T_{fr})}{\tau_s}$.

2°) Show that one can write the following equation for the fuel mass fraction in the PSR:

$$\frac{dY_{Fu}}{dt} = \tilde{R}_{Fu} + \frac{(Y_{Fu,fr} - Y_{Fu,br})}{\tau_s} \tag{9}$$

From now on, we will consider that the sytem is in steady state.

3°) Show that:

$$c_p(T_{br} - T_{fr}) = q_p(Y_{Fu,fr} - Y_{Fu,br}) \tag{10}$$

4°) Considering the condition to reach the adiabatic combustion temperature T_{ad} in the PSR, show that:

$$T_{ad} = T_{fr} + \frac{q_p}{c_p} Y_{Fu,fr} \tag{11}$$

5°) Show that:

$$\frac{c_p}{q_p}(T_{ad} - T_{br}) = Y_{fu,br} \tag{12}$$

6°) Show that the first term on the right hand side of Eq. (8) can be written as:

$$\frac{1}{\tau_{ch}}(T_{ad} - T_{br})e^{-\frac{T_{act}}{T}} \tag{13}$$

7°) Complete section Q7 of the provided Matlab script to graph the two terms on the right hand side of Eq. (8) with respect to temperature T_{br} and discuss the different behaviours for the five provided values of τ_s. Why is $\tau_s = 0.00615$ dubbed "ignition" and $\tau_s = 0.001035$ "extinction"?

The following values are used in the code:

T_{fr} (K)	T_{ad} (K)	T_{act} (K)	τ_{ch} (s)	τ_s (s)
800	2000	10 000	1e-6	0.0008 ; 0.001035 ; 0.002 ; 0.00615 ; 0.02

8°) Use section Q8 of the provided Matlab script to plot the possible steady state temperatures versus the residence time τ_s. This curve is called the S-curve.

What can you say about the central branch of the curve between the top and bottom branches?

How can this curve be used to explain the following statements:

- Blowing on a candle turns it off.
- It is better to place the spark plug of a gas turbine combustion chamber in a "calm" zone.

9°) Write the system of equations to solve to obtain T_br and τ_s at the ignition and extinction points for given values of T_{fr}, T_{ad}, T_{act} and τ_{ch}. Show that the ignition and extinction temperature values are the solutions of a second degree equation taking only into account the three temperatures T_{fr}, T_{ad} and T_{act}.

10°) The combustion adiabatic temperature is a function of the equivalence ratio ϕ. Using section Q10 of the provided Matlab script, plot the ratio $\frac{\tau_{ch}}{\tau_s}$ at extinction as a function of the equivalence ratio ϕ.

Discuss the different behaviours you observe.

Knowing that $\frac{\tau_{ch}}{\tau_s} \propto \frac{\dot{m}}{Vp^n}$, explain what can be done to increase the output power of a combustion system.