External aerodynamics

- Chapter 4: drag components and control (part A) -

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Outline of chapter 4

Part A: boundary layers

- Boundary layer
- Transition
- Separation

Part B: drag components and control devices

- Drag contributions
- Boundary layer and drag control

D'Alembert's paradox

In <u>inviscid</u> and <u>stationary</u> conditions, drag around a <u>2D</u> body is null.

By opposition, it can be inferred that drag is generated by:

- viscous effects (boundary layers);
- unsteadiness ("added mass" concept);
- 3D effects (induced drag).

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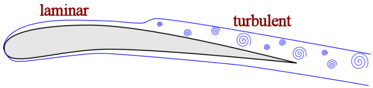
Boundary layer

The boundary layer is the flow region in the vicinity of solid walls, characterized by intense mean velocity gradients. It can be laminar or turbulent, and can evolve spatially from one regime to the other.



In aeronautical conditions, transition occurs at:

$$Re_x = \frac{\rho U_e x}{\mu} \approx 3 \cdot 10^5$$

where U_e is the external flow velocity and x the abscissa. However, this threshold is highly dependent on various parameters (see section 2).

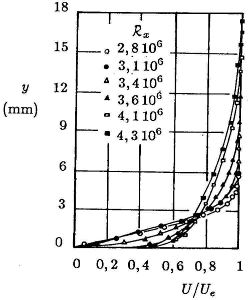
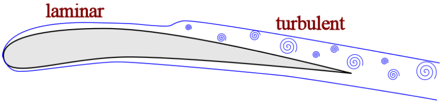


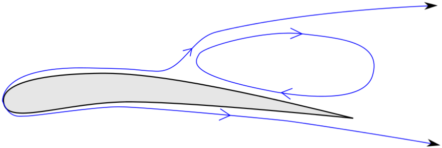
Figure: *Mean velocity profiles in boundary layer, from Schubauer and Klebanoff [1].*

Two phenomena must be distinguished:

Transition: at sufficiently high *Re*, an initially laminar boundary layer develops fluctuations referred to as turbulence.



Separation: inversion of the velocity direction (either in laminar or turbulent regime).



These phenomena will be discussed in specific sections. Previously, the laminar and turbulent regimes are considered separately.

[→ quiz]

Boundary layer
- Laminar BL velocity profiles: Falkner-Skan

2D incompressible laminar BL on a flat plate, *with pressure gradient*:

Prandtl's equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_e\frac{dU_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

Boundary conditions:

$$y = 0 : u = 0 \text{ and } v = 0$$

$$\lim_{y\to\infty} u(x,y) = U_e(x)$$

$$u(0,y) = U_e(x=0)$$

$$\frac{dP}{dx} = -\rho U_e \frac{dU_e}{dx}$$

For $U_e(x) = c_0 x^m$: $\dfrac{dP}{dx} = -m\dfrac{\rho U_e^2}{x}$, self similar solution:

Stream function: $\psi(x,y) = \sqrt{U_e \nu x} \cdot f(\eta)$, with: $\eta = \dfrac{y}{\sqrt{\nu x / U_e}}$

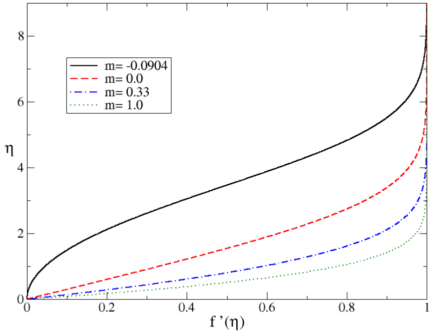
and: $u = \dfrac{\partial \psi}{\partial y} = U_e f'(\eta)$, $v = -\dfrac{\partial \psi}{\partial x}$

The problem yields:

$$f'''(\eta) + \frac{m+1}{2} f(\eta) f''(\eta) + m(1 - f'^2(\eta)) = 0$$

$$f(0) = 0,\ f'(0) = 0,\ \lim_{\eta \to +\infty} f'(\eta) = 1$$

$\rightarrow$ numerical solution $\rightarrow$ a velocity profile for each value of m.



- $m > 0$: favorable pressure gradient
- $m = 0$: no pressure gradient (Blasius equation)
- $m < 0$: adverse pressure gradient ($m = -0.0904$: separation)

$\rightarrow$ reference velocity profiles for laminar boundary layers with pressure gradient.

[$\rightarrow$ quiz]

friction: $\tau_w = \mu \dfrac{\partial u}{\partial y}(y=0) \propto \dfrac{df'}{d\eta}(\eta = 0)$

Falkner-Skan's integral parameters:

m	$f''(0) = 0.5C_f \cdot Re_x^{0.5}$	$\delta_1/x \cdot Re_x^{0.5}$	$\delta_2/x \cdot Re_x^{0.5}$	H_{12}
-0.0904	0	3.427	0.868	3.94
0	0.332	1.720	0.664	2.59
0.33	0.757	0.985	0.429	2.29
1	1.23	0.648	0.292	2.21

Boundary layer
- Turbulent BL velocity profiles: empirical

Reynolds decomposition: $\underline{u} = \overline{\underline{u}} + \underline{u}'$, where $\overline{\underline{u}}$: mean and $\underline{u}'$: fluctuating part.

Objective: determine the mean velocity profile. Equations?

Incompressible momentum equation:

$$\rho \frac{\partial \underline{u}}{\partial t} + \rho \underline{u} \cdot \nabla \underline{u} = -\nabla P + \mu \Delta \underline{u}$$

Averaged equation:

$$\rho \frac{\partial \overline{\underline{u}}}{\partial t} + \rho \overline{\underline{u}} \cdot \nabla \overline{\underline{u}} = -\nabla \overline{P} + \mu \Delta \overline{\underline{u}} - \rho \nabla . (\overline{\underline{u}' \otimes \underline{u}'})$$

$-\rho \overline{\underline{u}' \otimes \underline{u}'}$: Reynolds stresses $\rightarrow$ influence of turbulence on the mean flow.

Difficulty: determination of the mean flow requires a characterization of turbulence (e.g. direct numerical simulation).
$\Rightarrow$ an empirical description of the mean velocity profiles will be given here [2], on a flat plate in incompressible conditions.

2 mechanisms contribute to the formation of the turbulent boundary layer: viscosity and turbulence.

The boundary layer can be decomposed in 2 regions:

- internal region: influence of both viscosity and turbulence, universal law;
- external: influence of turbulence and pressure gradient.

Internal region

$$\text{mean wall friction: } \tau_w = \tau_{12} = \mu \left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x} \right) - \rho \overline{u' v'} = \mu \frac{\partial \overline{u}}{\partial y} \quad \text{(flat wall)}$$

$$u_* = \sqrt{\frac{\tau_w}{\rho}}$$

$$\text{definition of wall quantities: } y^+ = \frac{y \cdot u_*}{\nu} \quad ; \quad u^+ = \frac{u}{u_*}$$

Dimensional analysis: $\overline{u}\,(y, u_*, \rho, \nu)$.

Buckingham π theorem $\Rightarrow$ $\boxed{\overline{u}^+ = f(y^+)}$

- Very close to the wall: influence of turbulence is negligible.

$$\tau_{12} \approx \mu \frac{\partial \overline{u}}{\partial y} \approx \tau_w \Rightarrow \boxed{\overline{u}^+ = y^+}$$

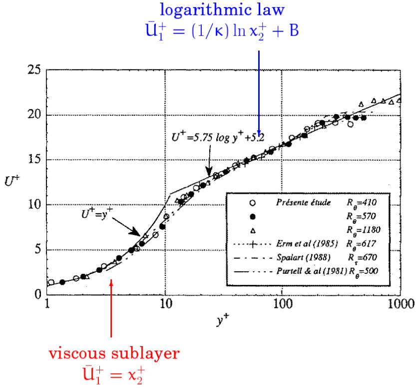
$\rightarrow$ viscous sublayer.

- When $y^+ \rightarrow +\infty$: influence of viscosity becomes negligible

$$\frac{\partial \overline{u}}{\partial y}(y, u_*, \rho) \Rightarrow \frac{\partial \overline{u}}{\partial y}\frac{y}{u^*} = cte = \frac{1}{\kappa}$$

$$\Rightarrow \boxed{\overline{u}^+ = \frac{1}{\kappa}\ln y^+ + B}$$

with $\kappa \approx 0.41$ and $B \approx 5$.



source: F. Laadhari (LMFA)

$\rightarrow$ universal law!

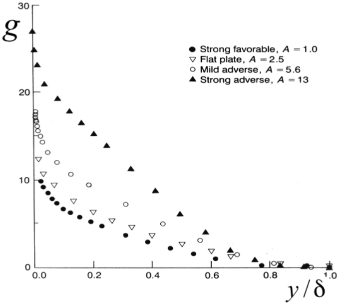
External region: controlled by turbulence and pressure gradient.

Velocity deficit: $U_e - \overline{u} = u_* \cdot g\left(\frac{y}{\delta}\right)$

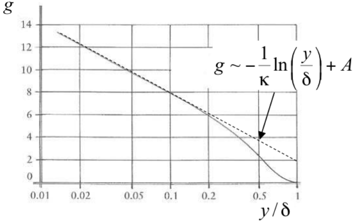
where g depends on the pressure gradient.

Coles' law: $g = A\cos^2\left(\frac{\pi y}{2\delta}\right) - \frac{1}{\kappa}\ln\left(\frac{y}{\delta}\right)$

$A \approx 2.5$: flat plate, no pressure gradient
$A < 2.5$: favorable (negative) pressure gradient
$A > 2.5$: adverse (positive) pressure gradient



flat plate case ($A = 2.5$):



Connection of the internal and external laws:

$$\frac{\overline{u}}{u_*} = \frac{1}{\kappa} \ln \left(\frac{y \cdot u_*}{\nu} \right) + B \quad \longleftrightarrow \quad \frac{U_e - \overline{u}}{u_*} = -\frac{1}{\kappa} \ln \left(\frac{y}{\delta} \right) + A$$

$$\text{which yields: } \frac{U_e}{u_*} = \frac{1}{\kappa} \ln Re_* + A + B, \text{ where: } Re_* = \frac{u_* \delta}{\nu}$$

Note: A depends on the pressure gradient, but B is a universal constant.

Interpretation of Re_*:

internal region thickness $\sim \nu / u_*$
external region thickness $\sim \delta$
$\Rightarrow Re_* \sim$ ratio of thicknesses of the external and internal regions.

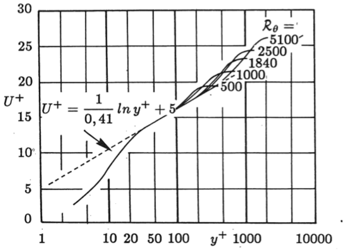


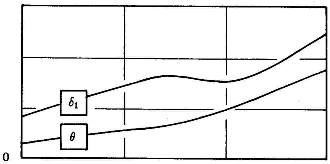
Figure: *Experimental velocity profiles, from Purtell et al.*

Boundary layer
- Integral parameters

The integral parameters characterize the boundary layer at a given abscissa (e.g. thickness, losses, shape). They provide a more synthetic information than the velocity profile.

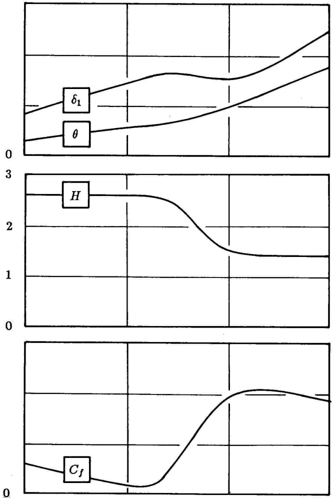
Boundary layer thickness: δ with $u(x, \delta) = 0.99 \cdot U_e$

However, the extraction of U_e can be difficult in practical cases where velocity varies outside of the boundary layer. An alternative criterion can be designed on vorticity: $\omega(x, \delta) = coef \cdot \omega(x, 0)$, with e.g. $coef = 0.01$.



Displacement thickness: characterize the mass flow deficit in the BL [→ quiz]

$$\delta_1 = \delta^* = \int_0^{\delta} 1 - \frac{\rho u}{\rho_e U_e} dy$$



Momentum thickness: characterize the momentum reduction in the BL

$$\delta_2 = \theta = \int_0^{\delta} \frac{\rho u}{\rho_e U_e} \left(1 - \frac{u}{U_e}\right) dy$$

Shape factor: characterize the shape of the velocity profile

$$H_{12} = \delta_1 / \delta_2$$

Friction coefficient:

$$C_f = \frac{\tau_w}{1/2 \rho U_e^2}$$

where τ_w is the wall friction.

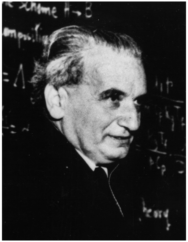
Can we derive equations on the integral parameters?

Integration of the momentum equation $\Rightarrow$
von Kármán's equation

For a body of revolution, in compressible conditions:

$$\frac{d\delta_2}{dx} = \frac{C_f}{2} - \delta_2 \left(\frac{H_{12}+2}{U_e} \frac{dU_e}{dx} + \frac{1}{\rho_e} \frac{d\rho_e}{dx} + \frac{1}{R} \frac{dR}{dx} \right)$$

where R is the curvature radius.



Theodore von Kármán (1881-1963) - source: NACA.

In 2D incompressible conditions: [→ quiz]

$$\frac{d\delta_2}{dx} = \frac{C_f}{2} - \delta_2 \left(\frac{H_{12}+2}{U_e} \frac{dU_e}{dx} \right)$$

δ_2 evolves under the influence of the friction and the pressure gradient. For example: $C_f > 0$ or $dP/dx > 0 \Rightarrow \delta_2 \nearrow$.

Proof of von Kármán's equation in 2D incompressible conditions:

Starting from Prandtl's equations p.8:

$$\text{momentum}_x + \text{incompressibility} \times (u - U_e)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right](u - U_e) = U_e\frac{dU_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u(u - U_e)}{\partial x} + \frac{\partial v(u - U_e)}{\partial y} + u\frac{dU_e}{dx} = U_e\frac{dU_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{d}{dx}\int_0^\delta u(u - U_e)dy + [v(u - U_e)]_0^\delta + \frac{dU_e}{dx}\int_0^\delta u - U_e dy = \left[\nu\frac{\partial u}{\partial y}\right]_0^\delta$$

$$\frac{-d}{dx}(U_e^2\delta_2) - \frac{dU_e}{dx}U_e\delta_1 = \frac{-\tau_w}{\rho} \Rightarrow 2U_e\frac{dU_e}{dx}\delta_2 + U_e^2\frac{d\delta_2}{dx} + \frac{dU_e}{dx}U_e\delta_1 = \frac{\tau_w}{\rho}$$

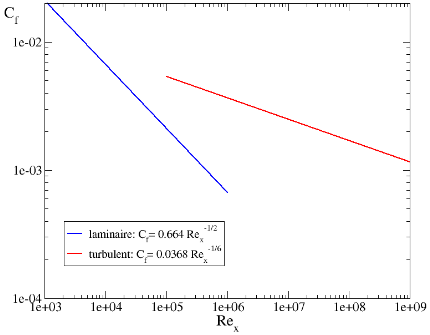
$$\Rightarrow \frac{d\delta_2}{dx} = \frac{C_f}{2} - \delta_2\left(\frac{H_{12} + 2}{U_e}\frac{dU_e}{dx}\right)$$

Empirical relations

Evolution of the integral parameters on a *flat plate in incompressible conditions*:

- Laminar (solution of Blasius equation):
  $\delta_1/x = 1.720 \cdot Re_x^{-1/2}$
  $\delta_2/x = 0.664 \cdot Re_x^{-1/2}$
  $H_{12} = 2.59$
  $C_f = 0.664 \cdot Re_x^{-1/2}$

- Turbulent (semi-empirical results, from R. Michel, $10^5 \leq Re \leq 10^9$):
  $\delta_1/x = 0.0309 \cdot Re_x^{-1/6}$
  $\delta_2/x = 0.0221 \cdot Re_x^{-1/6}$
  $H_{12} = 1.4$
  $C_f = 0.0368 \cdot Re_x^{-1/6}$

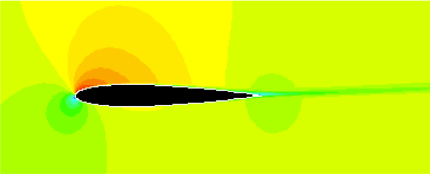


Boundary layer
- Numerical methods

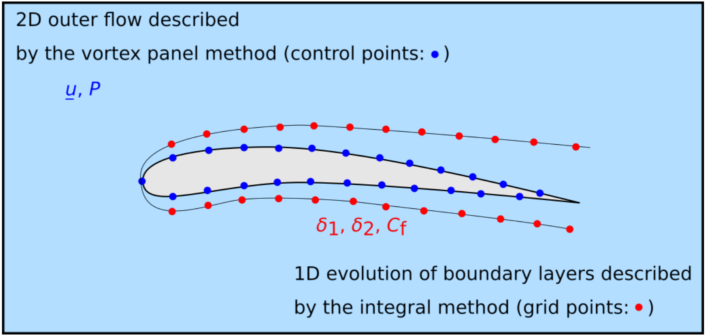
Numerical methods can be used to simulate flows, including the evolution of boundary layers, with few hypotheses (non planar, compressible...).

Two categories are identified:

- zonal methods: low computational cost, historic;
- CFD (computational fluid dynamics): commonly used in industry for design.

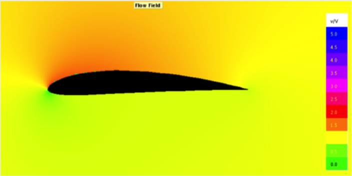


a) Zonal methods use different methods for the outer flow and the boundary layer.



(1) Outer flow

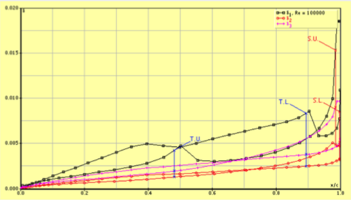
Non viscous: potential flow, panel methods (incompressible or weakly compressible)...



source: Javafoil

(2) Boundary layer

Integral methods. Unknowns: $\delta_2(x)$, $H_{12}(x)$, $C_f(x)$...



source: Javafoil

- The influence (1) $\rightarrow$ (2) corresponds to the terms with U_e in von Kàrmàn's equation p.18.
- The reverse coupling (2) $\rightarrow$ (1) can be neglected for attached flows at high Re, since $\delta/c \ll 1$. In this condition, the slip condition for (1) applies at the wall.

Set of equations for the integral method

The principal equation is von Kàrmàn's p.18. For example, for a body of revolution, in compressible condition:

$$\frac{d\delta_2}{dx} = \frac{C_f}{2} - \delta_2 \left(\frac{H_{12} + 2}{U_e} \frac{dU_e}{dx} + \frac{1}{\rho_e} \frac{d\rho_e}{dx} + \frac{1}{R} \frac{dR}{dx} \right)$$

where R is the curvature radius (geometry). U_e and ρ_e are provided by the solution of the outer flow (1).
The unknowns are: δ_2, H_{12} and C_f.

When more than 1 equation is required, empirical correlations can be added. *Exemple*: Eppler (implemented in Javafoil), uses in turbulent regime:

$$C_f = 0.045716 \left[(H_{12} - 1) \frac{U_e \delta_2}{\nu} \right]^{-0.232} \times e^{-1.260 H_{12}}$$

Note: instead of the integral paramaters ($\delta_2(x)$, $H_{12}(x)$, $C_f(x)$...), the unknowns of the integral method can be parameters of the velocity profile.

Exemple of parameterized velocity profiles: Kàrmàn and Polhausen

$$u(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3$$

Boundary conditions:

$$u(0) = 0 \quad ; \quad \mu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \frac{\partial p}{\partial x} \quad ; \quad u(\delta) = U_e \quad ; \quad \left.\frac{\partial u}{\partial y}\right|_{y=\delta} = 0$$

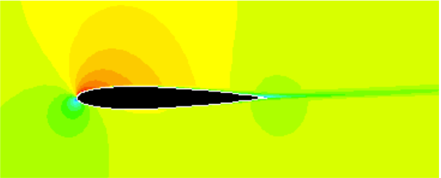
$$\text{For } \frac{\partial p}{\partial x} = 0 \rightarrow \frac{u(y)}{U_e} = 1.5\frac{y}{\delta} - 0.5\frac{y^3}{\delta^3} \quad \text{and: } \delta_2 = \frac{39}{280}\delta,\ C_f = \frac{3\nu}{\delta U_e}$$

1 unknown (δ) $\Rightarrow$ only 1 equation required: von Kàrmàn's

$$\frac{39}{280}\frac{d\delta}{dx} = \frac{3\nu}{2\delta U_e} \Rightarrow \frac{d\delta^2}{dx} = \frac{280}{13}\frac{\nu}{U_e} \Rightarrow \frac{\delta}{x} = 4.64 Re_x^{-1/2} \text{ and: } C_f = 0.646 Re_x^{-1/2}$$

Note: very close to the solution of Blasius equation (laminar), but not exactly.

a) Computational fluid dynamics (CFD) uses a unified viscous method for the whole flow domain.



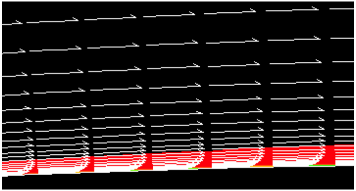


Figure: *velocity vectors above a wall, from a RANS simulation. Red: high vorticity region.*

5 scalar equations in compressible conditions: continuity, momentum (x3) and energy.

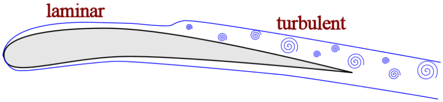
Turbulence?

- Reynolds averaged Navier-Stokes (RANS): additional equations (generally)
- Large-eddy simulation (LES)
- Direct numerical simulation (DNS)

Contents

Transition

Transition: at sufficiently high Re, an initially laminar boundary layer develops fluctuations referred to as turbulence.



In aeronautical conditions, transition occurs at:

$$Re_x = \frac{\rho U_e x}{\mu} \approx 3 \cdot 10^5$$

Appendix on the evolution of Re with altitude

This Re_x value for transition is an order of magnitude. The exact position of transition is influenced by various parameters:

[→ quiz]

- mean pressure gradient
- external fluctuations (bypass)
- roughness
- wall curvature
- 3D flow

They are investigated hereafter.

Transition

- Influence of mean pressure gradient

The momentum equation for mean velocity in incompressible condition shows the influence of the mean pressure gradient ($\nabla\overline{P}$) on mean velocity ($\underline{\overline{u}}$):

$$\frac{\partial\underline{\overline{u}}}{\partial t} + \underline{\overline{u}} \cdot \nabla\underline{\overline{u}} = -\frac{1}{\rho}\nabla\overline{P} + \mu\Delta\underline{\overline{u}} - \nabla.(\overline{\underline{u'} \otimes \underline{u'}})$$

Then, the equation for the turbulent kinetic energy ($k = 1/2\sum_i u_i'^2$) shows the influence of mean velocity ($\overline{u_i}$) on turbulence (k):

$$\frac{\partial k}{\partial t} + \overline{u_m}\frac{\partial k}{\partial x_m} = -\overline{u_i'u_m'}\frac{\partial\overline{u_i}}{\partial x_m} - \frac{1}{2}\frac{\partial\overline{u_i'u_i'u_m'}}{\partial x_m} - \frac{\partial}{\partial x_i}\left(\frac{\overline{p'u_i'}}{\rho}\right) - \nu\overline{\frac{\partial u_i'}{\partial x_m}\frac{\partial u_i'}{\partial x_m}} + \nu\frac{\partial^2 k}{\partial x_m^2}$$

- Adverse pressure gradient ($\nabla\overline{P} > 0$) $\Rightarrow$ momentum in the BL $\searrow$ and wall friction $\searrow$ $\Rightarrow$ promotes transition (stability reasons [1]).
- Conversely, favorable pressure gradient ($\nabla\overline{P} < 0$) can lead to relaminarization.

Experimental evidences of the influence of $\nabla\overline{P} > 0$ on transition:

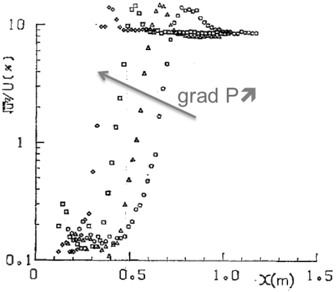


Figure: *Evolution of velocity fluctuations in a boundary layer with adjustable mean pressure gradient (Igarashi et al., Acta Mechanica 73, 1988).*

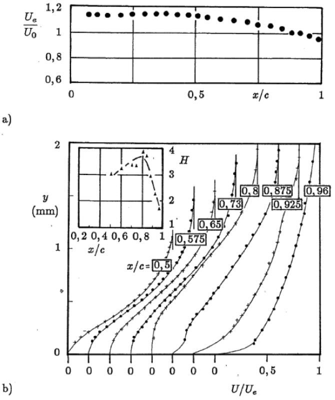


Fig 5 (1). Transition avec gradient de pression sur profil ONERA D (Cousteix, Pailhas 1979)
$\alpha = 0 \quad U_0 = 24\,ms^{-1} \quad C = 200\,mm$
a) distribution de la vitesse extérieure déduite de la pression statique à la paroi
b) profils de vitesses dans la couche limite.

Figure: *a) evolution of U_e on an ONERA D profile, and b) mean velocity profiles.*

Transition
- Influence of external fluctuations

→ promote transition (referred to as *bypass transition*).

The fluctuations can correspond to inflow turbulence.

Dependency on the frequency spectrum of the fluctuations (see figure).

[→ quiz]

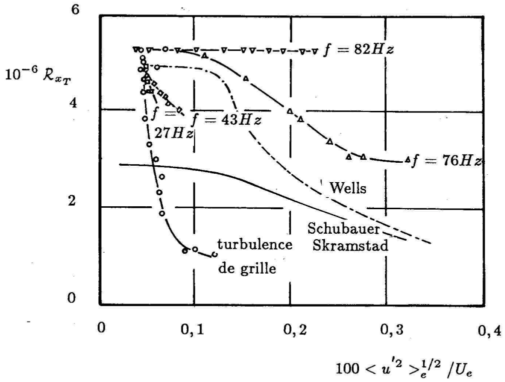


Figure: *Influence of the intensity and frequencies of the fluctuations on the transitional Reynolds number (from Spangler and Wells [1]).*

Transition

- Influence of roughness

Isolated roughness: possibly associated with rivet heads, plate junctions...

Perturbation of the mean velocity profile ⇒ transition.

Practical use (e.g. wind tunnel models): carborundum tape to trip transition.

[→ quiz]

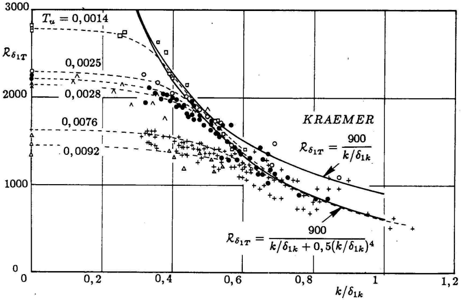
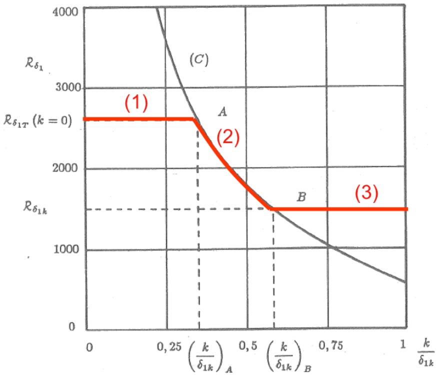


Fig 20. Déclenchement de la transition par une rugosité bidimensionnelle. Résultats regroupés par Dryden (1959)
○ ● plaque plane, fil (Tani, Hama, Mituisi)
□ △ plaque plane, fil (Tani, Inchi, Yamamoto)
+ cylindre circulaire, fil (Feindt)
∧ plaque plane, ruban de section rectangulaire (Quick).

Figure: *Influence of an isolated roughness on the transitional Reynolds number [1] (k: roughness height, δ_{1k}: displacement thickness in the case without roughness).*

Model:
(1) no influence of roughness;
(2) transition downstream of roughness, moving upstream when $k/\delta_{1k} \nearrow$;
(3) transition at the roughness position, with thickening:
$\Delta\delta_2 = 0.5 C_D k \left(U_k/U_e\right)^2$
(Arnal, 1984: $C_D \approx 0.5$)



Uniformly distributed roughness: no effect on transition if:

- $U_e k/\nu < 120$ (Feindt criterion)
- or: $U_k k/\nu < 25$ (according to Cousteix; U_k is the flow velocity in the unperturbed boundary layer at $y = k$).

Transition

- Influence of wall curvature

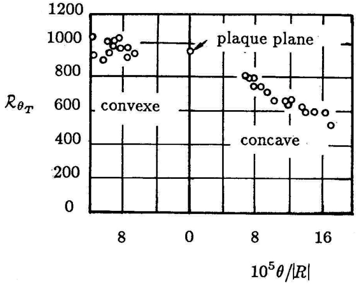


Figure: *Influence of wall curvature (left abscissa: convex, right abscissa: concave) on transitional Reynolds number (from Liepmann).*

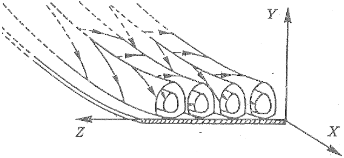


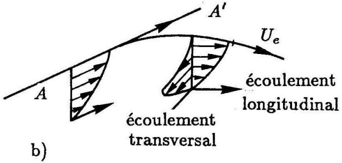
Figure: *Görtler vortices*

- No noticeable effect of convex walls.
- Transition favored by concave walls, through the generation of Görtler vortices aligned with the streamwise direction and counter-rotating.

Transition
- Influence of 3D flow

Swept wing, variation of chord length along span... $\Rightarrow$ 3D boundary layers, transition is favored.

Stronger curvature of the streamlines in the boundary layer, to compensate the lower velocity in the equilibrium between the centrifugal force ($\sim \rho||\underline{u}||^2/R$, where R is the curvature radius) and the cross-stream pressure gradient.



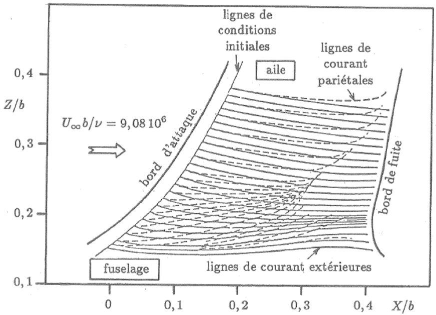


Figure: *Flow simulation on the suction side of a wing (solid curves: outer streamlines, dashed curves: friction lines at the wall), from Lindhout et al..*

Contents

Separation

Separation: inversion of the velocity direction (either in laminar or turbulent regime).

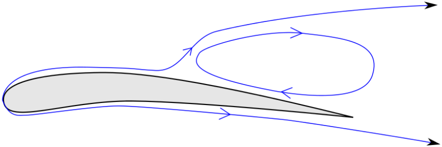


Illustration on a wing:

credits: pilottraining.ca

If $\nabla \overline{P} > 0$ is sufficiently large, the mean velocity can be reversed: flow separation.

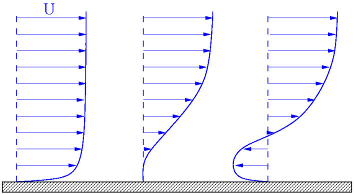


Figure: *Mean velocity profiles. Left: attached, middle: intermediate, right: separation.*

Separation criterion in 2D steady conditions:

$$C_f \approx \frac{\mu \frac{\partial \overline{u}}{\partial y}}{1/2 \rho U_e^2} < 0 \quad \text{(flat plate approximation)}$$

Separation
- Characterization on a NACA airfoil

Evolution of the flow around a NACA0012 airfoil with angle of attack:

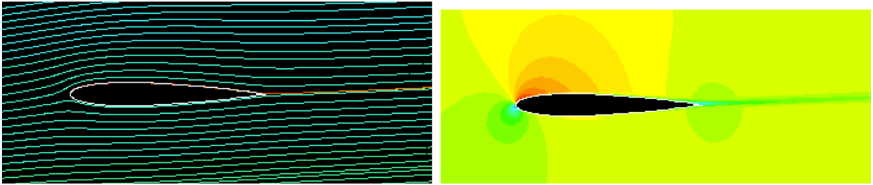


Figure: $\alpha = 4^\circ$: *boundary layers are attached.*

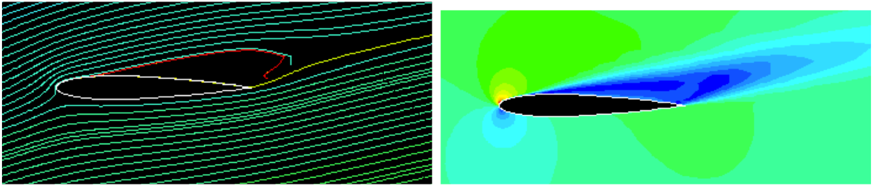
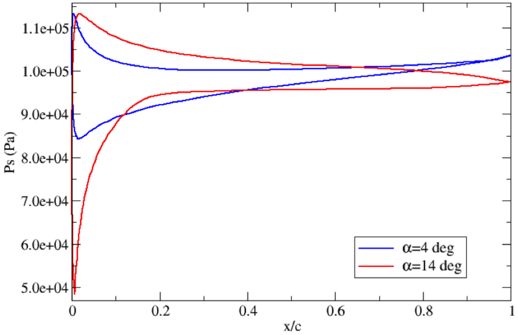


Figure: $\alpha = 14^\circ$: *suction side boundary layer is separated.*

Pressure distribution around the NACA0012 airfoil:



- Upper curves: pressure side, lower curves: suction side.
- $\nabla \overline{P} > 0$ on suction side, and increases significantly at $\alpha = 14°$ $\Rightarrow$ separation.
- Pressure plateau ($\nabla \overline{P} \approx 0$) inside the separation.

Separation
- Sensitivity to the regime

Due to the mixing properties of turbulence, a turbulent boundary layer is more resistant to separation than a laminar boundary layer.

In the figure ($\rightarrow$), the laminar velocity profile approaches separation, but the boundary layer recovers downstream of the transition to turbulence.

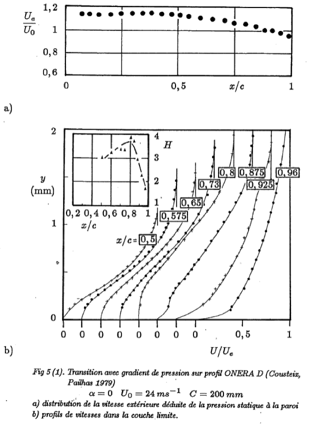
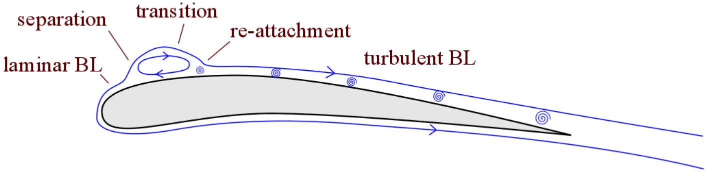
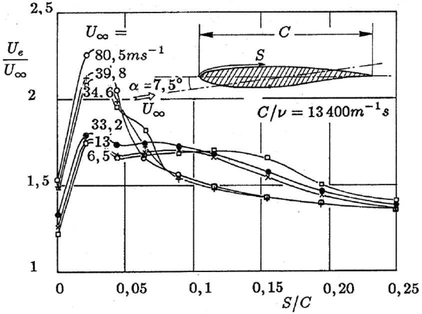


Figure: *a) evolution of U_e on an ONERA D profile, and b) mean velocity profiles.*

A singular phenomenon: separation bubble.



Mechanism: laminar boundary layer → separation → transition → re-attachment → turbulent boundary layer.



Plot: influence of a separation bubble on wall pressure [1].

Rem: in incompressible conditions

$$\frac{U_e}{U_\infty} = \sqrt{1 - \frac{P - P_\infty}{0.5\rho U_\infty^2}}$$

Separation
- Criteria

In **laminar regime**, the Falkner-Skan solutions yield $C_f = 0$ for $m = -0.0904$. In this condition: $\delta_2/x \cdot Re_x^{0.5} = 0.868$.

Introducing Pohlhausen parameter Λ_2:

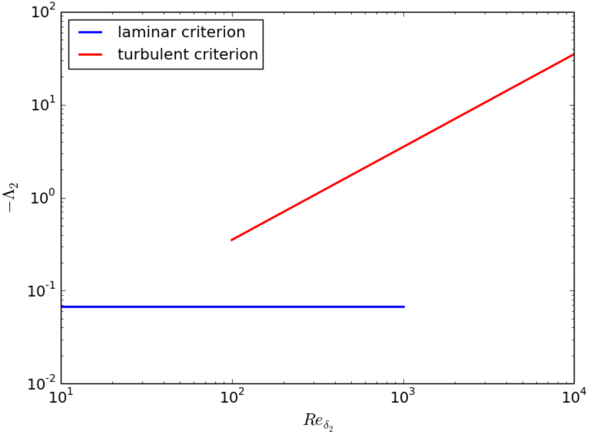
$$\Lambda_2 = \frac{\delta_2^2}{\nu}\frac{dU_e}{dx} \quad \text{with: } \delta_2 = 0.868 \cdot x \cdot Re_x^{-0.5} \quad \text{and: } \frac{dU_e}{dx} = c_0 m x^{m-1} = mU_e/x$$

$$\Lambda_2 = \frac{\delta_2^2}{\nu}\frac{dU_e}{dx} = -0.068$$

In **turbulent regime**, asymptotic analysis of equilibrium boundary layers [1] yields separation for:

$$\frac{\delta}{U_e}\frac{dU_e}{dx} = -0.0233 \Rightarrow \frac{\delta_2}{U_e}\frac{dU_e}{dx} = -0.0035 \Rightarrow \Lambda_2 = -0.0035 \cdot Re_{\delta_2}$$

Separation criteria:



Separation if $-\Lambda_2$ above these lines.

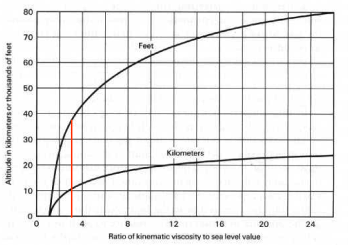
The turbulent boundary layer is more resistant to separation.

Contents

Appendix: evolution of Reynolds number with altitude?

Cruise altitude of commercial flights: $\sim 33\,000\,\text{ft}$, $\sim 10\,000\,\text{m}$.

International Standard Atmosphere (ISA):
$T = -50\,^\circ\text{C}$
$P = 26\,436\,\text{Pa}$
$\rho = 0.413\,\text{kg/m}^3 \sim \rho_0/3$



Sutherland's law:

$$\mu = \mu_{ref} \left(\frac{T}{T_{ref}} \right)^{3/2} \frac{T_{ref} + T_{Suth}}{T + T_{Suth}}$$

where: $\mu_{ref} = 1.716 \cdot 10^{-5}\,\text{kg/m-s}$,
$T_{ref} = 273.15\,\text{K}$ and $T_{Suth} = 110.6\,\text{K}$.
$\rightarrow \mu \approx 0.8 \cdot \mu_0$

Consequently, for given reference speed and length, $Re \approx Re_0/3$
$\Rightarrow$ delayed transition.

Glossary

English	**French**
boundary layer	couche limite
separation	décollement
transition	transition

Nomenclature

$\overline{\underline{u}}$	mean velocity vector ($\underline{u} = \overline{\underline{u}} + \underline{u}'$)
μ	dynamic viscosity $\mu = \rho \cdot \nu$
ν	kinematic viscosity $\nu = \mu / \rho$
$\underline{u}$	velocity vector
$\underline{u}'$	fluctuating velocity vector ($\underline{u} = \overline{\underline{u}} + \underline{u}'$)
u, v, w	Cartesian velocity components (eq. u_1, u_2, u_3)
u_1, u_2, u_3	Cartesian velocity components (idem u, v, w)
U_e	velocity outside of the boundary layer
BL	boundary layer
CFD	computational fluid dynamics

Bibliography

[1] J. Cousteix, *Turbulence et couche limite*. Cepadues editions, 1989.

[2] J. Scott, M. Gorokhovski, C. Corre, and C. Bailly, *Fluides et énergie*. Ecole Centrale de Lyon, 2017.