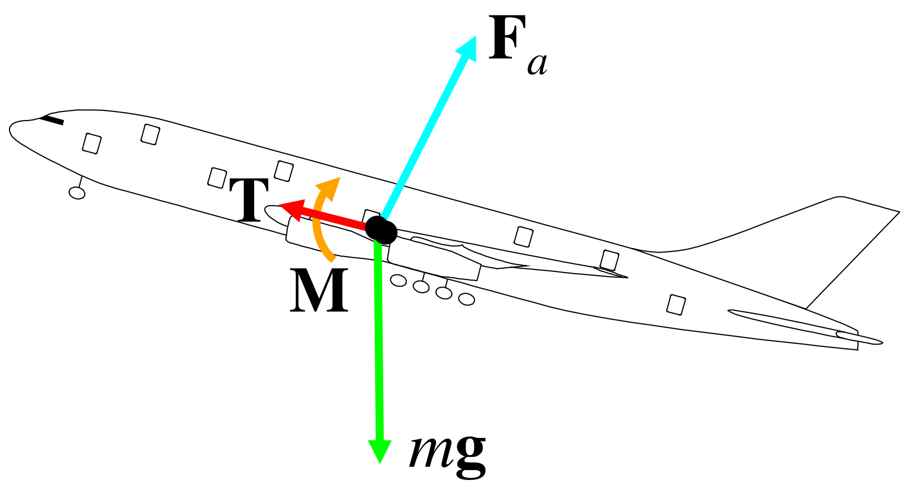
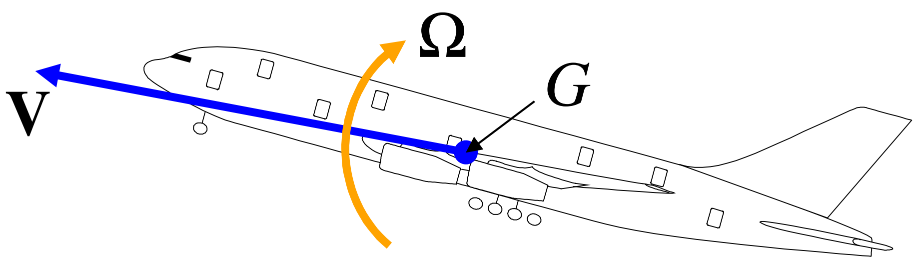
1. Flight Mechanics



Contents

1.1 Equations of motion

Aircraft considered as a rigid body:



V = air-relative velocity of centre of gravity G

Ω = aircraft angular velocity

$$m\dot{\mathbf{V}} = \sum \textbf{Forces} = \mathbf{F}$$

$$\dot{\mathbf{H}} = \sum \textbf{Moments} = \mathbf{M}$$

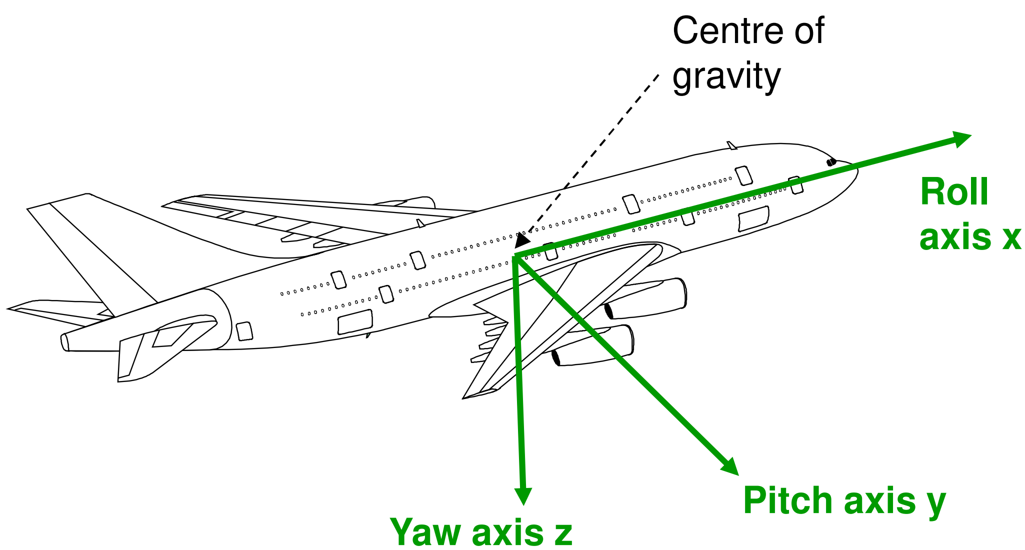
$$\mathbf{H} = \textbf{Angular momentum} = \mathfrak{I}_G . \mathbf{\Omega}$$

$\mathfrak{I}_G$ = moment of inertia tensor

Note: bold symbols = vectors.

Aircraft reference frame

Aerodynamic and thrust forces (as well as $\mathcal{I}_G$) are naturally expressed in aircraft frame of reference (x, y, z):



- aircraft symmetry plane: (x, z)
- aircraft axis: x
- roll, pitch and yaw moments: components of $\mathbf{M}$

Aircraft reference frame (contd)

• *a priori*, the time derivatives $\dot{\mathbf{V}}$ and $\dot{\mathbf{H}}$ in the equations of motion refer to a non-rotating frame (not the aircraft frame).

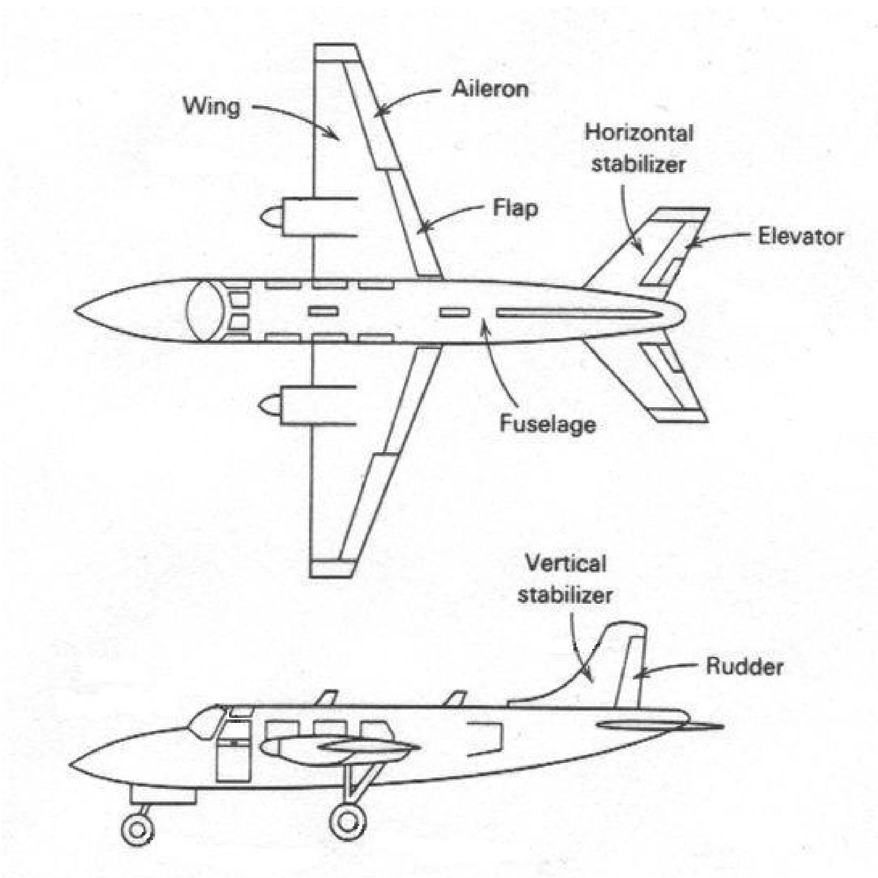
Note: If desired, one can use:

$$\dot{\mathbf{V}} = \left(\frac{d\mathbf{V}}{dt}\right)_{aircraft} + \mathbf{\Omega} \times \mathbf{V}$$

$$\dot{\mathbf{H}} = \left(\frac{d\mathbf{H}}{dt}\right)_{aircraft} + \mathbf{\Omega} \times \mathbf{H}$$

to express the equations in the aircraft frame. However, this powerful general formulation is not needed for the relatively simple flight regime (longitudinal flight) we will study in detail. It is mentioned here for completeness.

1.2 Aerodynamic elements and controls



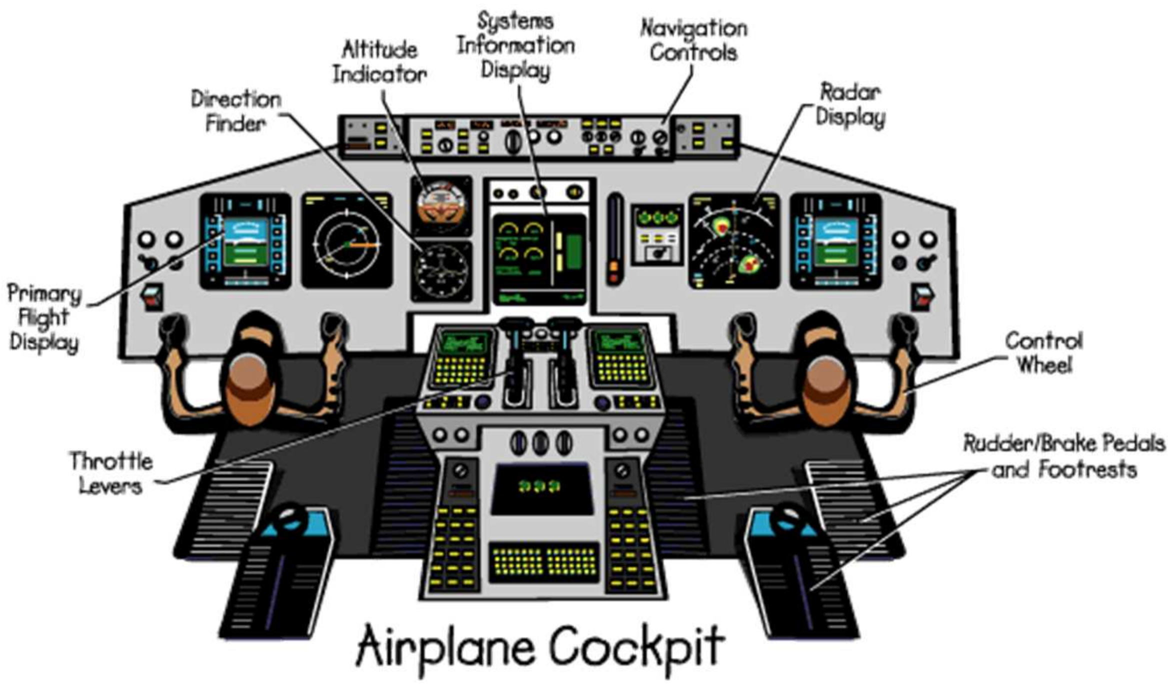
- ☞ Elevators
- ☞ Ailerons and rudder
- ☞ Flaps

control surfaces (moveable)

- ☞ Wings
- ☞ Stabilizers
- ☞ Fuselage

fixed surfaces

Main pilot controls



☞ Stick (aka control wheel; in front of pilot) ⇒ elevators and ailerons

☞ Pedals (on floor) ⇒ rudder

☞ Throttle levers (centre) ⇒ engine thrust

- the pilot controls the engine thrust and settings of the control surfaces...
- ...via the flight system ⇒ more precisely, pilot + flight system control the aircraft.

Functions of main aerodynamic elements

Principal function of fixed elements:

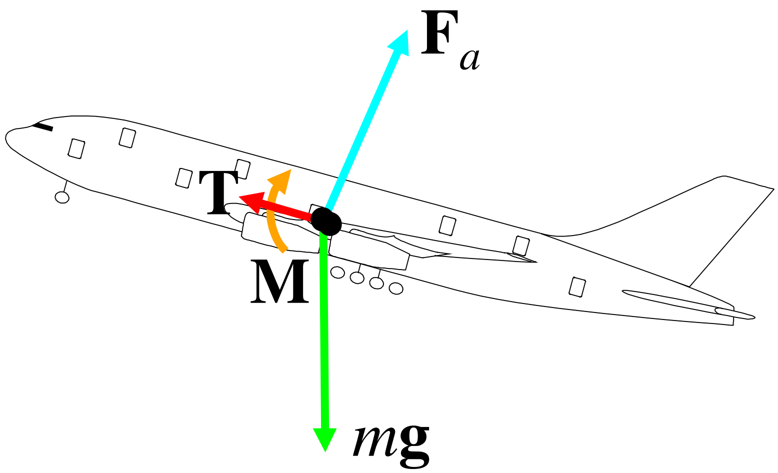
☞ Wings: provide lift
☞ Stabilizers: flight stability
☞ Fuselage: carry load

Principal function of control surfaces:

$$\left.\begin{array}{l}\text{☞ Elevators: pitching moment}\\ \text{☞ Ailerons: rolling moment}\\ \text{☞ Rudder: yawing moment}\end{array}\right\}\text{main controls}$$

☞ Flaps: increase lift at low speeds (takeoff and landing) to avoid stalling

1.3 Forces and moments



$m\mathbf{g}$ = weight

$\mathbf{T}$ = engine thrust

$\mathbf{F}_a$ = aerodynamic forces

$\mathbf{M}$ = moment about G

$= \mathbf{M}_a + \mathbf{M}_T$

Usually $m\mathbf{g}$ and $\mathbf{F}_a$ are the dominant forces and roughly in equilibrium.

Weight

- weight distribution $\Rightarrow \mathbf{x}_G$
- m and $\mathbf{x}_G$ vary *slowly* during flight (fuel consumption, etc)
- can be taken constant for flight dynamics
- $\mathbf{x}_G$ must respect safety limits (usually G located near wings).

Thrust

- e.g. propeller, pure jet, turbofan
- fixed line of application in aircraft frame $\Rightarrow \mathbf{T}$ and $\mathbf{M}_T$ determined by $T = |\mathbf{T}|$
- pilot controls T via throttle
- for a given throttle setting, T depends on airspeed $V = |\mathbf{V}|$ and atmospheric properties (density and temperature $\Rightarrow$ altitude).

Aerodynamic forces

Lift/drag decomposition:

$$\mathbf{F}_a = \underbrace{\mathbf{L}}_{Lift\,(\perp \mathbf{V})} \underbrace{-D\mathbf{e}_V}_{Drag}$$

- $\mathbf{e}_V = \mathbf{V} / |\mathbf{V}|$ = unit vector in flight direction
- usually lift >> drag

Energy equation:

$$\frac{d}{dt}\left(\frac{1}{2} m V^2\right) = m\mathbf{V.g} + \mathbf{V.T} - VD$$

$m\mathbf{V.g}$ = power required for climb (gravitational potential energy)

$\mathbf{V.T}$ = power delivered by engine thrust

VD = loss to drag

- lift does not contribute
- small drag generally better.

Aerodynamic coefficients

Nondimensional coefficients:

$$\mathbf{F}_a = \frac{1}{2}\rho S V^2 \mathbf{C}_F \qquad \mathbf{M}_a = \frac{1}{2}\rho S \ell V^2 \mathbf{C}_M$$

S = wing planform area
ℓ = length scale (in France, usually the wing mean chord)

- Reynolds number very large $\Rightarrow$ pressure variations of order $\rho V^2 \Rightarrow$ $\rho S V^2$ appropriate for aerodynamic forces
- the factors of ½ are conventional
- $\mathbf{C}_F = O(1)$ can be decomposed into lift and drag: $\mathbf{C}_L$ and C_D.

Aerodynamic coefficients (contd)

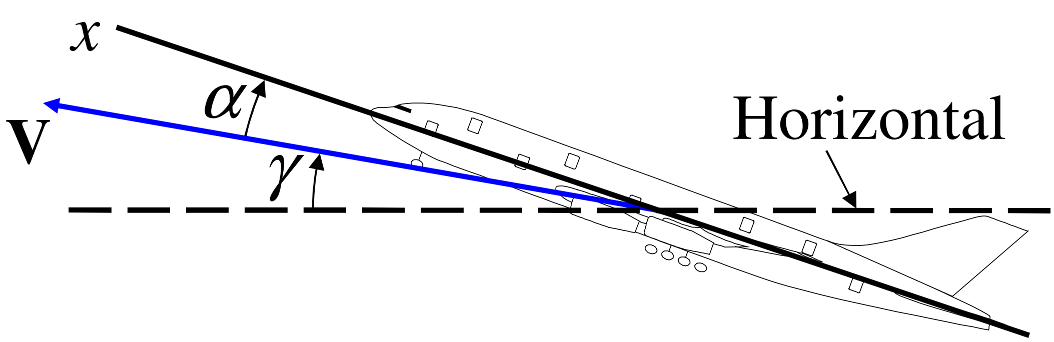
When expressed in the aircraft frame, $\mathbf{C}_F$ and $\mathbf{C}_M$ are functions of:

☞ settings of control surfaces
☞ flight-direction vector $\mathbf{e}_V$
☞ dimensionless angular velocity $\mathbf{\Omega}\ell/V$
☞ Mach number, $M = V/a$ (a = speed of sound; not to be confused with moment)
☞ Reynolds number, $Re = V\ell/\nu$

- *Re* dependence usually weak over speed range of aircraft and often neglected
- independent of *M* at low Mach ($M < 0.5$ say); *M* important at higher speeds
- care needed when applying small-scale experimental results to full scale.

1.4 Longitudinal flight

Longitudinal = aircraft symmetry plane fixed and vertical (allows climb and descent, but not turn or roll). Usually majority of flight time.



α = angle of attack
γ = angle of climb
$\theta = \alpha + \gamma$ = pitch angle

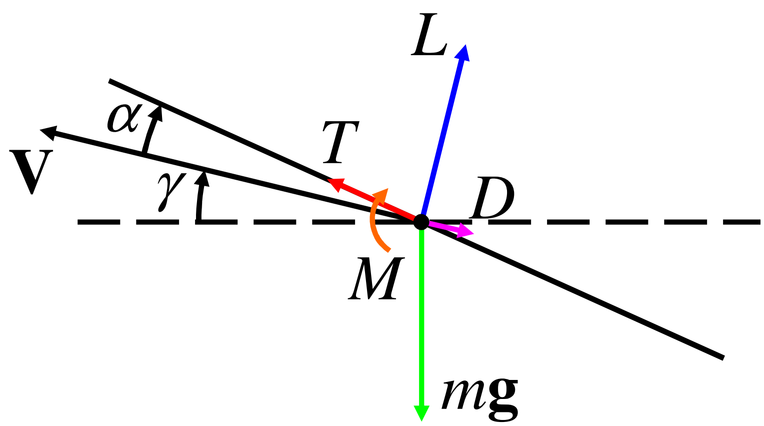
} important distinction

$$\mathbf{T} = (T, 0, 0) \qquad \mathbf{M} = (0, M, 0)$$
$$\mathbf{\Omega} = (0, \dot{\theta}, 0) \qquad \mathbf{H} = (0, I\dot{\theta}, 0)$$

} aircraft frame

I = moment of inertia about pitch axis

Equations of motion



$$\boxed{\begin{aligned} m\dot{V} &= T\cos\alpha - D - mg\sin\gamma \\ mV\dot{\gamma} &= T\sin\alpha + L - mg\cos\gamma \\ I\left(\ddot{\alpha}+\ddot{\gamma}\right) &= M = M_a + M_T \end{aligned}}$$

Three equations for V, α, γ: quantities *not* controlled directly by the pilot.

Longitudinal flight controls:

- ☞ throttle $\Rightarrow T$ (and hence M_T)
- ☞ stick $\Rightarrow$ elevator angle δ_e (downward)

Aerodynamic forces revisited

Closure requires aerodynamic forces:

$$L = \frac{1}{2}\rho S V^2 C_L \qquad D = \frac{1}{2}\rho S V^2 C_D$$

$$M_a = \frac{1}{2}\rho S \ell V^2 C_M$$

Coefficients C_L, C_D and C_M are functions of:

- ☞ δ_e
- ☞ α
- ☞ $\dot{\theta}\ell / V$
- ☞ M
- ☞ Re

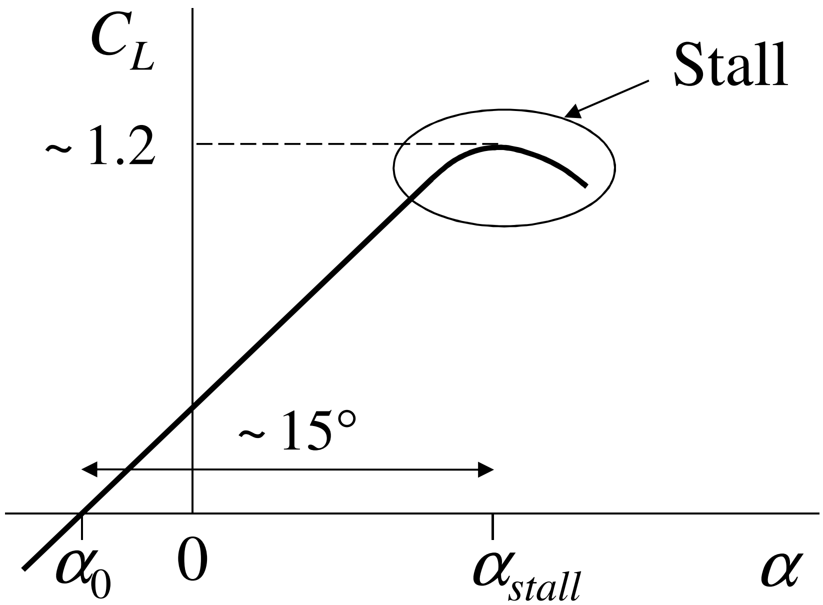
From here on, we neglect dependence on:

☞ Re : usual approximation

☞ M : valid for low enough Mach number

Aerodynamic coefficients

Typical lift coefficient versus angle of attack:



$$\delta_e = 0$$
$$\dot{\theta}\ell / V = 0$$

Stall: decrease in C_L, buffeting, pitch up/down, loss of control. Normal flight avoids stall.

Unstalled $\Rightarrow$ linear model for C_L and C_M:

$$C_L = C_{L\alpha}\left(\alpha - \alpha_0\right) + C_{L\delta}\delta_e + C_{L\dot{\theta}}\frac{\dot{\theta}\ell}{V}$$

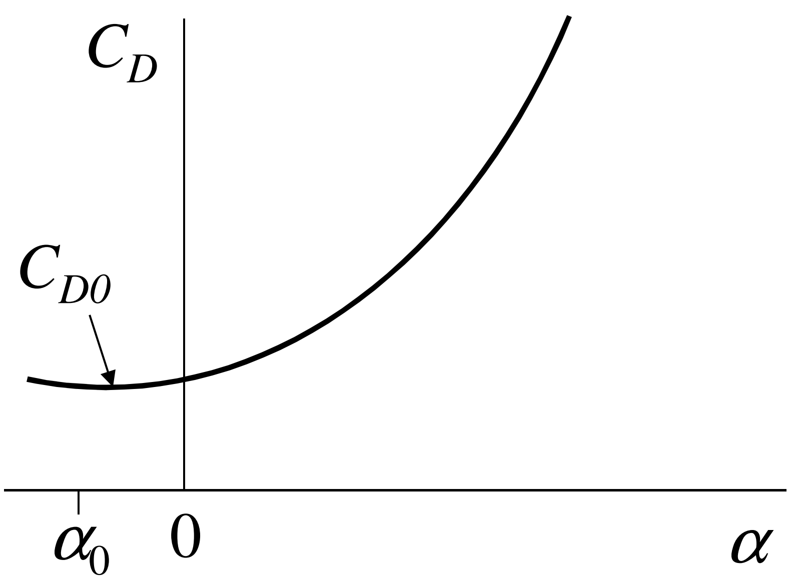
Aerodynamic coefficients (contd)

$$C_M = C_{M\alpha}\left(\alpha - \alpha_{M0}\right) + C_{M\delta}\delta_e + C_{M\dot{\theta}}\frac{\dot{\theta}\ell}{V}$$

Usual quadratic model for C_D:

$$C_D = C_{D0} + kC_L^2$$

- coefficients in models characterise aircraft aerodynamics
- more generally, coefficients depend on M, Re.



$$\delta_e = 0$$

$$\dot{\theta}\ell / V = 0$$

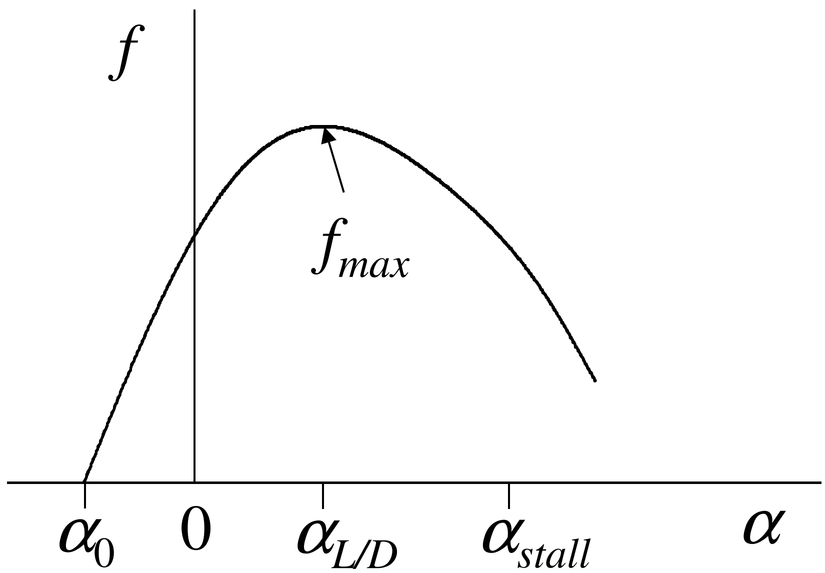
- C_D increases with α.

Aerodynamic coefficients (contd)

The lift/drag ratio:

$$\boxed{f = \frac{L}{D} = \frac{C_L}{C_D}}$$

is an important measure of aerodynamic efficiency.



$$\delta_e = 0$$

$$\dot{\theta}\ell / V = 0$$

- Maximum lift/drag $\Rightarrow$ optimal angle of attack according to this measure of performance. ($f_{max} \approx 12$)

1.5 Longitudinal equilibrium flight

V, α, γ constant $\Rightarrow$ aircraft flies in a straight line with constant speed and orientation (but may climb or descend). Commonest flight regime. Control settings fixed.

- left-hand sides of equations of motion zero (equilibrium)
- $\dot{\theta}\ell/V = 0 \Rightarrow \dot{\theta}\ell/V$ disappears as parameter in aerodynamic coefficients.

Simplifying assumptions:

☞ α and γ moderately small

$$\Rightarrow \cos\alpha \approx \cos\gamma \approx 1, \;\; \sin\gamma \approx \gamma$$

☞ terms M_T and $T\sin\alpha$ negligible.

Equilibrium equations

$$M_a = 0 \Rightarrow \qquad \boxed{C_M(\alpha;\delta_e) = 0} \qquad \text{(A)}$$

$$L = mg \Rightarrow \qquad \boxed{V^2 = \frac{2mg}{\rho S C_L(\alpha;\delta_e)}} \qquad \text{(B)}$$

$$T = D + mg\gamma \Rightarrow \qquad \boxed{\gamma = \frac{T}{mg} - \frac{1}{f(\alpha;\delta_e)}} \qquad \text{(C)}$$

where, as before, $f = C_L/C_D$ is lift/drag ratio.

(A) $\Rightarrow$ $\quad \alpha = \alpha_{eq}(\delta_e)$

(B) with (A) $\Rightarrow$ $\quad V = V_{eq}(\delta_e)$

Equilibrium airspeed and angle of attack controlled by stick (via elevator angle δ_e). No effect of throttle on equilibrium speed (or angle of attack), unlike car accelerator.

Control of airspeed and angle of attack

Using linear model of C_M and C_L (unstalled):

$$\alpha_{eq} = \alpha_{M0} - \frac{C_{M\delta}}{C_{M\alpha}}\delta_e$$

$$V_{eq}^2 = \frac{2mg}{\rho S} C_{Leq}^{-1}(\delta_e)$$

$$C_{Leq} = C_{L\alpha}(\alpha_{M0} - \alpha_0) + C_{L\delta}\left(1 - \frac{C_{L\alpha}C_{M\delta}}{C_{L\delta}C_{M\alpha}}\right)\delta_e$$

- $\alpha\uparrow \Rightarrow$ increased wing and tail lift $\Rightarrow C_{L\alpha} > 0$
- $\delta_e\uparrow \Rightarrow$ increased tail lift $\Rightarrow C_{L\delta} > 0,\ C_{M\delta} < 0$
- as we shall see, static stability $\Rightarrow C_{M\alpha} < 0$
- $C_{L\alpha}C_{M\delta} / C_{L\delta}C_{M\alpha} > 1$

$\Rightarrow$ $\alpha_{eq}(\delta_e),\ C_{Leq}(\delta_e)$ decrease if $\delta_e\uparrow$

$V_{eq}(\delta_e)$ increases if $\delta_e\uparrow$

Control of airspeed and angle of attack (contd)

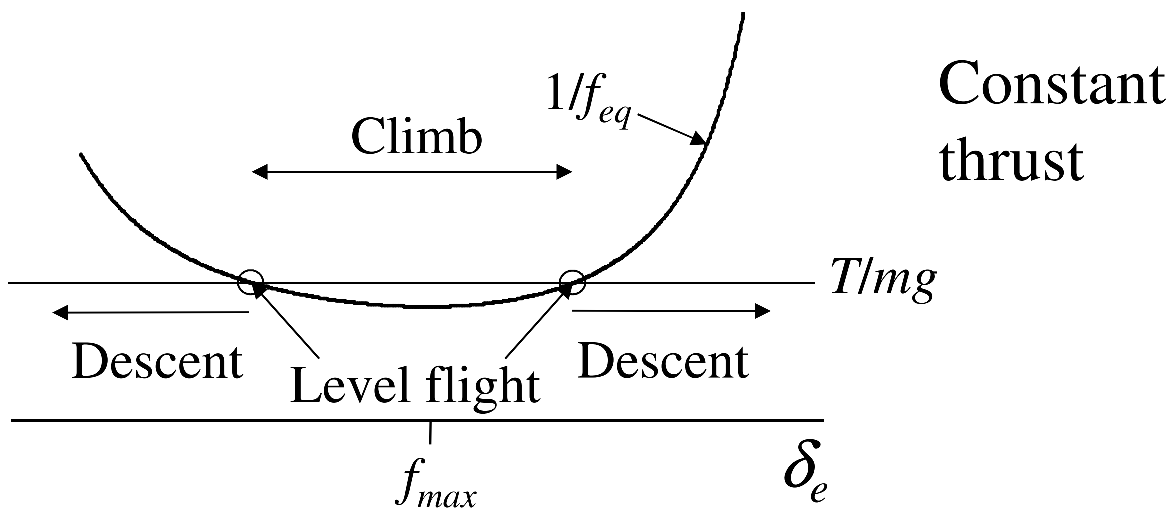
- pushing on stick $\Rightarrow \delta_e \uparrow \Rightarrow V_{eq} \uparrow$ and $\alpha_{eq} \downarrow$.
- low angle of attack associated with high airspeed and vice versa.
- stalling $\Rightarrow$ maximum $\alpha \Rightarrow$ minimum airspeed and δ_e. If pilot tries to fly too slowly, the aircraft stalls.
- particularly a problem when landing, because:
  - airspeed needs to be reduced
  - wind flow may be turbulent and affected by obstacles/topography
  - the ground is nearby, allowing less time and altitude for recovery.
- hence use of high-lift devices such as flaps on takeoff and landing. Not used in remainder of flight because non-optimal for e.g. fuel consumption.

Control of climb angle

(C) with (A) $\Rightarrow$ $\gamma = \gamma_{eq}\left(T, \delta_e\right)$

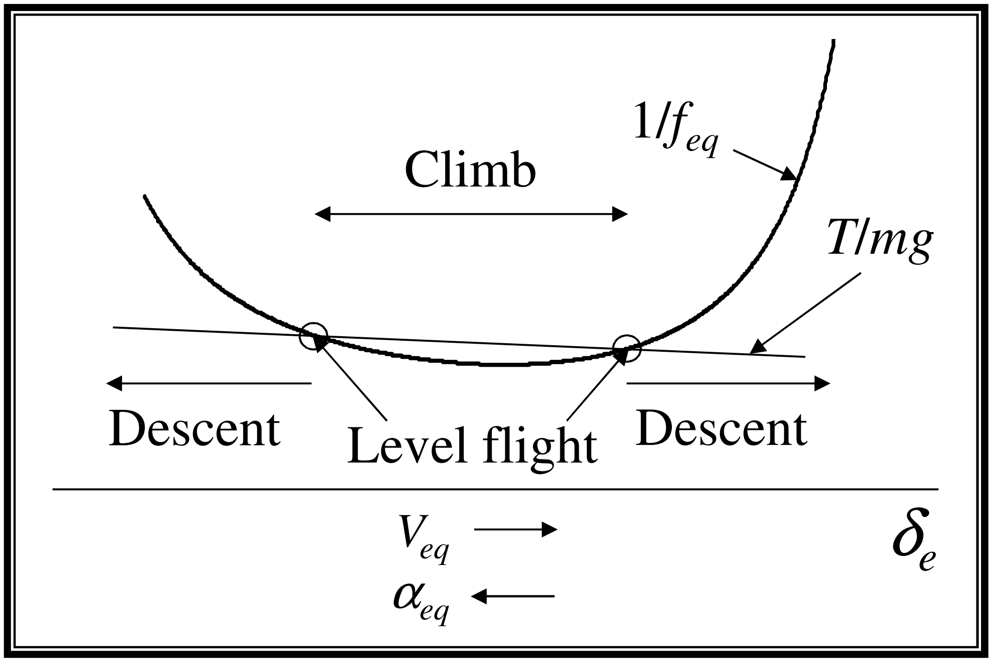
$$\gamma_{eq} = \underbrace{\frac{T}{mg}}_{Thrust} - \underbrace{\frac{1}{f_{eq}\left(\delta_e\right)}}_{Drag}$$

Climb angle controlled by both throttle and stick. Increase in thrust $\Rightarrow$ increased γ_{eq}.



- this assumes constant thrust. In fact, at constant throttle setting, thrust varies somewhat with airspeed...

Control of climb angle (contd)



Constant throttle setting

- details of T/mg curve depend on engine characteristics
- opening throttle causes the T/mg curve to move upwards $\Rightarrow$ widens range in which climbing occurs
- if T/mg curve below $1/f_{eq}$ for all δ_e with throttle fully open $\Rightarrow$ aircraft too heavy for engine (at least at given altitude: see below)
- stalling may occur to left of diagram.

Level flight and gliding

Level flight $\Rightarrow$ $$\frac{T}{mg} = \frac{1}{f_{eq}(\delta_e)}$$

relates throttle and stick settings. Still leaves one degree of freedom to control e.g. airspeed.

- for a given throttle setting, there can be two stick settings for level flight: low and high speed. Low-speed regime may stall.
- maximum speed for level flight set by engine.

Gliding $\Rightarrow T = 0 \Rightarrow$ $$\gamma_{eq} = -\gamma_g(\delta_e)$$

- glide angle $\gamma_g = 1/f_{eq}(\delta_e)$ controlled by stick; more precisely: $\tan\gamma_g = 1/f_{eq}(\delta_e)$
- minimum $1/f_{max} \Rightarrow$ smallest glide angle
- provides another interpretation of *L/D* ratio.

Effects of altitude and weight

Increasing altitude $h \Rightarrow$ density decreases.

Equilibrium equation (A) $\Rightarrow$

$$\alpha = \alpha_{eq}\left(\delta_e\right)$$

- fixed relation for a given aircraft, independent of altitude and weight.

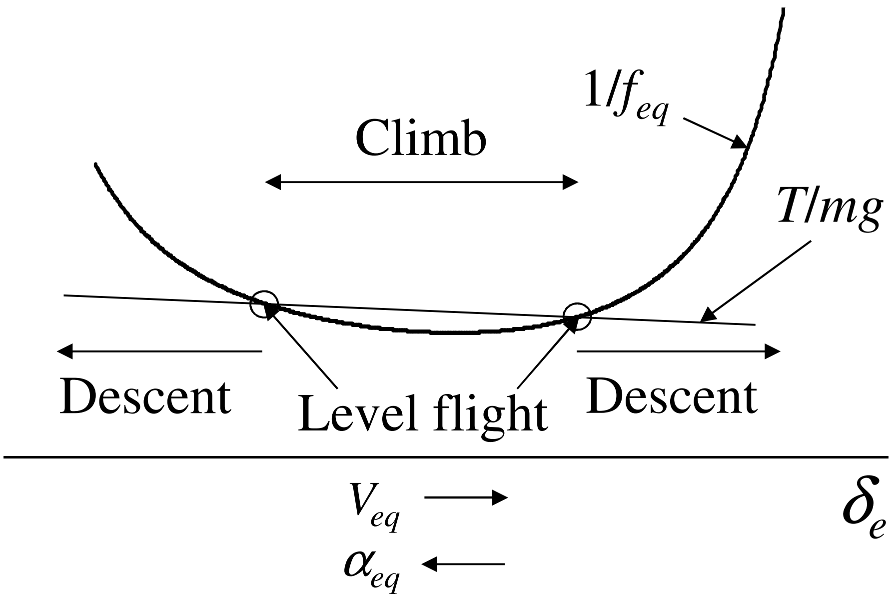
Equilibrium equation (B) $\Rightarrow$

$$V = V_{eq} = \left(\frac{mg}{\rho}\right)^{1/2} F\left(\delta_e\right)$$

- scaling factor depends on weight and altitude
- for a given stick setting δ_e, faster flight at altitude or with increased weight.

Effects of altitude and weight (contd)

Equilibrium equation (C) $\Rightarrow$



Constant throttle setting

- $1/f_{eq}$ curve fixed for a given aircraft
- at constant throttle and δ_e, engine thrust decreases with altitude $\Rightarrow T/mg$ curve drops with increasing altitude or weight
- at sufficiently high altitude, aircraft loses height for all δ_e even with maximum thrust
- "ceiling" (decreasing with weight) above which the aircraft cannot fly.

Indicated airspeed

From the pilot's point of view, the cockpit airspeed indicator is an important instrument. However, it does not give true airspeed, being based on dynamic pressure $\rho V^2/2$ assuming the density at the ground, $\rho_0 \Rightarrow$

$$V_{ind} = \left(\frac{\rho}{\rho_0}\right)^{1/2} V$$

$\Rightarrow$ indicated airspeed lower than true airspeed.

For equilibrium flight:

$$V_{ind} = \left(\frac{mg}{\rho_0}\right)^{1/2} F\left(\delta_e\right)$$

- for given weight, same relation between δ_e, α_{eq} and V_{ind} at all altitudes
- V_{ind} gives pilot a measure of angle of attack. High $V_{ind} \Rightarrow$ low α_{eq} and vice versa.

Performance

Different possible measures of performance, e.g.:

☞ maximum range
☞ maximum flight time (endurance)
☞ maximum climb rate or angle
☞ maximum or minimum airspeed.

Each of these implies a different flight strategy. Maximum range for level flight is perhaps the most important and illustrates the principles.

- maximum range for a given amount of fuel $\Leftrightarrow$ minimum fuel consumption over a given distance $\Leftrightarrow$ minimum $\mathcal{C}/V$, where $\mathcal{C}$ is fuel consumption rate.

Performance: minimising $\mathcal{C}/V$

Reminder: equilibrium, level flight $\Rightarrow$

$$V = \left(\frac{mg}{\rho}\right)^{1/2} F(\delta_e) \qquad T = D = \frac{mg}{f_{eq}(\delta_e)}$$

$\mathcal{C}$ depends on propulsion type:

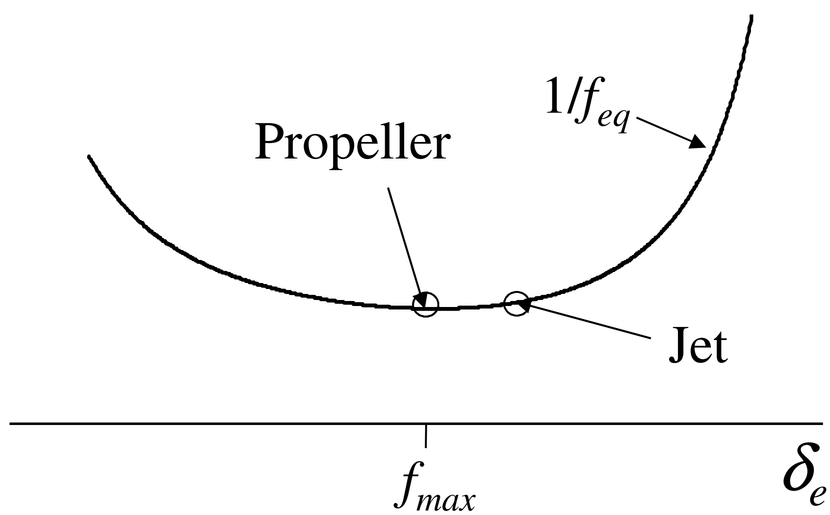
Propeller

- approximately $\mathcal{C} \propto$ power available for propulsion = $VT \Rightarrow$ minimum T
- optimal when stick setting gives $f_{eq} = f_{max}$ and throttle adjusted for level flight
- optimal altitude determined by propulsion system efficiency

Performance: minimising $\mathcal{C}/V$ (contd)

Pure jet

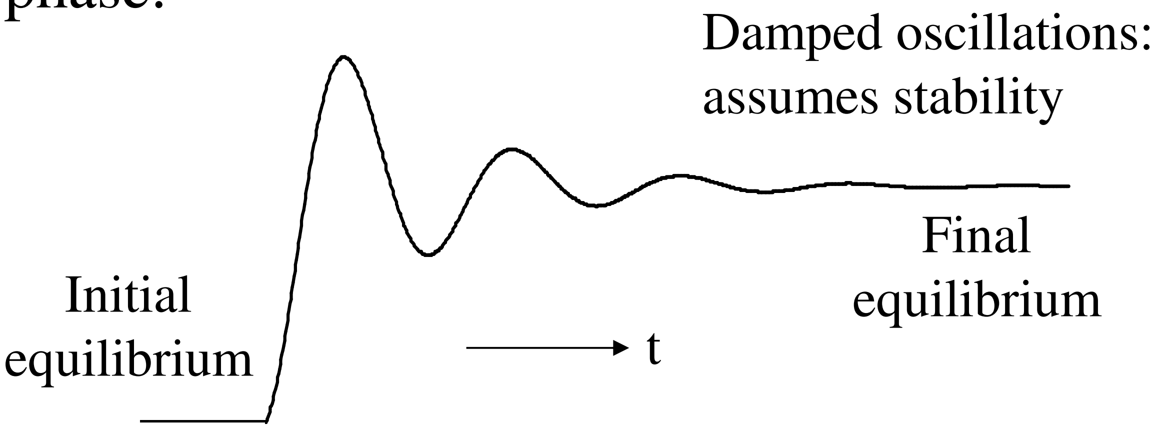
- approximately $\mathcal{C} \propto T \Rightarrow$ minimum
$$\frac{T}{V} = \frac{(\rho m g)^{1/2}}{F(\delta_e)\, f_{eq}(\delta_e)}$$
- optimal for given altitude when stick setting maximises $F(\delta_e)\, f_{eq}(\delta_e) \Rightarrow$ somewhat faster than for propeller
- since $T/V \propto \rho^{1/2}$, the higher the better



- turbofan lies between propeller and pure jet

1.6 Transients and stability

If the aircraft is initially in equilibrium flight and the controls are made to undergo a step change between constant settings $\Rightarrow$ transient phase:



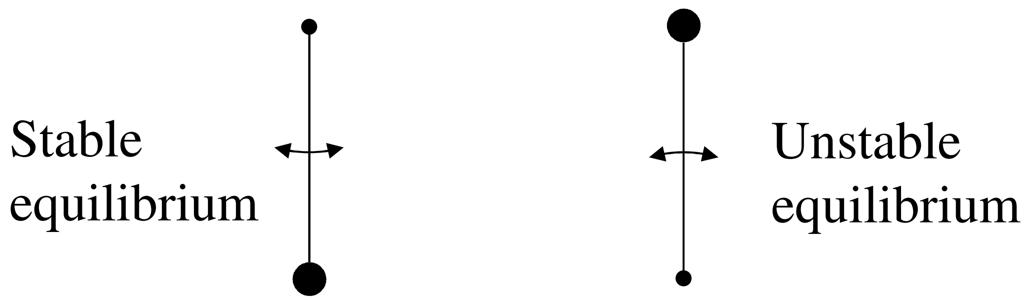
Example: sudden increase in thrust from level flight $\Rightarrow$

- initial increase in airspeed
- transient phase
- finally: climb and return to initial airspeed.

Stability

> An equilibrium is *stable* if small perturbations decay with time, *unstable* if they grow.

Example of pendulum:



Example with flight: paper airplanes are often unstable.

- the pilot and flight system can have important effects on overall stability ("control loop")
- inherent instability: pilot and flight system do nothing – stick-fixed or stick-free. This is what is usually meant by stability.

Stability versus controllability

- inherent instability is an important factor in the overriding requirement of *controllability*

Controllable = pilot can make aircraft do what he wants without undue effort

- controllability is subjective (even amongst "good" pilots); inherent stability is objective
- inherent instability is acceptable unless it is so violent as to imply control problems
- this usually means that the instability should not grow too rapidly
- inherent stability is just one factor in controllability, indeed the two characteristics may directly oppose each other
- from here on, stability = fixed-stick inherent stability

Linearisation

Writing $\mathbf{X}$ for the complete "state vector" of the aircraft (e.g. $\mathbf{X} = \left(V, \alpha, \gamma, \dot{\theta}\right)$ for longitudinal flight), the equations of motion take the form:

$$\frac{d\mathbf{X}}{dt} = \mathscr{F}\left(\mathbf{X};\boldsymbol{\delta}\right)$$

where $\boldsymbol{\delta}$ represents the (fixed) control settings.

Equilibrium $\Rightarrow$ $\mathscr{F}\left(\mathbf{X}_{eq};\boldsymbol{\delta}\right) = 0$
Small perturbation $\Rightarrow$ $\mathbf{X} = \mathbf{X}_{eq} + \mathbf{X}'$
Dropping nonlinear terms in Taylor's series $\Rightarrow$

$$\frac{d\mathbf{X}'}{dt} = \mathscr{A}\mathbf{X}'$$

$\mathscr{A}$ = matrix of partial derivatives of $\mathscr{F}\left(\mathbf{X};\boldsymbol{\delta}\right)$

Modes and stability

Solution = sum of modes:

$$\mathbf{X}' = \Re\left\{\boldsymbol{\xi}\, e^{\lambda t}\right\}$$

ξ, λ = complex eigenvectors and eigenvalues:

$$\mathcal{A}\,\boldsymbol{\xi} = \lambda\,\boldsymbol{\xi}$$

For each component of **X**:

$$X' = \Re\left\{\xi e^{\lambda t}\right\} = \left|\xi\right| e^{\lambda_r t} \cos\left(\lambda_i t + \phi\right)$$

λ_r : sign $\Rightarrow$ exponential growth or decay:

$\lambda_r > 0$: unstable mode

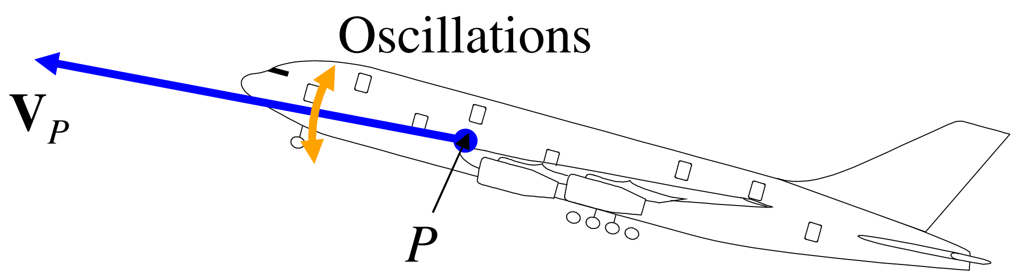
$\lambda_r < 0$: damped mode

λ_i : oscillation frequency

Stable if $\lambda_r \le 0$ for *all* eigenvalues of $\mathcal{A}$.

Longitudinal modes

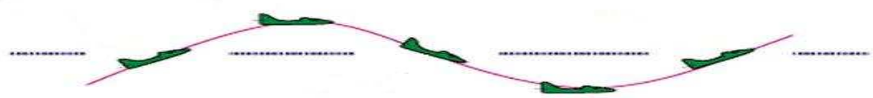
Pitch (short-period) mode



- visualised as pitch oscillations about point P: fixed in aircraft and of constant velocity $\mathbf{V}_P$
- unstable if centre of gravity too far back: important loading criterion
- typical period of oscillations ~ a few seconds
- growth/decay ~ second ⇒ hard to control if unstable ⇒ instability undesirable
- usually well-damped in practice
- reflects lack of equilibrium of moments ⇒ moment equilibrium reached following decay

Longitudinal modes (contd)

Phugoid (long-period) mode



- vertical oscillations of aircraft flight path, speed and pitch angle, but with constant angle of attack determined by equilibrium of moments
- exchange back and forth between kinetic and gravitational potential energy
- period $\approx 2^{1/2}\pi V_{eq}/g$ ~ 1-2 minutes $\Rightarrow$ easily controllable
- lightly damped: once excited, present for a relatively long time unless controlled
- reflects lack of force equilibrium

Static stability

Moment equilibrium:

$$C_M\left(\alpha_{eq};\delta_e\right)=0$$

Suppose angle of attack changes by a small amount $\Delta\alpha \Rightarrow$ aerodynamic moment:

$$M_a=\frac{1}{2}\rho S\ell V^2\frac{\partial C_M}{\partial\alpha}\left(\alpha_{eq};\delta_e\right)\Delta\alpha$$

provides a restoring force if

$$\boxed{\frac{\partial C_M}{\partial\alpha}\left(\alpha_{eq};\delta_e\right)<0}$$

which is the condition for *static stability*. Using the linear model for C_M, this condition gives

$$C_{M\alpha}<0$$

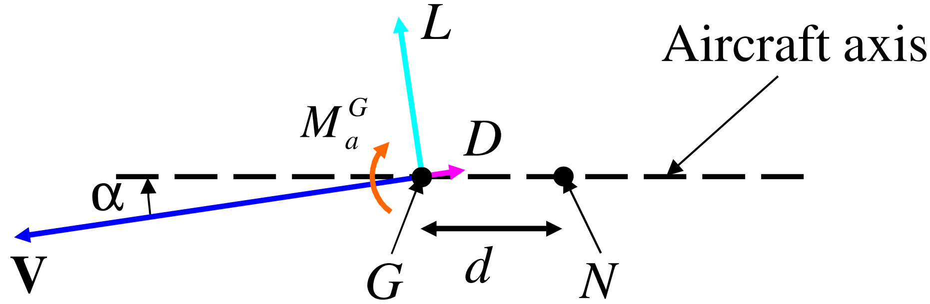
which is independent of the control settings, i.e. static stability is a property of the aircraft.

Static stability (contd)

- the term *static stability* is used in contrast to the (more fundamental) *dynamic stability* previously discussed
- static stability is really an approximation of the dynamic stability criterion for the pitch mode
- the idea of pitch instability as due to a negative restoring moment brings out its physical origin
- stable oscillations reflect a positive restoring moment (~ torsional spring)

Positioning of centre of gravity

Let N be a point in the aircraft obtained by moving rearwards a distance d from the centre of gravity. Aerodynamic forces and moments:



Aerodynamic moment about N:

$$M_a^N = M_a^G + d\left(L\cos\alpha + D\sin\alpha\right)$$

Small α and negligible $D\sin\alpha \Rightarrow$

$$M_a^N = M_a^G + dL$$

Introducing aerodynamic coefficients:

$$C_M^N = C_M^G + \frac{d}{\ell} C_L$$

Positioning of centre of gravity (contd)

Taking the derivative with respect to α and using linear models for the coefficients:

$$C_{M\alpha}^{N} = C_{M\alpha}^{G} + \frac{d}{\ell} C_{L\alpha}$$

Choose d according to

$$d = -\frac{C_{M\alpha}^{G}}{C_{L\alpha}} \ell \quad \Rightarrow \quad C_{M\alpha}^{N} = 0$$

defines *neutral point*: N = fixed point in aircraft for which aerodynamic moment independent of angle of attack.

Static stability criterion:

$$C_{M\alpha}^{G} = -C_{L\alpha} \frac{d}{\ell} < 0 \quad \Leftrightarrow \quad d > 0$$

Positioning of centre of gravity (contd)

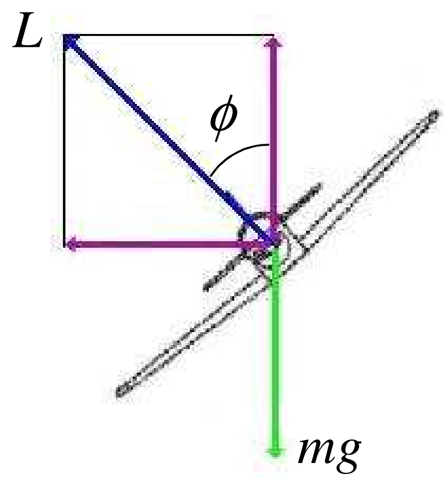
Aircraft is statically stable if centre of gravity located forwards of neutral point

- important practical constraint on aircraft loading distribution: needs checking before every flight
- static margin = d / ℓ, often expressed in percent (of mean chord): typically in the range 5%-40%.

Example: paper airplanes often need to be nose loaded to fly properly.

1.7 Non-longitudinal flight

Perhaps the most important example is turning. This is achieved principally by using the ailerons to bank:



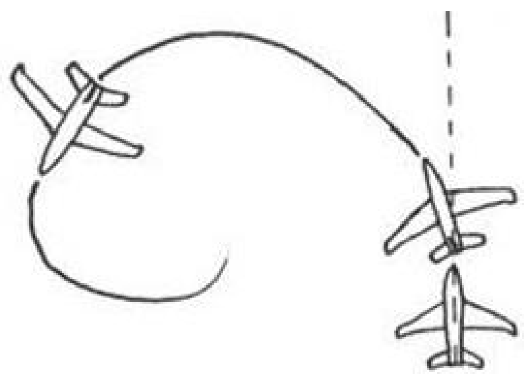
Turning to left

$$\tan\phi = \frac{V^2}{gr}$$

- horizontal component of lift $\Rightarrow$ centripetal force for turn and perhaps sideslip
- vertical component of lift supports weight
- correct (no sideslip) banking angle ϕ for given speed and turn radius may require rudder
- increased lift required in turn $\Rightarrow$ higher speed or lift coefficient $\Rightarrow$ elevators/throttle.

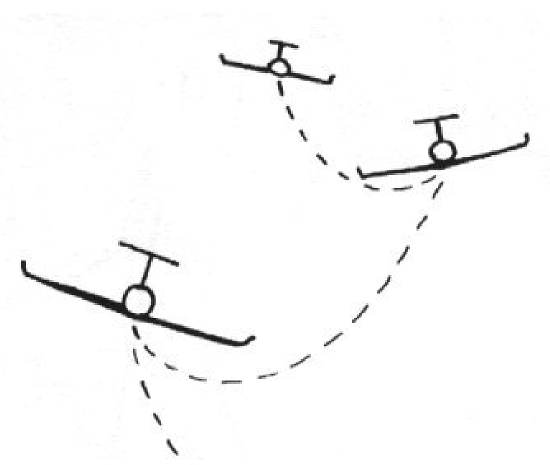
Potentially unstable lateral modes

Spiral divergence



- usually slow growth
  $\Rightarrow$ controllable
- if unchecked $\Rightarrow$ ever
  tightening spiral dive

Dutch roll



- oscillatory motion:
  aircraft “waddles”
  from side to side
- unpleasant for crew
  and passengers: to
  be avoided