



Act 6: Determinantes de Matrices

Aarón Mireles Barrón

Matrícula: 2132904

Profesor: Luis Ángel Gutiérrez Rodríguez

Materia: Inteligencia Artificial

Tenemos que obtener el determinante de una matriz 4x4 como se muestra a continuación:

$$\begin{vmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{vmatrix}$$

Usando la regla de Sarrus y la regla de Laplace podemos obtener una fórmula que resuelva directamente esta matriz, primero usamos la regla de Laplace para reducir esta matriz a tres de 3x3

$$\begin{vmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{vmatrix}$$

Es igual a

$$\begin{vmatrix} a_{22} & a_{32} & a_{42} \\ a_{23} & a_{33} & a_{43} \\ a_{24} & a_{34} & a_{44} \end{vmatrix} * a_{11} +$$

$$\begin{vmatrix} a_{12} & a_{32} & a_{42} \\ a_{13} & a_{33} & a_{43} \\ a_{14} & a_{34} & a_{44} \end{vmatrix} * (-a_{21}) +$$

$$\begin{vmatrix} a_{12} & a_{22} & a_{42} \\ a_{13} & a_{23} & a_{43} \\ a_{14} & a_{24} & a_{44} \end{vmatrix} * a_{31} +$$

$$\begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ a_{14} & a_{24} & a_{34} \end{vmatrix} * (-a_{41})$$

Ahora usamos la regla de Sarrus para obtener una fórmula que resuelva cualquier matriz de 4x4

La primera matriz:

$$\begin{vmatrix} a_{22} & a_{32} & a_{42} \\ a_{23} & a_{33} & a_{43} \\ a_{24} & a_{34} & a_{44} \end{vmatrix} * a_{11} +$$

Es igual a:

$$a_{11} * [(a_{22} * a_{33} * a_{44} + a_{32} * a_{43} * a_{24} + a_{42} * a_{23} * a_{34}) - (a_{22} * a_{43} * a_{34} + a_{32} * a_{23} * a_{44} + a_{42} * a_{33} * a_{24})]$$

La segunda matriz:

$$\begin{vmatrix} a_{12} & a_{32} & a_{42} \\ a_{13} & a_{33} & a_{43} \\ a_{14} & a_{34} & a_{44} \end{vmatrix} * (-a_{21}) +$$

Es igual a:

$$a_{21} * [-(a_{12} * a_{33} * a_{44} + a_{32} * a_{43} * a_{14} + a_{42} * a_{13} * a_{34}) + (a_{12} * a_{43} * a_{34} + a_{32} * a_{13} * a_{44} + a_{42} * a_{33} * a_{14})]$$

La tercera matriz:

$$\begin{vmatrix} a_{12} & a_{22} & a_{42} \\ a_{13} & a_{23} & a_{43} \\ a_{14} & a_{24} & a_{44} \end{vmatrix} * a_{31} +$$

Es igual a:

$$a_{31} * [(a_{12} * a_{23} * a_{44} + a_{22} * a_{43} * a_{14} + a_{42} * a_{13} * a_{24}) - (a_{12} * a_{43} * a_{24} + a_{22} * a_{13} * a_{44} + a_{42} * a_{23} * a_{14})]$$

La cuarta matriz:

$$\begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ a_{14} & a_{24} & a_{34} \end{vmatrix} * (-a_{41})$$

Es igual a:

$$a_{41} * [-(a_{12} * a_{23} * a_{34} + a_{22} * a_{33} * a_{14} + a_{32} * a_{13} * a_{24}) + (a_{12} * a_{33} * a_{24} + a_{22} * a_{13} * a_{34} + a_{32} * a_{23} * a_{14})]$$

Juntando todo obtenemos la fórmula para resolver una matriz de 4x4:

$$\begin{aligned} & a_{11}a_{22}a_{33}a_{44} + a_{11}a_{32}a_{43}a_{24} + a_{11}a_{42}a_{23}a_{34} - a_{11}a_{22}a_{43}a_{34} - a_{11}a_{32}a_{23}a_{44} - a_{11}a_{42}a_{33}a_{24} - a_{21}a_{12}a_{33}a_{44} - \\ & a_{21}a_{32}a_{43}a_{14} - a_{21}a_{42}a_{13}a_{34} + a_{21}a_{12}a_{43}a_{34} + a_{21}a_{32}a_{13}a_{44} + a_{21}a_{42}a_{33}a_{14} + a_{31}a_{12}a_{23}a_{44} + a_{31}a_{22}a_{43}a_{14} + \\ & a_{31}a_{42}a_{13}a_{24} - a_{31}a_{12}a_{43}a_{24} - a_{31}a_{22}a_{13}a_{44} - a_{31}a_{42}a_{23}a_{14} - a_{41}a_{12}a_{23}a_{34} - a_{41}a_{22}a_{33}a_{14} - a_{41}a_{32}a_{13}a_{24} + \\ & a_{41}a_{12}a_{33}a_{24} + a_{41}a_{22}a_{13}a_{34} + a_{41}a_{32}a_{23}a_{14} \end{aligned}$$