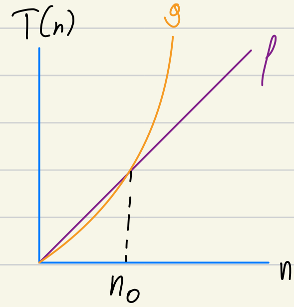
Week 3

a) $f(n) = 3n$ $\quad g(n) = 5n^3$ $\quad$ note g grows faster than f



$f \in O(g)$ is true as $f < cg$ & $\because \lim_{n\to\infty} \frac{f}{g} < \infty$

$f \notin \Omega(g(n))$ because g grows faster than f & $\lim_{n\to\infty} \frac{f}{g} < \infty$

$f \in o(g(n))$ because $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$ $\because \lim_{n\to\infty} \frac{3n}{5n^3} = \frac{3}{5n^2} = \frac{3}{\infty} = 0$

$f \notin \omega(g(n))$ because $f \notin \Omega(g(n))$

$f \notin \Theta(g)$ because $f \in \Theta(g)$ iff $f \in \Omega(g)$ & $f \in O(g)$ but $f \notin \Omega(g(n))$

$g \notin O(f)$ because $f \notin \Omega(f)$

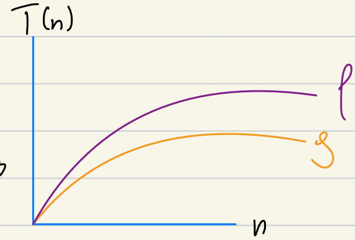
$g \in \Omega(f)$ because g grows faster than f &

$g \notin o(f)$ $\because \lim_{n\to\infty} \frac{g(n)}{f(n)} \neq 0$ $\because \lim_{n\to\infty} \frac{5n^3}{3n} \neq 0$ & $g \notin O(f)$

$g \notin \Theta(f)$ $\because$ $g \notin O(f)$

$g \in \omega(f)$ $\lim_{n\to\infty} \frac{5n^3}{3n} = \lim_{n\to\infty} \frac{5n^2}{3} = \infty$

b) $f(n) = 3n^{.8} + 2n^{.3} + 14\log n \qquad g(n) = n^{.5}$



$f \notin O(g)$ $\because$ f grows faster than g $\because$ $n^{.8} > n^{.5}$ as $n \to \infty$

$f \notin o(g)$ $\because$ $f \notin O(g)$ & $\lim_{n\to\infty} \frac{f}{g} = \infty$

$f \in \Omega(g)$ $\because$ g grows slower than f $\because$ $n^{.5} < n^{.8}$ as $n \to \infty$

$$f \in \omega(g) \because \lim_{n\to\infty} \frac{f}{g} = \lim_{n\to\infty} \frac{3n^{.8} + 2n^{.3} + 14\log n}{n^{.5}} = 3n^{.3} + 2n^{-.2} + \frac{14\log n}{n^{.5}}$$

$f \notin \Theta(g)$ $\because$ $f \notin O(g)$ $\qquad = \infty + 0 + 0$

$g \in O(f)$ $\because$ g grows slower than f $\because$ $n^{.5} < n^{.8}$ as $n \to \infty$ $\quad \lim_{n\to\infty} \frac{g}{f} < \infty$

$$g \in o(f) \because \lim_{n\to\infty} \frac{g}{f} = \frac{n^{.5}}{n^{.8}} = \frac{1}{n^{.3}} = \frac{1}{\infty} = 0$$

$g \notin \Omega(f)$ because $f \in \Omega(f)$ & $g \in o(f)$

$g \notin \Theta(f)$ $\because$ $g \notin \Omega(f)$ & $g \in o(f)$

C) $f(n) = \frac{n^2}{\log n}$ $\quad g(n) = n \log n$

$f \notin o(g) = \frac{f}{g} = \lim_{n\to\infty} \frac{n^2}{n\, 2\log n} \Rightarrow \lim_{n\to\infty} \frac{n}{2\log n} \Rightarrow \frac{\infty}{2\log \infty} = \infty \quad \infty \neq 0$

$f \notin O(g) \because f \notin o(g) \quad \lim_{n\to\infty} \frac{f}{g} \nless \infty$

$f \in \omega(g) = \frac{f}{g} \Rightarrow \lim_{n\to\infty} \frac{n}{2\log n} = \infty$

$f \in \Omega(g) \because f \in \omega(g)$ & f grows faster than g

$f \notin \Theta(g) \because f \notin O(g)$

$g \in o(f) \because \lim_{n\to\infty} \frac{g}{f} = \lim_{n\to\infty} \frac{2\log n}{n} = 0$

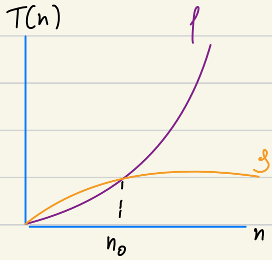
$g \in O(f) \because g \in o(f)$ & because g grows slower than f

$g \notin \omega(f) = \lim_{n\to\infty} \frac{g}{f} = \lim_{n\to\infty} \frac{2\log n}{n} = 0 \quad 0 \neq \infty \quad \therefore g \notin \omega(f)$

$g \notin \Omega(f) \because g \notin \omega(f)$

$g \notin \Theta(f) \because g \notin \Omega(f)$

d) $f(n) = (\log(3n))^3$ $\quad g(n) \; 3\log n$



$f \notin o(g) = \lim_{n\to\infty} \frac{f}{g} = \frac{(\log(3n))^3}{3\log n} \Rightarrow \infty \neq 0$

$(\log\infty)^3 > 3\log\infty$

$f \notin O(g) \because f$ grows faster than g

$f \in \omega(g) \because \lim_{n\to\infty} \frac{f}{g} = \infty$

$f \in \Omega(g) \because f \in \omega(g)$ & f grows faster than g

$f \notin \Theta(g) \because f \notin O(g)$

$g \notin \omega(f) \because \lim_{n\to\infty} \frac{g}{f} = 0$

$g \notin \Omega(f) \because \quad f \in \Omega(g)$ & $f \notin \Theta(g)$

$g \in o(f) \because \lim_{n\to\infty} \frac{g}{f} = 0$

$g \in O(f) \because g \in o(f)$ & g grows slower than f

$g \notin \Theta(f) \because f \notin \Theta(g)$.