Assignement 8

Question 1

$\neg(x \vee (y \wedge 1))$: prove it is a well-formed formula

Syntax of Boolean formula

Basis of inductive definition:

1a. Every Boolean variable X_i is a Boolean formula

1b. The two Boolean constants 0 and 1 are Boolean formulas

Induction step:

2a. if A and B are Boolean formulas, then $(A \wedge B)$ is a Boolean formula

2b. if A and B are Boolean formulas, then $(A \vee B)$ is a Boolean formula

2c. if A is a boolean formula, then $\neg A$ is a Boolean formula

Answer:

$$\dfrac{\neg(x \vee (y \wedge 1))}{\dfrac{(x \vee (y \wedge 1))}{\dfrac{x}{}\ 1a \qquad \dfrac{(y \wedge 1)}{\dfrac{y}{}\ 1a \quad \dfrac{1}{}\ 1b}\ 2a}\ 2b}\ 2c$$

thus $\neg(x \vee (y \wedge 1))$ is a well-formed formula

Question 2

$D = \{x_1, x_2, x_3\}$

$L(x_1) = 1,\ L(x_2) = 0,\ L(x_3) = 1$

determine semantics of the formula: $x_1 \vee \neg(x_2 \vee 0) \wedge x_3$

Semantics of Boolean formula:

Basis of induction step:

1a. For every Boolean variable $x \in D$, $L^*(x) = L(x)$

1b. For two Boolean constants 0 and 1, we set $L^*(0) = 0$ and $L^*(1) = 1$

induction step:

2a. $L^*((A \wedge B)) = \begin{cases} 1 & \text{if } L^*(A) = 1 \text{ and } L^*(B) = 1 \\ 0 & \text{otherwise} \end{cases}$

2b. $L^*((A \vee B)) = \begin{cases} 1 & \text{if } L^*(A) = 1 \text{ or } L^*(B) = 1 \\ 0 & \text{otherwise} \end{cases}$

2c. $L^*(\neg A) = \begin{cases} 1 & \text{if } L^*(A) = 0 \\ 0 & \text{if } L^*(A) = 1 \end{cases}$

CONTINUATION

Formula : $x_1 \vee \neg(x_2 \vee 0) \wedge x_3$

$$\underline{[x_1] \vee [\neg(x_2 \vee 0) \wedge x_3]}\ \text{2b}$$

$$\underline{\neg(x_2 \vee 0) \wedge x_3}\ \text{2a}$$

$$\underline{\neg(x_2 \vee 0)}\ \text{2c} \qquad \underline{x_3}\ \text{1a}$$

$$\text{1a}\ \underline{x_2} \vee \underline{0}\ \text{1b}$$

Final answer : $1 \vee \neg(0 \vee 0) \wedge 1$

$= (1 \vee 1) \wedge 1$

$= 1 \wedge 1$

$= 1$

Question 3

For simplicity let $(\varphi, \psi, \chi) \longrightarrow (A, B, C)$

$$\neg(A \wedge B \wedge C) \equiv \neg A \vee \neg B \vee \neg C$$

A	B	C	$\neg(A \wedge B \wedge C)$	$\neg A \vee \neg B \vee \neg C$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

Question 4

$(\neg x \wedge y) \vee (y \wedge \neg z) \vee (y \wedge z) \vee (x \wedge \neg y \wedge \neg z) \equiv y \vee (x \wedge \neg z)$

$(y \wedge \neg z) \vee (y \wedge z) \equiv y \wedge (\neg z \vee z)$: Distributivity law

$(\neg x \wedge y) \vee y \wedge (\neg z \vee z) \vee (x \wedge \neg y \wedge \neg z)$

$(\neg z \vee z) \equiv 1$: Complementation law

$(\neg x \wedge y) \vee y \wedge 1 \vee (x \wedge \neg y \wedge \neg z)$

$(\neg x \wedge y) \vee y \equiv Y$: Absorption law

$Y \wedge 1 \vee (x \wedge \neg y \wedge \neg z)$

$Y \wedge 1 \equiv Y$: identity

$Y \vee (x \wedge \neg y \wedge \neg z) \equiv (Y \vee x) \wedge (Y \vee \neg Y) \wedge (Y \vee \neg z)$: Distributivity law

$Y \vee \neg Y \equiv 1$: complementation law

$(Y \vee x) \wedge 1 \wedge (Y \vee \neg z)$

$(Y \vee x) \wedge 1 \equiv (Y \vee x)$: identity

$(Y \vee x) \wedge (Y \vee \neg z) \equiv Y \vee (x \wedge \neg z)$: distributivity law

hence proved

Question 5

Formula: $\neg x \vee \neg y \vee (x \wedge y \wedge \neg z)$ **DNF**

	X	Y	Z	$\neg x \vee \neg y \vee (x \wedge y \wedge \neg z)$
m_0	0	0	0	1
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	1
m_7	1	1	1	0

DNF : $m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6$

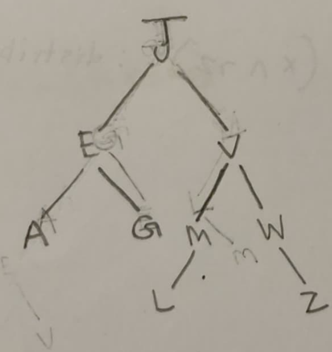
$\downarrow$

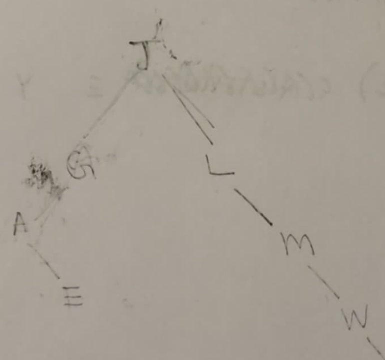
$(\neg x \wedge \neg y \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z) \vee$
$(\neg x \wedge y \wedge \neg z) \vee (\neg x \wedge y \wedge z) \vee (x \wedge \neg y \wedge \neg z)$
$\vee (x \wedge \neg y \wedge z) \vee (x \wedge y \wedge \neg z) \vee (x \wedge y \wedge z)$

Question 6

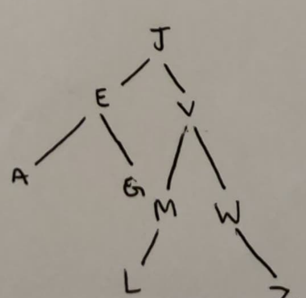
preorder : J, G, A, E, V, L, M, W, Z

inorder : A, E, G, J, L, M, V, W, Z





ANSWER :



Question 7

```
data Natural = Zero | Succ Natural

instance Show Natural where
    -- show :: Natural -> String
    show n = show (iterate n) where
        iterate zero     = 0
        iterate (succ m) = 1 + (iterate m)


-- Some constant
zero, one, two, three, four :: Natural
zero  = Zero
one   = Succ Zero
two   = Succ one
three = Succ two

--addition
infixl 6 <+>
(<+>) :: Natural -> Natural -> Natural
n <+> Zero     = n
n <+> (Succ m) = Succ (n <+> m)
```

ANSWER:

```
--multiplication
infixl 9 <.>
(<.>) :: Natural -> Natural -> Natural
n <.> Zero     = Zero
n <.> (Succ m) = Succ (n <.> m)
```