Assignment 4

Question 1)

$A \subseteq (B \cup C) \rightarrow \forall x . x \in A \rightarrow x \in B \vee x \in C$

$(A \cap B) \subseteq (A \setminus C)$

$A \cap B \rightarrow x \in A \wedge x \in B$

$A \setminus C \rightarrow x \in A \wedge x \notin C$

Final

$(x \in A \wedge x \in B) \rightarrow (x \in A \wedge x \notin C)$

Question 2)

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Let x be an arbitary element in $A \cup (B \cap C)$. this means x is in A OR in $B \cap C$

1st case:

if $x \in A$, then x is in $A \cup B$ and $A \cup C$ therefore x is in $(A \cup B) \cap (A \cup C)$

2nd case:

if $x \in B \cap C$, then x is in both B and C. There fore, x is in $A \cup B$ and $A \cup C$.

Hence x is in $(A \cup B) \cap (A \cup C)$

$\rightarrow$ hence $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Question 3)

$$\underbrace{\{n^2 \mid n \in \mathbb{N}\}}_{\text{All perfect sq where } n \in \mathbb{N}} \overset{\text{Intersection}}{\cap} \underbrace{\{n^3 \mid n \in \mathbb{N}\}}_{\text{All perfect cubes where } n \in \mathbb{N}} \underbrace{\neq \emptyset}_{\text{not equal to empty set}}$$

$\rightarrow$ There exists, atleast one number that can be expressed both as the square and the cube of a natural number

example:

$1^2 = 1$ hence $n = 1$

$1^3 = 1$

Question 4)

1. $I_N = \{(x,y) \in N \times N \mid x = y\}$

   - Reflexive → $N(x,x)$ belongs to R as $x = x$
   - Symmetric → $(x,y) \to x = y$ and $(y,x) \to y = x$ ~~hence~~
   - Transitive → $(x,y) \to x = y$ then for $(y,z) \to y = z$
     then $(x = y) \to (x,z)$

2. $L_R = \{(x,y) \in R \times R \mid x < y\}$

   - Irreflexive → (x,x) then $x < x$ cannot be hence NOT reflexive
   - Neither
   - Transitive → for $(x,y) \to x < y$, for $(y,z) \to y < z$
     then for sure $x < z$ hence (x,z) holds

Question 5)

P = Persons of certain family

$F \subseteq P$: All female persons in this family

1. $(S \cup D)^T \circ 1_x$

   represents female family members who are either mothers or fathers

2. $\Sigma \subseteq P \times P$ (sister relation) where $(x,y) \in \Sigma$ iff x is a sister of y

   x is female, and x is a daughter of y's parents but not equal to y

Question 6)

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intersect :: (Eq a) => [a] -> [a] -> [a]

intersect xs ys = [x | x <- xs, x 'elem' ys]
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