Assignment 5

Question 1

1. $f: R \rightarrow R$ with $f(x) = x^3 - x$

injective

↳ NO, $f(0) = 0$ AND $f(1) = 0$

thus two different inputs have the same output

Surjective

↳ yes, every real number y in the codomain has atleast one real no. x ~~y is mapped to by exactly one element of the domain x~~ in the domain such that $g(x) = y$

Example:

$x = 2$

$2^3 - 2 = 8 - 2 = 6$

$f(2) = 6$

Bijective

↳ No, since not injective

2. $g: N \rightarrow R$ with $g(x) = x + 0.01$

injective

↳ yes, no different natural numbers result to the same real number

surjective

↳ Every Natural number x has a real number y such that $g(x) = y$, yes

Bijective

↳ Yes, its both injective and surjective

3. $h: N \to Z$ with $h(n) = (-1)^n \times \left(\frac{n}{2} + \frac{1}{4}\right) - \frac{1}{4}$

injective

↳ Yes, Each natural number n maps to a unique integer

Surjective

↳ No, since the function alternates between +ve and -ve integers, it does not cover all integers

Bijective

↳ No, since not surjective

Question 2

1. $f \circ f$

↳ $f(f(x))$

thus $f(x) = x^3 - x$

$$f(f(x)) = (x^3-x)^3 - (x^3-x)$$

$$= (x^3-x)^2(x^3-x) - (x^3-x)$$

$$= (x^6 - 2x^4 + x^2)(x^3-x) - (x^3-x)$$

$$= x^9 - 2x^7 + x^5 - x^7 + 2x^5 - x^3 - x^3 + x$$

$$= x^9 - 3x^7 + 3x^5 - 2x^3 + x$$

2. $(f \circ g) \circ (h^{-1} \circ h)$

↓

$f(g(x))$

↓

$(x+0.01)^3 - (x+0.01)$

↳ $h(n) = (-1)^n \times \left(\frac{n}{2} + \frac{1}{4}\right) - \frac{1}{4}$

$y = (-1)^n \times \left(\frac{n}{2} + \frac{1}{4}\right) - \frac{1}{4}$

$n = (-1)^y \times \left(\frac{y}{2} + \frac{1}{4}\right) - \frac{1}{4}$

$n + \frac{1}{4} = (-1)^y \times \left(\frac{y}{2} + \frac{1}{4}\right)$

$2n + \frac{1}{2} = (-1)^y \left(y + \frac{1}{2}\right)$ → does not have inverse

estion 3

1. imp :: Boolean -> Boolean -> Boolean

imp = \x y -> x y false true

2. xor :: Boolean -> Boolean -> Boolean

xor = \x y -> x (y false true) (y true false)

Question 4

mystery function takes 3 Boolean values as inputs. It applies x to y and z, and then applies the result to the conjunction of y and z and the negation of z

mystery :: Boolean -> Boolean -> Boolean -> Boolean

mystery = \x -> \y -> \z -> (x y z) (conj y z) (neg z)

(conj y z) → Y AND Z

(neg z) → NOT Z

if x true → returns (Y AND Z)

if x false → returns (NOT Z)