Assignment 2

Question 1

$f, g : N \rightarrow R$

1. $f(n) = 7n + 10n\log_2(n) - 2n^3 + 42$

$f(a)\ f(b)\ f(c)\ f(d)\ f(e)\ f(f)\ f(g)\ f(h)$

$$f(n) = \underbrace{(f(a) \times f(b))}_{f(i)} + (f(c) \times f(d) \times f(e)) - \underbrace{(f(f) \times f(g))}_{f(j)} + f(h)$$

$f(i) = f(a) \times f(b)$

$f(a) = 7 \rightarrow O(1)$

$f(b) = n \rightarrow O(n)$

$f(i) \rightarrow n \times 1 \rightarrow O(n)$

$f(j) = f(f) \times f(g)$

$f(f) = 2 \rightarrow O(1)$

$f(g) = n^3 \rightarrow O(n^3)$

$f(j) = n^3 \times 1 \rightarrow O(n)^3$

Simplified

~~$10 \rightarrow O(1)$~~

$42 \rightarrow O(1)$

$10 \rightarrow O(1)$

$n\log n \rightarrow O(n\log n)$

$1 \times n\log n \rightarrow O(n\log n)$

$f(n) \rightarrow O(n) + O(n\log n) +$ ~~$O(1)$~~

rule 2: $(f_1 + f_2 + f_3) \in O(\max\{g_1, g_2, g_3, g_4\})$

thus $f(n) \rightarrow O(n^3)$ → Rank #6

2. $g(n) = 2\sqrt{n} + 3\log_2(n)$ n.b. $O(\log_2(n)) \subset O(n^{\frac{1}{2}})$

ANSWER: $O(n)$

Question 2

$n \in \{0, 1, 2, 3\}$ clubs, diamonds, hearts, spades

→ If magician claim is true turn over the card with a heart suit. If 2 is present then claim = True else false

→ To ensure magician claim, pick another card with hearts to see if the number is 2

so total flips (minimum) = 2

↳ 1st card to check if the claim was true

↳ 2nd card to ensure it was no fluke

Question 3

S: Lulu studies

$\neg S$: Lulu does not study

U: enjoys university life

$\neg U$: does not enjoy university life

M: good marks

$\neg M$: Not good marks

properties:

$S \longrightarrow M$

$\neg S \longrightarrow U$

$\neg M \longrightarrow \neg U$

Assuming the negation of the conclusion ($\neg M$), from premises:

1. if Lulu does not receive good marks, then she does not enjoy university life
2. If Lulu does not study, then she enjoys university life

combining both:

if $\neg S \longrightarrow \neg U$

From premise:

$S \longrightarrow M$

thus leads to contradiction as Lulu cannot both S and $\neg S$

thus, our assumption ($\neg M$) is false,

thus, Lulu receives good grades

Question 4

option(c), Since the premise states that if a ballon is not yellow, it cannot be of medium size, it implies that there cannot be any ballons of yellow colour, as they cannot be of medium size according to the premise

Question 5

```
prefix :: Eq a => a -> [a] -> [a]
prefix _ [] = []
prefix e (x:xs)
  | e == x = [x]
  | otherwise = x : prefix e xs
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