Assignment 3

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10/18/2022

1) Formulating Transportation using R

```
library(lpSolveAPI)
library(tinytex)
library(lpSolve)
```

This data has to be transformed into a table format.

```
#cost matrix
cost_1 <- matrix(c(22,14,30,600,100,
                  16,20,24,625,120,
                  80,60,70,"-","-","210/220"),ncol = 5,nrow = 3,byrow = TRUE)
```

```
## Warning in matrix(c(22, 14, 30, 600, 100, 16, 20, 24, 625, 120, 80, 60, : data
## length [16] is not a sub-multiple or multiple of the number of rows [3]
```

```
colnames(cost_1) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","ProductionCost","Production Capacity")

rownames(cost_1) <- c("Plant_A","Plant_B","Demand")

cost_1
```

```
##         Warehouse_1 Warehouse_2 Warehouse_3 ProductionCost Production Capacity
## Plant_A "22"        "14"        "30"        "600"          "100"
## Plant_B "16"        "20"        "24"        "625"          "120"
## Demand  "80"        "60"        "70"        "-"            "-"
```

The Objective function is to Minimize the TC

$$\text{Min } \; TC = 622x_{11} + 614x_{12} + 630x_{13} + 641x_{21} + 645x_{22} + 649x_{23}$$

Supply

$$X_{11} + X_{12} + X_{13} >= 100$$

$$X_{21} + X_{22} + X_{23} >= 120$$

Demand

$$X_{11} + X_{21} >= 80$$

$$X_{12} + X_{22} >= 60$$

$$X_{13} + X_{23} >= 70$$

Non-Negativity Constraints

$$X_{ij} >= 0$$

Where i = 1,2 and j= 1,2,3

```
#A "Dummy" row for Warehouse_4 Is added here
trans.cost_1 <- matrix(c(622,614,630,0,100,
                         641,645,649,0,120,
                         80,60,70,10,220), ncol = 5, nrow = 3, byrow = TRUE)
trans.cost_1
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  622  614  630    0  100
## [2,]  641  645  649    0  120
## [3,]   80   60   70   10  220
```

```
colnames(trans.cost_1) <- c("Warehouse_1","Warehouse_2","Warehouse_3","Dummy","Production Capacity")

rownames(trans.cost_1) <- c("Plant_1", "Plant_2","Monthly Demand")
trans.cost_1
```

```
##                Warehouse_1 Warehouse_2 Warehouse_3 Dummy Production Capacity
## Plant_1                622         614         630     0                 100
## Plant_2                641         645         649     0                 120
## Monthly Demand          80          60          70    10                 220
```

```
#costs matrix
costs <- matrix(c(622,614,630,0,
                  641,645,649,0), nrow = 2, byrow = TRUE)
costs
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  622  614  630    0
## [2,]  641  645  649    0
```

It is important to understand that the Supply function cannot be greater than the stated units, although the Demand function can.

```
#setting up constraint signs and right-hand sides(supply side)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

#Demand constraints#
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)
```

```
lptrans <- lp.transport(costs, "min", row.signs,row.rhs,col.signs,col.rhs)
```

```r
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

80 AEDs in Plant 2 - Warehouse_1

60 AEDs in Plant 1 - Warehouse_2

40 AEDs in Plant 1 - Warehouse_3

30 AEDs in Plant 2 - Warehouse_3

```r
lptrans$objval
```

```
## [1] 132790
```

The combined cost of production and shipping for the defibrilators is $132,790

```r
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

2)Formulate the dual of the transportation problem

Since the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA).

```r
cost_2 <- matrix(c(622,614,630,100,"u1",
                   641,645,649,120,"u2",
                   80,60,70,220,"-",
                   "v1","v2","v3","-","-"),ncol = 5,nrow = 4,byrow = TRUE)
colnames(cost_2) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","Production Capacity","Supply(Dual)")

rownames(cost_2) <- c("Plant_A","Plant_B","Demand","Demand(Dual)")
```

$$\text{Max } \ VA = 100P_1 + 120P_2 + 80W_1 + 60W_2 + 70W_3$$

Subject to the following constraints Total Profit Constraints

$$u_1 + v_1 <= 622$$

$$u_1 + v_2 <= 614$$

$$u_1 + v_3 <= 630$$

$$u_2 + v_1 <= 641$$

$$u_2 + v_2 <= 645$$

$$u_2 + v_3 <= 649$$

These are taken from the transposed matrix of the Primal of the LP. These are unrestricted where

$$u_k, v_l$$

where u=1,2 and v=1,2,3

```
#Objective function

f.obj <- c(100,120,80,60,70)

#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1), nrow = 6, byrow = TRUE)

f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=")

f.rhs <- c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

Z=139,120 and variables are:

$$u_1 = 614$$

$$u_2 = 633$$

$$v_1 = 8$$

$$v_3 = 16$$

So Z = $139,120 and variables are

$$u_1 = 614$$

which represents Plant A

$$u_2 = 633$$

which stands for Plant B

$$v_1 = 8$$

which stands for Warehouse_1

$$v_2 = 16$$

which stands for Warehouse_3

3) Economic Interpretation of the dual

minimal Z(Primal) = 132790 maximum Z(Dual) = 139120. We came to the conclusion that we shouldn't be shipping from Plants (A/B) to all three Warehouses. Where we ought to be shipping from:

$$60X_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40X_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80X_{13}$$

which is 60 Units from Plant B to Warehouse 1.

$$30X_{13}$$

which is 60 Units from Plant B to Warehouse 3.

We will Max the profit from each distribution to the respective capacity.

From the above using the Sensitivity and Duality, the shadow price can be tested. Change 100 to 101 and 120 to 121 in our LP Transport.

```r
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=",2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=",4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=",2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=",4)

lp.transport(costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```r
lp.transport(costs,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```r
lp.transport(costs,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

The 'min' of the specific function is taken here and we observe that the number goes down by 19. This indicates the shadow price is 19.

From the dual variable

$$v_1$$

where Marginal Revenue <= Marginal Cost. The equation was

$$u_2 \leq 645 - v_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$u_1^0 - v_1^0 \leq 622$$

then we subtract

$$v_1^0$$

to the other side to get

$$u_1^0 \leq 622 - v_1^0$$

The economic interpretation of the dual follows the universal rule of profit maximization which is MR >= MC

```
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

Warehouse1 >= Plant1 + 621 i.e. MR1 >= MC1

$$60X_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40X_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80X_{13}$$

which is 60 Units from Plant B to Warehouse 1.

$$30X_{13}$$

which is 60 Units from Plant B to Warehouse 3.

The condition is not satisfied by Plant B to Warehouse 2. As a result, no AED device will be shipped.