Topics: Confidence Intervals

1. For each of the following statements, indicate whether it is True/False. If false, explain why.

   I. The sample size of the survey should at least be a fixed percentage of the population size in order to produce representative results.

   Ans. False. For a sample size of more than 30, we can use z statistic to produce the results and for a lesser sample size, we can use a t statistic.

   II. The sampling frame is a list of every item that appears in a survey sample, including those that did not respond to questions.

   Ans. False. A sampling frame is a list of every item that appears in a survey sample, including those that will respond to questions.

   III. Larger surveys convey a more accurate impression of the population than smaller surveys.

   Ans. True

2. *PC Magazine* asked all of its readers to participate in a survey of their satisfaction with different brands of electronics. In the 2004 survey, which was included in an issue of the magazine that year, more than 9000 readers rated the products on a scale from 1 to 10. The magazine reported that the average rating assigned by 225 readers to a Kodak compact digital camera was 7.5. For this product, identify the following:

A. The population
B. The parameter of interest
C. The sampling frame
D. The sample size
E. The sampling design
F. Any potential sources of bias or other problems with the survey or sample

Ans. A. All the reader of the PC magazine (more than 9000)
B. Satisfaction rating with different brands of electronics
C. 9000
D. 225
E.
F. The sampling may not have been conducted in a random manner (biased and dependent samples). Also, it is not necessary that a random sample of the population have read the specific issue of the magazine in which the survey was floated. This means all the users of the electronics may not have read the magazine or participated in the survey.

3. For each of the following statements, indicate whether it is True/False. If false, explain why.

   I. If the 95% confidence interval for the average purchase of customers at a department store is $50 to $110, then $100 is a plausible value for the population mean at this level of confidence.

      Ans. True.

   II. If the 95% confidence interval for the number of moviegoers who purchase concessions is 30% to 45%, this means that fewer than half of all moviegoers purchase concessions.

      Ans. True.

   III. The 95% Confidence-Interval for μ only applies if the sample data are nearly normally distributed.

      Ans. False. It is not mandatory for the sample to follow near normal distribution. According to CLT, the samples from a population will follow normal distribution.

4. What are the chances that $\overline{X} > \mu$?

   A. ¼
   B. ½
   C. ¾
   D. 1

   Ans. B. 0.5 This is because, for a standard normal distribution, there is a 50% chance that a sample mean may be greater than the population mean and a 50% chance for the value to be less than the mean.

5. In January 2005, a company that monitors Internet traffic (WebSideStory) reported that its sampling revealed that the Mozilla Firefox browser launched in 2004 had grabbed a 4.6% share of the market.

   I. If the sample were based on 2,000 users, could Microsoft conclude that Mozilla has a less than 5% share of the market?

   II. WebSideStory claims that its sample includes all the daily Internet users. If that's the case, then can Microsoft conclude that Mozilla has a less than 5% share of the market?

   Ans. I. H_0: μ_0 >= 5. H_a: μ_0 < 5. Considering a standard normal distribution, the one sample z test for proportions will give

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.045 - 0.05}{\sqrt{\frac{0.05(1-0.05)}{2000}}} = -1.03$$

Using this value, we get p value of 0.1515 > 0.05. Hence we accept the null hypothesis that the mean is more than or equal to 5% of the market share.

II. If the sample includes all the observations, that means the data is the population data. In the population data, 4.6% of people use Firefox. This means Microsoft can conclude that less than 5% share of the market uses Mozilla.

6. A book publisher monitors the size of shipments of its textbooks to university bookstores. For a sample of texts used at various schools, the 95% confidence interval for the size of the shipment was 250 ± 45 books. Which, if any, of the following interpretations of this interval are correct?

   A. All shipments are between 205 and 295 books.

   B. 95% of shipments are between 205 and 295 books.

   C. The procedure that produced this interval generates ranges that hold the population mean for 95% of samples.

   D. If we get another sample, then we can be 95% sure that the mean of this second sample is between 205 and 295.

   E. We can be 95% confident that the range 160 to 340 holds the population mean.

   Ans. C is correct. There is a 95% chance that a randomly picked value lies between 205 and 295.

7. Which is shorter: a 95% z-interval or a 95% t-interval for μ if we know that σ =s?

   A. The z-interval is shorter
   B. The t-interval is shorter
   C. Both are equal
   D. We cannot say

   Ans. A. The 95% z-interval is shorter than t-interval for the same confidence leve even if we know that σ =s.

Questions 8 and 9 are based on the following: To prepare a report on the economy, analysts need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days.

8. How many randomly selected employers (minimum number) must we contact in order to guarantee a margin of error of no more than 4% (at 95% confidence)?

   A. 600
   B. 400

C. 550
D. 1000

Ans. A. Assuming a $\hat{p}$ value of 0.5, n = $\hat{p}$ x (1-$\hat{p}$) x $(z/E)^2$
We know z = 1.96 for 95% confidence. Also, E is given as 4% = 0.04
So, n = 0.5 x 0.5 x $(1.96/0.04)^2$ = 600.25

9. Suppose we want the above margin of error to be based on a 98% confidence level. What sample size (minimum) must we now use?

A. 1000
B. 757
C. 848
D. 543

Ans. C. Assuming a $\hat{p}$ value of 0.5, n = $\hat{p}$ x (1-$\hat{p}$) x $(z/E)^2$
We know z = 2.326 for 98% confidence. Also, E is given as 4% = 0.04
So, n = 0.5 x 0.5 x $(2.326/0.04)^2$ = 845.6