Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

A. 0.3875
B. 0.2676
C. 0.5
D. 0.6987

Ans. B. Value obtained is 0.26598552904870054 using python script:
"1 - stats.norm.cdf(50, loc = 45, scale = 8)"

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and 44.
B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. A) Statement is **False**. Probability of age > 38 = 0.5. number of people with age in range of 38 $\pm$ 6 is 65% = 0.65 probability. That means, for 38+6 = 44, probability is 0.325. So, probability that age > 44 is = 0.5 - 0.325 = 0.175. As the probability of age in range 38 to 44 is 0.325 which is greater than the probability of age being > 44, hence the statement is False.
B) The statement is **True**. This is because the probability that the age lies below 30 is 0.09121121972586788 from the python statement "stats.norm.cdf(30, loc = 38, scale = 6)". Now, for 400 employees, the approximate number of people becomes = 400 x P(one person) = 36.484487890347154

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans. If X_1 and X_2 are two independent random variables, then $X_1 + X_2 \sim N(\mu + \mu , \sigma^2 + \sigma^2)$. So $X_1 + X_2 \sim N(2\mu , 2\sigma^2)$. However, $2X_1 \sim N(2\mu , 4\mu^2)$. This is the difference. For $2X_1$, the standard deviation term has the coefficient value of 4 and in the case of $X_1 + X_2$, the value is 2.

4. Let X ~ $N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Ans. D. 48.5, 151.5
The probability that value takes a value 0.99 means, for a normal distribution, the probability that the value on either side of distribution is 0.01. This means, for one tail, the excluded probability is 0.005. Using python code "stats.norm.ppf(0.005, loc = 100, scale = 20)" , we get the minimum range as 48.5. The difference is 51.5 from 100 (mean). Adding this to the mean, we get the other limit (= 151.5)

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans. $P_1 \sim N(5, 3^2)$ and $P_2 \sim N(7, 4^2)$. So, $P = P_1 + P_2 \sim N(5+7,\ 3^2+4^2) \sim N(12, 5^2)$.

A. 95% probability means 5% outside the range. So, for one tail, 2.5% = 0.025. Using python code "stats.norm.ppf(0.025, loc = 12, scale = 5)", we get minimum value as 2.2. This means a difference of 9.8 from mean value (12). So, the maximum value is 12 + 9.8 = 21.8. So the interval is ($2.2, $21.8). Converting this into rupees, we get (Rs.99, Rs.981)
B. The fifth percentile can be found out by using the python code: "stats.norm.ppf(0.05, loc = 12, scale = 5)". The value comes out to be $3.7757318652426353. The equivalent amount in rupees is Rs. 169.9
C. Using the python code "stats.norm.cdf(0, loc = 5, scale = 3)" and "stats.norm.cdf(0, loc = 7, scale = 4)", we get the probabilities for making loss in P_1 as 0.04779 and P_2 as 0.040059 which means that the probability for making a loss is more for section with $P_1 \sim N(5, 3^2)$.