

Analysis of the Effect of Ball Type and Drop Height on Maximum Rebound Height

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Introduction

We did an experiment in order to see if the ball type and drop height affect the maximum rebound height. In order to complete this experiment successfully, we set up a meter stick against the wall and secured it. Then we followed a randomized trial order and dropped specific balls from specific heights. When the drop occurs, we made sure to record the drop and rebound height in slow motion. Finally, we noted down the maximum rebound height in an excel sheet based on the resulting video. Based on the results we got, we made a Two-way ANOVA model and performed Tukey-adjusted pairwise comparisons.

Experimental Design

For our experiment, the response variable was the maximum rebound height of the ball (cm). We had 2 factors which were the ball type and drop height with 3 levels in each of them.

Ball Type (Factor 1)

- Level 1: Tennis Ball
- Level 2: Golf Ball
- Level 3: Ping Pong Ball

Drop Height (Factor 2)

- Level 1: 50 cm
- Level 2: 75 cm
- Level 3: 100 cm

Treatment Combinations:

- Tennis Ball with 50 cm drop
- Tennis Ball with 75 cm drop
- Tennis Ball with 100 cm drop
- Golf Ball with 50 cm drop

- Golf Ball with 75 cm drop
- Golf Ball with 100 cm drop
- Ping Pong Ball with 50 cm drop
- Ping Pong Ball with 75 cm drop
- Ping Pong Ball with 100 cm drop

Our controlled variables were hard tile floor, measurement method, instrument, room conditions, ball releaser, ball condition and the testing day. Our experimental and observational unit is each individual ball drop. We had 4 replicates per treatment combination with 36 total number of trials. For our randomization, we randomized the order of trials in R and our tests were conducted in the specified order.

Statistical Methods

The statistical model is a Two-Way ANOVA in the form:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

What each of the terms in the model mean:

Y_{ijk} = maximum rebound height for replicate k at drop height i and type of ball j.

μ = overall mean of maximum rebound height

α_i = Drop Height effect

β_j = Type of ball effect

$(\alpha\beta)_{ij}$ = Interaction between drop height and type of ball effect.

ε_{ijk} = Error

i = 1,2,3; j = 1,2,3; k = 1,2,3,4

The assumptions for this model are:

$$\varepsilon_{ijk} \sim N(0, \sigma^2)$$

The errors are normal, independent with constant variance.

The constraints for this model are:

$$\sum \alpha_i = \sum \beta_j = \sum (\alpha\beta)_{ij} = 0$$

This is required to keep the model from being over-parameterized.

Data Analysis Process: First, rebound height data were collected for each trial of the experiment.

A Two-Way ANOVA model including the interaction between ball type and drop height was then fitted using R. A Global F-test was conducted to assess the significance of the interaction effect.

If the interaction was not statistically significant, the main effects of ball type and drop height were tested. Tukey-adjusted pairwise comparisons were used to find specific differences among our treatment levels. Throughout our analysis, we used a significance level of $\alpha = 0.05$.

Results

Before testing the factorial effects, the assumptions underlying the two-way ANOVA were first evaluated. Specifically, residual diagnostics from the fitted model were examined using a Normal Q-Q plot to assess normality and a Residuals versus Fitted Values plot to assess constant variance, as shown in Fig. 1a and b. Overall, the residuals closely followed the reference line in the Q-Q plot, indicating approximate normality. In addition, the residuals were randomly scattered around zero in the residuals versus fitted plot, with no apparent pattern or funnel shape.

Consequently, the assumptions of normality and homoscedasticity were considered reasonable, and no data transformation or corrective action was required.

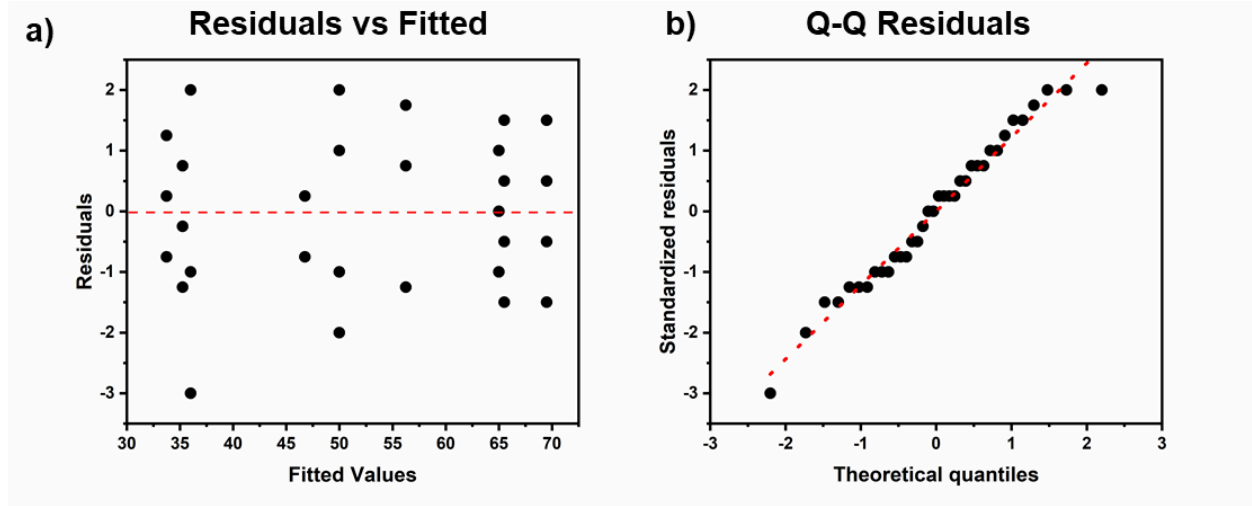


Fig 1: Residual diagnostics for the two-way ANOVA: (a) Residuals vs fitted values and (b) Normal Q–Q plot.

Furthermore, the experimental design was balanced, consisting of three ball types and three drop heights with four replicates per treatment combination, resulting in a total sample size of $n = 36$. This balanced structure supports the robustness of the ANOVA results.

Two-way ANOVA and interaction effects

After verifying the model assumptions, a Type I (sequential) two-way ANOVA was conducted to examine the effects of ball type, drop height, and their interaction on maximum rebound height at a significance level of $\alpha = 0.05$. The ANOVA results are summarized in Table 1.

Here is the hypothesis for interaction:

$$H_0: (\alpha\beta)_{ij} = 0 \text{ for all } i, j \quad H_a: (\alpha\beta)_{ij} \neq 0 \text{ for at least one pair } (i, j)$$

ANOVA Table					
Source of Variation	Df	Sum of Squares	Mean Square	F-Statistic	P-Value
Ball	2	109	54.4	27.69	2.88×10^{-7}
Height	2	6017	3008.4	1532.60	$<2 \times 10^{-16}$
Ball*Height	4	137	34.2	17.44	3.52×10^{-7}
Residuals	27	53	2.0		

Table 1: Type I two-way ANOVA results for rebound height.

Since the interaction between ball type and drop height was statistically significant, $F(4, 27) = 17.44$, $p = 3.52 \times 10^{-7} < 0.05$. As a result, the null hypothesis was rejected, demonstrating that the effect of drop height on rebound height depends on the type of ball.

This interaction is further supported by the interaction plot shown in Fig. 2. In this plot, the mean rebound-height lines across drop heights are non-parallel for different ball types, visually confirming that the relationship between drop height and rebound height varies among balls. Therefore, both factors jointly influence rebound behavior.

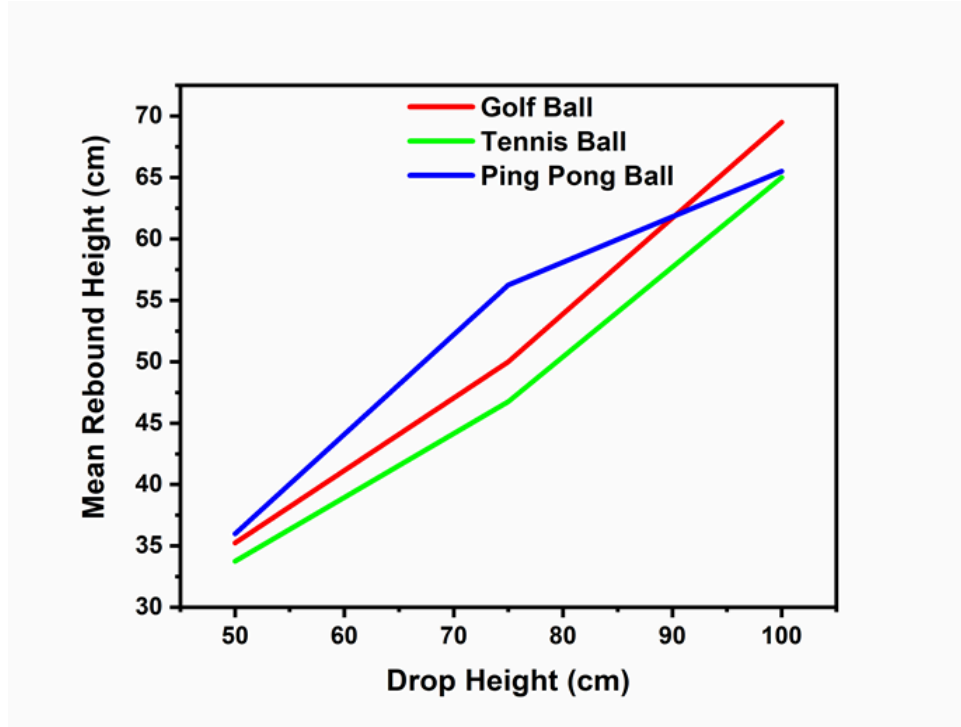


Fig 2: Interaction plot of mean rebound height versus drop height for each ball type.

Tukey-adjusted pairwise comparisons

Following the significant Ball Type \times Drop Height interaction observed in the two-way ANOVA, Tukey-adjusted pairwise comparisons were conducted within each drop height to examine differences among ball types while controlling the family-wise error rate at $\alpha = 0.05$. The results of these comparisons are presented in Table 2 and are interpreted separately for each drop height.

At a drop height of 50 cm, none of the pairwise comparisons among golf, ping pong, and tennis balls were statistically significant (all adjusted p-values > 0.05). This indicates that, at the lowest drop height, rebound heights did not differ significantly across ball types.

At a drop height of 75 cm, statistically significant differences were observed among all ball types. Ping pong balls rebounded significantly higher than golf balls (difference = 6.25 cm, $p <$

0.0001) and tennis balls (difference = 9.50 cm, $p < 0.0001$). In addition, golf balls rebounded significantly higher than tennis balls (difference = 3.25 cm, $p = 0.0078$). These results indicate a clear ordering of rebound performance at 75 cm, with ping pong balls exhibiting the highest rebound, followed by golf balls, and then tennis balls.

At a drop height of 100 cm, golf balls rebounded significantly higher than both ping pong balls (difference = 4.00 cm, $p = 0.0011$) and tennis balls (difference = 4.50 cm, $p = 0.0003$). However, no statistically significant difference was observed between ping pong and tennis balls at this height (difference = 0.50 cm, $p = 0.8697$). This suggests that, at the highest drop height, golf balls exhibit superior rebound performance, while ping pong and tennis balls behave similarly.

Overall, these height-specific Tukey comparisons further confirm the presence of a significant interaction effect. Specifically, the relative rebound performance of the ball types changes with drop height, demonstrating that the effect of ball type on rebound height depends strongly on the drop height.

Drop Height	Comparison	Difference	p-value	Significant ?
50 cm	Golf - Ping Pong	-.75	0.7320	No
	Golf - Tennis	1.50	0.3003	No
	Ping Pong - Tennis	2.25	0.0774	No
75 cm	Golf - Ping Pong	-6.25	<.0001	Yes
	Golf - Tennis	3.25	0.0078	Yes
	Ping Pong - Tennis	9.50	<.0001	Yes
100 cm	Golf - Ping Pong	4.00	0.0011	Yes
	Golf - Tennis	4.50	0.0003	Yes
	Ping Pong - Tennis	.50	0.8697	No

Table 2: Tukey-adjusted pairwise comparisons of ball types within each drop height.

Conclusion

Based on the results, drop height significantly affects the rebound height for all ball types. Higher drops produced higher rebounds. The ball type also affected the rebound height where the effect depends on the drop height (significant interaction). A challenge we had while conducting this experiment was the room lighting in the room we did the experiment making the video a bit unclear and so we addressed this by using a phone flashlight to illuminate the trial area. Another challenge we had while conducting this experiment was having consistent ball drops so we addressed this by making sure to practice the procedure before running the trials and also by

having the same person release the ball in all of the trials. Some potential improvements were to use a non-human mechanism to perfectly drop the ball and not have human error, a fixed stand to hold the recording device to ensure consistent readings and not have human error, and also just add more replicates per treatment.