Engineering Electromagnetics

Lecture 8

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by

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Example 1.13. Find the volume of a sphere of radius R.

Solution

$$V = \int d\tau = \int_{r=0}^{R} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \left(\int_0^R r^2 \, dr \right) \left(\int_0^{\pi} \sin\theta \, d\theta \right) \left(\int_0^{2\pi} d\phi \right)$$

$$= \left(\frac{R^3}{3} \right) (2)(2\pi) = \frac{4}{3} \pi R^3$$

Problem-2

Show that over the closed surface of a sphere of radius b, $\oint d\vec{s} = 0$.

EXAMPLE 2.14 Show that over the closed surface of a sphere of radius b, $\oint \vec{ds} = 0$.

Solution The outward unit normal to the surface of a sphere of radius b is in the direction of the unit vector $\vec{\mathbf{a}}_r$, as shown in Figure 2.25. Therefore,

$$\oint_s \vec{ds} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \vec{\mathbf{a}}_r b^2 \sin\theta \, d\theta \, d\phi$$

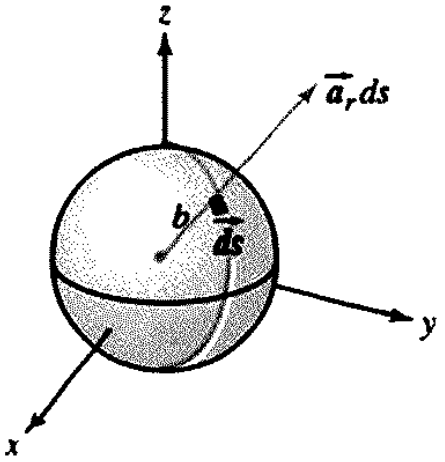

Figure 2.25

Because the unit vector $\vec{\mathbf{a}}_r$ is a function of both θ and ϕ, we must express it in terms of unit vectors in the rectangular coordinate system before integrating. From equation (2.43a,b) we have $\vec{\mathbf{a}}_r = \sin\theta\cos\phi\,\vec{\mathbf{a}}_x + \sin\theta\sin\phi\,\vec{\mathbf{a}}_y + \cos\theta\,\vec{\mathbf{a}}_z$. Thus,

$$\oint_s \vec{ds} = \vec{\mathbf{a}}_x b^2 \int_0^{\pi} \sin^2\theta \, d\theta \int_0^{2\pi} \cos\phi \, d\phi + \vec{\mathbf{a}}_y b^2 \int_0^{\pi} \sin^2\theta \, d\theta$$
$$\times \int_0^{2\pi} \sin\phi \, d\phi + \vec{\mathbf{a}}_z b^2 \int_0^{\pi} \sin\theta\cos\theta \, d\theta \int_0^{2\pi} d\phi$$
$$= 0$$

• • •

Problem-3

The electron density distribution within a spherical volume with radius of 2 meters is given as $n_e = (1000/r)\cos(\phi/4)$ electrons/meter^3. Find the charge enclosed if the charge on an electron is -1.6×10^{-19} coulomb.

EXAMPLE 2.16 The electron density distribution within a spherical volume with radius of 2 meters is given as $n_e = (1000/r)\cos(\phi/4)$ electrons/meter^3. Find the charge enclosed if the charge on an electron is -1.6×10^{-19} coulomb.

Solution Let N be the number of electrons in the region bounded by a sphere of 2-meter radius; then

$$N = \int_v n_e\,dv = \int_v \frac{1000}{r}\cos(\phi/4)\,dv$$
$$= \int_0^2 \frac{1000}{r} r^2\,dr \int_0^\pi \sin\theta\,d\theta \int_0^{2\pi} \cos(\phi/4)\,d\phi$$
$$= 16{,}000 \text{ electrons}$$

Thus, the total charge enclosed is $Q = 16{,}000(-1.6 \times 10^{-19}) = -2.56 \times 10^{-15}$ coulomb. • • •

“Ordinary” Derivatives

- Suppose we have a function of one variable: f (x).
- Question: What does the derivative, df/dx, do for us? Answer: It tells us how rapidly the function f (x) varies when we change the argument x by a tiny amount, dx:
- If we increment x by an infinitesimal amount dx, then f changes by an amount df ; the derivative is the proportionality factor.

$$df = \left(\frac{df}{dx}\right) dx$$

- The derivative df/dx → slope of the graph of f versus x.

Gradient

- Suppose, now, that we have a function of three variables→ T (x, y, z)
- "How fast does T vary?"

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

This tells us how *T* changes when we alter all three variables by the infinitesimal amounts *dx, dy, dz*. Notice that we do *not* require an infinite number of derivatives—*three* will suffice: the *partial* derivatives along each of the three coordinate directions.

$$dT = \left(\frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}\right) \cdot (dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}})$$

$$= (\nabla T) \cdot (d\mathbf{l}), \longleftarrow d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

$$\nabla T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

Gradient of **T**

Scalar/Vector?

Geometrical interpretation

where θ is the angle between ∇T and $d\mathbf{l}$

- Like any vector, the gradient has magnitude and direction.
- To determine its geometrical meaning ⇨ $dT = \nabla T \cdot d\mathbf{l} = |\nabla T||d\mathbf{l}|\cos\theta$
- Now, if we fix the magnitude |dl| and search around in various directions (that is, vary θ), the maximum change in T is when?
- for a fixed distance |dl|, dT is greatest when I move in the same direction as **∇T.**
- The gradient **∇T** points in the direction of maximum increase of the function T
- The magnitude |**∇T** | gives the slope (rate of increase) along this maximal direction
- <u>Gradient tells you how much something changes as you move from one point to another (such as the pressure in a stream). The gradient is the multidimensional rate of change of a particular function.*</u>
- Q: What would it mean for the gradient to vanish?
- f (x, y, z) = $x^2 + y^3 + z^4$ at a point (2,1,0)

$$\nabla T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

*https://qsstudy.com/physics/physical-significance-gradient#:~:text=Gradient%20tells%20you%20how%20much,change%20of%20a%20particular%20function.

Geometrical interpretation

- Gradient tells you how much something changes as you move from one point to another (such as the pressure in a stream). The gradient is the multidimensional rate of change of a particular function.*

⇨ $dT = \nabla T \cdot d\mathbf{l} = |\nabla T||d\mathbf{l}|\cos\theta$ where θ is the angle between ∇T and $d\mathbf{l}$

$$\nabla T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

Q: f (x, y, z) = $x^2 + y^3 + z^4$ find ∇f at a point (2,1,0)

Q: f(x, y, z) = $x^2 + y^2 + z^2$ find ∇f at (1,1,1)

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Solutions

- f (x, y, z) = $x^2 + y^3 + z^4$ at a point (2,1,0)
- $\boldsymbol{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} = 2x\ \hat{x} + 3y^2\hat{y} + 4z^3\hat{z} = 4\hat{x} + 3\hat{y}$
- f(x, y, z) = $x^2 + y^2 + z^2$ at (1,1,1)
- $\boldsymbol{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} = 2x\hat{x} + 2y\hat{y} + 2z\hat{z} = ?$

Dot product

$$\boxed{\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}.}$$

From the definition of ∇ we construct the divergence:

$$\begin{aligned}\nabla \cdot \mathbf{v} &= \left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) \cdot (v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}) \\ &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}.\end{aligned}$$

Vector or scalar?

Examples

- If $\mathbf{v} = x\hat{x} + y\hat{y} - \hat{z}$, $\nabla.\mathbf{v} =?$
- If $\mathbf{v} = 2\hat{y}$, $\nabla.\mathbf{v} =?$
- If $\mathbf{v} = x^2\hat{x}, \nabla.\mathbf{v} =?$

$$\boxed{\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}.}$$

Thank You