Engineering Optics

Lecture 34

06/06/2023

by

Debolina Misra

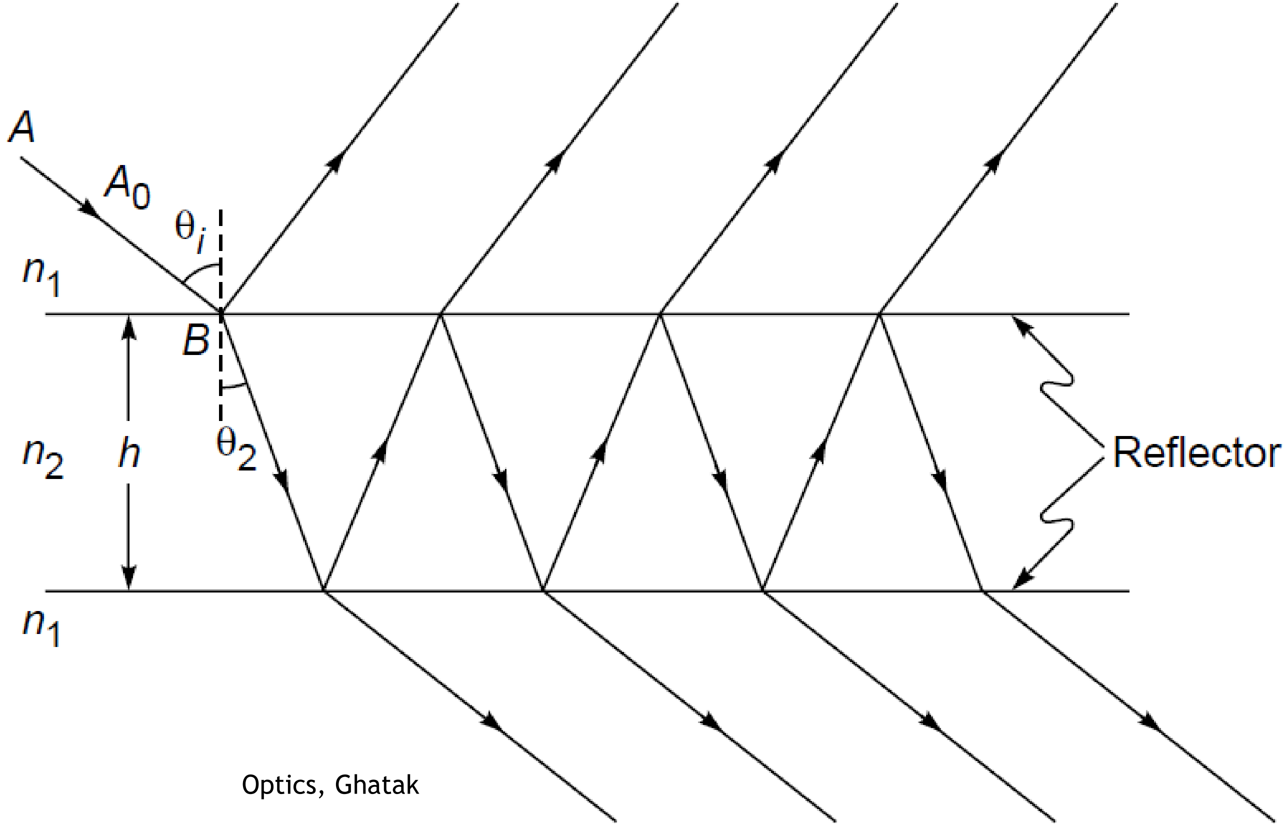
Dept of Physics
IIITDM Kancheepuram, Chennai, India

The Fabry-Perot interferometer

Multiple reflections from a plane parallel film

We consider the incidence of a plane wave on a plate of thickness h (and of refractive index n_2) surrounded by a medium of refractive index n_1

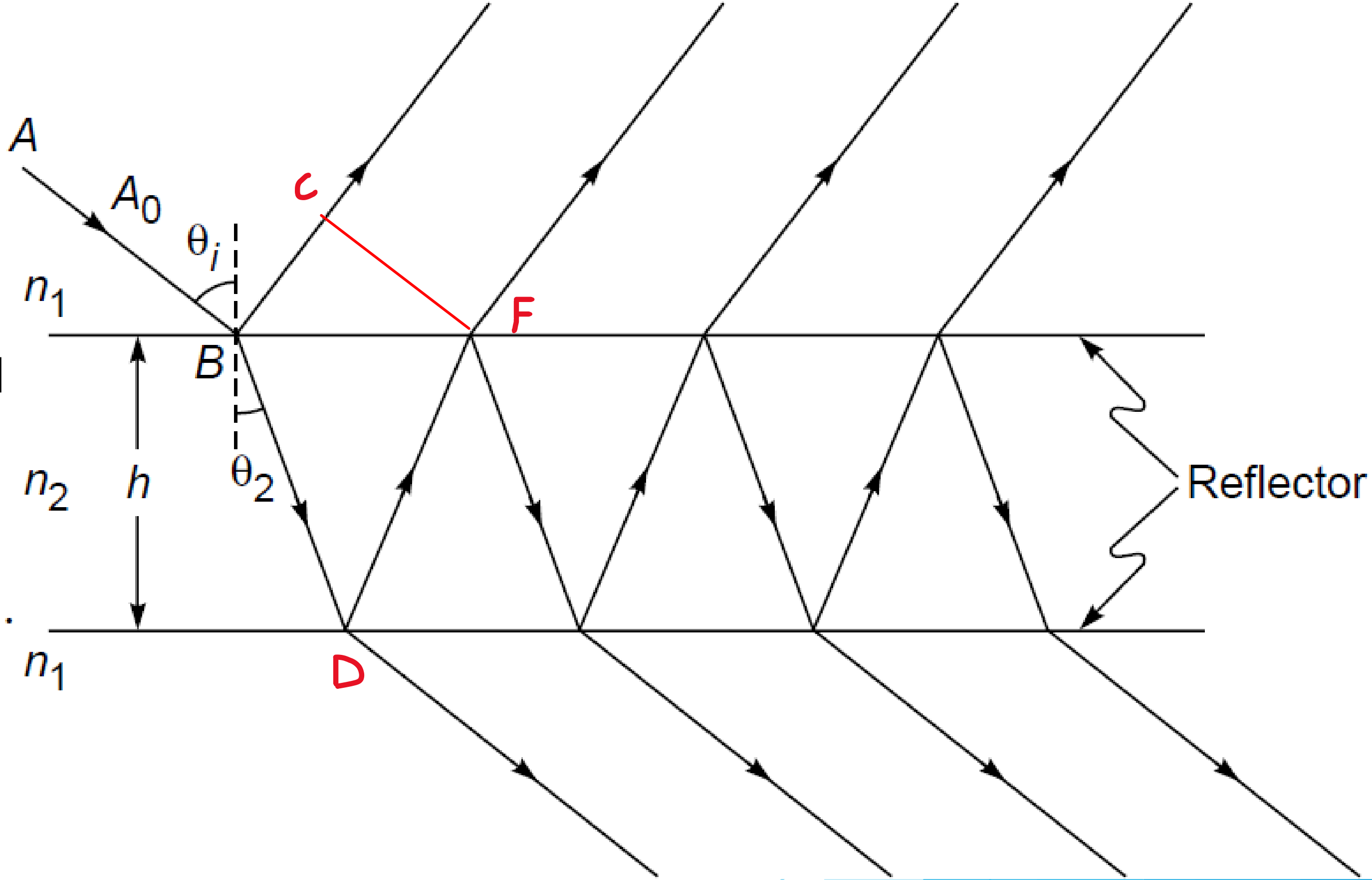
- Let A_0 be the (complex) amplitude of the incident wave.
- The wave will undergo multiple reflections at the two interfaces
- when the wave is incident from n_1 toward n_2:
- r_1 and t_1 represent the amplitude reflection and transmission coefficients, respectively
- When the wave is incident from n_2 toward n_1 → r_2 and t_2 represent the corresponding coefficients.



Optics, Ghatak

Multiple reflections from a plane parallel film

- A_0 → amplitude of the incident wave.
- When the wave is from n_1 toward n_2: r_1, t_1
- from n_2 toward n_1 → r_2 and t_2
- Thus the amplitude of the successive reflected waves will be



$$A_0 r_1,\ A_0 t_1 r_2 t_2 e^{i\delta},\ A_0 t_1 r_2^3 e^{2i\delta},\ \ldots$$

where $$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda_0}$$

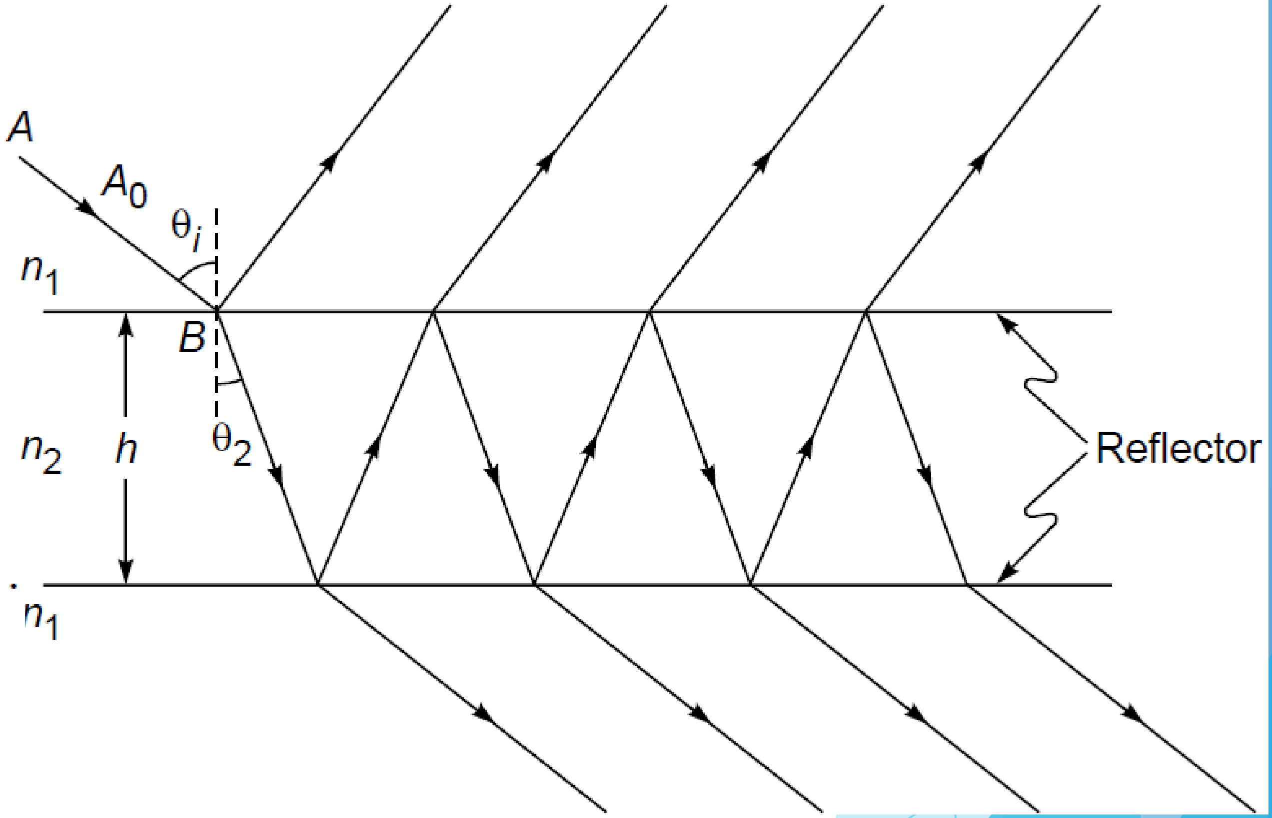
represents the phase difference (between two successive waves emanating from the plate)

Extra path = $n_2(BD+DF)-n_1BC$
Now check Δ
For the calculation see Section 15.3 “The Cosine Law”, Ghatak’s book

Optics, Ghatak

Multiple reflections from a plane parallel film

- A_0 → amplitude of the incident wave.
- When the wave is from n_1 toward n_2: r_1, t_1
- from n_2 toward n_1 → r_2 and t_2
- Thus the amplitude of the successive reflected waves will be



$$A_0 r_1,\ A_0 t_1 r_2 t_2 e^{i\delta},\ A_0 t_1 r_2^3 e^{2i\delta},\ \ldots$$

where $$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda_0}$$

represents the phase difference (between two successive waves emanating from the plate) due to the additional path traversed by the beam in the film

$$A_r = A_0[r_1 + t_1 t_2 r_2 e^{i\delta}(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots)]$$

$$= A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$

resultant amplitude of the reflected wave

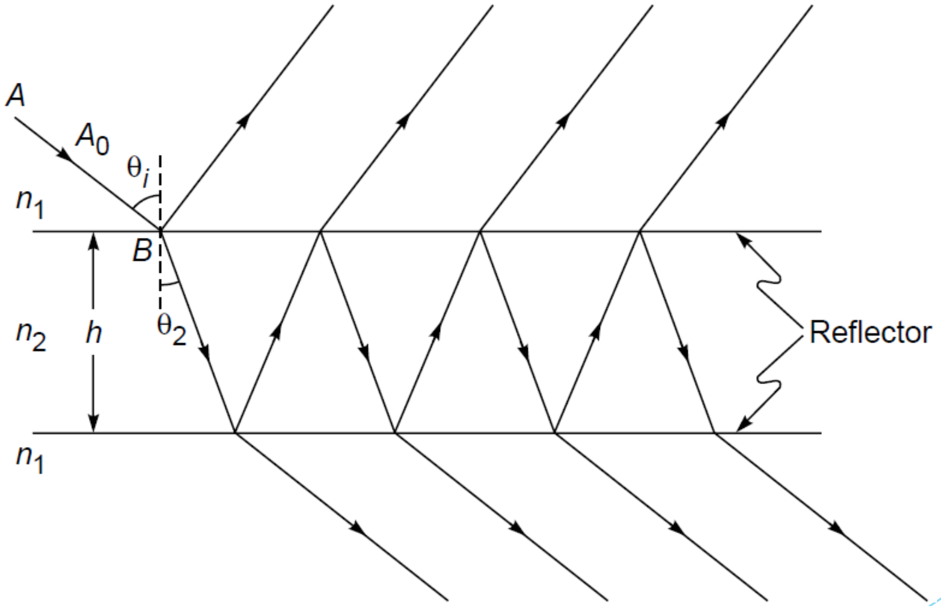
Optics, Ghatak

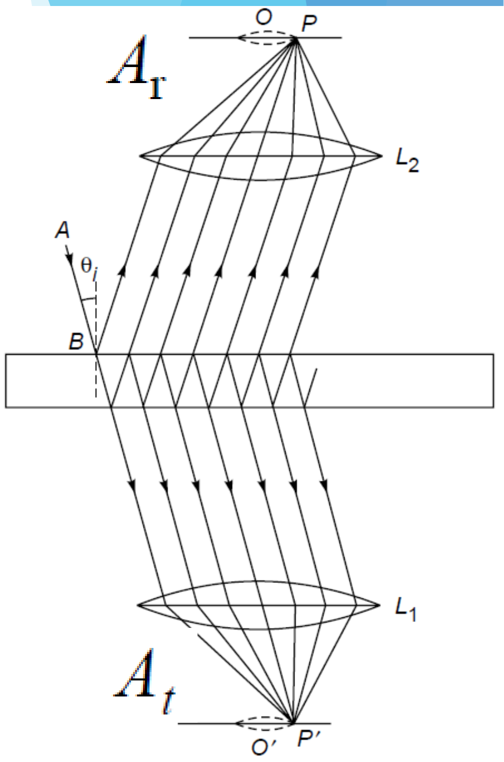
Resultant amplitude of reflected wave

$$A_r = A_0[r_1 + t_1 t_2 r_2 e^{i\delta}(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots)]$$

$$= A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$

$$\boxed{\frac{A_r}{A_0} = r_1\left[1 - \frac{(1-R)e^{i\delta}}{1 - Re^{i\delta}}\right]}$$





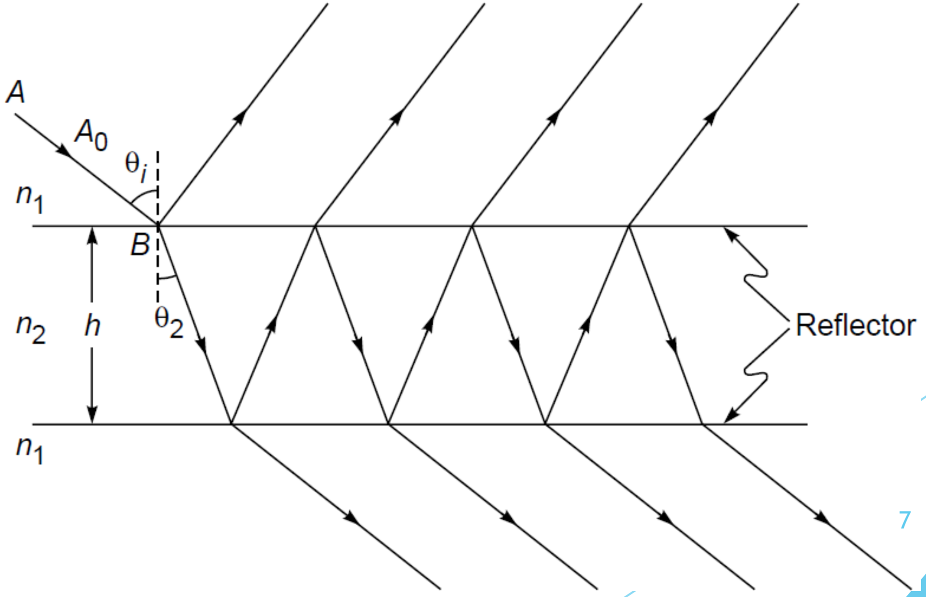
Optics, Ghatak

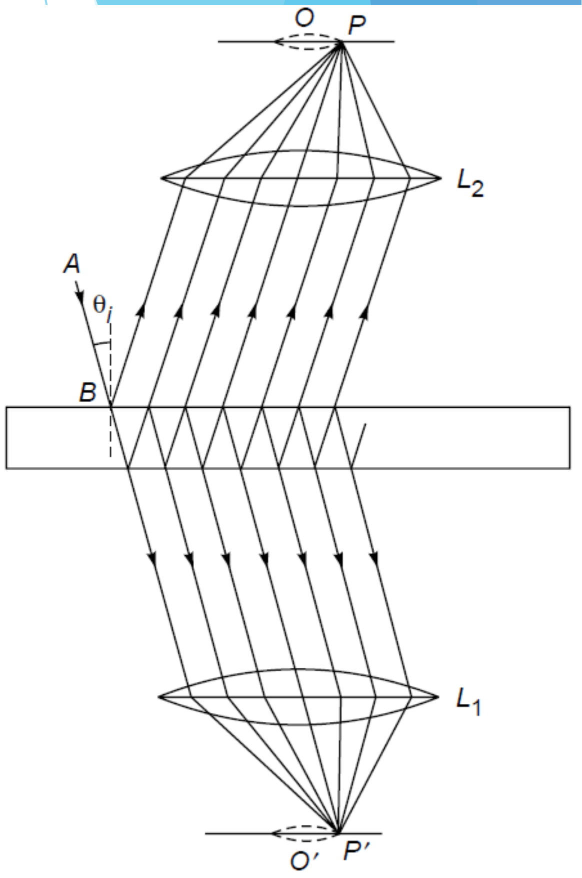
Resultant amplitude of transmitted wave

resultant amplitude of the transmitted wave

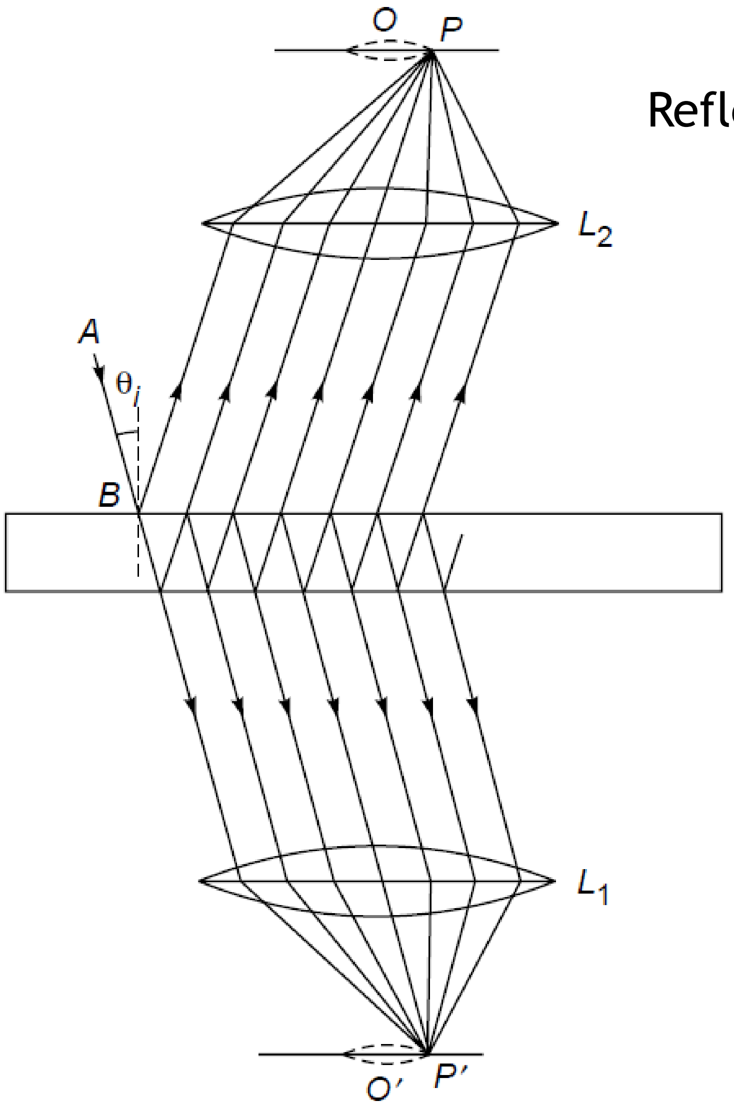
$$A_t = A_0 t_1 t_2 \left(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots\right)$$

$$= A_0 \frac{t_1 t_2}{1 - r_2^2 e^{i\delta}} = A_0 \frac{1 - R}{1 - R e^{i\delta}}$$





Optics, Ghatak



Reflectivity of the instrument using the interface

$$\mathcal{R} = \left|\frac{A_r}{A_0}\right|^2 = R\left|\frac{1-e^{i\delta}}{1-Re^{i\delta}}\right|^2$$

Reflectivity of the given interface

$$R = r_1^2 = r_2^2$$

$$\tau = t_1 t_2 = 1 - R$$

$$= R\,\frac{(1-\cos\delta)^2+\sin^2\delta}{(1-R\cos\delta)^2+R^2\sin^2\delta}$$

$$= \frac{4R\sin^2\delta/2}{(1-R)^2+4R\sin^2\delta/2}$$

$$\mathcal{R} = \frac{F\sin^2\delta/2}{1+F\sin^2\delta/2}$$

F=Coefficient of Finesse

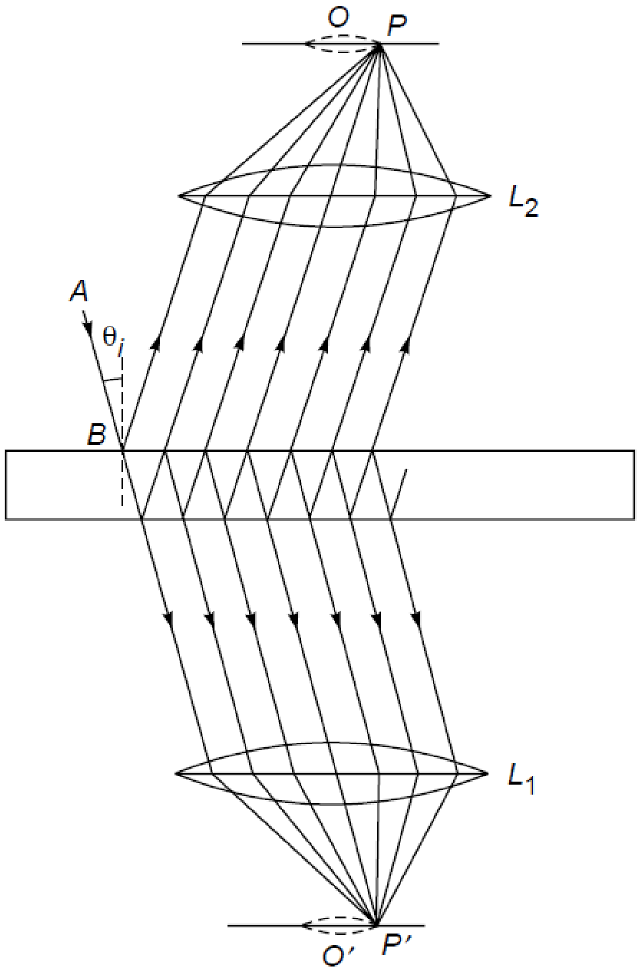
$$T = \left|\frac{A_t}{A_0}\right|^2 = \frac{(1-R)^2}{(1-R\cos\delta)^2+R^2\sin^2\delta}$$

$$T = \frac{1}{1+F\sin^2\delta/2}$$

Multiple reflections from a plane parallel film

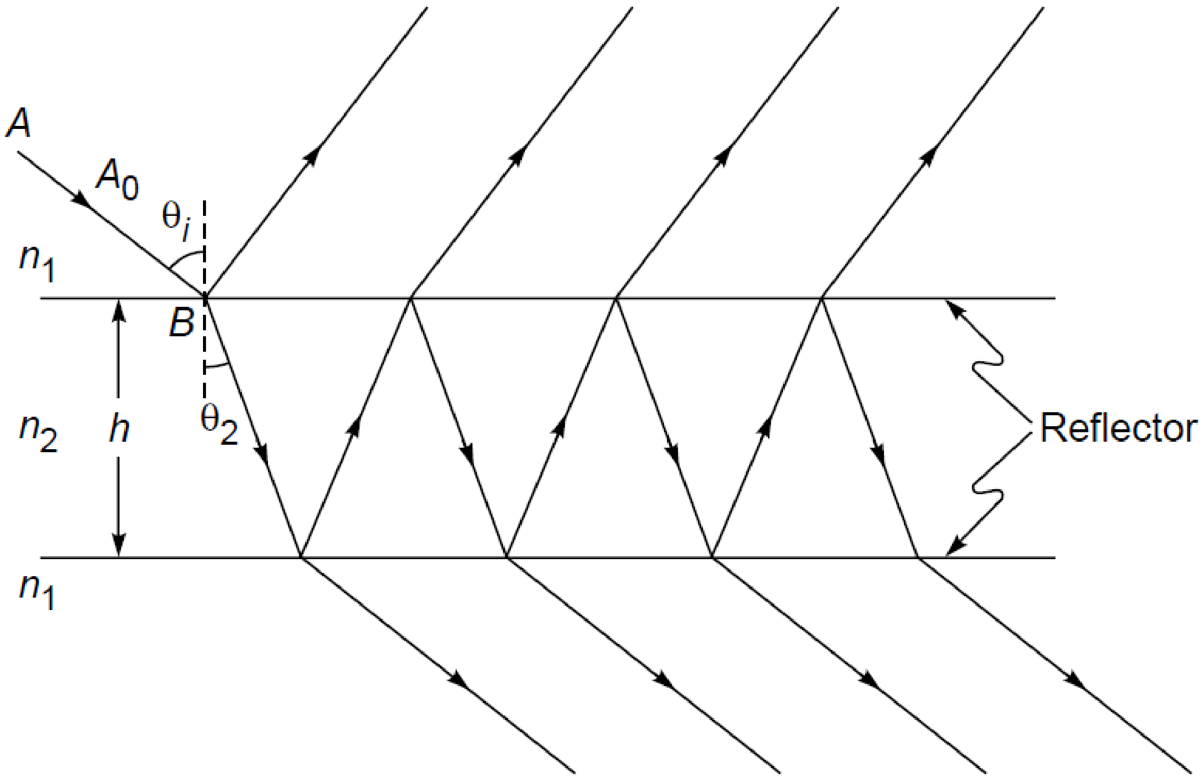
Any ray parallel to AB will focus at the same point P. If ray AB is rotated about the normal at B, then point P will rotate on the circumference of a circle centered at point O; this circle will be bright or dark depending on the value of θ_i .

Rays incident at different angles will focus at different distances from point O, and one will obtain concentric bright and dark rings for an extended source.



Optics, Ghatak

Multiple reflections from a plane parallel film



Q: *Do R and τ give any hint about single/multiple reflection?*
What else do you need for multiple reflection?

http://www1.udel.edu/chem/sneal/sln_tchng/CHEM620/CHEM620/Chi_4._Light_at_Interfaces.html

(1) We've 2 media (say air-glass/ air-diamond/ water-glass etc.).
2 different media → hence 2 different r.i. (n_1, n_2)

(2) For a particular interface/pair (air-glass/water-glass etc.) reflection coefficient

$$r = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

For upper and lower surfaces $r_1 = -r_2$

(3) For that particular interface:
Reflectivity $R = r^2$
Transmittivity $\tau = 1 - R$

(4) Generally T → here τ to avoid confusion between transmittivity for a given interface and transmittivity of a device having multiple reflection and transmission

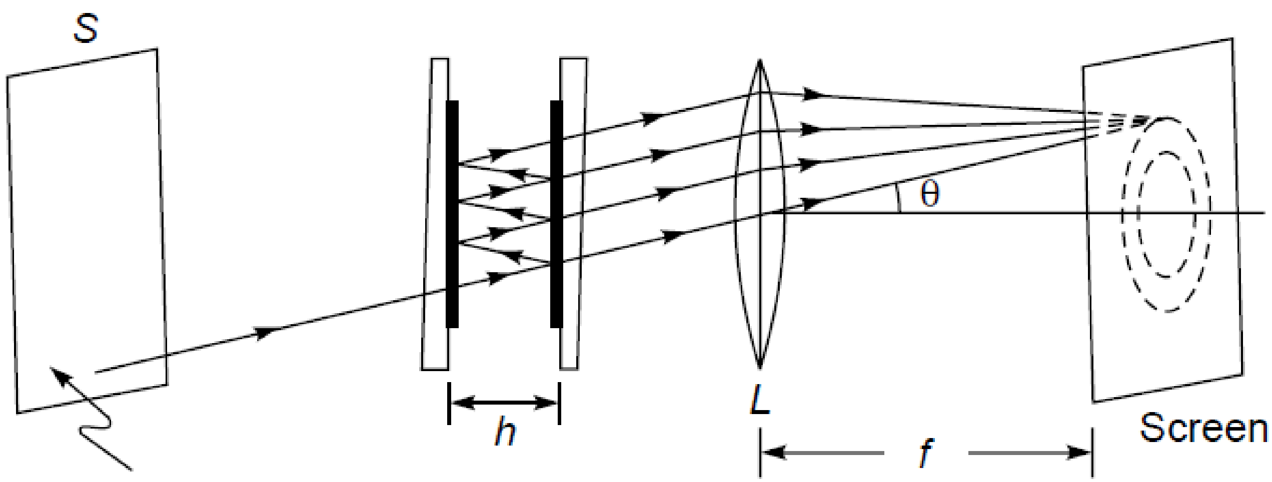
Fabry–Perot interferometer

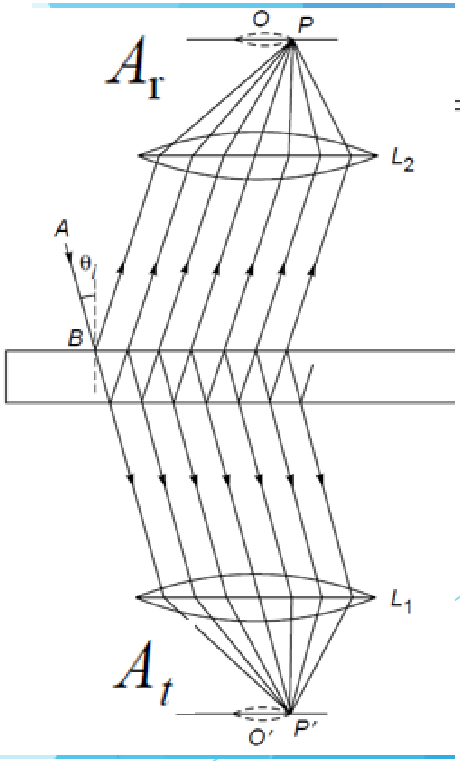
Reflectivity of the instrument

$$\mathcal{R} = \left| \frac{A_r}{A_0} \right|^2 = R \left| \frac{1 - e^{i\delta}}{1 - R e^{i\delta}} \right|^2$$

$$= R \frac{(1 - \cos\delta)^2 + \sin^2\delta}{(1 - R\cos\delta)^2 + R^2 \sin^2\delta}$$

$$= \frac{4R \sin^2 \delta/2}{(1-R)^2 + 4R \sin^2 \delta/2}$$





Transmittivity of the instrument (not τ → property of the 2 media forming interface)

$$T = \left| \frac{A_t}{A_0} \right|^2 = \frac{(1-R)^2}{(1 - R\cos\delta)^2 + R^2 \sin^2\delta}$$

Optics, Ghatak

Problem:1

Consider a monochromatic beam of wavelength 6000 Å incident (from an extended source) on a Fabry–Perot etalon with $n_2 = 1$, $h = 1$ cm, and $F = 200$. Concentric rings are observed on the focal plane of a lens of focal length 20 cm . Calculate the reflectivity of each mirror.

Answer:

$$F = \frac{4R}{(1-R)^2} \quad \Rightarrow \quad 200 = \frac{4R}{(1-R)^2}$$

$$50 = \frac{R}{(1+R^2-2R)}$$

$$50 + 50R^2 - 100R = R$$

$$50R^2 - 101R + 50 = 0$$

By solving above quadratic equation,

$$R = 1.15 \text{ or } 0.87$$

R=0.87 (Since R will be less than 1)

Thank You