Roll No: CS22B1069

Indian Institute of Information Technology, Design and Manufacturing, Kancheepuram
End Semester Exam, July 2023

Course Code: CS1004
Course Title: Data Structures and Algorithms
Batch: Batch A and B
Category: Core
Date of Examination: 03.07.2023
Instructors: Preeth, Muneeswaran
Duration: 3 Hours (180 Mins)
Maximum Marks: 40

---

1. Professor Banyan thinks he has discovered a remarkable property of binary search trees. Suppose that the search for key k in a BST ends up in a leaf. Consider three sets:(1) set A, the keys to the left of the search path,(2) set B, the keys on the search path and (3) set C, the keys to the right of the search path. Professor Banyan claims that any three keys a$\in$ A, b$\in$B, c$\in$ C must satisfy a $\leq$b$\leq$c. Justify it with example if his claim is true? If not justify it with suitable example. **(2 Marks)**

2. Nalini has been given with a random sequence (consisting of elements) examined in binary search tree with starting element as root.

$$1, 11, 111, 100, 90, 30, 89, 60, 75, 85, 88$$

Pranav has to search for the element 88 in the given binary search tree sequence. Is the given sequence is valid? Justify your answer and trace the Binary Search Tree in support of your answer **(2 Marks)**

3. Consider that monkeys are hanging in the branches of a mango tree which structurally resembles like a full binary tree. The monkeys that are having child monkeys are known as star. If 8 star monkeys are present in the mango tree, how many total monkeys are present in the entire mango tree? **(1 Mark)**

4. Zaheer attempts to construct a AVL tree with height 'h' which is less dense in nature. So, he wants to place minimal number of nodes in tree and it must also preserve the property of AVL tree. So, he attempts to derive a generalized equation. Write a generalized equation that satisfies this condition for any given value of 'h'. Justify your answer with explanation by constructing this type of AVL tree for any given 'h' value. **(2 Marks)**
[*Note: He considered that the depth and height of the root node as* 0]

5. Given a directed graph (V, E) where every edge has weight as either 1 or 2. The expected time complexity for determining the shortest path from a given source vertex 's' to a given destination vertex 't' is $O(V + E)$. How this can be achieved ? Trace the sequence from source to destination vertex. **(2 Marks)**

6. Consider the algebraic expression. Draw the expression tree corresponding to E. **(2 Marks)**

$$E = (5x + z) \times (3a - b)^2$$

7. How to implement an queue data structure that can hold Compact Disks(CD) of size 'N'? Rachel claims that "This can be done with atmost two vertical bin-case pouches,that resemble like a stack". Is Rachel's claim is true? Justify your answer. Analyze the running time of the *enQueue* and *deQueue* operation costly with respect to its implementation. **(2 Marks)**

8. For a given Tree T,

$$PREORDER: 1, 2, 4, 8, 9, 5, 3, 6, 7$$
$$POSTORDER: 8, 9, 4, 5, 2, 6, 7, 3, 1$$

All the non leaf nodes of T have two children. Can we uniquely construct the binary tree ? Justify your answer by tracing the steps in constructing binary tree T. **(2 Marks)**

9. Nancy claims that Radix sort does not work correctly (i.e., does not produce the correct output) if Insertion Sort is used to sort each individual digit instead of Counting Sort. If yes, Justify your answer with time complexity analysis? **(1 Mark)**

10. Suppose that the U (universe set) is consisting of possible keys {0, 1,...., n^n}.The hash function is *((2i+5) mod n)*. What is the maximum number of distinct keys the hash table can hold for the following collision resolution techniques viz., (a) Linear Probing, (b) Quadratic Probing, and (c) Separate Chaining? **(2 Marks)**

11. Given an array of n elements, What is the best case time complexity to convert the given array into Min-heap. Derive it in a step by step manner with careful consideration of all the steps . **(2 Marks)**

PTO...

12. Give an algorithm that runs on O(k) to compute the union (A U B) of two sets A and B of total size $|A|+|B|$ =k. Assume that elements of each set are stored separately by arrays in an arbitrary order. While computing the union, the algorithm should remove the duplicate elements that appear in both A and B.

(1 Mark)

13. Give a schematic representation for splitting and merging (or combine) for mergesort on the array (10, 2, 5, 3, 7, 13, 1, 6). Trace it through the steps involved in the sorting process. **(2 Marks)**

14. What is the worst-case running time of Quick sort? How would you order the elements in the input array to achieve the worst case? **(1 Mark)**

15. Solve $T(n) = 2T(\sqrt{n})+\log n$ **(2 Marks)**

16. While solving the recurrence relation of some cases of Quick Sort, one observes that leaf nodes are at the same depth. Write down the recurrence relation for such case of QuickSort. **(1 Mark)**

17. In QuickSort, if the partitioning algorithm always produces a 9 to 1 proportional split ($n>>100$), then what is the recurrence relation of QuickSort for n elements? Analyze its time complexity using the recurrence tree method. **(2 Marks)**

18. Suppose there are n^2 data elements stored in n chains (with every chain having n elements). A user wants to access elements from this list and one can guarantee that the data which user is requesting is always available either in anywhere in the first *log n* locations or last *log n* locations (hit) of any chain. Suggest, a suitable data structure for chaining that will allow the user to access the above mentioned data in *O(log n)* time complexity. **(2 Marks)**

19. Design median priority queue as follows: A median priority queue will be just like the normal priority queue but with a difference. In the normal priority queue, the *remove()* function will remove the smallest element (Element of highest priority), but in the median priority queue, the *remove()* function will remove the median element present in the queue. The same will be the case for the *peek()* function. The time complexity of these methods (median priority queue operations) should remain the same as normal priority queue i.e *peek()* should be *O(1)*, and insert and remove should be *O(log n)*. However, you are allowed to use any constant number of heaps (i.e. number of heaps <<<< n) but the entire space complexity should be the size of the input (n). **(3 Marks)**

20. Sachin is assigned with a task to create a B-Tree of order 4{M=4 ;t=2} in lexicographical order (i.e., $a < b < c < \ldots < y < z$). The keys in the input sequence are as follows $\{F, S, Q, K, C, L, H, T, V, W, M, R, N, P, A, B, X, Y, D, Z, E\}$. Trace through the steps involved in the construction of resultant B-Tree. **(3 Marks)**

21. Jerry mouse is outside a maze (as in Figure 1) and has to find the path to exit from the maze. Each path has pieces of cheese placed intelligently, to mark a door (opening or branch) inside the maze. Convert this problem into a proper computational problem and solve it using a standard approach. Explain your solution in detail. **(3 Marks)**

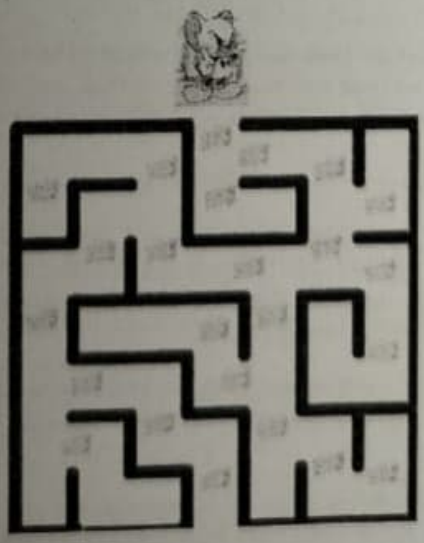

Figure 1: Maze

All The Best!!!