

Engineering Optics

Lecture 8

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by

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Division of wavefront

Young's double slit experiment

For an arbitrary point P (on line LL') to correspond to a maximum, we must have

$$S_2P - S_1P = n\lambda \quad n = 0, 1, 2, \dots$$

Now,

$$\begin{aligned} (S_2P)^2 - (S_1P)^2 &= \left[D^2 + \left(y_n + \frac{d}{2} \right)^2 \right] \\ &\quad - \left[D^2 + \left(y_n - \frac{d}{2} \right)^2 \right] \\ &= 2y_nd \end{aligned}$$

$$S_1S_2 = d \quad \text{and} \quad OP = y_n$$

$$y_n = \frac{n\lambda D}{d}$$

Thus

$$S_2P - S_1P = \frac{2y_nd}{S_2P + S_1P}$$

If $y_n, d \ll D$,
 $S_2P + S_1P \approx 2D$

distance between two consecutive bright fringes

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

fringe width $\beta = \frac{\lambda D}{d}$

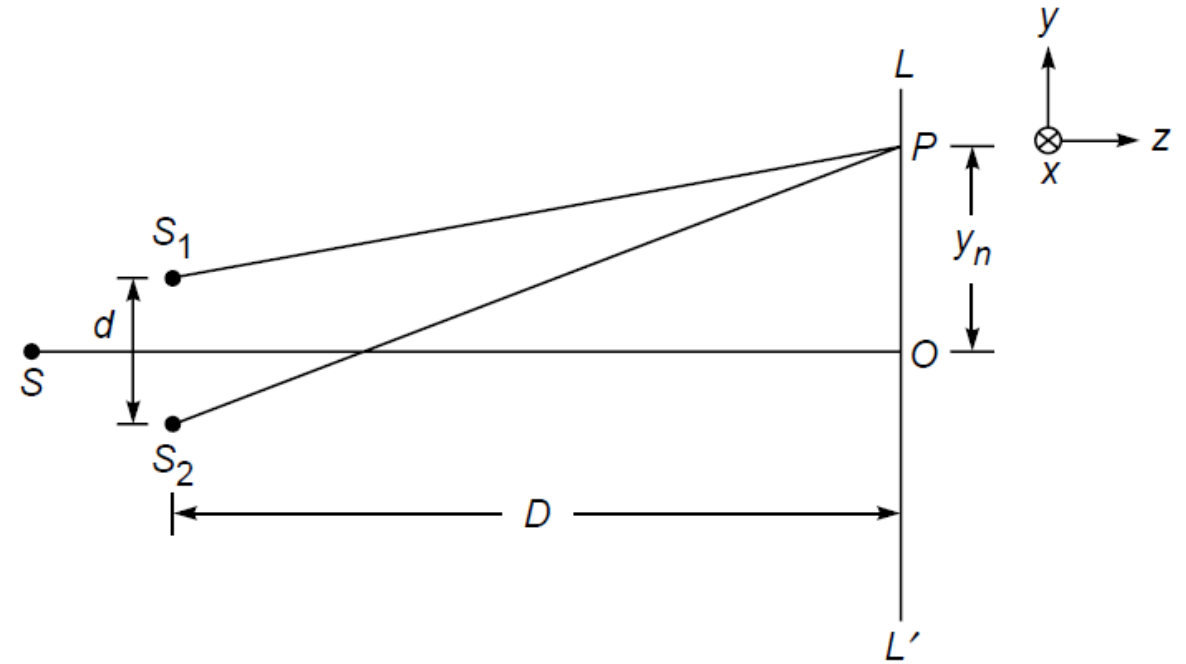


Fig. 14.8 Arrangement for producing Young's interference pattern.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$\delta = \frac{2\pi}{\lambda} (S_2P - S_1P)$$

Problem:1

Two parallel narrow horizontal slits in an opaque vertical screen are separated center-to-center by 2.644 mm. These are directly illuminated by yellow plane waves from a filtered discharge lamp.

Horizontal fringes are formed on a vertical viewing screen 4.5 m from the aperture plane. The center of the fifth bright band is 5.000 mm above the center of the zeroth or central bright band.

- (a) Determine the wavelength of the light in air
- (b) If the entire space is filled with clear soybean oil ($n = 1.4729$) where would the fifth fringe now appear?

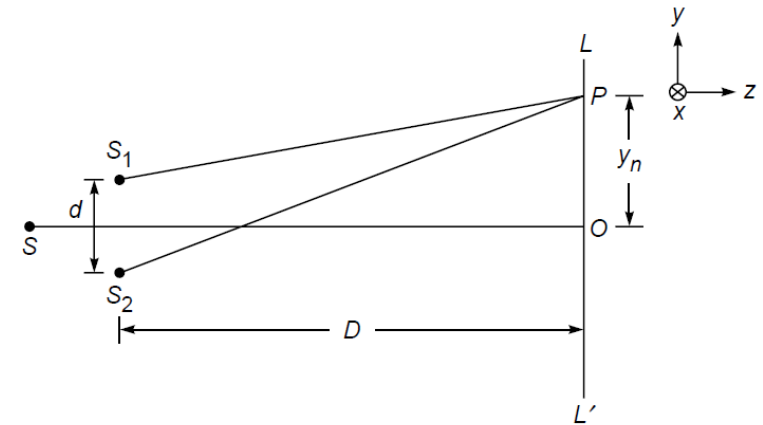
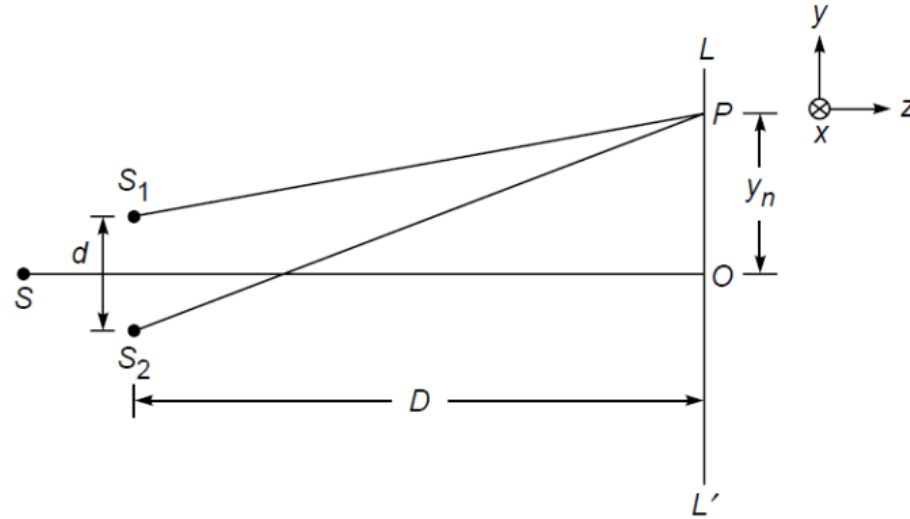


Fig. 14.8 Arrangement for producing Young's interference pattern.

Answer:

(a)



We know that $y_m = \frac{m\lambda D}{d}$ [comes from $S_2P - S_1P = m\lambda$, condition for maxima]

Given that: $D = 4.5 \text{ m}$, $d = 2.644 \text{ mm}$, $y_5 = 5.0 \text{ mm}$, $m = 5$

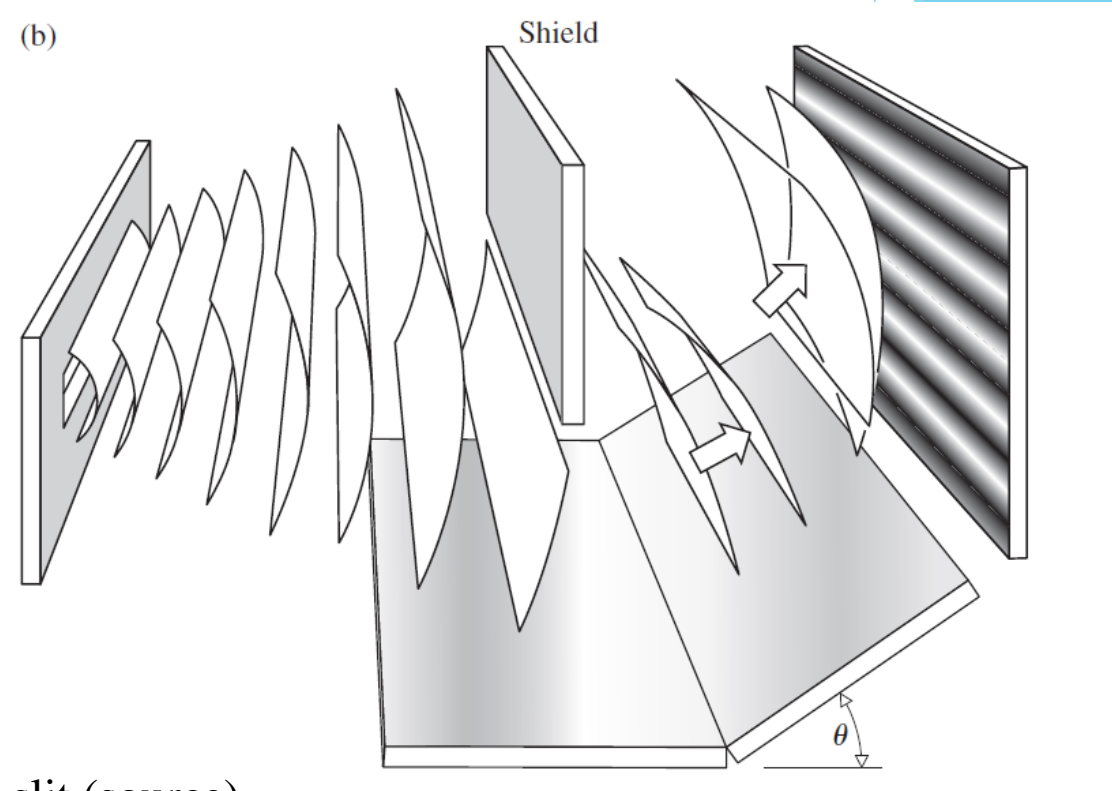
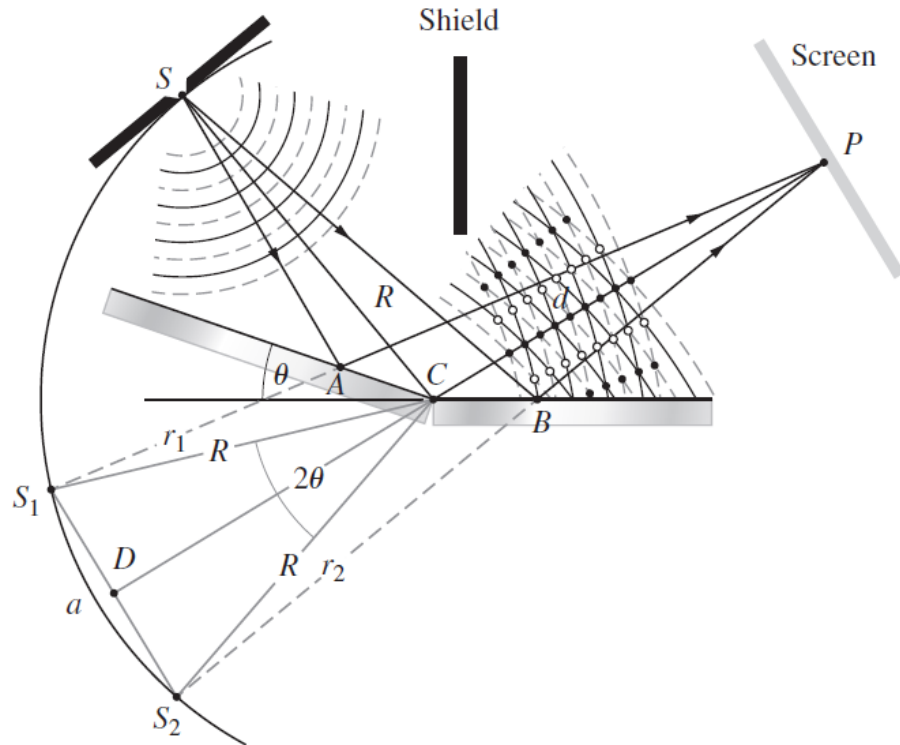
$$\lambda = \frac{y_n d}{m D} \Rightarrow \lambda = \frac{5.0 \times 10^{-3} \text{ m} \times 2.644 \times 10^{-3} \text{ m}}{5 \times 4.5 \text{ m}}$$
$$\lambda = 587.56 \text{ nm}$$

(b) When the space is filled with oil the wavelength will decrease and the new fringe location (y'_m) will be closer to the center of the apparatus.

$$y'_m = \frac{mD}{d} \left(\frac{\lambda}{n} \right) = \frac{y_m}{n}$$

$$y'_5 = \frac{5.0 \times 10^{-3} m}{1.4729} = 3.395 \text{ mm}$$

Fresnel Double Mirror



2 plane mirrors \rightarrow inclined by θ Point $S \rightarrow$ narrow slit (source)

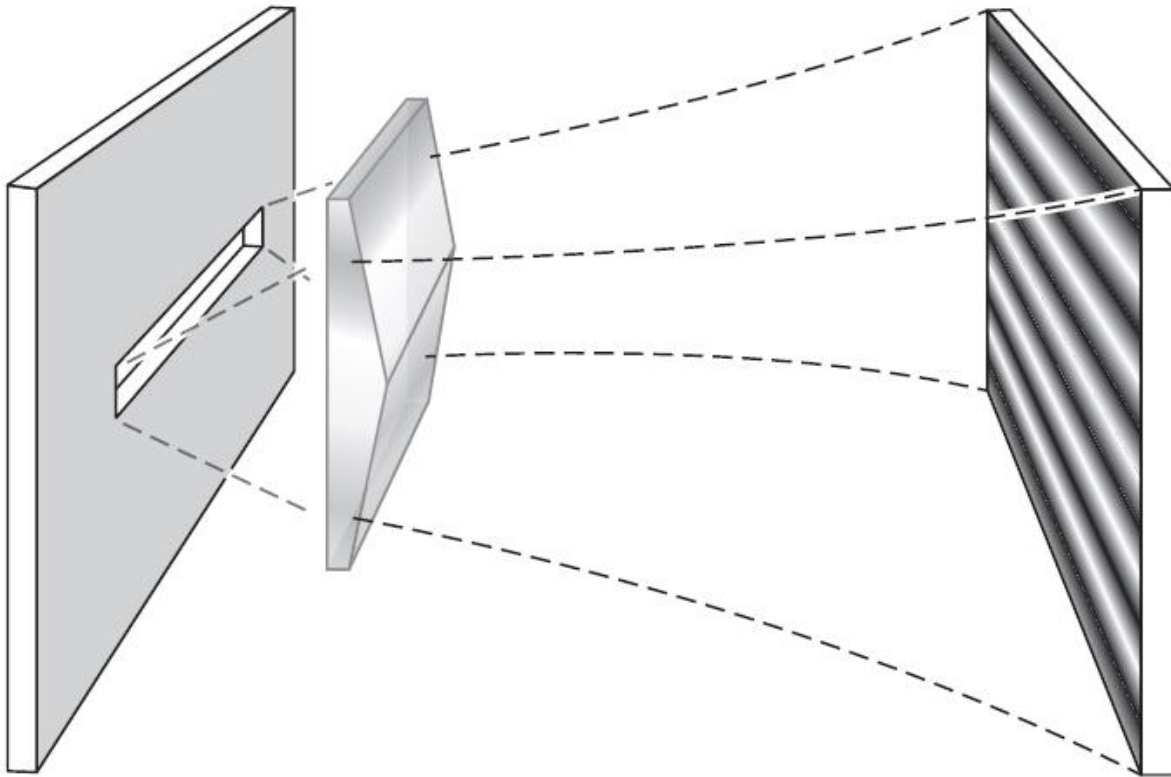
A portion of the wave front from $S \rightarrow$ reflected from *two mirrors*

reflected waves interfere \rightarrow **fringes on screen**

two wave fronts \rightarrow derived from $S \rightarrow$ coherent

wave fronts from the virtual sources S_1 and S_2 of S formed by mirrors

Fresnel Biprism

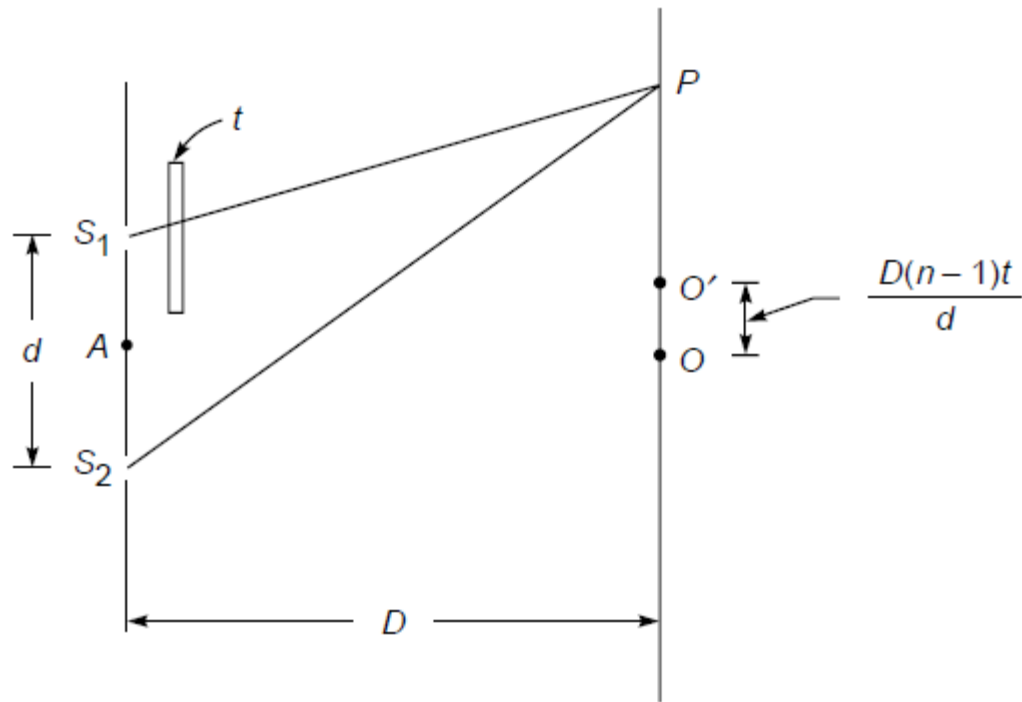


Another simple arrangement for interference pattern.

Prism with small base angles

Light from slit gets refracted → *interference*

Displacement of fringes



$$\begin{aligned}\frac{S_1P - t}{c} + \frac{t}{v} &= \frac{1}{c}(S_1P - t + nt) \\ &= \frac{1}{c}[S_1P + (n-1)t]\end{aligned}$$

optical path increases by $(n-1)t$. Thus, when the thin plate is introduced, the central fringe (which corresponds to equal optical path from S_1 and S_2) is formed at point O' where

$$S_1O' + (n-1)t = S_2O'$$

$$S_2O' - S_1O' \approx \frac{d}{D}OO'$$

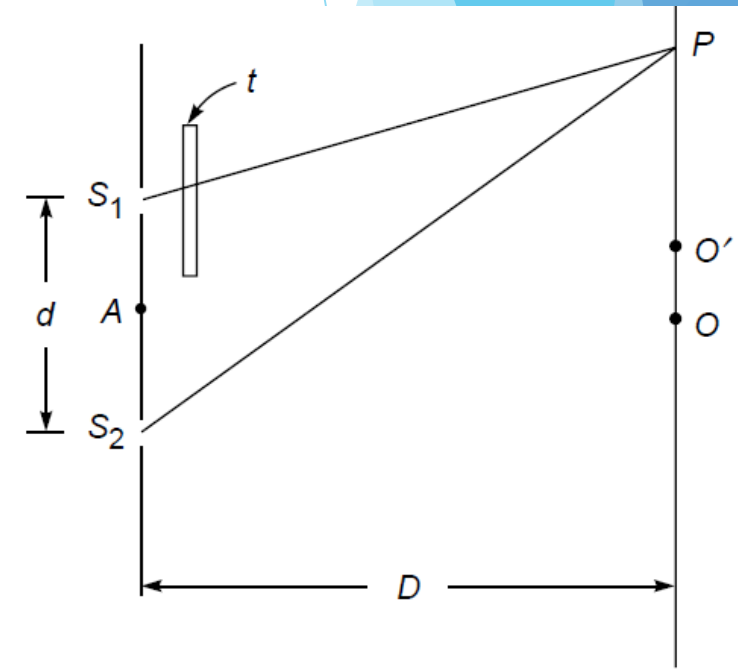
$$(n-1)t = \frac{d}{D}OO'$$

Thus the fringe pattern gets shifted by a distance Δ which is given by

$$\Delta = \frac{D(n-1)t}{d}$$

Problem-2

In a double-slit interference arrangement one of the slits is covered by a thin mica sheet whose refractive index is 1.58. The distances S_1S_2 and AO (see Fig. 14.20) are 0.1 and 50 cm, respectively. Due to the introduction of the mica sheet the central fringe gets shifted by 0.2 cm. Determine the thickness of the mica sheet.



Example 14.10 In a double-slit interference arrangement one of the slits is covered by a thin mica sheet whose refractive index is 1.58. The distances S_1S_2 and AO (see Fig. 14.20) are 0.1 and 50 cm, respectively. Due to the introduction of the mica sheet the central fringe gets shifted by 0.2 cm. Determine the thickness of the mica sheet.

Solution:

$$\Delta = 0.2 \text{ cm} \quad d = 0.1 \text{ cm} \quad D = 50 \text{ cm}$$

Hence

$$t = \frac{d\Delta}{D(n-1)} = \frac{0.1 \times 0.2}{50 \times 0.58} \\ \approx 6.9 \times 10^{-4} \text{ cm}$$

Problem-3

In an experimental arrangement similar to that discussed in Example 14.10, one finds that by introducing the mica sheet the central fringe occupies the position that was originally occupied by the eleventh bright fringe. If the source of light is a sodium lamp ($\lambda = 5893 \text{ \AA}$), determine the thickness of the mica sheet.

Example 14.11 In an experimental arrangement similar to that discussed in Example 14.10, one finds that by introducing the mica sheet the central fringe occupies the position that was originally occupied by the eleventh bright fringe. If the source of light is a sodium lamp ($\lambda = 5893 \text{ \AA}$), determine the thickness of the mica sheet.

Solution: The point O' (see Fig. 14.20) corresponds to the eleventh bright fringe, thus

$$S_2O' - S_1O' = 11\lambda = (n - 1)t = 0.58t$$

Division of amplitude

Amplitude-Splitting

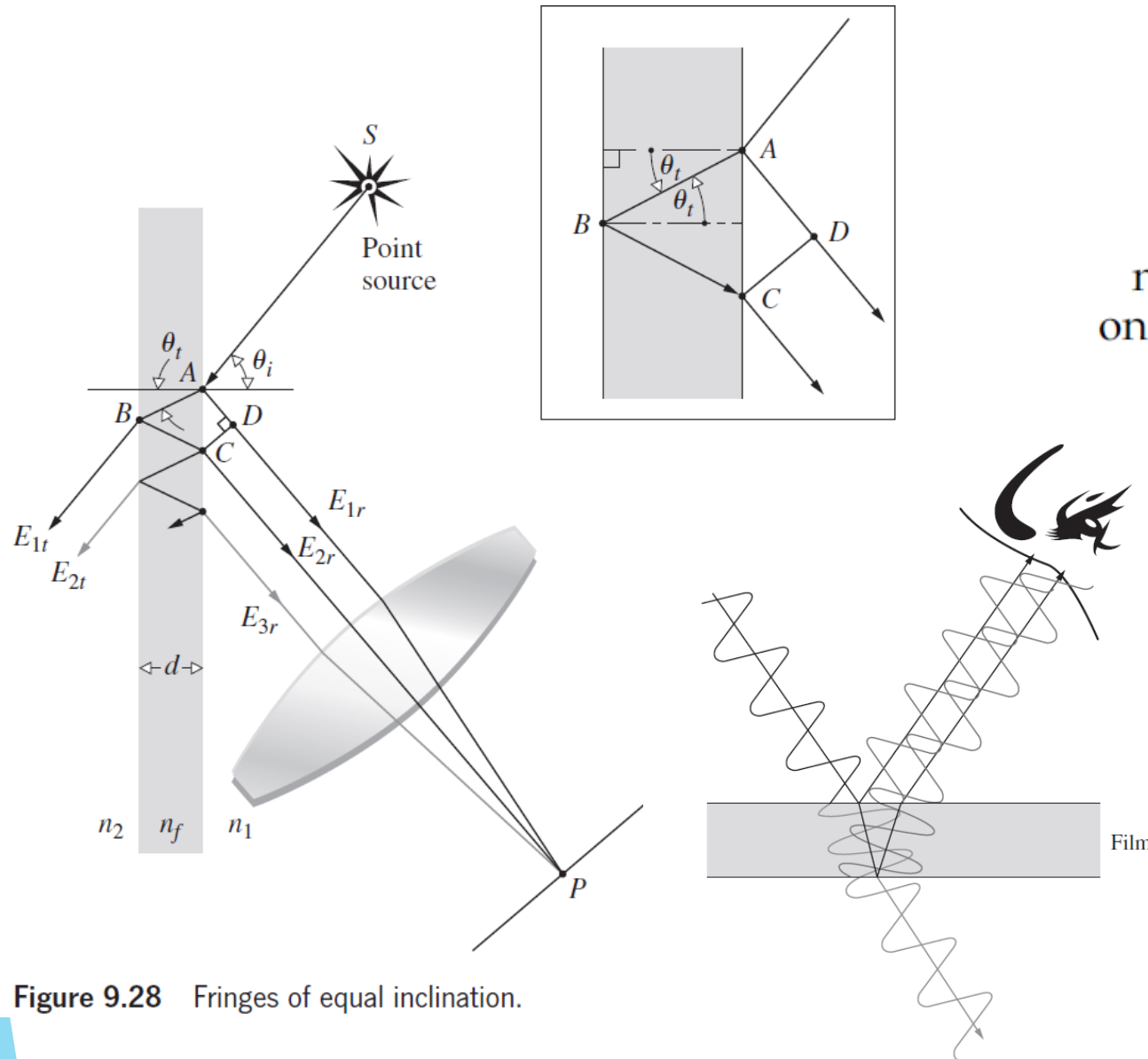
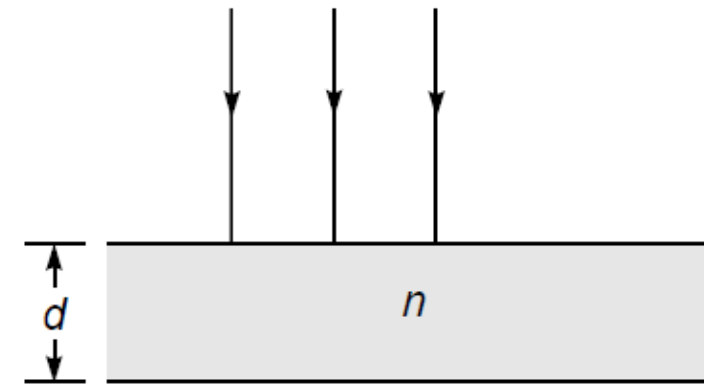


Figure 9.28 Fringes of equal inclination.



normal incidence of a parallel beam of light on a thin film of refractive index n and thickness d .

path length difference

$$\Lambda = 2n_f d \cos \theta_t$$

phase difference

$$\delta = k_0 \Lambda \pm \pi$$

Thank You