Electrical Circuits for Engineers (EC1000)

Lecture -4
Network Theorems
(Chapter 4)

Overview

- Thevenin's Theorem
- Norton's Theorem
- Superposition Theorem
- Maximum Power Transfer Theorem

2. Linear Circuits

- **Linearity** is the property of an element describing a linear relationship between *cause* and *effect.*
- we shall limit its applicability to resistors in this chapter.
- The property is a combination of both the homogeneity (scaling) property and the additivity property.

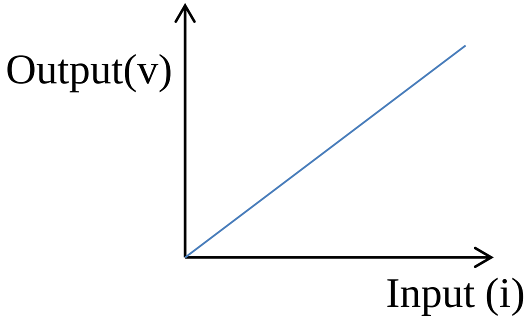
Homogeneity:

If the input (i.e the excitation) is multiplied by a constant, then the output (i.e response) is multiplied by the same constant.

For a resistor, for example,

Ohm's law relates the input *i* to the output *v*,

$$V \alpha\, i$$
$$V = iR$$



If the current is increased by a constant *k*, then the voltage increases correspondingly by *k*; that is,

$$kiR = k\,V$$

Network Theorem

Additivity property

Response to a sum of inputs is the sum of the responses to each input applied separately.

Using the voltage-current relationship of a resistor,

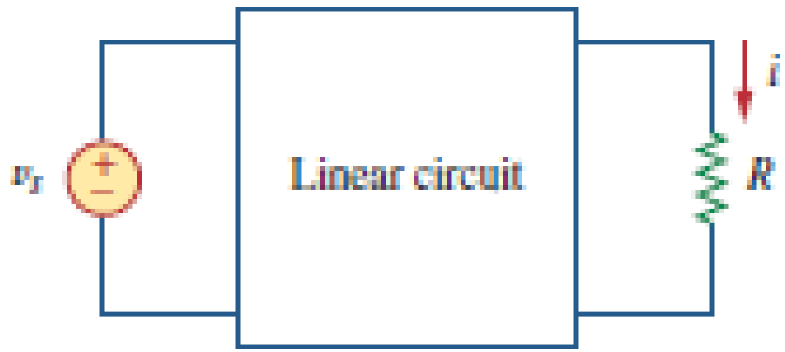
If, $V_1 = i_1 R, V_2 = i_2 R$

Then,

$$V = i_1 R + i_2 R = V_1 + V_2$$

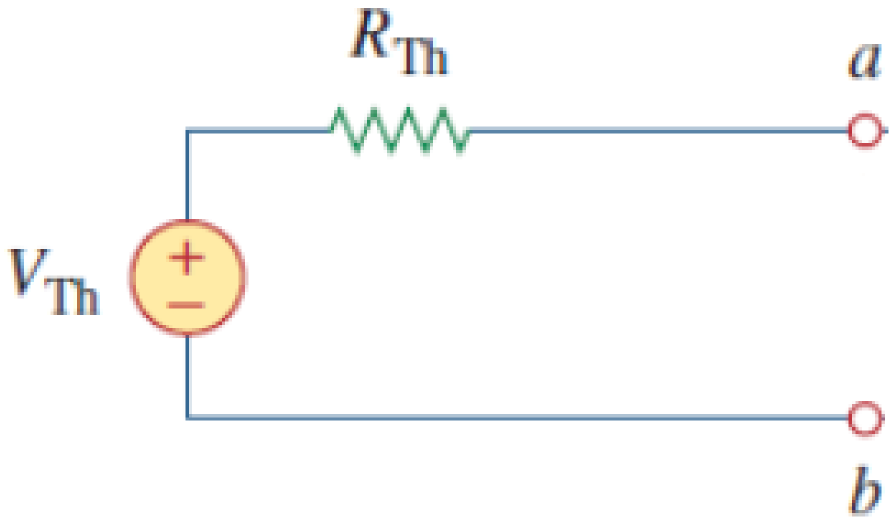
A **linear circuit** is one whose output is linearly related (or directly proportional) to its input.

Note that since $P=i^2/R$ or V^2/R (making it a quadratic function rather than a linear one), the relationship between power and voltage (or current) is Nonlinear. Therefore, the theorems covered in this chapter are not applicable to power.

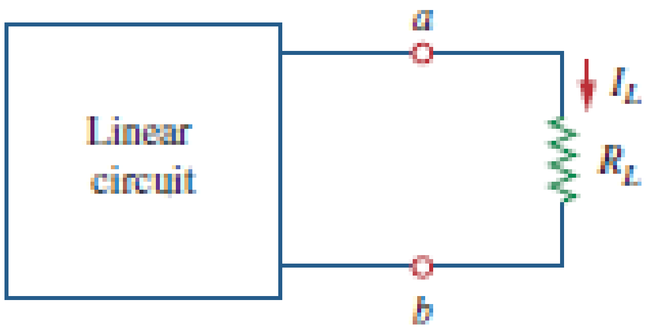


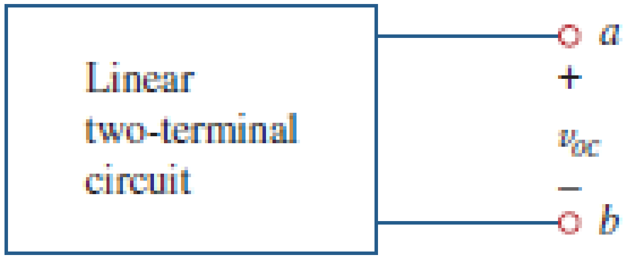
1. Thevenin's theorem

"*A linear BILATERAL electric circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off*".



1. Thevenin’s theorem

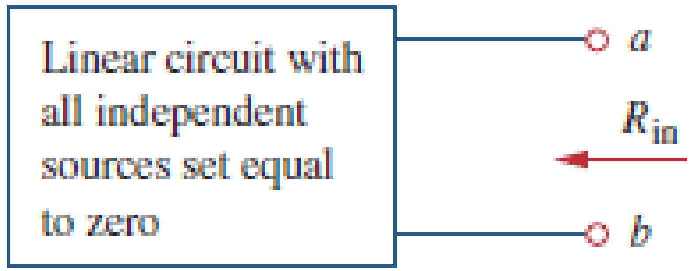




$V_{Th} = v_{oc}$

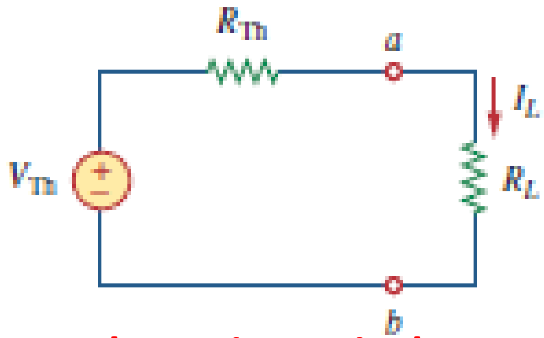
- Open circuit the terminals ‘a’ and ‘b’ by removing the load R_L connected to it.

- Find the voltage across the terminals ‘a’ and ‘b’.



$R_{Th} = R_{in}$

Set independent sources zero and use reduction techniques to find R_{Th}



Thevenin equivalent

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

Thevenin's theorem

Example 1

Find the Thevenin's voltage with respect to the load resistor R_L in circuit shown in Fig.

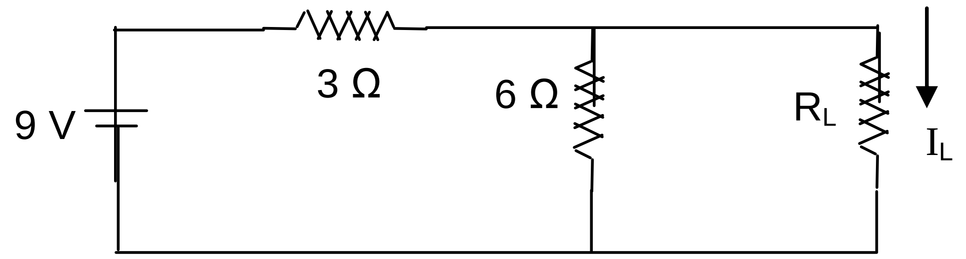
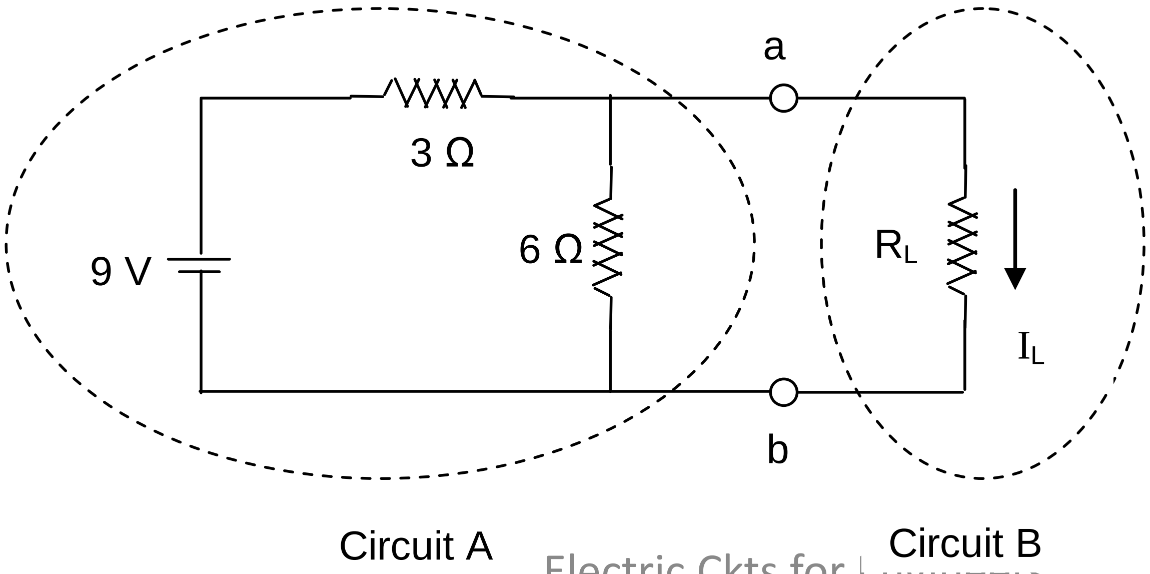


Fig. Circuit for Example1

Solution

The given circuit can be divided into two circuits as shown in Fig.



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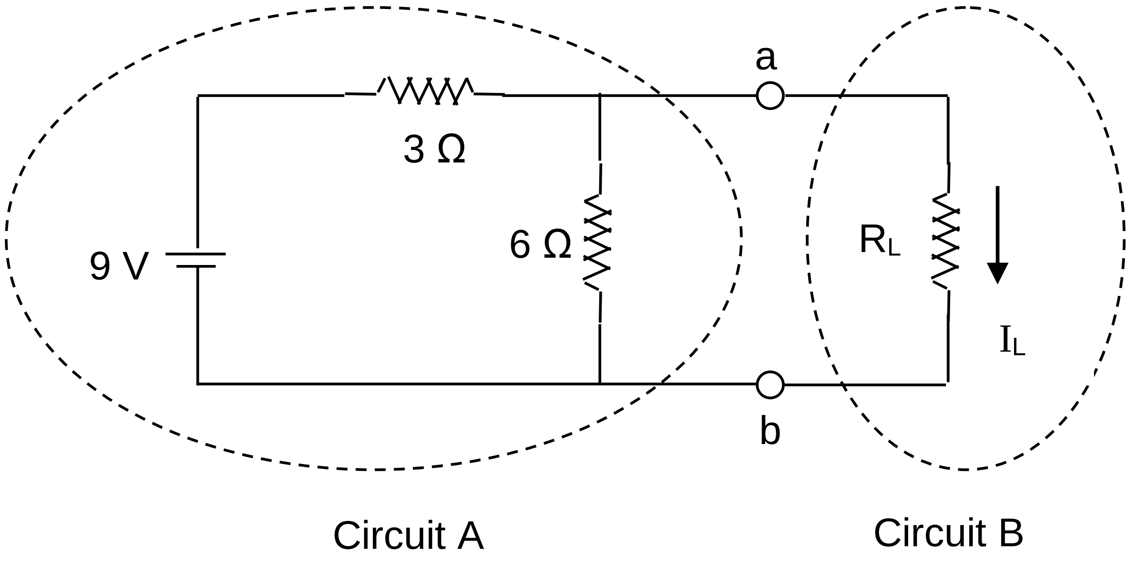


Thevenin's voltage of circuit A can be obtained from the circuit shown in Fig.

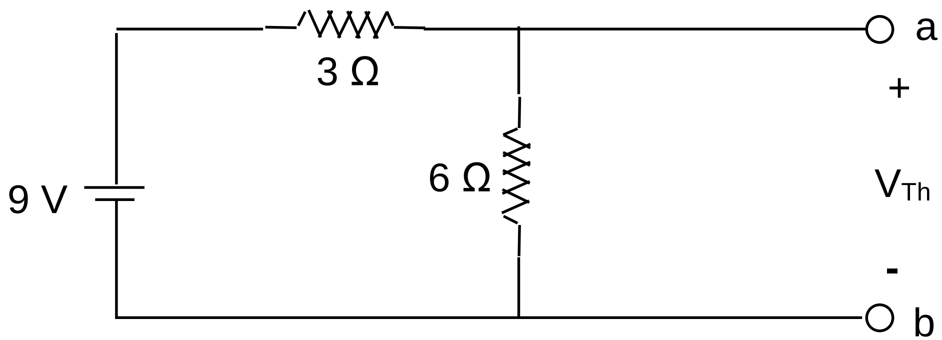
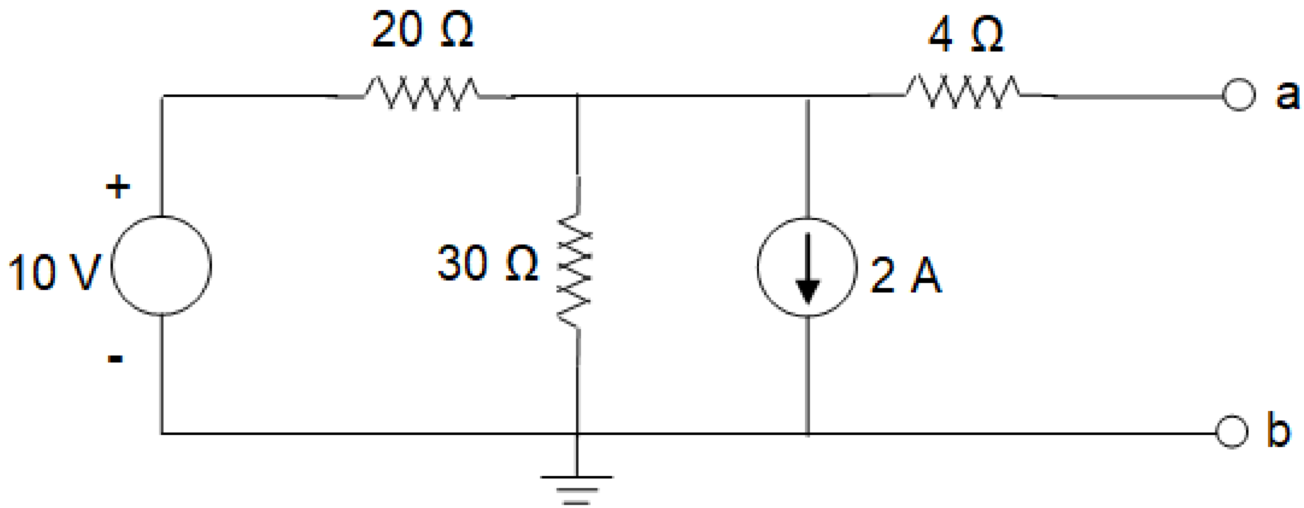


Fig. Circuit.

Using voltage division rule $V_{Th} = V_{6\Omega} = \frac{6}{9} \times 9 = 6\,V$

Example 2

Obtain the Thevenin's equivalent for the circuit shown in Fig.



Solution:

$$\frac{V_{Th} - 10}{20} + \frac{V_{Th}}{30} + 2 = 0$$

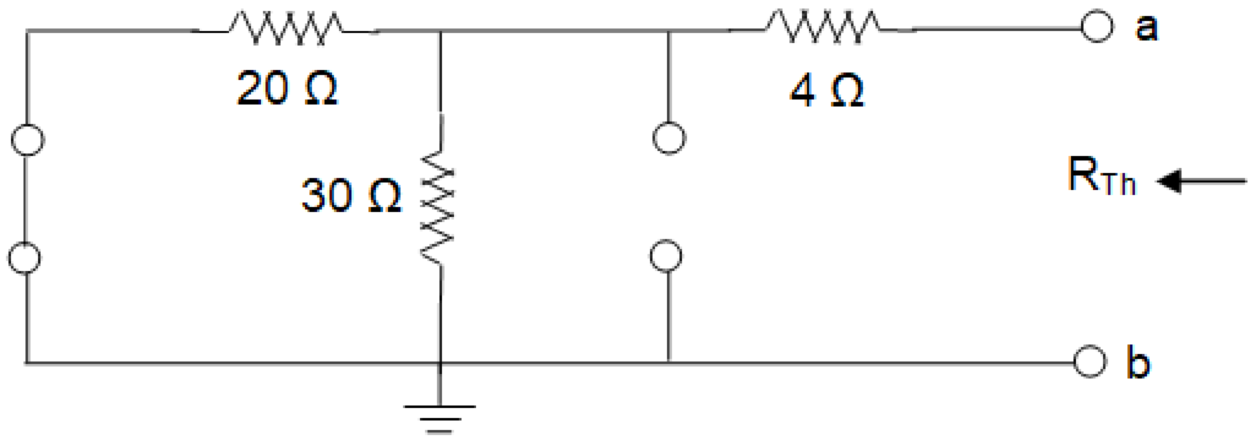
Open circuit voltage V_{ab} is the Thevenin's voltage V_{Th}.

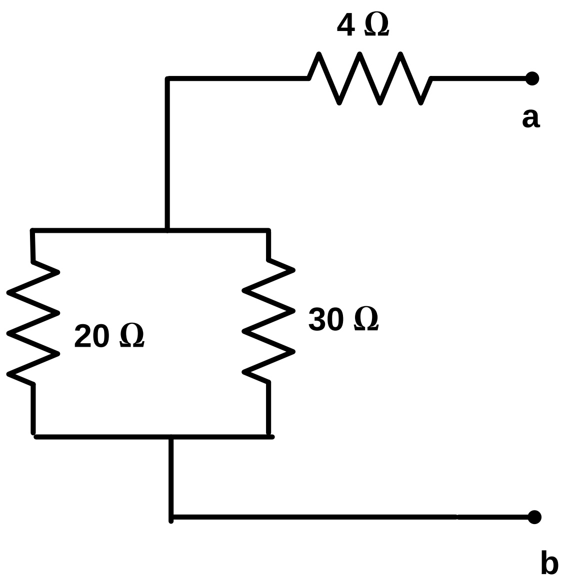
To find Thevenin's voltage:

Note that there is no current flow in resistor of 4 Ω. Therefore, voltage V_{Th} is same as the voltage across 30 Ω resistor. Then, the node voltage equation is

$$\boxed{V_{Th} = -18\ V}$$

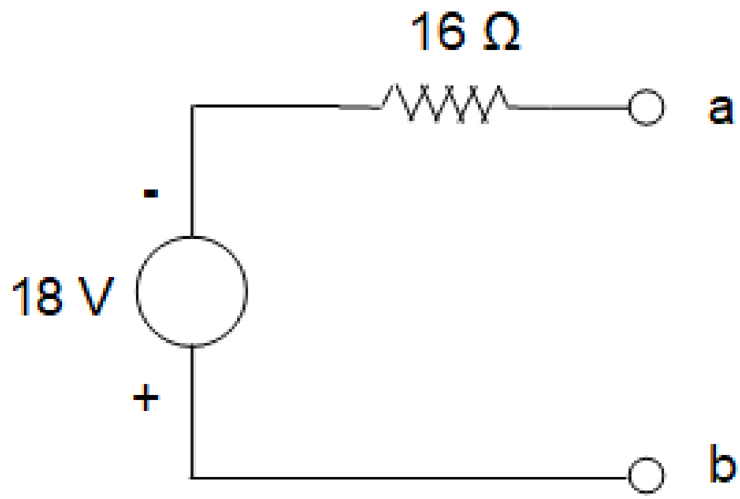
Reducing the sources to zero, the resulting circuit is shown in Fig.



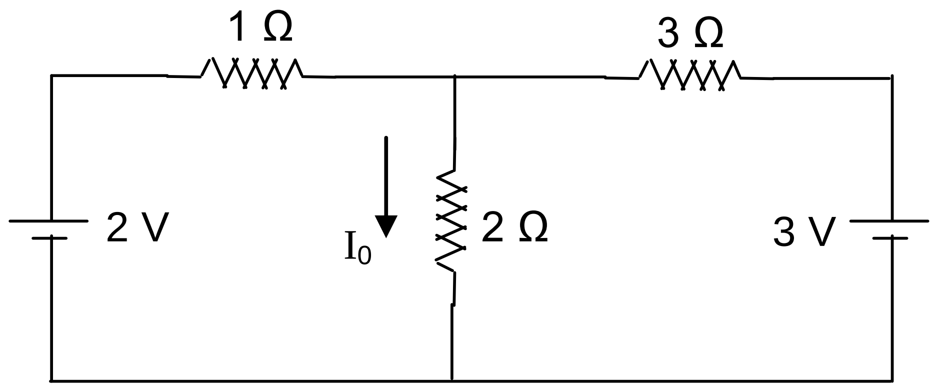


$R_{Th} = 4 + 20 || 30 = 16\ \Omega$

Thevenin's equivalent circuit



Example 3 Using Thevenin's equivalent circuit, calculate the current I_0 through the 2 Ω resistor in the circuit shown below.



Solution: Circuit by which V_{Th} and R_{Th} can be calculated are shown in Fig.

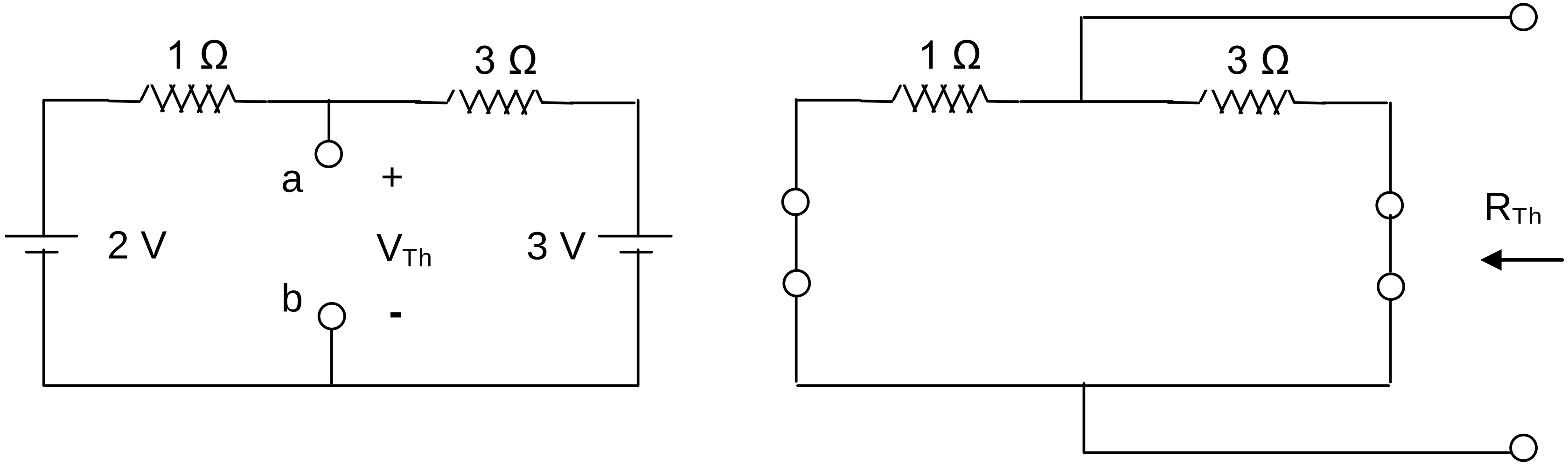


Fig. Circuits for V_{Th} and R_{Th} - Example 3.

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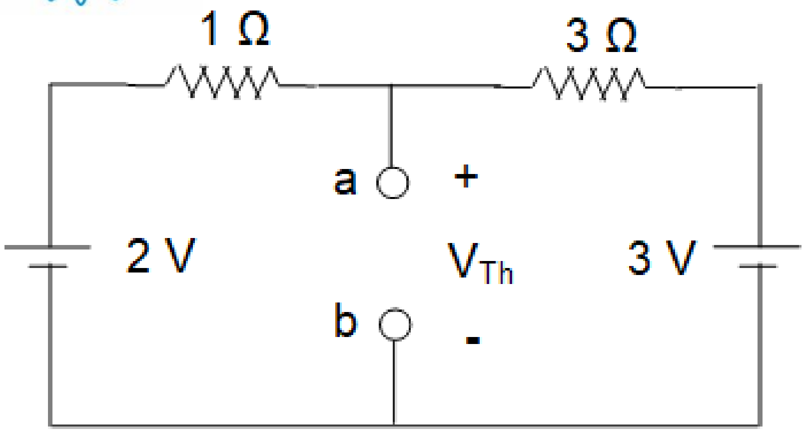
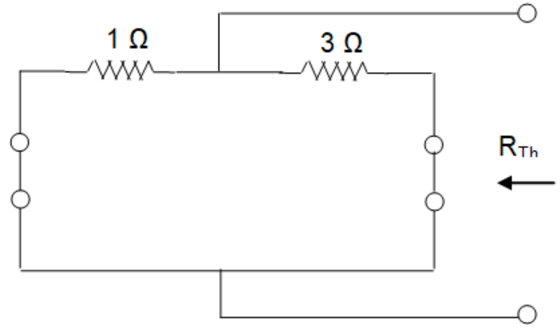


$$\frac{V_{Th}-2}{1}+\frac{V_{Th}-3}{3}=0$$

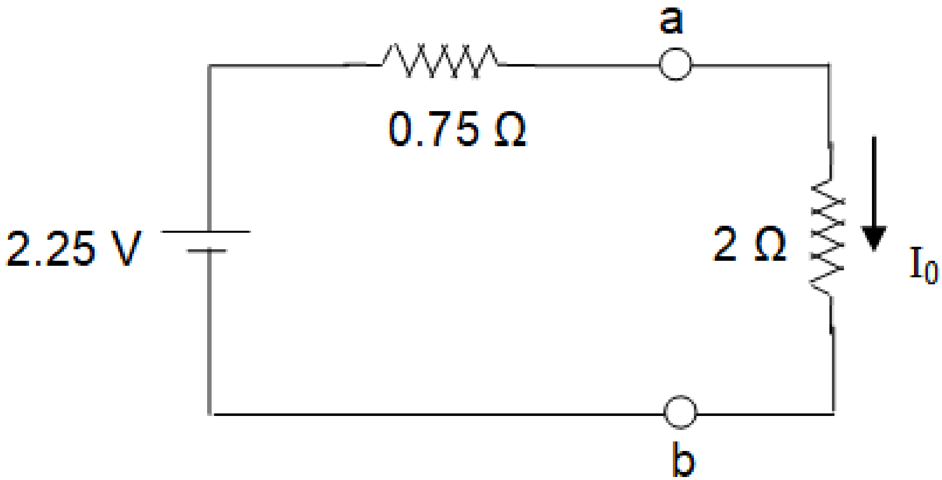
$$4V_{Th}=9$$

$$V_{Th}=2.25$$



$R_{Th} = 1||3 = 0.75\ \Omega$

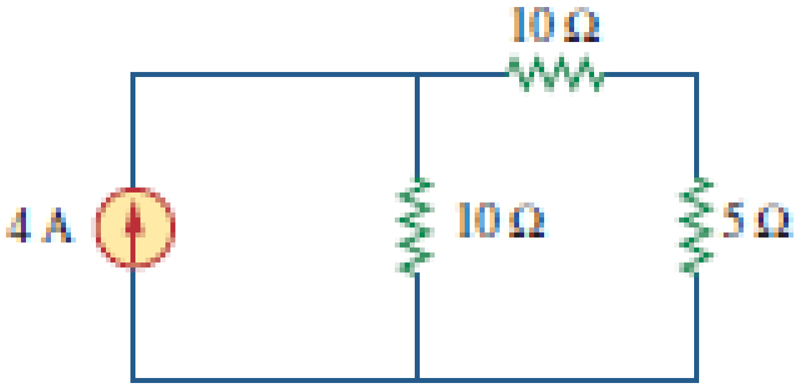
Thevenin's equivalent circuit becomes



Current $I_0 = 2.25 / 2.75 = 0.8182$ A

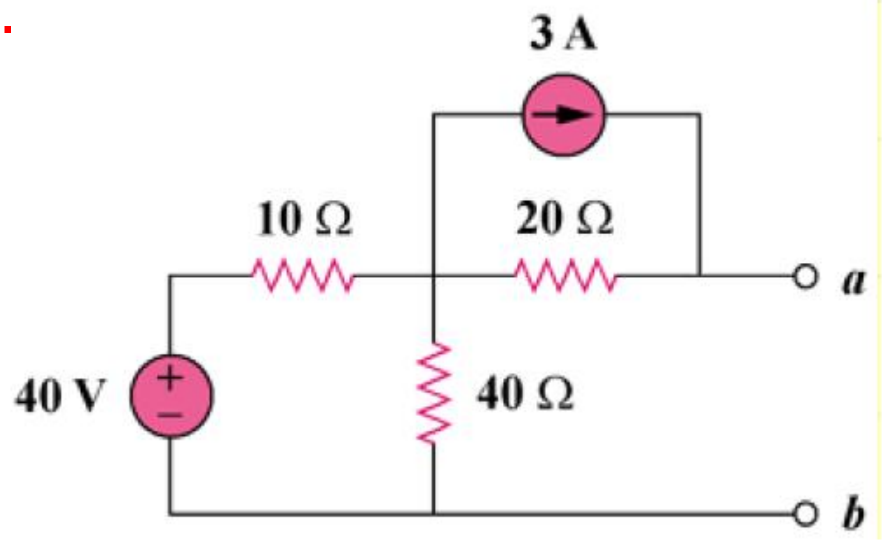
Practice Problems

1. Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit of Figure. Then find *I*.



Ans: 40 V, 20 Ohm

2. Find the Thevenin equivalent at terminals a-b of the circuit in Figure.

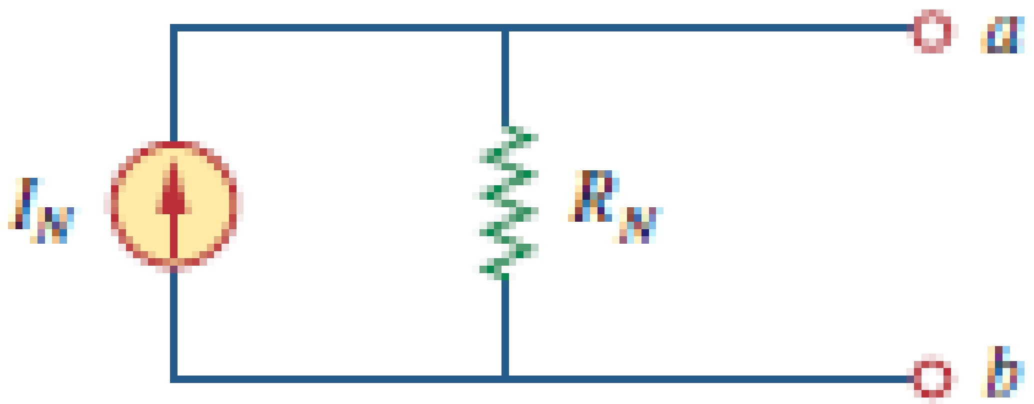


Ans: 92 V, 28 Ohm

2. Norton's Theorem

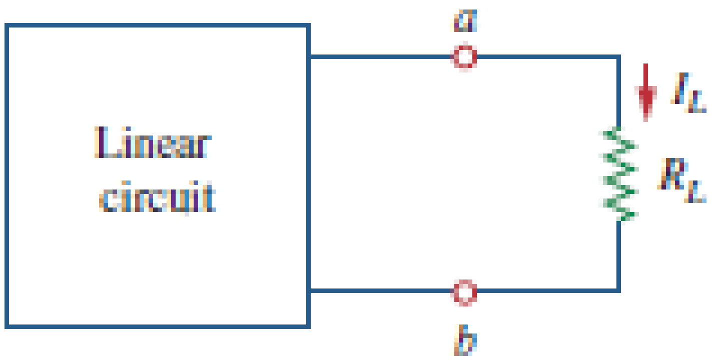
Statement:

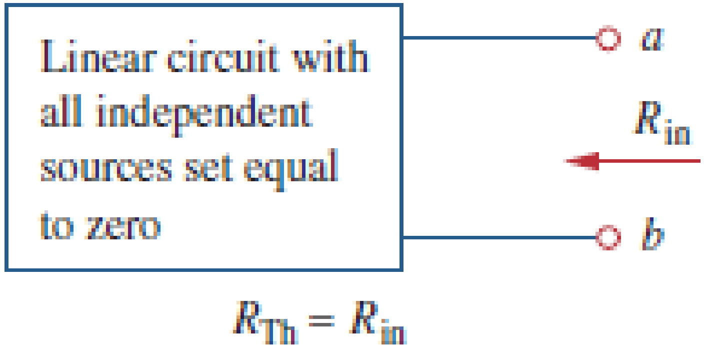
"*A linear BILATERAL circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off*".

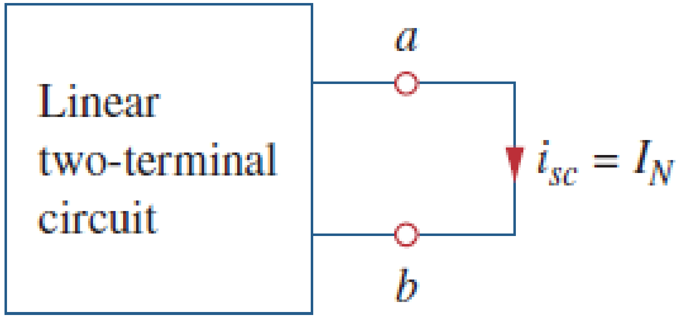


Norton's Equivalent Circuit

2. Norton's Theorem







$$I_N = \frac{V_{Th}}{R_{Th}}$$

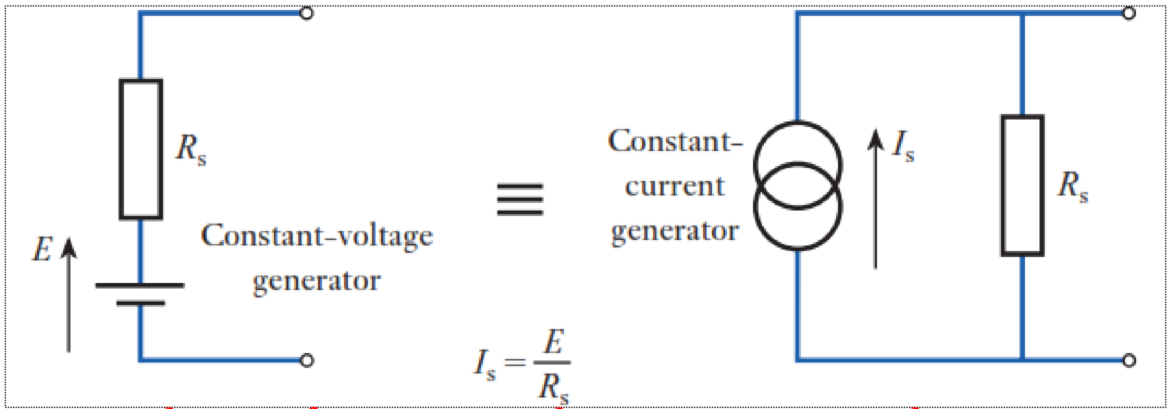
Set independent sources zero and use reduction techniques to find R_{Th}

$$V_{oc} = V_{Th}$$

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

$$I_N = i_{sc}$$

- Short circuit the terminals 'a' and 'b' by removing the load R_L connected to it.
- Find the current through the terminals 'a' and 'b'.



Equivalence of constant-voltage generator and constant-current generator forms of representation

2. Norton's Theorem (Examples)

Example 1

Using Norton's theorem, determine the current through the resistor R_L when R_L = 0.7, 1.2 and 1.6 Ω in the circuit shown in Fig.

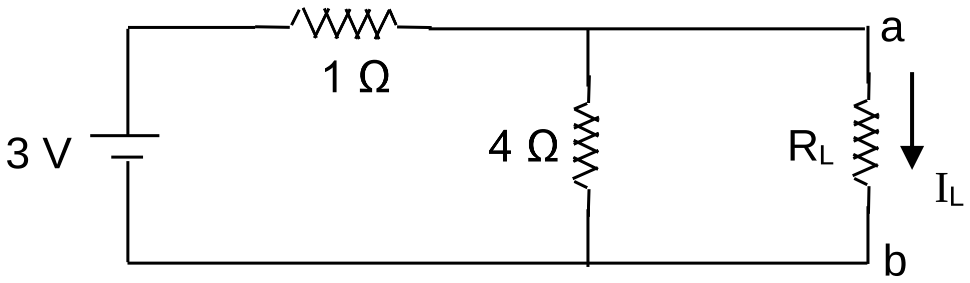
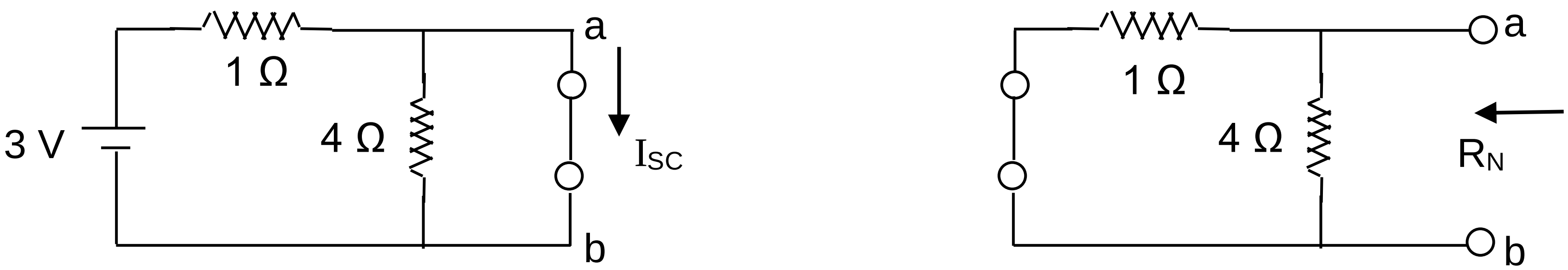


Fig. Circuit for Example 1.

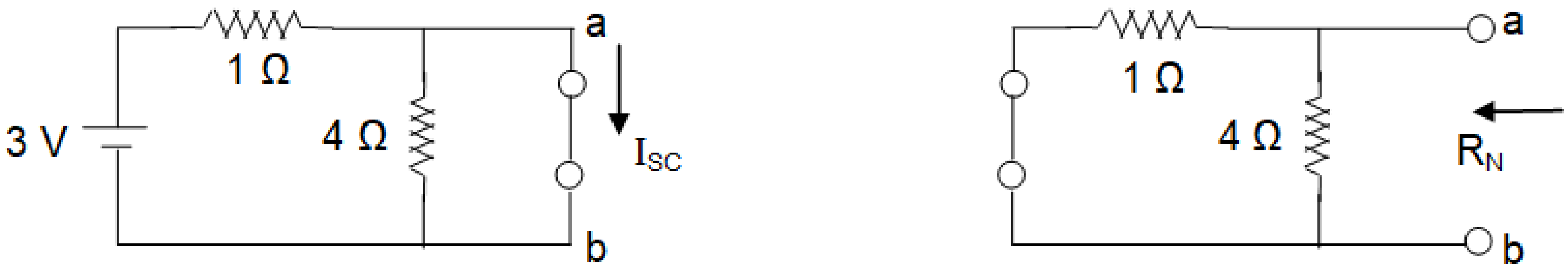
Solution:

Circuits to determine I_{SC} and R_N are shown in Fig. (a) and (b).



(a) (b)

Fig. Short circuit current and Norton's resistance.



(a) (b)

It is to be noted that since there is a short circuit parallel to 4 Ω no current flows in it.

Norton's current I_N = 3 A; Norton's resistance R_N = 1| |4 = 0.8 Ω

Norton's equivalent circuit is shown in Fig.

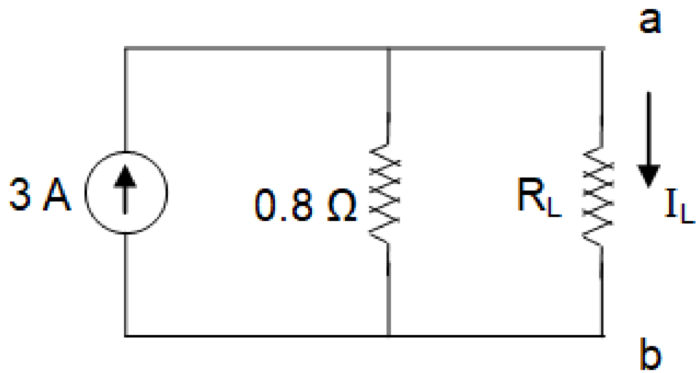


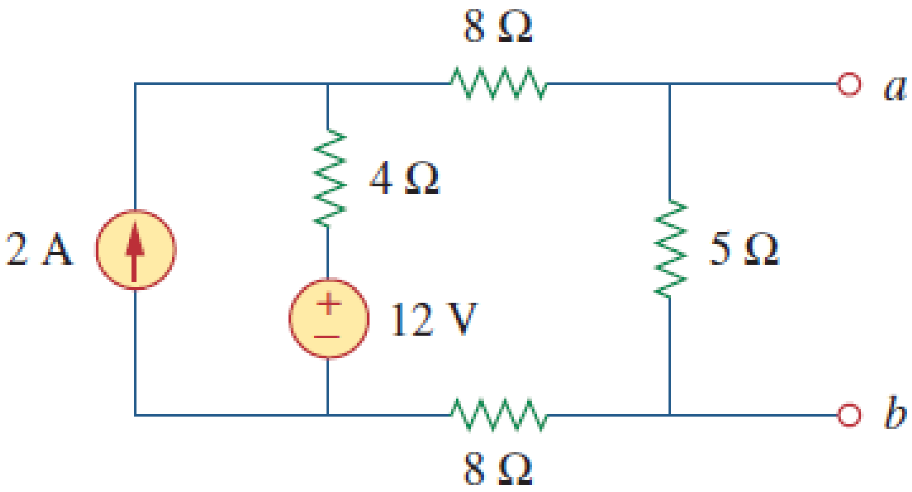
Fig. Norton's equivalent.

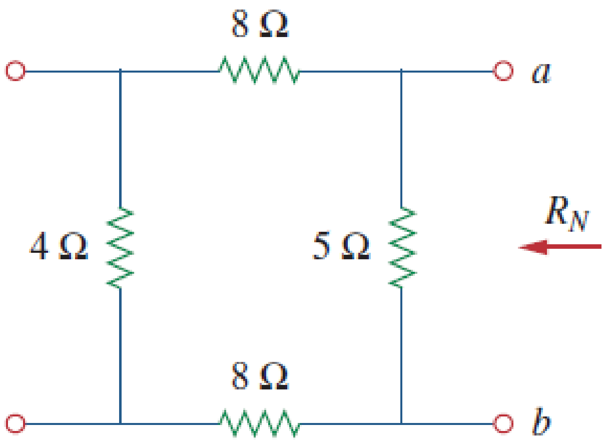
When R_L = 0.7 Ω, I_L = (0.8 / 1.5) x 3 = 1.6 A; When R_L = 1.2 Ω, I_L = (0.8 / 2) x 3 = 1.2 A

When R_L = 1.6 Ω, I_L = (0.8 / 2.4) x 3 = 1.0 A

Example Problems

2. Obtain the Norton's equivalent circuit for the below circuit.





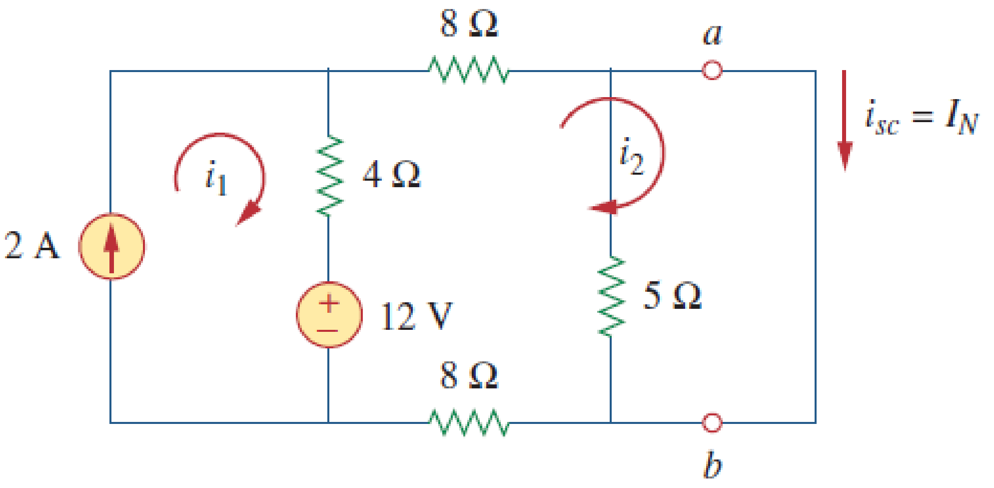
(a)

Solution

a) Find R_N

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

b) V_N



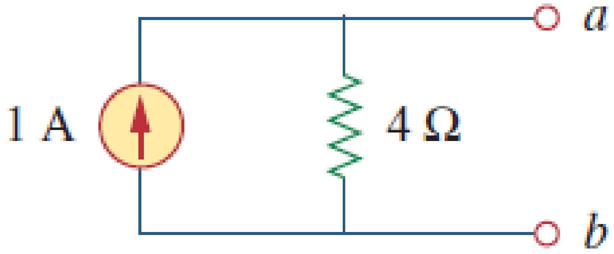
(b)

We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

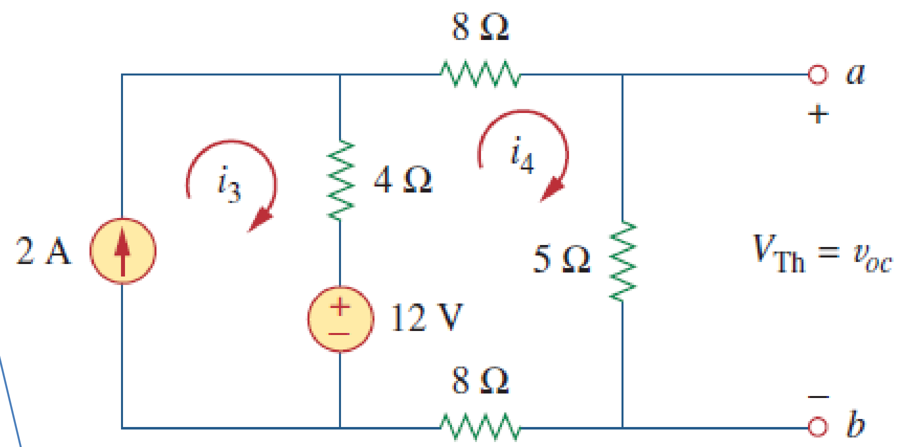
$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Norton's equivalent circuit



Alternate Method (Thevenin;s Voltage



(c)

$$i_3 = 2\text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8\text{ A}$$

$$v_{oc} = V_{Th} = 5i_4 = 4\text{ V}$$

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\text{ A}$$

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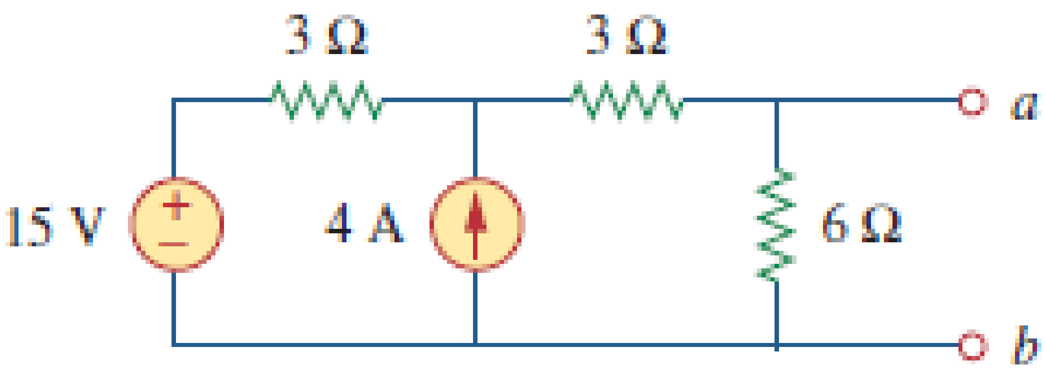
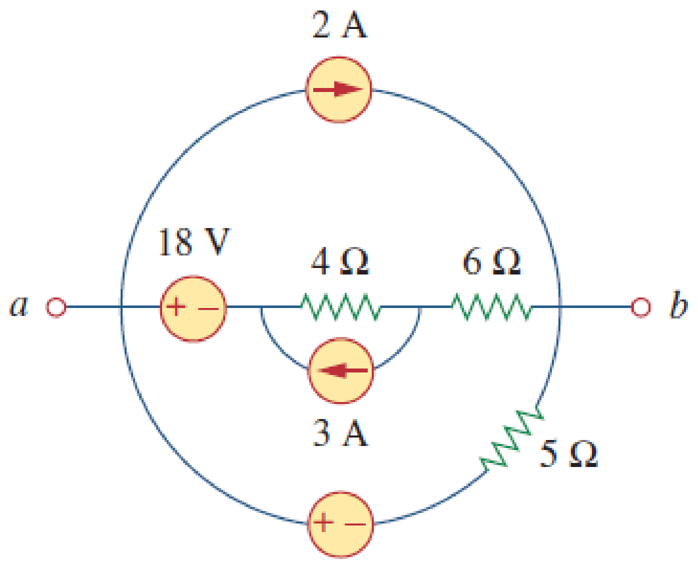
Practical Problems

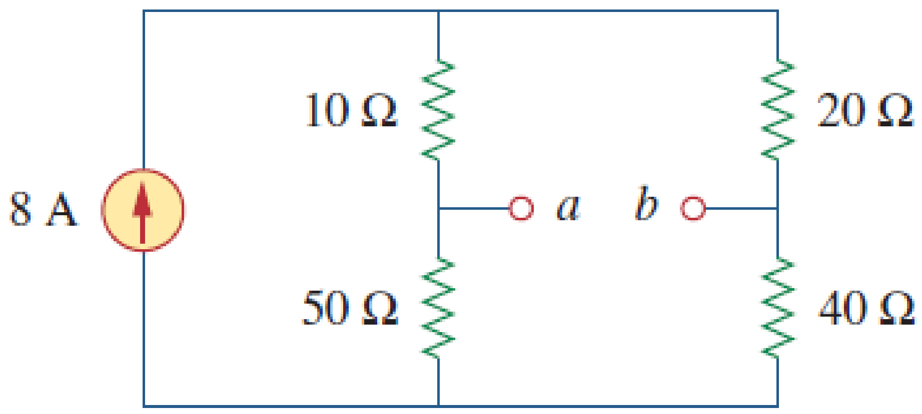
1. Obtain the Norton's equivalent circuit for the below circuit.



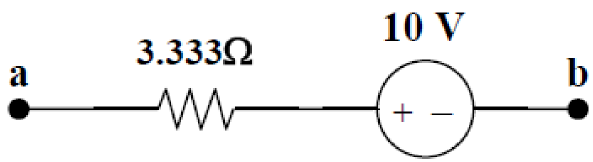
Answer: $R_N = 3\ \Omega, I_N = 4.5$ A.

2. Determine Thevenin’s and Norton's equivalent across a-b circuit for the below circuit





V_{Th} = **40V**, and $I_N = V_{Th}/R_{Th}$ = 40/22.5 = **1.7778 A**



4. Superposition Theorem

Statement:

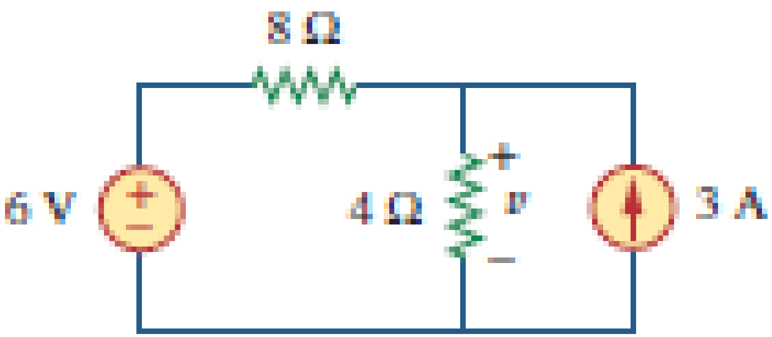
The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

- Superposition theorem is applicable to linear circuits having two or more independent sources.

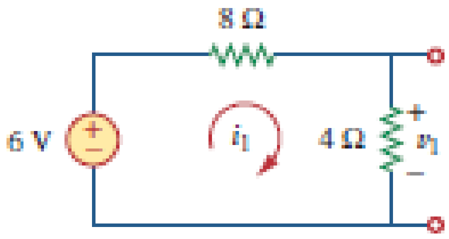
Note: When one source is acting alone, another source should be turned off (i.e Current source should be open circuited and Voltage sources should be short circuited)

Example Problems

Example 1. Use the superposition theorem to find v in the circuit of Figure.



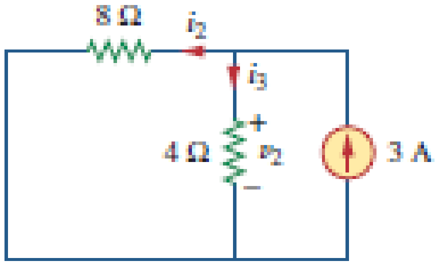
1. When 6 V source is acting



(a)

$$v_1 = \frac{4}{4+8}(6) = 2\text{ V}$$

2. When 3 A source is acting



(b)

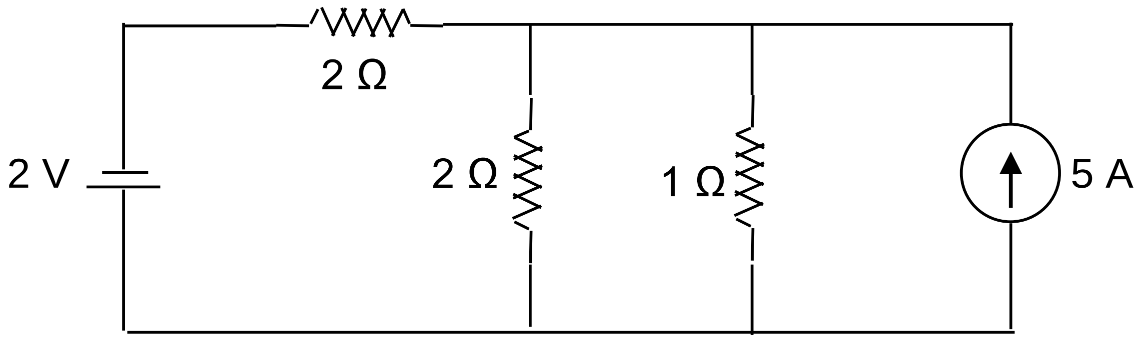
$$i_3 = \frac{8}{4+8}(3) = 2\text{ A}$$

$$v_2 = 4i_3 = 8\text{ V}$$

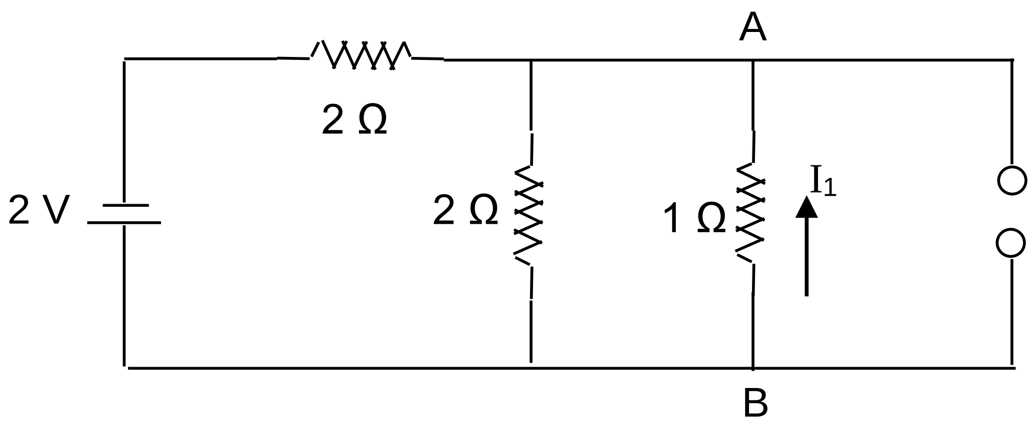
$V = v_1 + v_2 = 2\text{ v} + 8\text{ v} = 10\text{ V}$

Example Problems

Example 2 Calculate the current through the 1 Ω resistor in the circuit shown below.

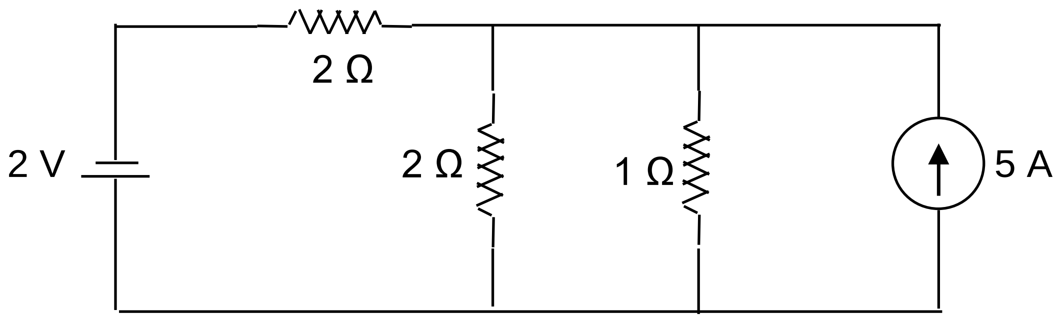


Solution: First calculate current I_1 due to voltage source alone. The current source is open circuited. The resulting circuit is shown below.



Total circuit resistance R_T = 2.6667 Ω. Circuit current $I_T = \frac{2}{2.6667} = 0.75$ A

$$\text{Current } I_1 = \frac{2}{3} \times 0.75 = 0.5 \text{ A from B to A}$$



Now calculate current I_2 due to current source alone. The voltage source is short circuited as shown in Fig.

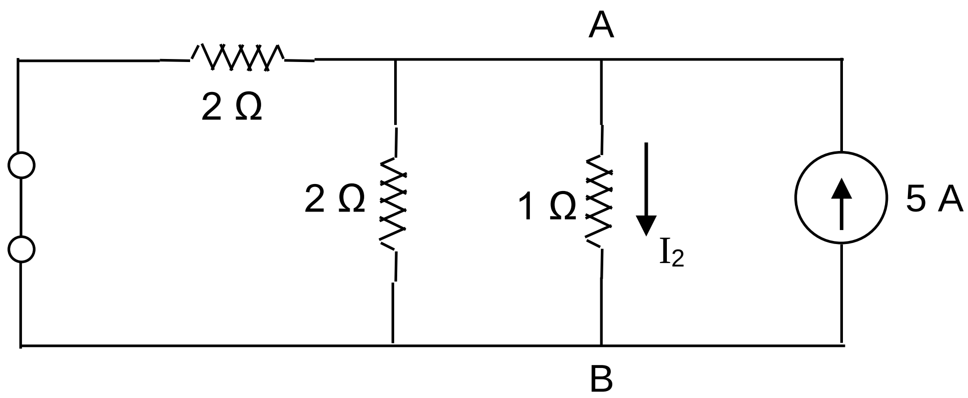


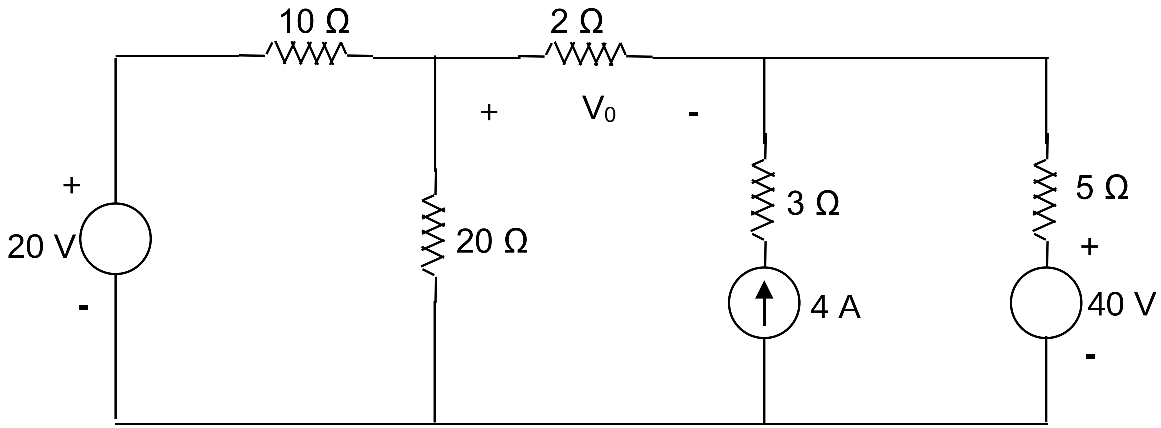
Fig. Circuit - Example 1

Noting that two 2 Ω resistors are in parallel, current I_2 = 2.5 A from A to B.

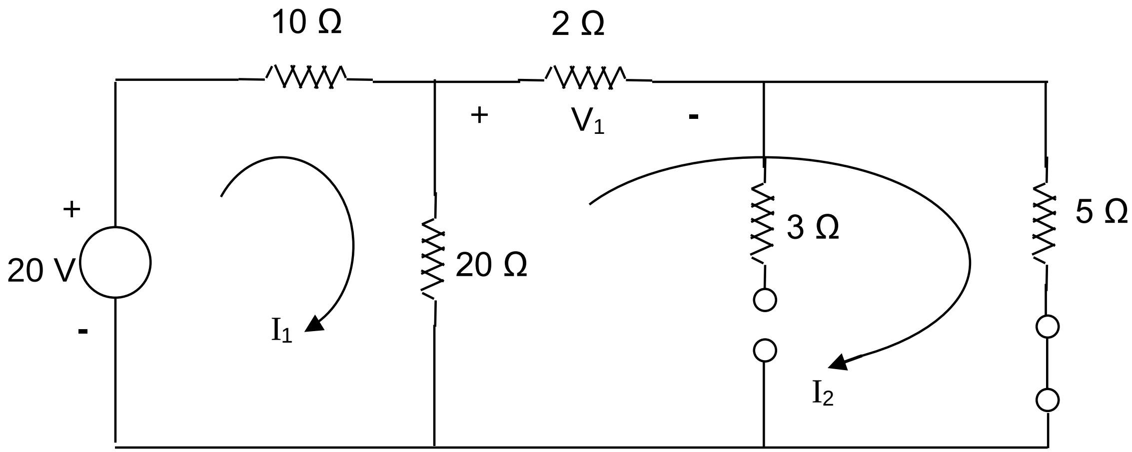
When both the sources are simultaneously present:

Current through 1 Ω resistor = 2.5 - 0.5 = 2 A from A to B.

Example 3 In the circuit shown, find the voltage drop, V_0 across the 2 Ω resistor using Superposition theorem.



Solution: **20 V source alone present:** The circuit will be as shown below.

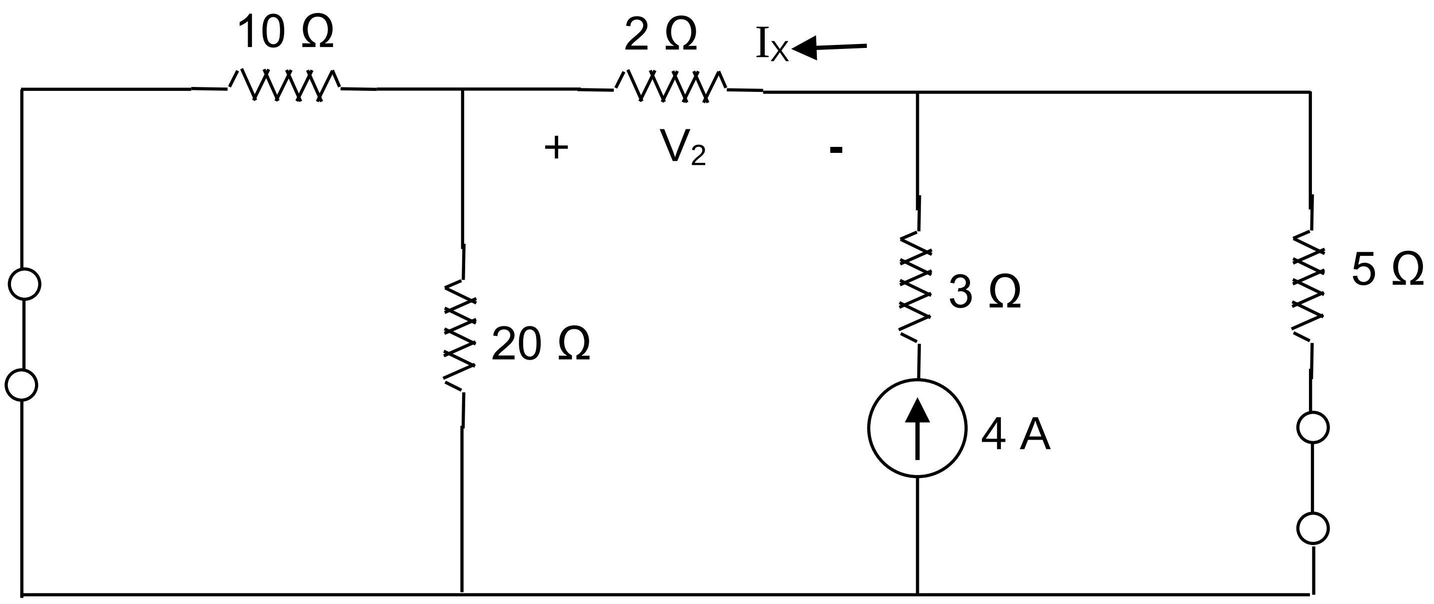


Mesh current equations : $\begin{bmatrix} 30 & -20 \\ -20 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$ On solving, $I_2 = 0.9756$ A

Thus voltage $V_1 = 2 \times 0.9756 = 1.9512$ V

4 A source alone present:

The circuit will be as shown below.



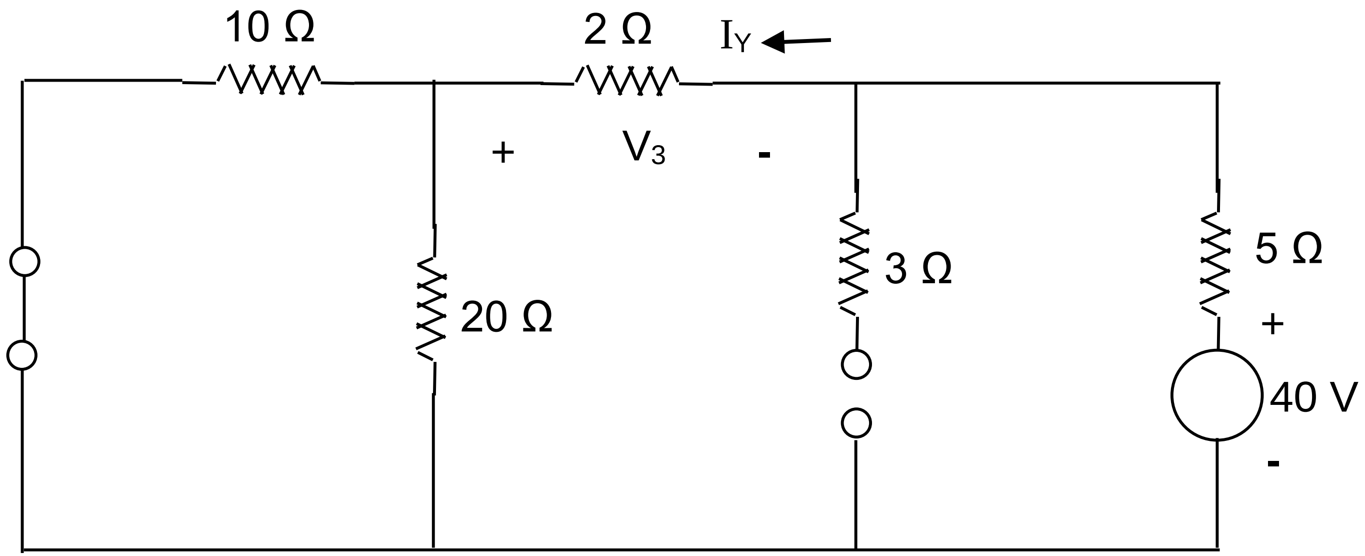
$$2 + 10 \,||\, 20 = 8.6667\ \Omega$$

Therefore current $I_X = \dfrac{5}{13.6667} \times 4 = 1.4634$ A

Thus voltage $V_2 = -\,2 \times 1.4634 = -\,2.9268$ V

40 V source alone present:

Resulting circuit is shown below.



Circuit resistance $R_T = 5 + 2 + (10||20) = 13.6667\ \Omega$

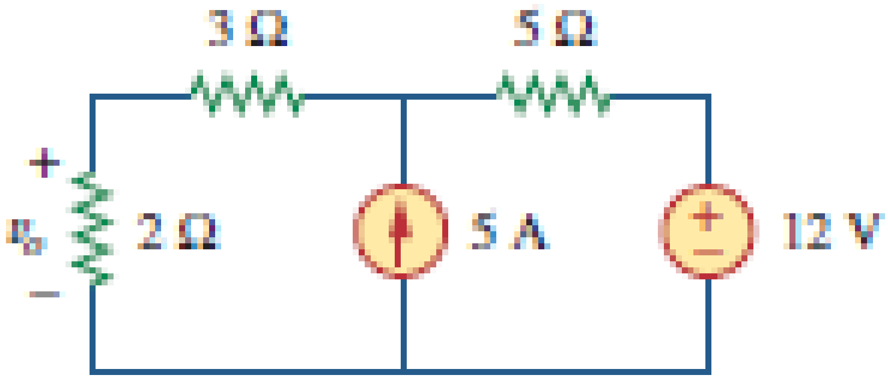
Current $I_Y = 40 / 13.6667 = 2.9268$ A; Thus voltage $V_3 = -\ 2 \times 2.9268 = -\ 5.8537$ V

When all the three sources are simultaneously present,

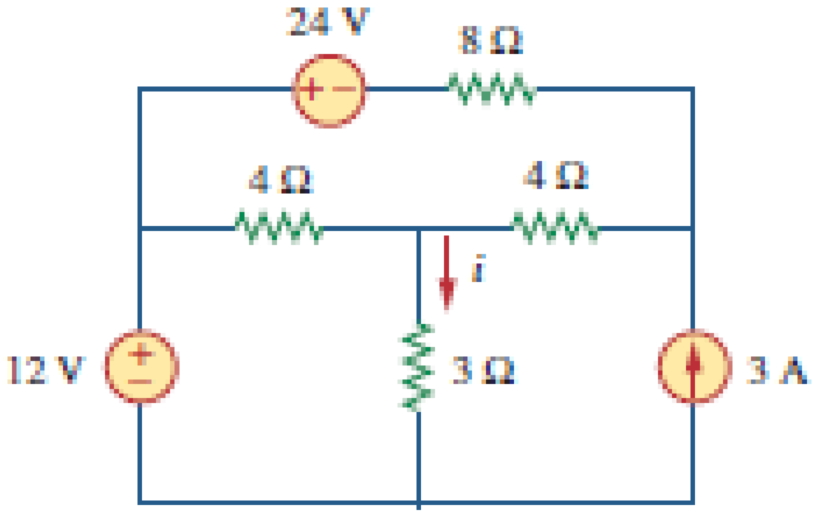
voltage across 2 Ω, i.e. $V_0 = V_1 + V_2 + V_3 = 1.9512 - 2.9268 - 5.8537 =$ **- 6.8293 V**

Practice Problems

1. Using superposition, find V_0 in the circuit of below Figure. **Ans:** 7.4 V



2. Using superposition, find i in the circuit of below Figure. **Ans:** 2 A



All the materials extracted from Fundamentals of Electric Circuits by Charles K. Alexander, Matthew N.O. Sadiku, 5th Edition, McGraw Hill, for the purpose of Teaching and Learning only.