

Roll No.: CS21B2045

Name: T.L. Srinivas



Indian Institute of Information Technology, Design and Manufacturing, Kanchi
End-Sem, Jan - May 2023

Course Code: MA2001

Date of Examination: 01.05.2023

Duration: 3 hour

Course Title: Probability and Statistics

Category: Elective

Maximum Marks: 50

Roll nos. of students appearing for the exam:

Instructions to students:

- Answer all questions.
- No partial marks will be given.

- i Passwords consist of six characters from the 10 digits and the 26 letters of the alphabet, and they are case sensitive.

 - (a) How many different passwords are possible?
 - (b) What is the probability that a password, chosen at random, consists of different characters?
 - (c) Using password cracking tools, it is claimed that passwords can be tested at the rate of a hundred million passwords in one second. How long would it take to try all of the passwords?

ii Suppose a valid password must begin with a letter, and contain at least one digit: how many passwords are now possible, and how long would it take to try all of the passwords?

(6)
- A series of 10 jobs arrives at a server farm with 10 processors. Assume that each of the jobs is equally likely to go through any of the processors. Find the probability that all processors are occupied.

(5)
- A PIN has four digits. Hence, there are 10^4 possible PINs. What is the probability that two people choosing the same 4-digit PIN? What is the minimum number of people that will ensure that there is at least a 50% probability that two or more will have the same PIN?

(5)
- Suppose that, after five years of use, 30% of tablet computers have problems with the camera, 20% have problems with the flash memory, and 10% have problems with both. What is the proportion of tablets that experience no problems with the camera or the flash memory after five years?

(5)
- As part of a promotional campaign, a store is offering a free Fitbit with the purchase of the new iPhone model. If 5% of the iPhones are faulty and 10% of the Fitbits are faulty, what is the probability that a customer gets both a faulty iPhone and a faulty Fitbit?

(5)
- In a university department, 50% of documents are written in Word, 30% in Latex, and 20% in Html. From past experience it is known that: 40% of the Word documents exceed 10 pages; 20% of the Latex documents exceed 10 pages; 20% of the Html documents exceed 10 pages. What is the overall proportion of documents containing more than 10 pages?

(6)
- A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95 and a transmitted 1 is correctly received with probability 0.75. It is also known that 70% of all signals are transmitted as a 0. If a signal is sent, determine the probability that

- i a zero is received;
- ii an error occurs.

(8)

8. Suppose there are n frames per packet, and the probability that a frame gets through without an error is 0.999. What is the maximum size that a packet can be so that the probability that it contains no frame in error is at least 0.9? (3)
9. It was reported in the International Journal of Circuit Theory and Applications, May-June 1990, that a switched-capacitor circuit for generating random signals had been designed, built, and tested by researchers at the University of California, Berkeley. The circuit's trajectory was shown to be uniformly distributed in the interval $(0, 1]$. Find the pdf and cdf of the random variable. (3)
10. A workstation contains ten chips each operating independently. Each has an average lifetime of 1,000 hours, and the lifetime is exponential. All chips must work for the workstation to work. What is the probability that the workstation will last more than 1,000? (4)

T.L. Saini Vol
CS 21B 2045



Course Code: MA2001

Date of Examination: 20.03.2023

Duration: 1 hour

Roll nos. of students appearing for the exam:

Instructions to students:

- Answer all questions.
- No partial marks will be given.

- ✓ 1. Let X be an Exponential random variable with parameter 3. Compute the function F_X . (3)
- ✓ 2. Let X be the number of heads when flipping three fair coins. Let $Y = 1$ if $X \geq 1$, with $Y = 0$ if $X = 0$. Find the density function of Y . (2)
- ✓ 3. Let X have density function $f_X(x) = (1/2) \sin(x)$ for $0 < x < \pi$, otherwise $f_X(x) = 0$. Let $Y = X^2$. Compute the density function $f_Y(y)$ for Y . (3)
- ✓ 4. Let X have the uniform distribution on $[0, 1]$, and let $Y = 3X$. What is the distribution of Y ? (2)
- ✓ 5. Let X be $N(0, 1)$. Let $Y = X^3$. Compute the density function $f_Y(y)$ for Y . (3)
- ✓ 6. Suppose $F_Y(y) = y^3$ for $0 \leq y < 1/2$, and $F_Y(y) = 1 - y^3$ for $1/2 \leq y \leq 1$. Compute each of the following. (6)
 1. $P(1/3 < Y < 3/4)$
 2. $P(Y = 1/3)$
 3. $P(Y = 1/2)$
7. Suppose X has density function f and Y has density function g . Suppose $f(x) > g(x)$ for $1 < x < 2$. Prove that $P(1 < X < 2) > P(1 < Y < 2)$. (4)
- ✓ 8. Let X be a normal random variable with mean -8 and $\text{variance } 4$. Compute each of the following:
 1. $P(X \geq -5)$
 2. ~~$P(X \leq -2)$~~
 $P(-2 \leq X \leq 7)$

Roll No: CS12B1064

Name: A. Arivindh

Indian Institute of Information Technology, Design and Manufacturing, Kanchi
Mid Semester - March 2024

Course Code: MA2001

Batch: CS

Date of Examination: 04.03.2024

Duration: 11:30 AM - 1:00 PM

Course Title: Probability and Statistics

Category: Elective

Instructor: Dr. Subramani

Maximum Marks: 25

1. (a) Passwords consist of six characters from the 10 digits and the 26 letters of the alphabet, and they are case sensitive. (1.5)
 - i. How many different passwords are possible? (1.5)
 - ii. What is the probability that a password, chosen at random, consists of different characters? (1.5)
 - iii. Using password cracking tools, it is claimed that passwords can be tested at the rate of a hundred million passwords in one second. How long would it take to try all of the passwords? (1.5)
- (b) Suppose a valid password must begin with a letter, and contain at least one digit; how many passwords are now possible, and how long would it take to try all of the passwords? (1.5)
2. A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95 and a transmitted 1 is correctly received with probability 0.75. It is also known that 70% of all signals are transmitted as a 0. If a signal is sent, determine the probability that
 - (a) a zero is received; (3)
 - (b) an error occurs. (3)
3. A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95 and a transmitted 1 is correctly received with probability 0.75. It is also known that 70% of all messages are transmitted as a 0. If a signal is received as a zero, what is the probability that it was transmitted as a zero? (4)
4. Suppose there are n frames per packet, and the probability that a frame gets through without an error is 0.999. What is the maximum size that a packet can be so that the probability that it contains no frame in error is at least 0.9? (4)
5. The average rate of job submissions in a computer center is four per minute. If it can be assumed that the number of submissions per minute interval is Poisson distributed, calculate the probability that:

- (a) at least two jobs will be submitted in any minute; (2)
- (b) no job will be submitted in any minute; (2)
- (c) no more than one job will be submitted in any one-minute interval (2)

Instructions to students:

- Answer all questions.
- No partial marks will be given.

2. Suppose that A and B are mutually exclusive events for which $P(A) = .3$ and $P(B) = .5$. What is the probability that

1. either A or B occurs?
2. A occurs but B does not?
3. both A and B occur?

$$\begin{aligned} A \cap B &= 0 \\ P(A \cup B) &= P(A) + P(B) \end{aligned}$$

2. (The birthday problem) Suppose there are C people, each of whose birthdays (month and day only) are equally likely to fall on any of the 365 days of a normal (i.e., non-leap) year. (a) Suppose $C = 2$. What is the probability that the two people have the same exact birthday? (b) Suppose $C \geq 2$. What is the probability that all C people have the same exact birthday? (c) Suppose $C \geq 2$. What is the probability that some pair of the C people have the same exact birthday? (d) What is the smallest value of C such that the probability in part (c) is more than 0.5? Do you find this result surprising

(2+2+3+3)

2. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability .4, whereas this probability decreases to .2 for a person who is not accident prone. If we assume that 30 percent of the population is accident prone, what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(4)

4. Consider rolling one fair six-sided die, so that $S = \{1, 2, 3, 4, 5, 6\}$, and $P(s) = 1/6$ for all $s \in S$. Let X be the number showing on the die, so that $X(s) = s$ for $s \in S$. Let $Y = X^2$. Compute the cumulative distribution function $F_Y(y) = P(Y \leq y)$, for all $y \in \mathbb{R}$.

(2)

5. Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win 2 rupees for each black ball selected and we lose 1 rupee for each white ball selected. Let X denote our winnings. What are the possible values of X , and find the pdf of X .

(4)

6. Each item produced by a certain manufacturer is, independently, of acceptable quality with probability 0.95. Approximate the probability that at most 10 of the next 150 items produced are unacceptable.

(2)