Problem Set - 1

January 29, 2025

1. Suppose that A_1 watches the six o'clock news $\frac{2}{3}$ of the time and watches the eleven o'clock news $\frac{1}{2}$ of the time, and watches both the six o'clock and eleven o'clock news $\frac{1}{3}$ of the time. For a randomly selected day, what is the probability that A_1 watches only the six o'clock news? For a randomly selected day, what is the probability that A_1 watches neither news?

2. Suppose that an employee arrives late 10% of the time, leaves early 20% of the time, and both arrive late and leave early 5% of the time. What is the probability that on a given day that employee will either arrive late or leave early (or both)?

3. Suppose your right knee is sore 15% of the time, and your left knee is sore 10% of the time. What is the largest possible percentage of time at least one of your knees is sore? What is the smallest possible percentage of time at least one of your knees is sore?

4. Suppose your team has a 40% chance of winning or tying today's game and has a 30% chance of winning today's game. What is the probability that today's game will be a tie?

5. Suppose we flip 100 fair independent coins. What is the probability that at least three of them are heads?

6. Suppose we keep dealing cards from an ordinary 52-card deck until the first jack appears. What is the probability that at least 10 cards go by before the first jack?

7. Suppose we roll three fair six-sided dice. What is the probability that two of them show the same value, but the third one does not?

8. Suppose we roll a fair six-sided die and flip three fair coins. What is the probability that the total number of heads is equal to the number showing on the die?

9. (The birthday problem) Suppose there are C people, each of whose birthdays (month and day only) are equally likely to fall on any of the 365 days of a normal (i.e., non-leap) year. (a) Suppose $C = 2$. What is the probability that the two people have the same exact birthday? (b) Suppose

$C \geq 2$. What is the probability that all C people have the same exact birthday? (c) Suppose $C \geq 2$. What is the probability that some pair of the C people have the same exact birthday? (d) What is the smallest value of C such that the probability in part (c) is more than 0.5? Do you find this result surprising?

10. Suppose we are dealt five cards from an ordinary 52-card deck. What is the probability that (a) we get all four aces, plus the king of spades? (b) all five cards are spades? (c) we get no pairs (i.e., all five cards are different values)? (d) we get a full house (i.e., three cards of a kind, plus a different pair)?

11. Suppose we pick a card at random from an ordinary 52-card deck and also flip 10 fair coins. What is the probability that the number of heads equals the value of the card (where we count jacks, queens, and kings as 10, and count aces as 1)?