DSCS

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ASSIGNMENT-1

Logic Questions

1: Write FOL for the following.

(a) You can fool all the people some of the time, and some of the people all the time, but you can't fool all the people all the time.

Ans UOD: Set of people

$f(x)$: You can fool x some of the time.

$g(x)$: You can fool x all the time.

FOL: $\forall x\, f(x) \wedge \exists x\, g(x)\, \neg\forall x\, g(x)$ $\quad x \in UOD$

(b) Everyone wants to get government job but no one wants to study in government school.

Ans UOD: Set of people.

$J(x)$: x wants to gets a government job

$S(x)$: x wants to study in government school.

FOL: $\forall x\,(J(x) \wedge \neg S(x))$ $\quad x \in UOD$

(c) There exist only two types of quantifiers universal quantification and existential quantification.

Ans UOD: Set of quantifiers

$U(x)$: x is an universal quantification

$E(x)$: x is existential quantification

FOL: $\exists! x\, U(x) \wedge [(x \neq y) \wedge \exists! y\, E(x)]$ $\quad x, y \in UOD$

(d) Everyday in our life may not be good, but there is something good in Everyday.

Ans UOD: Set of days.

$G(x)$: x is a Good day

$S(x)$: Something is good in day x.

$\exists x\, G(x) \wedge \forall x\, S(x)$ $\quad x \in UOD$

2. Write the FOL for the following.

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(a) Someone likes someone.

(b) Someone likes everyone

(c) None likes everyone.

(d) None likes All.

Ans UOD: Set of people

$P(x,y)$: x likes y

(a) $\exists x \exists y\, P(x,y)$

(b) $\exists x \forall y\, P(x,y)$

(c) $\neg(\cancel{\forall}\, \exists x \forall y\, P(x,y))$

(d) $\forall x \exists y\, \neg(P(x,y))$ $\qquad x, y \in UOD$

3. Write the defination of prime numbers in FOL.

Ans UOD: Set of all natural numbers

$D(x,y)$: x divides y

$P(x)$: x is a prime number. $\qquad x, y \in UOD$.

FOL: $\forall x[((x>1) \land \forall y(D(y,x))] \longrightarrow ((y=1) \lor (y=x))] \rightarrow P(x)$

4. Negate and Simplify: $\exists L \exists n \exists z(|z| \geq n \land \exists u \exists v \exists w((z=uvw, |uv| \leq n) \rightarrow \exists i(i \geq 0 \longrightarrow uv^i w \neq L)))$

Ans negation of given statement.

$\neg(\exists L \exists n \exists z(|z| \geq n \land \exists u \exists v \exists w((z=uvw, |uv| \leq n) \longrightarrow \exists i(i \geq 0 \longrightarrow uv^i w \neq L))))$

$= \neg(\exists L \exists n \exists z(|z| \geq n \land \exists u \exists v \exists w(\neg(z=uvw, |uv| \leq n) \lor \exists i(i \geq 0 \longrightarrow uv^i w \neq L))))$

$= \neg(\exists L \exists n \exists z(|z| \geq n \land \exists u \exists v \exists w((z \neq uvw, |uv| \leq n) \lor \exists i(i \geq 0 \longrightarrow uv^i w \neq L))))$

$= \forall L \forall n \forall z(|z| < n \lor \forall u \forall v \forall w \neg((z \neq uvw, |uv| \leq n) \lor \exists i(i \geq 0 \longrightarrow uv^i w \neq L)))$

$= \forall L \forall n \forall z(|z| < n\, \cancel{\forall} \lor \forall u \forall v \forall w(\neg(z \neq uvw, |uv| \leq n) \land \forall i\, \neg(i < 0 \lor uv^i w \neq L)))$

$= \forall L \forall n \forall z(|z| < n \lor \forall u \forall v \forall w((z = uvw, |uv| \leq n) \land \forall i(i \geq 0 \land uv^i w = L)))$

5. Prove or disprove

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(a) $\exists x(P(x) \leftrightarrow Q(x)) \rightarrow \neg\forall x Q(x) \vee \exists x P(x)$

ans $\underbrace{\exists x(P(x) \leftrightarrow Q(x))}_{A} \rightarrow \underbrace{\neg\forall x Q(x) \vee \exists x P(x)}_{B}$

Taking R.H.S.

$A = \exists x((P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow P(x)))$ $[\because P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)]$

$= \exists x((\neg P(x) \vee Q(x)) \wedge (\neg Q(x) \vee P(x)))$

$= \exists x((\neg Q(x) \vee P(x)))$ $[\because P \wedge Q \rightarrow P]$

$= \neg\forall x Q(x) \vee \exists x P(x) = B(R.H.S.)$

$\therefore A = B$

$\Rightarrow A \Rightarrow B$

hence given statement is valid.

(b) $\exists x(P(x) \leftrightarrow Q(x)) \rightarrow \neg\forall x P(x) \vee \exists x Q(x)$.

ans $\underbrace{\exists x(P(x) \leftrightarrow Q(x))}_{A} \rightarrow \underbrace{\neg\forall x P(x) \vee \exists x Q(x)}_{B}$

Taking L.H.S.

$A = \exists x((P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow P(x)))$ $[\because P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)]$

$= \exists x((\neg P(x) \vee Q(x)) \wedge (\neg Q(x) \vee P(x)))$

$= \exists x((\neg P(x) \vee Q(x)))$ $[\because P \wedge Q \rightarrow P]$

$= \neg\forall x P(x) \vee \exists x Q(x) = B(R.H.S.)$

$\therefore A = B$

$\therefore A \Rightarrow B$

Hence given statement is valid.

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(c) $\underbrace{\forall x \forall y (P(x,y))}_{A} \longleftrightarrow \underbrace{\forall y \forall x \, P(x,y)}_{B}$.

Ans
Taking L.H.S.

$A = \forall x \forall y (P(x,y))$

$= \forall y P(a,y)$ for any a (Universal instantiation)

$= P(a,b)$ for any a, b (universal instantiation)

$= \forall x P(x,b)$ (Universal generalisation)

$= \forall y \forall x P(x,y) = B$ (R.H.S) (Universal generalisation)

$A = B$

$\therefore A \Rightarrow B$

Taking R.H.S.

$B = \forall y \forall x P(x,y)$

$= \forall x P(x,b)$ for any b (Universal instantiation)

$= P(a,b)$ for any a, b (universal instantiation)

$= \forall y P(a,y)$ (Universal generalisation)

$= \forall x \forall y P(x,y) = A$ (L.H.S) (Universal generalisation)

$B = A$

$\therefore B \Rightarrow A$

$\because A \Rightarrow B$ &
$B \Rightarrow A$

$\Rightarrow \quad A \Leftrightarrow B$

Hence, it is proved.

(d) $\underbrace{\forall x \exists y \, P(x,y)}_{A} \longleftrightarrow \underbrace{\exists y \forall x \, P(x,y)}_{B}$.

Ans Taking L.H.S.

$A = \forall x \exists y P(x,y)$

$= \exists y P(a,y)$ for any a (Universal instantiation)

$= P(a,b)$ for any a and some b (Existential instantiation).

$= \forall x (P(x, b))$ (Universal generalisation)

$= \exists y \forall x (P(x, y))$ ~~two~~ (Existential generalisation)

$= B$ (R.H.S.) $A = B$

$\therefore A \Rightarrow B$

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Taking R.H.S.

$B = \exists y \forall x (P(x, y))$

$= \forall x (P(x, b))$ for some b (Existential instantiation)

$= P(a, b)$ for some b and all a (Universal instantiation)

$= \exists y P(a, y)$ (Existential generalisation)

$= \forall x \exists y P(x, y)$ (Universal generalisation)

$= A$ (L.H.S.)

$B = A$

$\therefore B \Rightarrow A$

$\because A \Rightarrow B$ & $B \Rightarrow A$

$\therefore A \Leftrightarrow B$

Hence it is proved.

6 Write four different expressions equivalent to $\forall x (P(x) \vee Q(x))$.

Ans $\forall x (\neg P(x) \rightarrow Q(x))$

$\neg(\exists x (\neg P(x) \wedge \neg Q(x)))$

$\forall x (\neg Q(x) \rightarrow P(x))$

$\forall x ((P(x) \rightarrow Q(x)) \rightarrow (P(x) \vee Q(x)))$ $[\because (P \rightarrow Q) \rightarrow (P \vee Q)]$

Check whether the following boolean expression is a tautology without using truth tables.

7. (a) $(p \wedge q \wedge r) \rightarrow (r \vee p)$

Ans $\neg(p \wedge q \wedge r) \vee (r \vee p)$

$= \neg p \vee \neg q \vee \neg r \vee r \vee p.$

$= (\neg p \vee p) \vee (\neg q) \vee (\neg r \vee r)$ $[\because P \vee (Q \vee R) = (P \vee Q) \vee R]$ associativity

$= T \vee \neg q \vee T$ $[\because P \vee T = T]$

$= T$ (tautology)

$\therefore$ Given Statement is tautology.

(b) $(p \leftrightarrow q) \rightarrow (\neg r \rightarrow p \wedge q)$

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Ans $(p \leftrightarrow q) \rightarrow (\neg r \rightarrow p \wedge q)$

$((p \rightarrow q) \wedge (q \rightarrow p)) \rightarrow (\neg r \rightarrow p \wedge q)$ $[\because p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)]$

$= ((\neg p \vee q) \wedge (\neg q \vee p)) \rightarrow (\neg(\neg r) \vee (p \wedge q))$

$= \neg((\neg p \vee q) \wedge (\neg q \vee p)) \vee (r \vee (p \wedge q))$

$= (p \wedge \neg q) \vee (q \wedge \neg p) \vee r \vee (p \wedge q)$ $(\because p \vee (q \vee r) = (p \vee q) \vee r)$ associativity

$= ((p \wedge \neg q) \vee (p \wedge q)) \vee (q \wedge \neg p) \vee r$

$= ((p \vee (p \wedge q)) \wedge (\neg q \vee (p \wedge q))) \vee (q \wedge \neg p) \vee r.$

$= (p \wedge ((\neg q \vee p) \wedge (\neg q \vee q))) \vee (q \wedge \neg p) \vee r$ $\left[\begin{array}{l} \because p \wedge (p \vee Q) = p \\ p \vee (p \wedge Q) = p \\ \text{absorption laws} \end{array}\right]$

$= (p \wedge ((\neg q \vee p) \wedge T)) \vee (q \wedge \neg p) \vee r$

$= (p \wedge (\neg q \vee p)) \vee (q \wedge \neg p) \vee r.$ $\left[\begin{array}{l} \because \neg q \vee q = T \\ a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \\ \text{distributive law} \end{array}\right]$

$= p \vee (q \wedge \neg p) \vee r$

$= ((p \vee q) \wedge (p \wedge \neg p)) \vee r$

$= ((p \vee q) \wedge T) \vee r$

$= p \vee q \vee r$

This is neither tautology nor contradiction.

$\therefore$ Given statement is not tautology.

8. Prove or disprove, ~~all~~ some students prepare for JEE and NEET. Some prepare for either NEET or JEE. Therefore there are students who have taken neither JEE nor NEET.

Ans UOD: Set of students

$J(x)$: x prepares for JEE

$N(x)$: x prepares for NEET

From given statements in the question

$\exists x (J(x) \wedge N(x))$ — ① Some students prepare for JEE and NEET

$\exists x ((J(x) \wedge \neg N(x)) \vee (\neg J(x) \wedge N(x)))$ — ② Some students prepare for either JEE or NEET

$\therefore \exists x (\neg J(x) \wedge \neg N(x))$ has to prove — There are students who have taken neither JEE nor NEET

$\exists x (J(x) \wedge N(x))$ — Eq. ①

$J(a) \wedge N(a)$ — for some a (existential instantiation)

$\left.\begin{array}{l} J(a) \\ N(a) \end{array}\right\}$ true for some a

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$\left.\begin{array}{l} \neg J(b) \\ \neg N(b) \end{array}\right\}$ true for some b $(a \neq b)$

$\Rightarrow \neg J(b) \wedge \neg N(b)$ true for some b

$\therefore \exists x (\neg J(x) \wedge \neg N(x))$ (Existential generalisation)

$\therefore$ There are students who have taken neither JEE nor NEET

9. Prove or disprove: all students of second btech are eligible for all internship. Some internships have specific requirements. Therefore, some second btechs are interning in a company with specific requirements.

Ans UOD: set of all second btech students

Let

$E(x)$: x is eligible for all internships.

$R(x)$: x has specific ~~internship~~ requirements for some internship.

$C(x)$: x is interning in a company with specific requirements.

From given statements

$\forall x\, E(x)$ — All students of second btech are eligible for all internship.

$\exists x\, R(x)$ — Some internship have specific requirements.

To prove: $\exists x\, C(x)$

we need to prove

$\exists x (E(x) \wedge C(x))$ — ①

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Proof by contradiction Rollno: CS22B2026

Let us assume that negation of eq① is true

$\neg \exists x(E(x) \wedge C(x)) = \forall x(\neg E(x) \vee \neg C(x))$ (De Morgan's law) — ②

But all students are eligible for internships

$\Rightarrow E(x)$ is true for all x

$\neg E(x)$ is false for all x.

$\Rightarrow$ Eq ② is true only if $\forall x(\neg C(x))$ becomes true

which implies that none of the students are interning in a company with specific requirements.

This contradicts our assumption.

$\therefore \exists x(E(x) \wedge C(x))$ is true

$\therefore$ There are some second btechs interning in a company with specific requirements.

10 Check the validity of the argument; Some functions are continuous or differentiable. All functions have ~~specific~~ a peculiar property. Therefore some continuous functions have a peculiar property.

Ans UOD: set of functions.

$C(x)$: x is continuous functions.

$D(x)$: x is differentiable functions

$P(x)$: x has a peculiar property.

Given statements

$\exists x(C(x) \vee D(x))$ — ① Some functions are continuous or differentiable.

$\forall x P(x)$ — ② All functions have a ~~sp~~ peculiar property.

We need to prove

$\exists x(C(x) \wedge P(x))$ Some continuous functions have a peculiar property.

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From eq(1)

$\exists x\,(C(x) \vee D(x))$

$C(a) \vee D(a)$ for some $a \in UOD$ (Existential instantiation)

From eq(2)

$\forall x P(x)$

$P(a)$ for all $a \in UOD$ (Universal instantiation)

Case(i) If $C(a)$ is true

$\Rightarrow$ a is a continuous function and since all functions have a peculiar property, a will also have a peculiar property.

$\therefore$ $C(a) \wedge P(a)$ is true

Case(ii) If $D(a)$ is true

$\Rightarrow$ a is a differentiable function ~~and is~~

$\Rightarrow$ a is also a continuous function

($\because$ All differentiable functions are continuous)

Since all function have a peculiar property.

a will also have a peculiar property

$\therefore$ $C(a) \wedge P(a)$ is true

$\therefore$ $C(a) \wedge P(a)$ is true for some $a \in UOD$

$\Rightarrow$ $\exists x\,(C(x) \wedge P(x))$ (Existential generalisation)

Therefore, some continuous functions have a peculiar property is a valid statement.

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Proof Techniques.

1. Show that every odd integer is the difference of two square numbers using direct proof.

Ans Let a be an odd number

$a = 2k+1$ k is any integer

add & substract k^2 on R.H.S.

$a = (2k+1+k^2) - k^2$

$= (k+1)^2 - k^2$

Let

$k+1 = b$

$k = c$

$\Rightarrow a = b^2 - c^2$

$\therefore$ Every odd integer is the difference of two squares.

2. Prove or disprove if x and y are rational, then x^y is rational.

Ans The statement can be disproved by giving a suitable counter example

Let $x = 3$ $y = 1/2$

where, x, y are rational (which are in form of P/q where $q \neq 0$ & (P,q) GCD $= 1$)

$x^y = 3^{1/2} = \sqrt{3}$

which is an irrational number.

hence, it is disproved.

3. If $n \in N$ and $2^n - 1$ is prime, then n is prime (proof by contraposition).

Ans Given

P: $2^n - 1$ is prime

Q: n is prime

$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$

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$\neg Q$: n is not prime
n is composite

$\neg P$: 2^n-1 is composite.

Let $n = xy$ $\quad (x, y \neq 1)$ ($\because$ n is composite)

$$2^n - 1 = 2^{xy} - 1$$
$$= (2^x)^y - 1$$
$$= \underbrace{(2^x - 1)}_{a}\underbrace{((2^x)^{y-1} + (2^x)^{y-2} + \dots + 1)}_{b}$$

Let $a = 2^x - 1$
$b = (2^x)^{y-1} + (2^x)^{y-2} + \dots + 1$ $\quad a, b \neq 1$ as $x, y \neq 1$

$\Rightarrow 2^n - 1 = ab$

$\Rightarrow 2^n - 1$ is not a prime.

$\therefore$ By proof by contraposition
$P \rightarrow Q$

$\therefore$ 2^{n-1} is prime when n is prime

4. Use a proof by contradiction to prove that the sum of rational and irrational number is irrational.

Ans Let us assume to the contrary that sum of an irrational and rational be rational.

Let R be a rational & I be an irrational

$R = P/q$ $\quad q \neq 0$ $\quad \overset{GCD}{(P, q)} = 1$

From assumption.

$R + I = \frac{a}{b}$ $\quad b \neq 0$ $\quad \overset{GCD}{(a, b)} = 1$

$\frac{P}{q} + I = \frac{a}{b}$

$I = \frac{a}{b} - \frac{P}{q}$.

$= \frac{aq - pb}{bq}$ $\quad bq \neq 0$

$GCD(aq - pb, bq) = 1$

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$\Rightarrow$ I is a rational number.

But this contradicts the fact that I is irrational

$\therefore$ our assumption is wrong

By proof by contradiction, sum of a rational and an irrational is irrational.

5. Show that $\sum_{i=1}^{n} n.n! = (n+1)! - 1$ using M.I.

Ans **Base case:**

For $n=1$

$$L.H.S = \sum_{i=1}^{1} 1.1! = 1.1! = 1$$

$$R.H.S = (1+1)! - 1 = 2! - 1 = 1$$

$$L.H.S. = R.H.S.$$

$\therefore$ Base case is proved.

Induction hypothesis

Let us assume that given statement is true for $n \geq 1$

$$\sum_{i=1}^{n} i.i! = (n+1)! - 1 \quad \text{—— (1)}$$

Induction step

Now we have to prove the statement for $n+1$

$$\sum_{i=1}^{n+1} i.i! = \sum_{i=1}^{n} i.i! + (n+1)(n+1)!$$

$$= (n+1)! - 1 + (n+1)(n+1)!$$

$$= (n+1)!(n+2) - 1$$

$$= (n+2)! - 1 = R.H.S.$$

$$L.H.S = R.H.S.$$

Hence, by M.I. $\sum_{i=1}^{n} i.i! = (n+1)! - 1$ is true.

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6. Consider the following four equations. Roll no: CS22B2026

(i) $1=1$

(ii) $2+3+4=1+8$

(iii) $5+6+7+8+9=8+27$

(iv) $10+11+12+13+14+15+16=27+64$.

Conjecture the general formula suggested by these four equations, and prove your conjecture by M.I.

Ans From the above equations,

General formula can be.

$$\sum_{k=1}^{2n+1} (n^2+k) = n^3+(n+1)^3$$

Base case:

For $n=0$

$$L.H.S. = \sum_{k=1}^{1} (0^2+k) = \sum_{k=1}^{1} k = 1$$

$$R.H.S. = (0)^3+(0+1)^3 = 1$$

$L.H.S. = R.H.S.$

$\therefore$ Base case is verified.

Induction hypothesis

Let us assume that given statement is true for $n \geq 0$

$$\sum_{k=1}^{2n+1} (n^2+k) = n^3+(n+1)^3 \quad \text{—— (1)}$$

Induction Step:

Let us prove that the statement is true for $(n+1)$

$$L.H.S. = \sum_{k=1}^{\cancel{2n+1}\ 2(n+1)+1} (n+1)^2+k$$

$$= \sum_{k=1}^{2n+3} n^2+2n+1+k$$

$$= \sum_{k=1}^{2n+3} n^2+k + \sum_{k=1}^{2n+3} (2n+1)$$

$$= \sum_{k=1}^{2n+1}(n^2+k) + n^2+2n+2+n^2+2n+3 + (2n+1)(2n+3)$$

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$$= n^3+(n+1)^3+2n^2+4n+5+4n^2+8n+3.$$

$$= n^2+6n^2+12n+8+(n+1)^3$$

$$= (n+2)^3+(n+1)^3 = R.H.S.$$

L.H.S. = R.H.S

$\therefore$ Given General equation is true for $n \geq 0$ by M.I.

7. Let F_n denote the n^{th} Fibonacci number. Prove that

$F_0+F_1+\ldots+F_n = F_{n+2}-1.$

Ans Fibonacci Series

$0, 1, 1, 2, 3, 5, 8, \ldots$

$F_0\ F_1\ F_2 \ldots$

Base case:

For $n=0$

L.H.S. $= F_0 = 0$

R.H.S. $= F_{0+2}-1 = 1-1 = 0$

L.H.S. = R.H.S.

hence base case is verified.

Induction hypothesis:

Induction hypothesis is true for $n \geq 0$

Let us assume that the given statement is true for $n \geq 0$

$$F_0+F_1+\ldots+F_n = F_{n+2}-1 \quad — ①$$

Induction Step:

We have to prove that the statement is true for $(n+1)$

we know that

$$\boxed{F_n+F_{n+1} = F_{n+2}} \quad \forall\, n \geq 0$$

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$$L.H.S. = \underbrace{F_0+F_1+ \ldots +F_n}+F_{n+1}$$

$$= F_{n+2}-1+F_{n+1} \quad \text{From eq①}$$

$$= \underbrace{F_{n+1}+F_{n+2}}-1$$

From eq②

$$= F_{n+3}-1 = R.H.S.$$

L.H.S. = R.H.S.

$\therefore$ By M.I. the given statement $F_0+F_1+\ldots+F_n = F_{n+2}-1$ is true for all $n \geq 0$.

8. Is it possible to produce change for n rupees by using rupees 5 and 6 such that the number of coins used is minimum. (use strong M.I.)

Ans Base cases:

$n = 20 = 4 \times R5$
$21 = 3 \times R5 + 1 \times R6$
$22 = 2 \times R5 + 2 \times R6$
$23 = 1 \times R5 + 3 \times R6$
$24 = 4 \times R6$
$25 = 5 \times R5.$

Induction hypothesis

Let us assume that given statement is true for $K, K-1, K-2, K-3, K-4$ for $K-4 \geq 20$

Induction Step

we have to prove that the statement is true for $K+1$.

$$K+1 = K-4 + 1 \times R5$$
$$= K+1$$

hence, the statement is true for $n \geq 20$ By Strong Mathematical Induction.

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9. Show that in any group of 20 people (where any two people are friends or enemies), there are either 4MF or 4ME.

Ans We know that in a group of 10 people there are (3MF(or)4ME) and (4MF(or)3ME). Let P_i be a person from 20 people and ~~in~~ remaining are 19 people.

There are two cases for this. First case P_i is friend with atleast 10 people and second case is P_i is enemy with atleast 10 people.

Case(i) $P_i \geq 10$ friends

P_i
|
10 friends
(3MF or 4ME)
and (4MF or 3MF)

For 3MF or 4ME including P_i there are 4MF or 4ME

For 4MF (or) 3ME including P_i there are 4MF

$\Rightarrow$ There are atleast 4MF (or) 4ME

Case(ii)

$P_i \geq 10$ enemies

P_i
⋮
10 enemies
(3MF or 4ME) and
(4MF or 3ME)

For 3MF(or)4ME including P_i there are. $\geq$ 4ME

For 4MF(or) 3ME including P_i there are. 4MF(or)4ME

$\Rightarrow$ There are atleast 4MF(or) 4ME

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10 A child watches TV at least one hour each day for seven weeks but, because of parential rules, never more than 11 hours in anyone week. Prove that there is some period of consecutive days in which the child watches exactly 20 hours of TV. (It is assumed that the child watches TV for a whole number of hours each day).

Ans Given:

Maximum no. of hours a child watches T.V. in a week = 11 hrs

Minimum no. of hours a child watches T.V. in a day = 1 hrs

=> Maximum no. of hours a child watches ~~the child watches~~ in 7 weeks = 11×7 = 77 hrs

Let a_i denotes the number of hours the child watches till day i

$a_1 = \#day1$

$a_2 = \#day1 + \#day2$

$\vdots$

$a_n = \#day1 + \#day2 + \ldots \ldots + \#day\,n.$

From above information

$1 \leq a_1 < a_2 < \ldots < a_{49} \leq 77$ —— 1

Adding 20 hours to each element in the above equation

$21 \leq a_1 + 20 < a_2 + 20 < \ldots < a_{49} + 20 \leq 97.$

here

Let the pigeons be $a_1, a_2, \ldots, a_{49}, a_1+20, \ldots a_{49}+20$ = 98 pigeons.

& pholes be 1, 2, . . . , 97 hours.

By pigeon hole principle, there exist atleast one amount of time when two sets of a_i falls in same amount of time.

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i.e. $i > j$

$$a_i = a_j + 20$$

$\therefore$ There exists consecutive days $j, j+1, \ldots, i$ such that amount of time child watches T.V. is exactly 20 hours.