Engineering Electromagnetics

Lecture 26

07/11/2023

by

Debolina Misra

Department of Physics
IIITDM Kancheepuram, Chennai, India

Infinite wire and integral of B along a path

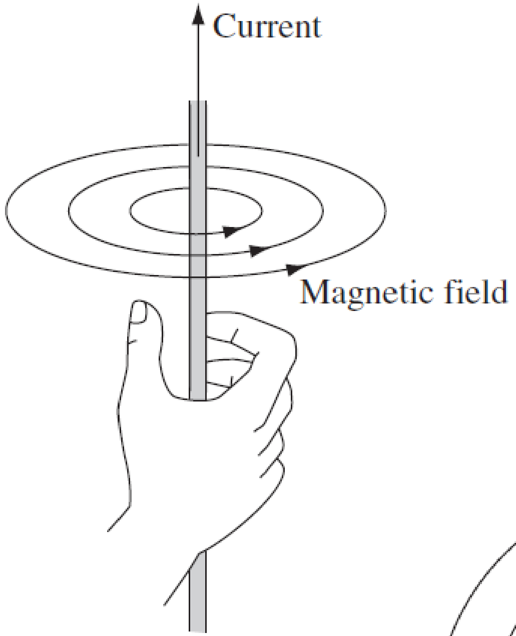
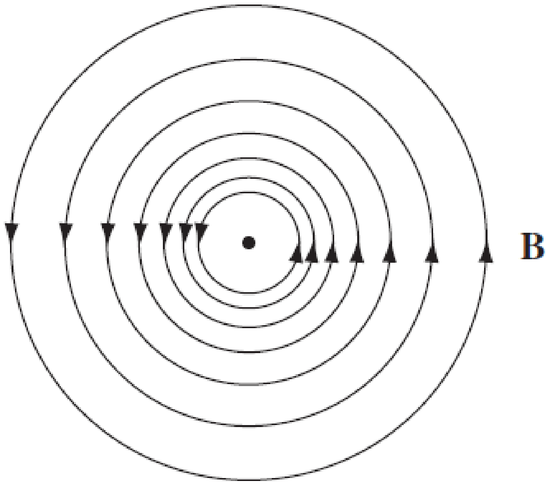


FIGURE 5.3

The magnetic field of an infinite straight wire is shown in Fig. 5.27 (the current is coming *out* of the page).

According to Eq. 5.38, the integral of **B** around a circular path of radius s, centered at the wire, is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint \frac{\mu_0 I}{2\pi s}\, dl = \frac{\mu_0 I}{2\pi s} \oint dl = \mu_0 I.$$

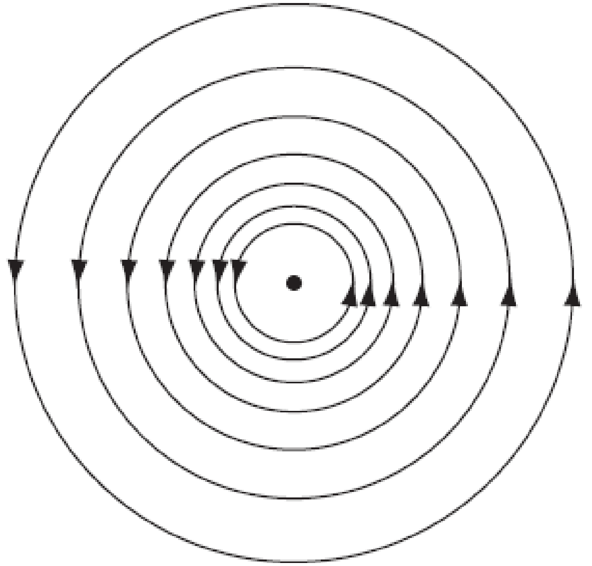


What do you think? ir/rotational

Not only along circular path, any loop that encloses the wire would give the same answer.
Try for cylindrical path

Cylindrical path

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint \frac{\mu_0 I}{2\pi s} \, dl = \frac{\mu_0 I}{2\pi s} \oint dl = \mu_0 I.$$

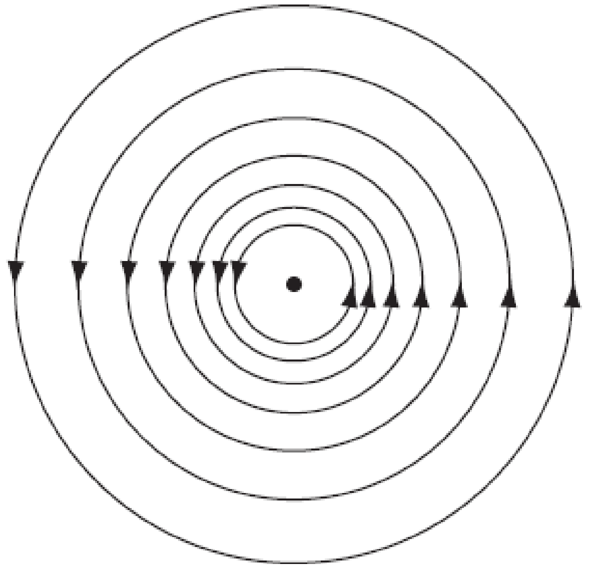


For if we use cylindrical
coordinates (s, ϕ, z), with the current flowing along the z axis, $\mathbf{B} = (\mu_0 I / 2\pi s)\hat{\boldsymbol{\phi}}$
and $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$, so

From Griffith's book

Cylindrical path

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint \frac{\mu_0 I}{2\pi s}\, dl = \frac{\mu_0 I}{2\pi s} \oint dl = \mu_0 I.$$



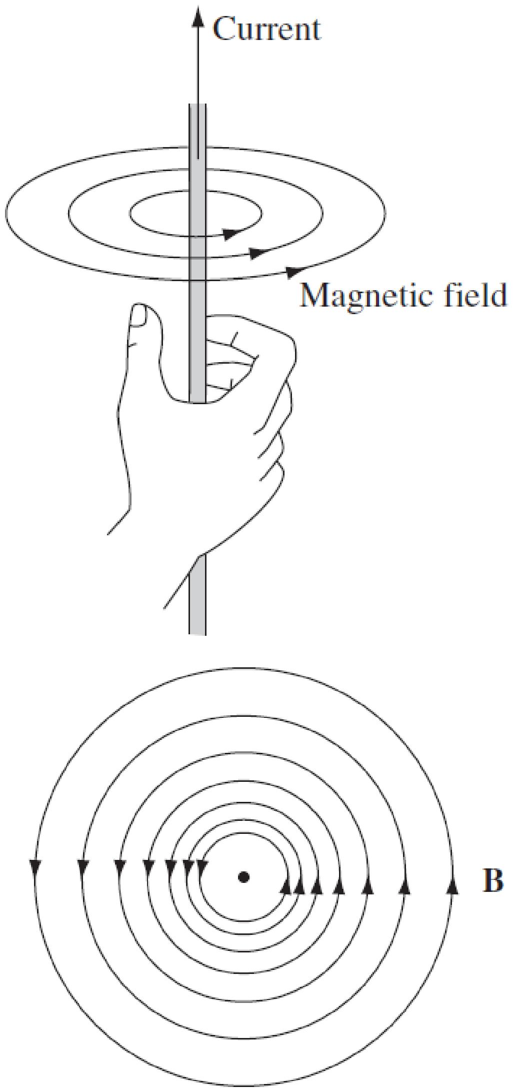
For if we use cylindrical
coordinates (s, ϕ, z), with the current flowing along the z axis, $\mathbf{B} = (\mu_0 I/2\pi s)\hat{\boldsymbol{\phi}}$
and $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$, so

$$\oint \mathbf{B} \cdot d\mathbf{l} = \frac{\mu_0 I}{2\pi} \oint \frac{1}{s}\, s\, d\phi = \frac{\mu_0 I}{2\pi} \int_0^{2\pi} d\phi = \mu_0 I.$$

This assumes the loop encircles the wire exactly once; if it went around twice, then ϕ would run from 0 to 4π

From Griffith's book

Infinite wire and integral of B along a path

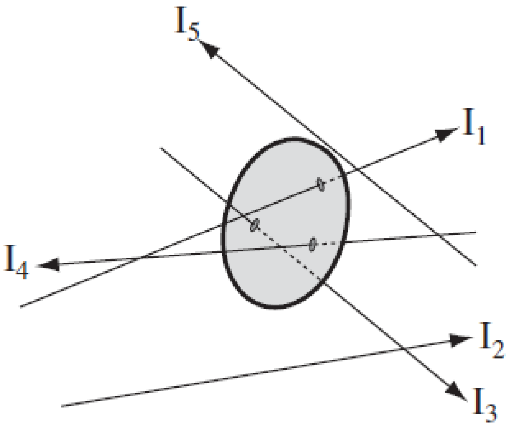


Now suppose we have a *bundle* of straight wires. Each wire that passes through our loop contributes $\mu_0 I$, and those outside contribute nothing (Fig. 5.29). The line integral will then be

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}, \tag{5.44}$$

where I_{enc} stands for the total current enclosed by the integration path. If the flow of charge is represented by a volume current density $\mathbf{J}$, the enclosed current is

$$I_{\text{enc}} = \int \mathbf{J} \cdot d\mathbf{a}, \tag{5.45}$$

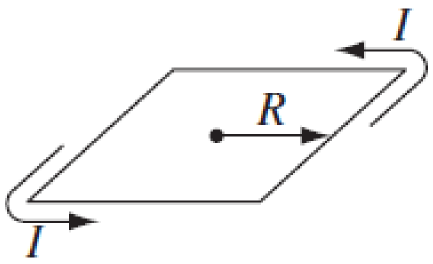


$$\int (\nabla \times \mathbf{B}) \cdot d\mathbf{a} = \mu_0 \int \mathbf{J} \cdot d\mathbf{a},$$

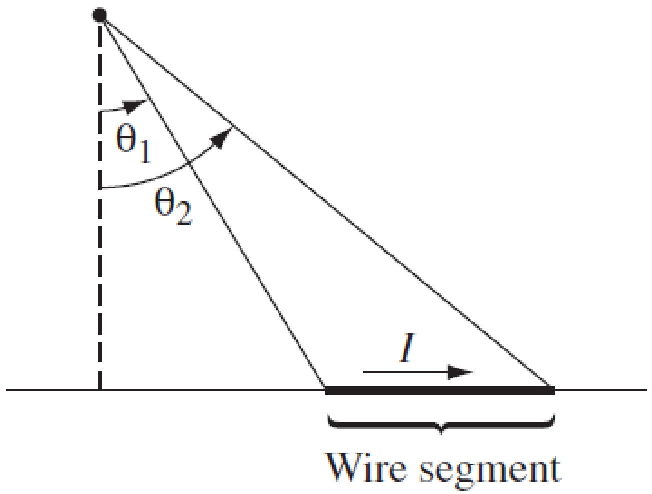
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Problem-1

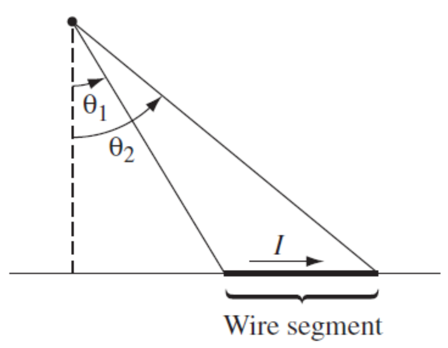
Find the magnetic field at the center of a square loop, which carries a steady current *I*. Let *R* be the distance from center to side



Hint: **B** for a straight wire?



The total field is then given by four times the contribution due to any side of square



$$\vec{B}_{tot} = 4\vec{B}_{side}$$

We know that

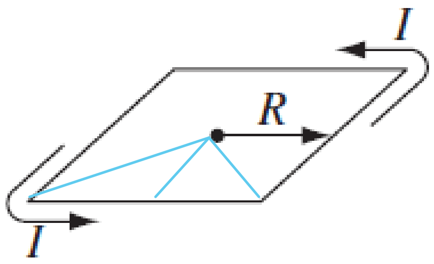
$$B = \frac{\mu_o I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$$

Here

$$\theta_1 = -45^o = -\frac{\pi}{4}$$

$$\theta_2 = 45^o = \frac{\pi}{4}$$

$$B_{side} = \frac{\mu_o I}{4\pi R}\left[2\frac{\sqrt{2}}{2}\right] = \frac{\sqrt{2}\mu_o I}{4\pi R}$$



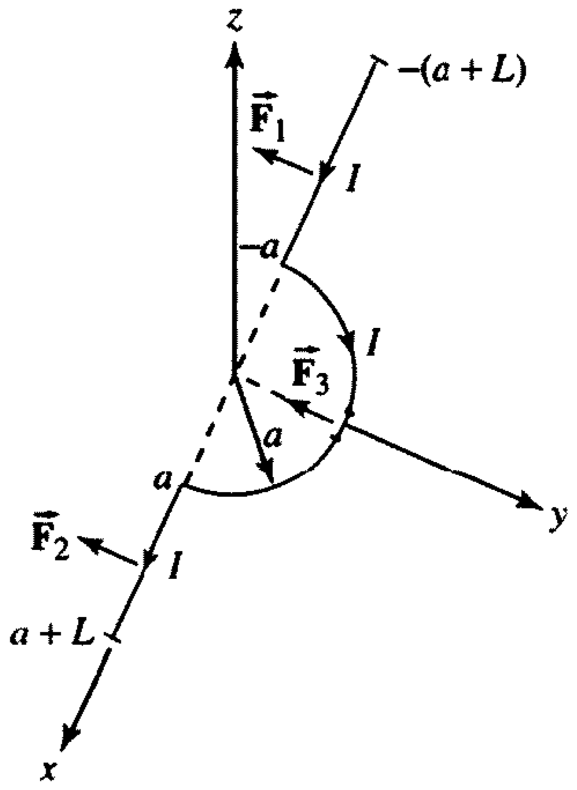
The field at the center is four times this value and directed out of the page.

$$B_{tot} = \frac{\sqrt{2}\mu_o I}{\pi R}$$

From Griffith's book

Problem-2

A wire bent as shown in Figure 5.10 lies in the xy plane and carries a current I. If the magnetic flux density in the region is $\vec{\mathbf{B}} = B\vec{\mathbf{a}}_z$, determine the magnetic force acting on the wire.



Solution-2

The magnetic force acting on the section of the wire from $x = -(a + L)$ to $x = -a$, from (5.12a), is

$$\vec{\mathbf{F}}_1 = \int_{-(a+L)}^{-a} IB(\vec{\mathbf{a}}_x \times \vec{\mathbf{a}}_z)\, dx = -BIL\vec{\mathbf{a}}_y$$

Similarly, the magnetic force experienced by the section of the wire from $x = a$ to $x = a + L$ is

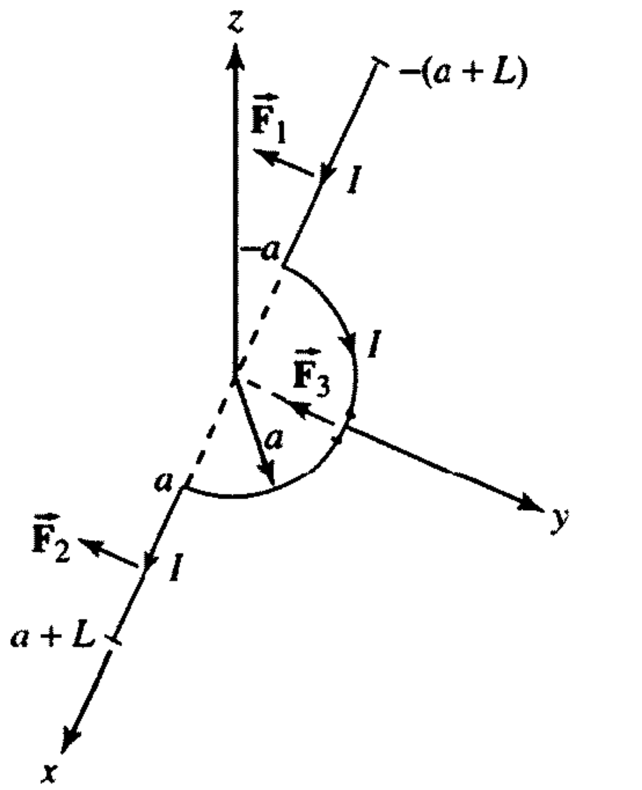
$$\vec{\mathbf{F}}_2 = -BIL\vec{\mathbf{a}}_y$$

The magnetic force acting on the semicircular arc of radius a is

$$\vec{\mathbf{F}}_3 = BIa \int_0^{\pi} [\vec{\mathbf{a}}_x \cos\phi + \vec{\mathbf{a}}_y \sin\phi]\, d\phi = -2IBa\vec{\mathbf{a}}_y$$

The resultant magnetic force on the whole wire is

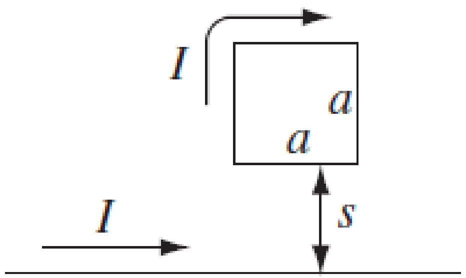
$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = -2IB(a + L)\vec{\mathbf{a}}_y$$



What about the force due to a straight wire of length 2(a+L)?

Problem-3

Find the force on a square loop placed as shown in Fig. 5.24(a), near an infinite straight wire. Both the loop and the wire carry a steady current *I* .



(a)

Solution-3

The forces on the two sides cancel.

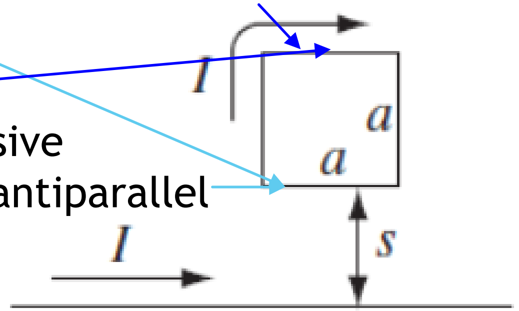
F= (force /length) x length
Here $I_1=I_2=I$

$$B = \frac{\mu_0 I}{2\pi s} \Rightarrow F_1 = \left[\left(\frac{\mu_0 I}{2\pi s}\right)\right] I a = \frac{\mu_0 I^2 a}{2\pi s} \quad \text{(up)}$$

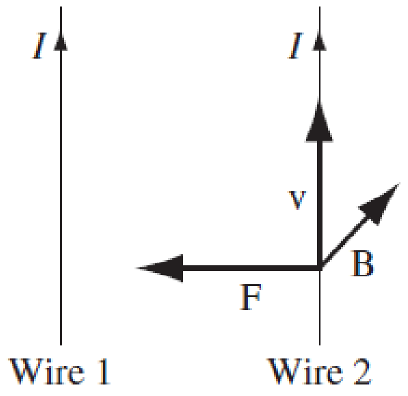
$$B = \frac{\mu_0 I}{2\pi (s+a)} \Rightarrow F_2 = \frac{\mu_0 I^2 a}{2\pi (s+a)} \quad \text{(down)}.$$

F is attractive
Currents parallel

F is repulsive
Currents antiparallel



Hint: Remember force for two current carrying wires



$$\mathbf{F}_{\text{mag}} = \int I\,(d\mathbf{l} \times \mathbf{B}).$$

The net force is $\dfrac{\mu_0 I^2 a^2}{2\pi s(s+a)}$ (up)

Since 1/s > 1/(s+a)

Magnetic flux

Flux through an open surface

$$\Phi = \int_s \vec{\mathbf{B}} \cdot \vec{ds}$$

For a closed surface

$$\oint_s \vec{\mathbf{B}} \cdot \vec{ds} = 0$$

- North, south poles → can't be separated
- No. of lines from north = to south
- Lines are concentric circles for long wires
- Magnetic flux is continuous
- Entering a closed surface = leaving the surface

Thank You