Engineering Optics

Lecture 23

10/05/2023

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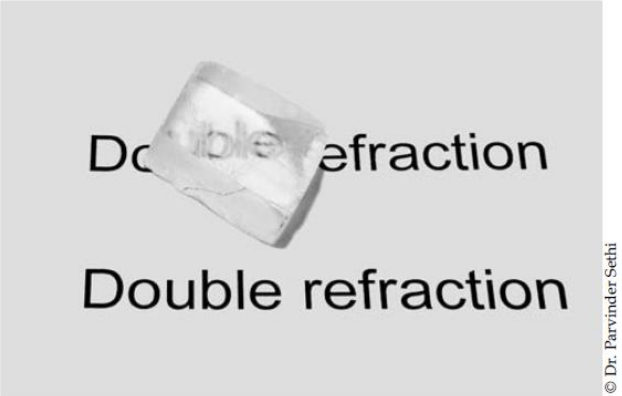
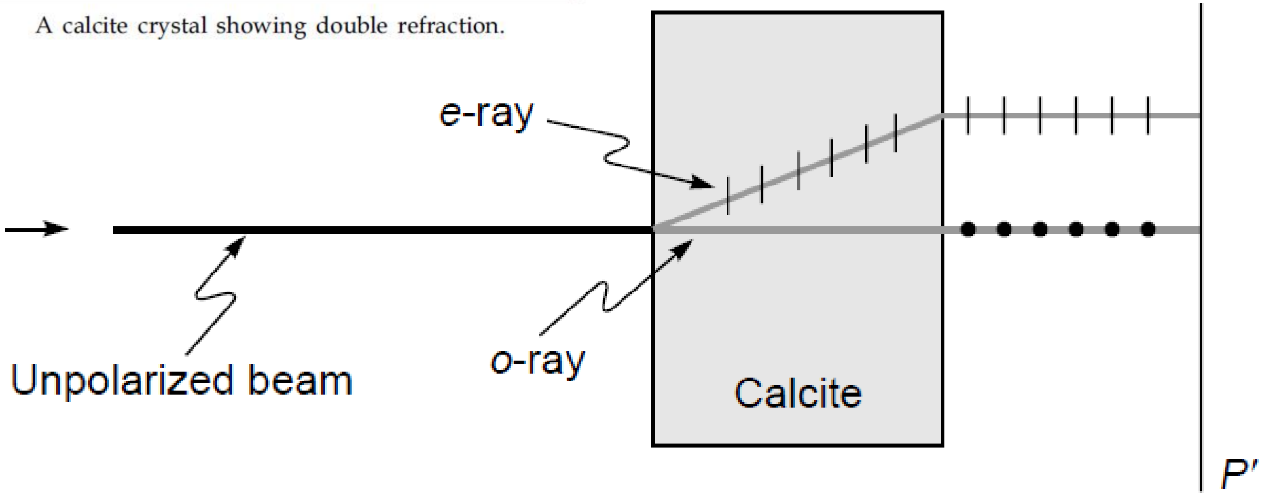
DOUBLE REFRACTION



A calcite crystal showing double refraction.



when an unpolarized beam enters an **anisotropic** crystal, it splits up into two linearly polarized beams, each has a certain state of polarization, different velocities, and different refractive indices.

The beam which travels undeviated is known as the ordinary ray (O-ray) obeys Snell's laws of Refraction

the second beam, does not obey Snell's laws, is known as the extraordinary ray (E-ray).

Anisotropic crystals: Calcite, Quartz etc.
Dichroic crystal: Tourmaline

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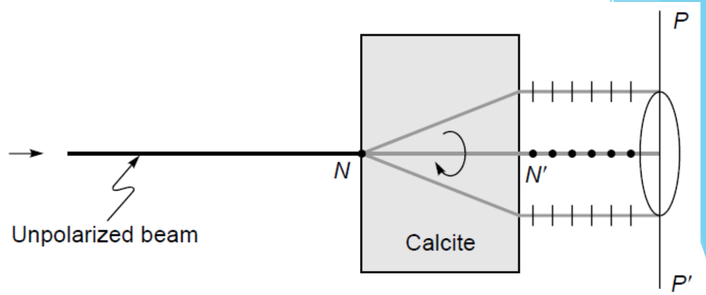
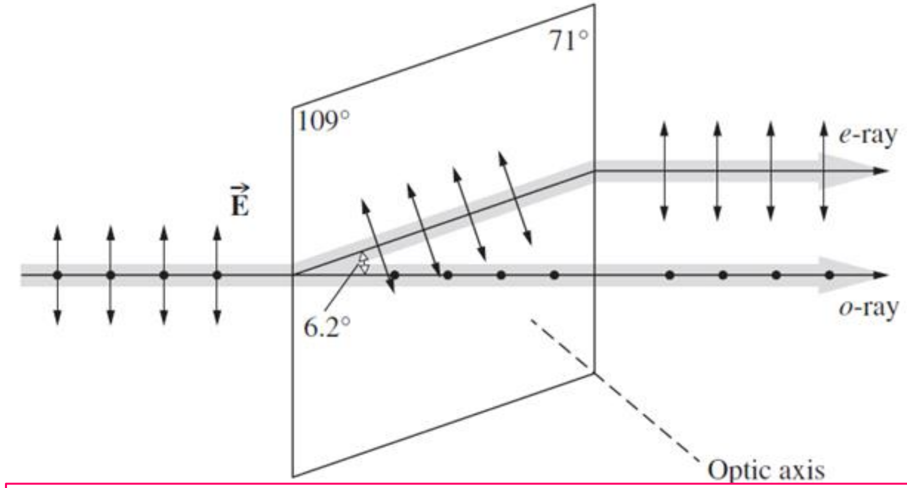
E-ray and O-ray continued



we rotate the crystal about $NN,'$ then the e-ray will rotate about NN'.



$$v_{ro} = \frac{c}{n_o} \qquad \text{ordinary ray}$$

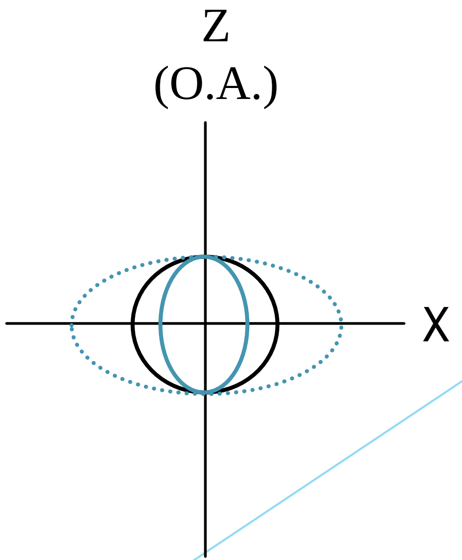
$$\frac{1}{v_{re}^2} = \frac{\sin^2\theta}{(c/n_e)^2} + \frac{\cos^2\theta}{(c/n_o)^2} \qquad \text{extraordinary ray}$$

$$\frac{z^2}{a^2} + \frac{x^2}{b^2} = 1 \quad \text{OR} \quad \frac{1}{\rho^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}$$

$$z = \rho\cos\theta \qquad x = \rho\sin\theta$$

n_o and n_e are constants of the crystal and θ is the angle that the ray makes with the optic axis (z) with the optic axis as the axis of revolution

- plot v_{re} as a function of θ
- plot v_{ro} as a function of θ



Which one is correct:
1. Sphere inside or
2. Ellipse inside ??

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Positive and negative crystals

Along the optic axis $$v_{ro} = v_{re} = \frac{c}{n_o}$$

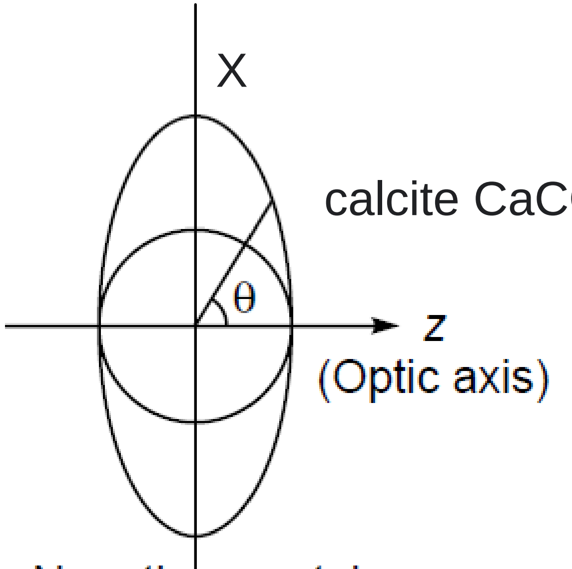
Along a direction perpendicular to optic axis ??

For a negative crystal $n_e < n_o$

$$v_{re}\left(\theta = \frac{\pi}{2}\right) = \frac{c}{n_e} > v_{ro}$$

On the other hand, for a positive crystal $n_e > n_o$

$$v_{re}\left(\theta = \frac{\pi}{2}\right) = \frac{c}{n_e} < v_{ro}$$

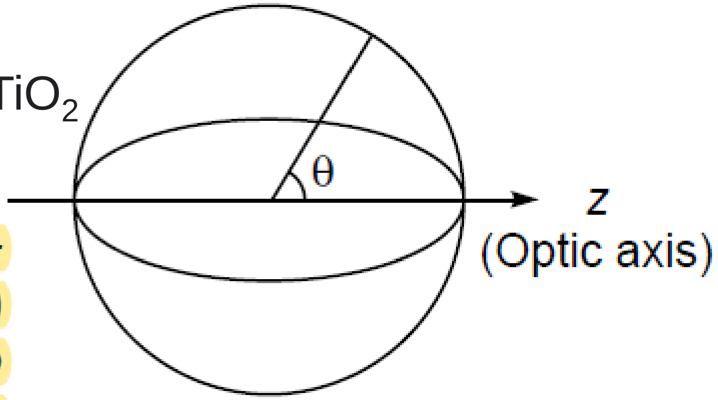


Negative crystal

(a)

calcite $CaCO_3$, ruby Al_2O_3

quartz SiO_2, rutile TiO_2



Positive crystal

(b)

(a) In a negative crystal, the ellipsoid of revolution (which corresponds to the extra ordinary ray) lies outside the sphere; the sphere corresponds to the ordinary ray. (b) In a positive crystal, the ellipsoid of revolution (which corresponds to the extraordinary ray) lies inside the sphere.

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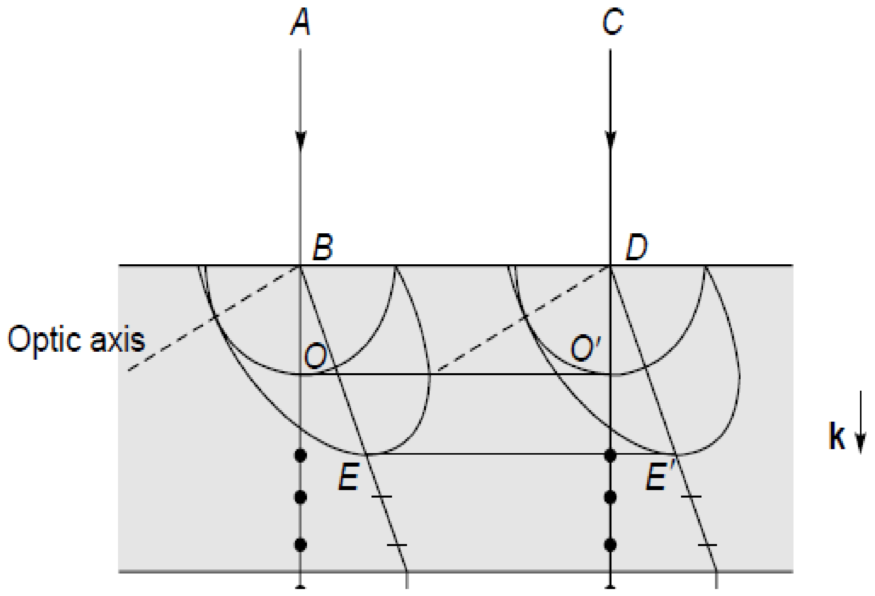
How do wavefronts travel?

- **Normal Incidence on negative crystal**
- **(1)**



A plane wave incident normally on a uniaxial crystal. Optic axis is shown as a dashed line.

Steps:

O-ray: with point B as the center, we draw a sphere of radius c/n_o.

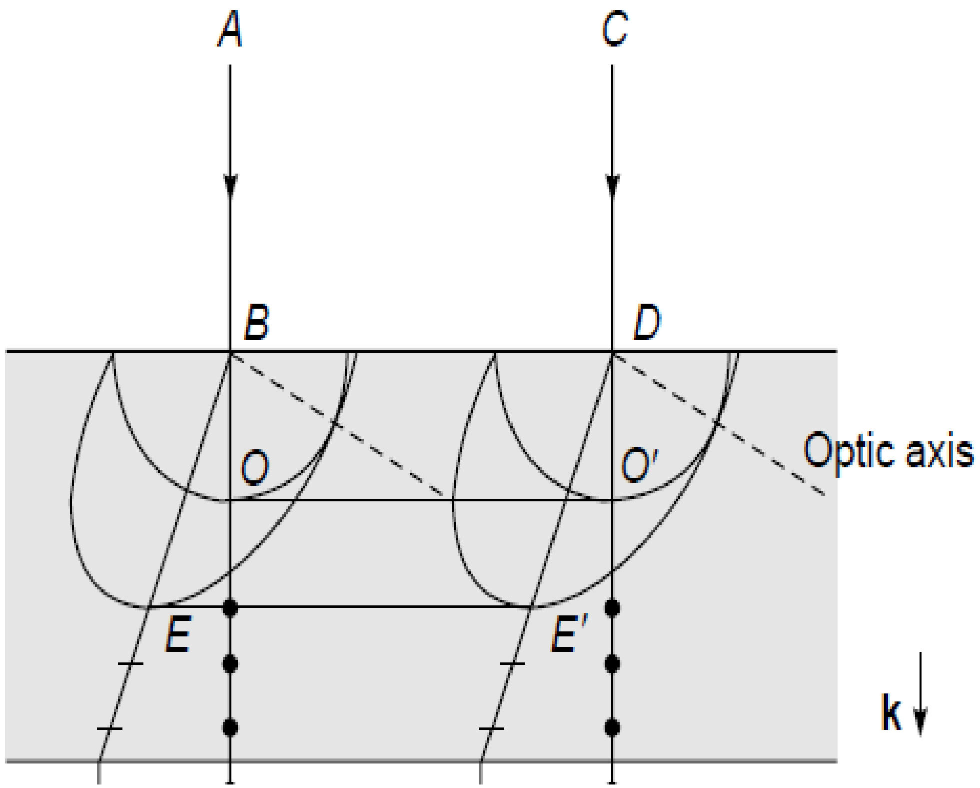
1. Similarly, we draw another sphere (of the same radius) from point D.
2. The common tangent plane to these spheres is shown as OO\` → wave front for O-ray.

E-ray: draw an ellipse centered at point B with its minor axis (= c/n_o) along the optic axis

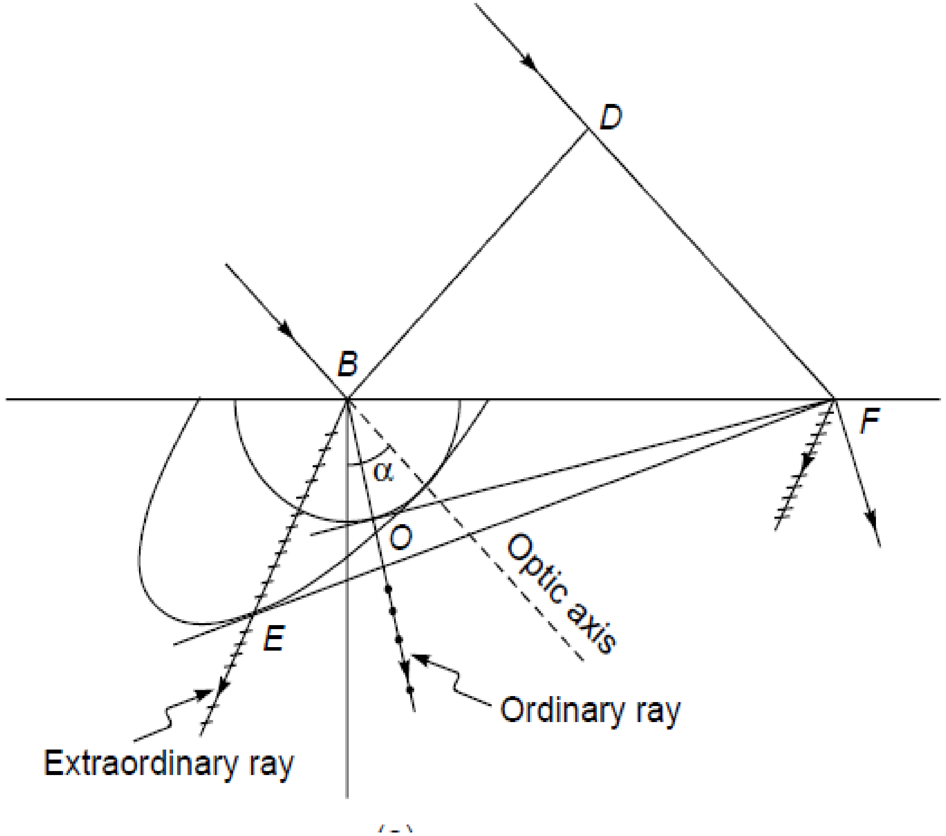
3. The ellipsoid of revolution is obtained by rotating the ellipse about the optic axis; major axis equal to c/n_e.
4. Similarly, we draw another ellipsoid of revolution from point D.
5. The common tangent plane to these ellipsoids is EE\`
6. BO and BE → directions in which O and E-rays move.

(2)

Wavefronts/envelopes for O-ray and E-ray for the given case of normal incidence.



2. For oblique incidence?



- Let *BD* represent the incident wave front.
- Time taken for the disturbance to reach point *F* from *D* is *t*
- With *B* as center we draw a sphere of radius (*c/no*)*t* and an ellipsoid of revolution of semiminor and semimajor axes (*c/no*)*t*, and (*c/ne*)*t*, respectively.
- From point *F* we draw tangent planes *FO* and *FE* to the sphere and the ellipsoid of revolution, respectively: refracted wave fronts corresponding to the ordinary and the extraordinary rays, respectively.
- If the points of contact are *O* and *E*, then the ordinary and extraordinary refracted rays will propagate along *BO* and *BE*, respectively

Thank You