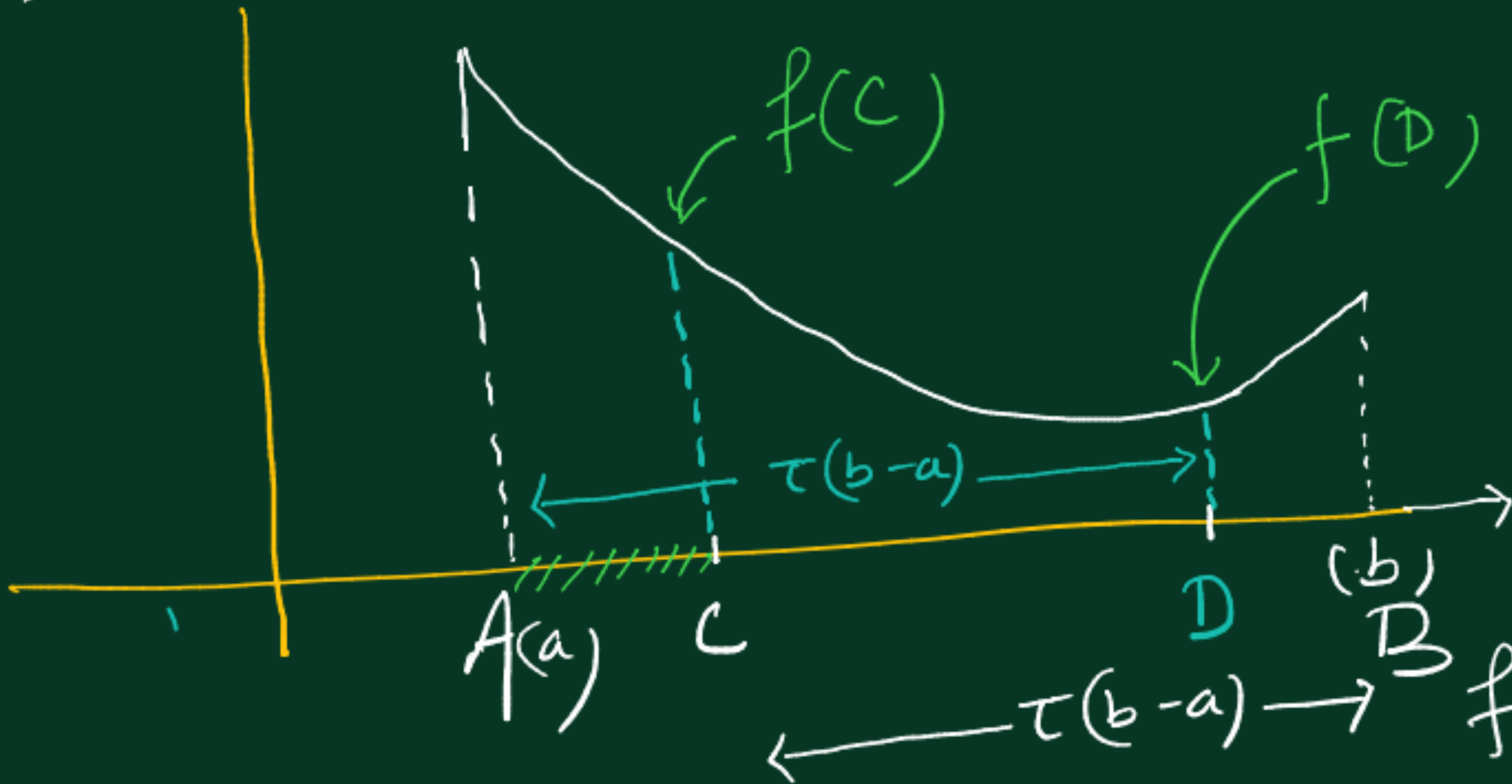
Golden Section Search Method

Case-I $\tau > 1/2$



$L = b - a$

$$\begin{cases} CB = \tau(b-a) \\ AD = \tau(b-a) \end{cases}$$

for $\tau \in (0,1)$

We have two points C & D ✓

$f(C) > f(D)$ then AC is eleminated.

$$AC = (1-\tau)(b-a)$$

$$\frac{\text{Total length} \to (b-a)}{\text{Large} \to \tau(b-a)} = \frac{\tau(b-a)}{(1-\tau)(b-a)} \leftarrow \text{Small}$$

$$\Rightarrow \quad \tau^2 = (1-\tau)$$

$$\Rightarrow \quad \tau^2 + \tau - 1 = 0 \quad \Rightarrow \tau = \frac{-1 \pm \sqrt{5}}{2}$$

$$\tau = \begin{cases} \frac{-1-\sqrt{5}}{2}; \\ \frac{-1+\sqrt{5}}{2}, \end{cases} \Rightarrow \tau = \begin{cases} -1.618 \\ 0.618 \text{ (accepted)} \end{cases}$$

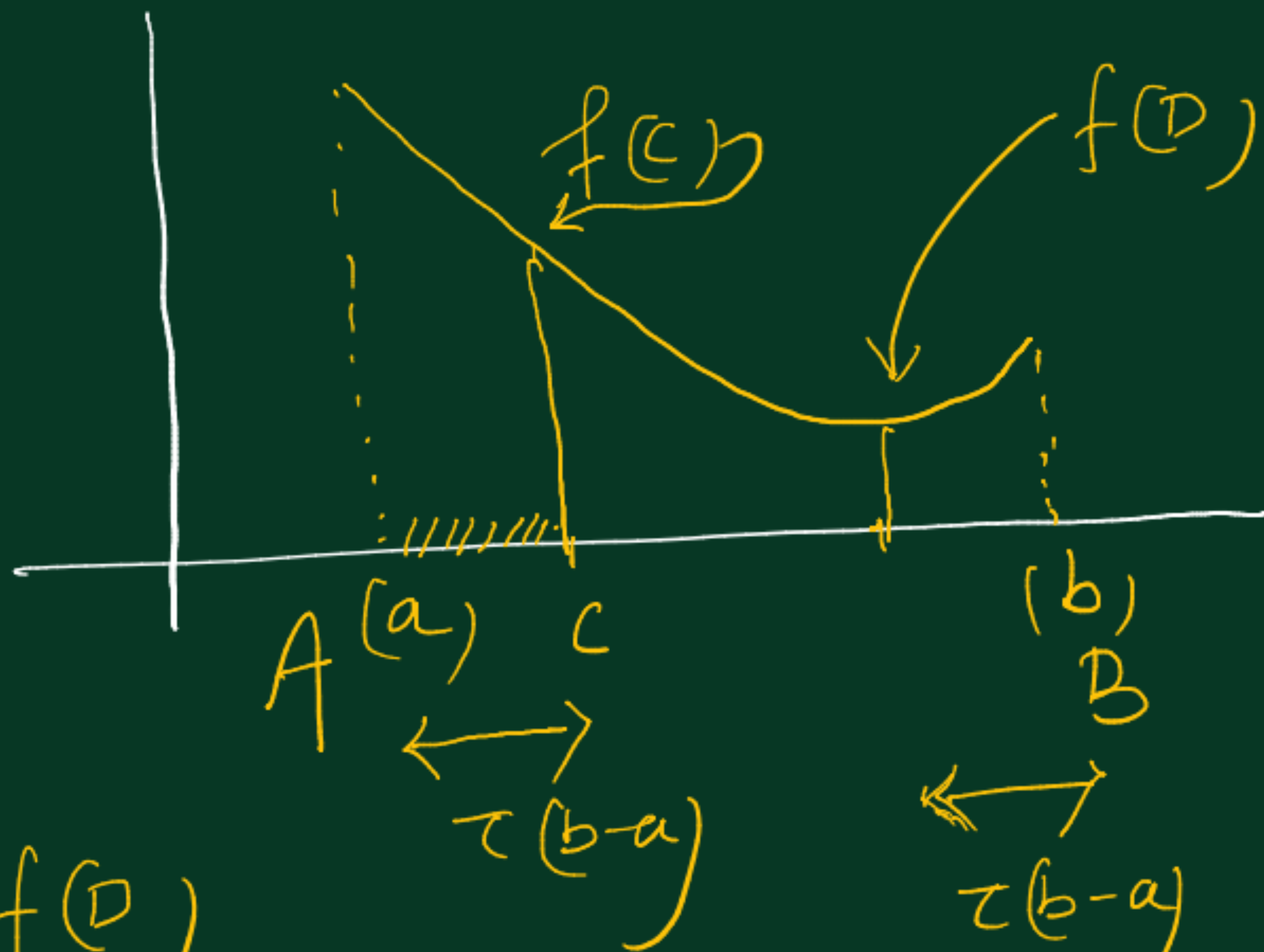
Then $(1-\tau) = 0.382$

Hence the eleminated part $AC = (1-\tau)(b-a)$

i.e. 38.2% of $(b-a)$.

Case – II   If $\tau < \frac{1}{2}$ then $\tau = 0.382$

then



$f(C) > f(D)$

then $AC = \tau(b-a)$ is removed or eleminated.

38.2 % of $(b-a)$.

After $\underline{\text{1st elemination}}$ →

$$CA = \tau(b-a) \quad \& \quad DA = (1-\tau)(b-a)$$

$$\begin{aligned} CD &= CA - DA \\ &= [\tau - (1-\tau)](b-a) \\ &= [\tau - \tau^2](b-a) \qquad \because \tau^2 = 1-\tau \\ &= \tau(1-\tau)(b-a) = (1-\tau)\, CA \end{aligned}$$

Then $DA = \tau\, CA$ from A to D is the old point

Again a New point E of TCA required for this 2nd iteration..

Here we prove that the 2nd iteration required only one function evaluation and uses one old function evalution from iteration one. $\square$