Engineering Optics

Lecture 27

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by

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The Michelson interferometer

S → a light source (may be a sodium lamp)

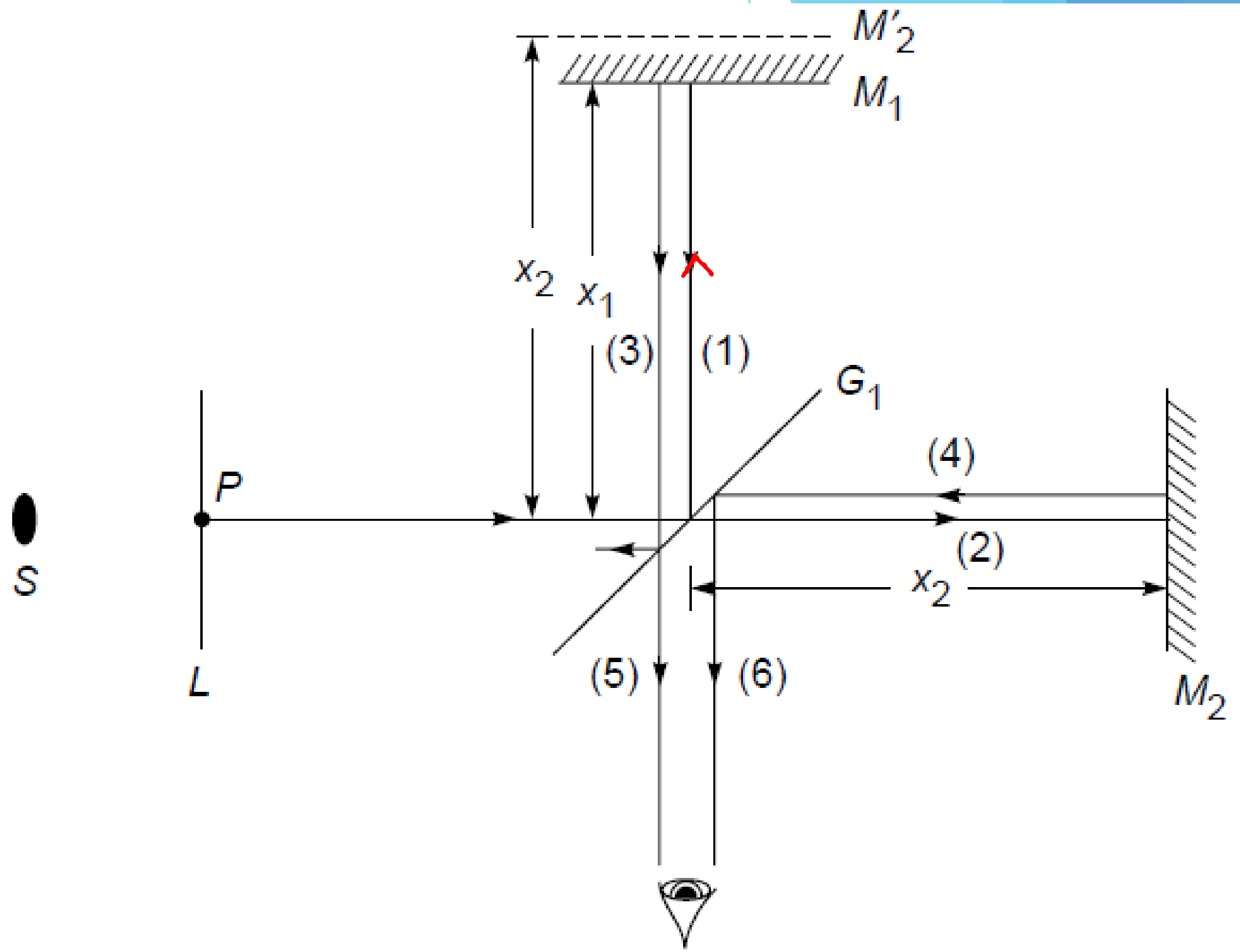
L → glass plate so that an extended source of almost uniform intensity is formed.

G_1 → a beam splitter
a beam incident on G1 gets partially reflected and partially transmitted

M1 and M2 → good-quality plane mirrors having very high reflectivity

One of the mirrors (M_2) is fixed and the other (usually M_1) is capable of moving away from or toward the glass plate G1 along an accurately machined track by means of a screw.

Usually mirrors M_1 and M_2 are perpendicular to each other and G_1 is at 45° to the mirror.



Schematic of the Michelson interferometer.

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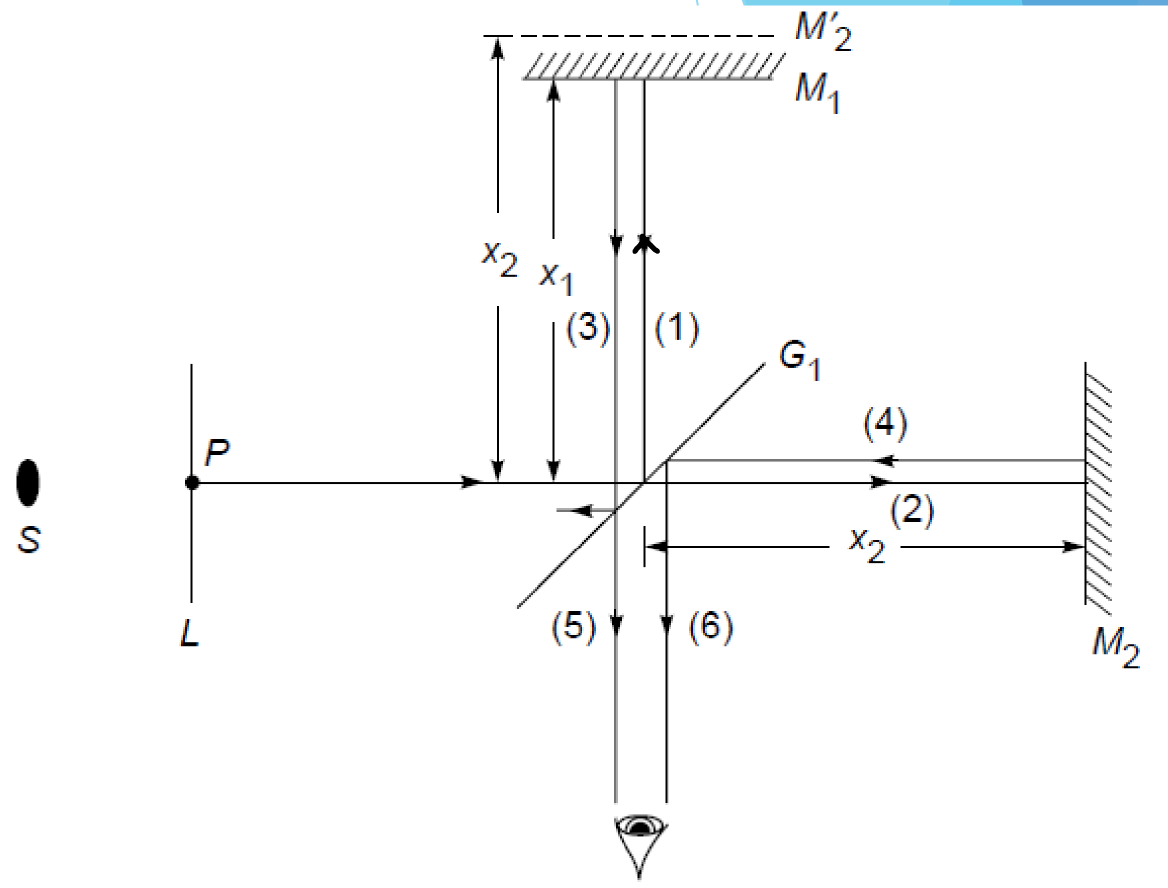
The Michelson interferometer

if x_1 and x_2 are the distances of mirrors M_1 and M_2 from the plate G_1, d = $x_1 \sim x_2$

To the eye the waves emanating from point P will appear to get reflected by two parallel mirrors (M_1 and M_2' — separated by a distance ($x_1 \sim x_2$).

if we use an extended source → if we have a camera, then on the focal plane we will obtain circular fringes, each circle corresponding to a definite value of θ



Schematic of the Michelson interferometer.

The Michelson interferometer

Now, if the beam splitter is just a simple glass plate, the beam reflected from mirror M_2 will undergo an abrupt phase change of π (when getting reflected by the beam splitter), and since the extra path that one of the beams will traverse will be $2(x_1 \sim x_2)$, the condition for destructive interference will be

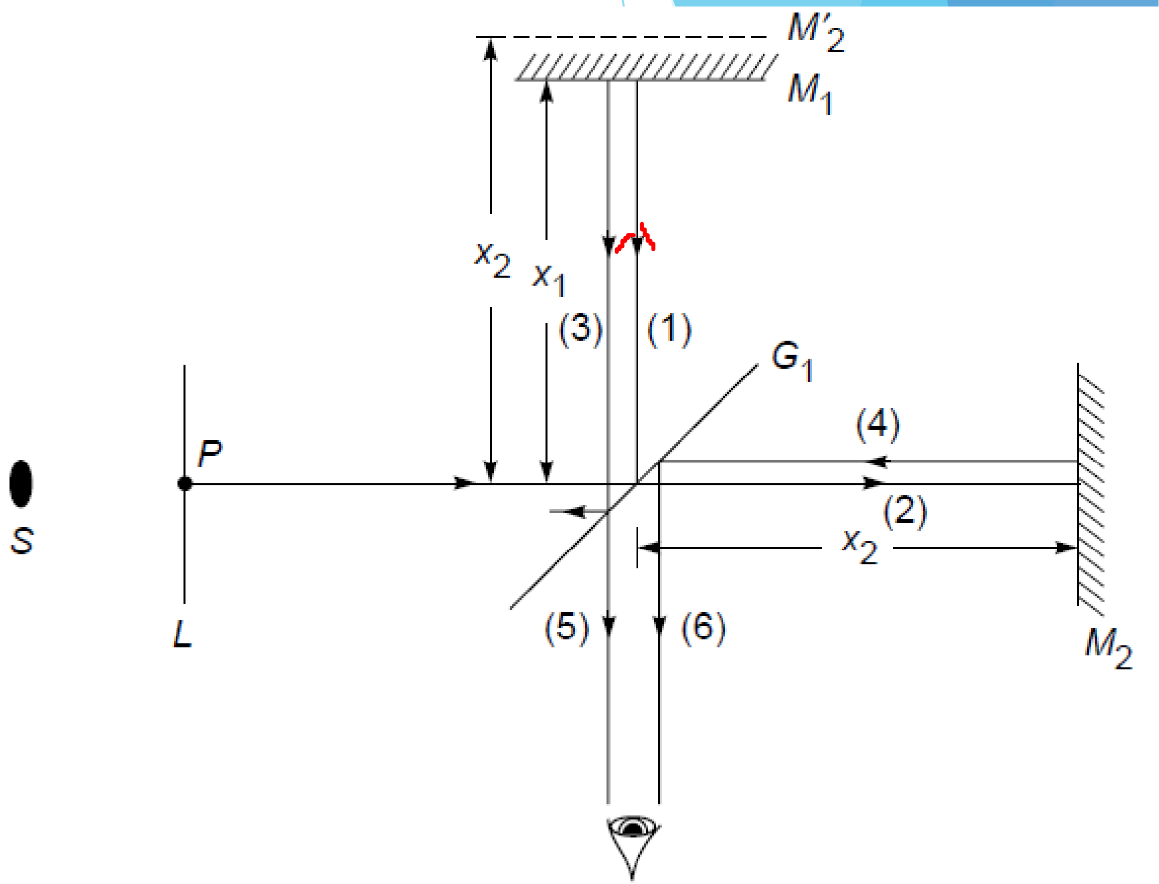
$$\boxed{2d \cos\theta = m\lambda}$$

where $m = 0, 1, 2, 3, \ldots$ and

$$d = x_1 \sim x_2$$

and the angle θ represents the angle that the rays make with the axis (which is normal to the mirrors as shown in Fig. 15.35). Similarly, the condition for a bright ring is

$$\boxed{2d \cos\theta = \left(m + \tfrac{1}{2}\right)\lambda}$$



Schematic of the Michelson interferometer.

The Michelson interferometer

Thus as we start reducing the value of *d*, the fringes will tend to collapse at the center

Conversely, if d is increased, the fringe pattern will expand.

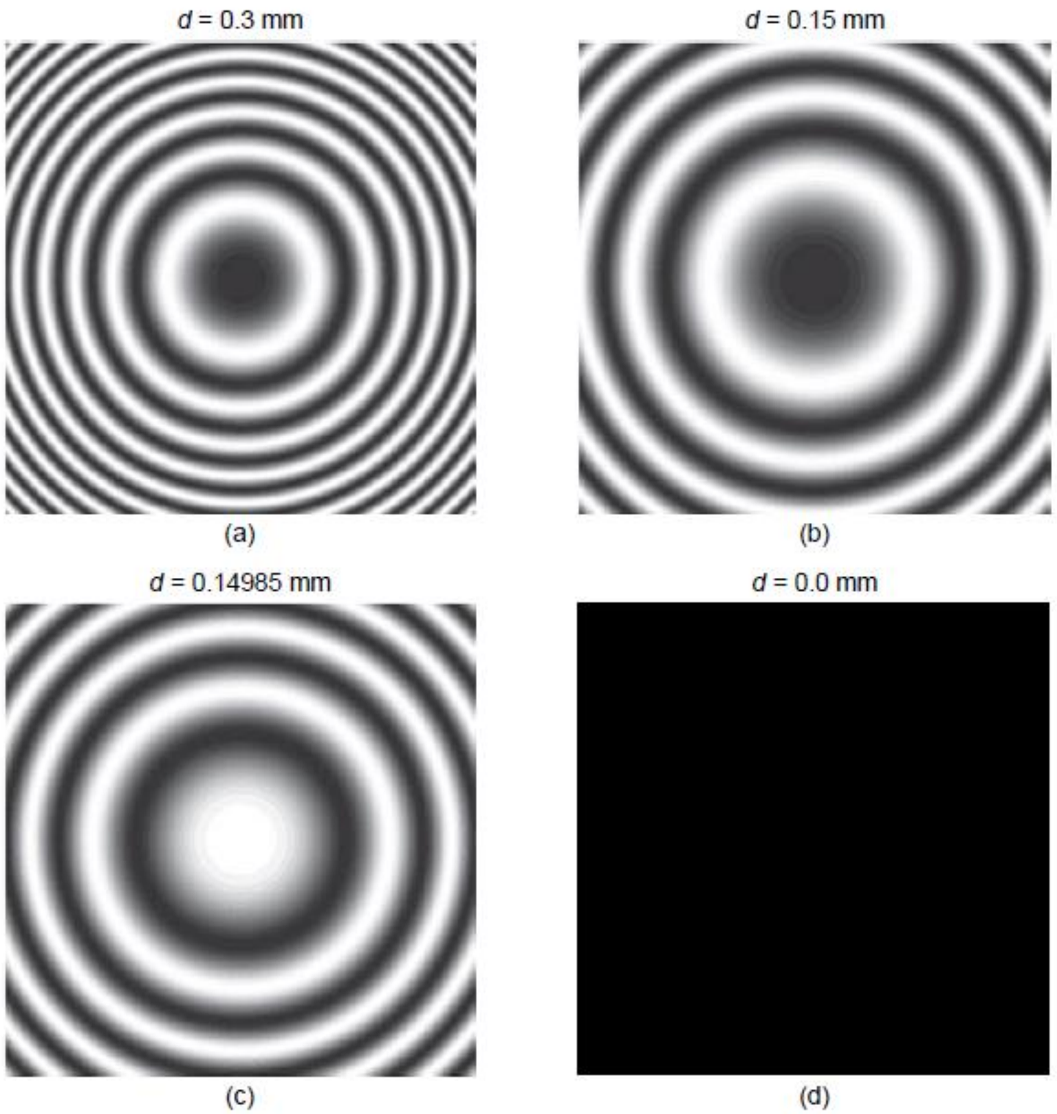
if N fringes collapse to the center as mirror M_1 moves by a distance d_0, then we must have

$$2d = m\lambda$$

$$2(d - d_0) = (m - N)\lambda$$

where we have set $\theta' = 0$ because we are looking at the central fringe. Thus

$$\lambda = \frac{2d_0}{N}$$



Computer-generated interference pattern produced by a Michelson interferometer.

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Wavelength measurement

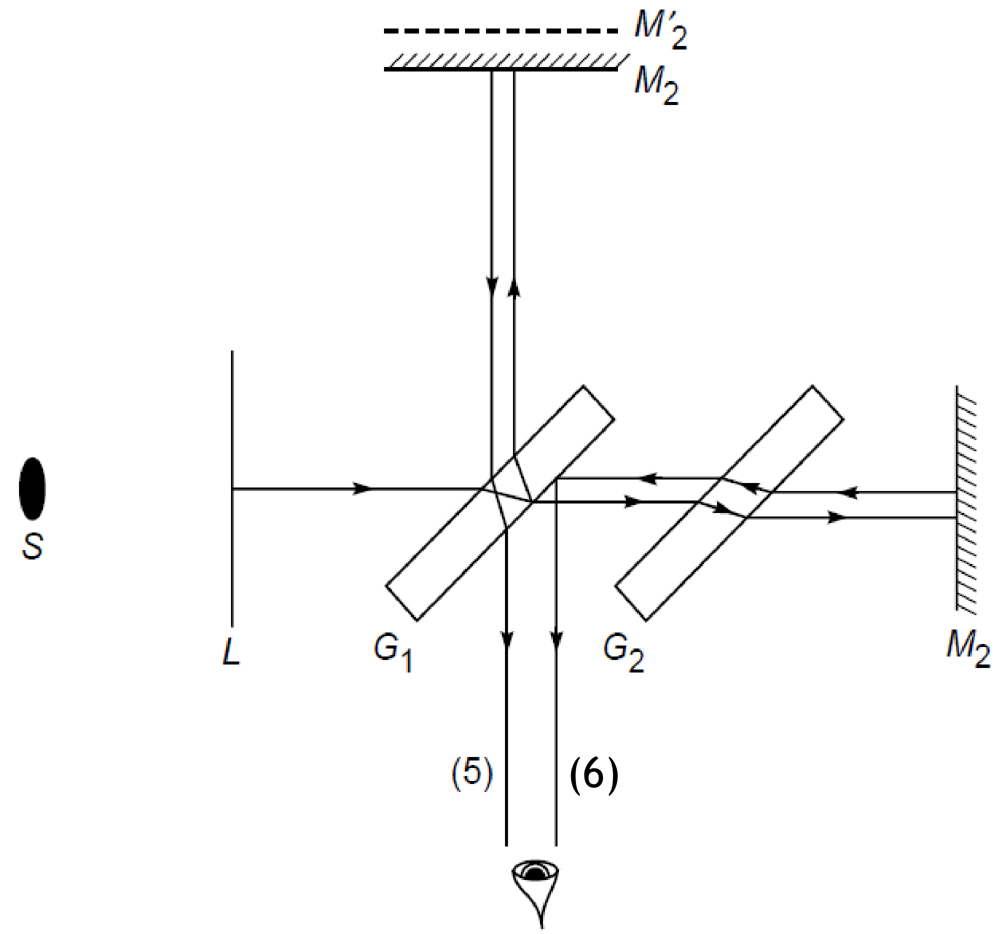
This provides us with a method for the measurement of the wavelength. For example, in a typical experiment, if 1000 fringes collapse to the center as the mirror is moved through a distance of 2.90×10^{-2} cm, then

$$\lambda = 5800 \text{ Å}$$

The above method was used by Michelson for the standardization of the meter. He found that the red cadmium line ($\lambda = 6438.4696$ Å) is one of the ideal monochromatic sources, and as such this wavelength was used as a reference for the standardization of the meter.

A point to note

- In an actual Michelson interferometer, the beam splitter G_1 consists of a plate (which may be about 1/2 cm thick),
- The back surface of which is partially silvered, and the reflections occur at the back surface
- The compensating plate is not really necessary for a monochromatic source because the additional path introduced by G_1 can be compensated by moving mirror M_1
- ***Q: How many time rays have crossed G_1?***
- **What about white light??**
- Difficult to adjust M_1 for each λ
- Hence, compensating plate G_2



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Application

- **Can be used in the measurement of two closely spaced wavelengths:**
- Sodium lamp → emits two closely spaced wavelengths 5890 and 5896 Å

If mirror M_1 is moved away from (or toward) plate G_1 through a distance d, then the maxima corresponding to the wavelength λ_1 will not, in general, occur at the same angle as λ_2. Indeed, if the distance d is such that

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = \frac{1}{2}$$

and if $2d \cos\theta' = m\lambda_1$, then $2d \cos\theta' = \left(m + \frac{1}{2}\right)\lambda_2$. Thus, the maxima of λ_1 will fall on the minima of λ_2, and conversely, and the fringe system will disappear.

Application continued

if $$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1$$

then interference pattern will again reappear. In general, if

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2, . . . , we will have disappearance of the fringe pattern; and if it is equal to 1, 2, 3, . . . , then the interference pattern will appear.

Few points to note

- When the mirrors of the interferometer are inclined with respect to each other, making a small angle (i.e., when M_1 and M_2 are not quite perpendicular), ***Fizeau fringes*** are observed. The resultant wedge-shaped air film between M_2 and M_1 creates a pattern of straight parallel fringes.
- by appropriate adjustment of the orientation of the mirrors-M_1 and -M_2, fringes can be produced that are straight, circular, elliptical, parabolic, or hyperbolic—this holds as well for the real and virtual fringes.

Experiment in brief

- https://www.youtube.com/watch?v=j-u3IEgcTiQ

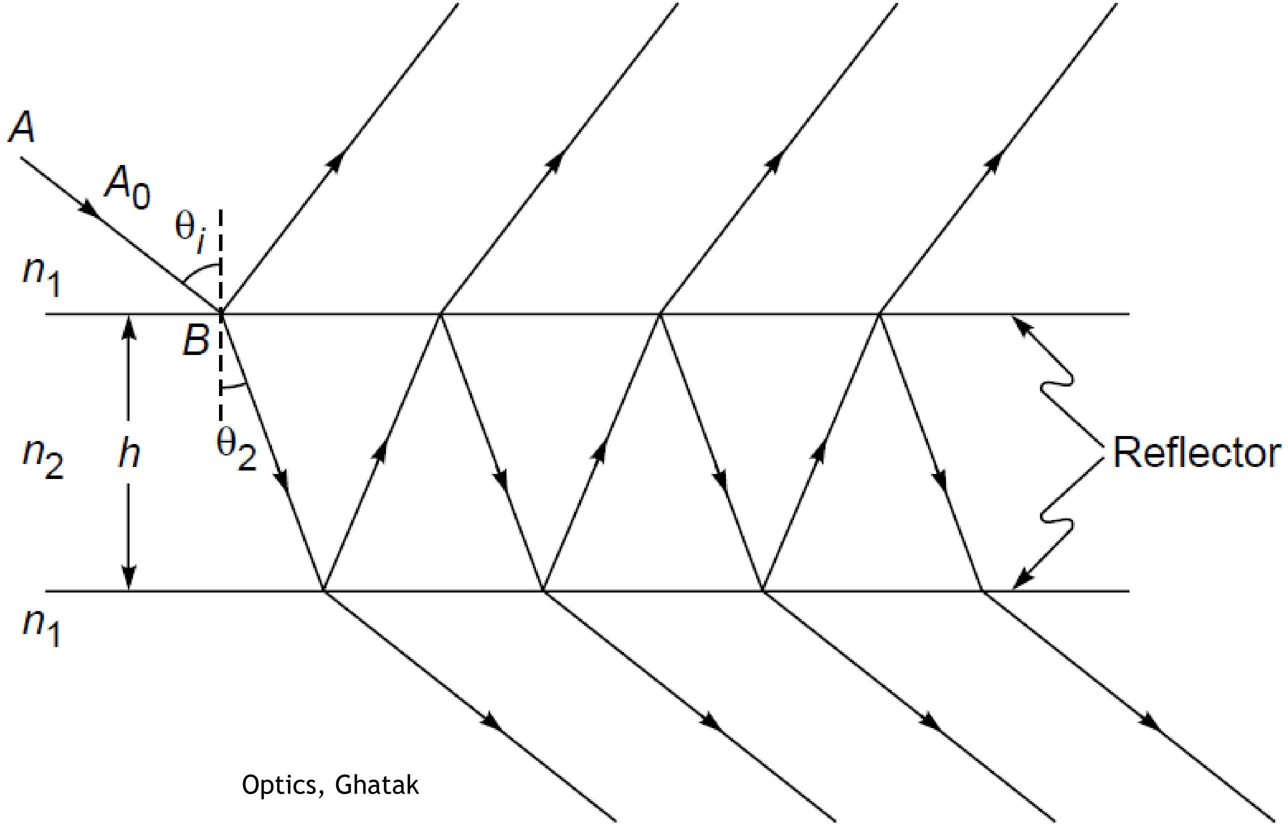
The Fabry-Perot interferometer

The Fabry-Perot *etalon*

Multiple reflections from a plane parallel film

We consider the incidence of a plane wave on a plate of thickness h (and of refractive index n_2) surrounded by a medium of refractive index n_1

- Let A_0 be the (complex) amplitude of the incident wave.
- The wave will undergo multiple reflections at the two interfaces
- when the wave is incident from n_1 toward n_2:
- r_1 and t_1 represent the amplitude reflection and transmission coefficients, respectively
- When the wave is incident from n_2 toward n_1 → r_2 and t_2 represent the corresponding coefficients.



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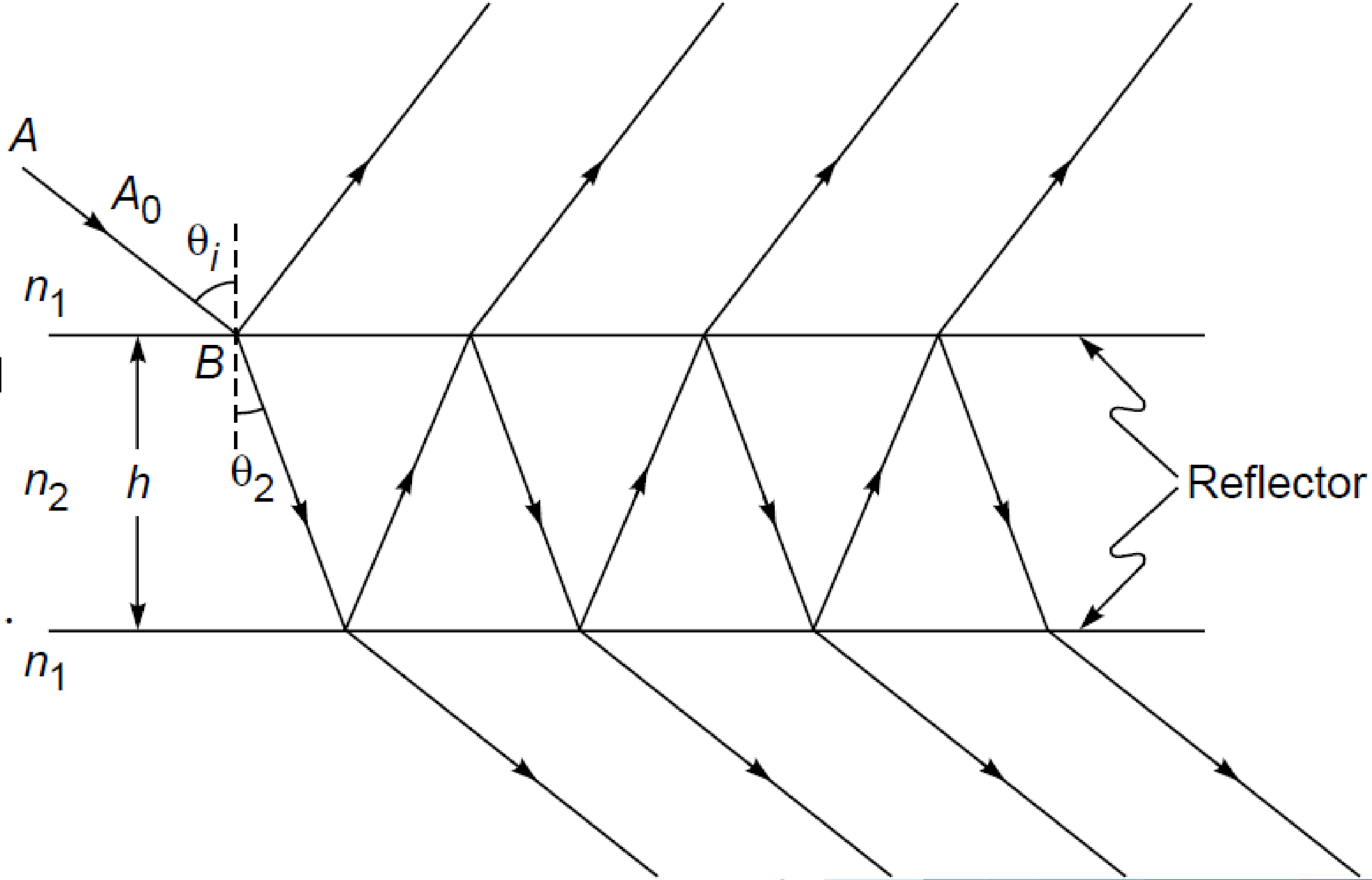
Multiple reflections from a plane parallel film

- $A_0 \rightarrow$ amplitude of the incident wave.
- When the wave is from n_1 toward n_2: r_1, t_1
- from n_2 toward $n_1 \rightarrow r_2$ and t_2
- Thus the amplitude of the successive reflected waves will be



$$A_0 r_1, A_0 t_1 r_2 t_2 e^{i\delta}, A_0 t_1 r_2^3 e^{2i\delta}, \ldots$$

where $$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda_0}$$

represents the phase difference (between two successive waves emanating from the plate) due to the additional path traversed by the beam in the film

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Multiple reflections from a plane parallel film

- A_0 → amplitude of the incident wave.
- When the wave is from n_1 toward n_2: r_1, t_1
- from n_2 toward n_1 → r_2 and t_2
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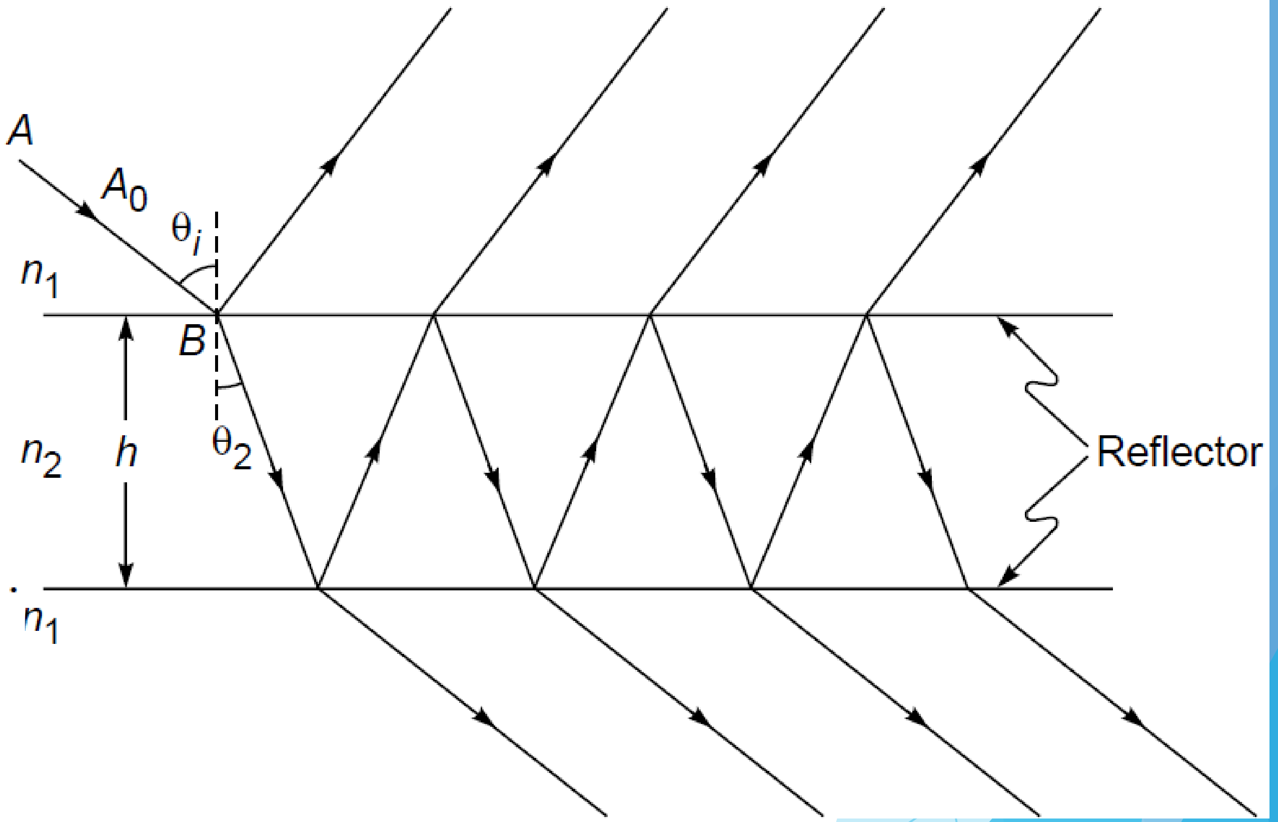

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$$A_r = A_0[r_1 + t_1 t_2 r_2 e^{i\delta}(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots)]$$

$$= A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$

resultant amplitude of the reflected wave

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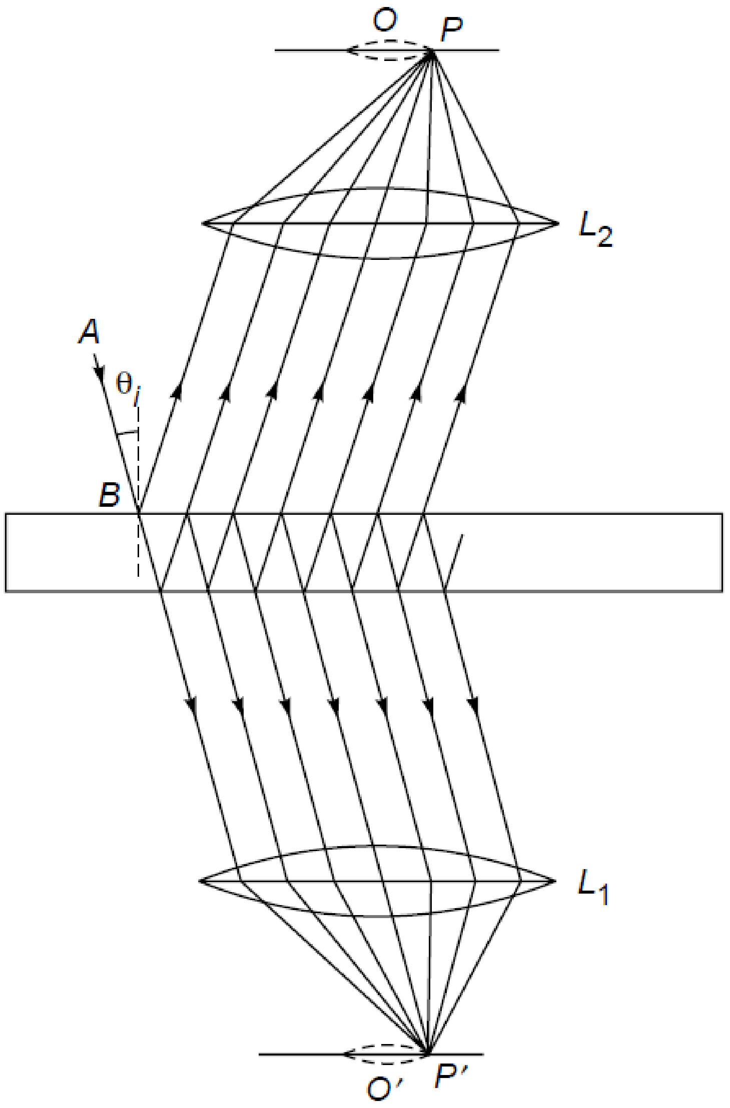

$$R = r_1^2 = r_2^2$$

$$\tau = t_1 t_2 = 1 - R$$

$$\mathcal{R} = \left| \frac{A_r}{A_0} \right|^2 = R \left| \frac{1 - e^{i\delta}}{1 - R e^{i\delta}} \right|^2$$

$$= R \, \frac{(1 - \cos\delta)^2 + \sin^2\delta}{(1 - R\cos\delta)^2 + R^2 \sin^2\delta}$$

$$= \frac{4R \sin^2 \delta/2}{(1 - R)^2 + 4R \sin^2 \delta/2}$$

$$\mathcal{R} = \frac{F \sin^2 \delta/2}{1 + F \sin^2 \delta/2}$$

$$T = \left| \frac{A_t}{A_0} \right|^2 = \frac{(1 - R)^2}{(1 - R\cos\delta)^2 + R^2 \sin^2 \delta}$$

$$T = \frac{1}{1 + F \sin^2 \delta/2}$$

Thank You