# Engineering Optics

## Lecture 33

31/05/2023

*by*

**Debolina Misra**

Assistant Professor in Physics
IIITDM Kancheepuram, Chennai, India

# The Michelson interferometer

$S$ → a light source (may be a sodium lamp)

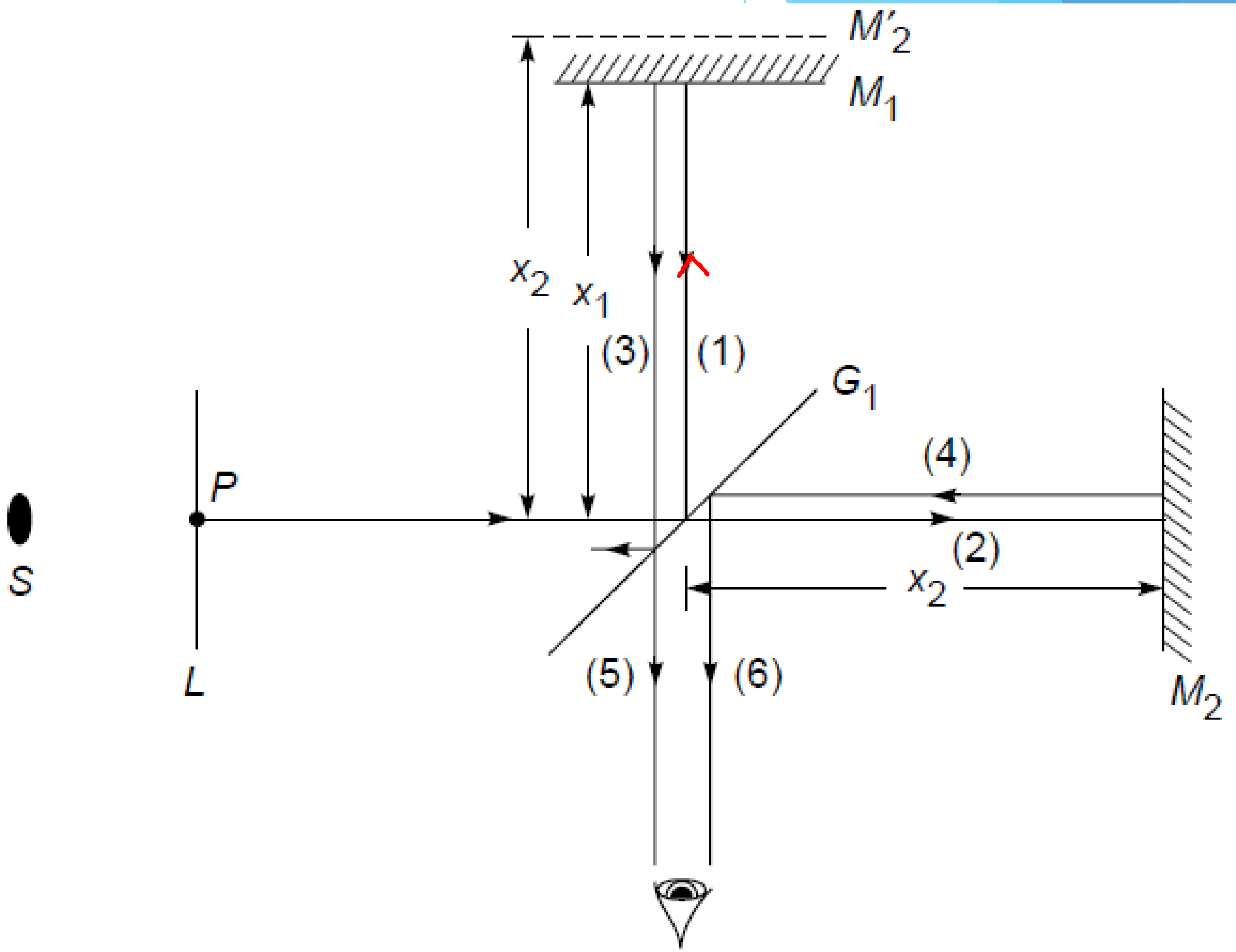
$L$ → glass plate so that an extended source of almost uniform intensity is formed.

$G_1$ → a beam splitter
a beam incident on G1 gets partially reflected and partially transmitted

M1 and M2 → good-quality plane mirrors having very high reflectivity

One of the mirrors ($M_2$) is fixed and the other (usually $M_1$) is capable of moving away from or toward the glass plate G1 along an accurately machined track by means of a screw.

Usually mirrors $M_1$ and $M_2$ are perpendicular to each other and $G_1$ is at 45° to the mirror.




Schematic of the Michelson interferometer.

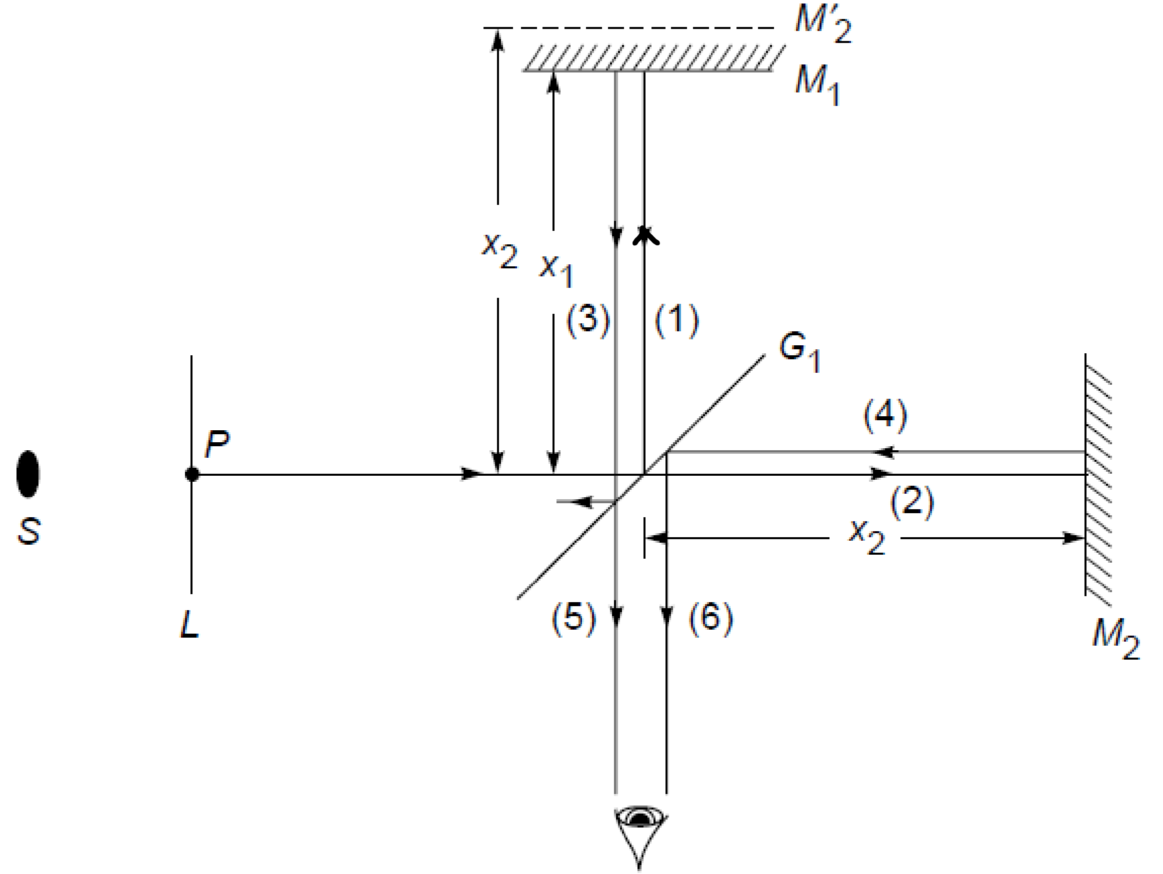
Optics, Ghatak

# The Michelson interferometer

**if $x_1$ and $x_2$ are the distances of mirrors $M_1$ and $M_2$ from the plate $G_1$, $d = x_1 \sim x_2$**

To the eye the waves emanating from point P will appear to get reflected by two parallel mirrors ($M_1$ and $M_2'$ — separated by a distance ($x_1 \sim x_2$).

if we use an extended source → if we have a camera, then on the focal plane we will obtain circular fringes, each circle corresponding to a definite value of θ




Schematic of the Michelson interferometer.

# The Michelson interferometer

Now, if the beam splitter is just a simple glass plate, the beam reflected from mirror $M_2$ will undergo an abrupt phase change of $\pi$ (when getting reflected by the beam splitter), and since the extra path that one of the beams will traverse will be $2(x_1 \sim x_2)$, the condition for destructive interference will be

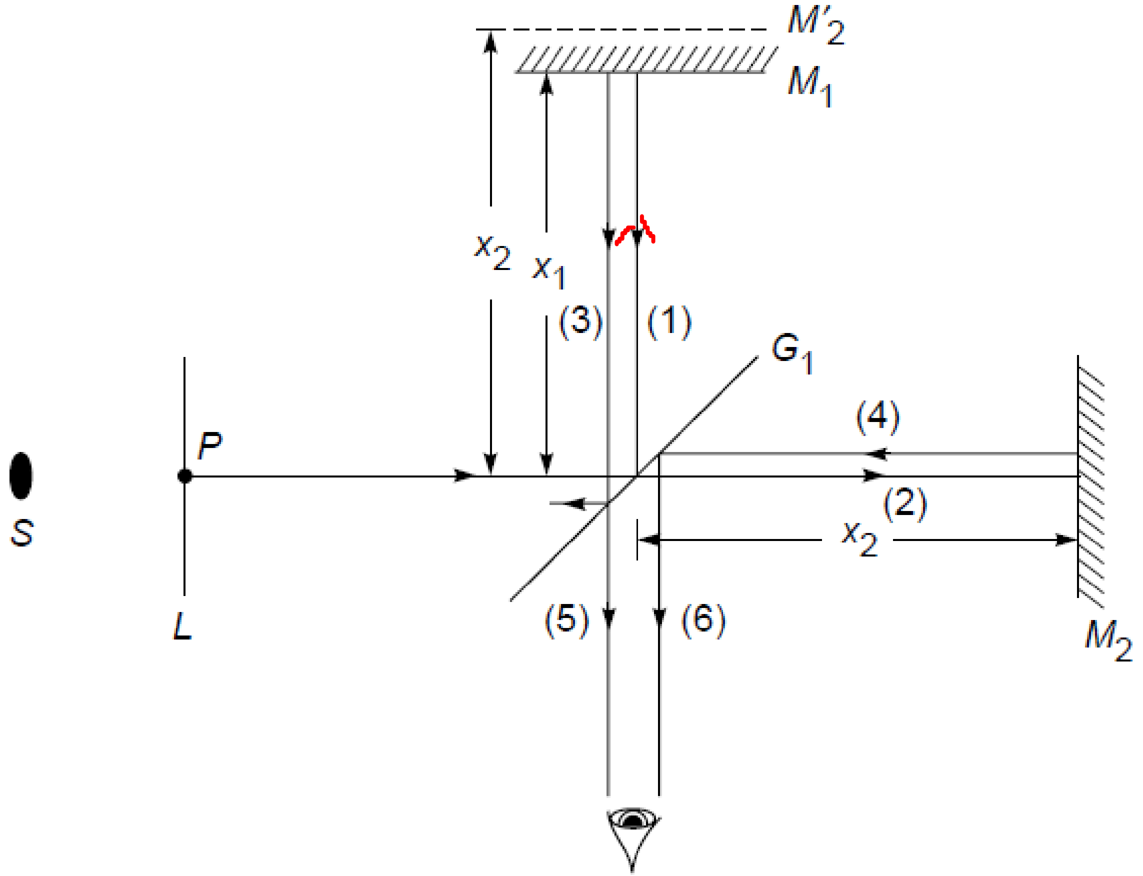
$$\boxed{2d \cos\theta = m\lambda}$$

where $m = 0, 1, 2, 3, \ldots$ and

$$d = x_1 \sim x_2$$

and the angle $\theta$ represents the angle that the rays make with the axis (which is normal to the mirrors as shown in Fig. 15.35). Similarly, the condition for a bright ring is

$$\boxed{2d \cos\theta = \left(m + \tfrac{1}{2}\right)\lambda}$$




Schematic of the Michelson interferometer.

# The Michelson interferometer

**Thus as we start reducing the value of *d*, the fringes will tend to collapse at the center**

Conversely, if d is increased, the fringe pattern will expand.

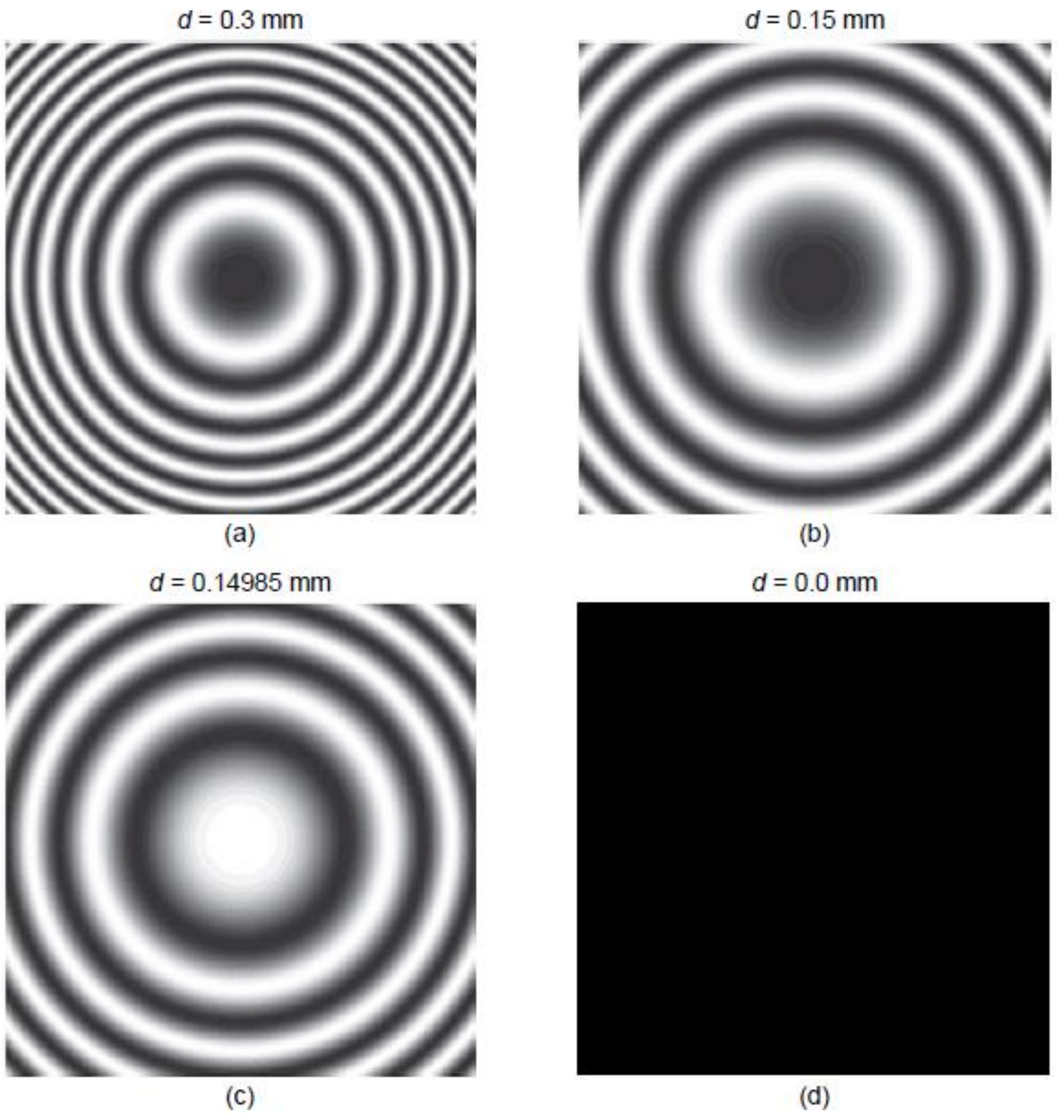
if N fringes collapse to the center as mirror $M_1$ moves by a distance $d_0$, then we must have

$$2d = m\lambda$$

$$2(d - d_0) = (m - N)\lambda$$

where we have set $\theta' = 0$ because we are looking at the central fringe. Thus

$$\lambda = \frac{2d_0}{N}$$




Computer-generated interference pattern produced by a Michelson interferometer.

Optics, Ghatak

# Wavelength measurement

This provides us with a method for the measurement of the wavelength. For example, in a typical experiment, if 1000 fringes collapse to the center as the mirror is moved through a distance of $2.90 \times 10^{-2}$ cm, then

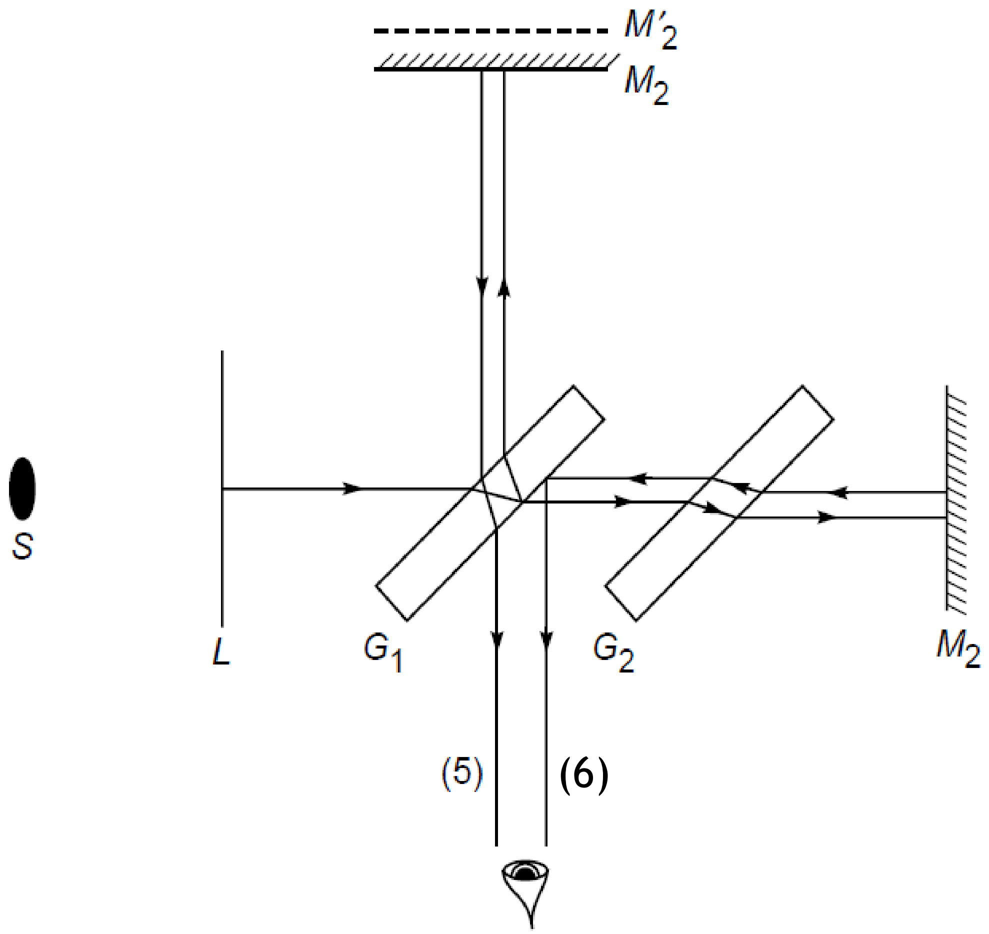
$$\lambda = 5800 \text{ Å}$$

The above method was used by Michelson for the standardization of the meter. He found that the red cadmium line ($\lambda = 6438.4696$ Å) is one of the ideal monochromatic sources, and as such this wavelength was used as a reference for the standardization of the meter.

In an actual Michelson interferometer, the beam splitter $G_1$ consists of a plate (which may be about $\frac{1}{2}$ cm thick), the back surface of which is partially silvered, and the reflections occur at the back surface as shown in Fig. 15.37. It is immediately obvious that beam 5 traverses the glass plate three times, and to compensate for this additional path, one introduces a "compensating plate" $G_2$ which is exactly of the same thickness as $G_1$. The compensating plate is not really necessary for a monochromatic source because the additional path $2(n - 1)t$ introduced by $G_1$ can be compensated by moving mirror $M_1$ by a distance $(n - 1)t$, where $n$ is the refractive index of the material of the glass plate $G_1$.

# A point to note

- In an actual Michelson interferometer, the beam splitter $G_1$ consists of a plate (which may be about 1/2 cm thick),
- The back surface of which is partially silvered, and the reflections occur at the back surface
- The compensating plate is not really necessary for a monochromatic source because the additional path introduced by $G_1$ can be compensated by moving mirror $M_1$
- ***Q: How many time rays have crossed $G_1$?***
- **What about white light??**
- Difficult to adjust $M_1$ for each $\lambda$
- Hence, compensating plate $G_2$




Optics, Ghatak

# Application

- **Can be used in the measurement of two closely spaced wavelengths:**
- Sodium lamp → emits two closely spaced wavelengths 5890 and 5896 Å

If mirror $M_1$ is moved away from (or toward) plate $G_1$ through a distance $d$, then the maxima corresponding to the wavelength $\lambda_1$ will not, in general, occur at the same angle as $\lambda_2$. Indeed, if the distance $d$ is such that

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = \frac{1}{2}$$

and if $2d \cos\theta' = m\lambda_1$, then $2d \cos\theta' = \left(m + \frac{1}{2}\right)\lambda_2$. Thus, the maxima of $\lambda_1$ will fall on the minima of $\lambda_2$, and conversely, and the fringe system will disappear.

# Application continued

if $$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1$$

then interference pattern will again reappear. In general, if

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2, . . . , we will have disappearance of the fringe pattern; and if it is equal to 1, 2, 3, . . . , then the interference pattern will appear.

# Few points to note

- When the mirrors of the interferometer are inclined with respect to each other, making a small angle (i.e., when $M_1$ and $M_2$ are not quite perpendicular), ***Fizeau fringes*** are observed. The resultant wedge-shaped air film between $M_2$ and $M_1$ creates a pattern of straight parallel fringes.
- by appropriate adjustment of the orientation of the mirrors-$M_1$ and -$M_2$, fringes can be produced that are straight, circular, elliptical, parabolic, or hyperbolic—this holds as well for the real and virtual fringes.

# Experiment in brief

- https://www.youtube.com/watch?v=j-u3IEgcTiQ

Problem:1

For a sodium lamp, the distance traversed by the mirror between two successive disappearances is 0.289 mm. Calculate the difference in the wavelengths of the $D_1$ and $D_2$ lines. Assume $\lambda$= 5890 Å

Answer:

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2, . . . , we will have disappearance of the fringe pattern; and if it is equal to 1, 2, 3, . . . , then the interference pattern will appear.

When the mirror moves through a distance 0.289 mm, the additional path introduced is 0.578 mm. Thus

$$\frac{0.578}{\lambda} - \frac{0.578}{\lambda + \Delta\lambda} = 1$$

$$\Delta\lambda \simeq \frac{\lambda^2}{0.578} = \frac{(5890 \times 10^{-7})^2}{0.578} \text{ mm}$$

$$\simeq 6 \text{ Å}$$

Assume $\Delta\lambda \times \lambda \ll \lambda^2$

## Problem:2

In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength

# Answer:

The wavelength $\lambda$ in Michelson interferometer is given by following equation.

$$\lambda = \frac{2d_0}{N}$$

Here, $d_0$ is the distance moved by the mirror, and $N$ is the number of fringes.

$$\lambda = \frac{2(0.08\,\text{mm})\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}{250}$$

$$= 6.4\times10^{-5}\,\text{cm}\left(\frac{10^{8}\,\text{A}^{\circ}}{1\,\text{cm}}\right)$$

$$= 6400\,\text{A}^{\circ}$$

# Thank You