Engineering Electromagnetics

Lecture 34

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by

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Induced emf

The process of inducing an emf in a coil is known as ***Electromagnetic induction***

How to achieve that?→ Any one of the following should be true

1. Flux through stationary coil is f(t)
2. Coil changes shape/position with t but B is uniform
3. Both 1 and 2 are true

$$\frac{d\Phi}{dt} = -BL\frac{dx}{dt} = -BLu$$

$$e = -\frac{d\Phi}{dt}$$

Maxwell's equation (Faraday's law)

A changing magnetic field induces an electric field.

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

then $\mathbf{E}$ is related to the change in $\mathbf{B}$ by the equation

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

This is **Faraday's law**

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

Q: Conservative?

For a static B →

Whenever (and for whatever reason) the magnetic flux through a loop changes, an emf

$$\mathcal{E} = -\frac{d\Phi}{dt}$$ **will appear in the loop**

Faraday's law

The *divergence* of $\mathbf{E}$ is still given by Gauss's law ($\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$). If $\mathbf{E}$ is a *pure* Faraday field (due exclusively to a changing $\mathbf{B}$, with $\rho = 0$), then

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0, \quad \boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

This is mathematically identical to magnetostatics,

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \quad \boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}.$$

Maxwell's equation general form

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot \nabla \times \mathbf{H} \equiv 0 = \nabla \cdot \mathbf{J}$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

can be true only if $\partial \rho_v / \partial t = 0$. This is an unrealistic limitation,

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{G}$$

$$0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{G}$$

$$\nabla \cdot \mathbf{G} = \frac{\partial \rho_v}{\partial t}$$

Replacing ρ_v with $\nabla \cdot \mathbf{D}$,

$$\nabla \cdot \mathbf{G} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{G} = \frac{\partial \mathbf{D}}{\partial t}$$

Conduction current density, $\boxed{\mathbf{J} = \sigma \mathbf{E}}$

$$\boxed{\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} + \mathbf{J}_d \\ \mathbf{J}_d &= \frac{\partial \mathbf{D}}{\partial t} \end{aligned}}$$

Displacement current density

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}}$$

A changing electric field induces a magnetic field

In integral form

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \int \left(\frac{\partial \mathbf{E}}{\partial t} \right) \cdot d\mathbf{a}}$$

Maxwell called his extra term the **displacement current**

$$\mathbf{J}_d \equiv \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

In a material with ε_r ?

J and conductivity σ in a conductor?

From Griffith book

Maxwell’s equation

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \text{(Gauss's law),}$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \qquad \text{(no name),}$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law),}$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad \text{(Ampère's law with Maxwell's correction).}$$

From Griffith book

Problem-1

- A piece of a matter has conductivity 0.11 S/m and relative permittivity 1.2. At t= 5 Sec, calculate (i) conduction current density (J_c), (ii) displacement current density (J_D), if the matter is placed in an electric field E= Cos0.1t (V/m)?

$$J_c = \sigma E \text{ and}$$
$$J_D = \frac{\partial D}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$$

Thank You