

IIITDM KANCHEEPURAM
MA1001 Differential Equations
Problem Set 6

¹ Solve the following differential equation with **variation of parameters method**.

(i) $\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$

(ii) $\frac{d^2y}{dx^2} + a^2y = \sec ax$

(iii) $\frac{d^2y}{dx^2} + (1 - \cot x)\frac{dy}{dx} - y \cot x = \sin^2 x$, where one solution is $y_1 = e^{-x}$

² Let $y_1(x) = e^{-x^2}$ be a solution of the differential equation

$$xy'' + \alpha y' + \beta x^3 y = 0$$

for some real number α and β . Find the value of α and β .

³ If $y = 3e^{2x} + e^{-2x} - \alpha x$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + \beta y = 4\alpha x, \quad y(0) = 4, \quad \text{and} \quad \frac{dy}{dx}(0) = 1, \quad \text{where } \alpha, \beta \in R,$$

find the value of α and β .

⁴ Find ordinary and singular (regular and irregular singular) points of the following differential equations.

(a) $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$

(b) $(x - 1)\frac{d^2y}{dx^2} + (\cot \pi x)\frac{dy}{dx} + (\csc^2 \pi x)y = 0$

(c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$

(d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$

(e) $x^4\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x^{-1}$

⁵ Find the roots of the indicial equation for following differential equations about $x = 0$.

(a) $4x^2\frac{d^2y}{dx^2} - 4xe^x\frac{dy}{dx} + 3\cos(x)y = 0$

(b) $2x^2\frac{d^2y}{dx^2} + x(x+1)\frac{dy}{dx} - 3\cos(x)y = 0$

(c) $x^4\frac{d^2y}{dx^2} - x^2\sin(x)\frac{dy}{dx} + 2(1 - \cos(x))y = 0$

⁶ Find the solution of the following differential equations about point $x = 0$.

(a) $2x(1 - x)\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} + 3y = 0$

(b) $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$

(c) $9x(1 - x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$

(d) $x(1 - x)\frac{d^2y}{dx^2} + (1 - 5x)\frac{dy}{dx} - 4y = 0$

(e) $x\frac{d^2y}{dx^2} + (p - x)\frac{dy}{dx} - y = 0$

(f) $(2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (1 + x)y = 0$