MA1002 Linear Algebra
Assignment 1
10 Marks

1. **Let** W **be the set of all** $(x_1, x_2, x_3, x_4, x_5)$ **in** R^5 **which satisfy** $2x_1 + \frac{3}{4}x_2 - x_3 - x_4 = 0$**,** $x_1 + \frac{2}{3}x_2 - x_5 = 0$ **and** $9x_1 + 6x_2 - 3x_3 - 3x_4 - 3x_5 = 0$**. Find a finite set of vectors which spans** W**. Justify your answer.**

2. **Let** A **be an** $m \times n$ **matrix over a field** F **and let** R **be a row-reduced echelon matrix row equivalent to** A**. Prove or disprove that the non-zero vectors of** R **forms a basis for the row space** A**.**

3. **Find a basis of** R^4 **that contains** $\{(1, 2, 3, 4), (4, 3, 2, 1)\}$**. Justify your answer.**

4. **Find an onto linear tranformation (if exists)** $T : R^4 \longrightarrow R^3$ **with** $N(T) = \{(4x, 3x, 2x, x) : x \in R\}$

5. **State and prove rank-nullity-dimension theorem.**

6. **Let us consider** $B = \{(1, 1), (1, 2)\}$ **is the order basis, then find an alternate order basis basis corresponding to the invertible matrix** $P = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$**. Again if** $B_1 = \{(2, 1), (1, 2)\}$ **then find the unique invertible matrix** Q **such that** $[\alpha]_B = Q[\alpha]_{B_1}$ **for any** $\alpha \in \mathbb{R}^2$**.**

7. **Let** x, y, z **be nonzero distinct vectors in a vector space** V **with x + y + z = 0. Show that span {x. y} = span {y. z} = span {z. x} .**

8. **Let** V **be a vector space. Suppose the vectors** $v_1, v_2, \cdots v_n$ **span** V**. Show that the** $v_1, v_2 - v_1, \cdots v_n - v_1$ **also span** V**. Further, show that if** $v_1, v_2, \cdots v_n$ **linearly independent, then** $v_1, v_2 - v_1, \cdots v_n - v_1$ **are linearly independent.**

9. **For matrices such that the product** AB **is defined, explain why each of the following statements is true. (a)** $R(AB) \subseteq R(A)$. **(b)** $N(AB) \supseteq N(B)$**. Note that** $R(A)$ **stands for row space of** A, $N(B)$ **stands for solution set of** $BX = 0$**.**

10. **Let** V **be a vector space of dimension** n **over a field** $\mathbb{F}$**. If** $T : V \to V$ **is linear, prove that the following are equivalent:**

    (a) $R(T) = N(T)$**.**

    (b) $ToT = 0$**,** $T \neq 0$**,** n **is even and Rank** $T = \dfrac{n}{2}$**. Note that** ToT **stands for composite function.**

Instructions : Submit a hard copy of the hand writeen assignment. Deadline is 18/11/2024 (Monday) 4.00PM.