Problem Set-4 Probability

January 2020

1. Consider rolling a fair six-sided die, so that $S = \{1, 2, 3, 4, 5, 6\}$. Let $X(s) = s$, and $Y(s) = s^3 + 2$. Let $Z = XY$. Compute $Z(s)$ for all $s \in S$.

2. Suppose a university is composed of 55% female students and 45% male students. A student is selected to complete a questionnaire. There are 25 questions on the questionnaire administered to a male student and 30 questions on the questionnaire administered to a female student. If X denotes the number of questions answered by a randomly selected student, then compute $P(X = x)$ for every real number x.

3. Consider flipping a fair coin. Let $Z = 1$ if the coin is heads, and $Z = 3$ if the coin is tails. Let $W = Z^2 + Z$.

   (a) What is the probability function of Z?

   (b) What is the probability function of W?

4. Let X be a Binomial random variable with parameter $(12, p)$. For what value of p is $P(X = 11)$ maximized?

5. An urn contains 4 black balls and 5 white balls. After a thorough mixing, a ball is drawn from the urn, its color is noted, and the ball is set aside. The remaining balls are then mixed and a second ball is drawn.

   (a) What is the probability distribution of the number of black balls observed?

   (b) What is the probability distribution of the number of white balls observed?

6. Suppose an urn contains 1000 balls — one of these is black, and the other 999 are white. Suppose that 100 balls are randomly drawn from the urn with replacement. Use the appropriate Poisson distribution to approximate the probability that five black balls are observed.

7. Suppose that there is a loop in a computer program and that the test to exit the loop depends on the value of a random variable X. The program exits the loop whenever $X \in A$, and this occurs with probability 1/3. If the loop is executed at least once, what is the probability that the loop is executed five times before exiting?

8. Let W be the uniform random variable in the interval $[1, 4]$. Compute each of the following.

   (a) $P(W \geq 5)$

   (b) $P(W \geq 2)$

   (c) $P(W^2 \leq 9)$

   (d) $P(W^2 \leq 2)$

9. Let X be the Exponential with parameter 3. Compute each of the following.

   (a) $P(0 < X < 1)$

   (b) $P(0 < X < 3)$

   (c) $P(0 < X < 5)$

   (d) $P(2 < X < 5)$

   (e) $P(2 < X < 10)$

   (f) $P(X > 2)$

10. Suppose X has density f and that $f(x) \geq 2$ for $0.3 < x < 0.4$. Prove that $P(0.3 < X < 0.4) \geq 0.2$.