Engineering Optics

Lecture 17

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by

Debolina Misra

Assistant Professor
Department of Physics
IIITDM Kancheepuram, Chennai, India

Summary of our earlier discussions

1-D differential wave equation

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

Spatial period → wavelength ***'λ'***

$$\psi(x, t) = \psi(x \pm \lambda, t)$$

Spatial frequency: wave number ***κ=1/λ***

Temporal period: τ

$$\psi(x, t) = \psi(x, t \pm \tau)$$

Temporal frequency: $\nu \equiv 1/\tau$

$$v = \nu\lambda$$

Sinusoidal / harmonic waves

$$\psi(x, t)\big|_{t=0} = \psi(x) = A \sin kx = f(x)$$

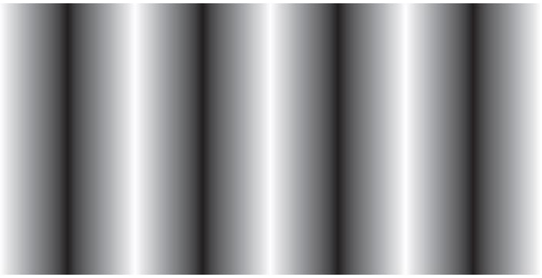
To transform it to a wave travelling with a speed *v*

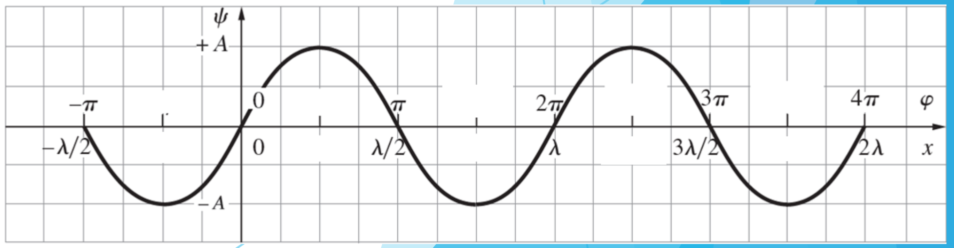
$$\psi(x, t) = A \sin k(x - vt) = f(x - vt)$$

Amplitude

?

$$k = 2\pi/\lambda$$





Brightness distribution → periodic

Reference: Optics *by* Hecht

Phase

- Consider a sinusoidal wave:

$$\psi = A \sin k(x - vt)$$

$[k(x-vt) = kx-kvt = kx - (2\pi/\lambda)(v\lambda)t = kx - (2\pi v)t = kx - \omega t]$

$$\boxed{\psi(x, t) = A \sin (kx - \omega t)}$$

Phase $\boxed{\varphi = (kx - \omega t)}$

At $t = x = 0$, $\psi(x, t)\big|_{\substack{x=0 \\ t=0}} = \psi(0, 0) = 0$

$$\psi(x, t) = A \sin (kx - \omega t + \varepsilon)$$

ε is the **initial phase**.

Initial phase → contribution from the generator.

Phase velocity

$$\varphi(x, t) = (kx - \omega t + \varepsilon)$$

Rate-of change of phase with time: $$\left|\left(\frac{\partial \varphi}{\partial t}\right)_x\right| = \omega \qquad (1)$$

Rate of change of phase with distance: $$\left|\left(\frac{\partial \varphi}{\partial x}\right)_t\right| = k \qquad (2)$$

(1)/(2) → $\dfrac{\omega}{k} = v$ → *phase velocity*

Superposition principle

Superposition principle

$$\frac{\partial^2 \psi_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2}$$

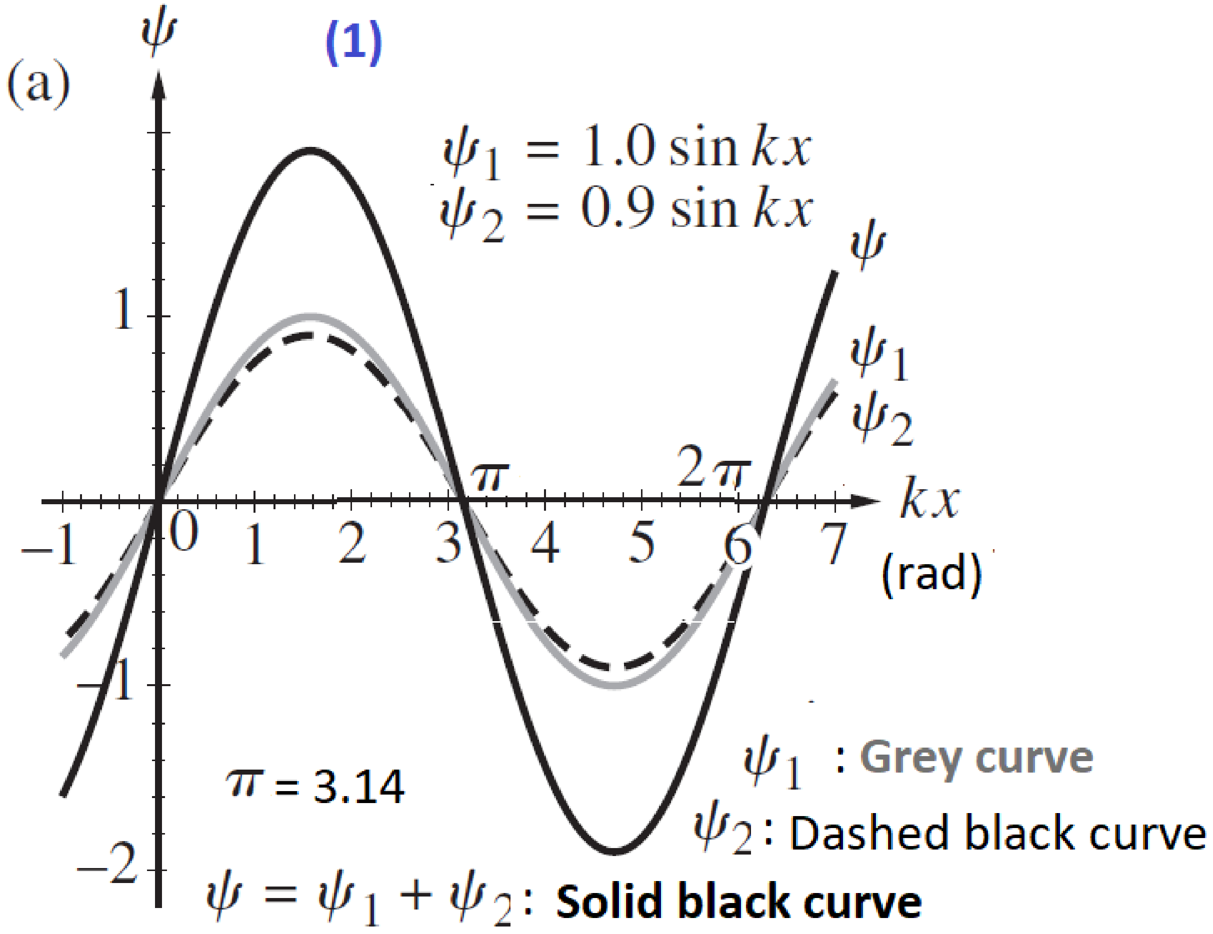
$$\frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2} + \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2}{\partial x^2}(\psi_1 + \psi_2) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(\psi_1 + \psi_2)$$

$$\boxed{\psi = \psi_1 + \psi_2}$$

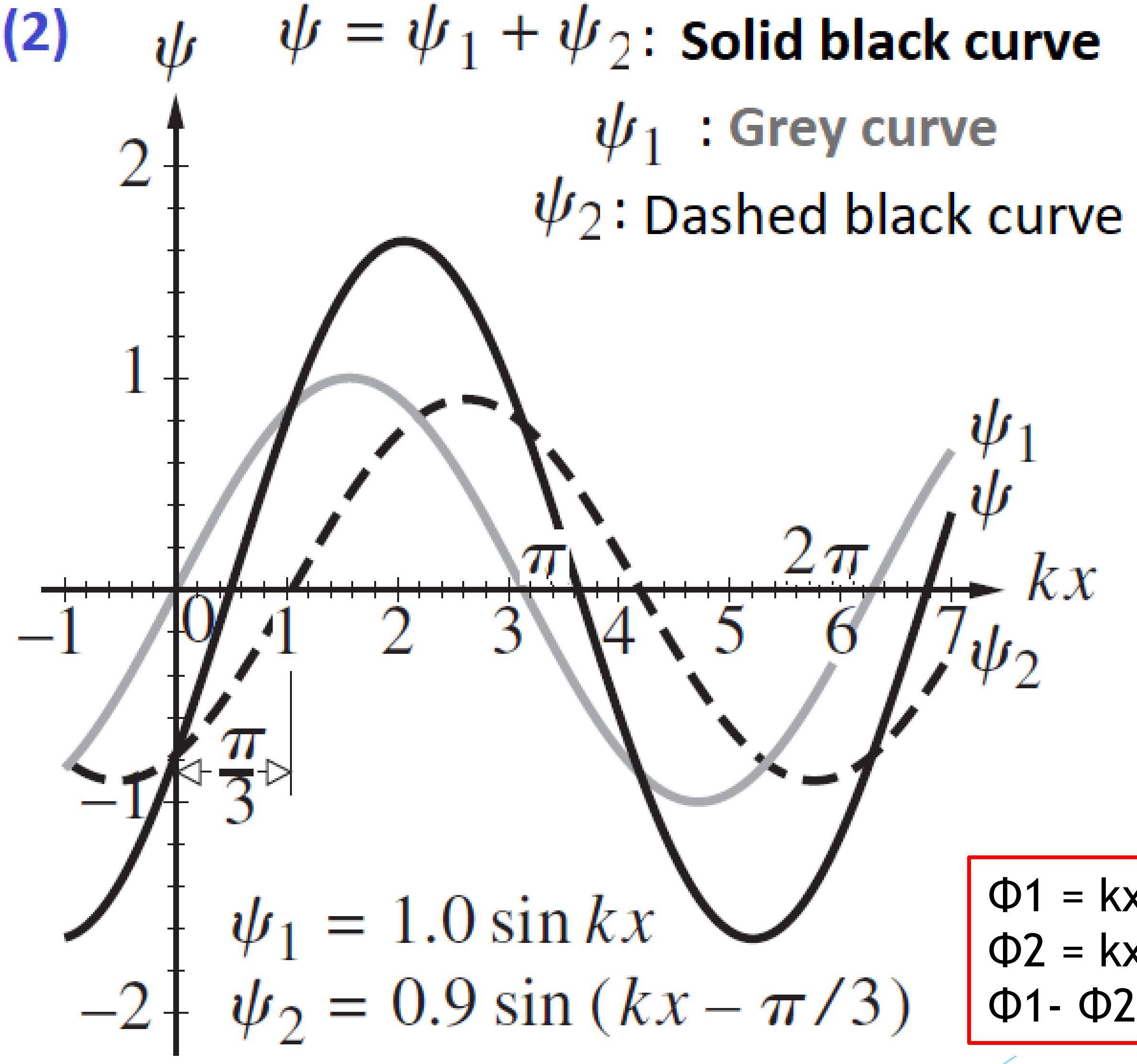
In-phase



***Constructive intereference**

Phase difference

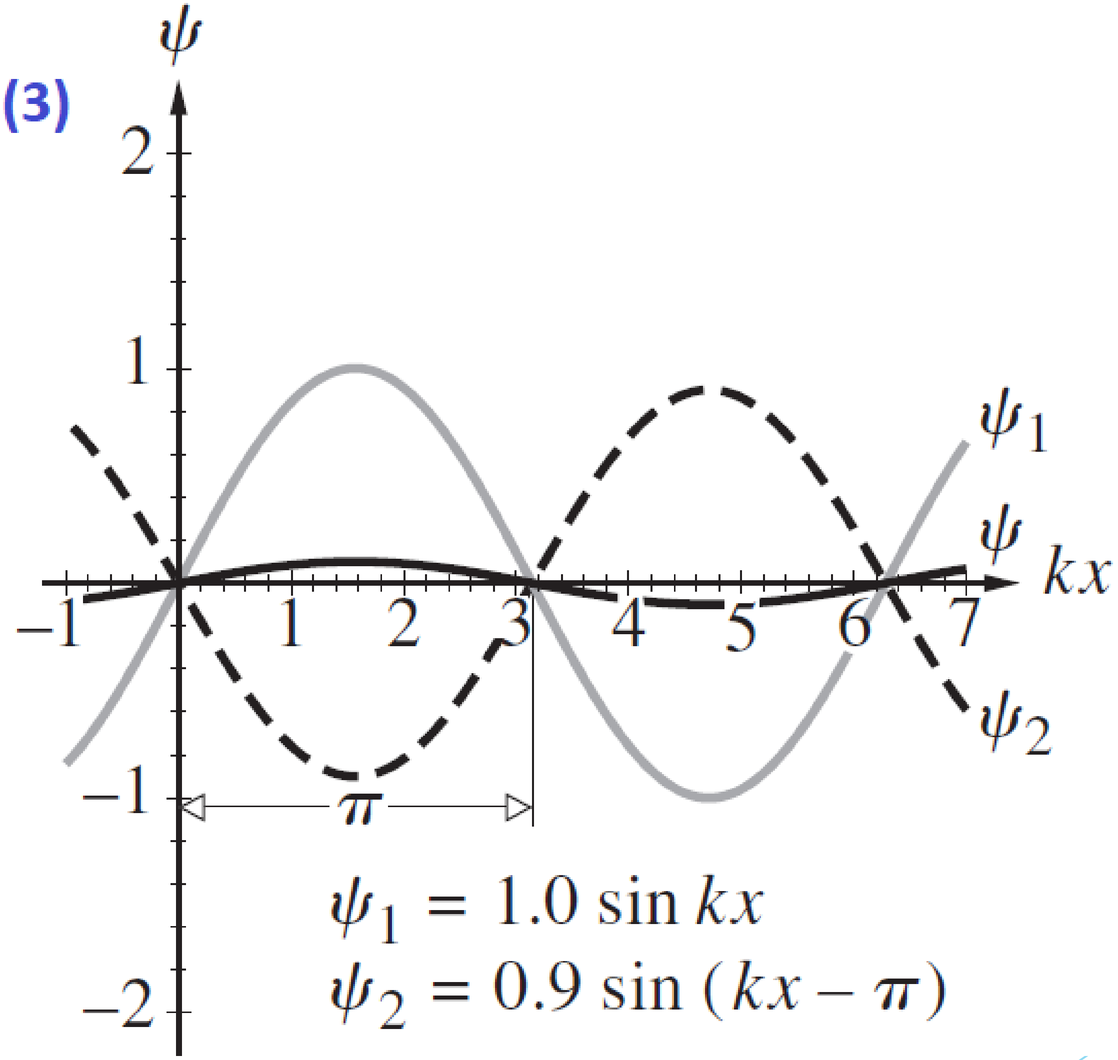
(2)



Φ1 = kx
Φ2 = kx −π/3
Φ1- Φ2 = π/3

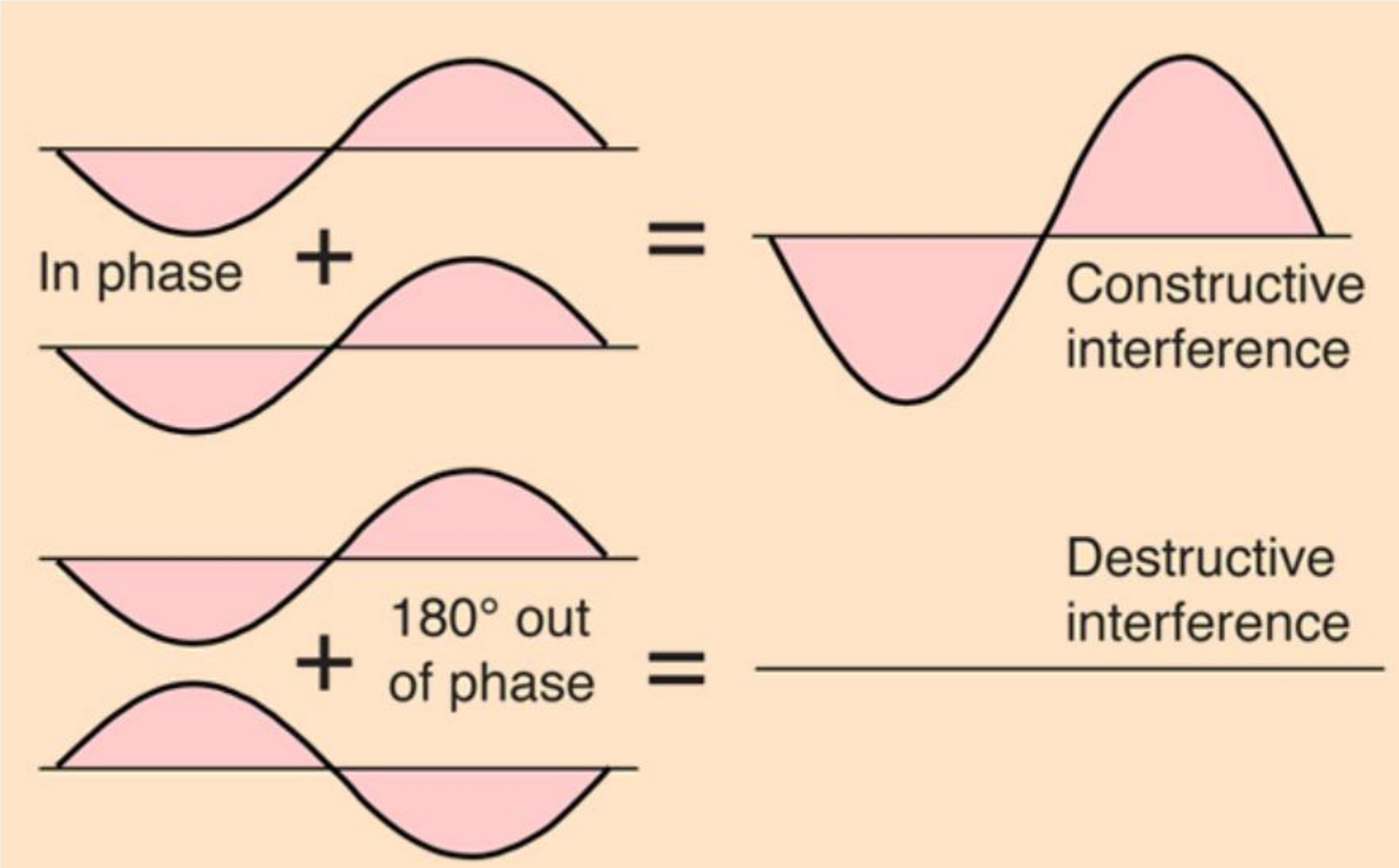
Out-of-phase

(3)



*Destructive intereference

Relative phase → Interference



Reference: http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/interf.html

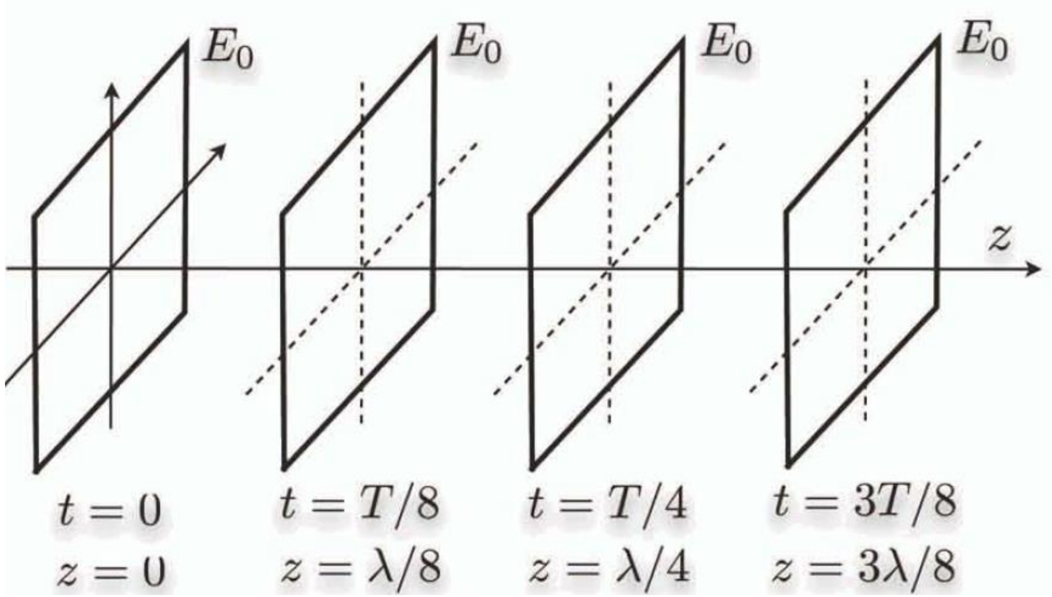
Wavefronts

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

Optical disturbance→ in space→ spatial distribution→ *wavefront*

at any t → *a surface of constant phase* → *wavefront (phase front)*

Plane wave:



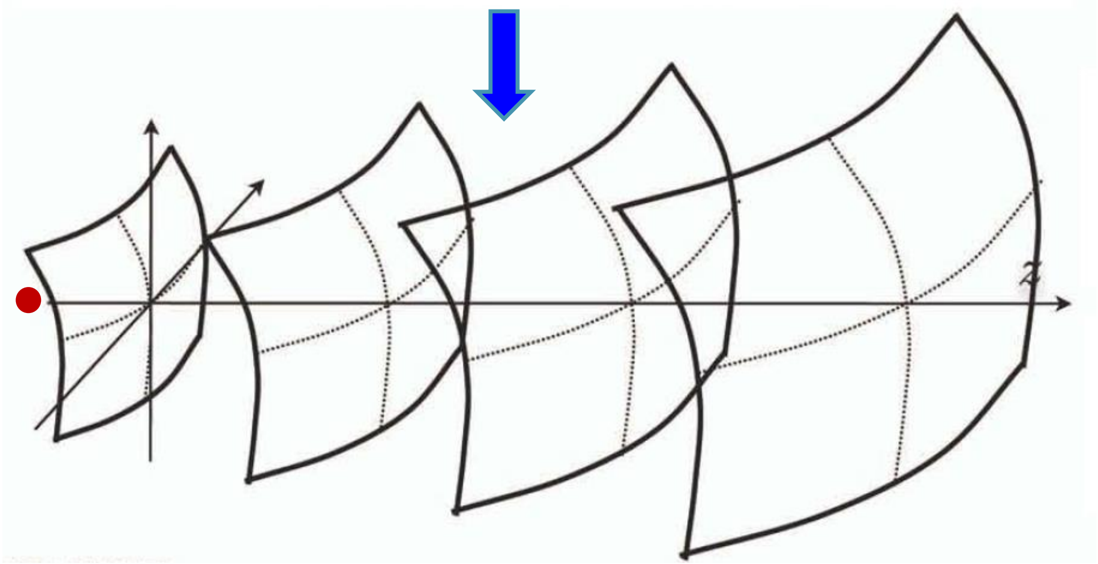
Optics, Hecht

Spherical waves

point source of light → radiating in all directions

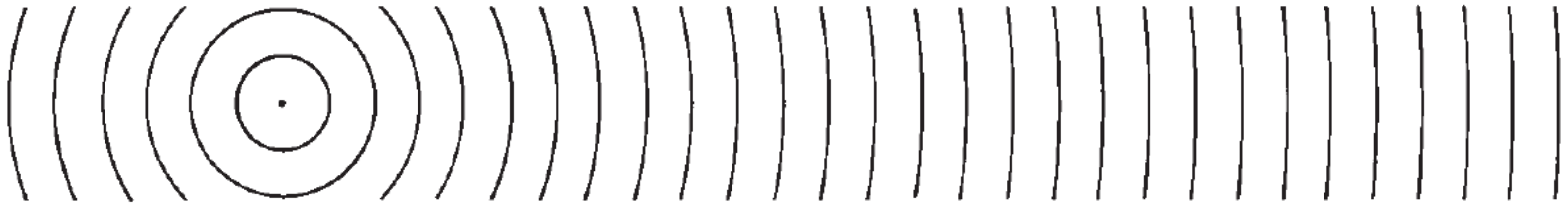
radius increases

Source



Flattening of spherical waves

Plane waves



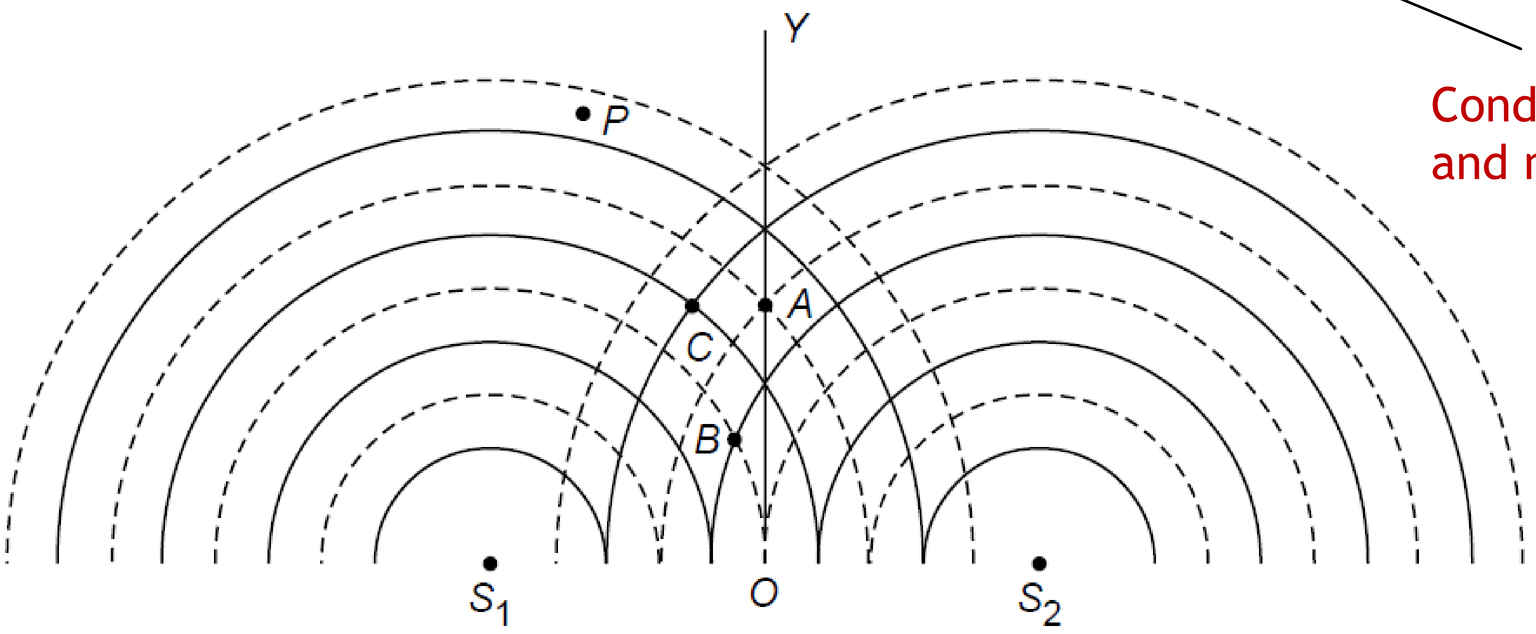
Optics, Hecht

Interference between two waves
e.g. on *surface of water*

Example-1: when the sources are vibrating in phase

Refer. '14.2 INTERFERENCE PATTERN PRODUCED ON THE SURFACE OF WATER'

Conditions for maxima and minima?

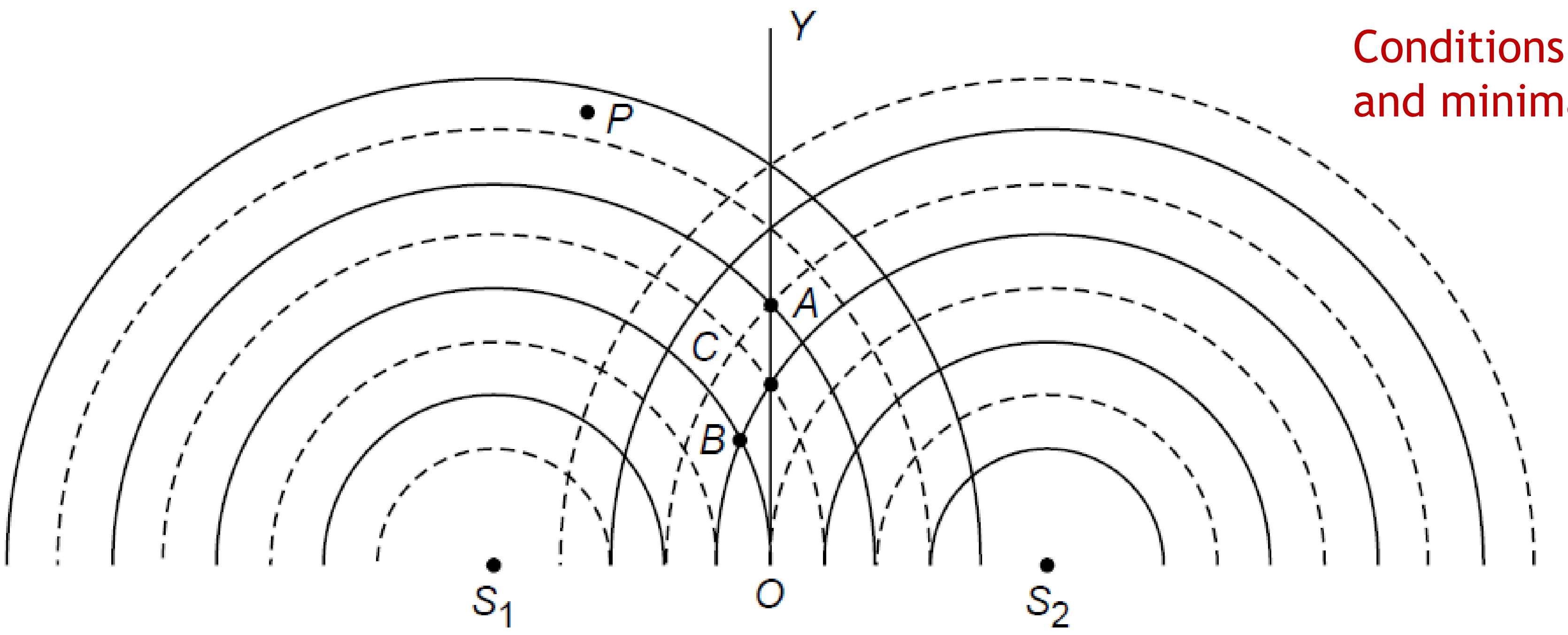


Waves emanating from two point sources S_1 and S_2 vibrating in phase. The solid and the dashed curves represent the positions of the crests and troughs, respectively.

Optics, Hecht

Interference between two waves

- Example-2: when the sources are vibrating out of phase
- Refer. ‘14.2 INTERFERENCE PATTERN PRODUCED ON THE SURFACE OF WATER’



Waves emanating from two point sources S_1 and S_2 vibrating out of phase.

Conditions for maxima and minima?

Optics, Hecht

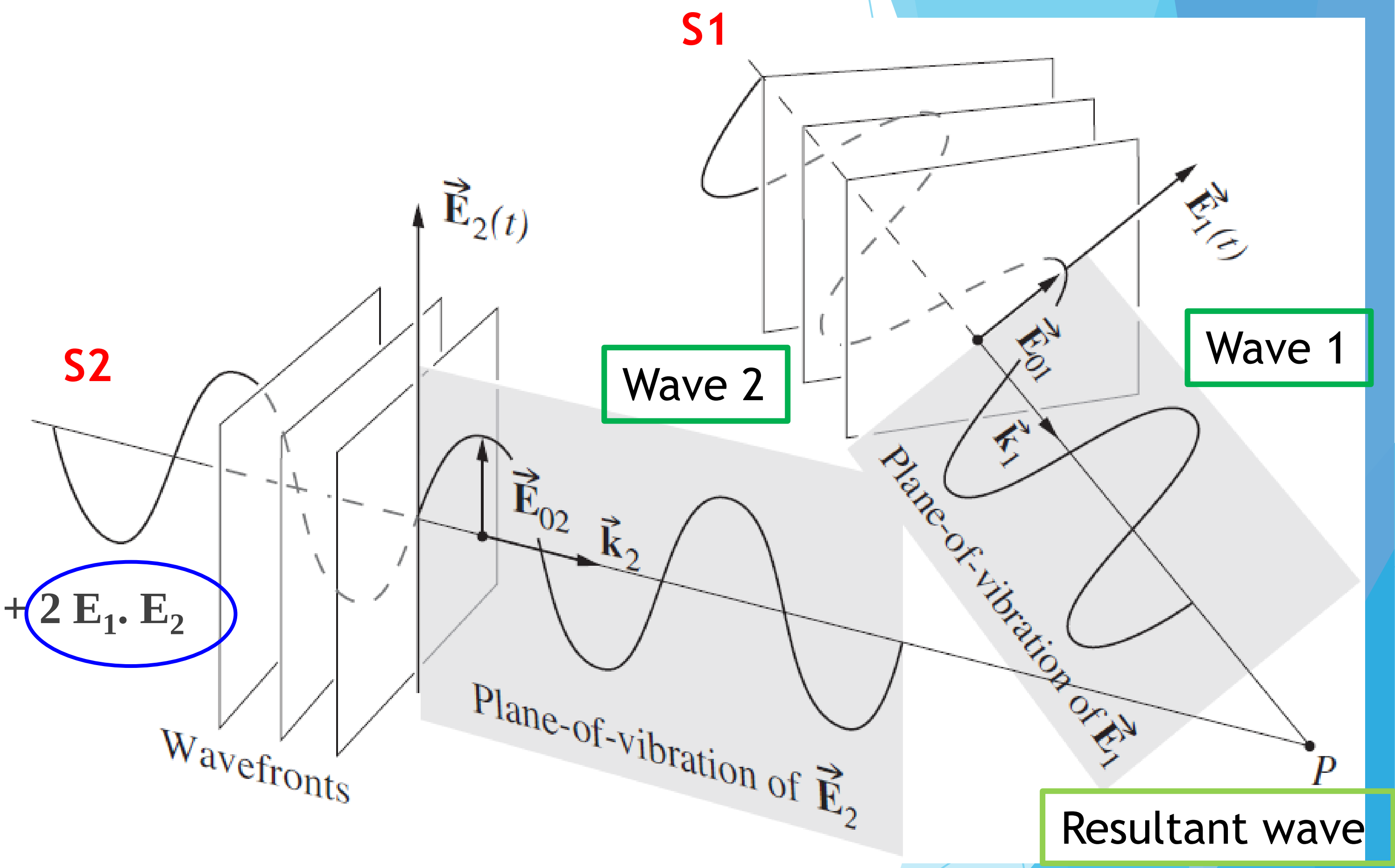
Light vectors and interference

Wave 1

- $\mathbf{E_1} = E_1 \, \mathrm{Sin}(\mathbf{k_1.r} - \omega t + \varepsilon_1)$

Wave 2

- $\mathbf{E_2} = E_2 \, \mathrm{Sin}(\mathbf{k_2.r} - \omega t + \varepsilon_2)$
- $\mathbf{E} = \mathbf{E_1} + \mathbf{E_2}$ Resultant wave
- $\mathbf{E.E} = (\mathbf{E_1} + \mathbf{E_2}) \cdot (\mathbf{E_1} + \mathbf{E_2}) = \mathbf{E_1 . E_1} + \mathbf{E_2 . E_2} + \mathbf{2\, E_1 . E_2}$
- **Intensity→ Irradiance** $= \langle E^2 \rangle_{\text{Time T}}$



Phase difference and interference

total irradiance is $$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

when $\cos\delta = 1$, $\delta = 0, \pm 2\pi, \pm 4\pi, \ldots$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

total constructive interference

disturbances are *in-phase.*

At $\delta = \pi/2$, $\cos\delta = 0$,

$$I = I_1 + I_2$$

When $0 < \cos\delta < 1$

$$I_1 + I_2 < I < I_{\max}$$

constructive interference

minimum irradiance

when $\cos\delta = -1$, $\delta = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

total destructive interference

Ranges

$0 > \cos\delta > -1$

$$I_1 + I_2 > I > I_{\min}$$

destructive interference

Interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

What will happen if

$$I_1 = I_2 = I_0.$$

$$I = 2I_0(1 + \cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

$$I_{\min} = 0$$

$$I_{\max} = 4I_0$$

Interference fringes

spherical wavefronts

$$\delta = k(r_1 - r_2) + (\varepsilon_1 - \varepsilon_2)$$

maxima $\delta = 2\pi m \qquad m = 0, \pm 1, \pm 2, \ldots$

$$(r_1 - r_2) = [2\pi m + (\varepsilon_2 - \varepsilon_1)]/k$$

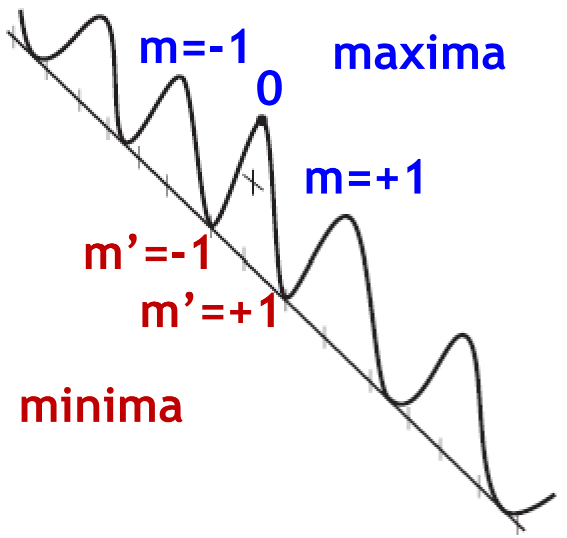
$$(r_1 - r_2) = 2\pi m/k = m\lambda$$

minima $\delta = \pi m' \qquad m' = \pm 1, \pm 3, \pm 5,$

or $m' = 2m + 1$

$$(r_1 - r_2) = [\pi m' + (\varepsilon_2 - \varepsilon_1)]/k$$

$$(r_1 - r_2) = \pi m'/k = \tfrac{1}{2}m'\lambda$$



Conditions for Interference

- Almost the same frequency
- Clearest pattern → amplitudes are almost same
- White lights from 2 sources → red with red, green with green etc.
- Sources → same initial phase? → not necessary
- Can have a phase difference (δ) → δ should not change with time
- <u>If δ between S1 and S2 = constant </u>→ ***Coherent sources***

Coherence

Coherence

Photos taken from google

Incoherence

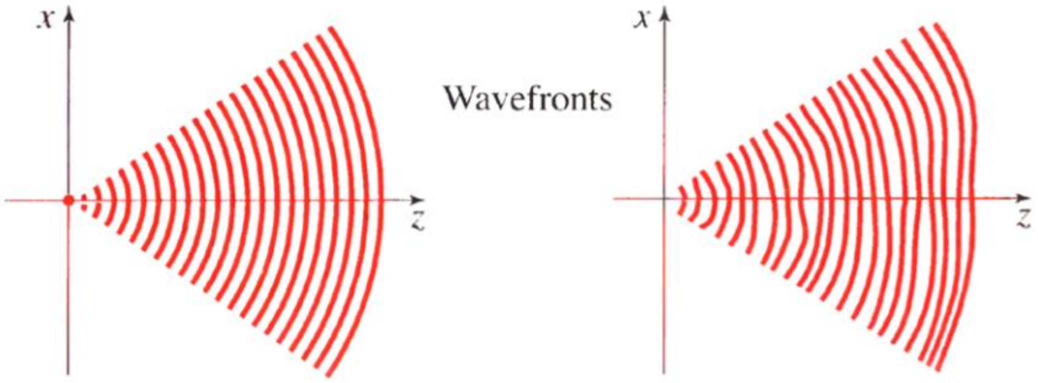
A measure of the phase correlation at different temporal and spatial points on a wave.

Spatial Coherence: at different points (transverse to k)→ how uniform the phase of a wavefront is

Pictures taken from web

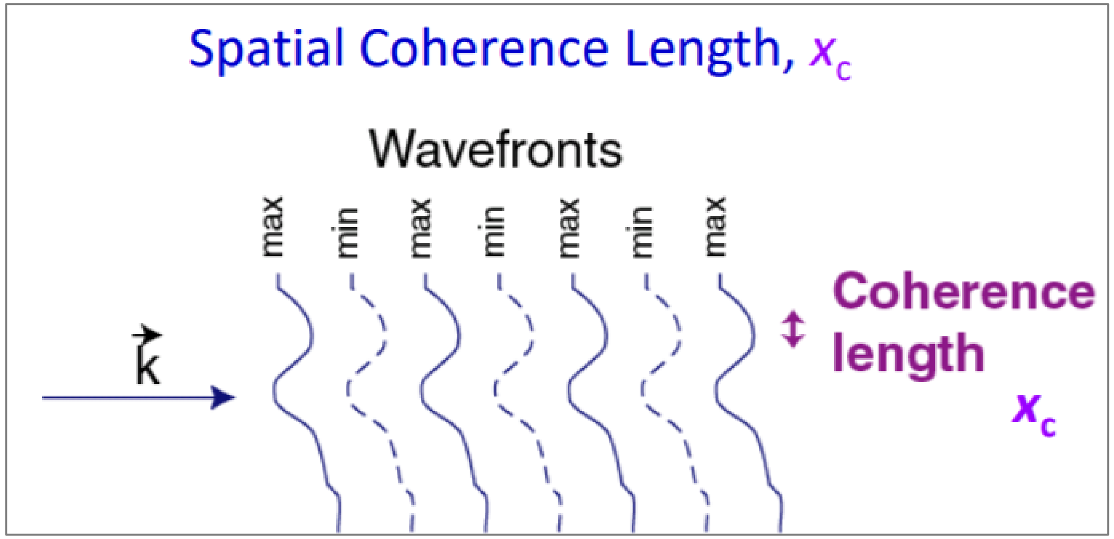
Spatial coherence

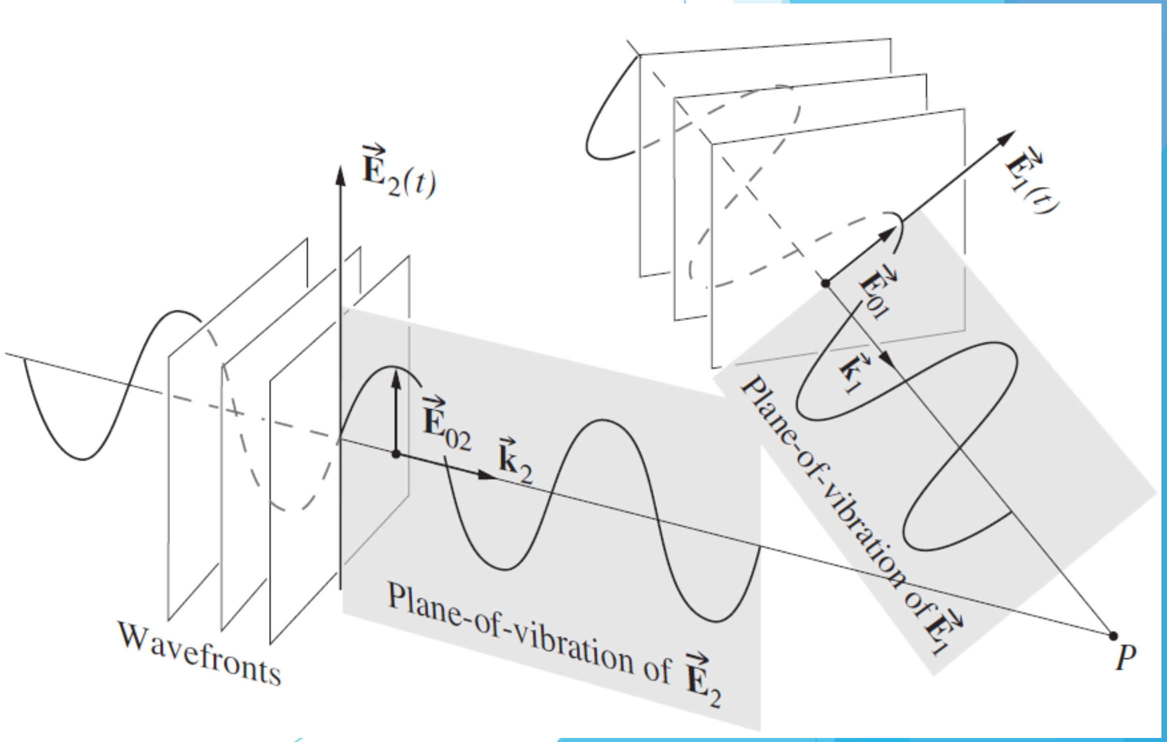
Spatial Coherence: at different points (transverse to k)→ how uniform the phase of a wavefront is



$$\mathbf{E_1} = \mathrm{E}_1\ \mathrm{Sin}(\mathbf{k_1.r} - \omega t + \varepsilon_1)$$

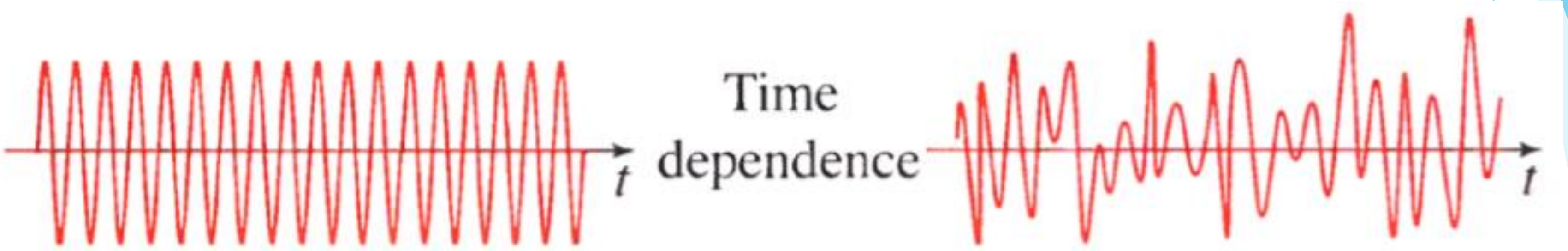
$$\mathbf{E_2} = \mathrm{E}_2\ \mathrm{Sin}(\mathbf{k_2.r} - \omega t + \varepsilon_2)$$

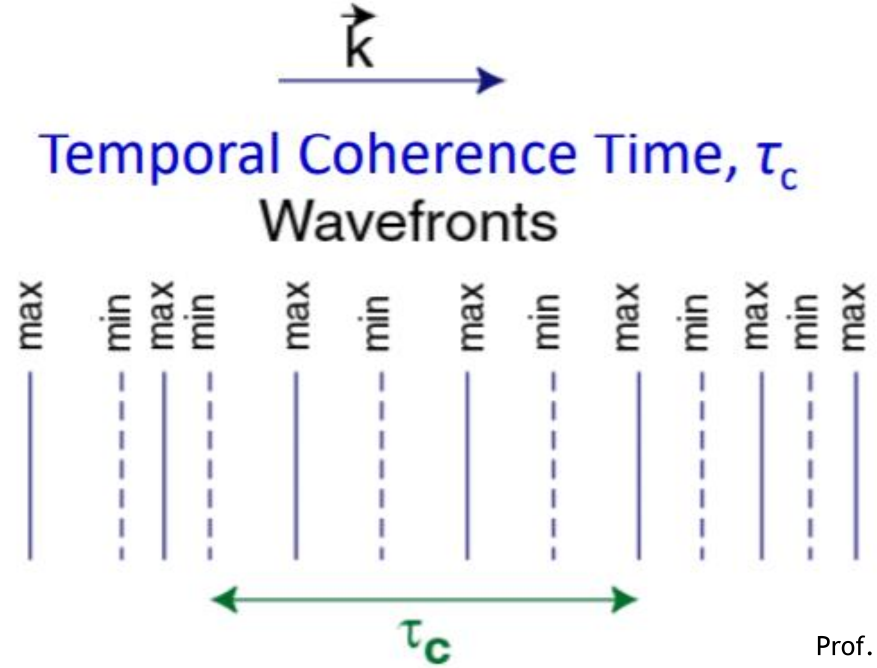




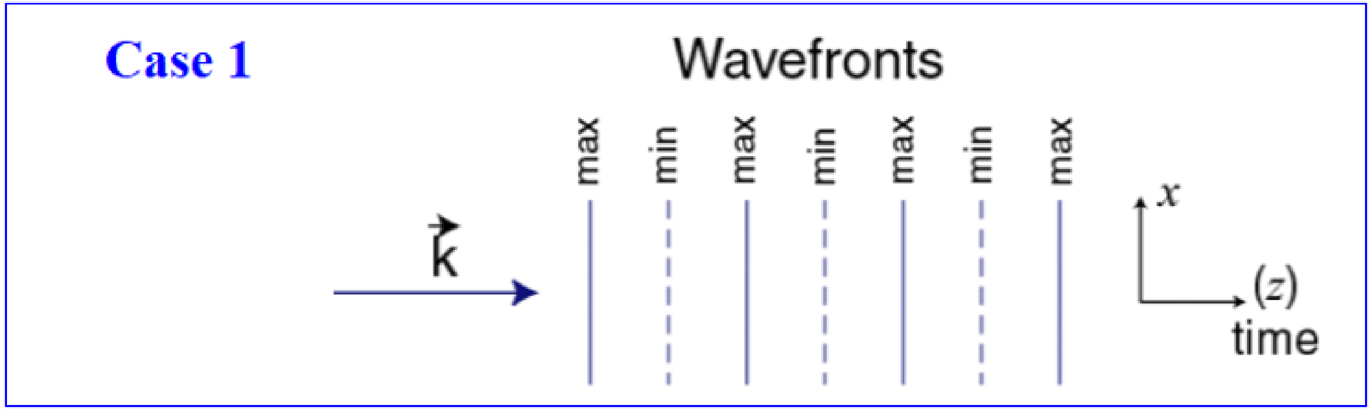
Temporal coherence

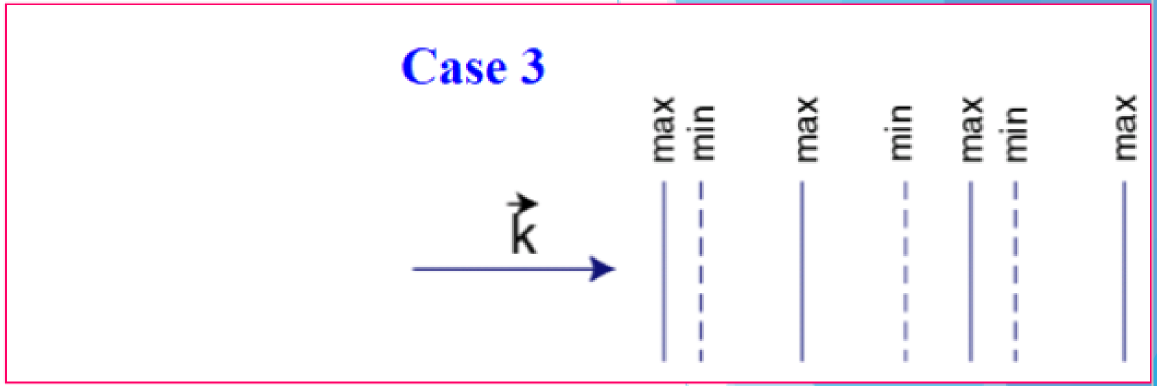
Phase correlation at different points along *k* – how monochromatic a source is

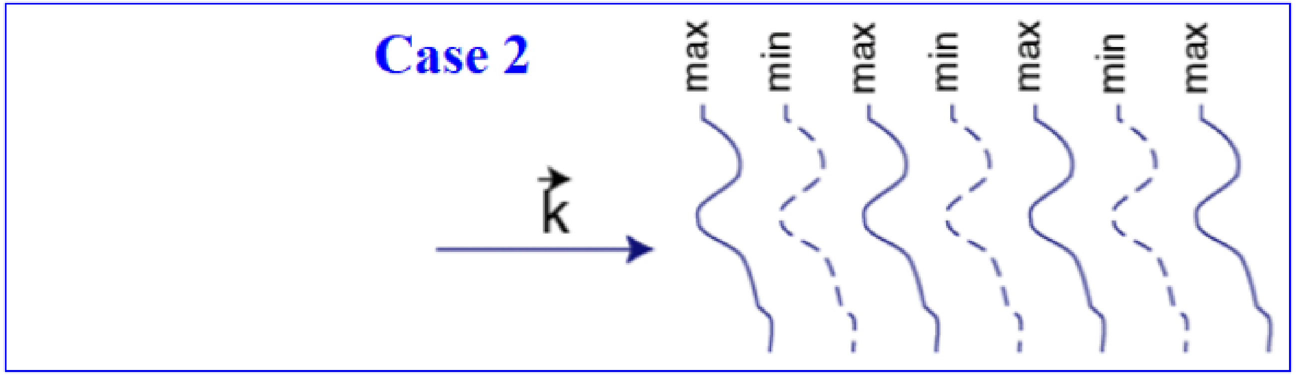


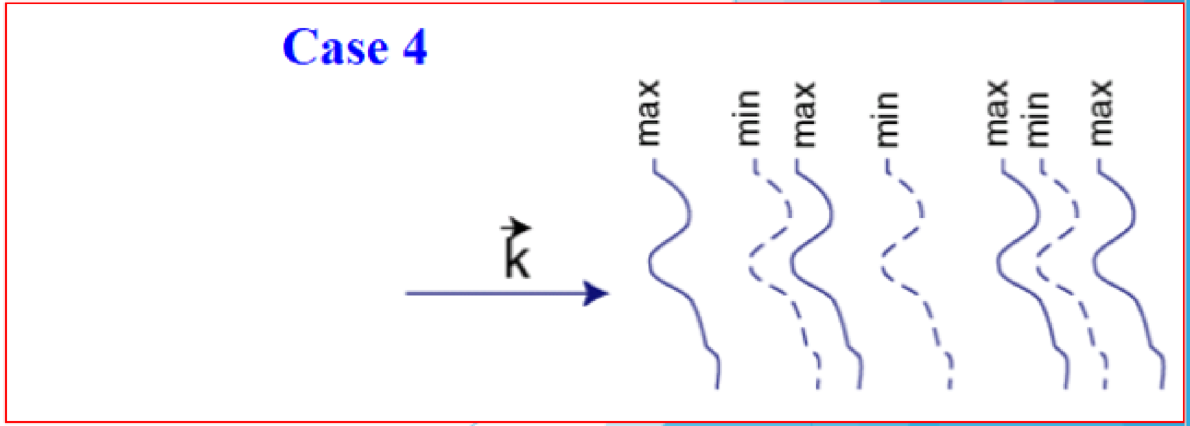


Coherence type??

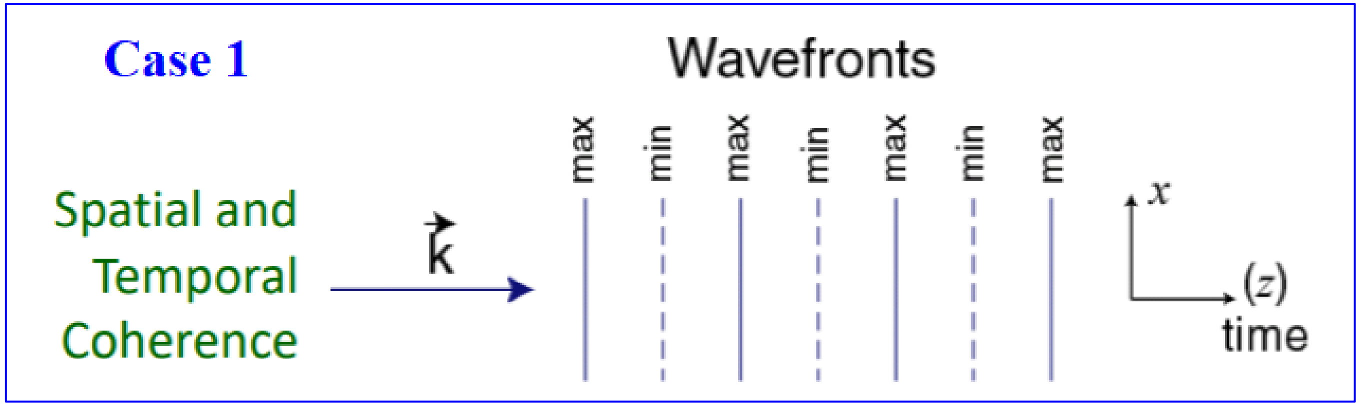


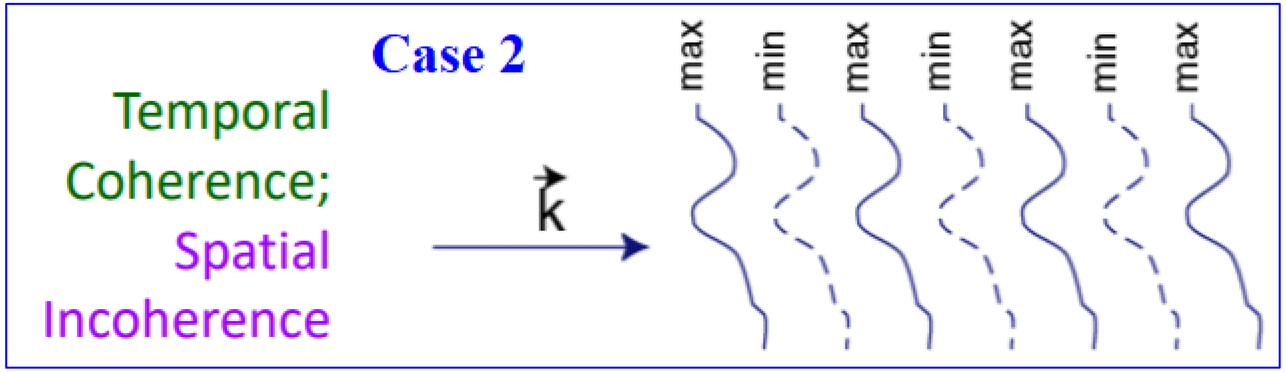


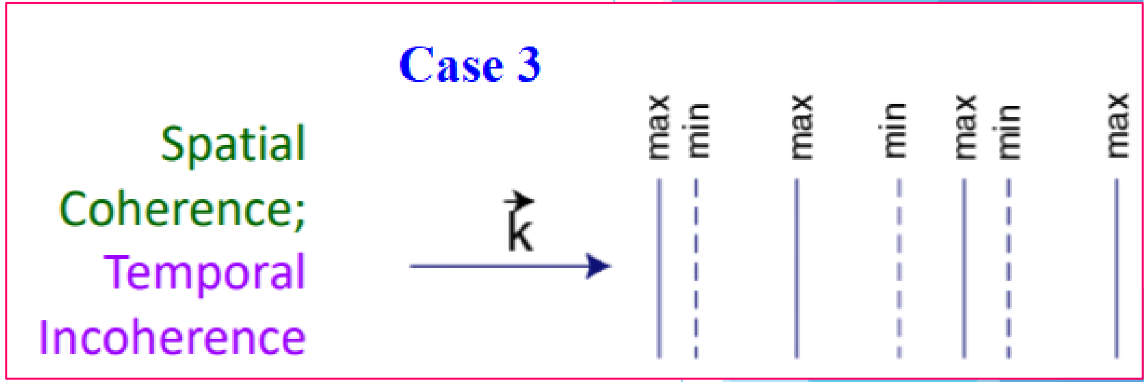


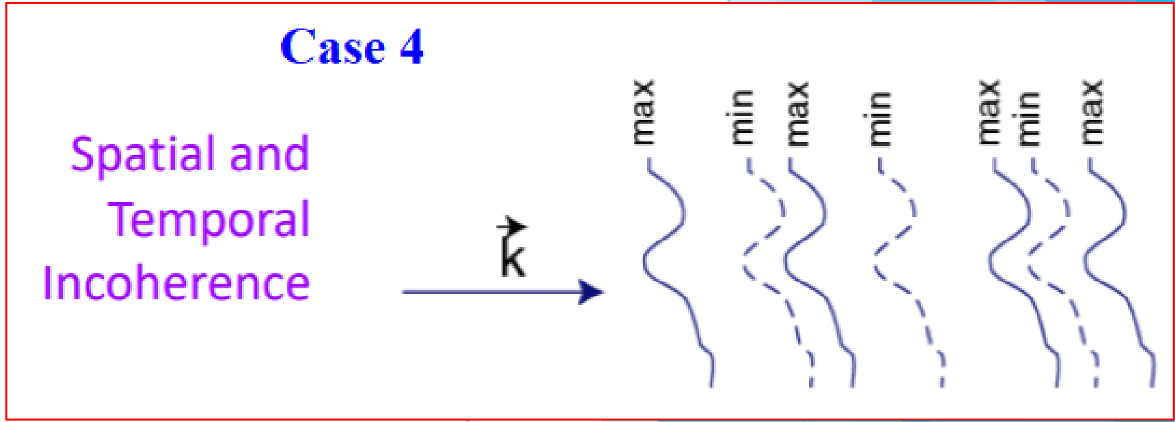


Spatial and temporal coherence









Interference *by* division of wavefronts

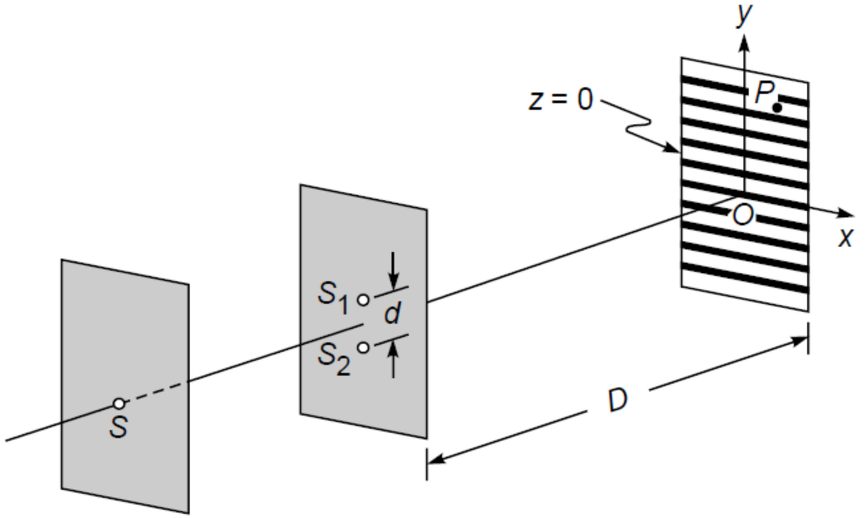
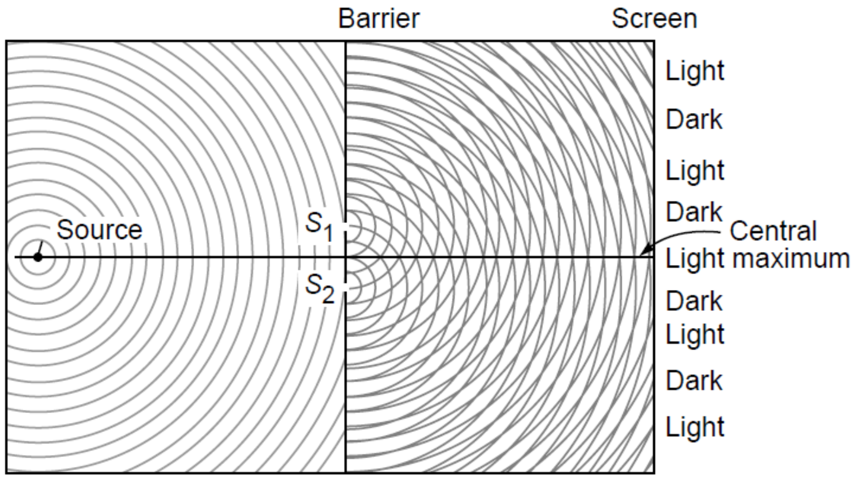


Fig. 14.6 Young's arrangement to produce interference pattern.



$(r_1 - r_2) = m\,\lambda$ **Maxima**

$= (m+\frac{1}{2})\lambda$ **Minima**

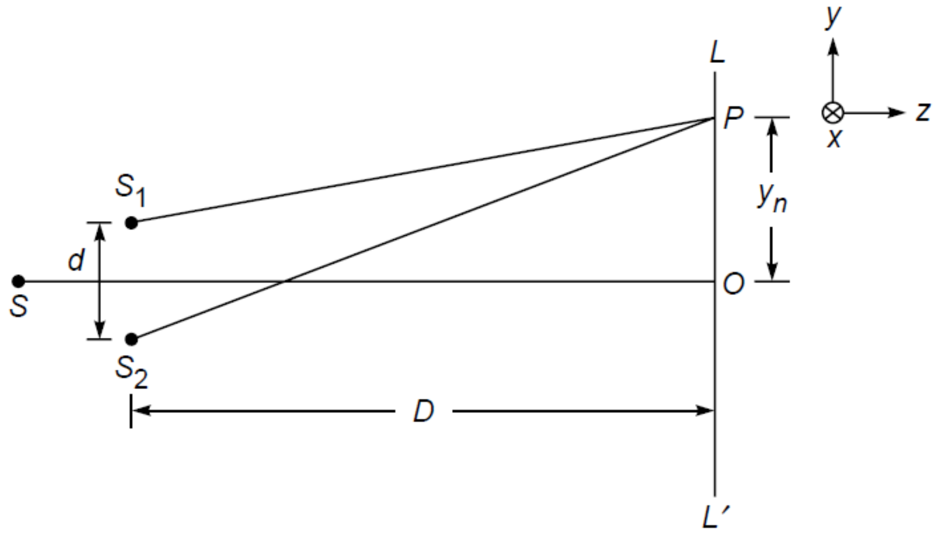


Fig. 14.8 Arrangement for producing Young's interference pattern.

Optics, by A. Ghatak

Young's double slit experiment

For an arbitrary point P (on line LL') to correspond to a maximum, we must have

$$S_2P - S_1P = n\lambda \qquad n = 0, 1, 2, \ldots$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y_n - \frac{d}{2}\right)^2\right] = 2y_n d$$

$$\boxed{y_n = \frac{n\lambda D}{d}}$$

$$S_1S_2 = d \qquad \text{and} \qquad OP = y_n$$

Thus

$$S_2P - S_1P = \frac{2y_n d}{S_2P + S_1P}$$

If y_n, $d << D$,
$S_2P + S_1P \approx 2D$

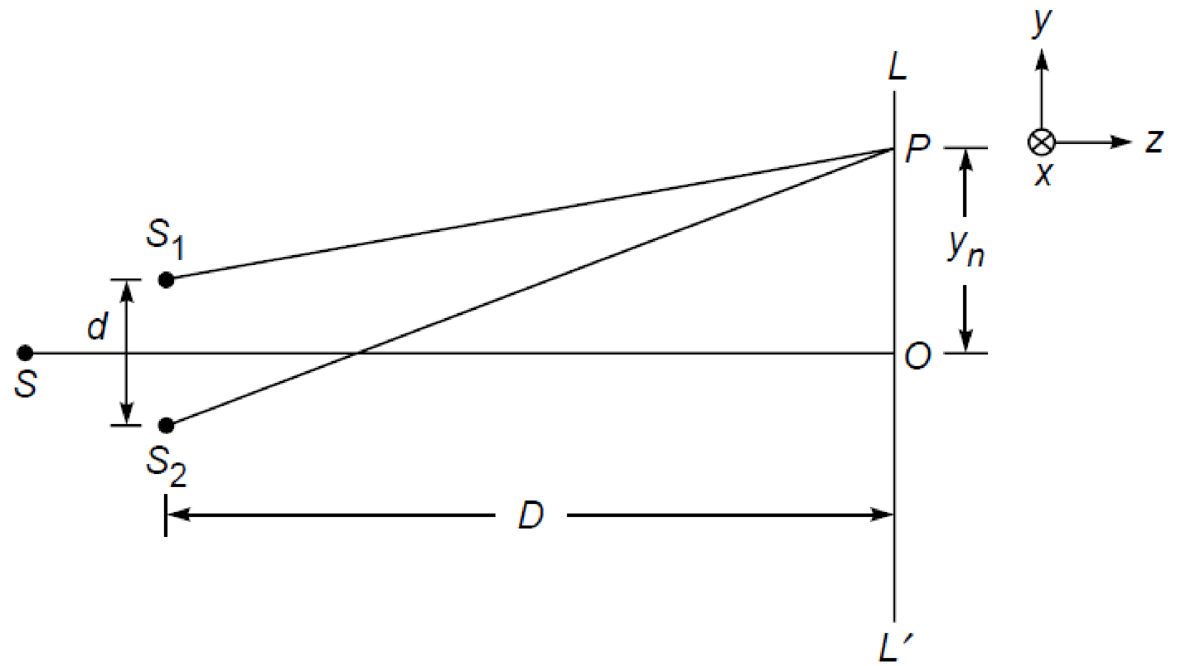


Fig. 14.8 Arrangement for producing Young's interference pattern.

distance between two consecutive bright fringes

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

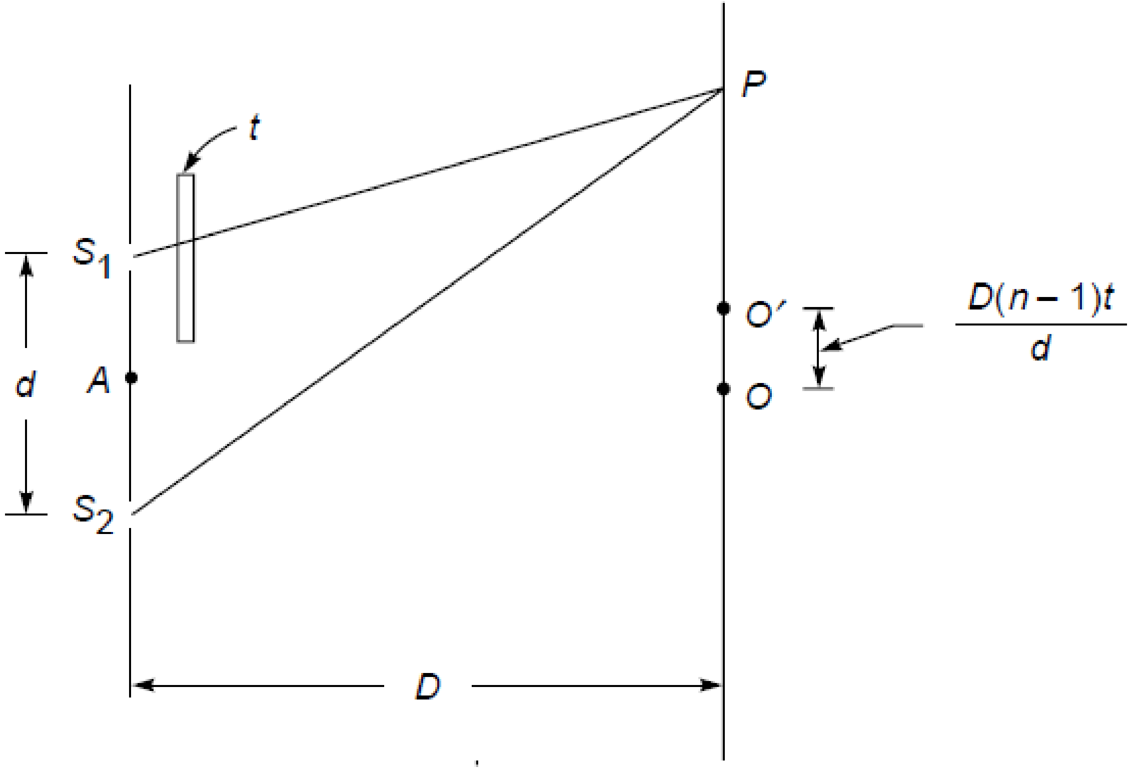
fringe width $\beta = \dfrac{\lambda D}{d}$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

Optics, by A. Ghatak

Displacement of fringes



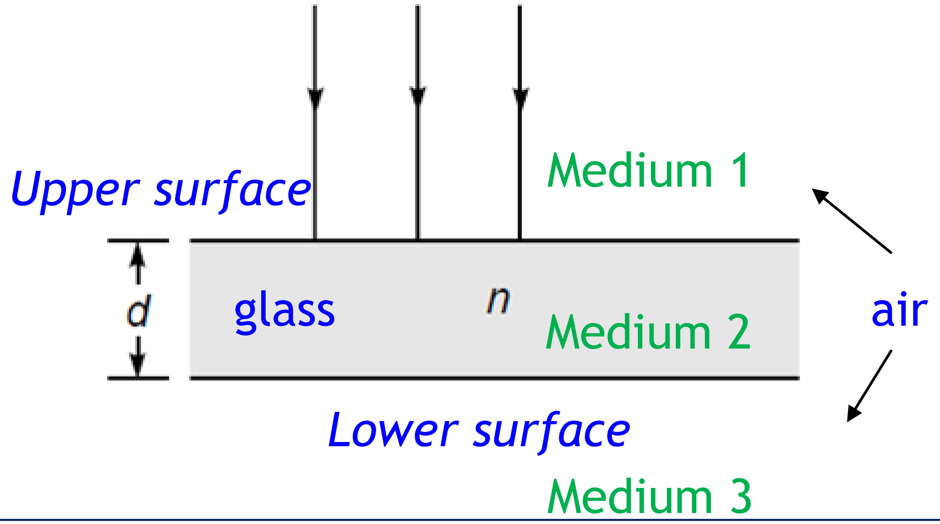
If a thin transparent sheet (of thickness t) is introduced in one of the beams, the fringe pattern gets shifted by a distance $(n - 1)tD/d$.

Optics, by A. Ghatak

Problem:
one finds that by introducing the mica sheet the central fringe occupies the position that was originally occupied by the eleventh bright fringe. If the source of light is a sodium lamp (λ = 5893 Å), determine the thickness of the mica sheet.

Solution:
The point O' corresponds to the eleventh bright fringe, thus
$S2O' - S1O' = 11\lambda = (n - 1)t = 0.58t$

Amplitude-Splitting (normal incidence)



Earlier

$(r_1 - r_2)$ or $S_2P-S_1P = m\lambda$ **Maxima**

$=(m+\frac{1}{2})\lambda$ **Minima**

$$2nd = m\lambda \qquad \text{destructive interference} \qquad \textbf{(1a)}$$

$$= \left(m+\tfrac{1}{2}\right)\lambda \qquad \text{constructive interference} \qquad \textbf{(1b)}$$

Now

where $m = 0, 1, 2, \ldots$ and λ represents the free space wavelength.

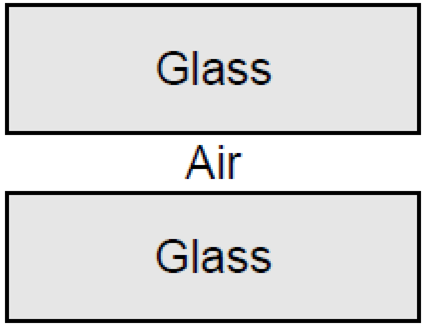
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

Because of additional π phase change due to reflection from denser medium

Optics, by Hecht; Ghatak

Interference conditions?

Ex-1



Thin film of air formed between two glass plates.

$$2nd = m\lambda \qquad \text{destructive interference} \qquad \textbf{(1a)}$$

$$= \left(m + \tfrac{1}{2}\right)\lambda \qquad \text{constructive interference} \qquad \textbf{(1b)}$$

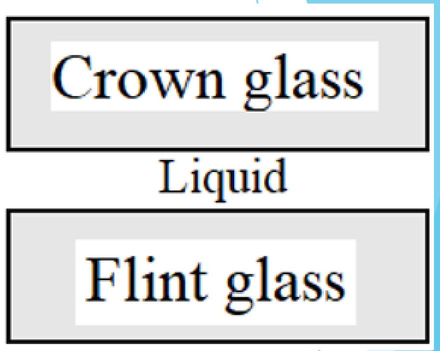
where $m = 0, 1, 2, \ldots$ and λ represents the free space wavelength.

Ex-2

$n1 = 1.52$

$n2 = 1.60$

$n3 = 1.66$

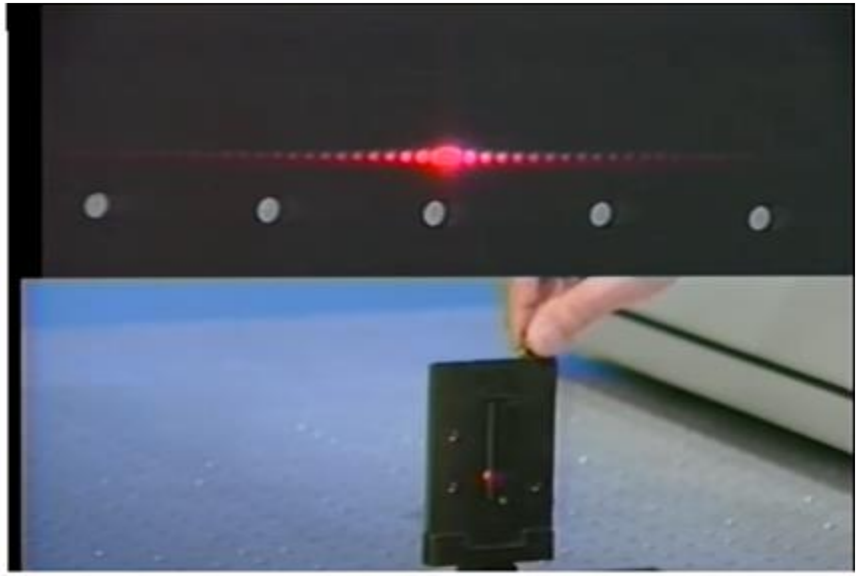


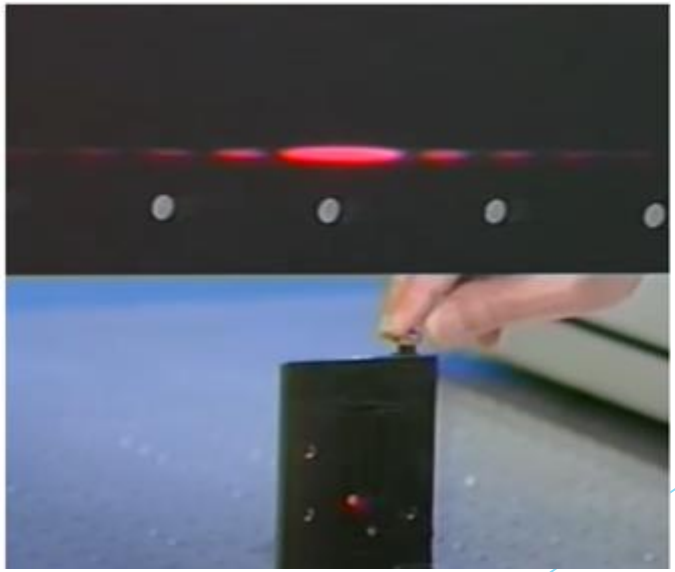
$2nd = m\lambda$ Maxima

$= (m+\frac{1}{2})\lambda$ Minima

Optics, Hecht; Ghatak

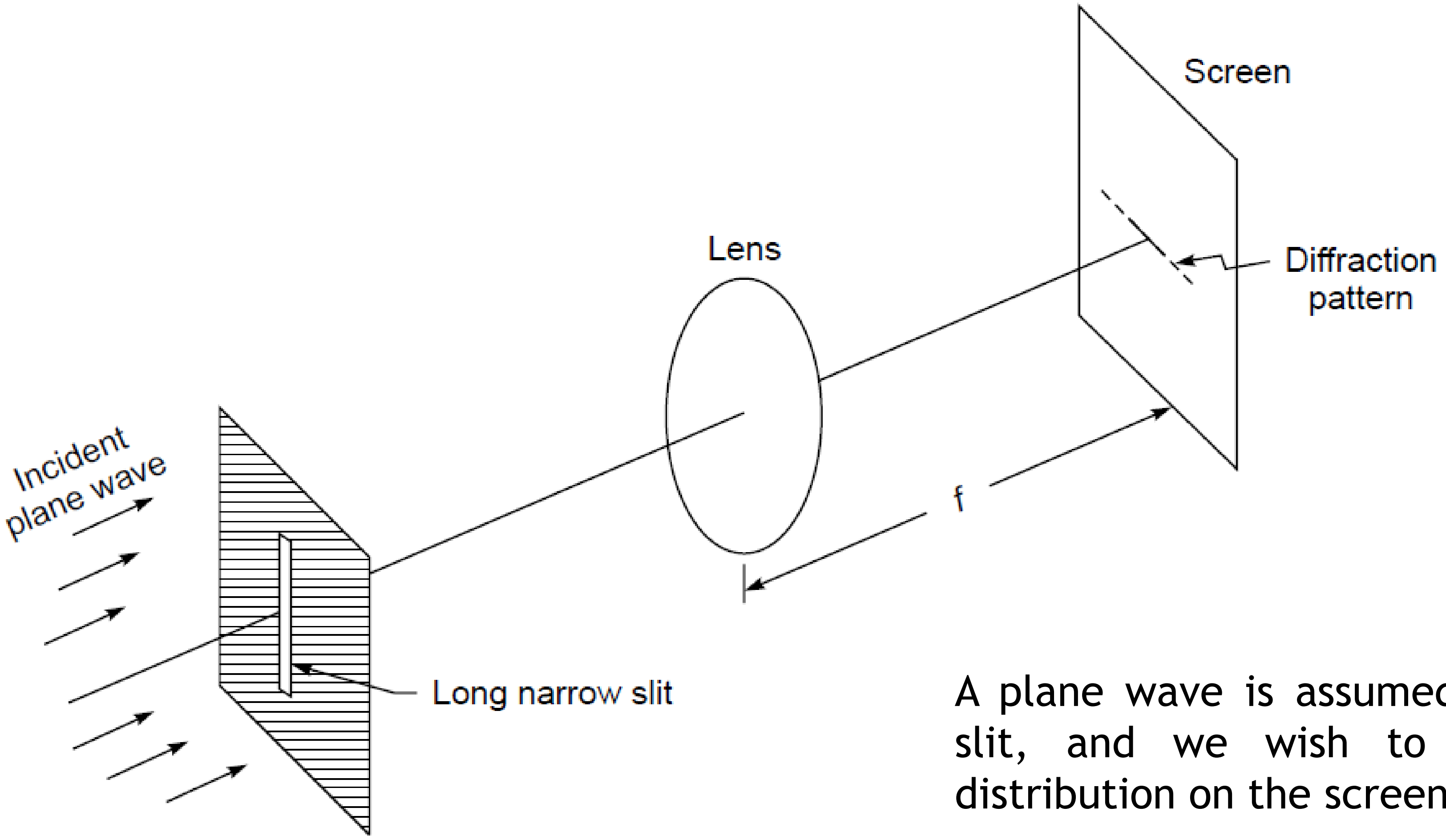
Single slit diffraction





https://ocw.mit.edu/resources/res-6-006-video-demonstrations-in-lasers-and-optics-spring-2008/demonstrations-in-physical-optics/fraunhofer-diffraction-2014-adjustable-slit/

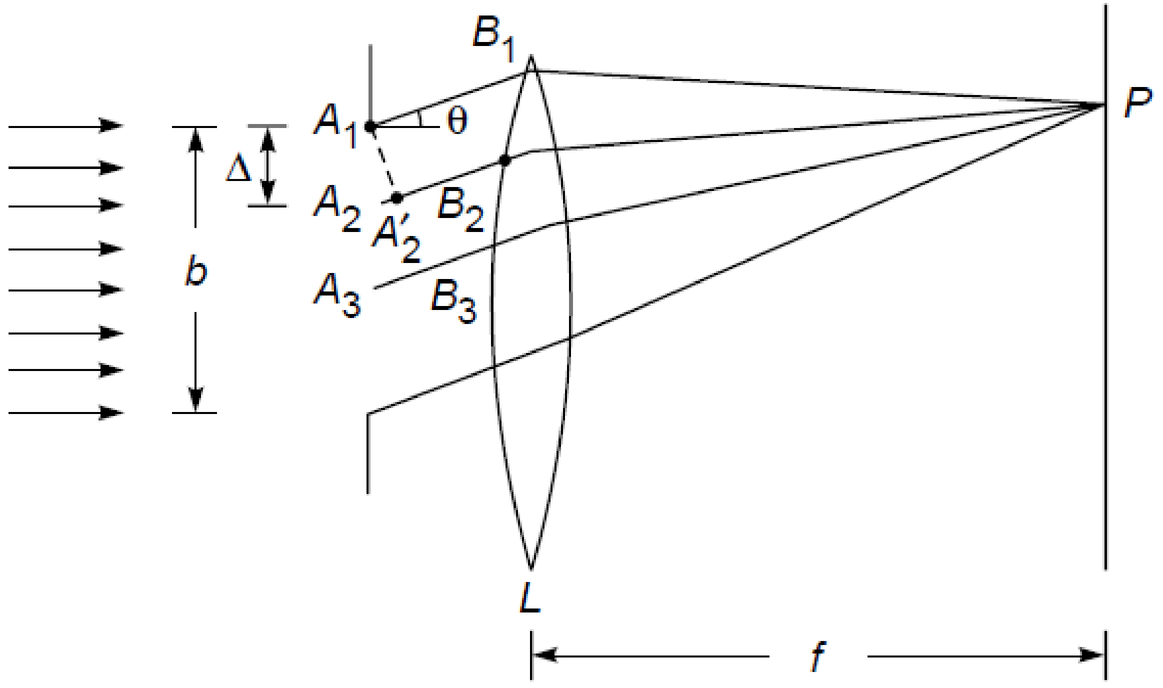
Single slit diffraction: Intensity distribution



A plane wave is assumed to fall normally on the slit, and we wish to calculate the intensity distribution on the screen

Optics, Ghatak

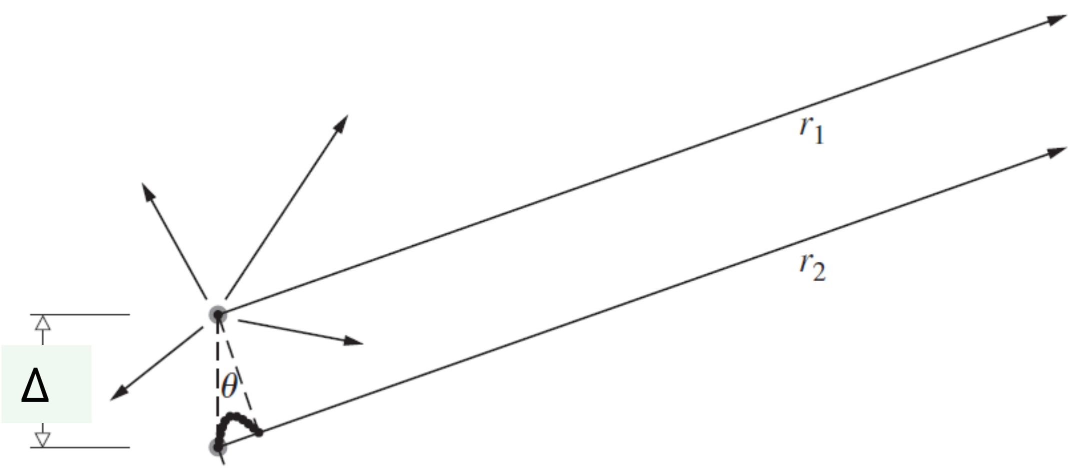
Single slit diffraction: Intensity distribution



- slit → large number of equally spaced point sources
- each point → source of Huygens' secondary wavelets
- 2ndary wavelets interfere
- A_1, A_2, A_3, . . ., → point sources
- Distance between two consecutive points → Δ
- number of point sources = n
- b = (n-1) Δ

Resultant field produced by these *n* sources at an arbitrary point P ?

Intensity distribution continued



- At P: $A_1 \approx A_2$; distance to P >> b
- slightly different path lengths → path diff → phase diff
- A_2A_2' → extra path; $A_1B_1P = A_2'B_2P$
- Path diff. $A_2A_2' = \Delta \sin\theta$
- Phase diff. $\varphi = k\, A_2A_2' = (2\pi/\lambda)\, \Delta \sin\theta$

$$E = a[\cos \omega t + \cos(\omega t - \varphi) + \ldots + \cos[(\omega t - (n-1)\varphi)]$$

$$\mathbf{E = E_0 \cos[(\omega t - \tfrac{1}{2}(n-1)\varphi)]}$$

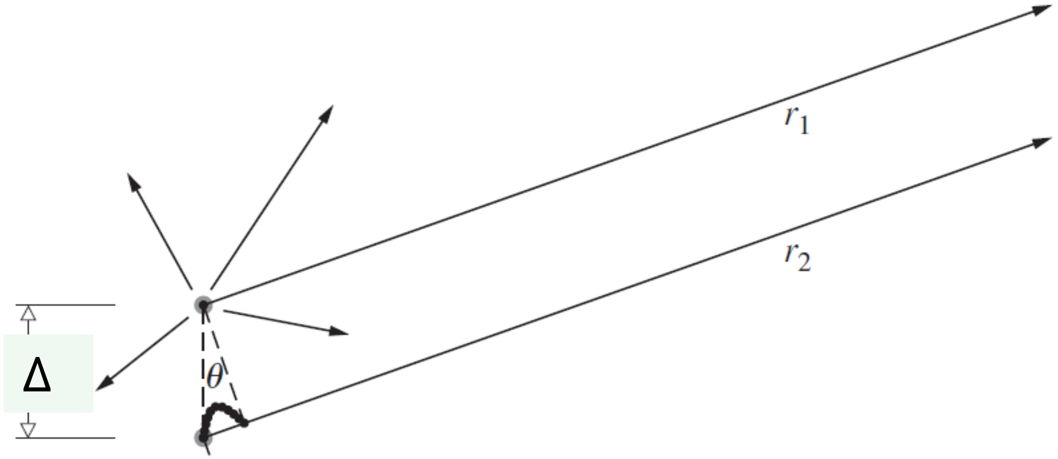
Where $$E_0 = a\frac{\sin(n\phi/2)}{\sin(\phi/2)}$$

if n → ∞ *and* Δ → *0*

Then n Δ → b

Optics, Hecht, Ghatak

Intensity distribution continued



$$E = a[\cos \omega t + \cos(\omega t - \varphi) + \ldots + \cos[(\omega t - (n-1)\varphi)]$$

$$E = E_0 \cos[(\omega t - \tfrac{1}{2}(n-1)\varphi)]$$

Where $$E_0 = a\frac{\sin(n\phi/2)}{\sin(\phi/2)}$$

if $n \to \infty$ *and* $\Delta \to 0$

Then $n\Delta \to b$

$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta = \frac{2\pi}{\lambda}\frac{b\sin\theta}{n}$$

$$\frac{n\phi}{2} = \frac{\pi}{\lambda}n\Delta\sin\theta \to \frac{\pi}{\lambda}b\sin\theta$$

$$\boxed{E_0 = A\frac{\sin\beta}{\beta}} \quad A = na \quad \beta = \frac{\pi b \sin\theta}{\lambda}$$

Amplitude of the resultant wave

Intensity? $\longrightarrow$ $$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)}$$

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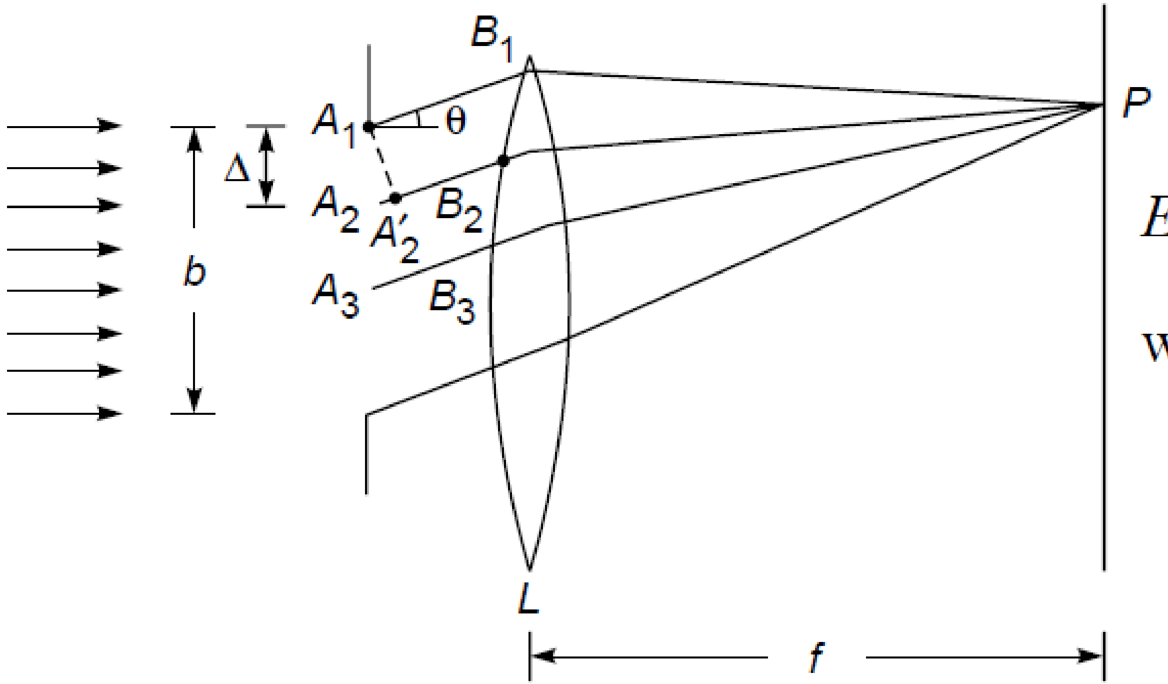
Single slit diffraction: Intensity distribution



$$E = a[\cos \omega t + \cos (\omega t - \phi) + \cdots + \cos [(\omega t - (n-1)\phi)]$$

where $\phi = \dfrac{2\pi}{\lambda} \Delta \sin \theta$

$$E = E_0 \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right]$$

$$E_0 = a\,\frac{\sin (n\phi/2)}{\sin (\phi/2)}$$

$n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to b$,

$$\boxed{E = A\,\frac{\sin \beta}{\beta} \cos (\omega t - \beta)}$$

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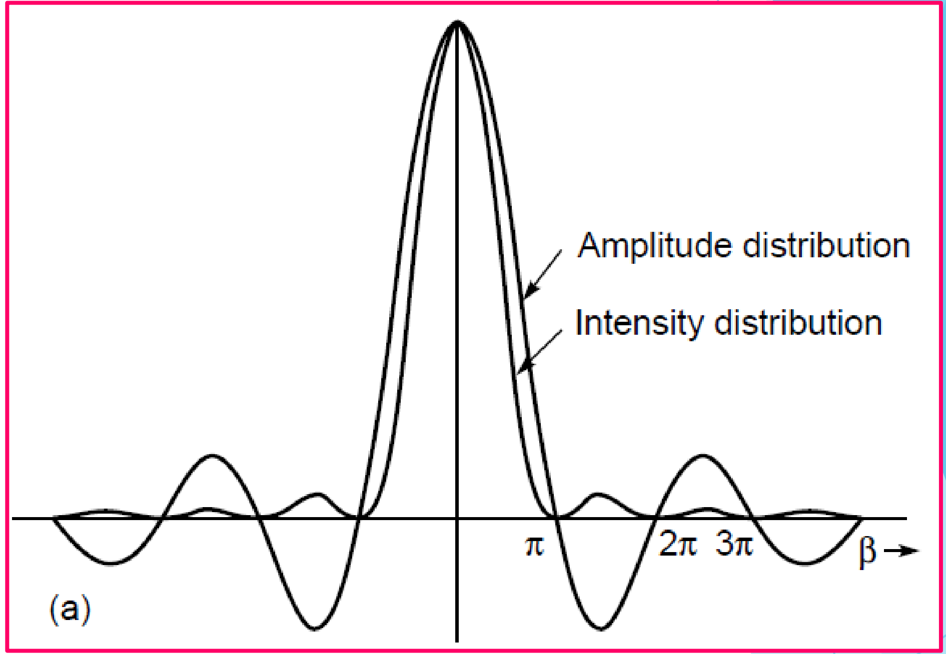
Single slit diffraction continued

$$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)} \quad \textbf{(1)}$$

$$A = na \quad \beta = \frac{\pi b\sin\theta}{\lambda} \quad \textbf{(2)}$$

$$\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}} \quad \textbf{(3)}$$



When is the intensity = 0 ?
Max.?

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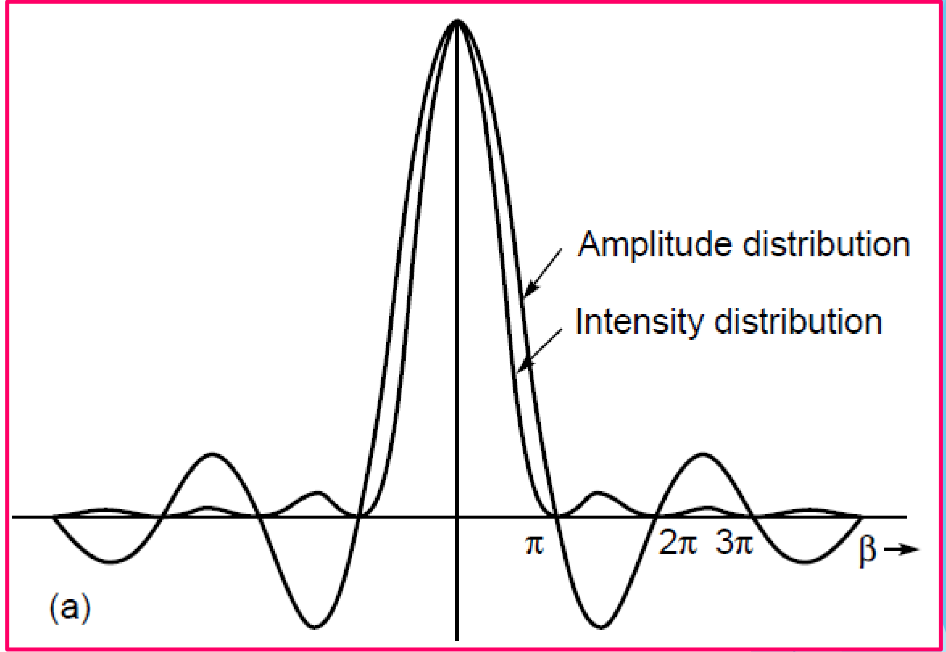
Single slit diffraction continued

$$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)} \quad \textbf{(1)}$$

$$A = na \qquad \beta = \frac{\pi b\sin\theta}{\lambda} \qquad \textbf{(2)}$$

$$\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}} \qquad \textbf{(3)}$$



Intensity = 0 if $\quad \beta = m\pi \qquad m \neq 0 \qquad$ **(4)**

Using (4) in (2):

$$\boxed{b\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3, \ldots \text{(minima)}}$$

first minimum

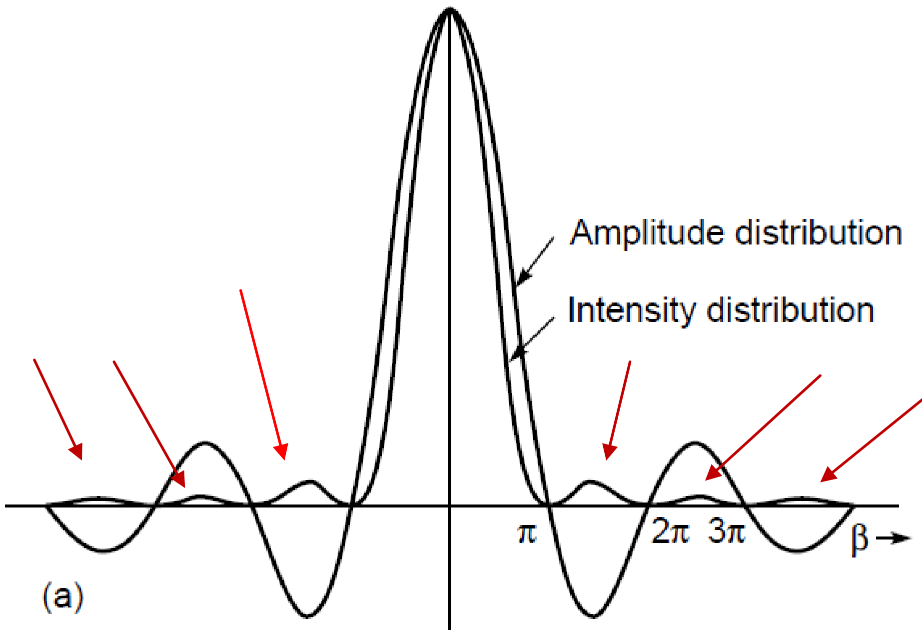
$$\theta = \pm\sin^{-1}(\lambda/b)$$

second minimum

$$\theta = \pm\sin^{-1}(2\lambda/b)$$

m closest to b/λ

What about other maxima?



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Single slit diffraction: maxima

maxima $\dfrac{dI}{d\beta} = I_0\left(\dfrac{2\sin\beta\cos\beta}{\beta^2} - \dfrac{2\sin^2\beta}{\beta^3}\right) = 0$

or $\quad \sin\beta\,(\beta - \tan\beta) = 0$

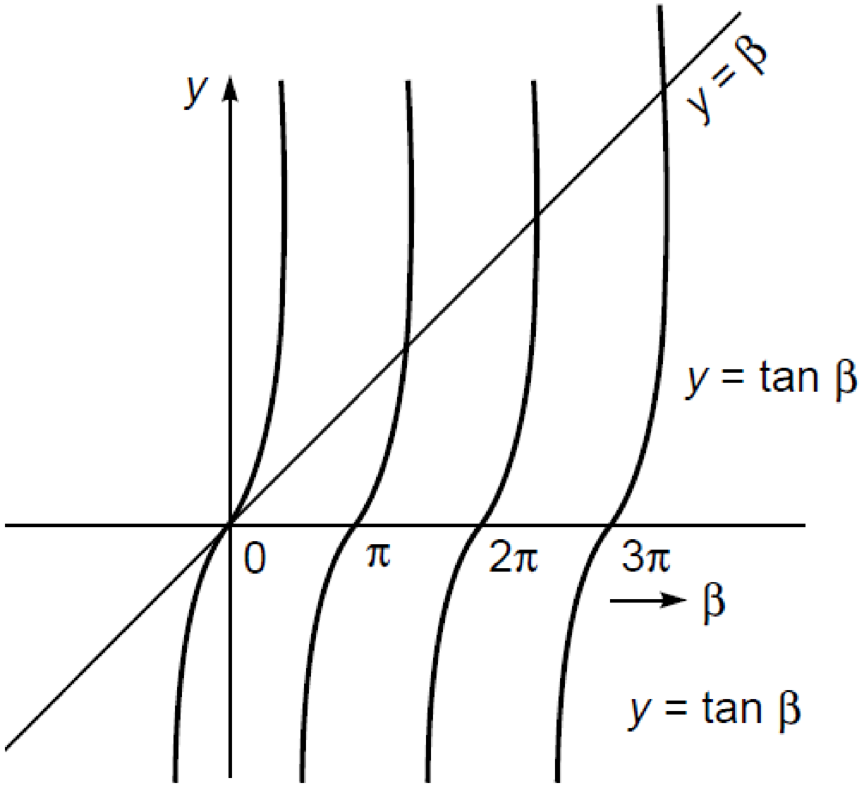
The condition $\sin\beta = 0$, or $\beta = m\pi$ $(m \neq 0)$, corresponds to minima. The conditions for maxima are roots of the equation

$$\tan\beta = \beta \qquad \text{(maxima)}$$

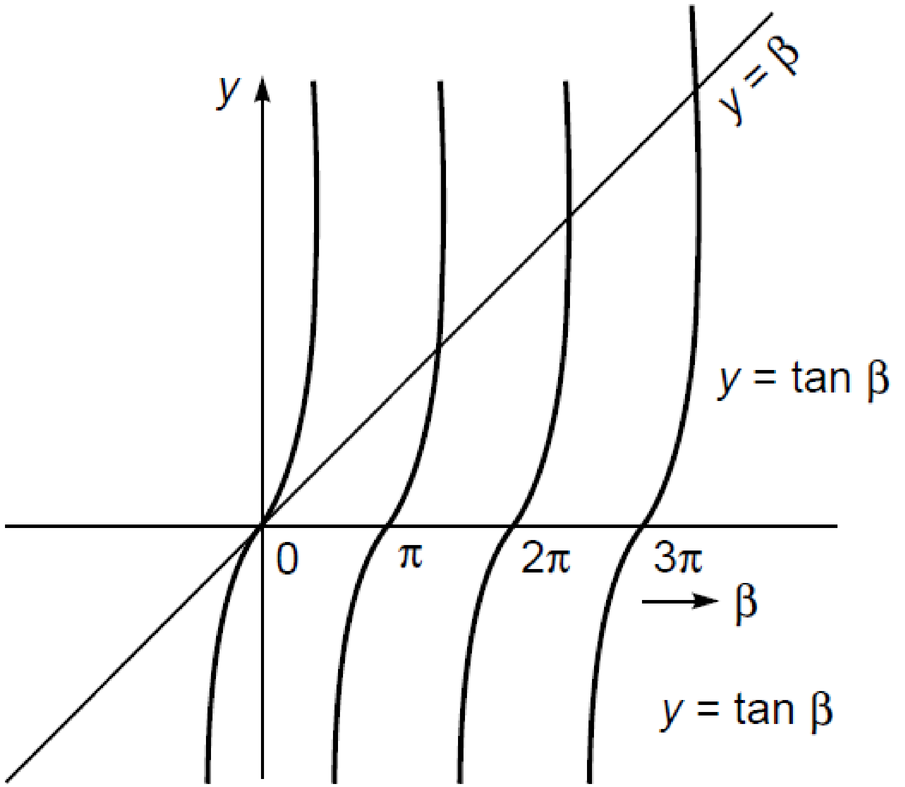
The root $\beta = 0$ corresponds to the central maximum.

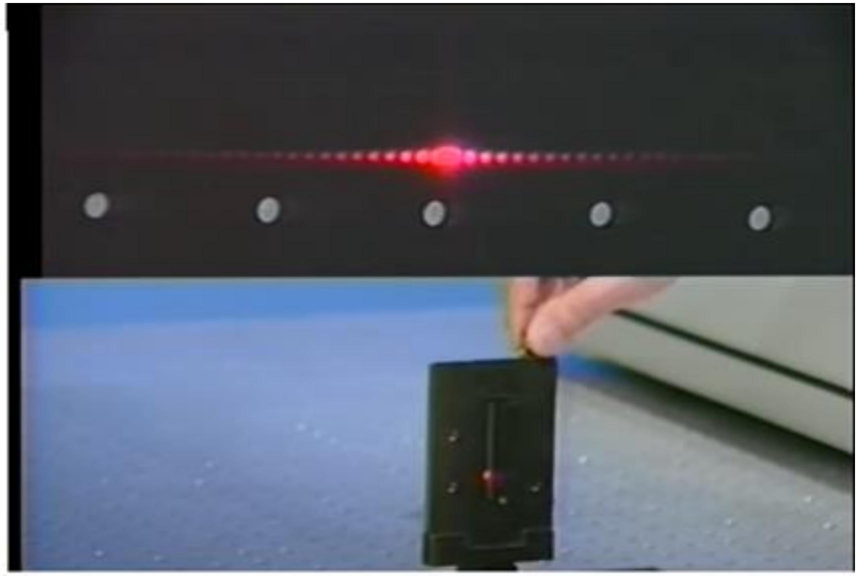
curves $y = \beta$ and $y = \tan\beta$ points of intersections

$$\beta = 1.43\pi,\ \beta = 2.46\pi$$



The central maxima is brightest!





https://ocw.mit.edu/resources/res-6-006-video-demonstrations-in-lasers-and-optics-spring-2008/demonstrations-in-physical-optics/fraunhofer-diffraction-2014-adjustable-slit/

The root $\beta = 0$ corresponds to the central maximum.

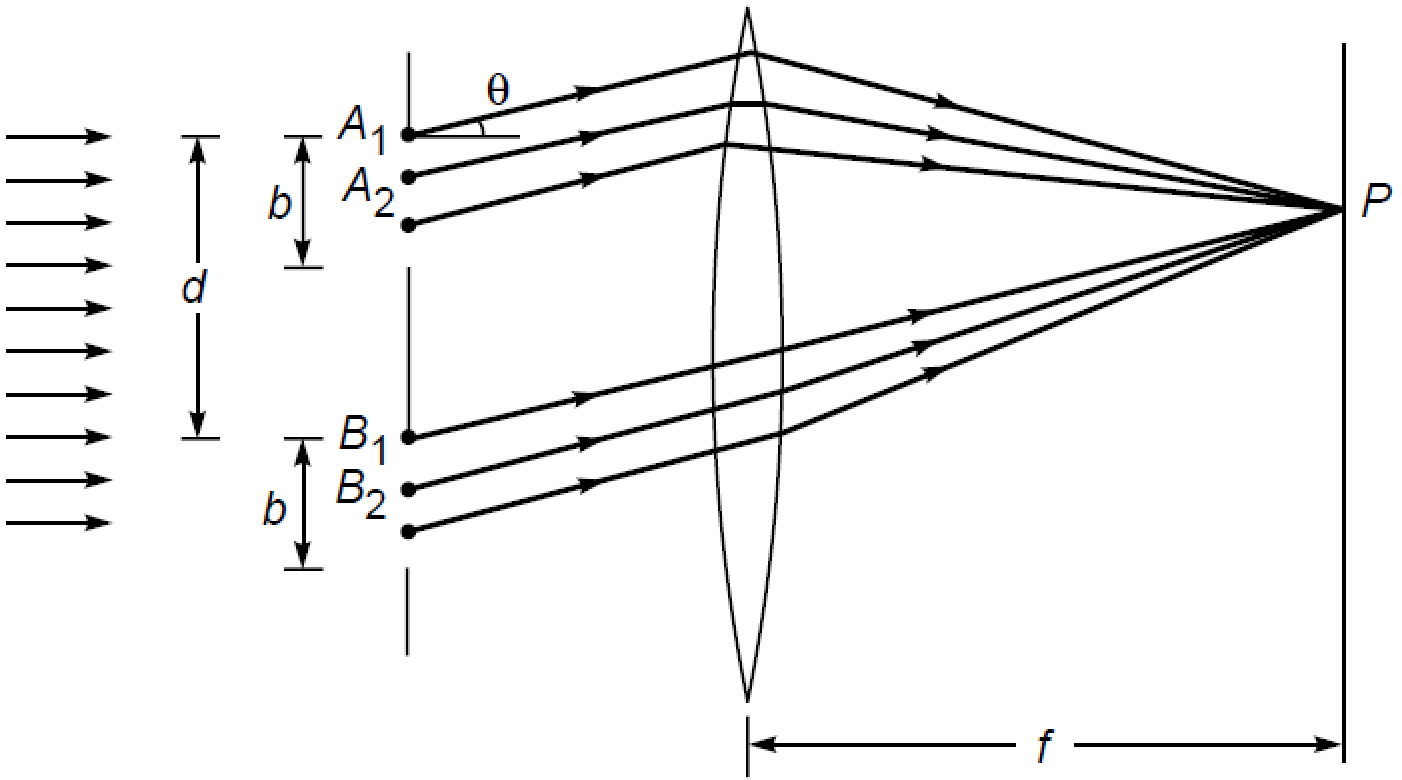
curves $y = \beta$ and $y = \tan\beta$ points of intersections

$\beta = 1.43\pi$, $\beta = 2.46\pi$,

1st maximum→ $\left(\dfrac{\sin 1.43\pi}{1.43\pi}\right)^2$

Check→ Utah State University by Professor Boyd F. Edwards
https://www.youtube.com/watch?v=uohd0TtqOaw

Double slit diffraction



Fraunhofer diffraction of a plane wave incident normally on a double slit.

Distance between two consecutive points in either of the slits is Δ

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Similarly, the second slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

at point P, where

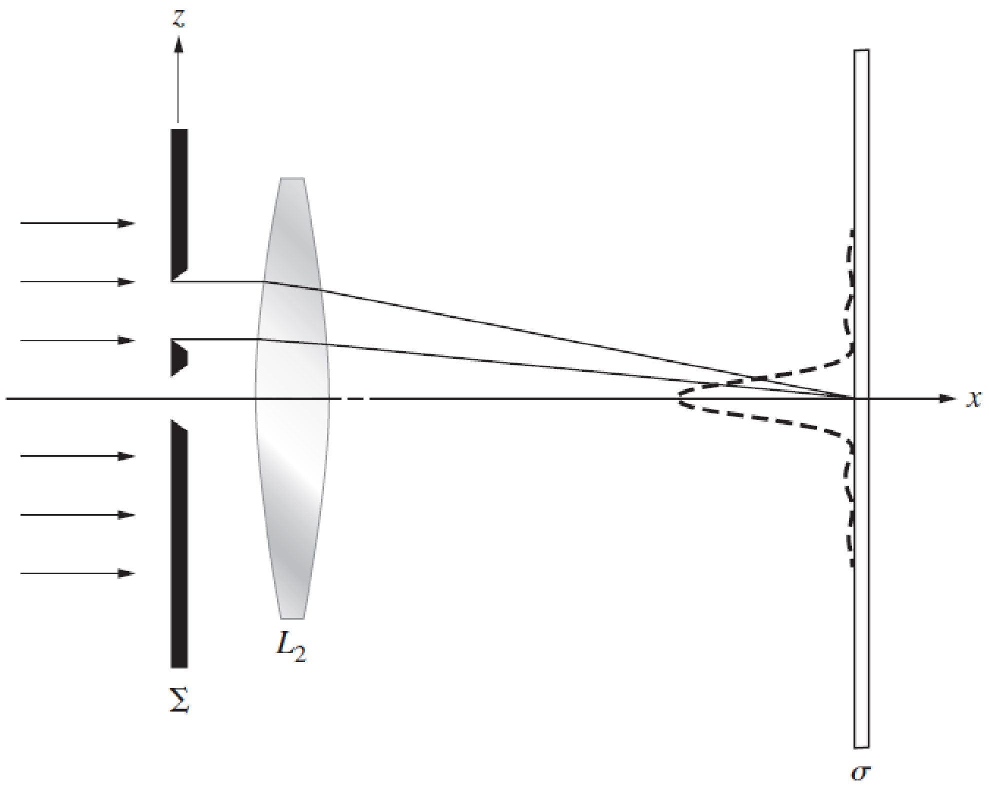
$$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

Summary of our discussion so far on Diffraction

- Double slit diffraction:

$$I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$$\boxed{\beta = \frac{\pi b \sin\theta}{\lambda}} \quad \boxed{\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta}$$



Minima

$$b \sin\theta = m\lambda \qquad m = 1, 2, 3, \ldots$$

$$d \sin\theta = \left(n + \tfrac{1}{2}\right)\lambda \qquad n = 0, 2, 3, \ldots$$

Maxima

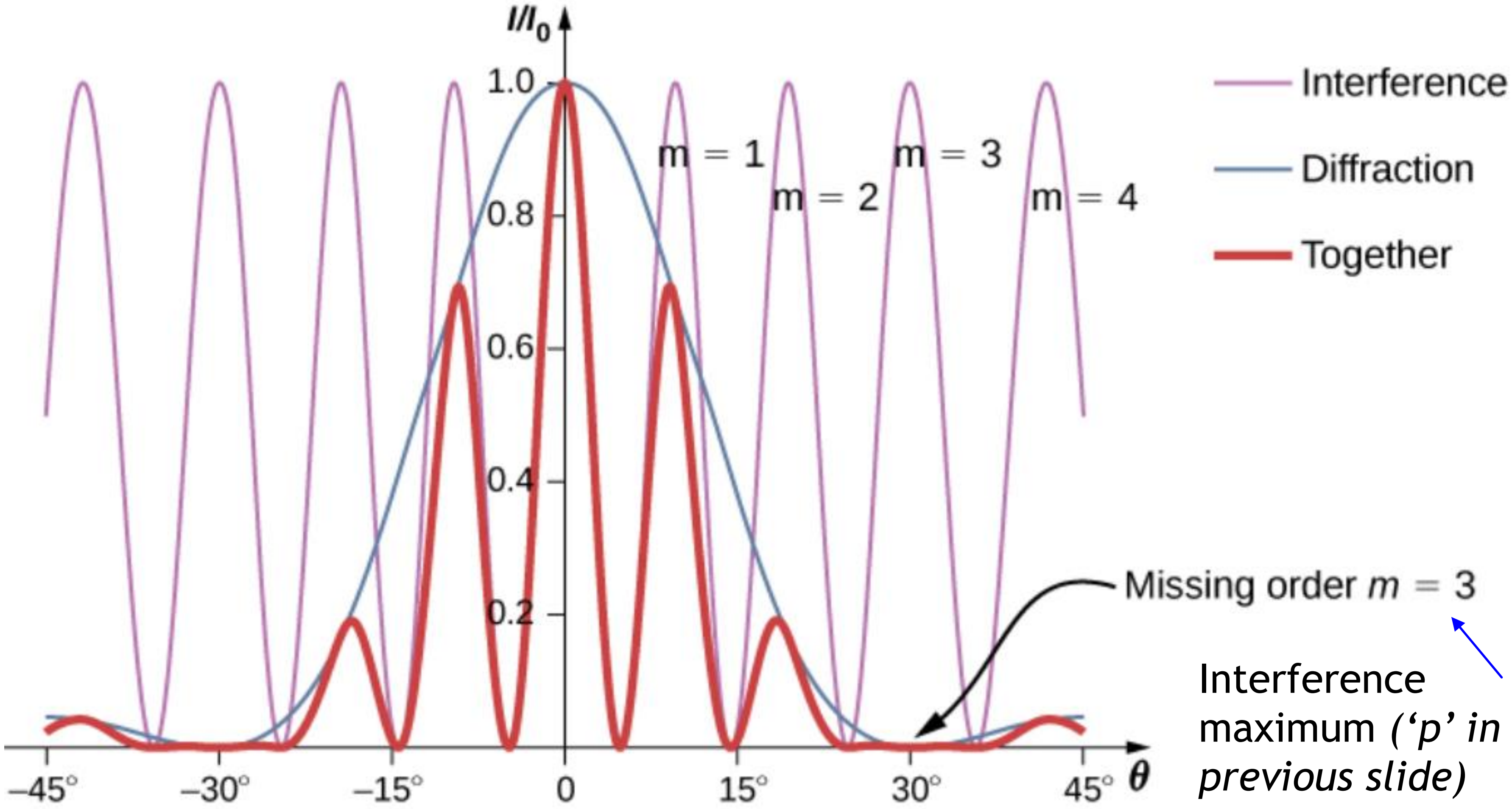
$$\tan\beta = \beta \qquad \text{(maxima)}$$

$$d \sin\theta = p\lambda$$

Problem-1

A parallel beam of light (λ = 5 x 10^{-5} cm) is incident normally on a long narrow slit of width 0.1 mm. A screen is placed at a distance of 5 m from the slit. Calculate the total width of the central maximum.

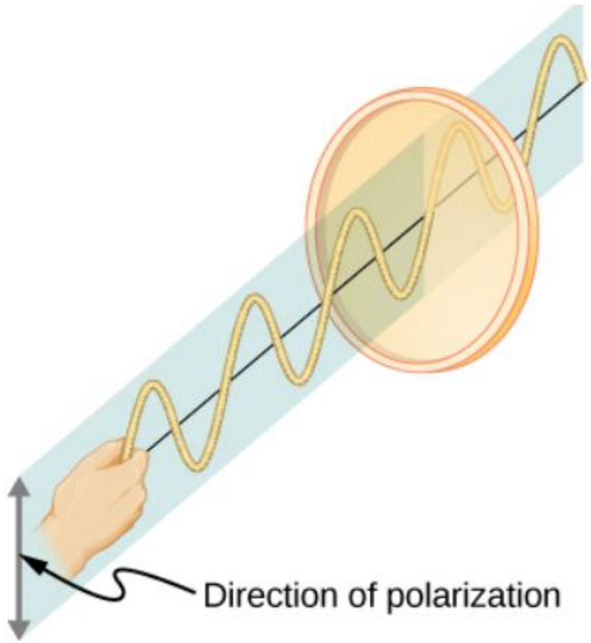
Double slit diffraction pattern



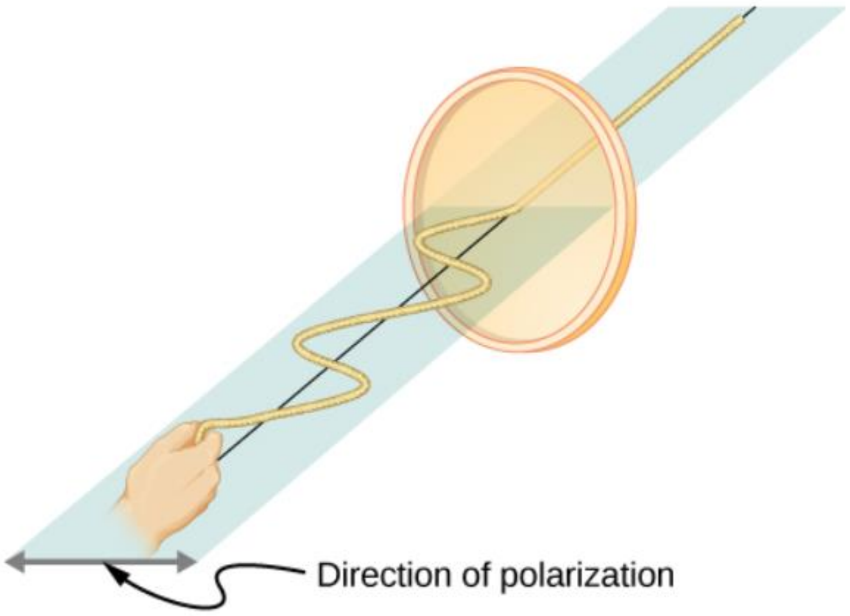
Interference maximum *('p' in previous slide)*

https://opentextbc.ca/universityphysicsv3openstax/chapter/double-slit-diffraction/

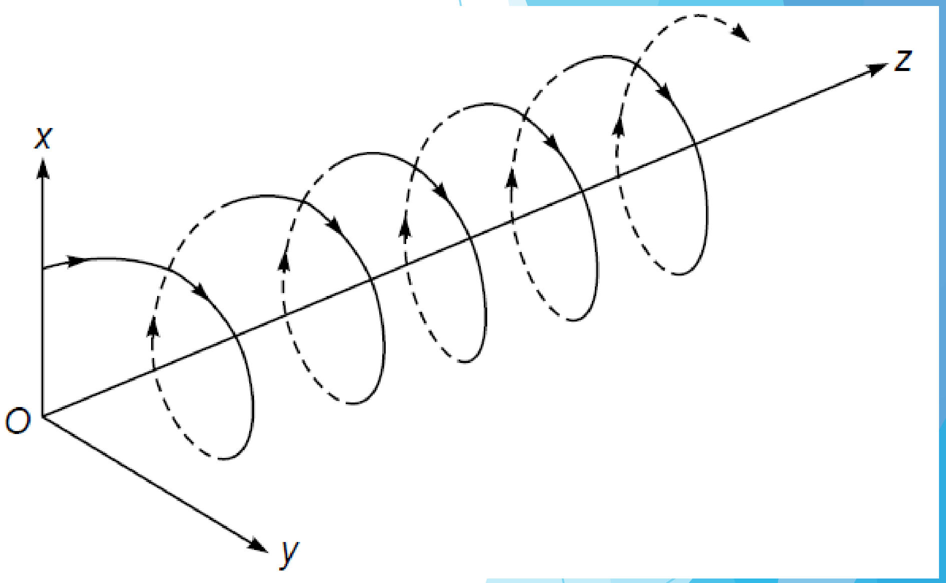
Polarized waves



$$x(z,t) = a\cos(kz - \omega t + \phi_1)$$
$$y(z,t) = 0$$



$$y(z,t) = a\cos(kz - \omega t + \phi_2)$$
$$x(z,t) = 0$$

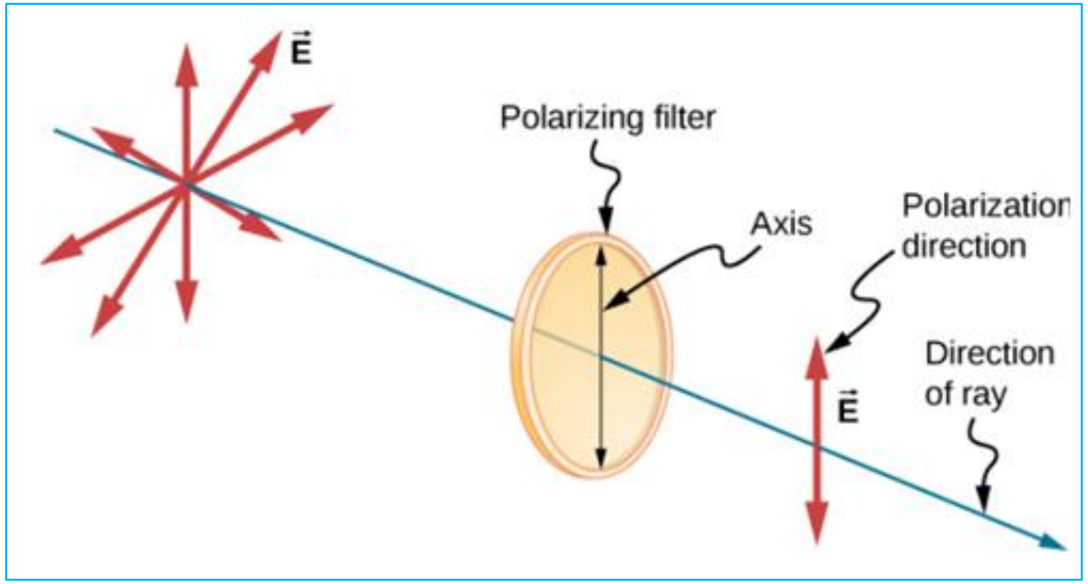


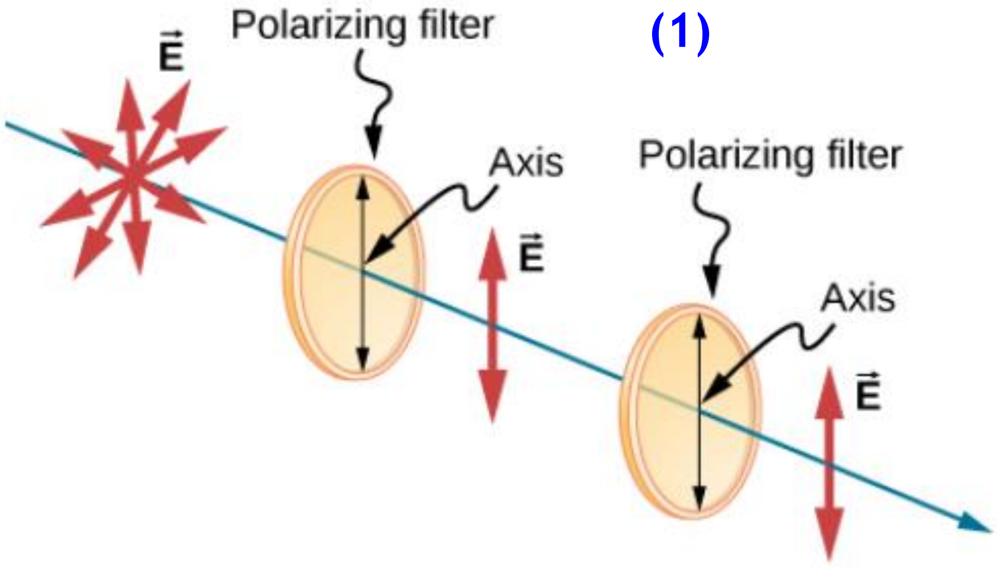
$$x(z,t) = a\cos(kz - \omega t + \phi)$$
$$y(z,t) = a\sin(kz - \omega t + \phi)$$

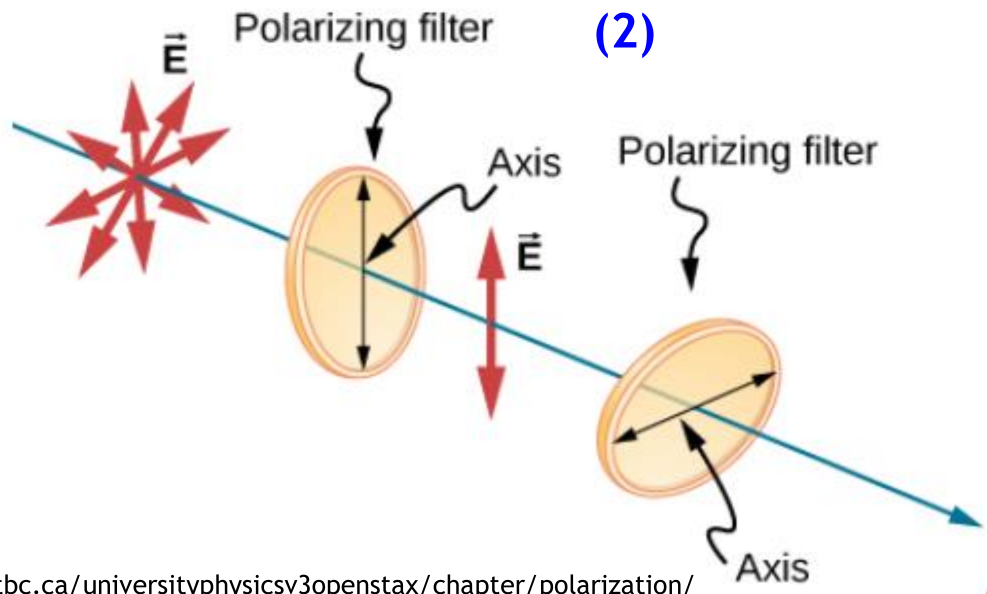
$$\boxed{x^2 + y^2 = a^2}$$

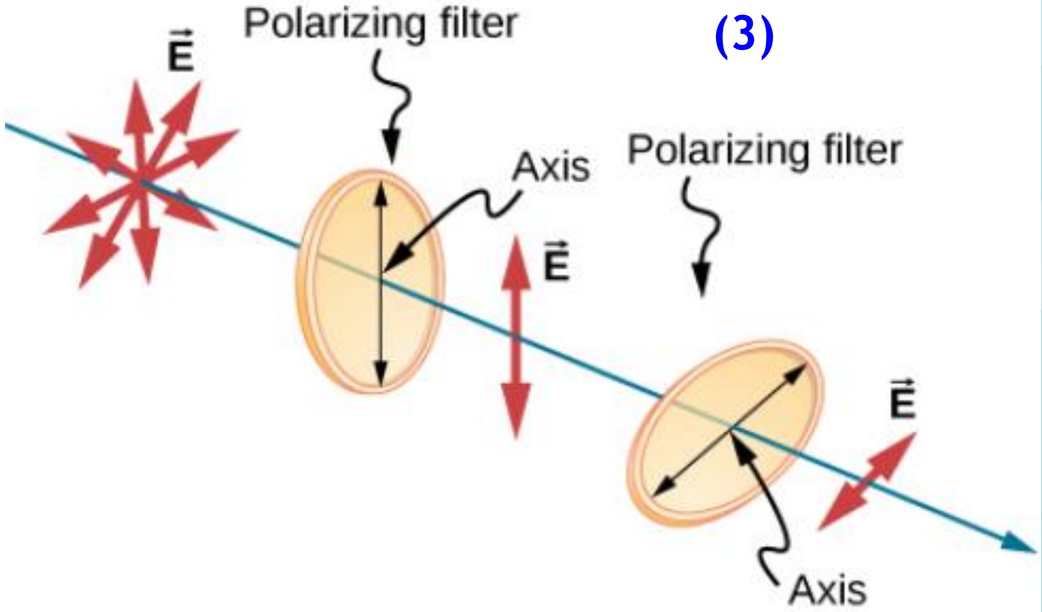
https://opentextbc.ca/universityphysicsv3openstax/chapter/polarization/

Polarization of light









https://opentextbc.ca/universityphysicsv3openstax/chapter/polarization/

Malus' Law

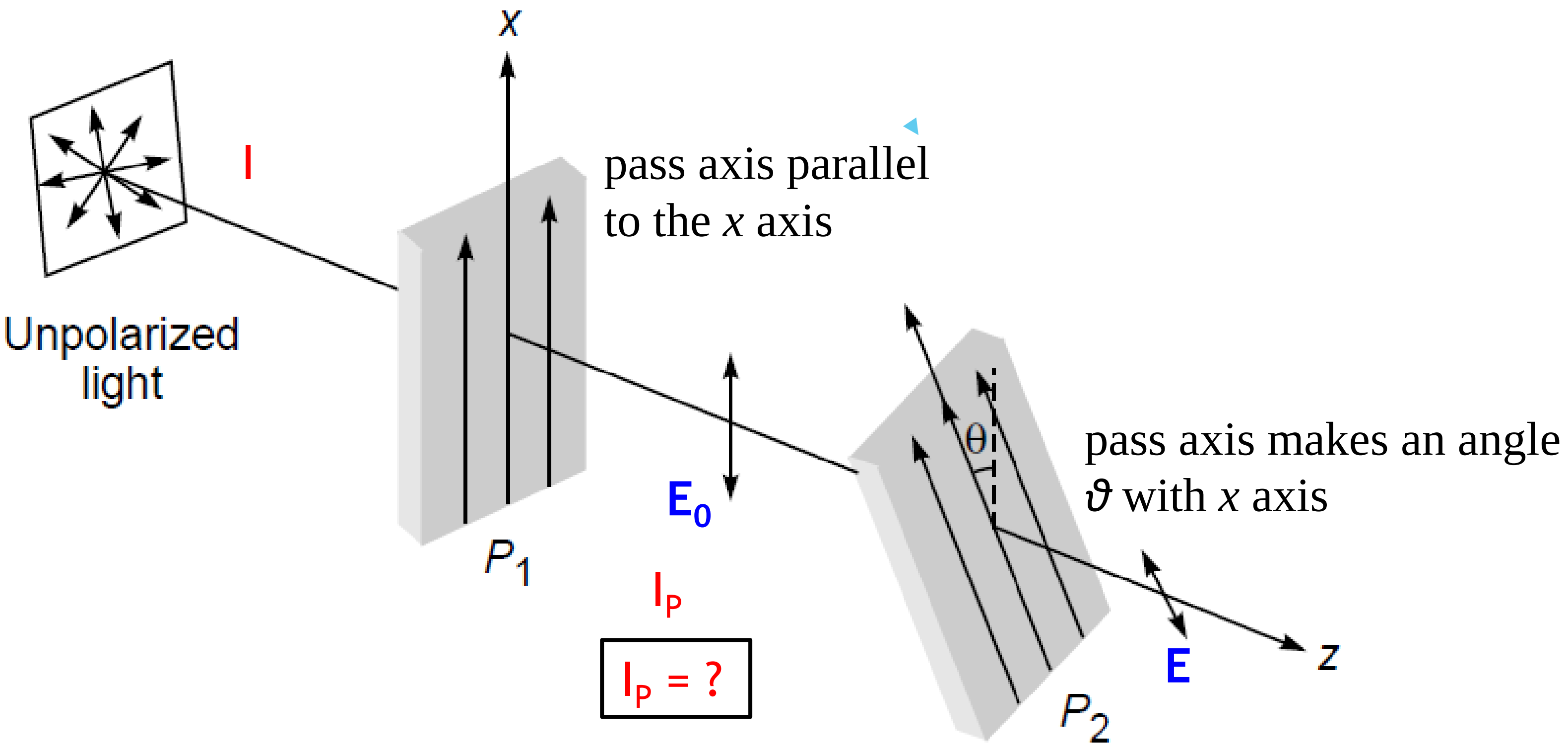


Fig. 22.15 An unpolarized light beam gets x-polarized after passing through the polaroid P_1, the pass axis of the second polaroid P_2 makes an angle θ with the x axis. The intensity of the emerging beam will vary as $\cos^2 \theta$.

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Amplitude

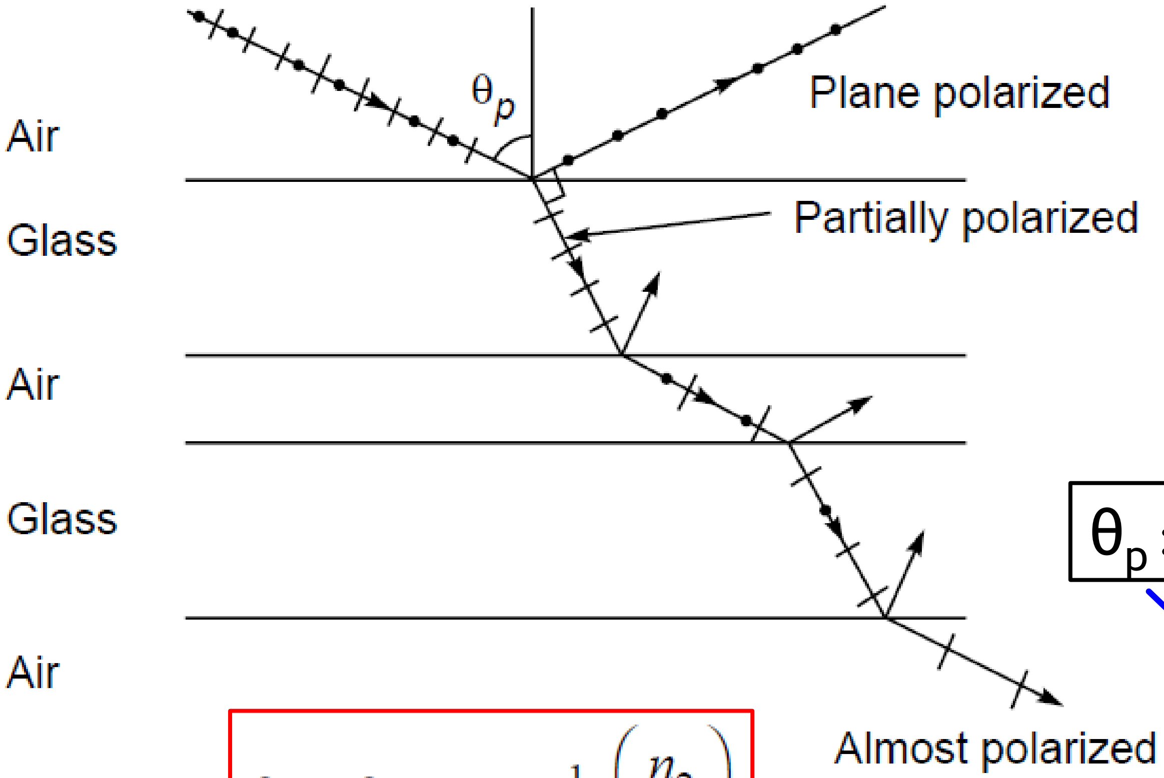
$E = E_0 \cos\theta$

Intensity

$$\boxed{I = I_0 \cos^2 \theta}$$

Malus' Law

Brewster's law



$$\boxed{\theta = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$$

Brewster's law

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If an unpolarized beam is incident with an angle of incidence equal to θ_p, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence. The transmitted beam is partially polarized, and if this beam is made to undergo several reflections, then the emergent beam is almost plane polarized with its electric vector in the plane of incidence.

θ_p : reflected and transmitted rays are at right angles

polarizing angle or the ***Brewster angle***

For the air-glass interface, $n_1 = 1$ and $n_2 \approx 1.5$, giving $\theta_p \approx 57^0$.

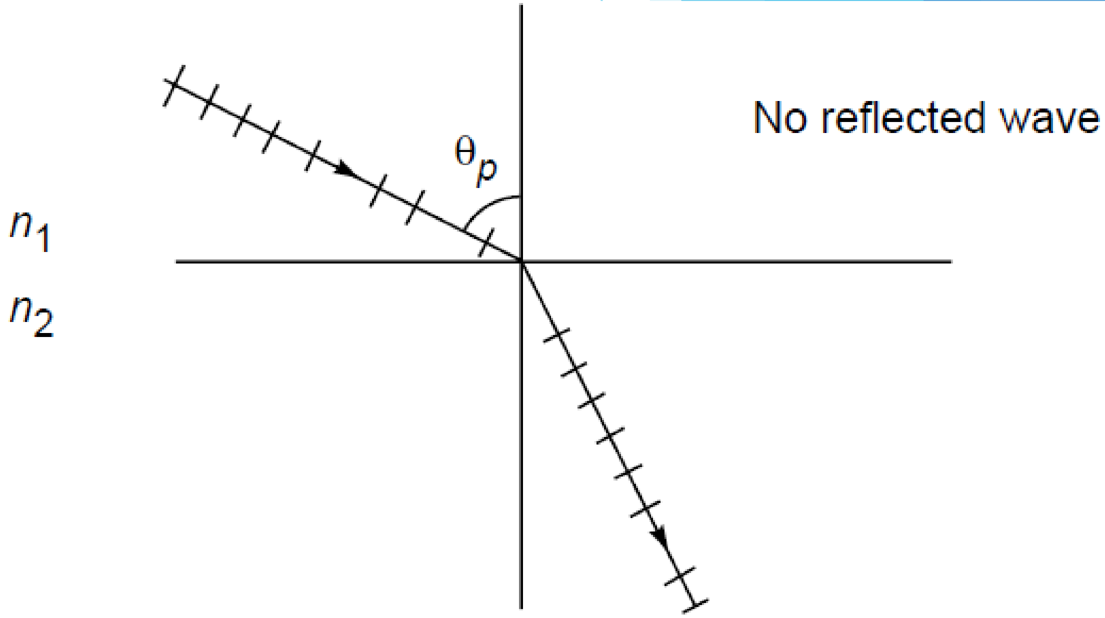
For the air-water interface, $n_1 \approx 1$ and $n_2 \approx 1.33$ and the polarizing angle $\theta_p \approx 53°$.

Polarization by Reflection

Let us consider the incidence of a plane wave on a dielectric. We assume that the electric vector associated with the incident wave lies in the plane of incidence as shown in Fig. 22.9. It will be shown in Sec. 24.2 that if the angle of incidence θ is such that

$$\theta = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

then the reflection coefficient is zero.



If a linearly polarized wave (with its **E** in the plane of incidence) is incident on the interface of two dielectrics with the angle of incidence equal to θ_p [$= \tan^{-1}(n_2/n_1)$], then the reflection coefficient is zero.

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Thank You