Engineering Electromagnetics

Lecture 36

01/12/2023

by

Debolina Misra

Department of Physics
IIITDM Kancheepuram, Chennai, India

Maxwell's equation

(i)	$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$	(Gauss's law),
(ii)	$\nabla \cdot \mathbf{B} = 0$	(no name),
(iii)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(Faraday's law),
(iv)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	(Ampère's law with Maxwell's correction).

From Griffith book

Poynting Theorem

power P supplied by the field

$$P = q(\vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot \vec{\mathbf{u}}$$

$$= q\vec{\mathbf{u}} \cdot \vec{\mathbf{E}}$$ *time-varying magnetic field does not supply any energy to the charged particle.*

Only the electric field

charge $\rho_v\, dv$ contained in volume dv $\quad dP = \rho_v\, dv\, \vec{\mathbf{E}} \cdot \vec{\mathbf{u}} = \vec{\mathbf{E}} \cdot \rho_v \vec{\mathbf{u}}\, dv$

As $\vec{\mathbf{J}} = \rho_v \vec{\mathbf{u}}$

power density p (power per unit volume) $\quad p = \dfrac{dP}{dv} = \mathbf{E.J}$

$$\vec{\mathbf{J}} = \nabla \times \vec{\mathbf{H}} - \frac{\partial \vec{\mathbf{D}}}{\partial t}$$

$$\nabla \cdot (\vec{\mathbf{E}} \times \vec{\mathbf{H}}) + \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} + \vec{\mathbf{H}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} + \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{D}}}{\partial t} = 0 \quad \Leftarrow \quad \vec{\mathbf{E}} \cdot (\nabla \times \vec{\mathbf{H}}) = \vec{\mathbf{H}} \cdot (\nabla \times \vec{\mathbf{E}}) - \nabla \cdot (\vec{\mathbf{E}} \times \vec{\mathbf{H}})$$

(differential) form of Poynting's theorem

$$\nabla \cdot (\vec{\mathbf{E}} \times \vec{\mathbf{H}}) + \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} + \vec{\mathbf{H}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} + \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{D}}}{\partial t} = 0$$

$$\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{H}}$$

$$\vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \text{ and } \vec{\mathbf{D}} = \epsilon \vec{\mathbf{E}}.$$

$$\vec{\mathbf{H}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} [\vec{\mathbf{B}} \cdot \vec{\mathbf{H}}] = \frac{1}{2} \frac{\partial}{\partial t} [\mu H^2]$$

$$\vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{D}}}{\partial t} - \frac{1}{2} \frac{\partial}{\partial t} [\vec{\mathbf{D}} \cdot \vec{\mathbf{E}}] - \frac{1}{2} \frac{\partial}{\partial t} [\epsilon E^2]$$

$$\nabla \cdot \vec{\mathbf{S}} - \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} + \frac{\partial}{\partial t} \left[\frac{1}{2} \mu H^2 \right] + \frac{\partial}{\partial t} \left[\frac{1}{2} \epsilon E^2 \right] = 0$$

$$w_m = \tfrac{1}{2} \vec{\mathbf{B}} \cdot \vec{\mathbf{H}} = \tfrac{1}{2} \mu H^2$$

$$w_e = \tfrac{1}{2} \vec{\mathbf{D}} \cdot \vec{\mathbf{E}} = \tfrac{1}{2} \epsilon E^2$$

$$\int_v \nabla \cdot \vec{\mathbf{S}} \, dv + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} \, dv + \int_v \frac{\partial}{\partial t} w_m \, dv + \int_v \frac{\partial}{\partial t} w_e \, dv = 0$$

or

$$\oint_s \vec{\mathbf{S}} \cdot \vec{ds} + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} \, dv + \frac{d}{dt} \int_v w_m \, dv - \frac{d}{dt} \int_v w_e \, dv = 0$$

$$\oint_s \vec{\mathbf{S}} \cdot \vec{ds} + \int_v \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} \, dv = \frac{d}{dt} \int_v (w_m + w_e) \, dv$$

where the volume v is bounded by a surface s.

Poynting vector

- $S = E \times H$
- Poynting vector
- Instantaneous power density
- Power flowing out per unit area
- Unit: Watt/m^2

Problem-3

The electric field intensity in a dielectric (perfect) medium is given as $\vec{\mathbf{E}} = E\cos(\omega t - kz)\vec{\mathbf{a}}_x$ V/m, where E is its peak value, and k is a constant quantity. Determine (a) the magnetic field intensity in the region, (b) the direction of power flow, and (c) the average power density.

Solution-2

$$E_x = E\cos(\omega t - kz)\ \text{V/m}$$

$$D_x = \epsilon E\cos(\omega t - kz)\ \text{C/m}^2$$

$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = -\nabla \times \vec{\mathbf{E}}$$

$$= -\frac{\partial}{\partial z}[E_x]\vec{\mathbf{a}}_y = -Ek\sin(\omega t - kz)\vec{\mathbf{a}}_y$$

$$B_y = \frac{Ek}{\omega}\cos(\omega t - kz)\ \text{T}$$

$$H_y = \frac{Ek}{\omega\mu}\cos(\omega t - kz)\ \text{A/m}$$

b) The instantaneous power density, or the Poynting vector, is

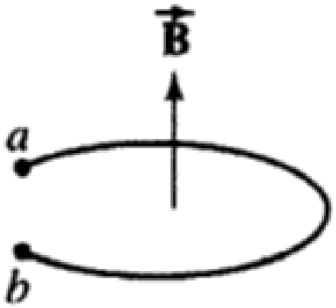
$$\vec{\mathbf{S}} - \vec{\mathbf{E}} \times \vec{\mathbf{H}}$$

$$= \frac{k}{\omega\mu}E^2\cos^2(\omega t - kz)\vec{\mathbf{a}}_z\ \text{W/m}^2$$

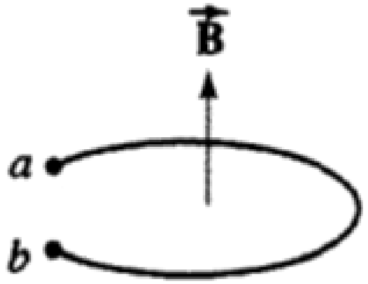
$$\langle S_z\rangle = \frac{1}{T}\int_0^T \frac{k}{\omega\mu}E^2\cos^2(\omega t - kz)\,dt$$

$$= \frac{k}{2\omega\mu}E^2\ \ \text{W/m}^2$$

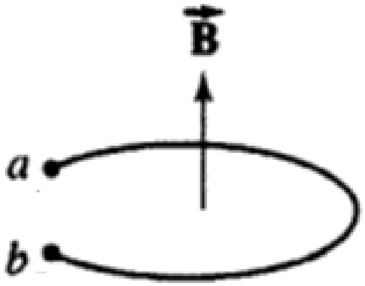
B and *e*



B is constant



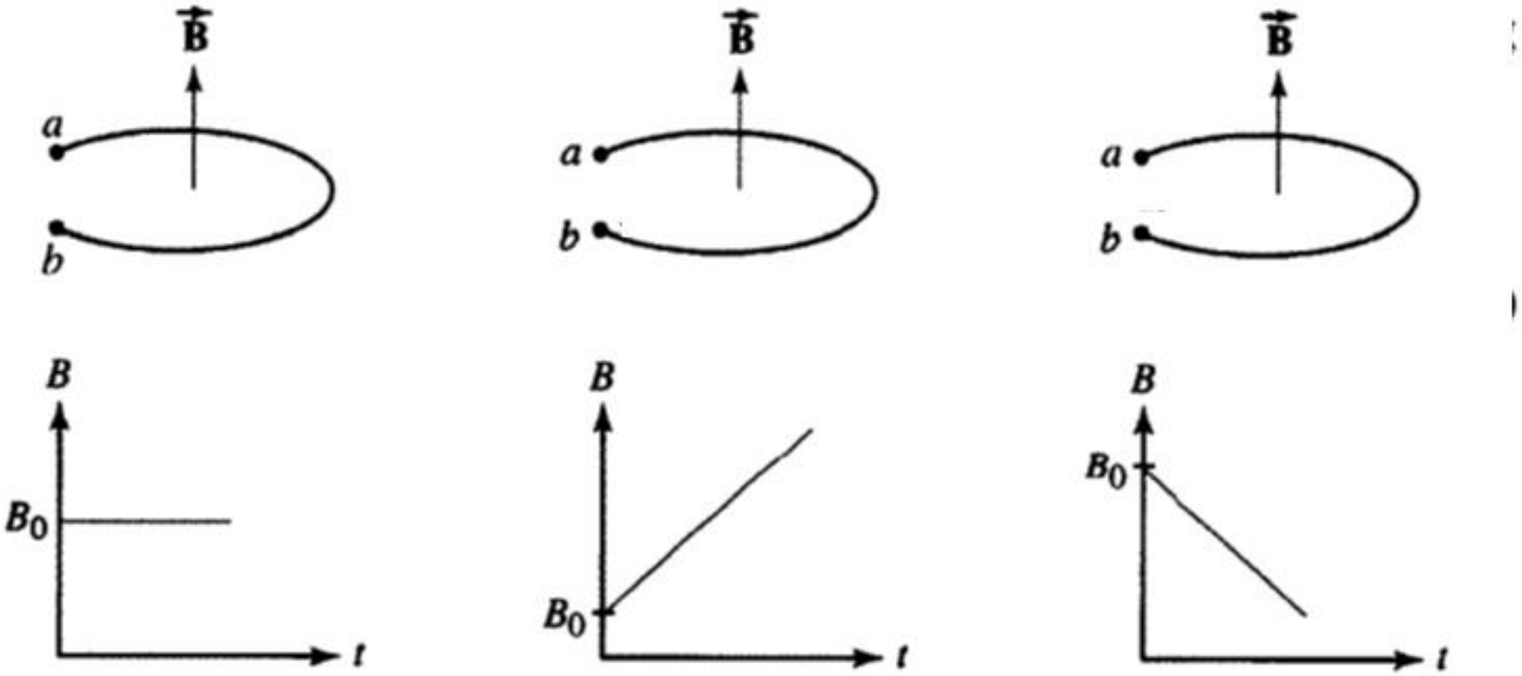
B increases with time

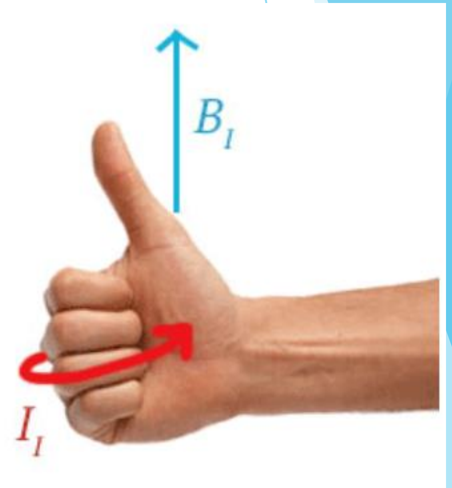


B decreases with time

Let us now consider an open loop placed in a magnetic field. When the magnetic flux density, and thereby the magnetic flux linking the loop, is of uniform strength, as shown in Figure 7.8a, the induced emf in the loop will be zero. When the magnetic flux density is increasing

B and *e*



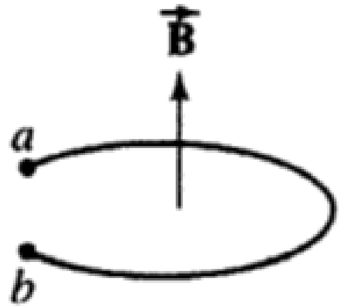


https://www.pasco.com/products/guides/right-hand-rule

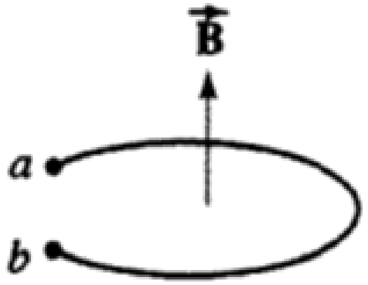
- B is constant
- B increases with time
- B decreases with time

Profile for e vs t ?

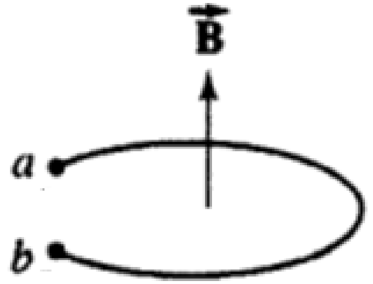
B and *e*



B is constant

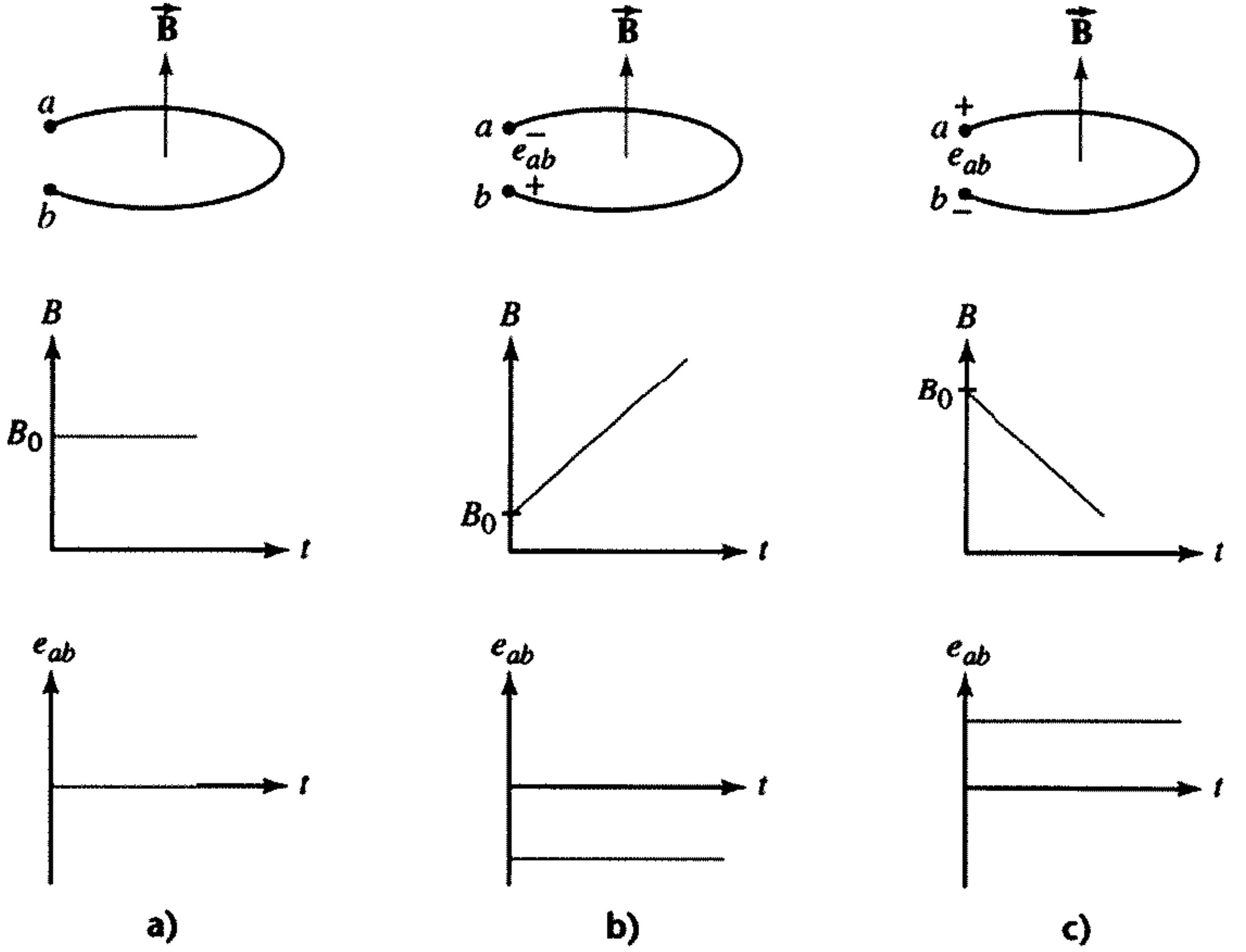


B increases with time



B decreases with time

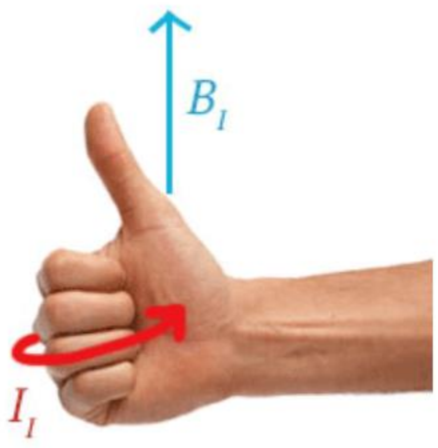
B and *e*



- B is constant
- B increases with time
- B decreases with time

Right hand rule to find polarity

- B_{ext} (Φ_{ext}) → emf in the circuit → i_{in}
- i_{in} → B_{in} → opposes change in Φ_{ext}
- To apply the right hand rule to Lenz's Law, first determine whether the magnetic field through the loop is increasing or decreasing.
- Accordingly try to find the direction of B_{in} , it should oppose any change in the rate of actual flux: → ***How?***
- B_{in} → what would have been the direction of i_{in} if a and b were closed?



Only this part matters → decides the direction of i_{in}

⇩

a to b or b to a

⇩

Decides polarity

https://www.pasco.com/products/guides/right-hand-rule

Thank You