Engineering Optics

Lecture 5

by

Debolina Misra
Department of Physics
IIITDM Kancheepuram, Chennai, India

Harmonic waves

1-D differential wave equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Simplest waveform: Sine or Cosine → ***Sinusoidal / harmonic waves***

$$\boxed{\psi(x, t)\big|_{t=0} = \psi(x) = A \sin kx = f(x)}$$

Any wave → superposition of harmonic waves

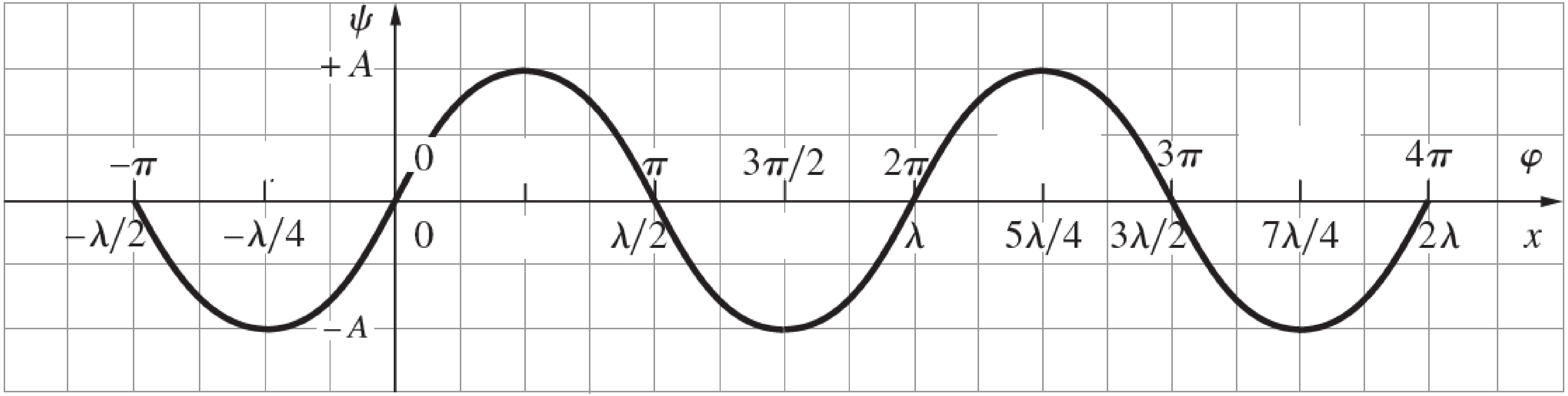
k **: propagation number** → a +*ve* constant

$|\psi(x)|_{max}$ = → *maximum disturbance* → ***amplitude***

Argument of Sine function → *'phase (φ)'*

Reference: Optics *by* Hecht

Harmonic waves continued



- *Spatial period* → wavelength ***'λ'*** → *meaning?* $\psi(x, t) = \psi(x \pm \lambda, t)$
- Units?

Spatial frequency: wave number (κ) = 1/λ

Reference: Optics *by* Hecht

Phase velocity

$$\varphi(x, t) = (kx - \omega t + \varepsilon)$$

Rate-of change of phase with time: $\left|\left(\frac{\partial \varphi}{\partial t}\right)_x\right| = \omega$ (1)

Rate of change of phase with distance: $\left|\left(\frac{\partial \varphi}{\partial x}\right)_t\right| = k$ (2)

(1)/(2) → $\frac{\omega}{k} = v$ → *phase velocity*

Superposition principle

$$\frac{\partial^2 \psi_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2}$$

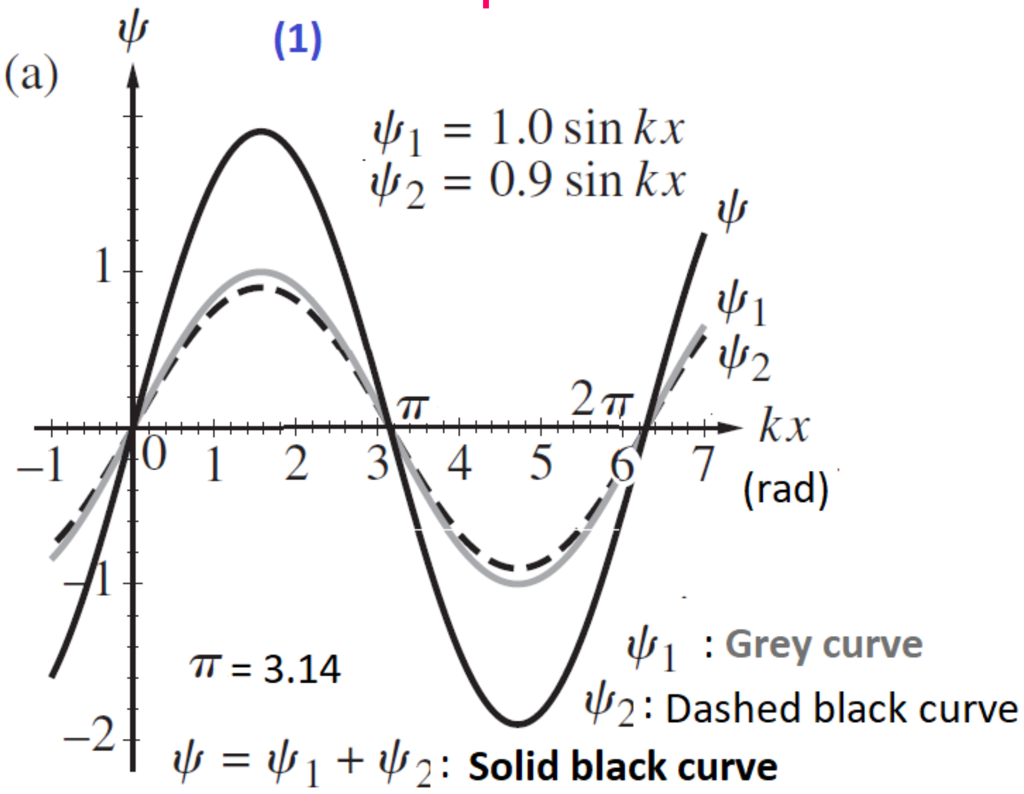
$$\frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2} + \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

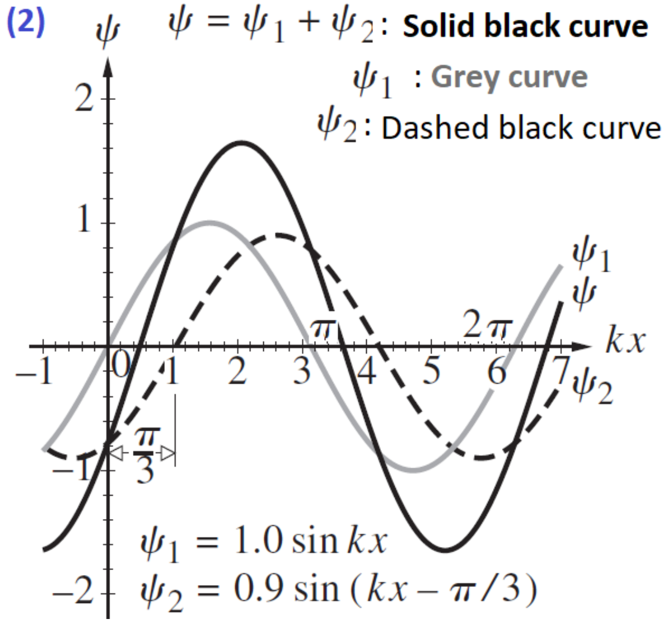
$$\frac{\partial^2}{\partial x^2}(\psi_1 + \psi_2) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(\psi_1 + \psi_2)$$

$$\boxed{\psi = \psi_1 + \psi_2}$$

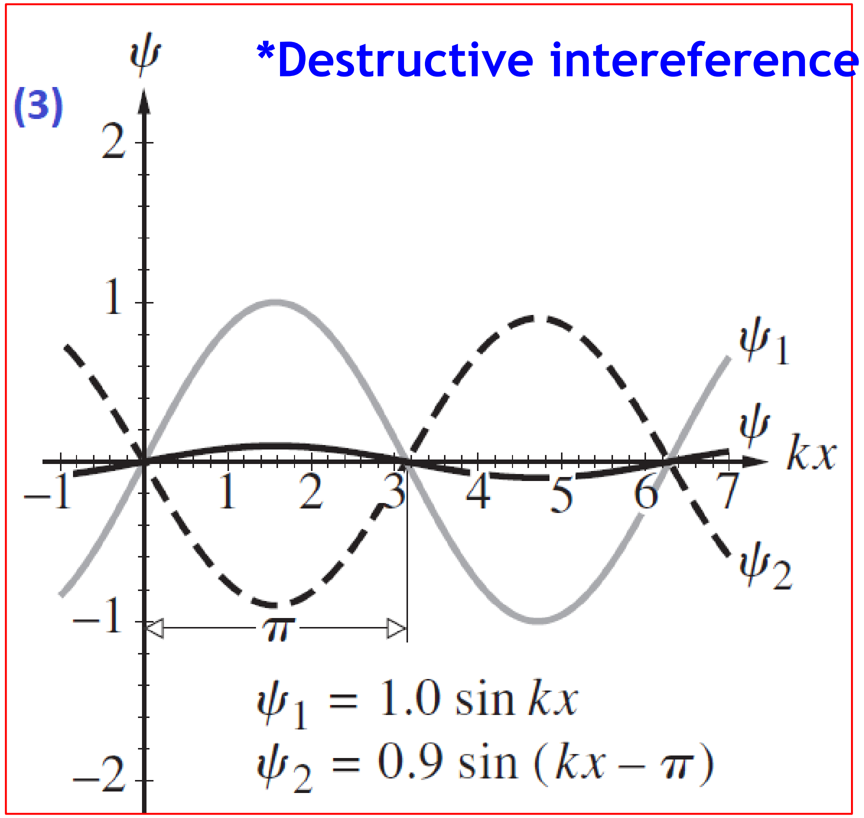
In-phase

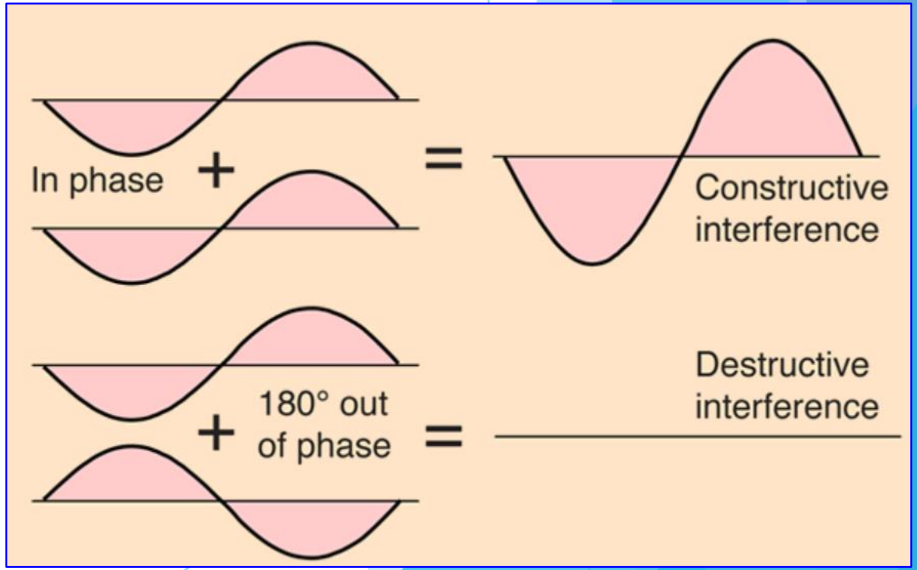


Phase difference



Optics, Hecht





Reference: http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/interf.html

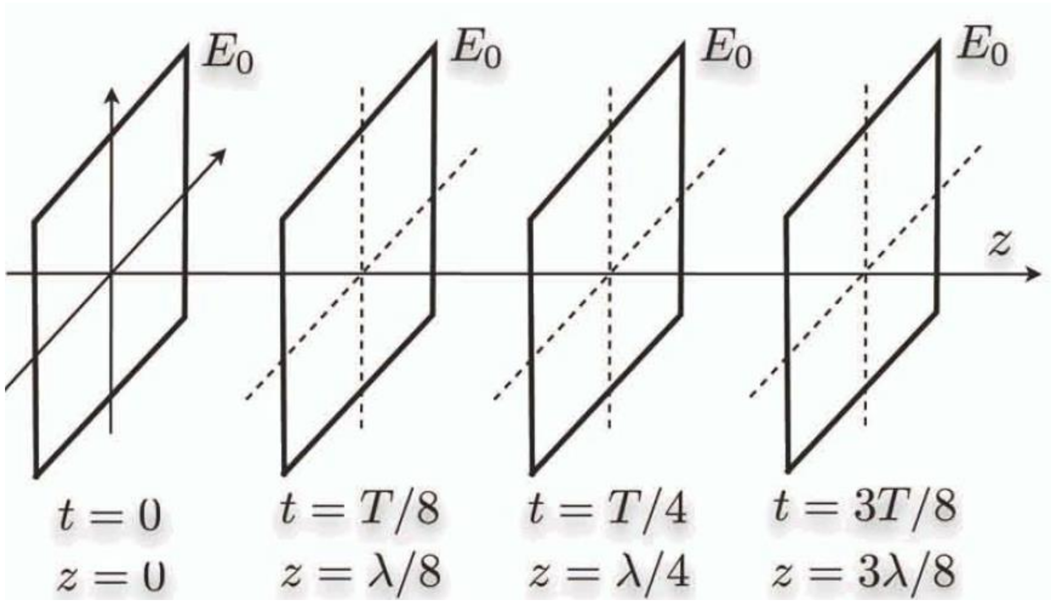
Wavefronts

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

Optical disturbance→ in space→ spatial distribution→ *wavefront*

at any t → *a surface of constant phase* → *wavefront (phase front)*

Plane wave:



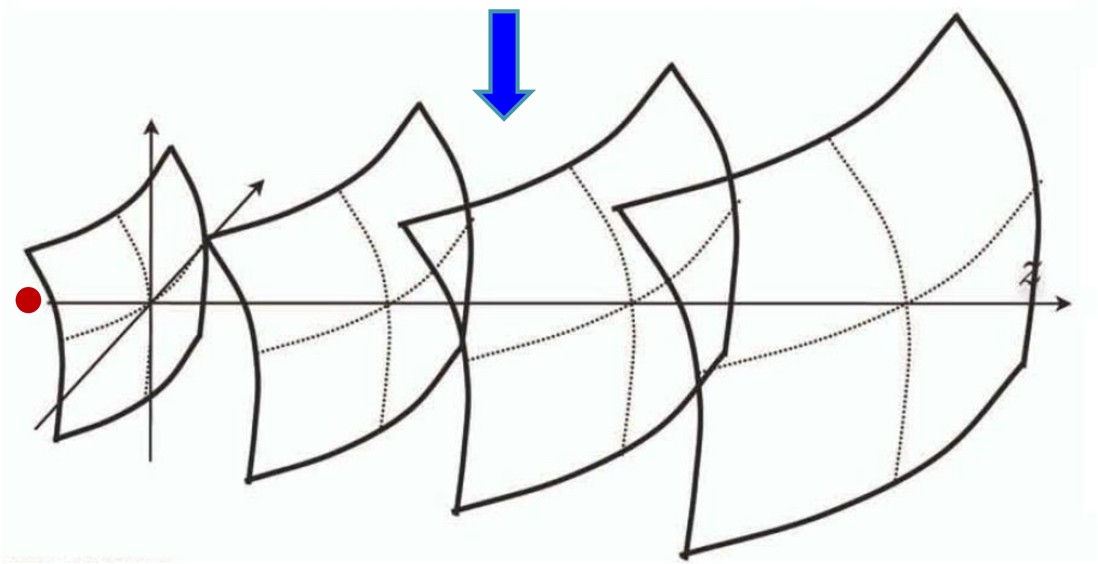
Optics, Hecht

Spherical waves

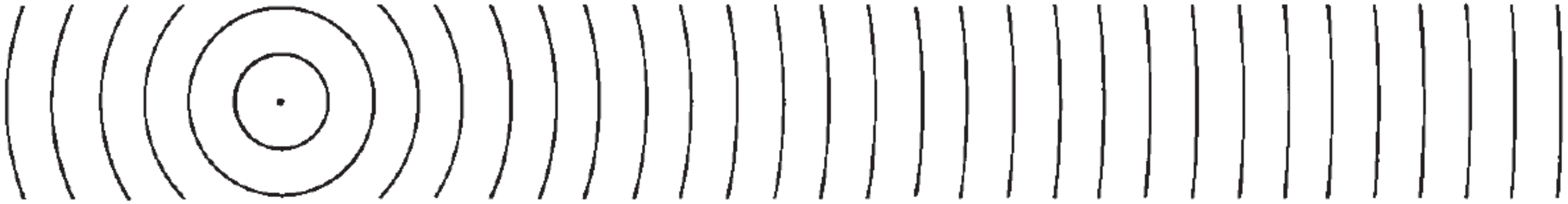
point source of light → radiating in all directions

radius increases

Source



Flattening of spherical waves *Plane waves*



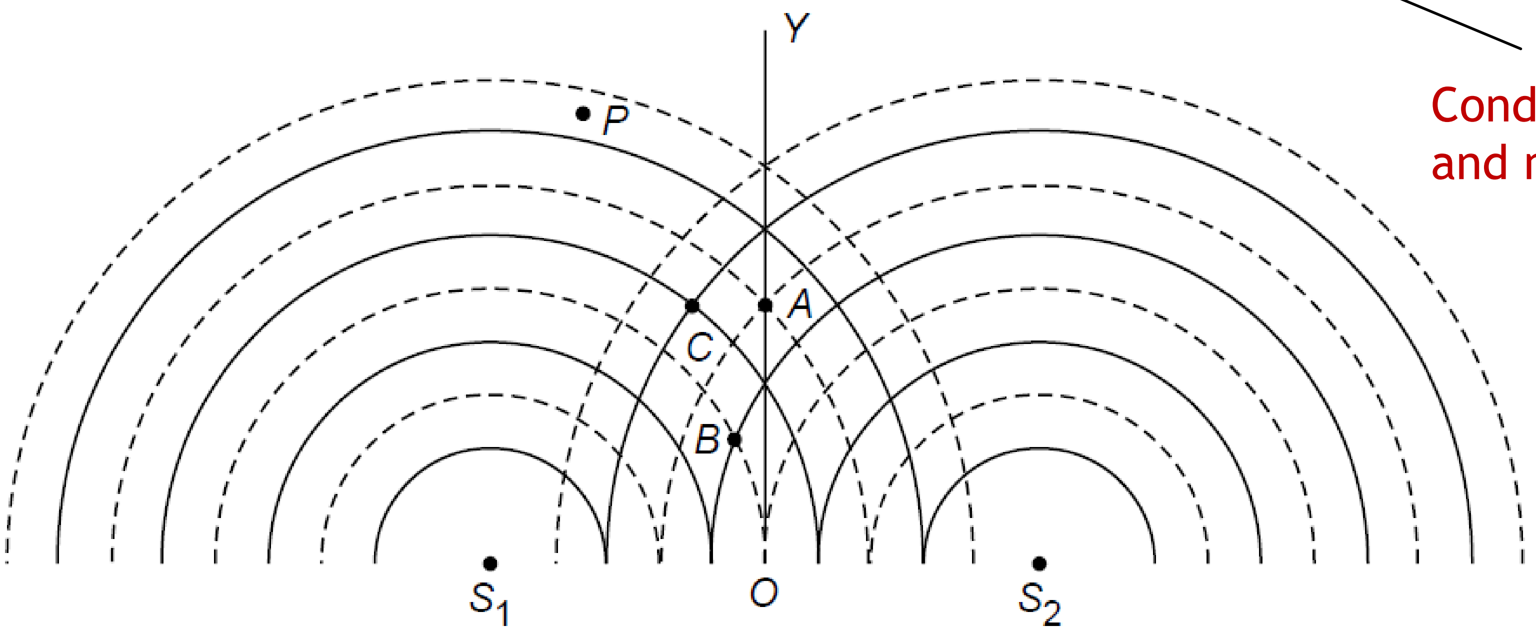
Optics, Hecht

Interference between two waves e.g. on *surface of water*

Example-1: when the sources are vibrating in phase

Refer. '14.2 INTERFERENCE PATTERN PRODUCED ON THE SURFACE OF WATER'

Conditions for maxima and minima?



Waves emanating from two point sources S_1 and S_2 vibrating in phase. The solid and the dashed curves represent the positions of the crests and troughs, respectively.

Optics, Ghatak

Answer

at A

$y = y_1 + y_2$

$= 2a \cos \omega t$

$S_2C - S_1C = \lambda$

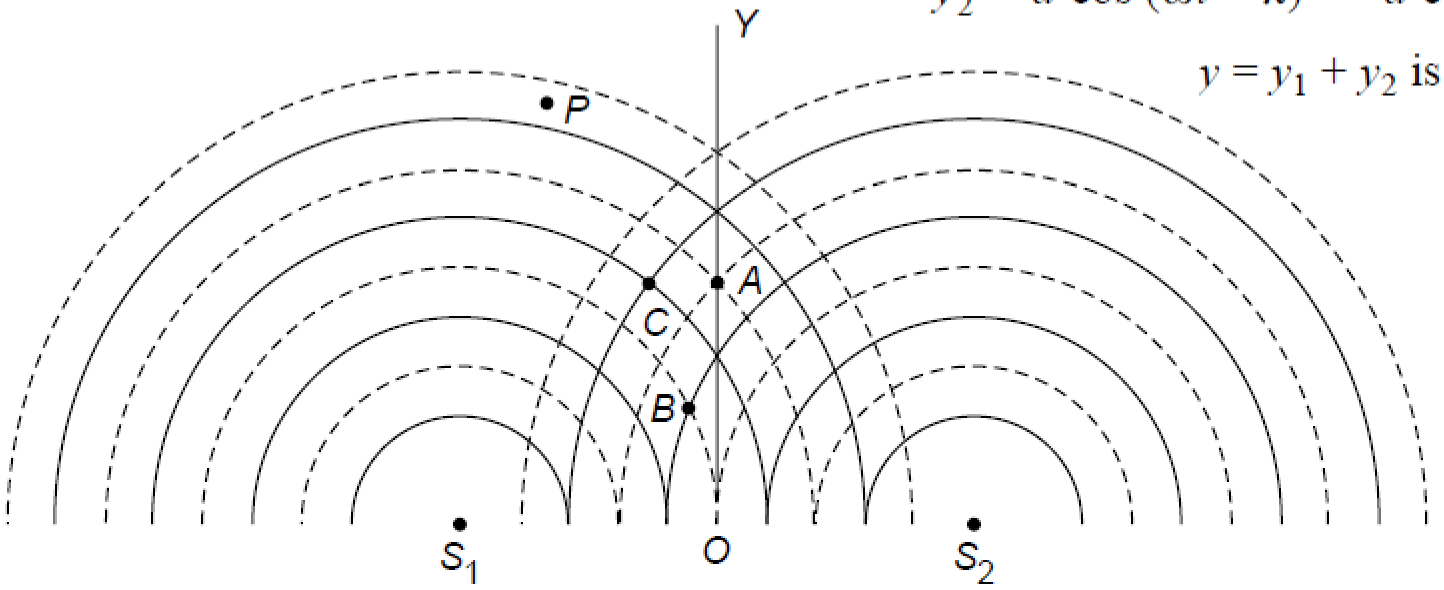
$S_2P \sim S_1P = n\lambda$ (maxima)

$S_2B - S_1B = \lambda/2$

$y_1 = a \cos \omega t$

$y_2 = a \cos (\omega t - \pi) = -a \cos \omega t$

$y = y_1 + y_2$ is zero at all times.



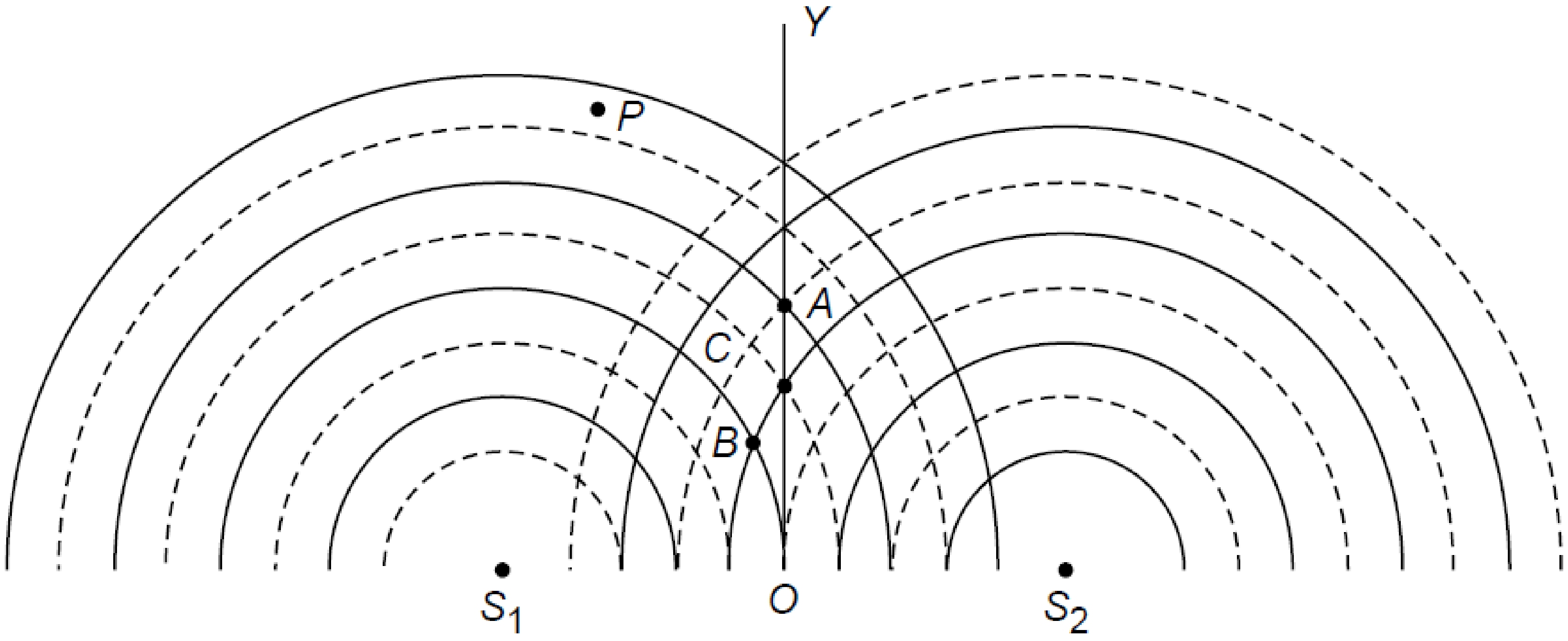
$S_2P \sim S_1P = n\lambda$ (maxima)

$S_2P \sim S_1P = \left(n + \frac{1}{2}\right)\lambda$ (minima)

Interference between two waves

- Example-2: when the sources are vibrating out of phase

Will you observe maxima and minima?



Waves emanating from two point sources S_1 and S_2 vibrating out of phase.

Thank You