CONIC SECTION

① Ellipse :

(i) Focus - Directrix Method :

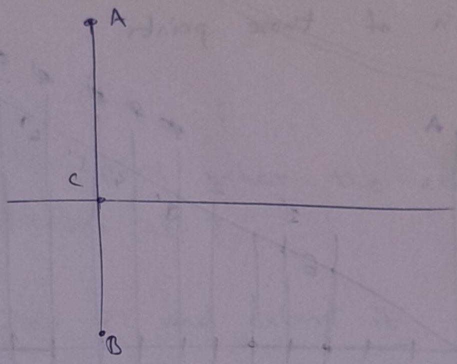
Given :

(a) distance of focus from directrix [(d)]

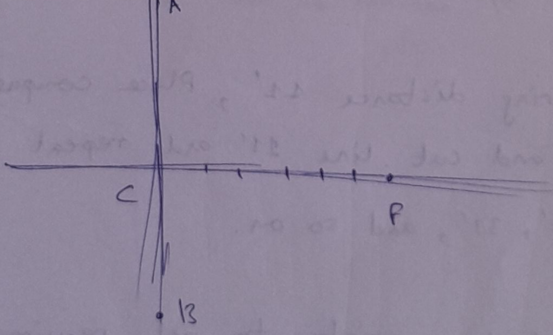
(b) eccentricity [(e) = a/b] [a<b]

Procedure :

(1) Draw vertical line AB, Mark C as midpoint and draw a Horizontal line from C.

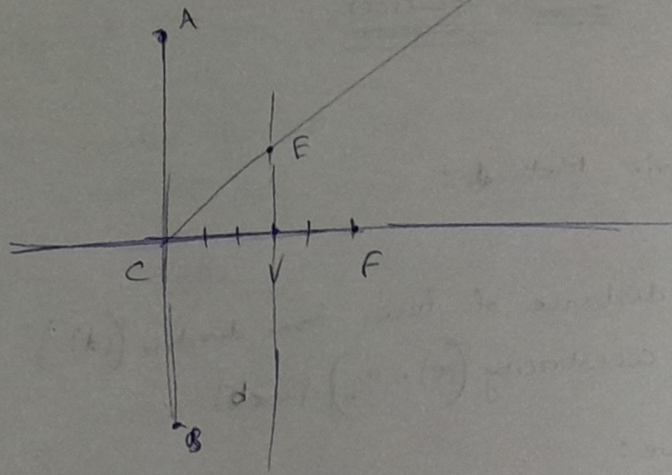


(2) Mark another point at a distance 'd' away from C and divide it into 'a+b' equal no. of parts

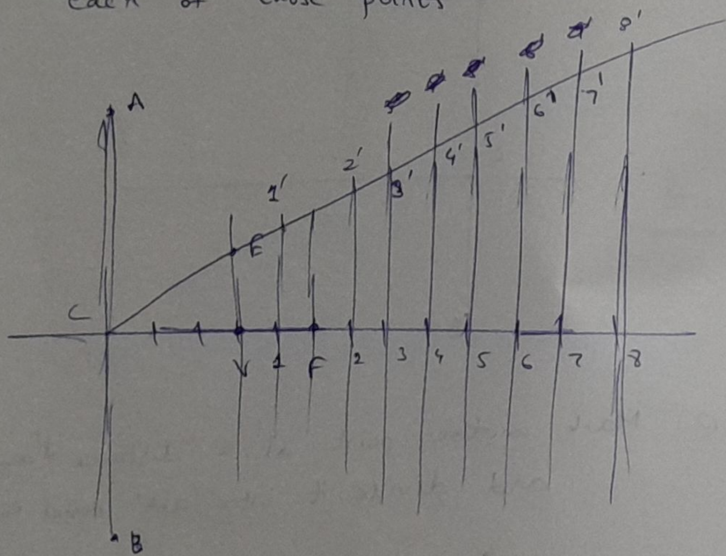


(3) Mark the vertex V a^{th} division from F and draw a vertical line from V and mark another point E on top of CF line such that

VF = VE & Now join CE and extend it.



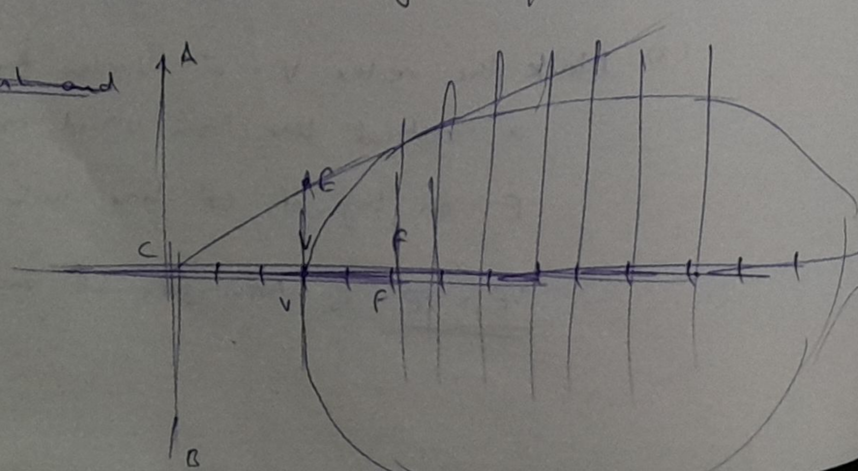
(4) Divide the line segment right of F in equal no. of parts and draw vertical lines from each of those points



(5) Measuring distance 11', Place compass needle on F and cut line 11' and repeat the same for 22', 33', and so on.

(6) Connect the points to get required Ellipse.

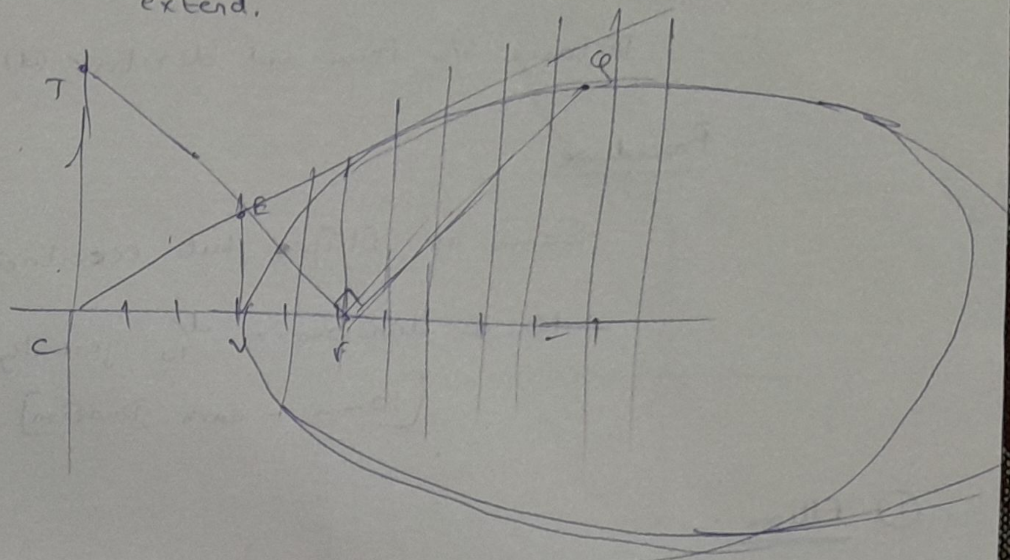
~~for Tangent and~~



For Tangent & Normal at any Point ?

(1) Choose any Point Q on ellipse and ~~point~~ join QF

(2) Construct 90° line from F and draw a line and extend.



Mark the point where the extended line cuts the Directrix as T.

(3) ~~Joint~~ Join QT and extend it.

This line is Tangent to Ellipse @ Point Q.

(4) From Point Q, Draw a Perpendicular Line ($\perp$ to TQ)

This line is normal to Ellipse @ Point Q

② Parabola:

(i) Focus-Directrix Method:

Given:

Distance b/w focus and directrix (d)

Procedure:

Same as Ellipse but eccentricity = 1

∴ No. of divisions = d/10 (generally)

[10mm - each Division]

① Ellipse:

(ii) Rectangle Method:

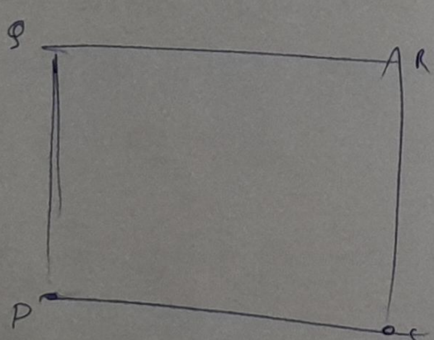
Given:

Major Axis length (2a)

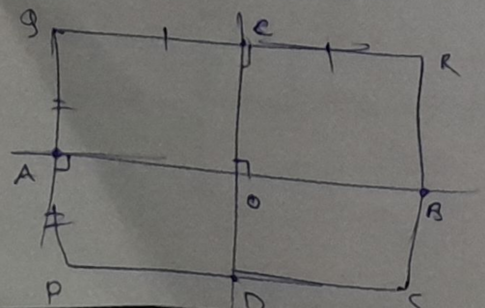
Minor Axis length (2b)

Procedure:

(1) Draw a Rectangle having length '2a' and breadth '2b'.



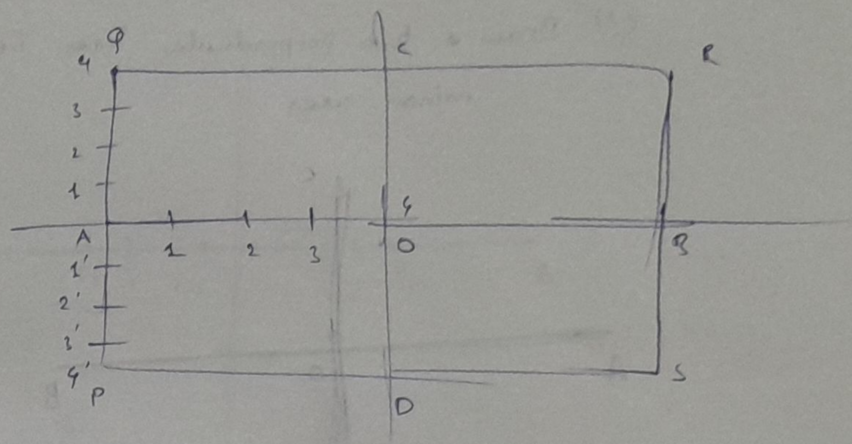
(2) Draw axes perpendicular bisectors of sides



Let Major axis – AB and
Minor axis – CD

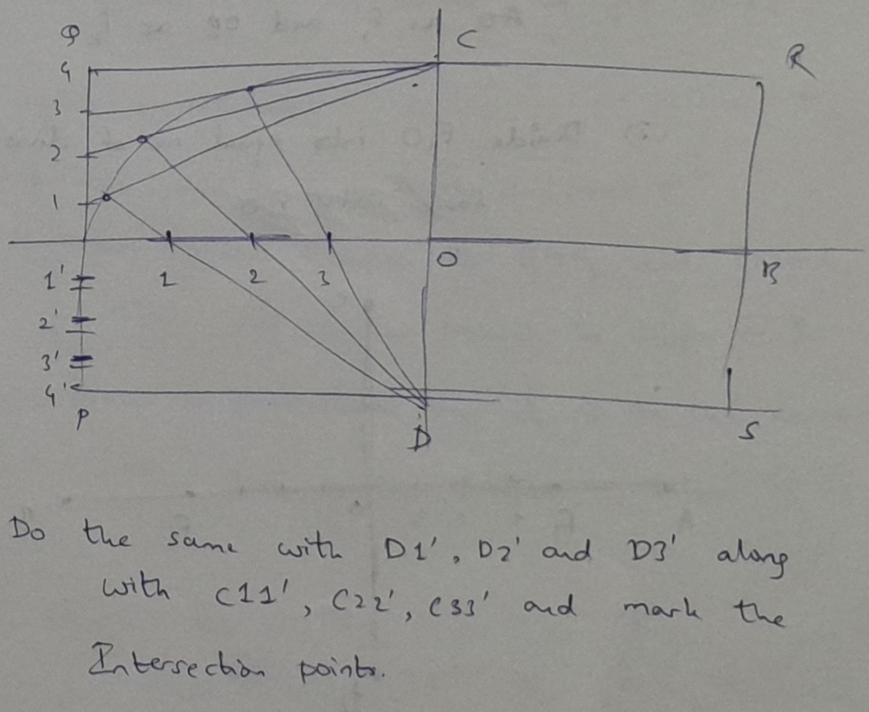
(3) Divide length AQ and AP into equal no. of parts, for say some 'd' divisions.

Divide AO also 'd' no. of divisions



(4) Join C with 1, 2 and 3 of AQ

Draw line from D towards AQ passing through 1, 2, 3 and extend with C1, C2 and C3 respectively.



5) Do the same with D1', D2' and D3' along with C11', C22', C33' and mark the Intersection points.

6) Join all points all the Ellipse is obtained.

(iii) Arcs of Circle Method :

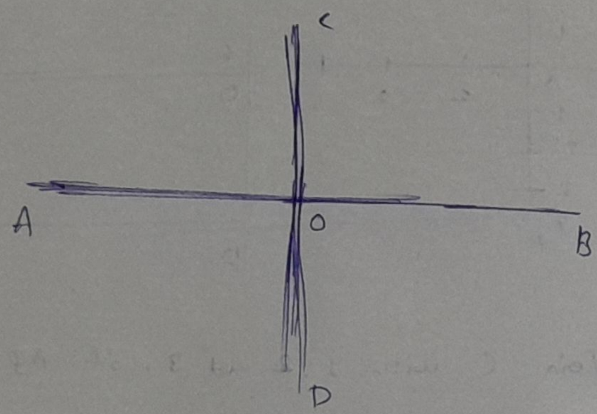
Given :

Major axis length (2a)

Minor axis length (2b)

Procedure :

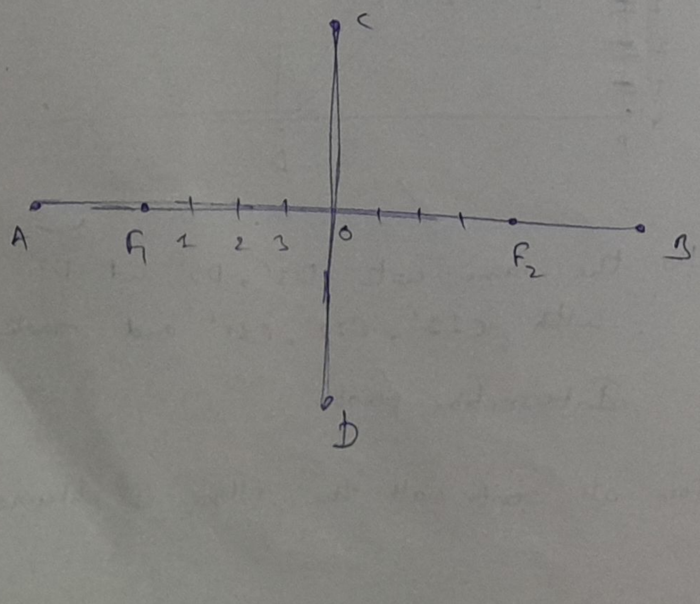
(1) Draw two perpendicular lines i.e. major and minor axes



(2) Take OA distance and by placing compass needle on C (or) D, mark a point on AO as F_1 and OB as F_2

(3) Divide F_1O into equal no. of divisions say d.

~~Same with F_2O~~



(4) Take distance as A1 and F_1 as centre,

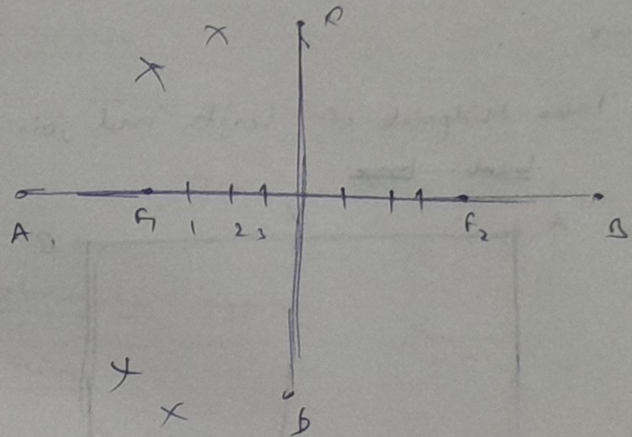
draw ~~an~~ arcs on either side of AB

~~And same~~

And Take distance as B1 and F_2 as centre,

draw arcs on either side of AB cutting

the previous arcs



Repeat the steps with A2 on F1 and B2 on F2,

A3 on F1 and B3 on F2 and so on.

(5) And connecting all points, we get Ellipse

for Tangent & Normal:

(1) Join point to F_1 and F_2 as point φ.

(2) Draw Angular Bisector to $F_1 \varphi F_2$ and Extend

This line is the Normal to Ellipse @ φ.

(3) Draw a perpendicular line at φ to the normal

This line is the tangent to Ellipse @ φ.

② Parabola.

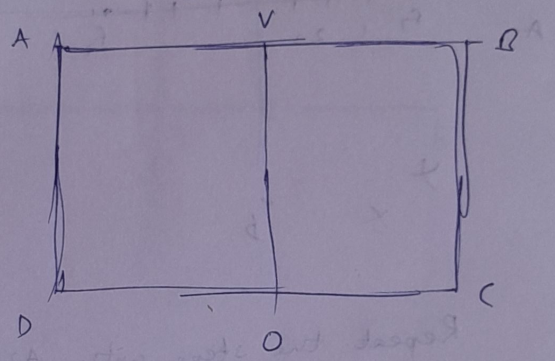
(ii) Rectangle Method

Given:

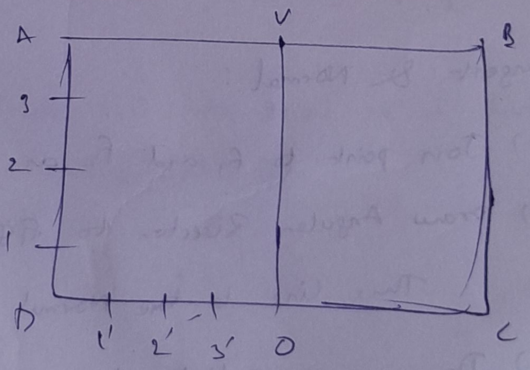
length &
breadth of rectangle (l, b)

Procedure :

(1) Take Midpoints of length and join them ~~back bage~~



(2) Divide lines AD and DO into equal no of divisions.



(3) Join V1, V2, V3

(4) Draw vertical lines from 1', 2' and 3' and wherever it cuts V1, V2 and V3 mark respectively

(5) Join Points we get Parabola

(iii) Parallelogram Method : Instead of Horizontal lines, draw inclined lines.

③ Hyperbole :

(i) focus - Directrix Method :

Given :

Dist. b/w focus and Directrix (d)

Eccentricity ($e = a/b$) [$a > b$]

Procedure :

Same as Ellipse.

Vertex → a^{th} Division from F

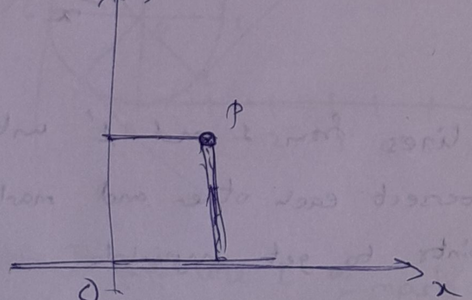
(ii) Rectangular hyperbola :

Given :

A point on hyperbola, at distances
'x' from the y-axis &
'y' from the x-axis

Procedure :

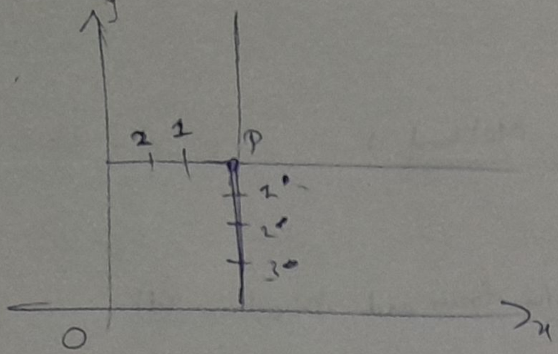
(1) Draw axes and mark point P(x, y)



(2) Divide the horizontal line into y/10 divisions
& the vertical line similarly into x/10 equal divisions.

[Conventionally, Each Division - 10 mm]

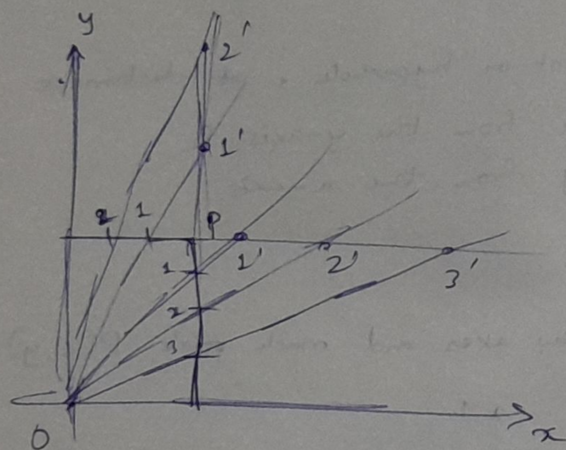
& Extend horizontal and vertical lines
from P.



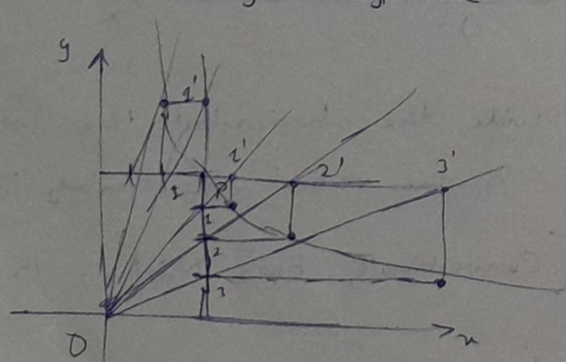
~~(2) Join O1 and O1'~~

3) Join O1 and extend & O1' and extend, intersecting the extended horizontal and vertical lines.

Similarly with O2 and O2



(4) Draw lines from 1 and 1' until they intersect each other and mark. Join all these points to get hyperbola



If Asymptotes are not 90° to each other, then all lines get inclined. No other difference

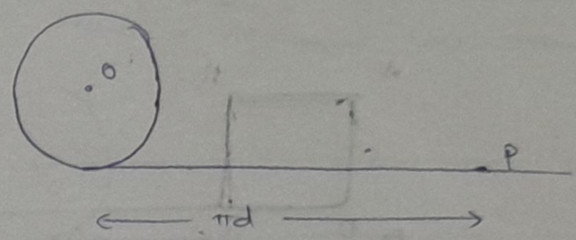
(4) Involute of a Circle :

Given:

radius of circle (r)

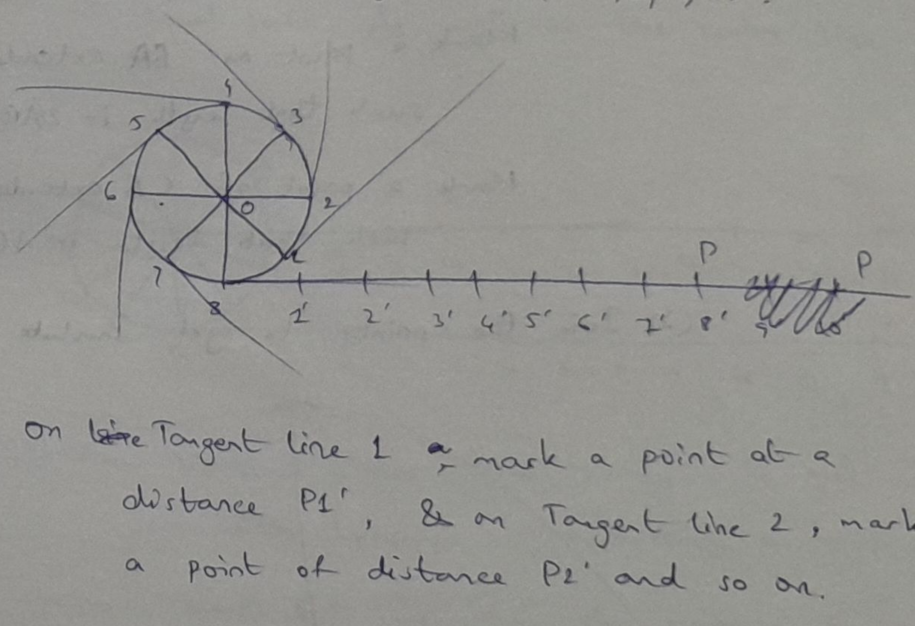
Procedure :

(1) Draw a ~~circle~~ circle of radius 'r' and draw a line of length '$2\pi r$' from base



(2) Divide the circle into ~~to equal~~ 8/10/12 equal parts and also the line into 8/10/12 equal parts

And Draw Tangents from 1, 2, 3, - - -



(3) On ~~line~~ Tangent line 1, mark a point at a distance P1', & on Tangent line 2, mark a point of distance P2' and so on.

(4) Connect all the Points to get Involute of a circle

12 equal parts when the length of string is equal to circumference of circle

8 equal parts when the length of string is not equal to circumference of circle

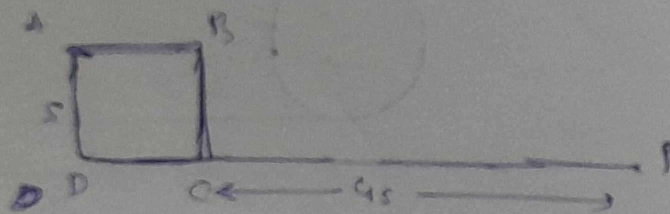
⑤ Involute of Square :

Given:

Side length (s)

Procedure :

(1) Draw a Square of side 's' and extend the base by distance '4s' Perimetre from the away end.



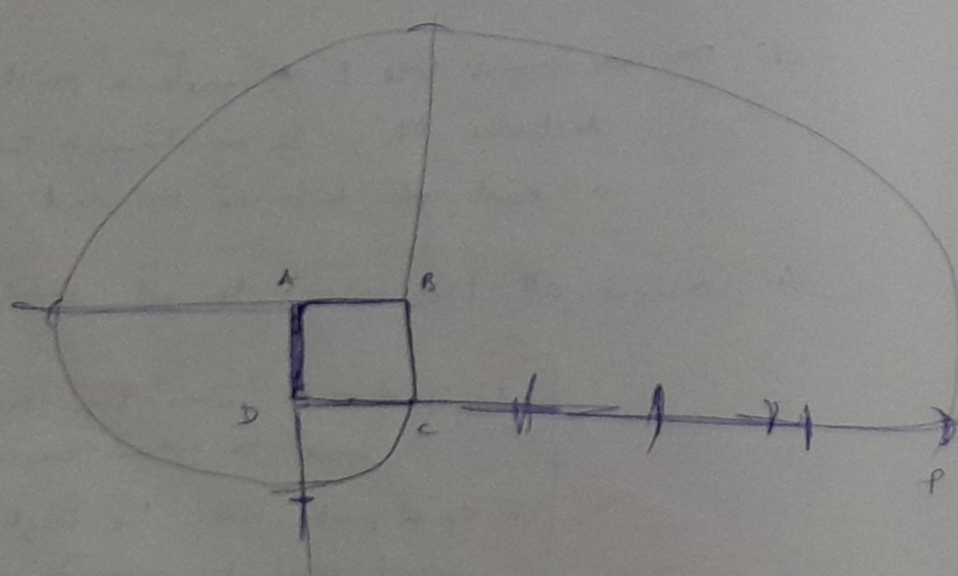
(2) Extend line CB, BA and AD as well.

Mark a point on DA line (extended) such that length is AB.

Mark a point on BA extended line such that length is 2(AB).

Mark a point of CB extended line such that length is 3(AB)

(3) Join the points to get Involute of Square



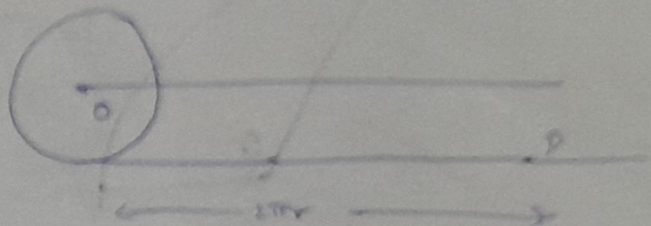
⑥ Cycloid :

Given :

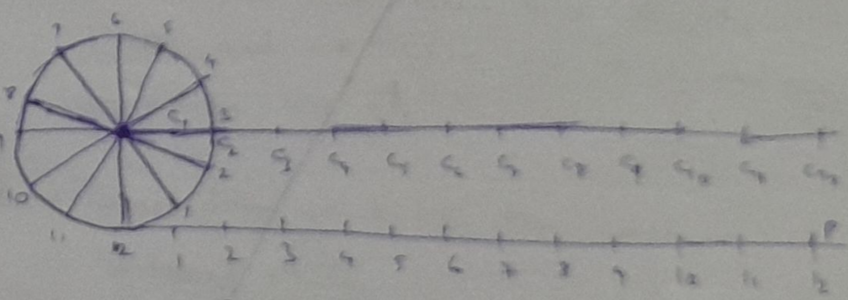
radius of circle (r)

Procedure :

(1) Draw circle and mark centre and draw a horizontal line from base, having length as circumference of circle. Also Draw a horizontal line from centre



(2) Divide the circle into 12 equal parts & Also divide the base horizontal line into 12 equal parts. As well as the centre line



(3) Draw vertical lines from $C_1, C_2, \dots C_n$ and join with 1, 2, 3, ... respectively on base line.

Draw horizontal lines from the points on the circle, i.e. from 6, from 7 joining 5, from 8 joining 4, etc.

(4) Take constant radius i.e. 'r' on compass and Place the needle on C_1 and cut 11-1 line, on C_2 and cut 10-2 line, etc & Join them all to get Cycloid.

⑦ Epicycloid :

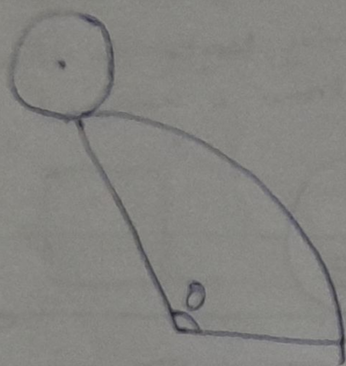
Given :

Radius of rolling circle : ~~a~~ r [r < R]

Radius of Directing circle : ~~b~~ R [redacted]

Procedure :

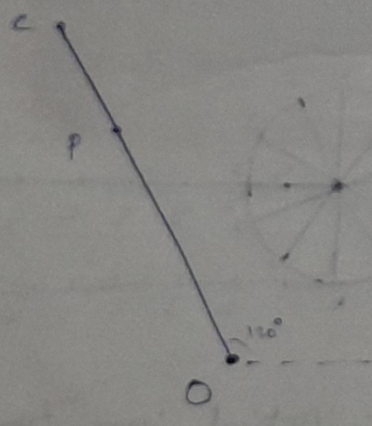
(1) Circle rolls for total distance over directing circle



$\theta = \frac{r}{R} \times 360^\circ$

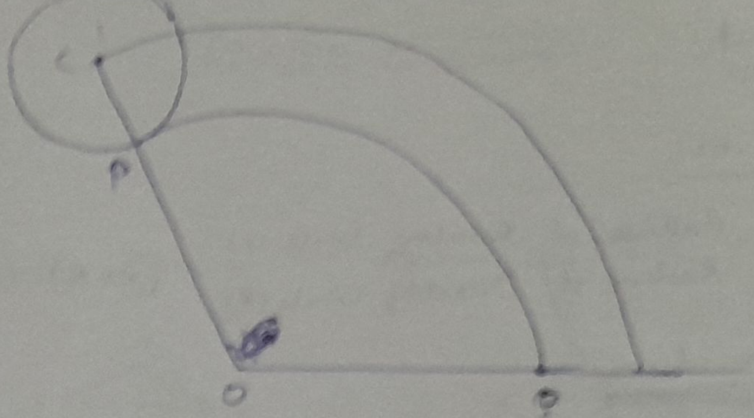
Generally, $\theta = 120^\circ$

(2) Draw line OPC, such that PC = r
OP = R

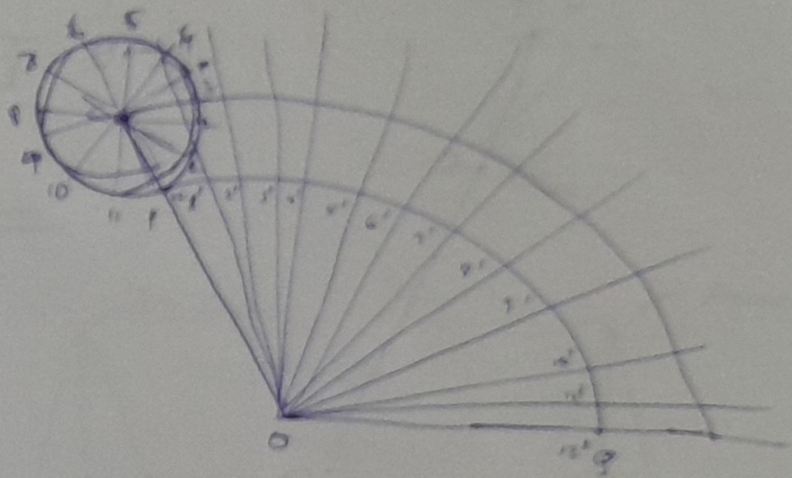


(3) Draw a circle centred at C going through P and ~~[illegible] centred at O passing through P~~ an arc touching the horizontal line from O and mark it as Q, such that ∠POQ = 120°.

Draw locus of centre C, take compass needle at O and radius OC and cut the horizontal line



(4) Divide circle into 12 equal parts and arc PQ into 12 equal parts (10° each)



Extend O1' and cut locus line at C1,
O2 and cut locus line at C2 and so-on till C12.

Make arcs from points on the circle,
centered at O,
Each arc - from 11-1, 10-2, 9-3, ... 7-5, 6

(5) Take Radius 'r' (Rolling circle) and cut the lines corresponding to the points, the compass needle is on
i.e. for C1, cut 11-1 arc
C2, cut 10-2 arc
C3, cut 9-3 and soo-on.

(6) Join all points, we get epicycloid.

For Normal @ distance 'd' from centre of Directing circle,
Cut Epicycloid from centre with distance d, Use it as centre and cut locus line with radius r. Join centre to that point. Where that line cuts the PQ arc, Join that point to the first point on epicycloid - NORMAL

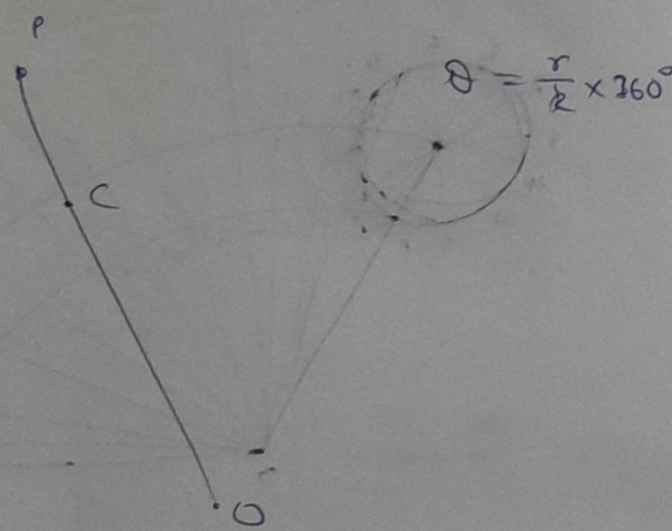
⑧ Hypocycloid :

Given :

Radius of Rotating Circle (r)
Radius of Directing Circle (R) [r<R]

Procedure :-

① line OPC , OP = R
CP = r



Rest,
Same procedure

14/2/24

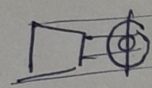
→ **Orthogonal Projection :**

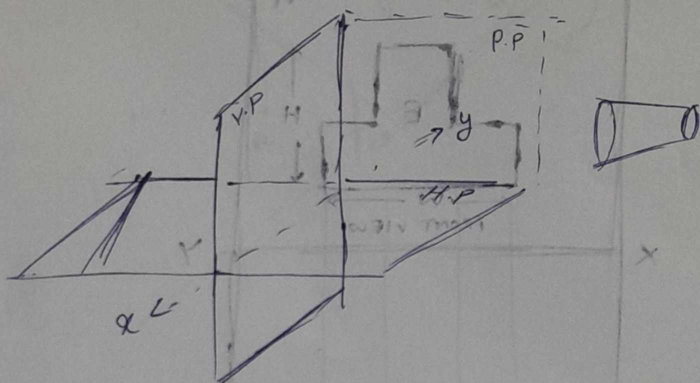
Three types :

(1) Horizontal Plane

(2) Vertical Plane

(3) Side Plane





~~Proect~~

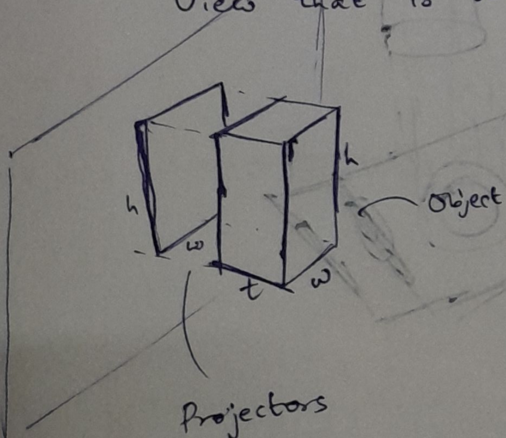
Projectors : Rays of sight

↳ Coming from ∞

∴ Object and Projected Image are of same size

→ **Front View :** (Elevation)

View that is obtained by projection on vertical plane



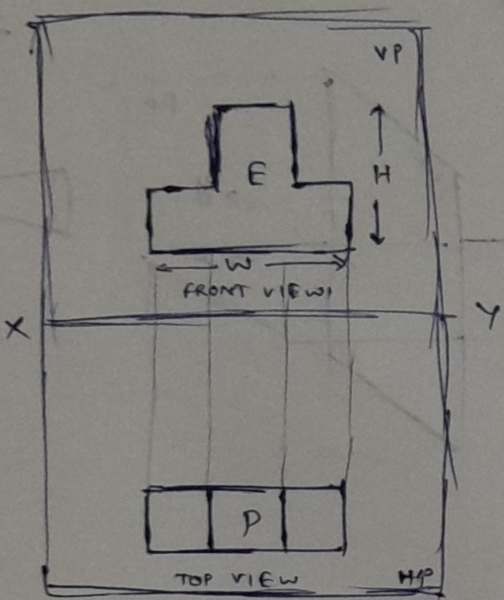
x-y line coincides with the ~~Horizontal~~ intersection of Horizontal and Vertical Plane.

x-y line AKA Reference line

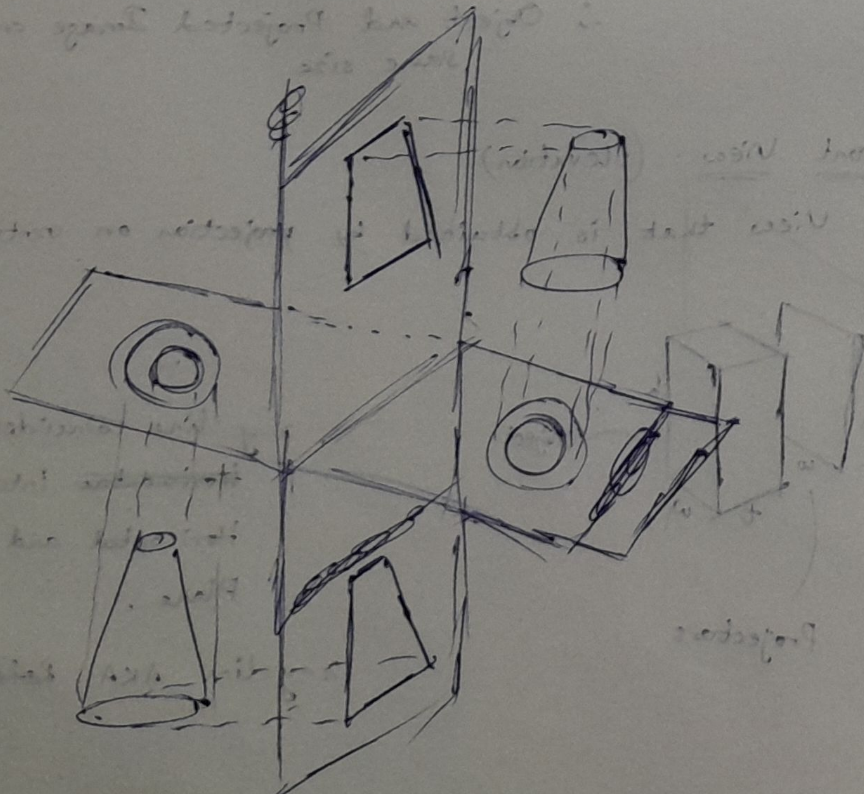
→ **Top View (or) Plan :**

View that is obtained by projection on the horizontal plane.

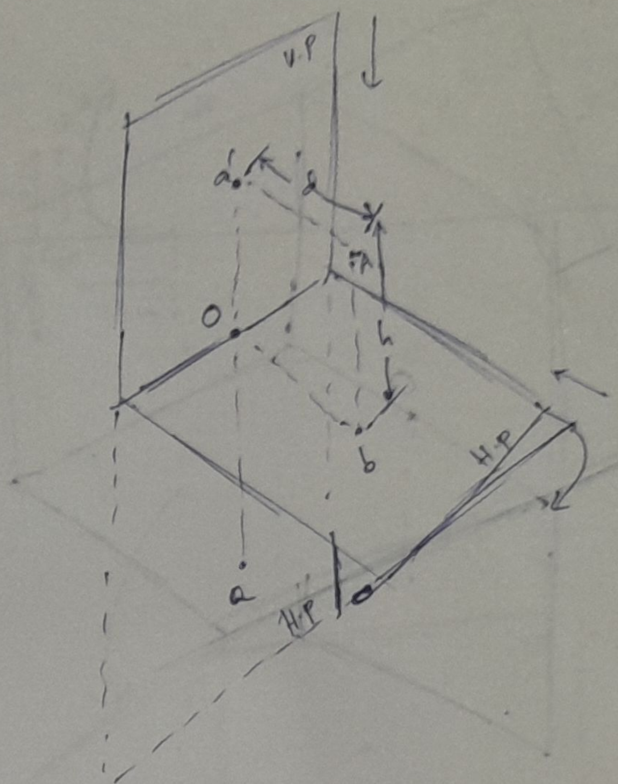
When looking from top, horizontal plane looks like a x-y plan.

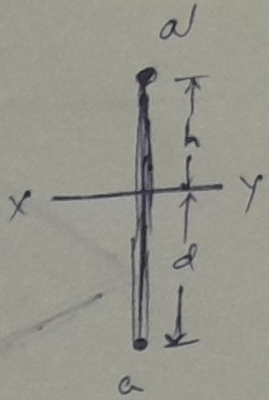


→ **First Angle Projection :**

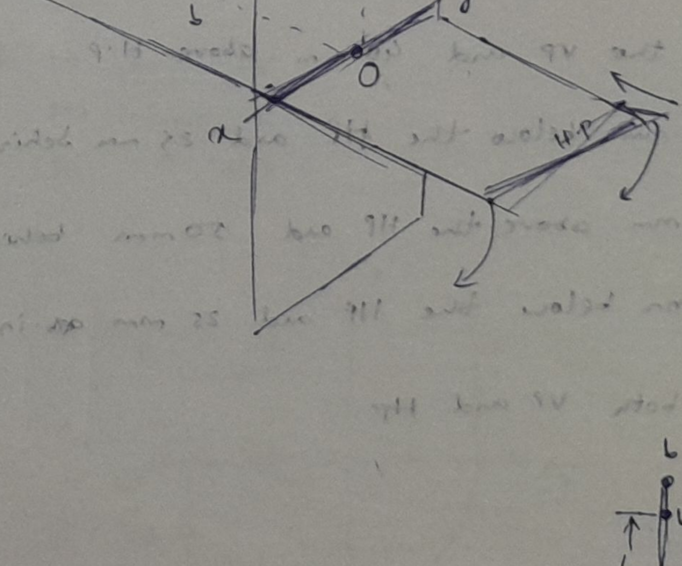


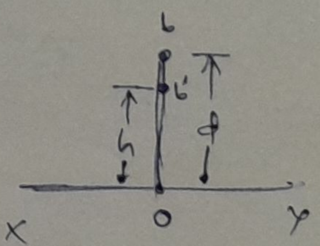
→ Point is in the first Quadrant :



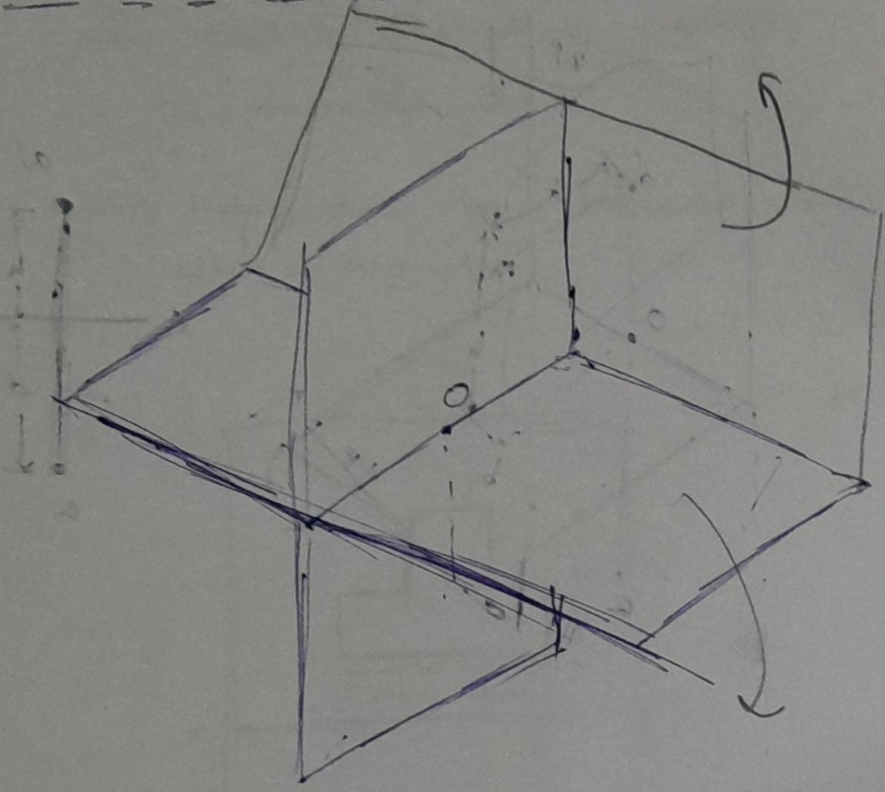


→ Point is in the Second Quadrant :





→ Point is in the Third Quadrant:



Q) Draw the projectors of the following on the same groundline, keeping the projector 25 mm apart

(i) A, in the H.P and 20 mm behind VP

(ii) B, 40 mm above H.P and 25 mm in front of VP

(iii) C, in the VP and 40 mm above H.P

(iv) D, 25 mm below the HP and 25 mm behind the VP

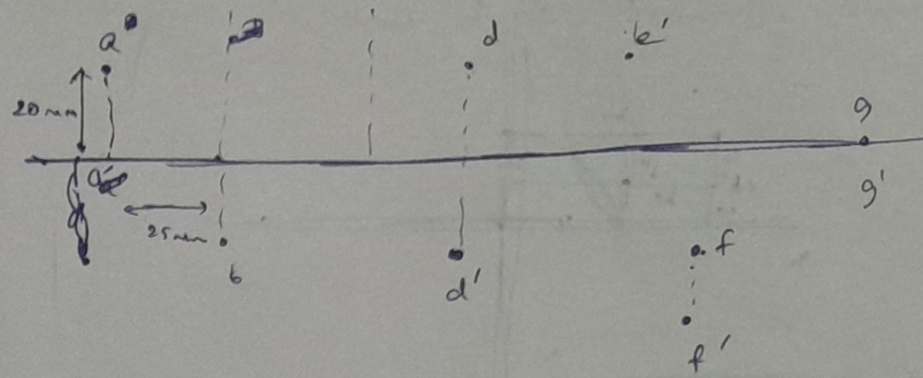
(v) E, 15 mm above the HP and 50 mm behind the VP

(vi) F, 40 mm below the HP and 25 mm ~~ab~~. in front of VP

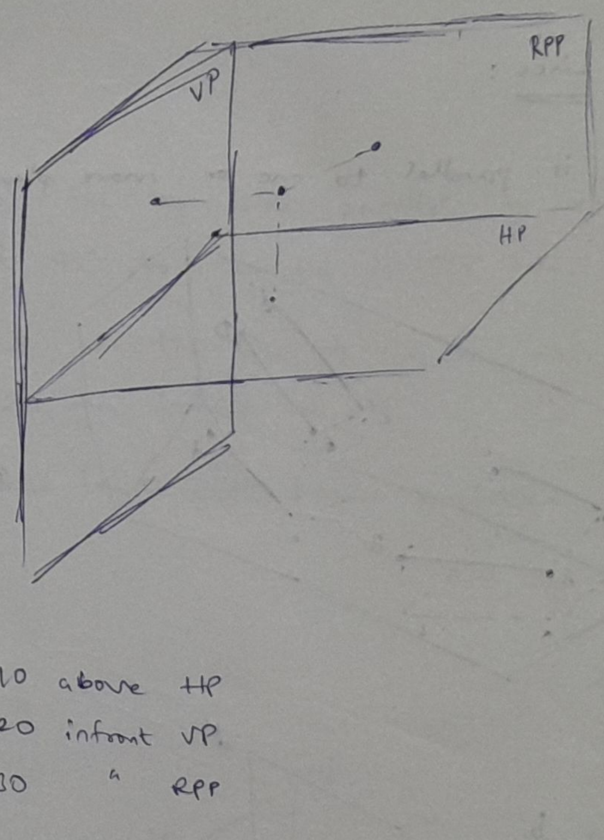
(vii) G, in both VP and HP

Sol.

above HP – draw up
below HP – draw below
behind VP – draw up
front of VP – draw below

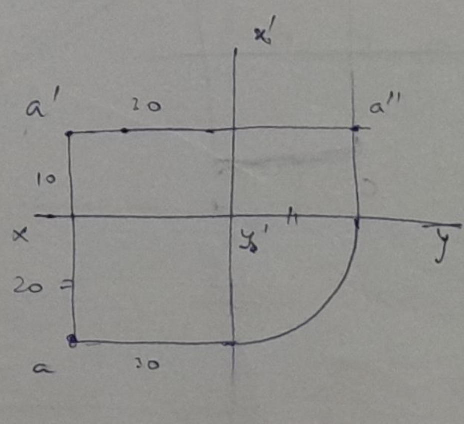


→ RPP:

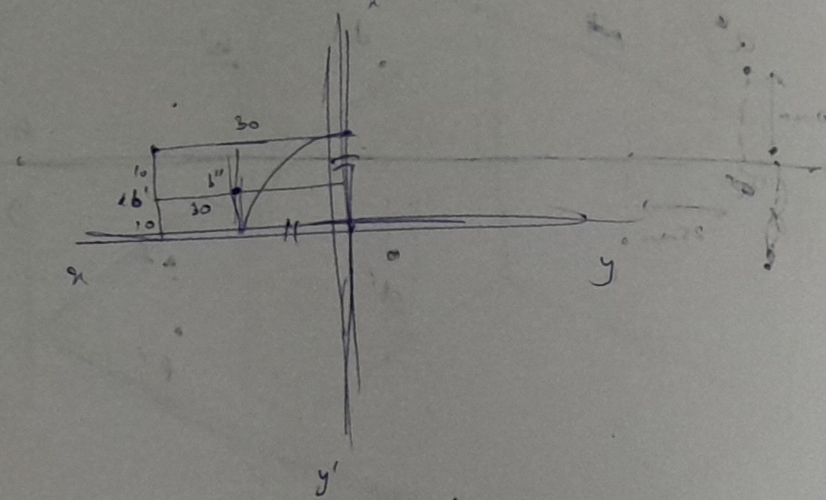


20/2/24

Ex. A 10 above HP
20 infront VP
30 " RPP

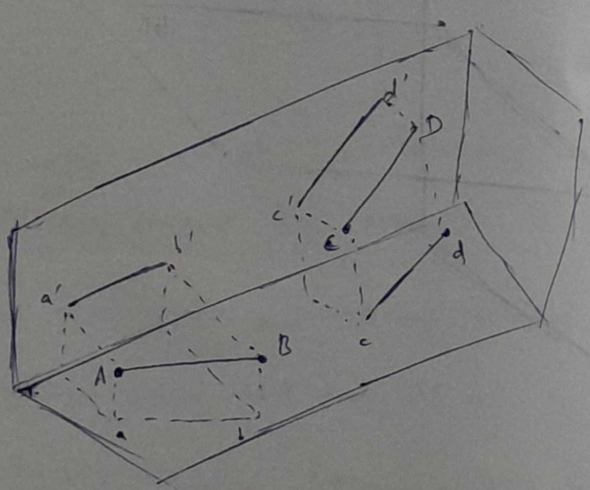


Second Quadrant:

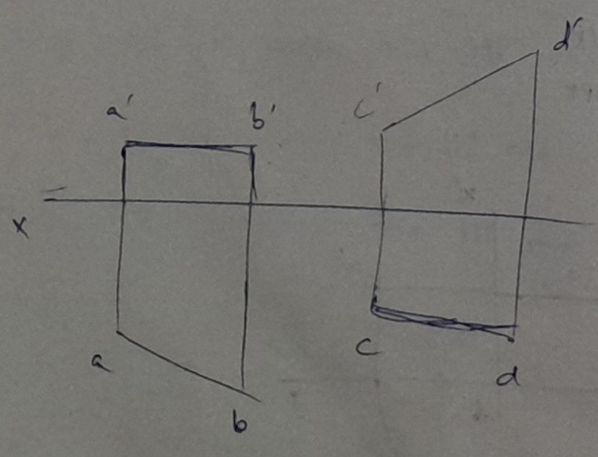


→ Projection of Lines:

When Line is parallel to one or more planes



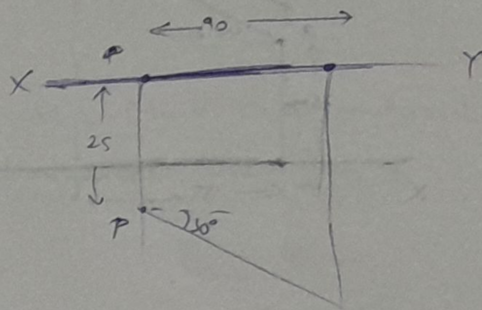
If line // HP, then true length will be visible in HP or top view only



Q) Line PQ — 90 mm long

is in HP & makes an angle with 30° with VP

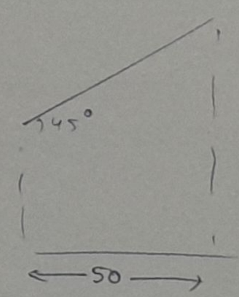
P is 25 mm in front of VP. Draw its Projection

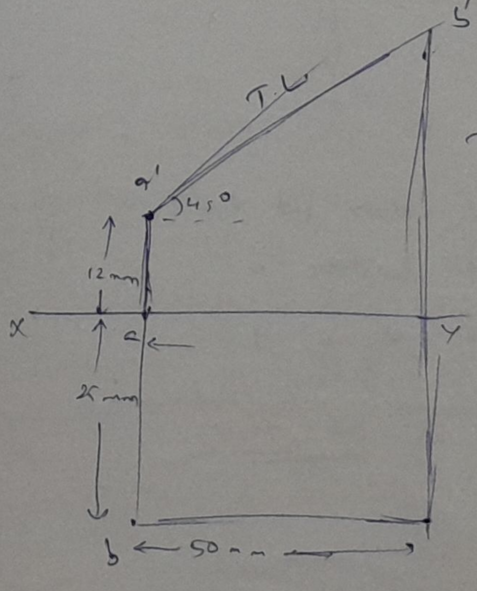


Q) Length of the top view parallel to VP and inclined at 45° to the HP is 50 mm.

One end of the line is 12 mm above HP and 25 mm infront of VP.

Find true length and Draw projection





True length : $\underline{50\sqrt{2}}$ mm

$= 70.7$ mm

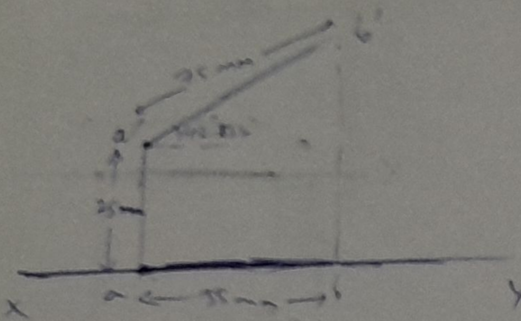
1.414

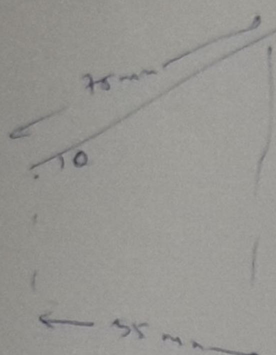
T.L = 70.7 mm

Q) Front view of 75 mm long line measures 55 mm.

Line - parallel to HP and one of its ends is in the VP and 25mm above HP

Draw Projection & Determine Inclination





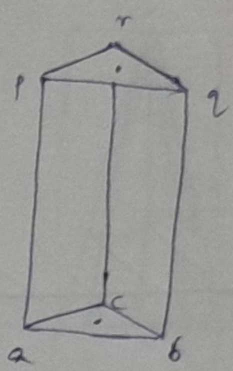
$$\theta = \cos^{-1}\left(\frac{55}{75}\right)$$

$$\underline{\theta = 42.835^\circ}$$

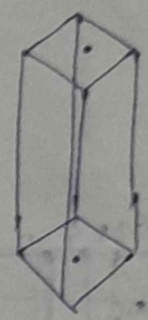
Projection of Solids

→ **Prisms:**

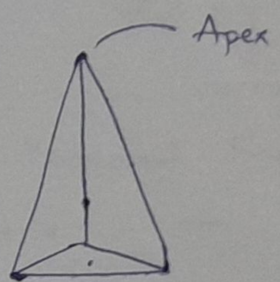
(1) Triangular Prism:

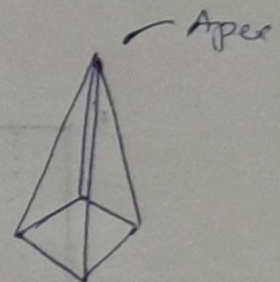


Rectangular Prism:



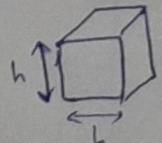
→ ~~Py~~ Pyramid:





→ **3D Shapes:**

Cube:

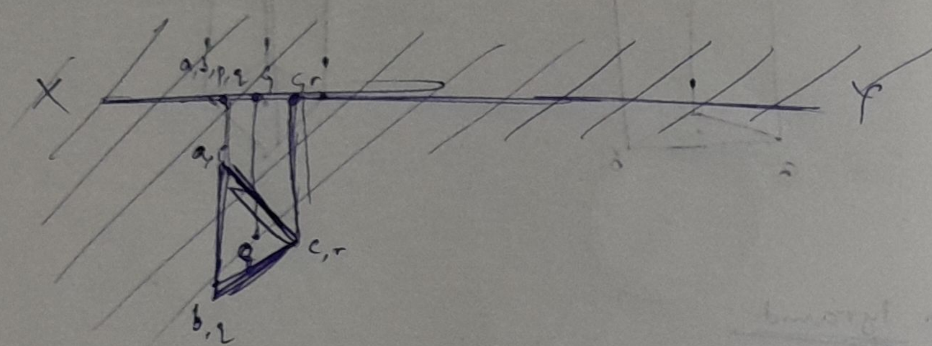


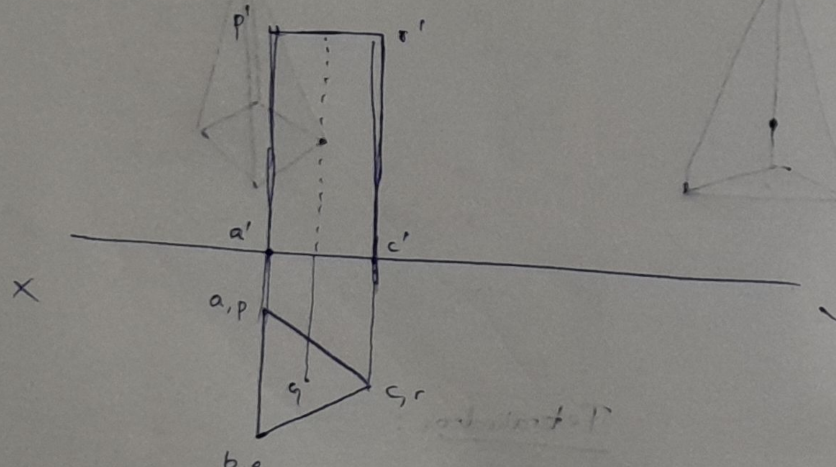
Tetrahedron:

Procedure: Draw Actual Shape of the base first

Q) Draw the projections of a Triangular Prism base 40 mm side and axis 50 mm long resting on one of ~~the~~ its bases on its H.P with a vertical face perpendicular to the VP.

Sol :



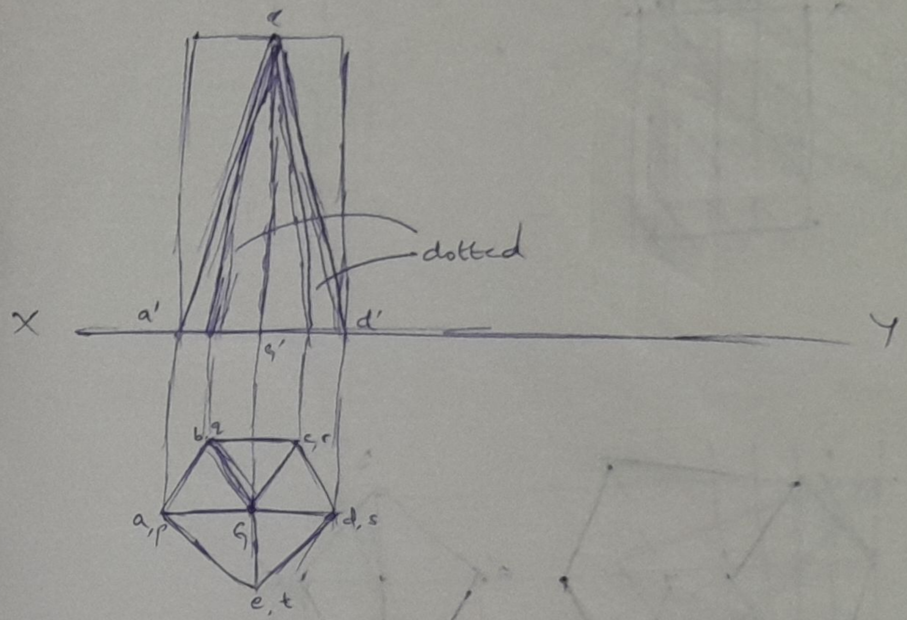


Q) Draw the projections of a Pentagonal ~~prism~~ pyramid

base length - 30 mm

Axis length - 50 mm

base on HP & parallel to VP



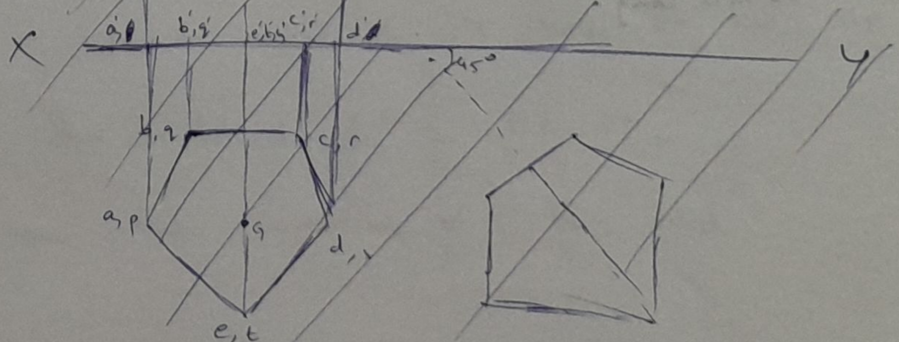
Q) Draw a Pentagonal Prism

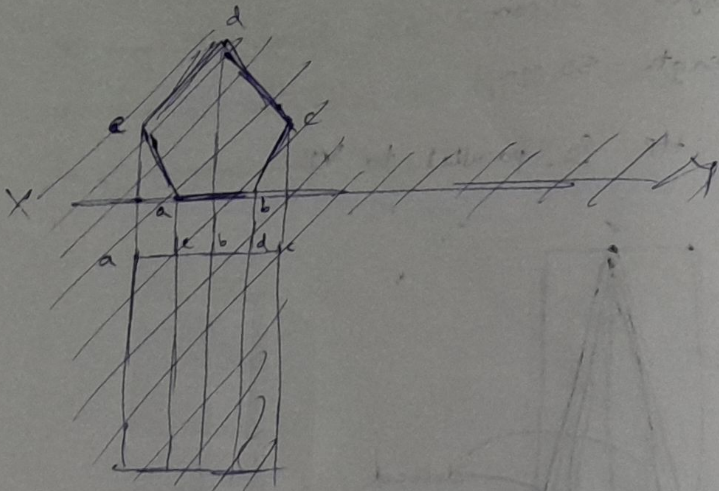
base - 25 mm

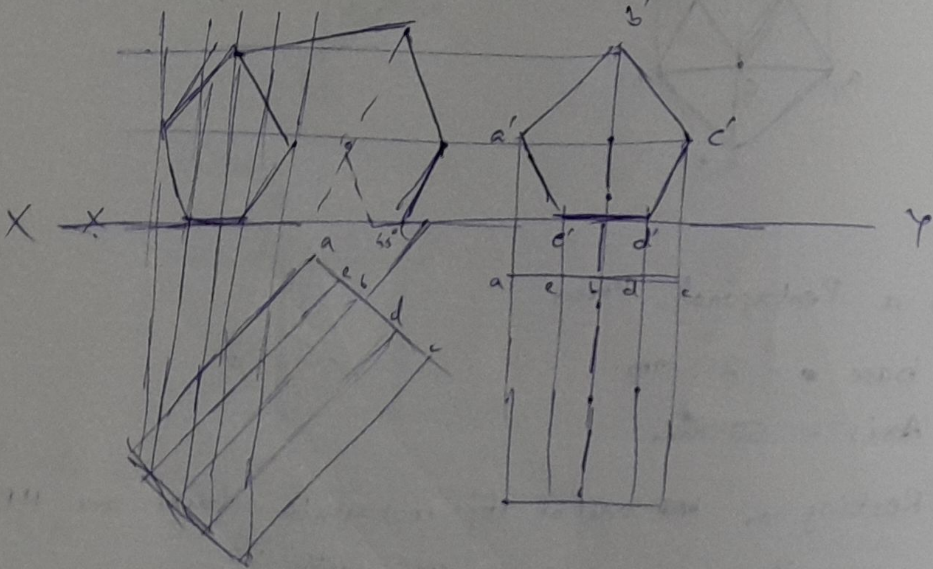
Axis - 50 mm

Resting on ~~HP~~ one of its rectangular faces on HP

with axis inclined 45° with VP





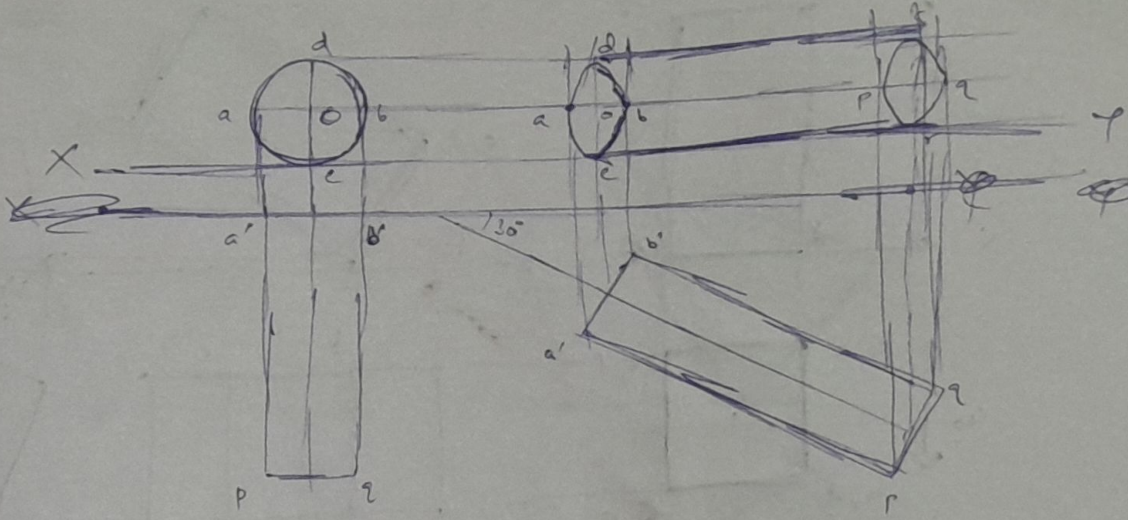


Q) Draw Projection of Cylinder

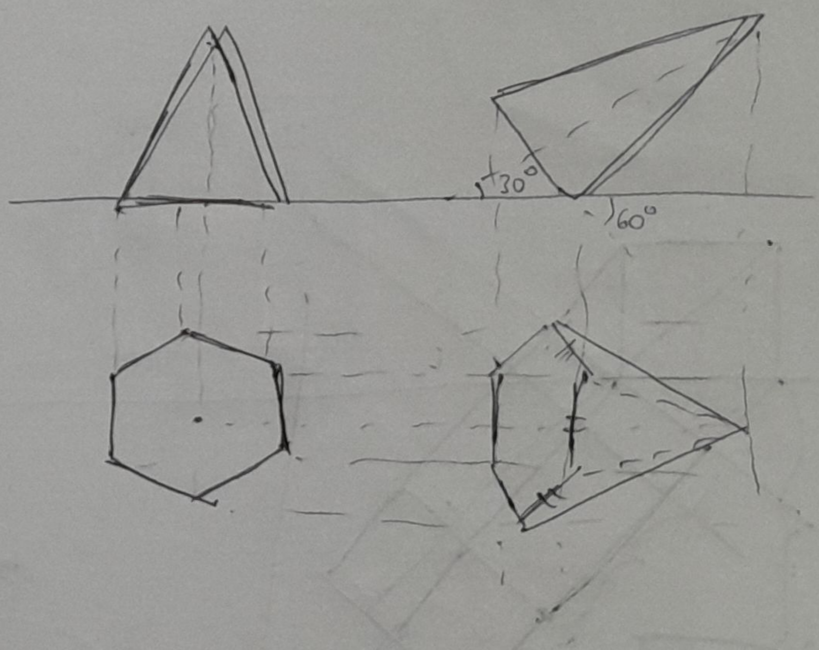
Lying on the ground axis 30° with VP & ∥ to ground

75 mm diametre

100 mm long



Q)

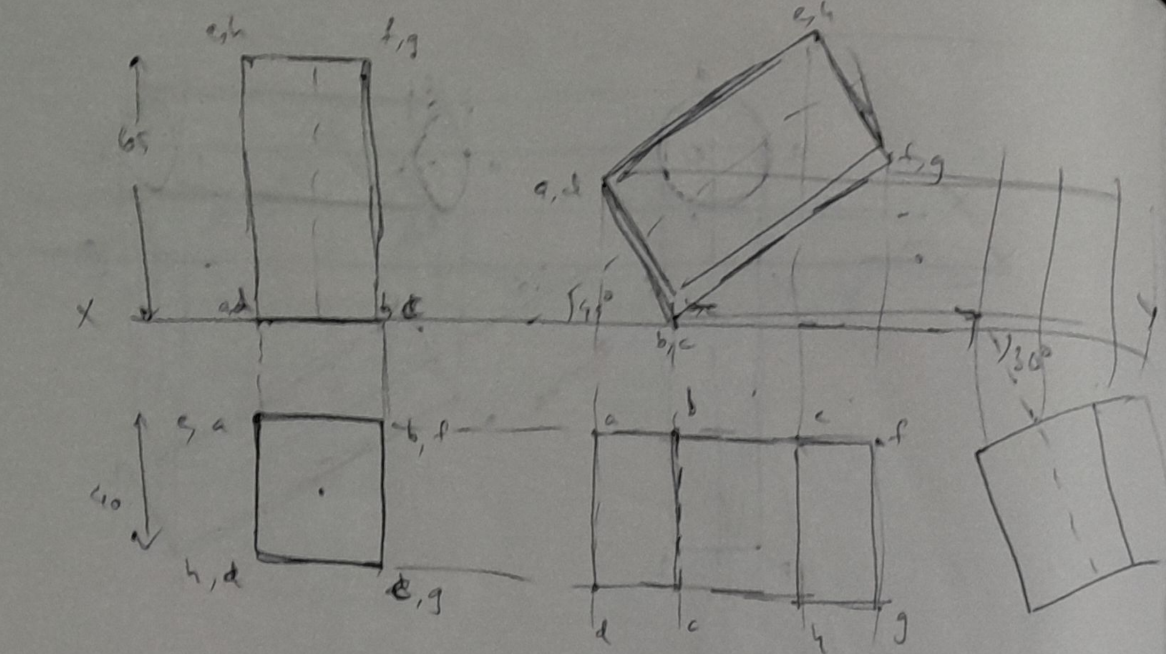


Q) A square Prism – 40 mm side – height 65 mm

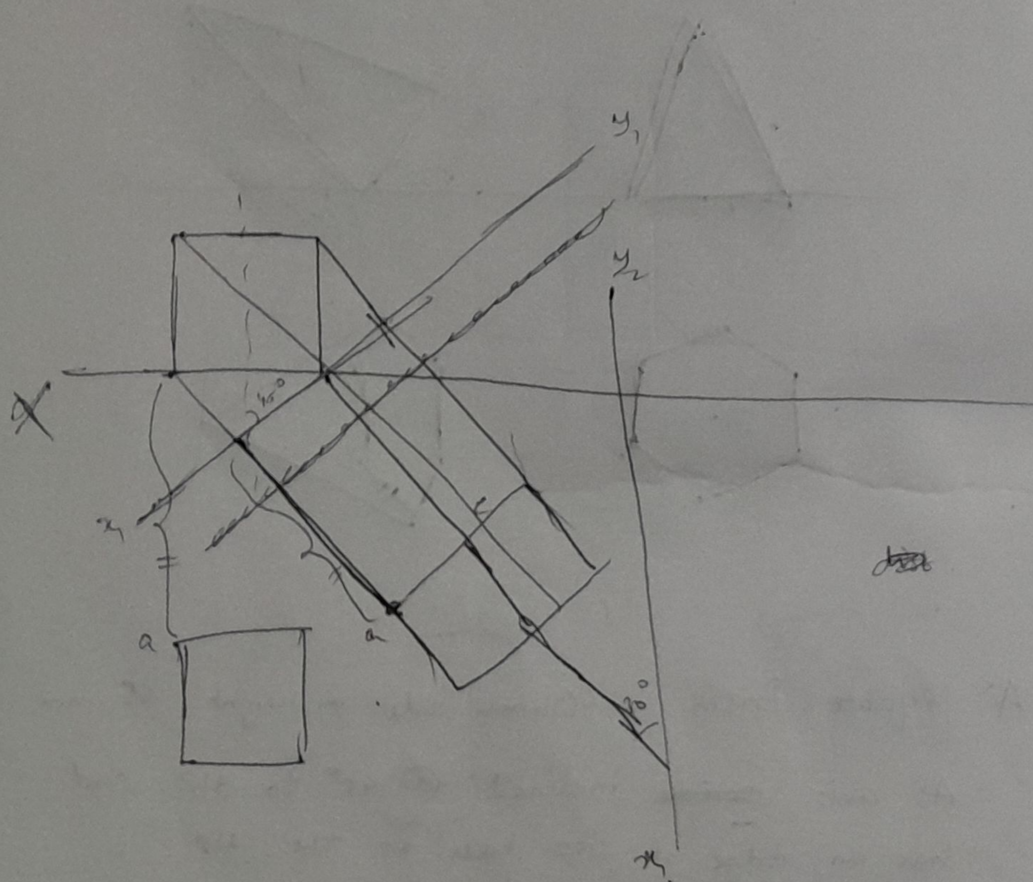
As axis ~~inclide~~ inclined @ 45° to HP and has an edge of its base on the HP

& Inclined 30° to the VP.

Draw the Projections of this solid.



→ Auxilliary Inclined Plane !

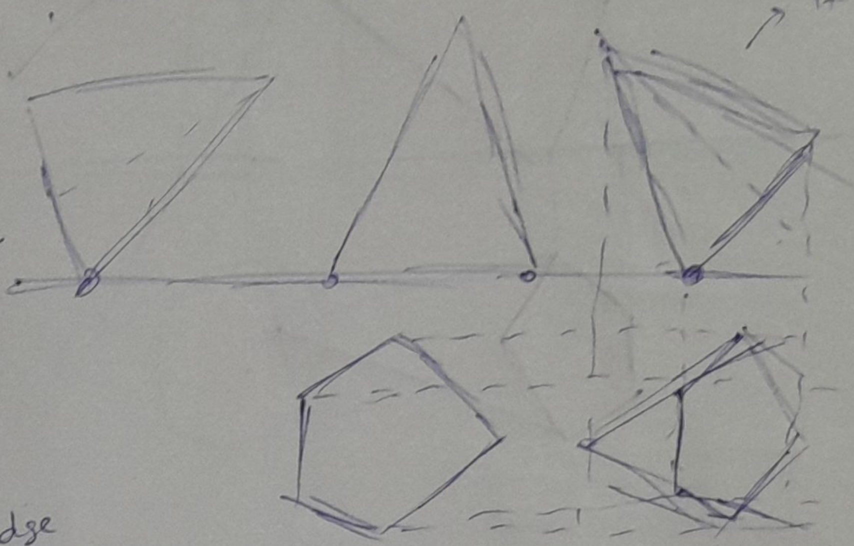


dist. b/w X-Y line to a

27/7/24

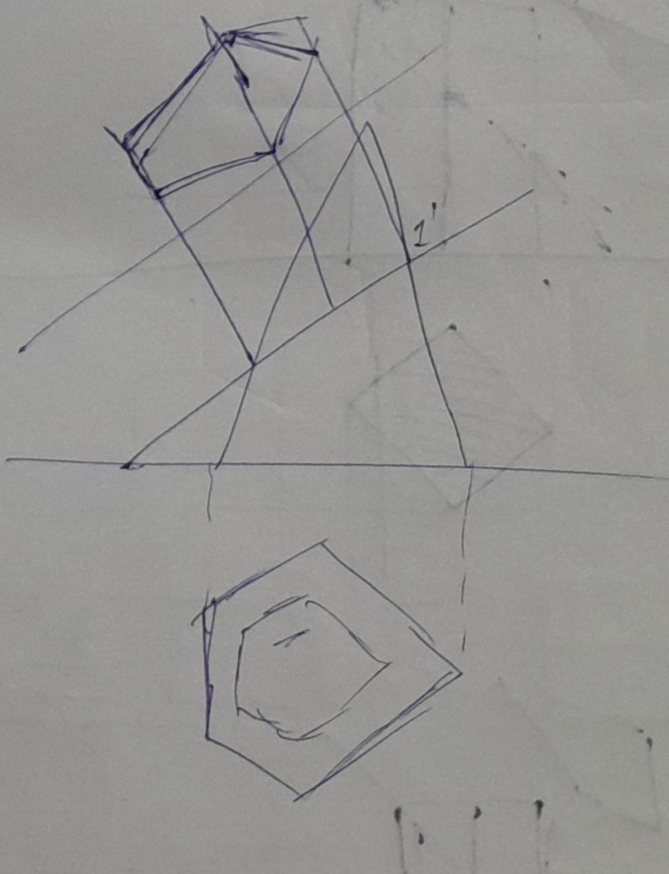
→ Pentagonal Pyramid:

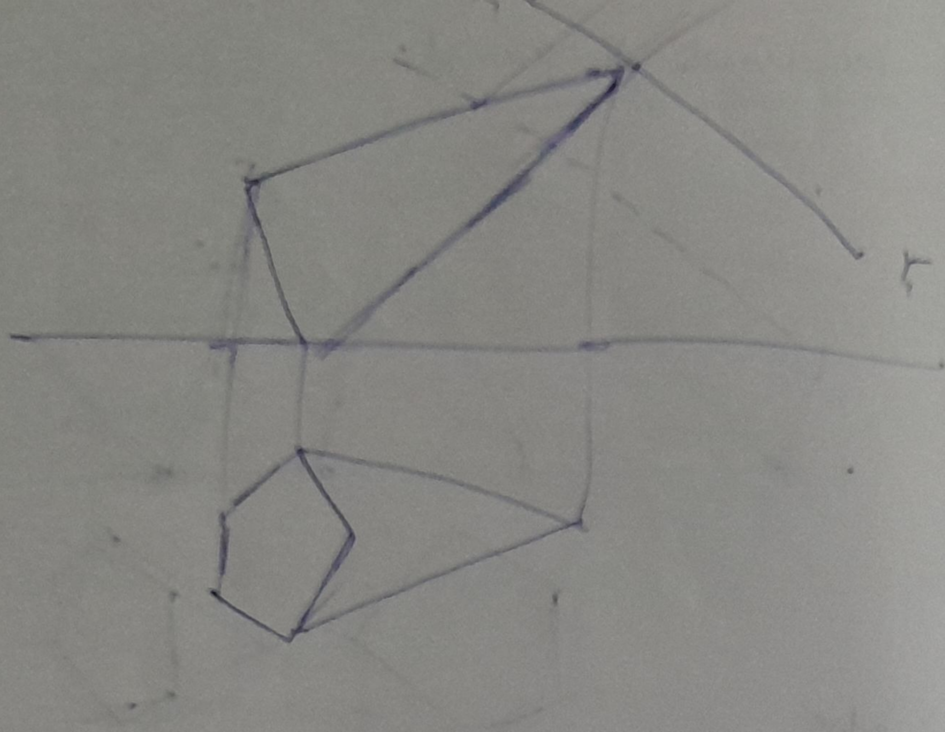
↗ If ~~edge~~ corner on ground

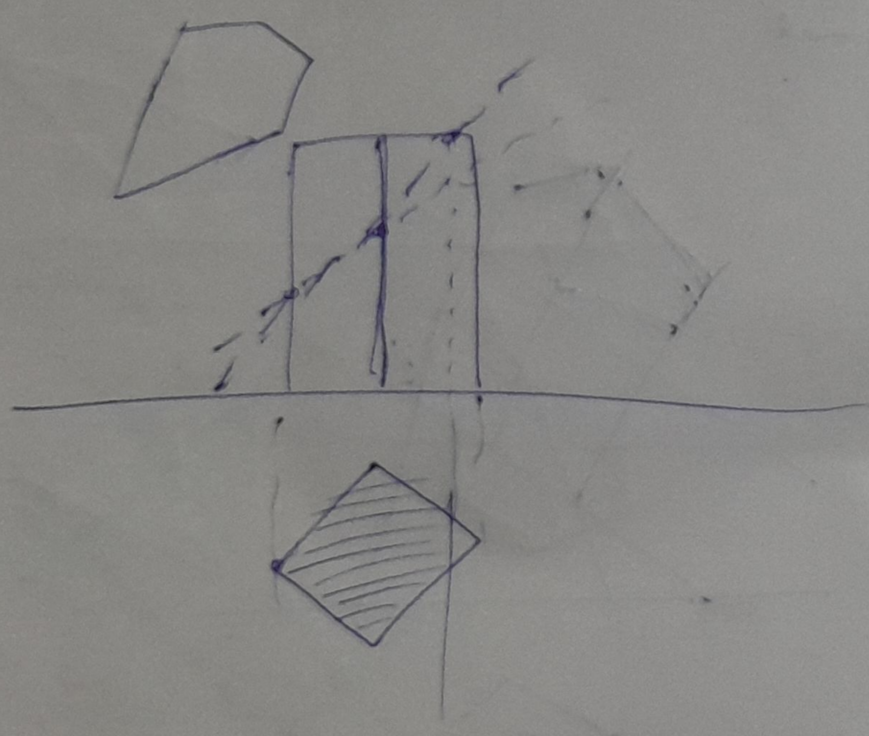


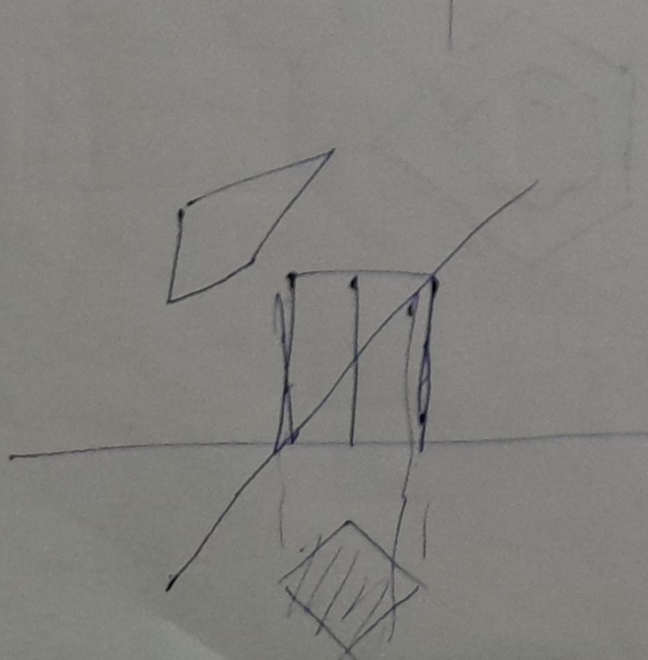
If ~~corner~~ edge on ground

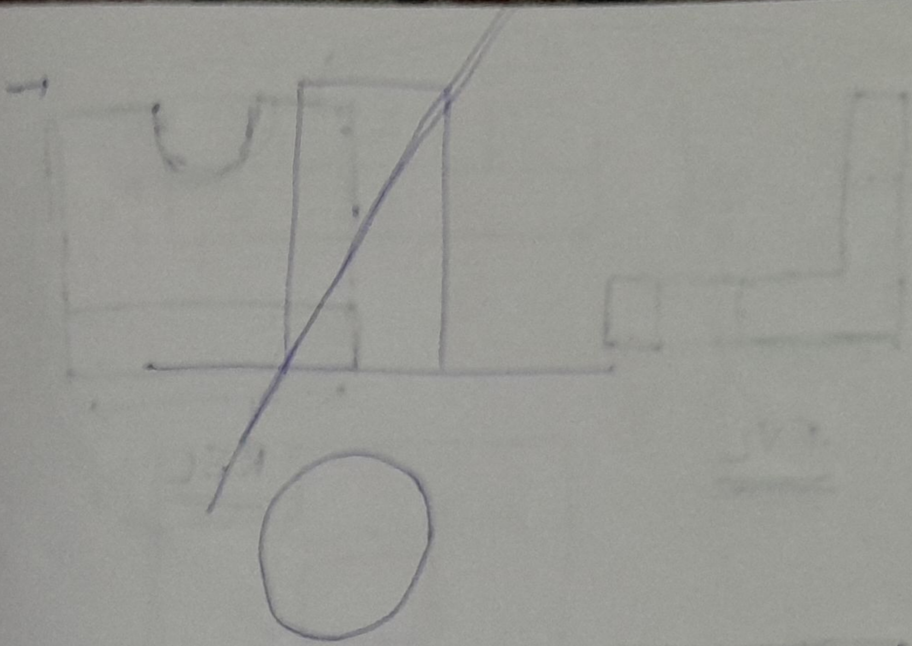
27/3/24

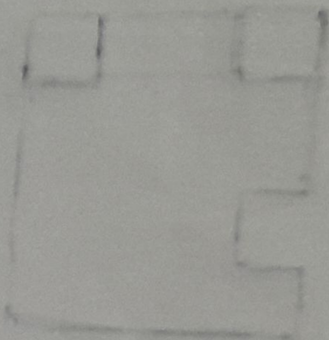




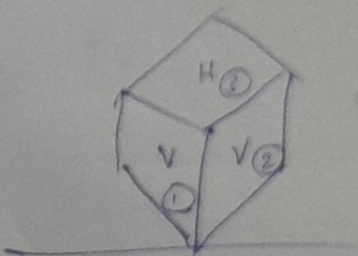


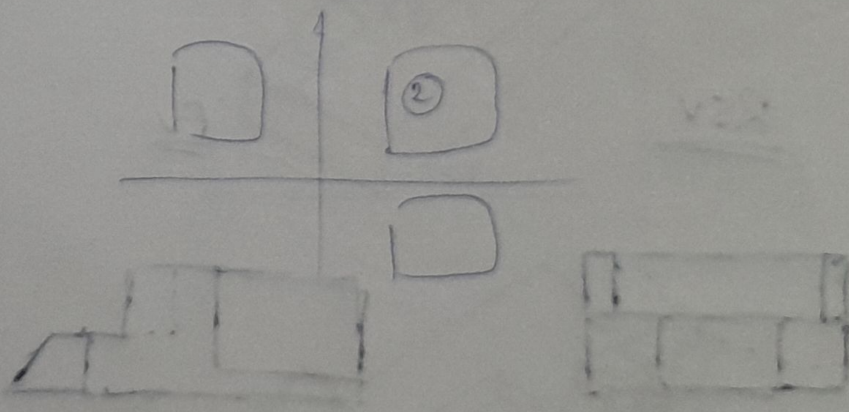


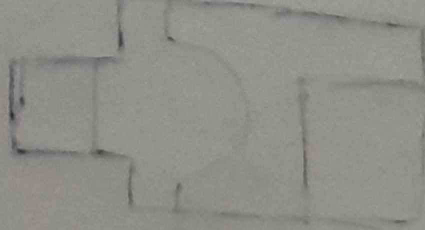




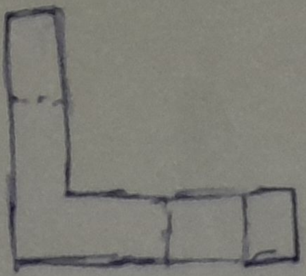
→ Isometric Projection :



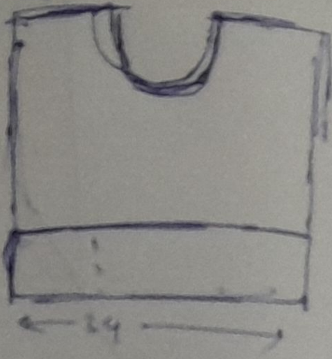




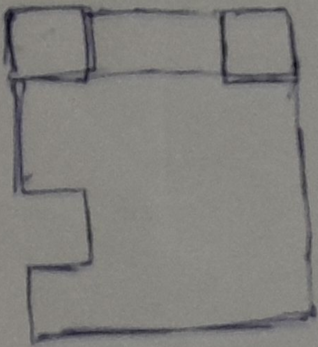
→



FVL



RSL



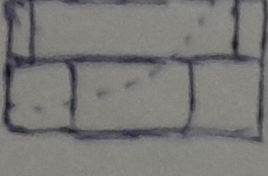
TV

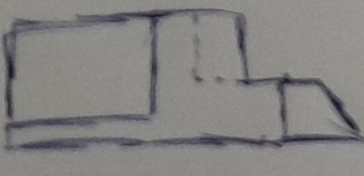
90°

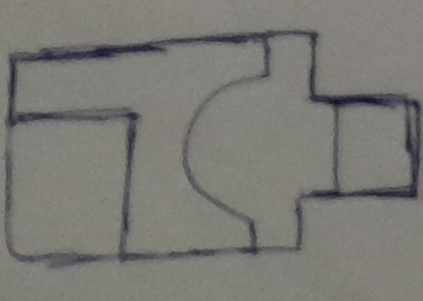
9)

RSV

FV

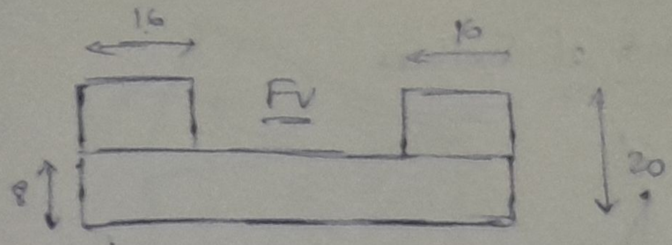


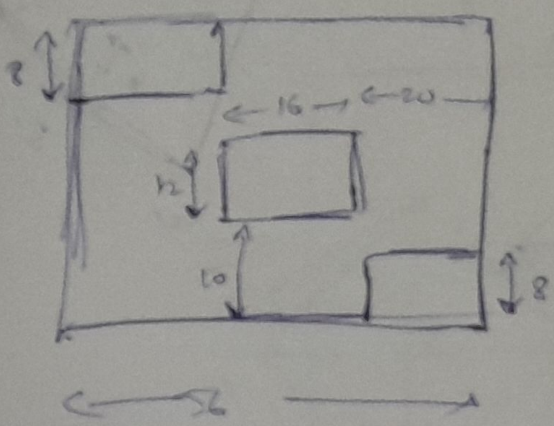




RS TV

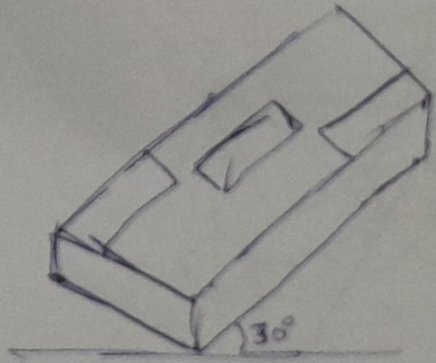
9)

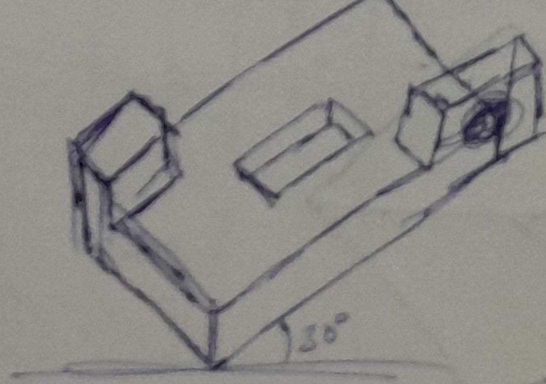




TV

Sol





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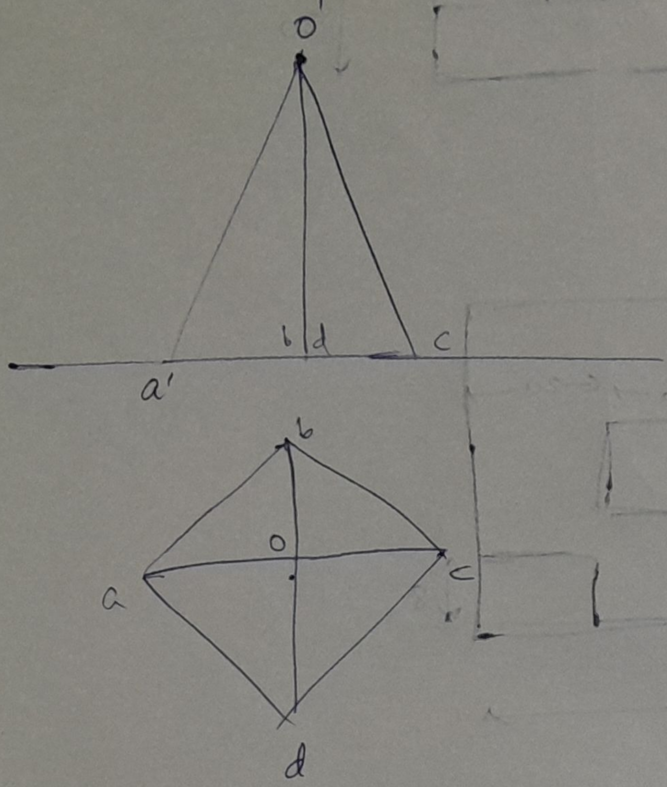
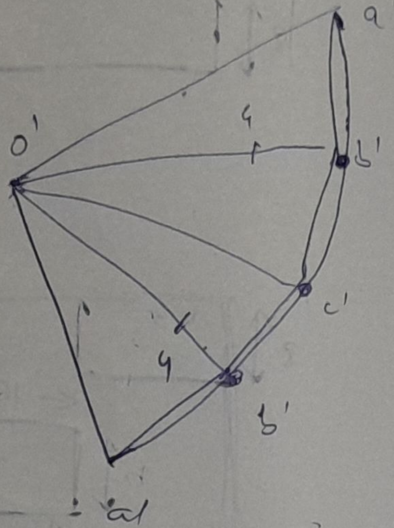
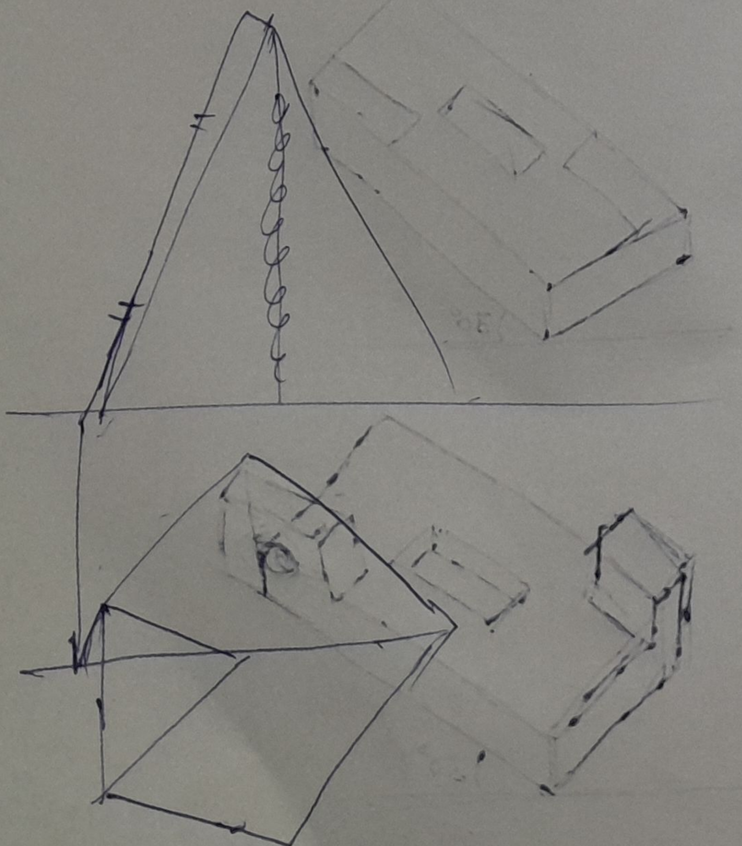
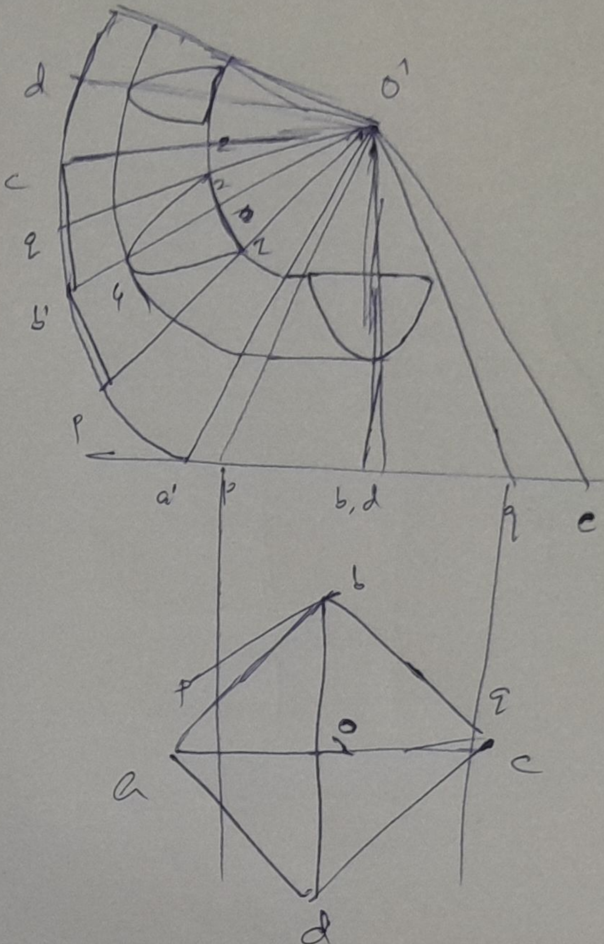
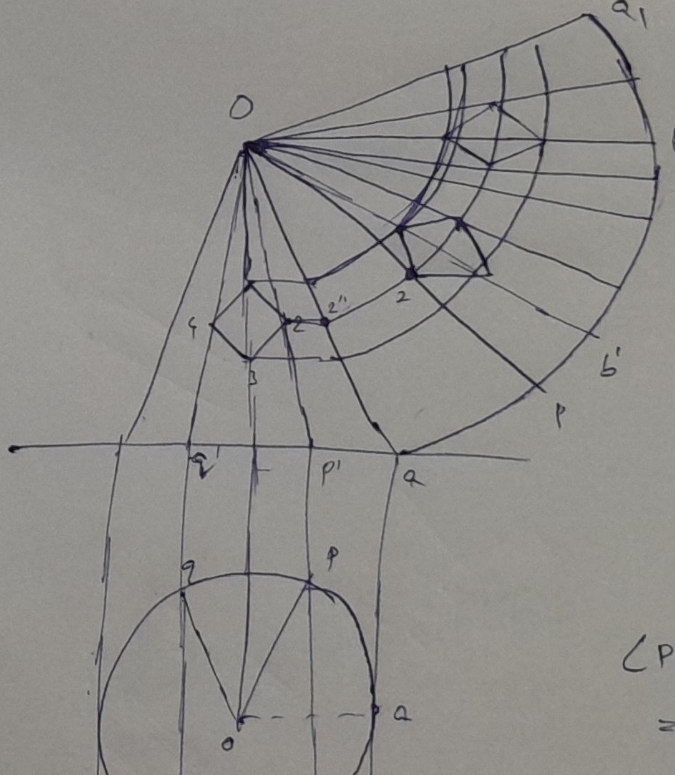
Development of Surfaces











$\angle POA$ on circle

$= \angle POA$ on curve ~~(down)~~

$|2''a| = |2p|$