Assignment-1

Waves and Vibrations (PH2001)

Marks: 5(2.5+2.5)

Last Date: 13/02/2024 (Till 24:00)

Groups 1:	CS20B1031, 44, 89, 98, 99	Question 1
Group 2:	CS23I1014, 29, 33, 37, 48	Question 2
Group 3:	EC23B1005, 15, 29, 36, 41	Question 3
Group 4:	EC23B1043, 44, 47, 48, 56	Question 4.
Group 5:	EC23B1060, 63, 64, 76, 83	Question 5
Group 6:	EC23B1091, 92, 95, 114, 118	Question 6
Group 7:	EC23B1123, EC23I1010, EC23I2001, ME20B1012, ME23B1004	Question 7
Group 8:	ME23B1008, 17, 26, 27, 29	Question 8
Group 9:	ME23B1031, 33, 42, 43, 44	Question 9
Group 10:	ME23B1062, 67, 80, 2002, 2019	Question 10
Group 11:	ME23B2020, 31, 37, 38	Question 11

Find the normal mode frequencies and its solutions for the given problem. Write a code (preferably in Python) to trace the locus of the particles.

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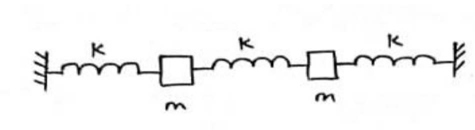
1. Consider two mass connected by three springs having k as spring constant, as shown in figure.



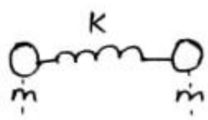
The equation of motion of the system is given by,

$$m\ddot{x}_1 + k(2x_1 - x_2) = 0$$

$$m\ddot{x}_2 + k(-x_1 + 2x_2) = 0$$

where x_1 & x_2 are the displacement of masses m from the equilibrium position. Find the normal mode frequencies?

2. Consider the following spring mass system of 'm' and spring constant 'k'.



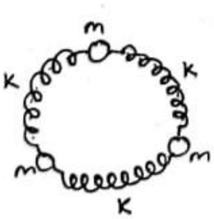
The equation of motion is given by,

$$m\ddot{x}_1 + \frac{kx_2}{2} = 0$$

$$m\ddot{x}_2 + \frac{k\mathrm{x}_1}{2} = 0$$

where 'x_1' & 'x_2' are the displacement of masses from the equilibrium position. Find the normal modes frequencies?

3. Three masses 'm' each, initially located equidistant from one another on a horizontal circle of radius 'R'. They are connected in pairs by three springs of force constant 'k' each and of unstretched length '2πR/3'. The spring threads the circular tract so that the mass is constrained to move on the circle.



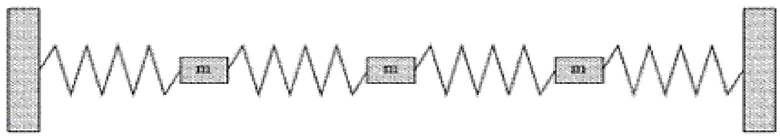
The equation of motion is given by,

$$mR^2\ddot{\theta}_1 + kR^2[(\theta_1 - \theta_2) - (\theta_3 - \theta_1)] = 0$$

$$mR^2\ddot{\theta}_2 + kR^2[(\theta_2 - \theta_3) - (\theta_1 - \theta_2)] = 0$$
$$mR^2\ddot{\theta}_3 + kR^2[(\theta_3 - \theta_1) - (\theta_2 - \theta_3)] = 0$$

Where θ_1, θ_2 and θ_3 are the angular displacements of the three masses from their equilibrium positions. Find the normal modes.

4. Consider a system of three equal masses 'm' and four springs, all with spring constant 'k', with the system fixed at the ends as shown in the figure below. The motion can only take place in one dimension, along the axes of the springs.



The equation of motion is given by,

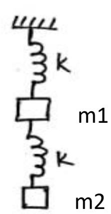
$$m\ddot{x}_1 + k(2x_1 - x_2) = 0$$

$$m\ddot{x}_2 + k(2x_2 - x_1 - x_3) = 0$$

$$m\ddot{x}_3 + k(2x_3 - x_2) = 0$$

where x_1, x_2 & x_3 are the displacement of masses 'm' from the equilibrium position. Find the normal mode frequencies?

5. Consider a spring-mass system of masses 'm_1' & 'm_2' and a spring constant 'k' as illustrated in Figure.



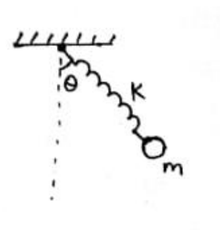
The equation of motion is given as,

$$m_1\ddot{x}_1 + k(x_1 - x_2) = 0$$

$$m_2\ddot{x}_2 + k(2x_2 - x_1) = 0$$

where x_1 & x_2 are the displacement of masses 'm' from the equilibrium position. Find the normal mode frequencies?

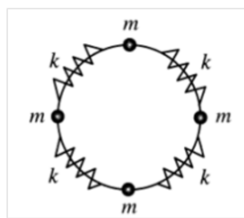
6. An elastic simple pendulum has 'b' as unextended length of spring, 'r' as extended length of spring, 'm' as mass of bob and 'k' as the spring constant as shown in figure. Find the normal mode frequency.



The equation of motion is given by,

$$m\ddot{r} - mr\dot{\theta}^2 - mgrcos\theta + kr - kb = 0$$
$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgrsin\theta = 0$$

7. Find the normal mode frequency for the given spring mass system arranged in a circle of radius of 'R'. Each mass is coupled to its two neighboring points by a spring constant 'k'.



The equation of motion is given by,

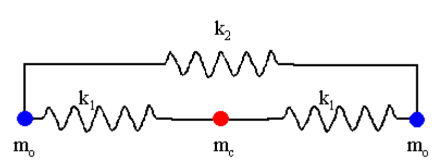
$$m\ddot{x_1} + \frac{k(x_2 + x_4)}{2} = 0$$

$$m\ddot{x_2} + \frac{k(x_1 + x_3)}{2} = 0$$

$$m\ddot{x_3} + \frac{k(x_2 + x_4)}{2} = 0$$

$$m\ddot{x_4} + \frac{k(x_3 + x_1)}{2} = 0$$

8. Find the normal modes frequencies for the given spring mass system shown below.



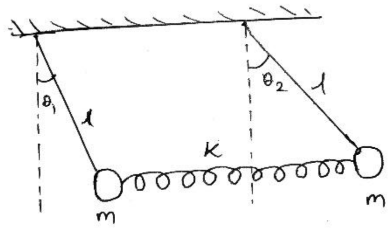
The equation of motion is given by,

$$m_0\ddot{x_1} - k_1(x_2 - x_1) - k_2(x_3 - x_1) = 0$$

$$m_c\ddot{x}_2 + k_1[\,(x_2 - x_1) - (x_3 - x_2)] = 0$$

$$m_0\ddot{x}_3 - k_1(x_3 - x_2) + k_2(x_3 - x_1) = 0$$

9. Consider two pendulums that are connected by a spring of spring constant ‘k’ as shown in figure.



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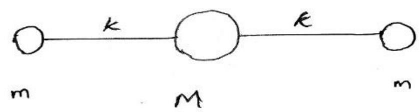
$$m\ddot{x}_1 + \frac{mgx_1}{l} - k\,(x_2 - x_1) = 0$$

$$m\ddot{x}_2 + \frac{mgx_2}{l} + k\,(x_2 - x_1) = 0$$

Find the normal mode frequency for the given system.

10. Consider a triatomic molecule, consisting of two molecules of masses ‘m’ connected to mass ‘M’ molecule. Assume that their bond is made by spring with spring constant ‘k’.



The equation of motion is given by,

$\eta_i = x_i - x_{i0}$ (x_{i0} is equilibrium position. x_i is stretched position)

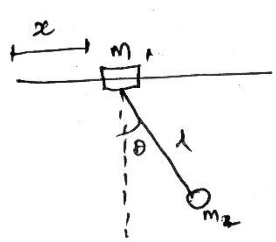
$$m\ddot{\eta}_1 + k\,(\eta_1 - \eta_2) = 0$$

$$M\ddot{\eta}_2 + k\,[\,(\eta_2 - \eta_1) + (\eta_2 - \eta_3)\,] = 0$$

$$m\ddot{\eta}_3 + k\,(\eta_3 - \eta_2) = 0$$

Find the normal mode frequency for the given system.

11. Consider a pendulum attached to mass ‘m_1’. Mass ‘m_1’ can move in x direction.



The equation of motion is given by,

$$(m_1 + m_2)\ddot{x} + m_2 l\ddot{\theta}\,\cos\theta - m_2 l\dot{\theta}^2 \sin\theta = 0$$

$$\ddot{x}\cos\theta + l\ddot{\theta} + g\sin\theta = 0$$

Find the normal mode frequency for the given system.