Tutorial for practice - Module 2

1.) Solve: $y'' + (1 - \cot x)y' - (\cot x)y = \sin^2 x$.

2) Find five distinct solutions of $y'' - 2y' + 2y = 3\sin x - \cos x$.

3.) Find the general solutions of the following equations

(a) $y'' + a^2 y = \sec ax$ (b) $y'' + 4y = \tan 2x$

4) Show that method of variation of parameters applied to the equation $y'' + y = f(x)$ leads to the particular solution

$$y_p(x) = \int_0^x f(t) \sin(x-t)\, dt$$

5.) Show that $x^2 y'' + p\,x\,y' + q\,y = 0$ can be transformed into differential equation with constant co-efficients with the transformation $x = e^z$.

6.) Let $f(x) = x^3$ and $g(x) = x^2|x|$ on the interval $[-1, 1]$ show that their wronskian is zero on $[-1, 1]$ and they are not linearly dependent. Do they contradict lemma on wronskian of two functions

7.) The equation $(1-x^2)y'' - xy' - a^2 y = 0$ has $y = e^{a \sin^{-1} x}$ as one solution. Find the general solution.

8.) Find the general solution of each of the following equations

(a) $y'' + y' = 0$; (b) $y'' + y = 0$; (c) $y'' - 2y' + y = 0$

9) State the following (a) Picard's theorem (b) The existence and uniqueness theorem for second order linear equations.