Tutorial -2
Signals and Systems

1. Compute and plot $y[n] = x[n] * h[n]$, where
   a. $x[n] = \begin{cases} 1, 3 \le n \le 8 \\ 0, otherwise \end{cases}$ and $h[n] = \begin{cases} 1, 4 \le n \le 15 \\ 0, otherwise \end{cases}$
   b. $x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$ and $h[n] = u[n-1]$
   c. $x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$ and $h[n] = u[n+2]$

2. Let $x[n] = \begin{cases} 1, 0 \le n \le 9 \\ 0, otherwise \end{cases}$ and $h[n] = \begin{cases} 1, 0 \le n \le N \\ 0, otherwise \end{cases}$ where $N \le 9$ is an integer.

   Determine the value of N given that $y[n] = x[n] * h[n]$ and $y[4] = 5$ and $y[14] = 0$.

3. Compute $y(t) = x(t) * h(t)$
   a. $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$
   b. $x(t) = \begin{cases} 1, 0 \le t \le 1 \\ 0, otherwise \end{cases}$ and $h(t) = x(t/\alpha)$, where $0 < \alpha \le 1$

4. The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
   a. $h[n] = (\frac{1}{3})^n u[n]$
   b. $h[n] = (5)^n u[3-n]$
   c. $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$
   d. $h[n] = n(\frac{1}{3})^n u[n-1]$

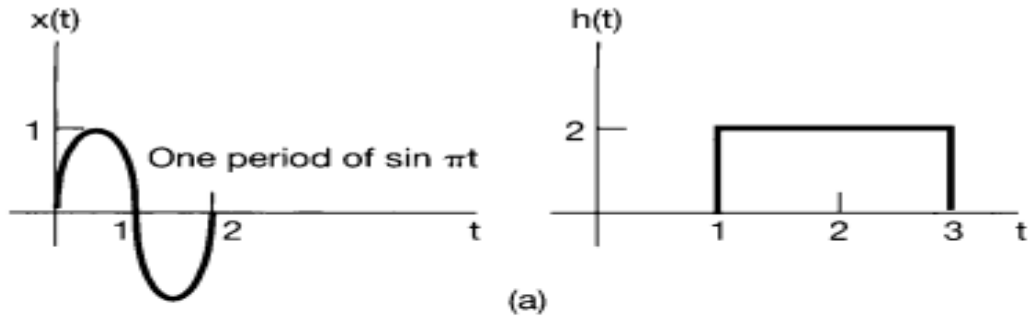
5. Which of the following impulse responses correspond(s) to stable LTI systems?
   a. $h(t) = e^{-(1-2j)t}u(t)$
   b. $h(t) = e^{-t}cos(2t)u(t)$
   c. $h[n] = ncos(\frac{\pi}{4}n)u[n]$
   d. $h[n] = 3^n u[-n+10]$

6. Consider an LTI system whose input *x(t)* and output *y(t)* are related by the differential equation

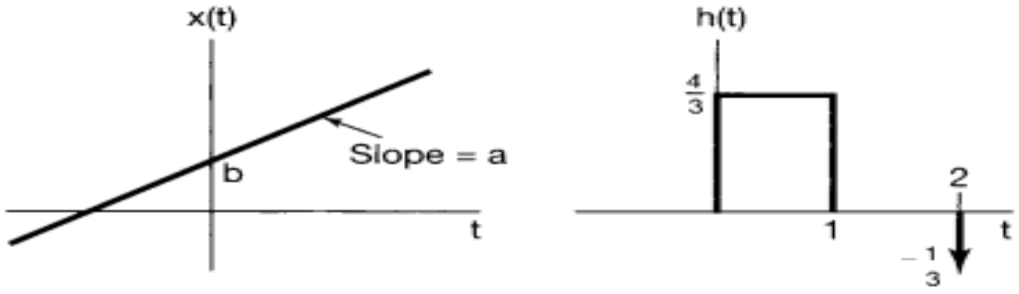
$$\frac{d}{dt}y(t) + 4\,y(t) = x(t)$$

   The system also satisfies the condition of initial rest.If $x(t) = e^{(-1+3j)t}u(t)$, what is $y(t)$?

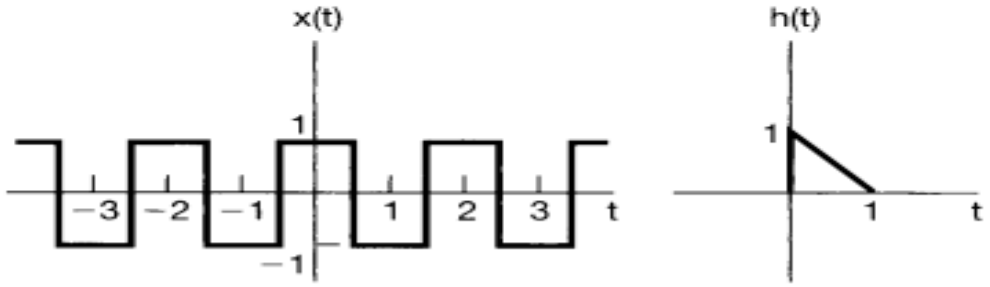
7. For each of the following pairs of waveforms, use the convolution integral to find the response *y(t)* of the LTI system with impulse response *h(t)* to the input *x(t).* Sketch your results.



(a)



(b)



(c)

8. Consider a causal LTI system whose input *x[n]* and output *y[n]* are related by the difference equation $y[n] = \frac{1}{4}y[n-1] + x[n]$. Determine $y[n]$ if $x[n] = \delta[n-1]$.

9. Suppose that the signal $x(t) = u(t+0.5) - u(t-0.5)$ is convolved with the signal $h(t) = e^{j\omega_0 t}$. Determine a value of ω_0 which ensures that $y(0) = 0$.

10. If $x_1[n] = (0.5)^n u[n]$, $x_2[n] = u[n+3]$ and $x_3[n] = \delta[n] - \delta[n-1]$
    a. Evaluate $(x_1[n] * x_2[n]) * x_3[n]$
    b. Evaluate $x_1[n] * (x_2[n] * x_3[n])$