IIITDM Kancheepuram

Discrete Structures for Computer Science

Assignment 1

Assigned Date: 7^{th} April 2024 Maximum Points: 20

General instructions:

1. Each question carries one point.
2. Write your **Roll number** and **Name** on each sheet of the paper.
3. Submit all the answers as a single PDF file named with your roll number.

Logic Questions

1. Write FOL for the following;
   (a) You can fool all the people some of the time, and some of the people all the time, but you cannot fool all the people all the time.
   (b) Everyone wants to get government job but no one wants to study in government school.
   (c) There exist only two types of quantifiers universal quantification and existential quantification.
   (d) Everyday in our life may not be Good, but there is something Good in Everyday.
2. Write FOL for the following;
   (a) Someone likes someone
   (b) Someone likes all
   (c) None likes everyone
   (d) None likes all
3. Write the definition of prime number in FOL.
4. Negate and Simplify. $\exists L \exists n \exists z(|z| \geq n \wedge \exists u \forall v \exists w((z = uvw, |uv| \leq n) \rightarrow \exists i(i \geq 0 \rightarrow uv^i w \neq L)))$
5. Prove or disprove
   (a) $\exists x(P(x) \leftrightarrow Q(x)) \rightarrow \neg \forall x Q(x) \vee \exists x P(x)$.
   (b) $\exists x(P(x) \leftrightarrow Q(x)) \rightarrow \neg \forall x P(x) \vee \exists x Q(x)$.
   (c) $\forall x \forall y P(x, y) \leftrightarrow \forall y \forall x P(x, y)$.
   (d) $\forall x \exists y P(x, y) \leftrightarrow \exists y \forall x P(x, y)$
6. Write four different expressions equivalent to $\forall x(P(x) \vee Q(x))$
7. Check whether the following Boolean expression is a tautology without using truth table
   (a) $(p \wedge q \wedge r) \rightarrow (r \vee p)$
   (b) $(p \leftrightarrow q) \rightarrow (\neg r \rightarrow (p \wedge q))$
8. Prove or disprove; some students prepare for JEE and NEET. Some prepare for either NEET or JEE. Therefore, there are students who have taken neither JEE nor NEET.
9. Prove or disprove; all students of second btech are eligible for all internships. Some internships have specific requirements. Therefore, some second btechs are interning in a company with specific requirements.
10. Check the validity of the argument; Some functions are continuous or differentiable. All functions have a peculiar property. Therefore, some continuous functions have a peculiar property.

(Turn the page)

Proof Techniques Questions

1. Show that every odd integer is the difference of two squares using direct proof.
2. Prove or disprove that if x and y are rational numbers, then x^y is also rational.
3. If $n \in N$ and $2^n - 1$ is prime, then n is prime(Proof by contraposition).
4. Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.
5. Show that $\sum_{i=1}^{n} n \cdot n! = (n+1)! - 1$ using M.I.
6. Consider the following four equations:
   i) $1 = 1$
   ii) $2 + 3 + 4 = 1 + 8$
   iii) $5 + 6 + 7 + 8 + 9 = 8 + 27$
   iv) $10 + 11 + 12 + 13 + 14 + 15 + 16 = 27 + 64$
   Conjecture the general formula suggested by these four equations, and prove your conjecture using M.I.
7. Let F_n denote the nth Fibonacci number. Prove that $F_0 + F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$.
8. Is it possible to produce change for n rupees by using rupees 5 and 6 such that number of coins used is minimum.(Use Strong MI)
9. Show that in any group of 20 people (where any two people are either friends or enemies), there are either four mutual friends or four mutual enemies
10. A child watches TV at least one hour each day for seven weeks but, because of parental rules, never more than 11 hours in anyone week. Prove that there is some period of consecutive days in which the child watches exactly 20 hours of TV. (It is assumed that the child watches TV for a whole number of hours each day.)

********** All the best **********