# Electrical Circuits for Engineers (EC1000)

## Lecture-9 (a)
## AC circuits
## Sinusoidal Steady State Analysis (Ch. 10)
## Mesh & Nodal Analysis

# 10. Sinusoidal Steady State Analysis

1. Introduction
2. Nodal Analysis
3. Mesh Analysis
4. Super Position Theorem
5. Thevenin/Norton Theorem

# 10.1 Introduction

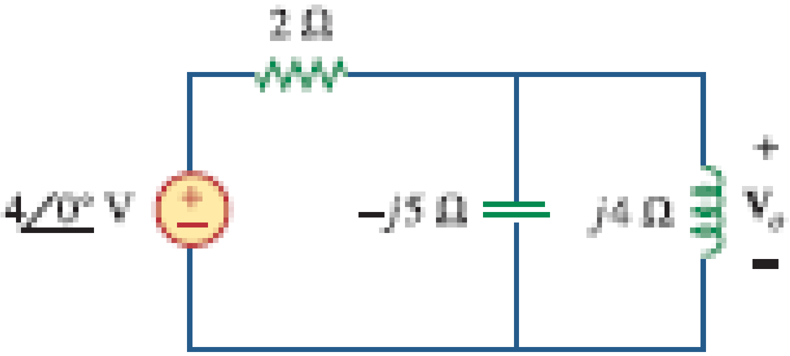
- We studied about steady state response of circuits response to sinusoidal inputs using Phasors.
- Kirchhoff's law and Ohm's law are applicable to ac circuits.
- In this chapter, we shall study about analysis of ac circuits using different techniques.

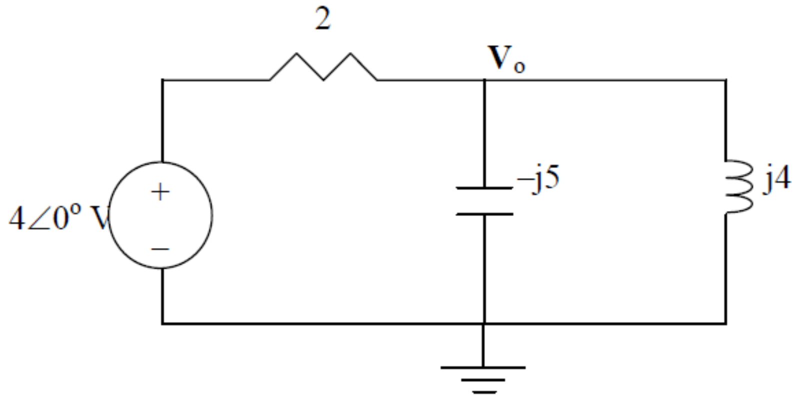
# Steps to Analyse AC circuits

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal, Mesh, theorem etc.,)
3. Transform the resulting phasor to the time domain

# 1. Nodal Analysis

10.1 Solve for $V_o$ in Figure, using nodal analysis.




**Solution**

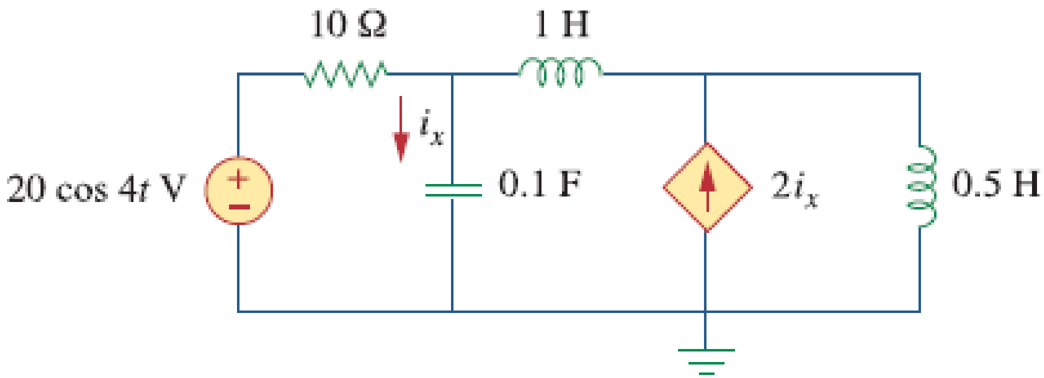
At the main node,

$$\frac{4-V_o}{2} = \frac{V_o}{-j5} + \frac{V_o}{j4} \longrightarrow 40 = V_o(10+j)$$

$$\mathbf{V_o} = 40/(10-j) = (40/10.05)\angle 5.71^\circ = \mathbf{3.98\angle 5.71^\circ\ V}$$

# 10.2.Nodal Analysis

10.2 Find $i_x$ in the circuit of Figure below using Nodal Analysis.




**Solution:**

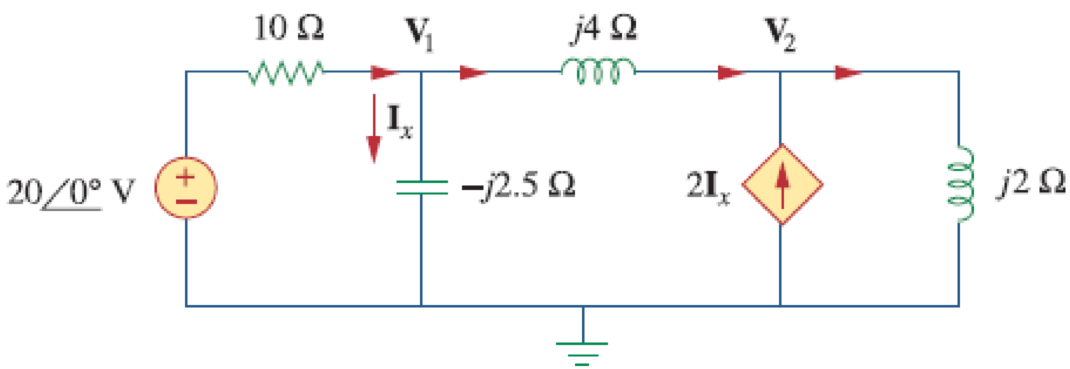
We first convert the circuit to the frequency domain:

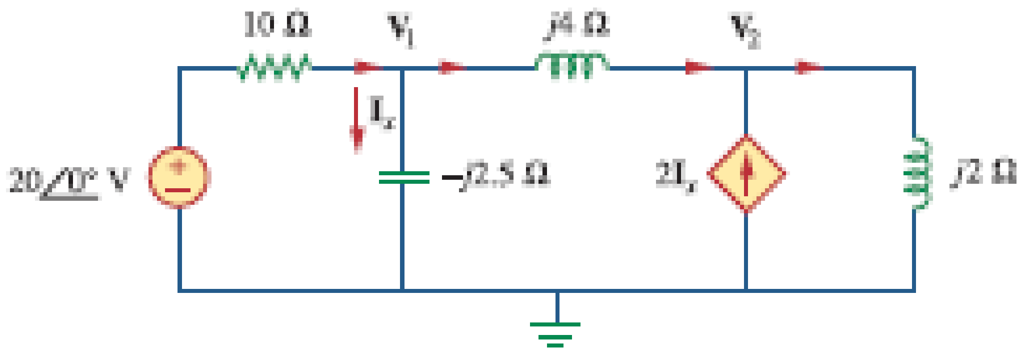
$$20\cos 4t \quad \Rightarrow \quad 20\angle 0^\circ, \qquad \omega = 4 \text{ rad/s}$$

$$1\text{ H} \quad \Rightarrow \quad j\omega L = j4$$

$$0.5\text{ H} \quad \Rightarrow \quad j\omega L = j2$$

$$0.1\text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$






Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

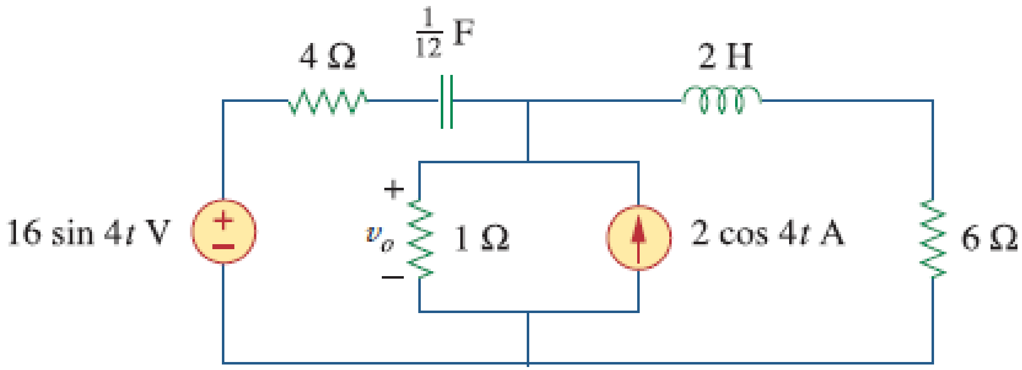
$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

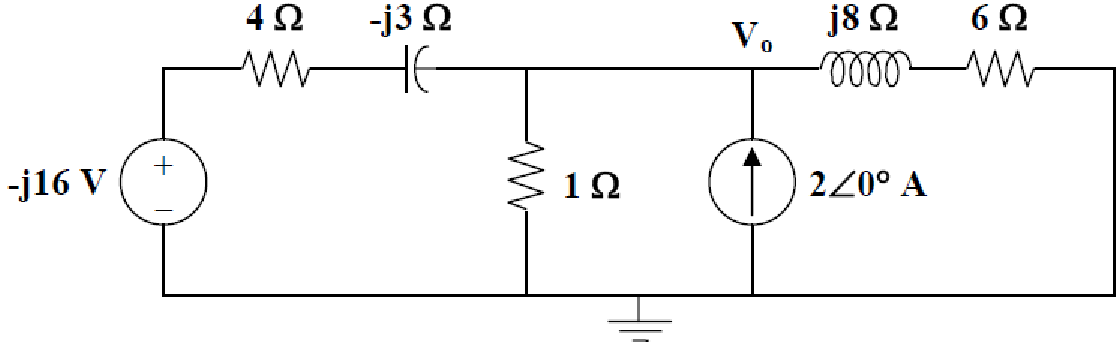
$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ}\text{ V} \\ \mathbf{V}_2 &= \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ}\text{ V} \end{aligned}}$$

# Practice Problem

10.3 Using Nodal Analysis, find $v_0$ for the given circuit.




**Solution**

$\omega = 4$

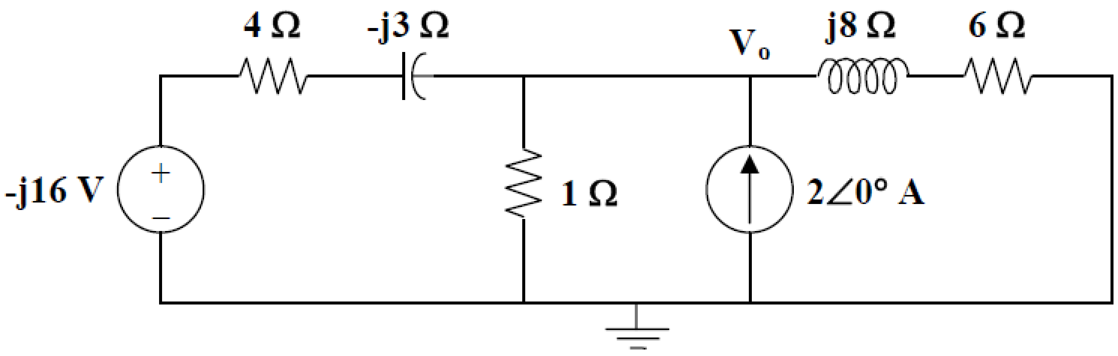
$$2\cos(4t) \longrightarrow 2\angle 0^\circ$$

$$16\sin(4t) \longrightarrow 16\angle -90^\circ = -j16$$

$$2\text{ H} \longrightarrow j\omega L = j8$$

$$1/12\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

Applying nodal analysis,




$$\frac{-j16 - \mathbf{V}_o}{4 - j3} + 2 = \frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{6 + j8}$$

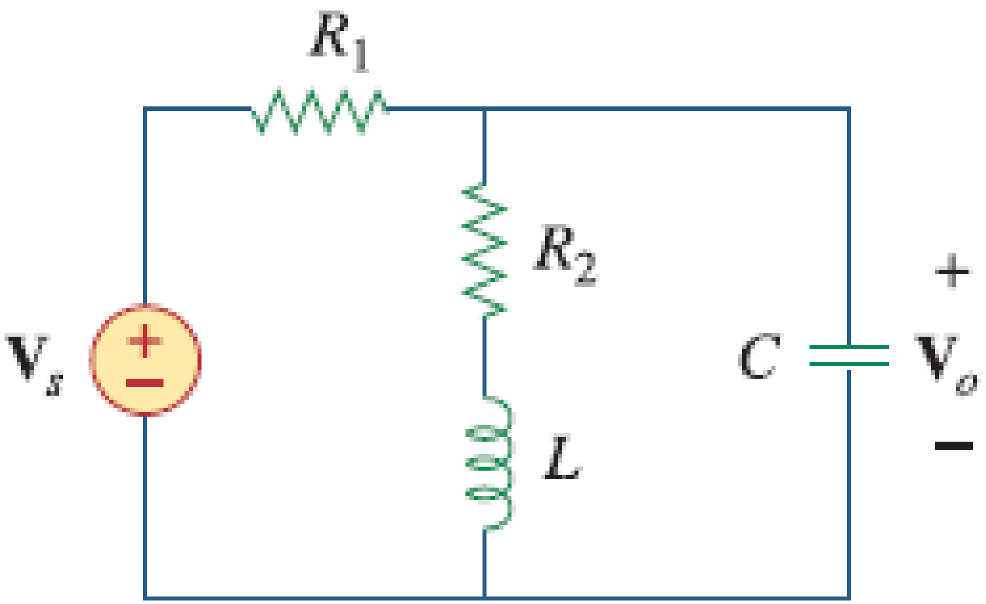
$$\frac{-j16}{4 - j3} + 2 = \left(1 + \frac{1}{4 - j3} + \frac{1}{6 + j8}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{3.92 - j2.56}{1.22 + j0.04} = \frac{4.682\angle -33.15°}{1.2207\angle 1.88°} = 3.835\angle -35.02°$$

$$\boxed{v_o(t) = \mathbf{3.835\cos(4t - 35.02°)\ V}}$$

# Practice Problem

1. For the given circuit find $V_o/V_s$ using Nodal Analysis

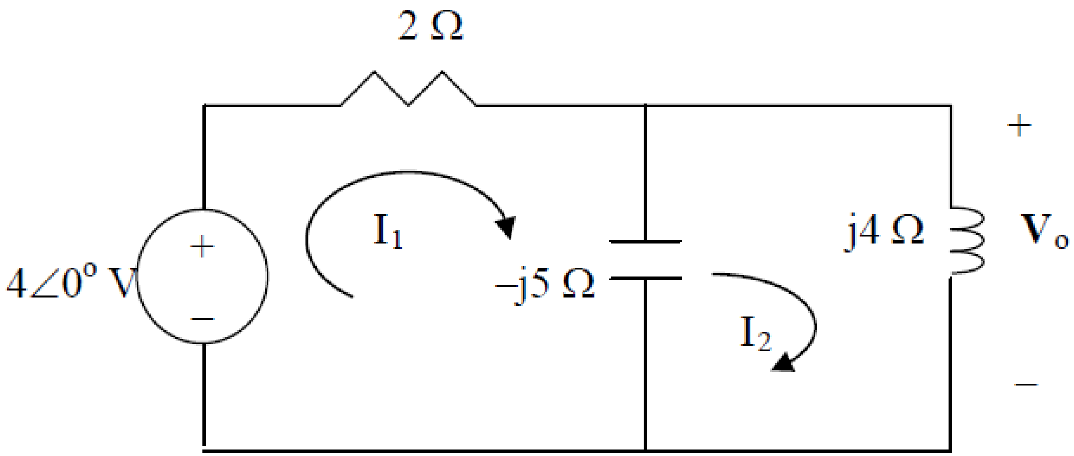



$$\frac{V_o}{V_s} = \frac{R_2 + j\omega L}{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1 R_2 C)}$$

# 2. Mesh Analysis

- **KVL forms the basis of Mesh Analysis**

1. Use mesh analysis to find $V_o$ in the circuit given below




For mesh 1,

$$4 = (2 - j5)I_1 + j5I_1 \qquad (1)$$

For mesh 2,

$$0 = j5I_1 + (j4 - j5)I_2 \longrightarrow I_1 = \frac{1}{5}I_2 \qquad (2)$$

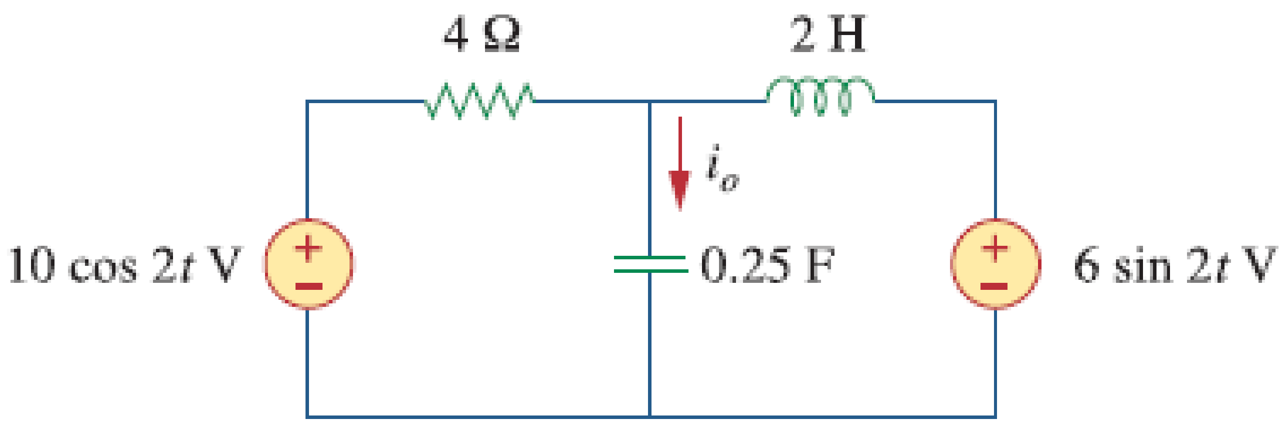
Substituting (2) into (1),

$$4 = (2 - j5)\frac{1}{5}I_2 + j5I_2 \longrightarrow I_2 = \frac{1}{0.1 + j}$$

$$\mathbf{V_o} = j4\mathbf{I_2} = j4/(0.1+j) = j4/(1.00499\angle 84.29°) = \mathbf{3.98\angle 5.71° \; V}$$

# Mesh Analysis

3. Find $I_0$ in the circuit of figure below using Mesh Analysis.




**Solution**

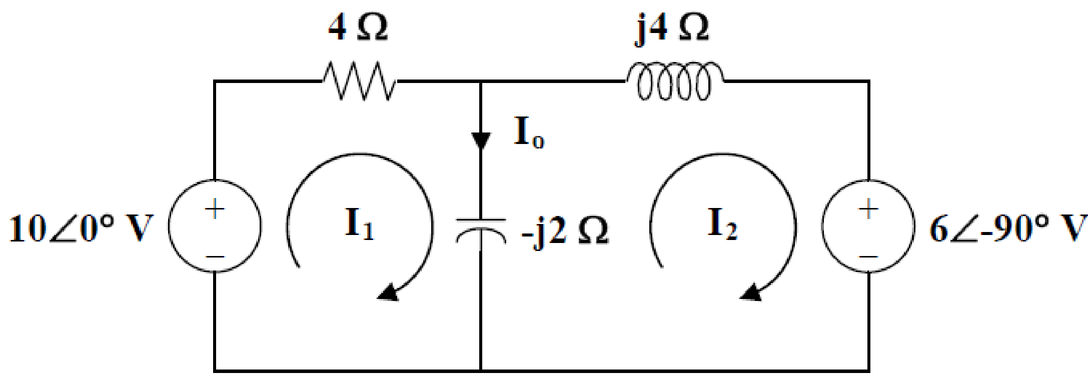
$\omega = 2$

$$10\cos(2t) \longrightarrow 10\angle 0^\circ$$

$$6\sin(2t) \longrightarrow 6\angle -90^\circ = -j6$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$



# Mesh Analysis

For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$
$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$
$$\mathbf{I}_1 + \mathbf{I}_2 = 3$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

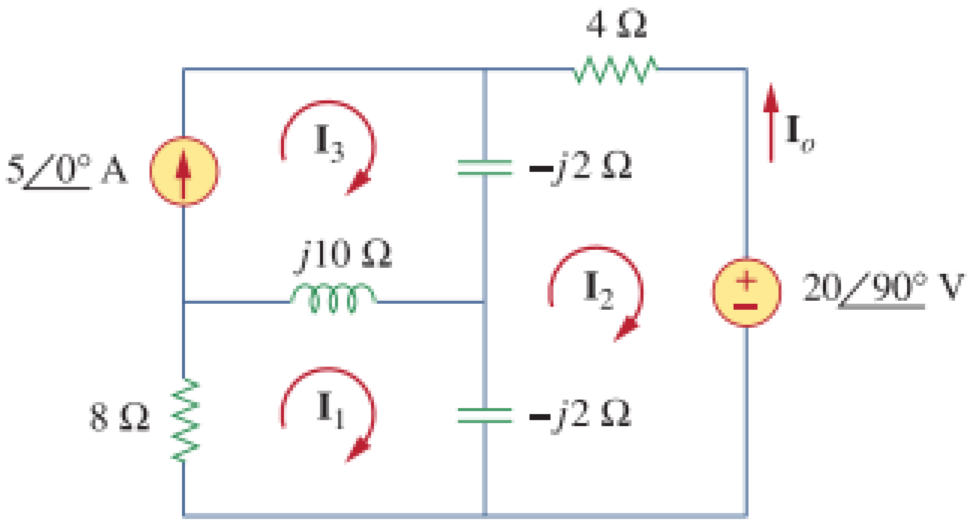
$\Delta = 2(1 - j)$, $\Delta_1 = 5 - j3$, $\Delta_2 = 1 - j3$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.4142\angle 45^\circ$$

$$\boxed{i_o(t) = \mathbf{1.4142\cos(2t + 45^\circ)\ A}}$$

# Mesh Analysis

4. Determine the current $I_0$ in the circuit of figure below using Mesh Analysis




## Solution

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$$

Substituting $I_3$=5 A

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$
$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\angle -35.22^\circ$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

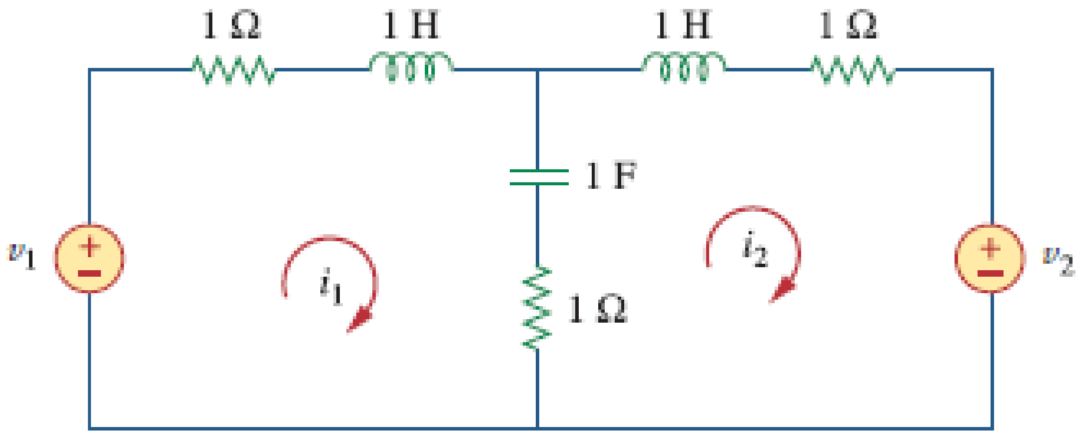
$$\boxed{\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}}$$

# Practice Problem

Determine the current $i_1$ and $i_2$ in the circuit of figure below using Mesh Analysis

Let $v_1 = 10 \cos 4t$ V

$v_2 = 20 \cos(4t - 30^\circ)$ V




**Ans:** $I_1 = 2.741\angle -41.07^\circ$, $I_2 = 4.114\angle 92^\circ$