1. Prove the following using the definition of convergence a sequence:

   (a) $\lim_{n\to\infty} \dfrac{\sqrt{n}-1}{\sqrt{n}+1} = 1.$

   (b) $\lim_{n\to\infty} n^{1/n} = 1.$

2. Prove that the sequence $\{a_n\}$, where $a_1 = 10$ and $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{10}{a_n}\right)$ for $n \geq 1$, converges. Also find the limit.

3. Find the limit of the sequence $\{a_n\}$ if $a_n = \dfrac{1}{(1+n)^2} + \dfrac{1}{(2+n)^2} + \ldots \dfrac{1}{(2n)^2}$.

4. Find the limit of the sequence $\{a_n\}$ if

   (a) $a_n = \left(1 + \dfrac{3}{4n}\right)^{\dfrac{8n}{3}}$.

   (b) $a_n = \sin(n!\alpha\pi)$, α where α is a rational number.

5. Let $a_n = \dfrac{1}{\ln(n+1)}$.

   (a) Show that limit of $\{a_n\}$ is zero.

   (b) Find a natural number N as required by the definition of convergence for each of (i) $\epsilon = 0.5$ and (ii) $\epsilon = 0.1$.