Waves and Vibrations (PH2001)

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Semester II

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Learning Objectives:

- To improve the conceptual, physical and mathematical comprehension of the phenomenon of waves and vibrations
- To Implement the understanding of waves and vibrations in real-time applications/device-design

Learning Outcome: You would be able to conceptualize the physical phenomenon of waves and/or vibrations for varieties of interdisciplinary product design applications

Evaluation: 100

***Continuous --- 25**

Assignment etc --- 15

Seminar --- 10

***MidSem --- 25**

***Sem. End --- 50**

Syllabus:

Module 1: Sources (electrical/mechanical/oceanic/optical) of waves and vibrations; Importance and applications of vibrations and waves in life; Free, damped, forced oscillations (Mathematical models)

Module 2: Wave equations, Classifications of Waves: transverse, longitudinal, plane, cylindrical, spherical, periodic, aperiodic, sinusoidal, square, triangular, saw tooth waves, polarization, circularly, plane, elliptically polarized waves with mathematical representation and examples/case studies from nature and real-time applications

Module 3: Superposition of waves, beats, wave packet, phase velocity, group velocity, dispersion, modulation, wave -plates, stationary and traveling waves, energy density

Module 4: Energy harvesting techniques along with basic electronic circuitry for product design applications

Module 5: Wave guiding and fiber Interferometers for smart sensing and measurement applications

References	1.Frank S Crawford Jr., Waves: Berkeley Physics Course Volume 3, McGraw Hill, 2008
	2. E. Hecht, Optics, Pearson, 5th edition, 2016 3. Shashank Priya and Daniel J Inman, Energy Harvesting Technologies, Springer, 2009 4. Daniele Tosi and Guido Perrone, Fiber-Optic Sensors for Biomedical Applications, Artech House, 2018

Introduction:

What?

Properties
Massive
Volume
Shape

States

Matter

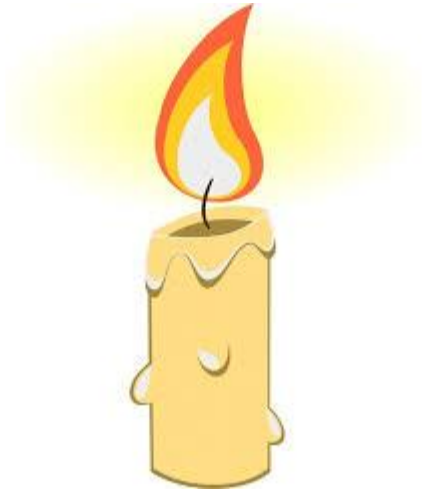
Universe

Energy

What?

Types

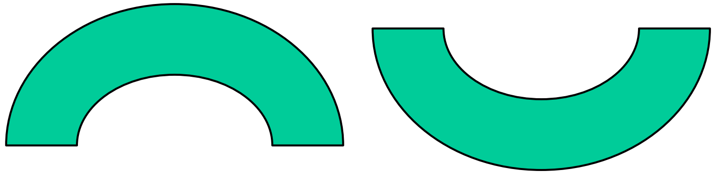
Conversion/source

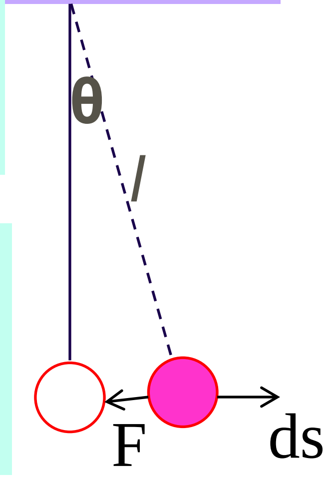


combustion -> hydrocarbons +oxygen;
carbon dioxide and **water**

Introduction:

Reason of oscillation : when a system is displaced from the equilibrium, a restoring force pulls it back and it moves to the other side because of the inertia.

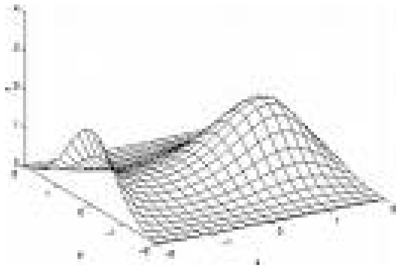




Equilibrium (Mechanical or Static): Mechanical state remain unchanged with time. Therefore, the net **force, moment of forces and torque** acting on the body are zero.

Stable Equilibrium: System comes back to its initial position after being displaced

Saddle Point: stable along a particular direction but unstable in other direction



$f_{xx}f_{yy}-f_{xy}^2 < 0$: SADDLE POINT.

$f_{xx}f_{yy}-f_{xy}^2 > 0$, and f_{xx} and f_{yy} are **both negative**, the point is a MAXIMUM.

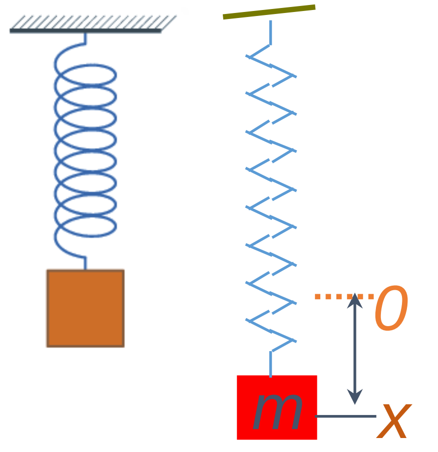
$f_{xx}f_{yy}-f_{xy}^2 > 0$ and f_{xx} and f_{yy} are **both positive**, the point is a MINIMUM.

In real world, most of the physical systems are not linear in nature

Introduction:

Simple Harmonic Motion (SHM) is a special case periodic motion where the restoring force acts towards the point of equilibrium

Mass attached to a spring (massless): Hooks law $\boxed{\vec{F} \propto x\hat{x}}$



For small displacement, $\boxed{\vec{F} = -kx\hat{x}}$

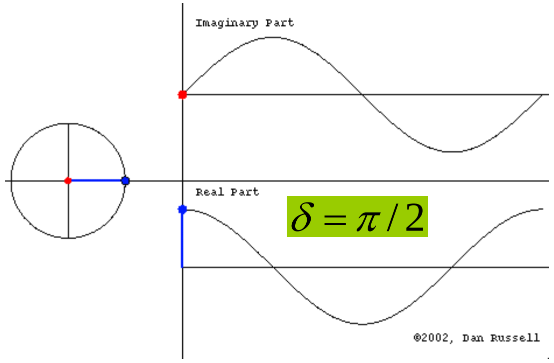
$$\boxed{\vec{F} = -\nabla V = -\frac{dV}{dx}}$$

$$\boxed{V(x) = \int \vec{F} \cdot \overrightarrow{dx} = \frac{1}{2}kx^2}$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \quad \omega = \sqrt{k/m} \rightarrow \text{frequency,}$$

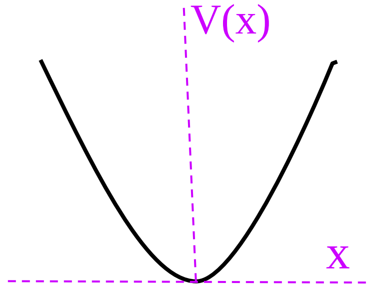
$$\boxed{x = A\sin(\omega t + \delta)}$$



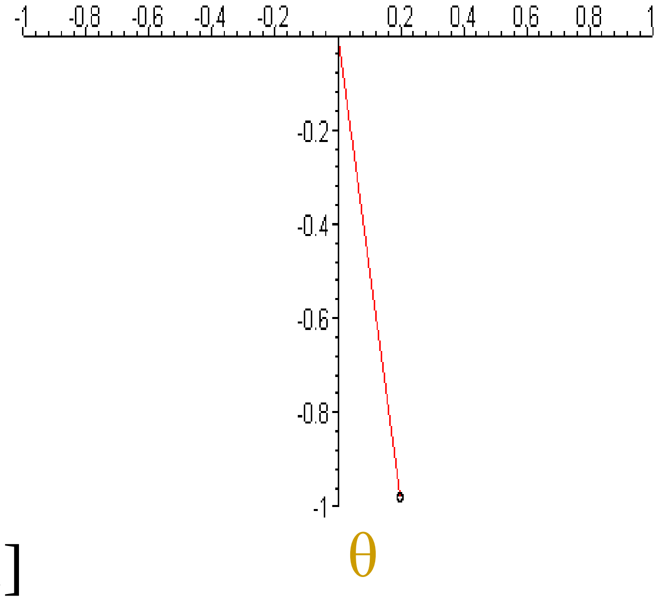
A →amplitude and δ → phase, which are evaluated from the initial/boundary conditions

cos{}/exp{} are also possible

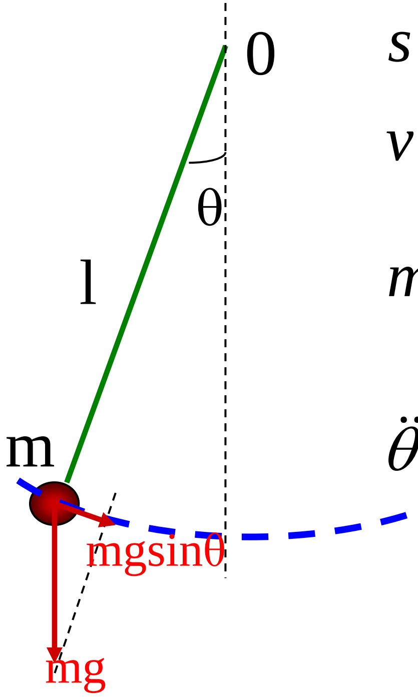
$$E = K.E.\{T\} + P.E\{V(x)\} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$



Introduction: Vibration



Pendulum: (Pendulus-'hanging' {Latin})



$$s = l\theta, \quad where,\ \theta \to 0$$

$$v = \dot{s} = l\dot{\theta}, \quad \Rightarrow a = l\ddot{\theta}$$

$$mg\sin\theta = ml\ddot{\theta}.\ \ For\ small\ angle, [\sin\theta \approx \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}\ldots]$$

$$\ddot{\theta} + \omega^2\theta = 0, \quad \to \theta = A\sin(\omega t + \delta), \qquad \omega = \sqrt{\frac{g}{l}}$$

Energy:

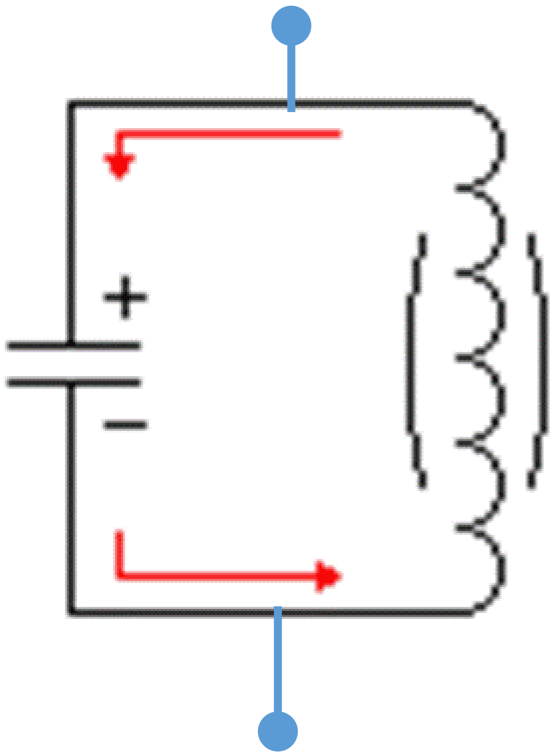
$$T = \frac{1}{2}mv^2 = \frac{1}{2}ml^2\dot{\theta}^2; \quad V(\theta) = mgh = mg(l - l\cos\theta)$$

$$As,\ \theta \to 0, \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}\cdots$$

$$V(\theta) = \frac{1}{2}mgl\theta^2$$

Introduction: Vibration

LC circuit:



$$V_c = Q/C$$

$$I = -\frac{dQ}{dt}, \quad \rightarrow Q = -\int I dt$$

$$V_L = -L\frac{dI}{dt}$$

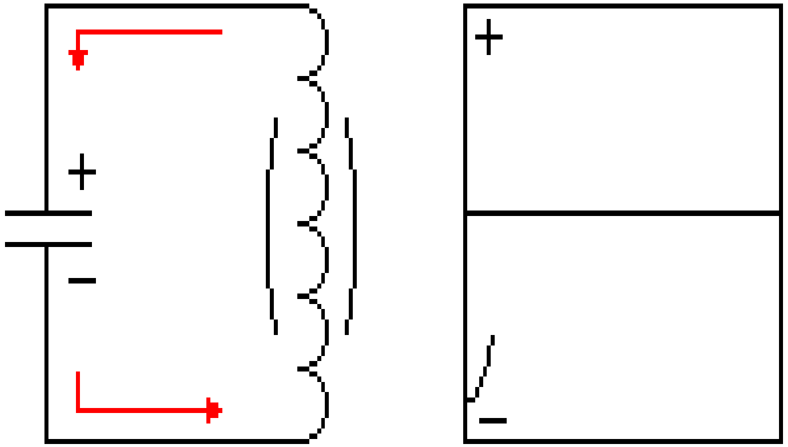
Sum of the voltags around the circuit is zero(0) $\Rightarrow -L\frac{dI}{dt} + \frac{Q}{C} = 0$

$$L\ddot{Q} + \frac{1}{C}Q = 0$$

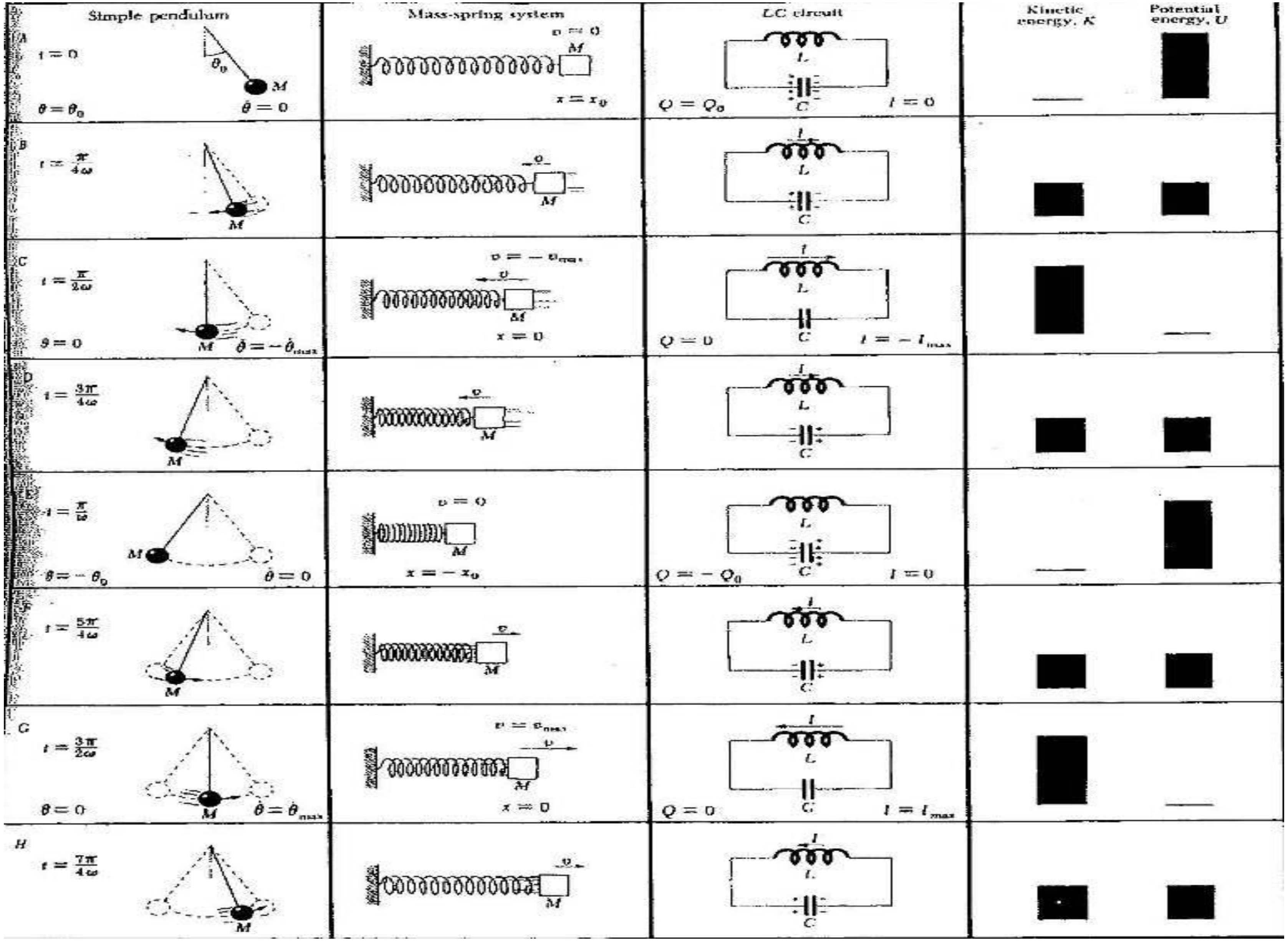
$$\ddot{Q} + \omega^2 Q = 0 \quad where, \omega = \sqrt{\frac{1}{LC}}$$

$$Q = A\sin(\omega t + \delta)$$

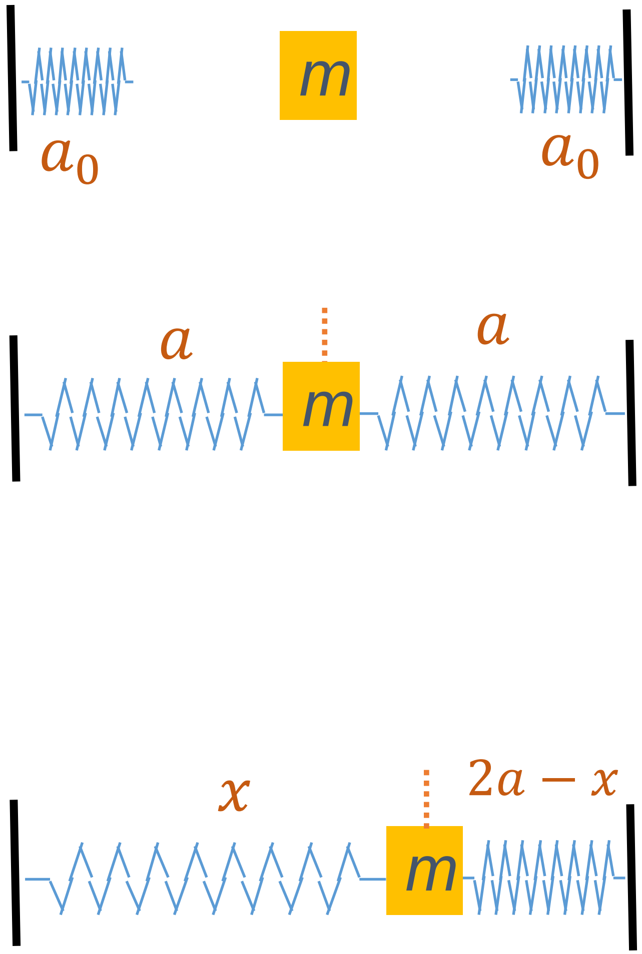
Q



Introduction: Vibration



Introduction: Vibration



$$\vec{F} = -k(x - a_0) + k(2a - x - a_0)$$
$$= -2k(x - a)$$

$$m\frac{d^2x}{dt^2} = -k(x - a)$$

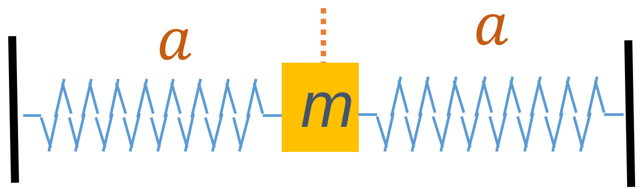
Displacement from the equilibrium, $\psi(t) = x - a$

$$\frac{d^2\psi}{dt^2} + \omega^2\psi = 0,$$

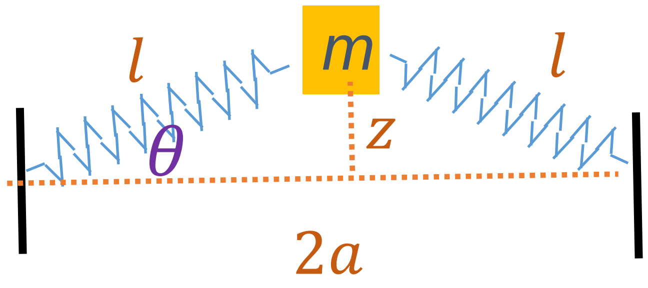
$$\omega = \sqrt{2k/m} \rightarrow \text{frequency},$$

$$\boxed{\psi(\mathrm{t}) = A\sin(\omega t + \delta)}$$

Introduction: Vibration



$$T_0 = k(a - a_0)$$



$$T = k(l - a_0)$$

$$m\frac{d^2z}{dt^2} = -2T\sin\theta$$

$$= -2k(l - a_0)\,\frac{z}{l}$$

$$= -2kz(1-\frac{a_0}{l})$$

Slinky approximation: $a \gg a_0 \Rightarrow \boldsymbol{\frac{a_0}{a}<<1}$

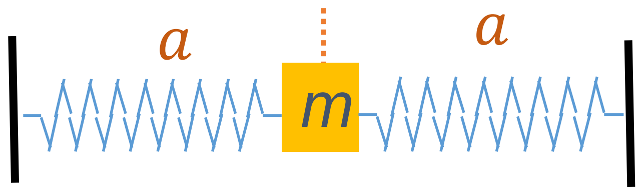
In this case : $\boldsymbol{\frac{a_0}{l}<<1}$

$$\frac{d^2z}{dt^2} + \omega^2 z = 0, \qquad \omega = \sqrt{2k/m} = \sqrt{\frac{2T_0}{ma}}$$

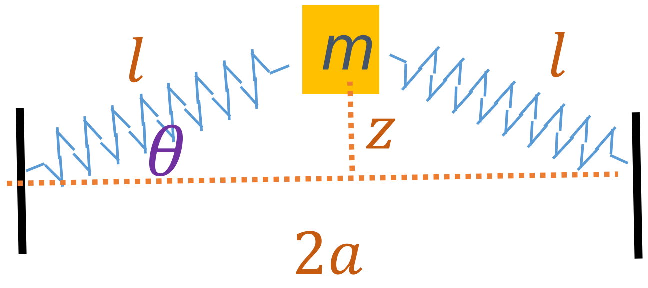
$$\boxed{IF, a \approx a_0 \text{?!}}$$

Introduction: Vibration

$$\omega = \sqrt{2k/m} = \sqrt{\frac{2T_0}{ma}}$$



$$T_0 = k(a - a_0)$$



$$T = k(l - a_0)$$

$$m\frac{d^2z}{dt^2} = -2T\sin\theta \quad = -2kz(1 - \frac{a_0}{l})$$

In this case : $\frac{a_0}{l}$ **is not small**

Small oscillation approximation:

$$l^2 = a^2 + z^2$$

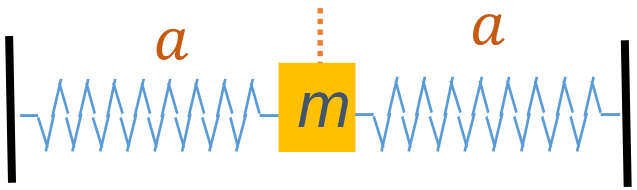
$$l^2 = a^2(1 + \varepsilon)$$

$$\varepsilon = \frac{z^2}{l^2}$$

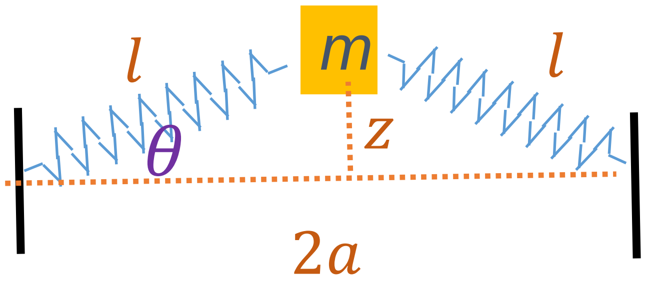
$$\frac{1}{l} = \frac{1}{a}(1 - \varepsilon)^{-1/2}$$

Introduction: Vibration

$$l^2 = a^2(1+\varepsilon) \qquad \varepsilon = \frac{z^2}{l^2} \qquad \omega = \sqrt{\frac{2T_0}{ma}}$$



$$T_0 = k(a - a_0)$$



$$T = k(l - a_0)$$

$$m\frac{d^2z}{dt^2} = -2T\sin\theta = -2kz(1-\frac{a_0}{l})$$

$$\frac{1}{l} = \frac{1}{a}(1 - \frac{1}{2}\varepsilon + \frac{3}{8}\varepsilon^2 - \ldots..)$$

For the first order approximation:

$$\frac{1}{l} = \frac{1}{a}(1 - \frac{1}{2}\frac{z^2}{l^2})$$

$$\frac{d^2z}{dt^2} = -\frac{2kz}{m}(1-\frac{a_0}{l}) = -\frac{2kz}{m}\left(1 - \frac{a_0}{a}\left\{1 - \frac{1}{2}\frac{z^2}{l^2}\right\} + \cdots,\right.$$

$$= -\frac{2k}{ma}(a-a_0)z + \frac{ka_0}{m}\left\{\frac{z^3}{a^3}\right\} + \cdots.),$$

$$\frac{d^2z}{dt^2} = -\frac{2k}{ma}(a-a_0)z = -\frac{2T_0}{ma}z$$

$$\frac{d^2z}{dt^2} + \omega^2 z = 0, \qquad \omega = \sqrt{\frac{2T_0}{ma}}$$