Engineering Electromagnetics

Lecture 22

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by

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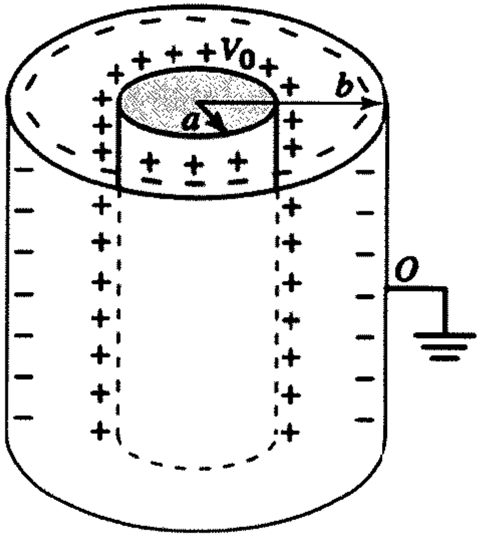
Problem 1

The inner conductor of radius a of a coaxial cable (see Figure 3.41) is held at a potential of V_0 while the outer conductor of radius b is grounded.



Determine (a) the potential distribution between the conductors, (b) the surface charge density on the inner conductor, and (c) the capacitance per unit length.

Since the two conductors of radii a and b form equipotential surfaces, we expect the potential V to be a function of ρ only. Thus, Laplace's equation reduces to

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dV}{d\rho}\right) = 0$$

Integrating twice, we obtain

$$V = c\ln\rho + d$$

where c and d are constants of integration.

At $\rho = b$, $V = 0 \Rightarrow d = -c\ln b$. Thus,

$$V = c\ln(\rho/b)$$

At $\rho = a$, $V = V_0 \Rightarrow c = V_0/\ln(a/b)$. Hence, the potential distribution within the region $a \le \rho \le b$ is

$$V = V_0\frac{\ln(\rho/b)}{\ln(a/b)}$$

The electric field intensity is

$$\vec{\mathbf{E}} = -\nabla V = -\frac{\partial V}{\partial\rho}\vec{\mathbf{a}}_\rho = \frac{V_0\vec{\mathbf{a}}_\rho}{\rho\ln(b/a)}$$

and the electric flux density is

$$\vec{\mathbf{D}} = \epsilon\vec{\mathbf{E}} = \frac{\epsilon V_0\vec{\mathbf{a}}_\rho}{\rho\ln(b/a)}$$

The normal component of $\mathbf{D}$ at $\rho = a$ yields the surface charge density on the inner conductor as

$$\rho_s = \frac{\epsilon V_0}{a\ln(b/a)}$$

The charge per unit length on the inner conductor is

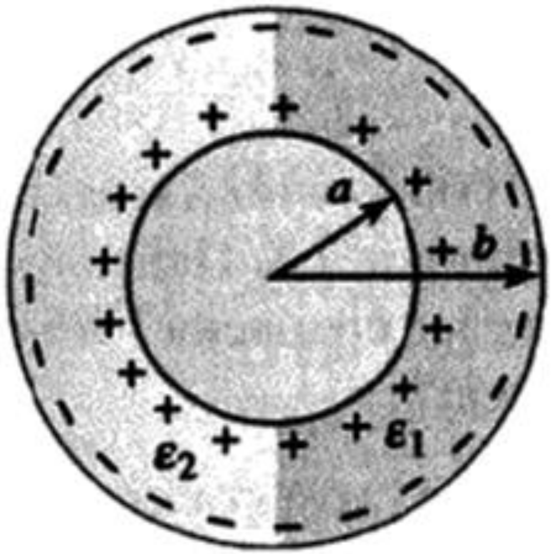
$$Q = \frac{2\pi\epsilon V_0}{\ln(b/a)}$$

Finally, we obtain the capacitance per unit length as

$$C = \frac{2\pi\epsilon}{\ln(b/a)}$$

Problem-3

The region between two concentric spherical shells is filled with two different dielectrics, as shown in Figure 3.39. Find the capacitance of the system.



Solution We expect the $\vec{\mathbf{E}}$ field to be in the radial direction and its tangential components to be continuous at the boundary between the two media. That is,

$$E_{r1} = E_{r2}$$

Since $\vec{\mathbf{D}} = \epsilon \vec{\mathbf{E}}$,

$$D_{r1} = \epsilon_1 E_{r1} \quad \text{and} \quad D_{r2} = \epsilon_2 E_{r2}$$

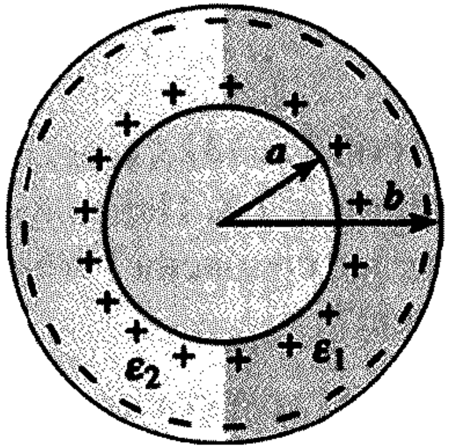


Figure 3.39

Therefore,

$$D_{r2} = \frac{\epsilon_2}{\epsilon_1} D_{r1} \tag{3.78}$$

From Gauss's law, at any closed surface r, $a \leq r \leq b$,

$$\oint_s \vec{\mathbf{D}} \cdot \vec{ds} = Q \tag{3.79}$$

Thus,

$$D_{r1} + D_{r2} = \frac{Q}{2\pi r^2} \tag{3.80}$$

From (3.78) and (3.80), we obtain

$$D_{r1} = \frac{Q\epsilon_1}{2\pi r^2(\epsilon_1 + \epsilon_2)} \quad \text{and} \quad E_{r1} = \frac{Q}{2\pi r^2(\epsilon_1 + \epsilon_2)}$$

The potential of the inner sphere with respect to the outer sphere is

$$\begin{aligned} V_{ab} &= -\frac{Q}{2\pi(\epsilon_1 + \epsilon_2)} \int_b^a \frac{1}{r^2}\, dr \\ &= \frac{Q}{2\pi(\epsilon_1 + \epsilon_2)} \left[\frac{b-a}{ab}\right] \end{aligned}$$

Hence, the capacitance of the system is

$$C = 2\pi(\epsilon_1 + \epsilon_2)\left[\frac{ab}{b-a}\right] = C_1 + C_2$$

where

$$C_1 = 2\pi\epsilon_1\left[\frac{ab}{b-a}\right] \quad \text{and} \quad C_2 = 2\pi\epsilon_2\left[\frac{ab}{b-a}\right]$$

Note that C_1 and C_2 are the capacitances of medium 1 and medium 2, respectively. Thus, the capacitance of the system is equivalent to the parallel combination of the two capacitances. You may have already used this result in the analysis of electrical circuits. • • •

Calculate the capacitance of a parallel-plate capacitor having a mica dielectric, $\epsilon_R = 6$, a plate area of $10\,\text{in}^2$, and a separation of 0.01 in.

Solution. We may find that

$$S = 10 \times 0.0254^2 = 6.45 \times 10^{-3} \quad \text{m}^2$$

$$d = 0.01 \times 0.0254 = 2.54 \times 10^{-4} \quad \text{m}$$

and therefore

$$C = \frac{6 \times 8.854 \times 10^{-12} \times 6.45 \times 10^{-3}}{2.54 \times 10^{-4}} = 1.349 \quad \text{nF}$$

Q: Find the dielectric constant of the material present between a parallel plate capacitor of area 0.2 m^2, separation is 10 micron, and Voltage between the plates is 20 V.

Can the problem be solved?

What if the energy is given? (0.1 mJ)

Displacement vector/ Electric Flux/ Displacement flux

His experiment, then, consisted essentially of the following steps:

1. With the equipment dismantled, the inner sphere was given a known positive charge.
2. The hemispheres were then clamped together around the charged sphere with about 2 cm of dielectric material between them.
3. The outer sphere was discharged by connecting it momentarily to ground.
4. The outer space was separated carefully, using tools made of insulating material in order not to disturb the induced charge on it, and the negative induced charge on each hemisphere was measured.

Faraday found that the total charge on the outer sphere was equal in *magnitude* to the original charge placed on the inner sphere and that this was true regardless of the dielectric material separating the two spheres. He concluded that there was some sort of "displacement" from the inner sphere to the outer which was independent of the medium, and we now refer to this flux as *displacement*, *displacement flux*, or simply *electric flux*.

Faraday's experiments also showed, of course, that a larger positive charge on the inner sphere induced a correspondingly larger negative charge on the outer sphere, leading to a direct proportionality between the electric flux and the charge on the inner sphere. The constant of proportionality is dependent on the system of units involved, and we are fortunate in our use of SI units, because the constant is unity. If electric flux is denoted by Ψ (psi) and the total charge on the inner sphere by Q, then for Faraday's experiment

$$\boxed{\Psi = Q}$$

$$\boxed{\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r}$$

D3.1. Given a 60-μC point charge located at the origin, find the total electric flux passing through: (*a*) that portion of the sphere $r = 26\,\text{cm}$ bounded by $0 < \theta < \frac{\pi}{2}$ and $0 < \phi < \frac{\pi}{2}$; (*b*) the closed surface defined by $\rho = 26\,\text{cm}$ and $z = \pm 26\,\text{cm}$; (*c*) the plane $z = 26\,\text{cm}$.

Ans. 7.5 μC; 60 μC; 30 μC

D3.2. Calculate **D** in rectangular coordinates at point $P(2, -3, 6)$ produced by: (*a*) a point charge $Q_A = 55\,\text{mC}$ at $Q(-2, 3, -6)$; (*b*) a uniform line charge $\rho_{LB} = 20\,\text{mC/m}$ on the x axis; (*c*) a uniform surface charge density $\rho_{SC} = 120\,\mu\text{C/m}^2$ on the plane $z = -5\,\text{m}$.

Ans. $6.38\mathbf{a}_x - 9.57\mathbf{a}_y + 19.14\mathbf{a}_z\ \mu\text{C/m}^2$; $-212\mathbf{a}_y + 424\mathbf{a}_z\ \mu\text{C/m}^2$; $60\mathbf{a}_z\ \mu\text{C/m}^2$

D3.3. Given the electric flux density, $\mathbf{D} = 0.3r^2\mathbf{a}_r\ \text{nC/m}^2$ in free space: (*a*) find **E** at point $P(r = 2, \theta = 25^\circ, \phi = 90^\circ)$; (*b*) find the total charge within the sphere $r = 3$; (*c*) find the total electric flux leaving the sphere $r = 4$.

Ans. $135.5\mathbf{a}_r$ V/m; 305 nC; 965 nC

Thank You