Formulae List

→ **Cylindrical:**

① $\rho = \sqrt{x^2+y^2}$

$\tan\phi = \dfrac{y}{x}$

$$\begin{bmatrix} x = \rho\cos\phi \\ y = \rho\sin\phi \end{bmatrix}$$

②
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

③

(i) $\vec{dl} = dl_\rho\,\hat{\rho} + dl_\phi\,\hat{\phi} + dl_z\,\hat{z}$

$\Rightarrow \vec{dl} = d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{z}$

(ii) $d\vec{S_\rho} = \rho\, d\phi\, dz\,\hat{\rho}$

$d\vec{S_\phi} = d\rho \cdot dz\,\hat{\phi}$

$d\vec{S_z} = \rho\, d\rho\, d\phi\,\hat{z}$

(iii) $d\tau = \rho\, d\rho\, d\phi\, dz$

④ $\vec{\nabla}\cdot\vec{A} = \dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}(\rho A_\rho) + \dfrac{1}{\rho}\dfrac{\partial}{\partial\phi}(A_\phi) + \dfrac{\partial}{\partial z}(A_z)$ [Divergence]

$$\vec{\nabla}\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$ [Curl]

→ Spherical :

① $r = \sqrt{x^2+y^2+z^2}$ $[\rho = r\sin\theta]$

②

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

③ (i) $\vec{dl} = dr\,\hat{r} + r\sin\theta\, d\phi\,\hat{\phi} + r\,d\theta\,\hat{\theta}$

(ii) $d\vec{S}_r = r^2\sin\theta\, d\theta\, d\phi\,\hat{r}$

$d\vec{S}_\phi = r\,d\theta\, dr\,\hat{\phi}$

$d\vec{S}_\theta = r\sin\theta\, dr\, d\phi\,\hat{\theta}$

(iii) $d\tau = r^2\sin\theta\, dr\, d\phi\, d\theta$

④ $\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(A_\phi)$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{A}_r & r\hat{A}_\theta & r\sin\theta\,\hat{A}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

Note : For sphere / Hemisphere, Limits

$\phi \rightarrow 0$ to 2π

$\theta \rightarrow 0$ to π [Sphere]

0 to $\pi/2$ [Hemisphere]

→ Theorems :

① Fundamental Theorem of Gradient :

$$\int_a^b dT = \int_a^b \vec{\nabla} T \cdot \vec{dl} = T(b) - T(a)$$

② Fundamental Theorem of Divergence :

$$\int_V (\vec{\nabla} \times \vec{v})\, d\tau = \oint_S \vec{v} \cdot d\vec{A}$$

③ Stoke's Theorem :

$$\int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = \oint \vec{F} \cdot \vec{dl}$$

Note :

① $\vec{\nabla} \cdot \vec{v} = 0 \Rightarrow \vec{v}$ is Solenoidal function

② $\vec{\nabla} \times \vec{v} = 0 \Rightarrow \vec{v}$ is Irrotational

$\vec{\nabla} \times \vec{v} \neq 0 \Rightarrow \vec{v}$ is Rotational

Electrostatics

① $\vec{F} = \dfrac{kQq}{r^2}\hat{r}$

② $\vec{E}(r) = k\displaystyle\int \frac{dq}{r^2}\hat{r}$

$\left[Q = \int \lambda\, dl = \int \sigma\, dA = \int \rho\, dV \right]$

$= k\displaystyle\int \frac{\lambda(r')}{r^2}\hat{r}\, dl'$

③ $\text{Flux} = \oint \vec{E}\cdot d\vec{S} = \dfrac{q_{enc}}{\varepsilon_0}$ → q_{free} enclosed

④ $E = -\vec{\nabla} V$

$\vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0}$ — ①

$\vec{\nabla}\times\vec{E} = 0$

$V_b - V_a = -\displaystyle\int_a^b \vec{E}\cdot d\vec{l} = \frac{W}{q}$

⑤ $\nabla^2 V = \dfrac{-\rho}{\varepsilon_0} \Longrightarrow \nabla^2 V = 0$

(Poisson's Equation) (Laplace Equation) → $[\rho = 0]$

⑥ $W = \dfrac{1}{2}\displaystyle\int \rho V\, d\tau$

$= \dfrac{1}{2}\displaystyle\int \sigma V\, dA$

$= \dfrac{1}{2}\varepsilon_0 \displaystyle\int E^2\, d\tau$

$W = \dfrac{1}{2}\displaystyle\sum_{i=1}^{n} q_i V_i$

$W = \dfrac{1}{2}(\vec{D}\cdot\vec{E})$

$= \dfrac{1}{2}\displaystyle\int_V \vec{D}\cdot\vec{E}\, d\tau$

⑦ Dipole :

$V = \dfrac{k(\vec{p}\cdot\hat{r})}{r^2}$

$\vec{p} = \alpha\vec{E}$

$\tau = \vec{p}\times\vec{E}$

$\vec{p} = \displaystyle\int \vec{P}\, d\tau$

⑨ Boundary conditions

$D_1^{\perp} - D_2^{\perp} = \sigma_f$

$E''_1 = E''_2$

⑧ Bound Charges

$\sigma_b = \vec{P}\cdot\hat{n}$

$\rho = \rho_b + \rho_f \begin{cases} \rho_b = -\vec{\nabla}\cdot\vec{P} \\ \rho_f = \vec{\nabla}\cdot\vec{D} \end{cases}$

→ $\oint D\cdot dA = Q_{free\,enc}$

$\left.\begin{aligned} \vec{D} &= \varepsilon\vec{E} \\ \vec{D} &= \varepsilon_0\vec{E} + \vec{P} \end{aligned}\right\} \Rightarrow \vec{P} = \varepsilon_0 \chi \vec{E}$

$\varepsilon_r = 1 + \chi$

10 $c = \frac{q}{V}$

$W = \frac{1}{2} c V^2$

MAGNETOSTATICS

1 $\vec{F} = q(\vec{v} \times \vec{B})$

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ $\quad [1\,T = 10^4 \text{ Gauss}]$

$\vec{F} = I(\vec{l} \times \vec{B})$

2 $\vec{F} = \int (\vec{J} \times \vec{B})\, d\tau$

$= \int (\vec{k} \times \vec{B})\, ds$

$\vec{\nabla} \times \vec{B} = \mu_0 J_{enc}$

$\vec{\nabla} \cdot \vec{B} = 0$ — 2

$\vec{B} = \vec{\nabla} \times \vec{A}$

3 $B(r) = \frac{\mu_0}{4\pi} \int \frac{i \times \hat{r}}{r^2}\, dl'$

4 $\vec{M} = i\vec{A}$

$\vec{\tau} = \vec{M} \times \vec{B}$

5 $\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} \Rightarrow \oint \vec{H} \cdot \vec{dl} = i_{enc}$ free current $\quad \boxed{\because B = \mu H}$

$\oint \vec{B} \cdot \vec{ds} = 0$

6 $\nabla^2 A = -\mu_0 J_{enc}$

$A = \frac{\mu_0}{4\pi} \oint \frac{i\, \vec{dl}'}{R}$

$= \frac{\mu_0}{4\pi} \oint_V \frac{J\, d\tau}{R}$

7 $\int \vec{B} \cdot \vec{ds} = \oint \vec{A} \cdot \vec{dl} = \text{Flux}$

8 $\vec{\nabla} \times \vec{H} = J_{free} = J_c + J_D = \sigma E + \frac{\partial \vec{D}}{\partial t}$ — 3

(Bound charges)

$J_b = \vec{\nabla} \times \vec{M}$ & $J_f = \vec{\nabla} \times \vec{H}$

$K_b = \vec{M} \times \hat{n}$

$\left.\right\} J = J_b + J_f$

$\vec{M} = \chi_m \vec{H}$

$\mu_r = 1 + \chi_m$

$\vec{B} = \mu_0 (1 + \chi_m) \vec{H}$

$M = \frac{m}{V}$, i.e. $m = \int_V M\, d\tau$

$A = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$

(9) Boundary Conditions :

$$B_1^{\perp} = B_2^{\perp}$$

$$\cancel{B_1^{\parallel} - B_2^{\parallel} = \mu_0 k}$$

$$H_1^{\parallel} - H_2^{\parallel} = k_f \times \hat{n}$$

Maxwells Equations : (1), (2), (3), (4)

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

(2) $\vec{\nabla} \cdot \vec{B} = 0$

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$

(4) $\vec{\nabla} \times \vec{H} = j + \frac{\partial \vec{D}}{\partial t}$

(10) $\mathcal{E} = -\frac{d\phi}{dt} = \frac{dW}{dq}$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t} \quad - (3)$$

$$\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

(11) $L = \frac{\lambda}{i}$ | $\mathcal{E} = N\frac{d\phi}{dt}$ | $[d\lambda = L\, di]$

$N\phi = Li$ | $\mathcal{E} = L\frac{di}{dt}$

$$\therefore \underbrace{N\phi = Li}_{=\lambda} \quad \& \quad \phi = B \cdot A$$

for Circular coil, $L = \frac{\mu_0 N^2 \pi r^2}{l}$

(12) $\phi_2 = M i_1$

$\phi_1 = M i_2$

$$\mathcal{E}_1 = -M\frac{d(i_2)}{dt}$$

$$\mathcal{E}_2 = -M\frac{d(i_1)}{dt}$$

(13) $W = \frac{1}{2} L i^2$

$$= \frac{1}{2\mu_0}\int_V B^2 \, d\tau \longrightarrow W = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\tau}\right)$$

(14) $\vec{S} = \vec{E} \times \vec{H}$

$$\frac{dP}{dV} = \vec{E} \cdot \vec{J}$$

$$\hookrightarrow \vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t}$$

(15) $\langle S_{avg} \rangle = \frac{1}{T}\int_0^T S \cdot dt$

EM Wave

(1) $\hat{n} \cdot \hat{z} = 0$

Polarisation Vector

(2) Plane Wave :

$$B_0 = \frac{E_0}{c}$$

$$E = E_0 \cos(\hat{k} \cdot \hat{r} - \omega t + \delta)\ \hat{n}$$

$$B = B_0 \cos(\hat{k} \cdot \hat{r} - \omega t + \delta)\ (\hat{k} \times \hat{n})$$

$$[\hat{k} \cdot \hat{n} = 0]$$

$$\vec{k} = \frac{2\pi}{\lambda}\ \overrightarrow{\text{wave}}$$

(3) $v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{n}$

$$n = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$$

For most materials,

$$\mu \approx \mu_0$$

$$\therefore n \cong \sqrt{\varepsilon_r}$$