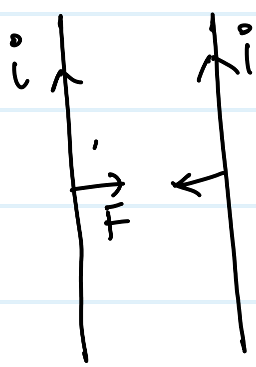
01/11/2023

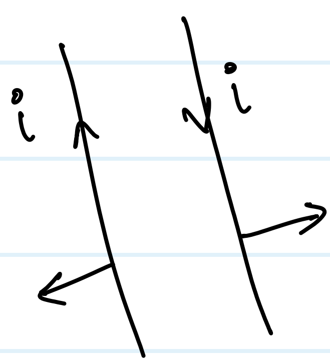
POST QUIZ-2

MAGNETOSTATICS

- Study of magnets at rest.
- bridging gap b/w electricity & magnetism: electricity can produce magnetism (Oersted expt)
- In electrostatics → charges at rest. What if, $v_q \neq 0$?
- $\frac{dq}{dt} \to i \to$ produces magnetism.
- steady current → studied in magnetostatics.



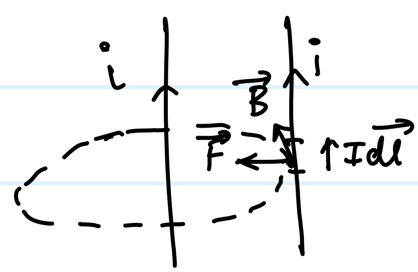
⟹ 2 straight, hanging wires, with current in same dir → get closer (Force of attrn)



Same wires with opp. dir of currents, repel each other (Force of repulsion)

$(\vec{F} \perp ar \ I\vec{dl})$

$(\vec{F} = I\vec{dl} \times \vec{B})$

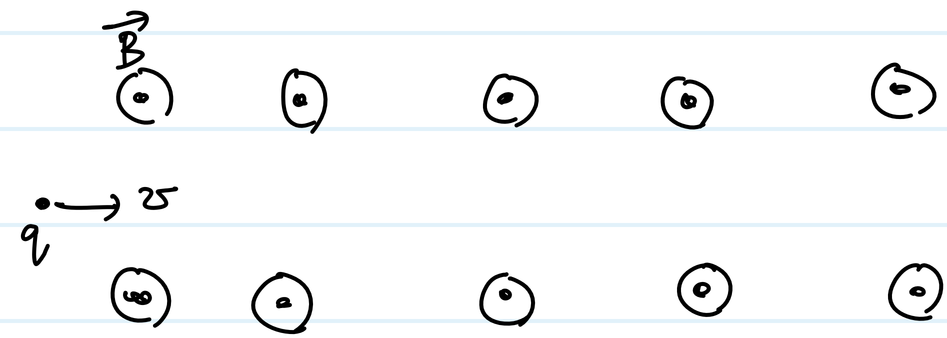


$$\vec{F}_{wire} = I_{wire} \times l_{wire} \times \vec{B}_{\substack{other\\wire}} \quad (\text{in } N)$$

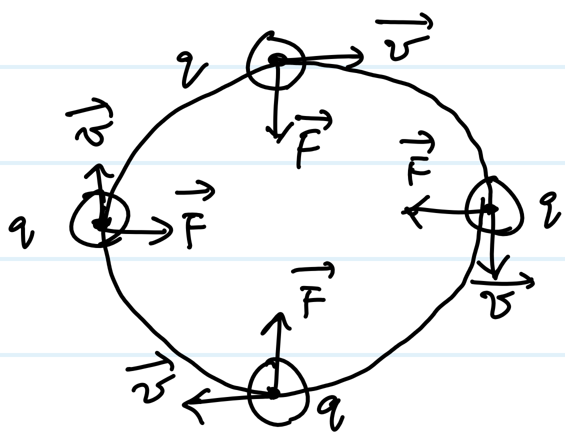
MAGNETIC FORCES

- $\vec{F}_{mag} = Q(\vec{v} \times \vec{B})$
- $\vec{F}_{Lorentz} = Q(\vec{E} + (\vec{v} \times \vec{B}))$

- Its effect?



$q \rightarrow$ follows circular trajectory.



$F_c = \dfrac{mv^2}{R}$; $F_m = BqV$

$(F_c = F_m)$

$F = BqV$

$Bq\cancel{V} = \dfrac{mV^{\cancel{2}}}{R}$

$R = \dfrac{mV}{Bq}$

if $v = 10^6\ m/s$

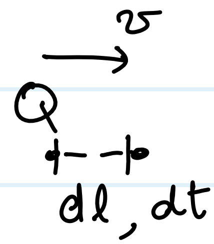
$B = 1T$

$R = \dfrac{\cancel{1.6} \times 10^{-27} \times 10^6}{1 \times \cancel{1.6} \times 10^{-19}}$

$= 10^{-8} \times 10^6$

$= 10^{-2}\ m = 1cm \Rightarrow \boxed{R = 1cm}$

WORK DONE BY $\vec{B}$



$$dW = \vec{F}_B \cdot \vec{dl} \qquad \vec{F}_B = Q(\vec{v}\times\vec{B})$$

$\vec{v}\times\vec{B}$ ⊥ar to $\vec{dl}$

$\Rightarrow \boxed{\theta = 90^\circ}$

$\Rightarrow dW = F_B \cdot dl \cdot \cos\theta$

$= F_B \cdot dl \cos 90$

$\Rightarrow dW = 0$

$\Rightarrow \int dW = W = 0$

if $\vec{v}$ ⊥ar to $\vec{B}$

∴ $\boxed{W_{mag} = 0}$

WORK DONE BY MAGNETIC FIELD, on a charge moving ⊥ar to it = ZERO.

$\vec{v}\times\vec{B}$ ⊥ar $\vec{dl}$

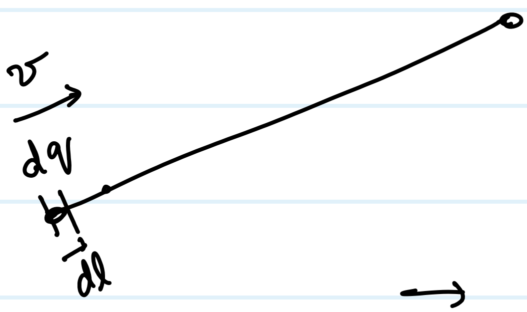
as $\vec{dl} = \vec{v}dt$

$\vec{v}dt$ is ALONG $\vec{v}$,

& $\vec{v}\times\vec{B}$ is ⊥ar to $\vec{v}$,

∴ $\vec{v}dt$, ∴ $\vec{dl}$.

CURRENT & FIELD due to LINE CHARGE



$dq = \lambda dl = \lambda v dt$

$\Rightarrow \boxed{\frac{dq}{dt} = i = \lambda v}$

$$\vec{F}_{avg} = \int (\vec{V}\times\vec{B})\,dq = \int (\vec{V}\times\vec{B})\lambda\, dl = \int (\vec{v}\lambda\times\vec{B})\, dl$$

fixed! → scalar

$$\Rightarrow \vec{F}_{avg} = \int (I\,\vec{dl} \times \vec{B})$$

$$\boxed{\vec{F}_{avg} = I \int (\vec{dl} \times \vec{B})}$$

FOR A VOLUME

$$i = \frac{dq}{dt} = \frac{\rho \cdot d\tau}{dt} = \frac{\rho \cdot A \cdot dl}{dt} \Rightarrow \rho A v$$

$$\Rightarrow \quad i/A = \rho v \quad \Rightarrow \boxed{J = \rho v}$$

$$\vec{F}_{avg} = \int (\vec{v} \times \vec{B}) \rho\, d\tau = \int (\vec{v}\rho \times \vec{B})\, d\tau = \int (\vec{J} \times \vec{B})\, d\tau$$

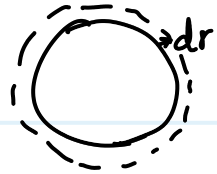
$$I = \int_S \vec{J} \cdot \vec{dA}_{\perp} = \int_S J\, dA \quad ; \quad \oint_S J \cdot dA = \int_V (\vec{\nabla} \cdot \vec{J})\, d\tau$$

$$= -\frac{d}{dt}\left(\int_V \rho\, d\tau\right)$$

$$= -\int_V \left(\frac{d\rho}{dt}\right) d\tau$$

$$\Rightarrow \int_V (\vec{\nabla} \cdot \vec{J})\, d\tau = \int_V \left(-\frac{d\rho}{dt}\right) d\tau$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}}$$



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FOR σ :

$$i = \frac{dq}{dt} = \frac{\sigma\, dA}{dt} = \frac{\sigma\, A\, dl}{dt} = \sigma A v$$

$$\Rightarrow i/A = \sigma v$$

$$\Rightarrow \boxed{\vec{K} = \sigma \vec{v}}$$

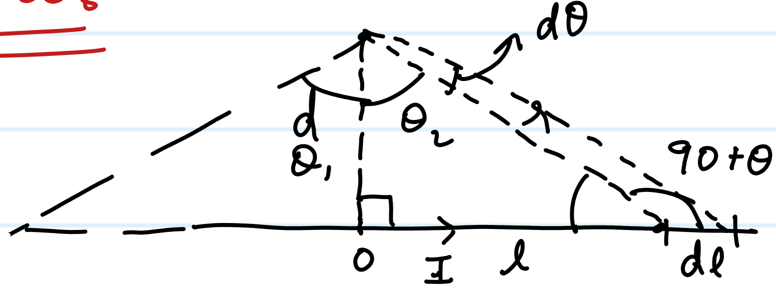
$$\vec{F}_{mag} = \int (\vec{v} \times \vec{B})\, \sigma\, dA = \int (\sigma \vec{v} \times \vec{B})\, dA = \int (\vec{K} \times \vec{B})\, dA$$

$$\Rightarrow \boxed{F_{mg} = \int (\vec{K} \times \vec{B})\, dA}$$

BIOT- SAVARTS LAW

$$B(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \vec{r}}{r^3}$$

EXAMPLE :



$$\vec{B}(d) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0 I}{4\pi} \int \frac{dl \cos\theta}{r^2}$$

$$\vec{B}(d) = \frac{\mu_0 I}{4\pi} \int \frac{d\sec^2\theta \, d\theta \cdot \cos^3\theta}{d^2}$$

$\tan\theta = \frac{l}{d}$

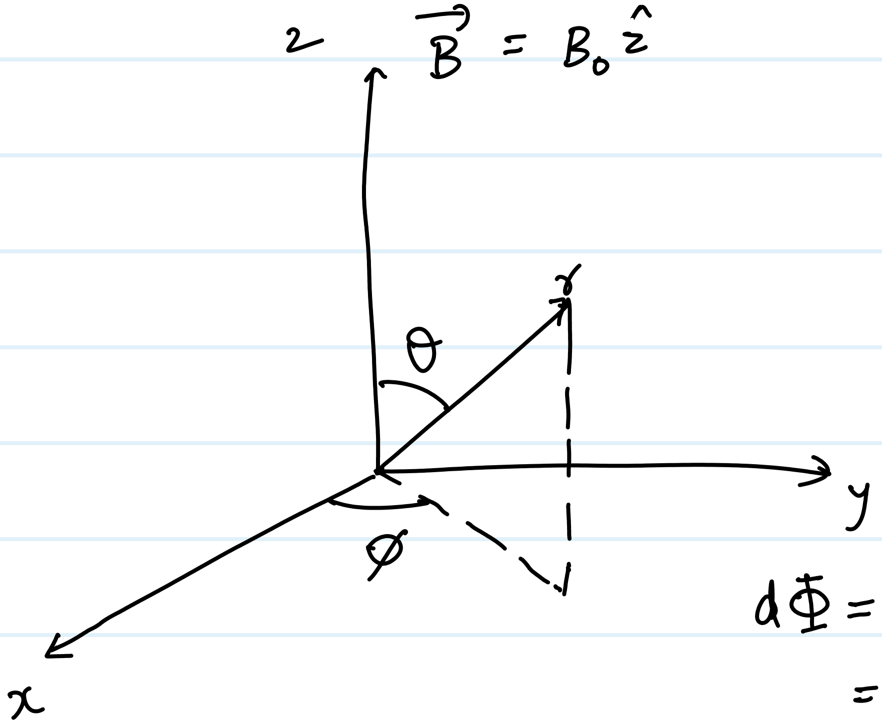
$\Rightarrow d\sec^2\theta \, d\theta = dl$

$r\cos\theta = d$

$$= \frac{\mu_0 I}{4\pi d} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$\vec{B}(d) = \frac{\mu_0 I}{4\pi d} \left[\sin\theta_2 - \sin\theta_1 \right]$$

$\vec{B} = B_0 \hat{z}$



$$\vec{ds} = dl_\theta \cdot dl_\phi \cdot \hat{r}$$
$$= r\,d\theta \cdot r\sin\theta \, d\phi \cdot \hat{r}$$
$$= r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

$$d\Phi = \vec{B} \cdot \vec{ds}$$
$$= B_0 \hat{z} \cdot r^2 \sin\theta \, d\theta \, d\phi \cdot \hat{r}$$
$$= B_0 r^2 \sin\theta \, d\theta \, d\phi \, (\hat{z} \cdot \hat{r})$$

$$d\Phi = B_0 r^2 \sin\theta \cos\theta \, d\theta \cdot d\phi$$

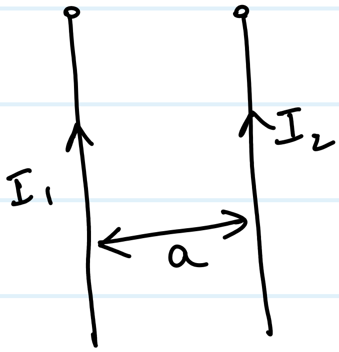
$$\Phi = \frac{B_0 r^2}{4} \int_0^{\pi/2} \sin 2\theta \, d2\theta \int_0^{2\pi} d\phi$$

$$= \frac{B_0 r^2}{4} \left[-\cos 2\theta \right]_0^{\pi/2} 2\pi$$

$$+ \frac{\pi B_0 r^2}{2} \left[+1 + 1 \right]$$

$$\underline{\underline{\pi B_0 r^2}}$$

$$\frac{\mu_0 i}{4\pi S}\left(\sin\phi_1 - \sin\phi_2\right) = B(r') \leftarrow \text{DERIVE LATER.}$$



$$B_{21} = \frac{\mu_0 I_1}{2\pi a}$$

$$\vec{F}_{21} = I_2 \vec{l}_2 \times \vec{B}_{21}$$

$$|\vec{F}_{21}| = \frac{I_2 l_2 \mu_0 I_1}{2\pi a}$$

$$\vec{F}_{12} = I_1 \vec{l}_1 \times \vec{B}_{12}$$

$$|\vec{F}_{12}| = \frac{I_1 l_1 \mu_0 I_2}{2\pi a}$$

$$\Rightarrow \left|\frac{\vec{F}_{21}}{l_2}\right| = f = \frac{\mu_0 I_1 I_2}{2\pi a} \;//$$

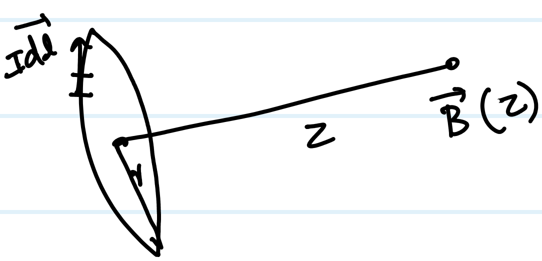
$$\Rightarrow \left|\frac{\vec{F}_{12}}{l}\right| = f = \frac{\mu_0 I_1 I_2}{2\pi a}$$

$$\Rightarrow \boxed{|\vec{F}_{21}| = |\vec{F}_{12}|}$$

BUT! $\rightarrow$ $\underbrace{\vec{F}_{21} = -\vec{F}_{12}}$

if $I_1 d\vec{l}_1 \parallel$ ~~el~~ $I_2 d\vec{l}_2 \rightarrow$ attractive

SOLVE QS ON PPT



$$\vec{B}(z) = \frac{\mu_0 I r^2}{2(z^2+r^2)^{3/2}}$$

$z \rightarrow \infty$, $z^2 + r^2 \rightarrow z^2$

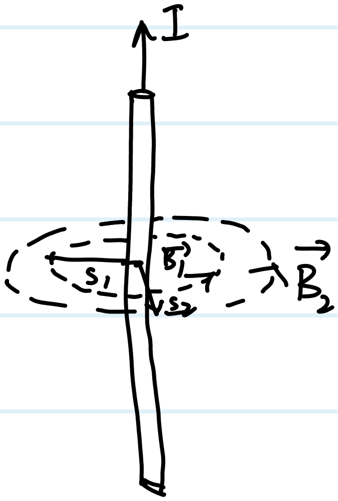
$$\Rightarrow \boxed{\vec{B}(z) = \frac{\mu_0 I r^2}{2z^3}}$$

$$\boxed{\vec{M} = IA\hat{n}}$$

$$\vec{B}(z) = \frac{\mu_0 I(\pi r^2)}{2\pi z^3}(\hat{z})$$

$$\vec{B}(z) = \frac{\mu_0 IA(\hat{z})}{2\pi z^3} = \frac{\mu_0 \vec{M}}{2\pi z^3}$$

For an ∞ly long wire



$$\oint \vec{B}\cdot\vec{dl} = \int \frac{\mu_0 I}{2\pi s}\cdot dl$$

$$= \frac{\mu_0 I}{2\pi s}\int dl = \frac{\mu_0 I}{\cancel{2\pi s}} \times \cancel{2\pi s}$$

$$\Rightarrow \boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$$

STOKES

$$\int (\vec{\nabla}\times\vec{B})\, ds = \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

if $I \neq 0$, $\boxed{\vec{\nabla}\times\vec{B} \neq 0}$

$\Rightarrow$ Curl of $\vec{B} \neq 0$

↳ rotational!

$$\int_S (\vec{\nabla}\times\vec{B})\, ds = \mu_0 I$$

$$= \mu_0 \int_S J\cdot ds$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 J}$$

$\mu_0 J \to$ non-zero

$\Rightarrow \vec{\nabla}\times\vec{B}$ is non-zero

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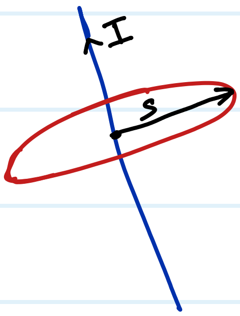
∞ wire and $\int$ of $\vec{B}$ along a path.

$$\oint \vec{B}\cdot\vec{dl} = \oint \frac{\mu_0 I}{2\pi s}\cdot dl = \frac{\mu_0 I}{2\pi s}\cdot\oint dl = \mu_0 I.$$



In cylindrical :

$$\vec{dl} = d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{z}$$

$$\Rightarrow \vec{dl} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z} \qquad (\rho = s)$$

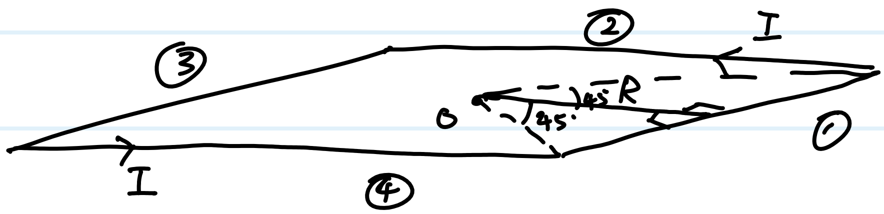
AND $$\vec{B} = \frac{\mu_0 I}{2\pi s}\cdot\hat{\phi}$$

$$\oint \vec{B}\cdot\vec{dl} = \oint \frac{\mu_0 I}{2\pi s}\hat{\phi}\cdot\left(ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}\right)$$

$$= \oint \frac{\mu_0 I}{2\pi \cancel{s}}\cdot\cancel{s}\,d\phi = \frac{\mu_0 I}{2\pi}\oint d\phi$$

$$= \mu_0 I$$

PROBLEM - 1 : find $\vec{B}_0$



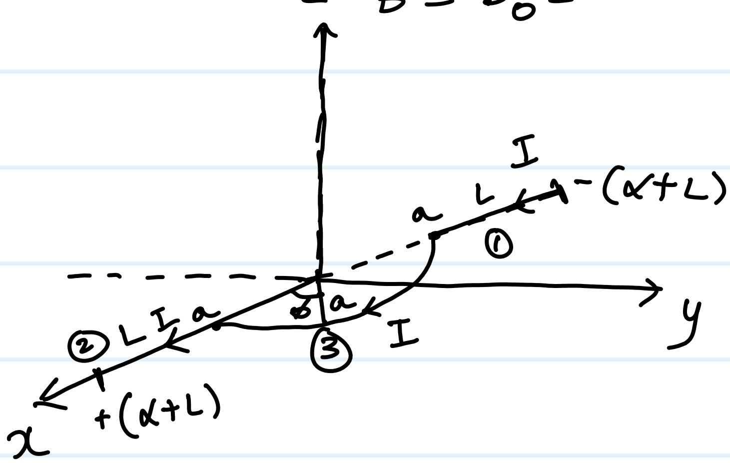
① $\vec{B}_{01} = \frac{\mu_0 I}{4\pi R}\left(\sqrt{2}\right)(\uparrow)$ ② $\vec{B}_{02} = \frac{\mu_0 I}{4\pi R}\left(\sqrt{2}\right)\uparrow$

③ $\vec{B}_{03} = \frac{\mu_0 I}{4\pi R}\left(\sqrt{2}\right)\uparrow$ ④ $\vec{B}_{04} = \frac{\mu_0 I}{4\pi R}\left(\sqrt{2}\right)\uparrow$

$$\vec{B} = \sum_{i=1}^{4} \vec{B}_{0i} = 4 \times \frac{\mu_0 I}{4\pi R}\sqrt{2} = \frac{\sqrt{2}\,\mu_0 I}{\pi R} \;//$$

PROBLEM-2

$z \quad \vec{B} = B_0 \hat{z}$



③ $\vec{F} = \int (I\, d\vec{l} \times \vec{B})$

$\vec{F}_{①} = I\left(\vec{l} \times \vec{B}\right)$

$= I\left(l B_0\right)\left(-\hat{y}\right) = -I l B_0\,\hat{y}$

$\vec{F}_{②} = -I l B_0\,\hat{y}$

③ $$IB_0 \int dl = IB_0 \int a\,d\phi = IB_0 \int (a\cos\phi + a\sin\phi)\,d\phi$$

$$= IB_0 a \left[\int \cos\phi\, d\phi + \int \sin\phi\, d\phi \right]$$

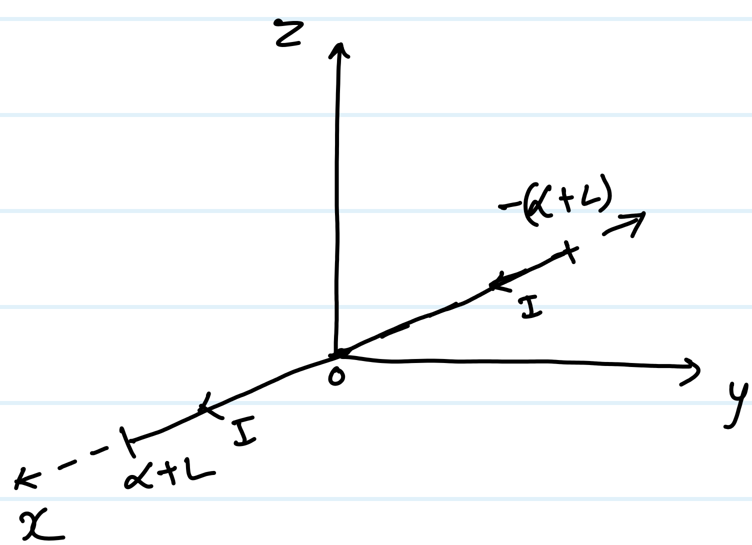
$$= IB_0 a \left[\cancel{(\sin\phi)_0^\pi} - (\cos\phi)_0^\pi \right]$$

$$= IB_0 a \left(- (-1-1) \right) = 2aIB_0 (-\hat{y})$$

$$\vec{F} = -Il B_0 \hat{y} - Il B_0 \hat{y} - 2IB_0 a \hat{y}$$

$$= (-2IlB_0 - 2IB_0 a)\hat{y}$$

$$\boxed{\vec{F} = -2IB_0 (l + a)\hat{y}}$$



$$\vec{F} = I(\vec{l} \times \vec{B})$$

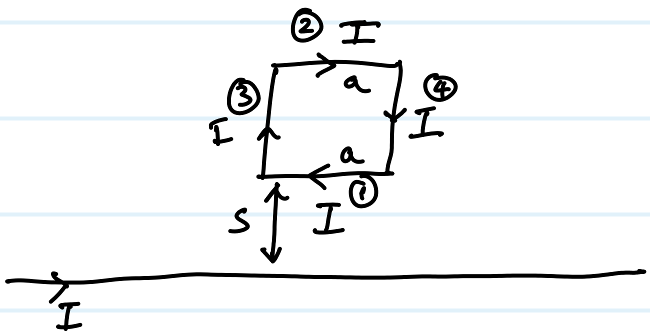
$$= I\left(2(\alpha + l)B_0\right)(-\hat{y})$$

$$= -2I(\alpha + L)B_0 \hat{y}$$

$$= -2IB_0(\alpha + L)\hat{y}$$

PROBLEM-3

find $\vec{F}$ on $\square$ loop.



① $F_1 = I(\vec{l} \times \vec{B})$

$$= I\left(a \frac{\mu_0 I}{2\pi s}\right)$$

$$F_1 = \frac{\mu_0 I^2 a}{2\pi s} \; (\uparrow)$$

② $F_2 = I\left(\frac{a \mu_0 I}{2\pi(a+s)}\right)$

$$= \frac{\mu_0 I^2 a}{2\pi(a+s)} \; (\downarrow)$$

③ $F_3 = I\int (d\vec{l} \times \vec{B})$

$$= I \int_s^{a+s} \frac{dl \cdot \mu_0 I}{2\pi l}$$

$$= \frac{\mu_0 I^2}{2\pi} \int_s^{a+s} \frac{dl}{l}$$

$$= \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{a+s}{s}\right) \leftarrow$$

④ $F_4 = \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{a+s}{s}\right) \rightarrow$

$\Rightarrow \boxed{③ + ④ = \vec{0}}$!

$\Rightarrow \boxed{\vec{F}_{net} = \vec{F}_1 + \vec{F}_2}$

$$F_{net} = \frac{\mu_0 I^2 a}{2\pi}\left(\frac{1}{s}\uparrow + \frac{1}{a+s}\downarrow\right)$$

$$= \frac{\mu_0 I^2 a}{2\pi}\left(\frac{1}{s} - \frac{1}{a+s}\right)\uparrow$$

$$= \frac{\mu_0 I^2 a}{2\pi}\left(\frac{a}{s(a+s)}\right)\uparrow$$

$$\vec{F}_{net} = \frac{\mu_0 I^2 a^2}{2\pi s(a+s)} \uparrow$$

MAGNETIC FLUX

$$\phi = \int_S \vec{B} \cdot \vec{ds}$$

for closed surface:

$$\boxed{\oint \vec{B} \cdot \vec{ds} = 0} \Rightarrow \int (\vec{\nabla} \cdot \vec{B}) d\tau = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{B} = 0$$

$$\Rightarrow \boxed{\vec{B} \rightarrow \text{SOLENOIDAL}}$$

- $n_{\text{field lines (origin)}}(\text{NORTH}) = n_{\text{field lines (entering)}}(\text{SOUTH})$
- NO Magnetic Monopoles!

08/11/2023

MAGNETIC VECTOR POTENTIAL

We know that: $\vec{\nabla} \cdot \vec{B} = 0$

$$\Rightarrow \vec{B} = \vec{\nabla} \times \vec{A}$$

$\vec{A}$: Magnetic Vector Potential

$\vec{A}$ should also be solenoidal

$$(\vec{\nabla} \cdot \vec{A} = 0)$$

EXAMPLE: Which $\vec{A}$ produces $\vec{B}$ along $-\hat{x}$

- $\frac{z}{2}\hat{y}$
- $z\hat{x}+x\hat{z}$
- $x\hat{x}+y\hat{y}$
- $-x\hat{x}-y\hat{y}-z\hat{z}$

$$\hookrightarrow \left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)\times\left(\frac{z}{2}\hat{y}\right)$$

$$= -\frac{1}{2}\hat{x}.$$

$$\hookrightarrow \left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)\times\left(x\hat{x}+y\hat{y}\right)$$

$$= 0$$

$$\hookrightarrow \left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)\times\left(z\hat{x}+x\hat{z}\right)$$

$$= +\hat{y}-\hat{y}=0.$$

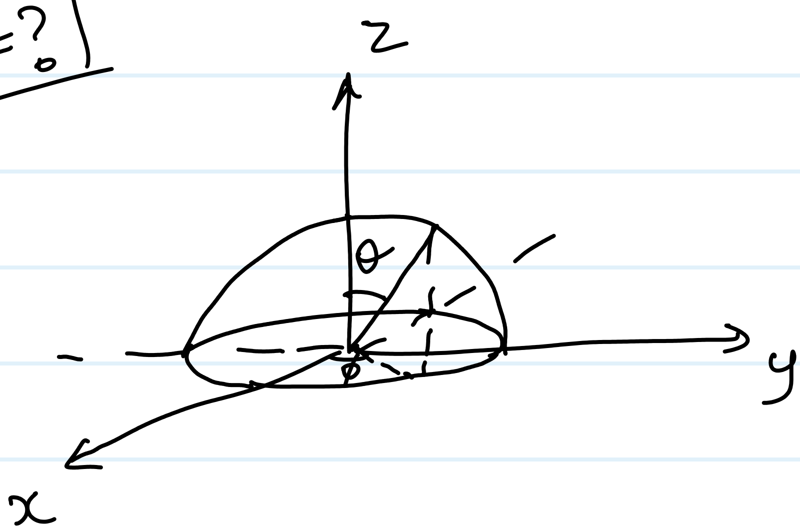
$$\hookrightarrow \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \times \left(-x\hat{x} - y\hat{y} - z\hat{z} \right)$$

$$= 0 .$$

EXAMPLE :

$$\vec{B} = B_0 \vec{a_z}$$

$$\boxed{\phi = ?}$$



$$\vec{ds} = (r\sin\theta\, d\phi)\, r\, d\theta$$

$$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$$

$$d\phi = \vec{B} \cdot \vec{ds}$$

$$B_0 (r^2 \sin\theta\, d\theta\, d\phi)\, \hat{z} \cdot \hat{r}$$

$$= B_0 r^2 \sin\theta \cos\theta\, d\theta\, d\phi$$

$$= \frac{B_0 r^2}{2} \sin 2\theta\, d\theta\, d\phi$$

$$\frac{B_0 r^2}{4} \int_0^{\pi/2} \sin 2\theta\, d2\theta \int_0^{2\pi} d\phi$$

$$= \frac{B_0 r^2}{\cancel{4}\,2} \left[-\cos 2\theta \right]_0^{\pi/2} \left[\cancel{2}\pi \right]$$

$$= \frac{\pi B_0 r^2}{2} \left[+1 + 1 \right]$$

$$= \pi B_0 r^2 \;//$$

$$\phi = \int_S \vec{B}\cdot\vec{dS} = \int_S (\vec{\nabla}\times\vec{A})\,ds = \int_l \vec{A}\cdot\vec{dl}$$

$$\Rightarrow \boxed{\phi = \int_l \vec{A}\cdot\vec{dl}}$$

$$\vec{B} = (\vec{\nabla}\times\vec{A})$$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A})$$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A$$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}(0) - \nabla^2 A$$

$$\vec{\nabla}\times\vec{B} = -\nabla^2 A$$

$$\Rightarrow \mu_0 J = -\nabla^2 A$$

$$\Rightarrow \boxed{\nabla^2 A = -\mu_0 J}$$

$$\int (\vec{\nabla}\times\vec{B})\,ds = \oint B\cdot dl = \mu_0 I_{enc}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A}$$

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EXPRESSION for $\vec{A}$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}' \times \vec{R}}{R^3}$$

$$\vec{R} = (x-x')\vec{a_x} + (y-y')\vec{a_y} + (z-z')\vec{a_z}$$

$$\vec{\nabla}\left(1/R\right) = \frac{-\vec{R}}{R^3}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi}\int \vec{\nabla}\left(\frac{1}{R}\right)\times \vec{dl}'$$

$$\vec{\nabla}\left(\frac{1}{R}\right)\times \vec{dl}' = \vec{\nabla}\times\left(\frac{\vec{dl}'}{R}\right) - \frac{1}{R}\left[\vec{\nabla}\times\vec{dl}'\right]$$

$\vec{\nabla}\times\vec{dl}' = 0$ (as curl is wrt unprimed)

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi}\int \vec{\nabla}\times\left(\frac{\vec{dl}'}{R}\right)$$

MAGNETIC FLUX I.T.O $\vec{A}$

$$\phi = \int_S \vec{B}\cdot\vec{ds} = \int_S (\vec{\nabla}\times\vec{A})\cdot\vec{ds} = \int_l \vec{A}\cdot\vec{dl}$$

$$\Rightarrow \boxed{\phi = \int_l \vec{A}\cdot\vec{dl}}$$

for an ∞ solenoid:

$$\int_l \vec{B}\cdot\vec{dl} = \mu_0 N I$$

$$\Rightarrow \vec{B}\cdot L = \mu_0 N I \Rightarrow \vec{B} = \mu_0\left(\frac{N}{L}\right)I \Rightarrow \boxed{\vec{B} = \mu_0 n I_{en}}$$

$$\phi = \int_S \vec{B}\cdot\vec{ds} \quad ; \quad \phi = \int \vec{A}\cdot\vec{dl}$$

$$\phi = \mu_0 n I(\pi s^2) \quad . \quad \phi = A\cdot 2\pi s$$

$$A = \frac{\mu_0 n I s}{2}$$

MAGNETIC FIELD INTENSITY

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

$$\Rightarrow \oint \mu_0 \vec{H} \cdot \vec{dl} = \mu_0 I_{enc} \Rightarrow \boxed{\oint \vec{H} \cdot \vec{dl} = I_{enc}}$$

$$\vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} \right) = \frac{1}{\mu_0} \cdot \vec{\nabla} \times \vec{B}$$

$$= \frac{1}{\mu_0} \left(\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) \right)$$

$$= \frac{1}{\mu_0} \left(\cancel{\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A})}^{\,0} - \nabla^2 A \right)$$

$$\vec{\nabla} \times H = \frac{-\nabla^2 A}{\mu_0}$$

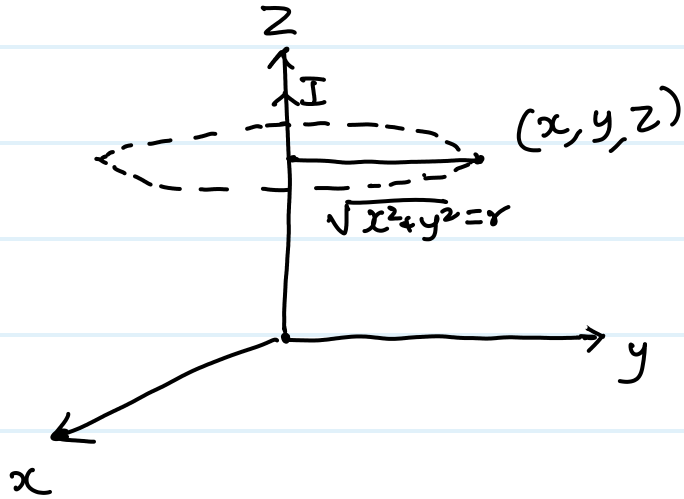
$$= \frac{+\cancel{\mu_0} J}{\cancel{\mu_0}} \Rightarrow \boxed{\vec{\nabla} \times \vec{H} = J}$$

$$\oint \vec{H}\cdot\vec{dl} = \int_s (\vec{\nabla}\times\vec{H})\cdot\vec{ds} = I_{enc} = \int_s \vec{J}\cdot\vec{ds}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{H} = \vec{J}}$$

$H \to$ irrotational

EXAMPLE



$$\int \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\int \vec{H}\cdot\vec{dl} = I$$

$$\Rightarrow H\cdot 2\pi r = I$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{2\pi r}(\hat{z}\times\hat{r})}$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}\hat{\phi}}$$