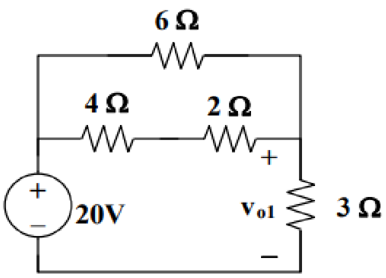
Solution set for chapter-04 (CIRCUITS THEOREMS)

SUPERPOSITION THEOREM:

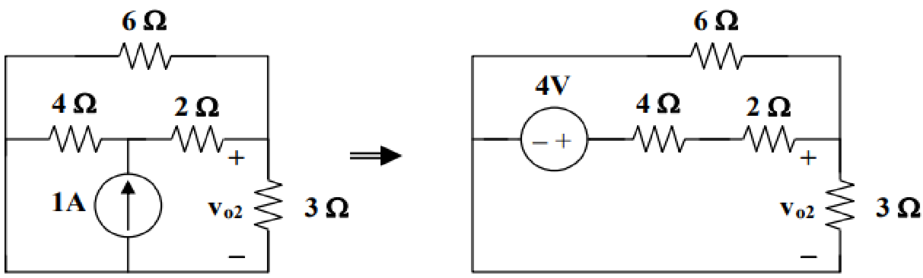
Q1.

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2}, and v_{o3}, are due to the 20-V, 1-A, and 2-A sources respectively. For v_{o1}, consider the circuit below.



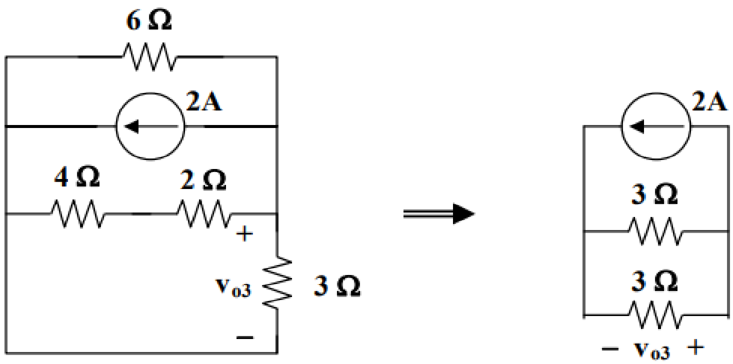
$$6||(4 + 2) = 3 \text{ ohms}, v_{o1} = (½)20 = 10 \text{ V}$$

For v_{o2}, consider the circuit below.



$$3||6 = 2 \text{ ohms}, v_{o2} = [2/(4 + 2 + 2)]4 = 1 \text{ V}$$

For v_{o3}, consider the circuit below.

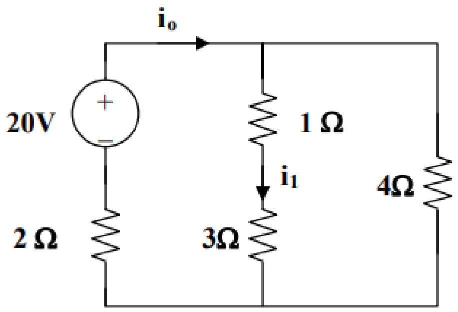


$$6||(4 + 2) = 3, v_{o3} = (-1)3 = –3$$

$$v_o = 10 + 1 – 3 = \mathbf{8\ V}$$

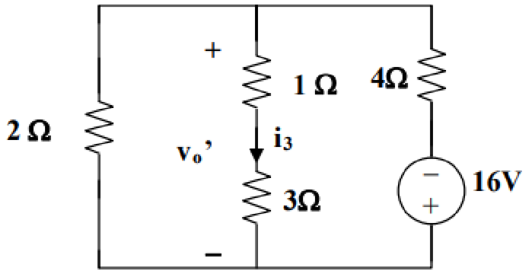
Q2.

Let $i = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are due to the 20-V, 2-A, and 16-V sources. For i_1, consider the circuit below.



$$4||(3 + 1) = 2 \text{ ohms}, \text{ Then } i_o = [20/(2 + 2)] = 5 \text{ A}, \; i_1 = i_o/2 = 2.5 \text{ A}$$

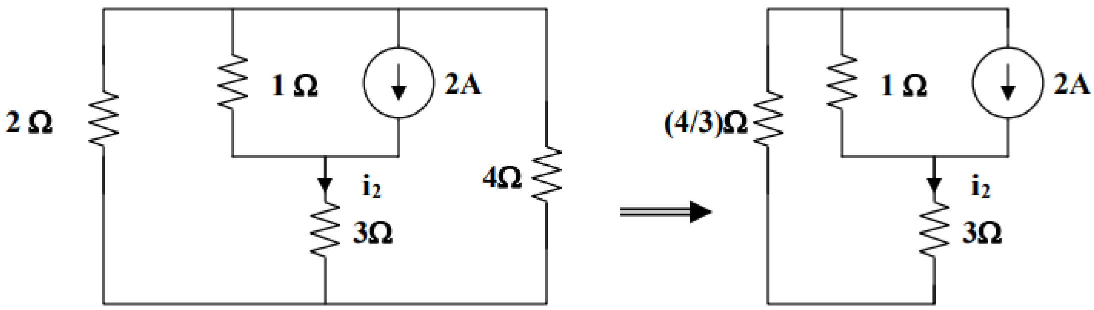
For i_3, consider the circuit below.



$$2||(1 + 3) = 4/3, \; v_o' = [(4/3)/((4/3) + 4)](-16) = -4$$

$$i_3 = v_o'/4 = -1$$

For i_2, consider the circuit below.



$$2||4 = 4/3, \; 3 + 4/3 = 13/3$$

Using the current division principle.

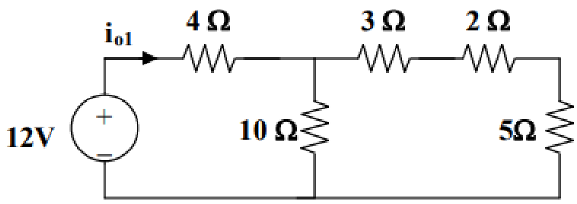
$$i_2 = [1/(1 + 13/3)]2 = 3/8 = 0.375$$

$$i = 2.5 + 0.375 - 1 = \mathbf{1.875\ A}$$

$$p = i^2R = (1.875)^2 3 = \mathbf{10.55\ watts}$$

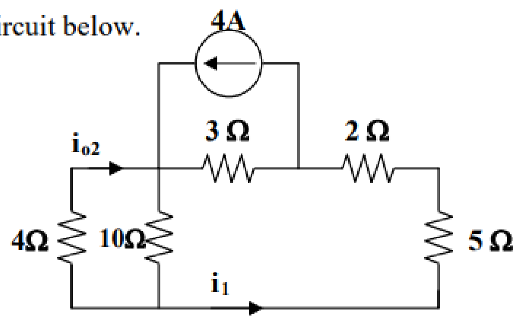
Q3.

Let $i_o = i_{o1} + i_{o2} + i_{o3}$, where i_{o1}, i_{o2}, and i_{o3} are due to the 12-V, 4-A, and 2-A sources. For i_{o1}, consider the circuit below.



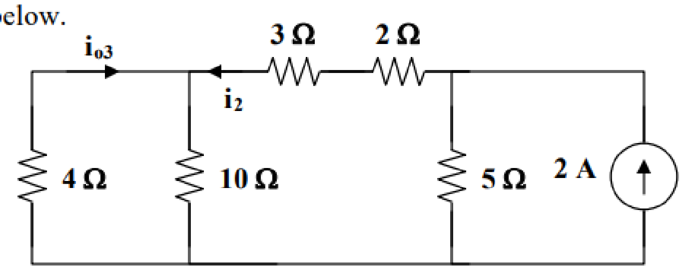
$$10||(3 + 2 + 5) = 5 \text{ ohms}, \quad i_{o1} = 12/(5 + 4) = (12/9) \text{ A}$$

For i_{o2}, consider the circuit below.



$$2 + 5 + 4||10 = 7 + 40/14 = 69/7$$
$$i_1 = [3/(3 + 69/7)]4 = 84/90, \quad i_{o2} = [-10/(4 + 10)]i_1 = -6/9$$

For i_{o3}, consider the circuit below.



$$3 + 2 + 4||10 = 5 + 20/7 = 55/7$$

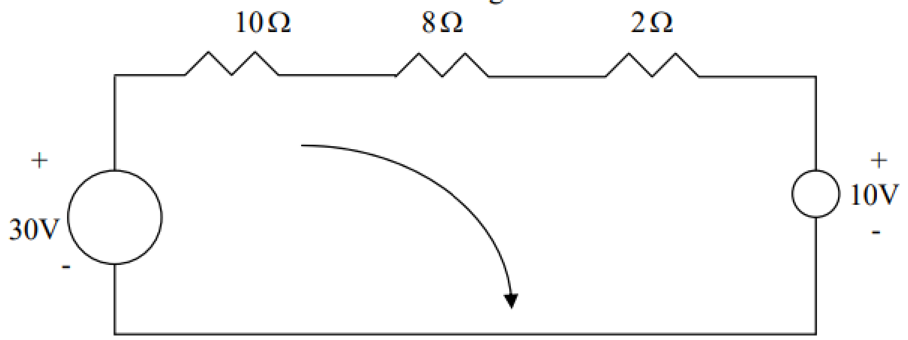
$$i_2 = [5/(5 + 55/7)]2 = 7/9, \quad i_{o3} = [-10/(10 + 4)]i_2 = -5/9$$

$$i_o = (12/9) - (6/9) - (5/9) = 1/9 = \mathbf{111.11\ mA}$$

SOURCE TRANSFORMATION:

Q1.

3//6 = 2-ohm. Convert the current sources to voltages sources as shown below.



Applying KVL to the loop gives

$$-30+10+I(10+8+2)=0 \longrightarrow I = \mathbf{1\ A}$$

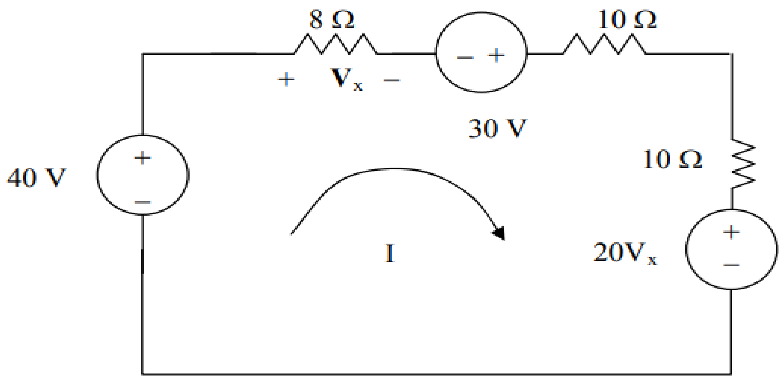
$$p = VI = I^2R = \mathbf{8\ W}$$

Q2.

Transform the two current sources in parallel with the resistors into their voltage source equivalents yield,

a 30-V source in series with a 10-Ω resistor and a $20V_x$-V sources in series with a 10-Ω resistor.

We now have the following circuit,



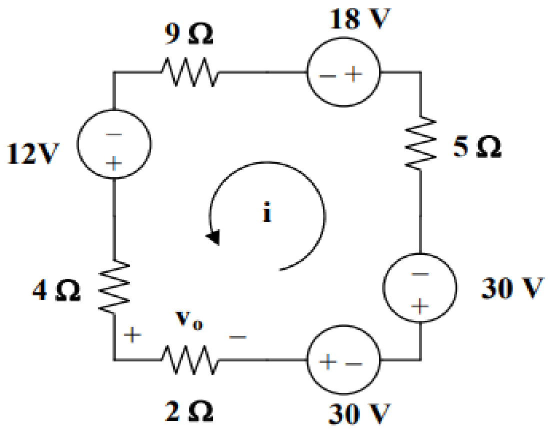
We now write the following mesh equation and constraint equation which will lead to a solution for V_x,

$28I - 70 + 20V_x = 0$ or $28I + 20V_x = 70$, but $V_x = 8I$ which leads to

$28I + 160I = 70$ or $I = 0.3723$ A or $V_x = \mathbf{2.978\ V}$.

Q3.

Transforming only the current source gives the circuit below.



Applying KVL to the loop gives,

$$-(4 + 9 + 5 + 2)i + 12 - 18 - 30 - 30 = 0$$

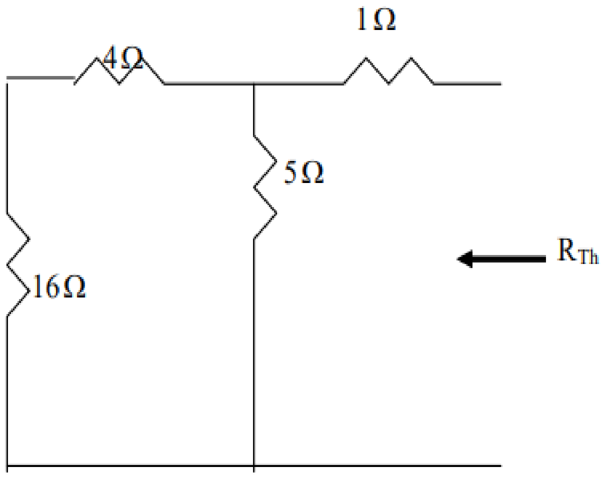
$$20i = -66 \text{ which leads to } i = -3.3$$

$v_o = 2i = \mathbf{-6.6\ V}$

THEVENIN AND NORTON THEOREM:

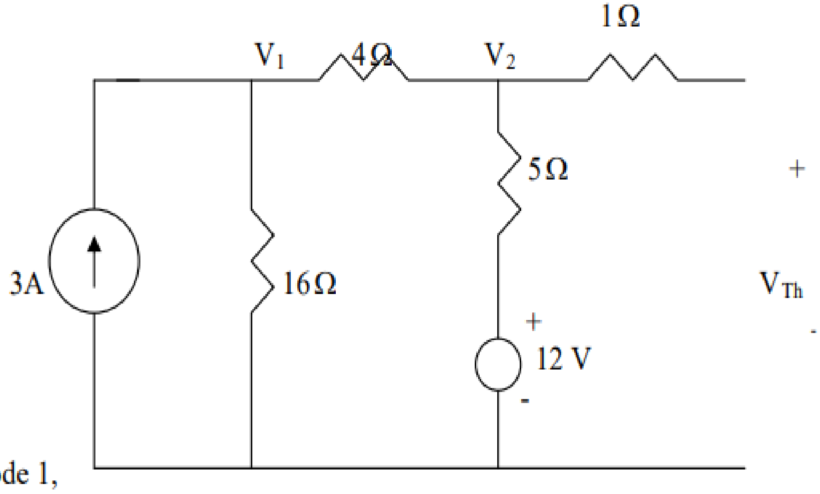
Q1.

We find Thevenin equivalent at the terminals of the 10-ohm resistor. For R_{Th}, consider the circuit below.



$$R_{Th} = 1 + 5 // (4 + 16) = 1 + 4 = 5\Omega$$

For V_{Th}, consider the circuit below.



At node 1,

$$3 = \frac{V_1}{16} + \frac{V_1 - V_2}{4} \longrightarrow 48 = 5V_1 - 4V_2 \qquad (1)$$

At node 2,

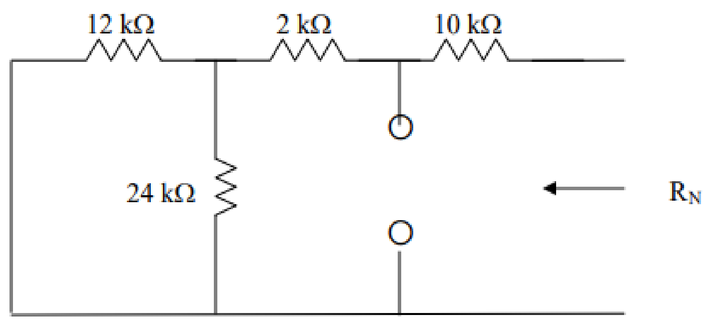
$$\frac{V_1 - V_2}{4} + \frac{12 - V_2}{5} = 0 \longrightarrow 48 = -5V_1 + 9V_2 \qquad (2)$$

Solving (1) and (2) leads to

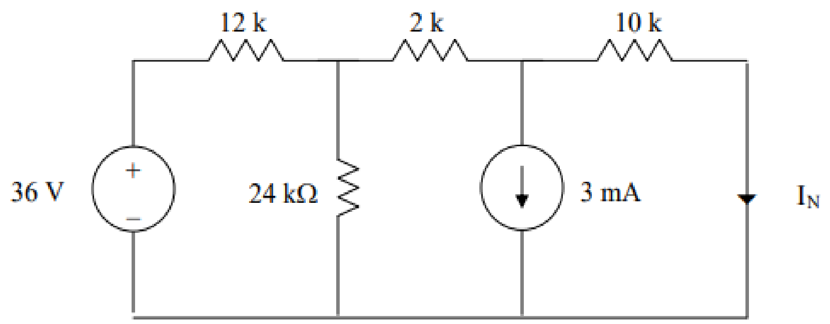
$$V_{Th} = V_2 = 19.2$$

Q2.

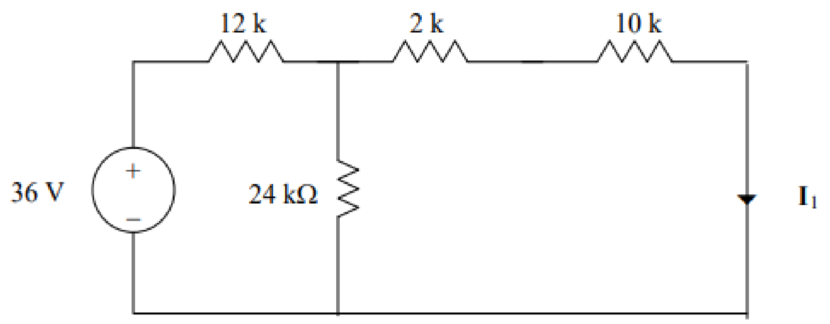
We remove the 1-kΩ resistor temporarily and find Norton equivalent across its terminals. R_{eq} is obtained from the circuit below.



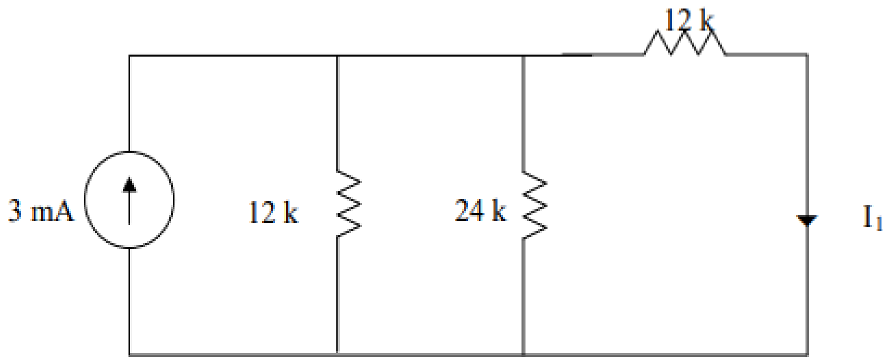
$R_{eq} = 10 + 2 + (12//24) = 12+8 = 20$ kΩ
I_N is obtained from the circuit below.



We can use superposition theorem to find I_N. Let $I_N = I_1 + I_2$, where I_1 and I_2 are due to 16-V and 3-mA sources respectively. We find I_1 using the circuit below.



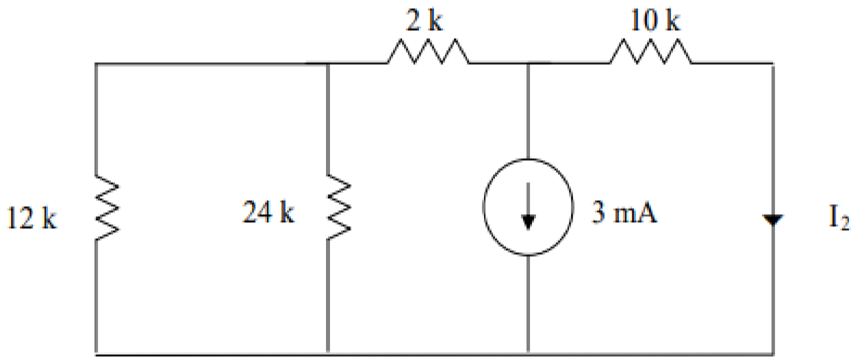
Using source transformation, we obtain the circuit below.



12//24 = 8 kΩ

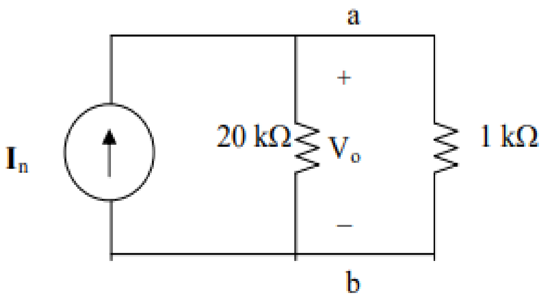
$$I_1 = \frac{8}{8+12}(3mA) = 1.2 \text{ mA}$$

To find I_2, consider the circuit below.



2k + 12k//24 k = 10 kΩ
I_2=0.5(-3mA) = -1.5 mA
I_N = 1.2 –1.5 = -0.3 mA

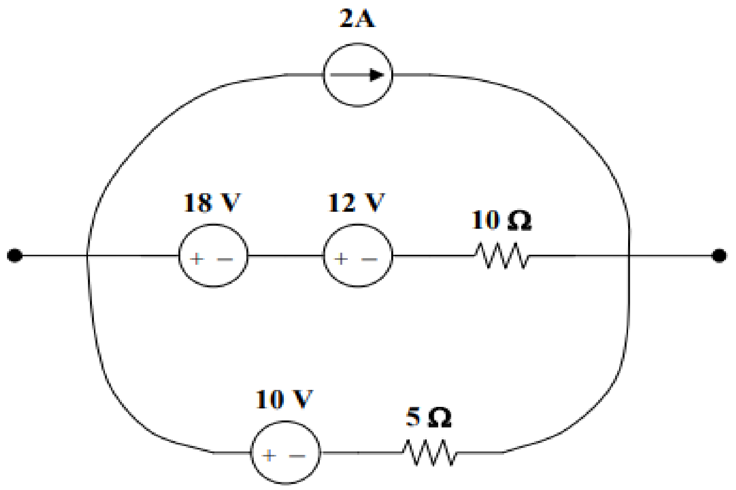
The Norton equivalent with the 1-kΩ resistor is shown below

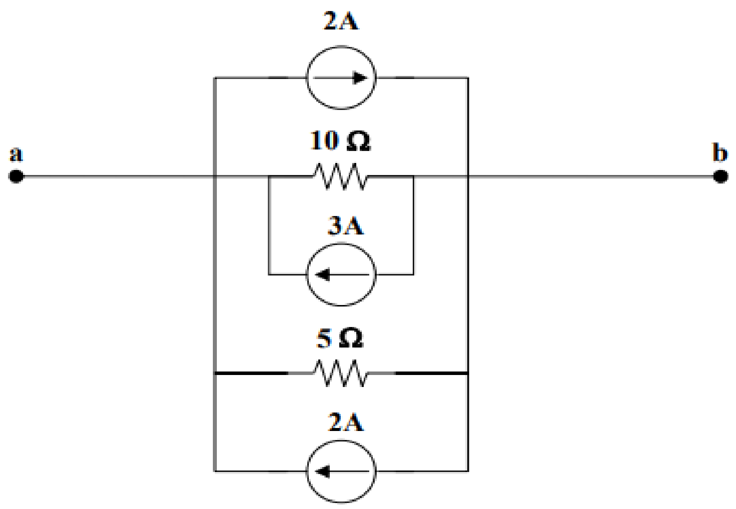


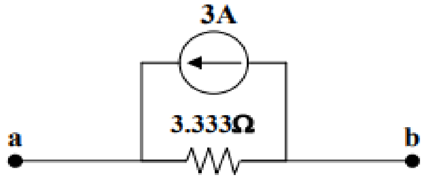
V_o = 1k(20/(20+1))(-0.3 mA) = **-285.7 mV**.

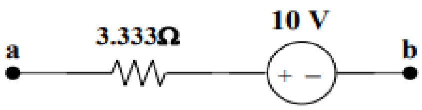
Q3.

The circuit can be reduced by source transformations.





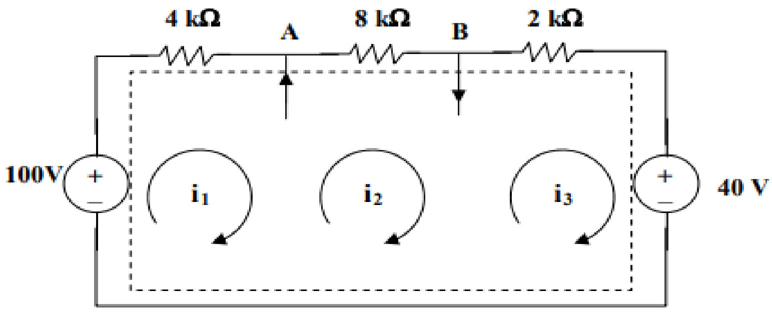




Norton Equivalent Circuit **Thevenin Equivalent Circuit**

SUPER MESH ANALYSIS:

Q1.



We have a supermesh. Let all R be in kΩ, i in mA, and v in volts.

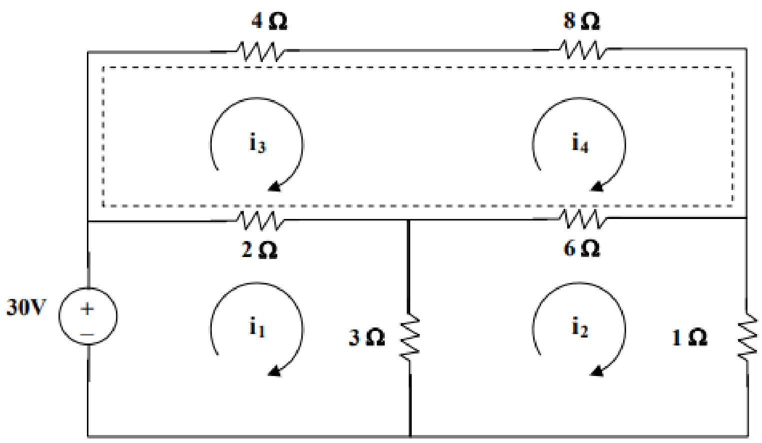
For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$ or $30 = 2i_1 + 4i_2 + i_3$ (1)

At node A, $i_1 + 4 = i_2$ (2)

At node B, $i_2 = 2i_1 + i_3$ (3)

Solving (1), (2), and (3), we get $i_1 =$ **2 mA**, $i_2 =$ **6 mA**, and $i_3 =$ **2 mA**.

Q2.



For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$ (1)

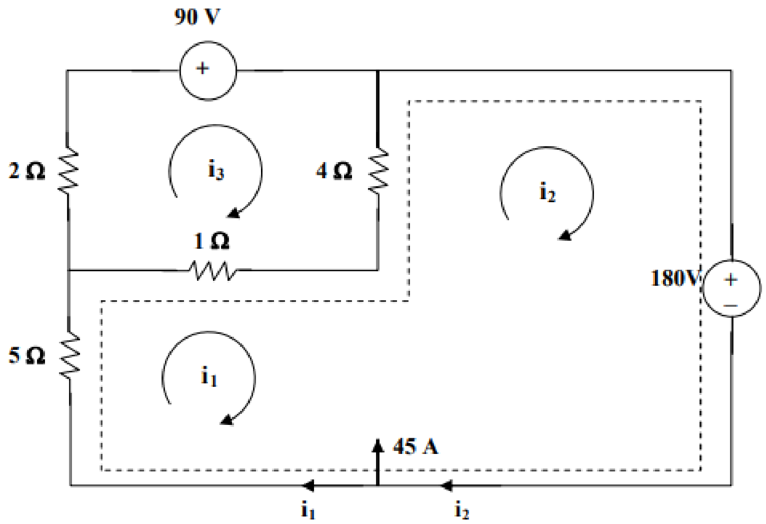
For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$ (2)

For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$ (3)

But $i_4 - i_3 = 4$ which leads to $i_4 = i_3 + 4$ (4)

Solving (1) to (4) by elimination gives $i = i_1 =$ **8.561 A**.

Q3.



Loop 1 and 2 form a supermesh. For the supermesh,

$$6i_1 + 4i_2 - 5i_3 + 180 = 0 \qquad (1)$$

For loop 3, $$-i_1 - 4i_2 + 7i_3 + 90 = 0 \qquad (2)$$

Also, $$i_2 = 45 + i_1 \qquad (3)$$

Solving (1) to (3), $i_1 = -46$, $i_3 = -20$; $i_o = i_1 - i_3 = \mathbf{-26\ A}$