Engineering Optics

Lecture 28

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by

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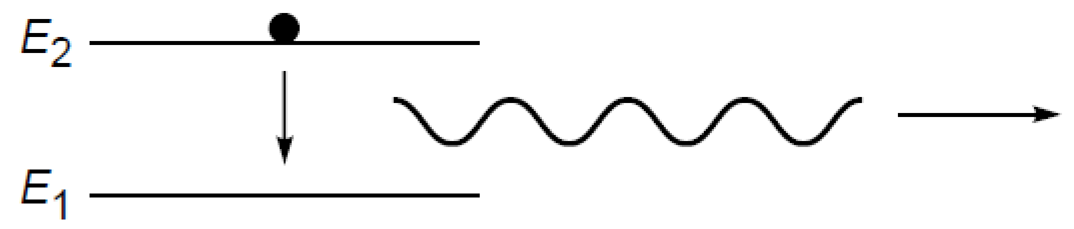
Spontaneous and Stimulated Emissions

$$\omega = \frac{E_2 - E_1}{\hbar}$$

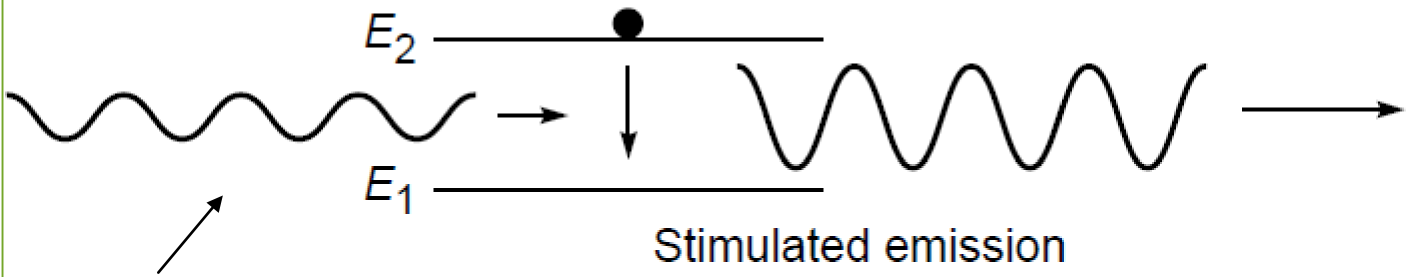
$$\hbar = \frac{h}{2\pi} \approx 1.0546 \times 10^{-34}\ \mathrm{J\ s}$$

- Atoms → discrete energy states
- **Q: How does an atom interact with electromagnetic radiation??**
- Ans: according to Einstein → 3 different ways



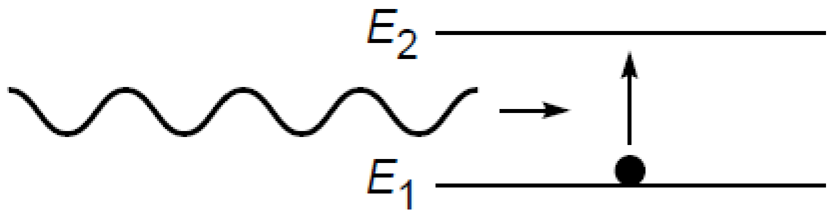
Spontaneous emission
(a)

The rate of spontaneous emission is proportional to the number of atoms in the excited state



Stimulated emission
(b)

The rate of stimulated emission depends on both the intensity of the external field and the number of atoms in the excited state



Absorption
(c)

The rate of stimulated absorption depends both on the intensity of the external field and on the number of atoms in the lower energy state.

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Population inversion

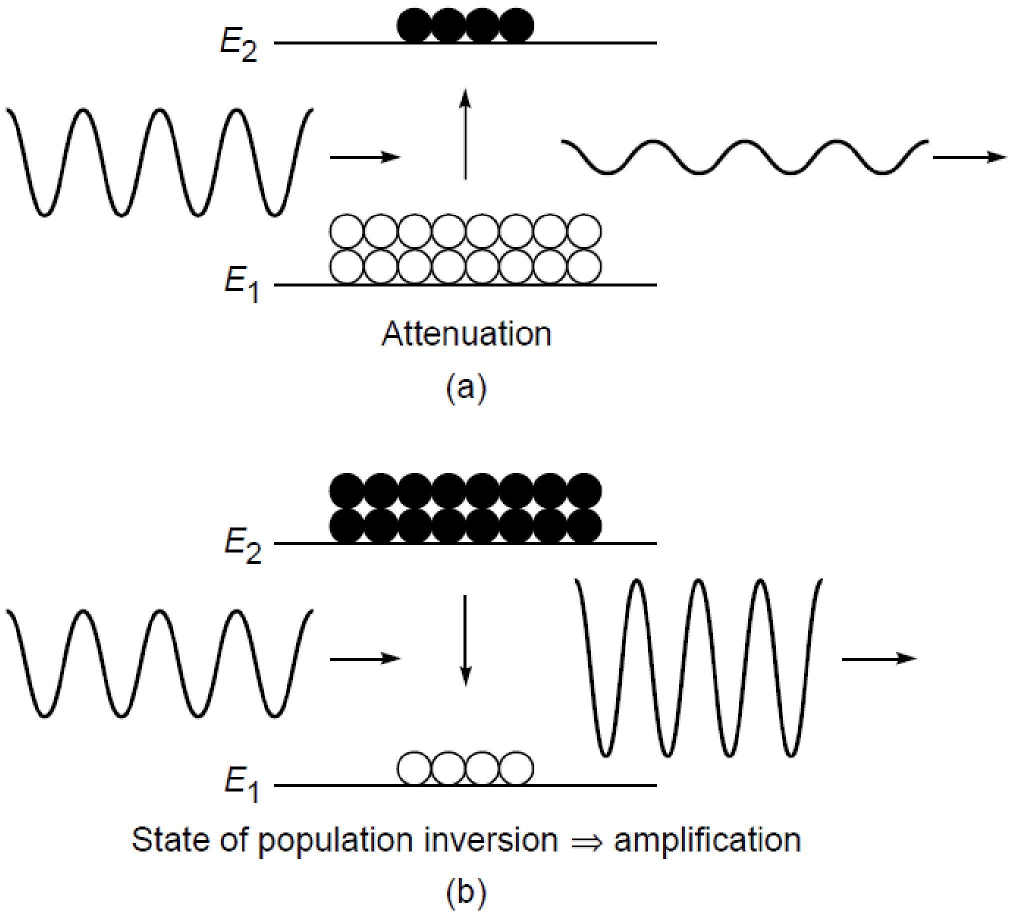

Fig. 26.2 (a) A larger number of atoms in the lower state result in the attenuation of the beam. (b) A larger number of atoms in the upper state (which is known as population inversion) result in the amplification of the beam.

When the atoms are in thermodynamic equilibrium, there are larger number of atoms in the lower state → **absorptions**

For **Stimulated emission** → more and more atoms need to be in the excited state ← ***Problem***

Solution → create a state of population inversion in which there are larger number of atoms in the upper state

Problem: lifetime of the excited state ~ 10^{-8} Sec

Solution??

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What is 'Population'?

- Imagine a chamber filled with a gas in equilibrium at some temperature T
- **If T is relatively low, most of the atoms will be in their ground states**, but a few will "rise" into an excited state
- Maxwell-Boltzmann distribution (N: number of atoms/volume)

$$N_i = N_0\, exp^{-E_i/k_BT}$$

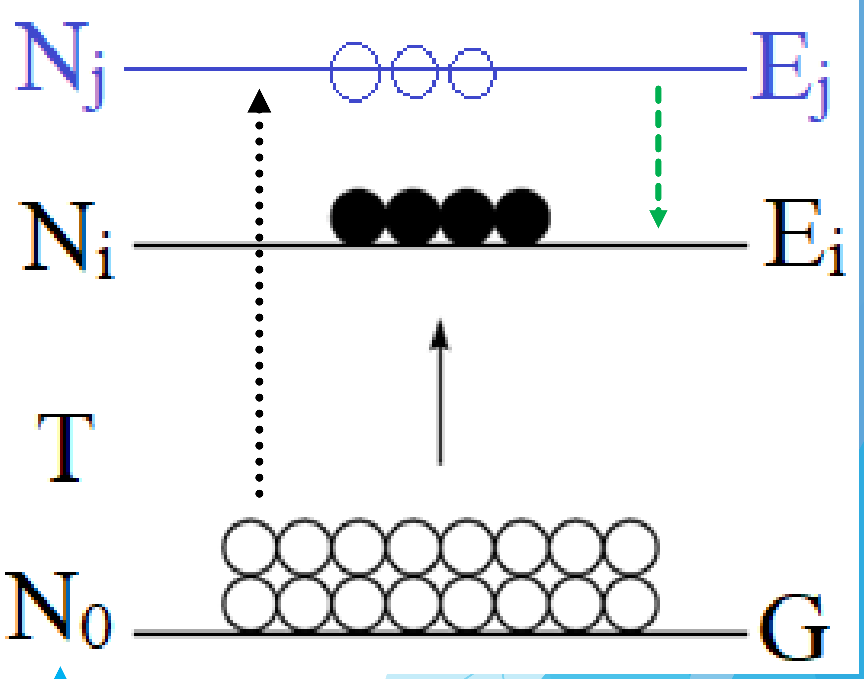
- higher E → fewer atoms there will be in that state

$$N_j = N_0\, exp^{-E_j/k_BT}$$

- Where $E_j > E_i$
- ratio of the populations occupying these two states

$$\frac{N_j}{N_i} = \frac{exp^{-E_j/k_BT}}{exp^{-E_i/k_BT}}$$

relative population, $N_j = N_i\, exp^{-(E_j-E_i)/k_BT}$



constant for a given T

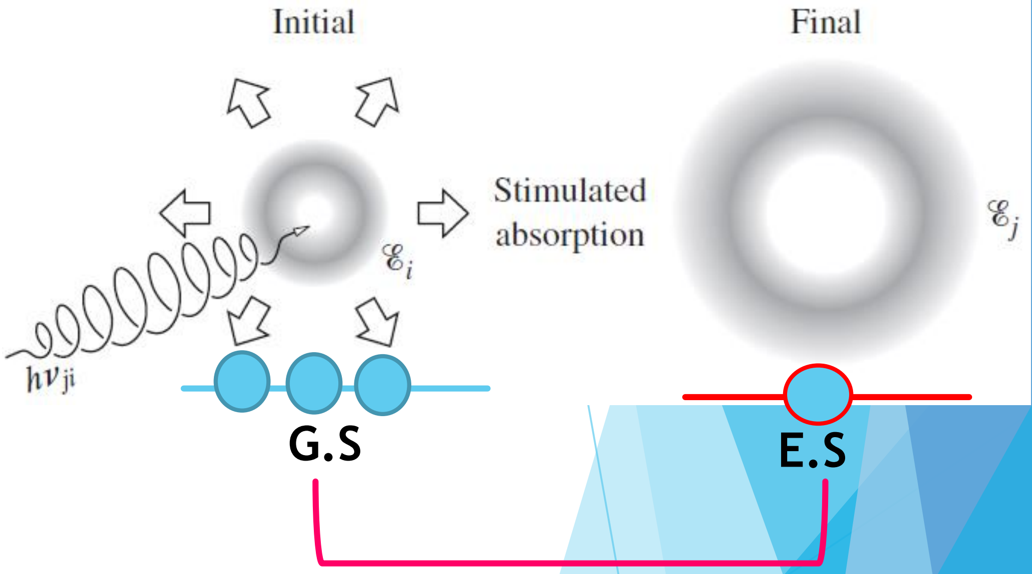
A transition from j^{th} to i^{th} state is also possible!

Optics, Hecht

The Einstein A and B Coefficients: Stimulated absorption

In 1916 Einstein devised an elegant and rather simple theoretical treatment of the dynamic equilibrium existing for a material medium bathed in electromagnetic radiation, absorbing and reemitting.

photon having an adequate amount of energy interacts with the atom, imparting that energy to the atom, thereby causing the electron cloud to take on a new configuration. The atom jumps into a higher-energy excited state



stimulated absorption, whereupon the transition rate is

$$\left(\frac{dN_i}{dt}\right)_{\text{ab}} = -B_{ij}N_i u_\nu$$

u_ν: spectral energy density = energy per unit volume per unit frequency interval ($J.s/m^3$)

Here B_{ij} is a constant of proportionality, the *Einstein absorption coefficient*, and the minus arises because N_i is decreasing.

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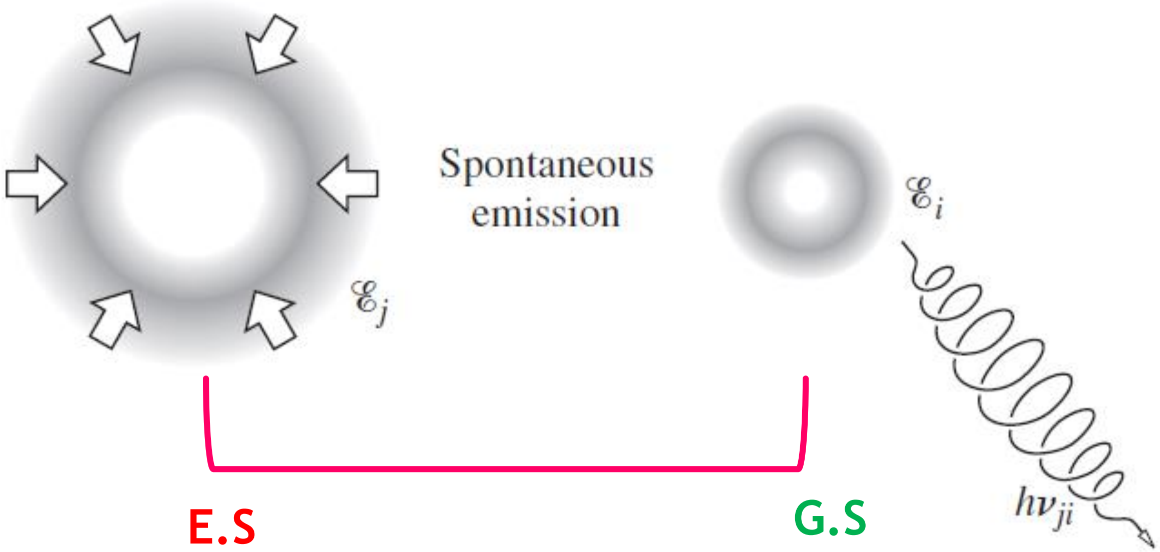
The Einstein A and B Coefficients: Spontaneous emission

Such an excess-energy configuration is usually (though not always) exceedingly short-lived, and in 10 ns or so, without the intercession of any external influence, the atom will emit its overload of energy as a photon. As it does, it reverts to a stable state in a process called *spontaneous emission*



[spontaneous emission] $$\left(\frac{dN_j}{dt}\right)_{sp} = -A_{ji}N_j$$

This is the rate of decrease of the higher-energy population, N_j, due to spontaneous emission. And A_{ji} is the *Einstein spontaneous emission coefficient* associated with a drop from energy level-j to level-i.

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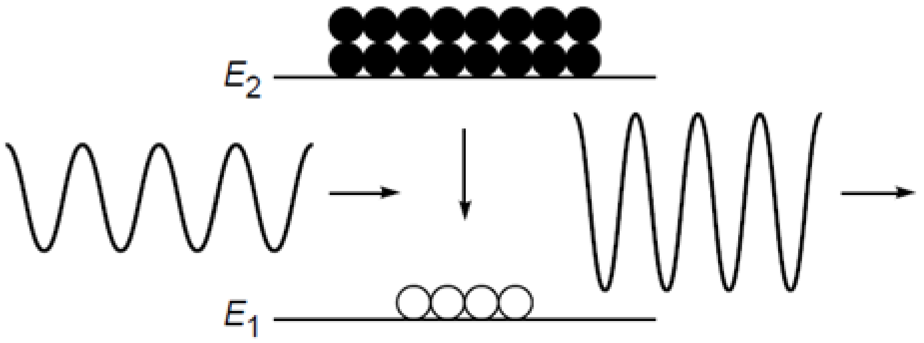
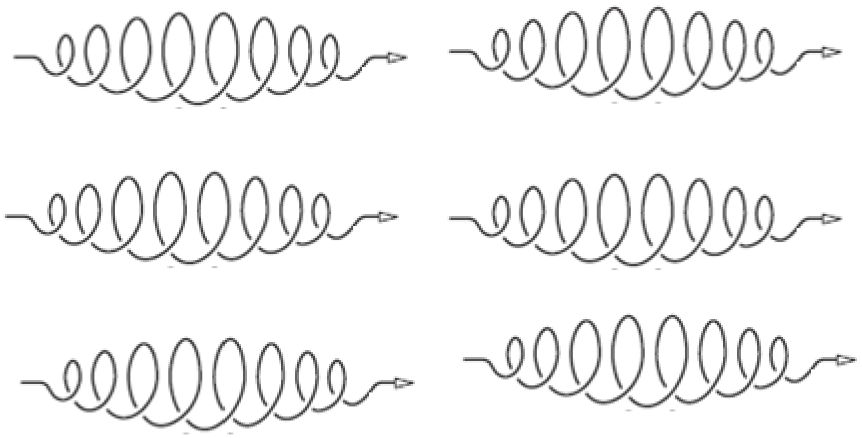
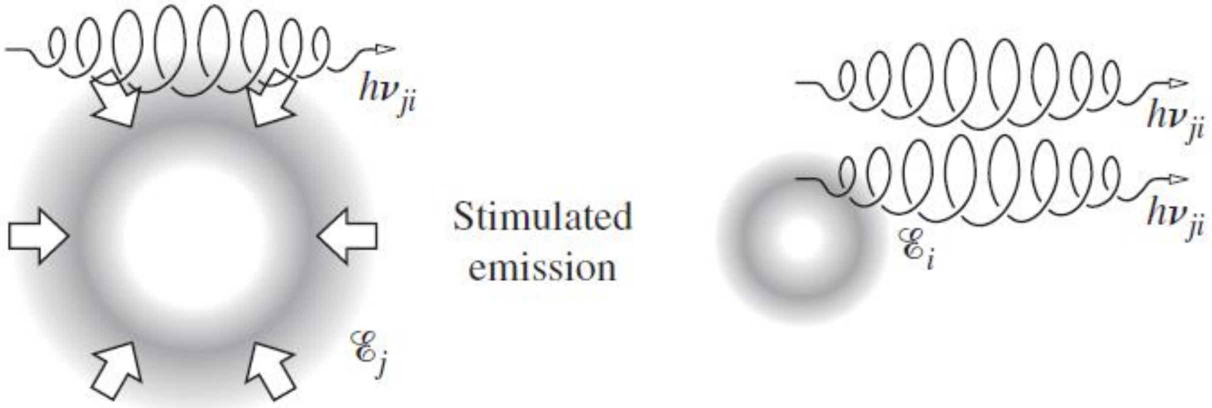
The Einstein A and B Coefficients: Stimulated emission



For a medium inundated with EM-radiation, it's possible for a photon to interact with an excited atom while that atom is still in its higher-energy configuration. The atom can then dump its excess energy in-step with the incoming photon, in a process now called **stimulated emission**



[stimulated emission]



$$\left(\frac{dN_j}{dt}\right)_{st} = -B_{ji}N_j u_\nu$$

The constant B_{ji} is the *Einstein stimulated emission coefficient*.

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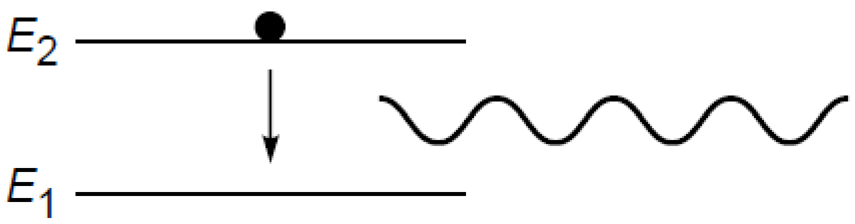
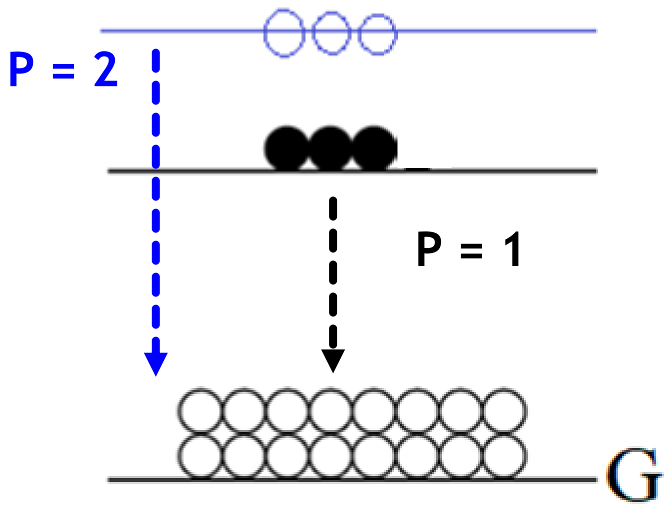
Probability and life time of a state

Keep in mind that the transition rate, the number of atoms making transitions per second, divided by the number of atoms, is the probability of a transition occurring per second, $\mathscr{P}$. Consequently, the probability per second of spontaneous emission is $\mathscr{P}_{sp} = A_{ji}$.





Q: most preferred state?

For a single excited atom making a spontaneous transition to a lower state, the inverse of the transition probability per second is the **mean life** or **lifetime** of the excited state τ. Thus (operating under conditions that exclude any other mechanism but spontaneous emission), if N atoms are in that excited state, the total rate of transitions, that is, the number of emitted photons per second, is $N\mathscr{P}_{sp} = NA_{ji} = N/\tau$. A low-transition probability means a long lifetime.

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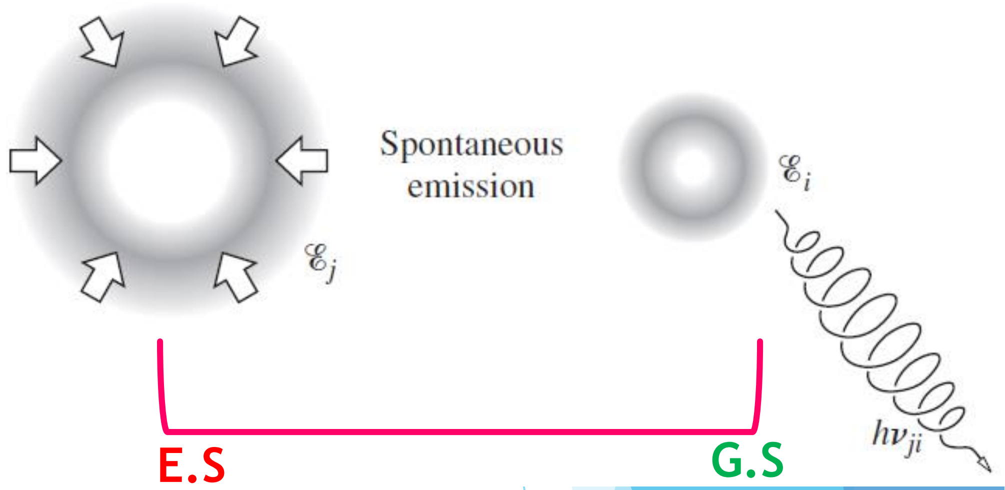
Problem-1

Q: Suppose a sample exists where there are N_j excited electrons per unit volume in energy level-j just above the ground state level-i. Show that the population of energy level-j falls exponentially as electrons leave via spontaneous emission. What can be said about the lifetime of level-j?



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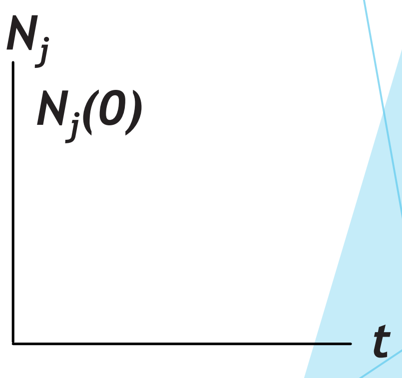
$$\frac{dN_j}{N_j} = -A_{ji}\,dt \;\Rightarrow\; \int \frac{dN_j}{N_j} = \int -A_{ji}\,dt + C$$

Say at $t = 0$, $N_j = N_j(0)$ $\longrightarrow$ $\boldsymbol{N_j = N_j(0)e^{-A_{ji}t}}$



Probability of transition occurrence/Sec = P = A_{ji}

→ Lifetime τ = 1/P = 1/A_{ji}

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QM facts

- A photon hits the matter
- If its energy is less than E_{12}? → scattering of photon
- If more and not within the energy width (ΔE) of the atomic energy level → scattering of photon
- If energy of the photon > the ionization energy of the atom, → electron will be kicked off and atom → ion (The photoelectric effect??)
- Quantum mechanics → probabilities (P) matter.
- photon carrying $E \approx E_{12}$, $P_{(absorption/emission)}$→ very high,
- P~0 for photon carrying energy different from E_{12}
- Photons→ bosons→ in presence of other bosons→ same phase→ clone each other

Thank You