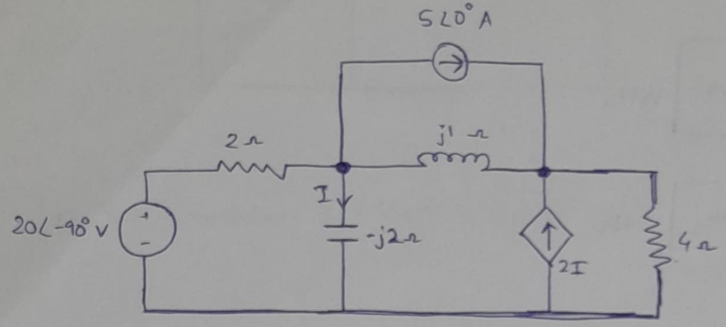
23/11/23

ASSIGNMENT - II

(EC1000)

Y. HARITH
CS23I1027
(B2)

Q1)



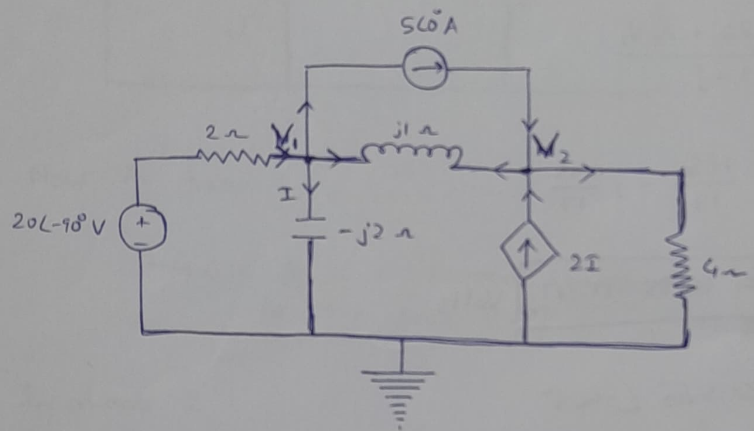
To Find :

I

Process :

Nodal Analysis

A1) Let's first Indicate two available nodes and the reference node.



Applying Nodal Analysis at Node 1 :

$$\frac{20\angle -90^\circ - V_1}{2} = 5\angle 0^\circ + \frac{V_1 - V_2}{j1} + \frac{V_1 - 0}{-j2}$$

where,

$$I = \frac{V_1 - 0}{-j2}$$

$$\therefore I = \frac{jV_1}{2}$$

$$\Rightarrow 20\angle -90^\circ - V_1 = 10\angle 0^\circ + (2V_2 - 2V_1)j + jV_1$$

$$\Rightarrow 20\angle -90^\circ - 10\angle 0^\circ = V_1(1-j) + V_2(2j)$$

$$\Rightarrow -20j - 10 = V_1(1-j) + V_2(2j)$$

$$\Rightarrow 10 + 20j = V_1(-1+j) + V_2(-2j) \quad - (1)$$

Applying Nodal Analysis at Node 2 :

$$2I + 5\angle 0^\circ = \frac{V_2 - V_1}{j1} + \frac{V_2 - 0}{4}$$

$$\Rightarrow \cancel{V_1 j} + 5 = \cancel{V_1 j} - V_2 j + \frac{V_2}{4}$$

$4\angle -30^\circ$ A

$$\Rightarrow 5 = V_2\left(\frac{1}{4} - j\right)$$

$$\therefore V_2 = \frac{5}{\frac{1}{4} - j} = \frac{20}{17} + j\,\frac{80}{17}$$

$$\boxed{\therefore V_2 = \frac{20}{\sqrt{17}} \angle 75.964^\circ} \text{ Volts}$$

(OR) $\boxed{V_2 = 1.176 + 4.706j}$ Volts

Now substituting in ①:

$$\Rightarrow 10 + 20j = V_1(-1+j) + V_2(-2j)$$

$$\Rightarrow V_1(-1+j) = 10 + 20j + 2j(V_2)$$

$$\Rightarrow V_1 = \frac{10 + 20j + 2jV_2}{-1+j}$$

$$\Rightarrow V_1 = \frac{185}{17} - j\,\frac{195}{17}$$

$$\boxed{\therefore V_1 = 10.88 - 11.47j} \text{ Volts}$$

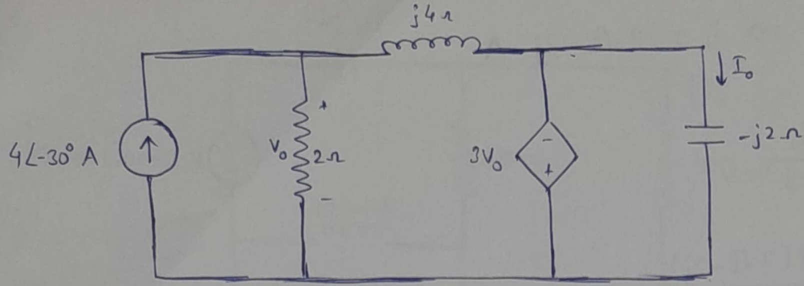
(OR) $V_1 = 5\sqrt{10} \angle -46.5^\circ$

$$\boxed{\therefore V_1 = 15.81 \angle -46.5^\circ} \text{ Volts}$$

Now $I = \frac{jV_1}{2}$

$$\boxed{\therefore I = 5.73 + 5.44j} \text{ A}$$

$$\boxed{\therefore I = 7.906 \angle 43.5^\circ} \text{ A}$$



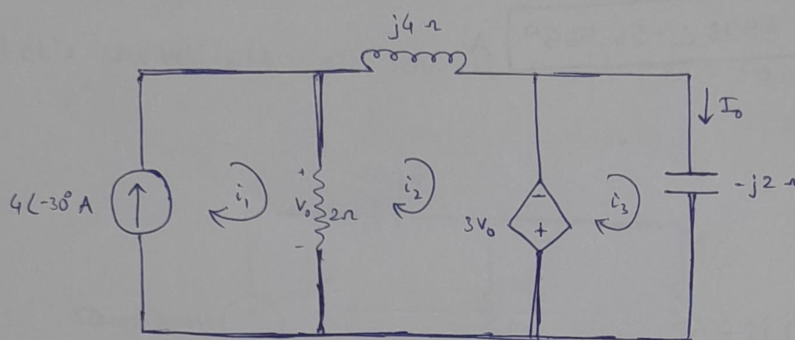
To find :

I_o and V_o

Process :

Mesh Analysis

A2) Let us consider 3 loops



Now in Loop 1 :

$-4\angle 30^\circ$ A is a Current source in the loop itself, $\therefore$ $\boxed{i_1 = 4\angle -30^\circ \text{ A}}$

In Loop 2 :

$$2(i_2 - i_1) + j4(i_2) - 3V_o = 0 \quad -①$$

~~But $V_o = 2(i_1 - i_2)$~~

But from Loop 1, $V_{2\Omega} = i_{2\Omega} \times 2$

$$\Rightarrow V_{2\Omega} = 2(i_1 - i_2)$$

$$\boxed{\therefore V_o = 2(i_1 - i_2)} \quad -②$$

Put ② in ① :

$$2(i_2 - i_1) + j4(i_2) + 6(i_2 - i_1) = 0$$

$$\Rightarrow 8(i_2 - i_1) + j4(i_2) = 0$$

$$\Rightarrow 2(i_2 - i_1) + ji_2 = 0$$

$$\Rightarrow 2i_1 - (2+j)\, i_2 = 0 \quad -③$$

But $i_1 = 4\angle{-30^\circ}$ A $\Rightarrow i_1 = 2\sqrt{3} - 2j$ A

$$\Rightarrow i_2 = \frac{2i_1}{2+j}$$

$$\therefore i_2 = \frac{2(2\sqrt{3}-2j)}{2+j}$$

$$\boxed{\therefore i_2 = 1.97 - 2.9856j} \text{ A}$$

(OR) $\boxed{i_2 = 3.578\angle{-56.565^\circ}}$ A

In Loop 3 :

$$\boxed{i_3 = I_0}$$

$$3V_0 + (-j2)\, i_3 = 0$$

But we know from (2) that

$$V_0 = 2(i_1 - i_2)$$

$$= 2(2\sqrt{3} - 2j - 1.97 + 2.9856j)$$

$$= 2(1.494 + 0.9856j)$$

$$\Rightarrow \boxed{V_0 = 2.988 + 1.971j} \text{ Volts}$$

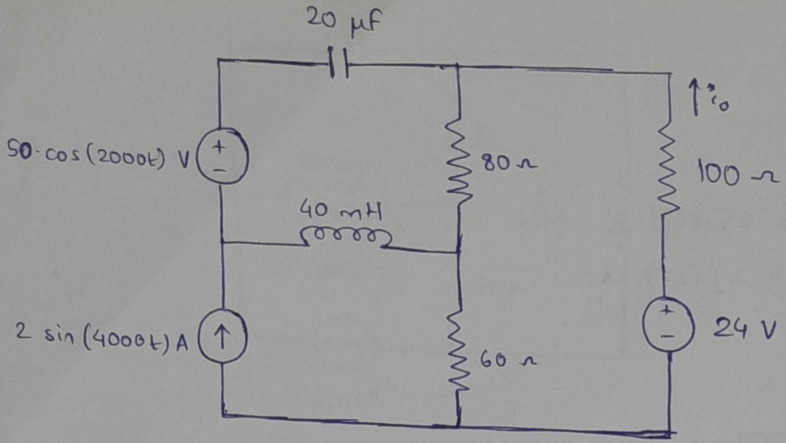
(OR) $\boxed{V_0 = 3.58\angle{33.41^\circ}}$ Volts

$$\Rightarrow i_3 = \frac{3V_0}{2j} = \frac{10.74\angle{33.41^\circ}}{2\angle{90^\circ}} = 5.22\angle{-56.6^\circ} \text{ A}$$

$$\boxed{\therefore I_0 = 5.22\angle{-56.59^\circ}} \text{ A}$$

$$\boxed{\therefore I_0 = 2.874 - 4.357j} \text{ A}$$

$$\boxed{\begin{array}{l} \therefore I_0 = 5.22\angle{-56.59^\circ} \text{ A} \\ V_0 = 3.58\angle{33.41^\circ} \text{ A} \end{array}}$$



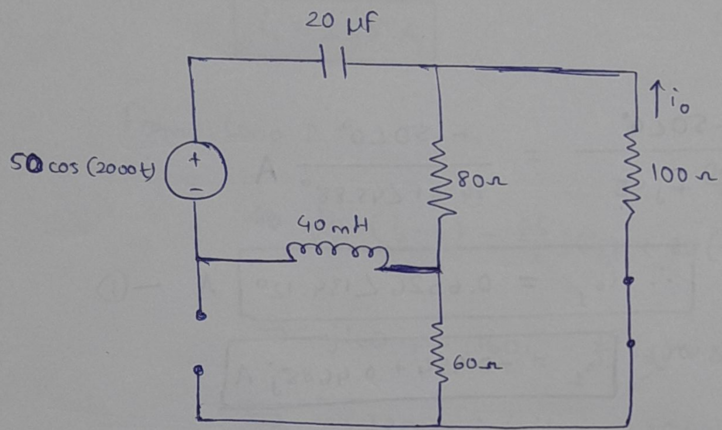
To find :

i_o

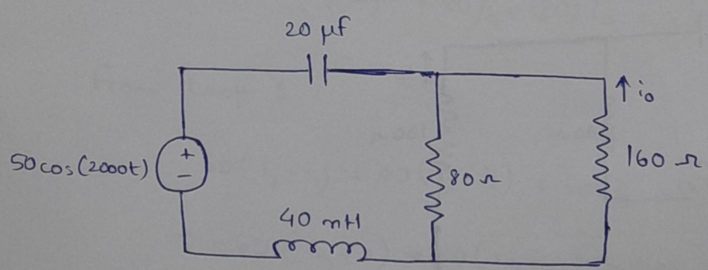
Process :

Superposition Theorem

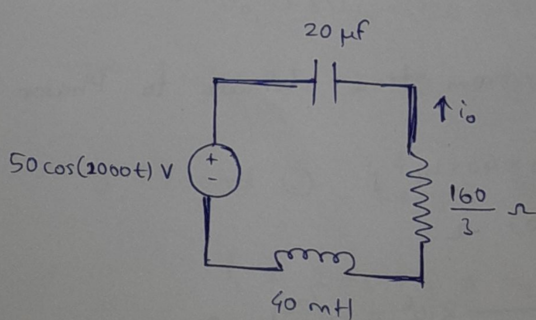
A3) Let's switch off all sources except $V_1 = 50 \cos(2000t)$ V



$\Downarrow$



$\Downarrow$



Quantities are given in time domain. Let us convert them to Phasor Domain, with

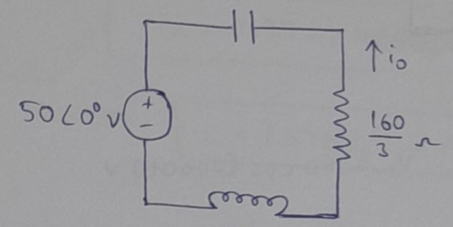
$\omega = 2000 \ s^{-1}$

$X_L = j\omega L = j(2000)(40\times10^{-3})$

$= j(80)\ \Omega$

$X_C = \frac{-j}{\omega C} = \frac{-j}{2000\times 20}\times10^6 = 25(-j)\ \Omega$

$= -j(25)\ \Omega$

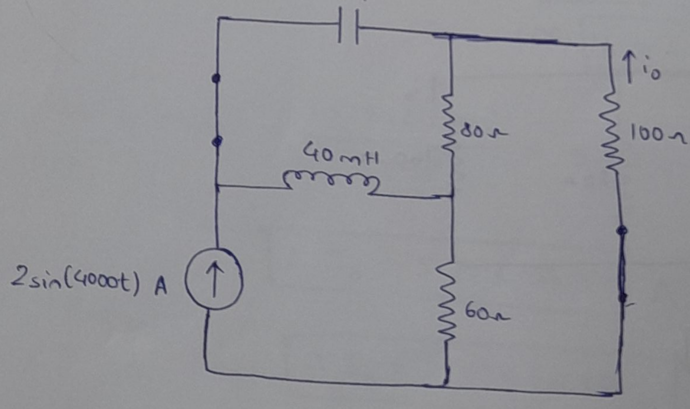


$\therefore\ i_{o_1} = \frac{-50\angle0^\circ}{\frac{160}{3}+j55} = \frac{-50\angle0^\circ}{76.61\angle45.88^\circ}\ A$

$\boxed{\therefore\ i_{o_1} = 0.6526\angle134.12^\circ}\ A$ — (1)

(OR) $\boxed{i_{o_1} = -0.454 + 0.4685j\ A}$

Now, Let's switch off all sources except $V_2 = 2\sin(4000t)$ A

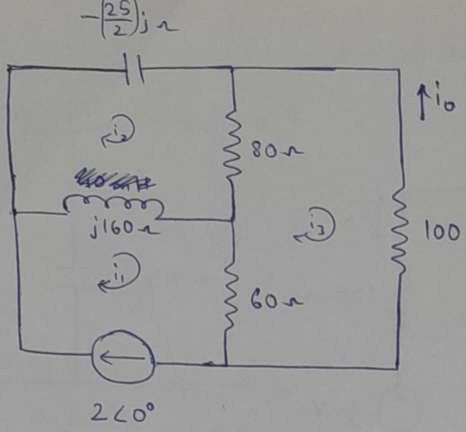


Let's convert quantities from time domain to Phasor Domain

$X_L = j\omega L = 4000\times40\times10^{-3}\ j$

$= j(160)\ \Omega$

$X_C = \frac{-j}{\omega C} = \frac{-j}{4000\times20}\times10^6 = -\left(\frac{25}{2}\right)j\ \Omega$



We can solve for i_{o_2} using Mesh Analysis.

from loop 1:

$$\boxed{i_1 = 2\angle 0^\circ\ A}$$

From Loop 2:

$$\cancel{40}\ j160(i_2 - i_1) - \frac{25}{2}j(i_2) + 80(i_2 - i_3) = 0 \qquad [i_3 = -i_{o_2}]$$

$$\Rightarrow j160\, i_2 - j160\, i_1 - \frac{25}{2} i_2 j + 80 i_2 + 80 i_{o_2} = 0$$

$$\Rightarrow -j320 + i_2\left(j160 - j\frac{25}{2}\right) + 80 i_2 + 80 i_{o_2} = 0$$

$$\Rightarrow i_2\left(j160 - j\frac{25}{2} + 80\right) + i_{o_2}(80) = j320$$

$$\Rightarrow \left(j\frac{295}{2} + 80\right) i_2 + (80)\, i_{o_2} = j320 \quad -②$$

From Loop 3:

$$60(i_3 - i_1) + 80(i_3 - i_2) + 100 i_3 = 0$$

$$\Rightarrow -60(i_{o_2} + 2) + 80(i_{o_2} + i_2) - 100 i_{o_2} = 0$$

$$\Rightarrow 60 i_{o_2} + 120 + 80 i_{o_2} + 80 i_2 + 100 i_{o_2} = 0$$

$$\Rightarrow 240 i_{o_2} + 80 i_2 + 120 = 0$$

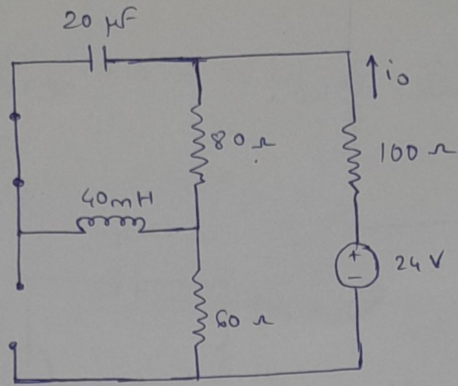
$$\Rightarrow 6 i_{o_2} + 2 i_2 + 3 = 0 \quad -③$$

Solving ② and ③, we get

$$i_2 = 2 + 0.45j\ A = 2.056\angle 12.754^\circ\ A$$

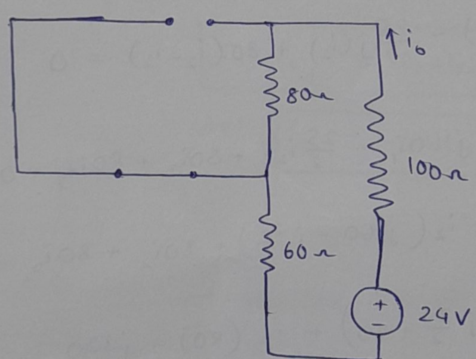
$$i_{o_2} = -1.168 - 0.15j\ A = 1.178\angle -172.62^\circ\ A \quad -④$$

Finally, Let's short 24V source.



In DC conditions,

Capacitors act as Open Circuit
Inductors act as Close Circuit (Short)



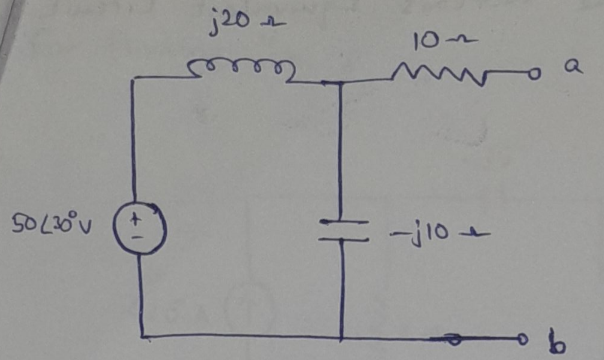
$$\therefore i_{o_3} = \frac{24}{240} = \frac{1}{10} \text{ A}$$

Now finally,

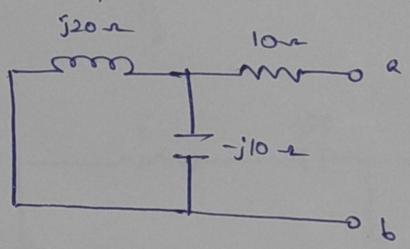
$$i_o = i_{o_1} + i_{o_2} + i_{o_3}$$

$$\therefore i_o = 0.6526 \cos(2000t + 134.12^\circ) + 1.178 \sin(4000t - 172.62^\circ) + \frac{1}{10} \text{ A}$$

We cannot Simplify further.



Let's find Z_{Th} by turning off the sources.



$$(j20 \parallel -j10) + 10 = Z_{Th}$$

$$\Rightarrow Z_{Th} = 10 + \frac{j20 \times (-j10)}{j10} = j(-20) + 10$$

$$\boxed{\therefore Z_{Th} = 10 - j20} \ \Omega$$

$$\boxed{\therefore Z_N = 10 - j20} \ \Omega \Rightarrow Z_N = 22.36\angle -63.435° \ \Omega$$

— (1)

For finding $V_{Th} = V_{ab}$:

V_{ab} = V across $-j10\,\Omega$ Capacitor

$$\Rightarrow V_{Th} = (50\angle 30°)\left(\frac{-j10}{j20 - j10}\right)$$

$$= 50\angle 30° \, (-1)$$

$$= (50\angle 30°)(1\angle 180°)$$

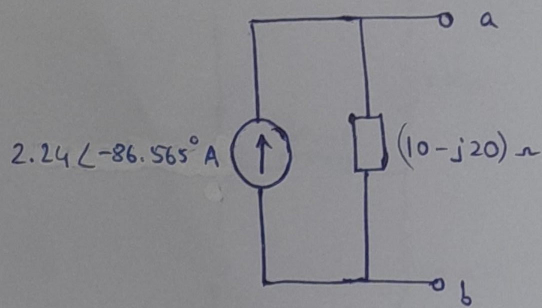
$$= 50\angle 210°$$

(OR) $\boxed{V_{Th} = 50\angle -150°}$ V

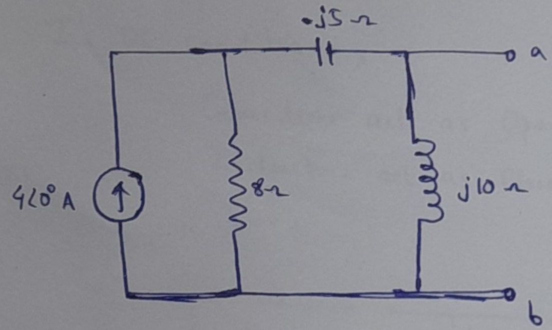
$$\therefore I_N = \frac{V_{Th}}{Z_{Th}} = \frac{50\angle -150°}{22.36\angle -63.435°} = 2.236\angle -86.565° \ A$$

$$\boxed{\therefore I_N = 2.236\angle -86.565° \ A}$$ — (2)

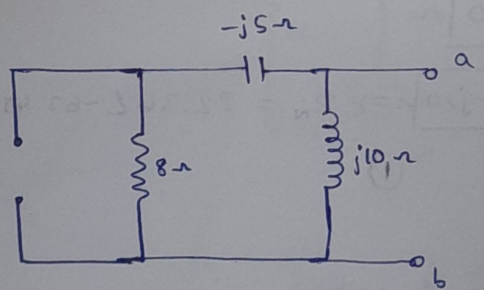
from ① and ②, we can draw the Norton's Equivalent Circuit



(b)



for finding Z_{Th}, let's switch off the sources



$$Z_{Th} = (j10) \parallel (8-j5)$$

$$= \frac{(j10)(8-j5)}{8+j5}$$

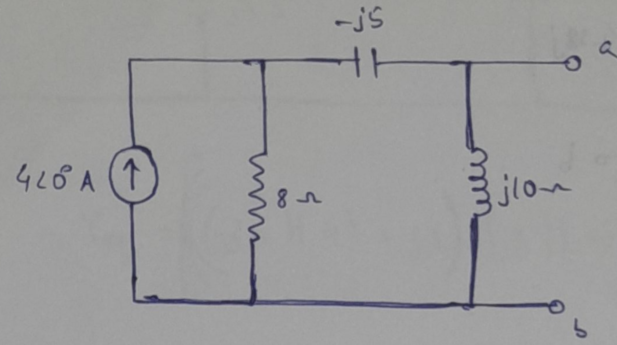
$$\Rightarrow Z_{Th} = \frac{800}{89} + j\frac{390}{89} \ \Omega$$

$$\therefore Z_{Th} = Z_N = (8.989 + 4.38j) \ \Omega$$

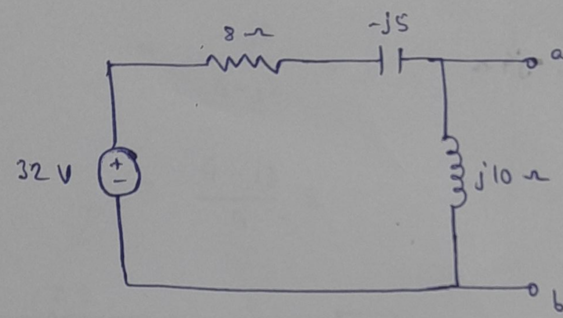
$$\boxed{Z_N = 10\angle 26^\circ \ \Omega}$$

(OR) $\boxed{Z_N = 9 + 4.38j \ \Omega}$ - ①

For finding V_{Th},



⇓



$$V_{ab} = V_{j10\Omega}$$

$$\therefore V_{Th} = (32\angle 0^\circ)\left(\frac{j10}{8-j5+j10}\right)$$

$$= (32\angle 0^\circ)(1.06\angle 58^\circ)\ V$$

$$\boxed{\therefore V_{Th} = 33.92\angle 58^\circ\ V}$$

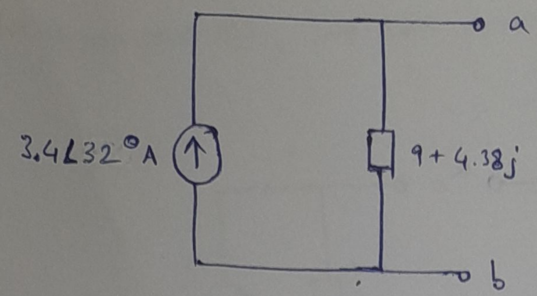
$$\therefore I_N = \frac{V_{Th}}{Z_{Th}} = \frac{33.92\angle 58^\circ}{10\angle 26^\circ}\ A$$

$$= 3.392\angle 32^\circ\ A$$

$$\boxed{\therefore I_N = 3.392\angle 32^\circ\ A} \quad - (2)$$

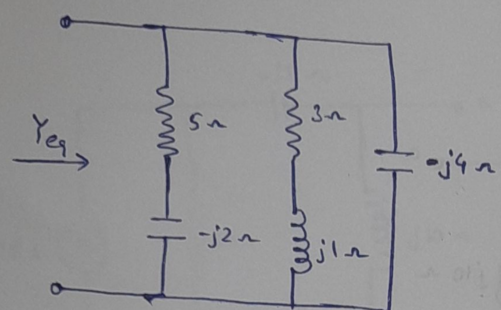
(OR) $\boxed{I_N = 2.88 + 1.8j}$ A

Let's draw Norton's Equivalent Circuit: [from (1) and (2)]



Q5) (a)

(i)



$$\text{Equivalent Admittance} = \frac{1}{\text{Eq. Impedence}}$$

$$Z_{eq} = (5-j2) \parallel (3+j) \parallel (-j4)$$

$$= \frac{(5-j2)(3+j)}{(8-j)} \parallel (-j4)$$

$$= \frac{\frac{(5-j2)(3+j)}{(8-j)}(-j4)}{\frac{(5-j2)(3+j)}{(8-j)} - j4} = \frac{(5-j2)(3+j)(-j4)}{(5-j2)(3+j) - j4(8-j)}\ \Omega$$

$$= 1.92\angle -24.87°\ \Omega = 1.74 - j0.8\ \Omega$$

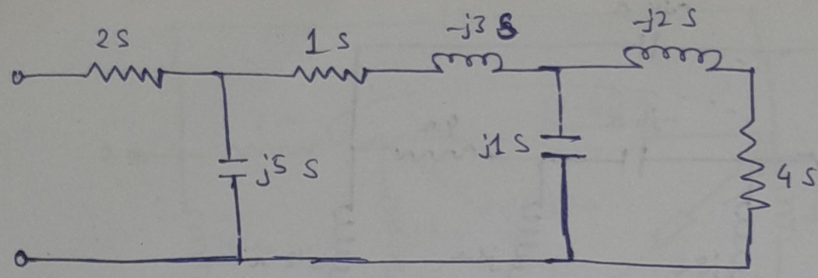
$$\therefore Y_{eq} = \frac{1}{Z_{eq}} = \frac{(5-j2)(3+j) - j4(8-j)}{(5-j2)(3+j)(-j4)} = \frac{13 - j33}{-4 - j68}\ S$$

$$= \frac{35.4683\angle -1.2°}{68.1175\angle -1.63°}$$

$$\boxed{Y_{eq} = 0.52\angle 0.434°}\ S$$

(OR) $\boxed{Y_{eq} = 0.472 + 0.22j}\ S$

(ii)



$$Y_{eq} = \left[\left(\left((-j2 \parallel 4) + j1\right) \parallel 1 \parallel -j3\right) + j5\right] \parallel 2$$

$$\Rightarrow Y_{eq_1} = \left(\frac{-j2 \times 4}{4-j2}\right) + j1$$

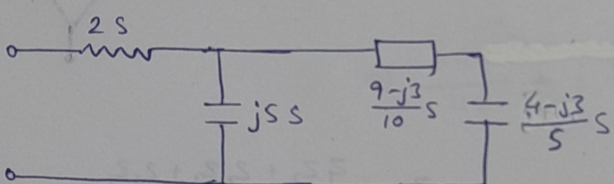
$$= \frac{-j8}{4-j2} + j$$

$$= \frac{4-j3}{5} \text{ S}$$

1 and $-j3$ → series

$$\frac{1(-j3)}{1-j3} = \frac{-j3}{1-j3}$$

$$= \frac{9-j3}{10} \text{ S}$$



Now $\frac{9-j3}{10}$ is series with $\frac{4-j3}{5}$ S

$$\Rightarrow \frac{(9-j3)(4-j3)}{\frac{50}{10}(9-j3+8-6j)} = \frac{81-j42}{185} \text{ S}$$

Now $\frac{81-j42}{185} + j5 = \frac{81+j883}{185}$ S

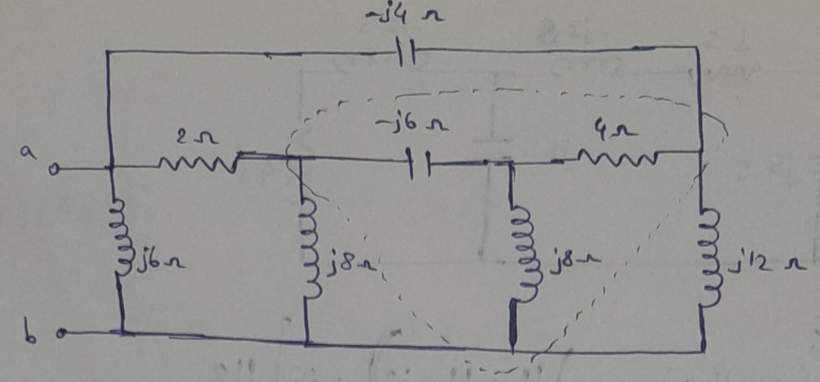
And $\frac{81+j883}{185}$ is in series with 2

$$\therefore Y_{eq} = \frac{\left(\frac{81+j883}{185}\right)2}{\frac{81+j883}{185}+2} = \frac{2(81+j883)}{451+j(883)} = (1.66+0.66j) \text{ S}$$

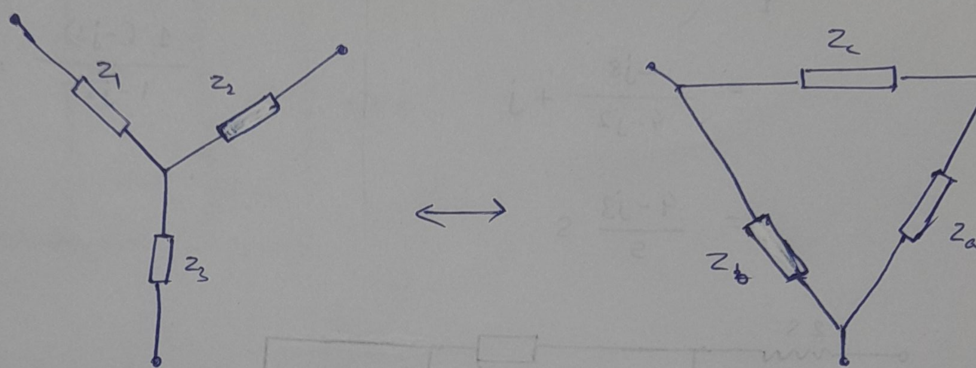
$$\boxed{\therefore Y_{eq} = 1.66+0.665j} \text{ S}$$

(OR) $\boxed{Y_{eq} = 1.79\angle 21.8^\circ}$ S

5) (b) (i)



As we know,



$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$Z_1Z_2 + Z_2Z_3 + Z_3Z_1$

$= (-j6)(4) + 4(j8) + (j8)(-j6)$

$= -j24 + j32 + 48$

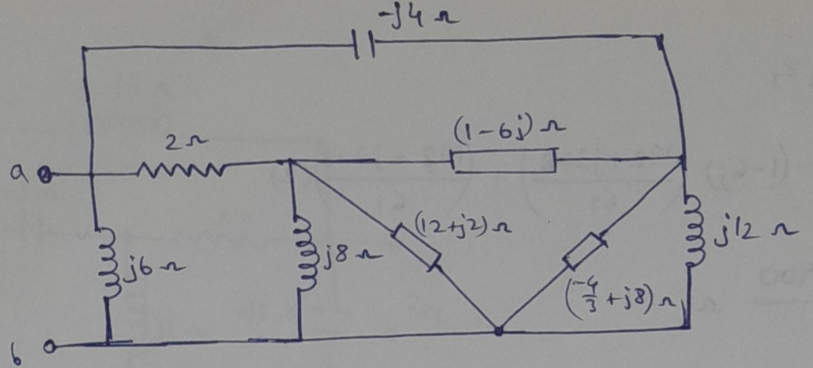
$= 48 + j8$

$$\therefore Z_a = \frac{48+j8}{-j6} = -\frac{4}{3} + j8 \ \Omega$$

$$Z_b = \frac{48+j8}{4} = 12 + j2 \ \Omega$$

$$Z_c = \frac{48+j8}{j8} = 1 - 6j \ \Omega$$

We can redraw the circuit using this Y to ∇ transformation.



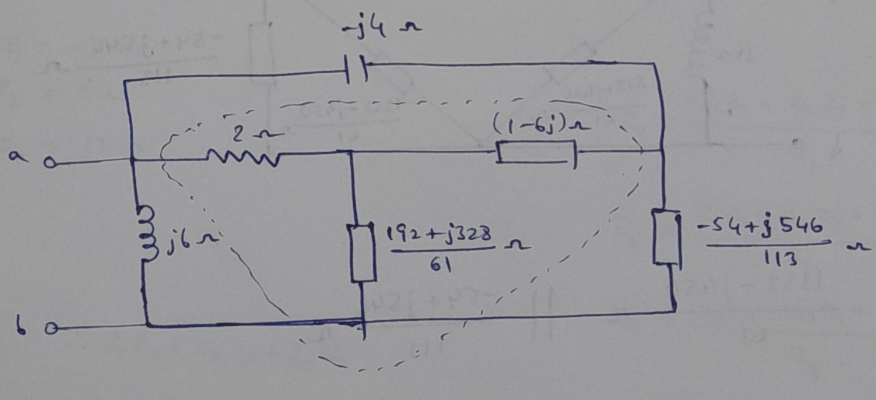
$$(12+j2) \parallel j8 \Rightarrow \frac{(12+j2)(j8)}{12+j10}\ \Omega$$

$$\Rightarrow \frac{(6+j)(j8)}{6+j5}\ \Omega$$

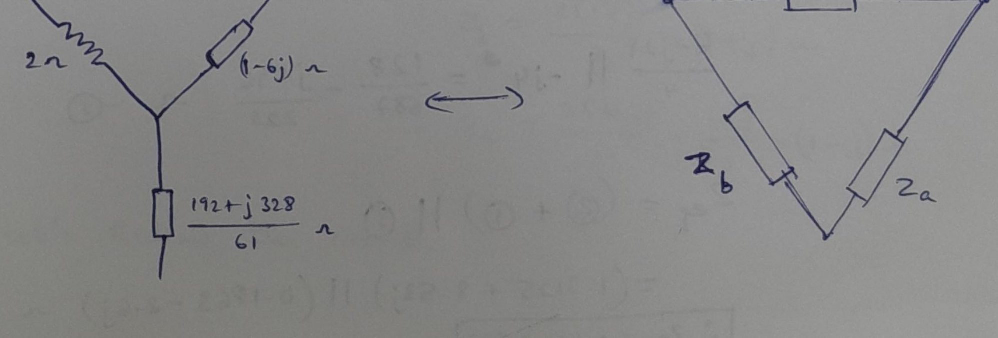
$$\Rightarrow \frac{192+j328}{61}\ \Omega$$

$$\left(\frac{-4}{3}+j8\right) \parallel j12 \Rightarrow \frac{(-4/3+j8)(j12)}{(-4/3+j20)}\ \Omega$$

$$\Rightarrow \frac{-54}{113} + j\frac{546}{113}\ \Omega$$



Applying Y to Δ transformation again:



$z_1 z_2 + z_2 z_3 + z_3 z_1$

$$= 2(1-6j) + (1-6j)\left(\frac{192+j328}{61}\right) + \left(\frac{192+j328}{61}\right)(2)$$

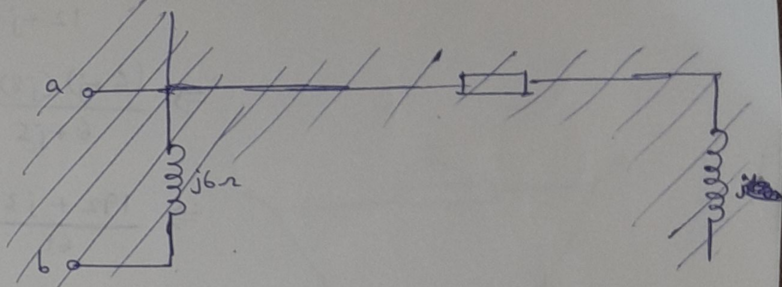
$$= \frac{2666 - j900}{61}\ \Omega$$

$$z_a = \frac{1333 - j450}{61}\ \Omega$$

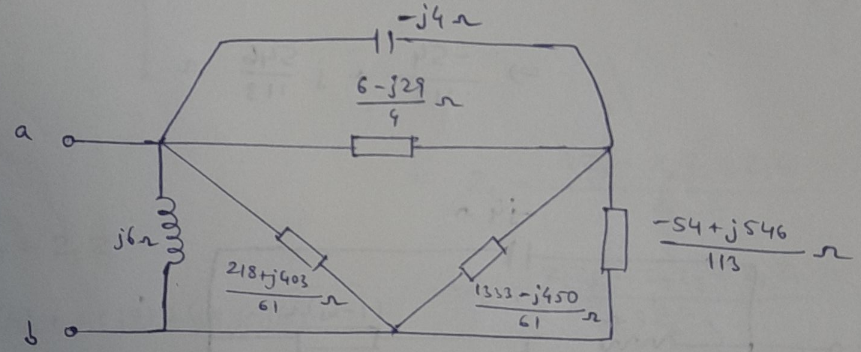
$$z_c = \frac{6 - j29}{4}\ \Omega$$

$$z_b = \frac{218 + j403}{61}\ \Omega$$

$\Leftrightarrow$



Let's Draw the circuit now :



$$\frac{1333 - j450}{61}\ \Omega \parallel \frac{-54+j546}{113}\ \Omega$$

$$= \frac{1314 + j12066}{2333}\ \Omega \quad -(1)$$

$$j6 \parallel \frac{218+j403}{61} = 0.75 + 3.357j\ \Omega \quad -(2)$$

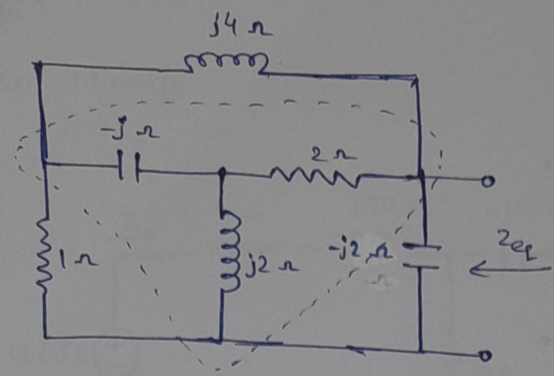
$$\frac{6-j29}{4} \parallel -j4 = \frac{128}{687} - \frac{j596}{229}\ \Omega \quad -(3)$$

$$z_{eq} = ((3) + (1)) \parallel (2)$$

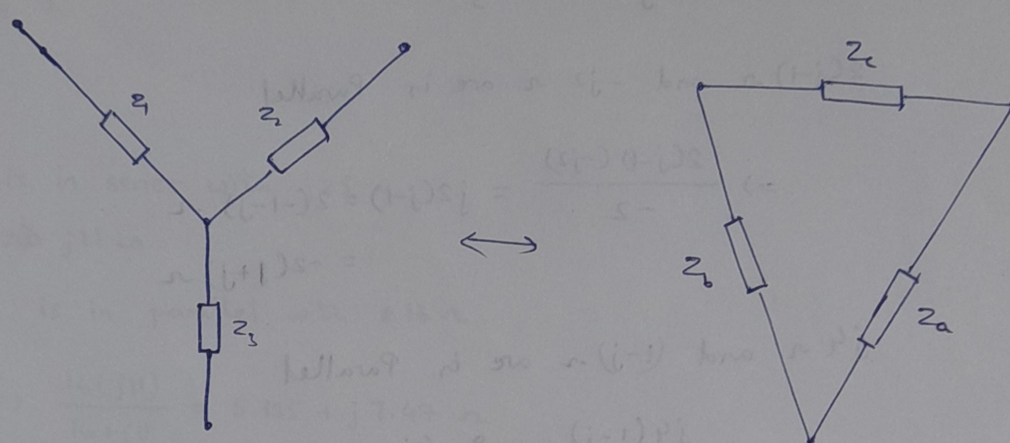
$$= (0.75 + 2.57j) \parallel (0.75 + 3.357j)\ \Omega$$

$$\boxed{\therefore z_{eq} = 0.38 + 1.457j\ \Omega} \Rightarrow \boxed{z_{eq} = 1.5\angle 75.34^\circ\ \Omega}$$

)



As We know, Conversion from Y to ∆ is as follows



$Z_1 = -j\ \Omega$

$Z_2 = 2\ \Omega$

$Z_3 = j2\ \Omega$

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$\therefore Z_1Z_2 + Z_2Z_3 + Z_3Z_1$

$= -j2 + j4 + 2$

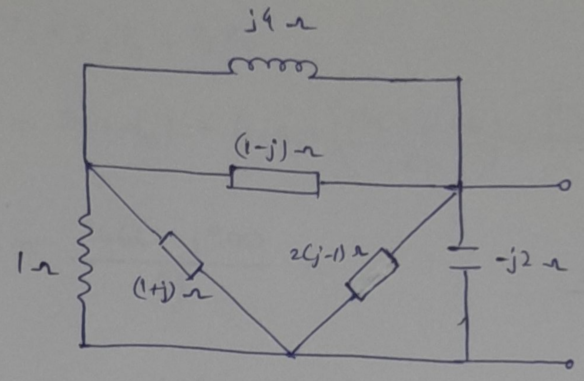
$= 2 + j2\ \Omega$

$$\therefore Z_a = \frac{2(1+j)}{-j} = \frac{-2+2j}{1} = -2(1-j)\ \Omega$$

$$Z_b = \frac{2(1+j)}{2} = (1+j)\ \Omega$$

$$Z_c = \frac{2(1+j)}{j2} = \frac{1+j}{j} = (1-j)\ \Omega$$

Now let's Redraw the circuit.



1 Ω and (1+j) Ω are in Parallel

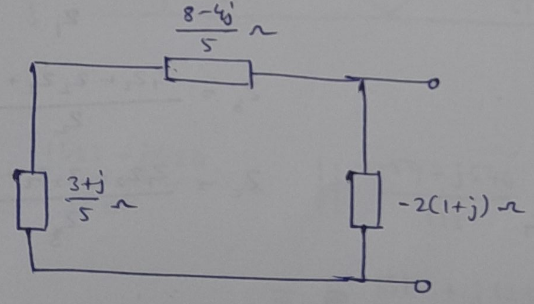
$$\Rightarrow \frac{1+j}{2+j}\,\Omega = \frac{3+j}{5}\,\Omega$$

2(j-1) Ω and -j2 Ω are in Parallel

$$\Rightarrow \frac{2(j-1)(-j2)}{-2} = j2(j-1) = 2(-1-j)\,\Omega$$
$$= -2(1+j)\,\Omega$$

j4 Ω and (1-j) Ω are in Parallel

$$\Rightarrow \frac{j4(1-j)}{1+j3} = \frac{8-4j}{5}\,\Omega$$



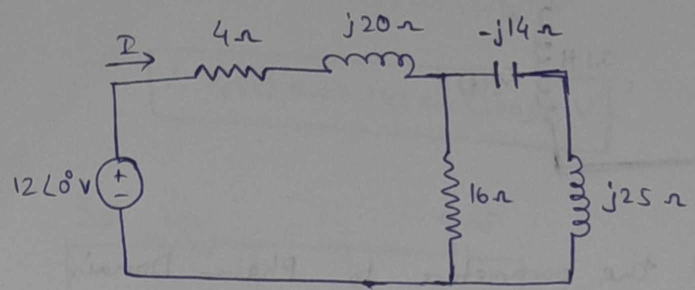
$$Z_{eff.} = \left(\frac{8-4j+3+j}{5}\right) \parallel (-2(1+j))$$

$$= \left(\frac{11-3j}{5}\right) \parallel (2(-1-j))$$

$$Z_{eq.} = \frac{18-j38}{17}\,\Omega \Rightarrow \boxed{\therefore Z_{eq} = 1.06 - 2.235j\,\Omega}$$

Given:

$\omega = 10$ rad/s



To find:

Z_{eq} and I

Solution:

$-j14$ is in series with $j25\ \Omega$

$\Rightarrow j11\ \Omega$

$j11\ \Omega$ is in parallel with $16\ \Omega$

$$\Rightarrow \frac{16(j11)}{16+j11} = 5.135 + j7.47\ \Omega$$

$4\ \Omega$ and $j20\ \Omega$ are in series

$\Rightarrow 4 + j20\ \Omega$

Now $Z_{eq.} = (4+j20) + (5.135 + 7.47j)\ \Omega$

$= (9.1353 + j\,27.47)\ \Omega$

$$\boxed{\therefore Z_{eq} = 9.1353 + j\,27.47\ \Omega}$$

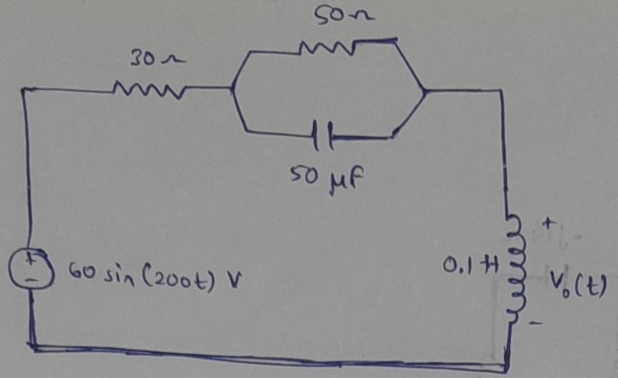
(OR) $\boxed{Z_{eq} = 28.95\angle 71.6^\circ}\ \Omega$

$$\therefore I = \frac{V}{Z_{eq}} = \frac{12\angle 0^\circ}{28.95\angle 71.6^\circ}\ A$$

$\therefore \boxed{I = 0.131 - 0.4j\ A}$

(OR) $\boxed{\therefore I = 0.4145\angle -71.6^\circ\ A} \Rightarrow \boxed{I = 0.4145\cos(10t - 71.6^\circ)\ A}$

7)



first Let's convert all the parameters to Phasor Domain

$\omega = 200$ rad/s

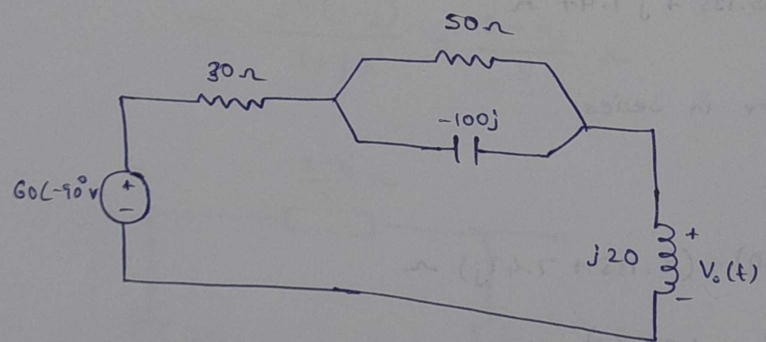
$30\,\Omega \longrightarrow 30\,\Omega$

$50\,\Omega \longrightarrow 50\,\Omega$

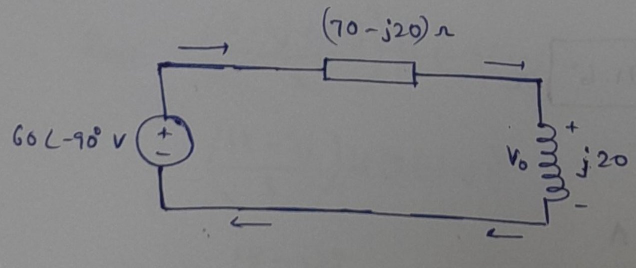
$60\sin(200t) \longrightarrow 60\angle -90^\circ$

$50\,\mu F \longrightarrow \dfrac{1}{j \times 50\times10^{-6} \times 200} = \dfrac{-j}{10^{-2}} = -100j$

$0.1\,H \longrightarrow j \times 200 \times 0.1 = 20j$



⇓



$\left[((50)\,\Omega) \,||\, ((-100j)\,\Omega)\right] + 30\,\Omega$

$= \dfrac{(50)\times(-100j)}{50-100j} + 30$

$= \dfrac{-100j}{1-2j} + 30$

$= 70 - j20\,\Omega$

$\therefore V_o(t) = (60\angle -90^\circ V)\left(\dfrac{j20}{70-\cancel{j20}+\cancel{j20}}\right)$

$= (60\angle -90^\circ)\left(\frac{2}{7}\angle 90^\circ\right) = \frac{120}{7}\angle 0^\circ$ V

But opposite direction, $V_o = \frac{120}{7}\angle 180^\circ$ V

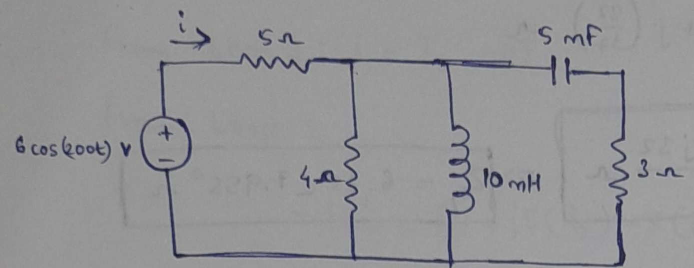
$\therefore V_o(t) = \frac{120}{7} \angle 180^\circ \, V$

$= 17.143 \angle 180^\circ \, V = 17.143 \cos(\omega t + 180^\circ)$

$\Rightarrow V_o(t) = -17.143(\cos(\omega t)) \, V$

$\boxed{\therefore V_o(t) = -17.143 \cos(200t) \, V}$

8)



All the quantities are present in time domain,
Let's convert to Phasor Domain with $\omega = 200$ rad/s

$6\cos(200t) \, V \longrightarrow 6\angle 0^\circ \, V$

$5\Omega \longrightarrow 5\Omega$

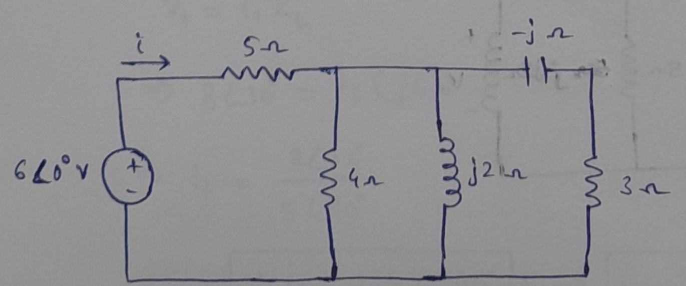
$4\Omega \longrightarrow 4\Omega$

$10\,mH \longrightarrow j \times 10^{-2} \times 200 = j2$

$3\Omega \longrightarrow 3\Omega$

$5\,mF \longrightarrow \frac{-j}{5 \times 10^{-3} \times 200} = -j$

Let's Redraw the circuit :



We can find $Z_{eq.}$ and then $i(t) = \frac{V(t)}{Z_{eq.}}$.

$-j\Omega$ and 3Ω are in Series : $3 - j \, \Omega$

$3-j$ and $j2\Omega$ are in Parallel :

$\frac{(3-j)(j2)}{(3+j)} = \frac{6+j8}{5} \, \Omega$

$\frac{6+j8}{5}\ \Omega$ and $4\ \Omega$ are in Parallel :

$$\frac{(6+j8)(4)}{(26+j8)} = \frac{44}{37} + j\frac{32}{37}\ \Omega$$

Now , in series with $5\ \Omega$,

i.e. $Z_{eq.} = \left(\frac{44}{37}+5\right) + j\left(\frac{32}{37}\right)\ \Omega$

$$\boxed{\therefore Z_{eq} = \frac{229+j32}{37}\ \Omega} \Rightarrow \boxed{Z_{eq} = 6.25\angle 7.955^\circ\ \Omega}$$

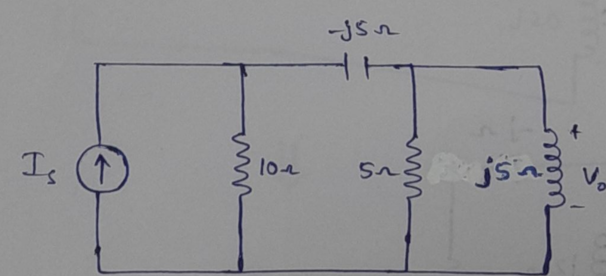
Now, $I = \frac{(6\angle 0^\circ)}{Z_{eq}} = \frac{6\angle 0^\circ}{6.25\angle 7.955^\circ}$ A

$$\boxed{\therefore I = 0.96\angle -7.955^\circ\ A}$$

$$\boxed{\therefore i(t) = 0.96\left(\cos(200t - 7.955^\circ)\right)\ A}$$

9) Given :

$V_0 = 8\angle 30^\circ$ V



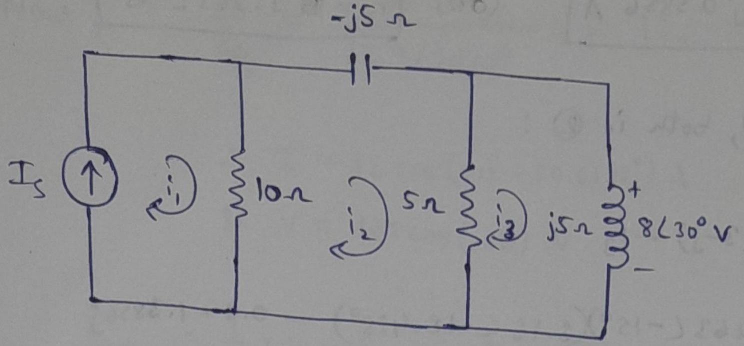
To find :

I_s

Solution :

Through Mesh Analysis , we can obtain I_s. Let's consider i_1, i_2 and i_3 in first, second and third loop respectively.

$$\boxed{\therefore i_1 = I_s}$$



From Loop 1: $i_1 = I_s$ - ①

From Loop 2 :

$$-10(i_2 - i_1) + j5(i_2) - 5(i_2 - i_3) = 0$$

$$\Rightarrow -10i_2 + 10i_1 + j5(i_2) - 5i_2 + 5i_3 = 0$$

$$\Rightarrow 10i_1 + i_2(-15 + j5) + 5i_3 = 0$$

$$\Rightarrow 2i_1 + i_2(-3 + j) + i_3 = 0 \quad - ②$$

From Loop 3 :

$$-5(i_3 - i_2) - V_o = 0$$

$$\Rightarrow V_o = 5(i_2 - i_3)$$

$$\Rightarrow 8\angle 30^\circ = 5(i_2 - i_3) \quad - ③$$

And

$$V_o = i_3 Z_L$$

$$\Rightarrow 8\angle 30^\circ = i_3(j5)$$

$$\Rightarrow i_3 = \frac{8\angle 30^\circ}{5\angle 90^\circ}$$

$$\boxed{\therefore i_3 = 1.6\angle -60^\circ \text{ A}} \Rightarrow \boxed{i_3 = 0.8 - 1.3856j \text{ A}} \quad - ④$$

Put ④ in ③ :

$$\Rightarrow 8\angle 30^\circ + 5i_3 = 5i_2$$

$$\Rightarrow 8\angle 30^\circ + 8\angle -60^\circ = 5i_2$$

$$\Rightarrow 6.93 + 4j + 4 - 6.93j = 5i_2$$

$$\Rightarrow 10.93 - 2.93j = 5i_2$$

$\boxed{\therefore i_2 = 2.1856 - j\,0.5856 \text{ A}}$ (OR) $\boxed{i_2 = 2.263 \angle -15^\circ}$ Now

Put i_2 value and ①, both in ② :

$$\Rightarrow 2I_s = i_2(3-j) - i_3$$

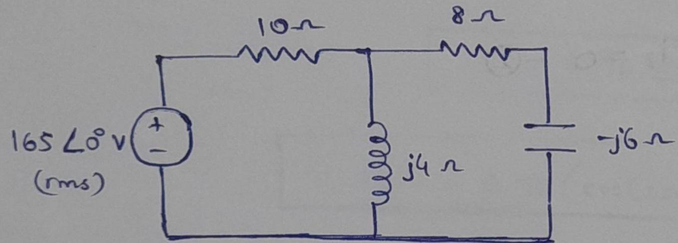
$$= (2.263\angle -15^\circ)(3.16\angle -18.435^\circ) - 0.8 + 1.3856j$$

$$\therefore I_s = \frac{5.17 - 2.557j}{2} \text{ A}$$

$$\boxed{\therefore I_s = 2.5856 - j1.278 \text{ A}}$$

(OR) $\boxed{I_s = 2.88 \angle -26.31^\circ \text{ A}}$

10)



To Find:

Avg. Power supplied by the source and Power factor

Solution:

Let's find Z_{eq}:

8Ω in series with $-j6\Omega \Rightarrow (8-j6)\Omega$

$$(8-j6)\Omega \parallel (j4)\Omega \Rightarrow \frac{(8-j6)(j4)}{(8-j2)} = \frac{32+j76}{17}\Omega$$

$\frac{32+j76}{17}\Omega$ series with $10\Omega \Rightarrow Z_{eq} = \frac{32+j76}{17} + 10$

$$\therefore Z_{eq} = \frac{202+j76}{17}\Omega$$

$$\boxed{\therefore Z_{eq} = 11.88 + 4.47j}\ \Omega$$

(OR) $\boxed{Z_{eq} = 12.7\angle 20.62^\circ\ \Omega}$

Now, $I = \frac{V}{Z_{eq.}} = \frac{165\angle 0^\circ}{12.7\angle 20.62^\circ} A = 13\angle -20.6182^\circ\ A$

$\therefore\ I = 13\cos(\omega t - 20.6182^\circ)\ A$

$\&\ V = 165\cos(\omega t)\ V$

$P_{avg.} = \frac{1}{2} \times V_m \times I_m \times \cos(\theta_v - \theta_i)$

$= \frac{1}{2} \times V_{rms} \times \sqrt{2} \times I_{rms} \times \sqrt{2} \times \cos(\theta_v - \theta_i)$

$\Rightarrow P_{avg} = V_{rms} \times I_{rms} \times \cos(\theta_v - \theta_i)$

$= 13 \times 165 \times \cos(0^\circ + 20.6182^\circ)$

$\boxed{\therefore\ P_{avg} = 2007.6\ W}$

And, $pf = \frac{P}{S} = \frac{V_{rms} \cdot I_{rms} \cdot \cos(\theta_v - \theta_i)}{V_{rms} \cdot I_{rms}}$

$\therefore\ pf = \cos(\theta_v - \theta_i)$ $\left[\begin{matrix}\theta_v = 0^\circ \\ \theta_i = -20.6182^\circ\end{matrix}\right]$

$\Rightarrow pf = \cos(20.6182^\circ)$

$\boxed{\therefore\ pf = 0.936}$ $\theta_v > \theta_i \Rightarrow$ Lagging pf

$\therefore$ Inductive,
which is proved from the circuit.

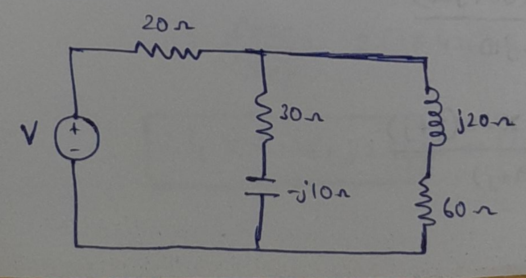
11) Given:

60 Ω Resistor ~~dravd~~ absorbs 240 W ($P_{avg.}$)

To find:

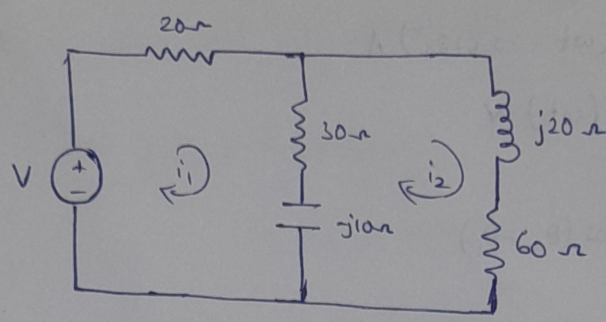
Complex Power (S) of each branch in circuit & Overall Complex Power, V.

Solution:



~~ten~~ Let's Consider i_1 flowing in Loop 1 and i_2 flowing in loop 2

$\therefore Ze$



In Loop 1 :

$$V - 20i_1 - 30(i_1 - i_2) + j10(i_1 - i_2) = 0$$

$$\Rightarrow V - 20i_1 - 30i_1 + 30i_2 + j10i_1 - j10i_2 = 0$$

$$\Rightarrow V - i_1(50 - j10) + i_2(30 - j10) = 0$$

$$\Rightarrow \boxed{V = i_1(50 - j10) - i_2(30 - j10)} \quad -①$$

In Loop 2 :

$$-j20(i_2) - 60i_2 + j10(i_2 - i_1) - 30(i_2 - i_1) = 0$$

$$\Rightarrow i_1(30 - j10) - i_2(90 + j10) = 0$$

$$\Rightarrow i_1(30 - j10) = i_2(90 + j10)$$

$$\Rightarrow \boxed{i_1(3 - j) = i_2(9 + j)} \quad -②$$

$$\boxed{V_{60\Omega} = i_2 \times 60} \quad -③$$

Finding Z_{eq} :

$$\left[(30 - j10) \,||\, (j20 + 60)\right] + 20$$

$$\Rightarrow Z_{eq} = 20 + \frac{(30 - j10)(60 + j20)}{90 + j10}$$

$$= 20 + \frac{(10)(3 - j) \times 20(3 + j)}{10(9 + j)}$$

$$= 20\left(1 + \frac{10}{9 + j}\right)$$

$$\boxed{\therefore Z_{eq} = 41.95 - 2.44j}\ \Omega$$

$$\Rightarrow \boxed{Z_{eq} = 42\angle -3.33^\circ\ \Omega}$$

(OR)

~~As~~

from ① and ② :

$$V = i_1(50 - j10) - i_1(8.05 - 7.561j)$$

$$\therefore V = i_1(41.95 - 2.44j)$$

$$[V = i_1 \times Z_{eq}]$$

$$\boxed{\therefore Z_{eq} = 41.95 - j2.44}\ \Omega$$

from ③,

$$V_{60\Omega} = i_2 \times 60\ V$$

$$i_{60\Omega} = i_2$$

$\therefore \cos(\theta_v - \theta_i) = 0$ for the Resistor as both voltage and current are in phase for the Resistor.

$$\therefore P_{avg} = \frac{1}{2} \times V_m \times I_m = V_{rms} \times I_{rms}$$

$$= \cancel{\tfrac{1}{2}}(60 i_2) \times i_2$$

$$\Rightarrow 60 i_2^2 = 240 \text{ (given)}$$

$$\Rightarrow i_2^2 = 4\ A^2$$

$$\boxed{\therefore i_2 = 2\ A} \Rightarrow i_2 = 2\angle \tan^{-1}\left(\frac{20}{60}\right) A$$

$$\boxed{\therefore i_2 = 2\angle 18.435^\circ A} \Rightarrow \boxed{i_2 = 1.897 + j0.632\ A}$$

$$\Rightarrow i_1 = i_2 \times \frac{9+j}{3-j}$$

$$= \cancel{\frac{(8+2j)}{(3+j)}}\ 2\angle 18.435^\circ \times \frac{9.055\angle 6.34^\circ}{3.162\angle -18.435^\circ}\ A$$

$$\boxed{\therefore i_1 = 4.174 + j(3.92)}\ A \Rightarrow \boxed{i_1 = 5.727\angle 43.21^\circ\ A}$$

And $V = i_1 \cdot Z_{eq}$.

$\therefore V = (4.17 + j3.92)(41.95 - j2.44)$

$= (5.727 \angle 43.21^\circ)(42 \angle -3.33^\circ)$

$= 240.54 \angle 39.88^\circ$

$\boxed{\therefore V = 240.54 \angle 39.88^\circ \text{ V}}$

Overall Complex Power $(S) = V_{rms} \cdot I_{rms} \angle \theta_v - \theta_i$

$= 240.54 \times 5.727 \angle 39.88^\circ - 43.21^\circ$

$= 1377.573 \angle -3.33^\circ$ VA

$\boxed{\therefore S = 1377.573 \angle -3.33^\circ \text{ VA}}$

$\Rightarrow \boxed{S = 1375.25 - 80j \text{ VA}}$

$S_{20\Omega} = (i_{rms} \times R) \times i_{rms} \angle \theta_v - \theta_i$

$= (5.727)^2 \times 20 \angle 0^\circ$

$\Rightarrow \boxed{S_{20\Omega} = 655.97 \text{ VA}}$

$S_{30\Omega - 70j\Omega} = (125.83)(4) \angle 36.85^\circ - 55.3^\circ$

$= 503.32 \angle -18.45^\circ$ VA

$\boxed{\therefore S_{30-70j\,\Omega} = 503.32 \angle -18.45^\circ \text{ VA}}$

$V - V_o = 20 i_1$

$\Rightarrow V_o = V - 20 i_1$

$= 125.83 \angle 36.85^\circ$ V

$\therefore V_o = 100.7 + j(75.46)$ V

$i_o = i_1 - i_2 = 2.277 + 3.289j$ A

$= 4 \angle 55.3^\circ$ A

[$V_o = V_{30\Omega}$ and $i_o = i_{30\Omega}$]

$S_{60+j20\Omega} = (125.83)(2) \angle 36.85^\circ - 18.435^\circ$

$= 251.66 \angle 18.415^\circ$

$\boxed{\therefore S_{60+j20\Omega} = 251.66 \angle 18.415^\circ \text{ VA}}$

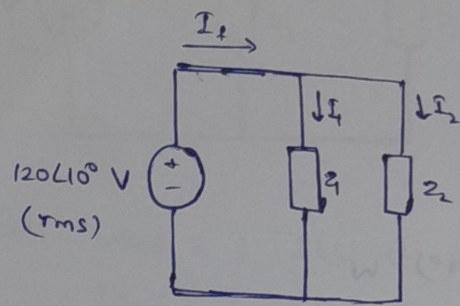
$\left.\begin{array}{l} S_{30-j70\Omega} = 477.45 - 159.3j \text{ VA} \\ S_{60+j20\Omega} = 238.77 + 79.5j \text{ VA} \\ S_{20\Omega} = 655.97 + 0j \text{ VA} \end{array}\right\}$ Adding gives $S = 1375.2 - 80j$ VA

i.e. Overall Complex Power

Given:

$Z_1 = 60\angle{-30^\circ}\ \Omega$

$Z_2 = 40\angle{45^\circ}\ \Omega$



To find:

(a) Apparant Power (c) Reactive Power

(b) Real Power (d) pf

Solution:

Apparant Power (S) $= V_{rms} \times I_{rms}$

For I_{rms}, We need to find Z_{eq} and $I_{rms} = \dfrac{V_{rms}}{Z_{eq}}$

$$Z_{eq} = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{(60\angle{-30^\circ})(40\angle{45^\circ})}{60\angle{-30^\circ} + 40\angle{45^\circ}}\ \Omega$$

$$\left[\begin{array}{l} 60\angle{-30^\circ} = 51.96 - j30 \\ 40\angle{45^\circ} = 28.284 + j28.284 \end{array}\right] \rightarrow Z_1 + Z_2 = 80.246 - 1.716j$$

$$\therefore Z_1 + Z_2 = 80.264\angle{-1.225^\circ}\ \Omega$$

$$\Rightarrow Z_{eq} = \frac{2400\angle{15^\circ}}{80.264\angle{-1.225^\circ}}\ \Omega$$

$$\boxed{\therefore Z_{eq} = 29.9\angle{16.225^\circ}\ \Omega} \Rightarrow \boxed{Z_{eq} = 28.71 + j8.355\ \Omega}$$

$$\therefore I_{rms} = \frac{V_{rms}}{Z_{eq}} = \frac{120\angle{10^\circ}}{29.9\angle{16.225^\circ}}\ A$$

$$\boxed{\therefore I_{rms} = 3.9895 - j0.435\ A}$$

$$\boxed{\therefore I_{rms} = 4.0132\angle{-6.225^\circ}}\ A$$

(a) Apparant Power $= V_{rms} \times I_{rms}$

$\Rightarrow S = 120 \times 4.0132$ VA

$\boxed{\therefore S = 481.584 \text{ VA}}$

(b) Real Power (or) Avg. Power (P)

$P = S \cos(\theta_v - \theta_i)$

$\Rightarrow P = 481.584 \times \cos(10^\circ + 6.225^\circ)$ W

$\boxed{\therefore P = 462.4 \text{ W}}$

(c) ~~pf~~ Reactive Power (Q)

Complex Power $(S) = V_{rms} \times I_{rms} \angle \theta_v - \theta_i$

$= 481.584 \angle 16.225^\circ$

$\Rightarrow S = 462.4 + 134.56j$

$\boxed{\therefore Q = 134.56 \text{ VAR}}$

(OR) $Q = V_{rms} \times I_{rms} \times \sin(\theta_v - \theta_i)$

$= 481.584 \times \sin(16.225^\circ)$

$= 134.56$ VAR

(d) pf (Power factor)

$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$

$= \cos(16.225^\circ)$

$= 0.96$

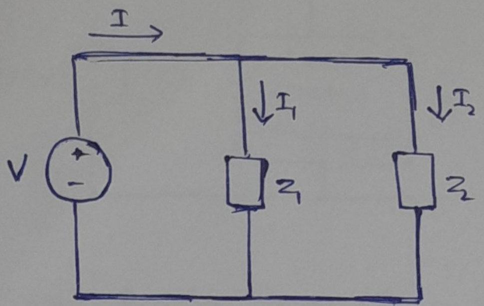
$\boxed{\therefore pf = 0.96}$

$\theta_v - \theta_i > 0 \Rightarrow$ Lagging pf

$\Downarrow$

Inductive Load

Given:



~~Load~~ Load 1 : 2kW @ pf = 0.75 leading

Load 2 : 4kW @ pf = 0.95 Lagging

Solution :

$P_1 = V \cdot I_1 \cdot \cos(\theta_v - \theta_{i_1})$

$P_2 = V \cdot I_2 \cdot \cos(\theta_v - \theta_{i_2})$

$$\left[Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} \right]$$

$pf_1 = \cos(\theta_v - \theta_{i_1}) = 0.75$

$pf_2 = \cos(\theta_v - \theta_{i_2}) = 0.95$

$$I_1 = I \cdot \frac{Z_2}{Z_1 + Z_2}$$

$$\left(\text{where } I = \frac{V}{Z_{eq}} \right)$$

$\Rightarrow 2000 = V \cdot I_1 \times 0.75$

$\Rightarrow \frac{8000}{3} = V \cdot I \cdot \frac{Z_2}{Z_1 + Z_2}$

$= V \times \frac{V}{Z_1 Z_2} \times \frac{(Z_1 + Z_2) Z_2}{(Z_1 + Z_2)}$

$= \frac{V^2 Z_2}{Z_1 Z_2} = \frac{V^2}{Z_1}$

$\theta_v - \theta_{i_1} < 0 \Rightarrow \theta_v - \theta_{i_1} = -41.41°$

$\theta_v - \theta_{i_2} > 0 \Rightarrow \theta_v - \theta_{i_2} = 18.195°$

$$\left[\begin{array}{l} \cos^{-1}(0.75) = -41.41° \\ \cos^{-1}(0.95) = 18.195° \end{array} \right]$$

$\Rightarrow \frac{8000}{3} = \frac{V_{rms}^2}{Z_1}$ — (1)

& $4000 = \frac{V^2}{Z_2} \times 0.95$

$\Rightarrow \frac{80000}{19} = \frac{V_{rms}^2}{Z_2}$ — (2)

~~Assume $\theta_v = 0$~~

~~$\therefore \theta_{i_1} = 41.41°$~~

~~$\theta_{i_2} = -18.195°$~~

$$\boxed{\therefore \frac{Z_1}{Z_2} = \frac{30}{19}}$$

~~$I_1 = I_1 \angle 41.41°$~~

~~$= 0.75 I_1 + j\, 0.66 I_1$~~

~~$I_2 = I_2 \angle -18.195°$~~

~~$= 0.95 I_2 + j\, 0.3125 I_2$~~

~~$P = P_1 + P_2 = V \cdot I \cos(\theta_v - \theta_i)$~~

~~$\Rightarrow 6000 = V I \cos(-\theta_i)$~~

$$S_1 = P_1 - jQ_1$$

$$= 2000 - j\left(\frac{8000}{3}\right)\sin(\cos^{-1}(0.75))$$

$$\boxed{\therefore S_1 = 2000 - j(1763.83)\ VA}$$

$$S_2 = P_2 + jQ_2$$

$$= 4000 + j\left(\frac{80000}{19}\right)\sin(\cos^{-1}(0.95))$$

$$\boxed{\therefore S_2 = 4000 + j(1314.74)\ VA}$$

$$\therefore S_{source} = S_1 + S_2$$

$$= 6000 + j(-449.1)\ VA$$

$$\boxed{\therefore S = 6000 - j(+449.1)\ VA}$$

$$P = V_{rms} \cdot I_{rms}\cos(\theta_v - \theta_i)$$

$$Q = V_{rms} \cdot I_{rms} \cdot \sin(\theta_v - \theta_i)$$

$$\tan(\theta_v - \theta_i) = \frac{Q}{P}$$

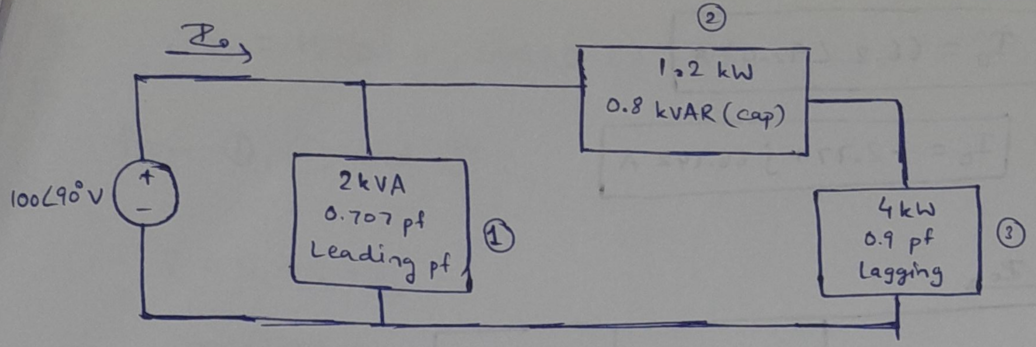
$$\Rightarrow \tan(\theta_v - \theta_i) = \frac{-449.1}{6000}$$

$$\Rightarrow \tan(\theta_v - \theta_i) = -0.07485$$

$$\Rightarrow \theta_v - \theta_i = \tan^{-1}(-0.07485)$$

$$pf = \cos(\theta_v - \theta_i) = \cos(\tan^{-1}(-0.07485))$$

$$\boxed{\therefore pf = 0.9972}$$



Overall Complex Power (S) $= S_1 + S_2 + S_3$

(i) $S_2 = 1200 + j(-800)$ VA — ② [$Q<0$ for Capacitive Loads]

(ii) $S_1 = 2000 \angle \theta_v - \theta_i$ — ①

$$\cos(\theta_v - \theta_i) = 0.707$$

$$\Rightarrow \theta_v - \theta_i = 45^\circ$$

But $\theta_v - \theta_i = -45^\circ$ [As $\theta_v - \theta_i < 0$ for Leading pf]

$$\therefore S_1 = 2000\angle -45^\circ \text{ VA}$$
$$= 1414.2 - j(1414.2) \text{ VA}$$

(iii) $S_3 = 4000 + jQ$

$$\tan(\theta_v - \theta_i) = \frac{Q}{P}$$

$\cos(\theta_v - \theta_i) = 0.9$

$$\Rightarrow \tan(\cos^{-1}(0.9)) = \frac{Q}{4000}$$

$$\Rightarrow Q = 4000 \times 0.484$$

$$\boxed{\therefore Q = 1937.288 \text{ VAR}}$$

$\therefore S_3 = 4000 + j\,1937.288$ VA — ③

$$S = S_1 + S_2 + S_3$$

$$\Rightarrow \boxed{S = 6614.21 - j\,276.925 \text{ VA}}$$

$\boxed{\therefore S = 6620\angle -2.4^\circ \text{ VA}}$ ← Overall Complex Power Supplied

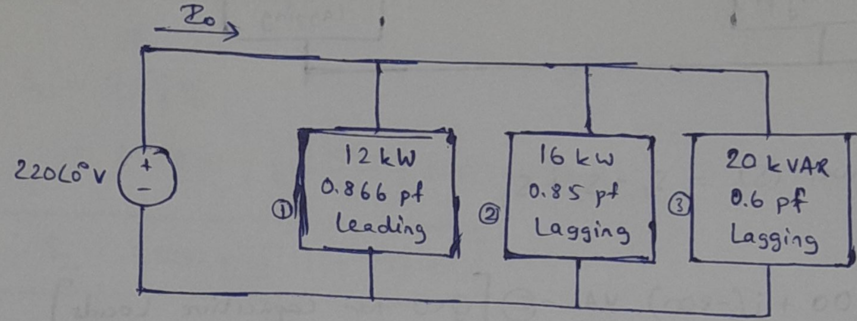
$$S = V_{rms} \cdot I_{rms}^*$$

$$\Rightarrow I_o^* = \frac{S}{V} = \frac{6620\angle -2.4^\circ}{100\angle 90^\circ} = 66.2\angle -92.4^\circ \text{ A}$$

$$\boxed{\therefore I_0 = 66.2\angle 92.4^\circ\ A}$$

(OR) $\boxed{I_0 = -2.77 + j\,66.142\ A}$

15)



$S = S_1 + S_2 + S_3$

$S = V I_0^*$ ⟹ Then we can find I_0

$S_1 = 12000 + jQ$ ($Q<0$ for Leading pf)

$\tan(\theta_v - \theta_i) = \frac{Q}{P}$ [$\because \theta_v - \theta_i < 0$ for leading pf]

$\Rightarrow \tan(\cos^{-1}(-0.866)) = \frac{Q}{12000}$

$\Rightarrow Q = 12000 \times (-0.577)$

$\therefore Q = -6928.2$ VAR

$\boxed{Q = -6928.2\ VAR}$

$\therefore S_1 = 12000 - j(6928.2)$ VA —①

$S_2 = 16000 + j(16000 \times \tan(\cos^{-1}(0.85)))$

$\therefore S_2 = 16000 + j(9915.91)$ VA —② ($Q>0$ for lagging pf)

$S_3 = P + j\,20000$

$\tan(\theta_v - \theta_i) = \frac{Q}{P}$

$\Rightarrow \tan(\cos^{-1}(0.6)) = \frac{20000}{P}$

$\Rightarrow P = \frac{20000}{4/3} = 15000$ W

$\therefore S_3 = 15000 + j20000$ VA — (3)

from (1), (2) and (3) :

$S = S_1 + S_2 + S_3$

$\Rightarrow S = (12000 + 16000 + 15000) + j(-6928.2 + 9915.91 + 20000)$ VA

$= 43000 + j\,22987.71$ VA

$\boxed{\therefore S = 43 + j\,22.988 \text{ kVA}}$

$\boxed{\therefore S = 43 + j\,23 \text{ kVA}}$

$S = V_{rms} \cdot I_{rms}^{*}$

$\Rightarrow I_{rms}^{*} = \dfrac{S}{V_{rms}} = \dfrac{48758.944 \angle 28.13^\circ}{220 \angle 0^\circ}$ A

$\Rightarrow I_o^{*} = 221.63 \angle 28.13^\circ$ A

$\boxed{\therefore I_o = 221.63 \angle -28.13^\circ}$ A

(OR) $\boxed{I_o = 195.45 - j\,104.5 \text{ A}}$