

Engineering Optics

Lecture 30

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by

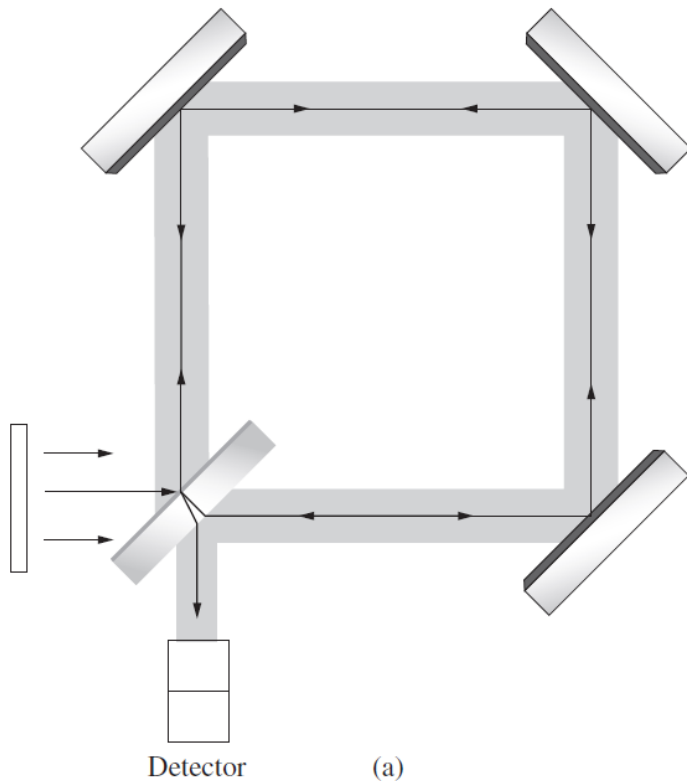
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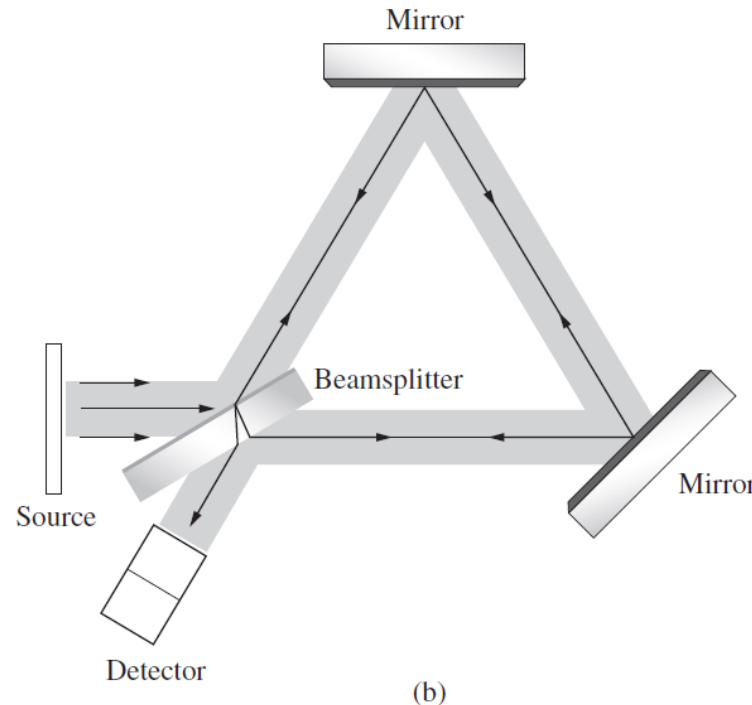
Sagnac Interferometer

- ▶ amplitude-splitting device
- ▶ very easy to align and quite stable
- ▶ Application of the device →

- ▶ main feature of the device:
- ▶ there are two identical but oppositely directed paths taken by the beams
- ▶ both form closed loops before they are united to produce interference



(a) A Sagnac Interferometer.



(b) Another variation of the Sagnac Interferometer.

A deliberate slight shift in the orientation of one of the mirrors will produce a path length difference and a resulting fringe pattern.

Since the beams are superimposed and therefore inseparable, the interferometer cannot be put to any of the conventional uses.

These in general depend on the possibility of imposing variations on only one of the constituent beams.

Problem:1

For a sodium lamp, the distance traversed by the mirror between two successive disappearances is 0.289 mm for Michelson interferometer. Calculate the difference in the wavelengths of the D_1 and D_2 lines.

Assume $\lambda = 5890 \text{ \AA}$

Answer:

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is $1/2, 3/2, 5/2, \dots$, we will have disappearance of the fringe pattern; and if it is equal to $1, 2, 3, \dots$, then the interference pattern will appear.

Answer:

When the mirror moves through a distance 0.289 mm, the additional path introduced is 0.578 mm. Thus

$$\frac{0.578}{\lambda} - \frac{0.578}{\lambda + \Delta\lambda} = 1$$

$$\begin{aligned}\Delta\lambda &\approx \frac{\lambda^2}{0.578} = \frac{(5890 \times 10^{-7})^2}{0.578} \text{ mm} \\ &= 6 \text{ \AA}\end{aligned}$$

Assume $\Delta\lambda \times \lambda \ll \lambda^2$

Problem:2

In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength.

Answer:

The wavelength λ in Michelson interferometer is given by following equation.

$$\lambda = \frac{2d_0}{N}$$

Here, d_0 is the distance moved by the mirror, and N is the number of fringes.

$$\begin{aligned}\lambda &= \frac{2(0.08 \text{ mm}) \left(\frac{1 \text{ cm}}{10 \text{ mm}} \right)}{250} \\ &= 6.4 \times 10^{-5} \text{ cm} \left(\frac{10^8 \text{ \AA}}{1 \text{ cm}} \right) \\ &= 6400 \text{ \AA}\end{aligned}$$

Problem:3

The Michelson interferometer experiment is performed with a source which consists of two wavelengths of 4882 and 4886 Å.

Through what distance does the mirror have to be moved between two positions of the disappearance of the fringes?

Answer:

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1 \text{ For appearance of fringe}$$

Then

$$d = \frac{1}{2} \left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \right)$$

$$\begin{aligned}d &= \frac{1}{2} \left(\frac{(4882 \text{ \AA})(4886 \text{ \AA})}{4886 \text{ \AA} - 4882 \text{ \AA}} \right) \\&= 2981681.5 \text{ \AA} \left(\frac{1 \text{ cm}}{10^8 \text{ \AA}} \right) \\&= 0.0298 \text{ cm} \left(\frac{10 \text{ mm}}{1 \text{ cm}} \right) \\&= 0.298 \text{ mm}\end{aligned}$$

Problem:4

Consider a monochromatic beam of wavelength 6000 \AA incident (from an extended source) on a Fabry–Perot etalon with $n_2 = 1$, $h = 1 \text{ cm}$, and $F = 200$. Concentric rings are observed on the focal plane of a lens of focal length 20 cm . Calculate the reflectivity of each mirror.

Answer:

$$F = \frac{4R}{(1-R)^2} \Rightarrow \begin{aligned} 200 &= \frac{4R}{(1-R)^2} \\ 50 &= \frac{R}{(1+R^2-2R)} \end{aligned}$$

$$50 + 50R^2 - 100R = R$$

$$50R^2 - 101R + 50 = 0$$

By solving above quadratic equation,

$$R = 1.15 \text{ or } 0.87$$

$R=0.87$ (Since R will be less than 1)

Thank You