Engineering Optics

Lecture 35

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by

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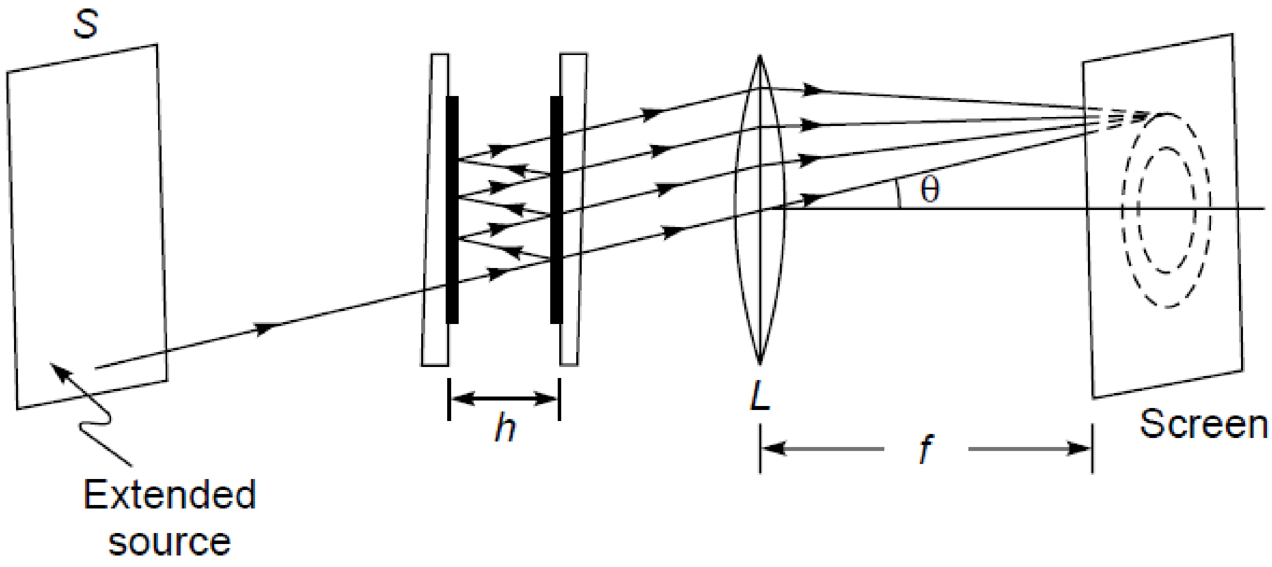
Fabry–Perot interferometer

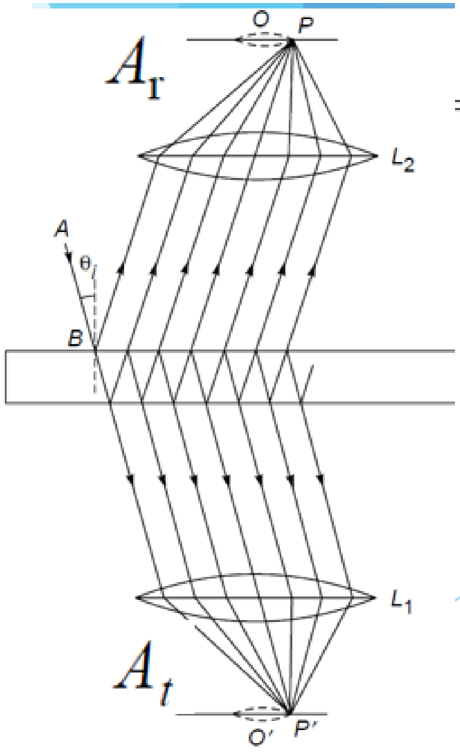
Reflectivity of the instrument

$$\mathcal{R} = \left|\frac{A_r}{A_0}\right|^2 = R\left|\frac{1-e^{i\delta}}{1-Re^{i\delta}}\right|^2$$

$$= R\,\frac{(1-\cos\delta)^2+\sin^2\delta}{(1-R\cos\delta)^2+R^2\sin^2\delta}$$

$$= \frac{4R\sin^2\delta/2}{(1-R)^2+4R\sin^2\delta/2}$$





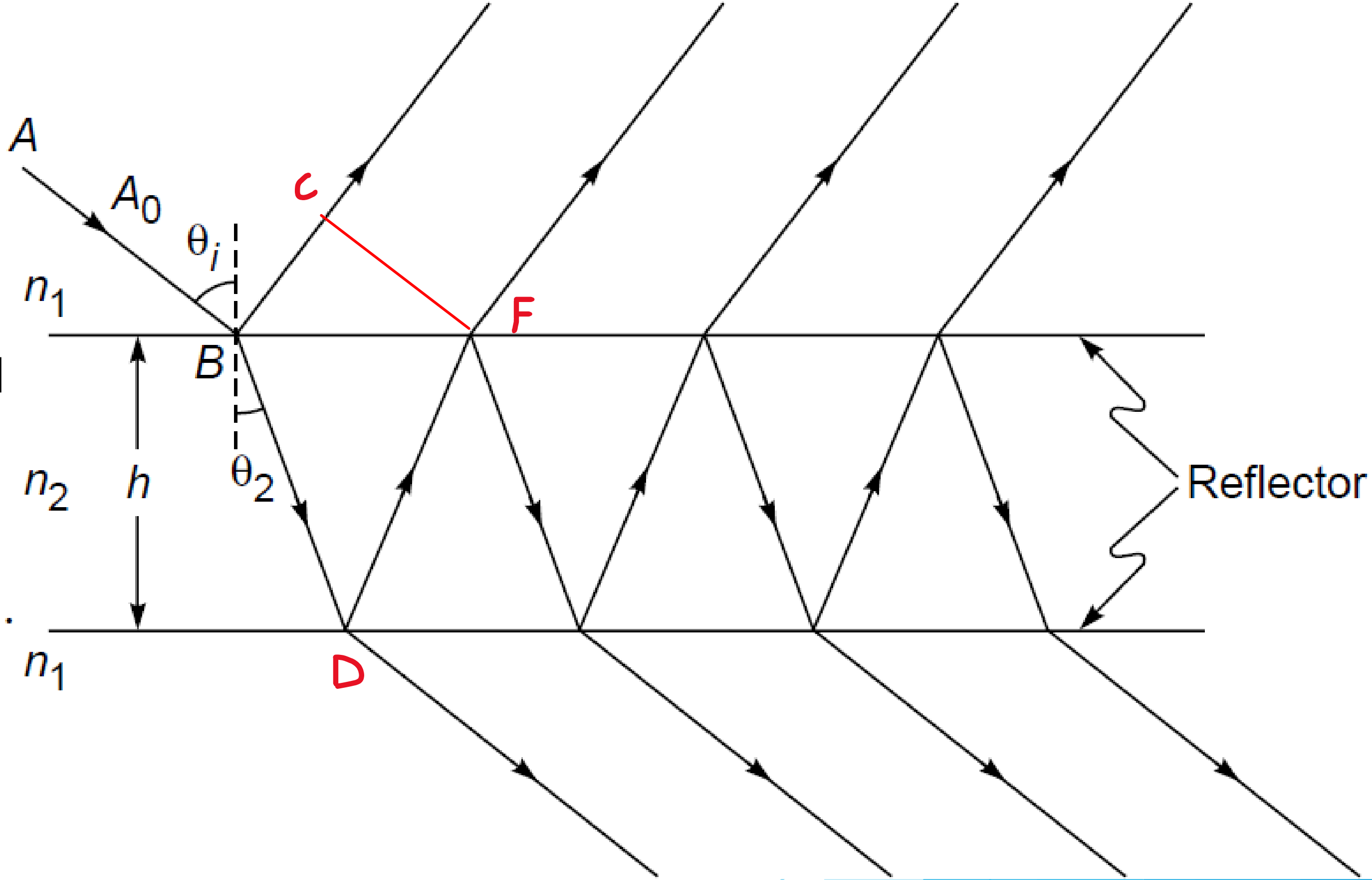
Transmittivity of the instrument (not τ → property of the 2 media forming interface)

$$T = \left|\frac{A_t}{A_0}\right|^2 = \frac{(1-R)^2}{(1-R\cos\delta)^2+R^2\sin^2\delta}$$

Optics, Ghatak

Multiple reflections from a plane parallel film

- A_0 → amplitude of the incident wave.
- When the wave is from n_1 toward n_2: r_1, t_1
- from n_2 toward n_1 → r_2 and t_2
- Thus the amplitude of the successive reflected waves will be



$$A_0 r_1,\ A_0 t_1 r_2 t_2 e^{i\delta},\ A_0 t_1 r_2^3 e^{2i\delta},\ \ldots$$

where
$$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda_0}$$

represents the phase difference (between two successive waves emanating from the plate)

Extra path = $n_2(BD+DF)-n_1BC$
Now check Δ
For the calculation see Section 15.3 “The Cosine Law”, Ghatak’s book

Optics, Ghatak

Measuring the wavelength of light

- Let the initial separation between the mirrors is d_1.
- vary the distance between the mirrors to d_2
- Count the number of fringes (say maxima) appearing or disappearing at the center ($\theta \approx 0$)
- then λ can be determined as follows:

$$2d_1 = m_1 \lambda,$$

$$m_2 - m_1 = \text{Number of maxima counted}$$

$$2d_2 = m_2 \lambda,$$

$$\boxed{\lambda = \frac{2(d_2 - d_1)}{m_2 - m_1}}$$

https://www.niser.ac.in/sps/sites/default/files/basic_page/Fabry-Perot%20Interferometer.pdf

Mach–Zehnder interferometer

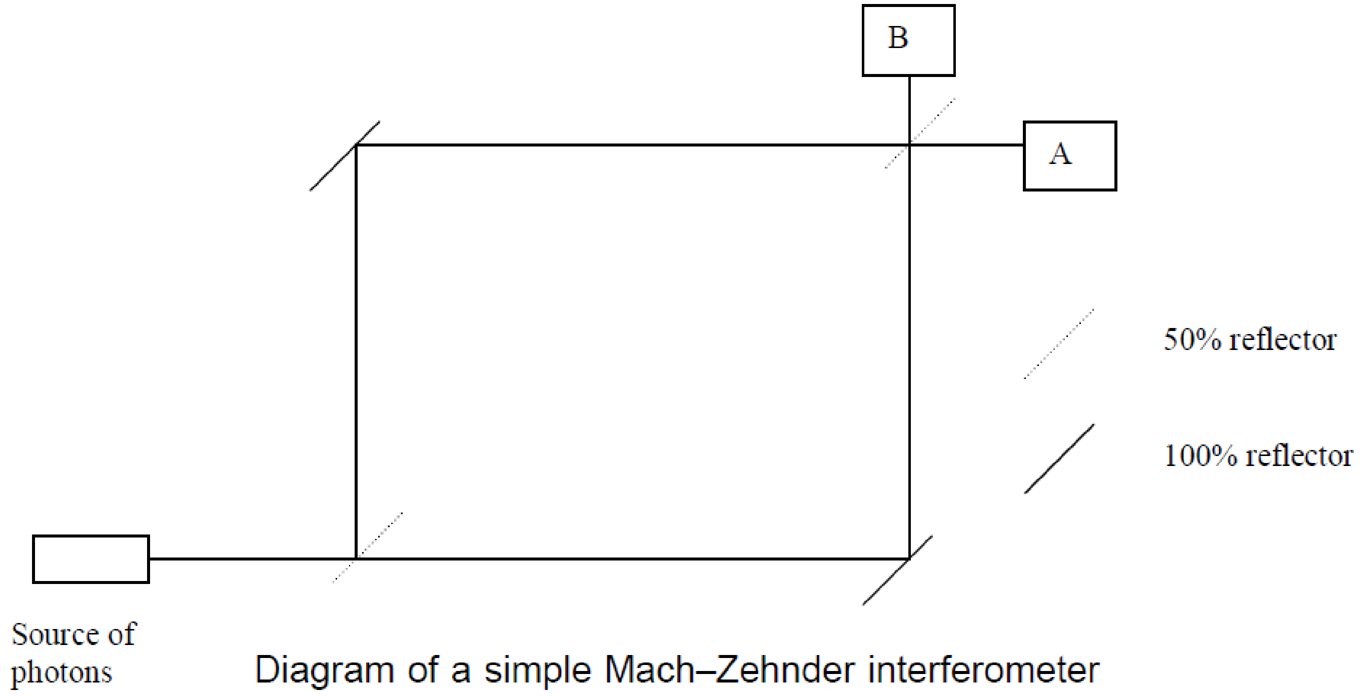


Diagram of a simple Mach–Zehnder interferometer

- used to determine the relative <u>phase shift</u> variations between two <u>collimated</u> beams derived by splitting light from a single source.
- caused by a sample or a change in length of one of the paths
- named after the physicists Ludwig Mach and Ludwig Zehnder
- How different from Michelson interferometer?

wikipedia

So, which path shows constructive interference, the path towards A or B?

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Mach–Zehnder interferometer

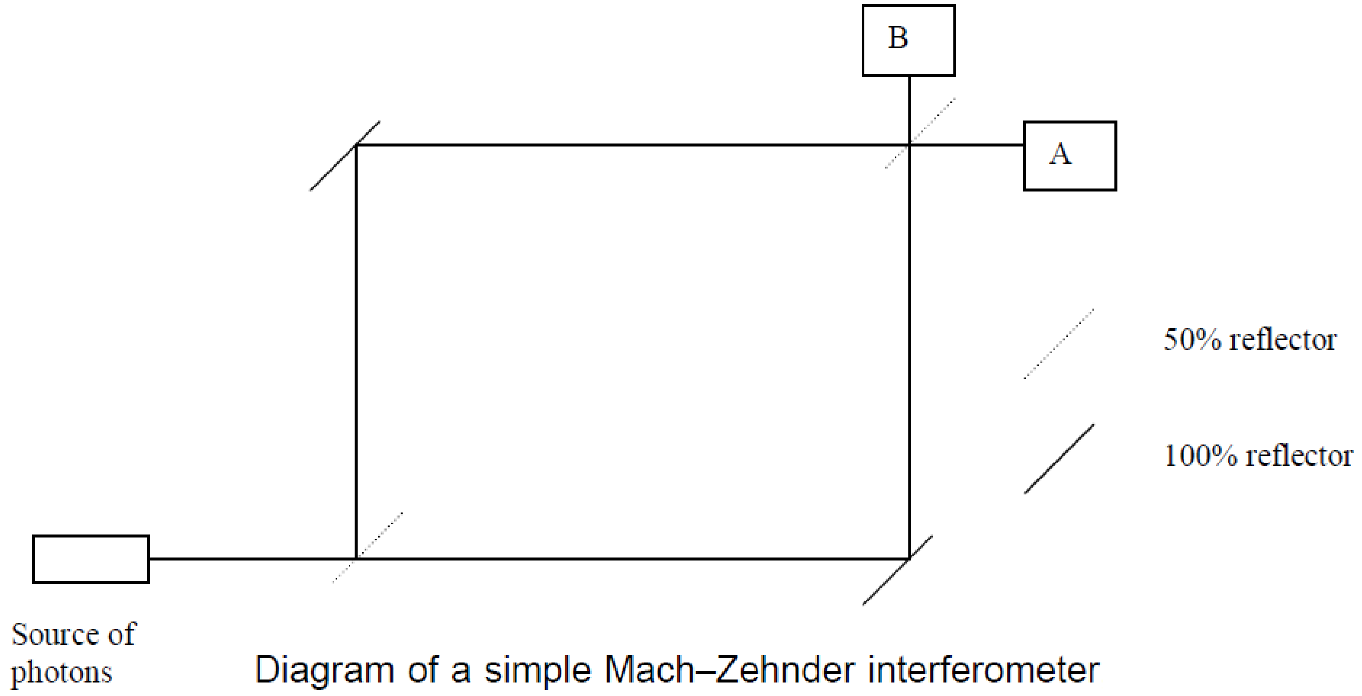

Diagram of a simple Mach–Zehnder interferometer

- used to determine the relative phase shift variations between two collimated beams derived by splitting light from a single source.
- caused by a sample or a change in length of one of the paths
- named after the physicists Ludwig Mach and Ludwig Zehnder
- In contrast to the well-known Michelson interferometer, each of the well-separated light paths is traversed only once

wikipedia

So, which path shows constructive interference, the path towards A or B?

the entire situation is symmetrical with respect to the two detectors and should one path allow constructive interference, so will the other
But is that allowed?

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Mach–Zehnder interferometer

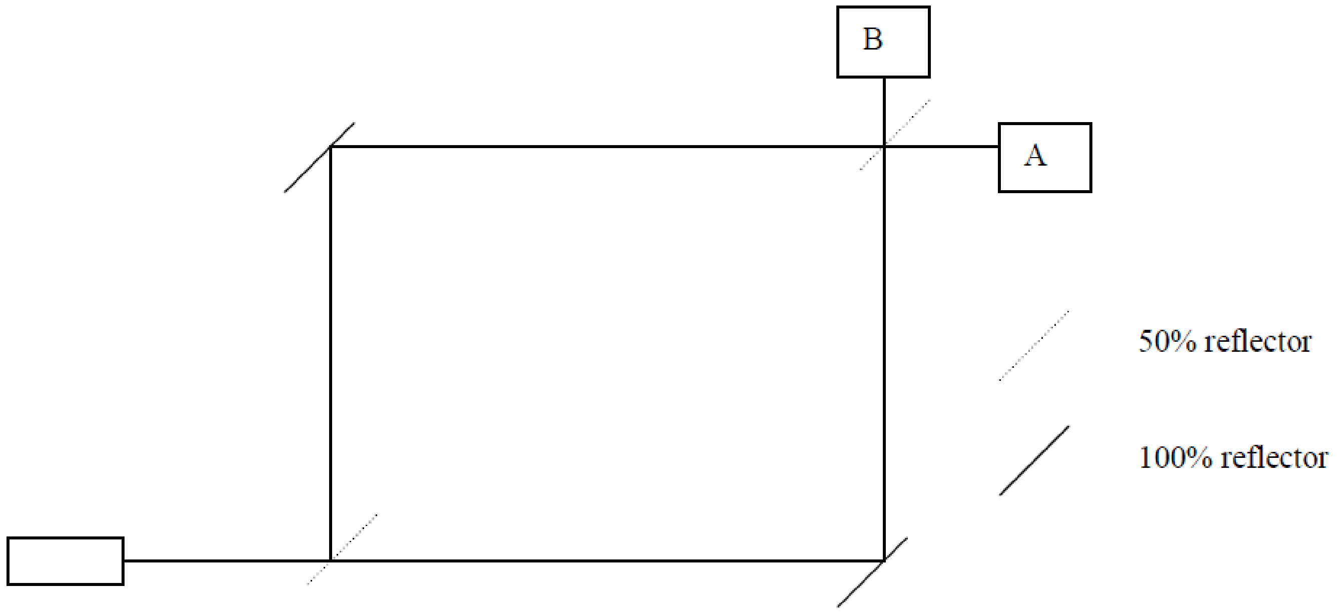

Diagram of a simple Mach–Zehnder interferometer

- look at the phase shift on reflection
- A standard piece of physics lore is that on transmission a wave picks up no phase shift, but on reflection it picks up a phase shift of π.
- So now let's investigate.
- We shall break the problem into two parts: (1) first the path from the source to the second beamsplitter, and then
- (2)the final stretch from the second beamsplitter to the detectors A and B.

- On the lower path: **Phase shift?**
- On the upper path: **phase shift?**

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Mach–Zehnder interferometer

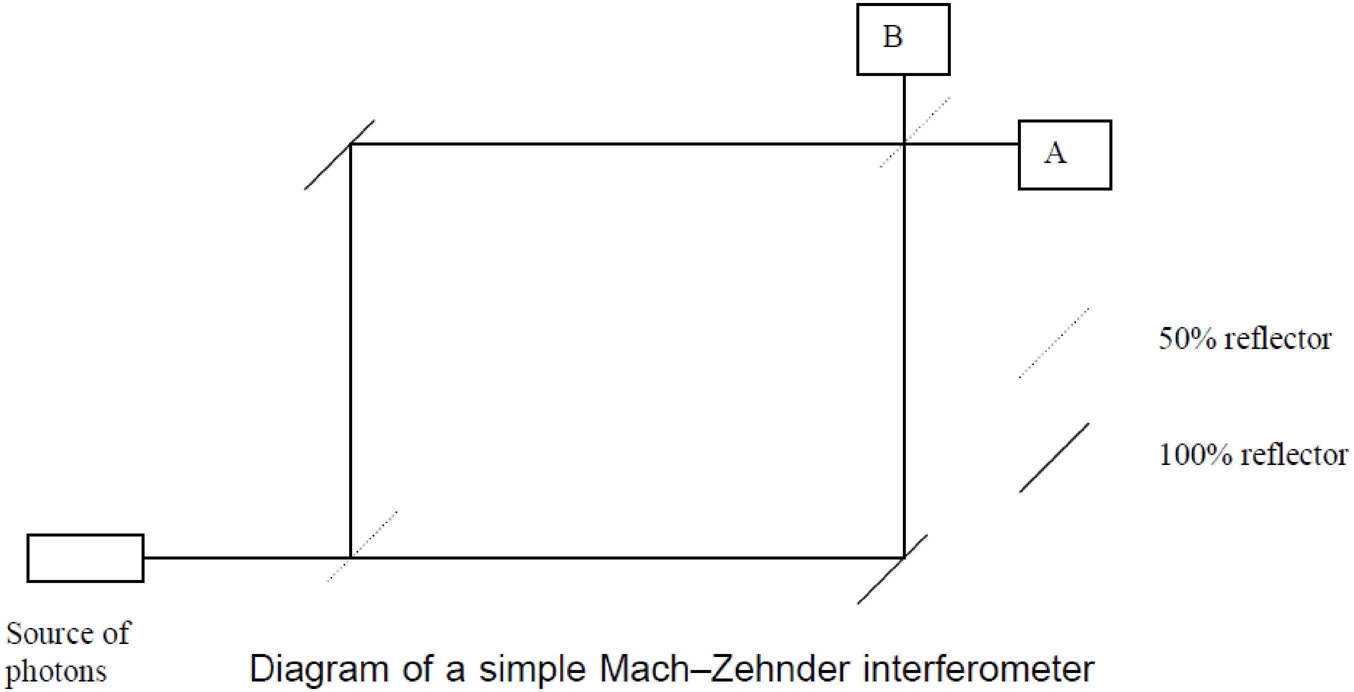

Diagram of a simple Mach–Zehnder interferometer

- Now if we continue on to **detector A**:
- the lower path makes one more reflection and the upper path one transmission.
- So now each path has a phase shift of 2π and they will **interfere constructively**.
- look at the path to **detector B**:
- the lower path makes one more transmission, picking up a total phase shift of π. The upper path makes a further reflection, so its total phase shift is 3 π.

- The puzzle/dilemma: difference is 2 π and **again we expect constructive interference.**
- So that is the problem. Energy conservation???

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1 photon interference??

- 1905 Einstein → hypothesis that light is made of quanta
- Light follows many paths simultaneously → merge→ interfere→ pattern depends on the optical path difference between those various paths
- **Can we say the same for a photon? How can a photon follow different paths simultaneously?**
- **1 incident photon → 1 (in A) + 1 (in B) → is that possible?**
- Q: does a photon interfere with itself? → *intriguing puzzle*
- **Where did we go wrong?**

Wave-particle duality

- In QM particles such as electrons, behave as matter waves→ the de Broglie wave → the famous wave-particle duality
- But what about the photons?
- Photons are not particles like electrons/neutrons as they have zero rest mass
- Let us revisit the very notion of wave particle duality in the context of optics when the primary concept is waves

Mach–Zehnder interferometer

- Let us understand what happens when *one photon* is sent to the two-armed Mach-Zender interferometer.
- Wave or particle?? → led to the very idea of quantum optics
- A beloved interferometer for theoretician
- A bit difficult in experiment→ all the components should be mounted with caution→ highly sensitive→ stability/accuracy should be ensured at the level of a fraction of micrometer
- In contrast→ widely used in integrated optics, optical waveguides

Mach–Zehnder interferometer

- The Mach–Zehnder interferometer is a particularly simple device for demonstrating interference by division of amplitude.
- A light beam is first split into two parts by a beam splitter and then recombined by a second beam splitter.
- **The operation of a Mach-Zehnder interferometer is often used as an example in quantum mechanics because it shows a clear path-choice problem.**
- However, it is not at all obvious at first glance that it works as claimed, until reflection phase shifts are considered properly

Mach–Zehnder interferometer

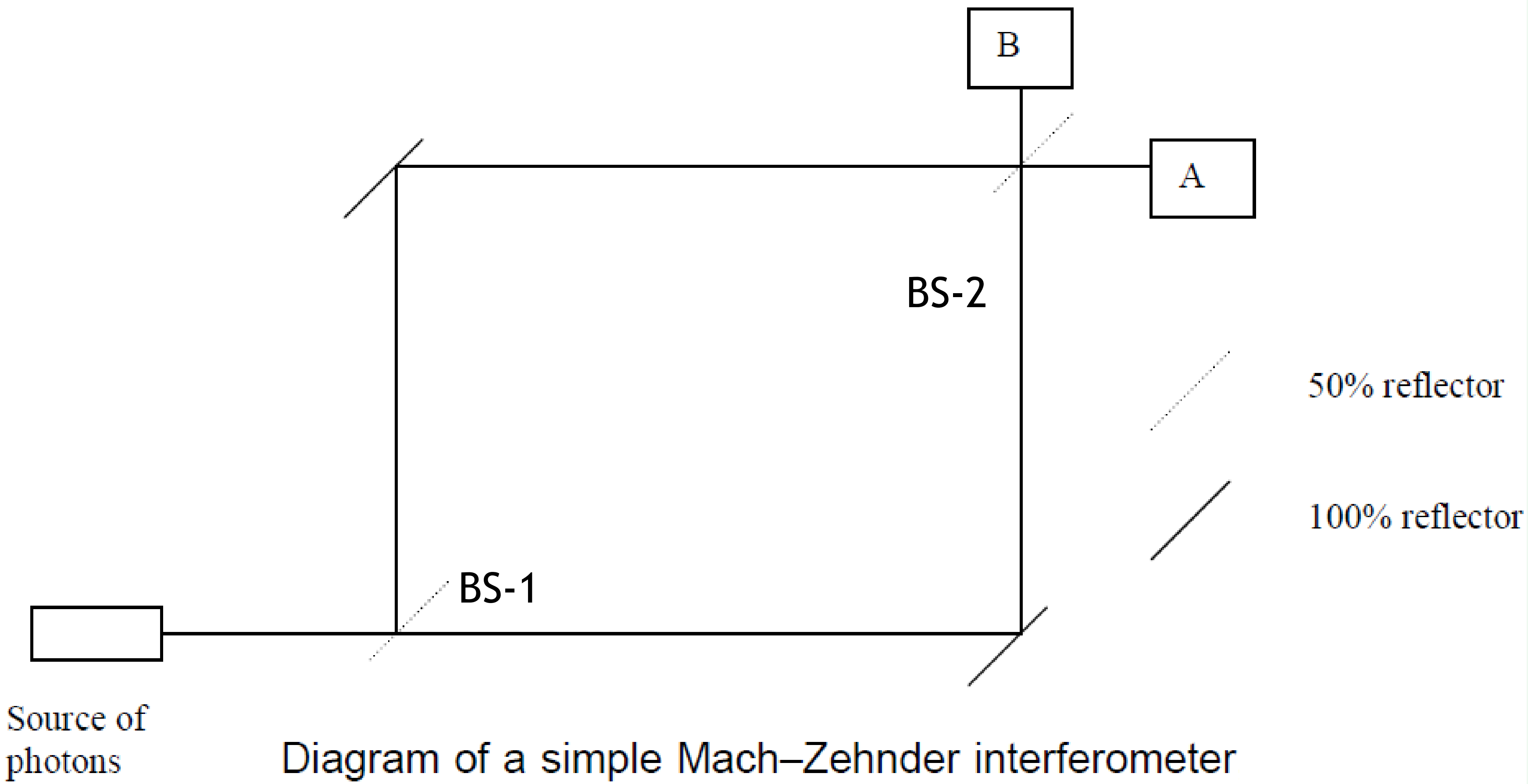

Diagram of a simple Mach–Zehnder interferometer

- on <u>transmission a wave picks up no phase shift</u>, but <u>on reflection it picks up a phase shift of π</u>. So now let's investigate.
- We shall break the problem into two parts: (1) the path from the source to the second beamsplitter,
- (2) from the second beamsplitter to the detectors A and B.

- **(1)** Source to beamsplitter:
- the lower path, net phase? = π
- The upper path, net phase = 2π

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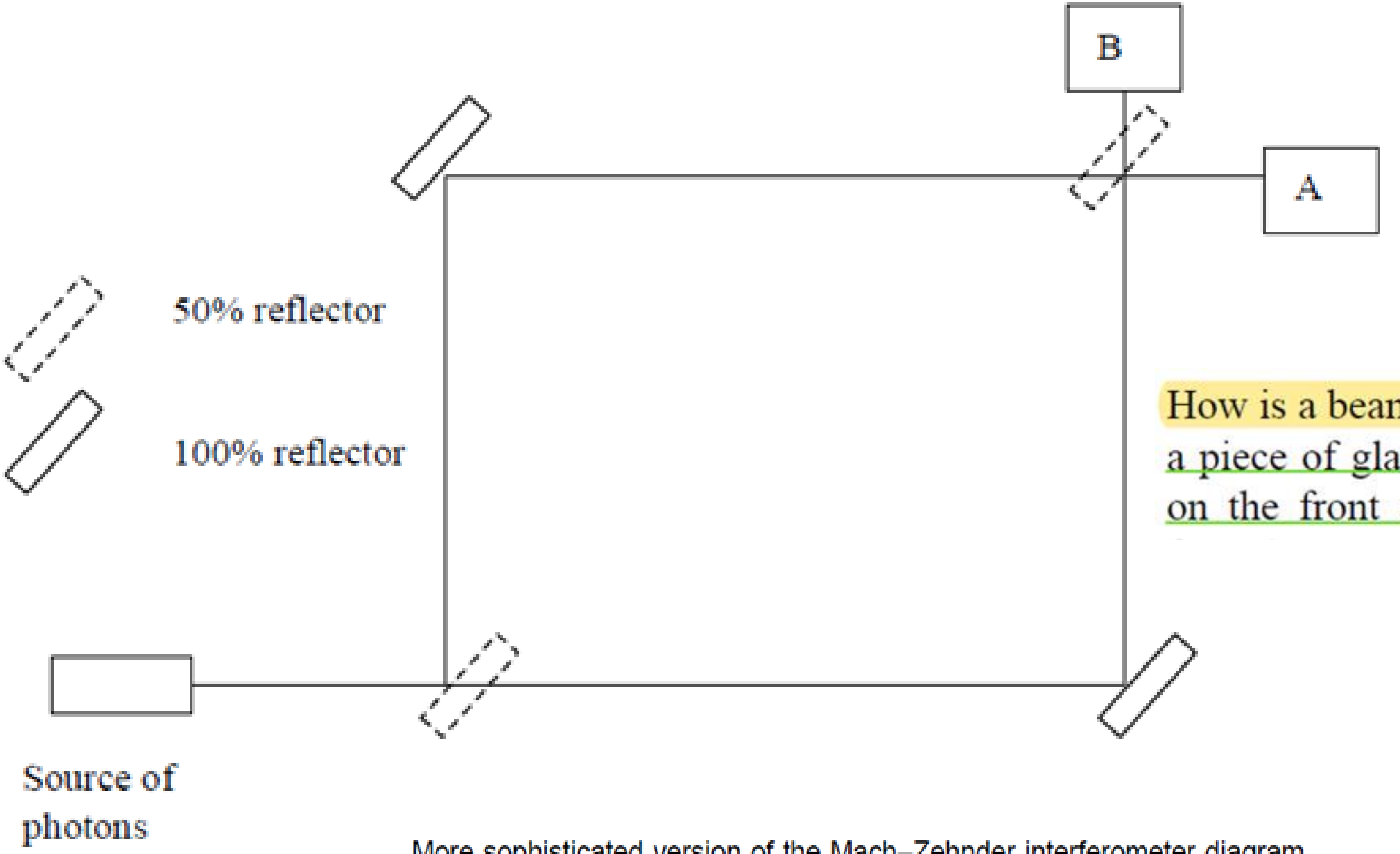
Resolution of the problem



How is a beamsplitter actually made? Usually it is a piece of glass with a dielectric or metal coating on the front surface.

More sophisticated version of the Mach–Zehnder interferometer diagram.
Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

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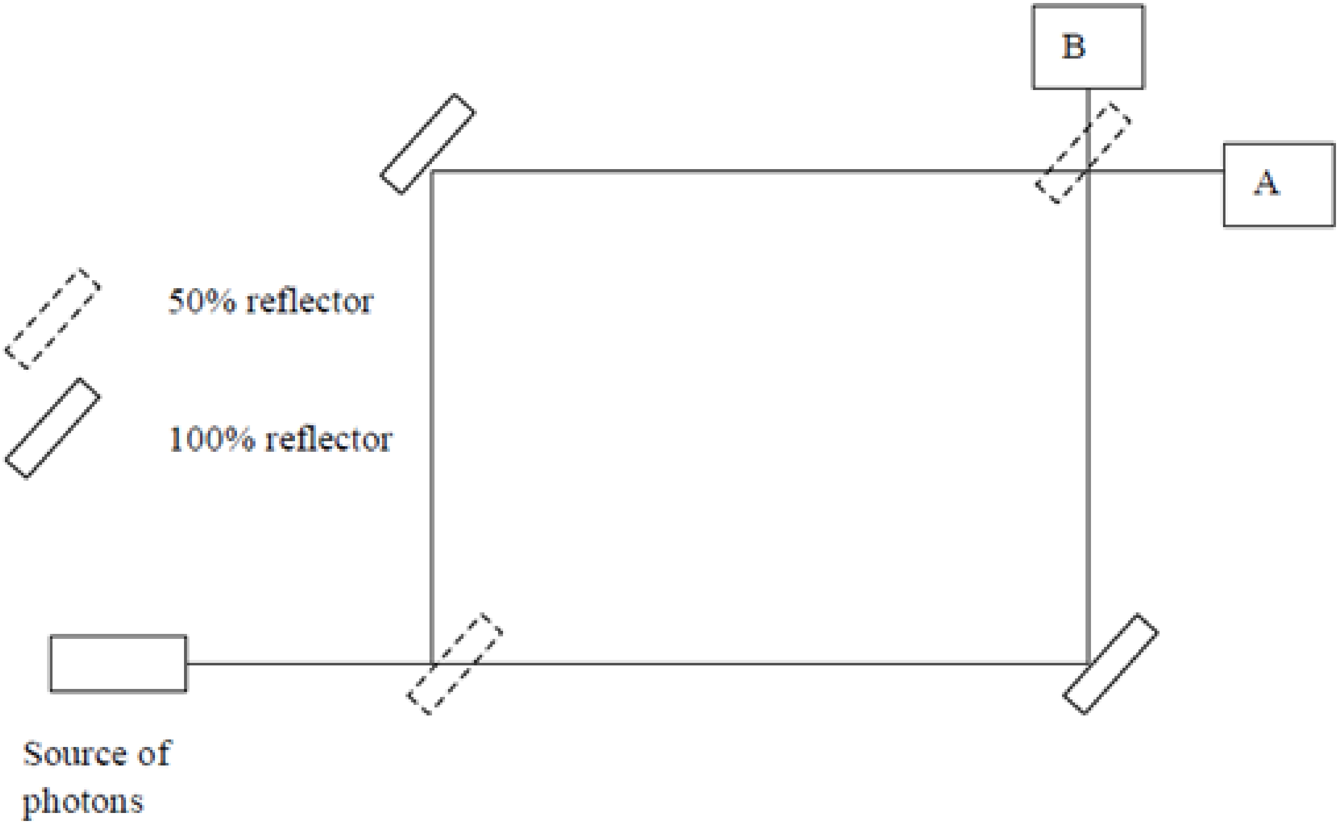
Resolution of the problem



More sophisticated version of the Mach–Zehnder interferometer diagram.
Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

Are we calculating the *phase* accurately?

However, the key to the problem lies in what happens to a photon approaching the beamsplitter from behind. There it first enters the glass (ignoring the small chance of reflection off the air–glass interface) and has a 50% chance of reflecting off the dielectric coating whilst within the glass. Here is the crux of the matter—*that* reflection does not induce a phase change. Given that, let us once again examine the phase shifts on the two paths.

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Resolution of the problem

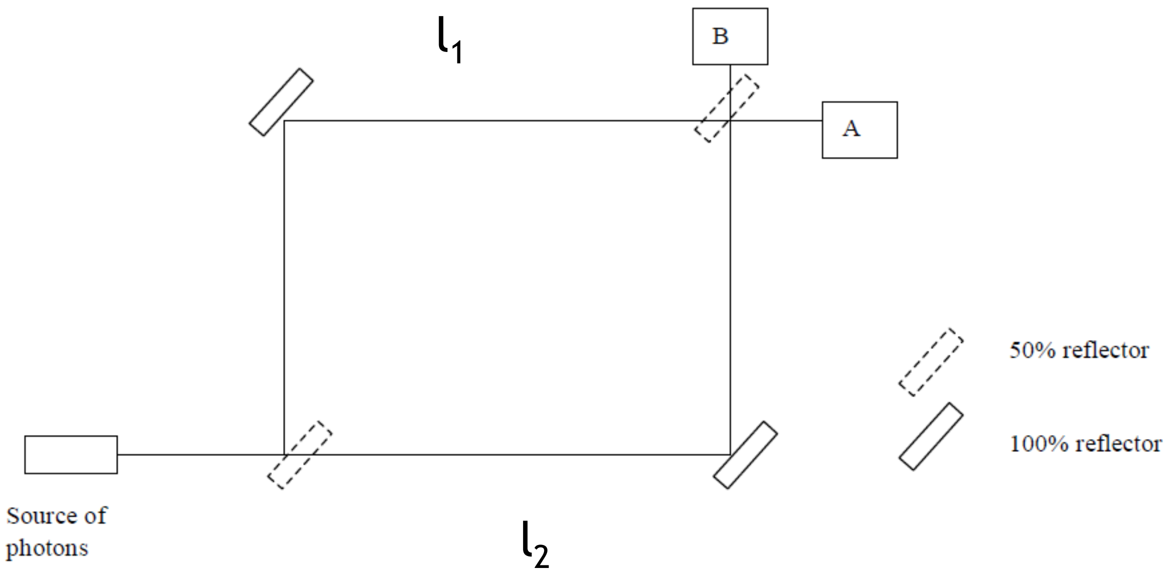

Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

l_1= total length from source to detector for upper path

l_2= total length from source to detector for lower path

Extra phase when the light passes through the glass of the beamsplitter $2\pi t/\lambda$

t: optical path length through the BS
Depends on length (thickness) and r.i.

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Resolution of the problem: detector A

To detector A

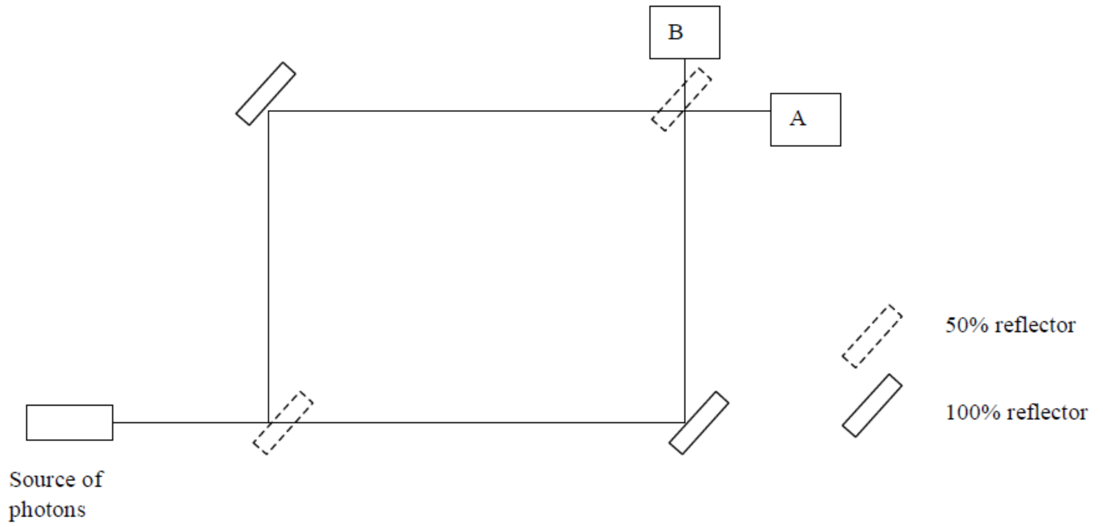

Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

The upper path picks up the following phase shifts on the way to detector A: π at the first reflection, π at the second (100%) reflection, nothing at the transmission, $2\pi l_1/\lambda$ for the distance travelled, and $2\pi t/\lambda$ for the extra phase picked up in traversing the glass substrates where the wavelength is reduced. This gives a total of

$$2\pi + 2\pi\left(\frac{l_1 + t}{\lambda}\right).$$

The lower path, also on its way to A, picks up a phase shift of π off the 100% reflector, π at the second beamsplitter, a phase shift of $2\pi l_2/\lambda$ for the distance travelled, and an extra phase shift of $2\pi t/\lambda$ from passing through the glass substrate at the first beamsplitter.

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Resolution of the problem: detector A

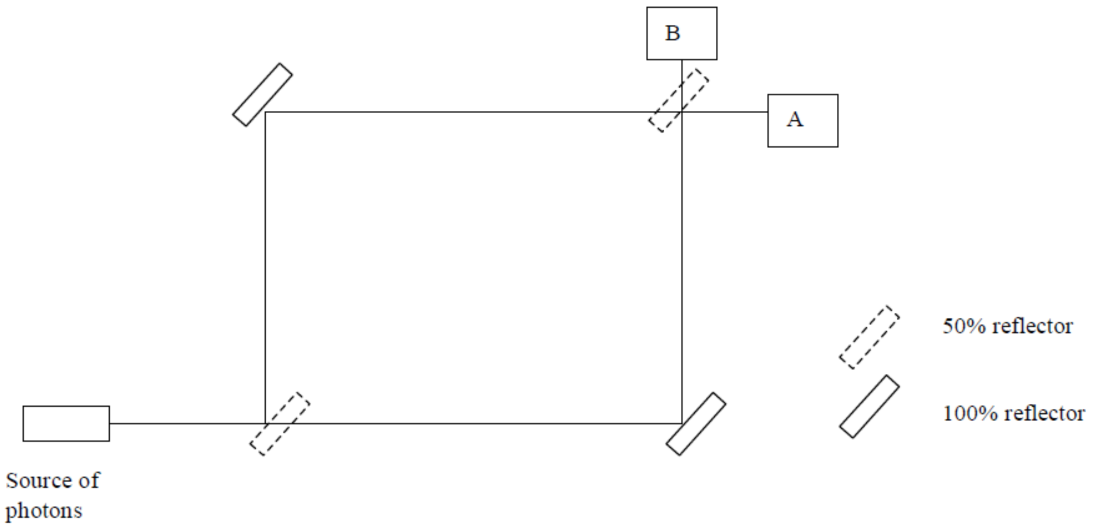

Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

The phase difference between the two paths is

$$2\pi + 2\pi\left(\frac{l_1 + t}{\lambda}\right) - 2\pi - 2\pi\left(\frac{l_2 + t}{\lambda}\right)$$

$$= 2\pi\left(\frac{l_1 - l_2}{\lambda}\right) = \delta$$

where δ is the phase shift due to the difference in the path lengths.

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Resolution of the problem : detector B

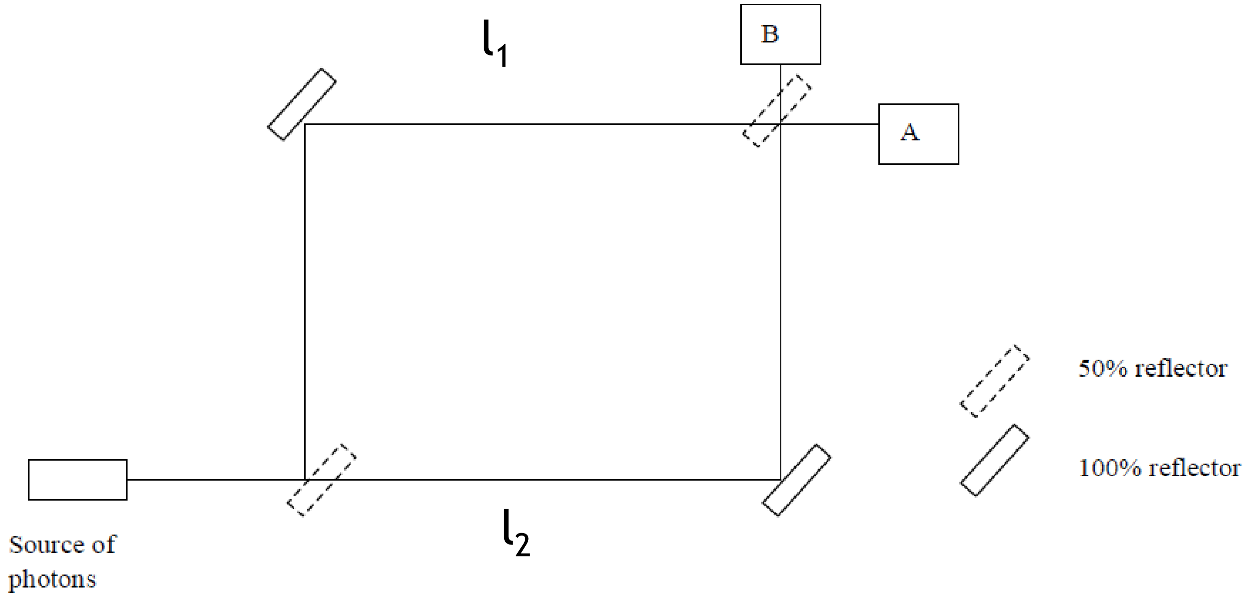

Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

Similarly, we can calculate the phase difference between the two paths on their way to detector B. We obtain

$$2\pi + 2\pi\left(\frac{l_1 + 2t}{\lambda}\right) - \pi - 2\pi\left(\frac{l_2 + 2t}{\lambda}\right)$$
$$= \pi + 2\pi\left(\frac{l_1 - l_2}{\lambda}\right) = \pi + \delta.$$

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Resolution of the problem : detector B

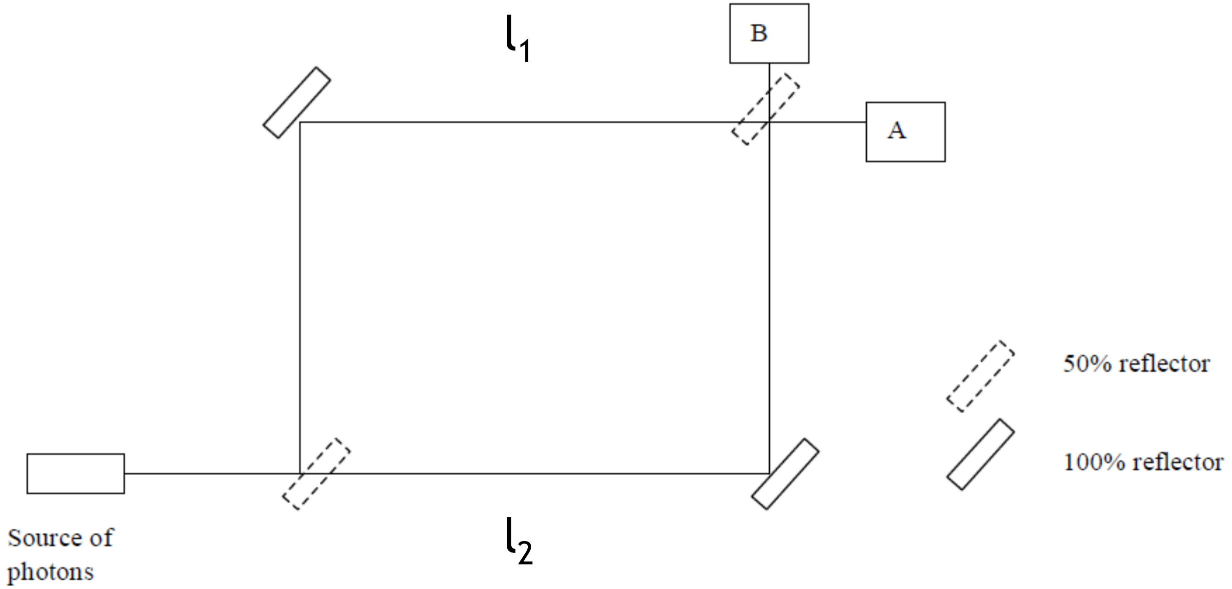

Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

Detector A: phase diff = δ
Detector B: phase diff. = $\pi+\delta$

When $\delta = 0$
Detector A→ Constructive interference
Detector B→ Destructive Interference

Change δ → pattern will vary
OR
Probability of detecting a photon at either detector: 0 to 1

Puzzle solved

All of the physics is contained in this analysis. In practice, the beamsplitters may be of different thicknesses but this will simply add a fixed phase difference, as will placing the second beamsplitter the other way around.

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Thank You