Chapter 12 - Heaps

Introduction

- Heaps are largely about priority queues.
- They are an alternative data structure to implementing priority queues (we had arrays, linked lists…)
- Recall the advantages and disadvantages of queues implemented as arrays
  - Insertions / deletions? O(n) … O(1)!
- Priority queues are critical to many real-world applications.

Introduction

- First of all, a heap is a **kind of tree** that offers both insertion and deletion in $O(\log_2 n)$ time.
- Fast for insertions; not so fast for deletions.

Introduction to Heaps

- Characteristics:
  - 1. A heap is 'complete' (save for the last row – going left to right – see figure 12.1, p. 580)
  - 2. **usually <u>implemented</u>** as an **array**
  - 3. Each node in a heap satisfies the 'heap condition,' which states that **<u>every node's key is larger than or equal to the keys of its children</u>**.
  - → The heap is thus an **<u>abstraction</u>**; we can draw it to **look** like a tree, but recognize that it is the **array** that **<u>implements</u> this abstraction** and that it is stored and processed in **primary memory (RAM)**.
  - No 'holes' in the array.

Priority Queues, Heaps, and ADTs

- Heaps are mostly used to implement priority queues.
- Again, a heap is usually <u>implemented</u> as an <u>array</u>.

"Weakly Ordered"

- We know how a <u>binary</u> <u>search</u> <u>tree</u> is developed – with lesser keys to the left; greater keys to the right as we descend.
  - Because of this, we have nice, neat algorithms for binary search trees.

- Here: No Strong Ordering:
  - But for nodes in a heap, <u>**we don't have this strong ordering**</u> - and this can cause us some difficulty.

- Cannot assert much:
  - We can **only assert** as one descends in the heap, nodes will be in descending order (see rule 3)
  - Equivalently, nodes <u>**below**</u> a node are <= the parent node.

- Weakly-ordered:
  - Heaps are thus said to be **weakly ordered**…

Weakly Ordered – More

- No Convenient Search Mechanism:
  - Because of weak ordering, there **is no convenient searching** for a specified key as we have in binary search trees.
  - Don't have enough info at a node to decide whether to descend left or right.
- Delete:
  - So to delete a node with a **specific key** there are issues
  - No real slick way to find it.
- Randomness:
  - So, a heap's organization approaches 'randomness.'
- Sufficient Ordering: Yet, there is 'sufficient ordering' to allow
  - **quick removal** (yes, a delete) of the **maximum** node and
  - **fast insertion** of new nodes.
- ➔ As it turns out, these are the only operations one needs in using a heap as a priority queue.
- We will discuss the algorithms later…

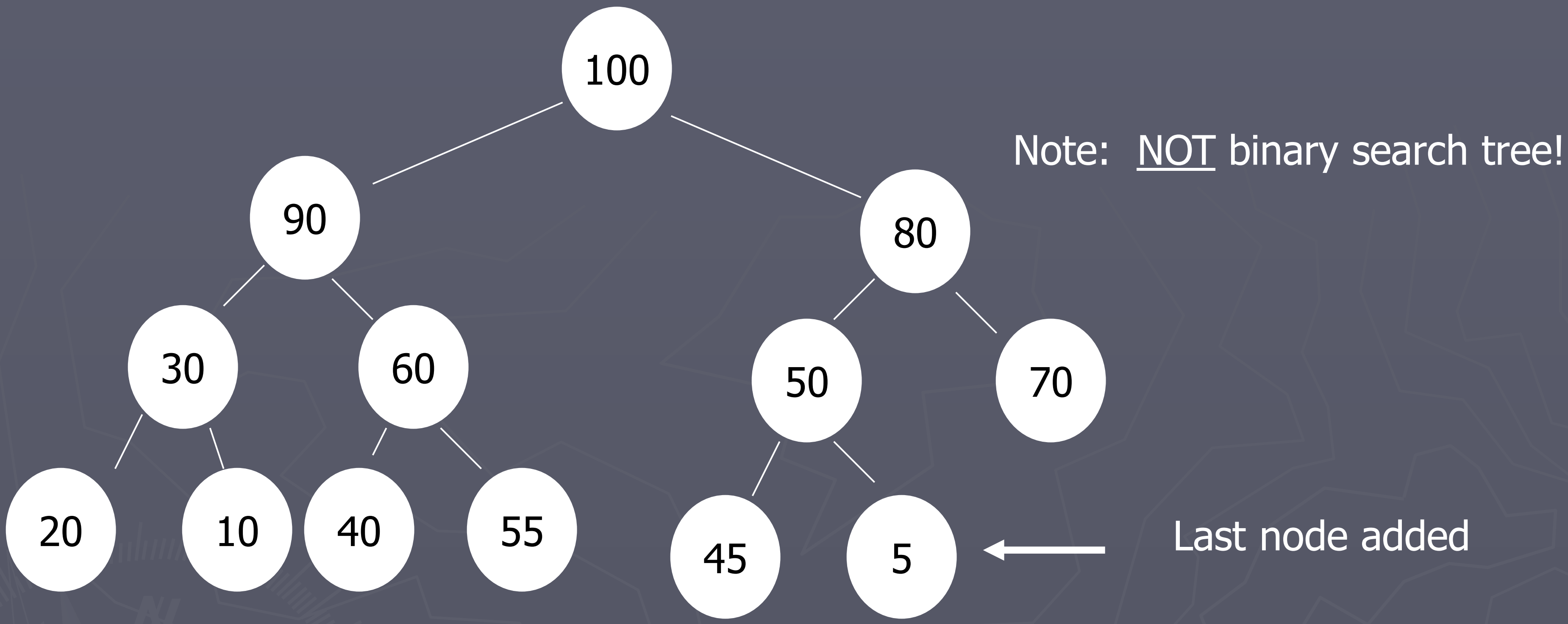
Removal

- Removal is easy:
  - When we remove from a heap, we always remove the node with the **largest** key.
  - Hence, removal is quite easy and has index 0 of the heap array.
  - maxNode = heapArray[0];
- But tree is then not complete:
  - But once root is gone, tree is not complete and we must fill this cell.
- Now this becomes interesting…

Removal of "maxNode"

- **Move 'last node' to root.**
  - Start by **moving** the 'last node' into the root.
  - The 'last' node is the rightmost node in the lowest occupied level of the tree.
  - This also corresponds to the **last filled cell** in the array (ahead).
- **Trickle-down**:
  - Then **trickle this last node <u>down</u>** until it is below a larger node and above a smaller one.

Removal (Delete)



Last node added is 5.

Delete Algorithm: heapArray[0] = heapArray [n-1]

n--; // size of array is decreased by one.

Heap is represented **logically** as a tree.

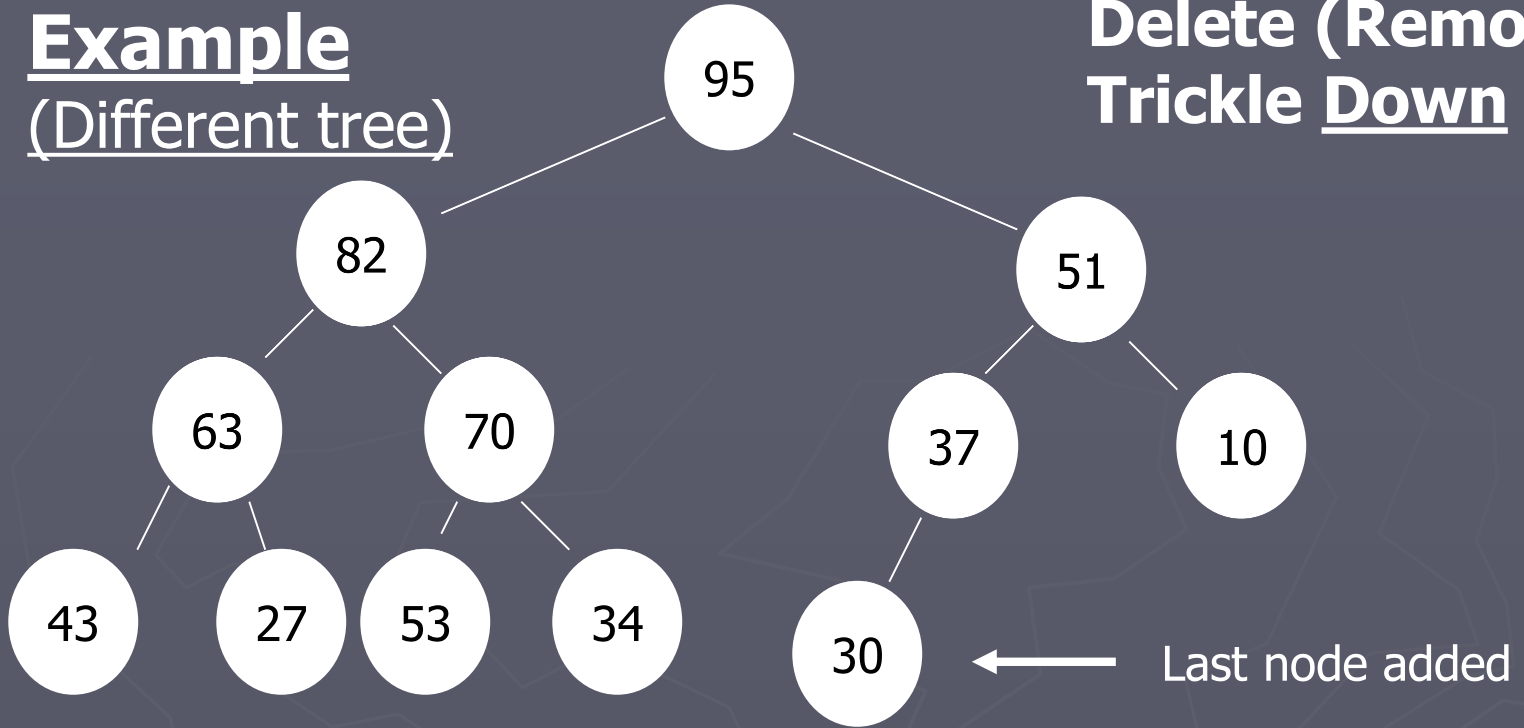
(Tree is organized to represent a priority queue. Easy to visualize.)

Heap is **physically implemented (here)** using an array.

<u>**Example**</u>

<u>(Different tree)</u>

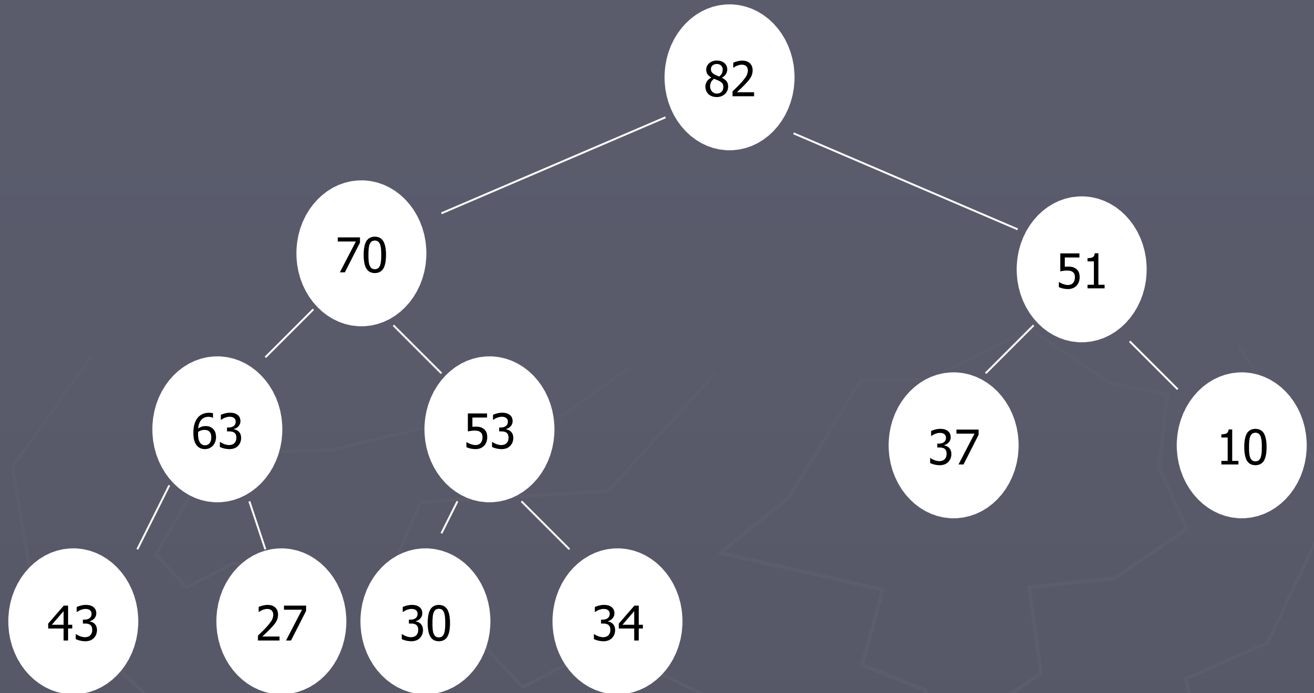
Delete (Removal)
Trickle <u>Down</u>



Trickle down swapping node w/larger child until node >= children

As we trickle down, the nodes swap out **always swapping the larger node** with the node we are trickling down (so as to maintain the larger nodes above…

We trickle down selecting **the largest child** with which to swap.
We must compare, but **always swap with the larger of the two**.
Note: we very well may <u>**NOT**</u> trickle down to a leaf node



The new arrangement via a delete would be as above.

Go through this…

Node 95 (the largest) is deleted.
Tree remains balanced after delete
and the rules for the heap are preserved.

(95 removed; compare 30 with 82; 82 selected and moved to root;
Compare 30 with 70. 70 moves up.
Compare 30 with 53; 53 moves up. 30 is a leaf node.)

Insertion – Trickle <u>Up</u>

- Pretty easy too.   Easier than Deletion.
- Insertions usually **<u>trickle up</u>**.

- Start at the **<u>bottom</u>** (first open position) via code:

```
heapArray[n] = newNode;
n++;
```

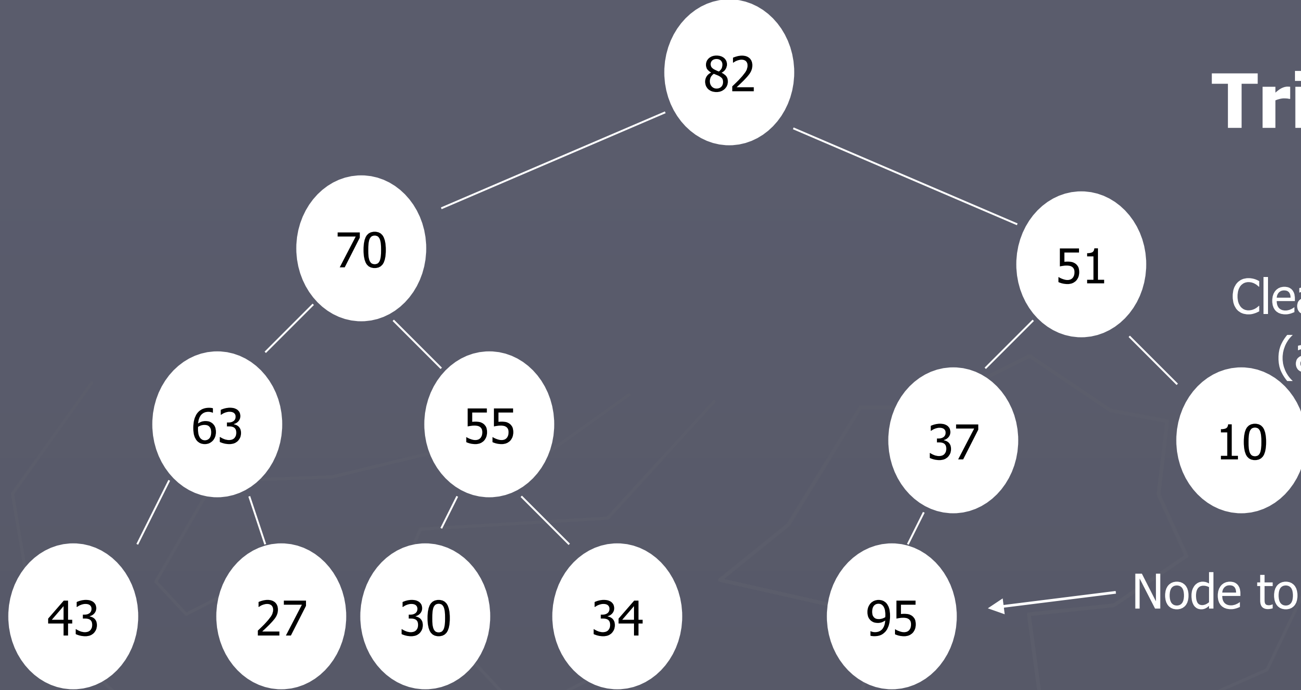
Inserting at bottom will likely destroy the heap condition.

This will happen when the new node is larger than its parent.

<u>Trickle</u> **<u>upwards</u>** until node is **<u>below</u>** a node larger than it and it is **<u>above</u>** a node smaller (or equal to) it.

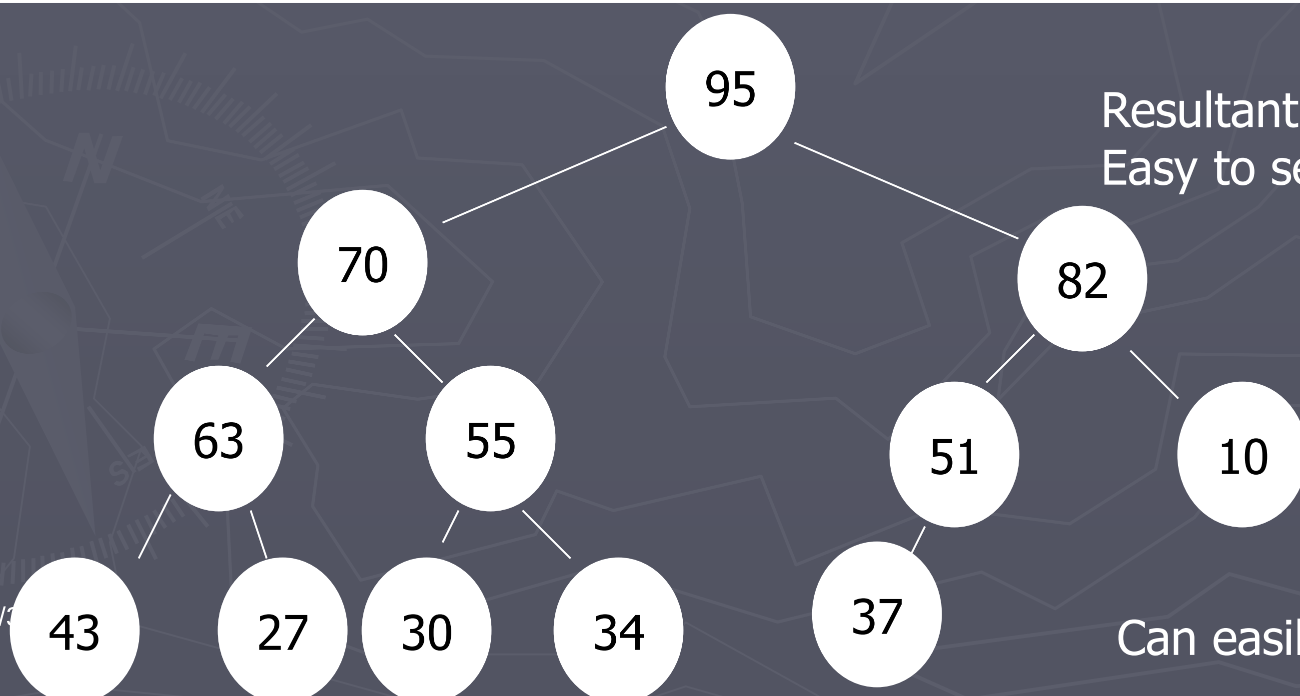
Consider the following slides:

Trickle Up



Clearly 95 must trickle up (and appears it will end up at the root!)

Node to be added…



Resultant heap:
Easy to see swaps…

Can easily see the swaps.

Insertion - more

- Note that this is easier because we don't have to **compare** which node to swap (as we do in going down).
- We only have **ONE** parent, and it is equal to or larger!
- Progress until parent is larger, at whatever level that might occur.

- A side point: Clearly, the same nodes may constitute different heaps depending upon their arrival.

Insert / Delete – a 'little bit' of Implementation

- While some implementations actually do the swap, there's a **lot of overhead** here for a large heap.

- A gain in efficiency is acquired if we substitute a '**copy**' for a '**swap**.' (like the selection sort…)

- Here, we **copy** the node to be trickled to a **temp area** and just do compares and copies (moves) until the right spot is found. Then we can **copy** (move) the node (temp) (to be trickled down) into that node location.

- More efficient than doing **swaps** repeatedly.

Sorting with a Heap: HeapSort: insert() and remove()

- Very easy to implement.
- We simply **insert()** all unordered items into the heap trickling **down** using an insert() routine….
- Then, repeatedly use the **remove**() items in sorted order….
- Both insert() and remove() operate in $O(\log_2 n)$ time and this must be applied n times, thus rendering an $O(n\log_2 n)$ time.
- **This is the same as a QuickSort**
- Not as fast because there are more operations in the trickledown than in the inner loop of a quicksort.
- Can be done both **iteratively** (using stacks) and **recursively.**
- Code is in your book.

Uses of Heaps

- Use of ***heap trees*** can be used to obtain improved running times for several network optimization algorithms.
- Can be used to assist in dynamically-allocating memory partitions.
- Lots of variants of heaps (see Google)

- A heapsort is considered to be one of the best sorting methods being in-place with no quadratic worst-case scenarios.
- Finding the min, max, both the min and max, median, or even the k-th largest element can be done in linear time using heaps.
- And more….