IIITDM KANCHEEPURAM
MAT1001 DIFFERENTIAL EQUATIONS
ASSIGNMENT 3
APRIL 23, 2024

DUE DATE: APRIL 26, 2024   MARKS: 7

1. Find a power series solution of the form $y = \sum a_n x^n$ for the differential equation $y' = \cos(x + y)$. (Assume that such a solution exists.)

2. Find the unique power series solution of the form $y = \sum a_n x^n$ for the initial value problem $y'' - (\sin x)\, y = 0, \quad y(\pi) = 1, \quad y'(\pi) = 0$.

3. Find a Frobenius series solution of the equation $xy'' + y' + xy = 0$.

4. Find the Laplace transform of $\dfrac{1 - \cos x}{x^2}$.

5. Let $f(x)$ be a function of period a. Prove that $L[f(x)] = \dfrac{1}{1 - e^{-ap}} \displaystyle\int_0^a e^{-px} f(x)\,dx$ for $p > 0$.

6. The equation $xy'' + y' + xy = 0$ has a single solution $y(x)$ for which $y(0) = 1$. Find it using Laplace transforms.

7. Let $f(x)$ be a function of period 2π such that

$$
f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ x, & 0 \le x < \pi \end{cases}.
$$

   (a) Sketch the graph of $f(x)$ on the interval $-3\pi \le x < 3\pi$.

   (b) Find the Fourier series representation of $f(x)$ on the interval $-\pi \le x < \pi$.

   (c) By giving appropriate values of x, show that

$$
\begin{aligned}
\frac{\pi}{4} &= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots. \\
\frac{\pi^2}{8} &= 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots.
\end{aligned}
$$

   Also conclude that $1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \cdots = \dfrac{\pi^2}{6}$.