NT ①    12/08/24

**Q1** Basic diff betn network & ckt.

ckt - loop or mesh - closed path

interconnections of ckt is n/w.

Building - Network

rooms - ckt

→ Basics : Passive, lumped R, L, C & Ohm's Law.

→ In presence of active sources (V & I), → Passive, lumped (R, L, C) will always absorb energy ⇒ current flows from +ve to -ve terminals

→ In the absence of V & I ⇒ the stored energy in L & C will be delivered to the memoryless Resistor ⇒ current flows from -ve to +ve terminals

**Q12** Ohm's Law basic

electromagnetic theory : $\vec{J} = \sigma \vec{E}$ → field developed

$\vec{J}$ → current density, $\sigma$ → conductivity

$$\vec{E} = \frac{v}{l}$$ $l$ = length of the conductor

$$\vec{J} = \frac{i}{S} \quad (A/m^2)$$

$$J = \sigma E \Rightarrow \frac{i}{S} = \sigma \frac{v}{l} \Rightarrow V = \left(\frac{l}{\sigma S}\right) \cdot i \Rightarrow V = R . i$$

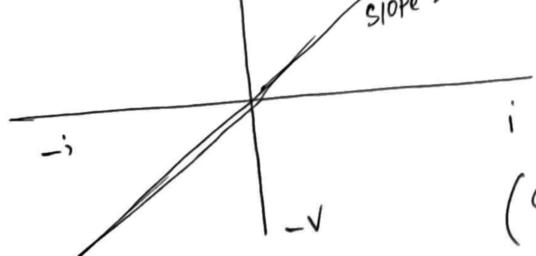
$$R = \left(\frac{l}{\sigma S}\right) = ohm = \Omega$$

→ Limitation : linear V & I relation

'R' constant i.e. temp. const.

$T\uparrow \Rightarrow l\uparrow, S\uparrow, \frac{l}{S} \approx$ almost const., $\sigma\downarrow$, so $R\uparrow$

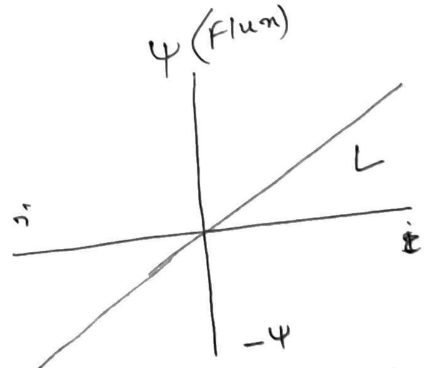
**Q)** unit of $\sigma$ & $\rho$ ($\Omega - m$)



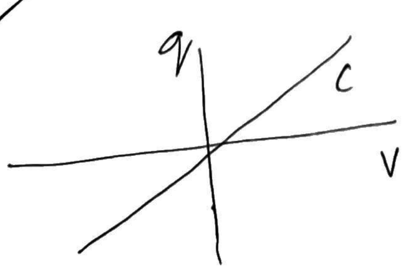

$v = Ri$

[Linear, Passive Bilateral, time invariant in V - I plane]

(Constant temp.)




$\psi = N\phi$ → flux per turn; $N$ → No. of turns

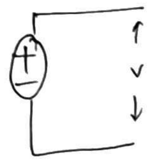


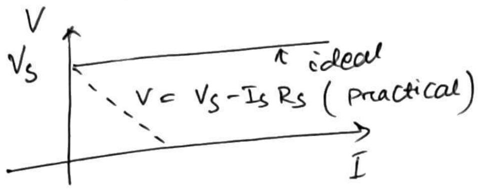
$v = L\frac{di}{dt}$ $\quad i = C\frac{dv}{dt}$ $\quad E = \frac{1}{2}CV^2$ (J)

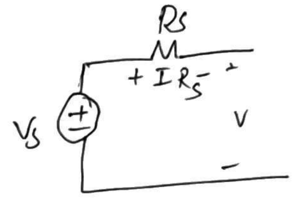
$E = \frac{1}{2}Li^2$ (J)

inductor will store the energy in magnetic field in the form of current and the capacitor will store the energy in electric field in the form of voltage

Q|| what is an ideal voltage source & practical





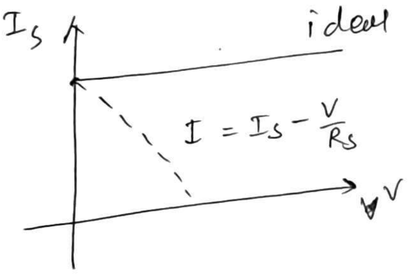



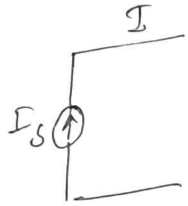

Load voltage is independent of load current

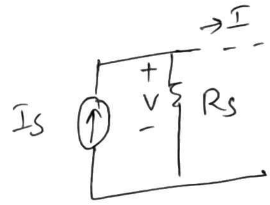
[Load voltage is a funcn of load current]

All sources : Active, unilateral.

Q|| ideal current source & practical



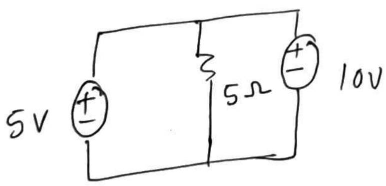




→ KCL + Ohm's Law = Nodal

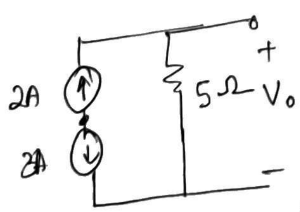
KVL + Ohm's Law = mesh.

→ Q|| Current through 5Ω resistor.




(X)

Q||



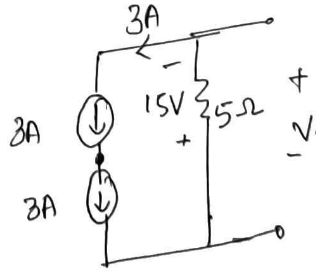

X

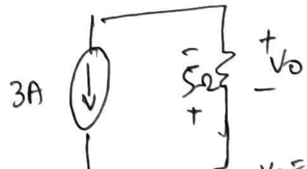
KCL

$2+3=0$

$\Rightarrow 5 = 0$ (X)

Q||




$V_o = -15V$

KCL

$-3+3=0$

→ Superposition is not applicable if violation of KCL

Total '8' forms of ohm's Law :

① $\vec{J} = \sigma \vec{E}$ ;

③ $I = GV$ conductance

② $V = RI$ ; $R = \frac{1}{\sigma}\frac{l}{A}$

④ $V = R\frac{dQ}{dt}$

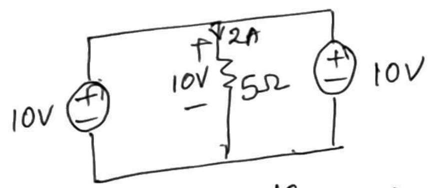
⑤ $V = L\frac{di}{dt}$

⑥ $i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$

⑦ $i = C\frac{dv}{dt}$

⑧ $v = \frac{1}{C}\int_{-\infty}^{t} i\,dt$

Q

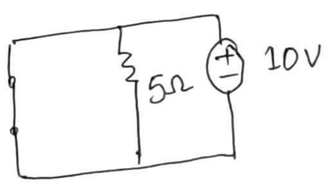



By KVL ⇒ $10 - 10 = 0$

⇒ $0 = 0$

$I_{5\Omega} = \frac{10}{5} = 2A$ / By ohm's Law

Superposition :

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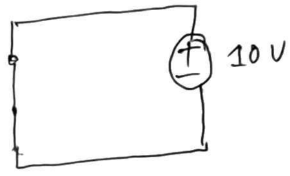



$0 \| 5\Omega = 0\,\Omega$

KVL ⇒ $0 - 10V = 0$

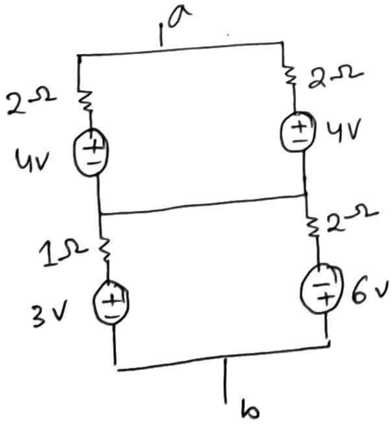
⇒ $10 = 0$ (×)

violation KVL

Note : ohm's Law is applicable to any ckt i.e. may be Lumped or distributed.

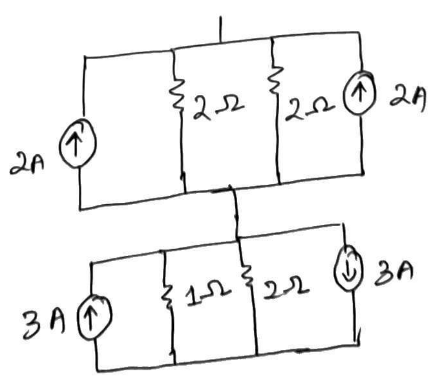
but K' Law is only for Lumped n/w.

→①

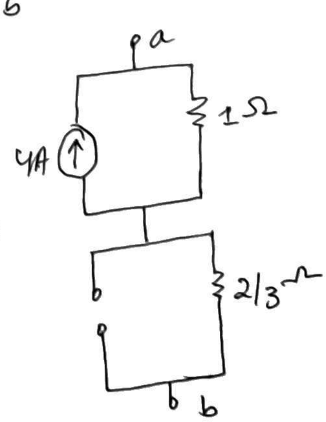



Simplify the n/w betn 'a' & 'b'

⇒




⇒

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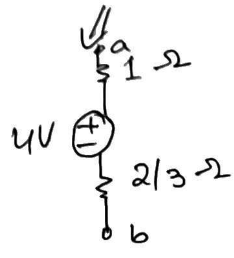
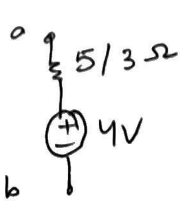
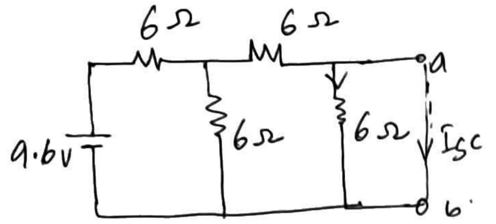



⇐




$L = \frac{\mu_0 N^2 A}{l}$

(2)

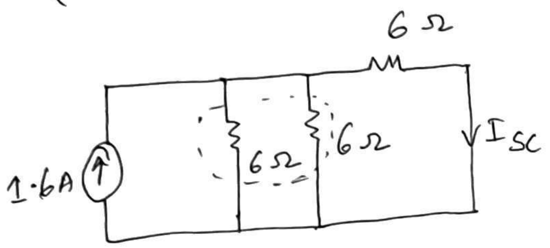



Determine the current through the ideal ammeter connected across 'a & b'

Note :

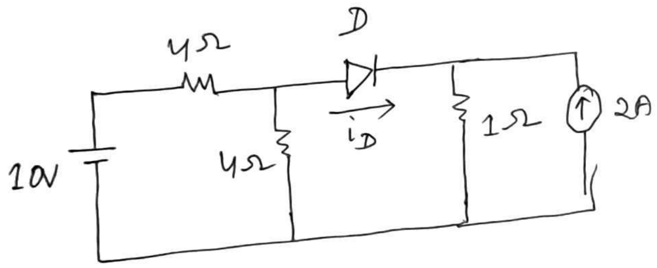

(A Practical Ammeter)

* For an ideal ammeter $R_i = 0$




$$I_{sc} = \frac{1.6 \times 3}{6+3} = \frac{1.6}{3} A$$

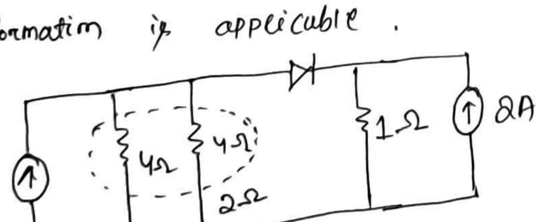
(3)




The current through the ideal diode 'D' is ?

Note : Since the diode is a Non-linear element, the n/w is non-linear and hence super position theorem is not applicable, so here the Source transformation is applicable.

$\frac{5}{2} = \frac{10}{4}$

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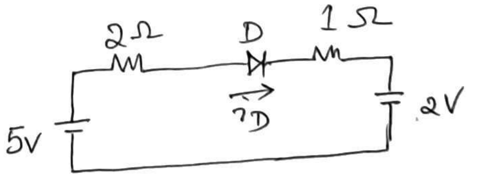
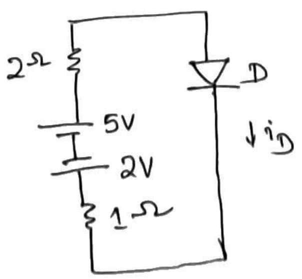



⇓

rotate and rearrange ;

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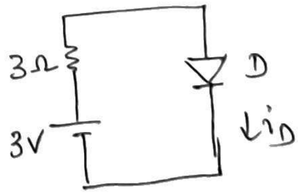


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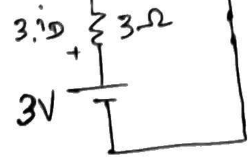



ideal case

By KVL ⇒ $3 - 3.i_D = 0$

⇒ $i_D = 1 A$.

## The Tellegen's Theorem

Defn : In an arbitrary n/w, the algebraic sum of powers at any given instant is zero.

i.e. Power delivered by some elements is equal to the power absorbed by remaining elements present in the n/w.

→ When the current enters at the -ve terminal of an element then that element will deliver the power, otherwise it will absorbs the power.

→ The sources can deliver the power or it can absorb the power, whereas the passive elements will always absorb the power, since the currents will enter at the +ve terminal in the respective R, L, C.

Properties -

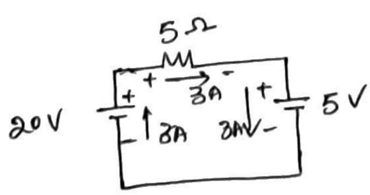
→ This theorem depends on the voltage and current product in an element, but not on the type of the element (linear, non-linear, active, passive, etc.) i.e. independent of the nature of the element like KCL & KVL)

→ The theorem expresses conservation of power (energy) in every lumped electric ckt.

{ KVL = Conservation of energy
{ KCL = Conservation of charge.

Note: While verifying the Tellegens theorem, do not disturb the original n/w given for evaluating the voltages & currents in each and every element of the n/w.

Q// verify

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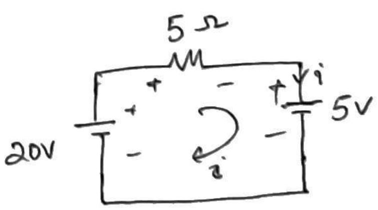



$P_{20V} = 20 \times 3 = 60W$ (del)

$P_{15V} = 15 \times 3 = 45W$ (abs) $= P_{5\Omega}$

$P_{5V} = 5 \times 3 = 15W$ (abs)

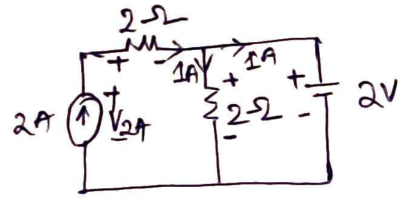
By KVL

$\Rightarrow 20 - 5i - 5 = 0$

$\Rightarrow i = \frac{20-5}{5} = 3A$

So, $P_{del} = 60W = P_{abs}$ i.e. conservation of power.

Q//




By KVL

$V_{2A} - 4 - 2 = 0$

$\Rightarrow V_{2A} = 6V$

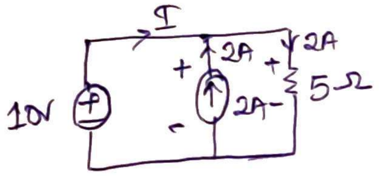
$P_{2A} = 2 \times 6 = 12$ (del)

$P_{2\Omega} = 2 \times 4 = 8W$ (abs)

$P_{2\Omega} = 2 \times 1 = 2W$ (abs)

$P_{2V} = 2 \times 1 = 2W$ (abs)

Q//




By KCL $\Rightarrow -I - 2 + 2 = 0$

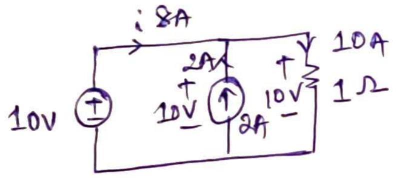
$\Rightarrow I = 0$

$P_{10V} = 10 \times 0 = 0W$

$P_{2A} = 10 \times 2 = \cancel{20} 20W$ (del)

$P_{5\Omega} = 10 \times 2 = 20W$ (abs)

Q//




By KCL $\Rightarrow -I - 2 + 10 = 0$

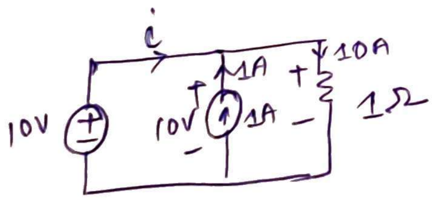
$\Rightarrow I = 8A$

$P_{10V} = 10 \times 8 = 80W$ (del)

$P_{2A} = 10 \times 2 = 20W$ (del)

$P_{1\Omega} = 10 \times 10 = 100W$ (abs)

Q//




By KCL $\Rightarrow -I - 1 + 10 = 0$

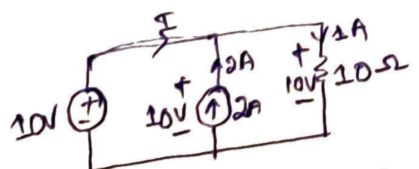
$\Rightarrow I = 9$

$P_{10V} = 10 \times 9 = 90W$ (del)

$P_{1A} = 10 \times 1 = 10W$ (del)

$P_{1\Omega} = 10 \times 10 = 100W$ (abs)

Q//




By KCL $\Rightarrow -I - 2 + 1 = 0$

$\Rightarrow I = -1$

$P_{10V} = 10 \times 1 = 10W$ (abs)

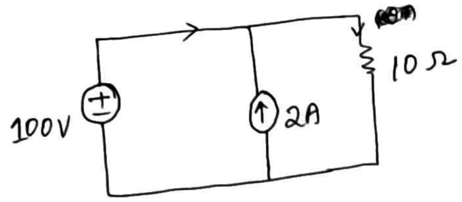
$P_{2A} = 10 \times 2 = 20W$ (del)

$P_{10\Omega} = 10 \times 1 = 10W$ (abs)

→ From the above 4 problems, the current through an ideal voltage source can be any value i.e. unknown, it is decided by the other elements (magnitudes) present in the n/w.

Some more example on Tellegen's thm :

Q)




KCL

$-I - 2A + 10A = 0$

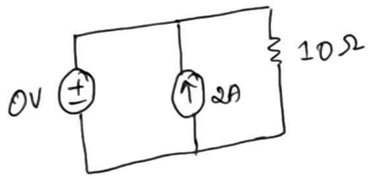
$\Rightarrow -I + 8 = 0 \Rightarrow I = 8A$

$P_{100V} = 100 \times 8 = 800\ W$ (del)

$P_{2A} = 100 \times 2 = 200\ W$ (del)

$P_{10\Omega} = 100 \times 10 = 1000\ W$ (abs)

Q)

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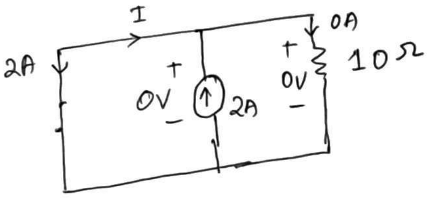



By KCL $\Rightarrow -I - 2 + 0 = 0 \Rightarrow I = -2A$

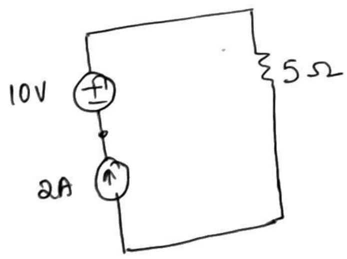
$P_{2A} = 0 \times 2 = 0\ W$

$P_{10\Omega} = 0 \times 0 = 0\ W$

$P_{del} = P_{abs} = 0\ W$

Note : voltage across an ideal current source can be any value i.e. unknown, it is decided by the other elements (magnitude) present in the n/w.

Q)



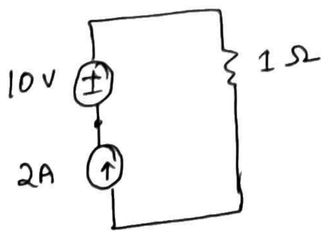

By KVL $\Rightarrow V_{2A} + 10 - 10 = 0$

$V_{2A} = 0V$

$P_{2A} = 0 \times 2 = 0W$ ; $P_{10V} = 10 \times 2 = 20W$ (del)

$P_{5\Omega} = 10 \times 2 = 20W$ (abs)

Q)




$V_{2A} + 10 - 2 = 0$

$\Rightarrow V_{2A} = -8V$

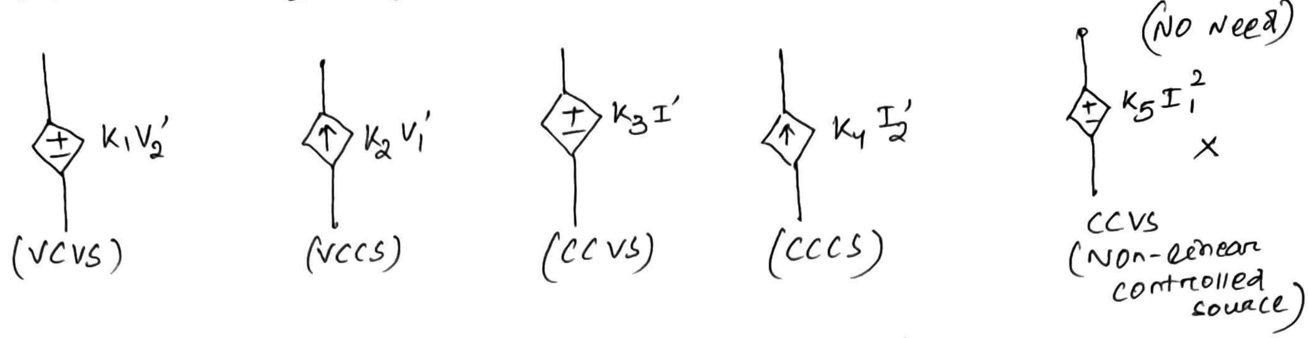
$P_{2A} = 8 \times 2 = 16W$ (abs)

$P_{10V} = 10 \times 2 = 20W$ (del)

$P_{1\Omega} = 2 \times 2 = 4W$ (abs)

→ The dependent or controlled sources

(No Need)




Linear controlled sources w.r.t the controlled variable, not w.r.t voltage and current relation.

⇒ Here $K_1, K_2 --- K_5$ are constants and $V_1, V_2, I_1, I_2$ are the controlled variables.

* W.r.t the controlled variable only the dependent sources said to be linear, active and bilateral. The presence of these elements make the n/w a linear, active and bilateral

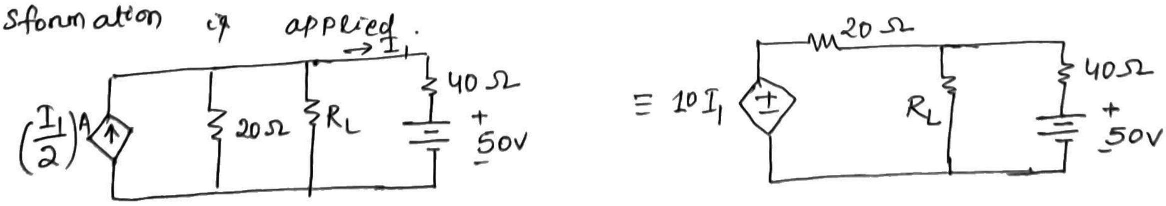
* The controlled sources are said to be the sources i.e. active elements only in the presence of atleast one independent source, then only the controlled variable are non-zero and hence their magnitudes are non-zero.

Source Transformation →

→ It is a simplification technique, which eleminates the extra nodes present in the n/w and it is applicable to only for the practical sources.

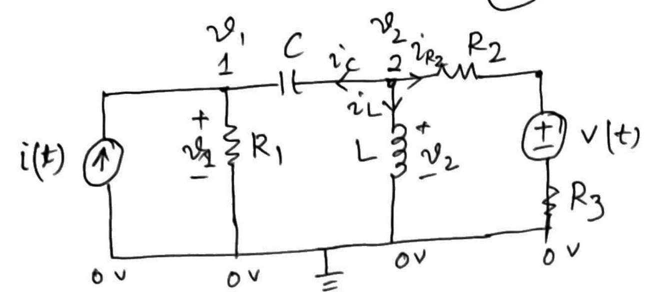
→ It is impossible to convert an ideal voltage source in to its equivalent current source and vice versa, since the violation of KCL & KVL

→ The source transformation is applicable to even for the dependent sources, provided the controlled variable is outside the branches, where the source transformation is applied.




Both will give same Thevenin's and Norton's equivalent.

## NODAL




→ identify the number of nodes

→ Assign the node voltages with ref to the ground node

→ KCL + Ohm's Law.

**Node (2)**

$v_2 > v_1$

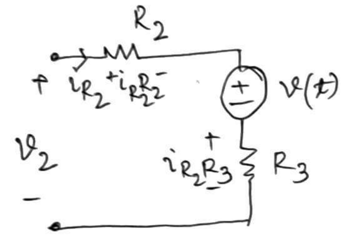
$v_2 > 0$

$v_2 > v(t)$

$$i_C + i_L + i_{R_2} = 0$$

$$\Rightarrow C\frac{d}{dt}(v_2 - v_1) + \frac{1}{L}\int_{-\infty}^{t} v_2\,dt + \frac{v_2 - v(t)}{R_2 + R_3} = 0$$

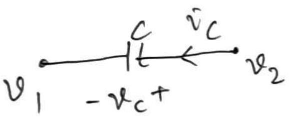
→




$$v_2 - i_{R_2}R_2 - v(t) - i_{R_2}R_3 = 0$$

$$\Rightarrow i_{R_2} = \frac{v_2 - v(t)}{R_2 + R_3} \quad A$$

→




$$i_C = C\frac{dv_C}{dt} \qquad \text{Ohm's Law}$$

$$v_1 + v_C - v_2 = 0 \quad \Rightarrow v_C = v_2 - v_1$$

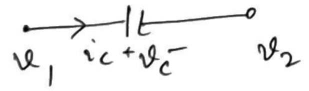
$$i_C = C\frac{d}{dt}(v_2 - v_1)$$

**Node (1)**

$v_1 > v_2$

$v_1 > 0$

$$-i(t) + \frac{v_1}{R_1} + C\frac{d}{dt}(v_1 - v_2) = 0$$

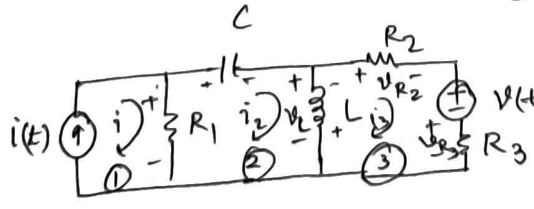



$$i_C = C\frac{dv_C}{dt}$$

$$v_1 - v_C - v_2 = 0 \Rightarrow v_C = v_1 - v_2 \Rightarrow$$

$$i_C = C\frac{d}{dt}(v_1 - v_2).$$

## MESH




* Equivalent ckt
* Star - delta (Y - Δ)

→ identify the no. of meshes

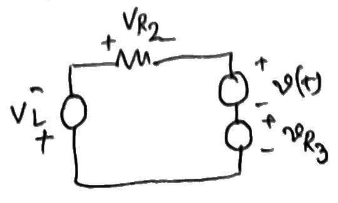
→ Assign the mesh currents in the clockwise direction

→ KVL + Ohm's Law

**Mesh (3)**

$i_3 > i_2$

$i_3 > i_1$



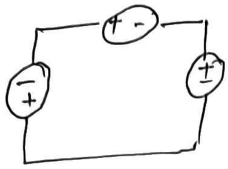
$v_L = L\frac{d}{dt}(i_3 - i_2)$ , $V_{R_2} = R_2 \cdot i_3$ , $V_{R_3} = R_3 \cdot i_3$

$-v_L - v_{R_2} - v(t) - V_{R_3} = 0$

$-L\frac{d}{dt}(i_3 - i_2) - R_2 i_3 - v(t) - R_3 \cdot i_3 = 0$

Mesh ②

$i_2 > i_1$
$i_2 > i_3$



$v_{R_1} = R_1(i_2 - i_1)$

$v_C = \frac{1}{C}\int_{-\infty}^{t} i_2 dt$

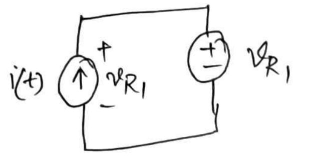
$v_L = L\frac{d}{dt}(i_2 - i_3)$

$-v_{R_1} - v_C - v_L = 0$

$-R(i_2 - i_1) - \frac{1}{C}\int_{-\infty}^{t} i_2 \cdot dt - L\frac{d}{dt}(i_2 - i_3) = 0$

Mesh ③

$i_1 > i_2$
$i_1 > i_3$




$v_{R_1} = R_1(i_1 - i_2)$

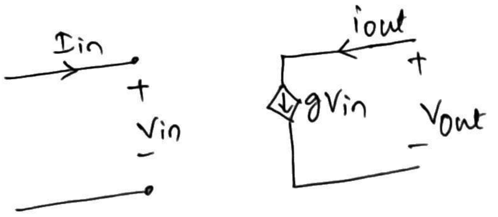
$v_{R_1} - v_{R_1} = 0$

$i_1 = i(t)$

The equivalent ckt :-

## (6) Another way Dependent Sources

(1)

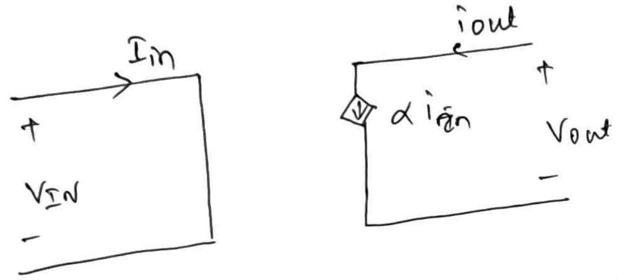



VCCS

'g' is the gain of the VCCS "transconductance" unit "A/V"

$I_{in} = 0$.

(2)



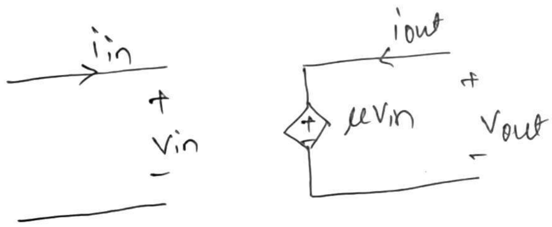

CCCS

$i_{out} = \alpha i_{in}$ α : current transfer ratio

unit A/A

$V_{in} = 0$

(3)



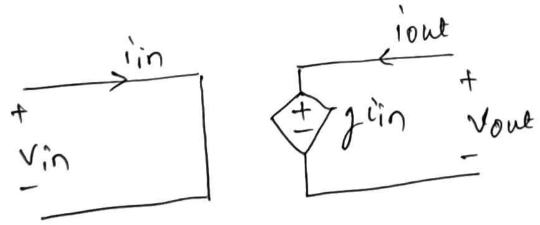

VCVS

$V_{out} = \mu V_{in}$ 'μ' is the gain with unit V/V

voltage transfer ratio

$i_{in} = 0$

(4)




CCVS

$V_{out} = \gamma i_{in}$ 'γ' = transresistance unit of resistance

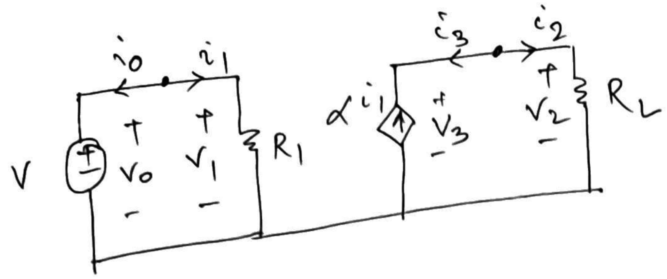
Note: The presence of the dependent sources does not alter the manner in which our approach to ckt analysis is applied.

first analyze the i/p side then O/P side.

-> In an idealized dependent source, the i/p port (control port) is an open ckt if the guiding variable is a voltage.

-> i/p is short ckt if the guiding variable is a current.

Ex.




$V_0 = V = V_1$

$V_1 = i_1 R_1$

$V_2 = i_2 R_2$

$i_3 = -\alpha i_1$

KCL

$i_0 + i_1 = 0$

$i_2 + i_3 = 0$

KVL

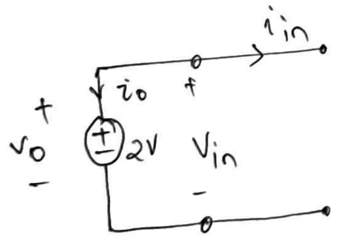
$V_0 = V_1$ $\Rightarrow -i_0 = i_1 = \frac{V}{R_1}$

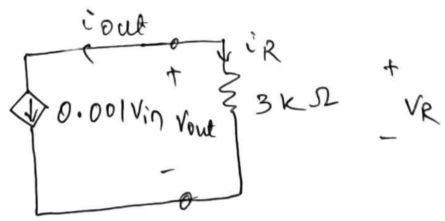
$V_2 = V_3$

$\Rightarrow -i_3 = i_2 = \frac{\alpha V}{R_1}$

$V_2 = V_3 = \frac{\alpha R_2 V}{R_1}$

eg.




Find out all the branch variables.

$0.001 V_{in} = 0.002$ A

$i_{out} = 0.002$ A

$i_R = -0.002$ A

$V_R = 3 \times 10^3 \times i_R = -6$ V

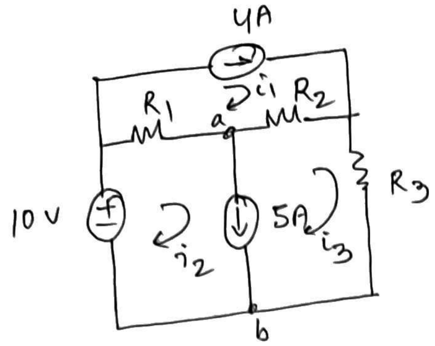
$V_{out} = V_R = -6$V

Power into the dependent current source

$V_{out} \times i_{out} = -6 \times 0.002 = -0.012$ W

Mesh

Ex




$R_1 = R_2 = 1\Omega$ and $R_3 = 2\Omega$

$i_1 = 4$

mesh (1)

$i_2 - i_3 = 5$

mesh (2) $\quad R_1 (i_2 - i_1) + V_{ab} = 10$

mesh (3) $\quad R_2 (i_3 - i_1) + R_3 i_3 = V_{ab}$

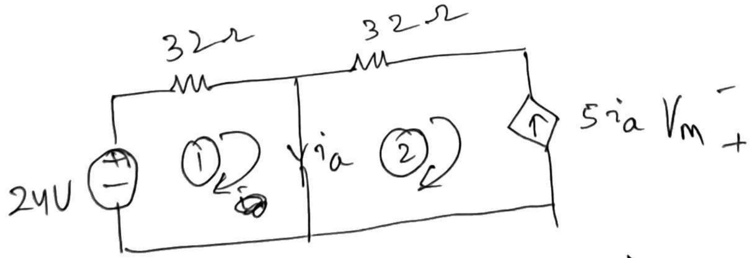
$$R_1 (i_2 - 4) + R_2 (i_3 - 4) + R_3 i_3 = 10$$

$$\Rightarrow R_1 (5 + i_3 - 4) + R_2 (i_3 - 4) + R_3 i_3 = 10$$

$$\Rightarrow i_3 = \frac{13}{4} \text{ A} \quad \& \quad i_2 = 5 + i_3 = \frac{33}{4} \text{ A}.$$

Note: When a CKt contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the mesh currents.

Ex




$i_a = i_1 - i_2 \quad \Rightarrow 5 i_a = -i_2.$

$i_2 = -5 i_a = -5(i_1 - i_2)$

$\Rightarrow -4 i_2 = -5 i_1 \quad \Rightarrow i_2 = \frac{5}{4} i_1$

$\Rightarrow 32 i_1 - 24 = 0 \quad \Rightarrow i_1 = \frac{3}{4}$ A

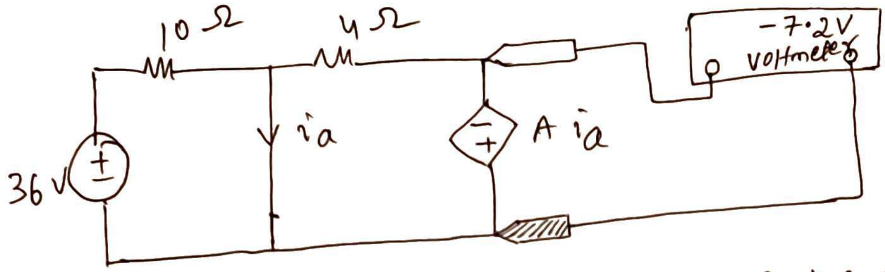
$i_2 = \frac{5}{4} i_1 = \frac{15}{16}$ A

mesh (2) $\quad 32 i_2 - V_m = 0 \quad \Rightarrow V_m = 32 i_2$

$\Rightarrow V_m = 32\left(\frac{15}{16}\right) = 30$ V

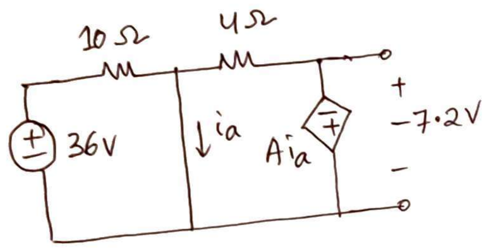
Ex (2)

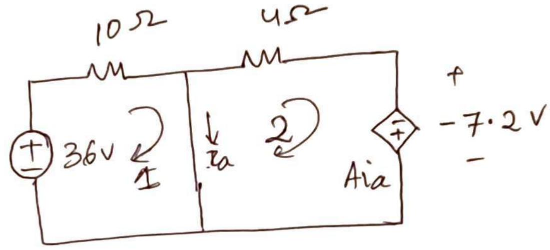
pending




Find the value of gain, A, of the CCVS.

Step (1) replace the voltmeter by an equivalent open ckt and labelling the voltage measured by the voltmeter.




$$Ai_a = -(-7.2) = 7.2V$$

Note: The controlling current of the dependent source, $i_a$, is the current in a short circuit. The short ckt current can be expressed in terms of the mesh current as

$$i_a = i_1 - i_2$$

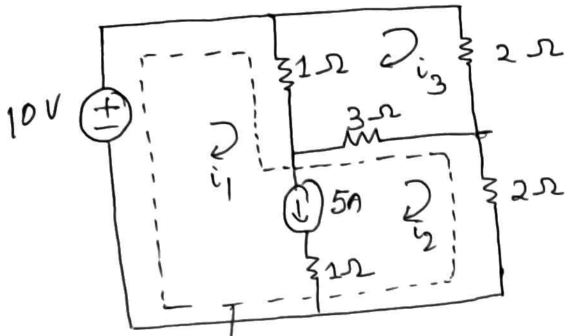
Apply KVL to mesh 1 to get $\quad 10i_1 - 36 = 0 \Rightarrow i_1 = 3.6\,A$

KVL to mesh 2 to get :-

$$4i_2 + (-7.2) = 0 \quad \Rightarrow i_2 = 1.8\,A$$

$$A = \frac{Ai_a}{i_a} = \frac{\cancel{A}\cancel{i_a}}{i_a} \quad \frac{Ai_a}{i_1 - i_2} = \frac{7.2}{3.6-1.8} = 4\ V/A$$

Super mesh :-




Supermesh.

It is impossible to write the mesh eq^n for the meshes ① and ③ independently, ⇒ supermesh.

$$i_1 - i_3 + 3i_2 - 3i_3 + 2i_2 = 10$$

$$\Rightarrow i_1 - 4i_3 + 5i_2 = 10.$$

$$\Rightarrow -10 + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0 \quad \text{(KVL for super mesh)}$$

mesh ③

$$1(i_3 - i_1) + 2i_3 + 3(i_3 - i_2) = 0$$

$$i_3 - i_1 + 2i_3 + 3i_3 - 3i_2$$

$$\Rightarrow 6i_3 - 3i_2 - i_1 = 0$$

$$\Rightarrow i_1 - i_2 = 5$$

$$\Rightarrow 1i_1 + 5i_2 - 4i_3 = 10$$

$$\Rightarrow -1i_1 - 3i_2 + 6i_3 = 0$$

Current source :- $1i_1 - 1i_2 = 5$

$i_2 = 2.5\,A$, $i_1 = 7.5\,A$ & $i_3 = 2.5\,A$

| Case | Method |
|---|---|
| ① A current source appears on the periphery of only one mesh 'n' | → Equate the mesh current on to the current source accounting for the direction of the current source. |
| ② A current source is common to two meshes | → Assume a voltage $V_{ab}$ across the terminal of the current source, write the KVL eq^n for the two meshes and add them to eliminate $V_{ab}$. Or create a supermesh as the |

Game

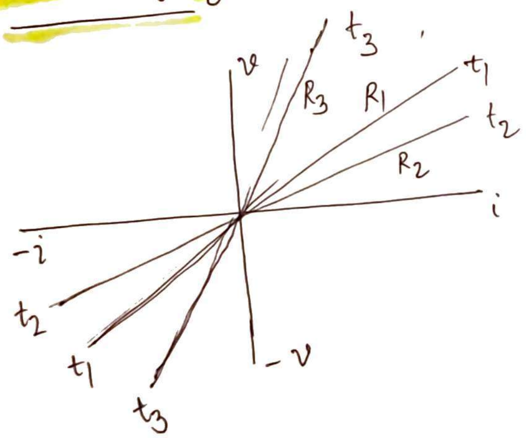
## Types of element

1. Active and passive
2. Linear and non-linear
3. Bilateral and unilateral
4. Distributed and lumped
5. Time invariant and time varying

**Linear** : * characteristic is a straight line through the origin, otherwise it is said to be non-linear.

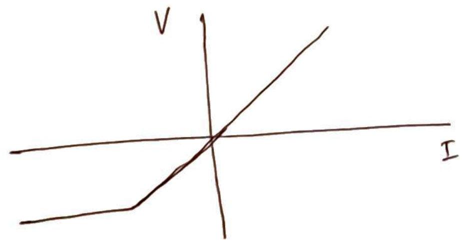
**Bilateral** : Same impedance for the different directions of the same current flow otherwise unilateral.

**Time varying** : Characteristic change with respect to time



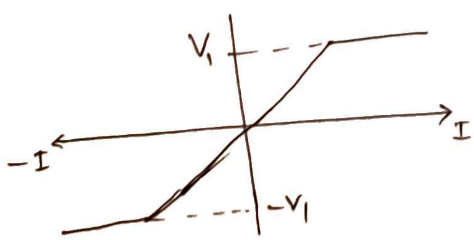

Linear, Passive & bi-lateral
time varying.

Q//



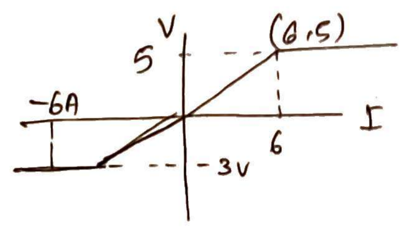

Non-linear
passive (slope is +ve)
unilateral.

Q//




→ Bilateral
→ Passive slope is +ve
→ Non-linear

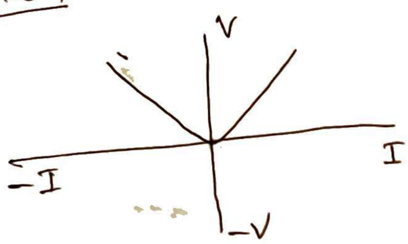
Q//




Non-linear
unilateral
passive

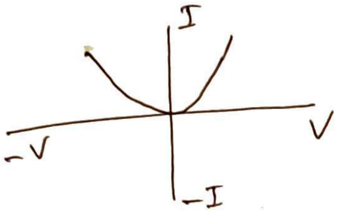
IES - 04

Q//



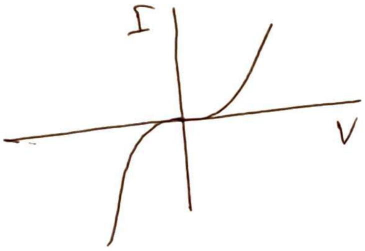

- Non-linear
- unilateral
- Active

Q/// $i = 2v^2$



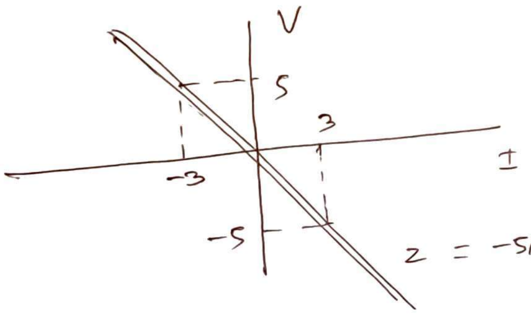

Non-linear
unilateral ($Z_1 \neq Z_2$)
Active

Q//




- Non-linear
- Passive
- Bilateral $Z_1 = Z_2$

Q//




Active
Bilateral
Linear

$Z = -5/3$

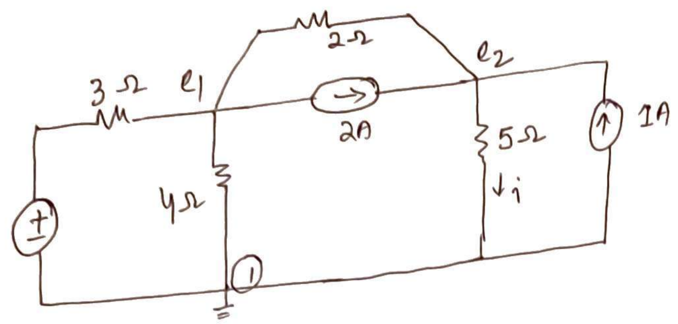
Note : all linear elements are bilateral but reverse is not true

## Node analysis

→ It is very simple, but complicated by the presence of floating independent voltage sources and by the presence of dependent sources.

→ Floating independent voltage source is a source that has neither terminal connected to ground nor through one or more other independent voltage sources.
⇓

Ex.




Determine the current 'i' through 5Ω resistor.

→ Use node method to solve the ckt

① step 1 :- Choose node 1 as gnd node

② step 2 :- Label the potentials of the remaining with respect to the ground node.

KCL

$$\frac{e_1 - 1}{3} + \frac{e_1}{4} + \frac{e_1 - e_2}{2} + 2 = 0 \quad \text{— (I)}$$

$$-2 + \frac{e_2 - e_1}{2} + \frac{e_2}{5} - 1 = 0 \quad \text{— (II)}$$

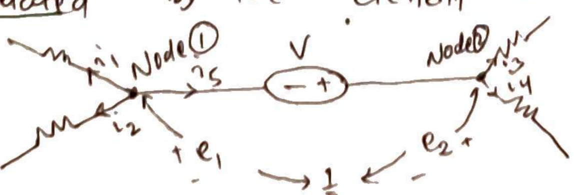
$e_1 = 0.65$ , $e_2 = 4.75$

$i = \frac{4.75}{5} = 0.95A$

Floating case

→ The reason node analysis does not work is that the element law for an independent voltage source does not relate its branch current to its branch voltage.

→ To apply node analysis to a ckt containing a floating voltage source we must realize that the node voltage at the terminals of the source are directly related by the element law for that source.



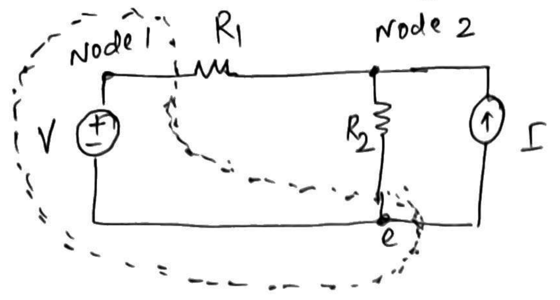

$\boxed{e_2 = V + e_1}$

$i_1 + i_2 + i_3 + i_4 = 0$ KCL for supernode

$i_3 + i_4 - i_5 = 0$

18

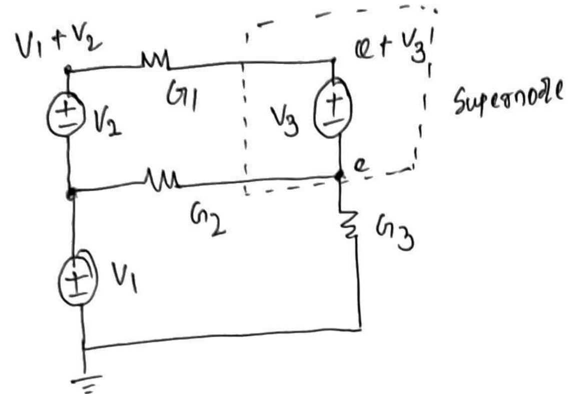
Ex ①




$$\frac{e+V}{R_1} + \frac{e}{R_2} + I = 0$$

$$e = \frac{-R_1 R_2}{R_1+R_2} I - \frac{R_2}{R_1+R_2} V$$

②




$$G_1\left[(e+V_3)-(V_1+V_2)\right] + G_2(e-V_1) + G_3 e = 0$$

$$\Rightarrow e = \frac{(G_1+G_2)V_1 + G_1 V_2 - G_1 V_3}{G_1+G_2+G_3}$$

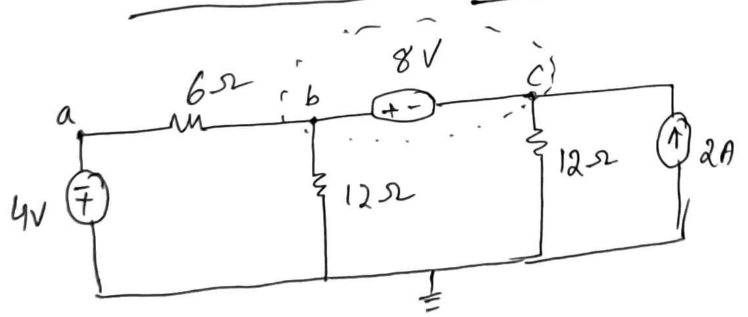
Cases ① The voltage source connects node q and the reference node.

→ Set $V_q$ equal to the source voltage accounting for the polarities and proceed to write the KCL at the remaining node.

Case ② - The voltage source lies betn two nodes 'a' and 'b'.

→ Create a supernode that incorporates 'a' and 'b' and equate the sum of all the currents into the super node to zero.

eg




$V_a = -4V$ , $V_b = V_c + 8$

KCL at supernode ⇒ $\dfrac{V_b - V_a}{6} + \dfrac{V_b}{12} + \dfrac{V_c}{12} = 2$ — ①

⇒ $3V_b + V_c = 24 + 2V_a$ ②

⇒ $V_a = -4V$ and $V_b = V_c + 8$ — ③

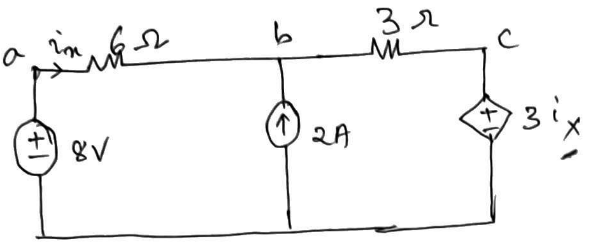
$3(V_c + 8) + V_c = 24 + 2(-4)$ ⇒ $V_c = -2V$

$V_b = V_c + 8 = -2 + 8 = 6V$

Node voltage with dependent sources :

→ When a ckt contains a dependent source, the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.

⇒ Eg.




VCCS

The controlling current '$i_x$'

$$i_x = \frac{V_a - V_b}{6}$$

$$V_a = 8V$$

$$i_x = \frac{8 - V_b}{6}$$

at 'c'

$$V_c = 3i_x = 3\left(\frac{8-V_b}{6}\right) = 4 - \frac{V_b}{2}$$

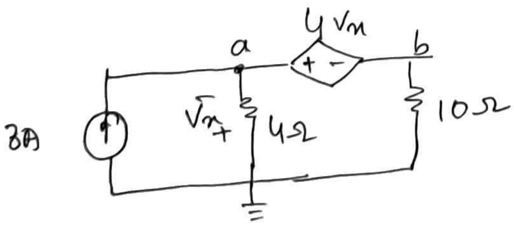
KCL at Node 'b'

$$\frac{8-V_b}{6} + 2 = \frac{V_b - V_c}{3}$$

$$\Rightarrow \frac{8-V_b}{6} + 2 = \frac{V_b - \left(4 - \frac{V_b}{2}\right)}{3} = \frac{V_b}{2} - \frac{4}{3}$$

$$V_b = 7V \quad ; \quad V_c = 4 - \frac{V_b}{2} = \frac{1}{2} V$$

Eg 2 VCVS




$$V_x = -V_a$$

$$V_a - V_b = 4V_x = 4(-V_a) = -4V_a$$

$$\boxed{V_b = 5V_a}$$

KCL at Supernode

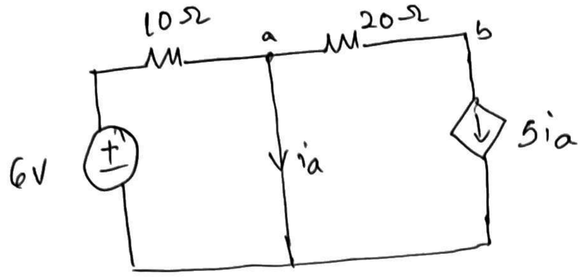
$$3 = \frac{V_a}{4} + \frac{V_b}{10} \Rightarrow 3 = \frac{V_a}{4} + \frac{5V_a}{10} \Rightarrow \frac{3}{4} V_a$$

$$\Rightarrow V_a = 4V. \qquad \Rightarrow V_b = 5V_a = 20V$$

(11)

(3)

<u>CCCS Eg.</u>




KCL at 'a'

$$\frac{6 - V_a}{10} = i_a + \frac{V_a - V_b}{20} \quad -(1)$$

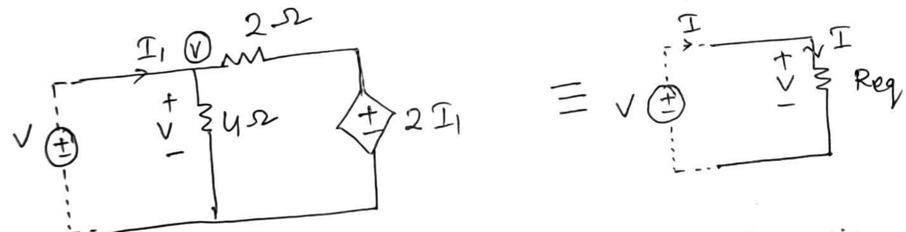
$$V_a = 0 \quad (2) \qquad i_a = \frac{12 + V_b}{20}$$

KCL at 'b'

$$\frac{0 - V_b}{20} = 5 i_a \Rightarrow \frac{0 - V_b}{20} = 5\left(\frac{12 + V_b}{20}\right)$$

$$\Rightarrow \boxed{V_b = -10V}$$

Q11




The ckt shown above will act as a load resistor of

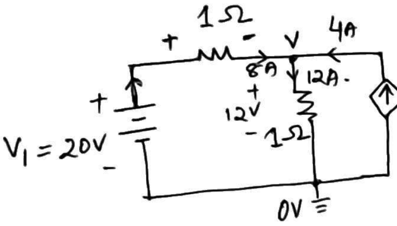
(a) $\frac{1}{3}\Omega$ (b) $\frac{2}{3}\Omega$ (c) $\frac{4}{3}\Omega$ (d) $\frac{8}{3}\Omega$

<u>Note:</u> Since no independent source in the N/W, the n/w is said to be unenergised, so called a dead n/w. The behavior of this n/w is a load resistor behavior.

$$-I_1 + \frac{V - 0}{4} + \frac{V - 2I_1}{2} = 0$$

$$\Rightarrow \frac{V}{I} = Req = \frac{8}{3}\Omega$$

Q11




$\left(\frac{V_1}{5}\right) A = \frac{20}{5} = 4A$

The dependent source shown in fig

(a) delivers 24 W

(b) absorbs 24 W

(c) delivers 48 W

(d) absorbs 48 W

$$\frac{V-20}{1} + \frac{V}{1} - 4 = 0$$

$\Rightarrow V = 12V$

$P_{\left(\frac{V_1}{5}\right)A} = 12 \times 4 = 48W$ (del)

<u>verify VTT</u>

$P_{20V} = 20 \times 8 = 160\,W$ (del)

$P_{1\Omega} = 8 \times 8 = 64\,W$ (abs)

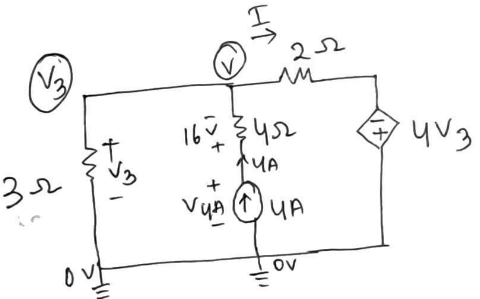
$P_{1\Omega} = 12 \times 12 = 144\,W$ (abs)

$P_{\left(\frac{V_1}{5}\right)} = 48W$ (del)

$P_{total}^{(abs)} = 64 + 144 = 208\,W$

$P_{total}$ (del) $= 160 + 48 = 208\,W$

Q11 Det I




$V_3 = V$

$$\frac{V_3}{3} - 4 + \frac{V_3 + 4V_3}{2} = 0$$

$\Rightarrow V_3 = \frac{24}{17}$ V

$$I = \frac{V_3 + 4V_3}{2} = \frac{60}{17} \text{ A}$$

KVL $\Rightarrow V_3 + 16 - V_{4A} = 0$

$V_{4A} = \left(16 + \frac{24}{17}\right) V = 17.41V.$

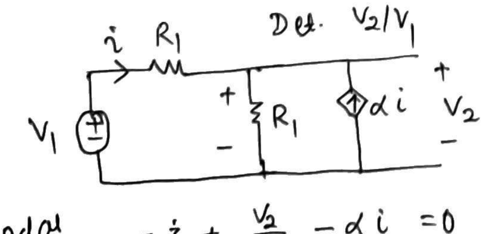
<u>H.W</u> V.T.T $P_{3\Omega} = \frac{8}{17} \times \frac{24}{17}$ (abs) $= 0.66\,w$ (abs) ; $P_{4\Omega} = 64\,w$ (abs)

$P_{2\Omega} = \frac{60}{17} \times \frac{120}{17} = 24.91\,w$ (abs) ; $P_{4V_3} = 19.93\,w$ (del)

$P_{4A} = 69.64\,w$ (del)

$P_{abs} = P_{del} = 89.57\,W$

Q11




Det. $V_2/V_1$

$i = \left(\frac{V_1 - V_2}{R_1}\right) A$

<u>Proof</u> KVL : $V_1 - R_1 i - V_2 = 0$

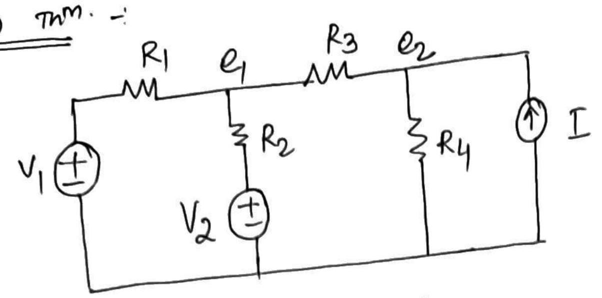
$\Rightarrow i = \left(\frac{V_1 - V_2}{R_1}\right) A$

<u>Nodal</u> $-i + \frac{V_2}{R_1} - \alpha i = 0 \Rightarrow (1+\alpha) i = \frac{V_2}{R_1} \Rightarrow (1+\alpha)\left(\frac{V_1 - V_2}{R_1}\right) = \frac{V_2}{R_1}$

$\Rightarrow (1+\alpha) V_1 - (1+\alpha) V_2 - V_2 = 0 \Rightarrow (1+\alpha) V_1 - (2+\alpha) V_2 = 0$

$\Rightarrow (1+\alpha) V_1 = (2+\alpha) V_2 \Rightarrow \boxed{\frac{V_2}{V_1} = \frac{1+\alpha}{2+\alpha}}$

## Superposition Thm. :-




A n/w with three sources.

After node analysis (in terms of conductance)

$$(V_1 - e_1) G_1 + (V_2 - e_1) G_2 + (e_2 - e_1) G_3 = 0$$

$$\Rightarrow (e_1 - e_2) G_3 - e_2 G_4 + I = 0$$

collecting the source terms on the left side

$$V_1 G_1 + V_2 G_2 = e_1 (G_1 + G_2 + G_3) - e_2 G_3$$

$$I = -e_1 G_3 + e_2 (G_3 + G_4)$$

write the eq$^n$ for $e_1$

$$e_1 = \frac{(V_1 G_1 + V_2 G_2)(G_3 + G_4) + I G_3}{(G_1 + G_2 + G_3)(G_3 + G_4) - G_3^2}$$

$$= \frac{V_1 G_1 (G_3 + G_4) + V_2 G_2 (G_3 + G_4) + I G_3}{G_1 G_3 + G_1 G_4 + G_2 G_3 + G_2 G_4 + G_3 G_4}$$

→ All denominator terms are of the same sign. Thus, the denominator can't be made zero for any non-zero value of conductance.

→ Each term on the right consists of one source term multiplied by a resistive (or conductive) factor.

→ Because of linearity, the first term remains unchanged if the other two sources are set to zero.

$$\begin{cases} \text{Additivity} \Rightarrow F(x_1 + x_2) = F(x_1) + F(x_2) \\ \text{Homogeneity} \Rightarrow F(ax) = a\,F(x) \end{cases}$$

→ Setting a voltage source to zero is equivalent to replacing that source by a short ckt.

→ Similarly, current source to zero means open ckt.

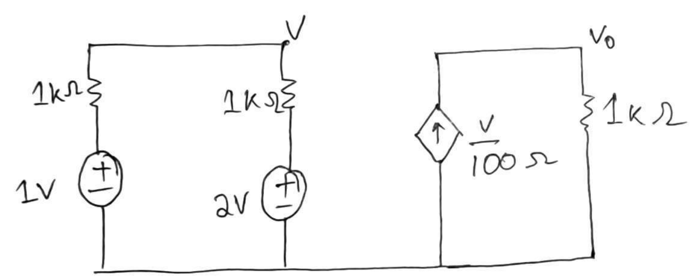
→ In a linear N/W : with a number of independent sources, the response can be found by summing the responses to each independent sources acting alone with all other independent sources set to zero.

Superposition Rules for dependent Sources :-

→ A practical way is to leave all the dependent sources in the ckt. The n/w can then be solved for one independent source at a time by setting all other independent sources to zero, and summing the individual responses.

→ If a dependent source is present, it is never deactivated and must remain active (unaltered) during the superposition process.

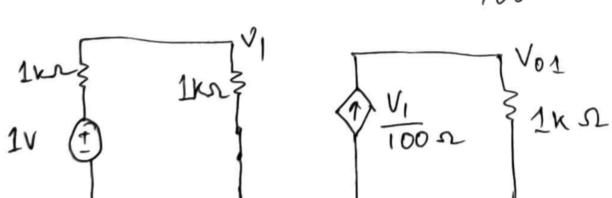
Ex



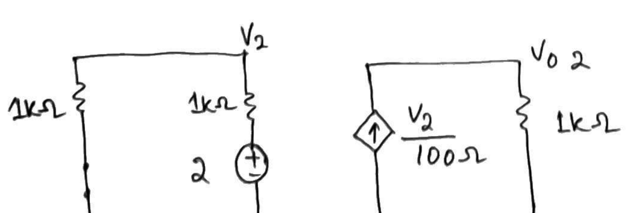

find $V_o$.

⇒ 1 V source acting alone :-

$V_1 = 0.5V$ ⇒ $V_{o1} = \frac{1}{100} V_1 \times 1k\Omega = 5V$




⇒




$V_2 = 1V$

$V_{o2} = \frac{1}{100} V_2 \times 1k\Omega$

$= 10V$

$V_o = V_{o1} + V_{o2} = \underline{15V}$

The need of thm

: In a complecated n/w in presence of several sources, nodes, and meshes if the response in a single element is designed, then the n/w thm are used.

Properties of SPT

(i) This thm is applicable only for linear n/w.

i.e n/w with R, L, C, transformer and linear controlled sources as element.

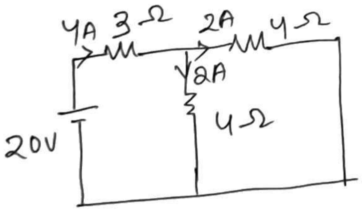
(ii) Both for active & passive.

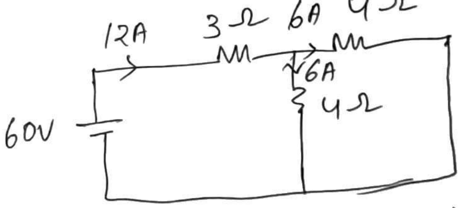
Homogenity principle :

It is a principle obeyed by all the linear n/w.

Defn :- In a linear n/w, if the excitation is multiplied with a constant 'K' then the responses in all the other branches in the n/w are also multiplied with same constant 'K'.

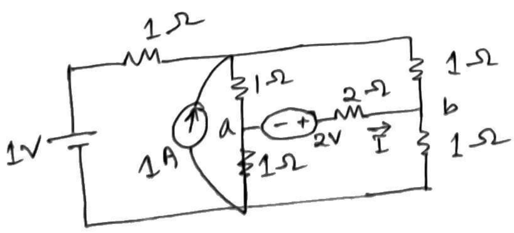
Proof



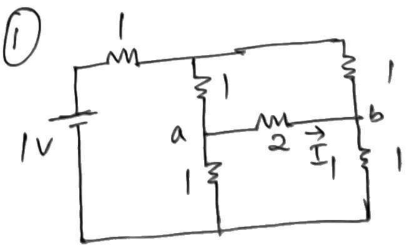




Excitation is multiplied by 3 and hence the response also.

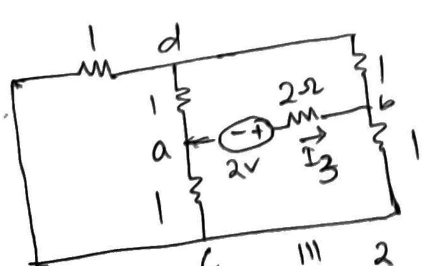
Q1 Det 'I'



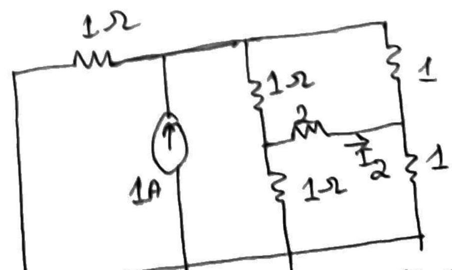

(A) (1)




$\Rightarrow I_1 = 0A$

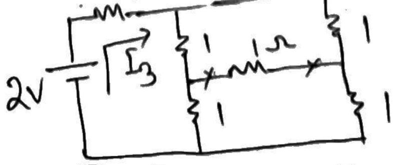



(2)




$\Rightarrow I_2 = 0A$

III

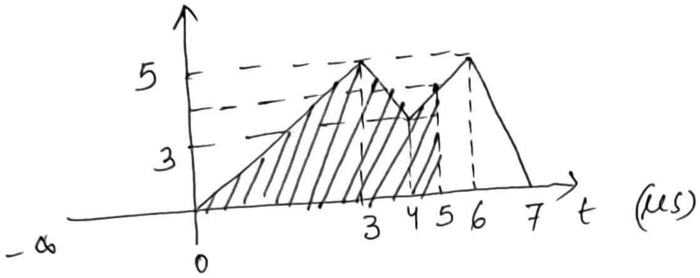



$I_3 = \frac{2}{2+(2\|2)} = \frac{2}{3}A$

Entrea

Problems on the power and Energy :-

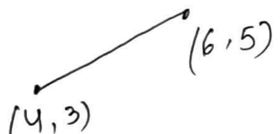
(1) Figure shows the current flowing through a capacitor. Determine the Charge acquired by the capacitor upto the first $5 \mu s$.




$$i(t) = \frac{dQ(t)}{dt} \text{ Amps}$$

$$dQ(t) = i(t)\,dt \Rightarrow Q = \int_{-\infty}^{t} i(t).dt$$

$$= \int_{0}^{5\mu s} i(t)\,dt = \text{Area under } i(t) \text{ upto } 5\mu s$$



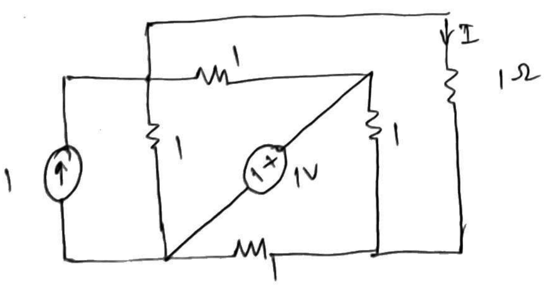

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{6-4} = 1$$

$$Q = Q_1 \Big|_{0-3\mu s} + Q_2 \Big|_{3-4\mu s} + Q_3 \Big/_{4-5\mu s}$$

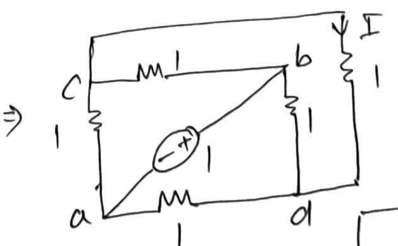
$$= \left(\frac{1}{2} \times 3 \times 5\right) + \left[\left(\frac{1}{2} \times 1 \times 2\right) + (1 \times 3)\right] + \left[\frac{1}{2} \times 1 \times 1 + 1 \times 3\right]$$

$$= 15 \mu s$$

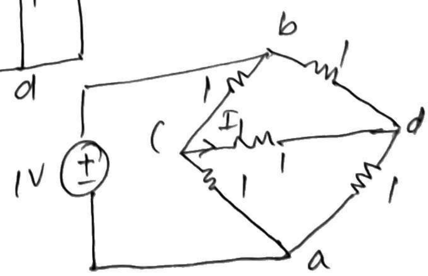
Q1



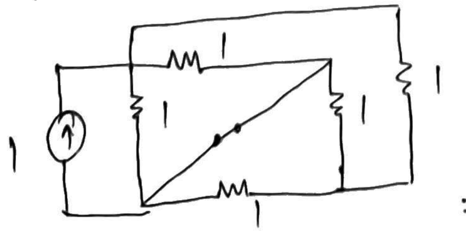

$\Rightarrow$



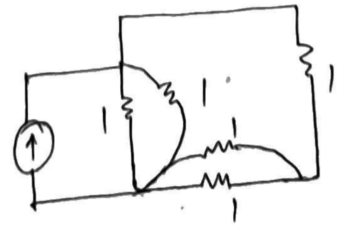

$\Rightarrow$




$I_1 = 0 A$



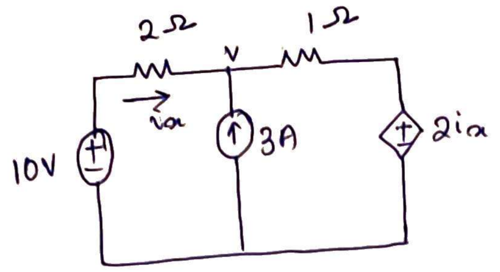

$\Rightarrow$




$$I_2 = \frac{1 \times 0.5}{0.5 + 1.5} = 0.25 A$$

$= I.$

**Q11** Det $I_x$

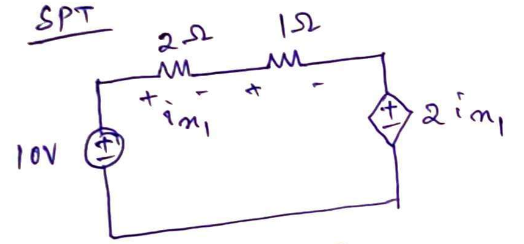



**Method ①** NDA

$$i_x = \frac{10-V}{2} \text{ A}$$

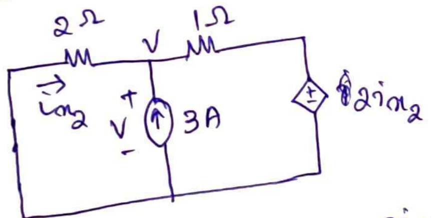
$$\frac{V-10}{2} - 3 + \frac{V-2i_x}{1} = 0 \quad \Rightarrow V = 7.2V, \quad i_x = 1.4A$$

**Method ②** SPT




$$10 - 2i_{x_1} - i_{x_1} - 2i_{x_1} = 0$$

$$\Rightarrow i_{x_1} = 2A$$




KVL

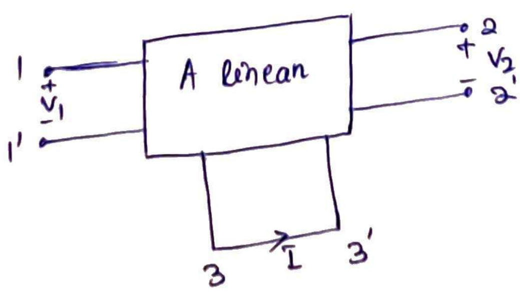
$$\Rightarrow \cancel{-2i_{x_2} - V = 0}$$

$$\cancel{\Rightarrow V = 2i_{x_2}}$$

NODAL $\quad -i_{x_2} - 3 + \frac{V - 2i_{x_2}}{1} = 0 \quad \Rightarrow i_{x_2} = -0.6A$

By SPT $\quad i_x = i_{x_1} + i_{x_2} = 2 + (-0.6) = 1.4A.$

**Q11**




Det I when $V_1 = 10V$ & $V_2 = -5V$

| $V_1$ | $V_2$ | I |
|---|---|---|
| 2 | 0 | 0.5A |
| 0 | 5V | −1A |

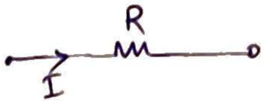
**Ans** $\quad I = K_1V_1 + K_2V_2$

$$\Rightarrow 0.5 = K_1 \cdot 2 + 0 \Rightarrow K_1 = \frac{1}{4}$$

$$\Rightarrow -1 = 0 + K_2 \cdot 5 \Rightarrow K_2 = -\frac{1}{5}$$

$$I = \frac{V_1}{4} - \frac{V_2}{5} \quad \Rightarrow \frac{10}{4} + \frac{5}{5} = 2.5 + 1 = \underline{3.5}$$

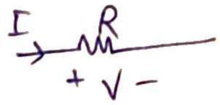
Q11 A n/w consistance several resistance, one of which is designated as 'R' and two dc sources. The power consumed by the 'R' is '$P_1$' when the +ve source is acting alone and $P_2$ when the second source is acting alone. If both the sources are acting simultaneously then the power dissipated by 'R' is.

(A) 


$I_1^2 R = P_1(\text{given}) \Rightarrow I_1 = \sqrt{\frac{P_1}{R}}$ — (1)

$I_2^2 R = P_2 \ (\text{given}) \Rightarrow I_2 = \sqrt{\frac{P_2}{R}}$ — (2)

By SPT $\quad I = I_1 + I_2 = \left(\sqrt{\frac{P_1}{R}} + \sqrt{\frac{P_2}{R}}\right) A$




When $V = IR$

By Tellegen's thm $\quad P = VI = RI \cdot I = I^2 R = \left(\sqrt{P_1} \pm \sqrt{P_2}\right)^2 W$

Q11 If $P_1 = 18W$ & $P_2 = 50W$ then $P_{max}$ & $P_{min}$ are

$P = \left(\sqrt{18} \pm \sqrt{50}\right)^2 = 2\ (3 \pm 5)^2$

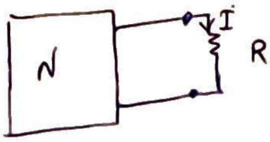
$P_{max} = 2\,(3+5)^2 = 128\,W$

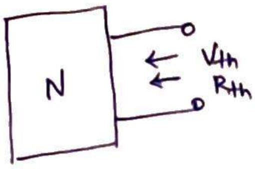
$P_{min} = 2\,(3-5)^2 = 8\,W$

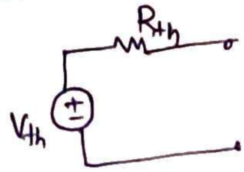
## The Thevenin's and Norton's Theorem

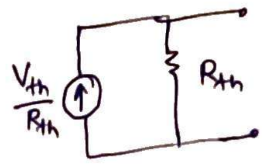
**Properties** : Same as SPT

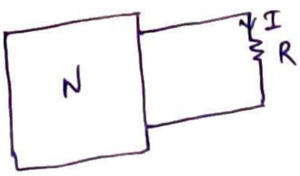
**Procedure** :

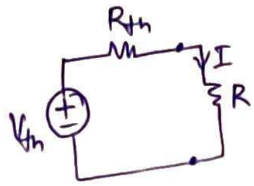


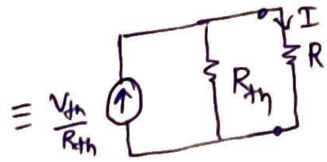

→ 


≡ 


≡ 


→ 

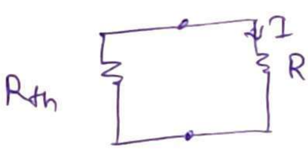

≡ 


≡ 


$$I = \frac{\frac{V_{th}}{R_{th}} \times R_{th}}{R_{th}+R} = \frac{V_{th}}{R_{th}+R}$$

**3 cases** → $V_{th}$ , $R_{th}$

1. All are independent sources
2. Atleast one independent source and atleast one dependent source
   $V_{th}$ , $I_{SC}$ ⇒ $R_{th} = \frac{V_{th}}{I_{SC}}$ ⇒ $I_{SC} = \frac{V_{th}}{R_{th}}$
3. All are dependent sources : $V_{th} = 0$ ; $I_N = 0\,A$ , but $R_{th} \neq 0$

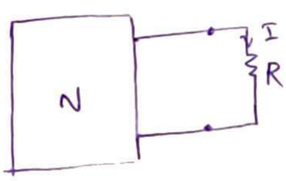


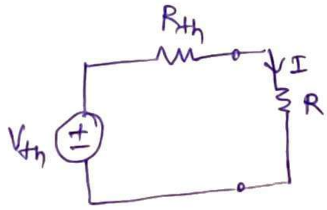

$I = 0$

Thevenin's = Norton's = A load resistor

**Case ①**

**Q**




≡ 


$I = 3A$ & $1.5A$ when $R = 0\,\Omega$ & $2\,\Omega$ , respectively

Det I when $R = 1\,\Omega$

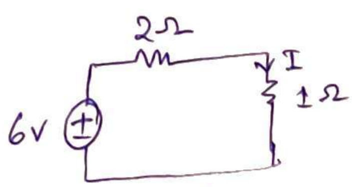
**A**

$\frac{V_{th}}{R_{th}} = 3$

$\frac{V_{th}}{R_{th}+2} = 1.5$

$V_{th} = 6\,V$

$R_{th} = 2\,\Omega$




$I = \frac{6}{3} = 2A$

Q11 An ideal voltage and current sources are connected in parallel, this combination will have

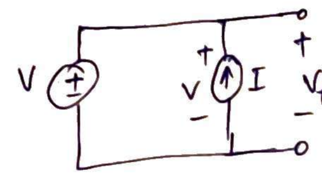
(a) Both Thevenin's and Norton's equivalent

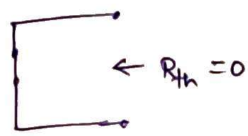
(b) Thevenin's but not Norton's

(c) Norton's but not Thevenin's

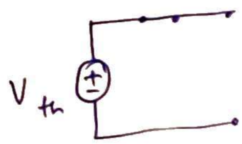
(d) Neither Norton nor Thevenin

(A)



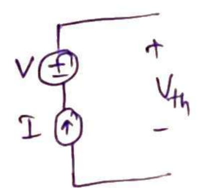

⇒




An ideal voltage source

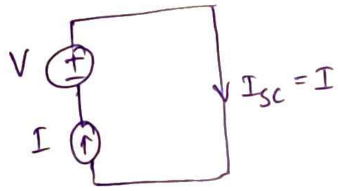
Note : It is impossible to convert an ideal voltage source into its equivalent current source and hence Norton's equivalent does not exist.

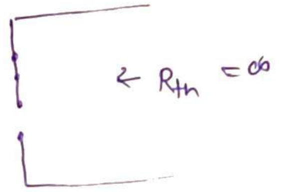
Q11 In the above case if the elements are connected in series.

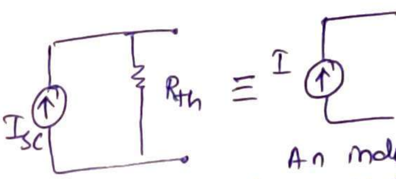



KVL ⇒ $V_{th} = V + \text{any value}$

$= \text{Any value} = \text{unknown}$

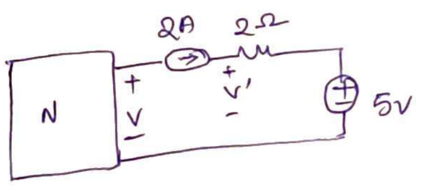



An ideal Current source

Q11



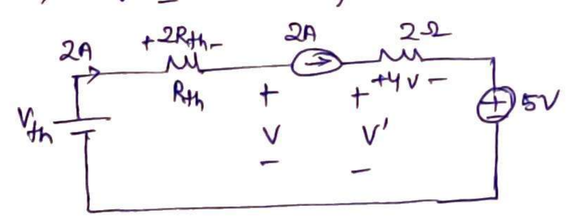

Det. 'V'

(a) 9V (b) 5V (c) 1V (d) None

KVL ⇒ $V' - 4 - 5 = 0$ ⇒ $V' = 9V$

KVL ⇒ $V = V' + \text{Any value} = 9 + \text{Any value} = \text{Any value} = \text{None}$

Thevenin's

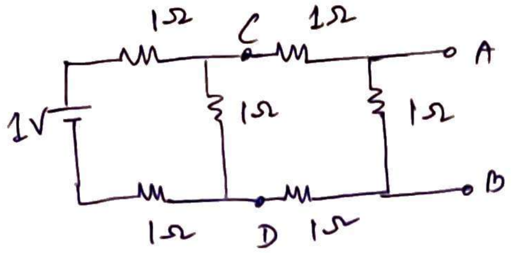



KVL

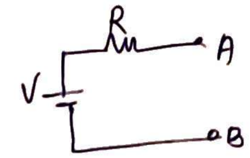
$V_{th} - 2R_{th} - V = 0$

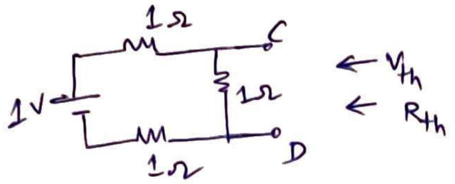
$\Rightarrow V = V_{th} - 2R_{th} = \text{un-known.}$

Q



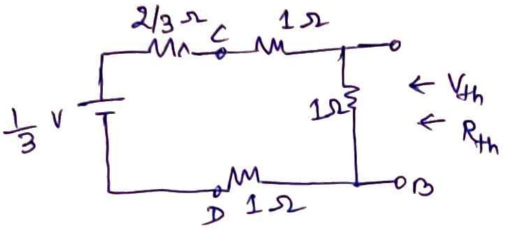
$\equiv$





$V_{th} = \frac{1}{3} \cdot 1 = \frac{1}{3}\,V$

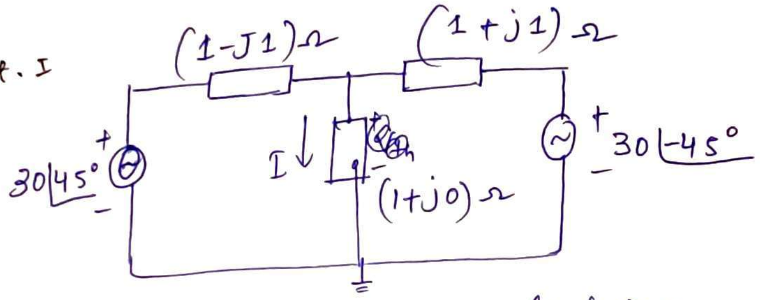
$R_{th} = 1 \parallel 2 = \frac{2}{3}\,\Omega$



$V_{th} = \left(\dfrac{\frac{1}{3}}{\frac{2}{3}+1+1+1}\right) \cdot 1 = \frac{1}{11}\,V$

$R_{th} = 1 \parallel \left(1+\frac{2}{3}+1\right) = \frac{8}{11}\,\Omega$

Q Det. I

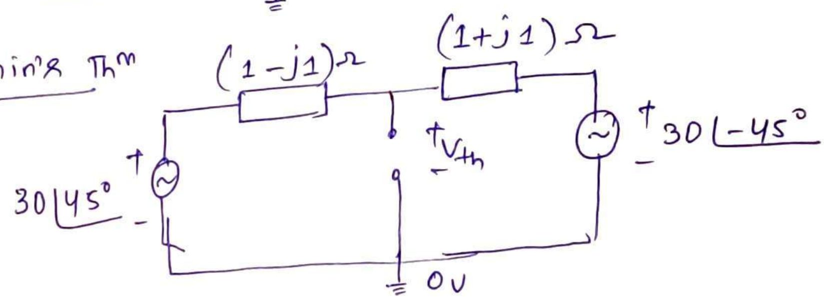


NDA

$\dfrac{V-30\angle 45^\circ}{1-j1} + \dfrac{V}{1+j1} + \dfrac{V-30\angle -45^\circ}{1+j1} = 0$

$\Rightarrow V(\quad) = 0 \Rightarrow V = 0 \Rightarrow \underline{I = 0\,A}$

By Thevenin's Thm

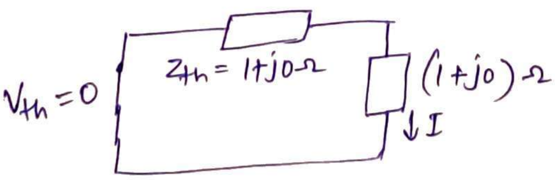


Nodal $\quad \dfrac{V_{th} - 30\angle 45^\circ}{1-j1} + \dfrac{V_{th} - 30\angle -45^\circ}{1+j1} = 0$

$\Rightarrow V_{th} = 0$

Q

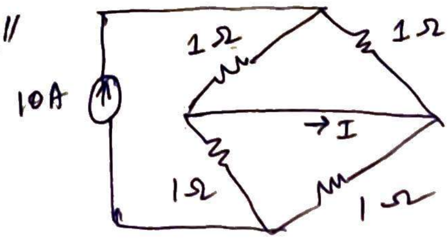
$Z_{th} = (1-j1) \parallel (1+j1) = (1+j0)\,\Omega$



$\Rightarrow I = 0\,A$

. . . are connected in parallel.

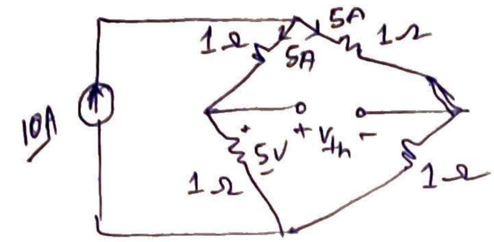
Q11




Prove $I = 0A$

$1\times 1 = 1 \times 1 \Rightarrow I = 0\,A$

Proof

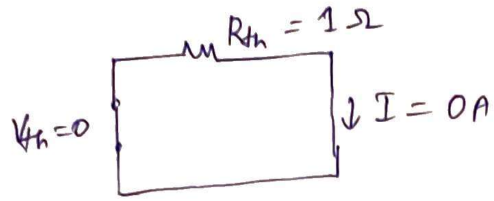



KVL

$5 - V_{th} - 5 = 0$

$\Rightarrow V_{th} = 0\,V$

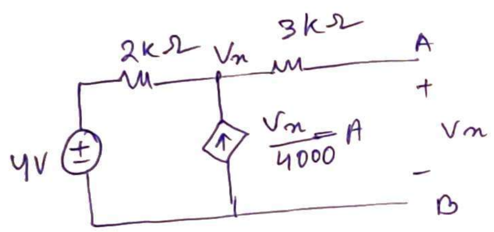
$R_{th} = 2 \| 2 = 1\,\Omega$




Obs: The open ckt voltage $V_{th}$ and the S.C. current $I_{sc}$ are independent of the load magnitude, since we are opening the load for $V_{th}$ and shorting the load for $I_{sc}$.

Case ②

Q11



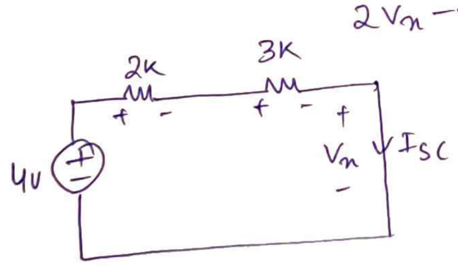

Determine the Thevenin's equivalent and Norton's equivalent across AB.

Ans $V_{th} = V_x$

Nodal $\frac{V_x - 4}{2k} = \frac{V_x}{4000} \Rightarrow \frac{V_x - 4}{\cancel{2000}} = \frac{V_x}{\cancel{4000}\,2}$

$2V_x - 8 = V_x \Rightarrow V_x = 8\,V = V_{th}$.

$I_{sc}$

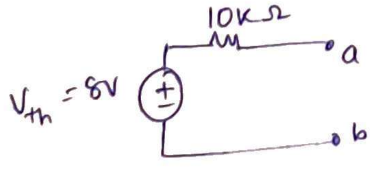



$V_x = 0$

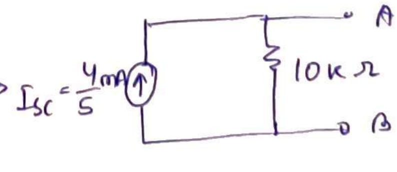
KVL $4 - 5\times 10^3 \cdot I_{sc} = 0$

$\Rightarrow I_{sc} = \frac{4}{5}\,mA$

$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{\frac{4}{5}\times 10^{-3}} = 10K\,\Omega$

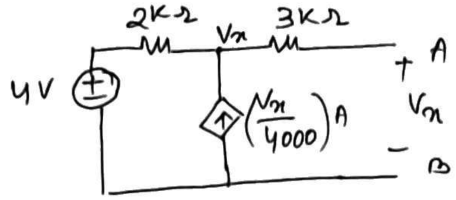



⟷



Case (2) (repeat)

Q) Determine the Thevenin's equivalent and Norton's equivalent across AB.

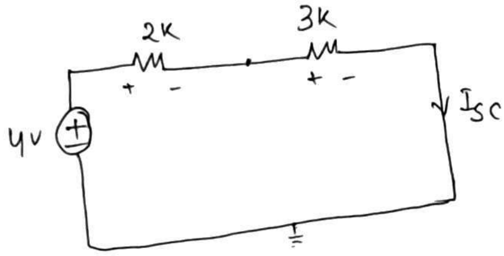



Ans $V_{th}$ : $V_{th} = V_x$

NODAL $\frac{V_x - 4}{2K} = \frac{V_x}{4000} \Rightarrow \frac{V_x - 4}{2000} = \frac{V_x}{4000}$

$\Rightarrow 2V_x - 8 = V_x \Rightarrow V_x = 8V = V_{th}$

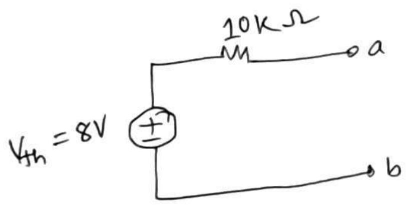
Norton



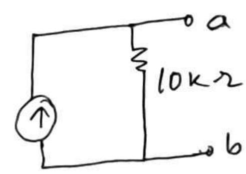

$V_x = 0$

KVL $4 - 5\times10^3 \cdot I_{sc} = 0 \Rightarrow I_{sc} = \frac{4}{5} mA$

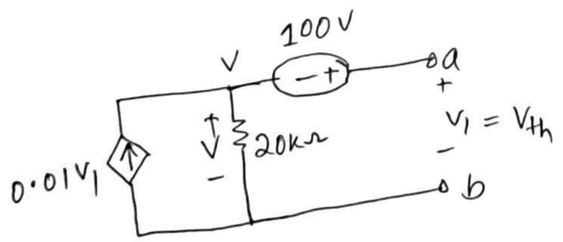
$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{\frac{4}{5}\times10^{-3}} = 10K\Omega$




$\longleftrightarrow$ $I_{sc} = \frac{4}{5} mA$




Q)




By KVL.

$\because V + 100 - V_1 = 0$

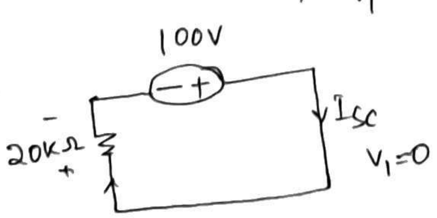
$\Rightarrow V = V_1 - 100$

$V_{th}$ : $V_1 = V_{th}$ Nodal $-0.01V_1 + \frac{V}{20K} + 0 = 0$

$\Rightarrow 200V_1 + V = 0 \Rightarrow -200V_1 + V_1 - 100 = 0$

$\Rightarrow V_1 = \frac{-100}{199} = -502.5 mV = V_{th}$

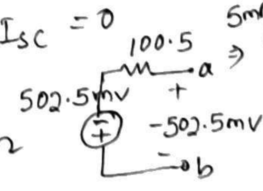
$I_{sc}$ :



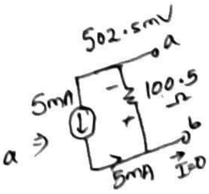

By KVL $\Rightarrow 100 - 20\times10^3 \, I_{sc} = 0$

$\Rightarrow I_{sc} = 5mA$

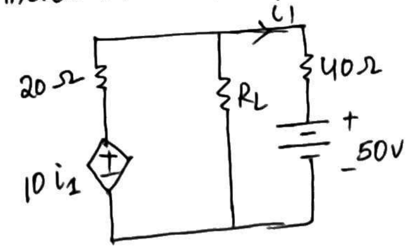
$R_{th} = \left|\frac{V_{th}}{I_{sc}}\right| = 100.5\Omega$



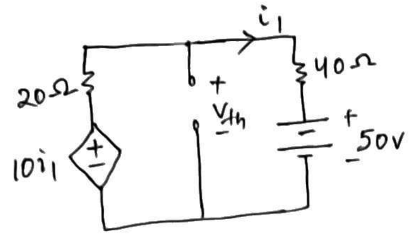

$\Rightarrow$



Q// Det Thevenin's & Norton's equivalent across $R_L$




(A)




$\Rightarrow \dfrac{V_{th} - 10 i_1}{20} + i_1 = 0$

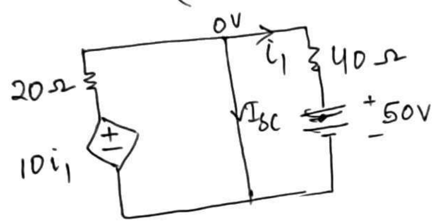
$\Rightarrow V_{th} - 10 i_1 + 20 i_1 = 0$

$\Rightarrow V_{th} + 10 i_1 = 0$

$\left(\because i_1 = \dfrac{V_{th} - 50}{40} A\right)$

$V_{th} + 10\left(\dfrac{V_{th} - 50}{40}\right) = 0 \Rightarrow V_{th} = 10V.$

$I_{sc}$ :

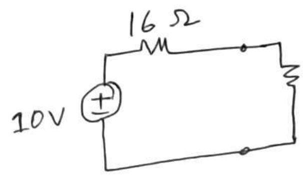



NODAL $\dfrac{0 - 10 i_1}{20} + i_{sc} + i_1 = 0$

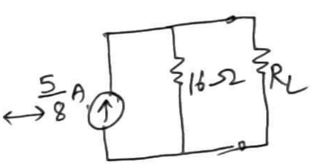
$\Rightarrow i_{sc} = -\dfrac{i_1}{2} \;\Big|\; i_1 = \dfrac{0 - 50}{40}$

$\Rightarrow i_{sc} = \dfrac{5}{8} A$

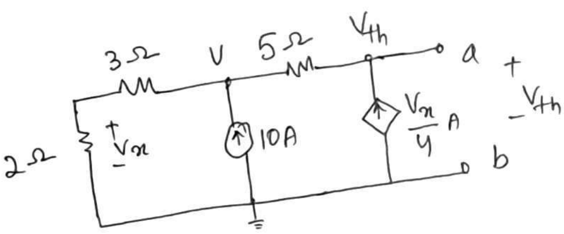
$R_{th} = \dfrac{V_{th}}{I_{sc}} = \dfrac{10}{5/8} = 16\,\Omega.$




$\longleftrightarrow$




Q//




$V_{th}$ :

$V_x = V_{2\Omega} = I_{2\Omega} \cdot 2 = \left(\dfrac{V - 0}{5}\right) 2 = \dfrac{2}{5} V$

Nodal $\dfrac{V}{5} - 10 + \dfrac{V - V_{th}}{5} = 0 \Rightarrow 2V - V_{th} = 50$ — ①

Nodal $\dfrac{V_{th} - V}{5} - \dfrac{V_x}{4} = 0 \Rightarrow \dfrac{V_{th} - V}{5} - \dfrac{2V}{20} = 0$ $\left(\because V_x = \dfrac{2}{5} V\right)$

$\Rightarrow V = \dfrac{2}{3} V_{th}$ — ②

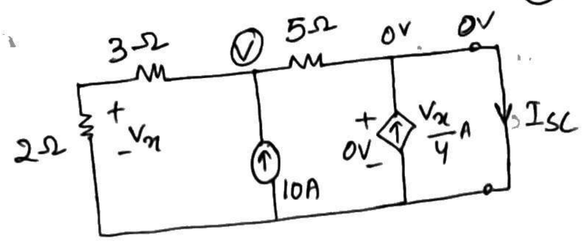
eqn ① & ②

$2V - V_{th} = 50$

$3V - 2V_{th} = 0$

$V = 100V$ $\qquad V_{th} = 150V$

$I_{sc}$ :




Nodal $\Rightarrow$ $\frac{V}{5} - 10 + \frac{V-0}{5} = 0$

$\Rightarrow V = 25V$

$V_x = \frac{2}{5} \times V = 10V$

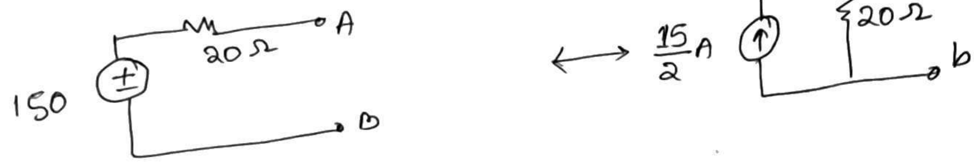
Nodal $\Rightarrow$

$\frac{0-V}{5} - \frac{V_x}{4} + I_{sc} = 0$

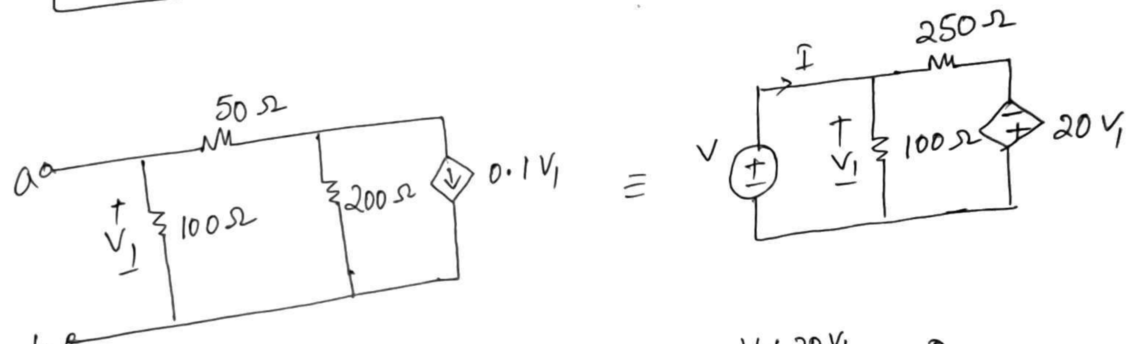
$\Rightarrow I_{sc} = \frac{V}{5} + \frac{V_x}{4} = \frac{25}{5} + \frac{10}{4}$

$= \frac{15}{2}$ A

$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{150}{(15/2)} = 20\Omega$

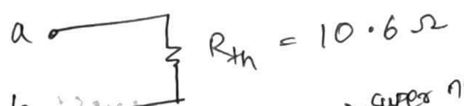



Case (III)



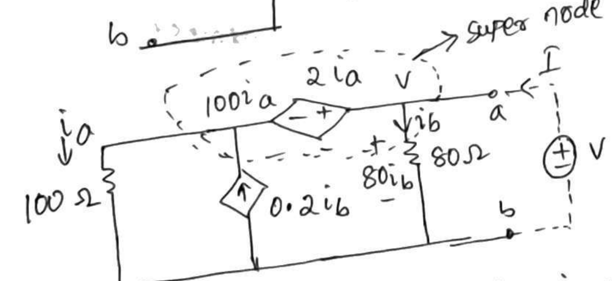

$V_1 = V$ $\Rightarrow$ Nodal $\Rightarrow$ $-I + \frac{V}{100} + \frac{V+20V_1}{250} = 0$

$\Rightarrow \frac{V}{I} = R_{th} = \frac{1}{\frac{1}{100} + \frac{21}{250}} = 10.6\,\Omega$




The Thevenin's & Norton's eq

Q11




Super node eqn $\Rightarrow$ $i_a - 0.2i_b + i_b - I = 0$

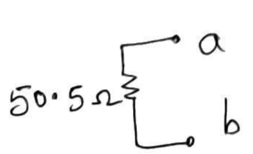
$\Rightarrow I = i_a + 0.8\,i_b$ ; $V = 80\,i_b \Rightarrow i_b = \frac{V}{80}$

Inside the super node always KVL is written $\Rightarrow$ $100\,i_a + 2i_a - V = 0$.

$\Rightarrow i_a = \frac{V}{102}$

eqn (1) $\Rightarrow I = \frac{V}{102} + 0.8 \times \frac{V}{80} \Rightarrow \frac{V}{I} = R_{th} = \frac{1}{\frac{1}{102} + \frac{1}{100}}$

$= 50.5\,\Omega$

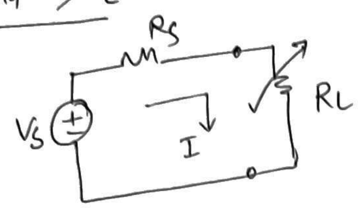


MPPT  same as SPT

→ This thm is applicable only when the load is a variable, otherwise choose the minimum internal impedance of the source, which results a maximum current through the fixed load and hence a maximum power is appeared across the load.

Under the variable load condn -

① $R_s$ and $R_L$ :-



$$I = \frac{V_s}{R_s + R_L}$$

$$P = I^2 R_L = \frac{V_s^2 \cdot R_L}{(R_s+R_L)^2} \text{ W.}$$

$$\frac{dP}{dR_L} = \frac{V_s^2\left[(R_s+R_L)^2 \cdot 1 - R_L \cdot 2(R_s+R_L)\right]}{(R_s+R_L)^4}$$

for MPT $\frac{dP}{dR_L} = 0 \Rightarrow R_L = R_s$

$$P_{max} = P\big|_{R_L = R_s} = \frac{V_s^2}{4R_L} \text{ W}$$

$P_{del} = P_{abs}$

$$P_T = I^2 R_s + I^2 \cdot R_L \big|_{R_L = R_s}$$

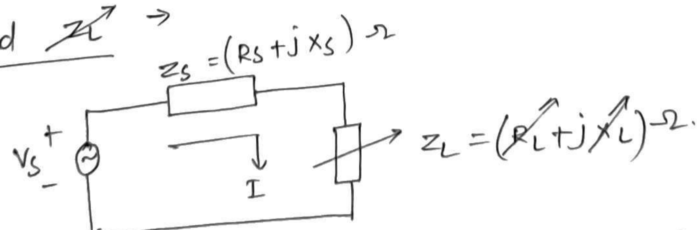
$$= \frac{V_s^2}{2R_L} \text{ W}$$

$$\eta \text{ (efficiency)} = \frac{\text{useful power}}{\text{Total power}} = \frac{\frac{V_s^2}{4R_L}}{\frac{V^2}{2R_L}} = \frac{1}{2} = 0.5.$$

$$\boxed{\% \, \eta = 50\%.}$$

→ So, the efficiency of the max power transform thm is atmost 50%.

② $Z_s$ and $Z_L$ →



$$I = \frac{V_s}{(R_s+R_L) + j(X_s+X_L)}$$

$$\Rightarrow P = |I|^2 R_L \text{ watt}$$

$$= \frac{V_s^2 \cdot R_L}{(R_s+R_L)^2 + (X_s+X_L)^2} \text{ watt}$$

2(i) : only $R_L$ is a variable.

$$\frac{dP}{dR_L} = \frac{V_S^2\left[(R_S+R_L)^2 + (X_S+X_L)^2 \cdot 1 - R_L \cdot 2(R_S+R_L)\right]}{(\quad)^2}$$

For MPT ; $\frac{dP}{dR_L} = 0 \Rightarrow R_L = \sqrt{R_S^2 + (X_S+X_L)^2}\ \Omega$

$P_{max} = P|_{R_L = \sqrt{R_S^2 + (X_S+X_L)^2}}$

$\%\ \eta < 50\%$.

2(ii) only $X_L$ is a variable

$$\frac{dP}{dX_L} = \frac{V_S^2\left[(R_S+R_L)^2 + (X_S+X_L)^2 \cdot 0 - R_L \cdot 2(X_S+X_L)\right]}{(\ )^2}$$

For MPT $\frac{dP}{dX_L} = 0 \Rightarrow X_S + X_L = 0$

$P_{max} = P|_{X_S+X_L=0}$

$\%\eta < 50\%$.

2(iii) Both $R_L$ and $X_L$ are varied simultaneously

In this case both the condn

$R_L = \sqrt{R_S^2 + (X_S+X_L)^2}$ & $X_S + X_L = 0$ are valid

$R_L = R_S$
$X_L = -X_S$

so, $Z_L = R_L + jX_L$
$= R_S - jX_S$

$$\boxed{Z_L = Z_S^*}\ \Omega$$

$P_{max} = P|_{R_L = R_S,\ X_L = -X_S} = \frac{V_S^2}{4R_L}$ W $\quad \boxed{\%\ \eta = 50\%}$

3) $Z_S$ and $R_L$ :- It is a special case of 2(i) with $X_L = 0$

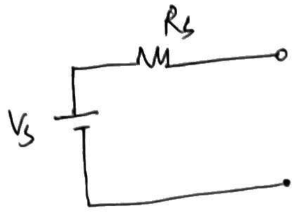
so, for MPT $\Rightarrow R_L = \sqrt{R_S^2 + (X_S+X_L)^2}\ \Big|_{X_L=0}$

$R_L = \sqrt{R_S^2 + X_S^2}\ \Omega$ $\quad R_L = |R_S + jX_S|\ \Omega = |Z_S|\ \Omega$

$P_{max} = P|_{R_L = |Z_S|\ \Omega}$ $\quad \boxed{\%\eta < 50\%}$

Q1. A practical d.c. source when it is short ckted, then supplies a current of 2.5 Amps and it can provide a power of 80 watts to a 20 Ω load. Determine the open ckt voltage and the value of $R_L$ for MPT.

(A)




$$\frac{V_s}{R_s} = 2.5A \quad \text{—— (1)}$$

$P_{20\Omega} = 80W$ given

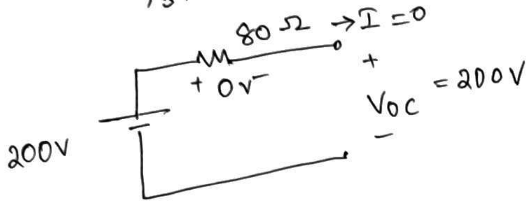
$I_{20\Omega}^2\, 20 = 80 \Rightarrow I_{20\Omega} = 2$

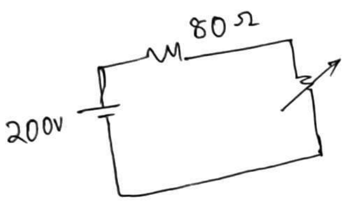
$$\Rightarrow \frac{V_s}{R_s+20} = 2 \quad \text{—— (2)}$$

$2.5 R_s = 2R_s + 40$

$\Rightarrow 0.5 R_s = 40 \Rightarrow R_s = 80\,\Omega$

$V_s = 2.5 \times 80 = 200\,V$

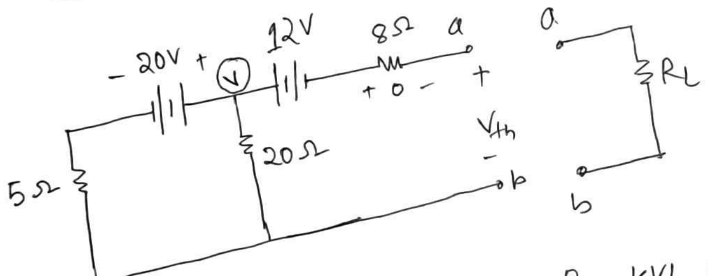



$R_L = R_S = 80\,\Omega$ for MPT

$$P_{max} = \frac{200^2}{4 \times 80} = \underline{125\,W}$$

Q1

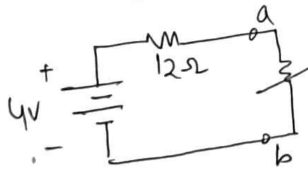



The MPT to RL is

$R_{th} = 8 + 5 \,||\, 20 = 12\,\Omega$ ; By KVL $\Rightarrow V - 12 - V_{th} = 0$

$\Rightarrow V = V_{th} + 12 \Rightarrow V_{th} = V - 12.$

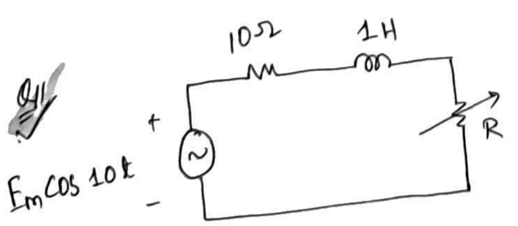
NODAL $\quad \dfrac{V-20}{5} + \dfrac{V}{20} + 0 = 0 \Rightarrow V = 16V \Rightarrow V_{th} = 4V$




$R_L = 12\,\Omega$ for MPT

$$P_{max} = \frac{4^2}{4 \times 12} = \frac{1}{3}\,W$$

Q1

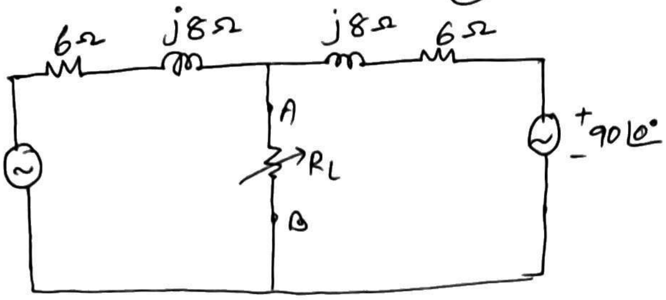



The value of 'R' for MPT

$R = |Z_s|\,\Omega = |10 + j\omega L|\,\Omega = |10 + j10|\,\Omega$

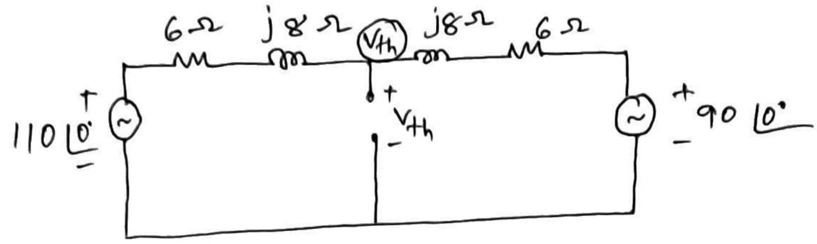
$= 10\sqrt{2} = 14.14\,\Omega$

Q11 H.W




The MPT to $R_L$ is

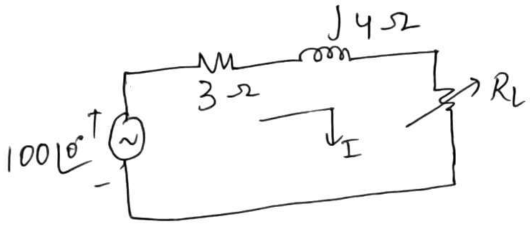
(A)




NODAL $\Rightarrow$ $\dfrac{V_{th} - 110\angle 0°}{6+j8} + \dfrac{V_{th} - 90\angle 0°}{6+j8} = 0$

$\Rightarrow 2V_{th} = 200\angle 0° \Rightarrow V_{th} = 100\angle 0°$

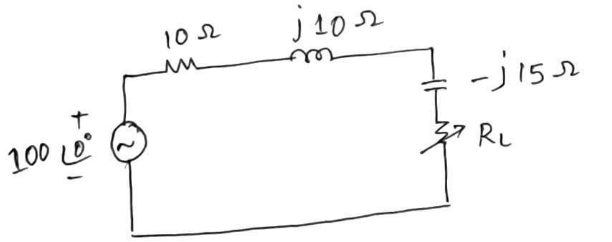
$Z_{th} = (6+j8) \parallel (6+j8) = Z \parallel Z = \frac{Z}{2} = (3+j4)\,\Omega$




$R_L = |Z_s|\,\Omega$
$= |3+j4| = 5\,\Omega$

$I = \dfrac{100\angle 0°}{3+j4+5}$ ; $P_{max} = |I|^2 . 5 = 625\,W$.

Q11



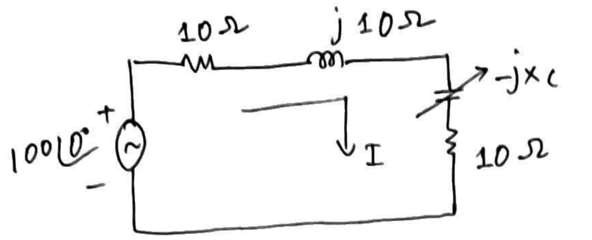

The MPT to $R_L$ is

(A) $Z_L = R_L - j15 = R_L + j(-15)$
$= R_L + jX_L \mid$ where $X_L = -15$

So, for MPT $R_L = \sqrt{R_S^2 + (X_S + X_L)^2} = \sqrt{10 + (10-15)^2} = 5\sqrt{5}\,\Omega$

$I = \dfrac{100\angle 0°}{10+j10-j15+5\sqrt{5}}$ ; $P_{max} = |I|^2 . 5\sqrt{5} = 236\,W$.

Q11 ✗




$$Z_L = 10 - jX_L = 10 + j(-X_C) = R_L + jX_L \quad | \text{ where } X_L = -X_C.$$

$$I = \frac{100\angle 0°}{10 + j10 - j10 + 10} = 5\angle 0° \text{ A}$$

For MPT

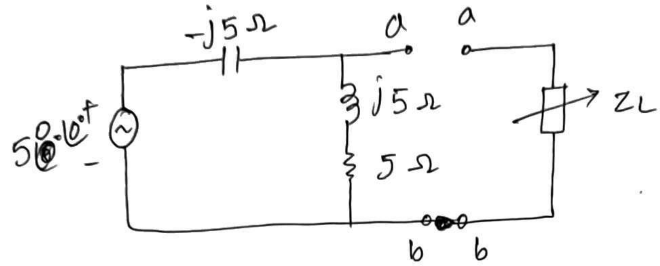
$$X_S + X_L = 0$$

$$\Rightarrow 10 - X_C = 0$$

$$\Rightarrow X_C = 10$$

$$P_{max} = |I|^2 \cdot R_L = 5^2 \times 10 = 250 \text{ W}.$$

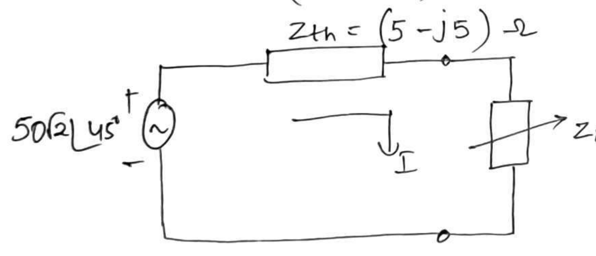
Q11 ✓




The maximum true power delivered to $Z_L$

$$V_{th} = \left(\frac{50\angle 0°}{-j5 + j5 + 5}\right) \cdot (j5 + 5) = 10\angle 0° \cdot 5\sqrt{2}\angle 45° = 50\sqrt{2}\angle 45°$$

$$Z_{th} = (-j5) \parallel (j5 + 5) = (5 - j5)\ \Omega$$

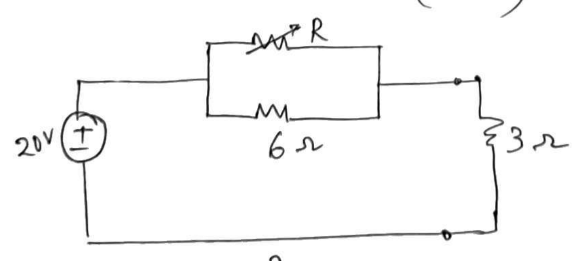



$$Z_L = Z_{th}^* = (5 + j5)\ \Omega$$

$$I = \frac{50\sqrt{2}\angle 45°}{5 - j5 + 5 + j5} = 5\sqrt{2}\angle 45°$$

$$P_{max} = |I|^2 R_L = (5\sqrt{2})^2 \cdot 5 = 250 \text{ W}.$$

Q11 ✓




(a) 3Ω (b) 6Ω (c) 9Ω (d) 12Ω

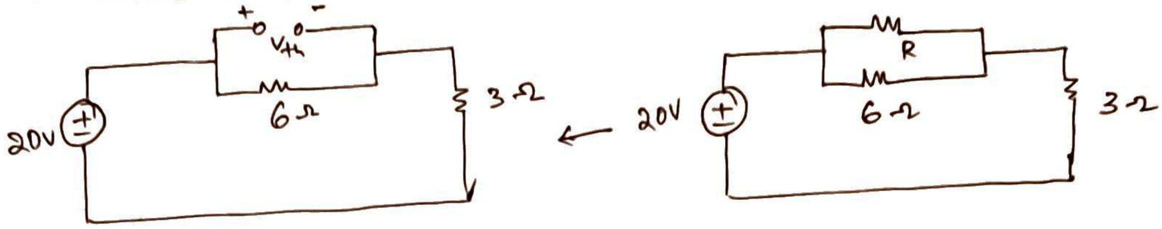
The value of 'R' for MPT to the 3Ω resistor is

$$P_{3\Omega} = I_{3\Omega}^2 \cdot 3 \text{ W} \Rightarrow I_{3\Omega} = \left(\frac{20}{R_{eq} + 3}\right) \text{A} \; ; \; R_{eq} = 6 \parallel R$$

if $R = 3\Omega \Rightarrow R_{eq} = 6 \parallel 3 = 2\Omega$

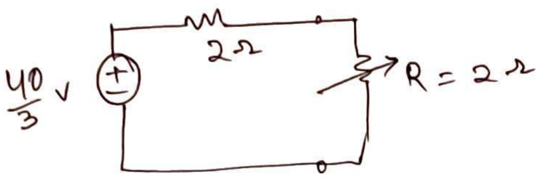
$$I_{3\Omega\,max} = \frac{20}{2 + 3} = 4\text{A} \; ; \; P_{3\Omega\,max} = 4^2 \times 3 = 48 \text{ W}.$$

Q11 In the above problem, the maximum power delivered to 'R' is




$$V_{th} = \left(\frac{20}{6+3}\right)\cdot 6 = \frac{40}{3}\ V$$

$$R_{th} = 6 \parallel 3 = 2\ \Omega$$




for MPT

$$P_{max} = \frac{\left(\frac{40}{3}\right)^2}{4\times 2}\ W.$$

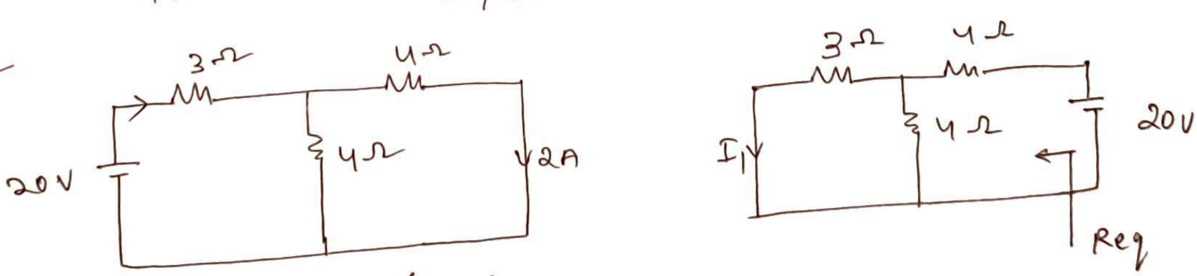
## The Reciprocity Thm.

⇒ In a linear, passive and bilateral n/w, the ratio of response to excitation is constant even though the source is interchange from the i/p terminals to the o/p terminals.


i.e. $\frac{I}{V}$ = constant ⇒ $\boxed{\frac{I_1}{V_1} = \frac{I_2}{V_2}}$

⇒ Not applicable to dependent sources.

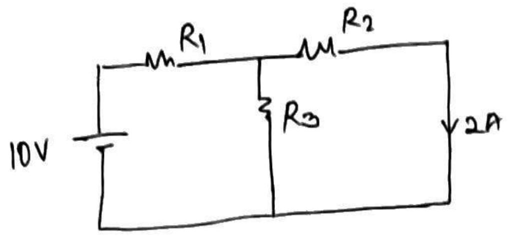
proof

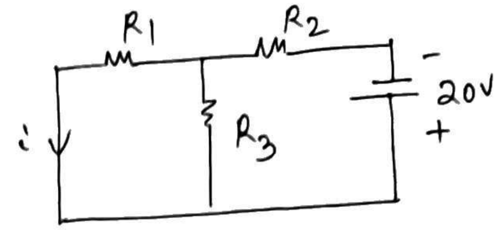



$$R_{eq} = 4 + (4 \parallel 3) = \frac{40}{7}\ \Omega$$

$$I = \frac{20}{40/7} = \frac{7}{2}\ A \qquad I_1 = \frac{\frac{7}{2}\cdot 4}{3+4} = 2A$$

Q//





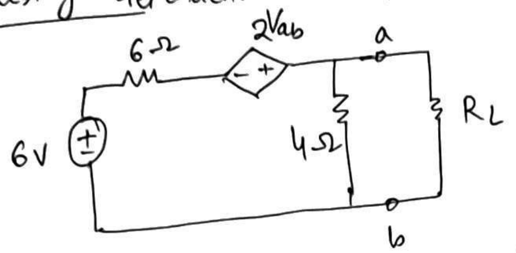
Find $i$

$$\frac{I_1}{V_1} = \frac{I_2}{V_2} \Rightarrow \frac{2}{10} = \frac{I}{-20} \Rightarrow I = -4A.$$

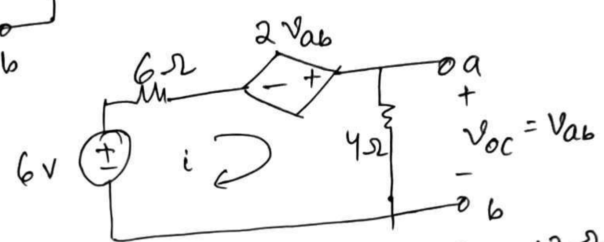
—o—

MPT using dependent source :-

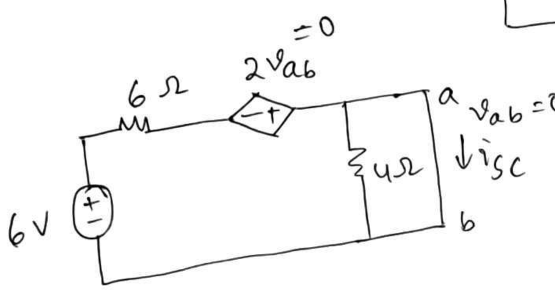
Q//
H.W

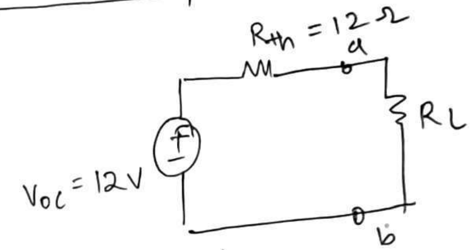


Find the load $R_L$ that will result in MPT & determine $P_{max}$.



Therenin eq.





$V_{th}$

$$-6 + 10i_1 - 2v_{ab} = 0 \Rightarrow 10i - 8i = 6$$

$$v_{ab} = V_{OC} = 4i.$$

$$i = 3A \quad ; \quad V_{OC} = 4i = 12V$$

$I_{sc}$

$$-6 + 6i_{sc} = 0 \Rightarrow i_{sc} = 1A$$

$$R_{th} = \frac{V_{OC}}{i_{sc}} = 12\Omega$$

MPT $\quad R_L = R_{th} = 12\Omega$

$$P_{max} = \frac{V_{OC}^2}{4R_L} = \frac{12^2}{4 \times 12} = \boxed{3W}$$

Q//

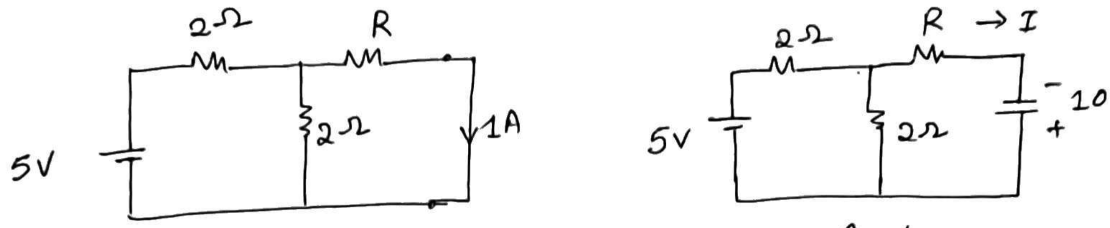


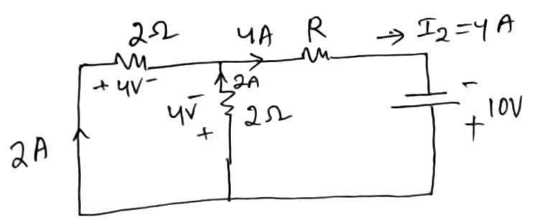

fig (a) fig (b)

Use the data given in fig (a)

Ans By SPT to fig (b) →

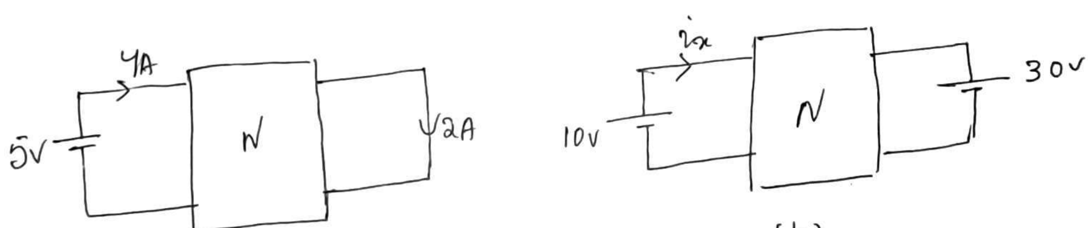
→ The current due to 5V source alone ⇒ $I_1 = 1$ Amp

→ By homogenity and reciprocity principles to fig (a), also by reversing the source polarities




By SPT $I = I_1 + I_2 = 1 + 4 = 5A$

Q//




(A) (b)

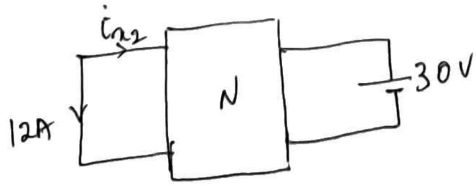
Network 'N' contains only resistors, use the data given in fig (a)

Find the current '$i_x$' in Fig (b)

(A) By SPT to fig (b) ⇒ The current due to 10V source alone

$= i_{x_1} = 2 \times 4 = 8$ Amps

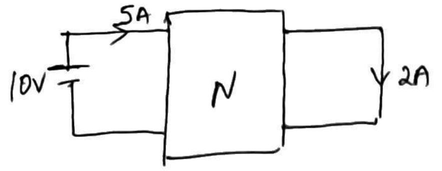
By homogenity & reciprocity principles to fig (a)



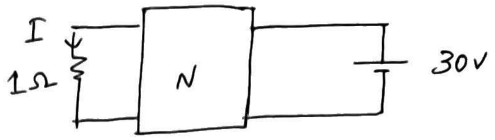

$i_{x_2} = -12A$

By SPT ⇒ $i_x = i_{x_1} + i_{x_2} = 8 - 12 = -4A$

Q11
H.W



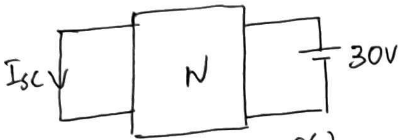

(a)




(b)

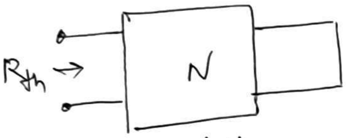
N/w 'N' contains only resistances, use the data given in fig.(a) find the current 'I' in fig (b)

(A)



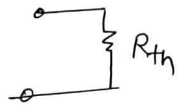

(c)

By homogenity & reciprocity principles to fig (a) $I_{SC} = 6\,AMP$

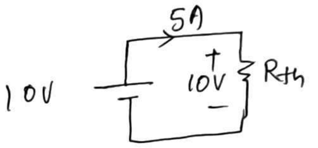



(d)

≡

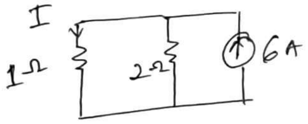



(a) → energized version of (d)



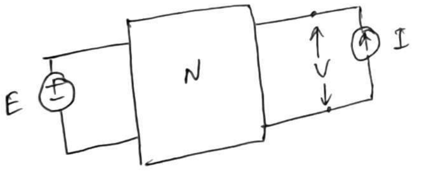

$R_{th} = \frac{10}{5} = 2\,\Omega$

(b) ≡




$I = \frac{6\times2}{2+1} = 4\,A$

Q11 H.W

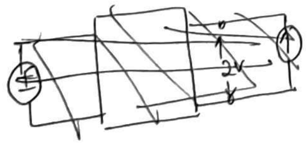


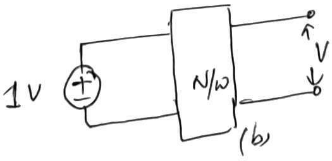

The 'N' contains only resistances.

E = 1 V & 0 V
I = 0 A & 2A respectively
V = 3 V & 2 V

If E = 10V and I is replaced by R = 2Ω then V = ?

(A)

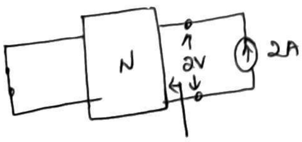





(b)

$V = 3V = V_{OC} = V_{th}$

$R_{th} = \frac{2V}{2A} = 1\,\Omega$

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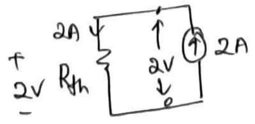


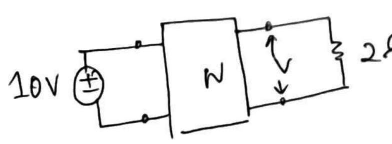


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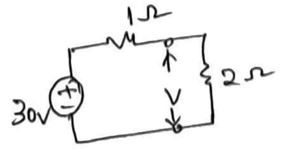
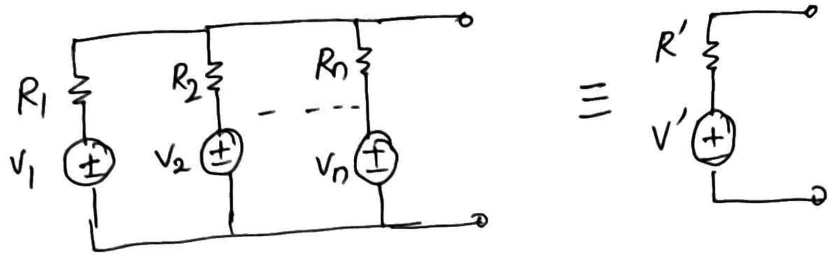



By homogenity to (b)

1 V → 3V
10 V → 30V

$V = \frac{30}{3} \times 2$
$= 20\,V$

The Milliman's thm. -




$$V' = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + \cdots V_n G_n}{G_1 + G_2 + G_3 + \cdots + G_n}$$

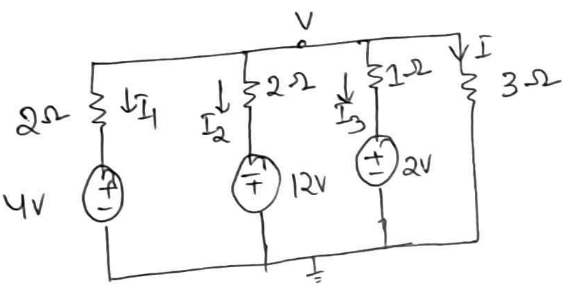
$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

$$G' = G_1 + G_2 + G_3 + \cdots + G_n$$

$$R' = \frac{1}{G'} = \frac{1}{G_1 + G_2 + \cdots G_n}$$

→ In the above case of the polarity of the source $V_2$ are reversed then $V_2$ is replaced by $-V_2$ in the expression of $V'$

Q/




Det I.

$$V' = \frac{4 \cdot \frac{1}{2} - 12 \cdot \frac{1}{2} + 2 \cdot \frac{1}{1}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{1}} = -1\,V$$

$$R' = \frac{1}{\frac{1}{2} + \frac{1}{2} + 1} = \frac{1}{2}\,\Omega$$

$$I = \frac{V'}{R' + 3} = \frac{-1}{\frac{1}{2} + 3} = -\frac{2}{7}\,A$$

Another method NODAL $\Rightarrow \frac{V-4}{2} + \frac{V+12}{2} + \frac{V-2}{1} + \frac{V}{3} = 0$

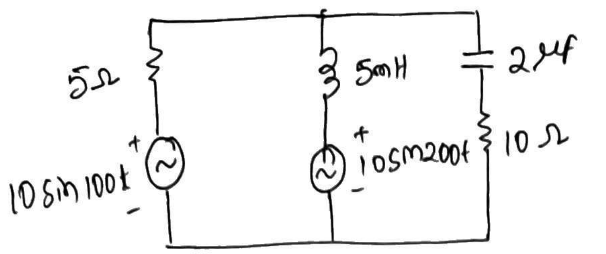
$$\Rightarrow V = -\frac{6}{7}\,V$$

$$I = \frac{V}{3} = -\frac{2}{7}A$$

$I_1 = \left(\frac{V-4}{2}\right) A = -\frac{17}{7}$, $I_2 = \left(\frac{V+12}{2}\right) A = \frac{39}{7}$, $I_3 = \left(\frac{V-2}{1}\right) A = -\frac{20}{7}$

VTT

Q11



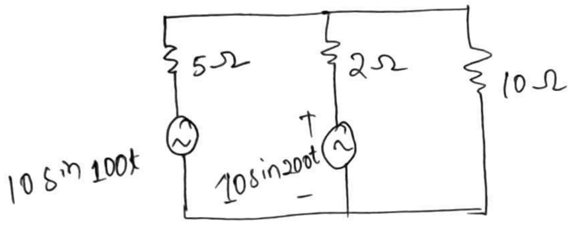

Which one of the following thm can be conviniently used to evaluate the response on the 10Ω resistor.

(a) Theve. Thm (b) MPTT (C) Milliman's thm (d) SPT.

Note: Since, two different freq. are operating on the n/w simultaneously, always the SPT is used to evaluate the response, since the reactive elements are freq. sensitive. $Z_L = j\omega L\ \Omega$ & $Z_C = \frac{1}{j\omega C}\ \Omega$

(2) In the above problem, both the sources are 100 radians/sec, then Millimann's thm is more conviniently used.

Q11




$$V' = \frac{10\sin 100t \cdot \frac{1}{5} + 10\sin 200t \cdot \frac{1}{2}}{\frac{1}{5} + \frac{1}{2}}$$

$$R' = \frac{1}{\frac{1}{5} + \frac{1}{2}}\ \Omega$$

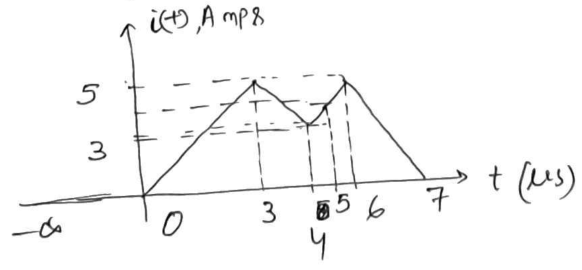
$$I_{10\Omega} = \left(\frac{V'}{R'+10}\right) A$$

→ So, here no reactive components then Millimann's thm is best suited.

— o —

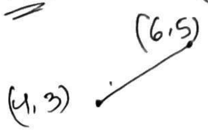
Problems on the Power and Energy:

Q11 Figure shows the current flowing through a capacitor. Determine the charge acquired by the capacitor upto first 5μs




Ans $i(t) = \frac{dq(t)}{dt}$ Amps $\Rightarrow dq(t) = i(t)\,dt$ $\Rightarrow q = \int_{-\infty}^{t} i(t)\,dt$

$= \int_{0}^{5\mu s} i(t)\,dt$ = Area under i(t) upto 5μs.



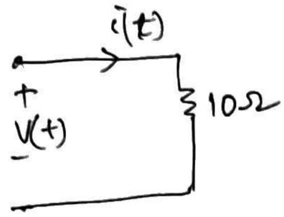

Slope $= \frac{5-3}{6-4} = 1$

$q = q_1\big|_{0-3\mu s} + q_2\big|_{3-4\mu s} + q_3\big|_{4-5\mu s} = \left(\frac{1}{2}\times 3\times 5\right) + \left[\left(\frac{1}{2}\times 1\times 2\right) + (1\times 3)\right] + \left[\left(\frac{1}{2}\times 1\times 1\right) + (1\times 3)\right] = 15$

5A

(24)

Q11



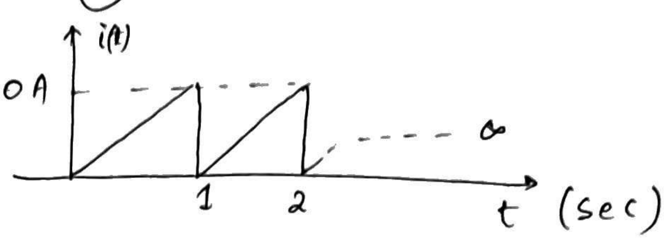




Fig. shows the current flowing through a $10\Omega$ resistor. Determine the average power dissipated by the resistor.

Soln The given signal is a periodic signal of period 1 sec. Let's assume 1 period.

$$\text{Slope} = \frac{10-0}{1-0} = 10$$

$$y = mx \Rightarrow i = 10t \text{ A} \quad \text{for } 0 \leq t \leq 1 \text{ sec.}$$

The ~~instanteneous~~ Power $\Rightarrow P(t) = i^2(t)\,R. = (10t)^2 \cdot 10$

$$= 1000t^2 \text{ W} \quad \text{for } 0 \leq t \leq 1 \text{ sec}.$$

$$P(0) = 0\text{W} \Rightarrow P_{min}$$

$$P(1) = 1000\text{ W} \Rightarrow P_{max}$$

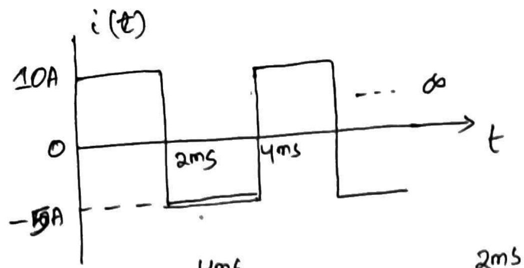
- Betn 0 & 1, the inst. power continuously varied in a non-linear fashion and hence is of no meaning, So, the unique average power

$$P_{avg} = \frac{\text{The energy absorbed over one period}}{\text{period}}$$

$$= \frac{\int_0^1 i^2 R\, dt}{1 \text{ sec}} = \frac{\int_0^t (10t)^2 \cdot 10\, dt}{1 \text{ sec}}$$

$$= \frac{\frac{1000}{3} \text{ J}}{1 \text{ sec}} = \frac{1000}{3} \text{ watt.}$$

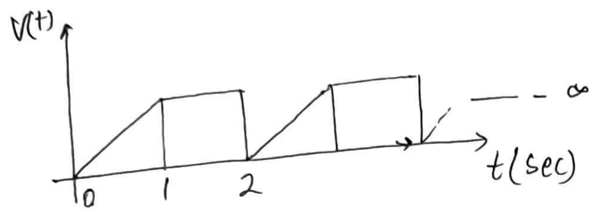
Q11




$$P_{avg} = \frac{\int_0^{4ms} i^2 \cdot R\, dt}{4ms} = \frac{\int_0^{2ms} 10^2 \cdot 10 \cdot dt + \int_{2ms}^{4ms} (-5)^2 \cdot 10 \cdot dt}{4ms}$$

$$= 625W$$

Q11




$$\text{slope} = \frac{1-0}{1-0} = 1$$

$$V(t) = \begin{cases} 1 \cdot t \ V & \text{for } 0 \le t \le 1 \text{ sec} \\ 1\ V & \text{for } 1 \le t \le 2 \text{ sec.} \end{cases}$$

$$P_{avg} = \frac{\int_0^2 \frac{V^2}{R} \cdot dt}{2 \text{ sec}} = \frac{\int_0^1 \frac{t^2}{10} dt + \int_1^2 \frac{1^2}{10} \cdot dt}{2} = \frac{1}{15} \text{ watt.}$$

Note: For any general period 'T'.

(i) In case of a current wave form

$$P_{avg} = \frac{\int_0^T i^2 R\, dt}{T} = \frac{1}{T}\int_0^T i^2 \cdot dt \cdot R.$$

$$\boxed{P_{avg} = I_{rms}^2 \cdot R} \ W$$

(ii) In case of a voltage wave form

$$P_{avg} = \frac{\int_0^T \frac{V^2}{R} \cdot dt}{T} = \frac{\frac{1}{T}\int_0^T V^2 \cdot dt}{R} \Rightarrow \boxed{P_{avg.} = \frac{V_{rms}^2}{R.}} \ W$$

## Network graph Theory

→ N/W may contain Passive as well as active elements.

→ An element is connected betn two terminals : branch

→ node : position at which two or more than two branches are connected together

→ loop : Any single closed Path of a N/W

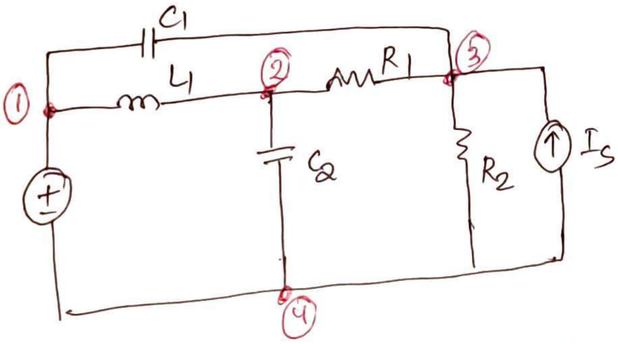
Note : R, L, C (elements) are replaced by lines.

voltage source : with S.C or internal impedance

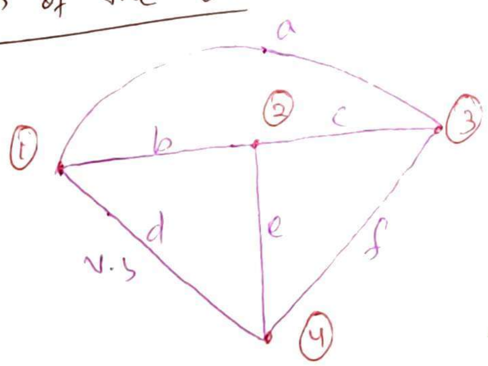
current source : " O.C

→ A graphical representation of electrical circuits, it is used to analyze complex ~~N/Ws~~ electric CKts into n/w graphs

Ex.




N/W graphs of the CKt.




Nodes & branches
4
(6)
Passive elements (R, L, C)
voltage - S.C
current - O.C.

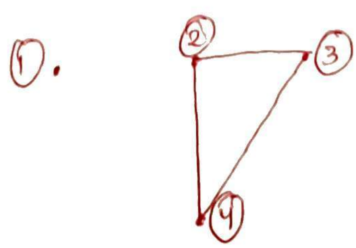
→ Line segments ~~with~~ of elements or V.S. will be branches

## Types :

→ connected & unconnected

→ Directed & undirected

Ex unconnected graph



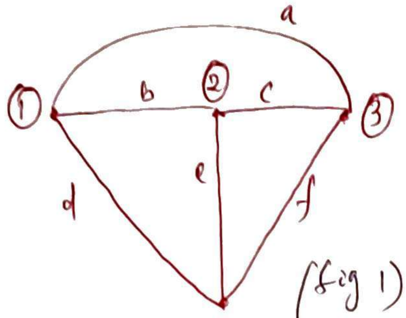

→ not a single branch is connected to node '1'

→ node 1 becomes an isolated node.

## Connected.

→ at least one branch is connected to a node.

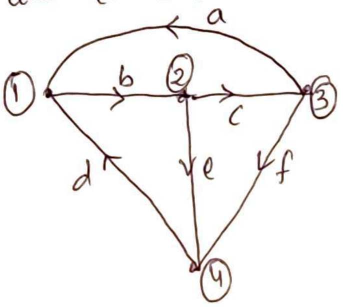
or no node should be left isolated.




(fig 1)

## Directed / oriented

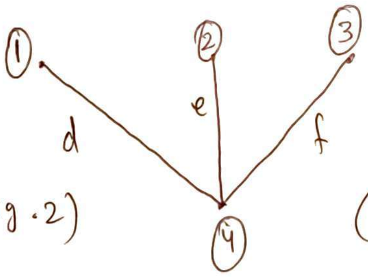
The branches are connected with arrows.




**Undirected / un oriented** : No arrows (fig 1)

**Subgraphs** : By removing some nodes or branches in a given graph, subset of that graph.

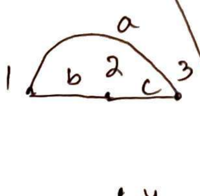
no. of branches are less




(Fig. 2) (Tree)

**Tree** → connected subgraph : { no loop, no nodes are left.

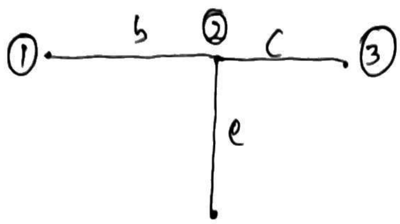
**Co-Tree** : complement of Tree.



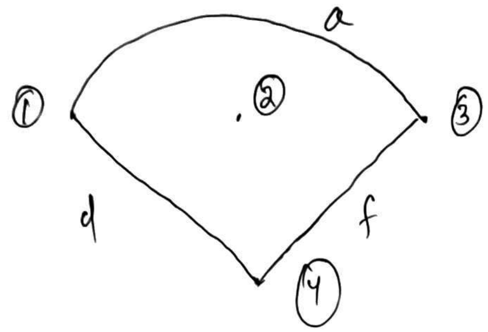


→ branches of Tree : Twigs. eg. Fig. 2 (d, e, f)

→ remaining branches : a, b, c links / Chords eg.

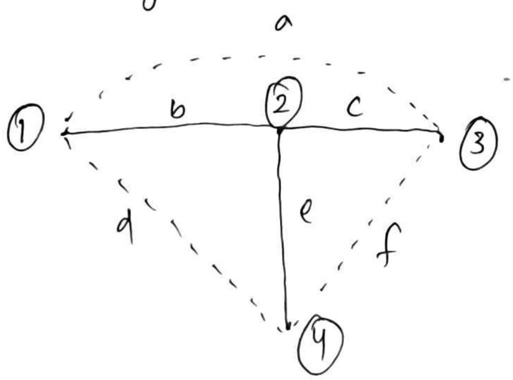



Tree – b, c, e twigs.




co-tree.

Links & twigs → combine → branches of the graph,




--- links

—— Twigs

Links + twigs = no. of branches.

$L + T = B$ — (1)

$T = N - 1$ — (2)

↓

Nodes in a graph

$L + N - 1 = B \Rightarrow \boxed{L = B - N + 1}$

## Incidence matrix :-

→ This matrix provides the information about the incidence of branches at different nodes.

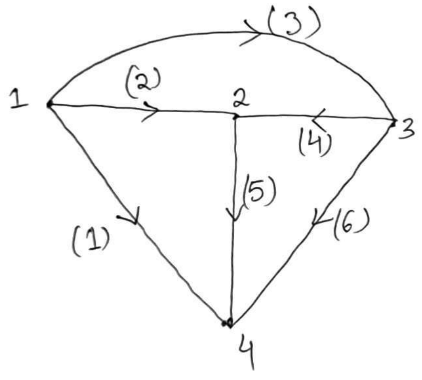
→ Incidence matrix : each row of the matrix → corresponding node of the graph
each column represents → branch of the graph.

→ If the graph has 'n' nodes and 'b' branches, the order of the matrix is '$n \times b$'. The elements $a_{ij}$ of the matrix are identified as:

(i) $a_{ij} = 1$ ; if the branch 'j' is incident at node 'i' and is oriented away from the node 'i'.

(ii) $a_{ij} = -1$, if the branch 'j' is incident at node 'i' and is directed towards the node 'i'.

(iii) $a_{ij} = 0$, if the branch 'j' is not incident at node 'i'.




The incident matrix

$$
A_i = \begin{array}{c} \text{Nodes} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \text{Branches} \\ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ -1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \end{array}
$$

→ The matrix $A_i$ is of the order $4 \times 6$ and satisfies the following

(i) The algebric sum of any column of $A_i$ is 'zero'

(ii) The determinant of $A_i$ of a closed loop is 'zero'

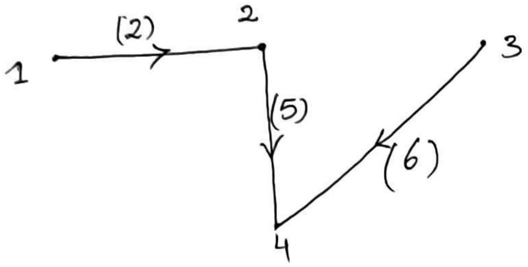
**Note :** If any row is removed from the incidence matrix $A_i$, the remaining matrix is known as a reduced incidence matrix 'A'. The order of the reduced matrix 'A' is $(n-1) \times b$ i.e. it has $(n-1)$ rows and 'b' columns. The row corresponding to the reference node is deleted.

eg. node '4' is called the reference node. the reduced incidence matrix 'A' is given by.

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & -1 & 1 & 0 \\ 3 & 0 & 0 & -1 & -1 & 0 & 1 \end{array}$$

-o-

eg. connected subgraph / Tree

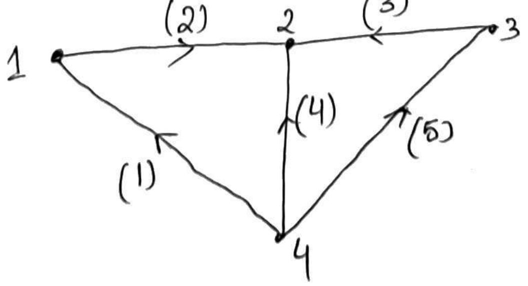



→ First the element of matrix 'A' for twigs and then the remaining for links.

| | Twigs | | | Links | | |
|---|---|---|---|---|---|---|
| Nodes | 2 | 5 | 6 | 1 | 3 | 4 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 2 | -1 | 1 | 0 | 0 | 0 | -1 |
| 3 | 0 | 0 | 1 | 0 | -1 | -1 |

Question

Q1 For the following graph, find the incidence matrix



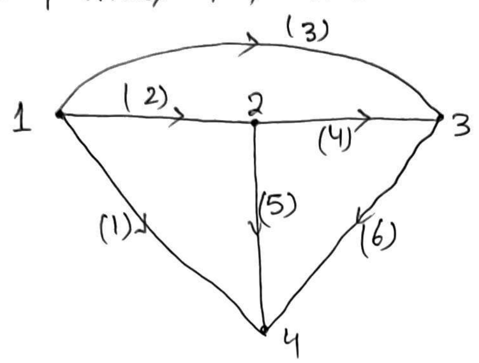

$$A_i = \begin{array}{c|ccccc} \text{Nodes} \backslash \text{Branches} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & -1 & 1 & 0 & 0 & 0 \\ 2 & 0 & -1 & -1 & -1 & 0 \\ 3 & 0 & 0 & 1 & 0 & -1 \\ 4 & 1 & 0 & 0 & 1 & 1 \end{array}$$

## Loop matrix :

→ From the incidence matrix we get the idea regarding the number of nodes and branches as well as their connection and orientation of the branches to the respective nodes.

→ However, it can't provide any idea regarding the interconnection of branches which form loops.

→ The loop matrix provides this idea




If a graph has 'n' nodes and 'b' branches, the loop matrix 'B' will have 'b' columns and the number of rows will be the number of possible loops.

$b_{ij}$ ⇒ the elements $b_{ij}$ of the loop matrix 'B' will satisfy the following conventions

$b_{ij}$ ⇒ 1 ; if branch 'j' is in ~~loo~~ loop 'i' and their directions coincide

= −1 , if branch 'j' is in loop 'i' and their directions don't coincide

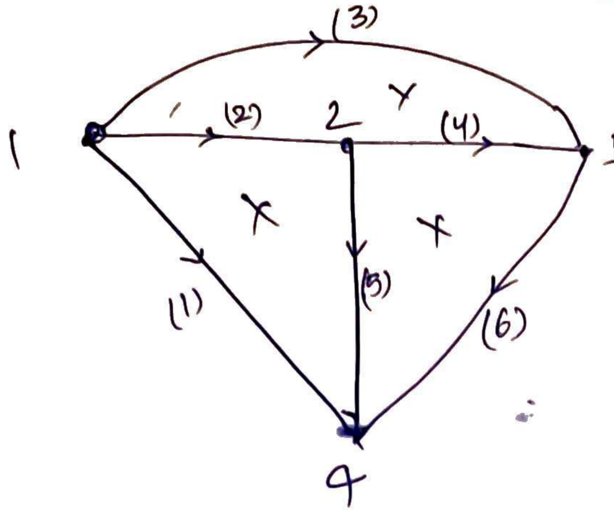
= 0 , if branch 'j' is not in loop 'i'.

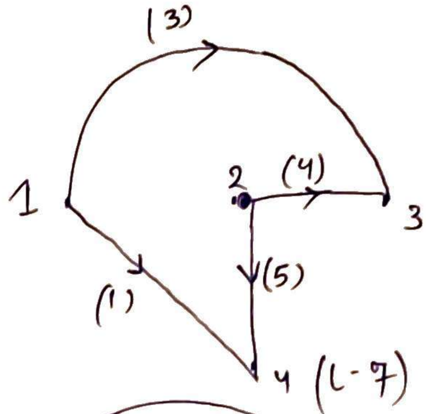
Seven possible loops :

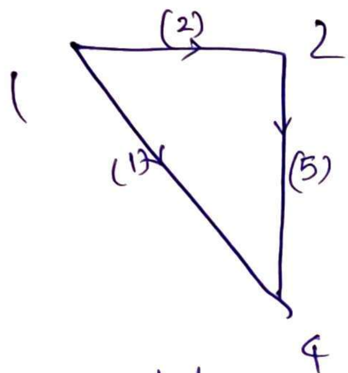
Branches

$$B = \begin{array}{c|c} \text{Loops} & 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\ \begin{matrix}1\\2\\3\\4\\5\\6\\7\end{matrix} & \left[\begin{matrix} \quad & \quad & \quad & \quad & \quad & \quad \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right] \end{array}$$

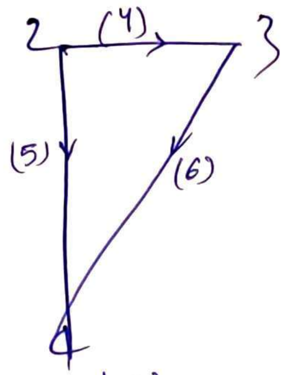
## (28) All possible loops



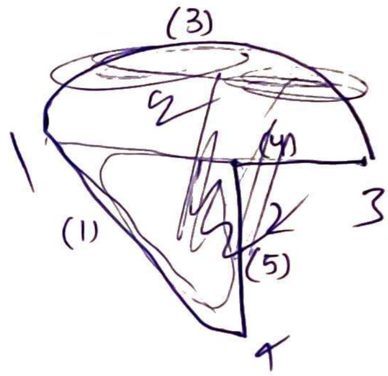



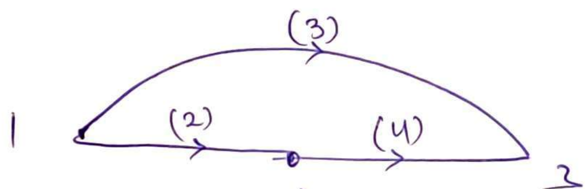




L-1



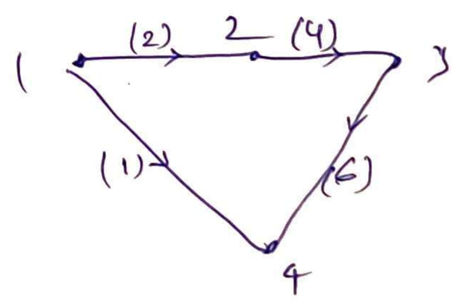

L-2



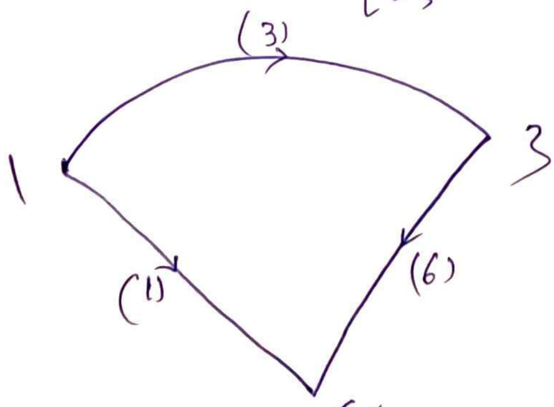




L-3

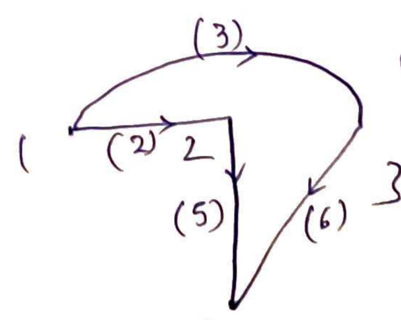
L-4




L-5




L-6

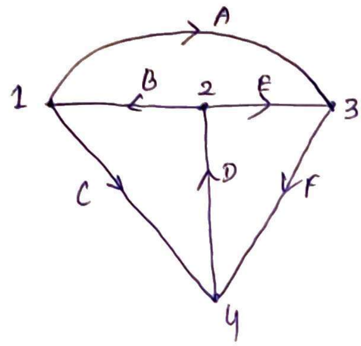


(After Tie set)

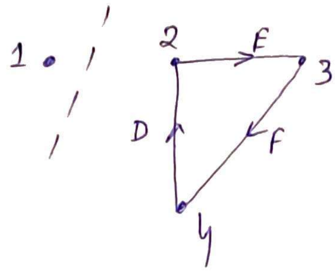
## Cut set

→ Here we remove some branches, which may or may not be identical to each other and it bisects graph into two divisions.

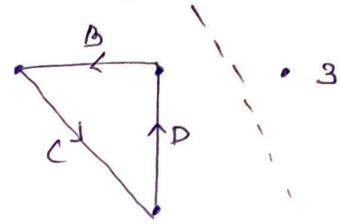
→ Find the cut set of given graph.




→ $C_2$ - A, B, C

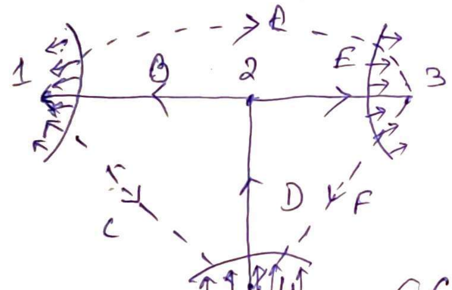



→ $C_2$ - A E F




## Fundamental cut set

- For a given Tree of graph, the cut set that contains one twig and rest link is referred fundamental cut set.




Twigs B E D

Links A C F

$$\begin{cases} C_1 = B\ A\ C \\ C_2 = E\ A\ F \\ C_3 = D\ C\ F \end{cases}$$

| Cut set | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $C_1$ | -1 | 1 | -1 | 0 | 0 | 0 |
| $C_2$ | 1 | 0 | 0 | 0 | 1 | -1 |
| $C_3$ | 0 | 0 | -1 | 1 | 0 | -1 |

(Branches)

## Cut set matrix

* It represents orientation of branches with respect to fundamental cut set.
* Direction of cut set is in the direction of Twig of fundamental cut set
* $a_{ij} = \begin{cases} +1 & \text{Direction of branch in cut set direction} \\ -1 & \text{" " " opposite to cut set direction} \\ 0 & \text{Branch is not connected in the cut set.} \end{cases}$
* Each fundamental cut set represents an equipotential surface.
* Cut set and Tie set are dual to each other.

## Fundamental loop matrix or Tie-set matrix

→ The n/w matrices will help to keep the data more organized when you keep them in rows & columns to make computation easier.

→ Complex calculation will be easier.

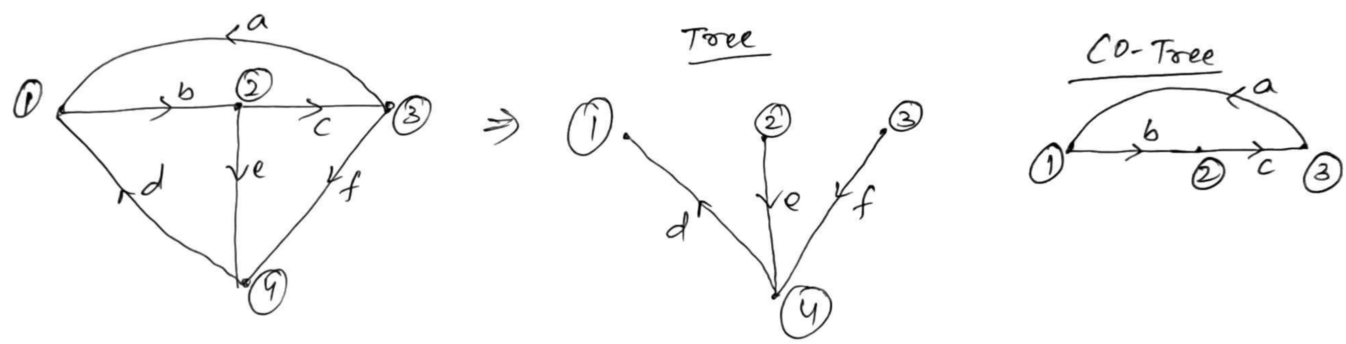
(i) Fundamental Loop matrix

(ii) Fundamental cut set matrix

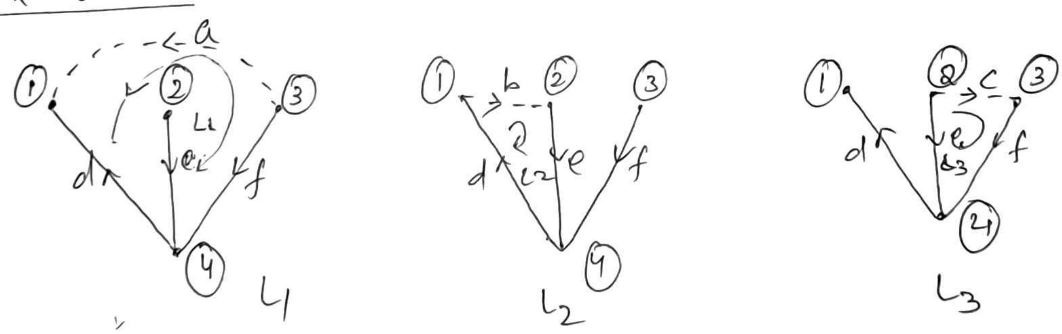
(iii) Incidence matrix.

(i) Fundamental loop matrix : It will help to calculate branch currents in a ckt.

f-loop : contains only one link & one/more twigs.




One link at a time.




How many fundamental loops = no. of links

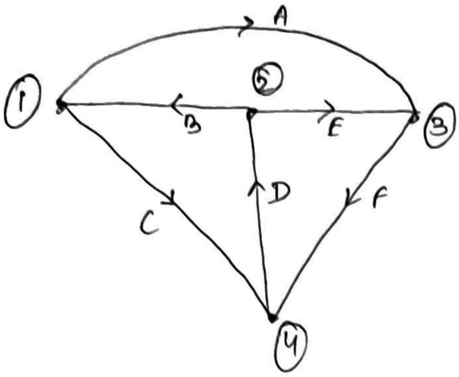
Note : The direction of loop will be the direction of link.

$$B = \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

For each tree there will be a fundamental matrix.

## Tie set

→ Tie set is a set of branches which forms a loop irrespective of direction of branch.



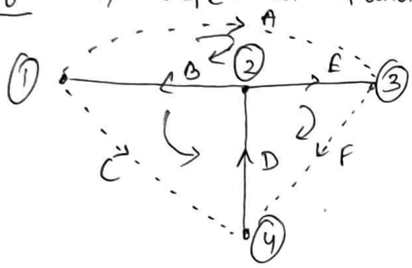

**Tie sets**

ABE, BCD, DEF, ACF, ACDE, ABDF

CBEF.

## Fundamental Tie set

→ For a given Tree of graph, the tie set that contains one link and rest twigs is referred fundamental tie set.




no. of branches of the tree ⇒ Total nodes −1 = 4−1 = 3

BED (twigs)

ACF links (consider one link at a time)

Fundamental tie set: $\underset{L_1}{ABE}$, $\underset{L_2}{CBD}$, $\underset{L_3}{FDE}$

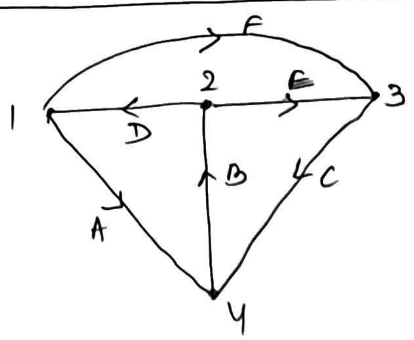
**Tie set matrix:** It represents orientation of branches w.r.t. tie set current. with fundamental tie set, assume tie set current in same direction of link.

$$a_{ij} = \begin{cases} +1, & \text{Direction of tie set current in the direction of branch} \\ -1, & \text{" " " " opposite direction of branch} \\ 0, & \text{branch is not connected in the tie set current} \end{cases}$$

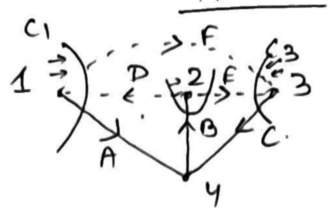
| Loops \ Branches | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $L_1$ | 1 | 1 | 0 | 0 | −1 | 0 |
| $L_2$ | 0 | 1 | 1 | 1 | 0 | 0 |
| $L_3$ | 0 | 0 | 0 | 1 | 1 | 1 |

Fundamental loop matrix

# Essential & key points of cut set matrix




Tree A, B, C




Links ⇒ D E F
Twig ⇒ A, B C

**Tie set** (one link & rest twig)  Direction of link = Direction of loop.

$L_1$ = D A B
$L_2$ = E C B
$L_3$ = F A C

$B =$

| Loops | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $L_1$ | 1 | 1 | 0 | 1 | 0 | 0 |
| $L_2$ | 0 | 1 | 1 | 0 | 1 | 0 |
| $L_3$ | -1 | 0 | 1 | 0 | 0 | 1 |

Branches: A B C = Twig; D E F = Link

**Cut set**  Fundamental cut set (one twig & rest links)  Direction of twig.

$C_1$ = A D F
$C_2$ = B D E
$C_3$ = C E F

$C =$

| Cut set | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $C_1$ | 1 | 0 | 0 | -1 | 0 | 1 |
| $C_2$ | 0 | 1 | 0 | -1 | -1 | 0 |
| $C_3$ | 0 | 0 | 1 | 0 | -1 | -1 |

Branches: A B C = twig; D E F = Link

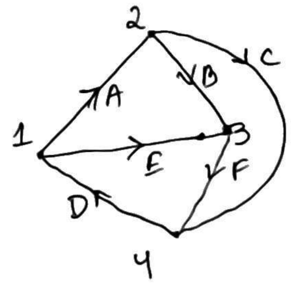
[Properties]

→ $[B]_{Link} = [I]$
→ $[C]_{Twig} = [I]$
→ $[B]_{Twig} = -[C]^T_{Link}$
→ $[C]_{Link} = -[B]^T_{Twig}$

* twig & Link are in sequence.

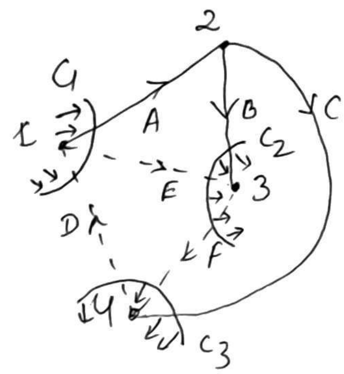
~~(2)~~ (31)

KVL & KCL eq. by cutset Matrix & Tie set matrix




Q) Derive KVL and KCL eq$^n$ using Tie set and cut set matrix, consider A, B, & C branches of tree for given graph.

A Tree = 4-1 = 3 branches




Twig [A, B, C]

Link [D, E, F]

Tie set matrix. (one Link & rest twigs)

Fundamental loops
Direction of loop = Direction of Link

| | | |
|---|---|---|
| D | AC | $L_1$ |
| E | BA | $L_2$ |
| F | BC | $L_3$ |

Branches

| Loops | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $L_1$ | 1 | 0 | 1 | 1 | 0 | 0 |
| $L_2$ | -1 | -1 | 0 | 0 | 1 | 0 |
| $L_3$ | 0 | 1 | -1 | 0 | 0 | 1 |

## Cut set matrix

Fundamental cutset [One twig & rest link]

Direction of twig.

$$\begin{array}{l} \quad\ \ \text{Twig} \ \text{Link} \\ C_1 = \underline{A}\ \underline{E\,D} \\ C_2 = \underline{B}\ \underline{E\,F} \\ C_3 = \underline{C}\ \underline{D\,F} \end{array}$$

| Cut set | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| $C_1$ | 1 | 0 | 0 | -1 | 1 | 0 |
| $C_2$ | 0 | 1 | 0 | 0 | 1 | -1 |
| $C_3$ | 0 | 0 | 1 | -1 | 0 | 1 |

(Branches)

$$[B] = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 1 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{matrix}}_{\text{Twigs}} & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\substack{\text{Link} \\ \downarrow \\ [I]}} \end{bmatrix} \quad ; \quad C = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\text{Twig}} & \underbrace{\begin{matrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{matrix}}_{\text{Link}} \end{bmatrix}$$

* $[B_{link}] = [I]$ ; $\rightarrow$ $[C_{Twig}] = [I]$ ; $[B]_{Twig} = -[C]^T_{link}$

$\rightarrow$ $[C_{link}] = -[B_{Twig}]^T$

* $\rightarrow$ KVL eqⁿ by Tie Set matrix.

$\nearrow$ voltage matrix.

$$[B] \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix} = [0]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

| | | | |
|---|---|---|---|
| $\Rightarrow V_a + V_c + V_d = 0$ — (1) | KVL eqⁿ | | $L_1$ |
| $\Rightarrow -V_a - V_b + V_e = 0$ — (2) | | Tie Set | $L_2$ |
| $\Rightarrow V_b - V_c + V_f = 0$ — (3) | | | $L_3$ |

Current eq$^n$ by Tie set.

$$\Rightarrow [B]^T [I_{set}] = [I_{branch}]$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{set_1} \\ I_{set_2} \\ I_{set_3} \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \\ I_f \end{bmatrix}$$

$$\Rightarrow \begin{aligned} I_a &= I_{set_1} - I_{set_2} \\ I_b &= I_{set_2} + I_{set_3} \\ I_c &= I_{set_1} - I_{set_3} \\ I_d &= I_{set_1} \\ I_e &= I_{set_2} \\ I_f &= I_{set_3} \end{aligned}$$

KCL by cut set matrix

$$\Rightarrow [C]\,[I_{branch}] = [0]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \\ I_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left\{ \begin{aligned} I_a - I_d + I_c &= 0 \quad -(I) \\ I_b + I_c + I_f &= 0 \quad -(II) \\ I_c - I_d + I_f &= 0 \quad -(III) \end{aligned} \right. \quad \left\{ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \right.$$

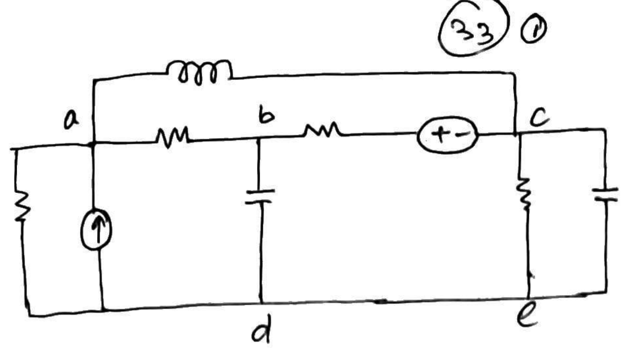
Voltage eq$^n$ by cut set matrix

$$\Rightarrow [C]^T [V_{set}] = [V_{branch}]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_{set_1} \\ V_{set_2} \\ V_{set_3} \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix}$$

$$\Rightarrow \quad \left.\begin{aligned} V_a &= V_{cut\,1} \\ V_b &= V_{cut\,2} \\ V_c &= V_{cut\,3} \\ V_d &= -V_{cut\,1} - V_{cut\,3} \\ V_e &= V_{cut\,1} + V_{cut\,2} \\ V_f &= -V_{cut\,2} + V_{cut\,3} \end{aligned}\right\}$$

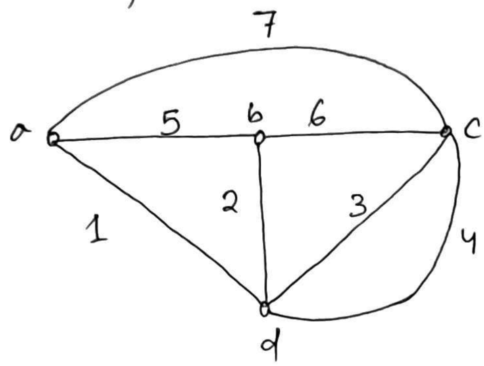
Q11

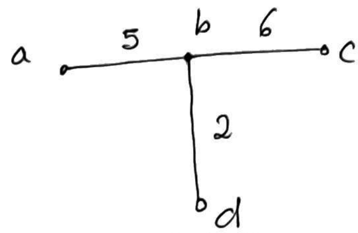


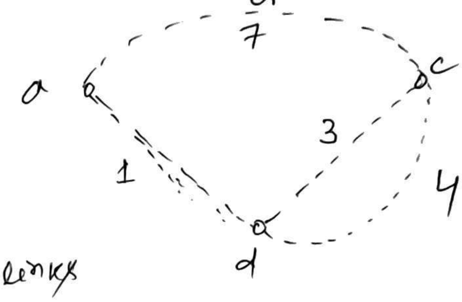

Draw the graph, one tree & co-tree

Ans

$N = 4$ , $B = 7$

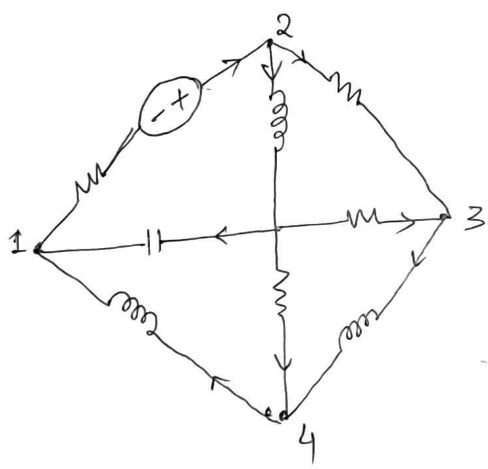


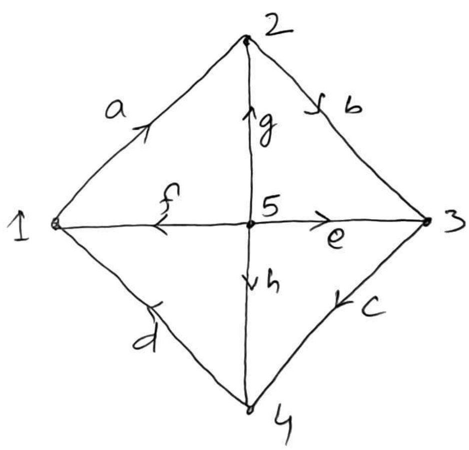




$L = B - N + 1 = 7 - 4 + 1 = 4$ links

Q11




(A) $n = 5$ ; $b = 8$

$a_k = 1$, if the current of branches $k$ leaves the node $i$

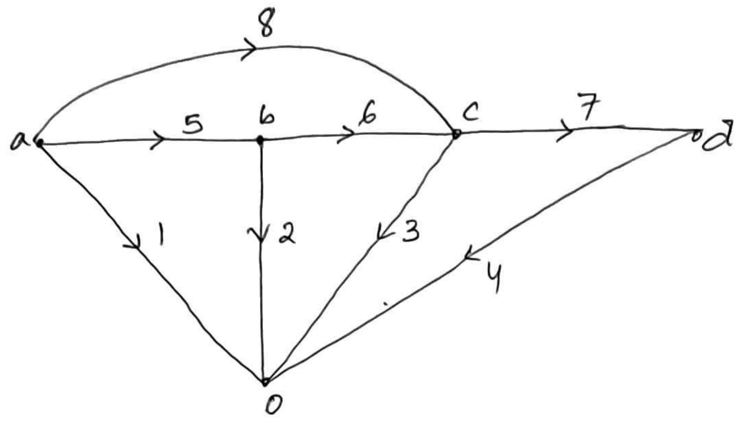
$= -1$, " enters node 'i'

$= 0$, $k$ is not connected with node 'i'.

| Node \ Branches | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|
| 1 | +1 | 0 | 0 | -1 | 0 | -1 | 0 | 0 |
| 2 | -1 | +1 | 0 | 0 | 0 | 0 | -1 | 0 |
| 3 | 0 | -1 | +1 | 0 | -1 | 0 | 0 | 0 |
| 4 | 0 | 0 | -1 | +1 | 0 | 0 | 0 | -1 |
| 5 | 0 | 0 | 0 | 0 | +1 | +1 | +1 | +1 |

Q// Incidence matrix

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
a & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
b & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
c & 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\
d & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0
\end{array}
$$




→ It can be seen that it is a reduced ~~mat.~~ incidence matrix.

Branches 1, 2, 3, & 4 are to be connected to the reference node.

→ Branch 5 betn a & b

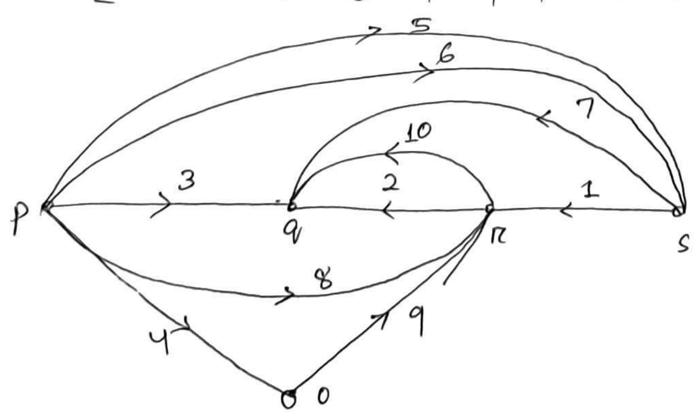
6 " b & c

7 " c & d

8 " a & c.

Q// Incidence matrix

$$
\begin{array}{c|cccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
p & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
q & 0 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\
r & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\
s & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0
\end{array}
$$

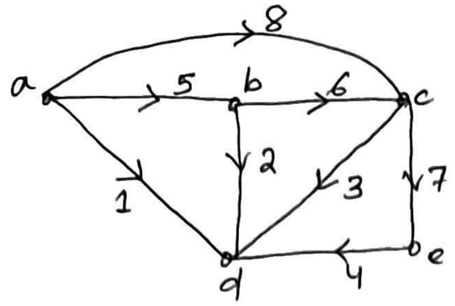



Sum of the elements in columns 4, 9 are not zero.
Therefore it is a reduced matrix.

A

34

Q11




write incidence matrix. Express branch voltage in terms of node voltages and then write a loop matrix and express branch current in terms of loop currents.

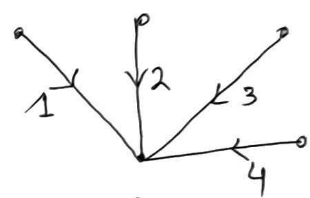
(A)

$$\begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline a & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ b & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ d & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ e & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{array}$$

Branch voltages in terms of node voltages are

$V_1 = e_a - e_d$

$V_2 = e_b - e_d$ etc.




(Tree)

For loop (tie-set) matrix,

$L = B - N + 1 = 8 - 5 + 1 = 4$

Twigs $= (1, 2, 3, 4)$ ; links $= (5, 6, 7, 8)$

one link at a time

$$\begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline i_1 = J_5 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ J_6 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ J_7 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 \\ J_8 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

twigs — Links

branch current in terms of loop currents

$J_1 = -i_1 - i_4$ ; $J_2 = i_1 - i_2$ etc.

Q11 In a singly connected n/w if there are 6 number of branches and 'n' no. of nodes then the no. of independent meshes M and independent nodes N are respectively

$L = 6 - n + 1$ How many current eqn..?

$N = n - 1 \Rightarrow$ How many nodal eqn ? or voltage eq.

Q4 Cutset matrix to graph

$$[C] = \begin{array}{c|ccccccc} & A & B & C & D & E & F & G \\ \hline C_1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ C_2 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ C_3 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ C_4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$[C] = \begin{array}{cccc|ccc|c} D & G & E & A & B & C & F & \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & C_1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & C_2 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & C_3 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & C_4 \end{array}$$

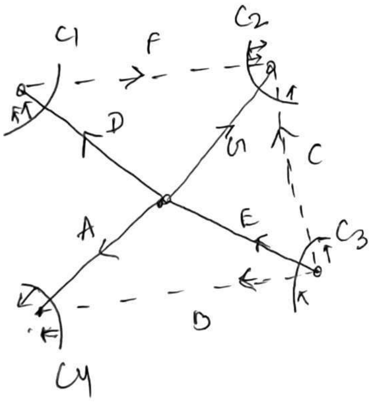
$\underbrace{\qquad\qquad}_{\text{Twigs}}$ $\underbrace{\qquad\qquad}_{\text{Links}}$

Links = Total loops = 3 = L

branches = 7 = b

$\Rightarrow L = b - n + 1 \Rightarrow 3 = 7 - n + 1 \Rightarrow n = 5$ (nodes)

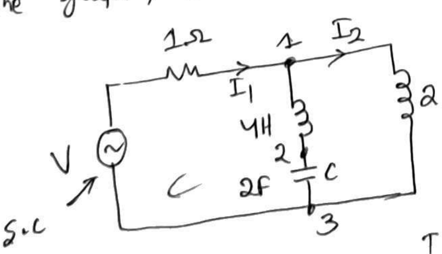
one twig others link

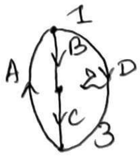



* Twig towards center so that it will not interfere with anyone.
* Links at edges.

Don't solve ✗

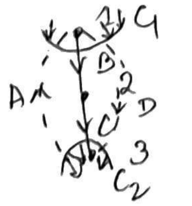
Q4 For the n/w, calculate the following data. The graph, one tree, cut set matrix, KCL equations, voltage eqn.




$n = 3$

Tree $T = n - 1 = 2$




T L

$C_1 = B\,\underline{AD}$

$C_2 = C\,\underline{AD}$

| | A | B | C | D |
|---|---|---|---|---|
| $C_1$ | -1 | 1 | 0 | 1 |
| $C_2$ | -1 | 0 | 1 | 1 |

$$[C] = \begin{bmatrix} -1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -I_a + I_b + I_d &= 0 \\ -I_a + I_c + I_d &= 0 \end{aligned}\right\} \text{KCL}$$

Voltage eqn by cut set.

$$\Rightarrow [C]^T [V_{cut}] = [V_{branch}]$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} [V_{cut_1} \quad V_{cut_2}] = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix}$$

$$\left.\begin{aligned} \Rightarrow V_a &= -V_{cut_1} - V_{cut_2} \\ V_b &= V_{cut_1} \\ V_c &= V_{cut_2} \\ V_d &= V_{cut_1} + V_{cut_2} \end{aligned}\right\} \underline{KVL}$$

Q11 A reduced incidence matrix of a graph is given by.

$$[A_r] = \begin{bmatrix} -1 & 0 & 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 & 0 & 0 \end{bmatrix}_{3\times6}$$

→ 1 row is removed, column wise summation is zero

The no. of possible trees are ____ [UPSC 2002]

Ans Total possible trees $= \text{Det}\left[[A_r][A_r]^T\right]$

$$= \text{Det}\begin{bmatrix} 4 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}_{3\times3}$$

1 row × 1 row
1 row × 2nd row
1 row × 3rd
2nd row × 3rd
3rd row × 3rd row
2nd row × 2nd row.

$$= 4(9-1) + 1(-3-2) - 2(1+6)$$

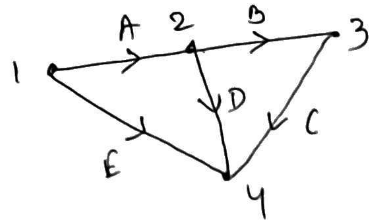
$$= 32 - 5 - 14$$

$$= \underline{13}$$

Some doubts; Total no. of $\left\{\begin{array}{l}\text{voltage} \\ \underline{\text{current}}\end{array}\right.$ eqn $= n-1$

$\left\{\begin{array}{l}\underline{\text{Voltage}} \\ \text{current}\end{array}\right.$ " $= b-n+1$

Q// Incidence matrix $\Rightarrow$ column wise $\Sigma = 0$

Q// 


Branches

| Nodes | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 1 |
| 2 | -1 | 1 | 0 | 1 | 0 |
| 3 | 0 | -1 | 1 | 0 | 0 |
| 4 | 0 | 0 | -1 | -1 | -1 |

$A_i =$ (table above)

reduced incidence matrix $\Rightarrow$ remove any row

$$[A_r] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}_{3\times 5}$$

Total no. Trees $= \text{Det}\left[[A_r][A_r]^T\right]$ Symmetrical $A_r A_r^T$

$$= \text{Det} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}_{3\times 3}$$

$$= 2(6-1) + 1(-2)$$

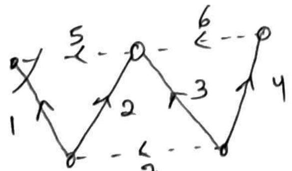
$$= 8$$

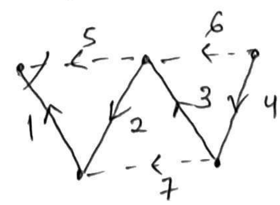
Q// f cut set matrix of a graph is given

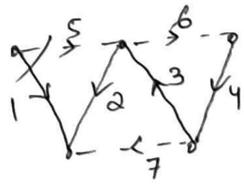
$$Q_F = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

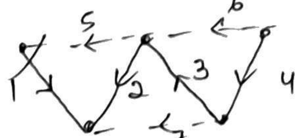
one Twig rest link

The oriented graph of the n/w is

× (a) 


× (b) 


(c) 

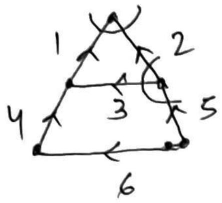

(d) ✓ 

Q11

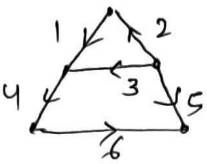
$$Q_F = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$Q_F = \left[\begin{array}{cccccc} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]\begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}$$

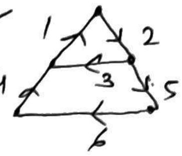
(a) ✗



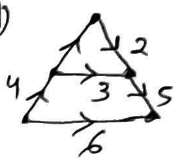

✗ (b)



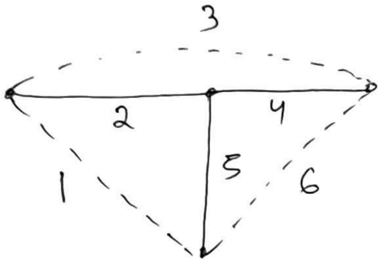

(c) ✓




✗ (d)




Q11

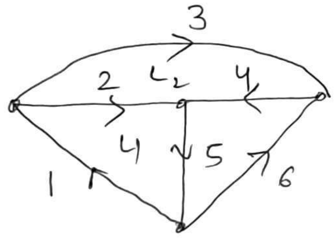



Fundament set matrix / Tie set matrix

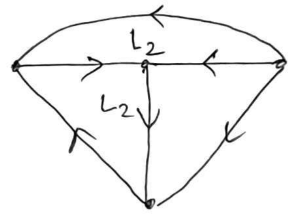
$$B_f = \left[\begin{array}{cccccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]\begin{array}{c} L_1 \\ L_2 \\ L_3 \end{array}$$

one link & rest twig.

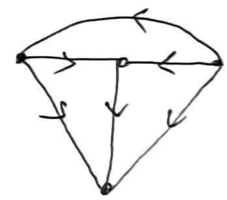
(a) ✓



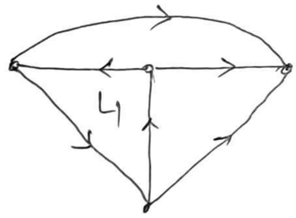

(b) ✗




(c) ✗



(d) ✗



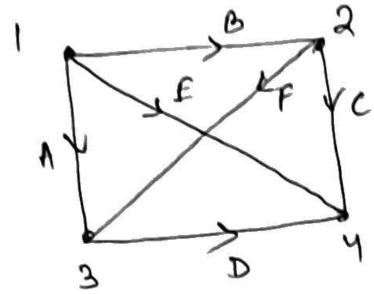

Q11 A reduced incidence matrix of a graph is given

$$A_n = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 \end{bmatrix}$$

The no. of possible trees ____ [UPSC 2015]

Ans '8'

(2)



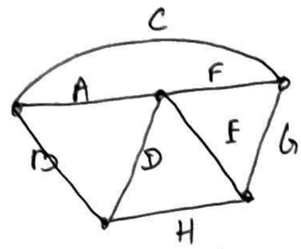

Set of Twig is

x (a) EF ~~(c)~~ CDE

(b) ABEF (d) None

Branches in Tree = $n-1 = 4-1 = 3$

(3)




Tree of the graph

$n-1 = 5-1 = 4$ branches

x (a) CDEH (b) ACFH ~~(c)~~ AFHG (d) AEFG

(4) A n/w has 7 nodes and 5 independent loops. The no. of branches in the n/w is [Gate 98]

$\Rightarrow L = b-n+1$

$\Rightarrow 5 = b-7+1 \Rightarrow b = 11$

(5) For a n/w of 11 branches and 6 nodes, what is the no. of independent loops? [UPSC, 2005]

$\Rightarrow L = b-n+1 = 11-6+1 = 6$

(6) 4 nodes & 3 ind. loops. The no. of branches [UPSC 07]

$L = b-n+1 \Rightarrow 3 = b-4+1 \Rightarrow b = 6$

(7) 10 nodes & 17 branches. The no. of node pair voltage would be [UPSC 2000]

For number of pair $= {}^{n}C_2 = \frac{n(n-1)}{2} = \underline{45}$

(8) The no. of edges in a complete graph of 'n' vertices is UPSC, 03

nodes

no of node pair = No. of edges $= {}^{n}C_2 = \frac{n(n-1)}{2}$

(9) For a connected graph of 'V' vertices and 'e' edges, the no. of meshes is [UPSC 03]

nodes of graph — branches

$L = b-n+1 \Rightarrow e-v+1$

(10) The graph of a n/w has six branches with three tree branches. The min. no. of eqn reqd. for the sol. of the n/w is UPSC 97

$\rightarrow$ Twig = 3, Links = $6-3 = \underline{3}$ (3 loop eqn)

(11) In a n/w, 12 ckt elements and five nodes. What is the min. no. of mesh eqn. UPSC 09

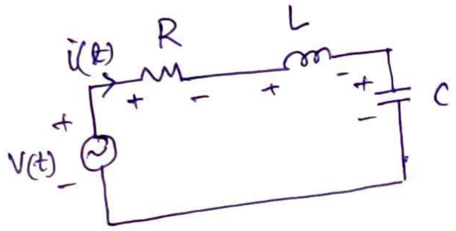
$b = 12$, $n = 5$ $\Rightarrow L = b-n+1 = 8$

→ **Duality Principle**

* The n/w and its dual are equal only w.r.t. the performance, but the elements and connection point of view they are not equal.

| | | | | | |
|---|---|---|---|---|---|
| $i(t)$ | $\longleftrightarrow$ | $v(t)$ | Series | $\longleftrightarrow$ | Parallel |
| $I$ | $\longleftrightarrow$ | $V$ | Star | $\longleftrightarrow$ | Delta |
| $R$ | $\longleftrightarrow$ | $G$ | Thevenin's | $\longleftrightarrow$ | Norton's |
| $L$ | $\longleftrightarrow$ | $C$ | Nodal | $\longleftrightarrow$ | mesh |
| $Z$ | $\longleftrightarrow$ | $Y$ | KCL | $\longleftrightarrow$ | KVL |
| O.C | $\longleftrightarrow$ | S.C | $L\frac{di(t)}{dt}$ | $\longleftrightarrow$ | $C\frac{dv(t)}{dt}$ |
| $R\,i(t)$ | $\longleftrightarrow$ | $G\,v(t)$ | $\frac{1}{C}\int i(t)dt$ | $\longleftrightarrow$ | $\frac{1}{L}\int v(t).dt$ |

Q.



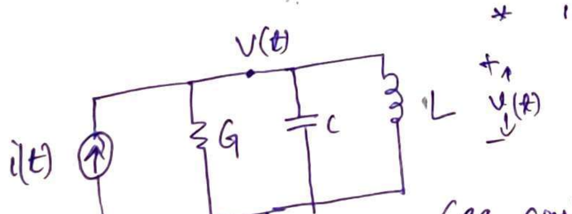

By KVL ⇒

* $V(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$

→ A mesh eqn.

* $i(t) = G\,v(t) + C\frac{d\,v(t)}{dt} + \frac{1}{L}\int v(t)\,dt$

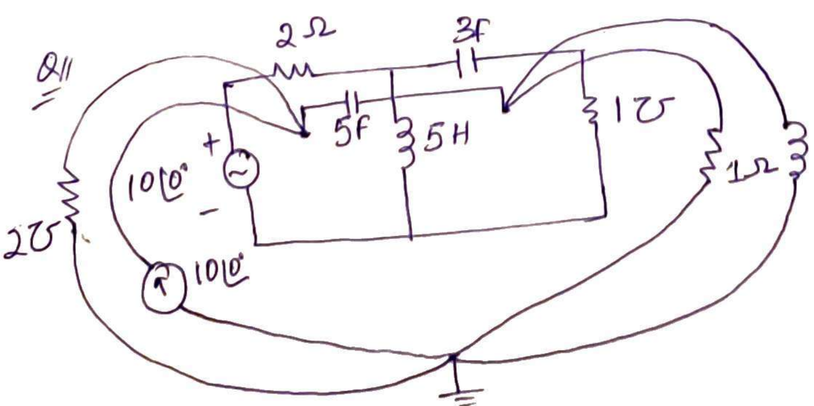
→ A Nodal eqn.




(eq. ckt)

Note : The no. of mesh eqn in the original n/w are equal to the no. of nodal eqns in its dual n/w and vice versa.

Q.

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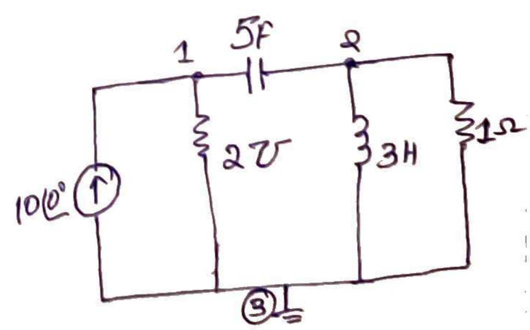
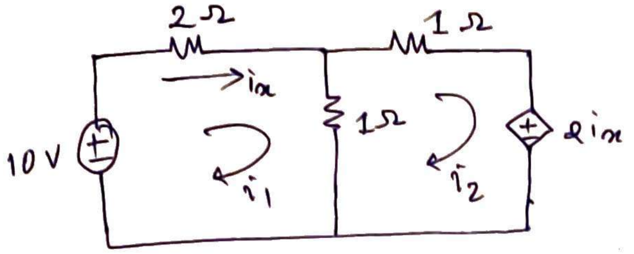


Q11



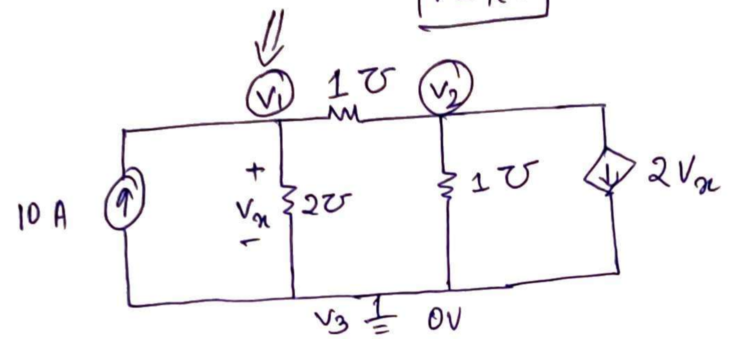

$i_x = i_1$

$\Rightarrow 10 = 2i_1 + 1(i_1 - i_2)$ — (1)

$0 = 1 \cdot i_2 + 2i_x + 1(i_2 - i_1)$ — (2)

mesh eqn

$\boxed{V = Ri}$




$V_x = V_1$

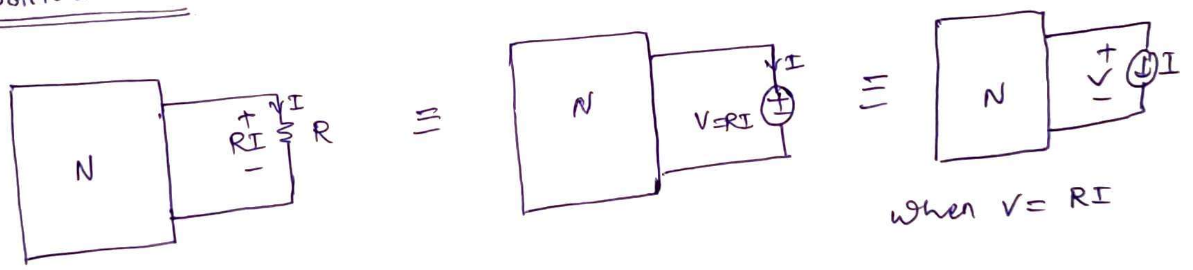
$\Rightarrow 10 = 2V_1 + 1 \cdot (V_1 - V_2)$ — (3)

$\Rightarrow 0 = 1 \cdot V_2 + 2V_x + 1(V_2 - V_1)$ — (4)

$\boxed{i = Gv}$

Obs : In the above problems, both independent sources are delivering the energy and dependent sources are absorbing energy.

(B) <u>The Substitution Thm.</u>

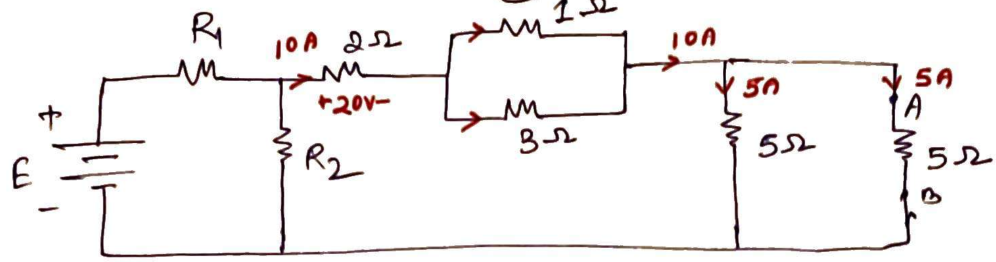



when $V = RI$

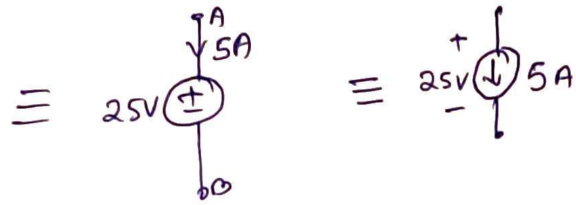
$P_R = P_V = P_I = VI = RI \cdot I = I^2R$ W (abs)

→ In a linear n/w any passive element can be equivalently substituted by an ideal voltage source or an ideal current source, provided all the other branch current and voltages are kept constant, which is possible only when the original passive element and the substituted active sources absorbs the same power.

Q//

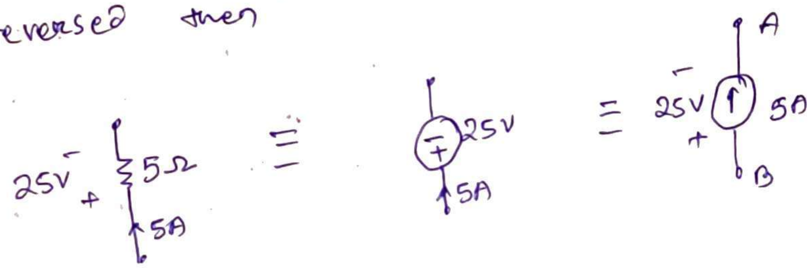



If the voltage across 2Ω is 20V, then 5Ω resistor been the terminals A, B can be replaced by



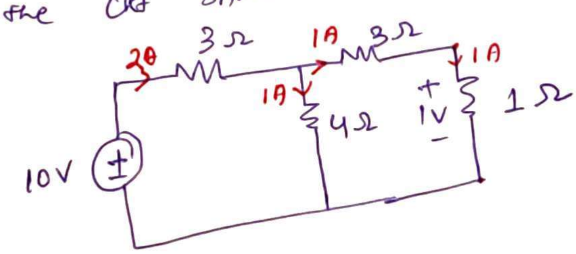

$P_{5\Omega} = P_{25V} = P_{5A} = 25 \times 5 = 125\,W$ (abs)

Q// In the above source if the power polarity of the source are reversed then



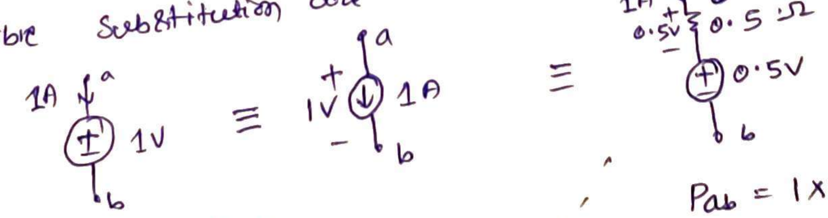

$P_{5\Omega} = P_{25V} = P_{5A} = 25 \times 5 = 125\,W$

Q// In the ckt shown below the 1Ω can be replaced by



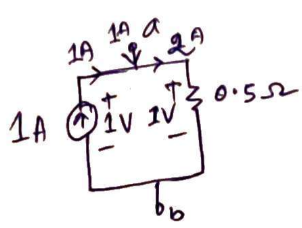

$P_{ab} = 1 \times 1 = 1W$ (abs)

The possible substitution are



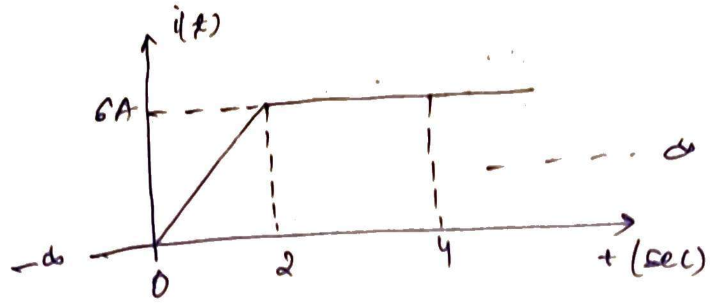

$P_{ab} = 1 \times 1 = 1W$ (abs)

$P_{ab} = 1 \times 0.5 + 1 \times 0.5$
$= 1\,W$ (abs)




$P_{ab} = 1 \times 1$ (del) $+ 2 \times 1$ (abs)
$= 1\,W$ (abs) from the ext 10V source

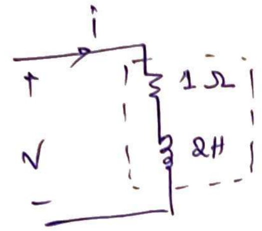
Q11




The current flowing through an inductor of resistance $1\,\Omega$ and inductance 2H. Determine the energy absorbed by the inductor upto the 1st 4 sec.

(a) 98 J (b) 132 J (c) 144 J (d) 168 J

(A)




A coil or a practical inductor or a ckt.

Or

Q11 on this problem, the energy stored up to 4 sec is

(A) only inductor will store the energy → 36 J

it is same up to '$\infty$'.

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{6-0}{2-0} = 3$$

(B)

$i = 3t$ A for $0 \le t \le 2$ sec

$i = 6$A for $2 \le t \le 4$ sec

$R = 1\,\Omega$ : $0 \le t \le 2$sec ; $i = 3t$

$$E_{R_1} = \int_0^2 i^2 R\,dt = \int_0^2 (3t)^2 \cdot 1\,dt = 24\text{ J}$$

$$E_{R_2} = \int_2^4 i^2 R\,dt = \int_2^4 6^2 \cdot 1 \cdot dt = 72\text{ J}$$

$L = 2H$ : $0 \le t \le 2$sec : $i = 3t$

$$E_{L_1} = \int_0^t L\,i\,\frac{di}{dt}\,dt = \int_0^2 2 \cdot 3t \cdot 3 \cdot dt = 36\text{ J}$$

$$E_{L_2} = \int_2^4 L\,i\left(\frac{di}{dt}\right) dt = \int_2^4 2 \cdot 6 \cdot 0\,dt = 0\text{ J}$$

$$E_{abs}\Big|_{t=4sec} = E_{R_1} + E_{R_2} + E_{L_1} + E_{L_2} = 24 + 72 + 36 + 0 = 132\text{ J}$$

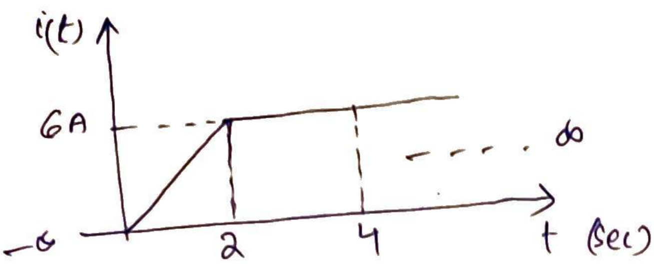
**Note:** ① $E_L\Big|_{t=2sec}$ or from $-\infty$ to 2sec $= E_{L_1} = \frac{1}{2} L \cdot i^2 = \frac{1}{2}\,2 \cdot 6^2 = 36$ J

② $E_L\Big|_{t=4sec}$ $-\infty$ to 4 sec $= E_{L_1} + E_{L_2} = \frac{1}{2} \cdot L \cdot i^2 = \frac{1}{2} \cdot 2 \cdot 6^2 = 36$ J ($E_{L_2} = 0$ J)

When the current through the ideal inductive coil (2H) is constant then the energy of it is zero, since the instantaneous power is zero. $P = Li\frac{di}{dt} = 0W$. Similarly by constant capacitor voltage $P = CV\frac{dv}{dt} = 0W$.

Tutorial (4) (39) 27/09.

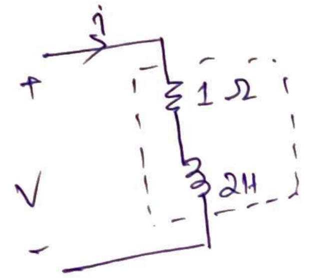
Q1)




The charact. is ⇒ the current flowing through an inductor of resistance 1 Ω and inductance 2H. Determine the energy absorbed by the inductor upto the first 4 sec.

(a) 98 J (b) 132 J (c) 144 J (d) 168 J.

Ans




A coil of practical inductor

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{2-0} = 3$$

$i = 3(t)$ A for $0 \le t \le 2$ sec

$i = 6$ A for $2 \le t \le 4$ sec.

$R = 1\,\Omega$ ; $0 \le t \le 2$ sec ; $i = 3t$

$$E_{R_1} = \int_0^2 i^2 R \, dt = \int_0^2 (3t)^2 \cdot 1 \cdot dt = 24 \text{ J}.$$

$$E_{R_2} = \int_2^4 i^2 R \, dt = \int_2^4 6^2 \cdot 1 \cdot dt = 72 \text{ J}.$$

$L = 2H$ ; $0 \le t \le 2$ sec ; $i = 3t$

$$E_{L_1} = \int_0^t L i \left(\frac{di}{dt}\right) dt = \int_0^2 2 \cdot 3t \cdot 3 \cdot dt = 36 \text{ J}$$

'0' (const. '6')

$$E_{L_2} = \int_2^4 L i \left(\frac{di}{dt}\right) dt = \cancel{2 \cdot 0} = 0 \text{ J}.$$

Total $E_{abs} \Big|_{t=4\text{sec}} = E_{R_1} + E_{R_2} + E_{L_1} + E_{L_2} = 24 + 72 + 36 + 0$

$= 132$ J.

$E_{R_3} = \int_4 i^2 \cdot R\,dt = 24$ J

Note.

① $E_L\Big|_{t=2sec} = E_{L_1} = \frac{1}{2} L \cdot i^2 = \frac{1}{2} \cdot 2 \cdot 6^2 = 36 J$

from $-\infty$ to 2 sec

② $E_L\Big|_{t=4sec} = E_{L_1} + \overset{\Rightarrow 0}{E_{L_2}} = \frac{1}{2} \cdot L \cdot i^2 = \cancel{\frac{1}{2}}\ 36 J$

or

$-\infty$ to 4 sec

→ When the current through the ideal inductive coil (2H) is constant, then the energy of ~~it~~ is zero, since the instanteneous power zero. $p = L i \frac{di}{dt} = 0 W$.

Similarly by constant capacitor voltage $p = C v \frac{dv}{dt} = 0 W$.

Q// In the above problem, the energy stored by the ckt (1 Ω, 2H) up to the first 4 sec is

A Only the inductive part (2H) in a practical inductor (1Ω, 2H) will store the energy, so it is 36 J.

The stored energy is same even upto '∞'

Q// In the above problem, the energy absorbed by the ckt upto '∞'

$$E\Big|_{t=\infty} = E_R\Big|_{t=\infty} + E_L\Big|_{t=\infty}$$

$$= E_{R_1} + \int_2^{\infty} i^2 \cdot R \cdot dt + 36$$

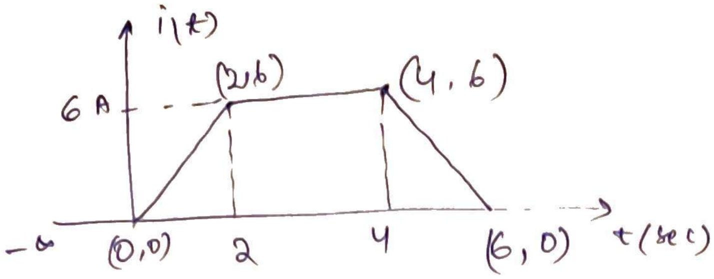
$$= 24 + \int_2^{\infty} 6^2 \cdot 1 \cdot dt + 36$$

$$= 60 + \left[36 t\right]_2^{\infty} = 60 + 36(\infty - 2) = \infty J$$

$$\boxed{\text{absorbed} = \underset{(\text{by } L \& C)}{\text{stored}} + \underset{(\text{by } R)}{\text{dissipated}}}$$

40

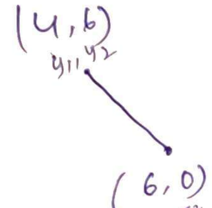
(2)




Energy stored by the ckt upto 6 sec.

Ans

$$\left[E_{stored}\Big|_{t=6\,sec} = E_L\Big|_{t=6\,sec} = \frac{1}{2}L\cdot i^2 = \frac{1}{2}\cdot 2\cdot 0^2 = \underline{0\,J}\right]$$




$$S = \frac{0-6}{6-4} = -3$$

Derivation / explanation

(6,0) $\quad Sl = -3$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow i - 0 = -3(t-6)$$

$$\Rightarrow i = -3(t-6)\ A \qquad (4 \le t \le 6\ sec)$$

$$E_{L_3} = \int_4^6 L\, i\, \frac{di}{dt}\, dt = \int_4^6 2\cdot -3\,(t-6)\cdot -3\, dt$$

$$= \frac{18}{9}\left[\frac{t^2}{2}\right]_4^6 = 36 - 16 - 6[\ldots]$$

$$= -36\,J$$

$$E_{stored}\Big|_{t=6\,sec} = E_{L_1} + E_{L_2} + E_{L_3} = 36J + 0 + (-36\,J)$$

$$= \underline{0\,J}$$

(ii) Energy absorbed by the ckt upto 6 sec.

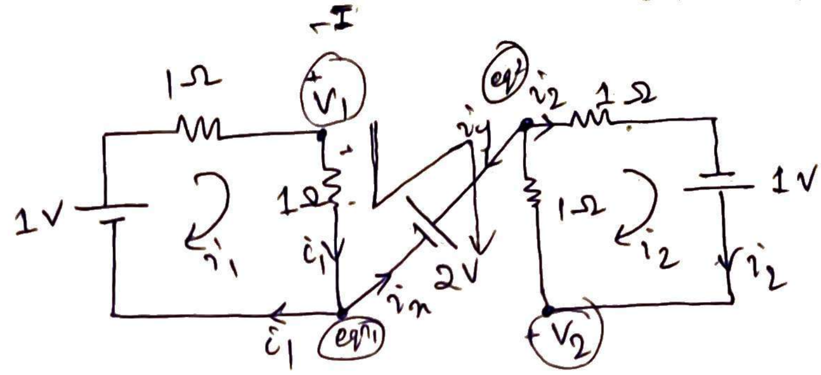
$$E_{abs}\Big|_{t=6sec} = E_{R_1} + E_{R_2} + E_{R_3} + E_{L_1} + E_{L_2} + E_{L_3}$$

$$= 24 + 72 + 24 + 36 + 0 - 36$$

$$= \underline{120\,J}$$

$$E_{R_3} = \int_4^6 i^2 \cdot R\, dt \Rightarrow \int_4^6 \{-3(t-6)\}^2 \cdot 1\, dt = \underline{24\,J}$$

Q11 Det. the relation betn $V_1$ & $V_2$




By KCL

$\Rightarrow -i_1 + i_1 + i_x = 0$ — (I)

$\Rightarrow -i_2 + i_2 + i_y = 0$ — (II)

so, $i_x = i_y = 0$

Since no current flows through 2V source, both the meshes are independent to each other.

By KVL

$\Rightarrow 1 - 1i_1 - 1i_1 = 0$ mesh (1)

$\Rightarrow 1 = 2i_1 \Rightarrow i_1 = 0.5A$

mesh (2) $1 - i_2 - i_2 = 0$

$i_2 = 0.5A$

By KVL

$V_1 - 1i_1 + 2 + 1i_2 - V_2 = 0$

$\Rightarrow \boxed{V_1 + 2 = V_2}$

So, $V_2$ is 2V higher than $V_1$.

Pending

→ The no. of possible ~~complete~~ ~~graph~~ trees of a complete graph with 'n' nodes

$= n^{n-2}$

→ The rank of incidence matrix = $n-1$ = rank of the graph

→ The rank of f-loop matrix = $b-n+1$ = links.

→ For a complete graph there are $n^{n-2}$ f-loop matrices

→ f-loop : minimum no. of loop or mesh eqn required to solve the corresponding n/w.

→ f-cutset : minimum no. of nodal eqns required to solve the corresponding n/w.

→ The rank of the f-cutset matrix = $n-1$ = twigs

→ For a complete graph there are $n^{n-2}$ $Q_C$ matrices.

Q11 A n/w consists of 17 branches and 10 nodes, Det. the no. of equations reqd. to solve the corresponding n/w.

(A) nodal eqn = f-cutsets = $n-1 = 9$

mesh eqn = f-loops = $b-n+1 = 8$

So, the no. of eqn reqd. = minimum of nodal, mesh eqn

= 8.

—o—

~~Coupled~~