Engineering Electromagnetics

Lecture 27

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by

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Magnetic vector potential

Just as $\nabla \times \mathbf{E} = \mathbf{0}$ permitted us to introduce a scalar potential (V) in electrostatics,

$$\mathbf{E} = -\nabla V,$$

so $\nabla \cdot \mathbf{B} = 0$ invites the introduction of a *vector* potential $\mathbf{A}$ in magnetostatics:

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}.} \tag{5.61}$$

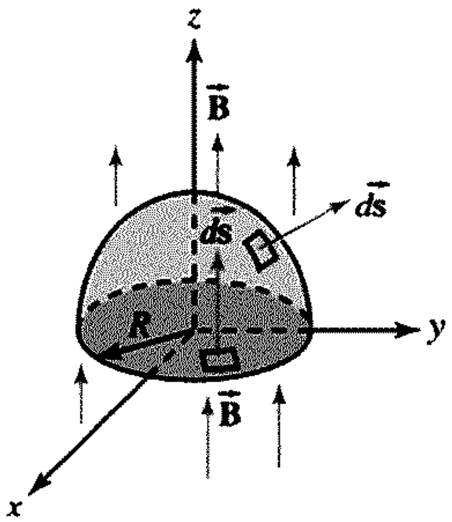
Problem-4

- Which **A** produces a **B** along negative x direction?
- $\frac{z}{2}\hat{y}$
- $z\hat{x} + x\,\hat{z}$
- $\mathrm{x}\hat{x} + y\hat{y}$
- $-x\hat{x} - y\hat{y} - z\hat{z}$

Problem-1

If $\vec{\mathbf{B}} = B\vec{\mathbf{a}}_z$, compute the magnetic flux passing through a hemisphere of radius R centered at the origin and bounded by the plane $z = 0$.

Solution The hemisphere and the circular disc of radius R form a closed surface, as illustrated in Figure 5.17; therefore, the flux passing through the hemisphere must be exactly equal to the flux passing through the disc. The flux passing through the disc is



$$\Phi = \int_s \vec{\mathbf{B}} \cdot \vec{ds} = \int_0^R \int_0^{2\pi} B\rho \, d\rho \, d\phi = \pi R^2 B$$

The reader is encouraged to verify this result by integrating over the surface of the hemisphere. • • •

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$$\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J}.$$

$$\boxed{\nabla \cdot \mathbf{A} = 0.}$$

$$\boxed{\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}.}$$

This *again* is nothing but Poisson's equation—or rather, it is *three* Poisson's equations, one for each Cartesian[19] component. Assuming $\mathbf{J}$ goes to zero at infinity, we can read off the solution:

$$\boxed{\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}} \, d\tau'.}$$

How??

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int_c \frac{\vec{d\ell}' \times \vec{\mathbf{R}}}{R^3}$$

$$\vec{\mathbf{R}} = (x - x')\vec{\mathbf{a}}_x + (y - y')\vec{\mathbf{a}}_y + (z - z')\vec{\mathbf{a}}_z$$

$$\nabla\left(\frac{1}{R}\right) = -\frac{\vec{\mathbf{R}}}{R^3} \Rightarrow \vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int_c \nabla\left(\frac{1}{R}\right) \times \vec{d\ell}'$$

$$\nabla\left(\frac{1}{R}\right) \times \vec{d\ell}' = \nabla \times \left[\frac{\vec{d\ell}'}{R}\right] - \frac{1}{R}[\nabla \times \vec{d\ell}']$$

Because the curl operation is with respect to the unprimed coordinates of point $P(x, y, z)$, $\nabla \times \vec{d\ell}' = 0$. Thus, from (5.25), we have

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int_c \nabla \times \left[\frac{\vec{d\ell}'}{R}\right]$$

The integration and the differentiation are with respect to two different sets of variables, so we can interchange the order and write the preceding equation as

$$\vec{\mathbf{B}} = \nabla \times \left[\frac{\mu_0 I}{4\pi} \int_c \frac{\vec{d\ell}'}{R}\right] \tag{5.26}$$

Comparing (5.24) and (5.26), we obtain an expression for the magnetic vector potential $\vec{\mathbf{A}}$ as

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \int_c \frac{I\,\vec{d\ell}'}{R} \tag{5.27a}$$

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \oint_c \frac{I\,\vec{d\ell}'}{R}$$

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \int_v \frac{\vec{\mathbf{J}}_v\,dv'}{R}$$

Magnetic flux i.t.o. A

We can also express the magnetic flux Φ in terms of $\mathbf{A}$ as

$$\Phi = \int_s \vec{\mathbf{B}} \cdot \overrightarrow{d\mathbf{s}} = \int_s (\boldsymbol{\nabla} \times \vec{\mathbf{A}}) \cdot \overrightarrow{d\mathbf{s}}$$

A direct application of Stokes' theorem yields

$$\Phi = \oint_c \vec{\mathbf{A}} \cdot \overrightarrow{d\ell}$$

where c is the contour bounding the open surface s.

Magnetic field intensity

- $D = \varepsilon E$
- Magnetic field intensity H in free space is $H = B/\mu_0$
- $B = \mu_0 H$
- What is Ampere's circuital law in terms of H then?

Thank You