Engineering Optics

Lecture 42

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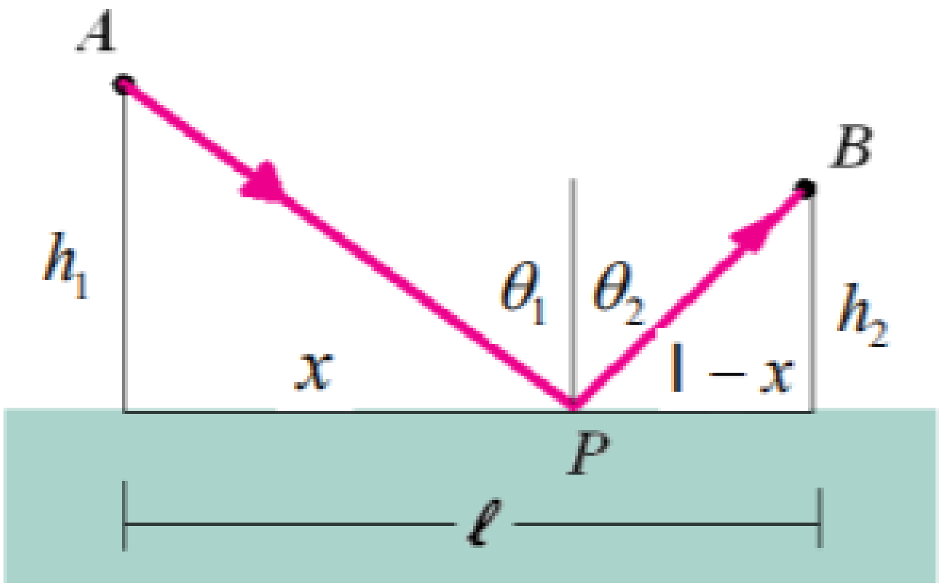
by

Debolina Misra

Assistant Professor in Physics
IIITDM Kancheepuram, Chennai, India

Fermat’s principle

- Fermat's principle →determines the path of the rays
- According to this principle the ray will correspond to that path for which the time taken is an extremum in comparison to nearby paths, i.e., it is either a minimum or a maximum or stationary.



$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c}$$

http://scipp.ucsc.edu/~haber/ph5B/fermat09.pdf

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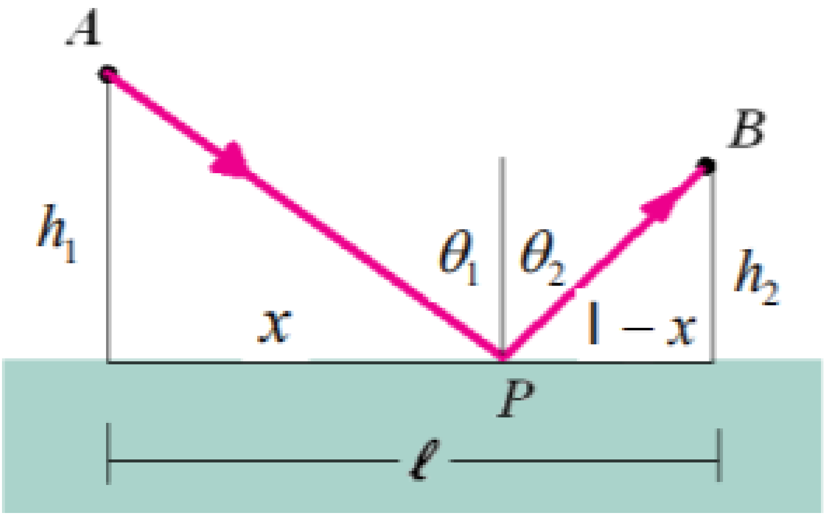

$$t = \frac{\sqrt{x^2+h_1^2}}{c} + \frac{\sqrt{(l-x)^2+h_2^2}}{c}$$

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2+h_1^2}} + \frac{-(l-x)}{c\sqrt{(l-x)^2+h_2^2}}$$

$$\frac{x}{\sqrt{x^2+h_1^2}} = \frac{(l-x)}{\sqrt{(l-x)^2+h_2^2}}$$

$$\sin\theta_1 = \sin\theta_2 \quad \rightarrow \quad \boxed{\theta_1 = \theta_2}$$

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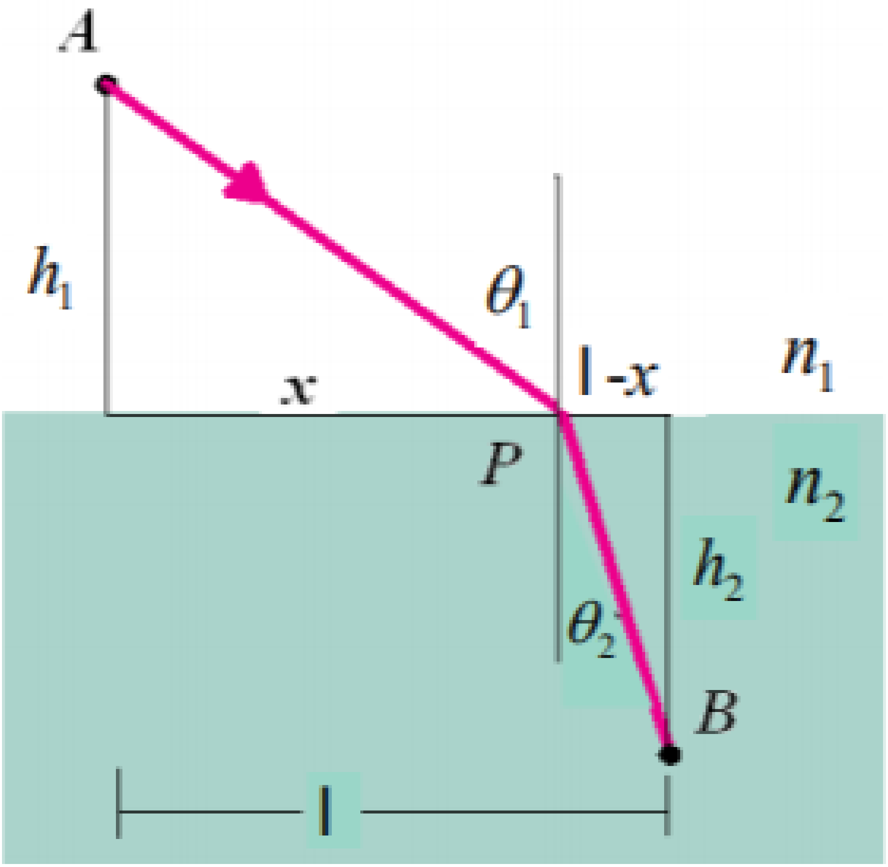
Fermat's principle



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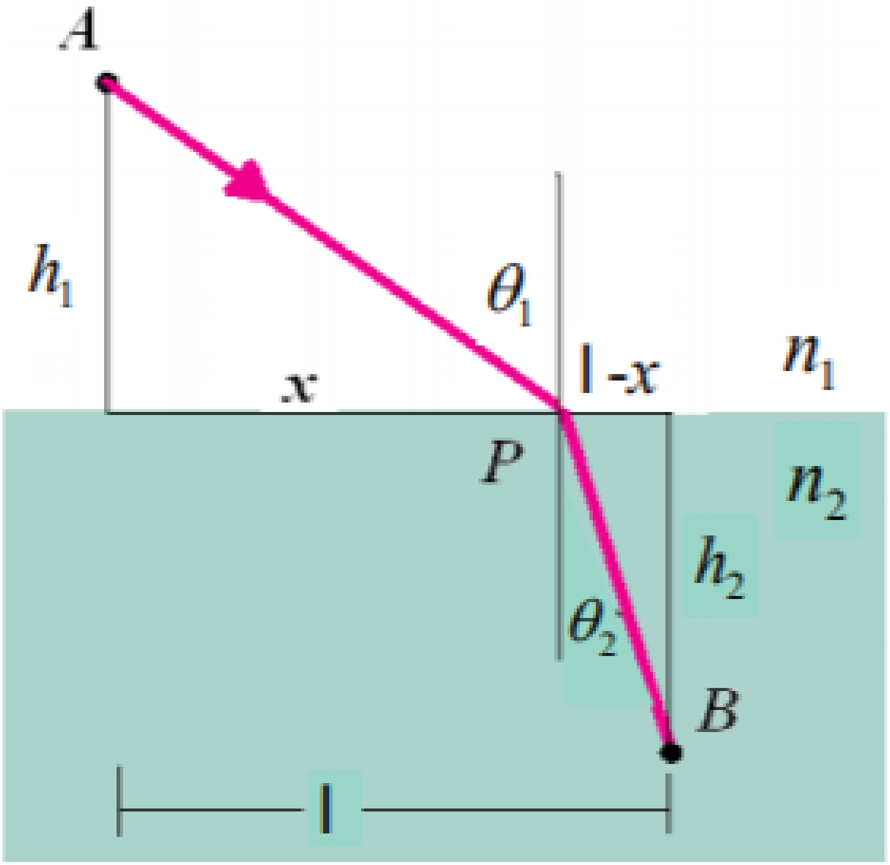
Fermat's principle



$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c/n_2}$$

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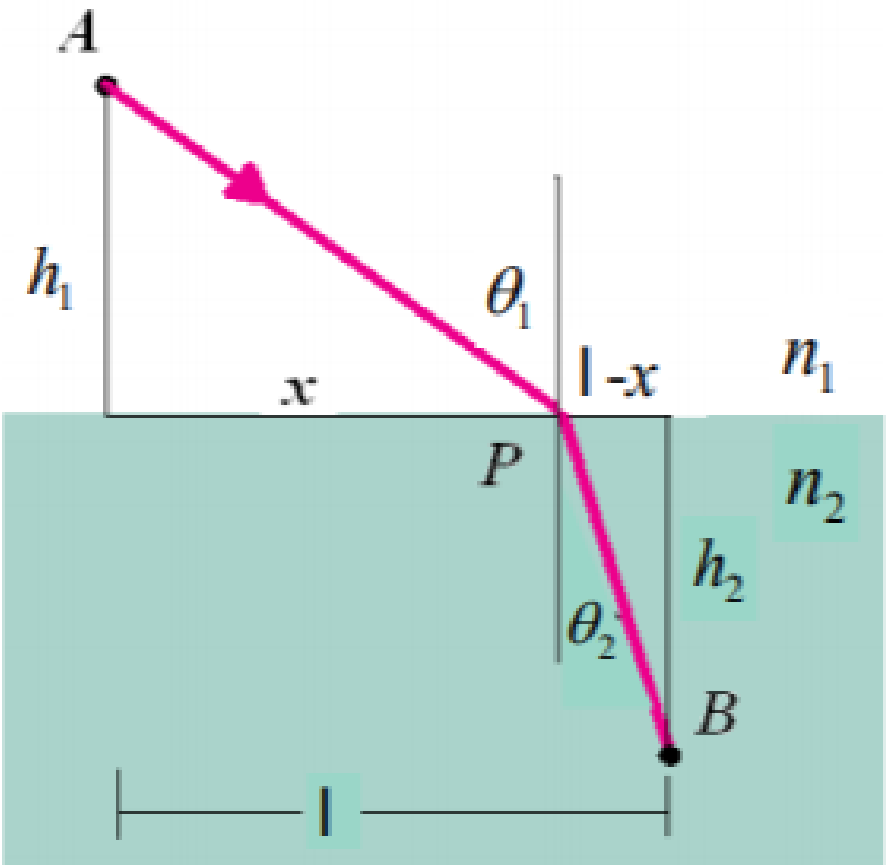
Fermat’s principle



$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c/n_2}$$

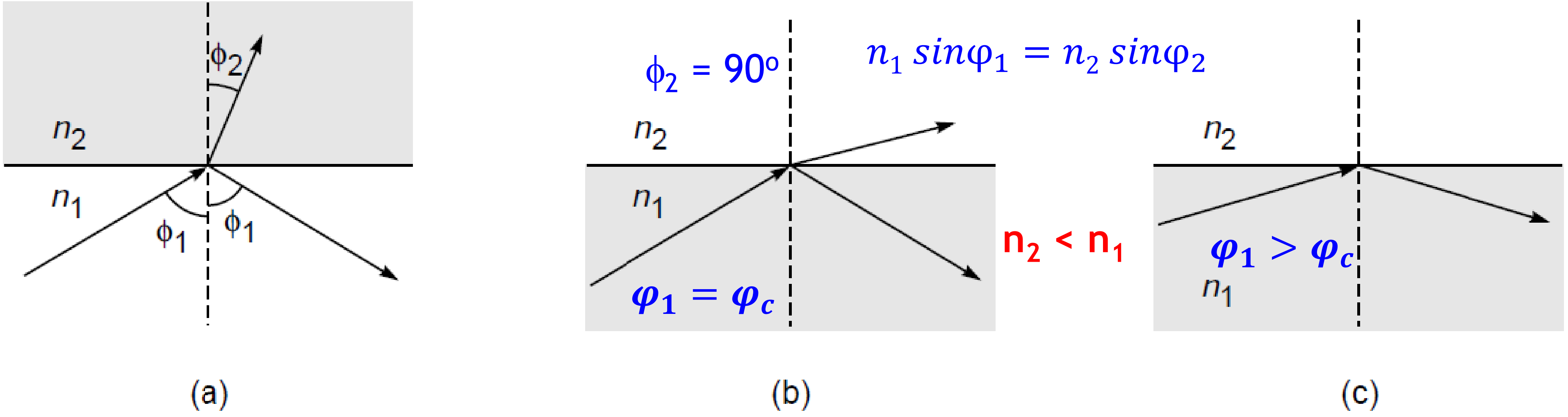
$$0 = \frac{d\,t}{dx} = \frac{n_1 x}{c\sqrt{x^2 + h_1^2}} + \frac{-n_2 (l - x)}{c\sqrt{(l - x)^2 + h_2^2}}$$

$$\frac{n_1 x}{\sqrt{x^2 + h_1^2}} = \frac{n_2 (l - x)}{\sqrt{(l - x)^2 + h_2^2}}$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

http://scipp.ucsc.edu/~haber/ph5B/fermat09.pdf

Total internal reflection



(a) For a ray incident on a denser medium ($n_2 > n_1$), the angle of refraction is less than the angle of incidence. (b) For a ray incident on a rarer medium ($n_2 < n_1$), the angle of refraction is greater than the angle of incidence. (c) If the angle of incidence is greater than critical angle, it will undergo total internal reflection.

- if a ray is incident at the interface of a rarer medium ($n_2 < n_1$), then the ray will bend away from the normal
- The angle of incidence, for which the angle of refraction is 90°, is known as the critical angle and is denoted by ϕ_c.
- When $\varphi_1 = \varphi_c = \sin^{-1}\frac{n_2}{n_1}$ → angle of refraction ϕ_2 = 90°
- If $\phi_1 > \phi_c$, there is no refracted ray and we have what is known as total internal reflection.

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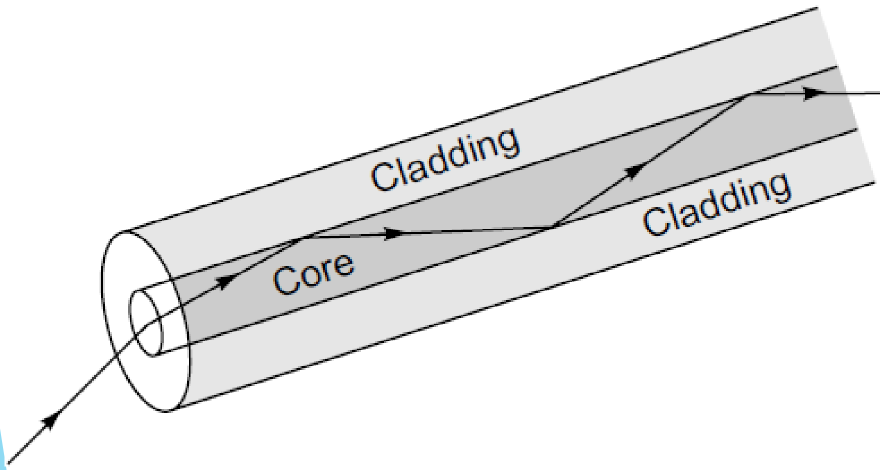
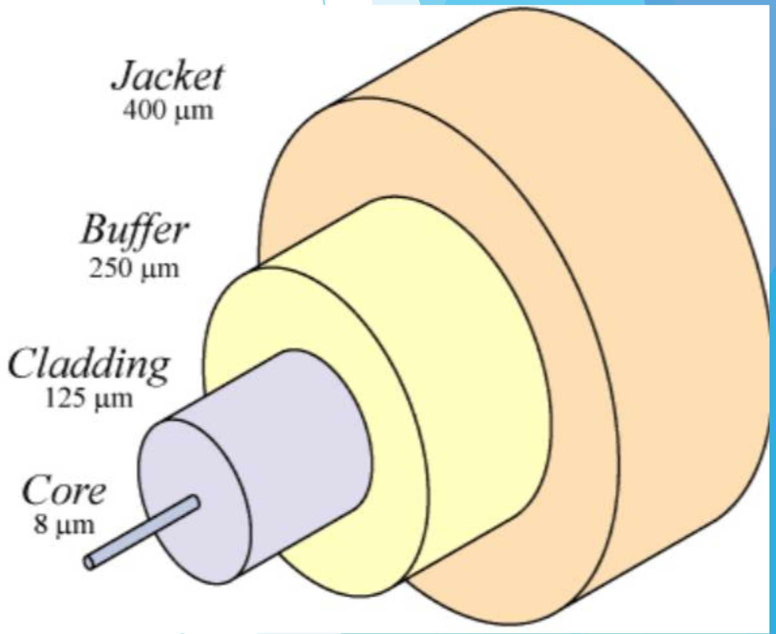
The optical fiber



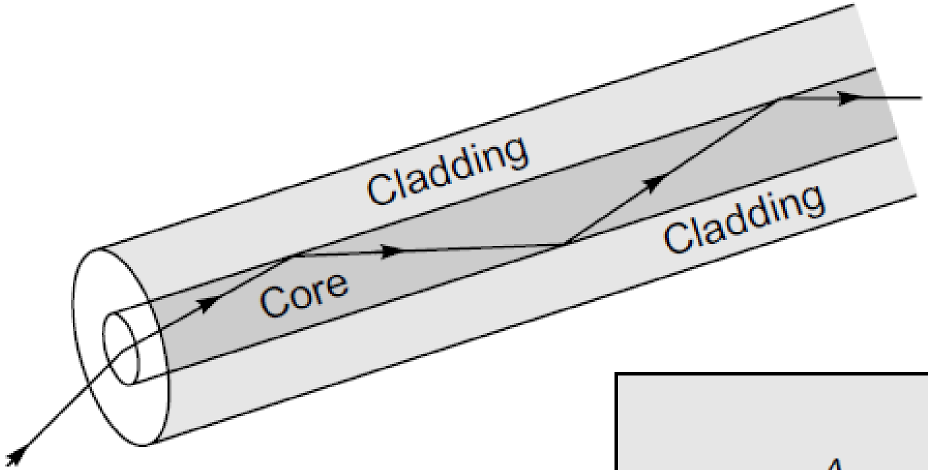
Optics, Hecht

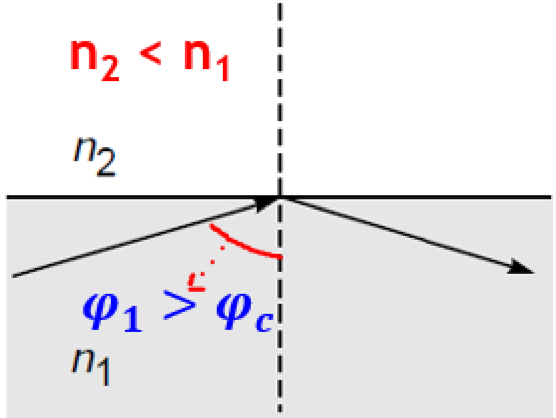
- **central dielectric core cladded by a material of slightly lower refractive index**
- Core/cladding: low loss light propagation
- Buffer/jacket: protection against mechanical damage and the environment (UV radiation, humidity, etc.)

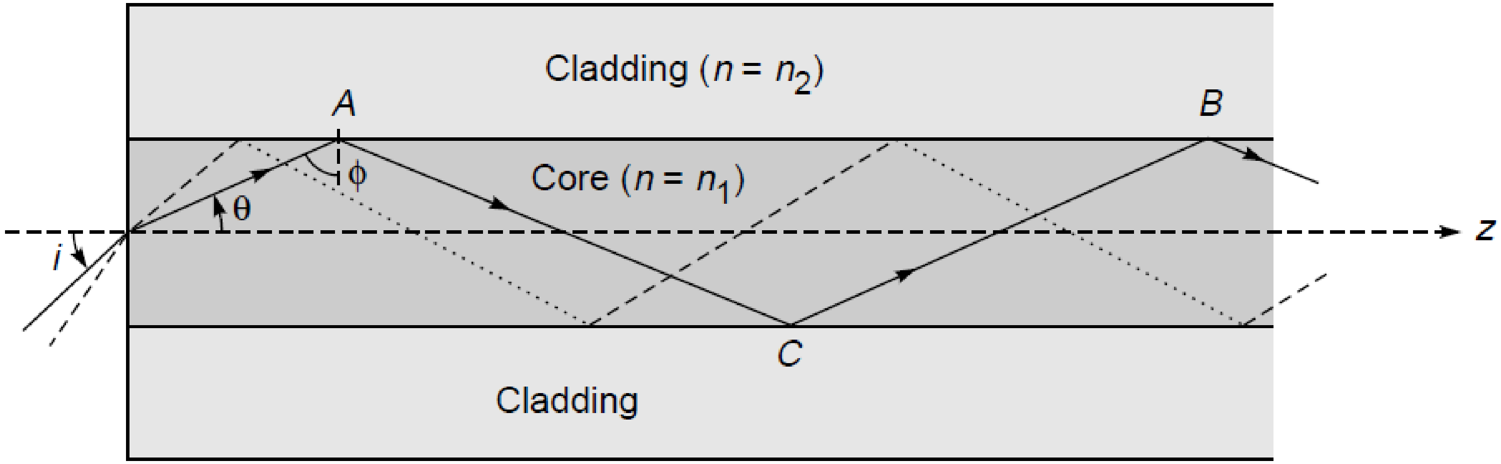


https://ocw.mit.edu/courses/materials-science-and-engineering/3-071-amorphous-materials-fall-2015/lecture-notes/MIT3_071F15_Lecture13.pdf

Optical fiber

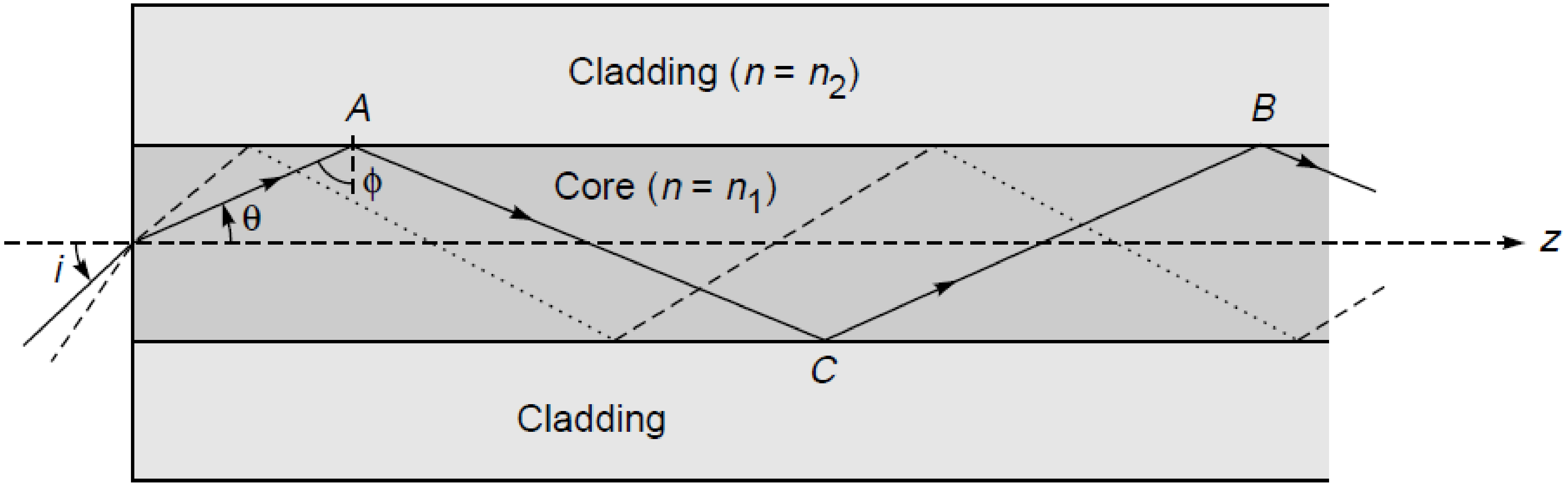






- a cylindrical central dielectric core **cladded by a material of slightly lower refractive index.**

Optical fiber



- Now, for a ray entering the fiber, if the angle of incidence (at the core-cladding interface) is greater than the critical angle ϕ_c, then the ray will undergo TIR at that interface.
- Thus, for TIR to occur at the core-cladding interface

$$\phi > \phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

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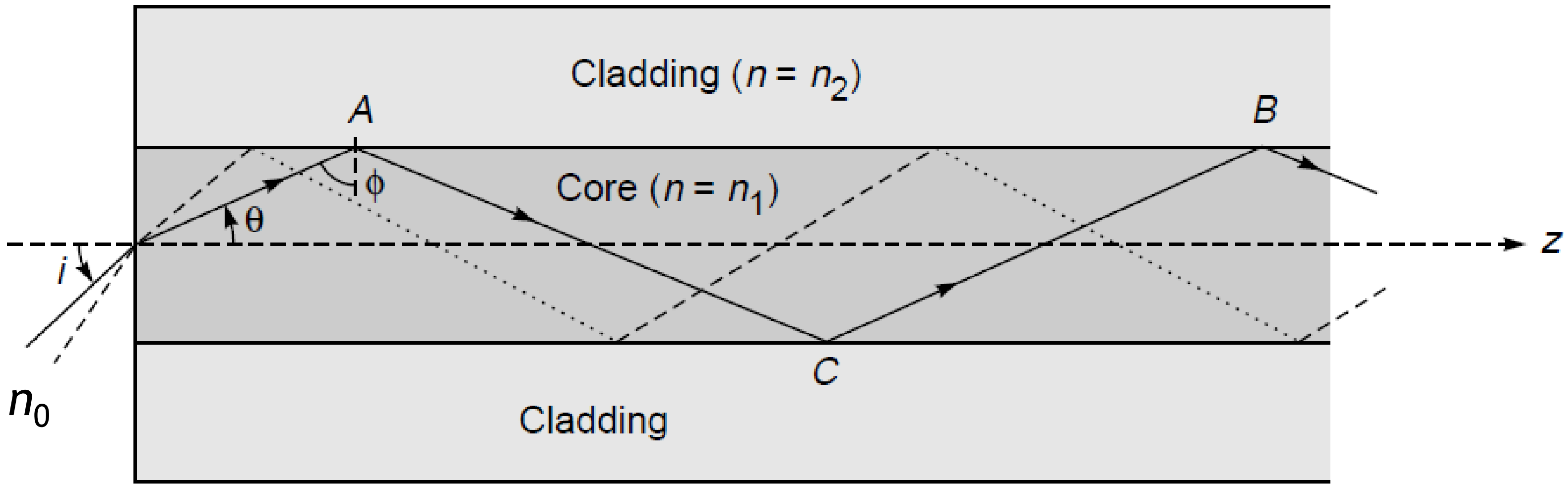
Solution



$$\sin\theta < \sqrt{1-\left(\frac{n_2}{n_1}\right)^2} \qquad \frac{\sin i}{\sin\theta} = \frac{n_1}{n_0}$$

$$\sin i < \frac{n_1}{n_0}\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \sqrt{\frac{\left(n_1^2 - n_2^2\right)}{n_0^2}}$$

In most cases, the outside medium is air, i.e., $n_0 = 1$

If max. value of i possible = i_m → $\boxed{\sin i_m = \sqrt{n_1^2 - n_2^2}}$

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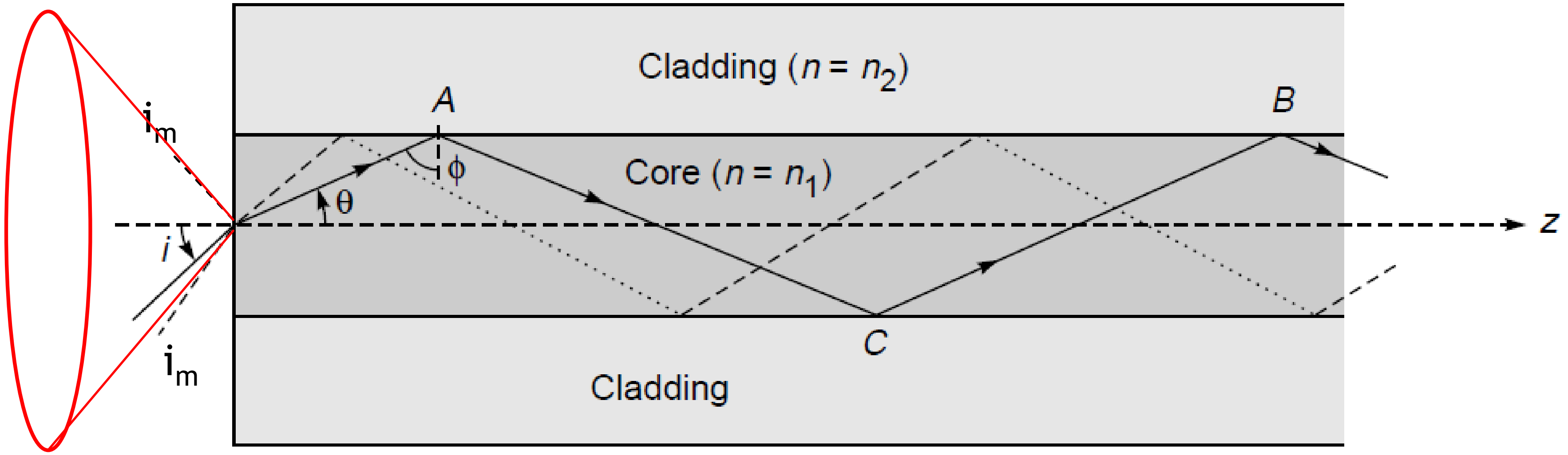
Numerical aperture



if light is incident on one end of the fiber, it will be guided through it provided $i < i_m$.

The quantity **sin i_m is known as the numerical aperture (NA)** of the fiber and is a measure of the light-gathering power of the fiber.

$$\boxed{\mathrm{NA} = \sqrt{n_1^2 - n_2^2}} \qquad \left(\sin i_m = \sqrt{n_1^2 - n_2^2}\right)$$

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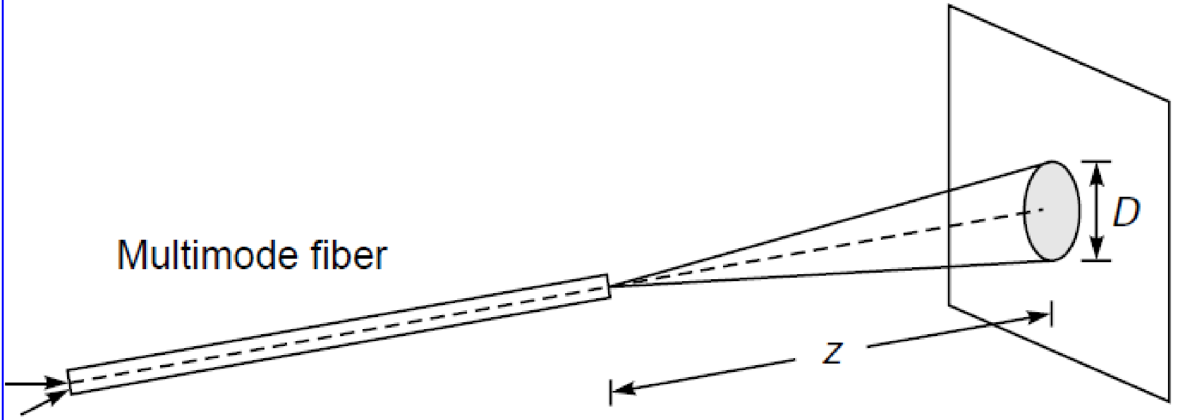
Calculation of N.A.

Now, in a short length of an optical fiber, if all rays between $i = 0$ and i_m are launched, then, the light coming out of the fiber will also appear as a cone of semiangle i_m emanating from the fiber end. If we now allow this beam to fall normally on a white paper (see Fig. 27.11) and measure its diameter, we can easily calculate the NA of the fiber. This allows us to estimate the NA of the optical fiber by a very simple experiment.



Measurement of the diameter D of the spot on a screen placed at a far-field distance z from the output end of a multimode fiber can be used to measure the NA of the fiber.

$$\boxed{\mathrm{NA} = \sin i_m = \sin\left[\tan^{-1}\left(\frac{D}{2z}\right)\right]}$$

Q: NA will then change with D and z ?

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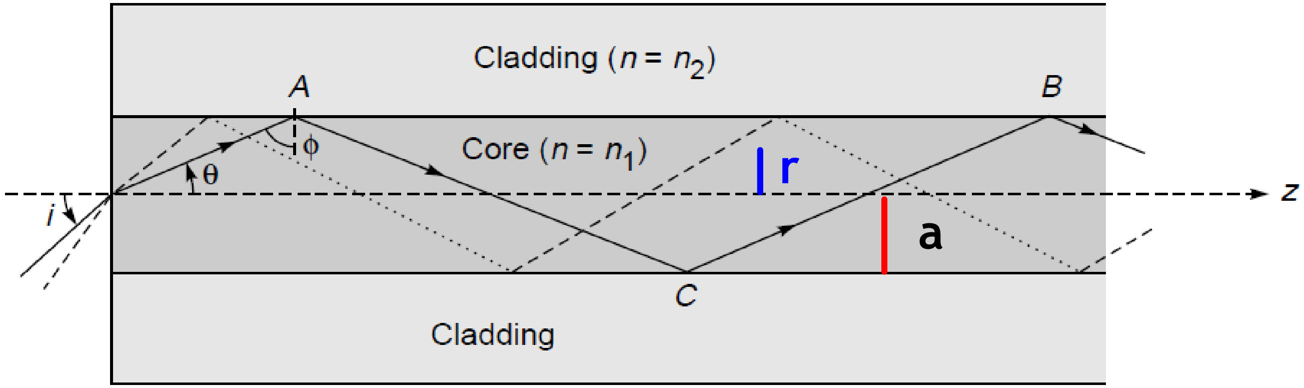
Optical fiber: refractive index



- refractive index distribution in optical fiber (in the transverse direction) is given by

$$n = \begin{cases} n_1 & 0 < r < a \\ n_2 & r > a \end{cases}$$

- where n_1 and n_2 (< n_1) → r.i. of core and cladding and 'a' represents the radius of the core.

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We define a parameter Δ

$$\boxed{\Delta \equiv \frac{n_1^2 - n_2^2}{2n_1^2}}$$

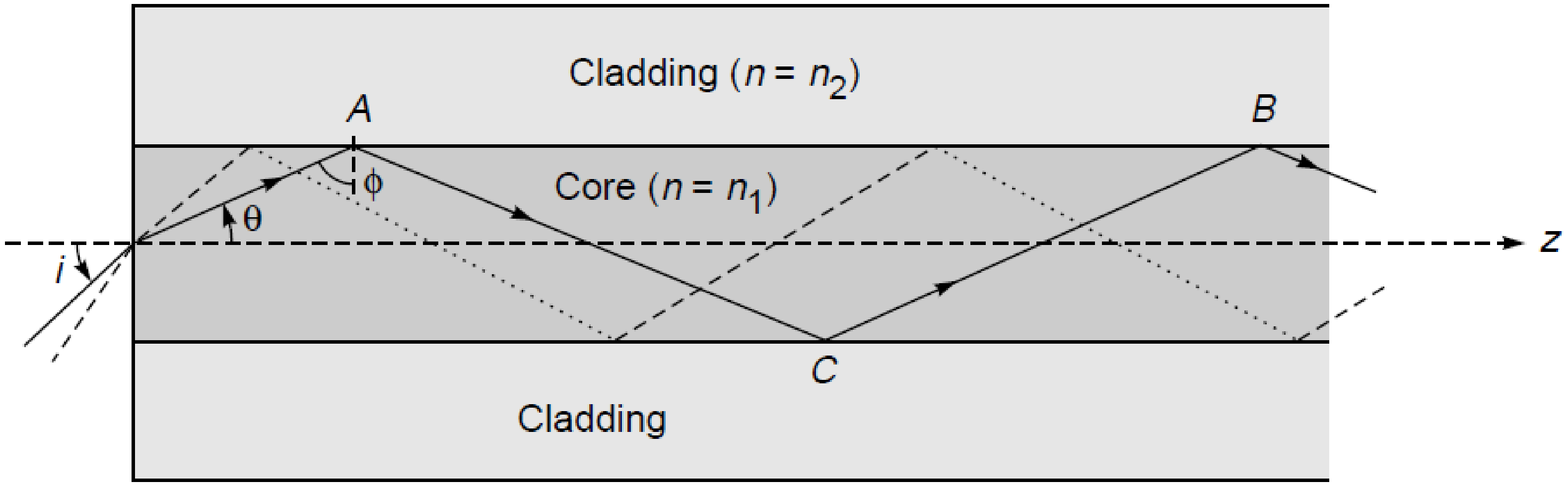
for most silica fibers, Δ <<1
as $n_1 \approx n_2$

$$\Delta = \frac{n_1 - n_2}{n_1} \frac{n_1 + n_2}{2\,n_1} \approx \frac{n_1 - n_2}{n_2} \approx \frac{n_1 - n_2}{n_1}$$

For a fiber, if 'a' ≈25 μm,
Cladding → pure silica with n_2 ≈1.45
Δ ≈ 0.01,
r.i. n_1 of the core = ?
core is usually silica doped with germanium;
doping by germanium increases refractive index.

Problem-1

- For an optical fiber with n_1=1.45 and Δ=0.01, calculate the NA.



Hint: if n_1 and Δ are given what is n_2 ?

Solution

$$\boxed{\mathrm{NA} = \sqrt{n_1^2 - n_2^2}}$$

$\Delta = 0.01$ and $n_1 = 1.45$

$\Delta = (n_1 - n_2)/n_1$

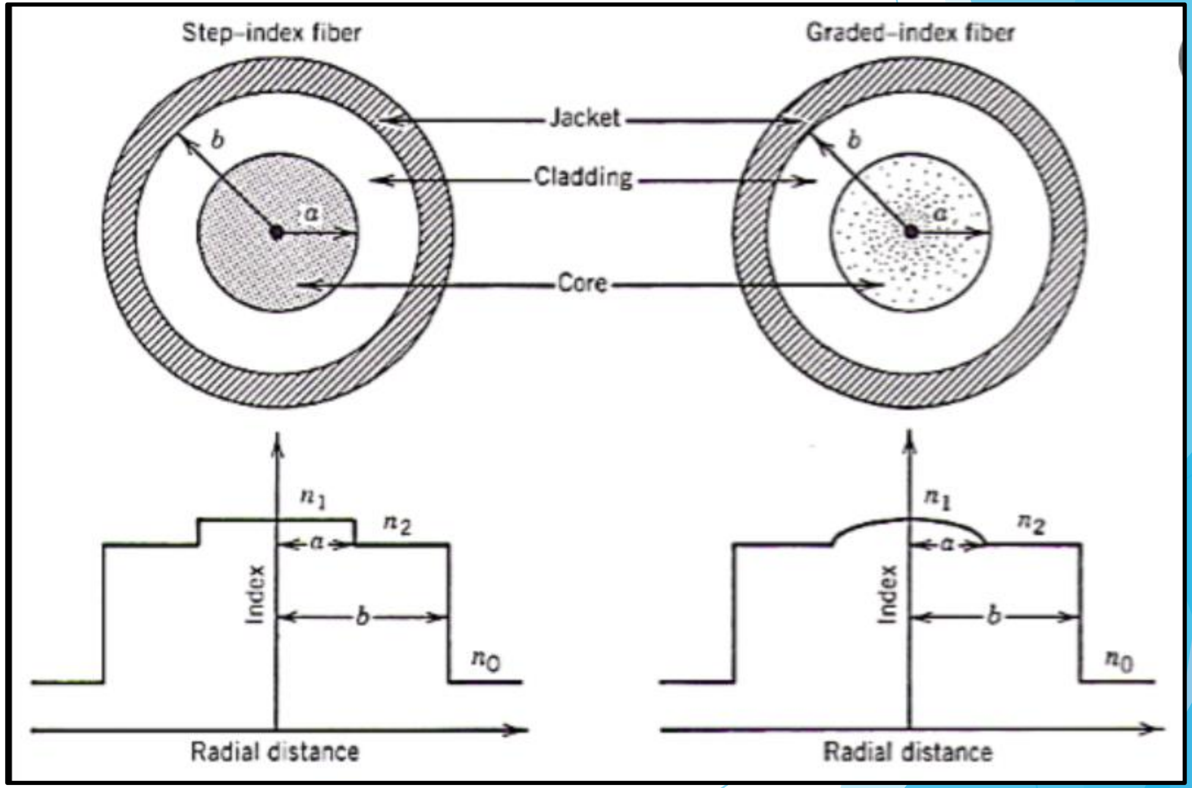
$n_2 = 1.4355$

NA = 0.2045

Step indexed and graded indexed fibers

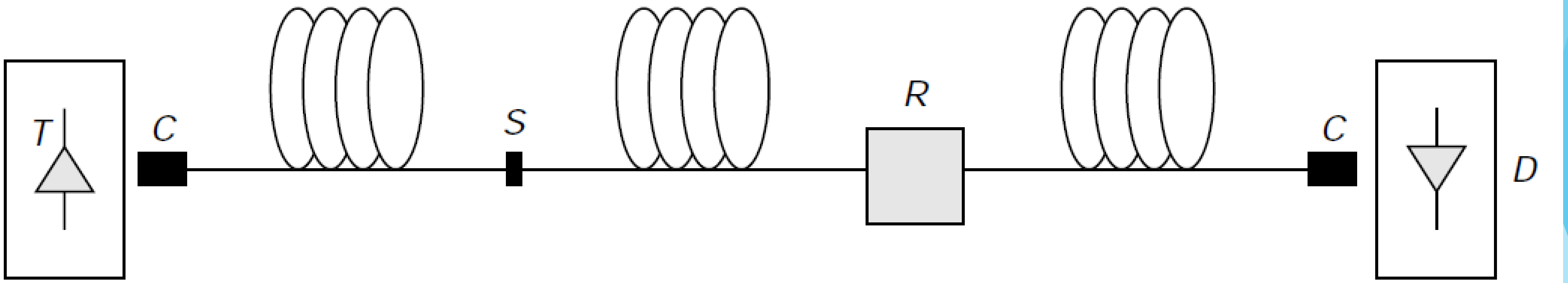
- Step index fiber → the core is of a uniform refractive index and there is a sharp decrease in the index of refraction at the cladding.

- Graded index fiber → refractive index of the core is maximum at the center core and then it decreases towards core-cladding interface.



https://www.fiberoptics4sale.com/blogs/wave-optics/step-index-optical-fibers

A typical optical fiber communication system



- It consists of a transmitter (T) → LED/ laser diode→ the light from which is coupled into an optical fiber through a connector C.
- Along the path of the optical fiber→ splices (S: permanent joints) between sections of fibers
- Repeaters R : to boost the signal and correct any distortion that may have accumulated along the path of the fiber.
- At the end of the link, the light is detected by a photodetector and electronically processed to retrieve the signal.

Attenuation (loss)

- **Attenuation/Loss** of an optical fiber → measured in *decibels (dB)*

- **If an input power P_1** results in an **output power P_2**, then the **loss** in decibels is given by

$$\boxed{\alpha = 10 \log \left(\frac{P_{\text{input}}}{P_{\text{output}}} \right)}$$

Attenuation (loss)

- **Problems:**

1. If the output power is the same as the input power, then the loss in dB is = ?
2. If the output power is one-tenth of the input power, then the loss is = ?
3. If the output power is one-hundredth of the input power, then the loss is = ?
4. If the output power is one-thousandth of the input power, then the loss is = ?

Answers

$$\alpha = 10 \log \left(\frac{P_{\text{input}}}{P_{\text{output}}} \right)$$

1. If the output power is the same as the input power, then the loss is = 0 dB.
2. If the output power is one-tenth of the input power, then the loss is = 10 dB.
3. If the output power is one-hundredth of the input power, then the loss is = 20 dB
4. If the output power is one-thousandth of the input power, then the loss is = 30 dB

Problem-5

(a) A 40 km fiber link has a loss of 0.4 dB per km. Each of the three connectors in its path has a loss of 1.8 dB, calculate the total loss.

(b) Another 40 km fiber link has a loss of 0.2 dB per km. Each of the three connectors in its path has a loss of 1.5 dB, calculate the total loss.

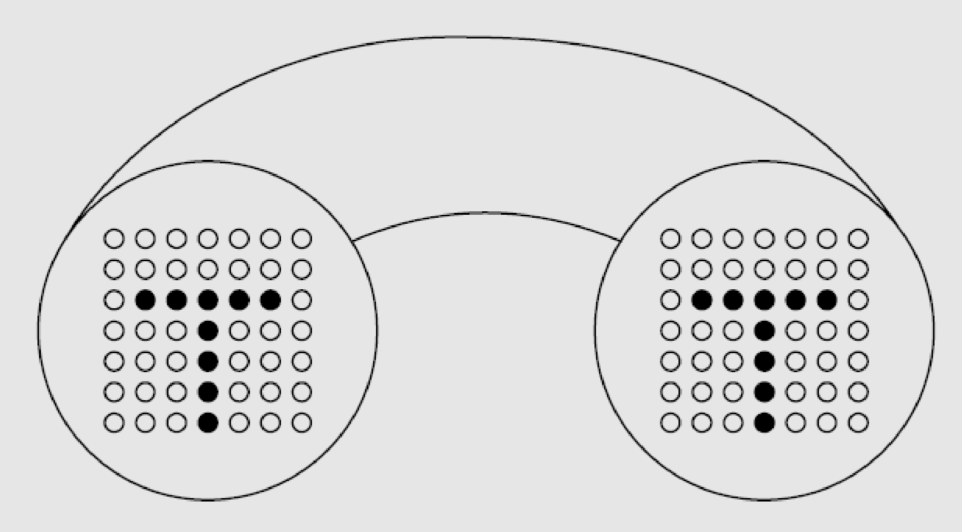
Answers

a) Total loss = (0.4 x 40) + (3 x 1.8) dB = 21.4 dB

b) Total loss = (0.2 x 40) + (3 x 1.5) dB = 12.5 dB

The coherent bundle

- If a large number of fibers are put together, it forms what is known as a *bundle*.
- If the fibers are not aligned, i.e., they are all jumbled up, the bundle is said to form an ***incoherent bundle***.
- However, if the fibers are aligned properly, i.e., if the relative positions of the fibers in the input and output ends are the same, the bundle is said to form a ***coherent bundle***.
- Usage: endoscope→ inserted through throat for detecting illnesses inside their stomach.
- At the top end →eyepiece and a lamp. The lamp shines its light down one part of the cable into the patient's stomach. When the light reaches the stomach, it reflects off the stomach walls into a lens at the bottom of the cable. Then it travels back up another part of the cable into the doctor's eyepiece.



A bundle of aligned fibers. A bright (or dark) spot at the input end of the fiber produces a bright (or dark) spot at the output end. Thus an image will be transmitted (in the form of bright and dark spots) through a bundle of aligned fibers.

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Incoherent bundle

- In an incoherent bundle the output image will be scrambled. Because of this property, an incoherent bundle can be used as a coder; the transmitted image can be decoded by using a similar bundle at the output end.

- In a bundle, since there can be hundreds of thousands of fibers, decoding without the original bundle configuration should be extremely difficult.

Why glass for optical fiber?

- Easier to control w.r.t variation in temperature. There is a wide range of accessible temperatures where its viscosity is variable and can be well controlled unlike most materials, like water and metals which remain liquid until they are cooled down to their freezing temperatures and then suddenly become solid. Glass, on the other hand, does not solidify at a discrete freezing temperature but gradually becomes stiffer.
- The second most important property is that highly pure silica is characterized with extremely low-loss; i.e., it is highly transparent. Today, in most commercially available silica fibers 96% of the power gets transmitted after propagating through 1 km of optical fiber. This indeed represents a truly remarkable achievement.
- The third most remarkable property is the intrinsic strength of glass. Its strength is about 2,000,000 lb/in2 so that a glass fiber of the type used in the telephone network and having a diameter (125 mm) of twice the thickness of a human hair can support a load of 40 lb.”

Thank You