Tutorial-3 SIGNALS AND SYSTEM EC23B1102
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1) $x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

$\cos\left(\frac{2\pi}{3}t\right) = \frac{e^{j\frac{2\pi}{3}} + e^{-j\frac{2\pi}{3}}}{2} = \frac{1}{2}e^{j2\pi/3} + \frac{1}{2}e^{-j2\pi/3}$.

$\sin\left(\frac{5\pi}{3}t\right) = \frac{e^{j\frac{5\pi}{3}} - e^{-j\frac{5\pi}{3}}}{2j} = \frac{1}{2j}e^{j\frac{5\pi}{3}} - \frac{1}{2j}e^{-j5\pi/3}$.

$x(t) = 2 + \frac{1}{2}e^{j2\pi/3} + \frac{1}{2}e^{-j2\pi/3} + \frac{2}{j}e^{j5\pi/3} - \frac{2}{j}e^{-j5\pi/3}$ [∵ using linear property].

∴ fundamental frequency $\omega_0 = \pi/3$

∴ $a_0 = 2$, $a_2 = \frac{1}{2}$, $a_{-2} = 1/2$, $a_5 = \frac{2}{j}$, $a_{-5} = -\frac{2}{j}$.

2) $x(t) \to$ real & $T_0 = 8 \Rightarrow \omega_0 = 2\pi/8 = \pi/4$. $x(t)$ in $\sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$.

$a_1 = a_{-1}^* = j \quad a_5 = a_{-5} = 2$.

$x(t) = x^*(t) = \sum a_{-k} e^{jk\omega_0 t}$.

$x(t) = a_1 e^{j\omega_0 t} + a_{-1}e^{-j\omega_0 t} + a_{-1}e^{-j\omega_0 t} + a_5 e^{5j\omega_0 t} + a_{-5}e^{-j5\omega_0 t}$.

$x^*(t) = a_1^* e^{-j\omega_0 t} + a_{-1}^* e^{j\omega_0 t} + a_5^* e^{-5j\omega_0 t} + a_{-5}^* e^{+j5\omega_0 t}$.

$= -je^{-j\omega_0 t} + je^{j\omega_0 t} + 2e^{-5j\omega_0 t} + 2e^{5j\omega_0 t}$.

as $x(t) = x^*(t)$.

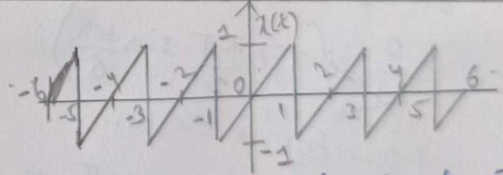
$x(t) = j(2j\sin\omega_0 t) + 2(2\cos 5\omega_0 t)$.

$x(t) = -2\sin\omega_0 t + 4\cos 5\omega_0 t$. $\Rightarrow x(t) = -2\cos(\omega_0 t - 90) + 4\cos(5\omega_0 t)$

$\Rightarrow x(t) = 2\cos\left(\frac{2\pi}{8}t + 90\right) + 4\cos\left(\frac{5\pi}{4}t\right)$

$A_1 = 2$, $A_5 = 4$.

3) a) $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$.



$a_0 = \frac{1}{T_0}\int_{-T}^{T} x(t)\,dt = \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{2}\left(\frac{t^2}{2}\right)_{-1}^{1} = 0$.

$a_k = \frac{1}{T_0}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} t e^{-jk\pi t}\,dt = \frac{1}{2(-jk\pi)}\left(t e^{-jk\pi t} - \frac{e^{-jk\pi t}}{-jk\pi}\right)_{-1}^{1}$

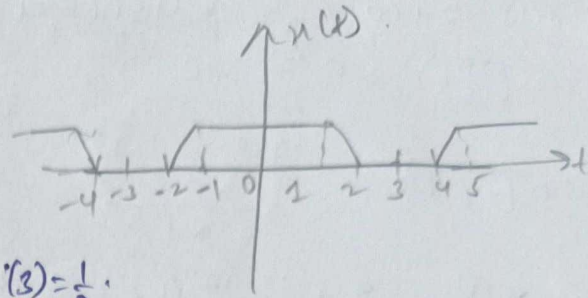
$a_k = \frac{1}{-2k\pi j}\left(e^{-jk\pi t}\left(t + \frac{1}{jk\pi}\right)_{-1}^{1}\right) = \frac{e^{-jk\pi}}{-2k\pi j}\left(1 + \frac{1}{jk\pi}\right] - \frac{e^{jk\pi}}{-2k\pi}\left(-1 + \frac{1}{jk\pi}\right)$

$a_k = \frac{-1}{2k\pi j}\left(e^{-jk\pi} + e^{jk\pi} + \frac{1}{jk\pi}\left(e^{-jk\pi} - e^{jk\pi}\right)\right] = \frac{-1}{2k\pi j}\left(2\cos k\pi - \frac{j}{jk\pi}(2\sin k\pi)\right)$

as $k \in \mathbb{Z}$ $\sin k\pi = 0$ $\Rightarrow a_k = \frac{-\cos k\pi}{jk\pi} = \frac{j(-1)^k}{k\pi}$.

$\therefore a_0 = 0$, $a_k = \frac{j(-1)^k}{k\pi}$ $k \neq 0$.

b) $T_0 = 6 \Rightarrow \omega_0 = \frac{2\pi}{6} = \pi/3.$



$$a_0 = \frac{1}{T_0}\int_{-T_1}^{T_1} x(t)\,dt = \frac{1}{6}\int_{-3}^{3} x(t)\,dt = \frac{1}{6}\int_{-2}^{2} x(t)\,dt.$$

$$a_0 = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)\,dt + \int_{-1}^{1}1\,dt + \int_{1}^{2}(2-t)\,dt\right].$$

$$= \frac{1}{6}\left[\left(\frac{t^2}{2}+2t\right)_{-2}^{-1} + t\Big)_{-1}^{1} + \left(2t - \frac{t^2}{2}\right)_{1}^{2}\right] = \frac{1}{6}(3) = \frac{1}{2}.$$

$\Rightarrow a_0 = 1/2.$

$$a_k = \frac{1}{T_0}\int_{T} x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{6}\int_{-3}^{3} x(t)e^{-jk\omega_0 t} = \frac{1}{6}\int_{-2}^{2} x(t)e^{-jk\omega_0 t}\,dt.$$

$$a_k = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-jk\omega_0 t}\,dt + \int_{-1}^{1}(1)e^{-jk\omega_0 t} + \int_{1}^{2}(2-t)e^{-jk\omega_0 t}\,dt.\right.$$

$$a_k = \frac{1}{6}\left[\frac{te^{-jk\omega_0 t}}{jk\omega_0} - \frac{2e^{-jk\omega_0 t}}{(jk\omega_0)^2} + \frac{2e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-2}^{1} + \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_{-1}^{1} + \frac{2e^{-jk\omega_0 t}}{-jk\omega_0 t}\Bigg|_{+2}^{2}$$

$$- \left(\frac{te^{-jk\omega_0 t}}{jk\omega_0 t} - \frac{e^{-jk\omega_0 t}}{(-jk\omega_0)^2}\right)_{1}^{2}$$

$$a_k = \frac{1}{6}\left[e^{-jk\omega_0}\left(\frac{t+2}{-jk\omega_0} + \frac{1}{(k\omega_0)^2}\right)\right]_{-2}^{1} + \left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_{-1}^{1} + \left(\left(\frac{2-t}{-jk\omega_0} - \frac{1}{(k\omega_0)^2}\right)e^{-jk\omega_0 t}\right)_{1}^{2}$$

$$= \frac{1}{6}\left(\frac{1}{k^2\omega_0^2}\left(e^{jk\omega_0} + e^{-jk\omega_0} \cancel{A(\ldots)} - e^{j2k\omega_0} - e^{-j2k\omega_0}\right)\right)$$

$$= \frac{1}{6k^2\omega_0^2}\left(2\cos k\omega_0 - 2\cos(2k\omega_0)\right) = \frac{1}{3k^2\omega_0^2}\left(\cos\frac{k\pi}{3} - \cos\frac{2k\pi}{3}\right)$$

as $k \in \mathbb{Z}$ $\sin k\pi = 0.$

$$= \frac{2}{3k^2\left(\frac{\pi}{3}\right)^2}\left(\sin\frac{k\pi}{2}\sin\frac{k\pi}{6}\right)$$

$$a_k = \begin{cases} 0 & \text{for } k = \text{even.} \\ \frac{6}{k^2\pi^2}\sin\frac{k\pi}{2}\sin\frac{k\pi}{6} & k = \text{odd.} \end{cases}$$

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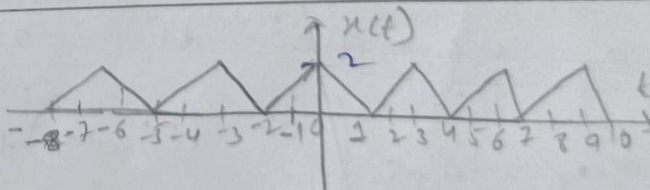
c) $T_0 = 3. \Rightarrow \omega_0 = \frac{2\pi}{3}$



$$a_0 = \frac{1}{T}\int x(t)\,dt = \frac{1}{3}\int_{-2}^{1} x(t)\,dt.$$

$$a_0 = \frac{1}{3}\int_{-2}^{0}(t+2)\,dt + \int_{0}^{1}\cancel{(t+2)\,dt \ldots}\, \int_{0}^{1}(\ldots t-2)\,dt = \frac{1}{3}\left[\int_{-2}^{0}(t+2)\,dt - \cancel{\int_{0}^{1}\frac{3}{2}t\,dt}\right]$$

$$\cancel{a_0 = \frac{1}{3}\left[\left(\frac{t^2}{2}+2t\right)_{-2}^{0} - \frac{3}{2}\left(\frac{t^2}{2}\right)_{0}^{1}\right] = \frac{5}{12}}. \quad a_0 = \frac{1}{3}\left(\left[\frac{t^2}{2}+2t\right]_{-2}^{0} - \left[2\left(\frac{t^2}{2}\right) - 2t\right]_{0}^{1}\right)$$

$\therefore a_0 = 1$

$$a_k = \frac{1}{T}\int x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{3}\int_{-2}^{1} x(t)e^{-jk\omega_0 t} = \frac{1}{3}\left[\int_{-2}^{0}(t+2)e^{-jk\omega_0 t}\,dt + \int_{0}^{1}(2-2t)e^{-jk\omega_0 t}\,dt\right]$$

$$a_k = \frac{1}{3}\left(\left[\frac{2e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{t\,e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{e^{-jk\omega_0 t}}{(-jk\omega_0)^2}\right]_{-2}^{0} + \left[\frac{(2-2t)e^{-jk\omega_0 t}}{-jk\omega_0} - \frac{2e^{-jk\omega_0 t}}{(-jk\omega_0)^2}\right]_0^1\right)$$

$$a_k = \frac{1}{3}\left(\frac{(-2+2)e^{-jk\omega_0(-2)}}{-jk\omega_0} + \frac{e^{+2jk\omega_0}}{k^2\omega_0^2}\right) + \left(\frac{2e}{-jk\omega_0} + \frac{1}{k^2\omega_0^2}\right) + \left(\frac{-2e^{-jk\omega_0}}{k^2\omega_0^2} - \frac{2e}{-jk\omega_0} + \frac{2}{k^2\omega_0^2}\right)$$

$$a_k = \frac{1}{3}\left(\frac{e^{2jk\omega_0}}{k^2\omega_0^2} + \frac{1}{k^2\omega_0^2} + \frac{2}{k^2\omega_0^2} + \cancel{\frac{2}{-jk\omega_0}} - \cancel{\frac{2}{-jk\omega_0}} - \frac{2e^{-jk\omega_0}}{k^2\omega_0^2}\right)$$

$$a_k = \frac{1}{3}\left(\frac{3}{k^2\omega_0^2} + \frac{-2e^{-jk\omega_0}}{k^2\omega_0^2} + \frac{e^{2jk\omega_0}}{k^2\omega_0^2}\right).$$

$$a_k = \frac{1}{3k^2\omega_0^2}\left(3 + -2e^{-jk\omega_0} + -e^{2jk\omega_0}\right)$$

$$a_k = \frac{1}{3k^2\omega_0^2}\left(1 - e^{2jk\omega_0} + 2 - 2e^{-jk\omega_0}\right) = \frac{1}{3k^2\omega_0^2}\left(e^{jk\omega_0}\left(e^{-jk\omega_0} - e^{jk\omega_0}\right)\right) + 2e^{-\frac{jk\omega_0}{2}}\left(e^{\frac{jk\omega_0}{2}} - e^{-\frac{jk\omega_0}{2}}\right)$$

$$a_k = \frac{1}{3k^2\omega_0^2}\left(e^{jk\omega_0}\cdot 2(-\sin k\omega_0) + 2\cdot e^{-\frac{jk\omega_0}{2}}(2j\sin\tfrac{k\omega_0}{2})\right)$$

$$a_k = \frac{2j}{3k^2\omega_0^2}\left[-e^{jk\frac{2\pi}{3}}\left(\sin\frac{2\pi}{3}k\right) + 2e^{-jk\frac{\pi}{3}}\left[\sin\frac{\pi}{3}k\right]\right]$$

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4) $T_0 = 2$, $x(t) = e^{-t}$ for $-1 < t < 1$.

$\omega_0 = \frac{2\pi}{2} = \pi$.

$$a_0 = \frac{1}{T}\int_{-T}^{T} x(t)\,dt = \frac{1}{2}\int_{-1}^{1} e^{-t}dt = \frac{-1}{2}\left(e^{-t}\right)_{-1}^{1} = \frac{-1}{2}\left(e^{-1} - e\right) = \frac{-1}{2}\left(\frac{1-e^2}{e}\right)$$

$$\Rightarrow a_0 = \frac{e^2-1}{2e}.$$

$$a_k = \frac{1}{T}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\int_{-1}^{1} e^{-t}\cdot e^{-jk\omega_0 t}dt = \frac{1}{2}\int_{-1}^{1} e^{-t(1+jk\omega_0)}dt.$$

$$= \frac{-1}{2(1+jk\omega_0)}\left(e^{-t(1+jk\omega_0)}\right)_{-1}^{1} = \frac{-1}{2(1+jk\omega_0)}\left(e^{-(1+jk\omega_0)} - e^{(1+jk\omega_0)}\right)$$

$$a_k = \frac{(-1)^k}{2(1+jk\pi)}\left(e - e^{-1}\right) \text{ for all } k.$$

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5) $\omega_0 = \pi \Rightarrow T = 2.$ $x(t) = \begin{cases} 1.5 & 0 \le t < 1 \\ -1.5 & 1 \le t < 2. \end{cases}$

$$a_0 = \frac{1}{T}\int_{-T}^{T} x(t)dt = \frac{1}{2}\left(\int_0^1 (1.5)dt + \int_1^2 (-1.5)dt\right) = \frac{3}{4}\left((t)_0^1 + (-t)_1^2\right) = \frac{3}{4}(1 + -2 + 1)$$

$$\therefore a_0 = \frac{0}{T}$$

$$a_k = \frac{1}{T}\int_{-T}^{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\left(\int_0^1 (1.5)e^{-jk\omega_0 t}dt + \int_1^2 -1.5e^{-jk\omega_0 t}dt.\right)$$

$$= \frac{1}{2}\left(\left(\frac{1.5e^{-jk\omega_0 t}}{-jk\omega_0}\right)_0^1 + \left(\frac{-1.5e^{-jk\omega_0 t}}{-jk\omega_0}\right)_1^2\right)$$

$a_k = \frac{-3}{4jk\omega_0}\left(e^{-jk\omega_0} - \frac{e^{-2jk\omega_0}}{2} + e^{-jk\omega_0}\right) \cancel{-\frac{1}{2}7 + \frac{7}{4jk\omega_0} e^{-2jk\omega_0}}$

$a_k = \frac{3j}{2k\pi}\left(e^{-jk\pi} - \frac{e^{-j2k\pi}}{2} - \frac{1}{2}\right) = \frac{3j}{2k\pi}\left(\cos k\pi - \frac{\cos 2k\pi}{2} - \frac{1}{2}\right)$

$\cancel{a_k = \frac{3j}{2k\pi}\left(2\sin\frac{k\pi}{2}\sin\frac{3k\pi}{2} - 1\right)}$ $a_k = \frac{3j}{2k\pi}(\cos k\pi - 1)$

$a_k = \frac{3}{2k\pi j}(1 - \cos k\pi) = \frac{3}{2k\pi j}\left(1 - e^{-jk\pi}\right) \cancel{= \frac{3}{2k\pi}\sin\frac{k\pi}{2}}$

$\therefore a_k = \frac{3}{2k\pi j}\left(1 - e^{-jk\pi}\right) = \frac{3}{2k\pi j}(1 - \cos k\pi)$

6) $T_0 = 1/2$ $x(t) = \cos(4\pi t)$ $y(t) = \sin(4\pi t)$.

a) coeff's of $x(t), y(t)$ of Fourier series

$\cancel{a_0 = \frac{1}{T}\int_{-T}^{T} x(t)\,dt = 2\int\left(\frac{e^{jk4\pi t} + e^{-jk4\pi t}}{2}\right)dt = \frac{e^{jk4\pi t}}{jk4\pi} + \frac{e^{-jk4\pi t}}{-jk4\pi}}$.

$\cancel{a_0 = \frac{1}{jk4\pi}\left(e^{jk4\pi t} - e^{-jk4\pi t}\right) = \frac{2}{jk4\pi}(\cos(4\pi t))} =$

$\cancel{a_k = \frac{1}{T}\int_{-T}^{T} x(t)e^{-jk4\pi t} = \frac{1}{T}\int_{-T}^{T}}$

$x(t) = \cancel{2\int}\frac{e^{jk4\pi t} + e^{-jk4\pi t}}{2} = \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}$.

$a_1 = a_{-1} = 1/2$.

$y(t) = \frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} = \frac{1}{2j}e^{j4\pi t} - \frac{1}{2j}e^{-j4\pi t}$.

$b_1 = \frac{1}{2j}$ $b_{-1} = \frac{-1}{2j}$

b) $\underset{c_k}{z(t)} = \underset{a_k}{x(t)} \cdot \underset{b_k}{y(t)}$. by property $c_k = T \cdot a_k b_k$.

$c_1 = \frac{1}{2}\left(\frac{1}{2}\cdot\frac{1}{2j}\right) = \frac{1}{8j}$ $c_{-1} = \frac{1}{2}\left(\frac{1}{2}\cdot\left(\frac{-1}{2j}\right)\right) = \frac{-1}{8j}$

7) $T_0 = T$, coeff a_k $x(t) \longrightarrow a_k$.

$\frac{d^2(x(t))}{dt^2} \longrightarrow ?$

By property $\frac{d^n}{dt^n}x(t) \longleftrightarrow (jk\omega_0)^n a_k \Rightarrow \frac{d^2(x(t))}{dt^2} = (jk\omega_0)^2 a_k$.

$x(t) = \sum_k a_k e^{jk\omega_0 t}$.

$\frac{d}{dt}((x(t))) = \frac{d}{dt}\sum_k a_k e^{jk\omega_0 t} = \sum_k a_k\left(\frac{d}{dt}e^{jk\omega_0 t}\right) = \sum_k\left(a_k e^{jk\omega_0 t}\, jk\omega_0\right)$

$\frac{d^2(x(t))}{dt^2} = \frac{d}{dt}\sum_k a_k jk\omega_0 e^{jk\omega_0 t} = \sum_k a_k jk\omega_0 \frac{d}{dt}e^{jk\omega_0 t} = \sum_k a_k (jk\omega_0)^2 e^{jk\omega_0 t}$

$\frac{d^2(x(t))}{dt^2} = (jk\omega_0)^2\sum a_k e^{jk\omega_0 t} = (jk\omega_0)^2 a_k$.

8) a) $e^{-2(t-1)}u(t-1)$

$$x(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} e^{-2(t-1)} u(t-1) e^{-j\omega t}\,dt \cdot \int_1^{\infty} e^{-2t} \cdot e^{2} e^{-j\omega t}\,dt.$$

$$x(j\omega) = e^{2} \int_1^{\infty} e^{-t(2+j\omega)}\,dt \cdot = \frac{e^{2}}{2+j\omega}\left(e^{-t(2+j\omega)}\right)_1^{\infty} = \frac{e^{2}}{2+j\omega}\left(-e^{-(2+j\omega)}\right)$$

$$x(j\omega) = -\frac{e^{-j\omega}}{2+j\omega} \Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-2j\omega}}{1+2j\omega} e^{j\omega t}\,d\omega.$$

b) $e^{-2|t-1|}$

$$x(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} e^{-2|t-1|} e^{-j\omega t}\,dt = \int_{-\infty}^{1} e^{2(t-1)-j\omega t}\,dt + \int_1^{\infty} e^{-2(t-1)-j\omega t}\,dt.$$

$$= e^{-2}\int_{-\infty}^{1} e^{t(2-j\omega)}\,dt + e^{2}\int_1^{\infty} e^{-t(2+j\omega)}\,dt = e^{-2}\left(\frac{e^{t(2-j\omega)}}{2-j\omega}\right)_{-\infty}^{1} + e^{2}\left(\frac{e^{-t(2+j\omega)}}{-(2+j\omega)}\right)_1^{\infty}$$

$$= e^{-2}\left(\frac{e^{2-j\omega}}{2-j\omega}\right) + e^{2}\left(\frac{e^{-(2+j\omega)}}{2+j\omega}\right)$$

$$= e^{-j\omega}\left(\frac{1}{2-j\omega} + \frac{1}{2+j\omega}\right) \Rightarrow x(j\omega) = e^{-j\omega}\left(\frac{4}{4+\omega^2}\right)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\omega}\left(\frac{4}{4+\omega^2}\right) e^{j\omega t}\,dt.$$

c) $\delta(t+1) + \delta(t-1)$

$$x(j\omega) = \int_{-\infty}^{\infty}\left(\delta(t+1) + \delta(t-1)\right) e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} \delta(t+1) e^{-j\omega t}\,dt + \int_{-\infty}^{\infty} \delta(t-1) e^{-j\omega t}\,dt$$

$$x(j\omega) = e^{j\omega} + e^{-j\omega} = 2\cos\omega. \quad F\{x(t)\} = 2\cos\omega.$$

d) $te^{-2t}u(t)$

$$x(j\omega) = \int_{-\infty}^{\infty} te^{-2t}u(t) e^{-j\omega t}\,dt = \int_0^{\infty} te^{-2t}e^{-j\omega t}\,dt = \int_0^{\infty} te^{-t(2+j\omega)}\,dt$$

$$x(j\omega) = \left(\frac{te^{-t(2+j\omega)}}{-(2+j\omega)} - \frac{e^{-t(2+j\omega)}}{(2+j\omega)^2}\right)_0^{\infty} = \frac{1}{(2+j\omega)^2}.$$

9) a) $x_1(t) = x(1-t) + x(-1-t) \qquad x(t) \xrightarrow{F.T} X(j\omega)$.

$x(-t) \xrightarrow{F.T} X(-j\omega)$

$x(-t+t_0) \longrightarrow e^{j\omega_a t_0} X(-j\omega)$

$\therefore X_1(t) = e^{j\omega} X(-j\omega) + e^{-j\omega} X(-j\omega)$

$x_1(t) = X(-j\omega) \cdot 2\cos\omega.$

b) $x_2(t) = x(3t-6) \qquad x(t) \xrightarrow{F.T} X(j\omega).$

$x(at) \rightarrow \frac{1}{|a|} X(j\omega/a) \qquad x(3t) \longrightarrow \frac{1}{3} X(j\omega/3)$

$x(3t-6) \longrightarrow \frac{1}{3} \cdot e^{-j\frac{\omega}{3}\times 6} X\left(\frac{j\omega}{3}\right).$

$\therefore F\{x_2(t)\} = \frac{1}{3} e^{-2j\omega} X(j\omega/3)$

c) $x_3(t) = \frac{d^2 x(t-1)}{dt^2} = \frac{d^2}{dt^2}x(t-1)$.

$x(t-1) \longrightarrow e^{-j\omega} X(j\omega)$

$\frac{d^2}{dt^2} x(t-1) \to (j\omega)^2 e^{-j\omega} X(j\omega)$

$F\left\{\frac{d^2 x(t-1)}{dt^2}\right\} = -\omega^2 e^{-j\omega} X(j\omega)$

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10) a) $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)$

$\downarrow \qquad \downarrow \qquad \downarrow$

$2\pi \cdot 1 \qquad \pi \frac{1}{2}\times e^{-j4\pi t} \qquad \pi\frac{1}{2}e^{+j4\pi t}$.

$x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi + 2\pi\left(\frac{e^{-j4\pi t} + e^{j4\pi t}}{2}\right) dt$.

$x_1(t) = \frac{\pi}{2\pi}\int_{-\infty}^{\infty} 2 + 2\cos 4\pi t \, dt$.

$x_1(t) = \frac{1}{2}[2 + 2\cos 4\pi t) \Rightarrow x_1(t) = 1 + \cos 4\pi t$.

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b) $X_2(j\omega) = \begin{cases} 2 & 0 \le \omega \le 2 \\ -2 & -2 \le \omega < 0 \\ 0 & |\omega| > 2 \end{cases}$

$x_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(j\omega)\, dt. = \int_{-2}^{0}(-2)e^{j\omega t} d\omega + \int_{0}^{2} 2e^{j\omega t} d\omega + 0.$

$= \frac{1}{2\pi}\left[-2\left(\frac{e^{j\omega t}}{j\omega}\right)_{-2}^{0}\right) + 2\left(\frac{e^{j\omega t}}{j\omega}\right)_0^2\Big]$

$= \frac{1}{2\pi}\left(-2\left(\frac{1}{j\omega} - \frac{e^{-2j\omega}}{j\omega}\right) + 2\left(\frac{e^{2j\omega}}{j\omega} - \frac{1}{j\omega}\right)\right]$

$= \frac{1}{\pi j\omega}\left(-1 + e^{-2j\omega} + e^{2j\omega} - 1\right) = +\frac{1}{\pi j\omega}\left(-2 + 2\cos 2\omega t\right)$

$x_2(t) = \frac{2}{\pi j\omega}(\cos 2\omega t - 1) = \frac{-2\cdot 2\sin^2 t}{\pi j\omega} = \frac{4j\sin^2 t}{\pi t}$.

$\therefore x_2(t) = \frac{4j\sin^2 t}{\pi t}$.