Engineering Optics

Lecture 19

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Debolina Misra

Assistant Professor
Department of Physics
IIITDM Kancheepuram, Chennai, India

Malus' Law

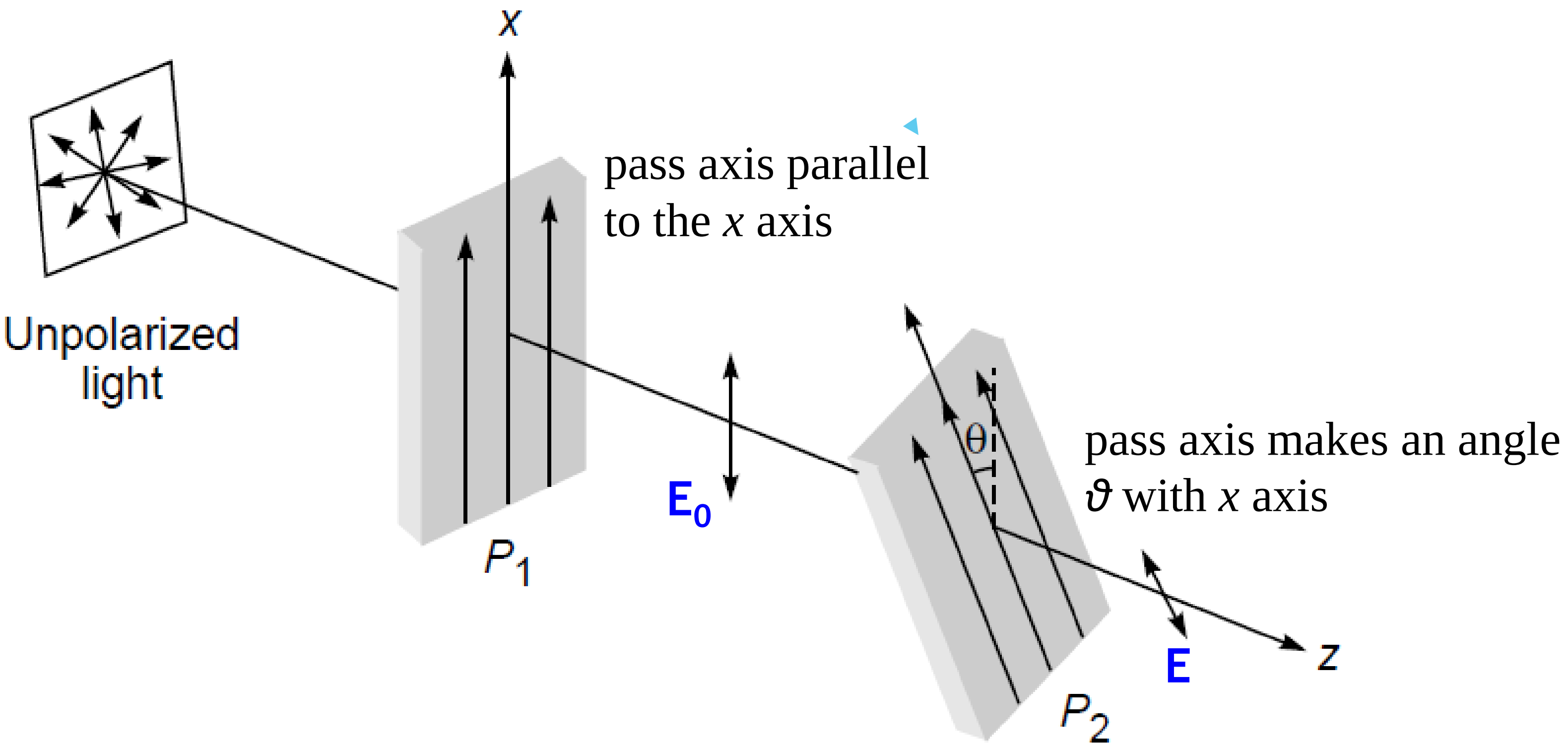

Fig. 22.15 An unpolarized light beam gets x-polarized after passing through the polaroid P_1, the pass axis of the second polaroid P_2 makes an angle θ with the x axis. The intensity of the emerging beam will vary as $\cos^2\theta$.

Amplitude

$$E = E_0 \cos\theta$$

Intensity

$$\boxed{I = I_0 \cos^2\theta}$$

Malus' Law

Problem:2

The electric field of a $1000\,\mathrm{W/m^2}$ linearly polarized lightbeam oscillates at $+10.0°$ from the vertical in the first and third quadrants. The beam passes perpendicularly through two consecutive ideal linear polarizers. The transmission axis of the first is at $-80.0°$ from the vertical in the second and fourth quadrants. And that of the second is at $+55.0°$ from the vertical in the first and third quadrants. (a) How much light emerges from the second polarizer? (b) Now interchange the two polarizers without altering their orientations and determine the amount of light that emerges. Explain your answers.

Answer:

(a) The incident light (at $+10°$) is perpendicular to the transmission axis of the first polarizer (at $-80°$) and so no light leaves it and no light leaves the second polarizer. (b) With the polarizers interchanged, the light now oscillates at $45.0°$ to the transmission axis of the first polarizer, which, via Malus's Law, passes (I_1) where

$$I(\theta) = I(0)\cos^2\theta$$

and so here

$$I_1 = (1000\ \mathrm{W/m^2})\cos^2 45.0°$$

Hence

$$I_1 = 500\ \mathrm{W/m^2}$$

This light, oscillating at +55.0°, makes an angle of 45.0° with the transmission axis of the new second polarizer. Therefore the irradiance emerging from it (I_2) is

$$I_2 = (500\ \mathrm{W/m^2})\cos^2 45.0°$$

$$I_2 = 250\ \mathrm{W/m^2}$$

More on Intensity after polarization

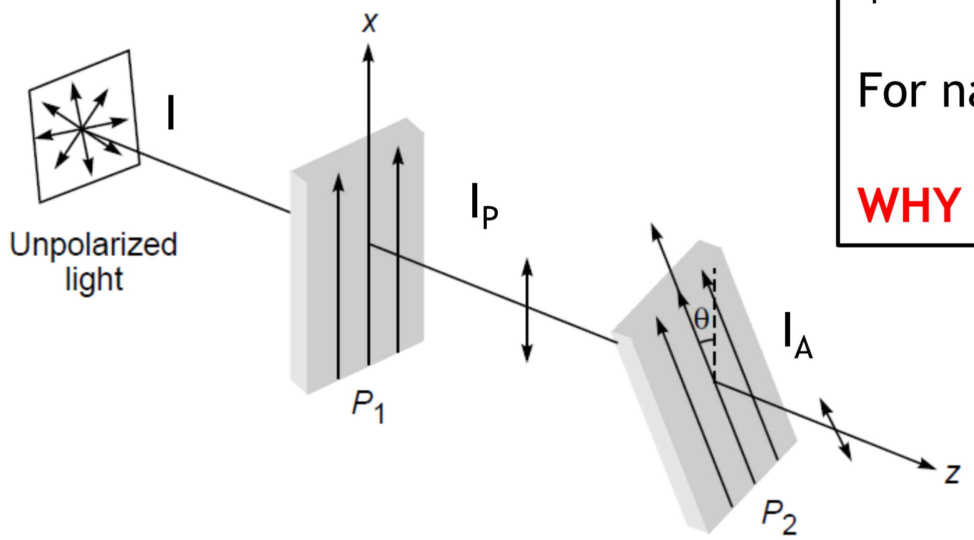

Fig. 22.15 An unpolarized light beam gets x-polarized after passing through the polaroid P_1, the pass axis of the second polaroid P_2 makes an angle θ with the x axis. The intensity of the emerging beam will vary as $\cos^2 \theta$.

$I_P = I/2$

For natural light

WHY ??

$$\langle (\cos\theta)^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} (\cos\theta)^2 \, d\theta = \frac{1}{2}$$

Now that we have some idea of what polarized light is, the next logical step is to develop an understanding of the techniques used to generate, change, and manipulate it to fit our needs. An optical device whose input is natural light and whose output is some form of polarized light is a **polarizer**. For example, recall that one possible representation of unpolarized light is the superposition of two equal-amplitude, incoherent, orthogonal $\mathscr{P}$-states. An instrument that separates these two components, discarding one and passing on the other, is known as a *linear polarizer*.

Optics, Ghatak, Hecht

Superposition of two disturbances

Case - 1

$$\mathbf{E}_1 = \hat{\mathbf{x}}\, a_1 \cos\,(kz - \omega t + \theta_1)$$
$$\mathbf{E}_2 = \hat{\mathbf{x}}\, a_2 \cos\,(kz - \omega t + \theta_2)$$

where a_1 and a_2 represent the amplitudes of the waves, $\hat{\mathbf{x}}$ represents the unit vector along the x axis, and θ_1 and θ_2 are phase constants. The resultant of these two waves is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

Superposition of two disturbances

Case - 2

We next consider the superposition of two linearly polarized electromagnetic waves (both propagating along the z axis) but with their electric vectors oscillating along two mutually perpendicular directions. Thus, we may have

$$\mathbf{E}_1 = \hat{\mathbf{x}} a_1 \cos(kz - \omega t)$$
$$\mathbf{E}_2 = \hat{\mathbf{y}} a_2 \cos(kz - \omega t + \theta)$$

If E_x and E_y represent the x and y components of the resultant field $\mathbf{E}\,(= \mathbf{E}_1 + \mathbf{E}_2)$, then

$$\boxed{E_x = a_1 \cos \omega t}$$

and

$$\boxed{E_y = a_2 \cos(\omega t - \theta)}$$

EXAMPLE -1

For $\theta = n\pi$, $E_x = a_1 \cos \omega t$

and $E_y = (-1)^n a_2 \cos \omega t$

from which we obtain

$$\frac{E_y}{E_x} = \pm \frac{a_2}{a_1} \qquad \text{(independent of } t\text{)}$$

Straight line; the angle ϕ that this line makes with the E_x axis depends on the ratio a_2/a_1

$$\boxed{\phi = \tan^{-1}\left(\pm \frac{a_2}{a_1}\right)}$$

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Problem-1

If E_x and E_y represent the x and y components of the resultant field $\mathbf{E}\,(=\mathbf{E}_1 + \mathbf{E}_2)$, then

$$E_x = a_1 \cos \omega t$$

and

$$E_y = a_2 \cos (\omega t - \theta)$$

$$\theta = n\pi$$

State of polarization for (i) $\theta = 0$ and $a_2 = 1.5\ a_1$ (ii) $\theta = \pi$ and $a_2 = 1.5\ a_1$

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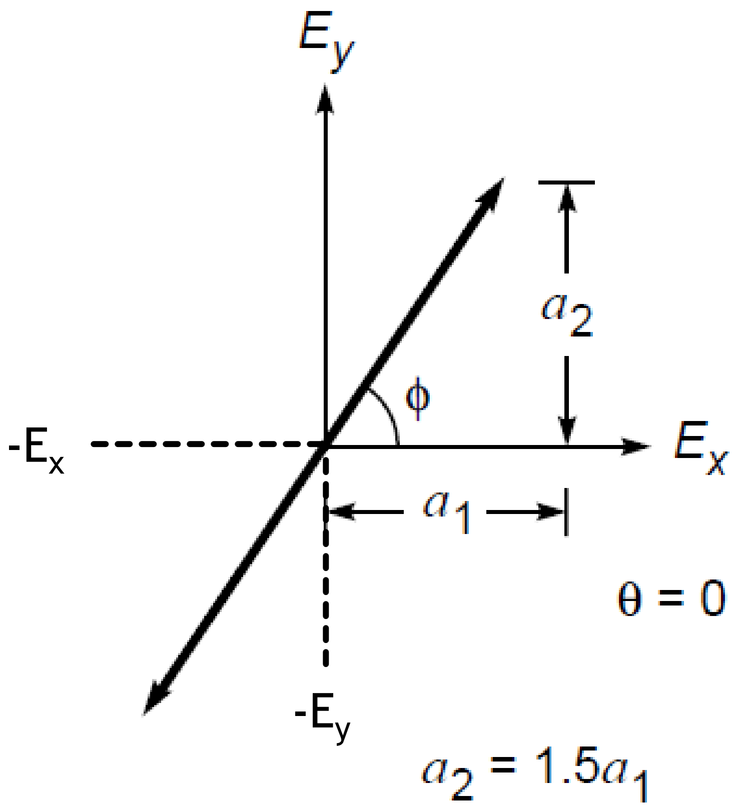
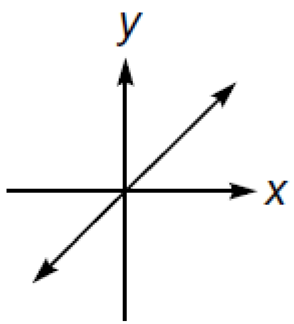
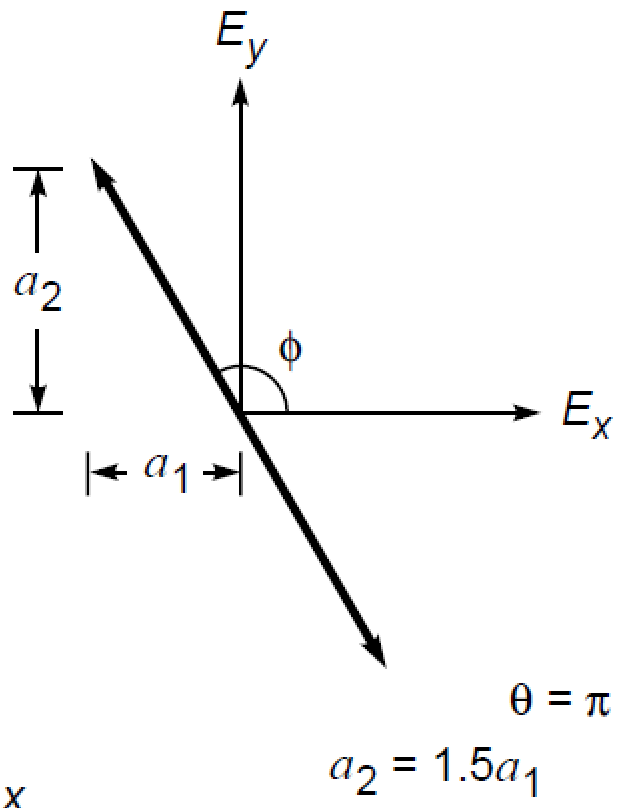
Case – 2: Examples

$\theta = n\pi$







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What if θ = π/2 ?

Now θ = π/2

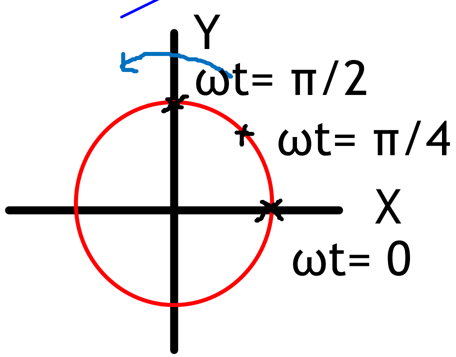
$$\boxed{\begin{aligned} E_x &= a_1 \cos \omega t \\ E_y &= a_2 \cos (\omega t - \theta) \end{aligned}}$$

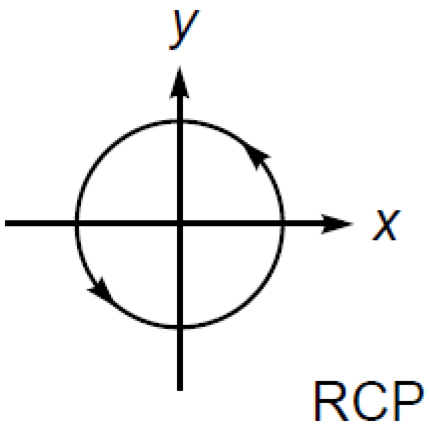
Q: Condition to get LCP light?

$$E_x = a_1 \cos \omega t$$

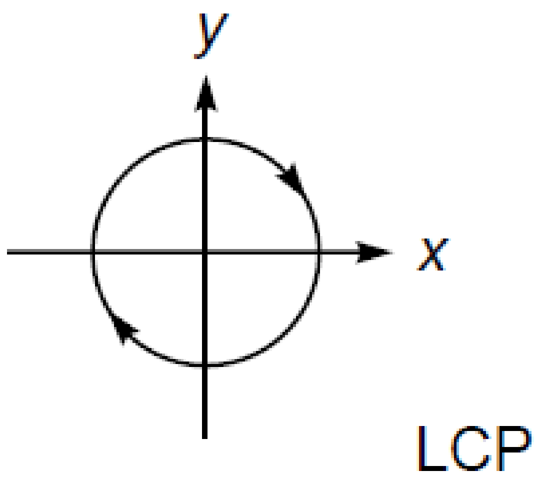
$$E_y = a_1 \sin \omega t$$

tip of the electric vector rotates on the circumference of a circle (of radius a_1) in the counterclockwise direction





θ = π/2



Q: What if a1 ≠ a2 ??

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Problem:2

Discuss the state of polarization when the *x* and *y* components of the electric field are given by the following equations:

(a) $E_x = E_0 \cos(\omega t + kz)$

$$E_y = \frac{1}{\sqrt{2}} E_0 \cos(\omega t + kz + \pi)$$

(b) $E_x = E_0 \sin(\omega t + kz)$

$$E_y = E_0 \cos(\omega t + kz)$$

(c) $E_x = E_0 \sin\left(kz - \omega t + \frac{\pi}{3}\right)$

$$E_y = E_0 \sin\left(kz - \omega t - \frac{\pi}{6}\right)$$

(d) $E_x = E_0 \sin\left(kz - \omega t + \frac{\pi}{4}\right)$

$$E_y = \frac{1}{\sqrt{2}} E_0 \sin(kz - \omega t)$$

(a) $E_x = E_0 \cos(\omega t + kz)$

$E_y = \frac{1}{\sqrt{2}} E_0 \cos(\omega t + kz + \pi)$

$\Rightarrow$Linearly polarized

(b) $E_x = E_0 \sin(\omega t + kz)$

$E_y = E_0 \cos(\omega t + kz)$

$\Rightarrow \theta = \frac{\pi}{2}, a_1 = a_2$

Left (?) circular polarization

(c) $E_x = E_0 \sin\left(kz - \omega t + \frac{\pi}{3}\right)$

$E_y = E_0 \sin\left(kz - \omega t - \frac{\pi}{6}\right)$

$\Rightarrow d\theta = \frac{\pi}{2}, a_1 = a_2$

Right(?) circular polarization

Thank You