Tutorial on Module 3_Riemann Integration

1. Compute $L(P, f)$ and $U(P, f)$ for the function $f(x) = x^2$ on $[0, 1]$, where $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$.

2. If a function f is Riemann integrable on $[a, b]$, then prove the following:

   (a) $|f|$ is Riemann integrable on $[a, b]$.

   (b) f^2 is Riemann integrable on $[a, b]$.

3. Determine whether f is Riemann-integral on [0,1] and justify your answer.

$$f(x) = \begin{cases} 2n & \text{if } x = \frac{1}{n}, \quad n = 1, 2, 3... \\ 0 & \text{otherwise} \end{cases}$$

4. Consider $f(x) = x^3$ on $[0, 1]$. Find its lower and upper Riemann sums, $L(P_n, f)$ and $U(P_n, f)$, corresponding to the partition $P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}, 1\right\}$. Also show that $\int_0^1 x^3 dx = \frac{1}{4}$.

5. Obtain $\int_{-1}^{1} |x|\, dx$ as the limit of Riemann sums.

6. Compute the derivative with respect to x using the fundamental theorem of calculus.

   (i) $F(x) = \int_3^{2x} \sqrt{t} \sin t\, dt$ (ii) $F(x) = \int_{-x^4}^{\sqrt{x}} \frac{t^2}{1+t^4} dt$

   (iii) $F(x) = \int_3^{\cos x} (1 + \sin t) dt$ (iv) $F(x) = \int_{x^2}^{x^3} (t) dt$

7. Let $G(x) = \int_2^x x \cos(t^3) dt$ for each $x \in \mathbb{R}$. Find $G''(x)$.

8. An object moves back and forth along a straight line with velocity $v(t) = (t - 1)^2$ during the time interval $[0, 3]$, where t is measured in seconds and $v(t)$ is measured in ft/s. Find the average velocity of the object.