ASSIGNMENT-2

Course: DSCS (CS1005)
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1. Let R be the relation on the set {1,2,3,4,5} containing the ordered pairs (1,1), (1,2), (1,3), (2,3), (2,4), (3,1), (3,4), (3,5), (4,2), (4,5), (5,1) and (5,2) and (5,4).
Find (a) R^2 (b) R^3 (c) R^4 (d) R^5
Represent them as directed graph.

Ans $R = \{(1,1), (1,2), (1,3), (2,3), (2,4), (3,1), (3,4), (3,5), (4,2), (4,5), (5,1), (5,2), (5,4)\}$

(a) $R^2 = R \circ R = \{$ (1,1) (2,1) (3,1) (4,1) (5,1)
(1,2) (2,2) (3,2) (4,2) (5,2)
(1,3) (3,3) (4,3) (5,3)
(1,4) (2,4) (3,4) (4,4) (5,4)
(1,5) (2,5) (3,5) (5,5) $\}$

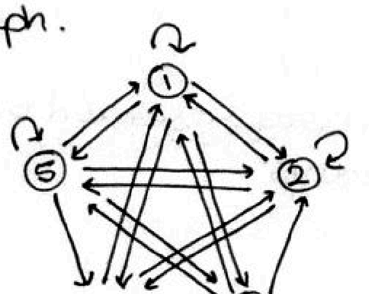
(b) $R^3 = R^2 \circ R = \{$ (1,1) (2,1) (3,1) (4,1) (5,1)
(1,2) (2,2) (3,2) (4,2) (5,2)
(1,3) (2,3) (3,3) (4,3) (5,3)
(1,4) (2,4) (3,4) (4,4) (5,4)
(1,5) (2,5) (3,5) (4,5) (5,5) $\}$

(c) $R^4 = R^3 \circ R = \{ R^3$
$\because R^3$ contains all elements in $A \times A$

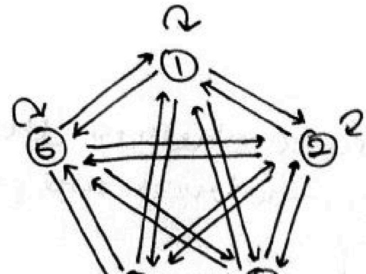
(d) $R^5 = R^4 \circ R = R^4$.

Directed graph.

R^2



R^3
R^4
R^5



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2. Let R be a nice reflexive symmetric binary relation defined on set A. The nice reflexive symmetric binary relation is a relation such that it is reflexive and contains exactly one symmetric pair. Count the number of nice reflexive symmetric binary relations.

Ans $\underbrace{(1,1)\,(2,2)\,\ldots\,(n,n)}_{n \text{ elements}}$ $\qquad \underbrace{\boxed{\begin{matrix}(1,2)\\(2,1)\end{matrix}} \;\ldots\ldots\; \boxed{\begin{matrix}(1,n)\\(n,1)\end{matrix}}}_{n^2-n \text{ elements.}}$

$\frac{n^2-n}{2}$ boxes

∵ relation is reflexive.

No. of ways to select first n elements $=1$

∵ relation should contain exactly one symmetric pair

No. of ways of selecting exactly one symmetric pair

$$= {}^{\frac{n^2-n}{2}}C_1 = \frac{n^2-n}{2}.$$

From the remaining boxes, we have three possibilities

- Select 1st element
- Select 2nd element
- Selecting none.

no. of ways for doing so $= 3^{\frac{n^2-n}{2}-1} = 3^{\frac{n^2-n-2}{2}}$

∴ The number of nice reflexive symmetric relations

$$= 1 \times \left(\frac{n^2-n}{2}\right) \times 3^{\frac{n^2-n-2}{2}}$$

$$= \frac{n^2-n}{2} \cdot 3^{\frac{n^2-n-2}{2}}$$

3. Determine whether the relation represented by these one-zero matrices are partial orders.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Ans $\because$ The diagonal entries of the matrix are 1

$\Rightarrow$ Given relation is a reflexive relation

Listing given relation, $R = \{(1,1), (2,2), (3,3), (1,3), (2,1)\}$

Given relation is antisymmetric

$$\because \forall a,b \in A \left\{ \begin{matrix} (a,b) \in R \\ \& \\ (b,a) \in R \end{matrix} \longrightarrow a=b \right\}$$

$(1,1) \,\&\, (1,3) \longrightarrow (1,3) \in R$

$(2,2) \,\&\, (2,1) \longrightarrow (2,1) \in R$

$(1,3) \,\&\, (3,3) \longrightarrow (1,3) \in R$

$(2,1) \,\&\, (1,1) \longrightarrow (2,1) \in R$

$(2,1) \,\&\, (1,3) \longrightarrow (2,3) \notin R$

$\therefore$ Given relation is not transitive

$\therefore$ Given relation is not Partial order.

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Ans $\because$ all diagonal entries are 1.

$\Rightarrow$ Relation is reflexive

Listing relation, $R = \{(1,1), (2,2), (3,3), (3,1)\}$

Given relation is antisymmetric

$$\because \forall a,b \in A \left\{ \begin{matrix} (a,b) \in R \\ \& \\ (b,a) \in R \end{matrix} \longrightarrow a=b \right\}$$

$(3,3) \,\&\, (3,1) \longrightarrow (3,1) \in R$

$(3,1) \,\&\, (1,1) \longrightarrow (3,1) \in R$

$\therefore$ Given relation is transitive.

$\therefore$ Given relation is partial order.

(c) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

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Ans ∵ all diagonal ~~...~~ entries are 1

$\Rightarrow$ Given relation is reflexive relation

Listing the given relation, $R = \{(1,1),(2,2),(3,3),(4,4),(1,3),(2,3),(3,4),(4,1),(4,2)\}$

$$\because \forall a,b \in A \left\{ \begin{matrix} (a,b)\in R \\ \& \\ (b,a)\in R \end{matrix} \longrightarrow a=b \right\}$$

$\Rightarrow$ Given relation is antisymmetric.

$(1,1)$ & $(1,3) \longrightarrow (1,3) \in R.$

$(2,2)$ & $(2,3) \longrightarrow (2,3) \in R$

$(3,3)$ & $(3,4) \longrightarrow (3,4) \in R$

$(4,4)$ & $(4,1) \longrightarrow (4,1) \in R$

$(4,4)$ & $(4,2) \longrightarrow (4,2) \in R$

$(1,3)$ & $(3,3) \longrightarrow (1,3) \in R$

$(1,3)$ & $(3,4) \longrightarrow (1,4) \notin R$

∴ Given relation is not transitive

∴ Given relation is not partial order.

4. (a) Prove or disprove. If R and S are equivalence relation on A, then RoS is an equivalence relation.

Ans Given statement is false

Let us disprove it by giving a counterexample.

Let $A = \{1,2,3\}$

$S = \{(1,1),(2,2),(3,3),(1,3),(3,1)\}$

$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$

$RoS = \{(1,1),(2,2),(3,3),(1,3),(3,1),(2,1),(1,2),(3,2)\}$

~~Given~~ ∵ $\forall a \in A\ (a,a) \in RoS$

$\Rightarrow$ RoS is a reflexive relation.

∵ $(3,2) \in RoS$ & $(2,3) \notin RoS$

$\Rightarrow$ RoS is not a symmetric relation

∴ RoS is not an equivalence relation

hence, disproved.

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(b) Prove $R = \{(x,y) \mid x+y$ is an even integer$\}$ is an equivalence relation on $\mathbb{Z}$. 8

Ans Sum of two integers is even in two cases
(i) both numbers are even
(ii) both numbers are odd.

if a is even or odd
$a+a = 2a$ which is even
$\Rightarrow \forall a \in \mathbb{Z}\ (a,a) \in R$
$\therefore$ Given relation is reflexive relation

if $(a,b) \in R$ ($a+b$ is even)
$\Rightarrow a,b$ both are either even or odd
$\Rightarrow (a,b) \in R \longrightarrow (b,a) \in R$ ($(b+a)$ is even)
hence, R is symmetric relation.

if $(a,b) \in R$ & $(b,c) \in R$
$\Rightarrow a,b$ both are either even or odd
$\Rightarrow b,c$ ~~is odd~~ are both odd if b is odd or even if b is even.
if a,b are even $\Rightarrow b,c$ are even
$\Rightarrow a,c$ are even $a+c$ is even
$(a,c) \in R$
if a,b are odd $\Rightarrow b,c$ are odd
$\Rightarrow a,c$ are odd $a+c$ is even
$(a,c) \in R$.
$\therefore$ R is a transitive relation
$\therefore$ R is an equivalence relation.

5. R is a relation defined on $A = \{0,1,2,3\}$. Let $R = \{(0,1),(0,2),(1,1),(1,3)$
$(2,2),(3,0)\}$.
Find the symmetric closure of R.
Find the transitive closure of R.
Find the reflexive closure of R.

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Ans Transitive closure of R

$$t(R) = \{(0,1),(0,2),(1,1),(1,3),(2,2),(3,0),(0,3),(1,0),(3,1),(3,2),(3,3),(0,0),(1,2)\}$$

Symmetric closure of R

$$S(R) = \{(0,1),(0,2),(1,1),(1,3),(2,2),(3,0),(1,0),(2,0),(3,1),(0,3)\}$$

Reflexive closure of R

$$r(R) = \{(0,1),(0,2),(1,1),(1,3),(2,2),(3,0),(0,0),(3,3)\}$$

6. Let S be a set with n elements and let a and b be distinct element of S. How many relation R are there on S such that
(a) $(a,b) \in R$

Ans 2^{n^2-1}

(b) $(a,b) \notin R$.

Ans 2^{n^2-1}

7. Let R be a relation from a set A to a set B. The complementary relation $\bar{R}$ is the set of ordered pairs $\{(a,b) | (a,b) \notin R\}$
Show that relation R on a set A is reflexive if and only if the inverse R^{-1} is reflexive.
Show that the relation R on a set A is reflexive if and only if the complementary relation $\bar{R}$ is irreflexive.

Ans $1 \Rightarrow 2$
Given R is reflexive
$\forall a \in A \ (a,a) \in R$
$a \in B$

By defination of R^{-1} we have $(a,a) \in R^{-1}$

$\forall a \in B \ (a,a) \in R^{-1}$
$a \in A$

$\therefore R^{-1}$ is reflexive.

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$2 \Rightarrow 1$

Given

R^{-1} is reflexive

$\forall a \in B$ $(a,a) \in R^{-1}$
$a \in A$

By defination of R, we have $(a,a) \in R$

$\forall a \in A$ $(a,a) \in R$
$a \in B$

$\therefore$ R is reflexive.

$1 \Rightarrow 2$

Given $\forall a \in A$ $(a,a) \in R$ ($\because$ R is reflexive)

By defination of $\bar{R}$

if $\forall a \in A$ $(a,a) \in R$
$a \in B$

$\Rightarrow \forall a \in A$ $(a,a) \notin \bar{R}$
$b \in B$

$\Rightarrow \bar{R}$ is irreflexive

$2 \Rightarrow 1$

Given $\forall a \in A$ $(a,a) \notin \bar{R}$ ($\because$ R is irreflexive)
$b \in B$

By defination of R

if $\forall a \in A$ $(a,a) \notin \bar{R}$
$a \in B$

$\Rightarrow \forall a \in A$ $(a,a) \in R$
$a \in B$

$\therefore$ R is reflexive

8. which of these are POSETS?

(a) $(R, =)$

Ans Given relation is reflexive

as $\forall a \in A$ $(a,a) \in R$ ($\because a = a$)

Given relation is antisymmetric

$\because$ $\forall a \in A$ $(a,a) \in R$ are present in the relation

$\Rightarrow$ R is antisymmetric

$\because$ $(1,1)$ & $(1,1) \longrightarrow (1,1) \in R$

$\vdots$

Similarly $\forall a \in A \cdot (a,a)$ & $(a,a) \longrightarrow (a,a) \in R$

$\Rightarrow$ R is transitive

$\Rightarrow$ R is POSET.

Let $A' = \{1,2,3,4\} \subseteq A$

Least element $= \phi$

$\therefore \exists A' \subseteq A$ such that no L.E. exist

$\Rightarrow$ A is not a total order

(b) $(R, <)$

Ans Given relation is not reflexive

$\because$ $a \not< a$

$\Rightarrow$ R is not a POSET

$\Rightarrow$ R is not total order.

(C) $(R, \leq)$

Ans Given relation is reflexive.

$\because \forall a \in A$ $(a,a) \in R$ $(\because a=a)$

if $(a,b) \in R \Rightarrow a \leq b$

$\Rightarrow (b,a) \notin R$ $(\because a<b)$

$\therefore$ Given relation is not symmetric

$\Rightarrow$ R is not a POSET

$\Rightarrow$ R is not a total order.

(d) $(R, \neq)$

Ans Given R is not reflexive

$\because (a,a) \notin R$ $(\because a=a)$

$\Rightarrow$ R is not POSET

$\Rightarrow$ R is not a total order.

Is there any total order relation among these?

Ans No, there is no total order relation among these.

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9. Prove or disprove. If R is an equivalence relation on A, then RoR is an equivalence relation on A.

Ans Given that R is an equivalence relation

Reflexivity:

$\because$ R is a reflexive relation

$\Rightarrow \forall a \in A \ (a,a) \in R$

$\Rightarrow (a,a) \in RoR \quad ((a,a) \in R \ \& \ (a,a) \in R \Rightarrow (a,a) \in RoR)$

Symmetric:

$\because$ R is a symmetric relation

$\forall a,b \in A \ ((a,b) \in R \longrightarrow (b,a) \in R\}$

here two cases can arises

(i) $(a,a) \ \& (a,b) \in R \Rightarrow (a,b) \in RoR$

$(b,a) \ \& (a,a) \in R \Rightarrow (b,a) \in RoR$

(ii) There exist a intermediate element x such that

(a,x) and $(x,b) \in R$

$\Rightarrow (a,b) \in RoR$

Similarily (x,a) and (b,x) are both in R.

$\Rightarrow (b,a) \in RoR$

$\therefore$ RoR is symmetric relation.

Transitivity:

$\because$ R is transitive relation

$$\forall a,b,c \in A \left\{ \begin{matrix} (a,b) \in R \\ \& \\ (b,a) \in R \end{matrix} \longrightarrow (a,c) \in R \right\}$$

By defination $\exists$ intermediate constants x, y such that

$(a,x), (x,b) \in R$ and $(b,y), (y,c) \in R$.

$\Rightarrow (a,b) \in RoR \ \& \ (b,c) \in RoR$

$\because$ R is transitive

$\Rightarrow (a,b) \in R \ \& (b,c) \in R \Rightarrow (a,c) \in R \ \& (a,c) \in RoR$

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$\therefore$ RoR is transitive relation

$\therefore$ RoR is equivalence relation.

10. Let R be the relation $\{(a,b) \mid a \neq b\}$ on the set of integers upto 15. What is the reflexive closure of R? Find the transitive closure of R. Find the ~~transitive~~ symmetric closure of R.

Ans $A = \{0, 1, 2, 3, \ldots 15\}$

$R = \{(a,b) \mid a \neq b$ on the set $A\}$

$r(R) = \{(a,b) \mid a \neq b\} \cup \{(a,a) \mid a \in A\}$

$S(R) = \{(a,b) \mid a \neq b\}$ $\quad \because$ if $a \neq b \Rightarrow (a,b) \in R$
$\Rightarrow b \neq a \quad \Rightarrow (b,a) \in R.$

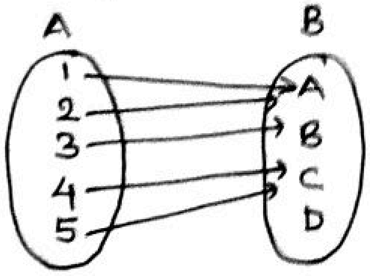
$t(R) = \{(a,b) \mid a \neq b\} \cup \{(a,a) \mid a \in A\}$ $\quad \because$ if $a \neq b$ & $b \neq c$
$\Rightarrow a \neq c.$
if $(a,b) \in R$
&
$(b,c) \in R$
$\Rightarrow (a,c) \in R.$

Functions.

11. Find an example of a function that is neither injective nor surjective.

Ans $A = \{1,2,3,4,5\}$

$B = \{A,B,C,D\}$



$f: A \to B$

f is not one-one $\because f(1) = f(2)$

$1 \neq 2$.

& f is not onto $\because$ There is no pre-image for D.

12. Define functions f, g and h as follows.

$f: \mathbb{R} \to \mathbb{R}, \forall x \in \mathbb{R}, f(x) = x^2$

$g: \mathbb{N} \to \mathbb{N}, \forall x \in \mathbb{N}, g(x) = x^2$

$h: A \to B, \forall x \in A, h(x) = x^2$

$A = \{0,1,2,3,4\}$ and $B = \{0,1,4,9,16\}$

which functions is one to one?

which functions are onto?

Ans (a) $f: \mathbb{R} \to \mathbb{R}$

$f(-1) = f(1) = 1$

$-1 \neq 1$

$\Rightarrow f$ is not one-one

$\because -1 \in \mathbb{R}$ but there is no preimage for -1

$\Rightarrow f$ is not onto.

(b) $g: \mathbb{N} \to \mathbb{N}$

$g(x_1) = g(x_2) \quad x_1, x_2 \in \mathbb{N}$

$x_1^2 = x_2^2$

$x_1 = \pm x_2$

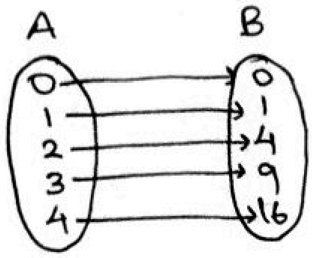
$x_1 = x_2 \quad (\because -x_2 \notin \mathbb{N})$

$\therefore$ g is one-one

$\because 3 \in \mathbb{N}$ but there is no preimage for 3

$\therefore$ g is not onto.

(c)



h is one-one.

$\because$ Every element in domain A has a unique image in co-domain.

h is onto

$\because$ Range = co-domain = B.

$f(0)=0 \in B$

$f(1)=1 \in B$

$f(2)=4 \in B$

$f(3)=9 \in B$

$f(4)=16 \in B$

13. Determine whether each of these functions from Z to Z is one to one.

(a) $f(n)=n-1$

Ans $n_1, n_2 \in Z$

$f(n_1) = f(n_2)$

$\Rightarrow n_1 - 1 = n_2 - 1$

$\therefore n_1 = n_2$

$\therefore f$ is one-one.

(b) $f(n) = n^2+1$

Ans $n_1, n_2 \in Z$

$f(n_1) = f(n_2)$

$n_1^2+1 = n_2^2+1$

$n_1^2 = n_2^2$

$n_1 = \pm n_2$

$\therefore f(n)$ is not one-one.

(C) $f(n) = n^3$

Ans $n_1, n_2 \in \mathbb{Z}$

$f(n_1) = f(n_2)$

$n_1^3 = n_2^3$

$n_1 = n_2$

$\therefore f(n)$ is a one-one.

14. If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are functions, then the functions $(f+g)\cancel{(x) = f(x)+g(x)} : \mathbb{R} \to \mathbb{R}$ is defined by the formula $(f+g)(x) = f(x) + g(x)$ for every real number x.

(a) $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ is one-one, IS $f+g$ is one-one? Justify your answer.

Ans Given statement is wrong

Let us disprove it by giving a counterexample

$f(x) = x$ & $g(x) = -x$ are two 1-1 functions.

$(f+g)(x) = f(x) + g(x) = x + (-x) = 0$

but $(f+g)(1) = (f+g)(100) = 0$

& $1 \neq 100$

$\Rightarrow f+g$ is not one-one.

$\therefore$ If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ is one-one then $f+g$ need not be a one-one function.

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(b) $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are both onto, Is $f+g$ also onto? Justify your answer.

Ans Given statement is wrong

Let us disprove it by giving a counterexample.

$f(x) = x$ & $g(x) = -x$ are two onto functions.

$(f+g)(x) = f(x) + g(x) = x + (-x) = 0$

$\Rightarrow$ Every element in $\mathbb{R}$ domain is mapped to 0 & element except 0 in the co-domain does not have a pre-image.

$\therefore$ $f+g$ is not onto.

hence disproved.

15. Suppose that f is a function from A to B where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if only if it is onto.

Ans $a \Rightarrow b$

Given that

$|A| = |B|$

$f: A \to B$ is one-one

This implies every element in domain has a unique image.

$\because |A| = |B|$

$\Rightarrow$ Every element in A has a ~~[illegible]~~ mapping with every element in B.

$\Rightarrow$ Every element in B has a preimage in B

$\therefore f: A \to B$ is onto.

$b \Rightarrow a$

Given

$|A| = |B|$

$f: A \to B$ is onto

This implies every element in co-domain (B) has a preimage in domain A

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$\because |A| = |B|$

$\Rightarrow$ Every element in B has a mapping with every element in A

$\because$ Two elements cannot have same preimage because then it won't be a function.

$\Rightarrow$ Every element in A has a image in B.

$\therefore f: A \longrightarrow B$ is one-one

hence proved.

16. Let D be the set of all finite subsets of all positive integers. and define $T: Z^+ \rightarrow D$ by the following rule: For every integer n, $T(n)$ = the set of all of the positive divisors of n.

(a) Is T one-one? Prove or give a counterexample.

<u>Ans</u> Let us take

$m, n \in Z^+$

$f(m) = f(n)$

$\Rightarrow$ divisors of m must be equal to divisors of n.

$\because$ set of divisors includes the number itself.

This is only possible if $m = n$

hence, f is one-one.

(b) Is T onto? Prove or give a counterexample.

<u>Ans</u> Let us disprove the statement by giving a counterexample.

~~Let~~ $\{1,2,3\} \in D$

The smallest positive integer having $\{1,2,3\}$ as divisor is 6 but 6 is also a divisor of itself. This implies $\{1,2,3\}$ does not ~~having~~ have a preimage.

hence, f is not onto.

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17. Determine whether each of the following functions from Z to Z is one-one.

(a) $f(n) = n + 7$.

Ans $n_1, n_2 \in Z$

$f(n_1) = f(n_2)$

$n_1 + 7 = n_2 + 7$

$\Rightarrow n_1 = n_2$

$\therefore$ f is one-one.

(b) $f(n) = 2n - 3$.

Ans $n_1, n_2 \in Z$

$f(n_1) = f(n_2)$

$2n_1 - 3 = 2n_2 - 3$

$\Rightarrow \cancel{2}n_1 = \cancel{2}n_2$

$\therefore n_1 = n_2$

$\therefore$ f is one-one

(c) $f(n) = \lceil n/2 \rceil$

Ans $f(1) = \lceil 1/2 \rceil = 1$

$f(2) = \lceil 1 \rceil = 1$

$f(1) = f(2)$

but $1 \neq 2$.

$\Rightarrow$ f is not one-one.

18. If X and Y are sets and $f : X \longrightarrow Y$ is one-one and onto, then $F^{-1} : Y \longrightarrow X$ is also one-one and onto.

Ans Given F

f is one-one and onto

f is bijective function.

$\Rightarrow |X| = |Y|$

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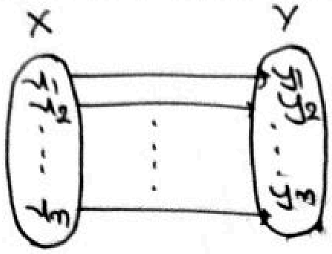

To show that F^{-1} is one-one
we need to show

$$\text{if } F^{-1}(y_1) = F^{-1}(y_2)$$
$$\Rightarrow y_1 = y_2$$
$$F(F^{-1}(y_1)) = F(F^{-1}(y_2))$$
$$y_1 = y_2$$

hence F^{-1} is one-one.

In order to show that F^{-1} is onto we need to prove that

$$\exists y \in Y \ \forall x \in X$$
$$F^{-1}(y) = x$$

$\because$ F is onto, $\exists x$ in X $\forall y \in Y$

~~for~~ $F(x) = y$

$$F^{-1}(y) = F^{-1}(F(x))$$
$$= x$$

$\therefore$ F^{-1} is onto

19. Suppose $f: X \longrightarrow Y$ is onto. Prove that for every subset $B \subseteq Y$. $F(F^{-1}(B)) = B$.

Ans B is every subset of Y

$\Rightarrow B \in P(Y)$ (powerset of X)

$\because$ ~~B~~ F is onto

$$F(F^{-1}(B)) = \{y \in Y : \exists x \in f^{-1}(B) [f(x) = y]\}$$
$$= \{y \in Y : \exists x \in \{z \in X : f(z) \in B\} [f(x) = y]\}$$
$$= \{y \in X : \exists x \in \{z X [f(x) = y \wedge f(x) \in B]\}$$
$$= B$$

$\therefore$ $F(F^{-1}(B)) = B$.

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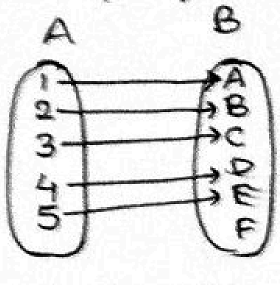
20. Give an example of finite sets A and B with $|A|, |B| \geq 4$ and a function $F: A \to B$ such that.

(a) F is one-one but not onto.

Ans $A = \{1,2,3,4,5\}$ $|A| = 5 > 4$

$B = \{A,B,C,D,E,F\}$ $|B| = 6 > 4$

$F: A \to B$



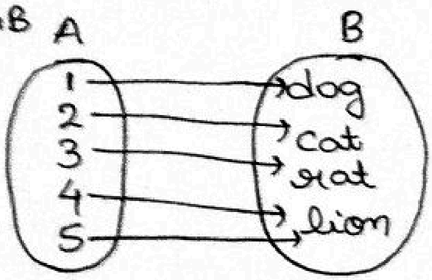
One-one but not onto ($\because$ F does not have a preimage).

(b) F is onto but not one-one.

Ans $A = \{1,2,3,4,5\}$ $|A| = 5 > 4$

$B = \{$~~1,2~~, dog, cat, rat, lion$\}$ $|B| = \cancel{5}4 = 4$

$F: A \to B$



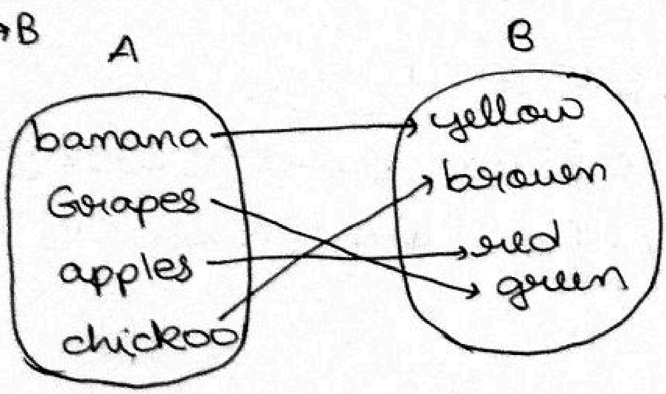
onto but not one-one ($\because$ ~~5 does not have a~~ $f(4) = f(5) =$ lion, $4 \neq 5$).

(C) F is onto and one-one

Ans A = {banana, grapes, apples, chickoo} $|A| = 4$

B = {yellow, brown, red, green} $|B| = 5$

$F: A \to B$



both one-one and onto.