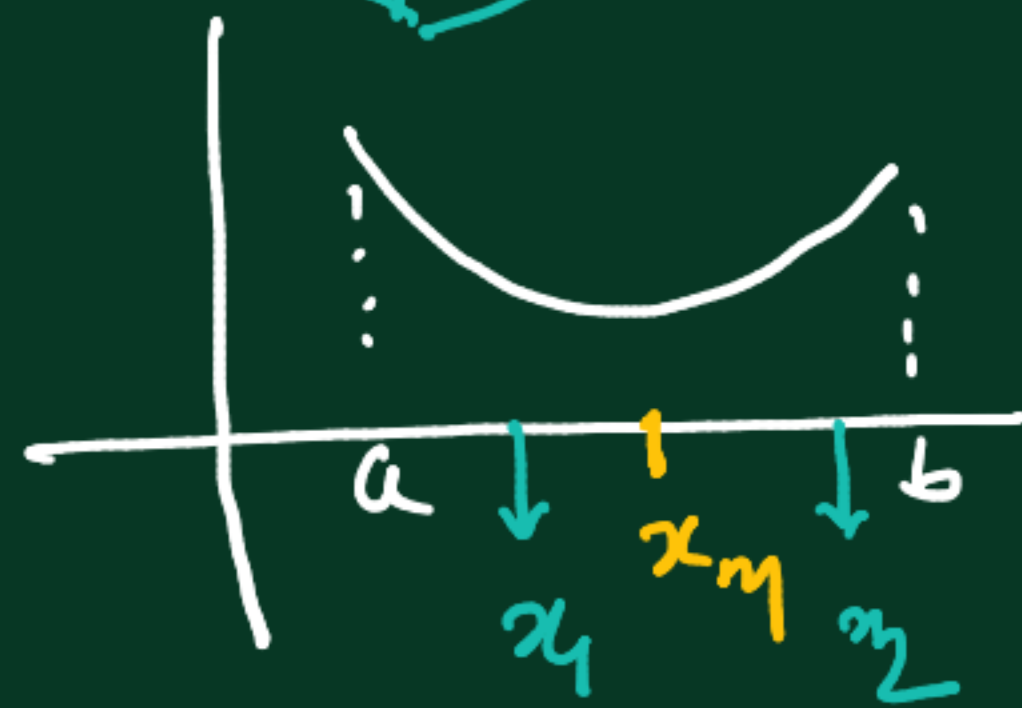
Dichotomous Search

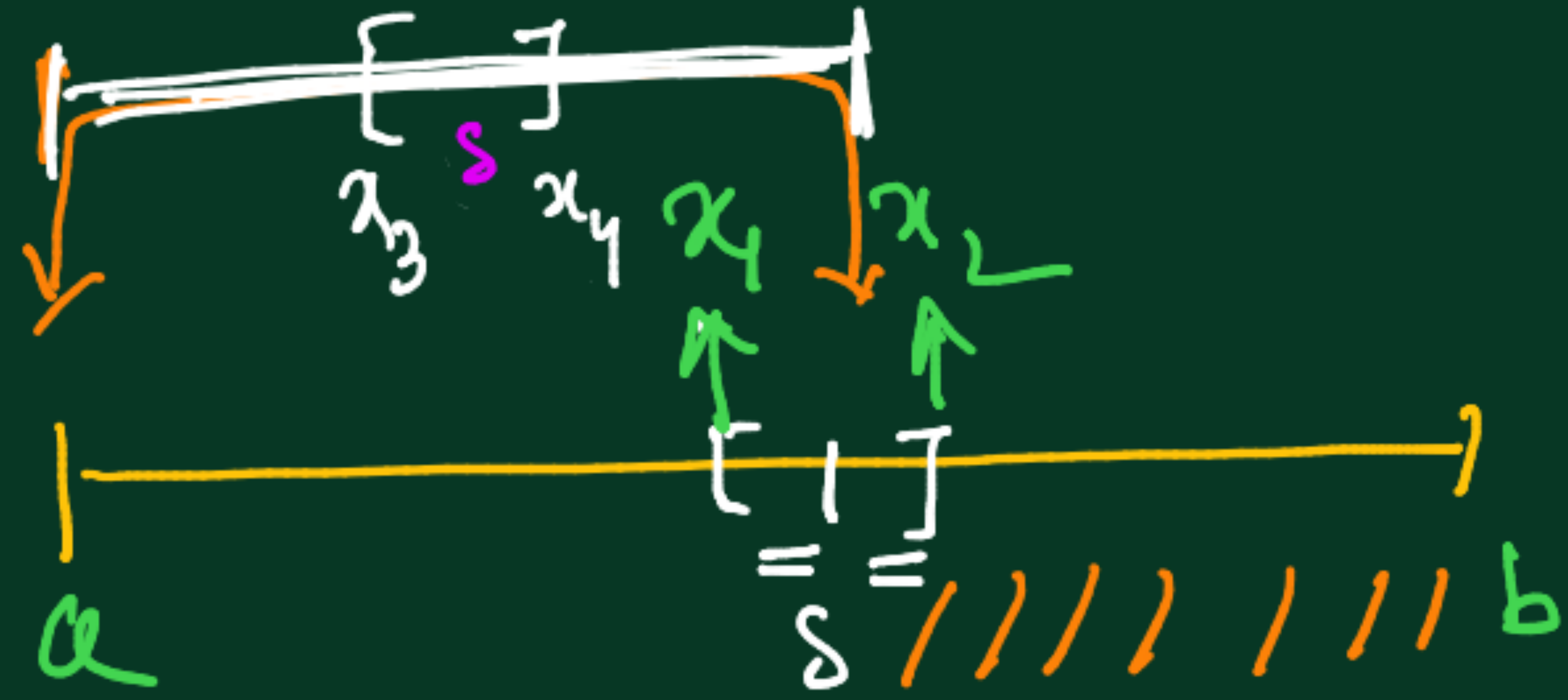
$\boxed{f(x)}$



$$f'(x) = e^x + \sin x = 0$$

50% per each iteration & 2-function evaluat

25% per funⁿ evalution



$$x_1 = \boxed{a + \frac{L}{2}} - \frac{\delta}{2}$$

$$x_2 = b - \frac{L}{2} + \frac{\delta}{2} = a + \frac{L}{2} + \frac{\delta}{2}$$

$$f(x_1) \overset{<}{\underset{>}{\longleftrightarrow}} f(x_2)$$

$\downarrow$

$$f(x_3) \longleftrightarrow f(x_4)$$

$$\frac{L}{2} - \frac{\delta}{2}$$

$< 50\%$

For each function evaluation

< 25 %.

X

$$\boxed{F_n = F_{n-1} + F_{n-2}}$$

$$\boxed{F_0 = F_1 = 1}$$

F_5

1 1 2 3 5 8 13 $= F_6$

F_2 F_3 F_4

$$\boxed{F_7 = 21}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \boxed{\frac{1+\sqrt{5}}{2}} = 1.618$$

$$\lim_{n \to \infty} \frac{F_n}{F_{n+1}} = \boxed{0.618} = \lim_{n \to \infty} \frac{F_{n-1}}{F_n}$$

Q:- $\boxed{\lim_{n \to \infty} \frac{F_{n-2}}{F_n}}$

Fibonacci Search Method

Step-1

Choose lower bound a
& upper Bound b

Set $\boxed{L = b - a}$

Assume No of function evaluation : n

Step.-2

Compute $L_k^* = \left(\frac{F_{n-k+1}}{F_{n+1}}\right) L$

Set $\boxed{x_1 = a + L_k^*}$ & $\boxed{x_2 = b - L_k^*}$

Step-3

Compute $f(x_1)$ & $f(x_2)$

Step-4

Is $\boxed{k = n}$?
- yes - stop
- No then go to **Step-2**

Example:-

Minimize the function

$$\boxed{f(x) = x^2 + \frac{54}{x}}$$

$a = 0, \quad b = 5$

$\boxed{n = 3}$, $\boxed{k = 2}$

$$L_2^* = \left(\frac{F_{3-2+1}}{F_{3+1}}\right) L = \left(\frac{F_2}{F_4}\right) 5$$

$$= \left(\frac{2}{5}\right) 5 = 2$$

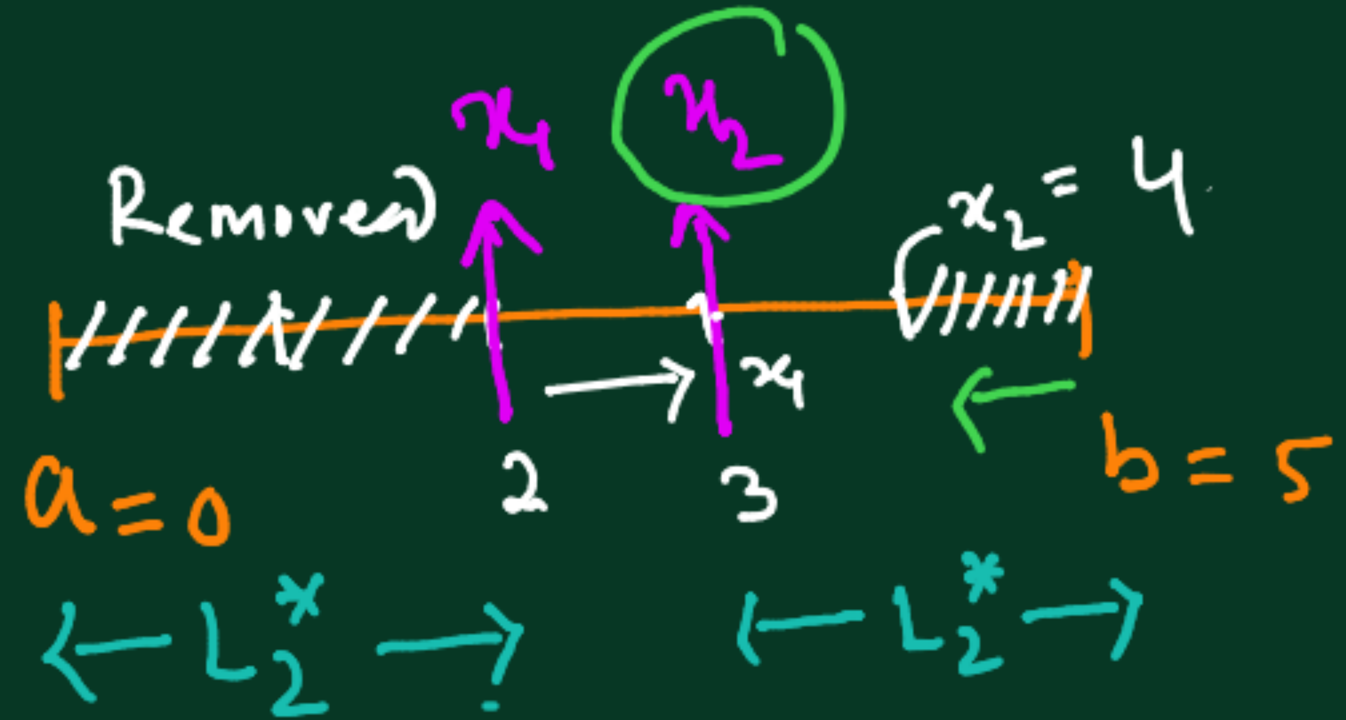
$$\left.\begin{aligned} x_1 &= a + L_2^* = 0 + 2 = 2 \\ x_2 &= b - L_2^* = 5 - 2 = 3 \end{aligned}\right\}$$

$\boxed{\text{Step - 3}}$

$$f(x_1) = x_1^2 + \frac{54}{x_1} = 4 + \frac{54}{2}$$

$$= \frac{62}{2} = 31$$

$$\boxed{f(x_2) = 27} \qquad f(x_1) > f(x_2)$$



$$= a + \frac{F_2}{F_4} L + \frac{F_3}{F_4} L$$

$$L_2 = L - L_2^* = 5 - 2 = \boxed{3}$$

$$f(x_4) > f(x_2)$$

$=$

$a = x_4 = 2, \quad b = 5, \quad , \quad x^* = 3.5$

Step-4

$k = 2 \neq 3 = n$ No, Go to Step-2 $\rightarrow k = k+1 = 3$

Step-2

$$L_3^* = \left(\frac{F_{n-k+1}}{F_{n+1}}\right) L = \left(\frac{F_1}{F_4}\right) L = \frac{1}{5} \cdot 5 = 1$$

Step-3

$x_1 = 3$, $x_2 = 4$

$f(x_1) = 27$, $f(x_2) = \boxed{29.5}$

$$f(x_1) < f(x_2)$$

$a = 2$, $b = 4$

$\hookrightarrow x^* = \frac{2+4}{2} = 3$

Step-4

$k = n = 3$

Stop

$$L_2 = L - \underbrace{\frac{F_{n-1}}{F_{n+1}} L}_{L_2^*} = \left(\frac{F_{n+1} - F_{n-1}}{F_{n+1}}\right) L$$

$$F_{n+1} = \underline{F_n + F_{n-1}}$$

$$L_2 = \left(\frac{F_n}{F_{n+1}}\right) L$$

$$L_3 = L_2 - L_3^* = \left(\frac{F_n}{F_{n+1}}\right) L - \left(\frac{F_{n-2}}{F_{n+1}}\right) L$$

$$= \left(\frac{F_n - F_{n-2}}{F_{n+1}}\right) L$$

$$= \left(\frac{F_{n-1}}{F_{n+1}}\right) L$$

$$\begin{cases} L_4 = \left(\dfrac{F_{n-2}}{F_{n+1}}\right) L \\ L_5 = \left(\dfrac{F_{n-3}}{F_{n+1}}\right) L \\ \vdots \qquad \ddots \qquad \ddots \\ L_n = L_{n-1} - L_n^* = \left(\dfrac{F_2}{F_{n+1}}\right) L \end{cases}$$

$$= \left(\frac{F_{n-n+2}}{F_{n+1}}\right) L$$

$$\boxed{\frac{2}{F_{n+1}} L < \epsilon}$$

$$\boxed{> 25\%}$$

For $\epsilon = 0.1$

$$|x^* - E| < \frac{0.1}{2}$$

$=$

$=$

$$\frac{2}{F_{n+1}} \times L < 0.1$$

$$\Rightarrow \frac{10}{F_{n+1}} < 0.1 \Rightarrow \frac{F_{n+1}}{10} > 10$$

$$\Rightarrow F_{n+1} > 100$$

$$\boxed{n = 10}$$

OR

$$\frac{1}{F_n} 5 < 0.1 \Rightarrow \frac{F_n}{5} > 10$$

$$\Rightarrow F_n > 50 \Rightarrow n = 9$$

$$\begin{cases} x^* \text{ is Numerical} \\ x \text{ is Exact} \end{cases}$$

$$\text{Error } E = |x - x^*| \leq \frac{L_n}{2}$$

$$\text{if } \frac{L_n}{2} < \epsilon \text{ then } E < \epsilon$$

$$\Rightarrow \frac{1}{F_{n+1}} L < \epsilon \Rightarrow \frac{1}{F_{n+1}} 5 < 0.1$$

$$\Rightarrow \frac{1}{F_{n+1}} < \frac{1}{50}$$

$$\Rightarrow \boxed{F_{n+1} > 50} \qquad n+1 = 9$$

$$\Rightarrow \boxed{n = 8}$$

Similarly

$$\frac{1}{2} L_n < \epsilon$$

$$\Rightarrow \quad \frac{1}{2} \times \frac{1}{F_n} L < \epsilon = 0.1$$

$$\Rightarrow \quad 1/F_n < \frac{2}{10} \times \frac{1}{5}$$

$$\Rightarrow \quad F_n > 25$$

$$n = 8$$

Golden Section Search Method

Golden Number $\tau = 0.618$

$$\tau^* = 1 - 0.618$$
$$= 0.382$$

Algorithm:-

$\boxed{\text{Step-1}}$ → Choose lower bounds a & b
Tolorence / error bound = ϵ

Normalize variable x by using the equation $\omega = \left(\frac{x-a}{b-a}\right)$

Thus $a_\omega = 0$, $b_\omega = 1$ and $L_\omega = 1$
set $K = 1$

$\boxed{\text{Step-3}} \rightarrow$

Set $w_1 = a_w + (0.618) L_w$

$w_2 = b_w - (0.618) L_w$

Compute $f(w_1)$ or $f(w_2)$, depending on whichever of the two was not evaluated earlier.

Use the fundamental rule of region of elemination rule to eleminate a region

Set New a_w & b_w

$\boxed{\text{Step-3}}$

is $|L\,w| < \epsilon$ small ?

If no, set $K = K+1$, Go To Step-2

In this algorithm

Interval reduces to $(0.618)^{n-1}$

after n function evaluation.

Thus No. of function evaluation

$$\boxed{(0.618)^{n-1}(b-a) = \epsilon}$$

Per function evaluation 38.2 % elimination

Exercise

Again $f(x) = x^2 + 54/x$

Step-1

$a = 0$ & $b = 5$

$$w = \frac{x-a}{b-a} = x/5 \quad (\Rightarrow \underline{x = 5w})$$

Thus, $\boxed{a_w = 0,}$ $\boxed{b_w = 1}$ and $\boxed{L_w = 1}$.

Now w is the variable

$$f(w) = 25w^2 + \frac{54}{5w}$$

In w-space lies at $w^* = 3/5 = 0.6$

We set iteration $k = 1$

$\boxed{\text{Step-2}}$ We set $w_1 = 0 + (0.618)1 = 0.618$

and $w_2 = 1 - (0.618)1 = 0.382$

$f(w_1) = 27.02$ $\qquad f(w_1) < f(w_2)$

$f(w_2) = 31.92$

We eleminate the region (a, w_2) or $(0, 0.382)$

Thus $\underline{aw = 0.382}$ and $\underline{bw = 1}$

$Lw = 0.618$

(Exact min 0.6 lies)

$\boxed{\text{Step-3}}$

Since $|L_w| \not< \epsilon = 10^{-5}$

Set $k = 2$, Goto Step 2

complete - 1 iteration

$\boxed{\text{Step-2}}$

$w_1 = 0.764$ | $f(w_1) = 28.73$

$w_2 = 0.618$ | $f(w_2) =$ known

Removed - $(0.764, 1)$

New interval - $[0.382, 0.764]$

$\boxed{\text{Step-3}}$

$\boxed{k = 3}$