Problem Set : 3

1. Show that $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges.

2. Show that $\sum_{n=1}^{1000} \frac{1}{n} \approx 7.49$

3. Explain why $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$ diverges.

4. Explain why $\sum_{n=1}^{\infty} \frac{5}{2^{1/n}+14}$ diverges.

5. Explain why $\sum_{n=1}^{\infty} \frac{3}{n}$ diverges.

4. Compute $\sum_{n=0}^{\infty} \left(\frac{4}{(-3)^n} - \frac{3}{3^n} \right)$.

5. Compute $\sum_{n=0}^{\infty} \left(\frac{3}{2^n} + \frac{4}{5^n} \right)$.

6. Determine whether each series converges or diverges.

(i) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi/4}}$ (ii) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ (iii) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(iv) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ (v) $\sum_{n=1}^{\infty} \frac{1}{e^n}$ (vi) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

(vii) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ (viii) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

7. Find an N so that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \sum_{n=1}^{N} \frac{1}{n^4} \pm 0.005$

8. Find an N so that $\sum_{n=0}^{\infty} \frac{1}{e^n} = \sum_{n=0}^{N} \frac{1}{e^n} \pm 10^{-4}$

9. Find an N so that $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2} = \sum_{n=1}^{N} \frac{\ln n}{n^2} \pm 0.005$

10. Find an N so that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} = \sum_{n=2}^{N} \frac{1}{n(\ln n)^2} \pm 0.005$.

11. Determine whether the following series converge or diverge.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+5}$ (ii) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{3n-2}$

(iii) $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n-3}}$ (iv) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$

(v) Approximate $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^3}$ to two decimal places.

(vi) Approximate $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^4}$ to two decimal places.

12. Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ converge?

13. Does $\displaystyle\sum_{n=2}^{\infty} \frac{|\sin n|}{n^2}$ converge?

14. Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-3}}$ converge?

15. Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3}}$ converge?

16. Determine whether the series converge or diverge.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^2+3n+5}$ (ii) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2n^2+3n-5}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^2 \,[illegible]\, 3n-5}$

(iv) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+4}{2n^2+3n+5}$ (v) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+4}{2n^2+3n+5}$ (vi) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(vii) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ (viii) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (ix) $\sum_{n=1}^{\infty} \frac{3^n}{2^n+5^n}$

(x) $\sum_{n=1}^{\infty} \frac{3^n}{2^n+3^n}$

17. Does $\sum_{n=2}^{\infty} \frac{\sin n}{n^2}$ converge?

18. Does $\sum_{n=0}^{\infty} (-1)^n \frac{3n+4}{2n^2+3n+5}$ converge?

19. Determine whether each series converge absolutely, converge conditionally, or diverges.

(i) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{2n^2+3n+5}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n^2+4}{2n^2+3n+5}$

(iii) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$ (iv) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n^3}$

(v) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$ (vi) $\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{2^n+5^n}$

(vii) $\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{2^n+3^n}$ (viii) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\arctan n}{n}$