

IIITDM KANCHEEPURAM
MA1001 Differential Equations
Problem Set 3

1. Determine which of the following equation are exact, and solve the ones that are.

(a) $(2y^2 - 4x + 5)dx = (4 - 2y + 4xy)dy$

(b) $(y + y \cos xy)dx + (x + x \cos xy)dy = 0$

2. Show that if $(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})/(-M)$ is a function of y alone, say $h(y)$, then

$$\mu = e^{\int h(y)dy}$$

is an integrating factor for $Mdx + Ndy = 0$.

3. Show that if $(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})/(Ny - Mx)$ is a function $g(z)$ of the product $z = xy$, then

$$\mu = e^{\int g(z)dz}$$

is an integrating factor for $Mdx + Ndy = 0$.

4. Solve each of the following equations by finding an integrating factor.

(a) $(xy - 1)dx + (x^2 - xy)dy = 0$

(b) $(x + 2) \sin y dx + x \cos y dy = 0$

5. Solve the following as linear equations.

(a) $y' + y = \frac{1}{1 + e^{2x}}$

(b) $(x \log x)y' + y = 3x^3$

6. A first order equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

is called a Bernoulli's equation. WKT, by the substitution $z = y^{1-n}$, it can be converted into a linear equation: $\frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x)$. Use this approach and solve the following Bernoulli's equations.

(a) $xy^2y' + y^3 = x \cos x$

(b) $xdy + ydx = xy^2dx$