Engineering Electromagnetics

Lecture 2

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by

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Course Content

Vectors - an introduction; Unit vectors in spherical and cylindrical polar co-ordinates; Concept of vector fields; Gradient of a scalar field; flux, divergence of a vector, Gauss's theorem, Continuity equation; Curl –rotational and irrotational vector fields, Stoke's theorem. (12)

Electrostatics:
Electrostatic potential and field due to discrete and continuous charge distributions, boundary condition, Energy for a charge distribution, Conductors and capacitors, Laplaces equation Image problem , Dielectric polarization, electric displacement vector, dielectric susceptibility , energy in dielectric systems. (10)

Magnetostatics:
Lorentz Force law Biot-Savart's law and Ampere's law in magnetostatics, Divergence and curl of B, Magnetic induction due to configurations of current-carrying conductors, Magnetization and bound currents, Energy density in a magnetic field Magnetic permeability and susceptibility. (10)

Electrodynamics:
Electromotive force, Time-varying fields, Faradays' law of electromagnetic induction, Self and mutual inductance, displacement current, Maxwell's equations in free space. Boundary condition, propagation in linear medium. Plane electromagnetic waves—reflection and refraction, electromagnetic energy density, Poynting vector. (10)

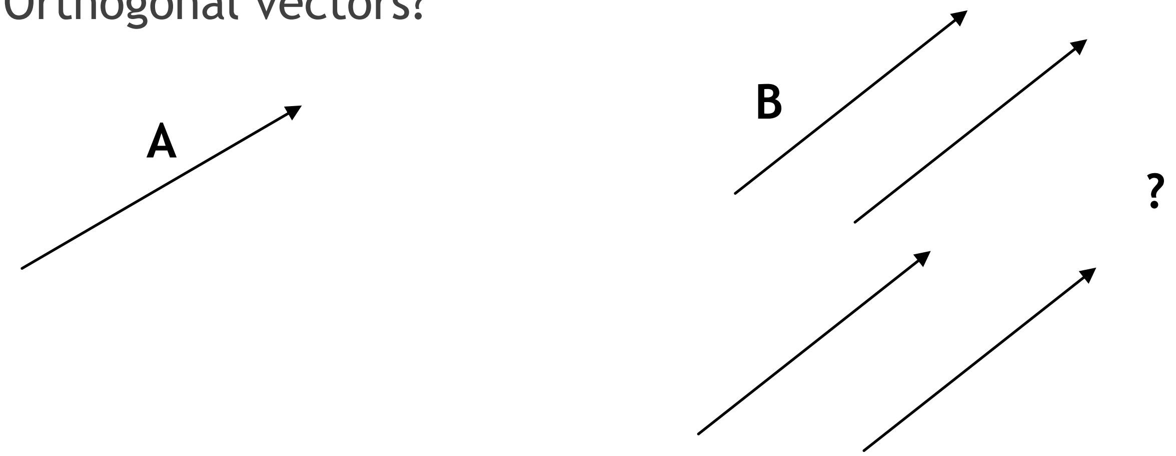
Few important points

- **Books:** **Introduction to Electrodynamics**, David J. Griffiths
- **Electromagnetic Field Theory Fundamentals** *by* BhagGuru, Hüseyin R. Hiziroglu

- Tentative Marks distribution and exam pattern
- Quiz 1: 15 marks
- Quiz 2: 15 marks
- Assignments: 2x10 = 20 marks
- End Semester: 50 marks

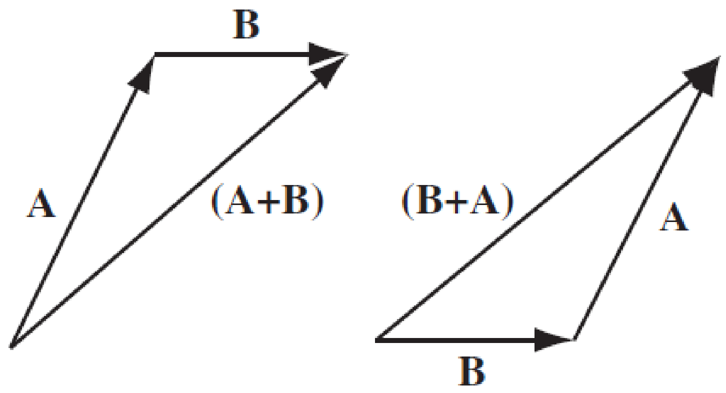
Scalars and Vectors

- Mass, Force, Temperature, Torque, Charge, Time, Work, Acceleration, Velocity, Height
- Null vector?
- Unit vector of **A**?
- Orthogonal vectors?

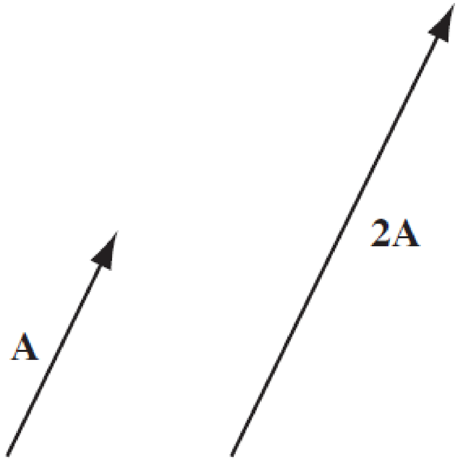


Vector operations

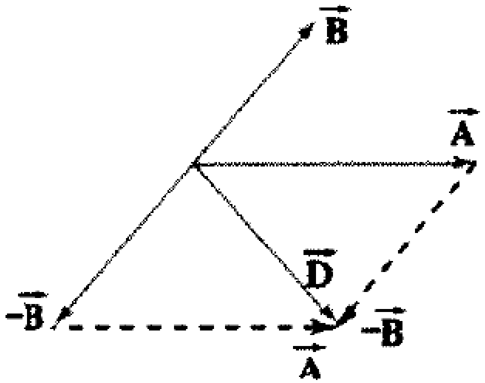
- **Addition:**



- **Scaling:**



Subtraction: A - B = A +(-B)



Vector operations: Dot product

- **Scalar product** $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$

- **Q:** What is *Bcosθ* in terms of $\hat{a}$? $(\hat{a}.\vec{B})$ → ***projection of B along A?***

Commutative: $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

Distributive: $\vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

Scaling: $k(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}) = (k\vec{\mathbf{A}}) \cdot \vec{\mathbf{B}} = \vec{\mathbf{A}} \cdot (k\vec{\mathbf{B}})$

Vector operations: Dot product

- If **P.Q = P.M** does that mean **Q** must always be = **M** ?
- **Ans:**
- **P. (Q-M)** = 0
- 1. **P** perpendicular to **Q-M**
- 2. **P** is null
- **Q-M** = 0 → then only **Q** = **M**

Vector operations: Cross product

- **Q: C = A x B,** If $\hat{\boldsymbol{n}}$ is unit vector along **C**, show that **n = ($\hat{\boldsymbol{a}}$ x $\hat{\boldsymbol{b}}$) /** Sinθ. θ = angle between **A** and **B .**

Ans:

$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} \qquad (1)$

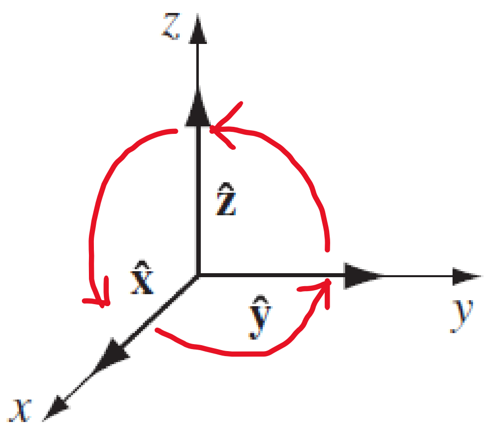
$C\hat{n} = \left(A\hat{a} \times B\hat{b}\right) = \left(\hat{a} \times \hat{b}\right)AB$

$\hat{n} = \left(\hat{a} \times \hat{b}\right)/Sin\theta$ (Hint: from eq(1) what is |**C**|?)

- **Triple products: A.(B x C) and A x (B x C)**

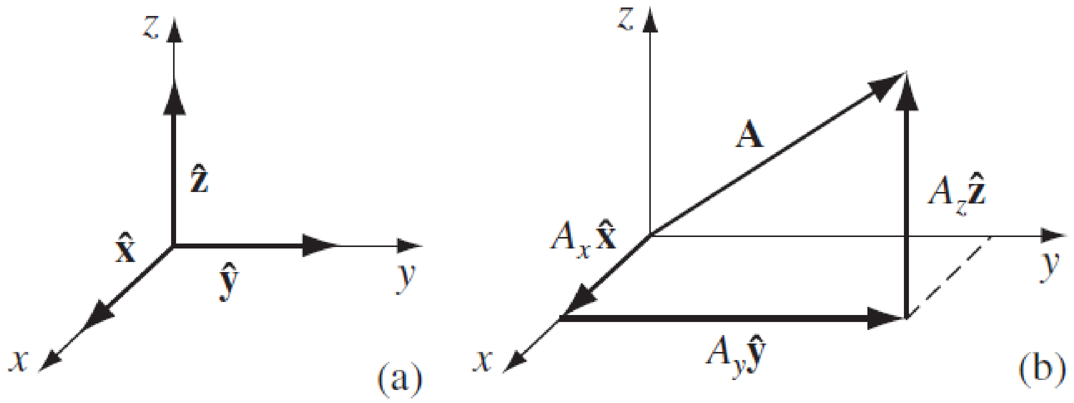
Vectors in cartesian coordinates

- So far discussion → general
- A vector → can always be expressed in terms of three components along three mutually orthogonal directions. But What precisely does "direction" mean?
- Here comes coordinate systems: Cartesian, Cylindrical, Spherical
- **Cartesian coordinates:** three mutually orthogonal axes (x, y, z) and their intersection → origin
- $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$ → **unit vectors** parallel to the *x*, *y*, and *z* axes
- Properties:
- $\hat{\boldsymbol{x}}.\hat{\boldsymbol{x}} = \hat{\boldsymbol{y}}.\hat{\boldsymbol{y}} = \hat{\boldsymbol{z}}.\hat{\boldsymbol{z}} = \ (?)$
- **Q:** $\hat{\boldsymbol{x}}.\ \hat{\boldsymbol{y}} = \hat{\boldsymbol{y}}.\ \hat{\boldsymbol{z}} = ?$
- $\hat{\boldsymbol{x}}$ x $\hat{\boldsymbol{y}}$ =; $\hat{\boldsymbol{y}}$ x $\hat{\boldsymbol{z}}$ =; $\hat{\boldsymbol{z}}$ x $\hat{\boldsymbol{x}}$ = (?)
- $\hat{\boldsymbol{x}}$ x $\hat{\boldsymbol{z}}$ =



Vectors in cartesian coordinates

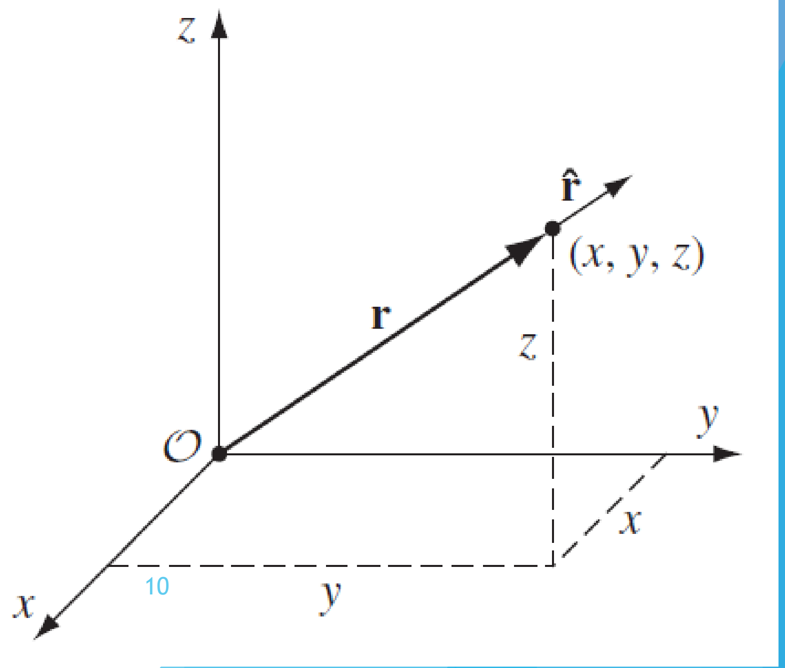
- any vector **A** can be expanded in terms of these basis vectors ($\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$)
- $\boldsymbol{A} = \boldsymbol{A_x}\hat{\mathbf{x}} + \boldsymbol{A_y}\hat{\mathbf{y}} + \boldsymbol{A_z}\hat{\mathbf{z}}$ and **B** $= \boldsymbol{B_x}\hat{\mathbf{x}} + \boldsymbol{B_y}\hat{\mathbf{y}} + \boldsymbol{B_z}\hat{\mathbf{z}}$
- Then $\boldsymbol{A} + \boldsymbol{B}$ =?



- Location of a point in 3D can be expressed in cartesian coordinates

$$\boxed{\mathbf{r} \equiv x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}$$

- Q: **r = (2,2,1)** → r (or |**r**|) =?



Problem 1:

Given $\vec{\mathbf{A}} = 3\vec{\mathbf{a}}_x + 2\vec{\mathbf{a}}_y - \vec{\mathbf{a}}_z$ and $\vec{\mathbf{B}} = \vec{\mathbf{a}}_x - 3\vec{\mathbf{a}}_y + 2\vec{\mathbf{a}}_z$, find $\vec{\mathbf{C}}$ such that $\vec{\mathbf{C}} = 2\vec{\mathbf{A}} - 3\vec{\mathbf{B}}$. Find the unit vector $\vec{\mathbf{a}}_c$ and the angle it makes with the z axis.

Answer

Solution

$$\vec{\mathbf{C}} = 2\vec{\mathbf{A}} - 3\vec{\mathbf{B}}$$
$$= 2[3\vec{\mathbf{a}}_x + 2\vec{\mathbf{a}}_y - \vec{\mathbf{a}}_z] - 3[\vec{\mathbf{a}}_x - 3\vec{\mathbf{a}}_y + 2\vec{\mathbf{a}}_z]$$
$$= 3\vec{\mathbf{a}}_x + 13\vec{\mathbf{a}}_y - 8\vec{\mathbf{a}}_z$$

The magnitude of vector $\vec{\mathbf{C}}$, from (2.26), is

$$C = \sqrt{3^2 + 13^2 + (-8)^2} = 15.556$$

The required unit vector is

$$\vec{\mathbf{a}}_c = \frac{\vec{\mathbf{C}}}{C} = 0.193\vec{\mathbf{a}}_x + 0.836\,\vec{\mathbf{a}}_y - 0.514\vec{\mathbf{a}}_z$$

The angle above unit vector makes with the z axis is

$$\theta_z = \cos^{-1}\left[\frac{C_z}{C}\right] = \cos^{-1}\left[\frac{-8}{15.556}\right] = 120.95^\circ$$

Problem 2

Show that the following vectors are orthogonal:

$\vec{\mathbf{A}} = 4\vec{\mathbf{a}}_x + 6\vec{\mathbf{a}}_y - 2\vec{\mathbf{a}}_z$ and $\vec{\mathbf{B}} = -2\vec{\mathbf{a}}_x + 4\vec{\mathbf{a}}_y + 8\vec{\mathbf{a}}_z$

Problem 3

Calculate the volume of a parallelepiped formed by vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ such that $\vec{\mathbf{A}} = 2\vec{\mathbf{a}}_x + \vec{\mathbf{a}}_y - 2\vec{\mathbf{a}}_z$, $\vec{\mathbf{B}} = -\vec{\mathbf{a}}_x + 3\vec{\mathbf{a}}_y + 5\vec{\mathbf{a}}_z$, and $\vec{\mathbf{C}} = 5\vec{\mathbf{a}}_x - 2\vec{\mathbf{a}}_y - 2\vec{\mathbf{a}}_z$.

Answer

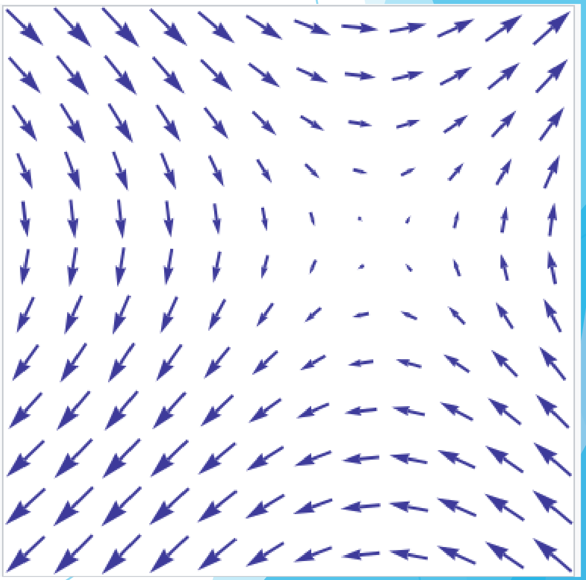
$$\text{volume} = \vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \begin{vmatrix} 2 & 1 & -2 \\ -1 & 3 & 5 \\ 5 & -2 & -2 \end{vmatrix} = 57$$

Concepts of scalar and vector fields

- Fields: Behaviour of a quantity in a given region in terms of a set of values

- A scalar field is an assignment of a scalar to each point in region in the space. E.g. the temperature at a point on the earth is a scalar field.

- A vector field is an assignment of a vector to each point in a region in the space. e.g. the velocity field of a moving fluid is a vector field as it associates a velocity vector to each point in the fluid.



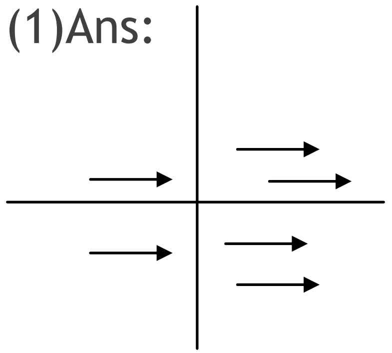
https://www.iitrpr.ac.in/MA101/MA101-Lecturenotes(2019-20)-Module%2013.pdf

Vector fields in cartesian coordinates

- System of vectors (2D/3D) → given by Vector functions
- Vector at every single point in the region
- $\boldsymbol{F}(x, y) = P\hat{\boldsymbol{\imath}} + Q\hat{\boldsymbol{\jmath}}$ in 2D (in 3D?)
- P, Q : defined functions in the region
- Example: $\boldsymbol{F}(x, y) = 3\hat{\boldsymbol{\imath}}$ (solution in the next slide)
- [Exercise:]
- Draw vector fields:
- $\boldsymbol{F}(x, y) = -x\hat{\boldsymbol{\imath}} - y\hat{\boldsymbol{\jmath}}$
- $\boldsymbol{F}(x, y) = xy\hat{\boldsymbol{\imath}} - 2y\hat{\boldsymbol{\jmath}}$
- $\boldsymbol{F}(x, y) = x\hat{\imath} + y\hat{\jmath}$
- $\boldsymbol{F}(x, y) = \frac{x}{\sqrt{(x^2+y^2)}}\hat{\boldsymbol{\imath}} + \frac{y}{\sqrt{(x^2+y^2)}}\hat{\boldsymbol{\jmath}}$

Answers

- (1)Ans:



- (2) Steps to follow to draw a vector field:
- Take a few points in all quadrants: (1,1), (3,2), (2, -3), (-2, -1)
- Draw the vectors at each point

Thank You