

Roll No.: C823I1056

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Indian Institute of Information Technology, Design and Manufacturing, Kancheeppuram
Mid Semester – Feb 2025

Course Code: MA2001

Date of Examination: 22.02.2025

Duration: 90 minutes

Course Title: Probability and Statistics

Category: Elective

Maximum Marks: 25

Roll nos. of students appearing for the exam:

Instructions to students:

- Answer all questions.
- No partial marks will be given.

✓ 1. Fred is answering a multiple-choice problem on an exam, and has to choose one of n options (exactly one of which is correct). Let K be the event that he knows the answer, and R be the event that he gets the problem right (either through knowledge or through ~~Conditional probability~~ ⁷⁵ luck). Suppose that if he knows the right answer he will definitely get the problem right, but if he does not know then he will guess completely randomly. Let $P(K) = p$.

1. Find $P(K|R)$ (in terms of p and n).
2. Show that $P(K|R) \geq p$

(2.5+2.5=5)

2. A hat contains 100 coins, where at least 99 are fair, but there may be one that is double headed (always landing heads); if there is no such coin, then all 100 are fair. Let D be the event that there is such a coin, and suppose that $P(D) = \frac{1}{2}$. A coin is chosen uniformly at random. The chosen coin is flipped 7 times, and it lands heads all 7 times.

1. Given this information what is the probability that one of the coins is double headed?
- ✓ 2. Given this information, what is the probability that one of the coins is double headed? ⁷⁵✗

(2.5+2.5=5)

3. 1. Suppose that in the population of college applicants, being good at baseball is independent of having a good math score on a certain standardized test (with respect to some measure of "good"). A certain college has a simple admissions procedure: admit an applicant if and only if the applicant is good at baseball or has a good math score on the test. Give an intuitive explanation of why it makes sense that among students that the college admits, having a good math score is negatively associated with being good at baseball, i.e., conditioning on having a good math score decreases the chance of being good at baseball.
2. Show that if A and B are independent and $C = A \cup B$, then A and B are conditionally dependent given C (as long as $P(A \cap B) > 0$ and $P(A \cup B) < 1$), with $P(A|B|C) < P(A|C)$

(5)

✓ 4. Let X be a random variable with cdf F , and $Y = \mu + \sigma X$, where μ and σ are real numbers with $\sigma > 0$. Find the cdf of Y , in terms of F .

(5)

5. A book has n typos. Two proofreaders, Prue and Frida, independently read the book. Prue catches each typo with probability p_1 and misses it with probability $q_1 = 1 - p_1$, independently, and likewise for Frida, who has probabilities p_2 of catching and $q_2 = 1 - p_2$ of missing each typo. Let X_1 be the number of typos caught by Prue, X_2 be the number caught by Frida, and X be the number caught by at least one of the two proofreaders. Find the distribution of X . (5)