Engineering Optics

Lecture 13

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by

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Double slit diffraction continued

$$\boxed{I = 4I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma}$$

intensity distribution produced
by one of the slits

Interference pattern produced by
two point sources separated by a distance *d*

**if the slit widths are very small* → *β* small

Young's interference pattern

Minima

$$\boxed{I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$

intensity is zero wherever

$$\beta = \pi, 2\pi, 3\pi, \ldots \quad \underline{b \sin\theta = m\lambda} \quad \textbf{(1)} \qquad m = 1, 2, 3, \ldots$$

or when

$$\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \; d \sin\theta = \left(n + \tfrac{1}{2}\right)\lambda \qquad n = 1, 2, 3, \ldots$$

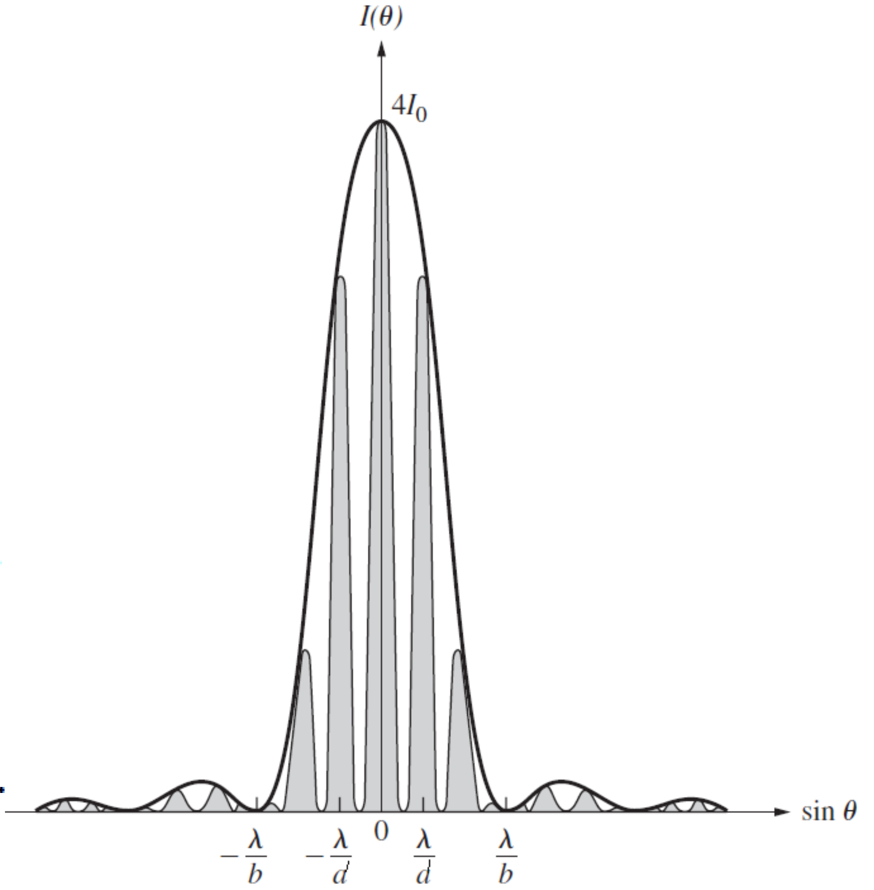
The interference maxima occur when

$$\gamma = 0, \pi, 2\pi, \ldots$$

or when

$p\lambda$

$$\underline{d \sin\theta} = 0, \lambda, 2\lambda, 3\lambda, \ldots \quad \textbf{(2)}$$



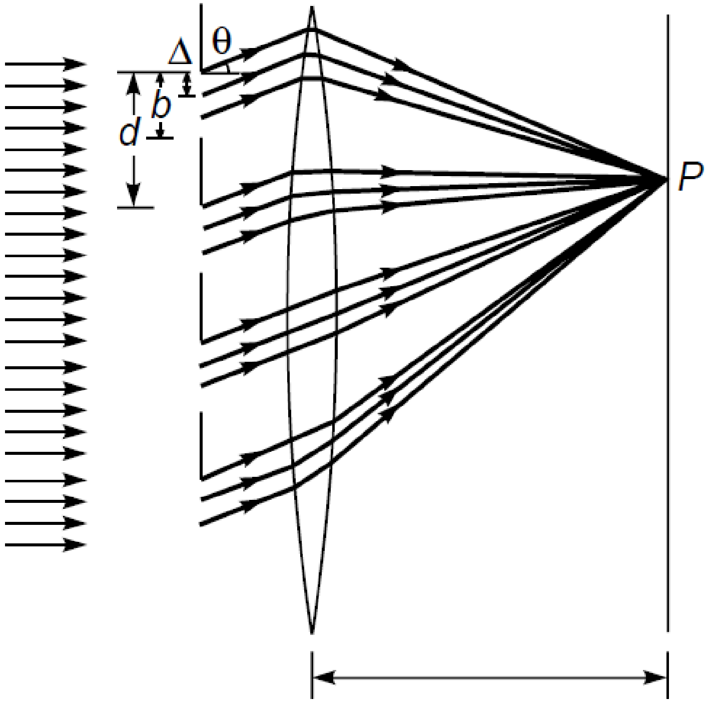
An interference maximum and a diffraction minimum (zero) may correspond to the same θ-value

(d/b = p/m) missing order

Optics, Ghatak; Hecht

N-slit diffraction

Equivalent to multiple slit Fraunhofer Diffraction



Fraunhofer diffraction of a plane wave incident normally on a multiple slit.

- When a plane wave is incident normally on N parallel slits, the Fraunhofer diffraction pattern is given by

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \cdot \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

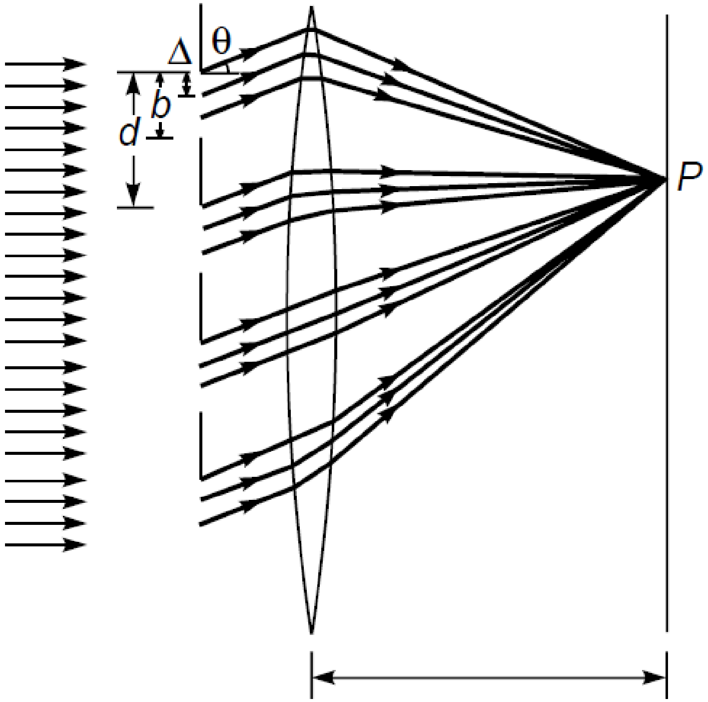
where

$$\beta = \frac{\pi b \sin\theta}{\lambda} \qquad \gamma = \frac{\pi d \sin\theta}{\lambda}$$

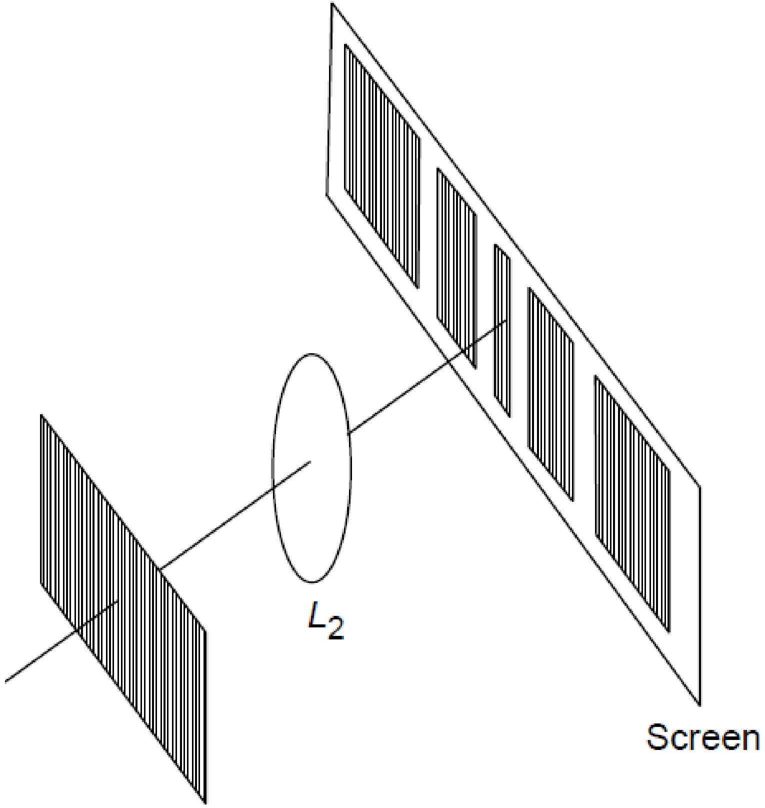
Optics by Ghatak

Grating equation

Equivalent to multiple slit Fraunhofer Diffraction



Fraunhofer diffraction of a plane wave incident normally on a multiple slit.



grating equation

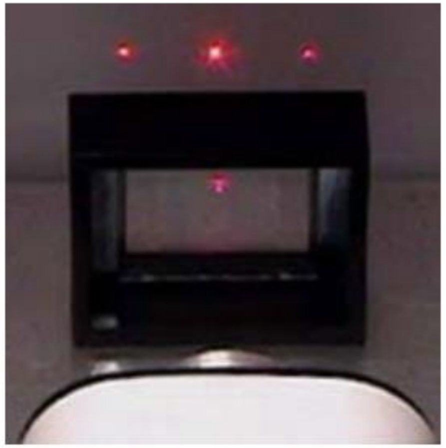
$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, \ldots$$

Optics by Ghatak

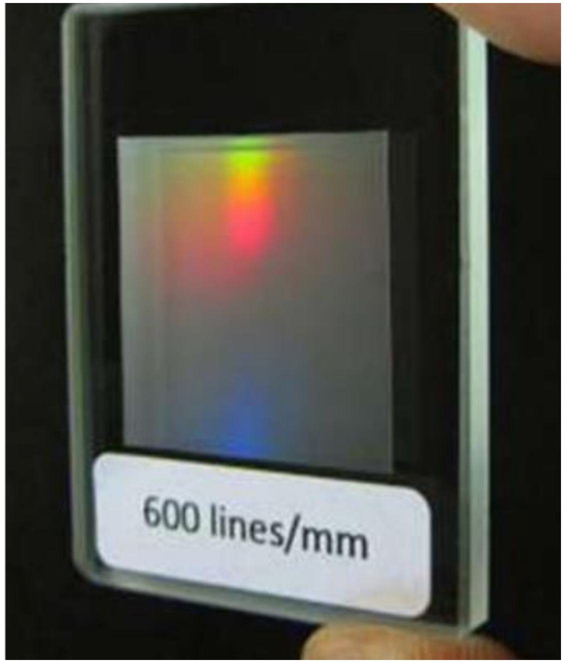
Diffraction Grating

10.2.8 The Diffraction Grating

A repetitive array of diffracting elements, either apertures or obstacles, that has the effect of producing periodic alterations in the phase, amplitude, or both of an emergent wave is said to be a **diffraction grating**.



Diffraction Grating



Use:

Spectroscopy

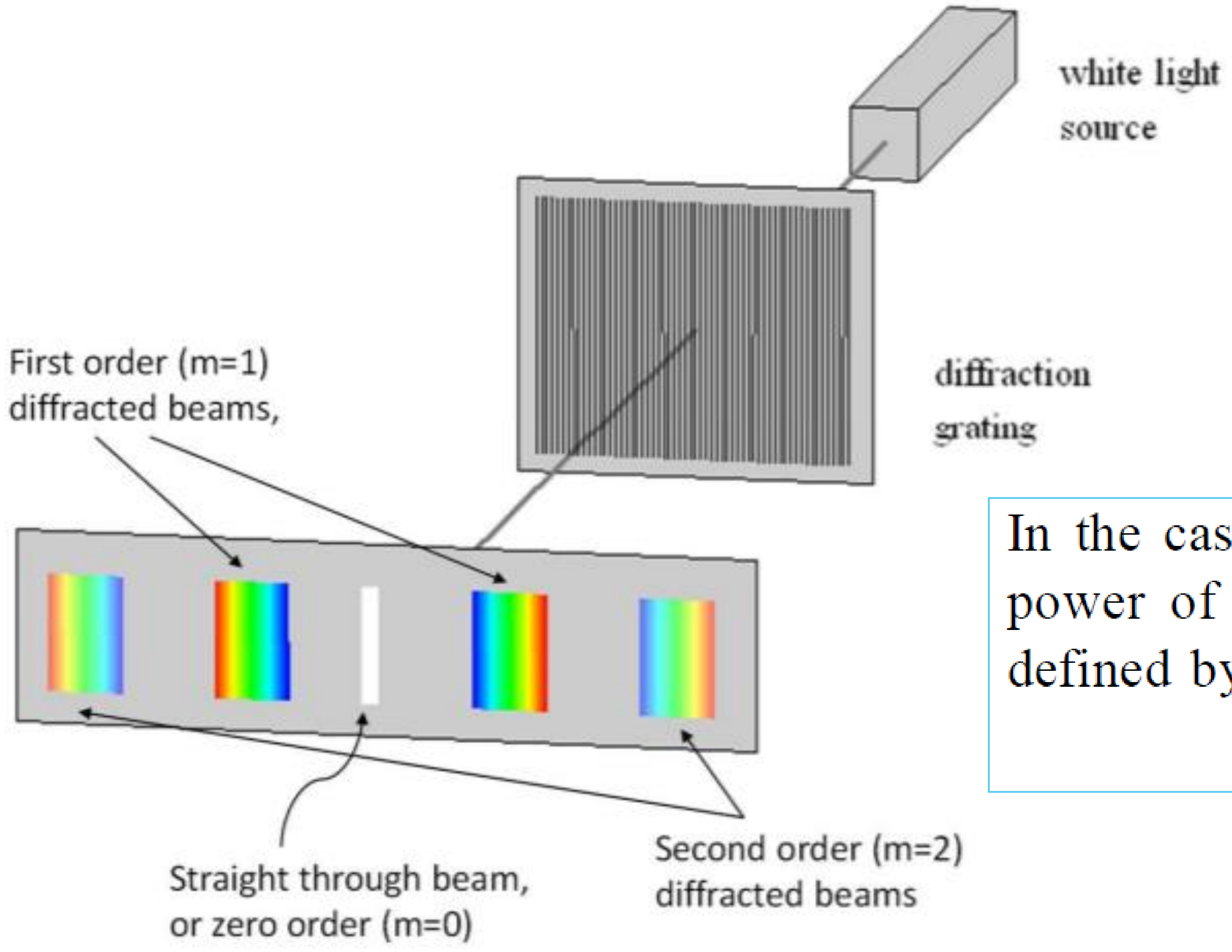
Astronomy

Monochromatic light production

Wavelength tuning of LASER etc.

References: Optics by Hecht
Photos taken from Web

Grating spectrum



$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots$$

Grating element = 1/(no. of lines/cm)

Grating constant = *b* + *d*

In the case of a grating, the resolving power refers to the power of distinguishing two nearby spectral lines and is defined by the

$$R = \frac{\lambda}{\Delta\lambda}$$

Optics by Ghatak

https://www.physics.purdue.edu/irnanodev/docs/outreach/Diffraction%20grating%20experiments.pdf

Grating spectrum

What should be the order after central white fringe?
Red to blue/violet or the reverse? Why?

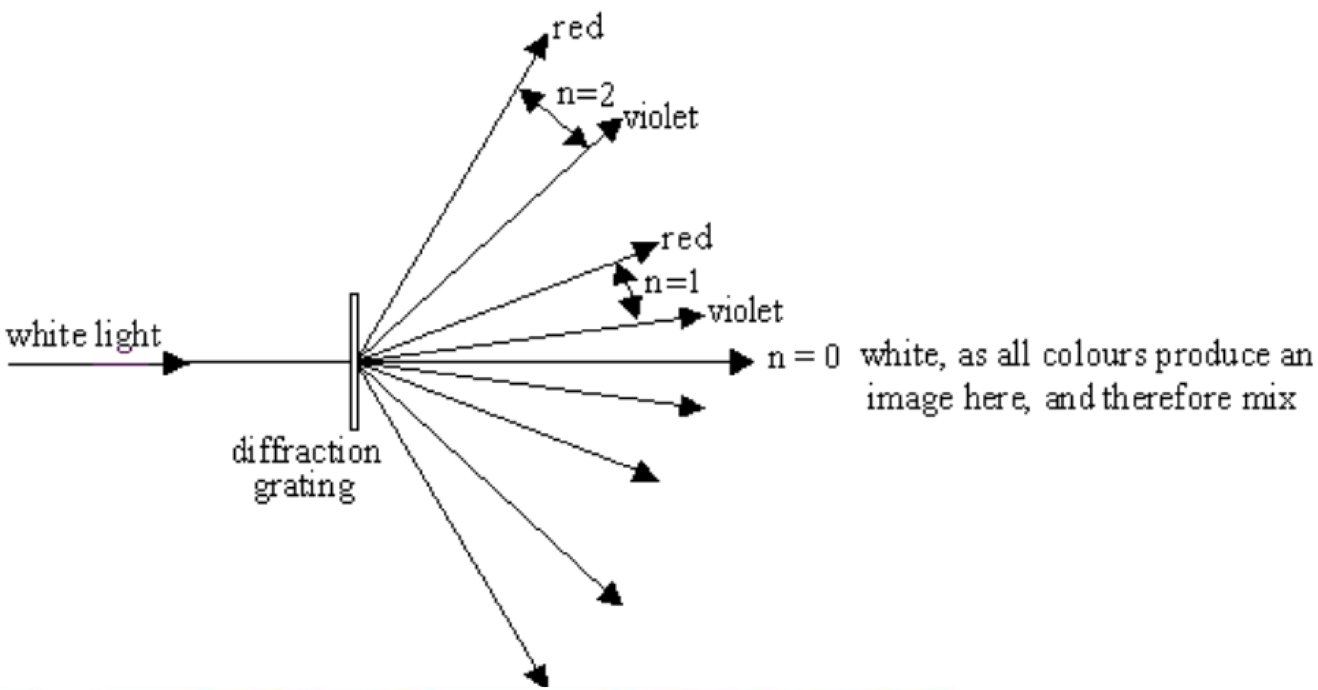
https://www.animatedscience.co.uk/ks5_physics/general/Oscillations%20&%20Waves,%20Reflection%20&%20Refraction/Waves%20(Part%202).htm

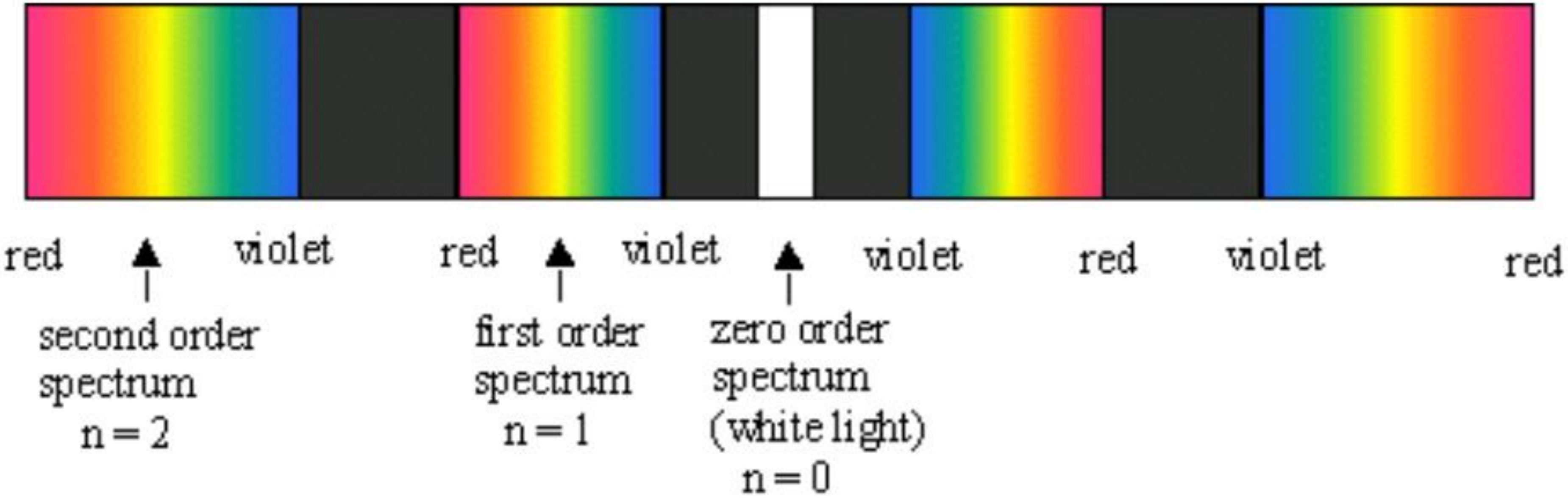
Grating spectrum

$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, \ldots .$$

$\lambda_R > \lambda_V$

$\theta_R > \theta_V$ for any given order n

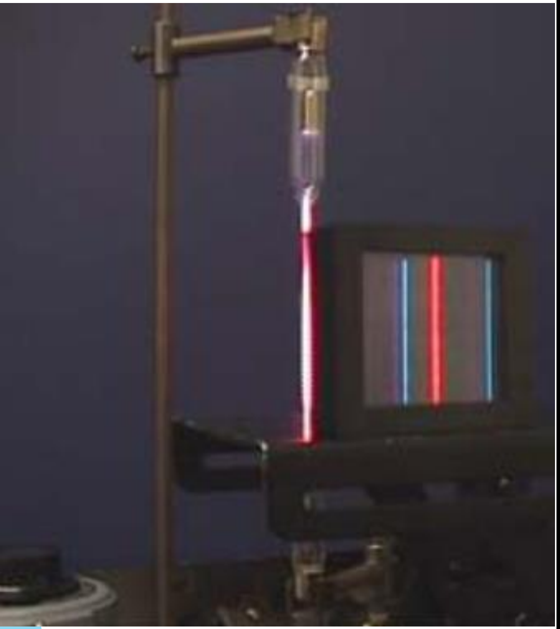


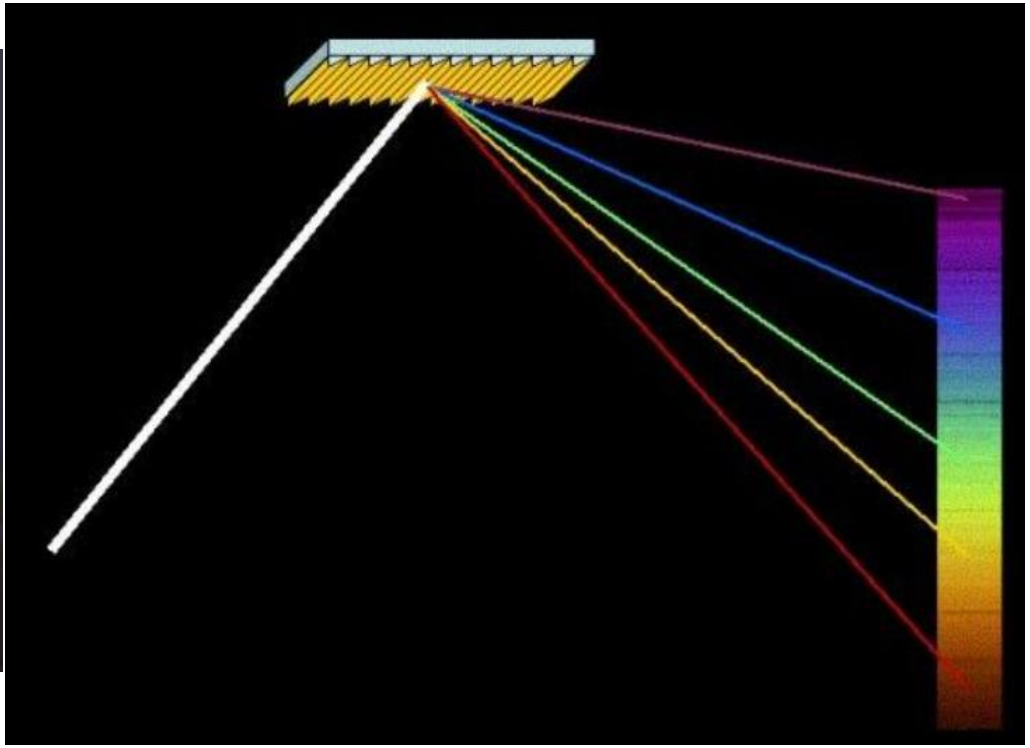


https://www.animatedscience.co.uk/ks5_physics/general/Oscillations%20&%20Waves,%20Reflection%20&%20Refraction/Waves%20(Part%202).htm

Few applications

1. Identify elements

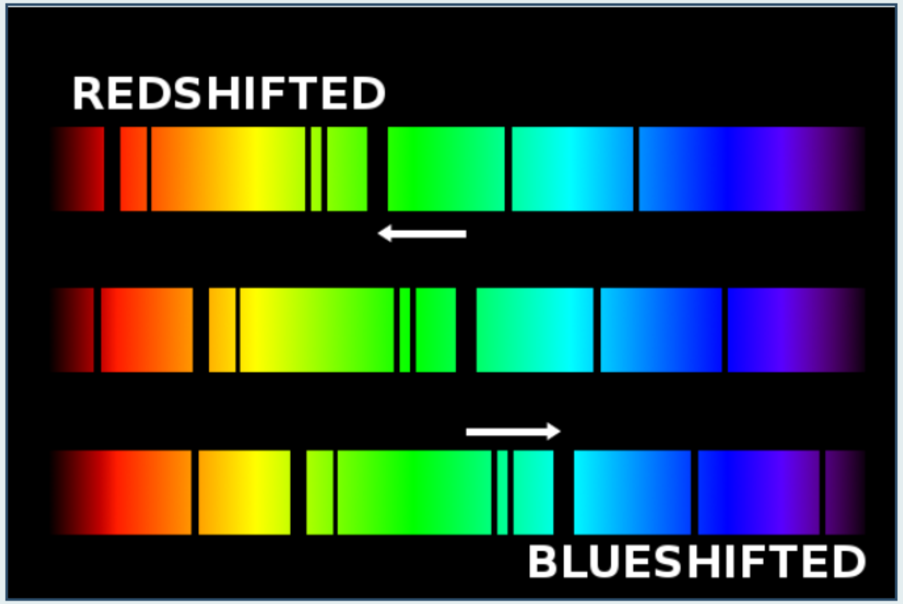




Credit: *NASA*
Dispersion of light from a diffraction grating

https://www.atnf.csiro.au/outreach/education/senior/astrophysics/spectrographs.html#:~:text=Diffracti on%20gratings%20are%20more%20efficient,ruled%20onto%20a%20glass%20surface.

2. Red/Blue shift



A blue shift does not mean that the object ends up blue. It just means that the entire spectrum is shifted up in frequency. Note that this is a schematic diagram and not actual data. Public Domain Image, source: Christopher S. Baird.

https://wtamu.edu/~cbaird/sq/2013/06/27/have-astronomers-ever-observed-a-violet-shift-like-they-have-blue-shifts-and-red-shifts/

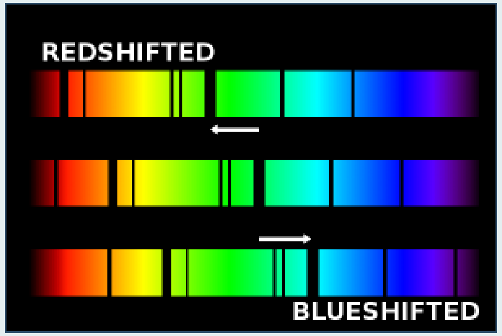
Science Questions with Surprising Answers

Answers provided by
Dr. Christopher S. Baird

Recent Questions | Biology | Chemistry | Earth Science | Health | Physics | Society | Space

Have astronomers ever observed a violet shift like they have blue shifts and red shifts?

Category: Space   Published: June 27, 2013



A blue shift does not mean that the object ends up blue. It just means that the entire spectrum is shifted up in frequency. Note that this is a schematic diagram and not actual data. Public Domain Image, source: Christopher S. Baird.

Violet shifts happen all the time. We call them blue shifts. When a star emits light, the color of its light as observed on earth depends on its motion relative to earth. If a star is moving *towards* the earth, its light is shifted to *higher* frequencies on the color spectrum (towards the green/blue/violet/ultraviolet/x-ray/gamma-ray end of the spectrum). A higher frequency shift is called a "blue shift". The faster a star moves towards the earth, the more its light is shifted to higher frequencies. In contrast, if a star is moving *away* from the earth, its light is shifted to *lower* frequencies on the color spectrum (towards the orange/red/infrared/microwave/radio end of the spectrum). A lower frequency shift is called a "red shift". The faster a star moves away from the earth, the more its light is shifted to lower-frequency colors. This effect is known as the "Doppler shift". It is the same principle at work when an ambulance siren coming towards you is high pitched and then switches to a lower pitch sound once the ambulance passes you and is traveling away from you. The Doppler shift is also used in police radar guns to measure how fast your car is going based on how the radio wave shifts in frequency when it bounces off your car.

Submit a Science Question

To submit a question, email me:
chrisbaird.ma@gmail.com

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https://wtamu.edu/~cbaird/sq/2013/06/27/have-astronomers-ever-observed-a-violet-shift-like-they-have-blue-shifts-and-red-shifts/

Problem-1

Consider a diffraction grating with 15,000 lines per inch. (a) Show that if we use a white light source, the 2nd and 3rd order spectra overlap. (λ_v and λ_r are 4 x 10^{-5} and 7 x 10^{-5} cm respectively)

Solution: (a) The grating element is

$$d = \frac{2.54}{15,000} = 1.69 \times 10^{-4} \text{ cm}$$

Let θ_{mv} and θ_{mr} represent the angles of diffraction for the mth-order spectrum corresponding to the violet and red colors, respectively. Thus

$$\theta_{2v} = \sin^{-1}\left(\frac{2\times 4\times 10^{-5}}{1.69\times 10^{-4}}\right) \approx \sin^{-1} 0.473 \approx 28.2^\circ$$

$$\theta_{2r} = \sin^{-1}\left(\frac{2\times 7\times 10^{-5}}{1.69\times 10^{-4}}\right) \approx \sin^{-1} 0.828 \approx 55.90^\circ$$

and

$$\theta_{3v} = \sin^{-1}\left(\frac{3\times 4\times 10^{-5}}{1.69\times 10^{-4}}\right) \approx \sin^{-1} 0.710 \approx 45.23^\circ$$

where we have assumed the wavelengths of the violet and red colors to be 4×10^{-5} and 7×10^{-5} cm, respectively. Since $\theta_{2r} > \theta_{3v}$, the second- and third-order spectra will overlap. Further since $\sin \theta_{3r} > 1$, the third-order spectrum for the red color will not be observed.

Thank You