Engineering Electromagnetics

Lecture 35

29/11/2023

by

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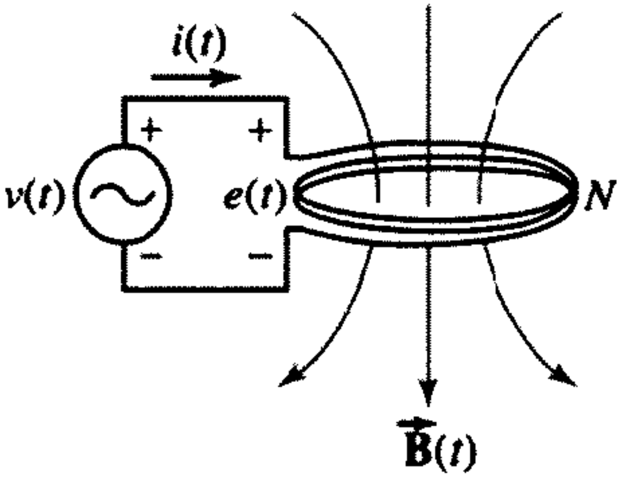
Self-inductance



a)

- Time varying source produces time varying current i(t)
- Passes through coil with N turns
- Creates time-varying flux
- Induces emf → that creates a current i_{in} → opposes the very cause of it i.e. i(t)

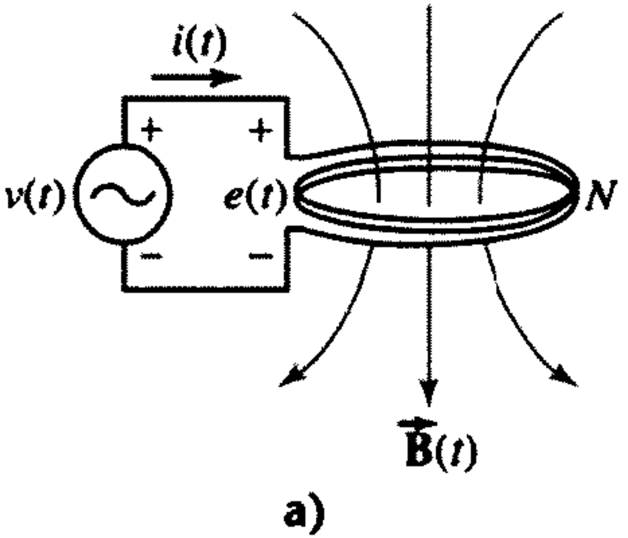
$$v = e = N\frac{d\Phi}{dt}$$

- Number of flux linkage: $\lambda = N\Phi$

The rate of change of flux linkages per unit change in the current is called the *self-inductance* or *inductance* of the coil and is usually symbolized by L. Thus

$$L = \frac{d\lambda}{di}$$

Self-inductance



$$Li = N\Phi$$

$$\boxed{L\frac{di}{dt} = N\frac{d\Phi}{dt}}$$

$$\boxed{L = \frac{N\Phi}{i}}$$

- Flux = B. A
- $\boldsymbol{B} = \boldsymbol{\mu_0 ni} = \mu_0 Ni/l$
- $\boldsymbol{A} = \boldsymbol{\pi r^2}$
- $L = \frac{N\left(\mu_0 Ni.\frac{\pi r^2}{l}\right)}{i} = \frac{\mu_0 N^2 \pi r^2}{l}$
- $\boxed{L = \frac{\mu_0 N^2 \pi r^2}{l}}$

Problem-3

Two solenoids have number of turns in a ratio of 1:1. However, their lengths and radii are in the ratios 2:1 and 3:2 respectively. The ratio of their self-inductance will be

- a) 2:3
- b) 1:2
- c) 4:9
- d) **9:8**
- e) 2:4

$$\boxed{L = \frac{\mu_0 N^2 \pi r^2}{l}}$$

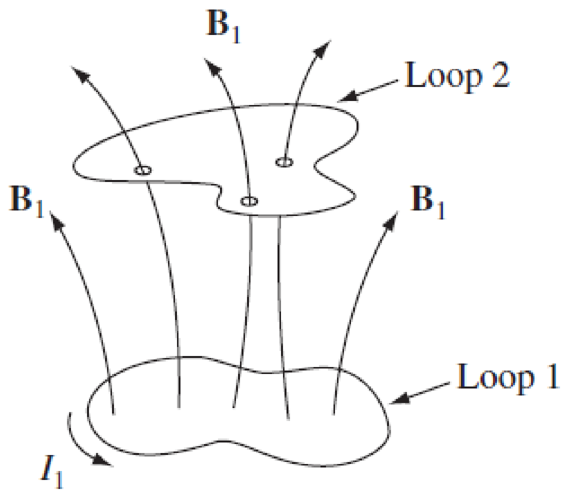
Suppose you have two loops of wire, at rest (Fig. 7.30). If you run a steady current I_1 around loop 1, it produces a magnetic field $\mathbf{B}_1$. Some of the field lines pass through loop 2; let Φ_2 be the flux of $\mathbf{B}_1$ through 2. You might have a tough time actually *calculating* $\mathbf{B}_1$, but a glance at the Biot-Savart law,

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\mathbf{l}_1 \times \hat{\mathscr{r}}}{\mathscr{r}^2},$$

reveals one significant fact about this field: *It is proportional to the current I_1.* Therefore, so too is the flux through loop 2:

$$\boxed{\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2. \quad \Phi_2 = M_{21} I_1,}$$

where M_{21} is the constant of proportionality; it is known as the **mutual inductance** of the two loops.



Suppose, now, that you *vary* the current in loop 1. The flux through loop 2 will vary accordingly, and Faraday's law says this changing flux will induce an emf in loop 2:

$$\boxed{\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}.} \quad (7.25)$$

From Griffith book

Energy in Magnetic Fields

It takes a certain amount of energy to start a current flowing in a circuit.

is the work you must do *against the back emf* to get the current going.

The work done on a unit charge, against the back emf, in one trip around the circuit is $-\mathcal{E}$ (the minus sign records the fact that this is the work done *by you against* the emf, not the work done by the emf). The amount of charge per unit time passing down the wire is I. So the total work done per unit time is

$$\frac{dW}{dt} = -\mathcal{E} I = L I \frac{dI}{dt}.$$

$$\mathcal{E} = -\frac{d\phi}{dt} = L\frac{dI}{dt}$$

If we start with zero current and build it up to a final value I, the work done (integrating the last equation over time) is

$$\boxed{W = \frac{1}{2} L I^2.}$$

$$\boxed{W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau}$$

From Griffith book

Problem-1

A very long cylinder of radius 20 cm is closely and tightly wound with 200 turns per unit length to form an air-core inductor (solenoid). If the current in the coil is constant, determine its inductance.

Solution-1

The magnetic flux density inside a very long cylinder is

$$\vec{\mathbf{B}} = \mu_0 n I \vec{\mathbf{a}}_z$$

where n is the number of turns per unit length. The flux enclosed by a cylinder of radius b is

$$\Phi = \int_s \vec{\mathbf{B}} \cdot \vec{ds} = \mu_0 n I \int_0^b \rho \, d\rho \int_0^{2\pi} d\phi = \mu_0 n I \pi b^2$$

$$L = \frac{N\Phi}{I}$$

$$\frac{L}{l} = \frac{n\Phi}{I}; \; n = N/l$$

The inductance of the solenoid per unit length, from (7.25), is

Actually L/l

$$L = \mu_0 \pi n^2 b^2$$

Substituting the values, we get

$$L = 4\pi \times 10^{-7} \times \pi \times 200^2 \times 0.2^2$$

$$= 6.32 \text{ mH/m}$$

…

Maxwell's equation

(i)	$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$	(Gauss's law),
(ii)	$\nabla \cdot \mathbf{B} = 0$	(no name),
(iii)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(Faraday's law),
(iv)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	(Ampère's law with Maxwell's correction).

From Griffith book

Problem-2

The magnetic field intensity in free space is given as $\vec{\mathbf{H}} = H_0 \sin\theta \vec{\mathbf{a}}_y$ A/m, where $\theta = \omega t - \beta z$, and β is a constant quantity. Determine (a) the displacement current density and (b) the electric field intensity.

Solution-2

The conduction current density in free space is zero. Thus, from (7.67) the displacement current density is equal to $\nabla \times \vec{\mathbf{H}}$. That is,

$$\frac{\partial \vec{\mathbf{D}}}{\partial t} = \begin{vmatrix} \vec{\mathbf{a}}_x & \vec{\mathbf{a}}_y & \vec{\mathbf{a}}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & H_0 \sin\theta & 0 \end{vmatrix}$$

$$= -\frac{\partial}{\partial z}[H_0 \sin\theta]\vec{\mathbf{a}}_x + \frac{\partial}{\partial x}[H_0 \sin\theta]\vec{\mathbf{a}}_z$$

$$= \beta H_0 \cos\theta\, \vec{\mathbf{a}}_x \ \text{A/m}^2$$

What is the inst. Power density?

$$\vec{\mathbf{D}} = \frac{\beta}{\omega} H_0 \sin\theta\, \vec{\mathbf{a}}_x \ \text{C/m}^2$$

Finally, the electric field intensity in free space is

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{D}}}{\epsilon_0} = \frac{\beta}{\omega\epsilon_0} H_0 \sin\theta\, \vec{\mathbf{a}}_x \ \text{V/m}$$

Poynting vector

- $S = E \times H$
- Poynting vector
- Instantaneous power density
- Power flowing out per unit area
- Unit: Watt/m^2

Thank You