Engineering Optics

Lecture 10

10/04/2023

by

Debolina Misra
Department of Physics
IIITDM Kancheepuram, Chennai, India

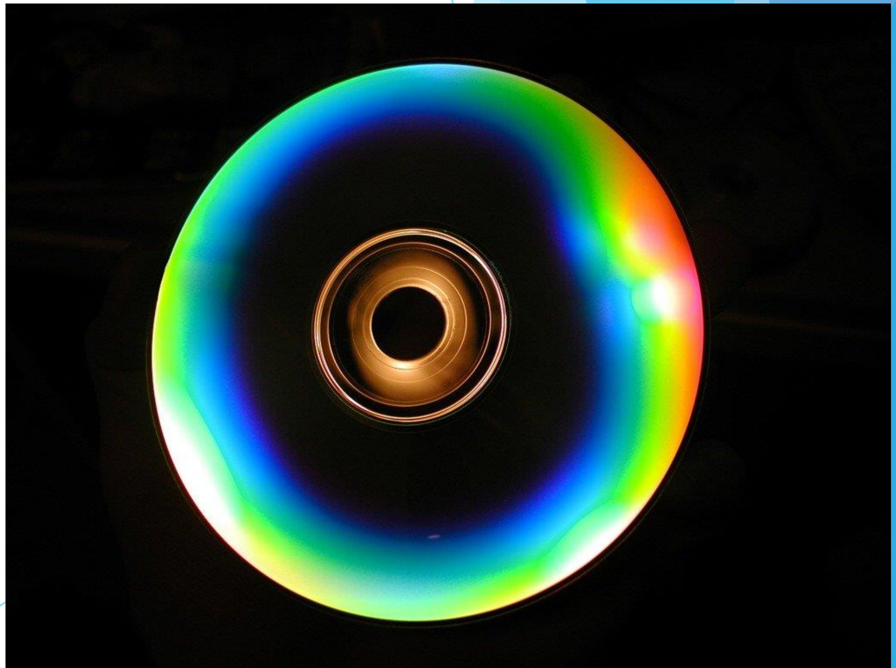
Diffraction of light

diffraction grating

x-ray diffraction studies of crystals

holography

slowly closing two fingers while observing the **light** transmitted between them

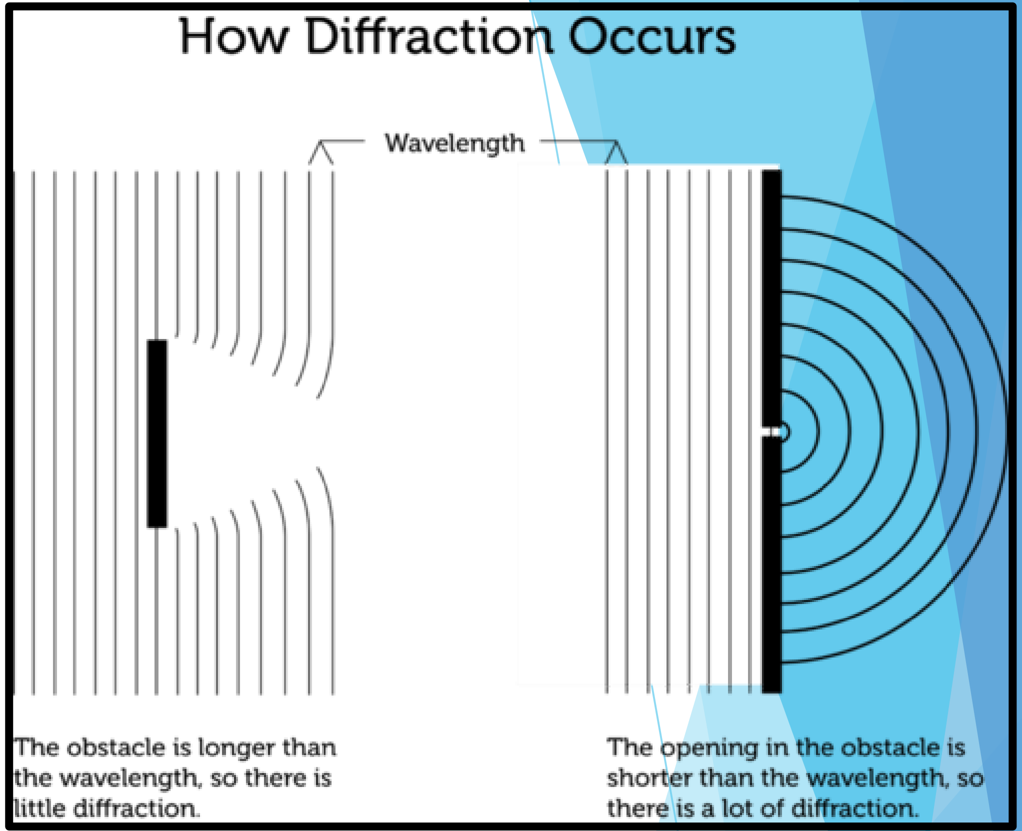


Diffraction

- **What is diffraction of light?**

Light encounters an obstacle or opening.

(It is defined as the bending of waves around the corners of an obstacle or through an aperture into the region of geometrical shadow of the obstacle/aperture.)



https://www.ck12.org/book/ck-12-physical-science-for-middle-school/section/19.3/

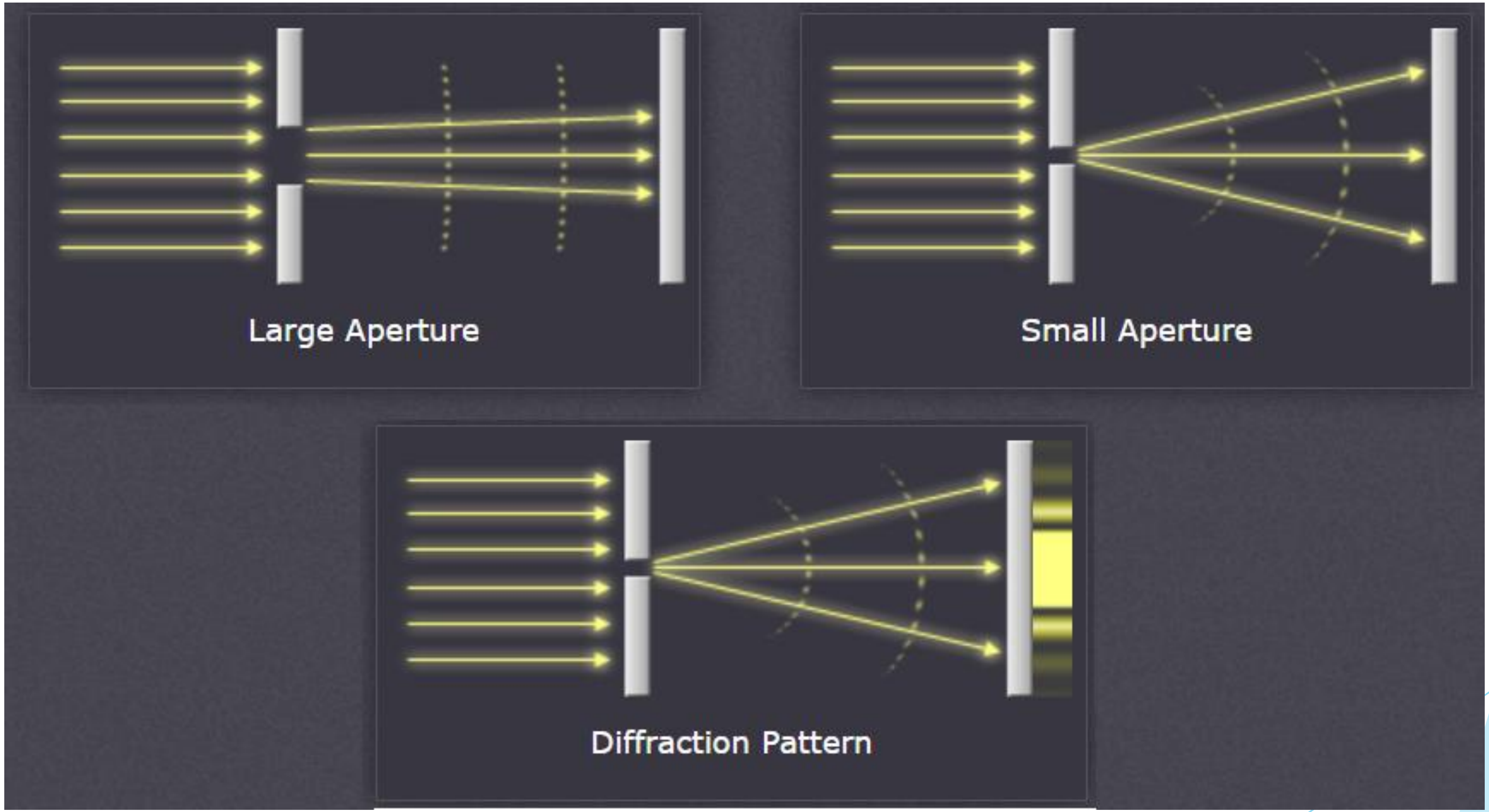
- **Diffraction or Interference?**

No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do is, roughly speaking, is to say that when there are only a few sources, say two, interfering, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used.

—Richard Feynman, Feynman Lectures on Physics, Vol. 1

Optics, Hecht

Diffraction by aperture

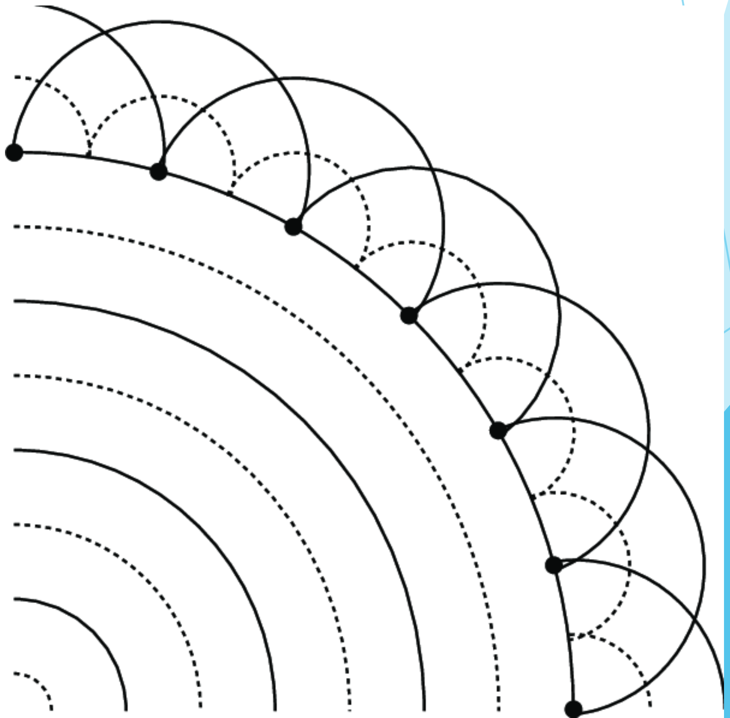


https://www.cambridgeincolour.com/tutorials/diffraction-photography.htm

Huygens–Fresnel Principle

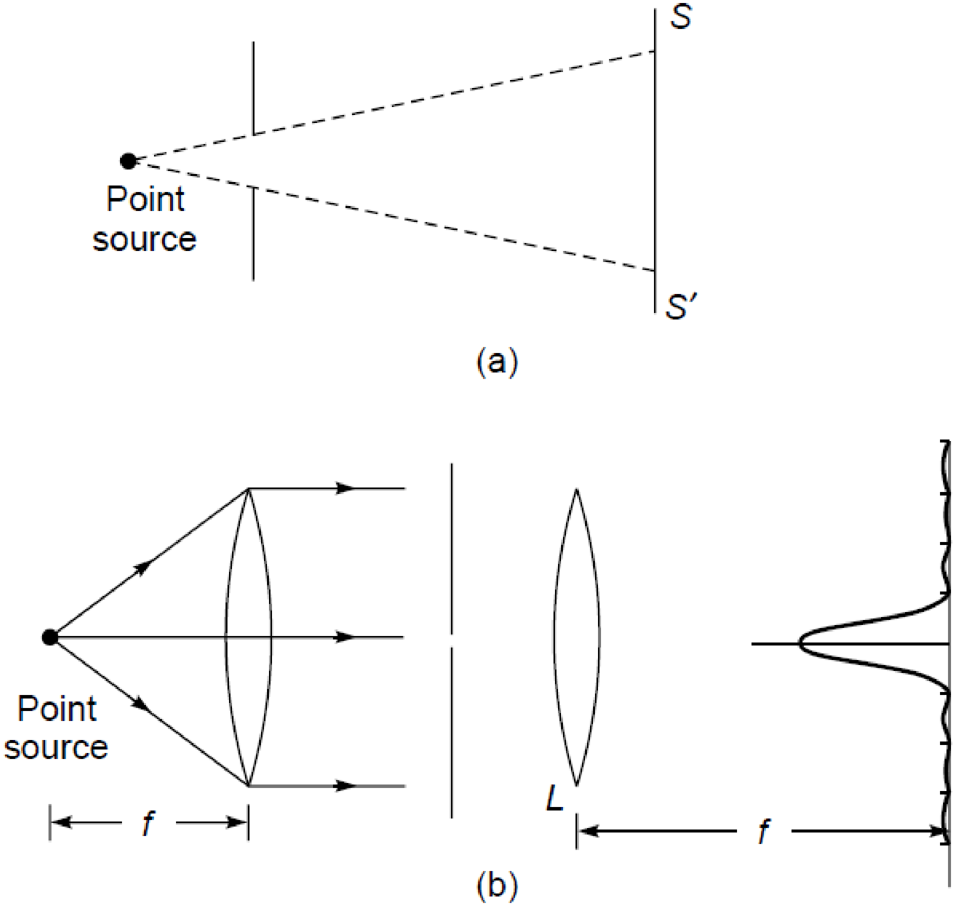
Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave).

The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases).



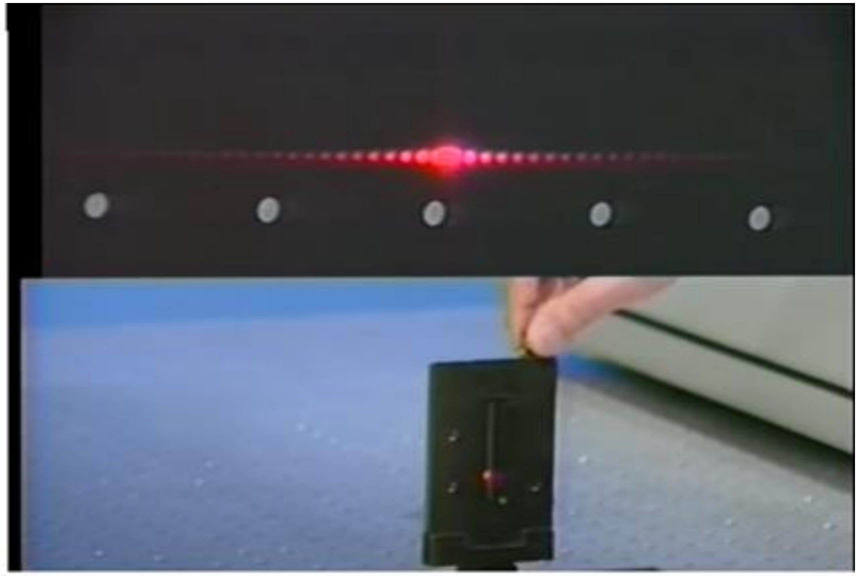
Optics, Hecht

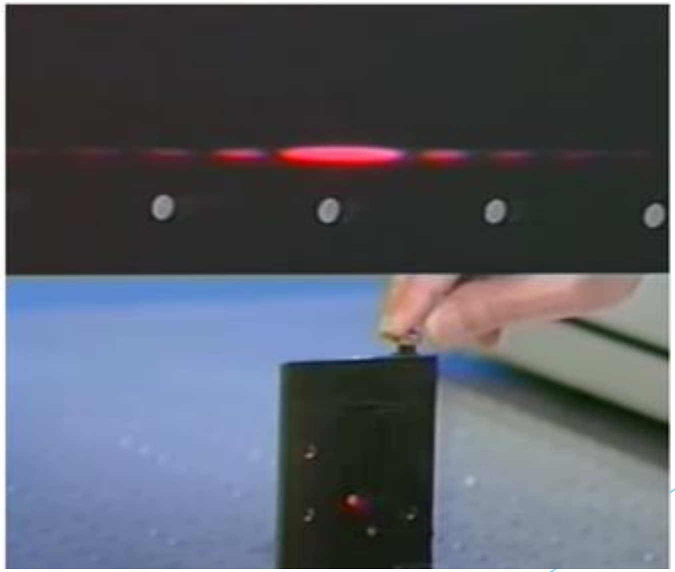
Fresnel diffraction and Fraunhofer diffraction



(a) When either the source or the screen (or both) is at a finite distance from the aperture, the diffraction pattern corresponds to the Fresnel class. (b) In the Fraunhofer class both the source and the screen are at infinity.

Single slit diffraction

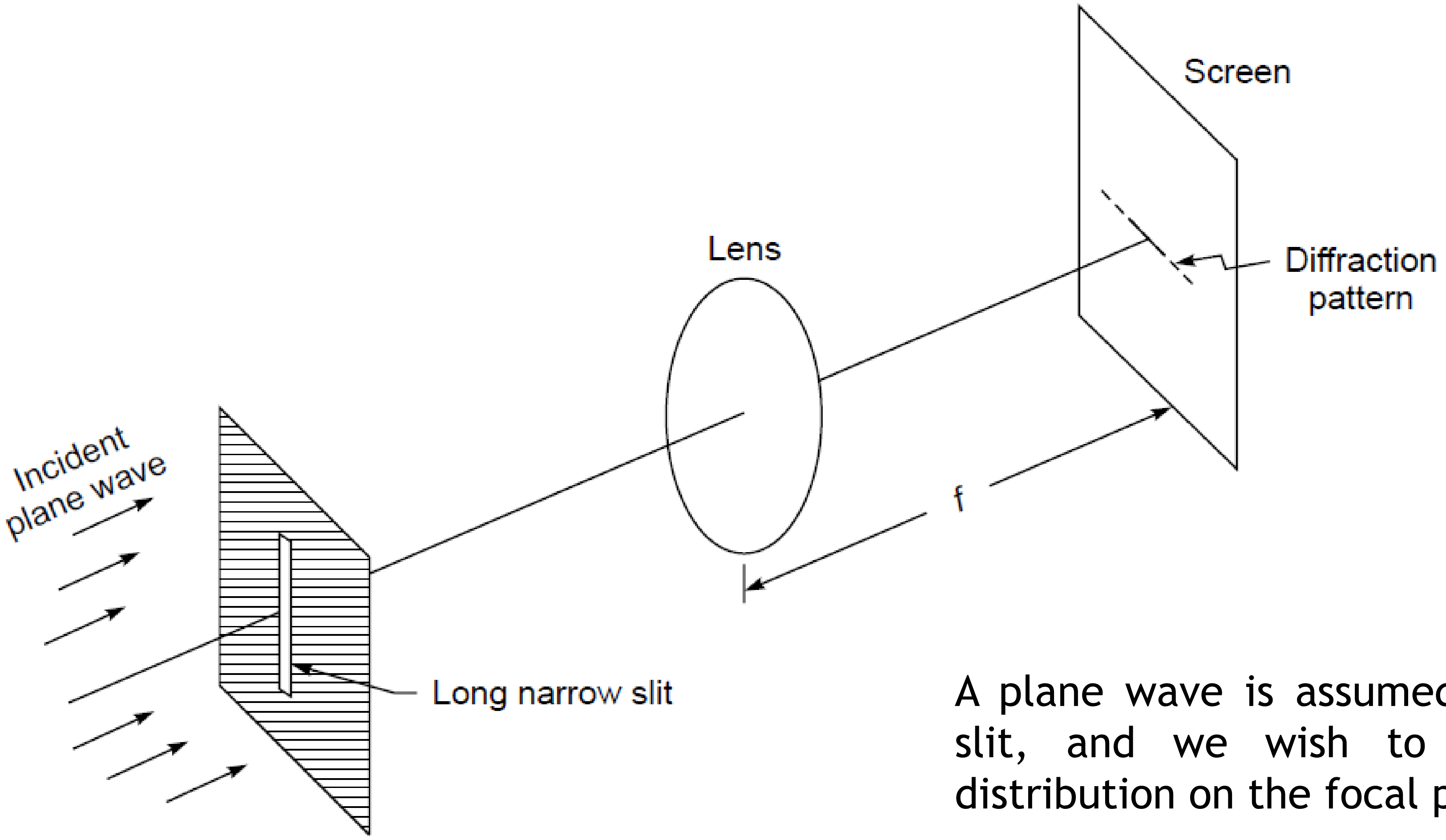




Single slit diffraction

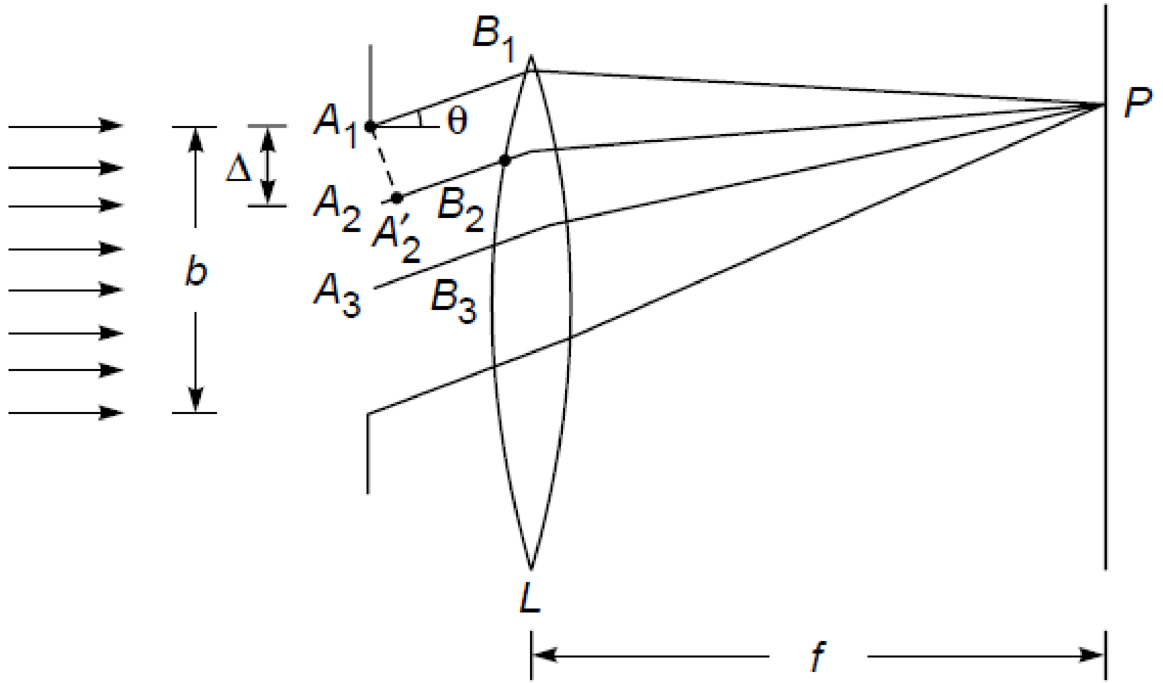
https://ocw.mit.edu/resources/res-6-006-video-demonstrations-in-lasers-and-optics-spring-2008/demonstrations-in-physical-optics/fraunhofer-diffraction-2014-adjustable-slit/

Single slit diffraction: Intensity distribution



A plane wave is assumed to fall normally on the slit, and we wish to calculate the intensity distribution on the focal plane of lens *L*

Single slit diffraction: Intensity distribution

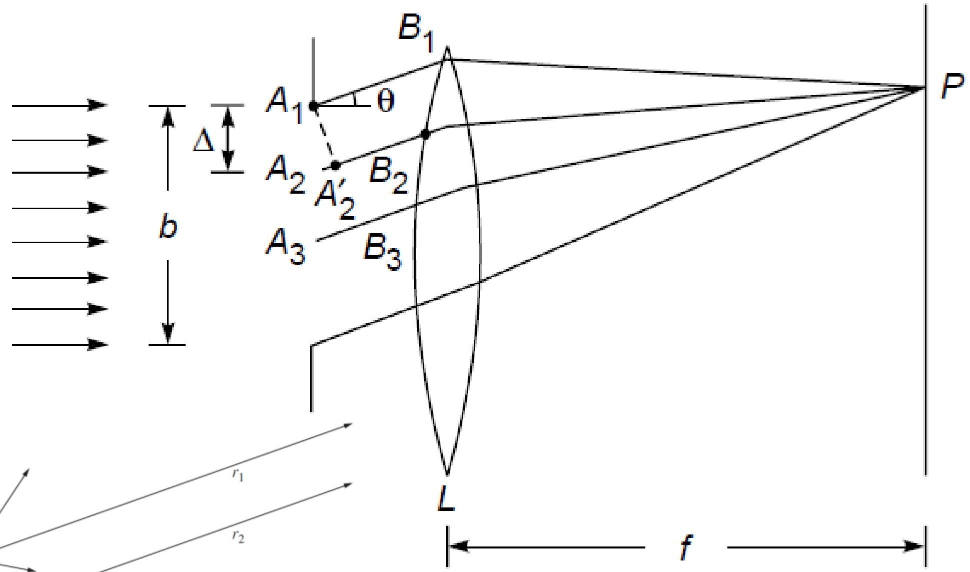


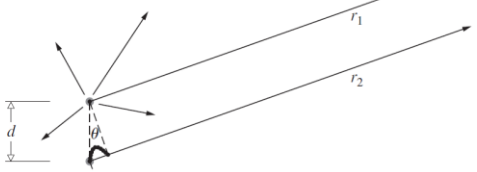
- slit → large number of equally spaced point sources
- each point → source of Huygens' secondary wavelets
- 2ndary wavelets interfere
- A_1, A_2, A_3, . . ., → point sources
- Distance between two consecutive points → Δ
- number of point sources = n
- $b = (n-1)\,\Delta$

Resultant field produced by these *n* sources at an arbitrary point P ?

Optics, Ghatak

Intensity distribution continued





- At *P*: $A_1 \approx A_2$; distance to *P* >> b
- slightly different path lengths → path diff → phase diff
- A_2A_2' → extra path; $A_1B_1P = A_2'B_2P$
- Path diff. $A_2A_2' = \Delta \sin\theta$
- Phase diff. $\varphi = k\, A_2A_2' = (2\pi/\lambda)\, \Delta \sin\theta$

$$E = a[\cos \omega t + \cos (\omega t - \varphi) + \ldots + \cos [(\omega t - (n-1)\varphi)]$$

$$E = E_0 \cos [(\omega t - \tfrac{1}{2}(n-1)\varphi)]$$

Where $E_0 = a \dfrac{\sin (n\phi/2)}{\sin (\phi/2)}$

if n → ∞ and Δ → 0

Then n Δ → b

Amplitude of the resultant wave

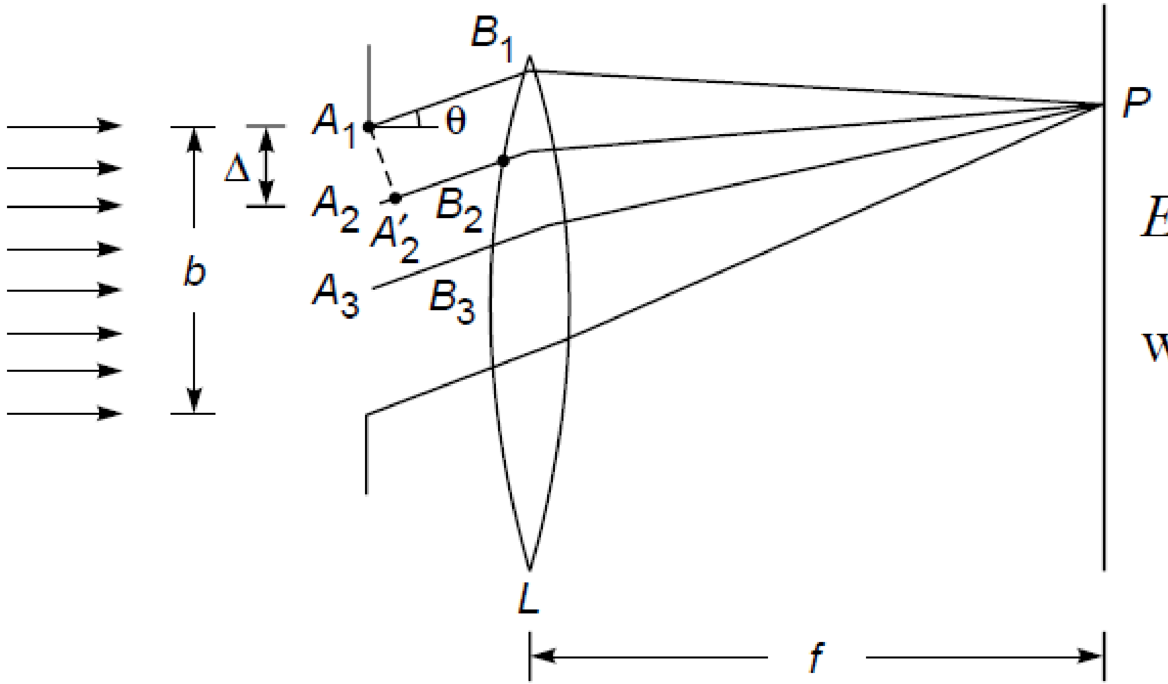
$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta = \frac{2\pi}{\lambda}\frac{b \sin\theta}{n}$$

$$\frac{n\phi}{2} = \frac{\pi}{\lambda} n\Delta \sin\theta \rightarrow \frac{\pi}{\lambda} b \sin\theta$$

$$\boxed{E_0 = A\frac{\sin\beta}{\beta}} \quad A = na \quad \beta = \frac{\pi b \sin\theta}{\lambda}$$

Optics, Hecht, Ghatak

Single slit diffraction: Intensity distribution



$$E = a[\cos \omega t + \cos(\omega t - \phi) + \cdots + \cos[(\omega t - (n-1)\phi)]$$

where $\phi = \dfrac{2\pi}{\lambda}\Delta \sin\theta$

$$E = E_0 \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right]$$

$$E_0 = a\frac{\sin(n\phi/2)}{\sin(\phi/2)}$$

$n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to b$,

$$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)}$$

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2}}$$

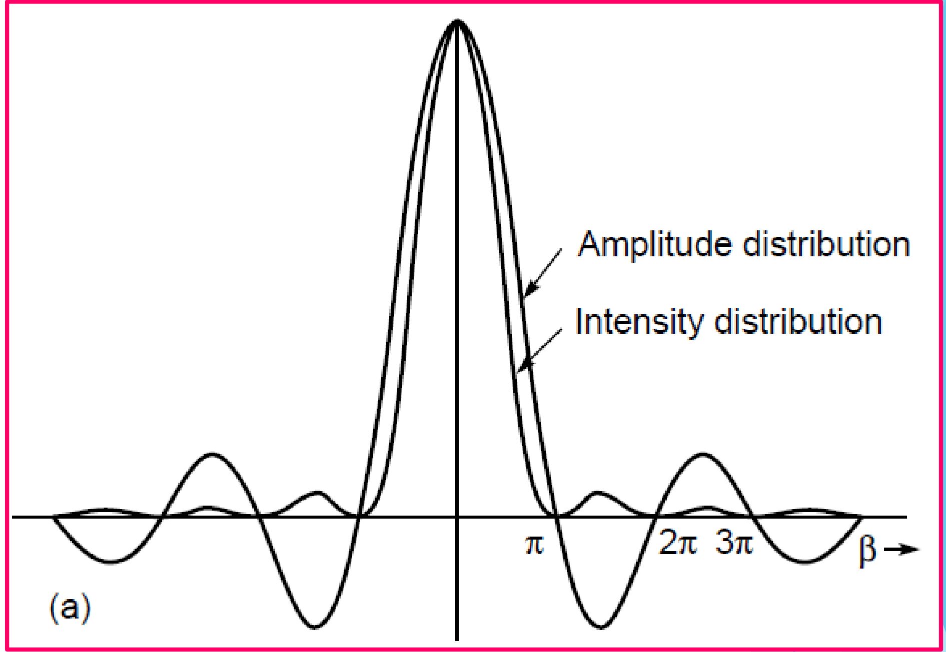
Optics, Ghatak

Single slit diffraction continued

$$\boxed{E = A \frac{\sin \beta}{\beta} \cos(\omega t - \beta)} \quad \textbf{(1)}$$

$$A = na \quad \beta = \frac{\pi b \sin \theta}{\lambda} \quad \textbf{(2)}$$

$$\boxed{I = I_0 \frac{\sin^2 \beta}{\beta^2}} \quad \textbf{(3)}$$



Intensity = 0 if _???___

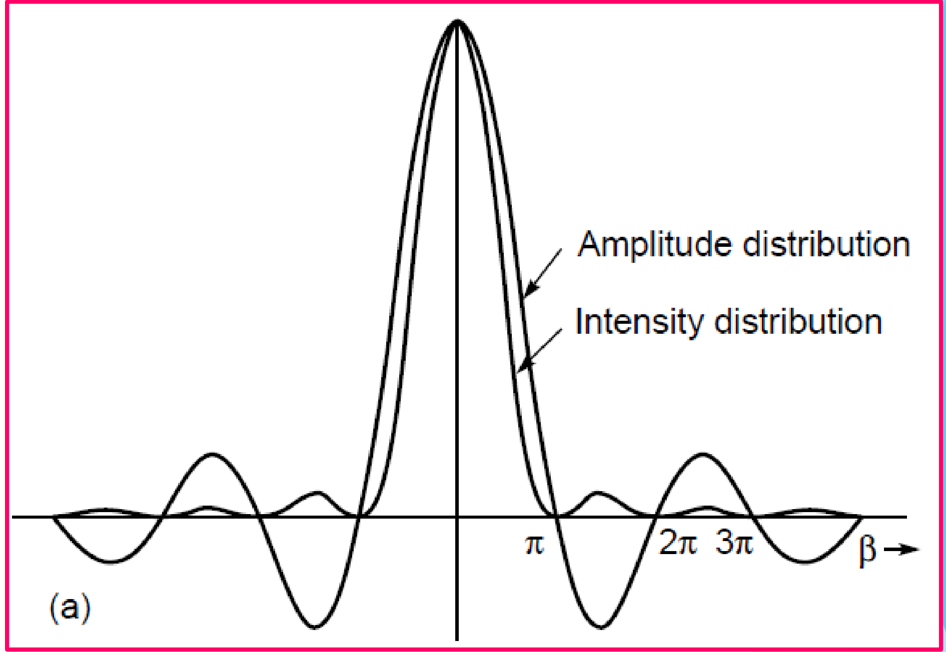
$$\beta = \frac{\pi b \sin \theta}{\lambda}$$

Single slit diffraction continued

$$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)} \quad \textbf{(1)}$$

$$A = na \qquad \beta = \frac{\pi b \sin\theta}{\lambda} \qquad \textbf{(2)}$$

$$\boxed{I = I_0\frac{\sin^2\beta}{\beta^2}} \qquad \textbf{(3)}$$



Intensity = 0 if $\quad \beta = m\pi \qquad m \neq 0 \qquad$ **(4)**

Using (4) in (2):

$$\boxed{b\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3, \ldots \text{ (minima)}}$$

first minimum

$$\theta = \pm\sin^{-1}(\lambda/b)$$

second minimum

$$\theta = \pm\sin^{-1}(2\lambda/b)$$

m closest to b/λ

Single slit diffraction: maxima

maxima $$\frac{dI}{d\beta} = I_0\left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right) = 0$$

or $$\sin\beta\,(\beta - \tan\beta) = 0$$

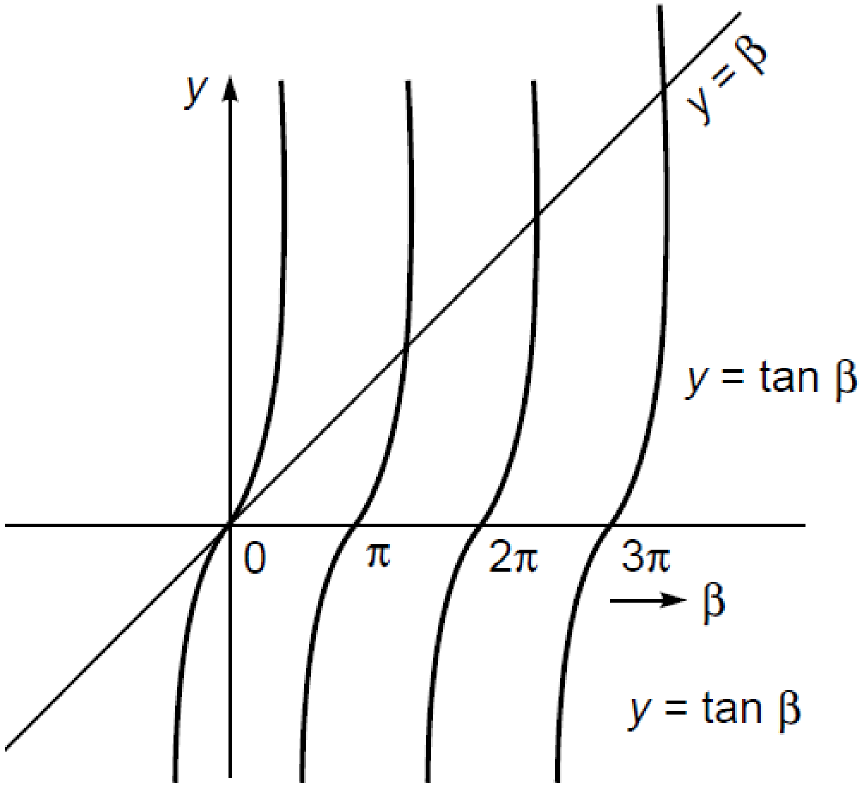
The condition $\sin\beta = 0$, or $\beta = m\pi$ ($m \neq 0$), corresponds to minima. The conditions for maxima are roots of the equation

$$\tan\beta = \beta \qquad \text{(maxima)}$$

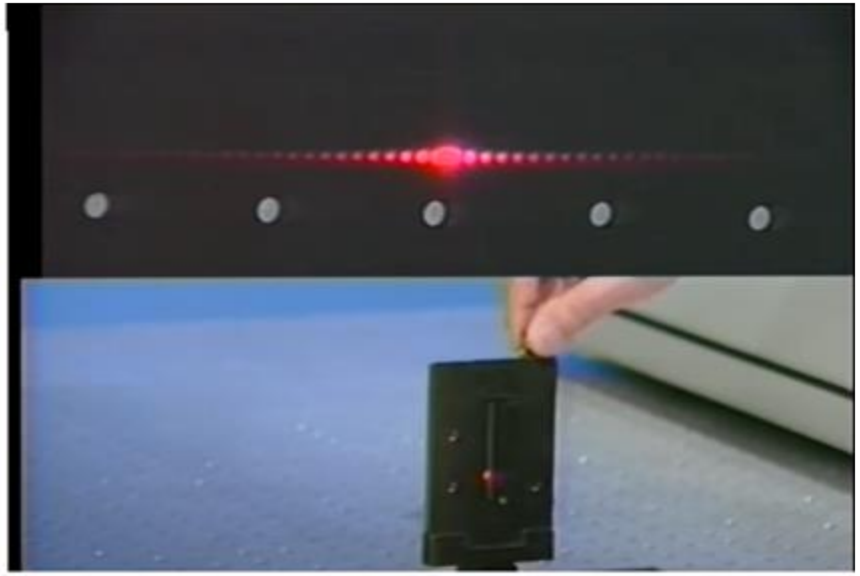
$\boxed{\text{The root } \beta = 0 \text{ corresponds to the central maximum.}}$

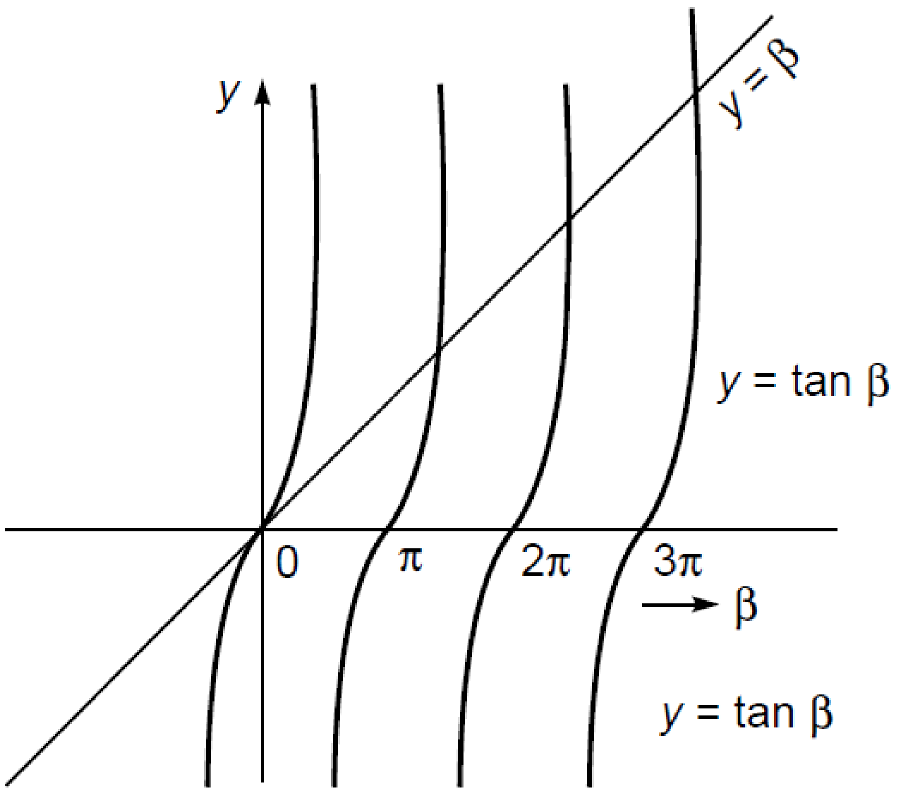
curves $y = \beta$ and $y = \tan\beta$ points of intersections

$$\beta = 1.43\pi,\ \beta = 2.46\pi$$



The central maxima is brightest!





The root $\beta = 0$ corresponds to the central maximum.

curves $y = \beta$ and $y = \tan\beta$ points of intersections

$\beta = 1.43\pi$, $\beta = 2.46\pi$

1st maximum→ $$\left(\frac{\sin 1.43\pi}{1.43\pi}\right)^2$$

https://ocw.mit.edu/resources/res-6-006-video-demonstrations-in-lasers-and-optics-spring-2008/demonstrations-in-physical-optics/fraunhofer-diffraction-2014-adjustable-slit/

Thank You