# Engineering Electromagnetics

## Lecture 24

01/11/2023

*by*


**Debolina Misra**

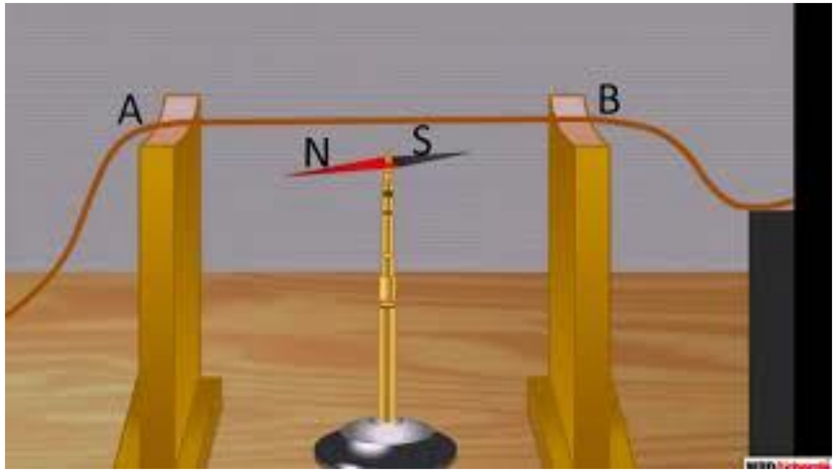
Dept. of Physics
IIITDM Kancheepuram, Chennai, India

# Magnetostatics

# Magnetostatics

- Till early nineteenth century → magnetic materials can be magnetized and can be used as magnets

- In 1820: Hans Christian Oersted → a magnetic needle gets deflected by a current carrying wire



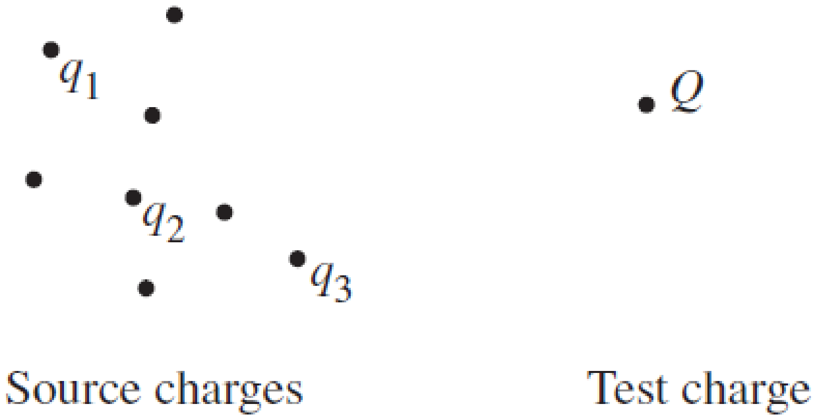

https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3DqS361iadCPA&psig=AOvVaw1OHDPqWbczHckHoKFkP8Yi&ust=1644604975877000&source=images&cd=vfe&ved=0CAgQjRxqFwoTCKjTiY3l9fUCFQAAAAAdAAAAABAu

- Bridged the gap between electricity and magnetism: electricity can also be the source of magnetism

"Magnetostatics"?

# Magnetostatics

- Remember for a collection of charges $q_1$, $q_2$, $q_3$, . . . (the "source" charges), and we want to calculate the force they exert on some other charge Q (the "test" charge).




- principle of superposition → sum of all the individual forces.

- Up to now, we have confined our attention to the simplest case, electrostatics, in which the source charge is at rest.

- What happens if the charges are in motion?



From Griffith Book

# Charges in motion

- A simple apparatus demonstrates that something weird happens when charges are in motion:
- Two wires hanging from the celling and a few cm away:
- If we run currents next to one another in parallel, we find that they are attracted when the currents run in the same direction;
- they are repulsed when the currents run in opposite directions.
- Due to electric field? → wires are completely neutral: if we put a stationary test charge near the wires, it feels no force.

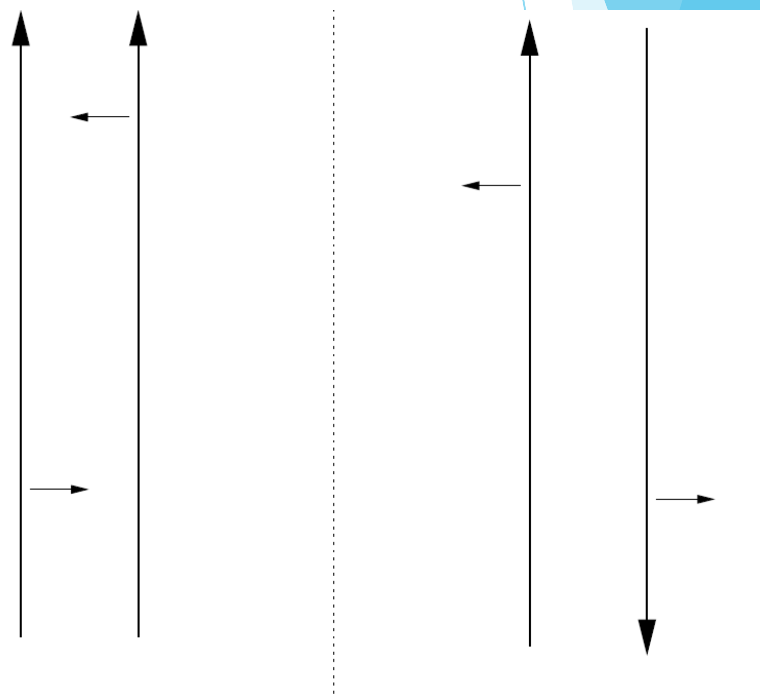


Figure 1: Left: parallel currents attract. Right: Anti-parallel currents repel.

https://web.mit.edu/sahughes/www/8.022/lec10.pdf

# Magnetic field B

- Furthermore, experiments show that the force is proportional to the currents → double the current in one of the wires, and you double the force.
- Double the current in both wires, and you quadruple the force.
- This all indicates a force that is proportional to the velocity of a moving charge; and that points in a direction perpendicular to the velocity.
- These conditions are screaming for a force that depends on a cross product.
- This is ? field" → arises from ?.



https://web.mit.edu/sahughes/www/8.022/lec10.pdf

# Magnetic field B

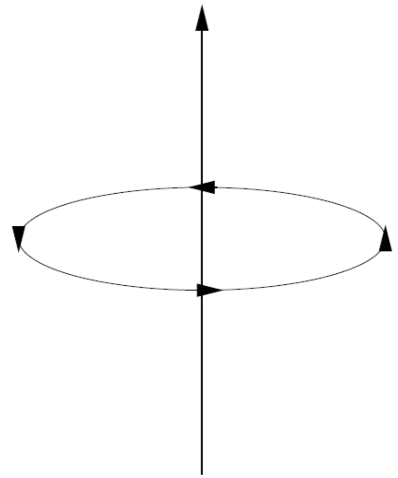
- Furthermore, experiments show that the force is proportional to the currents → double the current in one of the wires, and you double the force.
- Double the current in both wires, and you quadruple the force.
- This all indicates a force that is proportional to the velocity of a moving charge; and that points in a direction perpendicular to the velocity.
- These conditions are screaming for a force that depends on a cross product.
- we say is that some kind of field **B** → the “magnetic field” → arises from the current.



https://web.mit.edu/sahughes/www/8.022/lec10.pdf

# Direction of B

- The direction of this field is kind of odd: it wraps around the current in a circular fashion, with a direction that is defined by the right-hand rule: We point our right thumb in the direction of the current, and our fingers curl in the same sense as the magnetic field.
- if you hold up a tiny compass in the vicinity of a current-carrying wire, you quickly discover a very peculiar thing: The field does not point toward the wire, nor away from it, but rather it circles around the wire. In fact, if you grab the wire with your right hand—thumb in the direction of the current—your fingers curl around in the direction of the magnetic field (Fig. 5.3).
- It's going to take a strange law to account for these directions.



https://web.mit.edu/sahughes/www/8.022/lec10.pdf

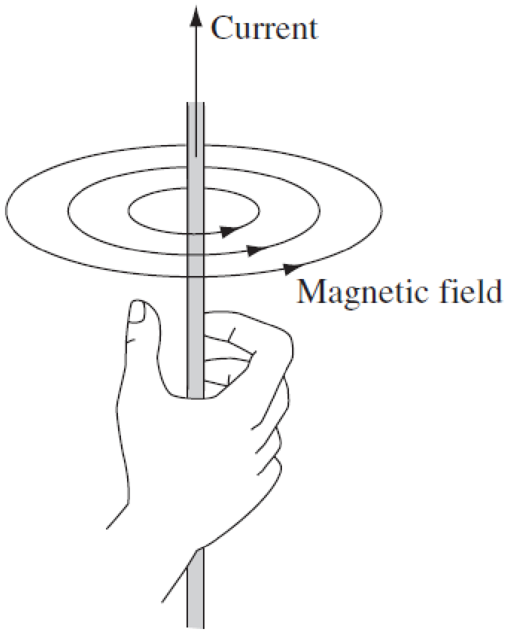



FIGURE 5.3

From Griffith Book

# Magnetic Forces

In fact, this combination of directions is just right for a cross product: the magnetic force on a charge $Q$, moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, is[2]

$$\boxed{\mathbf{F}_{\mathrm{mag}} = Q(\mathbf{v} \times \mathbf{B}).} \qquad (5.1)$$

This is known as the **Lorentz force law.**[3]

What if there is electric field **E** too?



From Griffith Book

# Magnetic Forces

In fact, this combination of directions is just right for a cross product: the magnetic force on a charge $Q$, moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, is[2]

$$\boxed{\mathbf{F}_{\text{mag}} = Q(\mathbf{v} \times \mathbf{B}).} \tag{5.1}$$

This is known as the **Lorentz force law**.[3] In the presence of both electric *and* magnetic fields, the net force on $Q$ would be

$$\mathbf{F} = Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]. \tag{5.2}$$

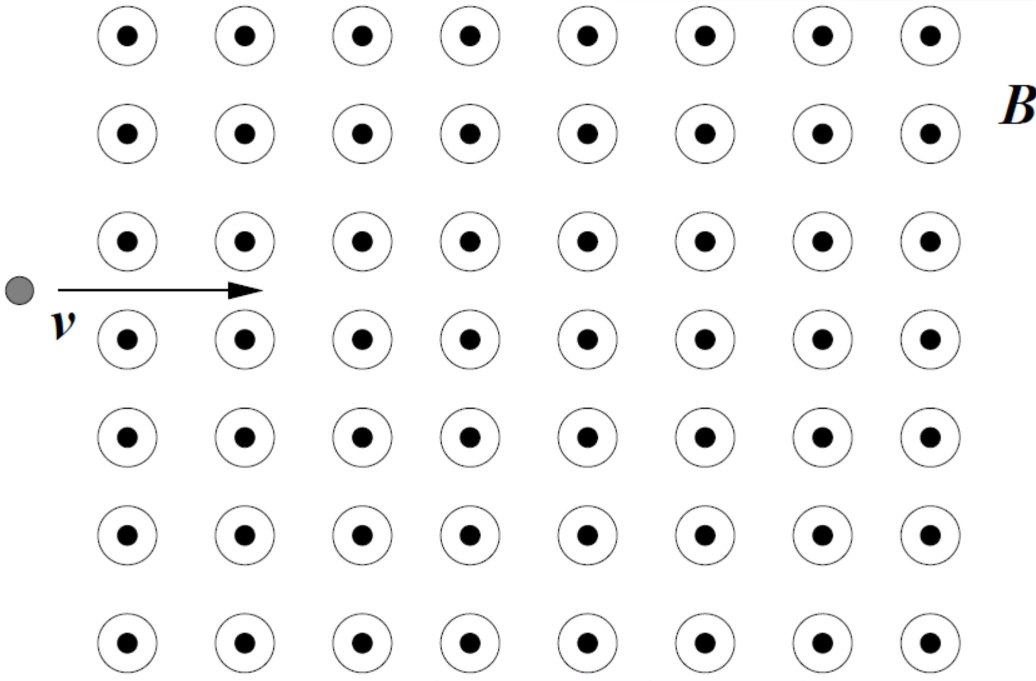
The SI unit of magnetic field is called the Tesla (T): the Tesla equals a Newton/(coulomb × meter/sec).
To convert: 1T = $10^4$ Gauss (CGS)



From Griffith Book

# Consequences of magnetic force

- Suppose I shoot a charge into a region filled with a uniform magnetic field:
- The magnetic field B points out of the page; the velocity v initially points to the right. What motion results from the magnetic force?
- At every instant, the magnetic force points perpendicular to the charge's velocity → exactly the force needed to cause circular motion. It is easy to find the radius of this motion:
- if the particle has charge q and mass m, then
- $F_{mag} = F_{centripetal}$
- ? = ?

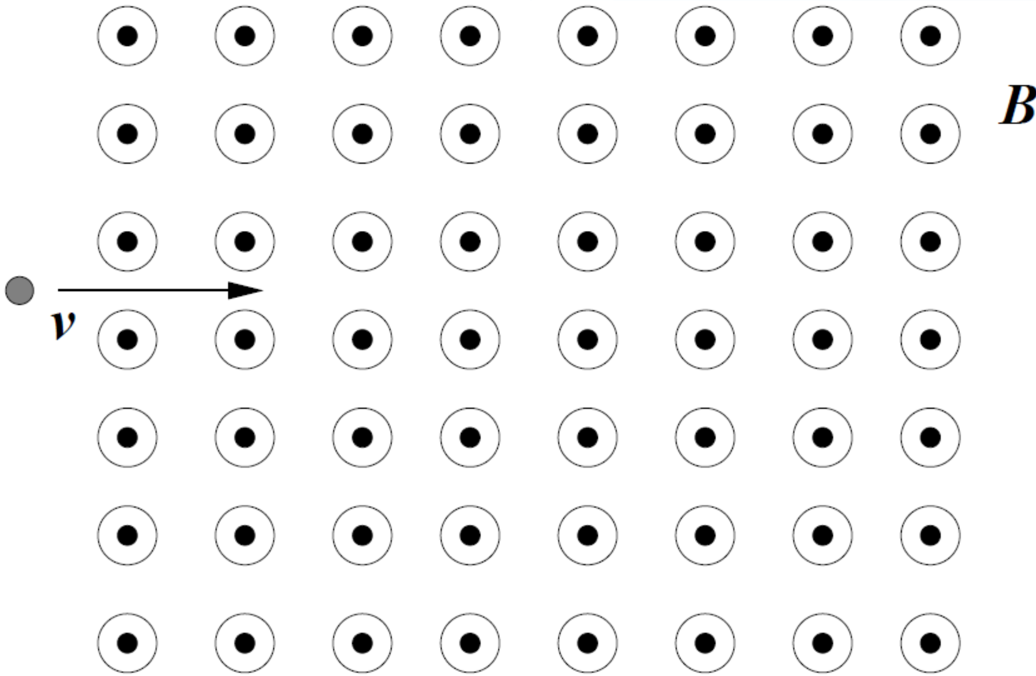





https://web.mit.edu/sahughes/www/8.022/lec10.pdf

# Consequences of magnetic force

- Suppose I shoot a charge into a region filled with a uniform magnetic field:
- The magnetic field B points out of the page; the velocity v initially points to the right. What motion results from the magnetic force?
- At every instant, the magnetic force points perpendicular to the charge's velocity → exactly the force needed to cause circular motion. It is easy to find the radius of this motion:
- if the particle has charge q and mass m, then
- $F_{mag} = F_{centripetal}$
- $qvB = mv^2/R$






https://web.mit.edu/sahughes/www/8.022/lec10.pdf

# Problem-1

- A proton is moving with a speed of $10^6$ m/Sec in a direction perpendicular to a uniform magnetic field of 1 T. What kind of motion it undergoes? Radius of the circle traced out?

- $qvB = mv^2/R$
- Put all the values for proton.
- Ans: ~$10^{-2}$ m

# Magnetic force and work done

- If charge Q with velocity ‘v’ moves by ‘dl’ in time ‘dt’ then work done under the Magnetic force $F_{mag}$= ?

# Magnetic force and work done

- If charge Q with velocity ‘**v**’ moves by ‘**dl**’ in time ‘dt’ then work done under the Magnetic force $F_{mag}$= ?
- dW = **F**. **dl** = q(**v** x **B**) .**v** dt = 0
- Magnetic force do not work on moving charges: Magnetic forces may alter the direction in which a particle moves, but they cannot speed it up or slow it down.
- Current ⟺ Magnetic Field ⟺ Force
- https://www.youtube.com/watch?v=z2kY_Q_b634

# Current and field due to line charge

Line charge λ travelling down a wire with velocity **v**.

$$\boldsymbol{I} = \lambda\,\mathbf{v}$$

$$\mathbf{F}_{\text{mag}} = \int (\mathbf{v} \times \mathbf{B})\, dq = \int (\mathbf{v} \times \mathbf{B})\lambda\, dl = \int (\mathbf{I} \times \mathbf{B})\, dl$$

Inasmuch as **I** and $d\mathbf{l}$ both point in the same direction, we can just as well write this as

$$\boxed{\mathbf{F}_{\text{mag}} = \int I\,(d\mathbf{l} \times \mathbf{B}).}$$

Typically, the current is constant (in magnitude) along the wire, and in that case $I$ comes outside the integral:

$$\mathbf{F}_{\text{mag}} = I \int (d\mathbf{l} \times \mathbf{B}).$$

# Due to volume charge density ρ

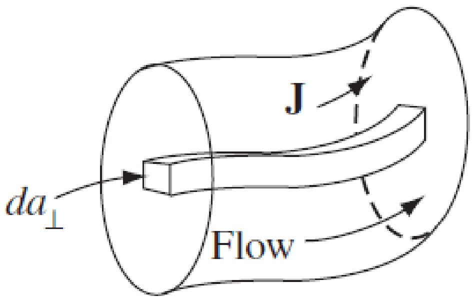
Current density ***J*** = ρ **v**

**J** = current/area; Volume charge density = ρ and velocity is **v**

The magnetic force on a volume current is therefore

$$\boxed{\mathbf{F}_{\text{mag}} = \int (\mathbf{v} \times \mathbf{B}) \rho \, d\tau = \int (\mathbf{J} \times \mathbf{B}) \, d\tau}$$

# Continuity Equation




$$\mathbf{J} \equiv \frac{d\mathbf{I}}{da_\perp}$$

$$I = \int_{\mathcal{S}} J \, da_\perp = \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}. \tag{5.28}$$

the charge per unit time leaving a volume $\mathcal{V}$ is

$$\oint_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a} = \int_{\mathcal{V}} (\nabla \cdot \mathbf{J}) \, d\tau.$$

*Because charge is conserved*, whatever flows out through the surface must come at the expense of what remains inside:

$$\int_{\mathcal{V}} (\nabla \cdot \mathbf{J}) \, d\tau = -\frac{d}{dt} \int_{\mathcal{V}} \rho \, d\tau = -\int_{\mathcal{V}} \left( \frac{\partial \rho}{\partial t} \right) d\tau.$$

(The minus sign reflects the fact that an *outward* flow *decreases* the charge left in $\mathcal{V}$.) Since this applies to *any* volume, we conclude that

$$\boxed{\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.} \tag{5.29}$$

This is the precise mathematical statement of local charge conservation; it is called the **continuity equation**.

# Thank You