CHAPTER-1

Digital Systems and Binary Numbers

Digital Design (with an introduction to the Verilog HDL) 6th Edition,
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Digital Systems

- Digital age or information age
- Digital computers
  - general purposes
  - many scientific, industrial and commercial applications
- Digital systems
  - telephone switching exchanges
  - digital camera
  - electronic calculators
  - digital TV

Signal

- An information variable represented by physical quantity
- For digital systems, the variable takes on discrete values
  - Two level, or binary values are the most prevalent values
- Binary values are represented abstractly by:
  - digits 0 and 1
  - words (symbols) False (F) and True (T)
  - words (symbols) Low (L) and High (H)
  - and words On and Off.
- Binary values are represented by values or ranges of values of physical quantities

Binary Numbers

- Decimal number (*base* or *radix*: 10)

$\ldots a_5 a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3} \ldots$

Decimal point

a_j

Coefficients (0, 1, 2,..9)

Place value

$$\cdots + 10^5 a_5 + 10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10^1 a_1 + 10^0 a_0 + 10^{-1} a_{-1} + 10^{-2} a_{-2} + 10^{-3} a_{-3} + \cdots$$

Example:

$$7,329 = 7 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0$$

- General form of base-r system

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r^1 + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

Coefficient: $a_j = 0$ to $r - 1$

Binary Numbers

Name	Radix	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

- The six letters (in addition to the 10 integers) in hexadecimal represent: 10, 11, 12, 13, 14, and 15, respectively.

Binary Numbers

Example: Base-2 number **(Binary)**

$$(11010.11)_2 = (26.75)_{10}$$

$$= 1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0 + 1\times 2^{-1} + 1\times 2^{-2}$$

Example: Base-5 number **(Quinary)**

$$(4021.2)_5$$

$$= 4\times 5^3 + 0\times 5^2 + 2\times 5^1 + 1\times 5^0 + 2\times 5^{-1} = (511.5)_{10}$$

Example: Base-8 number **(Octal)**

$$(127.4)_8$$

$$= 1\times 8^3 + 2\times 8^2 + 1\times 8^1 + 7\times 8^0 + 4\times 8^{-1} = (87.5)_{10}$$

Example: Base-16 number **(Hexadecimal)**

$$(\mathrm{B65F})_{16} = 11\times 16^3 + 6\times 16^2 + 5\times 16^1 + 15\times 16^0 = (46,687)_{10}$$

The letters A, B, C, D, E, and F are used for the digits 10, 11, 12, 13, 14, and 15

Binary Numbers

Convert $(110101)_2$ to decimal

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$

Special Powers of 2

- 2^{10} (1024) is Kilo, denoted "K"
- 2^{20} (1,048,576) is Mega, denoted "M"
- 2^{30} (1,073, 741,824)is Giga, denoted "G"

Binary Numbers

Table 1.1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Binary Arithmetic

Arithmetic operations with numbers in base *r* follow the same rules as decimal numbers.

❑ Addition

Augend: 101101

Addend: +100111

Sum: 1010100

Rules of Binary Addition

- 0 + 0 = 0
- 0 + 1 = 1
- 1 + 0 = 1
- 1 + 1 = 0, and carry 1 to the next higher significant bit

❑ Subtraction

Minuend: 10 2 Ø2 1 0 1

Subtrahend: –100111

Difference: 000110

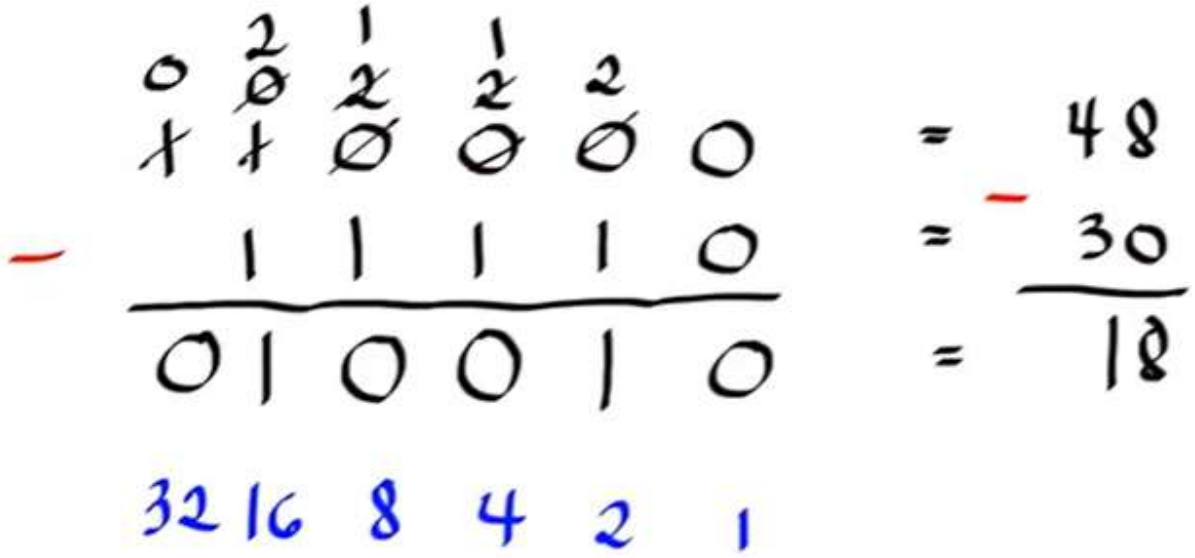
Rules of Binary Subtraction

- 0 - 0 = 0
- 0 - 1 = 1, and borrow from the next higher significant bit
- 1 - 0 = 1
- 1 - 1 = 0

(borrow in a given significant position adds 2 to a minuend digit)

Binary Arithmetic

- Practice



Binary Arithmetic

❑ Multiplication

Multiplicand:	1011
Multiplier:	× 101
	1011
	0000
	1011
Product:	110111

Rules of Binary Multiplication

- 0 x 0 = 0
- 0 x 1 = 0
- 1 x 0 = 0
- 1 x 1 = 1, and no carry or borrow bits

Number-Base Conversions

Example1.1

Convert decimal 41 to binary. The process is continued until the integer quotient becomes 0.

	Integer Quotient		Remainder	Coefficient
$41/2 =$	20	+	$\frac{1}{2}$	$a_0 = 1$
$20/2 =$	10	+	0	$a_1 = 0$
$10/2 =$	5	+	0	$a_2 = 0$
$5/2 =$	2	+	$\frac{1}{2}$	$a_3 = 1$
$2/2 =$	1	+	0	$a_4 = 0$
$1/2 =$	0	+	$\frac{1}{2}$	$a_5 = 1$

$$(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$$

Number-Base Conversions

- The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

101001 = answer

Conversion from decimal integers to any base-r system is similar to this example, except that division is done by r instead of 2.

Number-Base Conversions

Example 1.2

Convert decimal 153 to octal. The required base *r* is 8.

Integer	Remainder
153	
19	1
2	3
0	2

$= (231)_8$

The *conversion of a decimal fraction to binary* is accomplished by a method similar to that used for integers. However, multiplication is used instead of division, and integers instead of remainders are accumulated.

Number-Base Conversions

Example1.3 **Convert $(0.6875)_{10}$ to binary.**

The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

$$(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$$

- To convert a decimal fraction to a number expressed in base *r*, a similar procedure is used. However, multiplication is by *r* instead of 2, and the coefficients found from the integers may range in value from 0 to *r* – 1 instead of 0 and 1.

Number-Base Conversions

Example1.4

Convert $(0.513)_{10}$ to octal.

$$0.513 \times 8 = 4.104$$
$$0.104 \times 8 = 0.832$$
$$0.832 \times 8 = 6.656$$
$$0.656 \times 8 = 5.248$$
$$0.248 \times 8 = 1.984$$
$$0.984 \times 8 = 7.872$$

$$(0.513)_{10} = (0.406517\ldots)_8$$

- From Examples 1.1 and 1.3: $(41.6875)_{10} = (101001.1011)_2$
- From Examples 1.2 and 1.4: $(153.513)_{10} = (231.406517)_8$

Number-Base Conversions

Table 1.2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Number-Base Conversions

- **Conversion from binary to octal can be done by positioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right.**

$$(10\ 110\ 001\ 101\ 011\ .\ 111\ 100\ 000\ 110)_2 = (26153.7406)_8$$
$$2\quad 6\quad 1\quad 5\quad 3\qquad 7\quad 4\quad 0\quad 6$$

- **Conversion from binary to hexadecimal is similar, except that the binary number is divided into groups of four digits:**

$$(10\ 1100\ 0110\ 1011\ \cdot\ 1111\ 0010)_2 = (2C6B.F2)_{16}$$
$$2\quad C\quad 6\quad B\qquad F\quad 2$$

- **Conversion from octal or hexadecimal to binary is done by reversing the preceding procedure.**

$$(673.124)_8 = (110\ 111\ 011\ \cdot\ 001\ 010\ 100)_2$$
$$6\quad 7\quad 3\qquad 1\quad 2\quad 4$$

$$(306.D)_{16} = (0011\ 0000\ 0110\ \cdot\ 1101)_2$$
$$3\quad 0\quad 6\qquad D$$

Number-Base Conversions

- **Convert octal 673.124 to hex**

Convert every octal digit to 3 binary digits, then convert every 4 binary digits to 1 hex digit:

$(673.124)_8$

= 6 7 3.1 2 4

= 110 111 011.001 010 1

= 1 1011 1011.0010 101

= 1 B B.2 A

= **$(1BB.2A)_{16}$**

Complements of Numbers

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.

- There are two types of complements for each base-r system:
  1. Radix complement → r's complement
  2. Diminished radix complement → (r - 1)'s complement

- For binary numbers (r=2):
  Radix complement → 2's complement
  Diminished radix complement → 1's complement

- For decimal numbers (r=10):
  Radix complement → 10's complement
  Diminished radix complement → 9's complement

Complements of Numbers

Diminished Radix Complement → (r - 1)'s complement

- " Given a number N in base r having n digits, the (r - 1)'s complement of N, i.e., its diminished radix complement, is defined as $(r^n - 1) - N$.

Example:

♣ **For decimal numbers, *r* = 10 and *r* – 1 = 9, 9's complement of *N* is $(10^n - 1) - N$.**

$$(10^6 - 1) - 546700 = (1000000 - 1) - 546700 = 999999 - 546700$$

The 9's complement of 546700 is 999999 – 546700 = 453299.

The 9's complement of 012398 is 999999 – 012398 = 987601.

♣ **For binary numbers, *r* = 2 and *r* – 1 = 1, so the 1's complement of *N* is $(2^n - 1) - N$.**
→2^n is represented by a binary number that consists of a 1 followed by n 0's.

$$(2^7 - 1) - 1011000 = ((10000000)_2 - 1) - 1011000 = 1111111 - 1011000$$

The 1's complement of 1011000 is 0100111

The 1's complement of 0101101 is 1010010

1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's

Complements of Numbers

- **Radix Complement → r 's complement**

The *r*'s complement of an *n*-digit number *N* in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. Comparing with the $(r - 1)$'s complement, we note that the *r*'s complement is obtained by adding 1 to the $(r - 1)$'s complement, since $r^n - N = [(r^n - 1) - N] + 1$.

⟹ **r 's complement = (r –1)'s complement + 1**

Example: Base-10

The 10's complement of 012398 is 987602
The 10's complement of 246700 is 753300

"leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9"

Example: Base-2

The 2's complement of 1101100 is 0010100
The 2's complement of 0110111 is 1001001

"leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits"

Subtraction with Complements

Subtraction with Complements

The subtraction of two *n*-digit unsigned numbers *M* – *N* in base *r* can be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N. Mathematically, $M + (r^n - N) = M - N + r^n$.
2. If $M \geqq N$, the sum will produce and end carry r^n, which can be discarded; what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front.

Subtraction with Complements

Example 1.5: **Using 10's complement, subtract 72532 – 3250.** $M \geq N$

	$M =$	72532
10's complement of	$N =$	+96750
	Sum =	169282
	Discard end carry $10^5 =$	−100000
	Answer =	69282

← 10's complement of 03250 (N)

Example 1.6: **Using 10's complement, subtract 3250 – 72532** $M < N$

	M =	03250
10's complement of	N =	+27468
	Sum =	30718

⇒ **There is no end carry.**

⇒ **Therefore, the answer is – (10's complement of 30718) = – 69282.**

Subtraction with Complements

Example 1.7

Given the two binary numbers *X* = 1010100 and *Y* = 1000011, perform the subtraction (a) X – Y and (b) *Y* – *X* by using 2's complement.

(a)		
	X =	1010100
	2's complement of Y =	+0111101
	Sum =	10010001
	Discard end carry 2^7 =	–10000000
	Answer. X – Y =	0010001

(b)		
	Y =	1000011
	2's complement of X =	+ 0101100
	Sum =	1101111

There is no end carry. Therefore, the answer is *Y* – *X* = – (2's complement of 1101111) = – 0010001.

Subtraction with Complements

- **Subtraction of unsigned numbers can also be done by means of the (*r* – 1)'s complement. Remember that the (*r* – 1) 's complement is one less then the *r*'s complement.**

Example 1.8 **Repeat Example 1.7, but this time using 1's complement.**

(a) $X - Y = 1010100 - 1000011$

$X =$	1010100
1's complement of $Y =$	+ 0111100
Sum =	10010000
End-around carry =	+ 1
Answer. $X - Y =$	0010001

Removing the end carry and adding 1 to the sum is referred to as an *end-around carry*.

(b) $Y - X = 1000011 - 1010100$

$Y =$	1000011
1's complement of $X =$	+ 0101011
Sum =	1101110

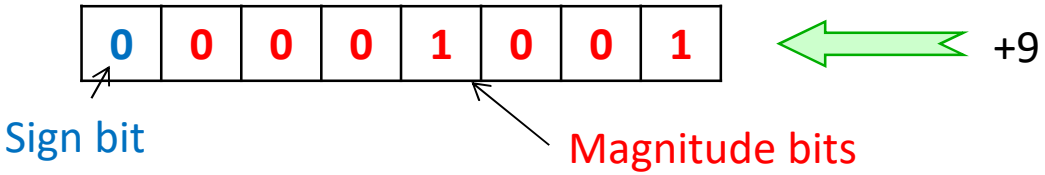
There is no end carry, Therefore, the answer is *Y* – *X* = – (1's complement of 1101110) = – 0010001.

Signed Binary Numbers

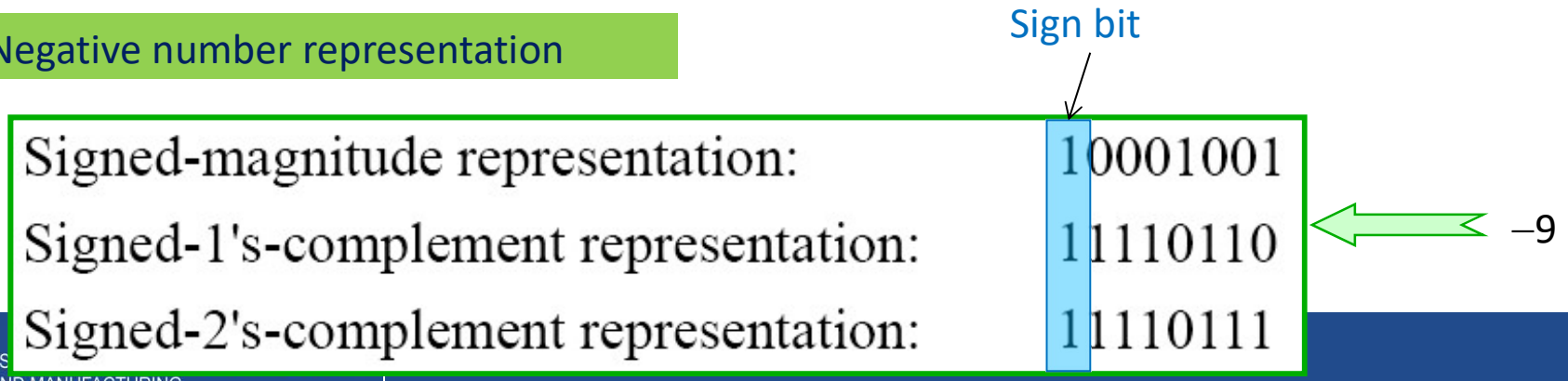
- In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign. Because of hardware limitations, computers must represent everything with binary digits.
- It is customary to represent the sign with a bit placed in the leftmost position of the number.
- The convention is to make the sign bit **0 for positive** and **1 for negative**.

Positive number representation

signed-magnitude representation:



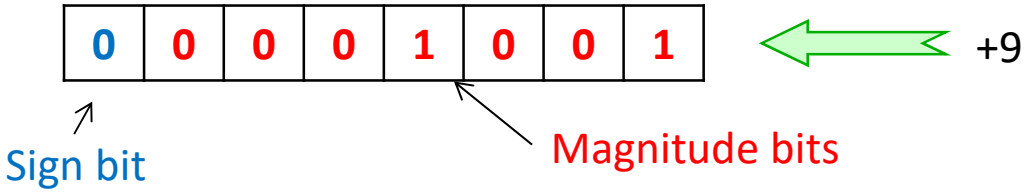
Negative number representation



Signed Binary Numbers

- **In signed magnitude, –9 is obtained from +9 by changing only the sign bit in the leftmost position from 0 to 1.**
- **In signed-1's-complement, –9 is obtained by complementing all the bits of +9, including the sign bit.**
- **The signed-2's-complement representation of –9 is obtained by taking the 2's complement of the positive number, including the sign bit.**

signed-magnitude representation:



Negative number representation

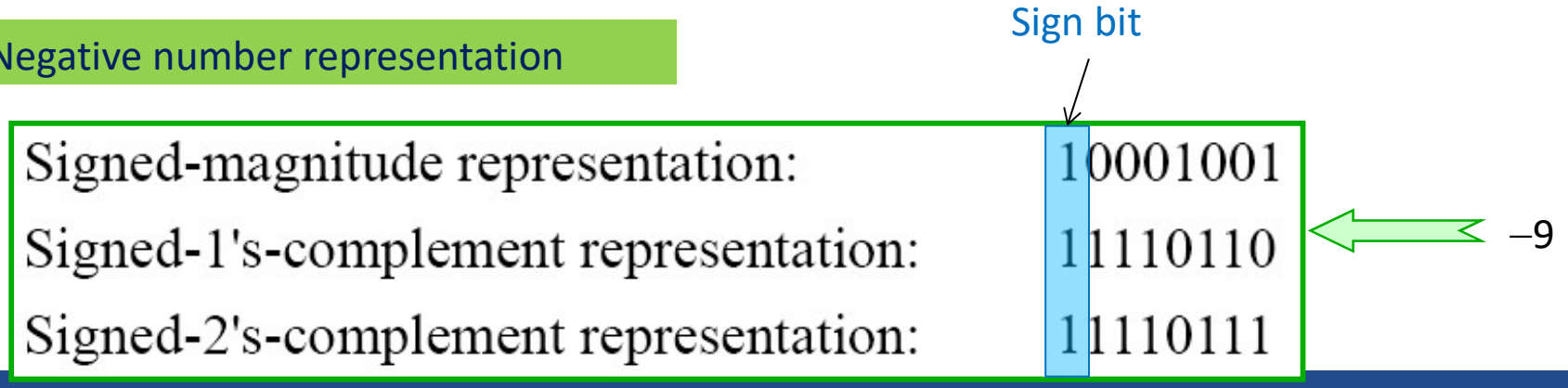


Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Signed Binary Numbers Arithmetic

Addition

- The addition of two signed binary numbers with **negative numbers represented in signed-2's-complement form** is obtained from the addition of the two numbers, including their sign bits.
- A carry out of the sign-bit position is discarded.
- The result is in signed form.

Example:

+ 6	00000110	− 6	11111010
+13	00001101	+13	00001101
+ 19	00010011	+ 7	00000111
+ 6	00000110	− 6	11111010
−13	11110011	−13	11110011
− 7	11111001	− 19	11101101

Discard carry

Note that negative numbers must be initially in 2's-complement form and that if the sum obtained after the addition is negative, it is in 2's-complement form. For example, -7 is represented as 11111001, which is the 2s complement of +7.

Signed Binary Numbers Arithmetic

Subtraction

- Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit).
- A carry out of the sign-bit position is discarded.

$$(\pm A)-(+B)=(\pm A)+(-B)$$
$$(\pm A)-(-B)=(\pm A)+(+B)$$

Example:

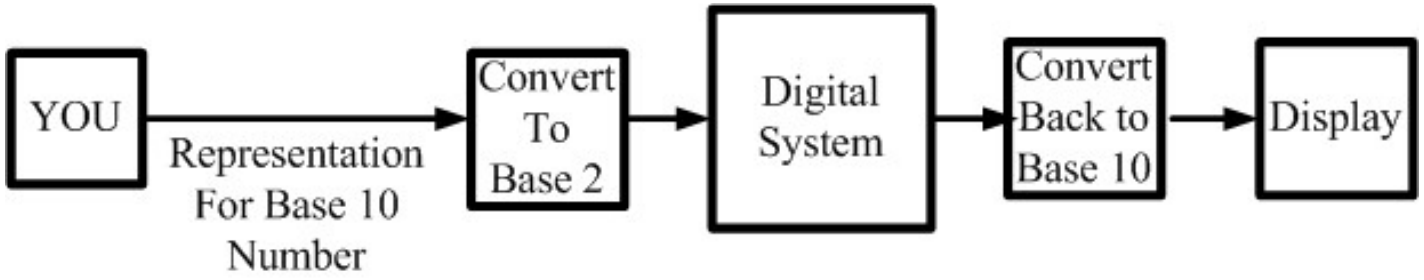
00000110	(+6)	11111010	(–6)	00000110	(+6)	11111010	(–6)
00001101	–(+13)	00001101	–(+13)	11110011	–(–13)	11110011	–(–13)

Take the 2's complement of the negative number and add:

00000110	(+6)	11111010	(–6)	00000110	(+6)	11111010	(–6)
11110011	(–13)	11110011	(–13)	00001101	(+13)	00001101	(+13)
11111001	**(–7)**	**~~1~~11101101**	**(–19)**	**00010011**	**(+19)**	**~~1~~00000111**	**(+7)**

Binary-Coded Decimal (BCD) code

- We naturally live in a base 10 environment
- Computer exist in a base 2 environment
- So give the computer/digital system the task of doing the conversions for us.



- The binary-coded decimal (BCD) is an encoding for decimal numbers in which each digit is represented by its own binary sequence.

Binary-Coded Decimal (BCD) code

Table 1.4
Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- **A number with *k* decimal digits will require 4*k* bits in BCD.**
- **Decimal 396 is represented in BCD with 12 bits as 0011 1001 0110,** with each group of 4 bits representing one decimal digit.
- **A decimal number in BCD is the same as its equivalent binary number only when the number is between 0 and 9.**
- **A BCD number greater than 10 looks different from its equivalent binary number, even though both contain 1's and 0's. Moreover,** the binary combinations 1010 through 1111 are not used and have no meaning in BCD.

Binary-Coded Decimal (BCD) code

$$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

- **The BCD value has 12 bits to encode the characters of the decimal value, but the equivalent binary number needs only 8 bits.**
  - representation of a BCD number needs more bits than its equivalent binary value

- **However, there is an advantage in the use of decimal numbers, because computer input and output data are generated by people who use the decimal system.**

- **BCD numbers are decimal numbers and not binary numbers, although they use bits in their representation.**

Binary-Coded Decimal (BCD) code

BCD Addition

- When the binary sum is equal to or less than 1001 (without a carry), the corresponding BCD digit is correct.
- However, when the binary sum is greater than or equal to 1010, the result is an invalid BCD digit.
- The addition of 6 = $(0110)_2$ to the binary sum converts it to the correct digit and also produces a carry as required.
- This is because a carry in the most significant bit position of the binary sum and a decimal carry differ by 16 - 10 = 6.

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Binary-Coded Decimal (BCD) code

BCD Addition

Example:

Consider the addition of 184 + 576 = 760 in BCD:

BCD	1	1		
	0001	1000	0100	184
	+0101	0111	0110	+576
Binary sum	0111	10000	1010	
Add 6		0110	0110	
BCD sum	0111	0110	0000	760

The first, least significant pair of BCD digits produces a BCD digit sum of 0000 and a carry for the next pair of digits. The second pair of BCD digits plus a previous carry produces a digit sum of 0110 and a carry for the next pair of digits. The third pair of digits plus a carry produces a binary sum of 0111 and does not require a correction.

Binary-Coded Decimal (BCD) code

Decimal Arithmetic

plus → 0 → 0000
minus → 9 → 1001

Addition is done by summing all digits, including the sign digit, and discarding the end carry. This operation assumes that all negative numbers are in 10's-complement form.

Consider the addition (+375) + (-240) = +135, done in the signed-complement system:

0	375
+9	760
0	135

9760 is the 10's complement of 0240

Other Decimal Codes

Table 1.5
Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused	1011	0110	0001	0010
bit	1100	0111	0010	0011
combi-	1101	1000	1101	1100
nations	1110	1001	1110	1101
	1111	1010	1111	1110

In a weighted code, each bit position is assigned a weighting factor in such a way that each digit can be evaluated by adding the weights of all the 1's in the coded combination.

- BCD and the 2421 code are examples of weighted codes.
- The 2421 and the excess-3 codes are examples of self-complementing codes.

(9's complement of a decimal number is obtained directly by changing 1's to 0's and 0's to 1's)

- Excess-3 is an unweighted code in which each coded combination is obtained from the corresponding binary value plus 3.
- The 8, 4, -2, -1 code is an example of assigning both positive and negative weights to a decimal code.

Gray Code

Table 1.6
Gray Code

Gray Code	Decimal Equivalent	Binary (base 2)
0000	0	0000
0001	1	0001
0011	2	0010
0010	3	0011
0110	4	0100
0111	5	0101
0101	6	0110
0100	7	0111
1100	8	1000
1101	9	1001
1111	10	1010
1110	11	1011
1010	12	1100
1011	13	1101
1001	14	1110
1000	15	1111

- Gray code is an unweighted code
- The advantage of the Gray code over the straight binary number sequence is that only one bit in the code group changes in going from one number to the next.
- The Gray code is used in applications in which the normal sequence of binary numbers generated by the hardware may produce an error or ambiguity during the transition from one number to the next.

ASCII Character Code

Table 1.7
American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	−	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	−	o	DEL

- The standard binary code for the alphanumeric characters is the **American Standard Code for Information Interchange (ASCII),** which uses seven bits to code 128 characters.
- ASCII is a seven-bit code, but most computers manipulate an eight-bit quantity as a single unit called a byte.
- Therefore, ASCII characters most often are stored one per byte. The extra bit is sometimes used for other purposes, depending on the application.

ASCII Character Code

- ASCII Character Code

Control Characters

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

Error-Detecting Code

- To detect errors in data communication and processing, an eighth bit is sometimes added to the ASCII character to indicate its parity.

- A *parity bit* is an extra bit included with a message to make the total number of 1's either even or odd.

Example:

Consider the following two characters and their even and odd parity:

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

Error-Detecting Code

- Redundancy (e.g. extra information), in the form of extra bits, can be incorporated into binary code words to detect and correct errors.
- A simple form of redundancy is parity, an extra bit appended onto the code word to make the number of 1's odd or even. Parity can detect all single-bit errors and some multiple-bit errors.
- A code word has even parity if the number of 1's in the code word is even.
- A code word has odd parity if the number of 1's in the code word is odd.

Binary Storage and Registers

Registers

- **A *binary cell* is a device that possesses two stable states and is capable of storing one of the two states.**
- **A *register* is a group of binary cells. A register with *n* cells can store any discrete quantity of information that contains *n* bits.**

n cells ⇒ 2^n possible states

- **A binary cell**
  - **two stable state**
  - **store one bit of information**
  - **examples: flip-flop circuits**
- **A register**
  - **a group of binary cells**
  - **AX in x86 CPU**

Binary Storage and Registers

Register Transfer

- a transfer of the information stored in one register to another
- one of the major operations in digital system

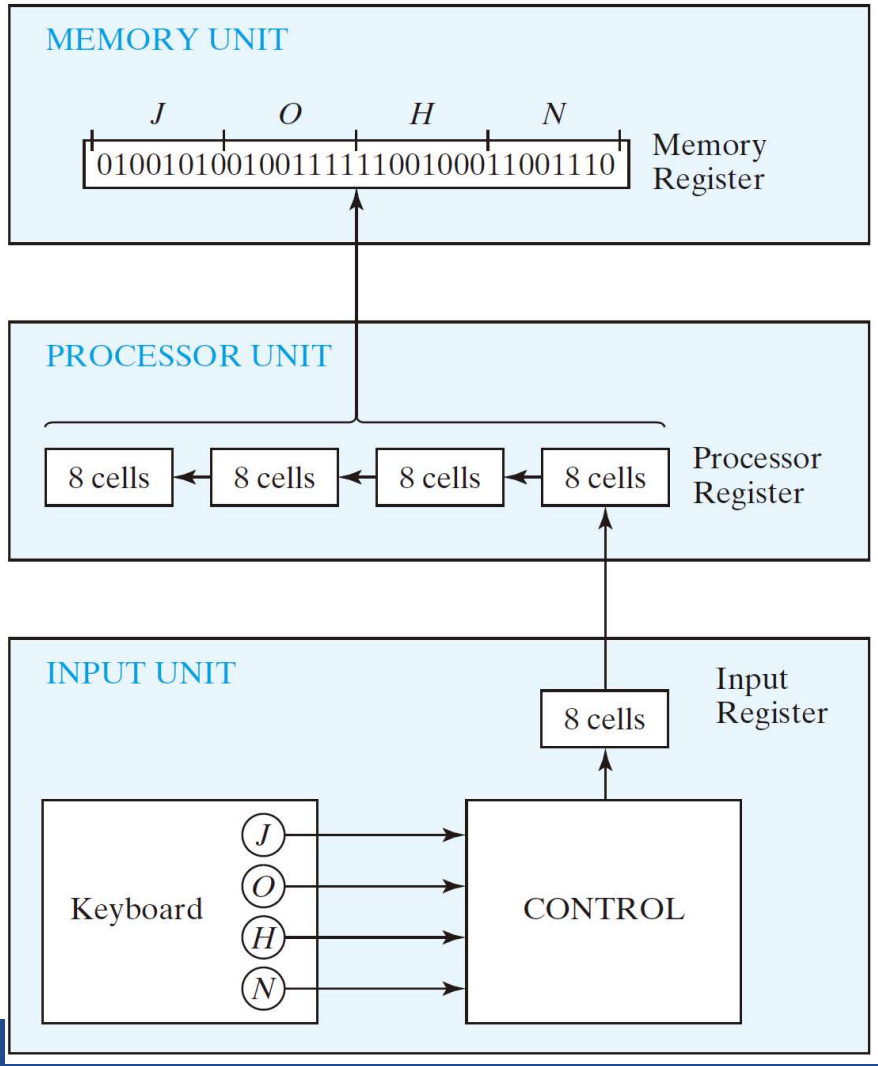
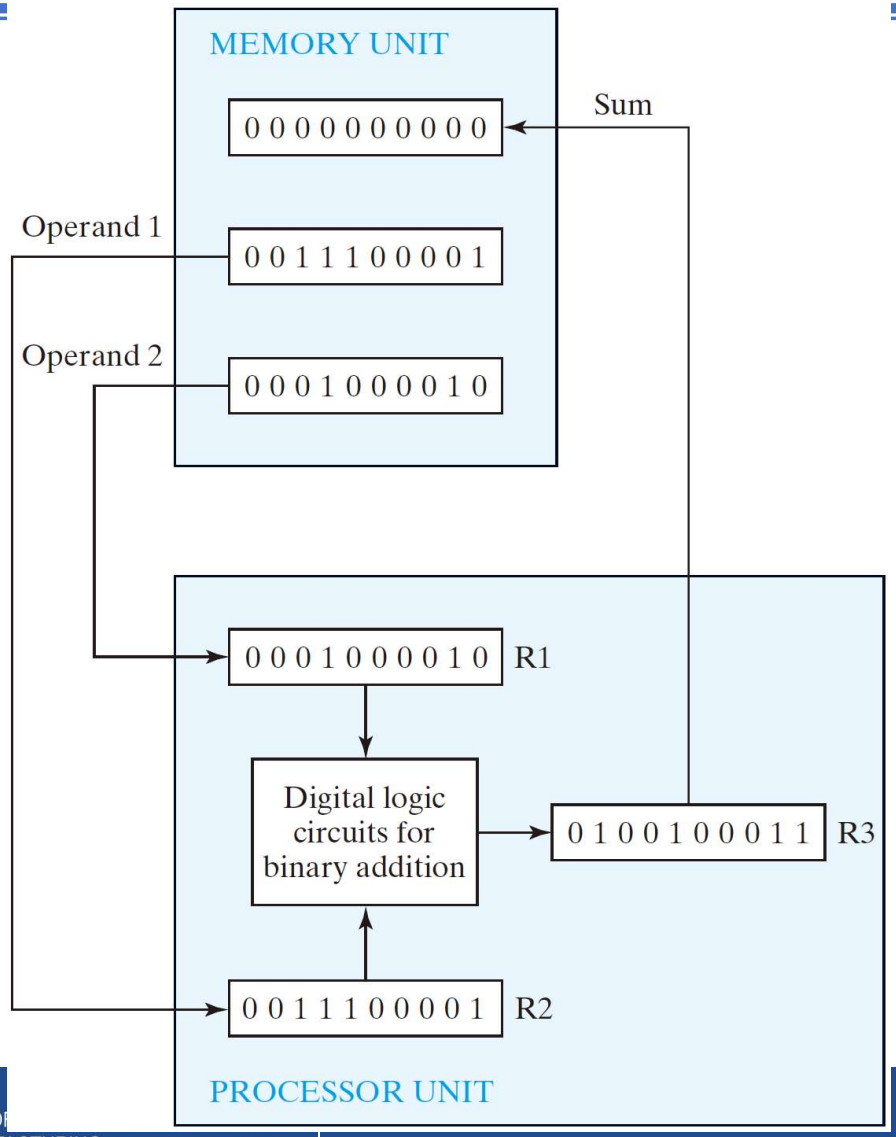


FIGURE 1.1
Transfer of information among registers

Binary Storage and Registers



The device most commonly used for holding data is a register.

FIGURE 1.2
Example of binary information processing

Binary Logic

- **Binary logic consists of binary variables and a set of logical operations. The variables are designated by letters of the alphabet, such as A, B, C, x, y, z, etc., with each variable having two and only two distinct possible values: 1 and 0, There are three basic logical operations: AND, OR, and NOT.**

1. **AND:** This operation is represented by a dot or by the absence of an operator. For example, $x \cdot y = z$ or $xy = z$ is read "***x* AND *y* is equal to *z*.**" The logical operation AND is interpreted to mean that $z = 1$ if and only if $x = 1$ and $y = 1$; otherwise $z = 0$. (Remember that x, y, and z are binary variables and can be equal either to 1 or 0, and nothing else.) The result of the operation $x \cdot y$ is z.

2. **OR:** This operation is represented by a plus sign. For example, $x + y = z$ is read "***x* OR *y* is equal to *z*,**" meaning that $z = 1$ if $x = 1$ or if $y = 1$ or if both $x = 1$ and $y = 1$. If both $x = 0$ and $y = 0$, then $z = 0$.

3. **NOT:** This operation is represented by a prime (sometimes by an overbar). For example, $x' = z$ (or $\bar{x} = z$) is read "**not *x* is equal to *z*,**" meaning that z is what x is not. In other words, if $x = 1$, then $z = 0$, but if $x = 0$, then $z = 1$. The NOT operation is also referred to as the complement operation, since it changes a 1 to 0 and a 0 to 1, i.e., the result of complementing 1 is 0, and vice versa.

Binary Logic

- The truth tables for AND, OR, and NOT are given in Table 1.8.

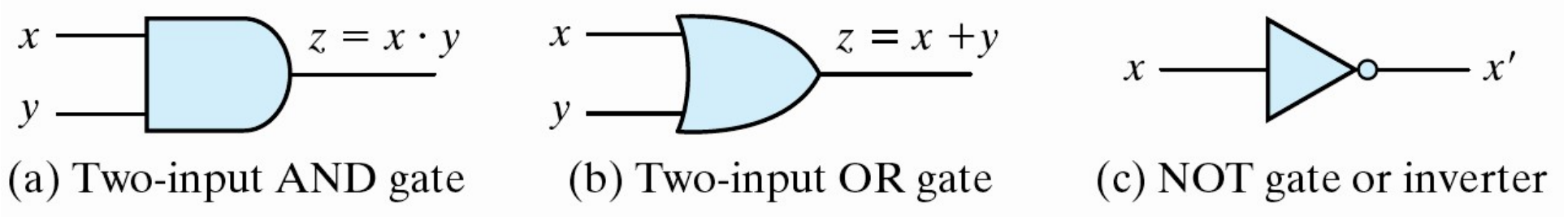
Table 1.8
Truth Tables of Logical Operations

AND			OR			NOT	
x	y	$x \cdot y$	x	y	$x + y$	x	x'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

Binary Logic

Logic gates

♣ Graphic Symbols and Input-Output Signals for Logic gates:



(a) Two-input AND gate (b) Two-input OR gate (c) NOT gate or inverter

Fig. 1.4 Symbols for digital logic circuits

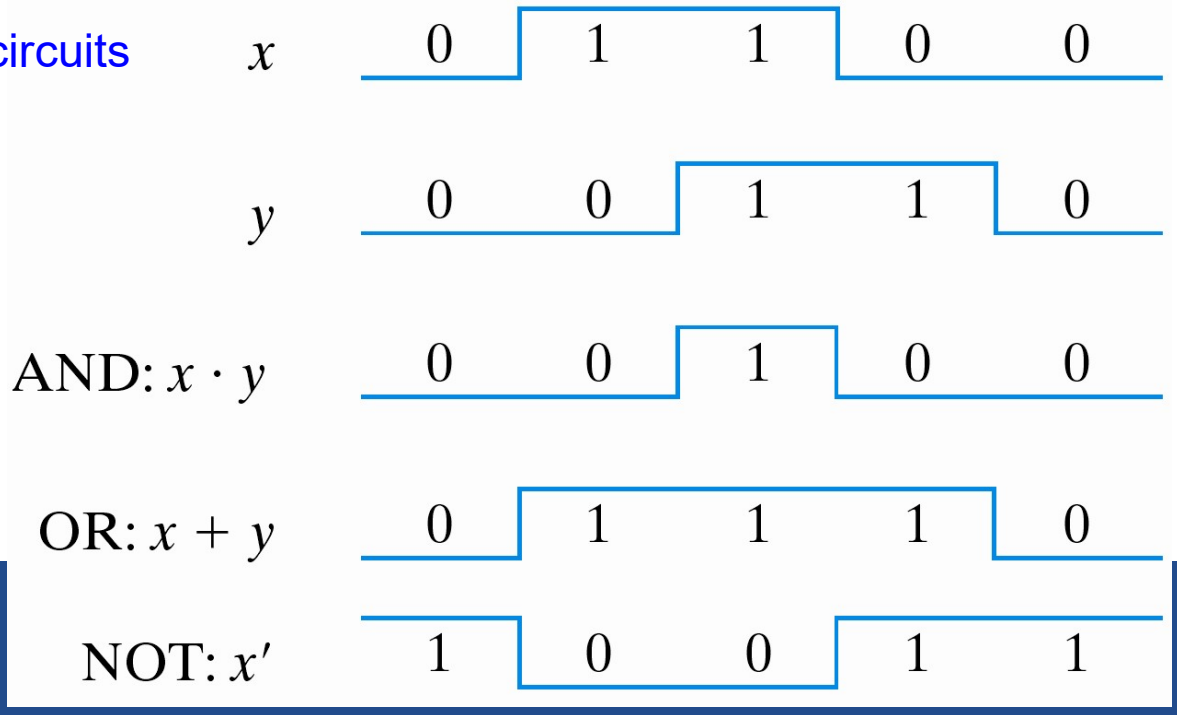
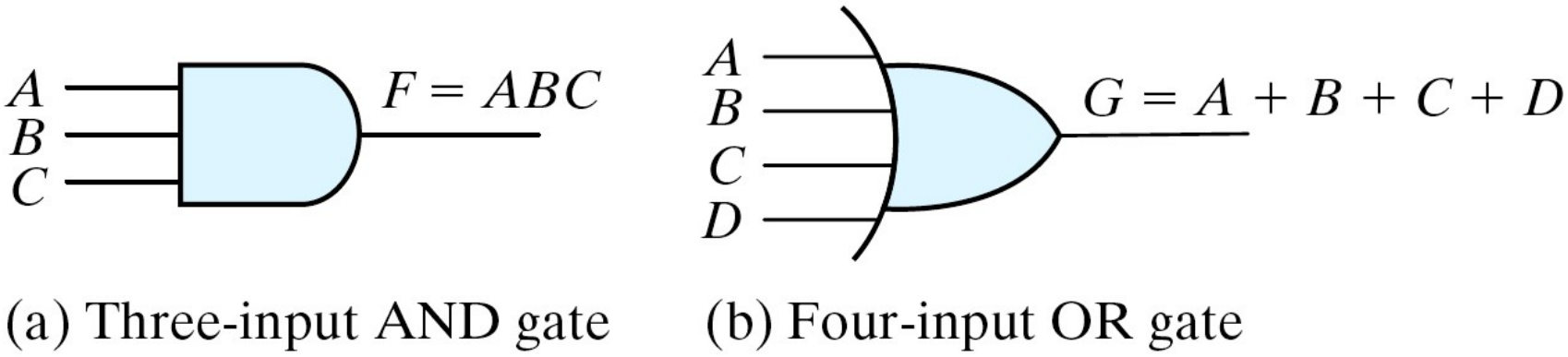


Fig. 1.5
Input-Output signals
for gates

Binary Logic

Logic gates

♣ Graphic Symbols and Input-Output Signals for Logic gates:



(a) Three-input AND gate (b) Four-input OR gate

Fig. 1.6 Gates with multiple inputs

- The three-input AND gate responds with logic 1 output if all three inputs are logic 1. The output produces logic 0 if any input is logic 0.
- The four-input OR gate responds with logic 1 if any input is logic 1; its output becomes logic 0 only when all inputs are logic 0.

Binary Logic

- Electrical signals such as voltages or currents exist as analog signals having values over a given continuous range, say, 0 to 3 V, but in a digital system these voltages are interpreted to be either of two recognizable values, 0 or 1.
- Voltage-operated logic circuits respond to two separate voltage levels that represent a binary variable equal to logic 1 or logic 0.
- For example, a particular digital system may define logic "0" as a signal equal to 0 V and logic "1" as a signal equal to 3 V.
- In practice, each voltage level has an acceptable range.

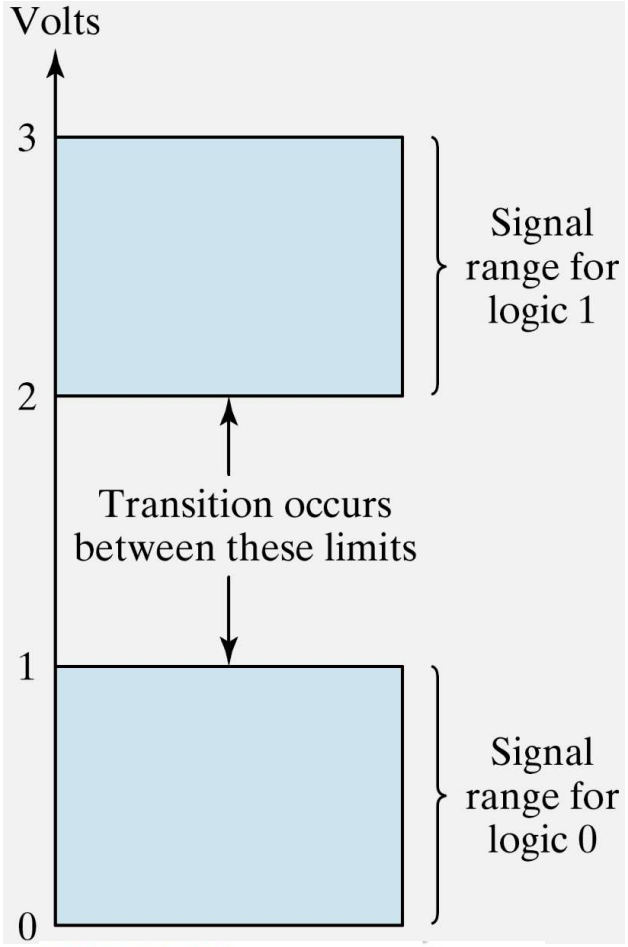


FIGURE 1.3
Signal levels for binary logic values

The End

Reference:

1. **Digital Design (with an introduction to the Verilog HDL) 6th Edition, M. Morris Mano, Michael D. Ciletti**