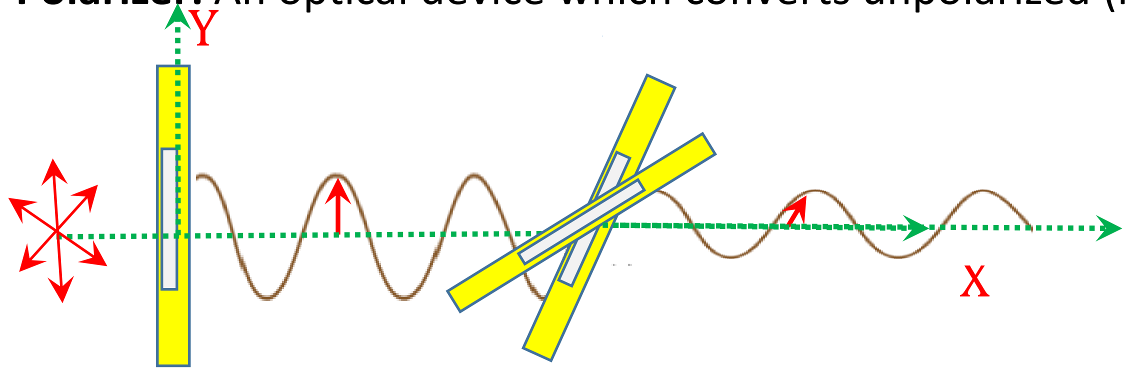
Polarization: **Polarizers**

$\vec{E} = \hat{\imath}\, E_0 sin\omega t + \hat{\jmath}\, E_0 cos\omega t$

Polarizer: An optical device which converts unpolarized (randomly polarized) light to some form of polarized light.

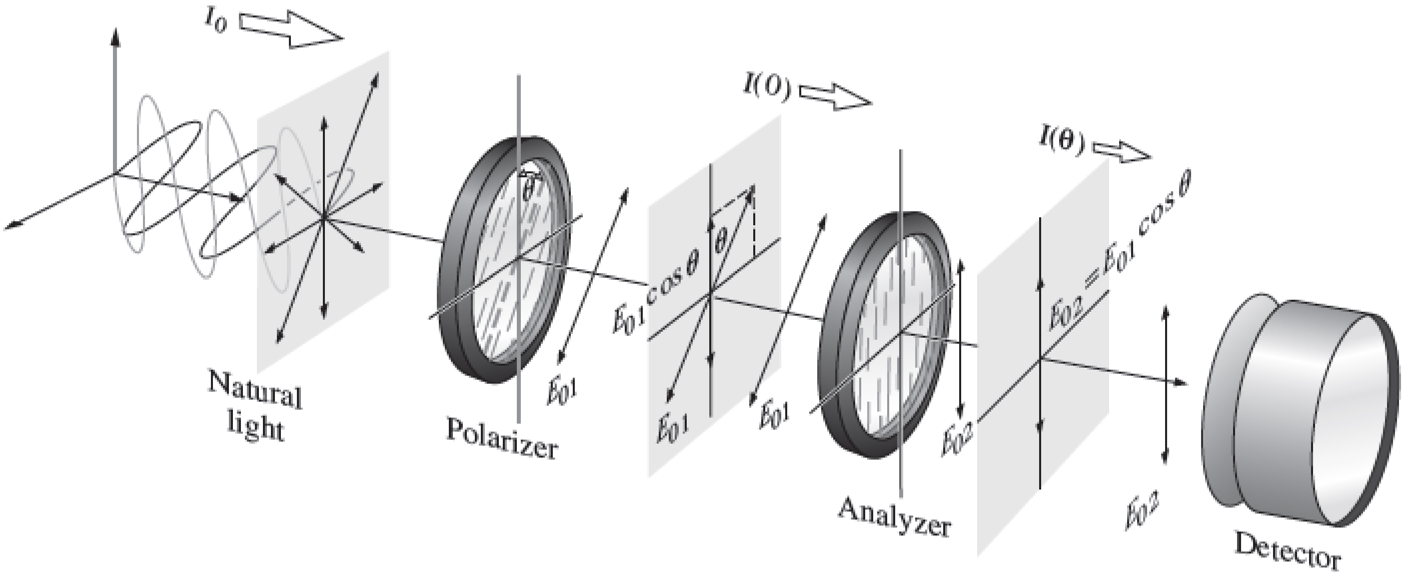


$$\vec{E} = E_0 \cos(kz - \omega t + \delta)$$

$$I = \langle |\vec{S}| \rangle_t = \left\langle \frac{1}{\mu_0} \frac{E^2}{v} \right\rangle, \quad as\ \vec{B} \perp \vec{E}, and\ |B| = \frac{|E|}{v}$$

$$I_0 = \frac{1}{2} v \varepsilon_0 E_0^2$$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) ---$$



Output after polarizer,

$$\vec{E} cos\theta = E_0 \cos(kz - \omega t + \delta)\, cos\theta$$

$$\boxed{I_\theta = \frac{1}{2} c\epsilon_0 E_0^2 \cos^2\theta = I_0 \cos^2\theta}$$

Malus's Law determines whether a device is a linear polarizer.

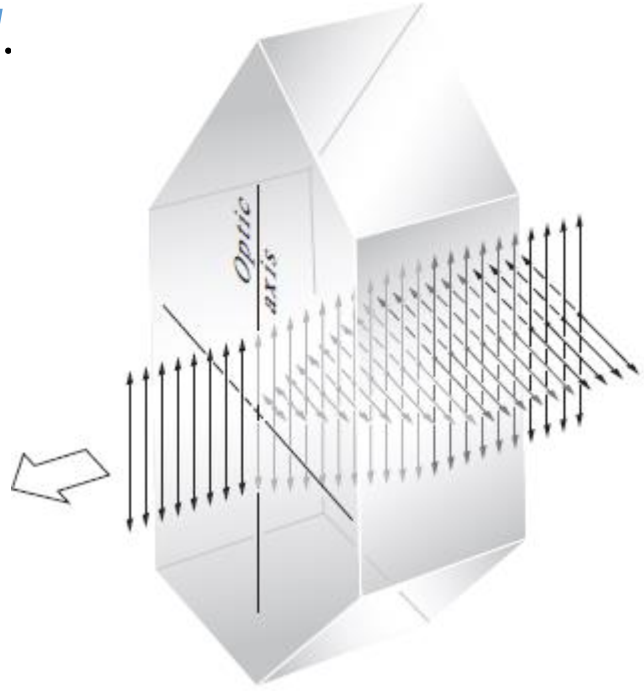
Polarization: **Polarizers**

Polarizer: An optical device which converts unpolarized (randomly polarized) light to some form of polarized light.

Process associated with some form of asymmetry are usually used.

Fundamental mechanisms:

- Selective absorption (*dichroism*),
- Reflection;
- Scattering
- Double refraction (*birefringence*).



Dichroism refers to the selective absorption of one of the two orthogonal polarization-state components of an incident beam.

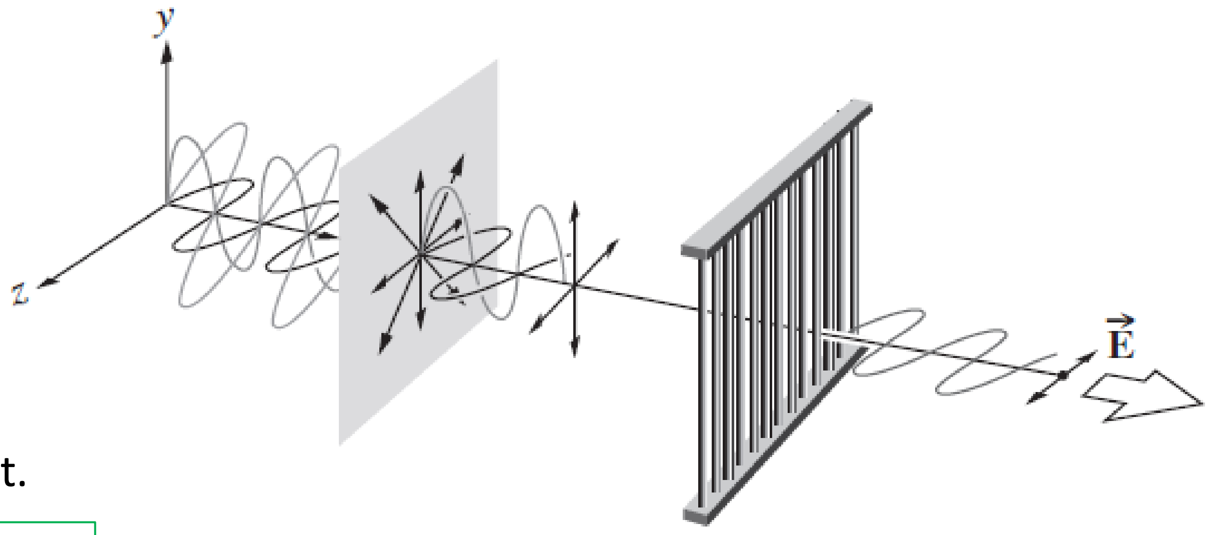
Natural Crystal: Materials having an anisotropy in their respective crystalline structures exhibits dichroism. Tourmaline (a naturally available stone used in jewelry). Tourmalines are boron silicates of different chemical composition [e.g., $NaFe_3B_3Al_6Si_6O_{27}(OH)_4$].

Polarization: Polarizers ➢ Selective absorption (*Wire-grid*):

Wire-grid Polarizer

Grid of very thin Cu wires polarizes microwaves easily

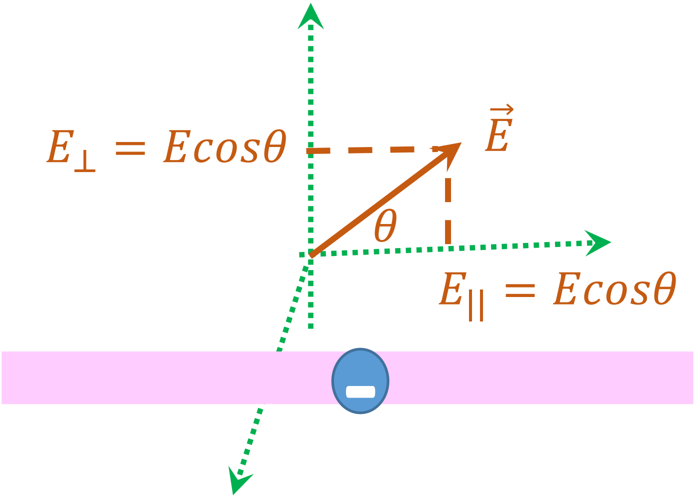
The transmission axis of the grid is perpendicular to the wires.



Fabrication of such polarizer for visual light (λ~400nm-700nm) for which the wires to be placed at a gap of λ is extremely difficult.

Bird and Parrish (1950) put 30,000 wires in 2.54in.

A polymer, H-sheet, invented by Land (1938) containing long-chain molecules are being used to make Polaroid.
A sheet of clear polyvinyl alcohol is heated and stretched in a given direction, its long hydrocarbon molecules becoming aligned in the process. The sheet is then dipped into an ink Solution rich in iodine. The iodine impregnates the plastic and attaches to the straight long-chain polymeric molecules, effectively forming a chain of its own. The conduction electrons associated with the iodine can move along the chains as if they were long thin wires.



Polarization: **Polarizers** ➢ Reflection

The glare from the wet road in night, glass window etc, are all generally due to partially polarized light coming from them.

A common source of polarized light is reflection from **dielectric media.**

$$n_1 sin\theta_i = n_2 sin\theta_t$$

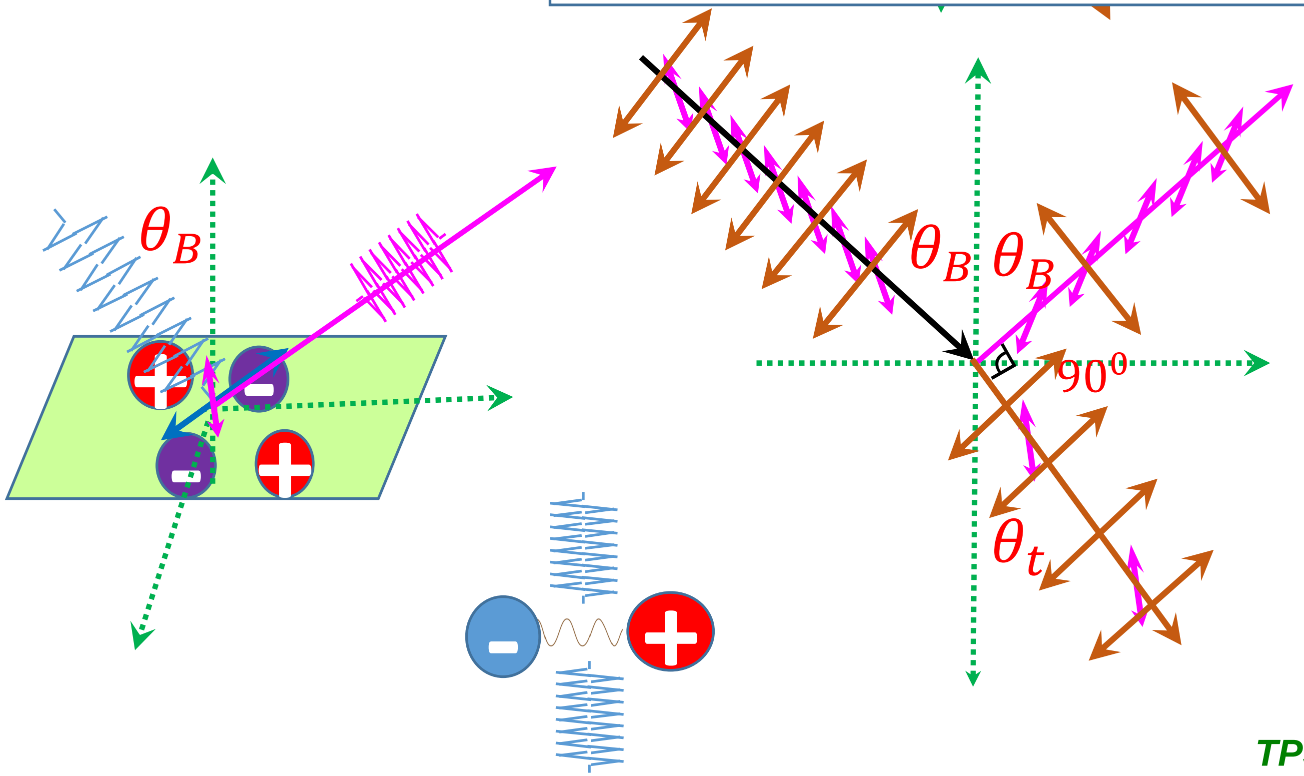
$$\theta_i = \theta_B;$$

$$\theta_t + \theta_B = 90^0$$

$$n_1 sin\theta_B = n_2 cos\theta_B$$

$$tan\theta_B = \frac{n_2}{n_1}$$

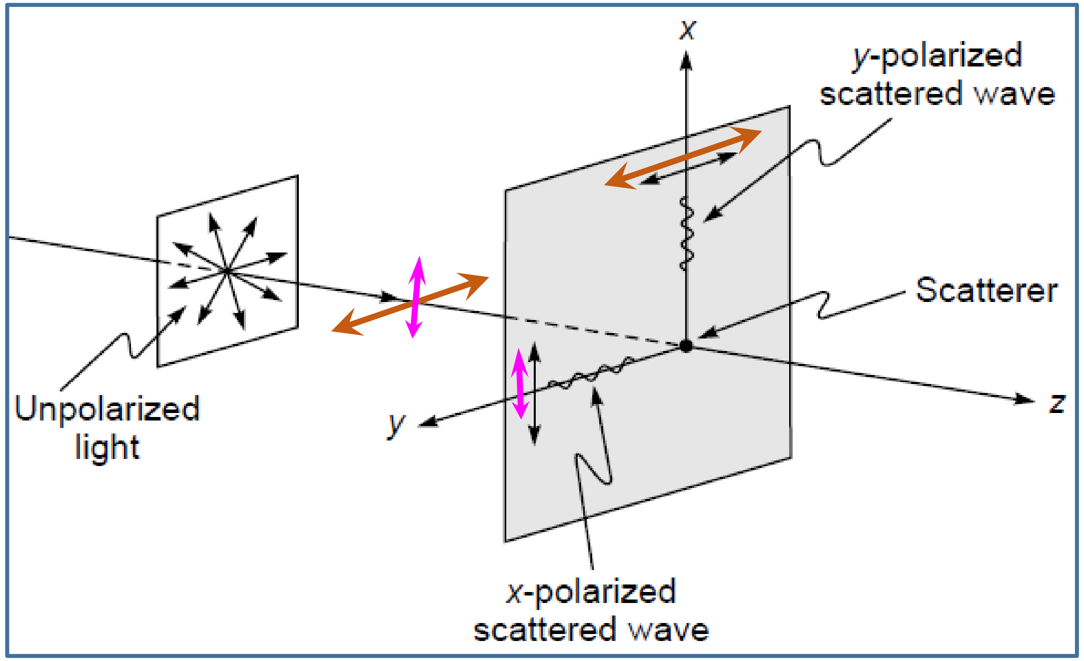
Brewster's law

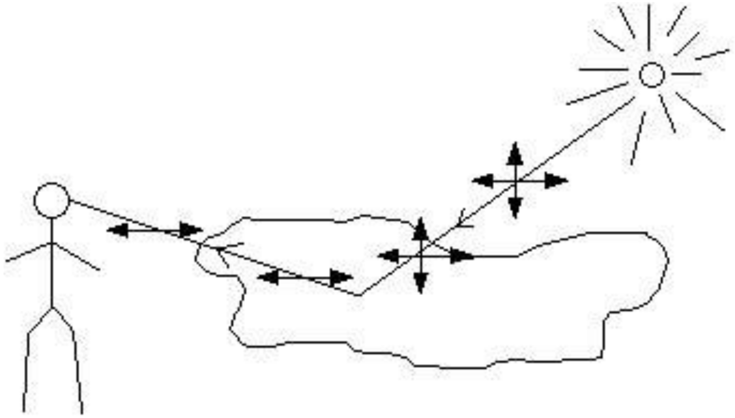


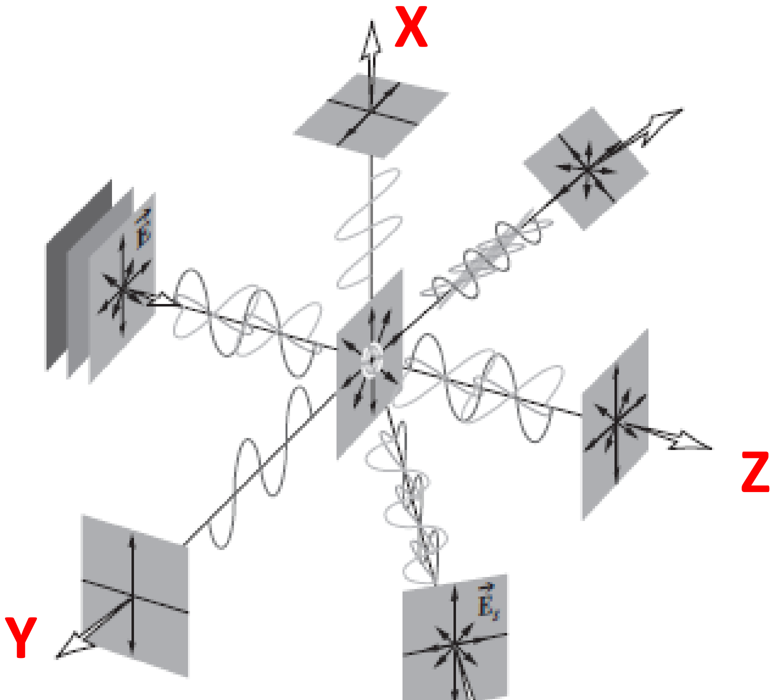
For the air-water interface, n_1 a 1 and n_2 a 1.33 and the polarizing angle (Brewster's angle) is 53°.

Polarization: Polarizers ➤ Scattering

A linearly polarized plane wave incident on an air.
The vibrations induced in the atom are parallel to the E-field of the incoming light wave and so are perpendicular to the propagation direction. The dipole does not radiate in the direction of molecule.

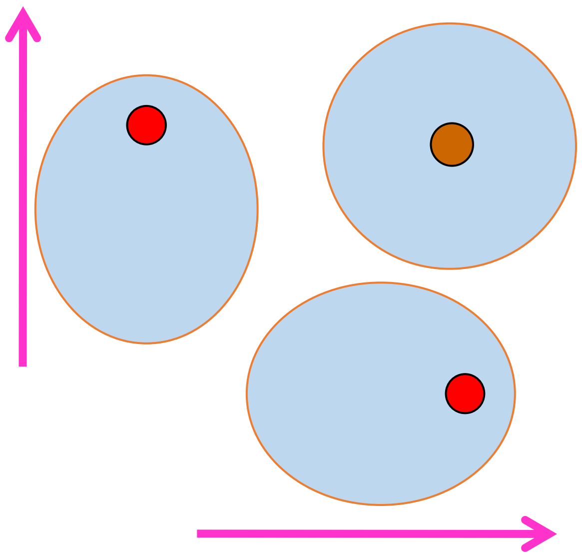






If the incident wave is unpolarized (can be represented by superposition of two orthogonal plane polarization states), the scattered Evidently. The scattered light in the forward direction is completely unpolarized; off that axis it is partially polarized, becoming increasingly more polarized as the angle increases. When the direction of observation is normal to the primary beam, the light is completely linearly polarized.

Polarization: Polarizers ➢ Double refraction (*birefringence*).



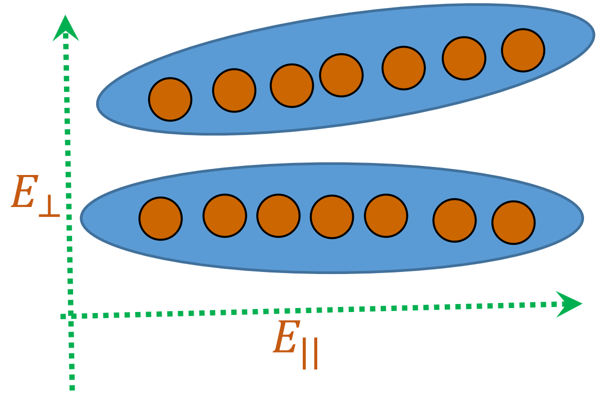
EM field propagating in a medium (composed of atoms, molecules etc) induces a time-varying response in charged particles.

Most of the motion of these charged particles are elastic and energy is transferred temporarily to the particles and returned. But is takes time.

Response➔Polarization=Dipole Moment/volume:
More charged particle per unit or/and more they move in response to the EM wave, the EM wave will move slower.

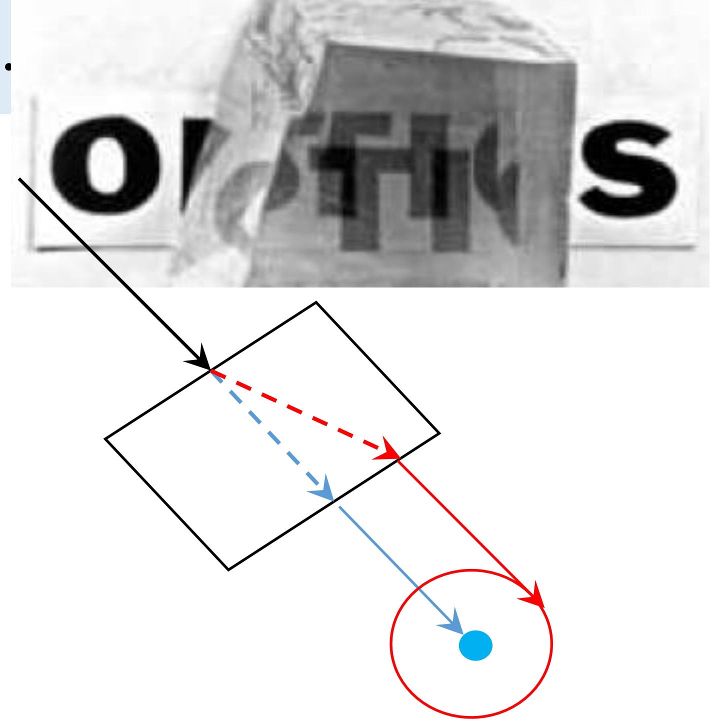
$$n = \sqrt{\epsilon_r \mu_r} = \sqrt{\epsilon_r} \qquad v = \frac{c}{n}$$

Response to the EM field-----Long molecules (natural crystals/polymers)



Polarization: **Polarizers**

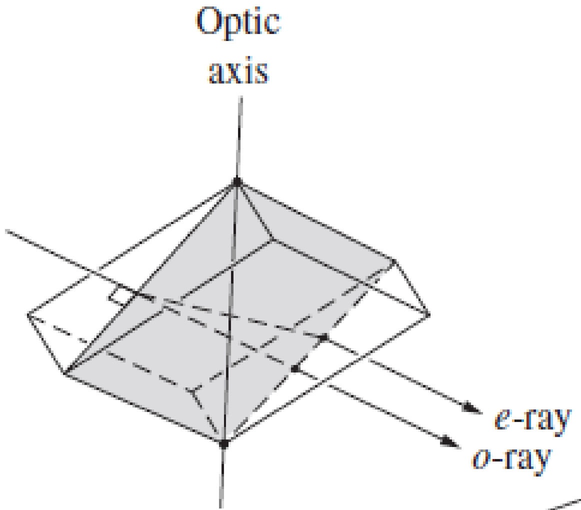
➢ Double refraction (*birefringence*).



A material of this sort, which displays two different indices of refraction in two different direction, is said to be **birefringent**

As a rule, an *o*-ray comports travels at the same speed in every direction as if the medium were isotropic.

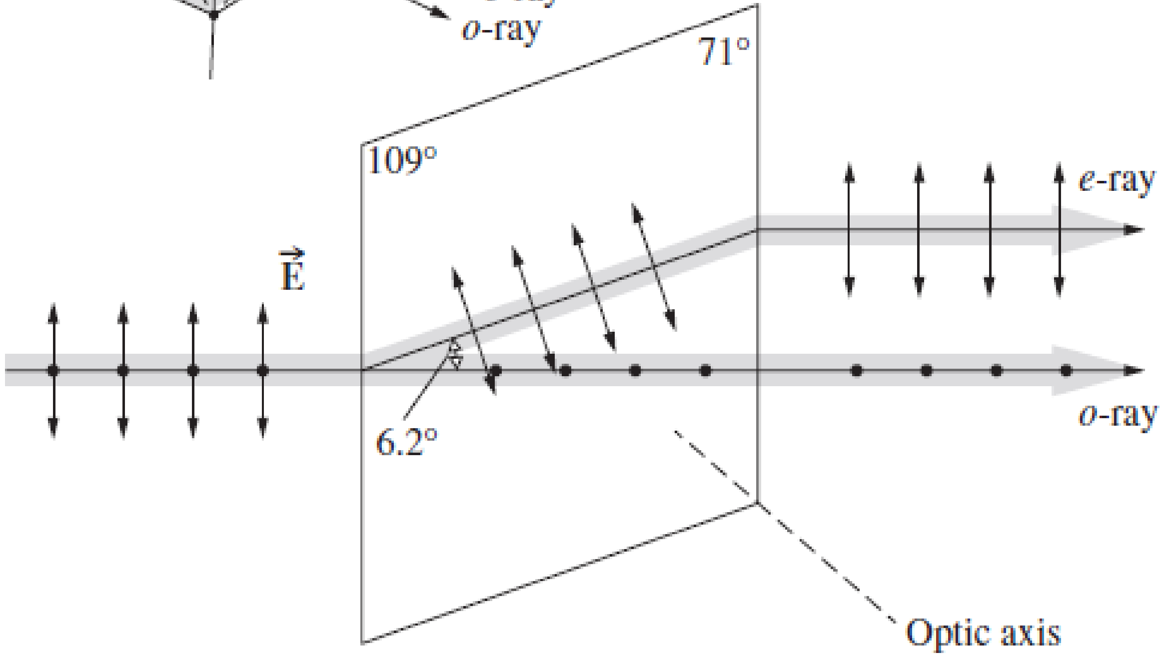
An *e*-wave does not travel through a birefringent crystal at the same speed in all directions



Along a particular direction (fixed in the crystal), the two velocities are equal; this direction is known as the **Optic axis** of the crystal. (Uniaxial, biaxial etc)

Refractive Indices of Some Uniaxial Birefringent Crystals $\lambda_0 = 589.3nm$

	n_o	n_e	
Tourmaline	1.669	1.638	
Calcite	1.6584	1.4864	
Quartz	1.5443	1.5534	(+)
Sodi nitrate	1.5854	1.3369	
Ice	1.309	1.313	(+)
Rutile(TiOz)	2.616	2.903	(+)



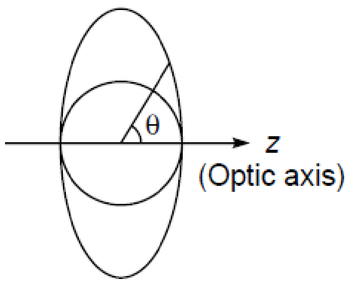
positive crystal $n_e > n_o$

negative crystal $n_e < n_o$

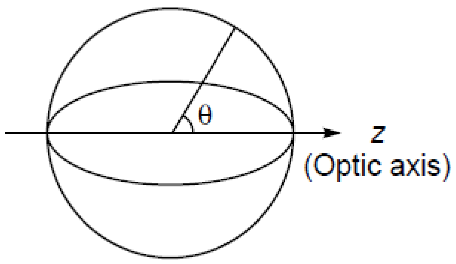
Polarization: *Birefringence*-- Velocities

$$v_o = \frac{c}{n_o}, \quad \frac{1}{v_e} = \frac{\cos^2\theta}{c/n_e} + \frac{\sin^2\theta}{c/n_o},$$

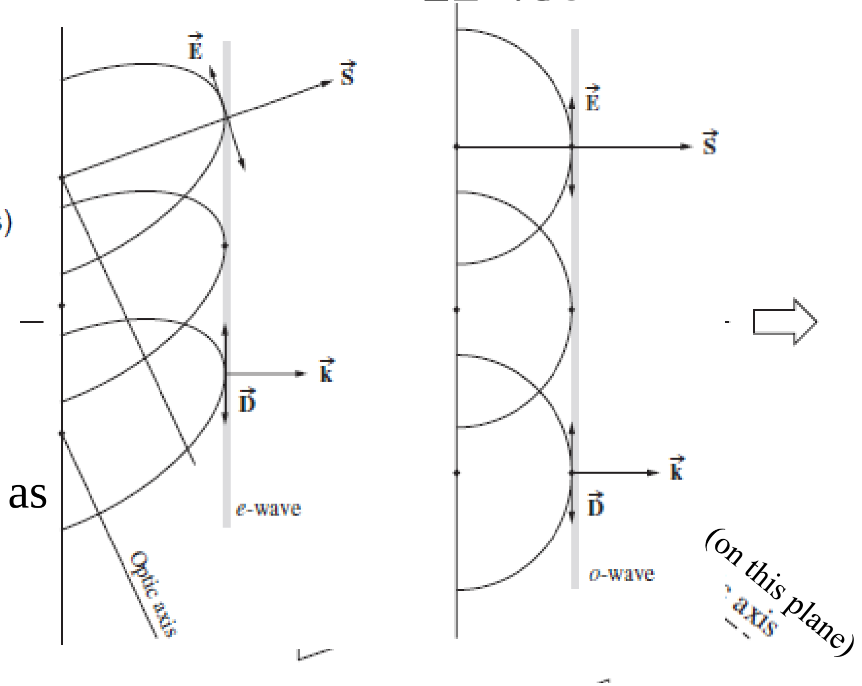
← *EM theory*



Negative crystal

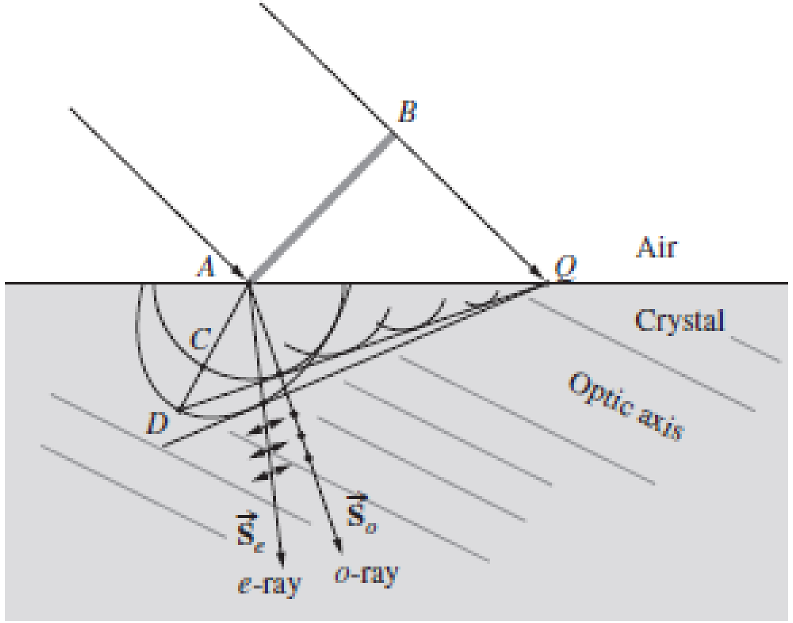


Positive crystal



Huygens's Principle of wavelet:

$\vec{E}$-field is parallel to the principal section, $\vec{E}$ has a component normal to the optic axis, as well as a component parallel to it. Since the medium is birefringent, light of a given frequency polarized parallel to the optic axis propagates with a speed $v_{||}$,
where $v_{||} \neq v_{\perp}$. In particular for calcite (-ve) and λ= 589 nm, **1.486 $v_{||}$ = 1.658 $v_{\perp}$ = c.**
Huygens's wavelets now----- $v_{||} > v_{\perp}$
So the wavelet will elongate in all directions normal to the optic axis. The secondary wavelets associated with the *e*-wave are ellipsoids of revolution about the optic axis with semiminor and semimajor axes $\left(\frac{c}{n_o}\right)t$, and $\left(\frac{c}{n_e}\right)t$. Semi-minor axis is along optic axis.



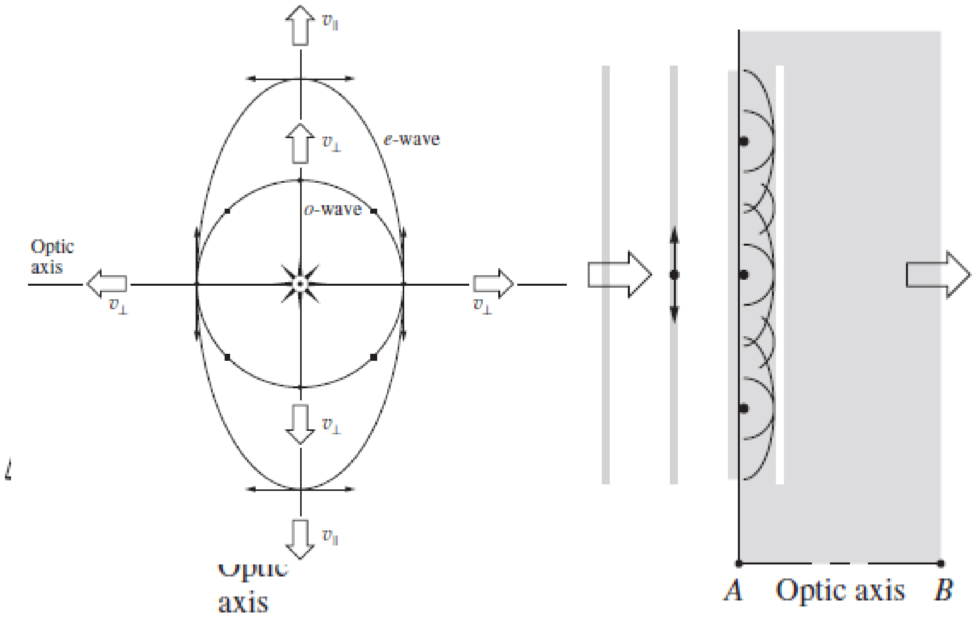
o-ray follows the Snell's Law because it travels at the same speed in every direction as if the medium were isotropic.
e-wave does not travel at the same speed in all directions and so does not generally obey Snell's Law.

Polarization: **Wave Plate (Retarders)**

Causes a relative phase difference between two constituent coherent polarization states by a predetermined amount.

Enable to convert any given polarization state into any other state (even circular and elliptic polarization)

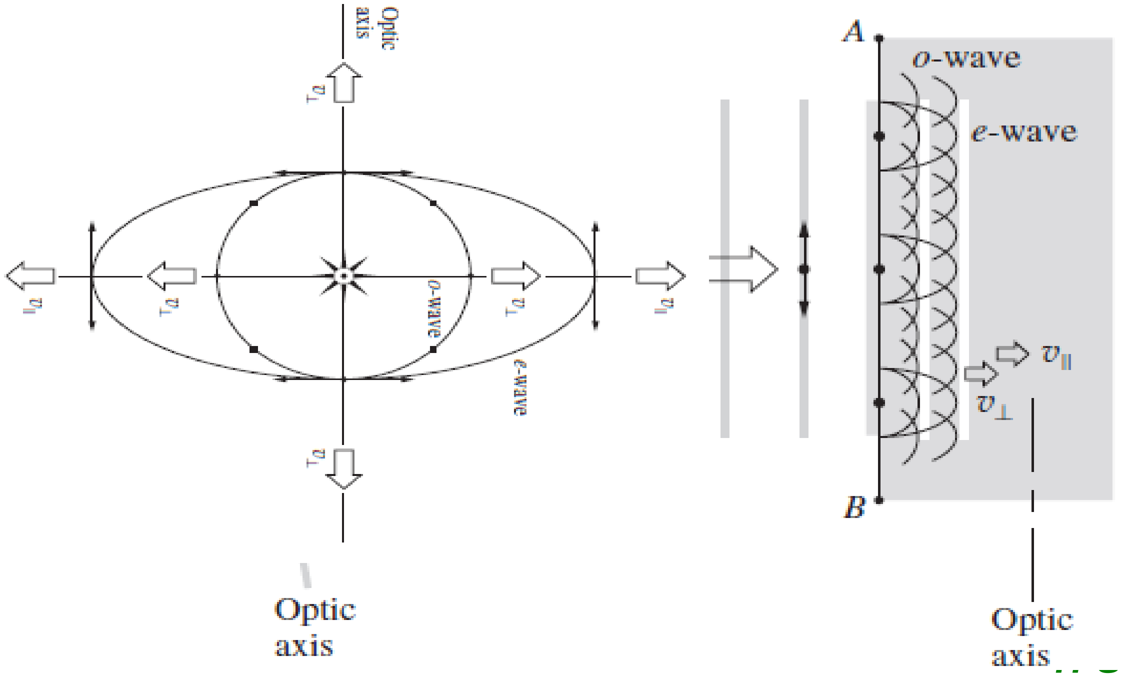
We can cut and polish a calcite crystal so that its optic axis will be normal to both the front and back surfaces. A normally incident plane wave can only have its $\vec{E}$ field perpendicular to OA.



Now, the direction of the optic axis is arranged to be parallel to the front and back surfaces. $\vec{E}$ -field of an incident **monochromatic** plane wave has components parallel and perpendicular to the OA, two separate plane waves will propagate through the crystal. Since $\boldsymbol{v_{||} > v_{\perp}}$ $(n_o > n_e; -ve\ crystal)$, and **the *e*-wave will move across the specimen more rapidly than the *o*-wave**. After traversing a plate of thickness *d*, the resultant EM wave is the superposition of the *e*- and *o*-waves, which now have a relative phase difference of

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right) \times optical\ path\ diff$$

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right) d(|n_o - n_e|)$$



Polarization: Wave Plate

$$\vec{E} = \vec{E}_0 \cos(kz - \omega t) \qquad E_x = E_0 \cos\theta \cos(kz - \omega t)$$

$$E_y = E_0 \sin\theta \cos(kz - \omega t)$$

$at\ z = 0$ $\qquad E_x = E_0 \cos\theta \cos(\omega t)$

$$E_y = E_0 \sin\theta \cos(\omega t)$$

$at\ z = d$ $\qquad E_x = E_0 \cos\theta \cos(k n_e d - \omega t)$

$$E_y = E_0 \sin\theta \cos(k n_o d - \omega t)$$

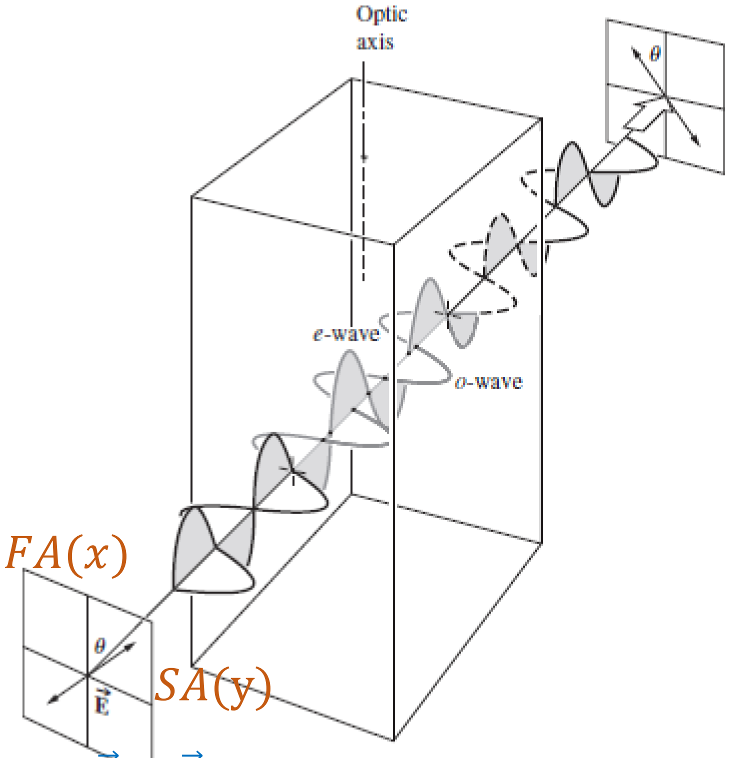
Which can be written as $\qquad E_x = E_0 \cos\theta \cos(\omega t - \Delta\phi)$

$$E_y = E_0 \sin\theta \cos(\omega t)$$

$$\boxed{\Delta\phi = \left(\frac{2\pi}{\lambda}\right) d(|n_o - n_e|); \quad a_1 = E_0 \cos\theta; \quad a_2 = E_0 sin\theta}$$

$at\ z > d$ $\qquad E_x = a_1 \cos(kz - \omega t)$

$$E_y = a_2 \cos(kz - \omega t + \Delta\phi)$$



$FA(x)$

$SA(y)$

$\vec{E} = \vec{E}_0 \cos(kz - \omega t)$

$$\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2 - 2\left(\frac{\psi_1}{a_1}\right)\left(\frac{\psi_2}{a_2}\right) cos\psi = \sin^2\psi$$

$$tan2\alpha = \frac{2a_1 a_2}{a_1^2 - a_2^2} cos\psi$$

$$\boxed{\theta = \pi/4} \qquad \boxed{a_1 = a_2 = E_0/\sqrt{2}}$$

$$\psi = \Delta\phi = \pi/2 \Rightarrow \psi_2^2 + \psi_1^2 = a^2$$

$$\boxed{\alpha = \pi/4} \qquad \boxed{a_1 = a_2 = E_0}$$

$$\pi \Rightarrow (\psi_1 + \psi_2)\text{^}2 = 0 \Rightarrow \psi_2 = -\psi_1$$

$$\boxed{E_x^2 + E_y^2 - 2E_x E_y cos\psi = E_0^2 \sin^2\psi}$$

$$2\pi \Rightarrow (\psi_1 - \psi_2)\text{^}2 = 0 \Rightarrow \psi_2 = \psi_1$$

Polarization: Wave Plate

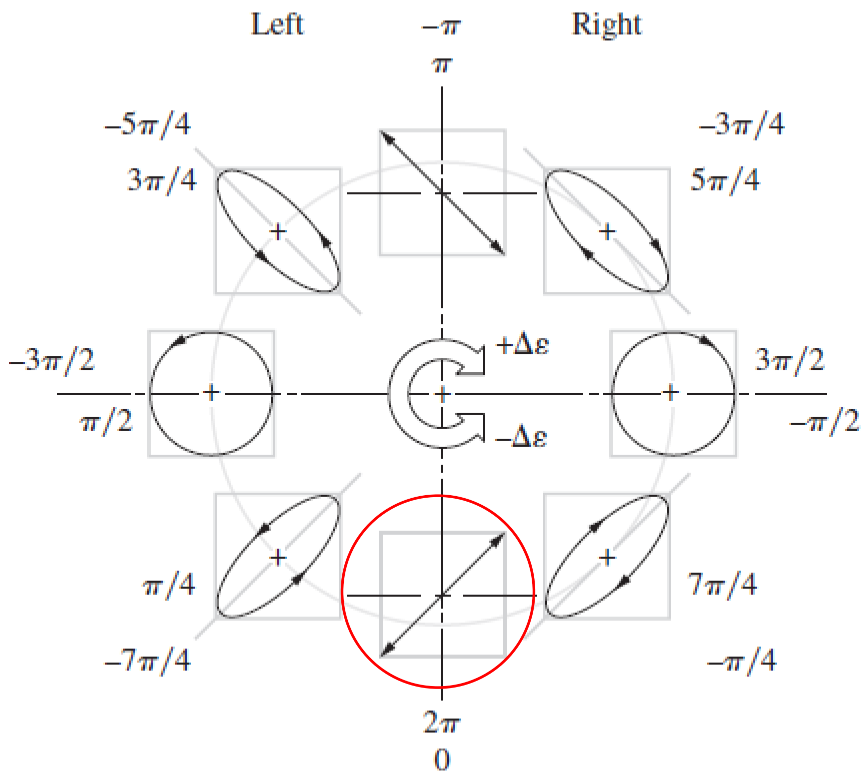
$$\vec{E} = \hat{i}\,\vec{E}_0\cos(kz - \omega t) + \hat{j}\,\vec{E}_0\cos(kz - \omega t + \Delta\phi)$$

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right) d(|n_o - n_e|)$$

The Full-Wave Plate

If $\Delta\phi$ is equal to 2π , the ***relative retardation*** is one wavelength

If such a device is placed at some arbitrary orientation between crossed linear polarizers, all the light entering it (in this case let it be white light) will be linear. Only the one wavelength that satisfies with $\Delta\phi = 2\pi$ will pass through the retarder unaffected, thereafter to be absorbed in the analyzer. All other wavelengths will undergo some retardance and will accordingly emerge from the wave plate as various forms of elliptical light. Some portion of this light will proceed through the analyzer, finally emerging as the complementary color to that which was extinguished.



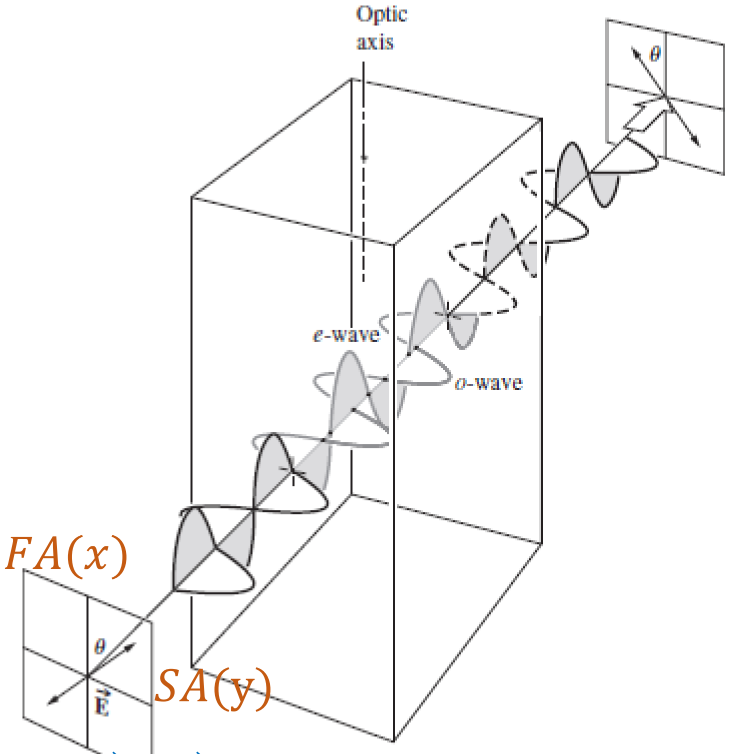
If, instead, the analyzer is positioned with its transmission axis Parallel to the transmission axis of the first polarizer, with the full- wave plate between them, the system acts as a filter. Stacking several such arrangements produces a narrow-wavelength filter.

The full-wave retarder is often used to eliminate unwanted changes in the polarization state of light passing through an Optical system ----- linear light reflected from a metal surfaced mirror will have phase shifts introduced that cause it to emerge as elliptical light. This can be corrected by passing the beam through a full-wave plate

Polarization: **Wave Plate**

The Half-Wave Plate

A retardation plate that introduces a relative phase difference $\Delta\phi$ of π radians, between the *o*- and *e*-waves is known as a **half-wave plate** or **half-wave retarder**.



$\vec{E} = \vec{E}_0\cos(kz - \omega t)$

When the waves emerge from the plate, there will be a relative phase shift of $\lambda/2$ (π radians), with the effect that $\vec{E}$ will have rotated through 2θ.

A half-wave plate will similarly flip elliptical light. In addition, it will invert the handedness of circular or elliptical light, changing right to left and vice versa.

$$E_x = E_0\cos\theta\cos(kz - \omega t)$$

$$E_y = E_0\sin\theta\cos(kz - \omega t + \Delta\phi)$$

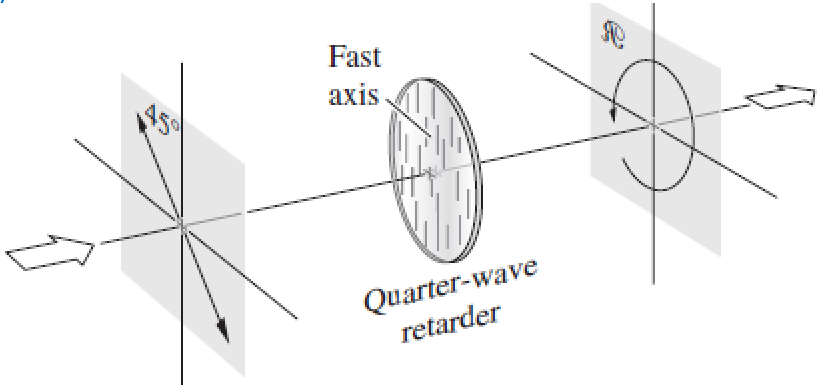
$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)d(|n_o - n_e|) = \pi$$

$\boxed{E_x^2 + E_y^2 - 2E_xE_y\cos\theta = E_0^2\sin^2\theta}$ $\boxed{\alpha = \pi/4}$ $\pi \Rightarrow (E_x + E_y)^2 = 0 \Rightarrow E_x = -E_y$

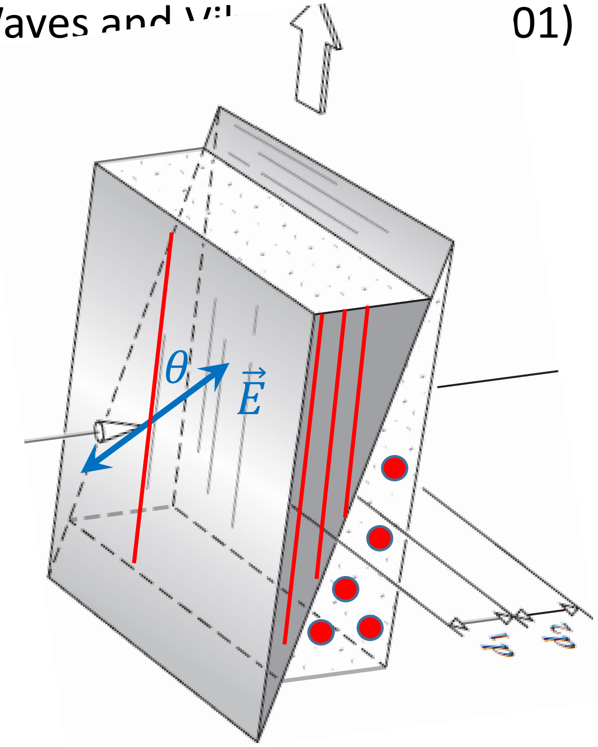
The Quarter-Wave Plate

A retardation plate that introduces a relative phase difference $\Delta\phi$ of $\pi/2$ radians, between the *o*- and *e*-waves is known as a **quarter-wave plate** or **quarter-wave retarder**.

$$\pi/2 \Rightarrow \psi_2^2 + \psi_1^2 = a^2$$



Polarization: **Compensators**



Compensator: *an optical device that is capable of setting in a controllable retardance on a wave*

Babinet compensator: consists of two independent calcite or quartz, wedges whose optic axes are indicated by the red lines and dots in the figure.
in the bottom wedge. The wedges are very thin, so the separation between o and e rays are negligible. In quartz, e-ray moves slower than o-ray ($n_e > n_o$).

$$\Delta\phi_1 = \left(\frac{2\pi}{\lambda}\right) d_1(n_e - n_o)$$

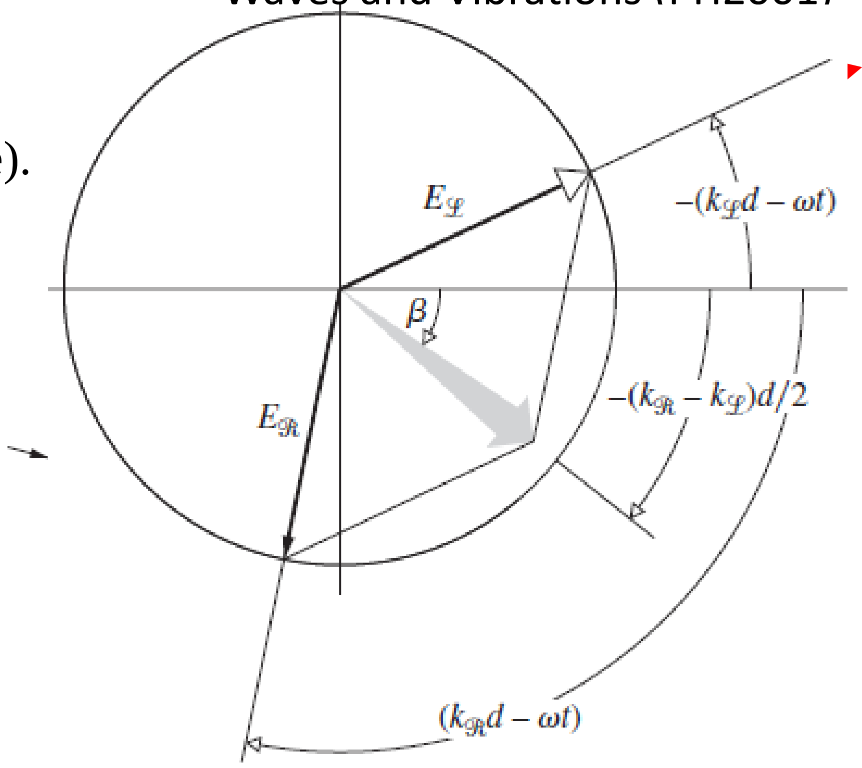
The *o*- and *e*-rays in the upper wedge become the *e*- and *o*-rays, respectively,

$$\Delta\phi_2 = \left(\frac{2\pi}{\lambda}\right) d_2(n_o - n_e)$$

Total Phase difference, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(d_1 - d_2)(|n_e - n_o|)$

Polarization: Optical Activity

The plane of vibration of a beam of linearly polarized light undergoes a continuous rotation as it propagates along the optic axis of some materials (quartz plate, turpentine).



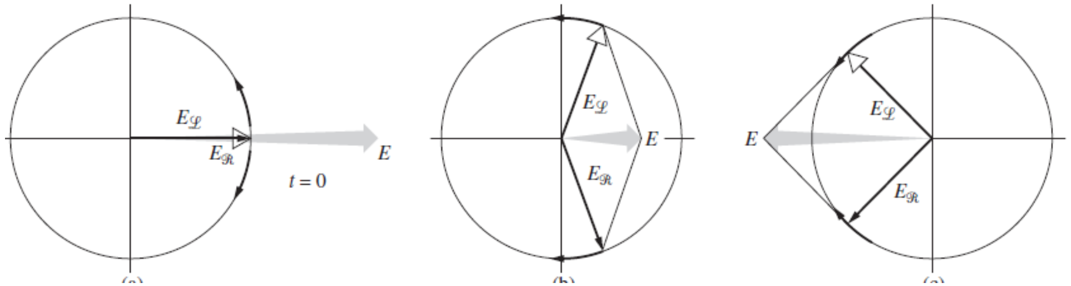
Material that causes the $\vec{E}$-field of an incident linear plane wave to appear to rotate is said to be *optically active*

Dextrorotatory, or *d-rotatory* (right) *substance*: causes the rotation of the plane-of-vibration clockwise, Alternatively, *Levorotatory*, or *l-rotatory* (left) *substance*: anticlockwise rotation.

Fresnel's Description: linear wave can be represented as a superposition of R- and L-states (RCP, LCP). An active material shows *circular birefringence*; that is, it possesses two indices of refraction, n_R and n_L. In traversing an optically active specimen, the two circular waves would get out-of-phase, and the resultant linear wave would appear to have rotated. $\vec{E}=\vec{E}_0\cos(kz-\omega t)$

$$\vec{E}=\vec{E}_R+\vec{E}_L=\hat{\imath}\,(E_0/2)cos(k_Rz-\omega t)+\hat{\jmath}\,(E_0/2)sin(k_Rz-\omega t)$$
$$+\hat{\imath}\,(E_0/2)cos(k_Lz-\omega t)-\hat{\jmath}\,(E_0/2)sin(k_Lz-\omega t)$$

$$\vec{E}=E_0\cos\left(\frac{k_R+k_L}{2}z-\omega t\right)\left[\hat{\imath}\cos\left(\frac{k_R-k_L}{2}z\right)+\hat{\jmath}\sin\left(\frac{k_R-k_L}{2}z\right)\right]$$



$at\ z=0,\ \vec{E}=\hat{\imath}E_0\cos(\omega t)$

$at\ z=d,\ E_R=\frac{E_0}{2}e^{i(k_Rd-\omega t)},$

$E_L=\frac{E_0}{2}e^{-i(k_Ld-\omega t)}.$

$$\beta=-\frac{k_R-k_L}{2}d$$

$$\beta=\frac{\pi}{\lambda}d(n_L-n_R)$$

Moreover, if $n_R>n_L$ $(k_R>k_L)$ will rotate anticlockwise, whereas if $k_R<k_L$, the rotation is clockwise (looking toward the source). Traditionally, the angle β through which $\vec{E}$ rotates is defined as positive when it is clockwise.

Applications of Optical Rotation

- Optical rotation is a function of time and it is used to determine kinetic reactions.
- this helps in analyzing molecular structure
- If the specific rotation of a sample is known its concentration in the solution can be estimated.
- The technique may be extended to the determination of optical substances in the presence of optically inactive species.

Polarization: Electro-optic effect

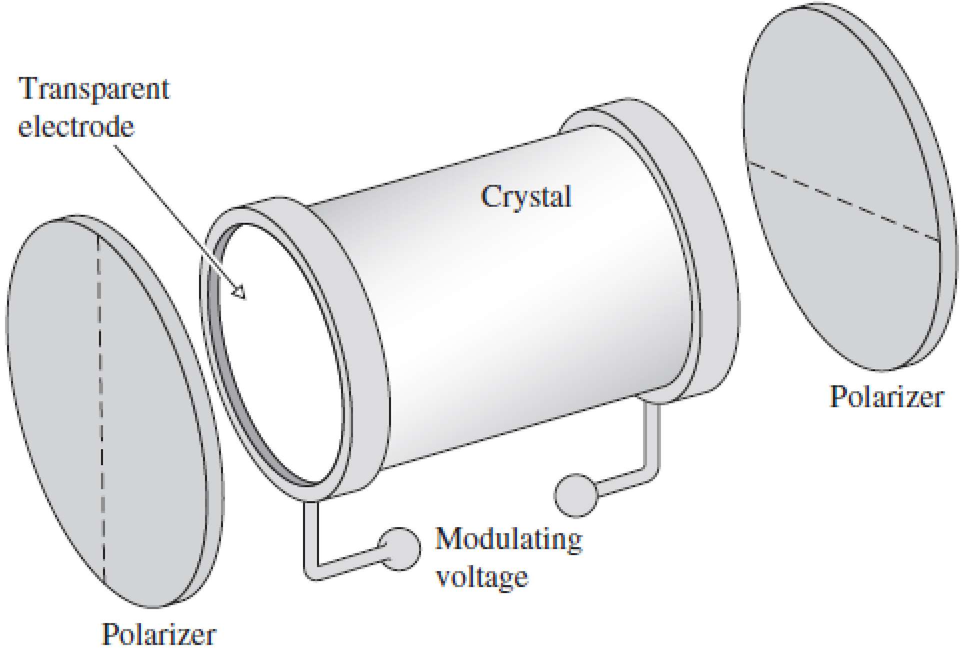
Kerr Effect:

Isotropic transparent substance becomes birefringent when placed in an electric field $\vec{E}$.

Pockels Effect: The Pockels effect is the linear electro-optic effect, where the refractive index of a medium is modified in proportion to the applied electric field.
This effect can occur only in non-centrosymmetric materials. The most important materials of this type are crystal materials such as lithium niobate (LiNbO3), lithium tantalate (LiTaO3), potassium di-deuterium phosphate (KD*P), β-barium borate (BBO), potassium titanium oxide phosphate (KTP), and compound semiconductors such as gallium arsenide (GaAs) and indium phosphide (InP).

Electro-optic Constants (Room Temp, $\lambda_0 = 546.1nm$)

Material	r_{63} (units of 10^{12} m/V)	n_o (approx.)	$V_{\lambda/2}$ (in kV)
ADP (NH4H2PO4)	8.5	1.52	9.2
KDP (KH2PO4)	10.6	1.51	7.6
KDA (KH2AsO4)	13.0	1.57	6.2
KD*P (KD2PO4)	23.3	1.52	3.4



The crystal itself is generally uniaxial in the absence of an applied field, and it is aligned such that its optic axis is along the beam's propagation direction. For such an arrangement the retardance,

$$\Delta\phi = \frac{2\pi n_o^3 r_{63} V}{\lambda} = \frac{\pi V}{V_{\lambda/2}}$$

The half-wave voltage corresponds to a value of $\Delta\phi = \pi$

where r_{63} is the *electro-optic constant* in m/V, n_o is the *ordinary* index of refraction, V is the potential difference in volts, and λ_0 is the vacuum wavelength in meters

Polarization: Electro-optic effect

Kerr Effect:

Isotropic transparent substance becomes birefringent when placed in an electric field $\vec{E}$.

Their difference, n, is the birefringence, and it is found to be

$$\Delta n = |n_{\parallel} - n_{\perp}| = \lambda_0 K E^2$$

where K is the ***Kerr constant***. When K is positive, Δn which can be thought of as $n_{\parallel} - n_{\perp}$, is positive, and the substance behaves like a positive uniaxial crystal.

If the plates functioning as the electrodes have an effective length of l cm and are separated by a distance d, and potential V the retardation is given by

$$\Delta\phi = \frac{2\pi K l V^2}{d^2}$$

Thus a nitrobenzene cell in which $d = 1cm$ and l is several centimeters will require a rather large $V = 3 \times 10^4 V$, in order to respond as a half-wave plate. This is a characteristic quantity known as the *half-wave voltage*, $V_{\lambda/2}$.

- They serve as shutters in high-speed photography
- *Q*-switches in pulsed laser systems

Kerr Constants for Some Selected Liquids (20°C, $\lambda_0 = 589.3$ nm)

Material	K(units of 10^{-7}cm statvolt^{-2})
Benzene C_6H_6	0.6
Carbon disulfide CS_2	3.2
Chloroform $CHCl_3$	-3.5
Water H_2O	4.7
Nitrotoluene $C_5H_7NO_2$	123
Nitrobenzene $C_6H_5NO_2$	220