Engineering Optics

Lecture 14

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by

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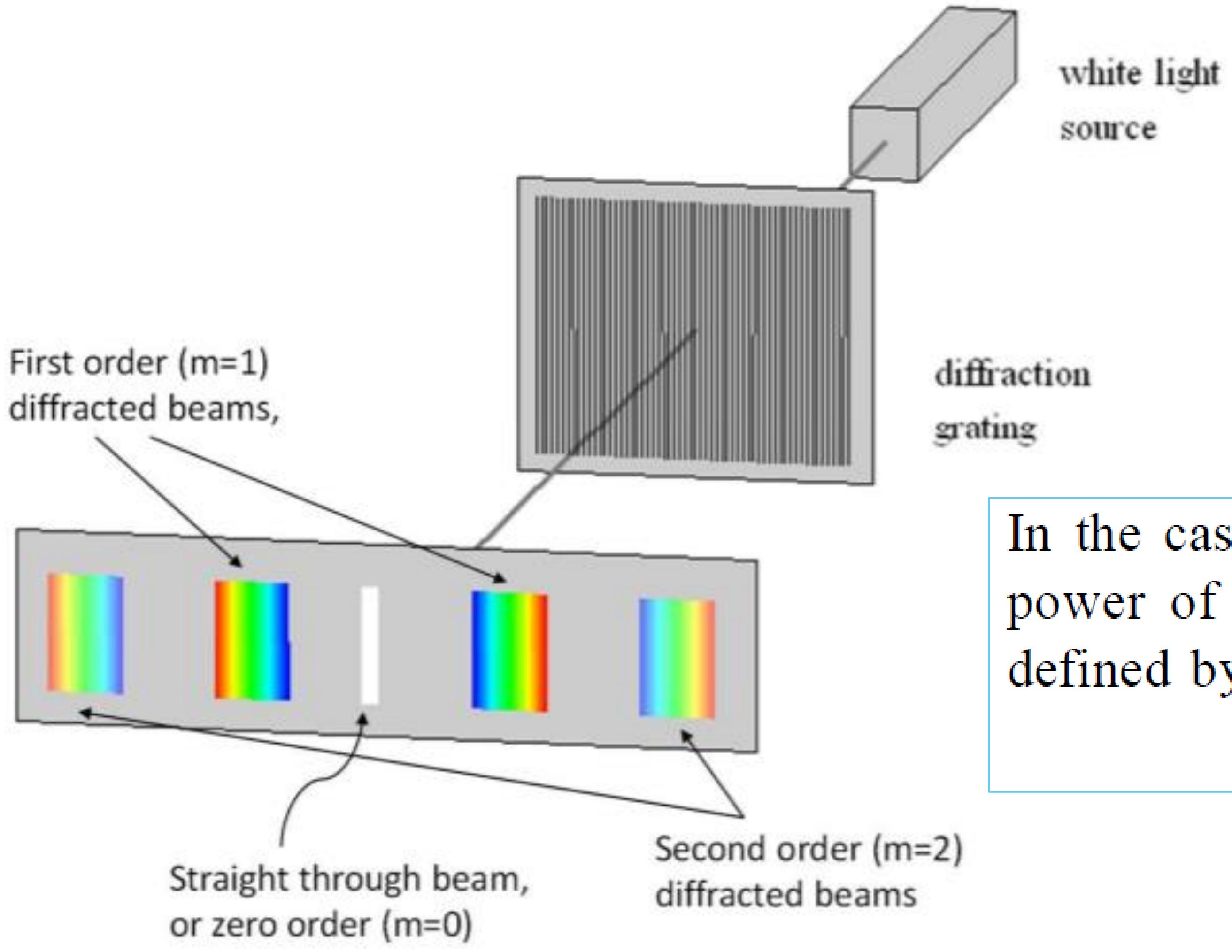
Grating spectrum



$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots$$

Grating element = 1/(no. of lines/cm)

Grating constant = *b* + *d*

In the case of a grating, the resolving power refers to the power of distinguishing two nearby spectral lines and is defined by the

$$R = \frac{\lambda}{\Delta\lambda} = \text{m N}$$

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https://www.physics.purdue.edu/irnanodev/docs/outreach/Diffraction%20grating%20experiments.pdf

Problem-1

We wish to resolve the two bright yellow sodium lines (589.592 3 nm and 588.995 3 nm) in the second-order spectrum produced by a transmission grating. How many slits or grooves must the grating possess at minimum?

Answer-1

SOLUTION The resolving power of the grating is $\lambda/(\Delta\lambda)_{\text{min}}$, where λ is the mean wavelength, or $\frac{1}{2}(589.5923 + 588.9953)$ nm $= 589.2938$ nm.

$(\Delta\lambda)_{\text{min}} = (589.5923 - 588.9953)\,\text{nm} = 0.597\,\text{nm}.$
with $m = 2$,

$$\frac{\lambda}{(\Delta\lambda)_{\text{min}}} = mN$$

and

$$N = \frac{589.2938\ \text{nm}}{2(0.597\,\text{nm})}$$

$$N = 493.5$$

To see the two lines we need a grating with at least 494 slits.

Fresnel Diffraction

- Either the source or the screen (or both) is at a finite distance from the diffracting aperture.

[Fraunhofer class of diffraction → wave incident is a plane wave and the diffraction pattern is observed on the focal plane of a convex lens → screen is far away from the aperture. diffracting system was relatively small, and the point of observation was very distant.]

- But we are now going to deal with the near-field region, which extends right up to the diffracting element itself.
- Huygens–Fresnel principle : Each point on a wave front is a source of secondary

disturbance, and the secondary wavelets emanating from different points mutually interfere

1. Fresnel Half-period Zones
2. Zone plates and applications

Fresnel half-period zones

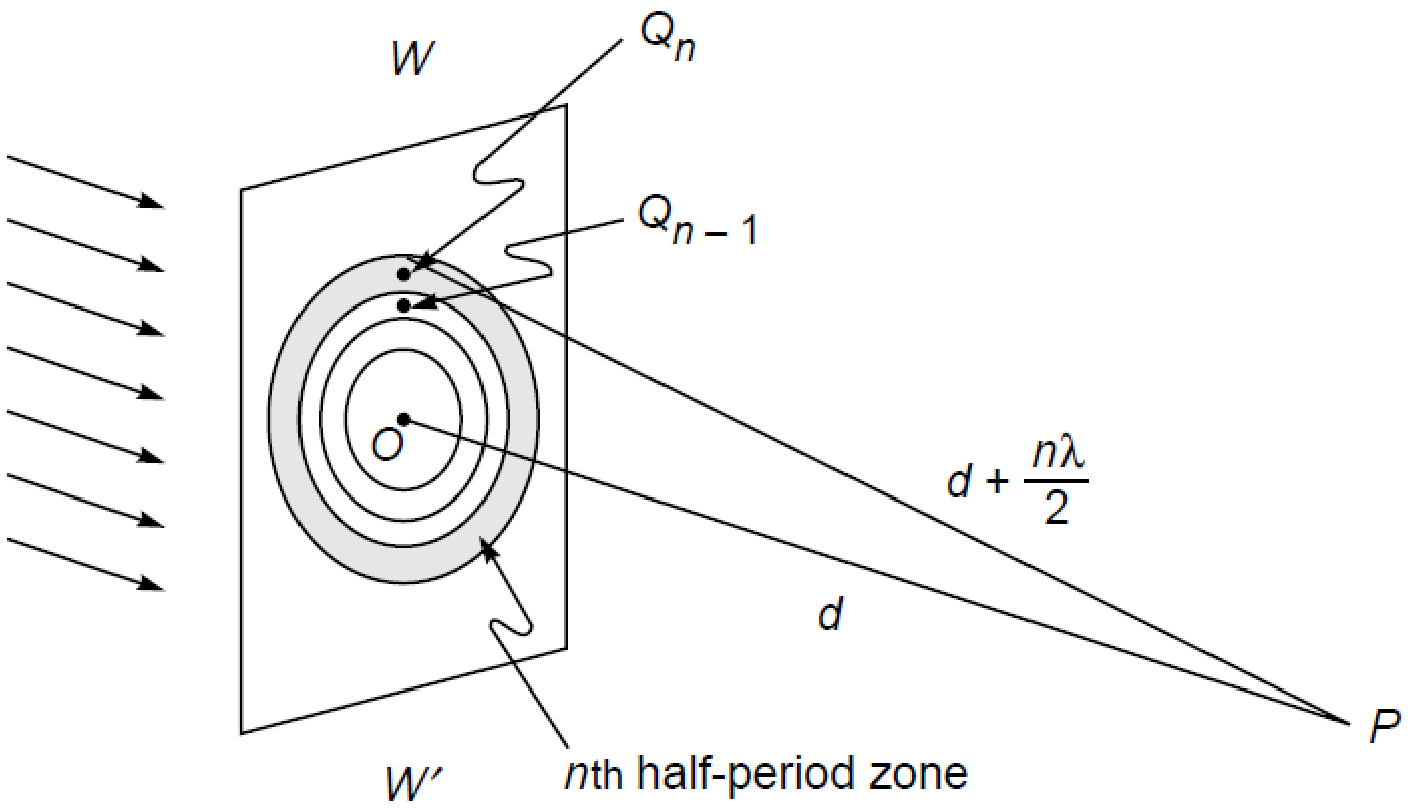

Fig. 20.2 Construction of Fresnel half-period zones.

From point P a perpendicular PO on the wave front. $PO = d$,

With point P as center draw spheres of radii
$d + \lambda/2, d + 2\lambda/2, d + 3\lambda/2, \ldots,$

these spheres will intersect WW' in circles

Fresnel half-period zones

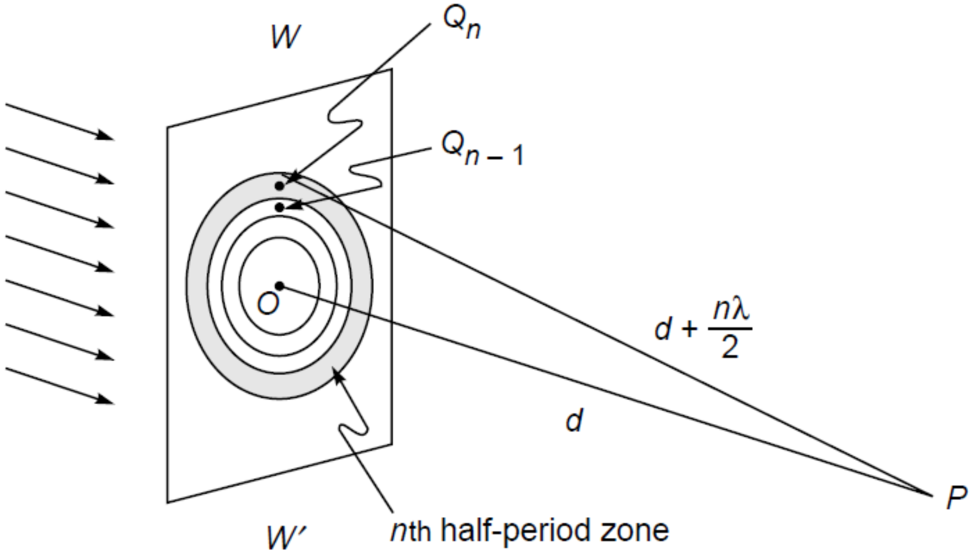

Fig. 20.2 Construction of Fresnel half-period zones.

$$r_n = \left[\left(d + n\frac{\lambda}{2}\right)^2 - d^2\right]^{1/2}$$

$$= \sqrt{n\lambda d}\left(1 + \frac{n\lambda}{4d}\right)^{1/2}$$

or

$$r_n \approx \sqrt{n\lambda d}$$

where we have assumed $d >>> \lambda$; this is indeed justified for practical systems using visible light. Of course, we are assuming that n is not a very large number. The annular region between the nth circle and $(n - 1)$st circle is known as the nth half-period zone;

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Question

- What is the area of the *n* th zone?

Area of a half-period zone

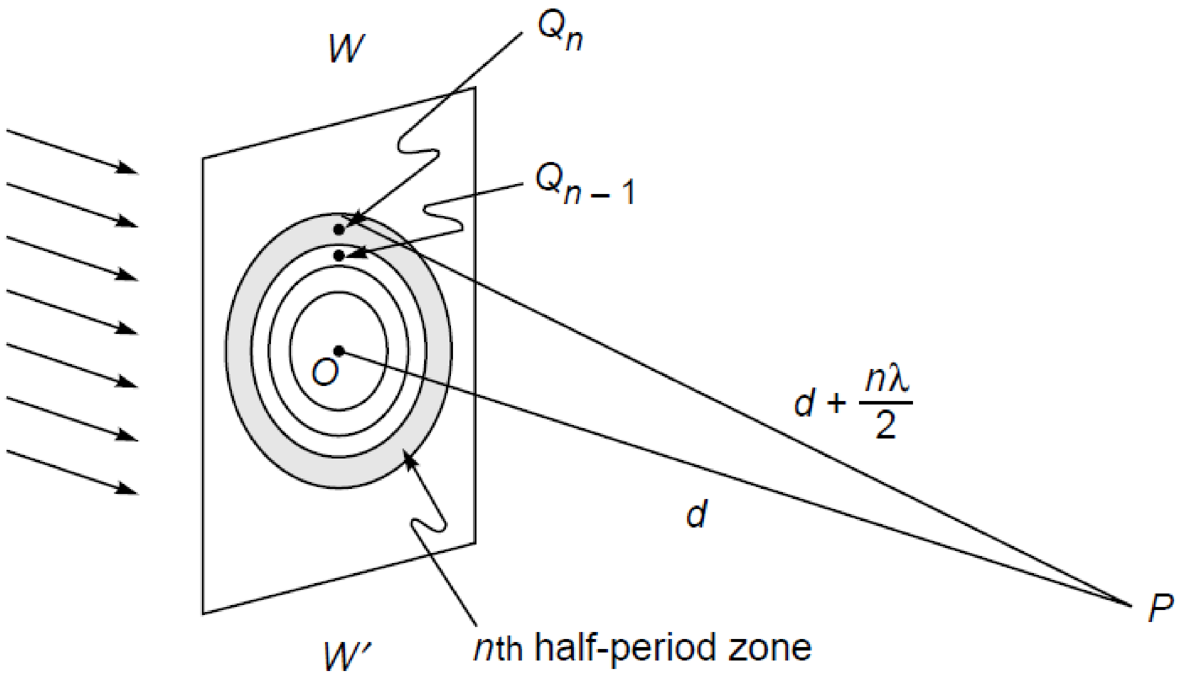


Fig. 20.2 Construction of Fresnel half-period zones.

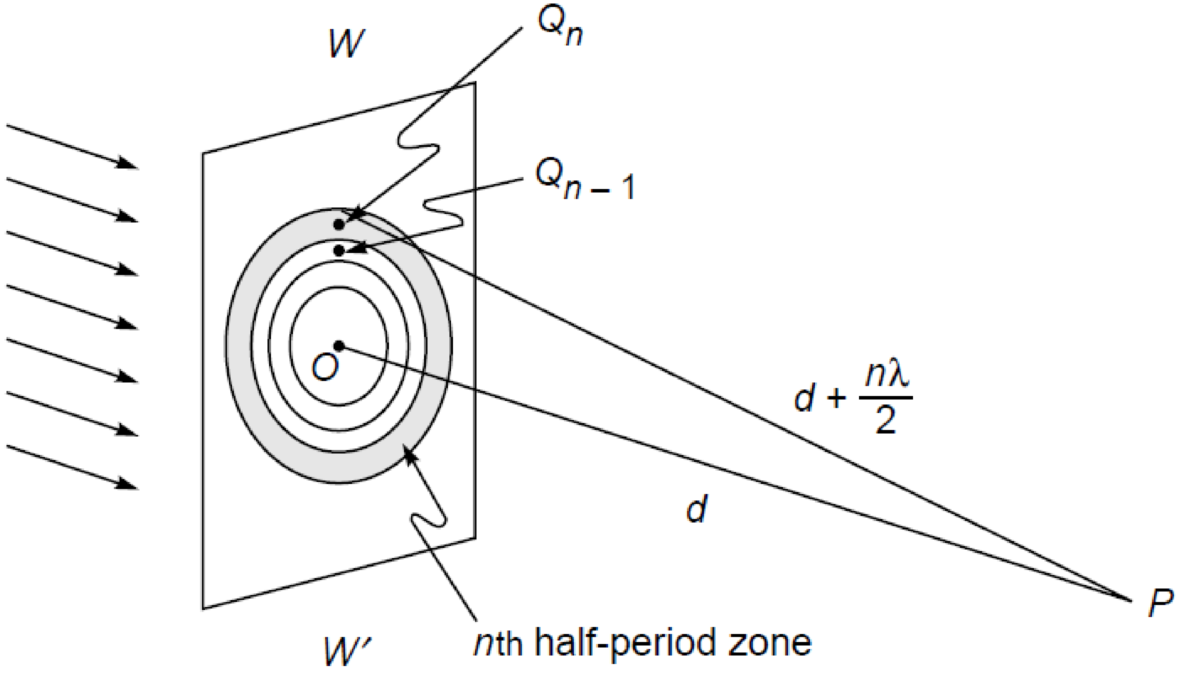
area of the nth half-period zone is given by

$$A_n = \pi r_n^2 - \pi r_{n-1}^2$$

$$\approx \pi [n\lambda d - (n-1)\lambda d] = \pi\lambda d$$

Net amplitude '*u*' at P due to all the zones?

Area of a half-period zone



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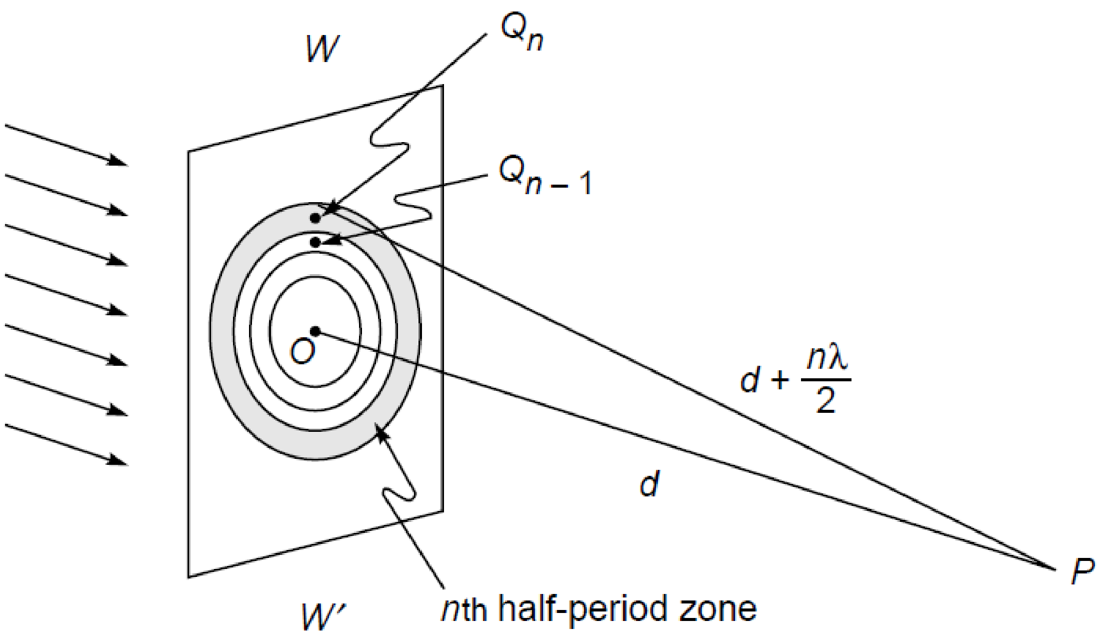
Thus, the resultant amplitude at point P can be written as

$$u(P) = u_1 - u_2 + u_3 - u_4 + \cdots + (-1)^{m+1} u_m + \cdots$$

$$Q_nP - Q_{n-1}P = \frac{\lambda}{2}$$

Amplitude at the point *P*

- Amplitude at P $\propto A_n$
- $\propto$ 1/ distance of the zone from *P*



20.2 Construction of Fresnel half-period zones.

$$\boxed{\text{obliquity factor } \tfrac{1}{2}(1 + \cos\chi)}$$

$$u(P) = u_1 - u_2 + u_3 - u_4 + \cdots + (-1)^{m+1}u_m + \cdots$$

$$u(P) = \frac{u_1}{2} + \left[\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right] + \left[\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right] + \cdots$$

$$u(P) \approx \frac{u_1}{2} + \frac{u_m}{2} \quad m \text{ odd}$$

$$u(P) \approx \frac{u_1}{2} - \frac{u_m}{2} \quad m \text{ even}$$

$$u(P) \approx \frac{u_1}{2}$$

implying that *the resultant amplitude produced by the entire wave front is only one-half of the amplitude produced by the first half-period zone.*

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Thank You