Engineering Optics

Lecture 3

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by

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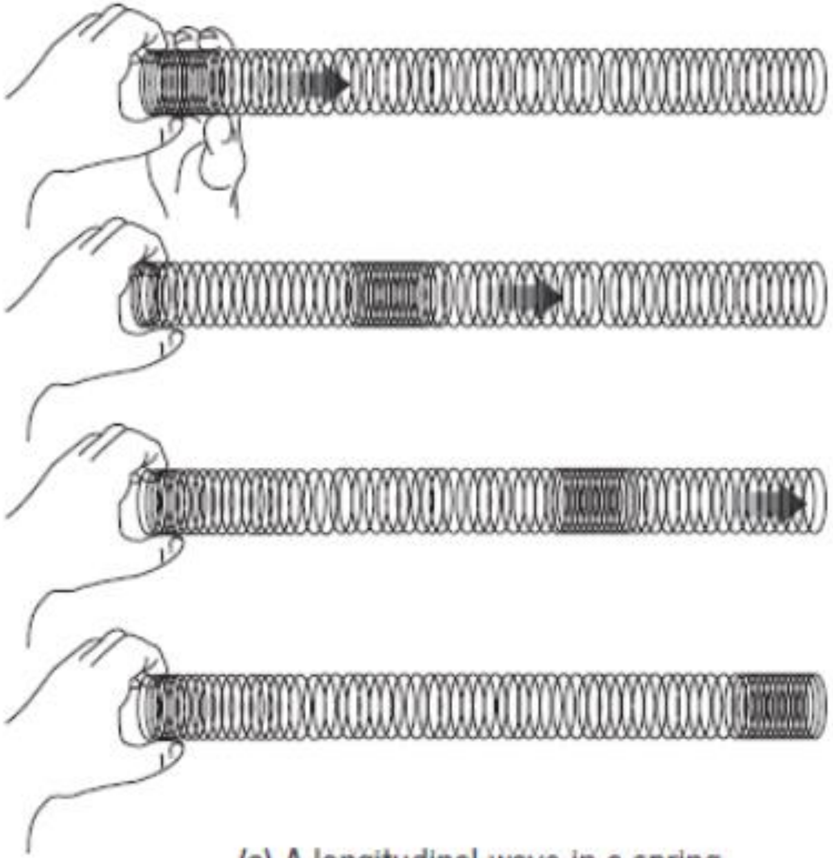
Wave-particle dilemma

- "*Is light a wave phenomenon or a particle phenomenon?*" → at the heart of Optics→ far more complicated.
- **Particle nature:** ball or a pebble and shrink it→ vanishingly small → particle
- pebble interacts with its environment → gravitational field → spreads out into space—an inextricable part of the ball
- Real particles interact via fields → ***the field is the particle and the particle is the field***

Wave-particle dilemma

- **Wave nature:** the essential feature of a wave is its non-localization.
- A classical traveling wave is a self-sustaining disturbance of a medium, which moves through space transporting energy and momentum.
- Conceptually, the classical EM wave → continuous entity→ wave. not particle.
- But in the past century we found that classical formulation of the EM wave good at macroscopic level, particle nature at microscopic level (Einstein)
- Both classical and wave treatment of light uses **mathematical description of waves**.

Longitudinal wave

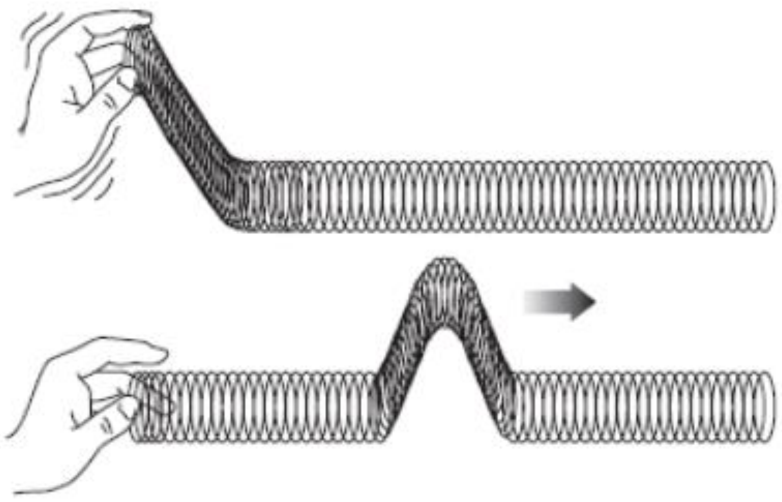


(a) A longitudinal wave in a spring.

The medium is displaced in the direction of motion of the wave

Optics, Hecht

Transverse waves



(b) A transverse wave

Medium is displaced in a direction perpendicular to that of the motion of the wave

In all cases, energy-carrying disturbance advances → individual participating atoms remain in the vicinity of their equilibrium positions:

- the disturbance advances, not the material medium.
- Difference from a stream of particles.
- Leonardo da Vinci → first recognized → wave does not transport the medium → waves propagate at very great speeds.

Optics, Hecht

How to describe a wave?

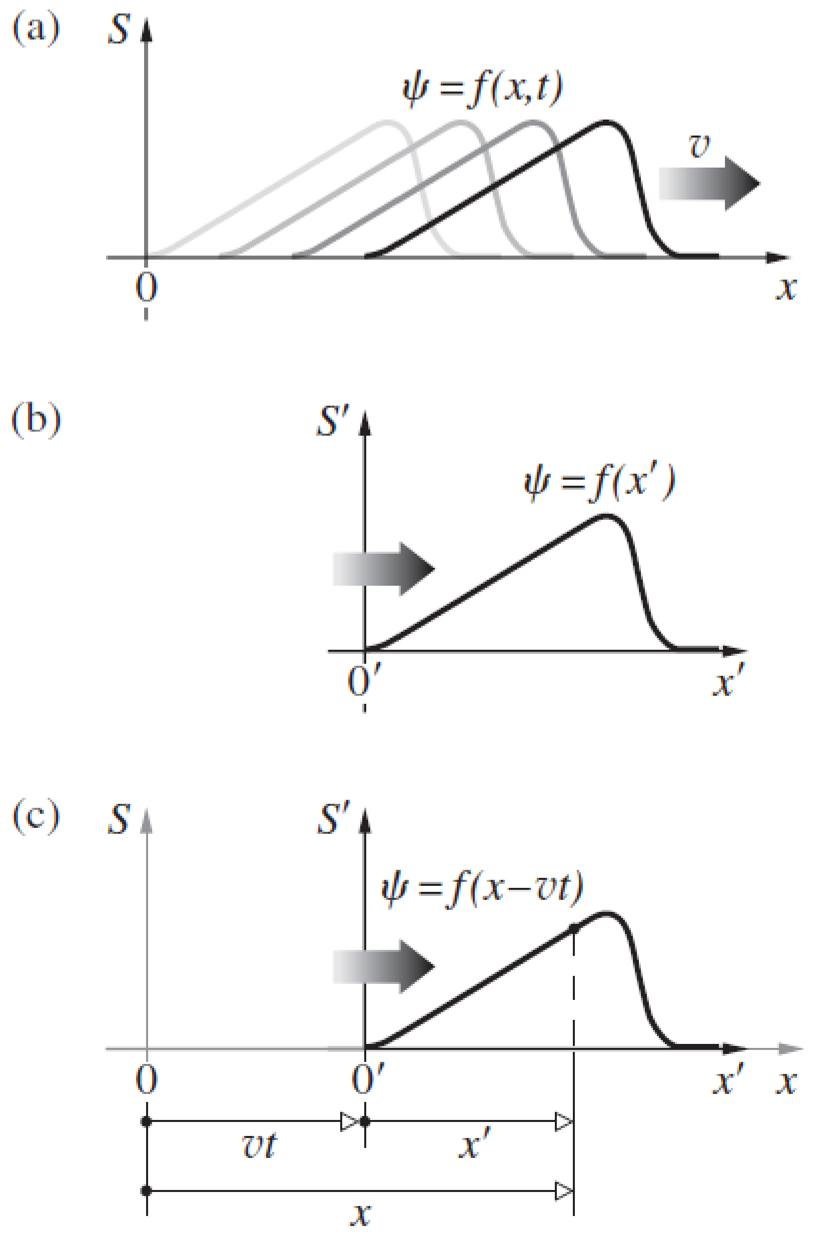
1D wave function

- A moving disturbance $\psi(x, t) = f(x, t)$
- Assume→ wave does not change its shape as it progresses through space.
- After t the pulse has moved vt along x, but in all other respects it remains unaltered.
- introduce a coordinate system S', that travels along with the pulse at the speed v.
- In this system ψ is no longer a function of time →stationary constant profile
- The disturbance looks the same at any t in S' as it did at t = 0 in S (S and S' → a common origin)
- How the observer at S will see the disturbance now? → rewrite disturbance in terms of x

$$\psi = f(x')$$

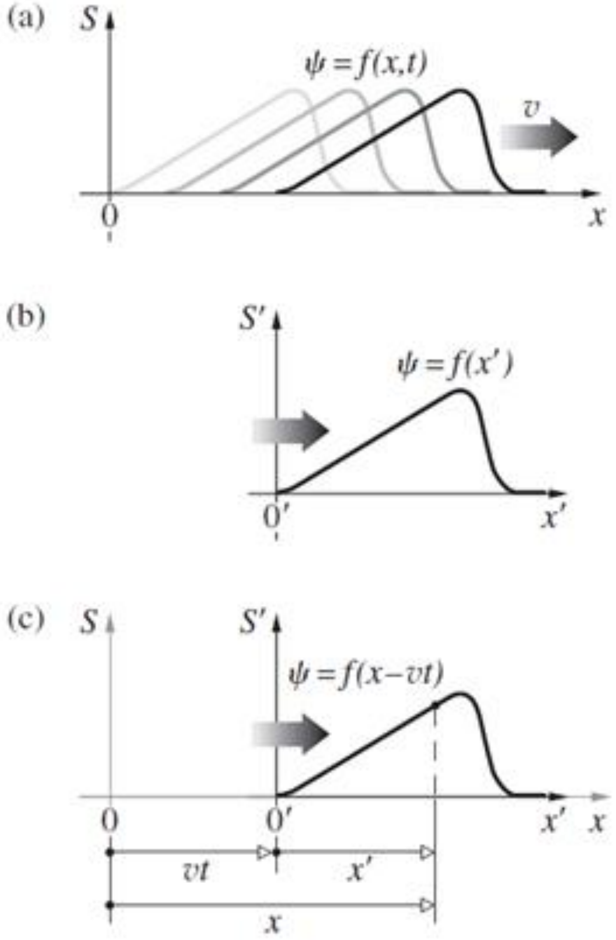
$$x' = x - vt$$

$$\psi(x, t) = f(x - vt)$$

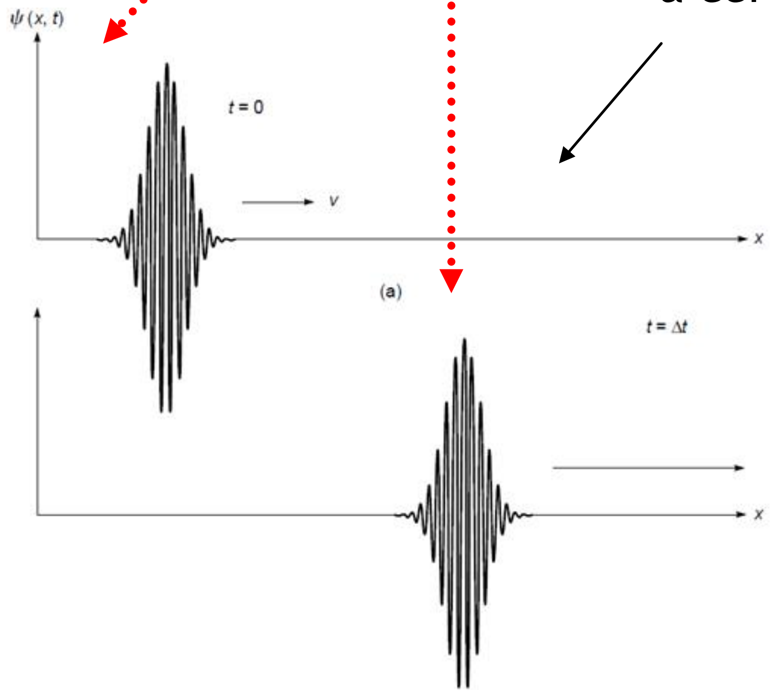


1D Wave

Shape can be found at t=0→ wavefunction



Shapes→ same



disturbance has traveled through a certain distance= vΔt.

if the equation describing the rope at t = 0 is y(x),
then the equation of the curve is y(x – vt), at a later instant t,

which simply implies a shift of the origin by a distance vt.

Similarly, for a disturbance propagating in the –x direction, equation of the curve is y(x+vt)

Optics, Hecht; Ghatak

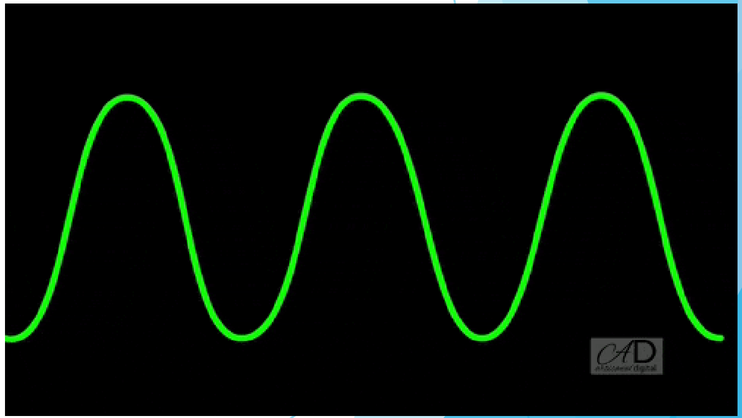
1D Wave continued

- $\psi(x, t) = f(x\text{-}vt)$ → choose the shape → there is a wave moving in the positive x-direction with a speed v.
  - Example: **f(x) = exp(-ax^2)** → which function is this? (how does it look like?)

- ***<u>Differential wave equation</u>***

Important→ different kinds of waves, each described by own $\psi(x)$

→ *all satisfy the same wave equation*



Optics, Hecht; Ghatak

SINUSOIDAL WAVES: FREQUENCY AND WAVELENGTH

$$y(x, t) = a \cos k(x - vt)$$

- **Shape and time profile.**

??

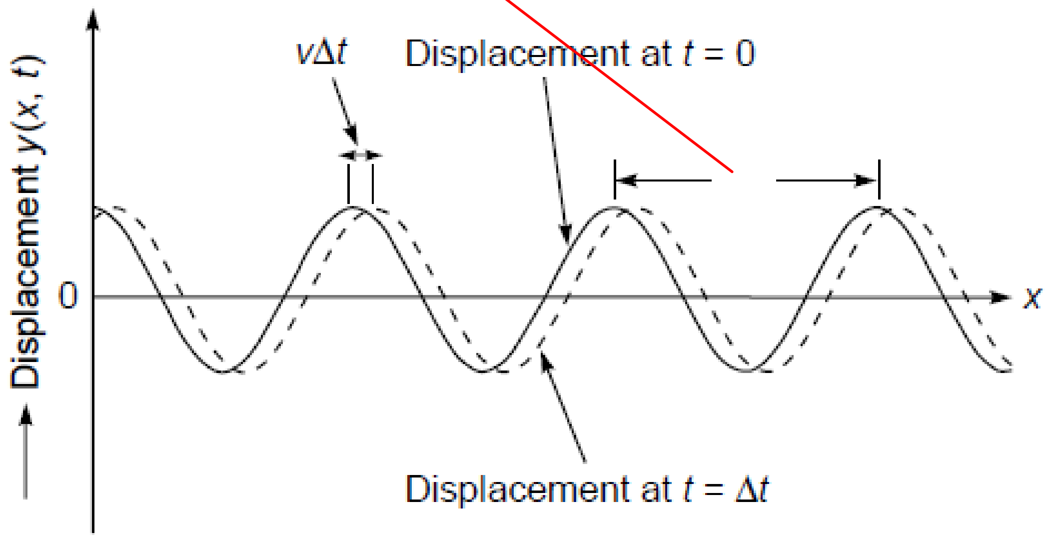


Fig. 11.4 The curves represent the displacement of a string at $t = 0$ and $t = \Delta t$, respectively, when a sinusoidal wave is propagating in the $+x$ direction.

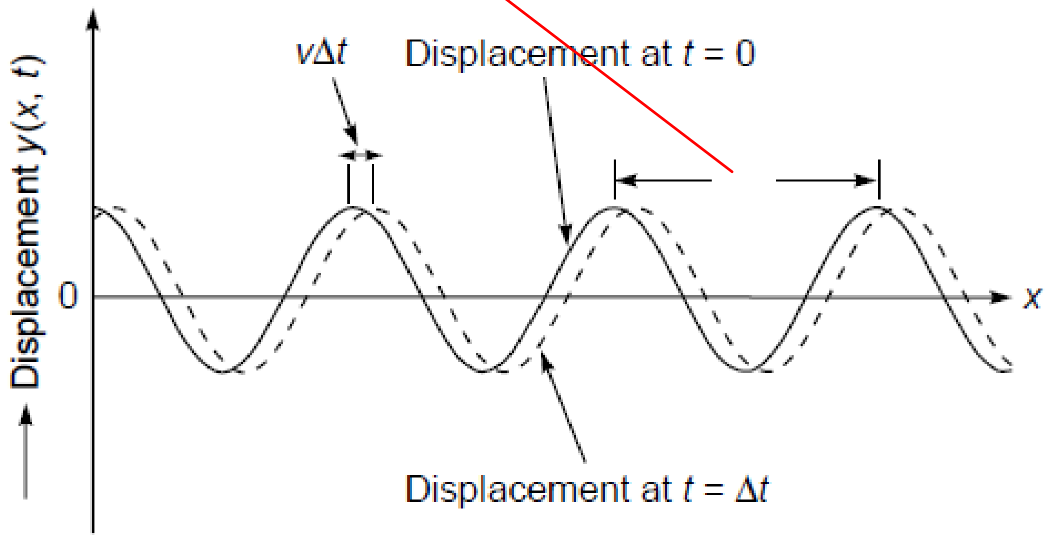
Optics by Ghatak

SINUSOIDAL WAVES: FREQUENCY AND WAVELENGTH

$$y(x, t) = a \cos k(x - vt)$$

- A wave propagating along _?_ direction.
- Can two points separated by a distance have same displacement?

??



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Optics by Ghatak

SINUSOIDAL WAVES: FREQUENCY AND WAVELENGTH

$$y(x, t) = a \cos k(x - vt)$$

- It can be seen from the figure that, at a particular instant, any two points separated by a distance $\lambda \rightarrow$ same displacement
- $\lambda \rightarrow$ wavelength
- maximum displacement of the particle (from its equilibrium position) is ?
- which is known as the ___ of the wave.

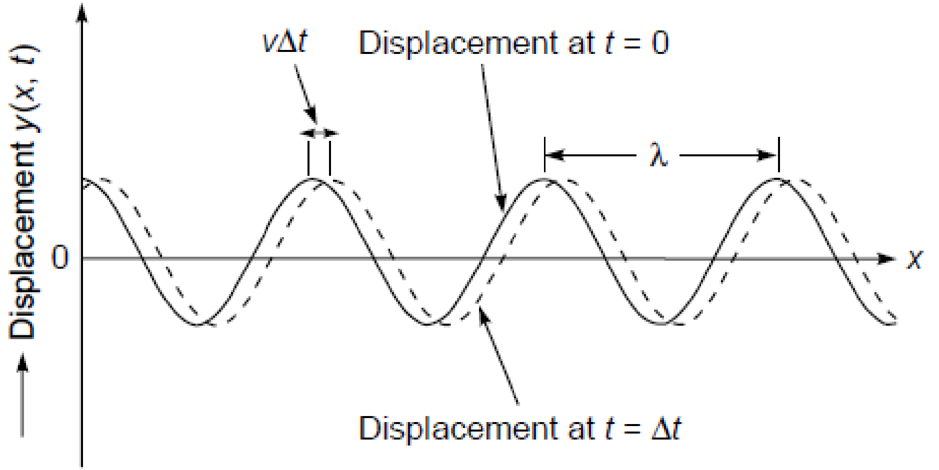

Fig. 11.4 The curves represent the displacement of a string at $t = 0$ and $t = \Delta t$, respectively, when a sinusoidal wave is propagating in the $+x$ direction.

Optics by Ghatak

SINUSOIDAL WAVES: FREQUENCY AND WAVELENGTH

$$y(x, t) = a \cos k(x - vt)$$

- It can be seen from the figure that, at a particular instant, any two points separated by a distance λ → same displacement
- λ → wavelength
- maximum displacement of the particle (from its equilibrium position) is → 'a'
- which is known as the amplitude of the wave.

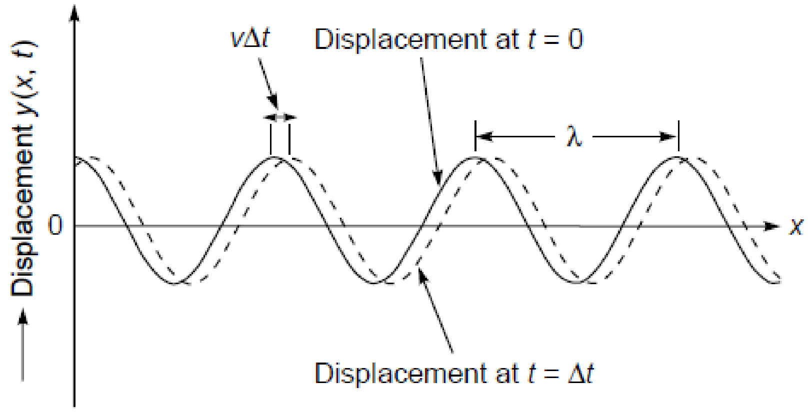

Fig. 11.4 The curves represent the displacement of a string at $t = 0$ and $t = \Delta t$, respectively, when a sinusoidal wave is propagating in the $+x$ direction.

SINUSOIDAL WAVES: Time dependence

$$y(x, t) = a \cos k(x - vt)$$

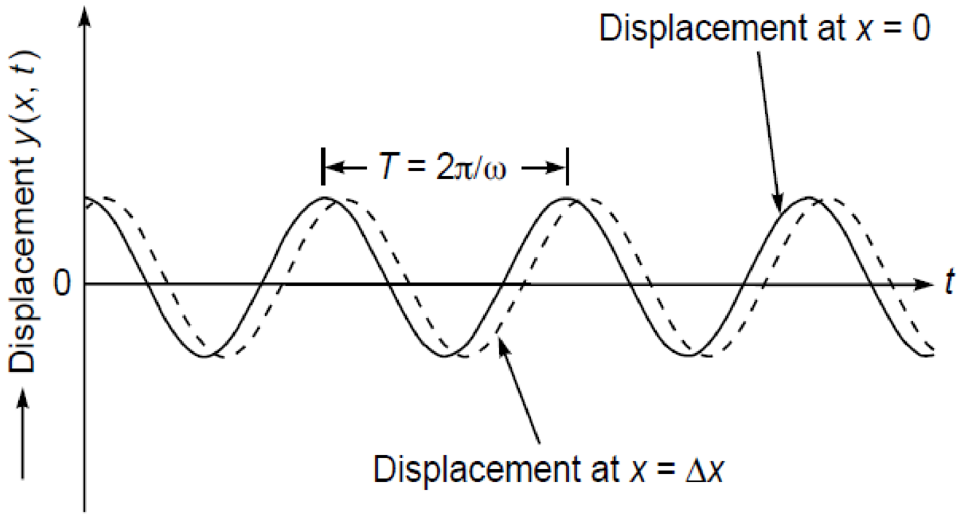

Fig. 11.5 The curves represent the time variation of the displacement of a string at $x = 0$ and $x = \Delta x$, respectively, when a sinusoidal wave is propagating in the +x direction.

Optics by Ghatak

$$y(t) = a \cos \omega t \qquad \text{at } x = 0$$

$$y(t) = a \cos(\omega t - k\Delta x) \qquad \text{at } x = \Delta x$$

where

$$\omega = kv$$

- Corresponding to a particular point, the displacement repeats itself after a time __?
- Called Time period of the wave $T = 2\pi/\omega$
- How is T related to v?
- No. of oscillation a particle carries out in 1s.

1D differential wave Equation

$$\psi(x, t) = f(x')$$
$$x' = x \mp vt,$$

taking the partial derivative w.r.t x, keeping t constant

$$\frac{\partial \psi}{\partial x} = \frac{\partial f}{\partial x}$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x} = \frac{\partial f}{\partial x'} \qquad (1)$$

$$\frac{\partial x'}{\partial x} = \frac{\partial (x \mp vt)}{\partial x} = 1$$

partial derivative w.r.t time and keeping x constant

$$\frac{\partial \psi}{\partial t} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t} = \frac{\partial f}{\partial x'}(\mp v) = \mp v \frac{\partial f}{\partial x'} \qquad (2)$$

combining (1) & (2) $\quad \frac{\partial \psi}{\partial t} = \mp v \frac{\partial \psi}{\partial x} \qquad (3)$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2} \qquad (4)$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\mp v \frac{\partial f}{\partial x'}\right) = \mp v \frac{\partial}{\partial x'}\left(\frac{\partial f}{\partial t}\right) \qquad (5)$$

from (2) : $\frac{\partial \psi}{\partial t} = \frac{\partial f}{\partial t}$

Hence, (5) becomes $\frac{\partial^2 \psi}{\partial t^2} = \mp v \frac{\partial}{\partial x'}\left(\frac{\partial \psi}{\partial t}\right)$

Using (2) again, $\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x'^2}$

Or,

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}}$$

1-D differential wave equation

Optics, Hecht

Harmonic waves

1-D differential wave equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

x → x - vt
What about damping?

Simplest waveform: Sine or Cosine → ***Sinusoidal / harmonic waves***

$$\boxed{\psi(x, t)\big|_{t=0} = \psi(x) = A \sin kx = f(x)}$$

Any wave → superposition of harmonic waves

k **: propagation number** → a +*ve* constant; why do we need *k*?

$|\psi(x)|_{max} = ?$ → *maximum disturbance* → ***amplitude***

Argument of Sine function → *'phase (φ)'*

Reference: Optics *by* Hecht

Thank You