Engineering Optics

Lecture 11

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by

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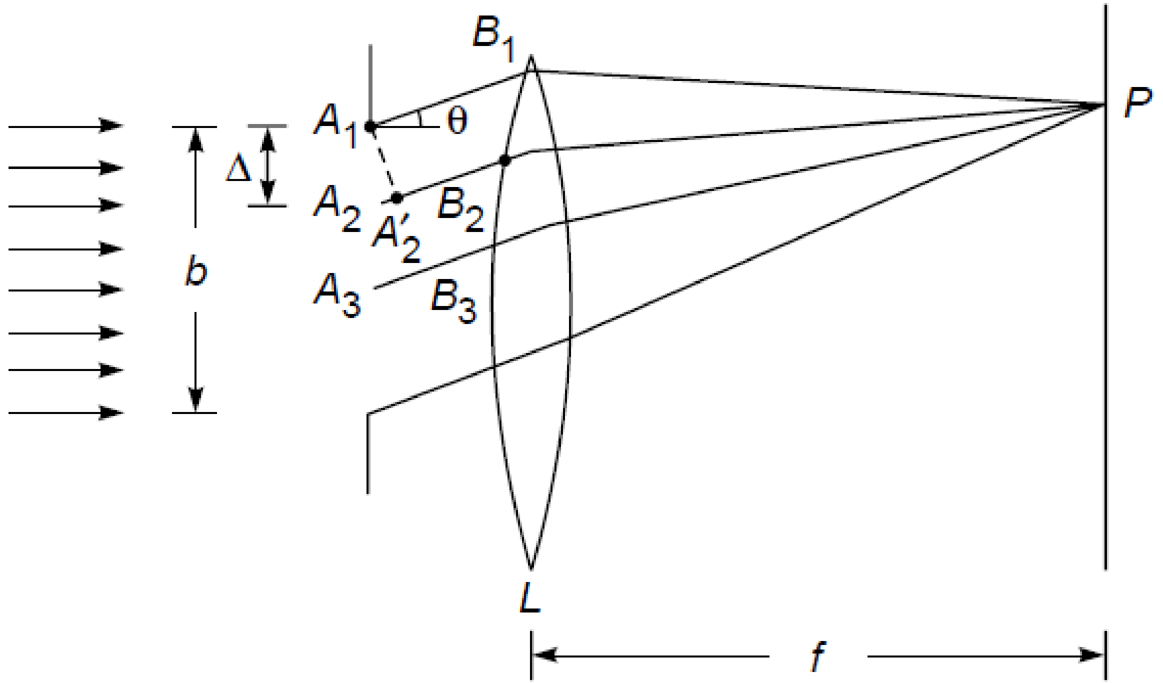
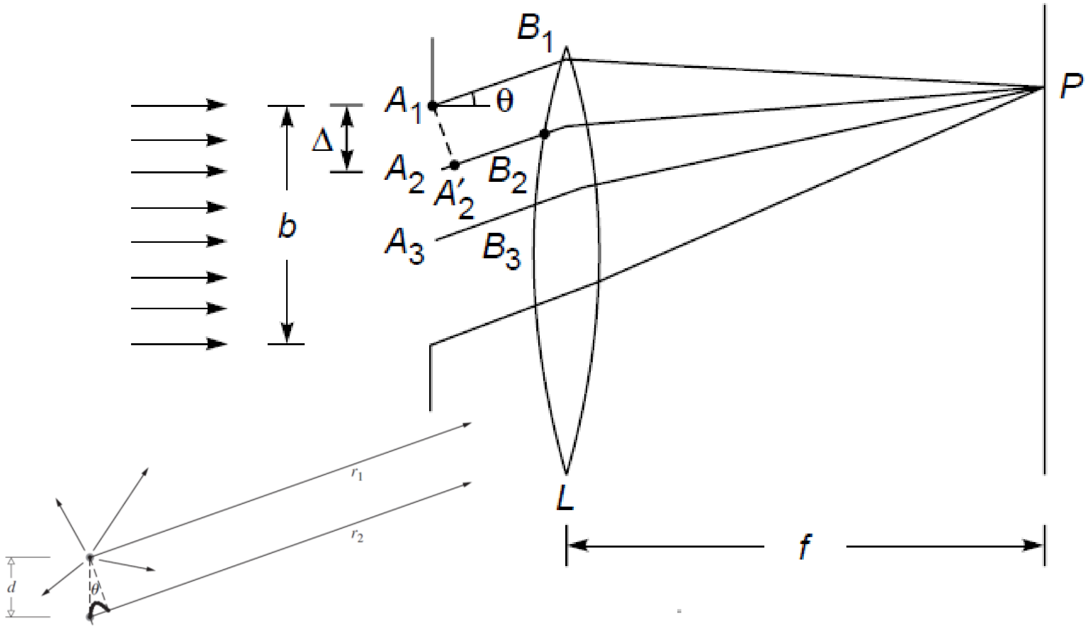
Single slit diffraction: Intensity distribution



- slit → large number of equally spaced point sources
- each point → source of Huygens' secondary wavelets
- 2ndary wavelets interfere
- A_1, A_2, A_3, ..., → point sources
- Distance between two consecutive points → Δ
- number of point sources = n
- $b = (n-1)\,\Delta$

Resultant field produced by these *n* sources at an arbitrary point P ?

Intensity distribution continued



$$\phi = \frac{2\pi}{\lambda}\Delta \sin\theta = \frac{2\pi}{\lambda}\frac{b\sin\theta}{n}$$

$$\frac{n\phi}{2} = \frac{\pi}{\lambda} n\Delta\sin\theta \to \frac{\pi}{\lambda} b\sin\theta$$

$$\boxed{E_0 = A\frac{\sin\beta}{\beta}} \quad A = na \quad \beta = \frac{\pi b\sin\theta}{\lambda}$$

- At P: $A_1 \approx A_2$; distance to P >> b
- slightly different path lengths → path diff → phase diff
- A_2A_2' → extra path; $A_1B_1P = A_2'B_2P$
- Path diff. $A_2A_2' = \Delta \sin\theta$
- Phase diff. $\varphi = k\, A_2A_2' = (2\pi/\lambda)\, \Delta \sin\theta$

$$E = a[\cos \omega t + \cos(\omega t - \varphi) + \dots + \cos[(\omega t - (n-1)\varphi)]$$

$$E = E_0 \cos[(\omega t - \tfrac{1}{2}(n-1)\varphi)]$$

Where $E_0 = a\dfrac{\sin(n\phi/2)}{\sin(\phi/2)}$

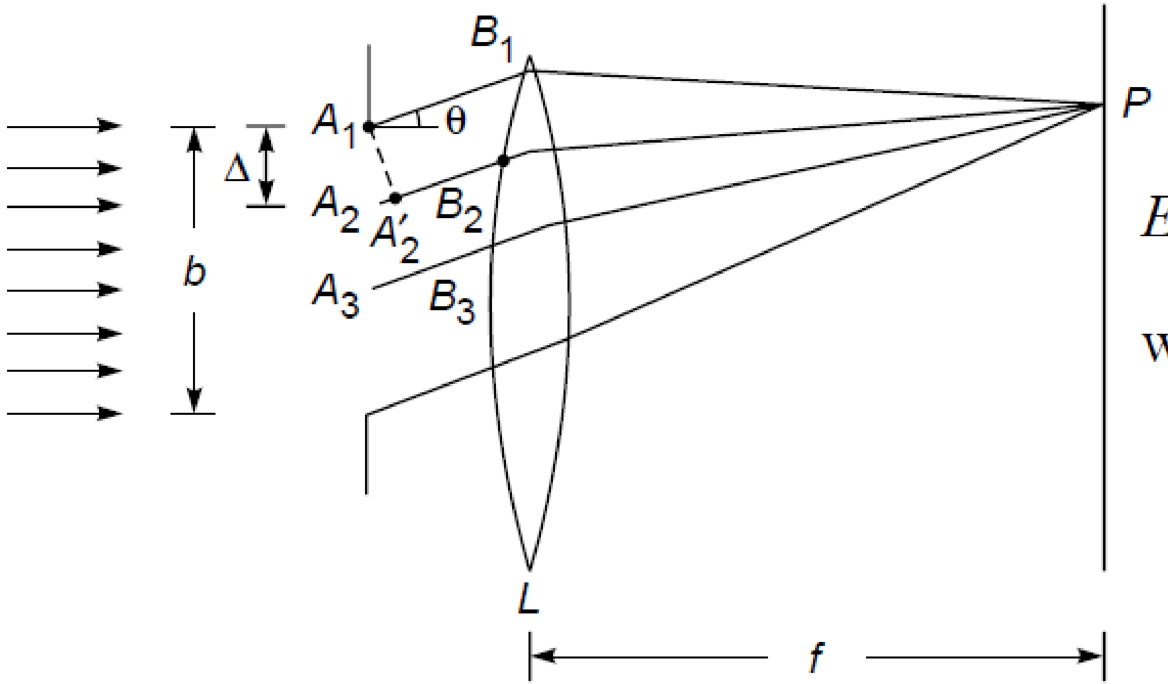
if $n \to \infty$ and $\Delta \to 0$

Then $n\Delta \to$ b

Amplitude of the resultant wave

Optics, Hecht, Ghatak

Single slit diffraction: Intensity distribution



$$E = a[\cos \omega t + \cos (\omega t - \phi) + \cdots + \cos [(\omega t - (n-1)\phi)]$$

where $\phi = \dfrac{2\pi}{\lambda}\Delta \sin\theta$

$$E = E_0 \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right]$$

$$E_0 = a\frac{\sin(n\phi/2)}{\sin(\phi/2)}$$

$n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to b$,

$$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)} \qquad \boxed{I = I_0\frac{\sin^2\beta}{\beta^2}}$$

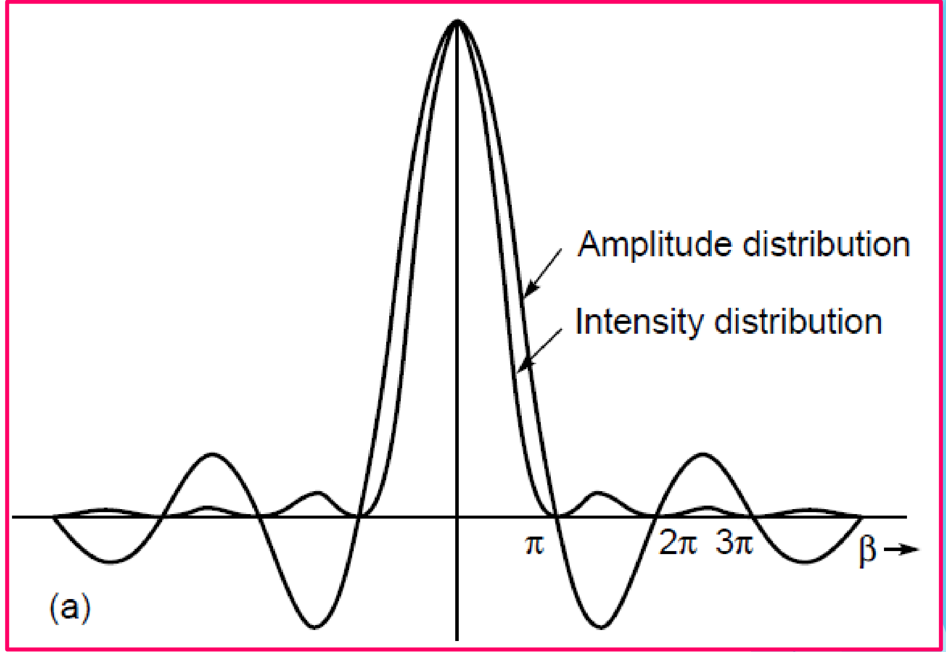
Optics, Ghatak

Single slit diffraction continued

$$\boxed{E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)} \quad \textbf{(1)}$$

$$A = na \quad \beta = \frac{\pi b \sin\theta}{\lambda} \quad \textbf{(2)}$$

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2}} \quad \textbf{(3)}$$



Intensity = 0 if $\beta = m\pi \qquad m \neq 0$ **(4)**

Using (4) in (2):

$$\boxed{b\sin\theta = m\lambda \qquad m = \pm1, \pm2, \pm3, \ldots \text{(minima)}}$$

first minimum

$$\theta = \pm\sin^{-1}(\lambda/b)$$

second minimum

$$\theta = \pm\sin^{-1}(2\lambda/b)$$

m closest to b/λ

Single slit diffraction: maxima

$$\text{maxima}\quad \frac{dI}{d\beta} = I_0\left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right) = 0$$

or

$$\sin\beta\,(\beta - \tan\beta) = 0$$

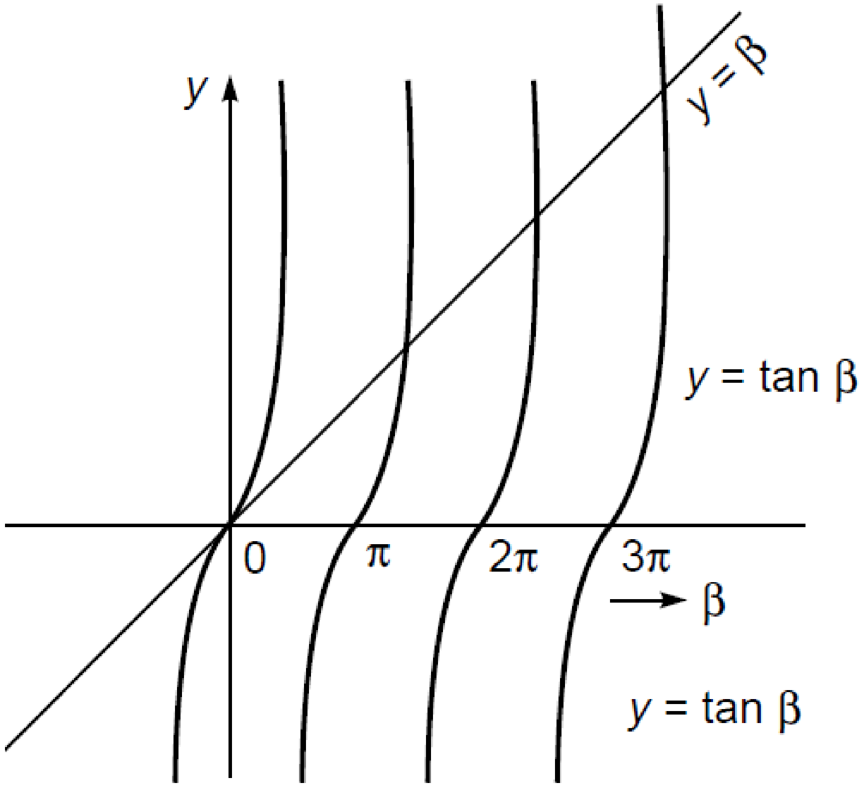
The condition $\sin\beta = 0$, or $\beta = m\pi$ ($m \neq 0$), corresponds to minima. The conditions for maxima are roots of the equation

$$\tan\beta = \beta \qquad \text{(maxima)}$$

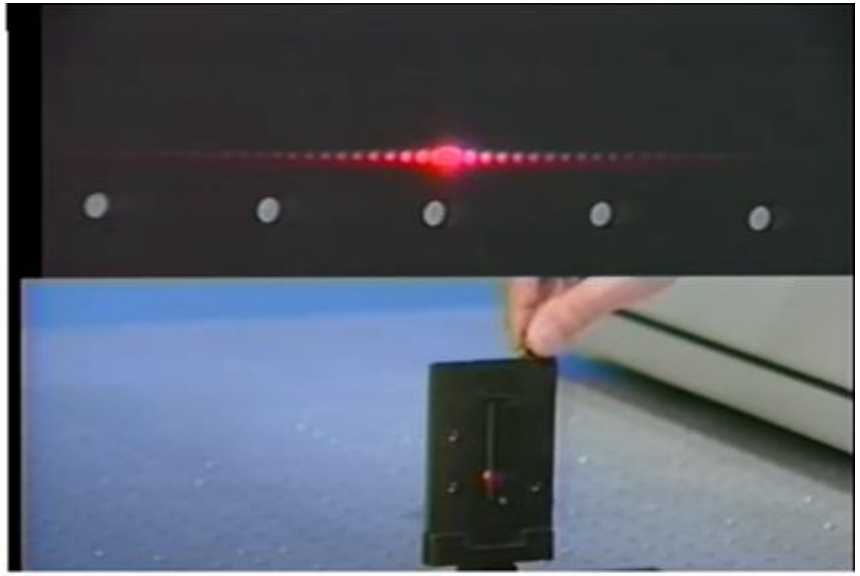
The root $\beta = 0$ corresponds to the central maximum.

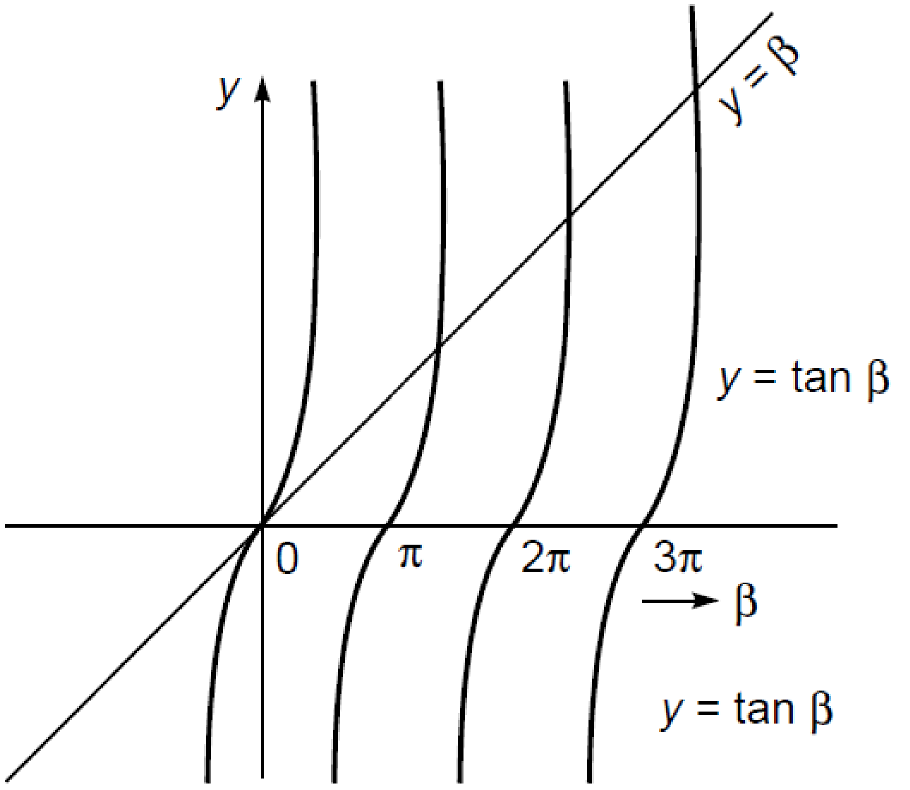
curves $y = \beta$ and $y = \tan\beta$ points of intersections

$$\beta = 1.43\pi,\ \beta = 2.46\pi$$



The central maxima is brightest!





The root $\beta = 0$ corresponds to the central maximum.

curves $y = \beta$ and $y = \tan\beta$ points of intersections

$\beta = 1.43\pi,\ \beta = 2.46\pi$

1st maximum→ $\left(\dfrac{\sin 1.43\pi}{1.43\pi}\right)^2$

https://ocw.mit.edu/resources/res-6-006-video-demonstrations-in-lasers-and-optics-spring-2008/demonstrations-in-physical-optics/fraunhofer-diffraction-2014-adjustable-slit/

Problem-1

A parallel beam of light is incident normally on a narrow slit of width 0.2 mm. The Fraunhofer diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens whose focal length is 20 cm. Calculate the distance between the first two minima and the first two maxima on the screen. Assume that $\lambda = 5 \times 10^{-5}$ cm and that the lens is placed very close to the slit.

Solution

$$\frac{\lambda}{b} = \frac{5\times10^{-5}}{2\times10^{-2}} = 2.5 \times 10^{-3}$$

Now, the conditions for diffraction minima are given by $\sin\theta = m\lambda/b$. We assume θ to be small (measured in radians) so that we may write $\sin\theta \approx \theta$ (an assumption which will be justified by subsequent calculations); thus, on substituting the value of λ/b, we get

$$\theta \simeq 2.5 \times 10^{-3} \text{ and } 5 \times 10^{-3} \text{ rad}$$

as the angles of diffraction corresponding to the first and second minima, respectively. Notice that since

$$\sin(2.5\times10^{-3}) = 2.4999973\times10^{-3}$$

the error in the approximation $\sin\theta \simeq \theta$ is about 1 part in 1 million! These minima will be separated by a distance $(5 \times 10^{-3} - 2.5 \times 10^{-3}) \times 20 = 0.05$ cm on the focal plane of the lens. Similarly, the first and second maxima occur at

$$\beta = 1.43\pi \quad \text{and} \quad 2.46\pi$$

respectively. Thus

$$b \sin\theta = 1.43\lambda \quad \text{and} \quad 2.46\lambda$$

or

$$\sin\theta = 1.43 \times 2.5 \times 10^{-3} \quad \text{and} \quad 2.46 \times 2.5 \times 10^{-3}$$

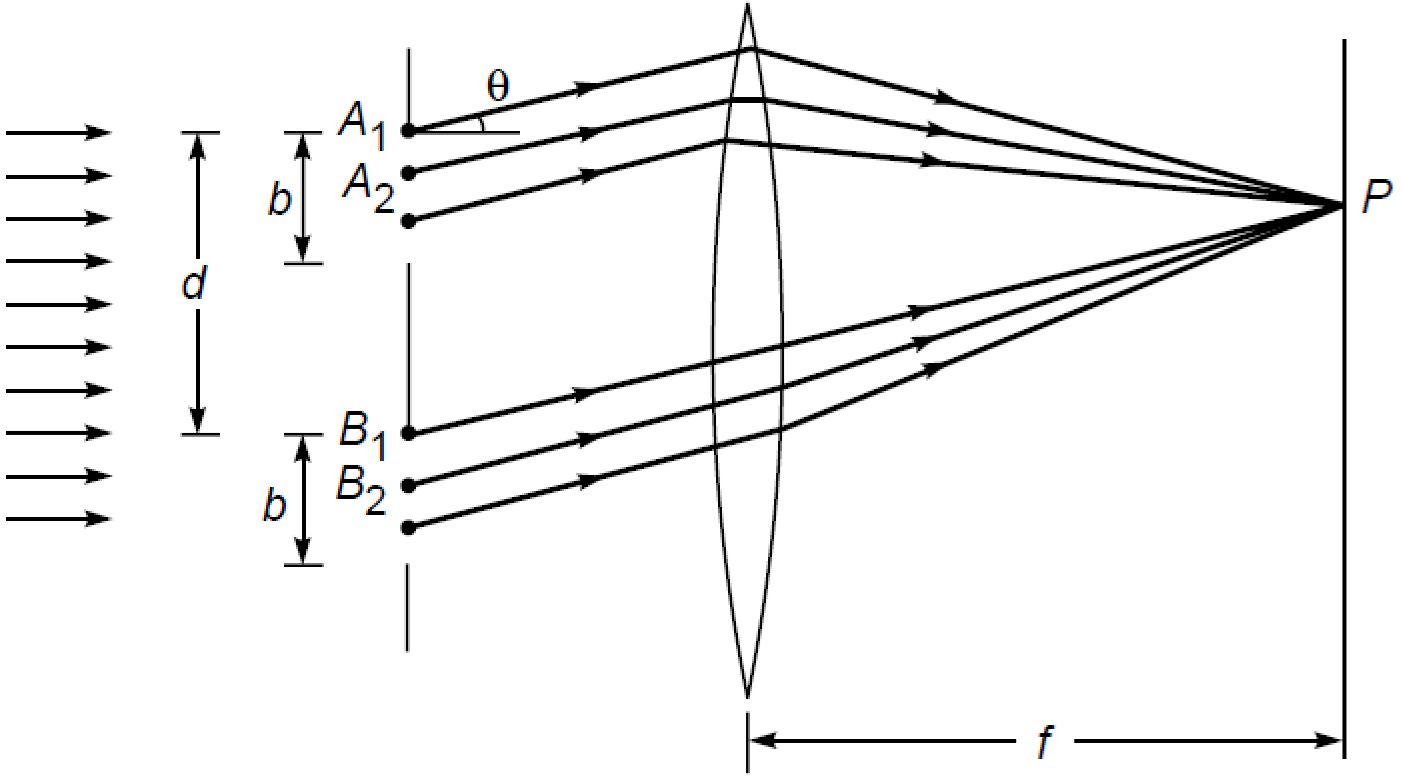
Consequently, the maxima will be separated by the distance given by

$$(2.46 - 1.43) \times 2.5 \times 10^{-3} \times 20 \simeq 0.05 \text{ cm}$$

Problem-2

Consider, once again, a parallel beam of light ($\lambda = 5 \times 10^{-5}$ cm) to be incident normally on a long narrow slit of width 0.2 mm. A screen is placed at a distance of 3 m from the slit. Assuming that the screen is so far away that the diffraction is essentially of the Fraunhofer type, calculate the total width of the central maximum.

Double slit diffraction



Fraunhofer diffraction of a plane wave incident normally on a double slit.

Distance between two consecutive points in either of the slits is Δ

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Similarly, the second slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

at point P, where

$$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$$

Double slit diffraction continued

$$E = E_1 + E_2$$

$$= A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)]$$

$$E = 2A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{1}{2}\Phi_1\right)$$

where

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda}d\sin\theta$$

The intensity distribution will be of the form

$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$$\boxed{I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

Meaning?

Interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

What will happen if

$$I_1 = I_2 = I_0.$$

$$I = 2I_0(1 + \cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

$$I_{\text{min}} = 0$$

$$I_{\text{max}} = 4I_0$$

Double slit diffraction continued

$$I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

intensity distribution produced by one of the slits

Interference pattern produced by two point sources separated by a distance *d*

**What will happen when the slit widths are very small* ??

Double slit diffraction continued

$$\boxed{I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

intensity distribution produced
by one of the slits

Interference pattern produced by
two point sources separated by a distance *d*

**if the slit widths are very small* → *β* small

Young's interference pattern

Thank You