

IIITDM KANCHEEPURAM
MA1001 Differential Equations
Problem Set 8

- ¹ Find the Fourier series of the periodic function $f(x) = \begin{cases} -\frac{\pi}{2} - \frac{1}{2}x, & \text{if } -\pi < x < 0 \\ \frac{\pi}{2} - \frac{1}{2}x, & \text{if } 0 \leq x < \pi \end{cases}$.

Also sketch the graph of $f(x)$ in the interval $-2\pi < x < 2\pi$.

- ² Find the Fourier series for the periodic function on the interval $[-\pi, \pi]$

$$f(x) = \begin{cases} -1, & \text{if } -\pi < x \leq -\pi/2 \\ 0, & \text{if } -\pi/2 \leq x \leq \pi/2 \\ 1, & \text{if } \pi/2 \leq x \leq \pi \end{cases}$$

- ³ A sinusoidal voltage $E \sin \omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function with period $p = 2\ell = 2\pi/\omega$

$$u(t) = \begin{cases} 0, & -\ell < t < 0, \\ E \sin \omega t, & 0 < t < \ell. \end{cases}$$

- ⁴ Find the Fourier series expansion of $f(x) = x^2$, $-2 \leq x \leq 2$. Also show that

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$.

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

- ⁵ Find the Fourier cosine series of the function $f(x) = \begin{cases} x^2, & \text{if } 0 \leq x \leq 2 \\ 4, & \text{if } 2 \leq x \leq 4. \end{cases}$

- ⁶ Find the Fourier series expansion of the following periodic function of period $p = 2\ell = 4$.

$$f(x) = \begin{cases} 2 + x, & \text{if } -2 \leq x \leq 0 \\ 2 - x, & \text{if } 0 \leq x \leq 2. \end{cases}$$

- ⁷ Find the Fourier series of the given function $f(x)$ with period 2π

$$f(x) = \begin{cases} x^2, & -\pi/2 < t < \pi/2 \\ \pi^2/4, & \pi/2 < t < 3\pi/2. \end{cases}$$

- ⁸ Find the Fourier series expansion of the following periodic function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 \leq x < \pi \end{cases}$.

Also find the sum of the following series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

- ⁹ Expand the periodic function $f(x) = x^2$, $0 \leq x \leq l$ in a series of sine functions only. Also find the sum of the series at $x = 0$.

- ¹⁰ Find the Fourier series of a periodic function $f(x) = |\sin(x)|$ in $[\pi, \pi]$.
- ¹¹ Show that the Fourier series of the 2π periodic function $f(x) = x + x^2$, $-\pi \leq x \leq \pi$ converges to π^2 at $x = \pi$.
- ¹² Show that Fourier series expansion of the periodic function $f(x) = \begin{cases} -x, & -4 < x < 0 \\ x, & 0 \leq x < 4 \end{cases}$ does not contain the sine terms.