Engineering Electromagnetics

Lecture 21

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by

Debolina Misra

Dept. of Physics
IIITDM Kancheepuram, Chennai, India

Problem-1

The plane $z = 0$ marks the boundary between free space and a dielectric medium with a dielectric constant of 40. The $\vec{\mathbf{E}}$ field next to the interface in free space is $\vec{\mathbf{E}} = 13\vec{\mathbf{a}}_x + 40\vec{\mathbf{a}}_y + 50\vec{\mathbf{a}}_z$ V/m. Determine the $\vec{\mathbf{E}}$ field on the other side of the interface.

Let $z > 0$ be the dielectric medium 1 and $z < 0$ be the free space medium 2. Then

$$\vec{\mathbf{E}}_2 = 13\vec{\mathbf{a}}_x + 40\vec{\mathbf{a}}_y + 50\vec{\mathbf{a}}_z$$

The unit vector $\vec{\mathbf{a}}_n$ normal to the interface is $\vec{\mathbf{a}}_z$. Because the tangential components of the $\vec{\mathbf{E}}$ field are continuous, then

$$E_{x1} = E_{x2} = 13 \quad \text{and} \quad E_{y1} = E_{y2} = 40$$

For a dielectric–dielectric interface, the normal components of the $\vec{\mathbf{D}}$ field are also continuous. That is, As $\rho_s = 0$

$$\epsilon_1 E_{z1} = \epsilon_2 E_{z2}$$

However, $\epsilon_2 = \epsilon_0$ and $\epsilon_1 = 40\epsilon_0$. Therefore,

$$E_{z1} = \frac{E_{z2}}{40} = \frac{50}{40} = 1.25$$

Thus, the $\vec{\mathbf{E}}$ field in medium 1 is

$$\vec{\mathbf{E}} = 13\vec{\mathbf{a}}_x + 40\vec{\mathbf{a}}_y + 1.25\vec{\mathbf{a}}_z \text{ V/m}$$

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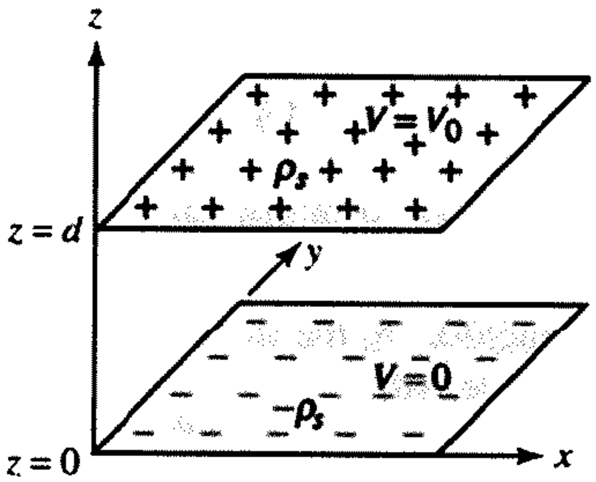
Laplace's equation

The two metal plates of Figure 3.40 having an area A and a separation d form a parallel-plate capacitor. The upper plate is held at a potential of V_0, and the lower plate is grounded. Determine (a) the potential distribution, (b) the electric field intensity, (c) the charge distribution on each plate, and (d) the capacitance of the parallel-plate capacitor.



Since the two metal plates (conductors) form equipotential surfaces in the xy plane at $z = 0$ and $z = d$, we expect that the potential V must be a function of z only. For the charge-free region between the plates, Laplace's equation reduces to

$$\frac{\partial^2 V}{\partial z^2} = 0$$

with a solution

$$V = az + b$$

where a and b are constants to be evaluated from the knowledge of boundary conditions.

When $z = 0$, $V = 0 \Rightarrow b = 0$. The potential distribution within the plates now becomes

$$V = az$$

However, when $z = d$, $V = V_0$ suggests that $a = V_0/d$. Thus, the potential varies linearly in a parallel-plate capacitor as

$$V = \frac{z}{d} V_0$$

We can now compute the electric field intensity as

$$\vec{\mathbf{E}} = -\nabla V = -\vec{\mathbf{a}}_z \frac{\partial V}{\partial z} = -\frac{V_0}{d}\,\vec{\mathbf{a}}_z$$

and the electric flux density is

$$\vec{\mathbf{D}} = \epsilon\vec{\mathbf{E}} = -\frac{\epsilon V_0}{d}\,\vec{\mathbf{a}}_z$$

Since the normal component of the $\vec{\mathbf{D}}$ field must be equal to the surface charge density on a conductor, the surface charge density on the lower plate is

$$\rho_s|_{z=0} = -\frac{\epsilon V_0}{d}$$

and that on the upper plate is

$$\rho_s|_{z=d} = \frac{\epsilon V_0}{d}$$

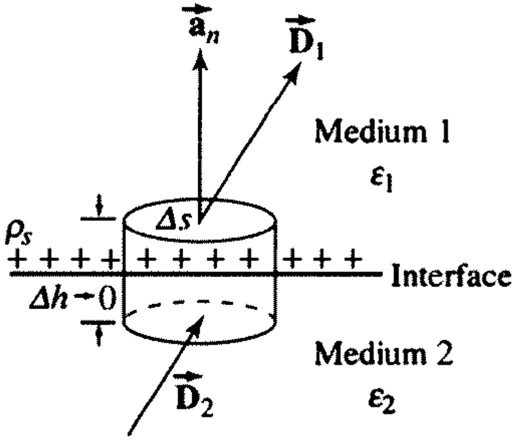
The total charge on the upper plate is

$$Q = \frac{\epsilon V_0 A}{d}$$

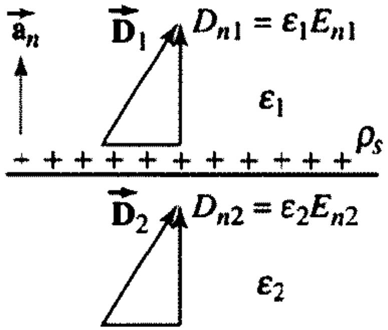
Thus, the capacitance of the parallel plate capacitor is

$$C = \frac{Q}{V_0} = \frac{\epsilon A}{d} \qquad \bullet\bullet\bullet$$

If one medium is a conductor



a) Boundary conditions



b) Normal components

If medium 2 is a conductor, the electric flux density $\vec{\mathbf{D}}_2$ must be zero under static conditions. For the normal component of the electric flux density $\vec{\mathbf{D}}_1$ to exist in medium 1, there must be a free surface charge density on the conductor's surface in harmony with (3.70). That is,

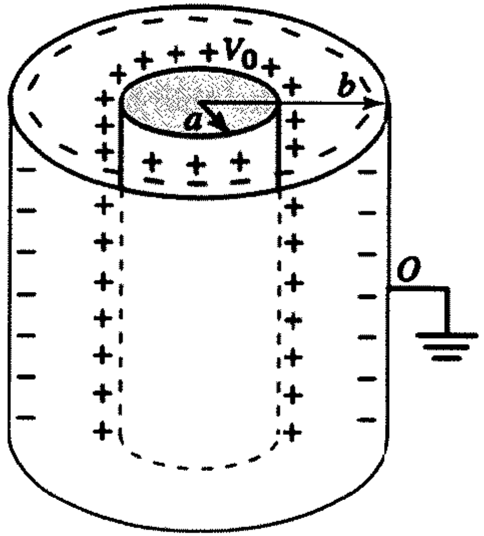
$$\vec{\mathbf{a}}_n \cdot \vec{\mathbf{D}}_1 = D_{n1} = \rho_s \tag{3.72a}$$

$$\epsilon_1 E_{n1} = \rho_s \tag{3.72b}$$

The normal component of the electric flux density in a dielectric medium just above the surface of a conductor is equal to the surface charge density on the conductor.

Problem 2

The inner conductor of radius a of a coaxial cable (see Figure 3.41) is held at a potential of V_0 while the outer conductor of radius b is grounded.



Determine (a) the potential distribution between the conductors, (b) the surface charge density on the inner conductor, and (c) the capacitance per unit length.

Thank You