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MA1001 Differential Equations

Problem Set 4

1. By eliminating the constants c_1 and c_2, find the differential equation of each the following 2-parameter families of curves:
(a) $y = c_1x + c_2x^2$; (b) $y = c_1x + c_2 \sin x$.

2. Consider the functions $f(x) = x^3$ and $g(x) = x^2|x|$ on the interval $[-1, 1]$.
   (a) Show that their Wronskian is identically zero on $[-1, 1]$.
   (b) Show that $f(x)$ and $g(x)$ are not linearly dependent.
   (c) Do (a) and (b) contradict a result (figure out the result and state it) known to you? If not, why not?

3. The equation $(1 - x^2)y'' - xy' - a^2y = 0$ has $y = e^{a \sin^{-1} x}$ as one solution. Find the general solution.

4. $y = e^{-x^2}$ is a solution of the differential equation $xy'' + \alpha y' + \beta x^3y = 0$ for some real numbers α and β. Find α and β.

5. Solve the following differential equations:
(a) $\dfrac{d^4y}{dx^4} - \dfrac{d^2y}{dx^2} = 0$; (b) $\dfrac{d^4y}{dx^4} - y = 0$; (c) $\dfrac{d^4y}{dx^4} + y = 0$.

6. The equation $x^2y'' + pxy' + qy = 0$ is called Euler's equidimensional equation. Show that the change of independent variable given by $x = e^z$ transforms it into an equation with constant coefficients. Apply this technique to find the general solution of $x^2y'' + 3xy' + 10y = 0$.