IIITDM KANCHEEPURAM

MA1001 Differential Equations

Problem Set 2

1. Verify that the following equations are homogeneous and solve them:

   (a) $x \sin \frac{y}{x} \dfrac{dy}{dx} = y \sin \frac{y}{x} + x$

   (b) $xy' = y + 2xe^{-y/x}$.

2. Find the orthogonal trajectories of the family of all circles tangent to the y-axis at the origin. (The given family is $x^2 + y^2 = 2cx$.)

3. Show that the substitution $z = ax + by + c$ changes

   $$y' = f(ax + by + c)$$

   into an equation with separable variables, and apply this method to solve

   (a) $y' = (x + y)^2$

   (b) $y' = \sin^2(x - y + 1)$

4. (a) If $ae \neq bd$, show that constants h and k can be chosen in such a way that the substitutions $x = z - h$, $y = w - k$ reduce

   $$\frac{dy}{dx} = F\left(\frac{ax + by + c}{dx + ey + f}\right)$$

   to a homogeneous equation.

   (b) If $ae = bd$, discover a substitution that reduces the equation in (a) to one in which the variables are separable.

5. Solve the following equations:

   (a) $\dfrac{dy}{dx} = \dfrac{x + y - 1}{x + 4y + 2}$;

   (b) $\dfrac{dy}{dx} = \dfrac{x + y + 4}{x + y - 6}$.