Probability - Problems

February 19, 2025

1. A poker hand means a set of five cards selected at random from usual deck of playing cards.

   (a) Find the probability that it is a Royal Flush - means that it consists of ten, jack, queen, king, ace of one suit.

   (b) Find the probability that it is four of a kind - means that there are four cards of equal face value.

   (c) Find the probability that it is a full house - means that it consists of one pair and one triple of cards with equal face values.

   (d) Find the probability that it is a straight - means that it consists of five cards in a sequence regardless of suit.

   (e) Find the probability that it consists of three cards of equal face value and two other cards but not a full house.

   (f) Find the probability that it consists of two distinct pairs and another card but does not fall into previous categories.

   (g) Find the probability that it consists a pair and three other cards but does not fall into previous categories.

2. A bridge distribution means a distribution of the usual deck of playing cards among four persons to be called N, E, S, W, each getting 13 cards.

   (a) Show that the probability p of W receiving exactly k aces is same as the probability that an arbitrary hand of 13 cards contains excatly k aces.

   (b) What is the probability that N and S together get k aces? Here $k = 0, 1, 2, 3, 4$.

   (c) Find the probability that N, S, E, W get a, b, c, d spades respectively

3. I have n sticks. Each is broken into two pieces - one long and one short piece. These $2n$ pieces are paired at random two form n sticks.

   (a) What is the probability that they are joined to form original sticks?

   (b) Find the probability that all long parts are paired with short parts.

4. In how many ways can two rooks of different colours be put on a chess board so that they can take each other?

5. Show that it is more probable to get at least one ace with four dice than at least one double ace in 24 throws of two dice. This is apparently known as de Mere's paradox. There is, of course, no paradox – it just 1 so happens that Chevalier de Mere thought that both probabilities are equal.

6. If n balls are placed at random into n cells, find the probability that exactly one cell remains empty.

7. A man is given n keys, in a random order, of which only one fits the door. He tries the keys, one after the other, to open the door. This procedure may require $1, 2, 3 \cdots, n$ trials. Show that each of these n has probability $\frac{1}{n}$.

8. A box contains 90 good and 10 defective items. If 10 items are selected then what is the probability that none of them is defective?

9. If n men among whom are A and B, stand in a row what is the probability that there are exactly r men between A and B? What if they stand in a ring (what does this mean?) and the clock-wise direction is used for counting the number between A and B?

10. What is the probability that two throws with three dice each will show the same configuration, if the dice are distinguishable? What if the dice are not distinguishable?