CHAPTER – 4

CURVES

4.1 CONIC SECTIONS

Cone is formed when a right angled triangle with an apex and angle θ is rotated about its altitude as the axis. The length or height of the cone is equal to the altitude of the triangle and the radius of the base of the cone is equal to the base of the triangle. The apex angle of the cone is 2θ. When a cone is cut by a plane, the curve formed along the section is known as a conic section.

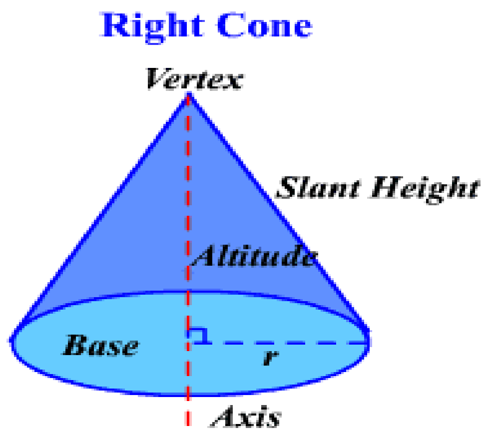


Fig 4.1

Following are some important conic sections.

1. Circle
2. Ellipse
3. Parabola
4. Hyperbola

4.1.1 CIRCLE :

When a cone is cut by a section plane A-A making an angle α= 90° with the axis, the section obtained is a circle.

4.1.2 ELLIPSE :

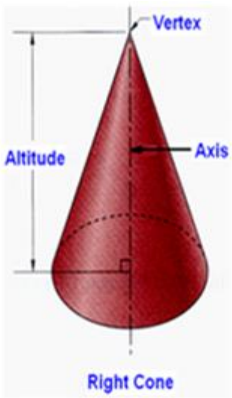
When a cone is cut by a section plane B-B at an angle, αmore than half of the apex angle i.e., θ and less than 90°, the curve of the section is an ellipse. Its size depends on the angle α and the distance of the section plane from the apex of the cone.

4.1.3 PARABOLA :

If the angle α is equal to θ i.e., when the section plane C-C is parallel to the slant side of the cone the curve at the section is a parabola. This is not a closed figure like circle or ellipse. The size of the parabola depends upon the distance of the section plane from the slant side of the cone.

4.1.4 HYPERBOLA:

If the angle α is less than θ (section plane D-D), the curve at the section is hyperbola. The curve of intersection is hyperbola, even if $\alpha = \theta$, provided the section plane is not passing through the apex of the cone. However if the section plane passes through the apex, the section produced is an isosceles triangle.



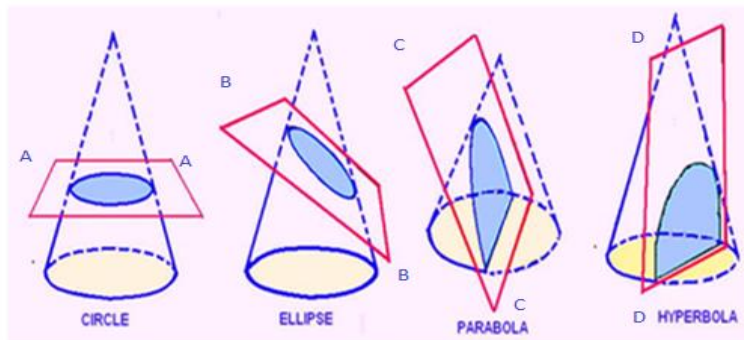


Fig 4.2

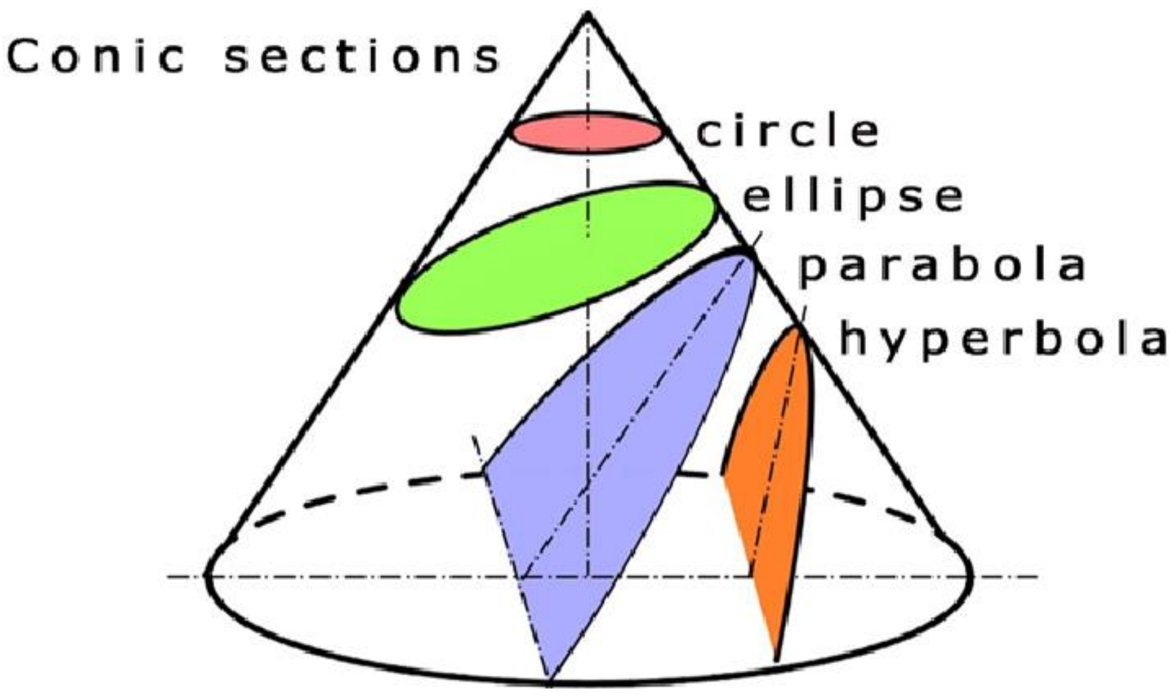


Fig 4.3

4.2 SOME IMPORTANT DEFINITIONS:

4.2.1 Major axis : It is the longest distance which passes through the centre, at right angle to the fixed lines called the directrix. AB is the major axis.

4.2.2 Minor axis : It is the maximum distance which bisects the major axis at right angle. It will be parallel to the directrix. CD is the minor axis.

4.2.3 Directrix : It is a straight line perpendicular to the major axis.

4.2.4 Focus: When an arc is drawn with C or D as centre and radius equal to half of the major axis i.e. AB/2 it is cut at two points F_1 and F_2 on the major axis. F_1 and F_2 are the focal points of an ellipse F_1 or F_2 is the focus. The sum of the distances from F_1, F_2 to any point on the curve i.e., $F_1P + F_2P$ is always constant and equal to the major axis.

4.2.5 Focal radii : The distances from point P on the curve to the focal points F_1 and F_2 are called focal radii. Sum of the focal radii is equal to the major axis.

4.2.6 Eccentricity: The ratio between the distances from the vertex to focus and vertex to the directrix is called the eccentricity.

a. If e=1, it is parabola

b. If e>1, it is hyperbola

c. If e<1, it is an ellipse

4.2.7 Vertex : The end points of the major axis on the curve are called vertex. (A, B)

4.2.8 Tangent and normal to an ellipse : Normal is the line bisecting the angle F_1 P F_2 in Fig 4. Tangent is a line at 90° to the normal and touching the ellipse.

Directrix, axis, focus, vertex and tangent are the elements common to ellipse, parabola and hyperbola.

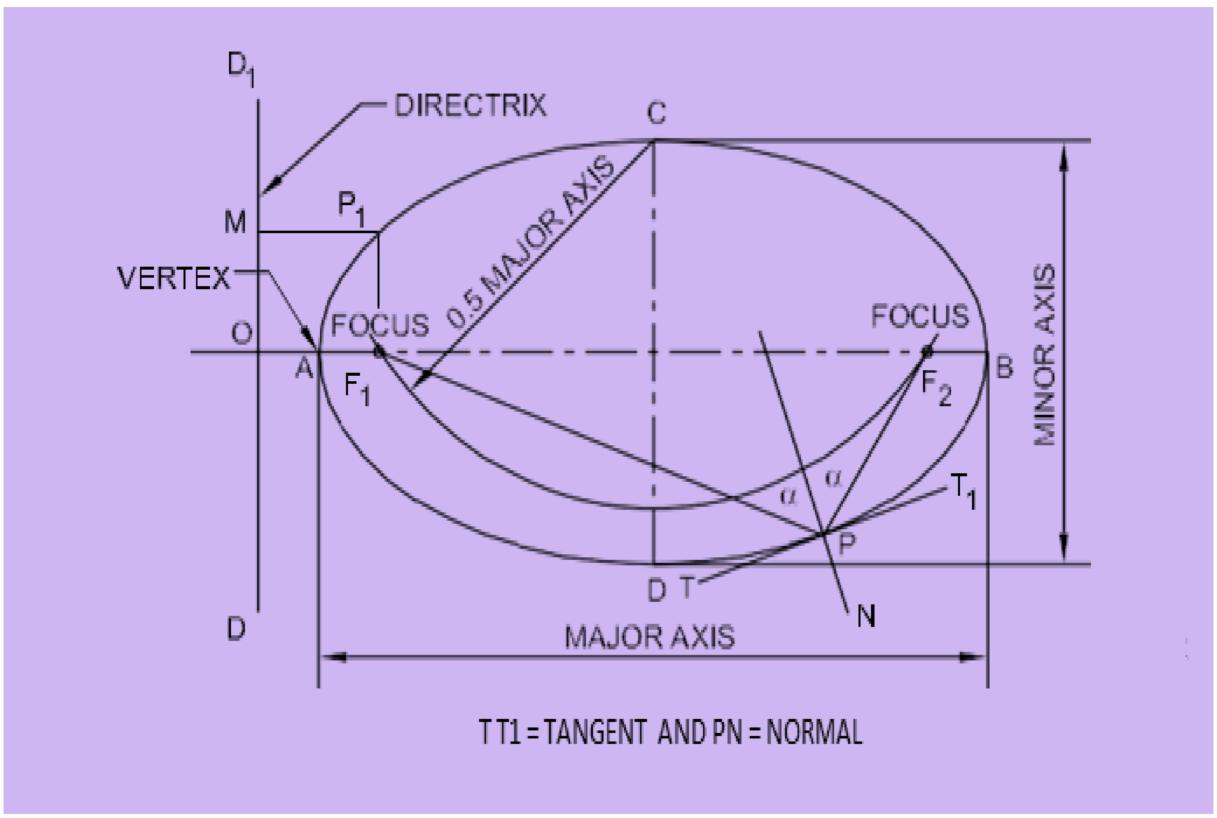


Fig 4.4

Q. No. 1.

To draw a parabola with the distance of the focus from the directrix at 60mm (Eccentricity method)

Construction :

Draw the directrix AB and the axis CC’ at right angles to it:

Mark the focus F on the axis at 60 mm.

Locate the vertex V on CC’ such that CV = VF

Draw a line VB perpendicular to CC’ such that VB = VF

Join C, B and extend. Now, VF / VC = 1, the eccentricity.

Locate number of points 1, 2, 3, ….., on VC’ and erect perpendiculars through them meeting CB produced at 1',2',3'…..

With F as a centre and radius equal to 1-1’ cut two arcs on the perpendicular through 1 to locate P1 and P1’. Similarly with F as center and radius 2-2’, 3-3’ etc cut arcs on the corresponding perpendiculars to locate P2 and P2’, P3 and P3’ etc.

Draw a smooth curve passing through V,P1, P2, P3….P3’,P2’,P1’.

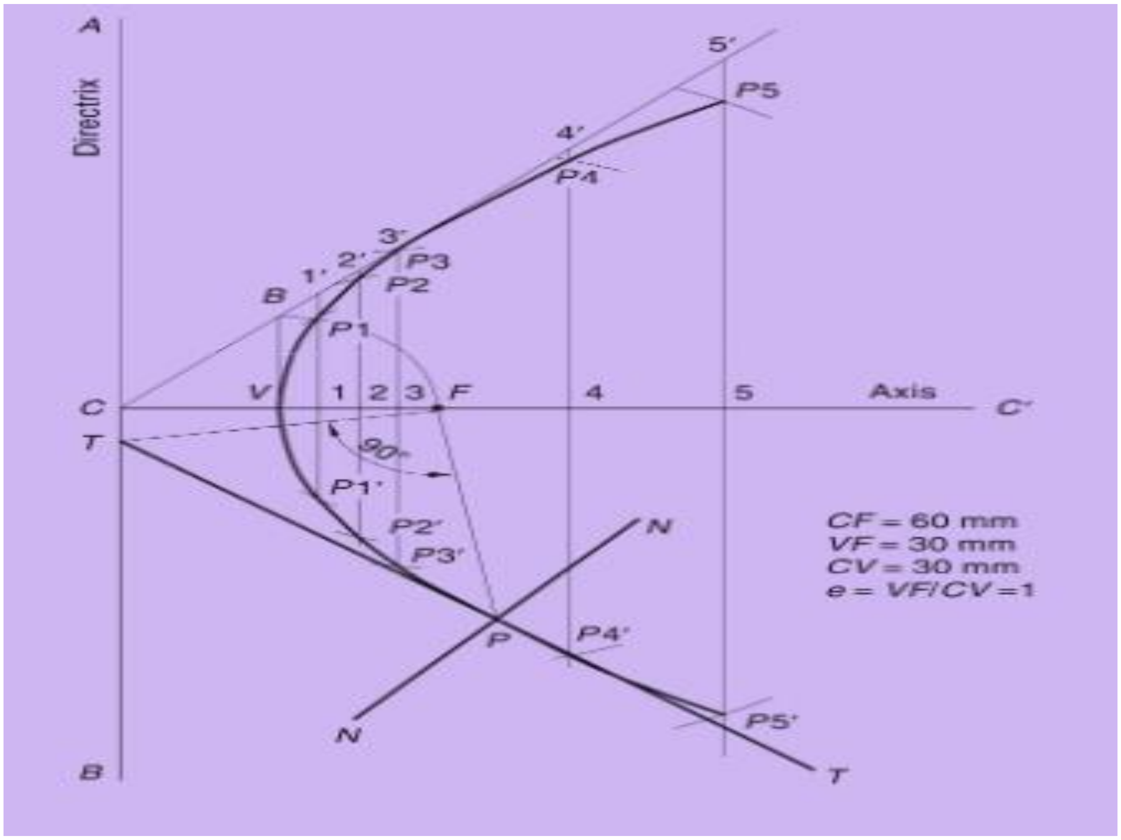


Fig 4.5

All dimensions are in mm

To draw a normal and tangent through a point 40mm from the directrix.

To draw a tangent and normal to the parabola. locate the point P which is at 40 mm from the directrix. Then join P to F and draw a line through F, perpendicular to PF to meet the directrix at T. The line joining T and P and extended is the tangent and a line NN, through P and perpendicular to TP is the normal to the curve.

Q. No. 2.

To draw an ellipse with the distance of the focus from the directrix at 50mm and eccentricity = 2/3 (Eccentricity method).

Construction :

1. Draw any vertical line AB as directrix and mark appoint C on it.
2. Draw a horizontal line CD of any length from point C as axis.
3. Mark a point F on line CD at 50mm from C.
4. Divide CF in 5 equal divisions.
5. Mark V on 2nd division from F.
6. Draw a perpendicular on V and mark a point E on it at a distance equal to VF.
7. Join CE and extend it.
8. Mark points 1,2,3…..on CF beyond V at uniform distance and draw perpendiculars on each of them so as to intersect extended CE at 1’,2’,3’………..

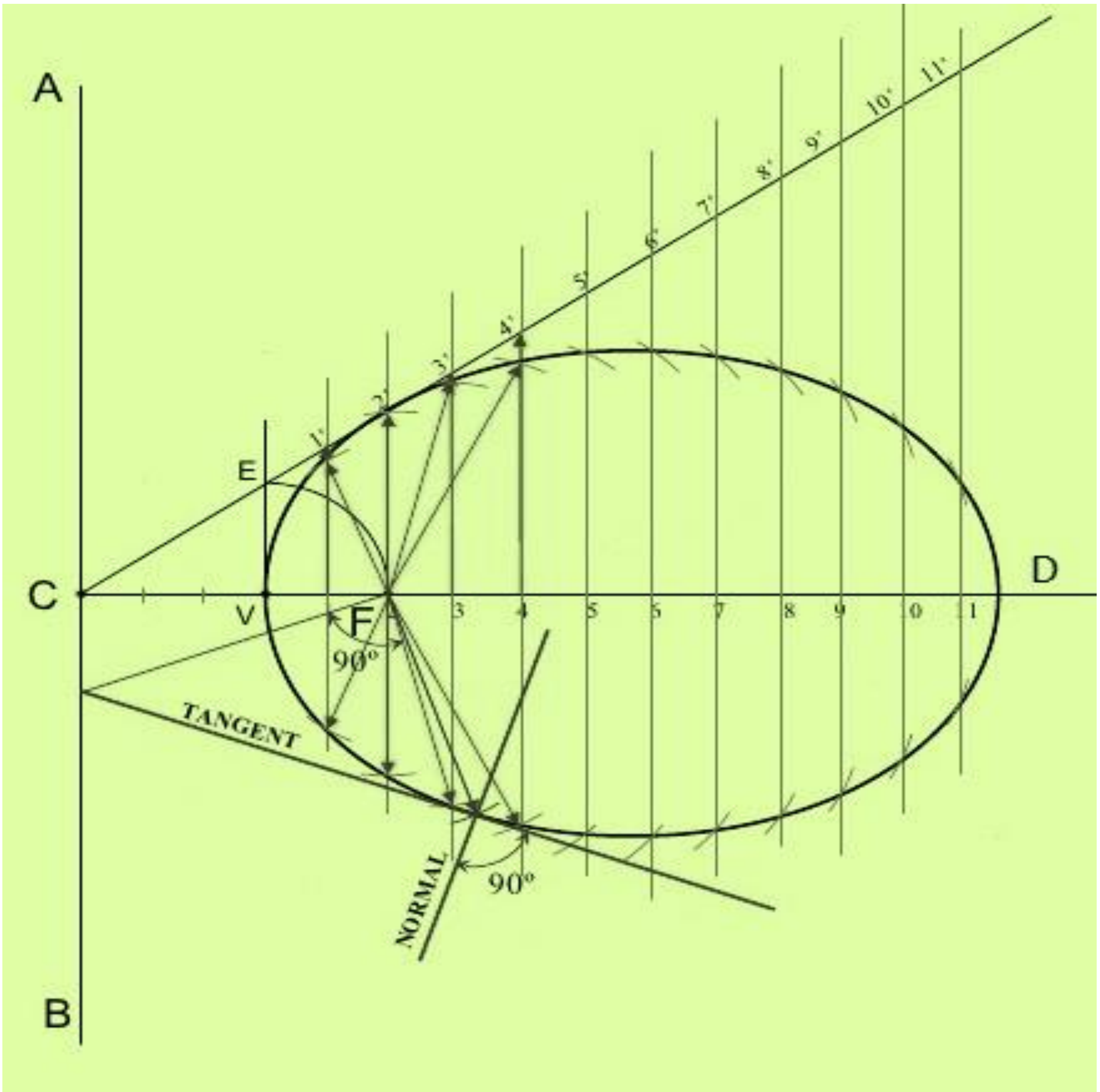


Fig 4.6

All dimensions are in mm

Q. No. 3.

To draw a hyperbola with the distance of the focus from the directrix at 50mm and e=3/2 (Eccentricity method).

Construction :

1. Draw the directrix AB and the axis CC'.
2. Mark the focus F on CC' and 50mm from C.
3. Divide CF into 5 equal divisions and mark V the vertex, on the second division from C.
4. Draw a line VE perpendicular to CC' such that VE=VF. Join C and E.
5. Mark any point 1 on the axis and through it, draw a perpendicular to meet CE produced at1'.
6. With centre F and radius equal to 1-1', draw arcs intersecting the perpendicular through 1 at P1 and P1'.

   Similarly mark a number of points 2, 3 etc and obtain points P2 and P2',etc.

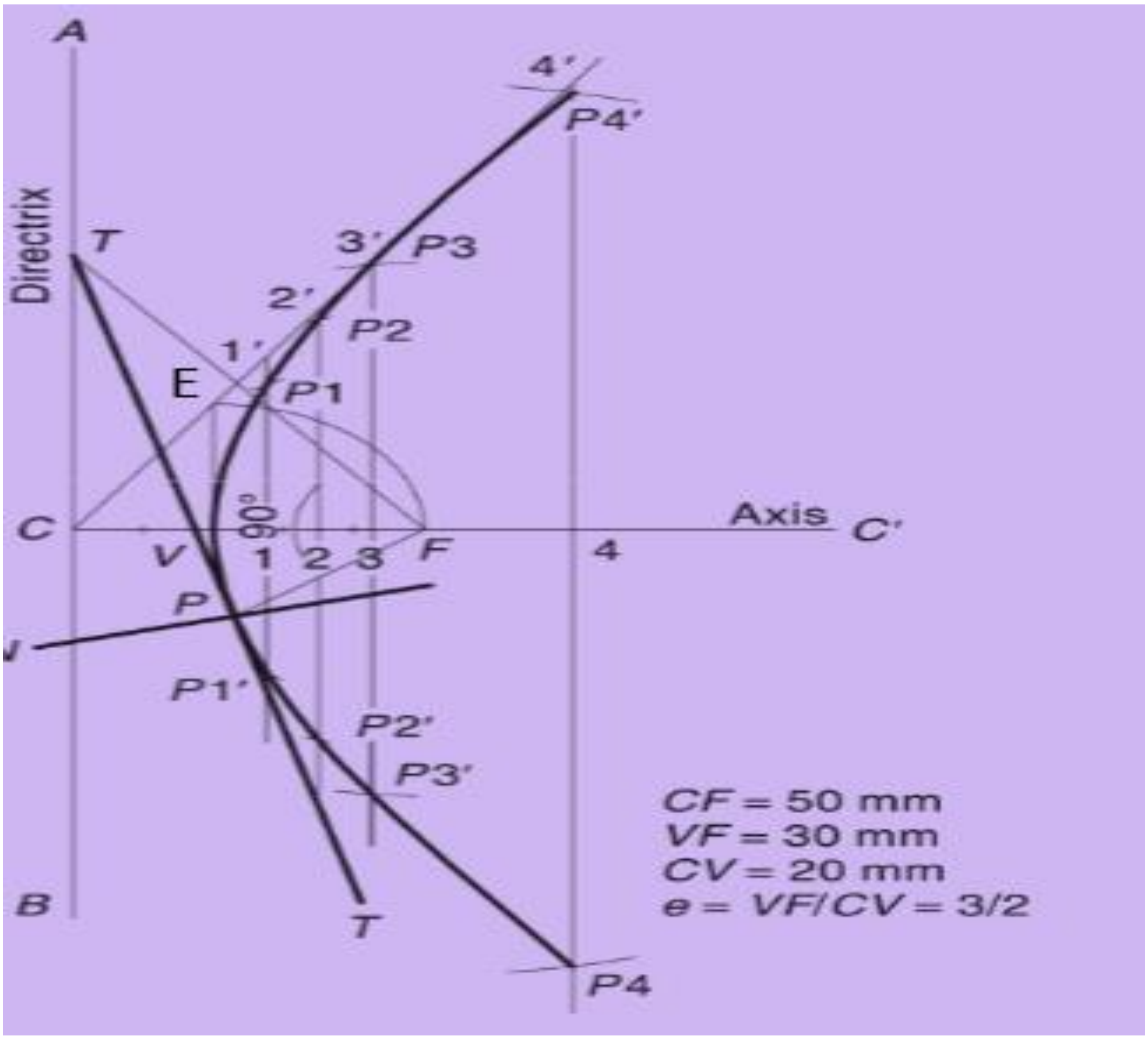


Fig 4.7 All dimensions are in mm