Engineering Optics

Lecture 15

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by

Debolina Misra
Department of Physics
IIITDM Kancheepuram, Chennai, India

Fresnel half-period zones

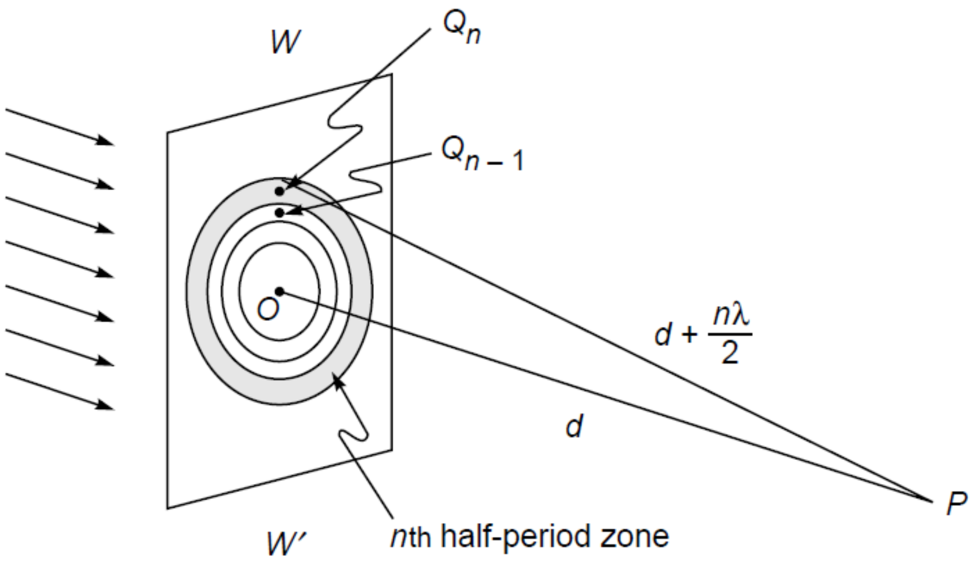

Fig. 20.2 Construction of Fresnel half-period zones.

$$r_n = \left[\left(d + n\frac{\lambda}{2}\right)^2 - d^2\right]^{1/2}$$

$$= \sqrt{n\lambda d}\left(1 + \frac{n\lambda}{4d}\right)^{1/2}$$

or

$$r_n \approx \sqrt{n\lambda d}$$

where we have assumed $d >>> \lambda$; this is indeed justified for practical systems using visible light. Of course, we are assuming that n is not a very large number. The annular region between the nth circle and $(n - 1)$st circle is known as the nth half-period zone;

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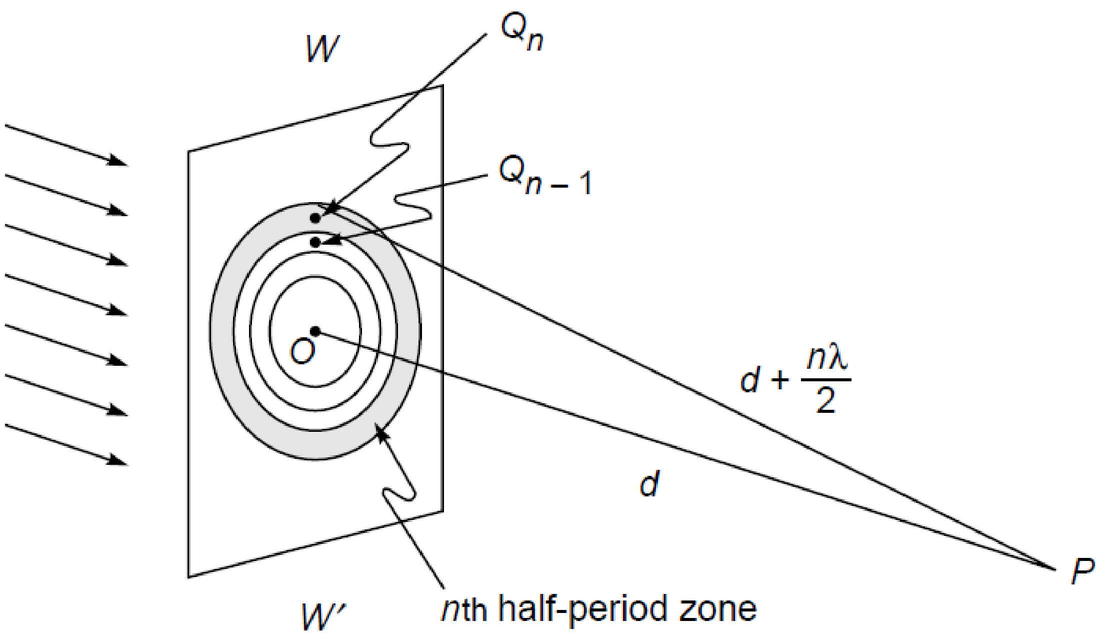
Amplitude at the point *P*

- Amplitude at P $\propto A_n$
- $\propto$ 1/ distance of the zone from *P*

$$\boxed{\text{obliquity factor } \tfrac{1}{2}(1 + \cos\chi)}$$



20.2 Construction of Fresnel half-period zones.

$$u(P) = u_1 - u_2 + u_3 - u_4 + \cdots + (-1)^{m+1}u_m + \cdots$$

$$u(P) = \frac{u_1}{2} + \left[\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right] + \left[\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right] + \cdots$$

$$u(P) \approx \frac{u_1}{2} + \frac{u_m}{2} \quad m \text{ odd}$$

$$u(P) \approx \frac{u_1}{2} - \frac{u_m}{2} \quad m \text{ even}$$

$$u(P) \approx \frac{u_1}{2}$$

Suppose intensity→ I_0

*What if only 1st zone is open?

Intensity I = ?

implying that *the resultant amplitude produced by the entire wave front is only one-half of the amplitude produced by the first half-period zone.*

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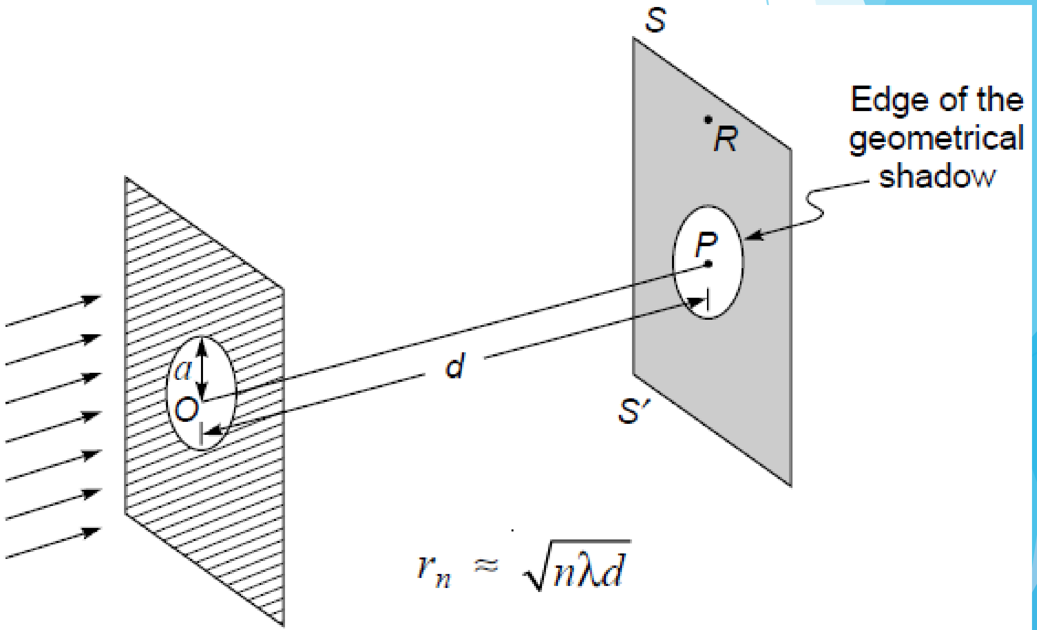
Problem-1

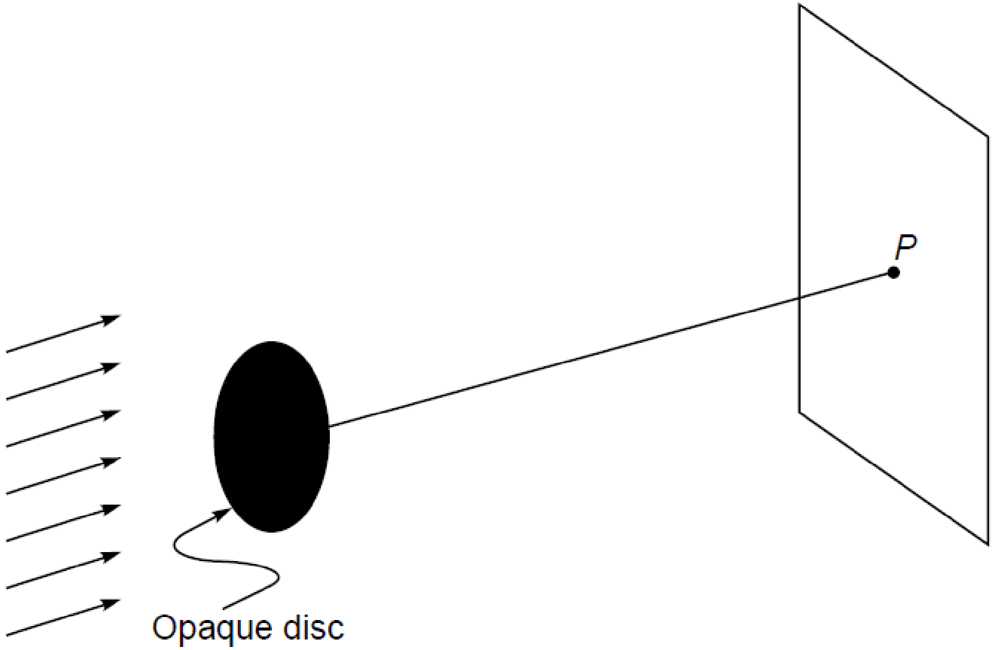
- Assume a plane wave ($\lambda = 5 \times 10^{-5}$ cm) to be incident on a circular aperture of radius 0.5 mm. Calculate the positions of the brightest and darkest points on the axis.

Solution-1

- For the brightest point, the aperture should contain only the first zone, and thus we must have
- $(0.05)^2 = OP(5 \times 10^{-5})$ $[r_1=?]$.
- Thus $OP = 50$ cm.
- Similarly the darkest point would be at a distance
- $= (0.05)^2 / (2 \times 5 \times 10^{-5}) = 25$ cm.



Poisson’s spot



When a plane wave is incident normally on an opaque disc, a bright spot is always formed on an axial point. This spot is known as the Poisson spot.

Fresnel Zone plate

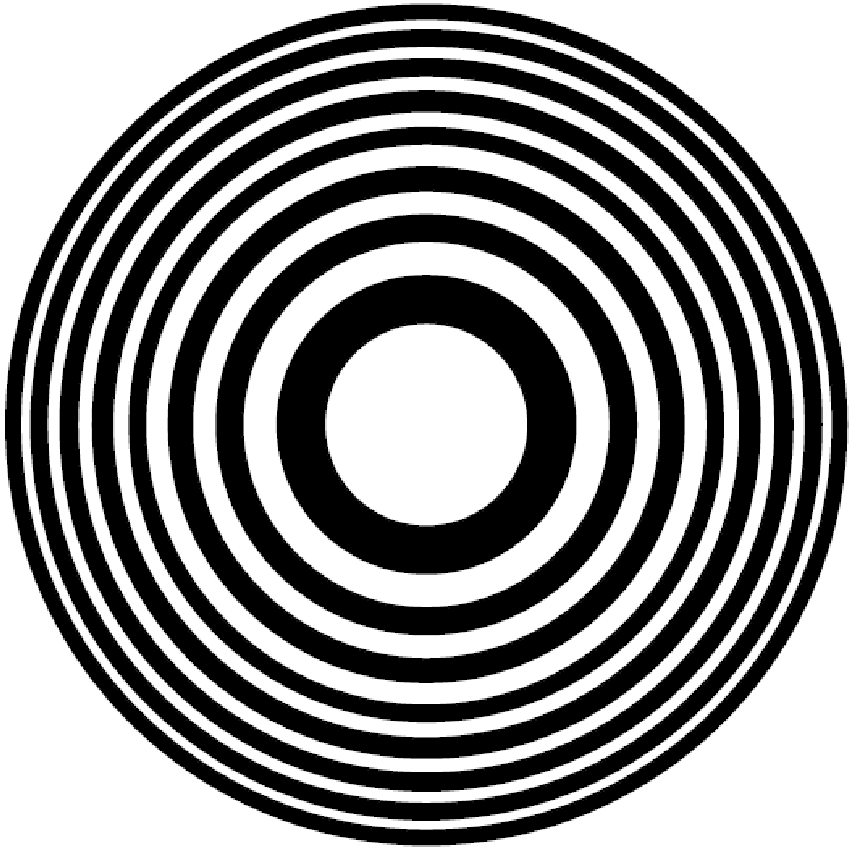


Fig. 20.4 The zone plate.

20.3 THE ZONE PLATE

A beautiful application of the concept of Fresnel half-period zones lies in the construction of the zone plate which consists of a large number of concentric circles whose radii are proportional to the square root of natural numbers and the alternate annular regions of which are blackened (see Fig. 20.4). Let the radii of the circles be $\sqrt{1}\,K$, $\sqrt{2}\,K$, $\sqrt{3}\,K$, $\sqrt{4}\,K, \ldots$ where K is a constant and has the dimension of length.

Positive and Negative Zone plates

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Zone plate

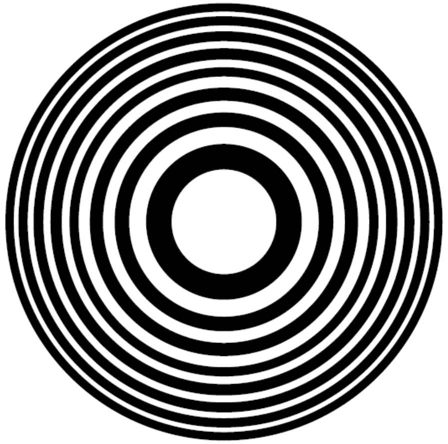
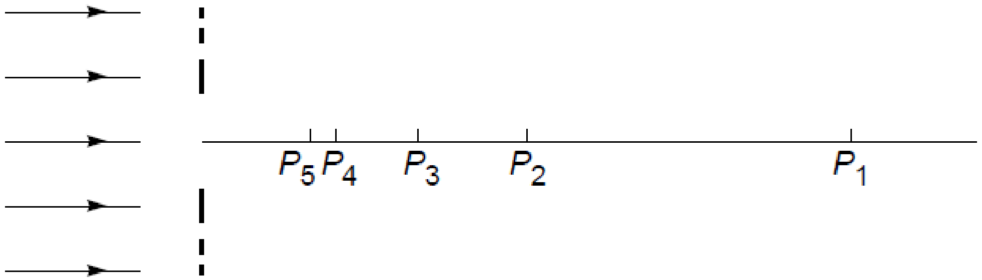
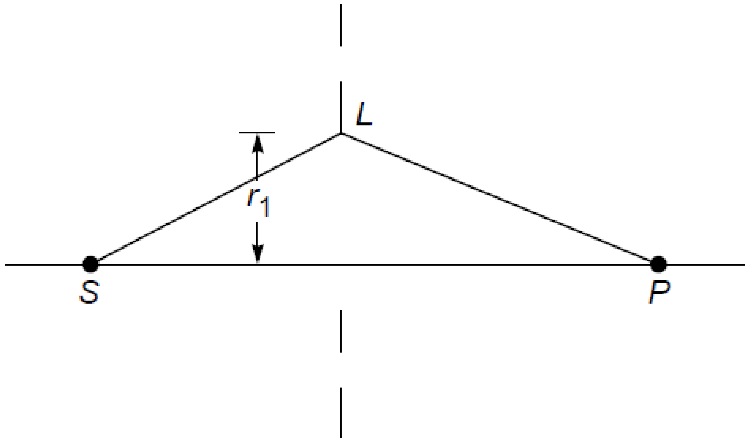


Fig. 20.4 The zone plate.

- At P_1 at a distance K^2/λ from the zone plate, blackened rings are 2nd, 4th, 6th, . . Resultant amplitude *u= ? (max or min?)*
- For P_3 at $K^2/3\lambda$ the first blackened ring contains the 4th, 5th, 6th zones, the second blackened ring contains the 10th, 11th, and 12th zones, etc.; $u = (u1 - u2 + u3) + (u7 - u8 + u9) + \ldots$ (max, less intense than 1st)
- For P_2 at $K^2/2\lambda$ the first blackened ring contains the 3rd and 4th half-period zones… $u = (u1 - u2) + (u5 - u6) + \ldots$ (min)
- If a plane wave is incident normally on a zone plate, then the corresponding focal points are at K^2/λ, $K^2/3\lambda$, $K^2/5\lambda$...
- +ve and –ve Zone plates



(a)



(b)

(a) For a plane wave incident on a zone plate, the maximum intensity occurs at points P_1, P_3, etc. The minima occur at P_2, P_4, ... (b) Imaging of a point object by a zone plate.

Zone plate

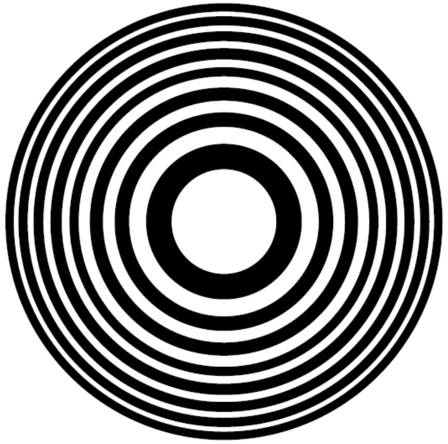
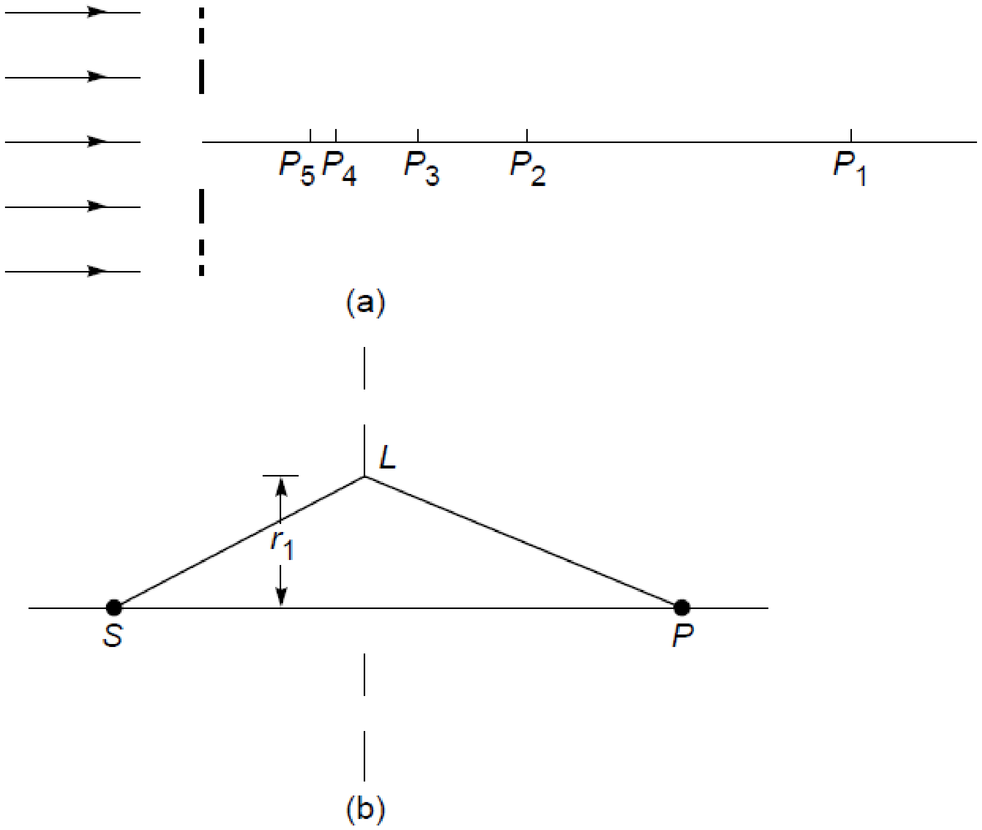


Fig. 20.4 The zone plate.



(a) For a plane wave incident on a zone plate, the maximum intensity occurs at points P_1, P_3, etc. The minima occur at P_2, P_4, ... (b) Imaging of a point object by a zone plate.

- At P_1 at a distance K^2/λ from the zone plate, blackened rings are 2nd, 4th, 6th, . . Resultant amplitude $u = u1 + u3 + u5 + \ldots$ (max)
- For P_3 at $K^2/3\lambda$ the first blackened ring contains the 4th, 5th, 6th zones, the second blackened ring contains the 10th, 11th, and 12th zones, etc.; $u = (u1 - u2 + u3) + (u7 - u8 + u9) + \ldots$ (max, less intense than 1st)
- For P_2 at $K^2/2\lambda$ the first blackened ring contains the 3rd and 4th half-period zones… $u = (u1 - u2) + (u5 - u6) + \ldots$ (min)
- If a plane wave is incident normally on a zone plate, then the corresponding focal points are at K^2/λ, $K^2/3\lambda$, $K^2/5\lambda$...
- +ve and –ve Zone plates

Applications

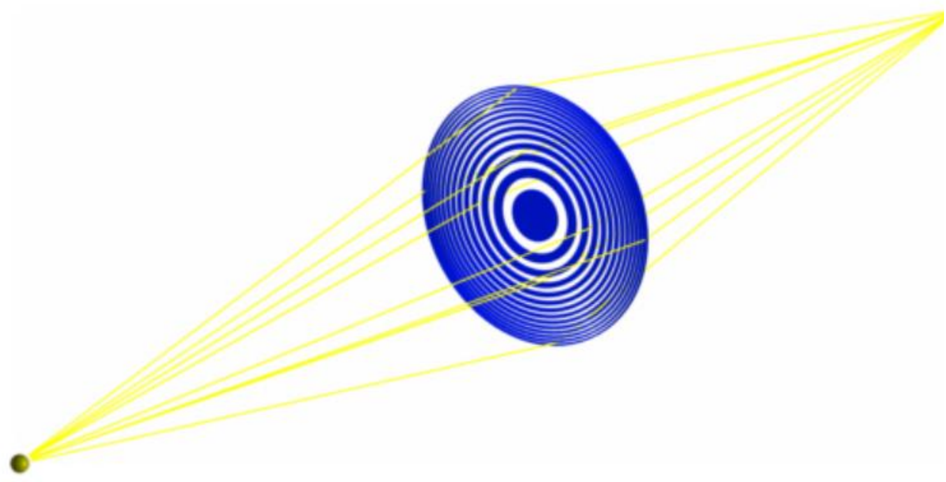
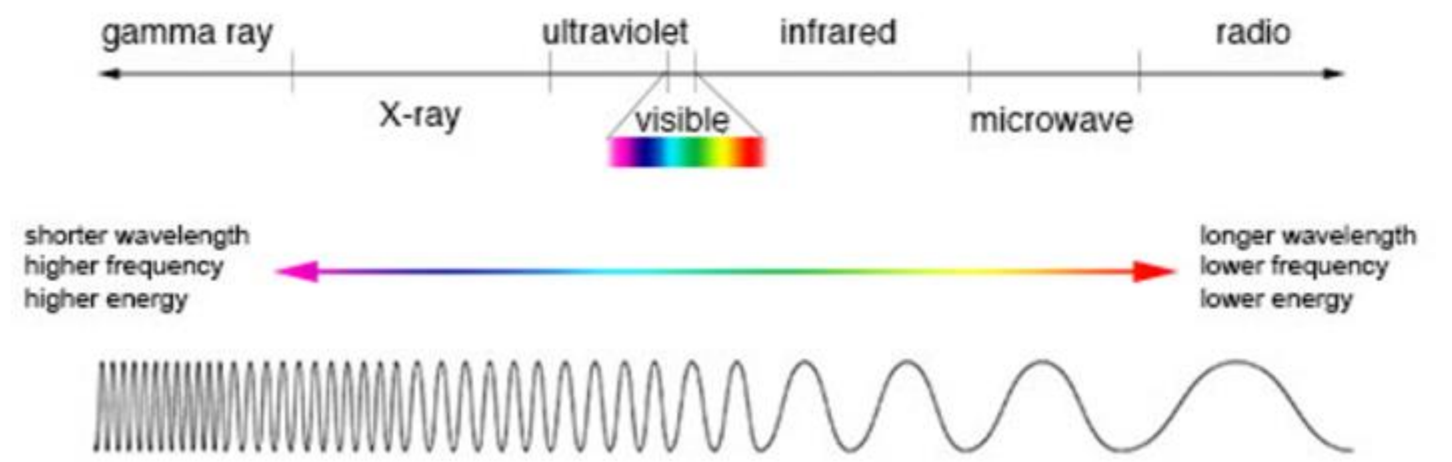


Fig. 1: Focusing X-rays with a diffracting zone plate



Comparison of wavelength, frequency and energy for the electromagnetic spectrum. (Credit: NASA's Imagine the Universe)

There are many wavelengths of light outside of the visible area of the electromagnetic spectrum where traditional lens materials like glass are not transparent, and so lenses are more difficult to manufacture. Zone plates eliminate the need for finding transparent, refractive, easy-to-manufacture materials for every region of the spectrum.

Photography: soft focus image

Fresnel zone antenna

http://www.x-ray-optics.de/index.php/en/types-of-optics/diffracting-optics/fresnel-zone-plates

https://en.wikipedia.org/wiki/Zone_plate

Problem-2

Consider a zone plate with radii

$$r_n = 0.1\sqrt{n} \quad \text{cm}$$

For $\lambda = 5 \times 10^{-5}$ cm, calculate the positions of various foci.

Solution-2

The most intense focal point will be at a distance

$$\frac{r_1^2}{\lambda} = \frac{0.01}{5 \times 10^{-5}} = 200 \text{ cm}$$

The other focal points will be at distances of 200/3, 200/5,and 200/7 cm, etc. Between any two consecutive foci there will be dark points on the axis corresponding to which the first circle will contain an even number of half-period zones.

Thank You