ASSIGNMENT-3

Course: DSCS (CS1005)
Name: P. Veerish
Rollno: CS22B2026

Infinite and finite sets.

1. Consider the following statements

S1: There exists infinite sets A, B, C such that $A \cap (B \cup C)$ is finite

S2: There exists two irrational numbers x and y such that (x+y) is rational

Which of the above statements are true?

Ans S1:

Given A, B, C are infinite set

Let us prove it by taking an example

$A = \{\cancel{8} 2, 4, 6, 8, \ldots\}$ even numbers

$B = \{1, 3, 5, 7, \ldots\}$ odd numbers

$C = \{\cancel{1}, 2, 3, 5, 7, 11, 13, \ldots\}$ prime numbers

$B \cup C = \{1, 2, 3, 5, 7, 9, 11, 13, \ldots\}$

$A \cap (B \cup C) = \{2\}$

which is a finite set.

hence S1 is true

S2:

Let us prove it by taking ~~two~~ one example

Let $x = 3 + \sqrt{5}$ & $y = 3 - \sqrt{5}$

$x + y = 3 + \sqrt{5} + 3 - \sqrt{5} = 6$ which is rational.

hence S2 is true

$\therefore$ Both the statements S1 and S2 are true

2. If S is an infinite set and $S_1, \ldots S_n$ be sets such that

$S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_n = S$ then

(a) atleast one of the set S_i is a finite set

(b) not more than one of sets S_i can be finite

(c) atleast one of the set S_i is an infinite set

(d) not more than one of the sets S_i can be infinite

Name: P.Veeresh
Rollno: CS22B2026

Ans In the question it is given that S is infinite

~~In~~ option A is wrong because it say atleast one of the set S_i is finite it means if all sets are finite then S is finite.

Option D is wrong because it says that not more than one set is ~~f~~ infinite it takes us to the previous option A when all are finite S is also finite.

In option B ~~atleast~~ not more than one of the sets S_i can be finite it means at maximum one of the S_i's will be finite

$\Rightarrow$ S is an infinite set.

In option C atleast one of the sets S_i is an infinite sets means if one set is infinite then S is infinite.

So, option is also true.

But option C is a superset of all the possibilities of option B

Hence option C is most appropriate solution.

$\therefore$ option C is the correct answer.

3. If x is a set and thset contains an integer which is neither positive nor negative then set x is ——

Ans an integer which neither positive nor negative is $\{0\}$.

$\Rightarrow x = \{0\}$

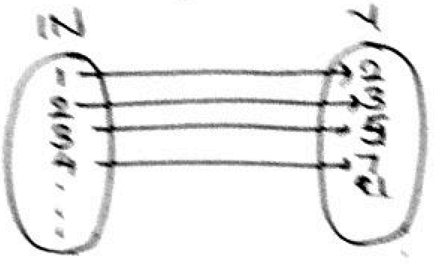
$\therefore$ Given set is non-empty and finite.

Name: P Veerush
Rollno: CS22B2026

4. If $x \in \mathbb{N}$ and x is prime, then x is ___ set

Ans $x = \{2, 3, 5, 7, \ldots\}$

Let us build a bijection from $\mathbb{N} \to x$.



$\because \exists$ bijection from $\mathbb{N} \to x$

$\therefore$ x is an infinite set.

5. If x is a set and the set contains the real number between 1 and 2, then the set is.

Ans Proof by contradiction

Assume that $[0,2]$ is a countable set

$\Rightarrow \exists$ enumeration

$a_1 = 0.a_{11}a_{12}\ldots$ $\quad\quad \forall_{i,j}\ a_{ij} \in [0,9]$ 0-9

$a_2 = 0.a_{21}a_{22}\ldots$

$\vdots$

Let $b = 0.y_1y_2\ldots$

$y_1 = 0$ if $a_{11} \neq 0$

$y_1 = 1$ if $a_{11} = 0$

$y_i = 0$ if $a_{ii} \neq 0$

$y_i = 1$ if $a_{ii} = 0$

$\Rightarrow \nexists$ enumeration for $b \in [0,2]$

$\therefore$ Our assumption is wrong

$\therefore$ $[0,2]$ is uncountable set.

As we know all uncountable sets are infinite

$\therefore$ $[0,2]$ is an infinite set.

Name: P. Veeresh
Roll no: CS22B2026

6. State which of the following sets are finite or infinite:

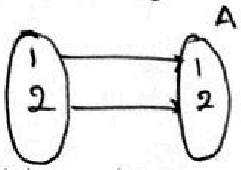
(i) $\{x : x \in N \text{ and } (x-1)(x-2)=0\}$

Ans $(x-1)(x-2)=0$

$x = 1, 2.$

So, given set $=\{1,2\}$ has only two elements (countable)

~~Hence~~,



$\because \exists$ bijection from $\{1,2\} \longrightarrow \{1,2\}$ $k=|A|=2.$
hence, given set is finite.

(ii) $\{x : x \in \mathbb{N} \text{ and } x^2=4\}$

Ans Given $x^2=4$

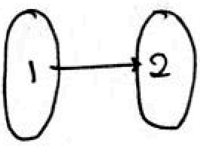
$(x+2)(x-2)=0$

$x = 2, -2$

$\because x \in \mathbb{N} \Rightarrow x=2.$

So, given set $=\{2\} = A$

$\because \exists$ bijection $\{1\} \longrightarrow A$ $k=|A|=2$



$\therefore$ Given set is finite.

(iii) $\{x : x \in N \text{ and } 2x-1=0\}$

Ans Given $2x-1=0$

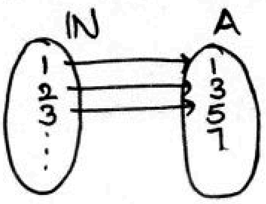
$x = 1/2$

$\because x \in \mathbb{N} \Rightarrow$ given set $=\{\phi\} = A$

$\therefore$ Given set is finite

(iv) $\{x : x \in \mathbb{N} \text{ and } x \text{ is } \text{~~prime~~ odd}\}$

Ans



$\because \exists$ bijection $\mathbb{N} \longrightarrow A$

$\therefore$ A is a countable infinite set

Name: P.Veeresh
Rollno: CS22B2026

7. Given $\Sigma = \{a, b\}$, which one of the following sets is not countable?

(a). Set of all strings over Σ

(b) Set of all languages over Σ

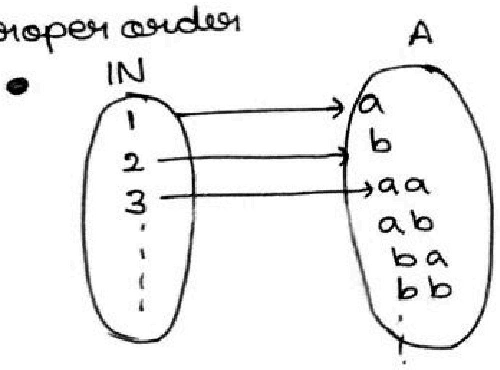
(c) Set of all regular languages over Σ

(d) Set of all languages over Σ accepted by Turing machine.

Ans (a) A set Σ^* is countable because each element of this set can be generated in the following order (called proper order).

Let $\Sigma = \{a, b\}$

So, proper order



$\because \exists$ bijection from $\mathbb{N} \longrightarrow A$

$\therefore$ Set of all strings over Σ is countable infinite set.

(d) The set of all languages accepted by Turing machine (TM) is the set of all TMs basically, which is countable because each TM can be represented by a binary string and each binary string can be obtained in a proper order (as stated over) and checked whether it's TM or not.

(c) The set of all regular languages is a subset of the set of all recursively enumerable languages. And a subset of a countable set is always countable. This is because all ~~strings~~ elements of the countable set can be written in a specific order and each of that element can be checked for its membership in the other set.

Name: P.Veeresh
Roll no: CS22B2026

(b) Power of set of an infinite set is uncountable. Set of languages over Σ is the power set of set of strings over Σ which is an infinite set. Hence set of all languages becomes an uncountable set.

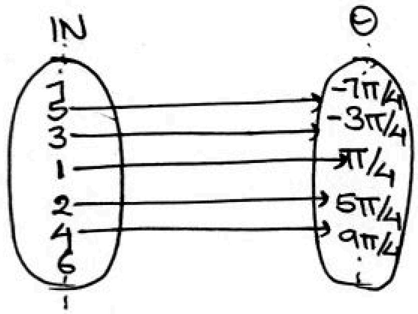
$\therefore$ option (b) is correct.

8. If $\tan\theta = 1$ then the solution set of the equation is?

Ans $\tan\theta = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \theta = n\pi + \frac{\pi}{4} \quad n \in \mathbb{Z}$

$\theta = \{x : x \in (n\pi + \pi/4), n \in \mathbb{Z}\}$



$\because \exists$ bijection $\mathbb{N} \longrightarrow \theta$

$\therefore$ θ is a ~~uncou~~ countable infinite set.

9. If $A = \{x : x \in R$ such that $x^2 - 7x + 12 = 0\}$ then A is a/an?

Ans Given $x^2 - 7x + 12 = 0$

$x^2 - 3x - 4x + 12 = 0$

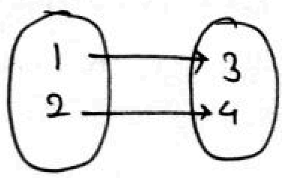
$x(x-3) - 4(x-3) = 0$

$(x-4)(x-3) = 0$

$x = 3, 4$

$A = \{3, 4\}$

$\because \exists$ bijection $\{1,2\} \longrightarrow A \quad k = |A| = 2.$



$\therefore$ A is a finite set.

Name: P. Veeresh
Roll no: CS22B2026

10. Show the R is infinite set.

Ans To prove R is infinite, we need to establish a function $f: R \to R$ such that f is 1-1 and $f(R) \subset R$.

Consider the function $f(x) = x+2$ if $x \geq 0$ and
$f(x) = x+1$ if $x < 0$

This function is 1-1 but not onto ($\because$ 1 does not have a pre image) and $f(R) \subset R$.

Hence, R is infinite.

Graph Theory.

1. Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is.

Ans To get the maximum edges, take one vertex each for each component, except the last component. Now, (k-1) components have 1 vertex each and the last component has n-(k-1) vertices. Make the last component complete i.e. it has

$$= {}^{n-(k-1)}C_2 = \frac{(n-(k-1))(n-(k-1)-1)}{2} = \frac{(n-k)(n-k+1)}{2}$$

Number of edges(e) = $\frac{(n-k)(n-k+1)}{2}$

From given data

$k=2$ $n=10$

$$e = \frac{(10-2)(10-2+1)}{2} = \frac{\overset{4}{\cancel{8}} \times 9}{\cancel{2}} = 36$$

$\therefore$ Maximum number of edges is 36 edges.

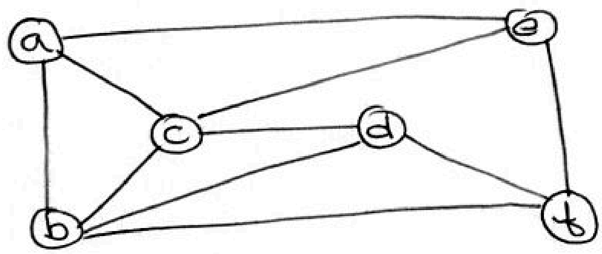
Name : P.Veerush
Roll no: CS22B2026

2. Let G be an undirected complete graph on n vertices, where n>2, Then the number of different Hamiltonian cycles in G is equal to

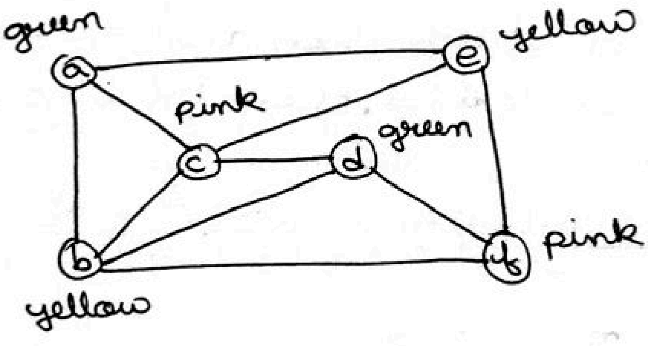
Ans no. of different hamiltonion cycles in undirected complete graph is equal to no. of cyclic graph possible with n vertices.

∴ number of different hamiltonian cycles = $\frac{(n-1)!}{2}$.

3. The chromatic number of the following graph is



Ans chromatic number (X): minimum number of colours required to properly colour a graph



∴ $\chi(G) = 3$.

Name: P.Veeresh
Roll no: CS22B2026

4. The maximum number of edges in a bipartiate graph on 12 vertices is

ans Let V_1 and V_2 are the bipartition of graph where

$|V_1| = k$ and $|V_2| = n-k$.

Total no. of edges $= k(n-k) \cdot e(k) = kn - k^2$

no. of edges is maximum when

$e(k)$ is maximum

$e'(k) = 2k - n = 0$

$\Rightarrow k = n/2$.

when $k = n/2$ then the bipartiate graph will have the maximum number of edges.

$\Rightarrow$ maximum number of edges $= \frac{n}{2}(n - n/2) = \frac{1}{4}n^2$ if n is even

$\frac{n^2-1}{4}$ if n is odd.

$\because n = 12$.

maximum no. of edges $= \frac{1}{4}(12)^2 = \frac{144}{4} = 386$

5. Let G be a simple undirected planar on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G in the plane is equal is

ans From Euler's rule

$v + f = e + 2$

where

v = vertices

f = faces

e = edges

$v = 10 \quad e = \cancel{10}\,15\cancel{+2}$

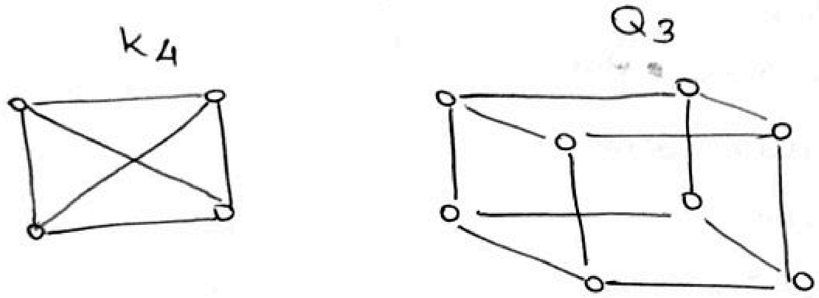
$10 + f = \cancel{10+17}\,15 + 2$

$f = 7$

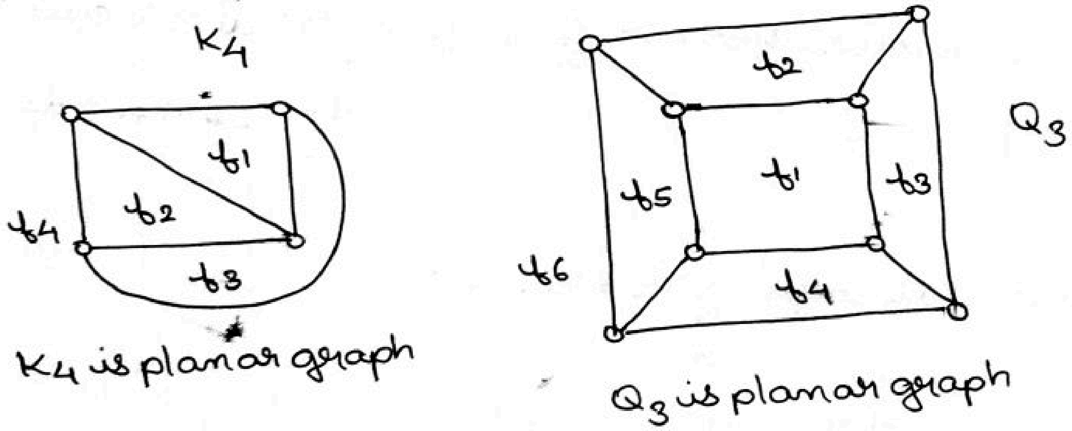
Out of 7, there will be always one unbounded face.

So, the number of faces is 6.

6. Identify the planar graphs from the below two graphs.

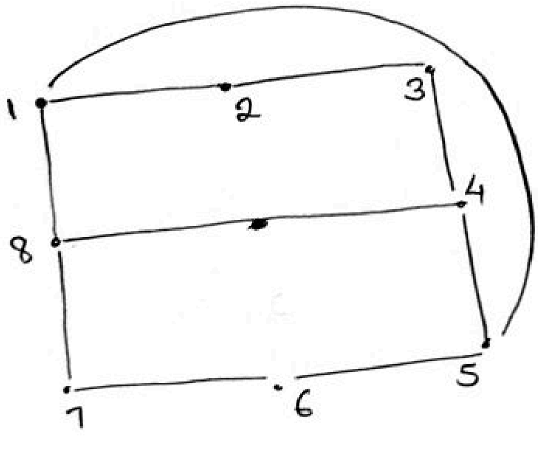


Ans Planar graph: A graph with planar drawing such that no two edges are crossed.



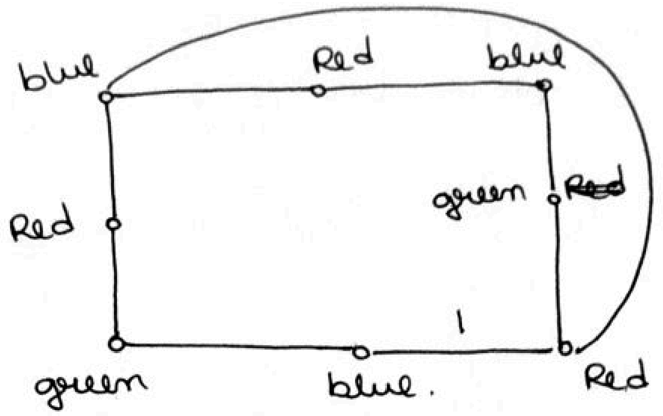
∴ Both K_4 & Q_3 are planar graphs.

7. What is the chromatic number of following graph?



Name : P.Veeresh
Roll no : CS22B2026

Ans

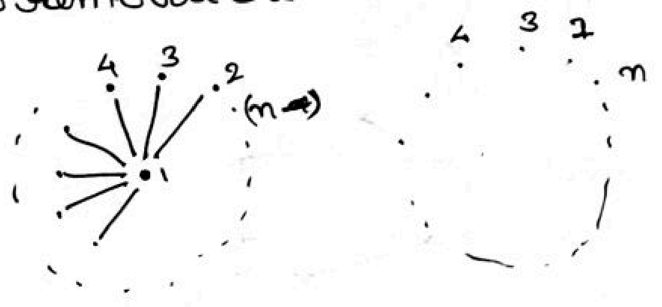


$\chi(G) = 3.$

8. Let G be an arbitary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between.

Ans Minimum : The removed vertex itself is a separate connected component. So removal of a vertex creates K-1 components.

Maximum : It may be possible that the removed vertex disconnects all components. For example the removed ~~components~~ vertex is center of a star. So removal creates n-1 components.



(n-1) components.

Name: P.Veeresh
Roll no: CS22B2026

9. Graph is self complementary if it is isomorphic to its complement. For all self complementary graph on n vertices, n is.

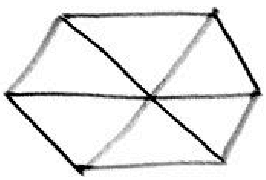
(a) a multiple of 4
(b) Even
(c) Odd
(d) Congruent to 0 mod 4, or, 1 mod 4.

Ans An n-vertex self complementary graph has exactly half number of edges of the complete graph i.e. $\frac{{}^nC_2}{2} = \frac{n(n-1)}{2}$ edges

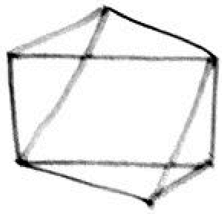
Since n(n-1) must be divisible by 4, n must be congruent to 0 mod 4 or 1 mod 4.

10. Which one of the following graph is not planar?

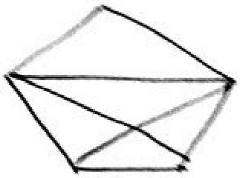
G1:



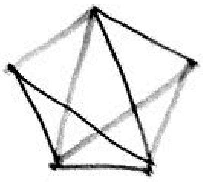
G2:



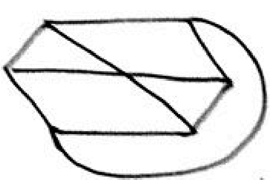
G3:



G4:

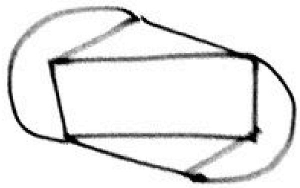


Ans G1:



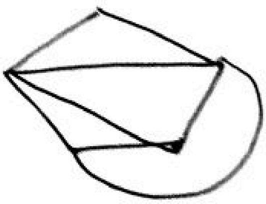
not planar

G2:



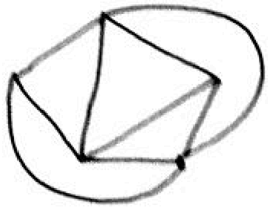
planar.

G3:



planar

G4:



planar.

∴ G1 is not planar.