Electrical Circuits for Engineers (EC1000)

Lecture-9 (b)
AC circuits

Sinusoidal Steady State Analysis (Ch. 10)

Network Theorems

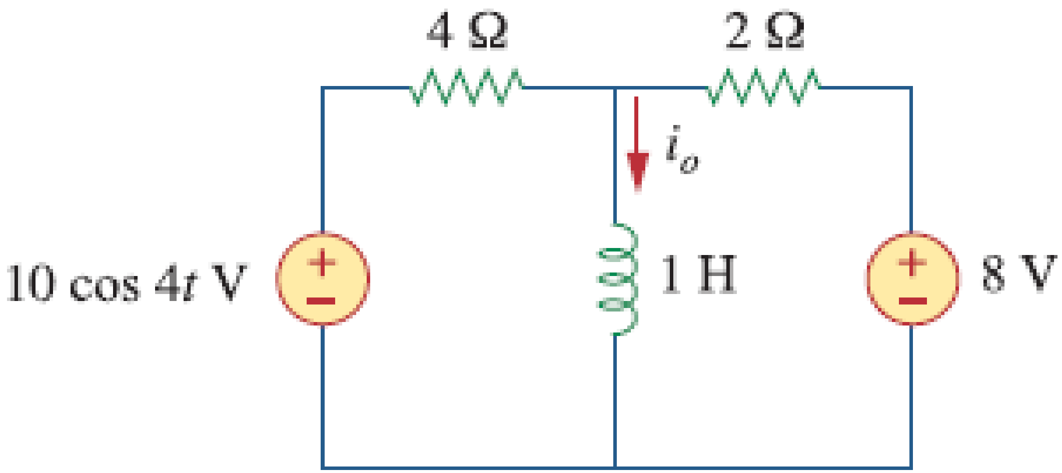
Steps to Analyse AC circuits

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal, Mesh, theorem etc.,)
3. Transform the resulting phasor to the time domain

3.Superposition Theorem

Since ac circuits are linear, superposition theorem applies to ac circuits.

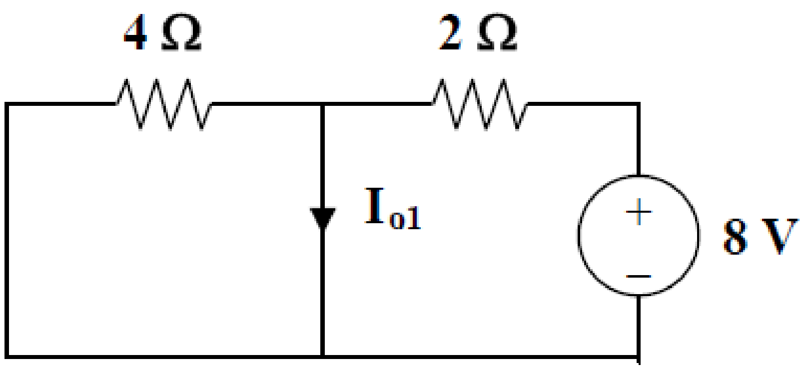
1. Determine the current I_0 in the circuit of figure below using Super Position Theorem.



3.Superposition Theorem

Let $I_o = I_{o1} + I_{o2}$, where I_{o1} is due to the dc source and I_{o2} is due to the ac source. For I_{o1}, consider the circuit in Fig. (a).

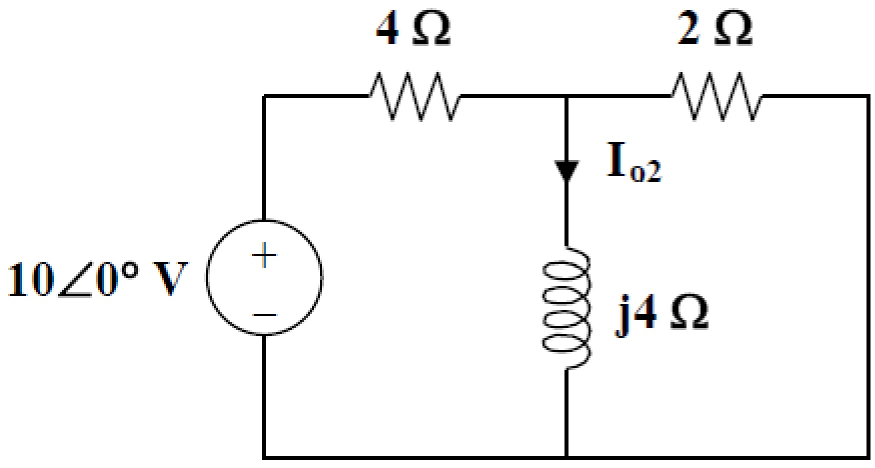
Clearly,



(a)

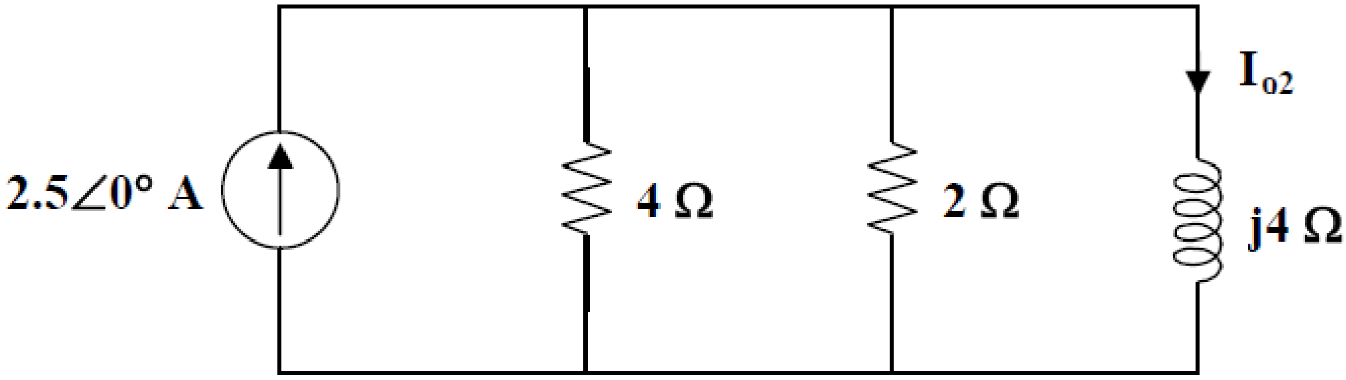
$$I_{o1} = 8/2 = 4 \text{ A}$$

For I_{o2}, consider the circuit in Fig. (b).



(b)

3. Superposition Theorem



(c)

By the current division principle,

$$\mathbf{I}_{o2} = \frac{4/3}{4/3 + j4}(2.5\angle 0^\circ)$$

$$\mathbf{I}_{o2} = 0.25 - j0.75 = 0.79\angle -71.56^\circ$$

Thus, $$I_{o2} = 0.79\cos(4t - 71.56^\circ)\ A$$

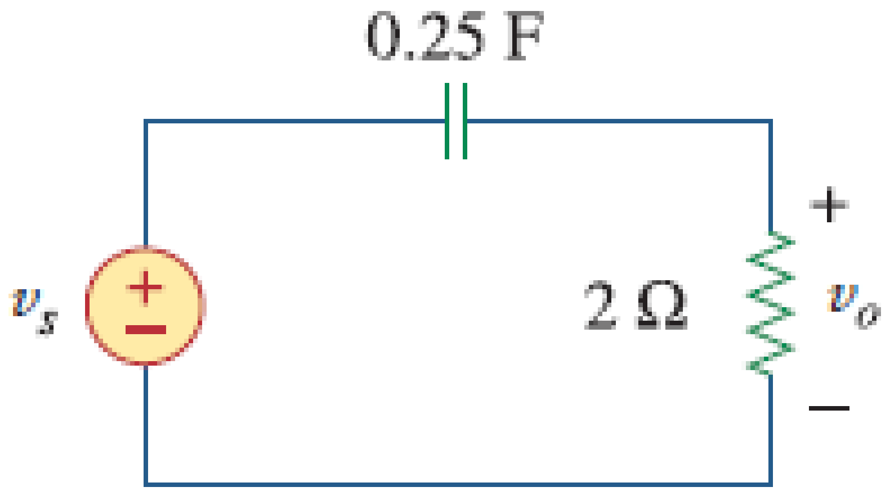
Therefore,

$$\mathrm{I_o} = \mathrm{I_{o1}} + \mathrm{I_{o2}} = \mathbf{[4 + 0.79\cos(4t - 71.56^\circ)]\ A}$$

Example Problem

Find v_o for the circuit in Fig

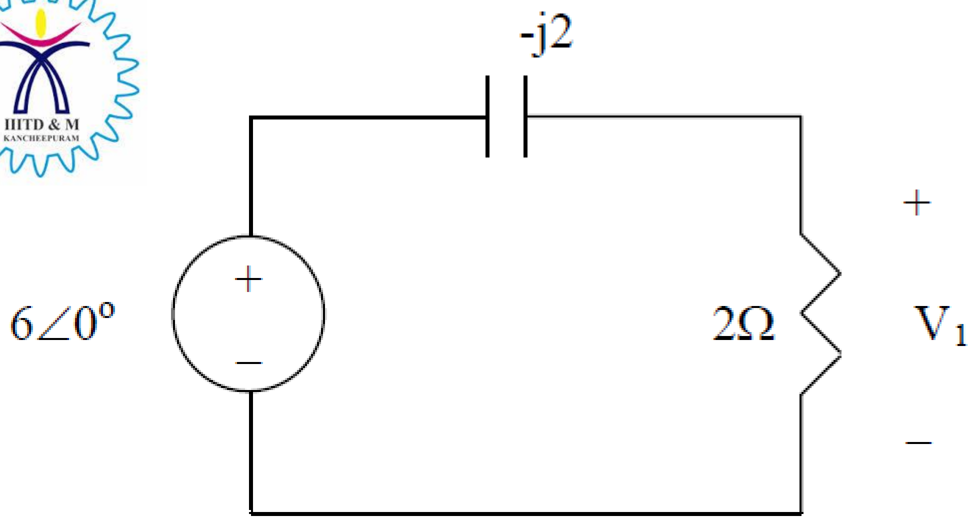
$v_s = 6\cos 2t + 4\sin 4t$ V.

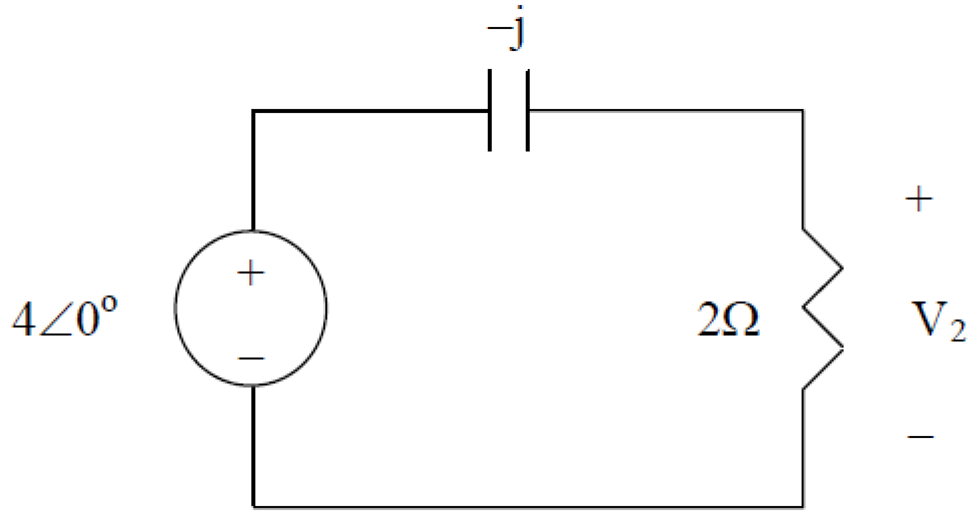


We apply superposition principle. We let

$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to the sources 6cos2t and 4sin4t respectively.





$$\mathbf{V_1} = \frac{2}{2 - j2} V\ (6) = 3 + j3 = 4.243\angle 45^\circ$$

$v_1(t) = \mathbf{4.243\cos(2t+45^\circ)\ volts}.$

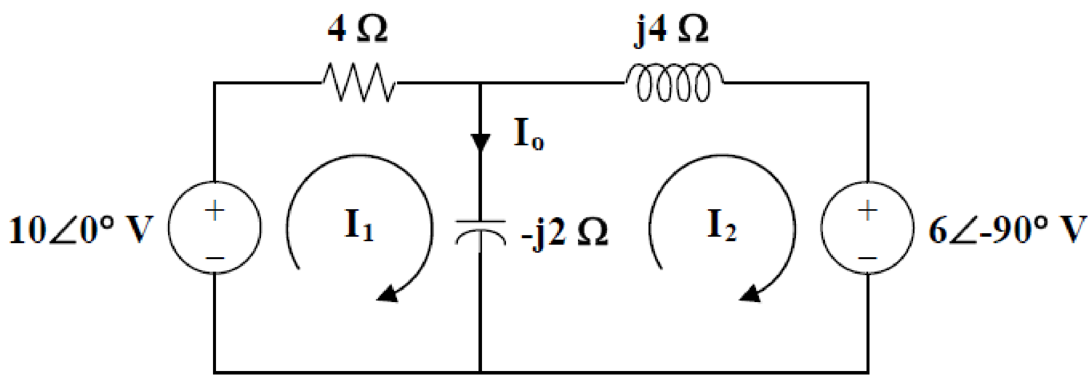
$$\mathbf{V_2} = \frac{2}{2 - j}(4) = 3.2 + j11.6 = 3.578\angle 26.56^\circ$$

$v_2(t) = 3.578\sin(4t+25.56^\circ)$ volts.

$$v_o = \mathbf{[4.243\cos(2t+45^\circ) + 3.578\sin(4t+25.56^\circ)]\ volts}.$$

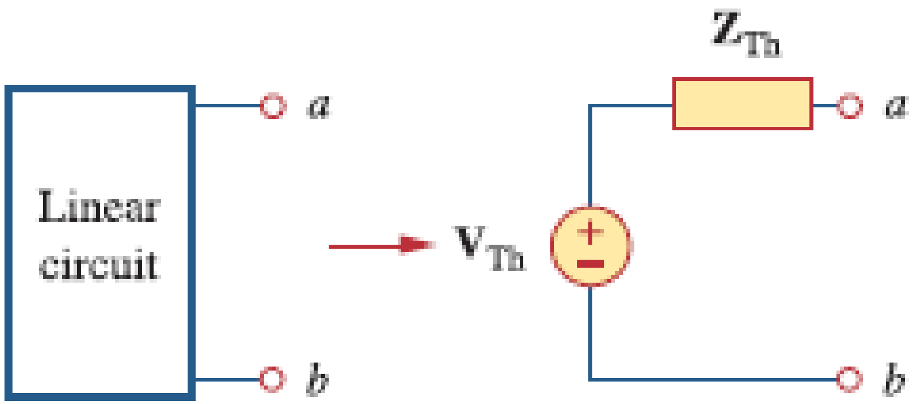
Practice Problem

1. Find I_0 in the circuit of figure below using Super Position Theorem

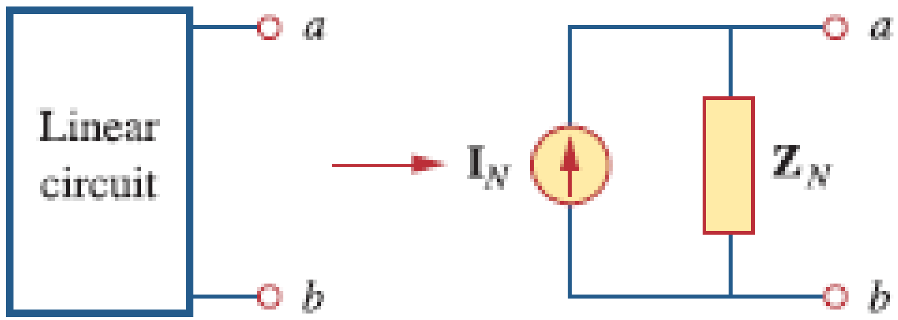


$$\boxed{i_o(t) = \mathbf{1.4142\cos(2t + 45°)\ A}}$$

4. Thevenin & Norton Equivalent Circuit



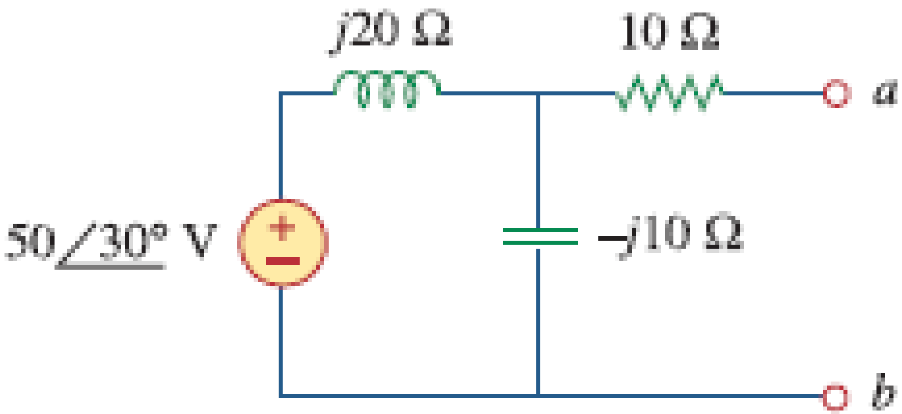
Thevenin Equivalent Circuit



Norton Equivalent Circuit

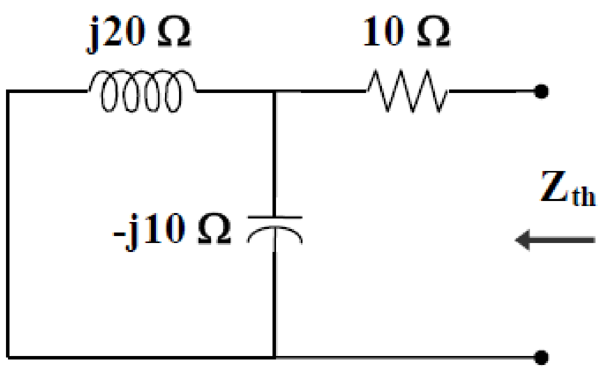
Example Problem

1. Find Thevenin and Norton Equivalent Circuits at the terminals a-b



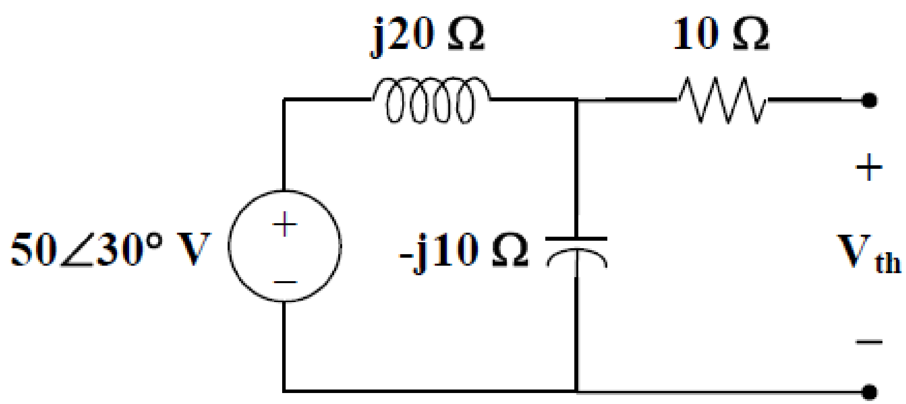
Solution:

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 10 + j20 \,||\, (-j10) = 10 + \frac{(j20)(-j10)}{j20 - j10}$$

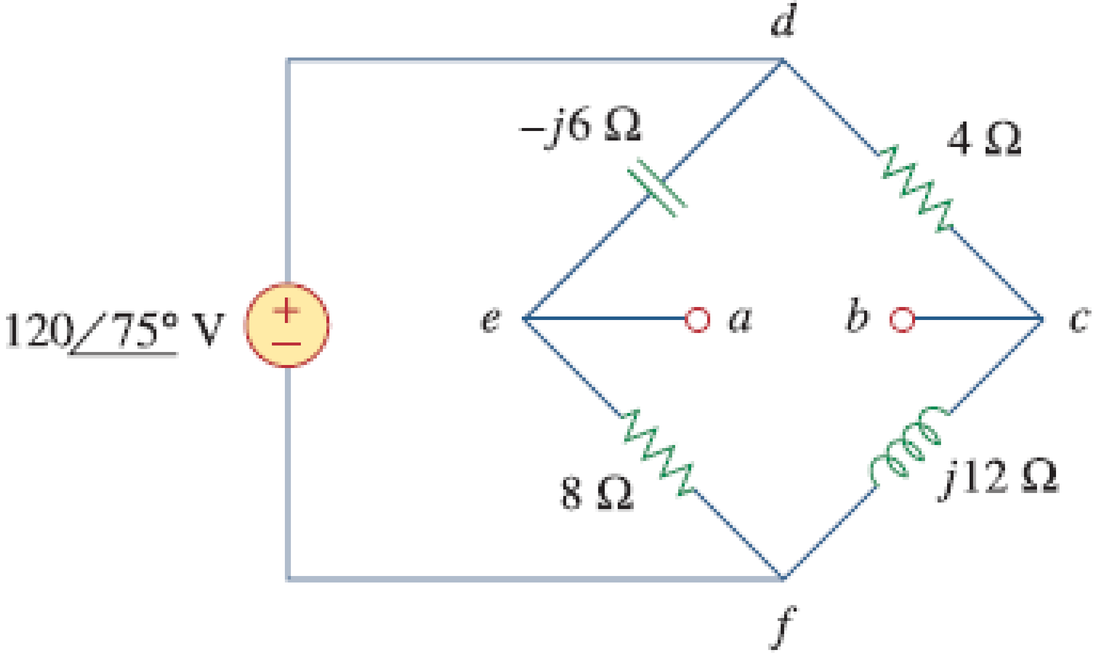
$$= 10 - j20 = \mathbf{22.36\angle\text{-}63.43° \;\Omega}$$

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(50\angle 30°) = \mathbf{-50\angle 30° \;V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{-50\angle 30°}{22.36\angle -63.43°} = \mathbf{2.236\angle 273.4° \;A}$$

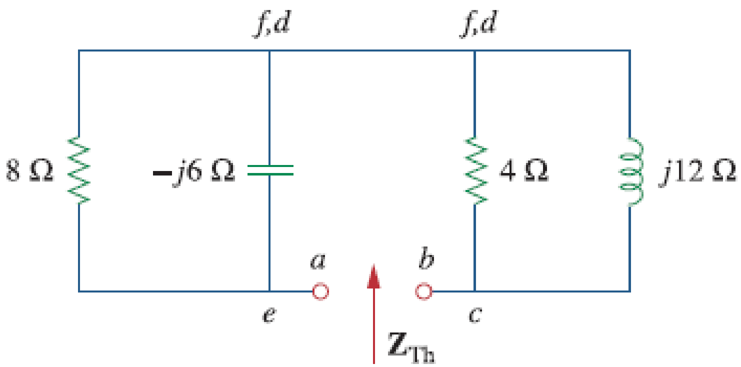
Example Problem

2. Obtain the Thevenin Equivalent at terminals a-b for the below circuit



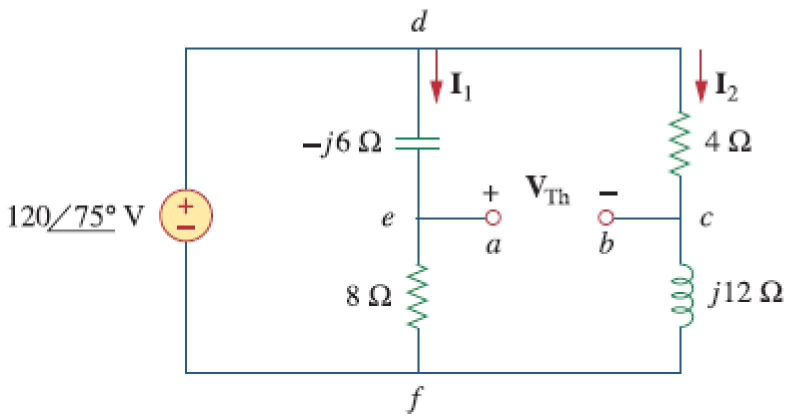
Solution

Z_{th}



(a)

V_{th}



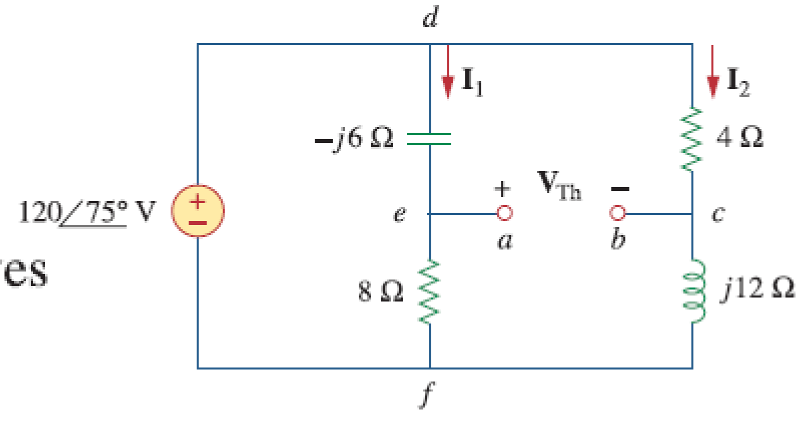
(b)

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{\text{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{\text{Th}}$, consider the circuit in Fig. 10.23(b). Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\underline{/75^\circ}}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\underline{/75^\circ}}{4 + j12}\ \text{A}$$



(b)

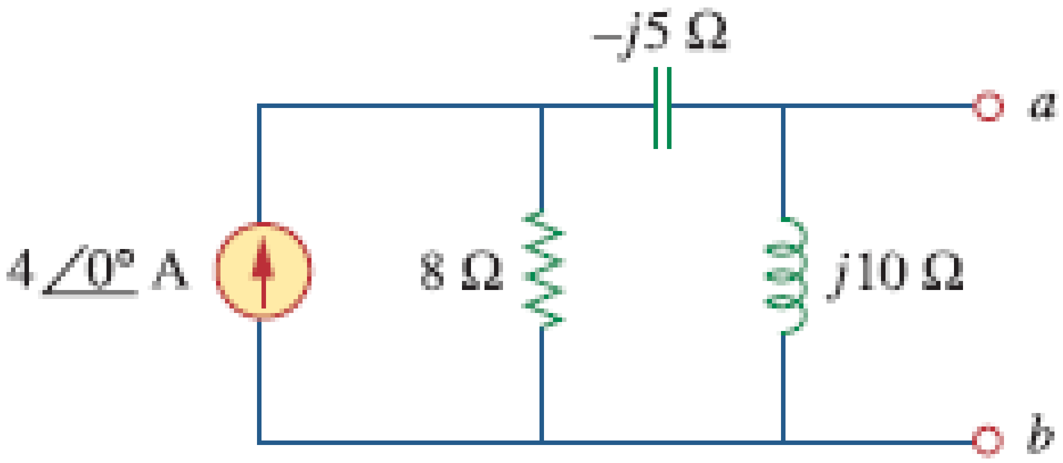
Applying KVL around loop *bcdeab* in Fig. 10.23(b) gives

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

$$\begin{aligned}\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= \frac{480\underline{/75^\circ}}{4 + j12} + \frac{720\underline{/75^\circ + 90^\circ}}{8 - j6} \\ &= 37.95\underline{/3.43^\circ} + 72\underline{/201.87^\circ} \\ &= -28.936 - j24.55 = 37.95\underline{/220.31^\circ}\ \text{V}\end{aligned}$$

Practice Problem

1. Obtain the Thevenin Equivalent at terminals a-b for the below circuit



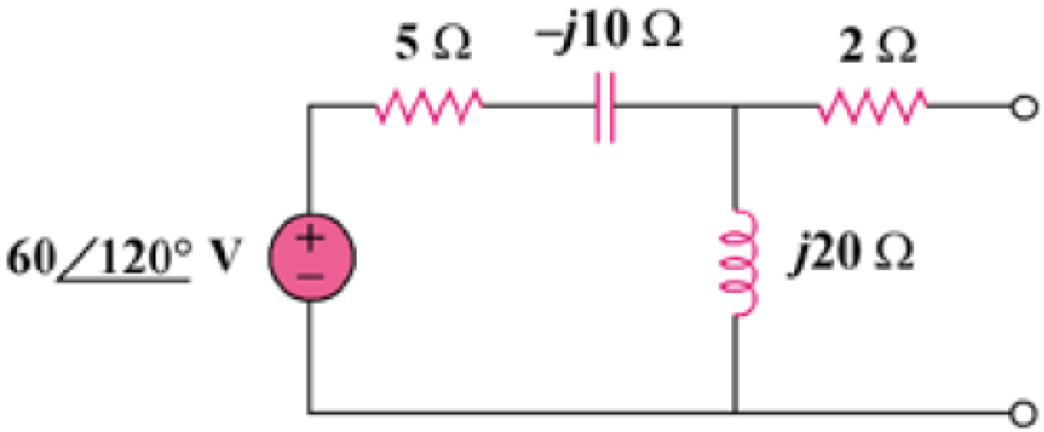
$$\mathbf{Z}_N = \mathbf{Z}_{th} = j10 \,||\, (8 - j5) = \frac{(j10)(8 - j5)}{j10 + 8 - j5} = \mathbf{10\angle 26^\circ\ \Omega}$$

$$\mathbf{V}_{th} = j10\mathbf{I}_o = \frac{j320}{8 + j5} = \mathbf{33.92\angle 58^\circ\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{33.92\angle 58^\circ}{10\angle 26^\circ} = \mathbf{3.392\angle 32^\circ\ A}$$

Practice Problem

2. Find the Thevenin and Norton equivalent circuits for the circuit shown in



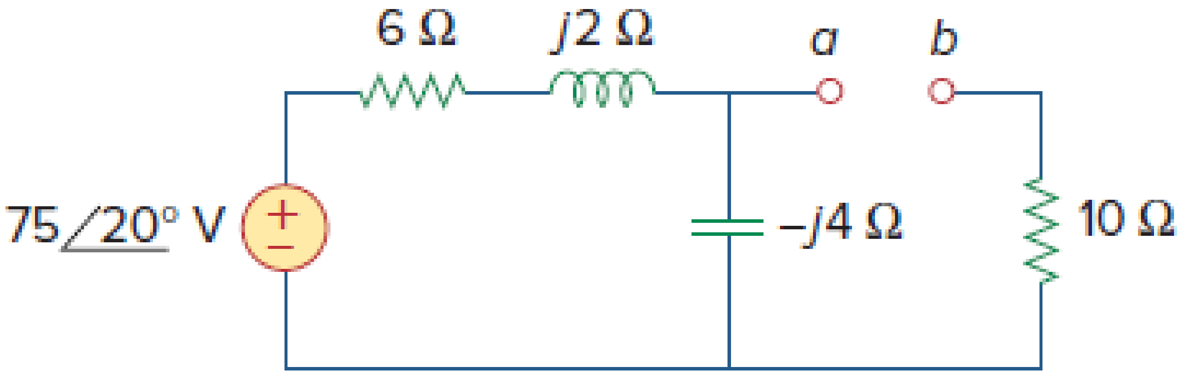
$$Z_{th} = \mathbf{21.633\angle{-33.7^\circ}\ \Omega}$$

$$V_{th} = \frac{j20}{5 - j10 + j20}(60\angle 120^\circ) = \frac{j4}{1 + j2}(60\angle 120^\circ)$$
$$= \mathbf{107.3\angle 146.56^\circ\ V}$$

$$I_N = \frac{V_{th}}{Z_{th}} = \frac{107.3\angle 146.56^\circ}{21.633\angle -33.7^\circ} = \mathbf{4.961\angle{-179.7^\circ}\ A}$$

Practice Problems

1. Find the Thevenin's equivalent circuit for the circuit shown below.



Answer: $\mathbf{Z}_{Th} = 12.4 - j3.2\ \Omega$, $\mathbf{V}_{Th} = 47.43\underline{/-51.57^\circ}$ V.

2. Find the Norton equivalent circuits for the circuit shown below.