Linear Algebra

Tutorial - 1

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(1) Are the following two systems of linear equations equivalent?

$-x_1 + x_2 + 4x_3 = 0$	$x_1 - x_3 = 0$
$x_1 + 3x_2 + 8x_3 = 0$	$x_2 + 3x_3 = 0$
$0.5x_1 + x_2 + 2.5x_3 = 0$	

(2) Let F be a set which contains exactly three elements $0, 1, 2$. Define an addition and multiplication by the tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Prove or disprove that $(F, +, \times)$ is a field.

(3) Define (i) a subfield and (ii) the characteristic of a field.

(4) Prove that each subfield of the field of complex numbers contains every rational number.

(5) Prove that if two homogeneous systems of linear equations in two unknowns have the same solutions, then they are equivalent.

(6) Prove or disprove that if A is an $m \times n$ matrix, B is an $n \times m$ matrix and $n < m$, then AB is not invertible.

(7) Let

$$A = \begin{bmatrix} \frac{1}{3} & 2 & -6 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \\ -\frac{7}{3} & 2 & -\frac{8}{3} \end{bmatrix}.$$

Does there exist a 3×4 matrix B such that (i) $AB = 0$ and (ii) $B \neq 0$?

(8) Prove of disprove that A is invertible and find A^{-1} if it exists where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$