# IIITDM KANCHEEPURAM

## MA1000 Calculus
## Problem Set 1

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1. Prove using the definition of convergence that the following sequences converge:
   (a) $\left\{\dfrac{1}{n^2}\right\}$; (b) $\left\{\dfrac{(-1)^{n+1}}{n}\right\}$.

2. Prove: $\dfrac{2^n}{n!} \longrightarrow 0$.

3. Find the limit of the sequence $\{a_n\}$ if (a) $a_n = \left(1+\dfrac{1}{n}\right)^n$; (b) $a_n = \left(1+\dfrac{3}{4n}\right)^{\frac{8}{3}n}$.

4. If $\alpha$ is a rational number, find $\lim\limits_{n\to\infty} \sin(n!\alpha\pi)$.

5. Calculate $\lim\limits_{n\to\infty}(\sqrt{n^2+n}-n)$.

6. Find the limit of the sequence $\{a_n\}$, where $a_1 = \sqrt{2}$, $a_n = \sqrt{2+\sqrt{a_{n-1}}}$ for $n \geq 2$.

7. Prove that the sequence $\{a_n\}$, where $a_1 = 10$ and $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{10}{a_n}\right)$ for $n \geq 1$, converges. Also find its limit.

8. Let $a_1 = 1$ and $a_2 = 1$. For $n \geq 3$, let $a_n = a_{n-2} + a_{n-1}$. Then the sequence $\{a_n\}$ is called the Fibonacci sequence. Find $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n}$.

9. Find the limit of the sequence $\{a_n\}$ if $a_n = \dfrac{1}{n+1} + \dfrac{1}{n+2} + \cdots + \dfrac{1}{2n}$.

10. Find the limit of the sequence $\{a_n\}$ if $a_n = \dfrac{1}{(n+1)^2} + \dfrac{1}{(n+2)^2} + \cdots + \dfrac{1}{(2n)^2}$.

11. Let $\epsilon > 0$ be given. Find a natural number $N$ as required by the definition of convergence of a sequence for proving the following limits:
    (a) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n}-1}{\sqrt{n}+1} = 1$; (b) $\lim\limits_{n\to\infty} n^{1/n} = 1$

12. Let $a_n = \dfrac{1}{\ln(n+1)}$.
    (a) Show that the sequence $\{a_n\}$ converges to zero.
    (b) Find a natural number as required of the definition of convergence for
    (i) $\epsilon = 0.5$; (ii) $\epsilon = 0.1$.