Engineering Electromagnetics

Lecture 33

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by

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Motional electric field

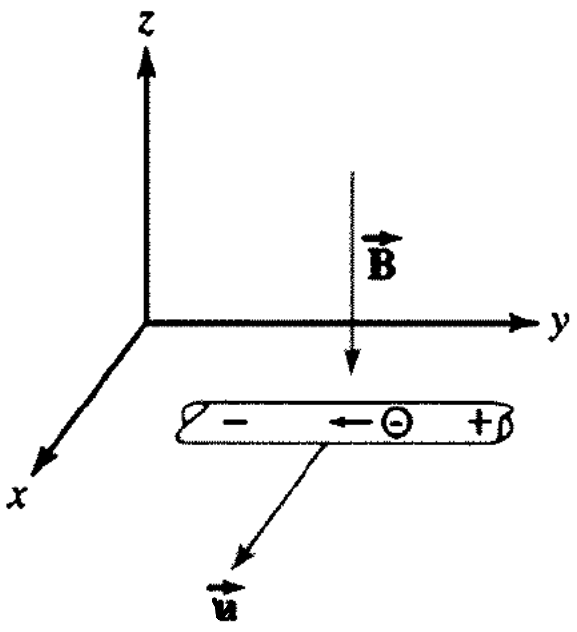


Figure 7.1 A conductor moving in a uniform magnetic field

uniform flux density $\vec{\mathbf{B}}$ such that $\vec{\mathbf{B}} = -B\vec{\mathbf{a}}_z$. The magnetic force acting on each of the free electrons in the conductor is

$$\begin{aligned} \vec{\mathbf{F}} &= q_e\vec{\mathbf{u}} \times \vec{\mathbf{B}} \\ &= q_e u B\vec{\mathbf{a}}_y \end{aligned} \tag{7.1}$$

$$\vec{\mathbf{E}} = \vec{\mathbf{u}} \times \vec{\mathbf{B}} = uB\vec{\mathbf{a}}_y \qquad \textit{motional electric field.}$$

The induced electric field is tangential to the surface of the conductor.

field.

When both v and B(t) exist?

$\frac{d\Phi}{dt} \neq 0$ when (i) B is f(t)

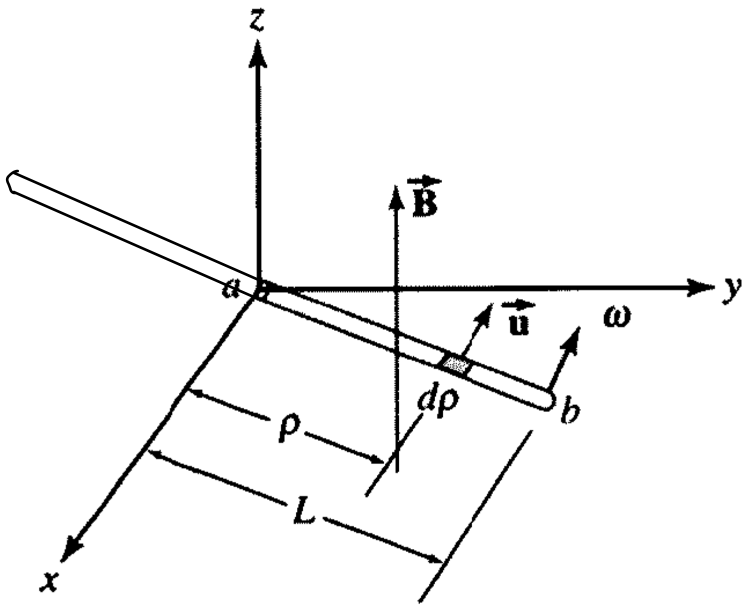
(ii) conductor is in motion

We now define the *electromotive force* or the *induced emf* as the amount of work done per unit positive charge by the external force:

$$e = \frac{dW}{dq} = BLu \tag{7.5}$$

Problem-2

A copper strip of length $2L$ pivoted at the midpoint is rotating with an angular velocity ω in a uniform magnetic field, as illustrated in Figure 7.4. Determine the induced emf between the midpoint and one of the ends of the strip. What is the induced emf between the two ends?



Solution-2

Let us imagine that the copper strip of length $2L$ is made of two copper strips each of length L but joined together at the pivoted end. From Example 7.1, it is then evident that the far end of each strip is at a higher potential with respect to its pivoted end. As both strips are of equal length and are rotating with the same angular velocity in a common uniform magnetic field, the magnitude of the induced emf between the free and the pivoted end of each strip must be the same. Thus, the induced emf between one end of the strip and the midpoint is

$$e_{ba} = \frac{1}{2} B\omega L^2$$

The induced emf between the two far ends of the copper strip is zero.

For a circuit?

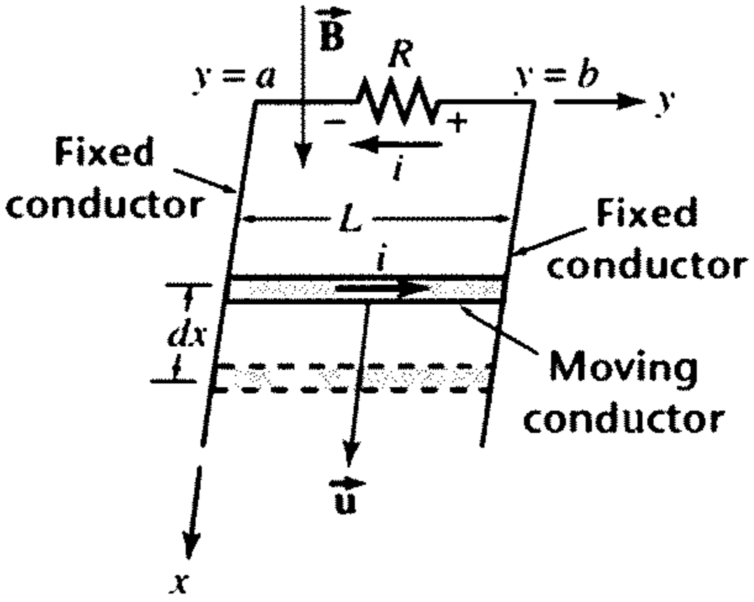


Figure 7.2 A sliding conductor

Let us now examine the situation when the conductor is sliding freely over a pair of stationary conductors, as illustrated in Figure 7.2. A resistor is connected between the far ends of the two stationary conductors.

Where will the e’s get accumulated?

Force

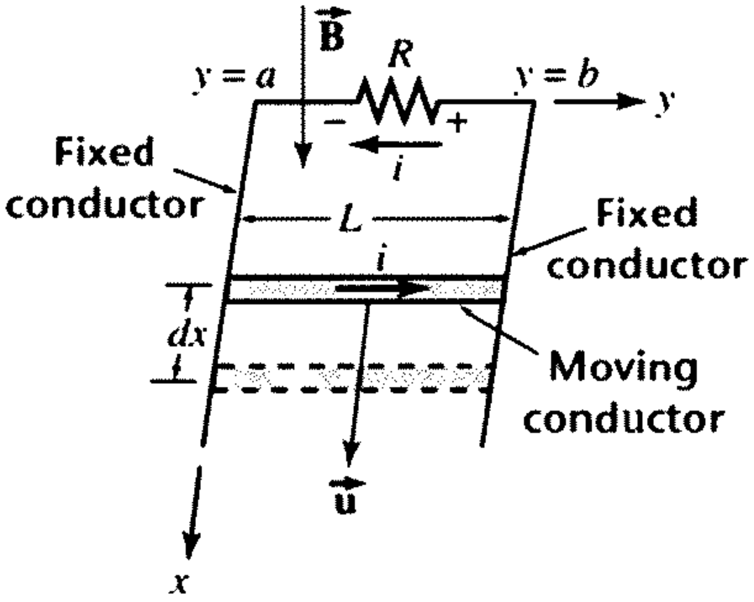


Figure 7.2 A sliding conductor

In accordance with the Lorentz force equation, the magnetic force experienced by the sliding conductor is

$$\vec{\mathbf{F}}_m = i\vec{\mathbf{L}} \times \vec{\mathbf{B}} = -BiL\vec{\mathbf{a}}_x \tag{7.3}$$

where i is the current in the sliding conductor and L is its effective length. As expected, the magnetic force is in a direction that opposes the motion of the conductor. Therefore, we must apply an external force in the x direction to keep the conductor moving in that direction. The external force that must be applied to keep the conductor in motion with a uniform velocity must be

$$\vec{\mathbf{F}}_{\text{ext}} = -\vec{\mathbf{F}}_m = BiL\vec{\mathbf{a}}_x \tag{7.4}$$

For a circuit?

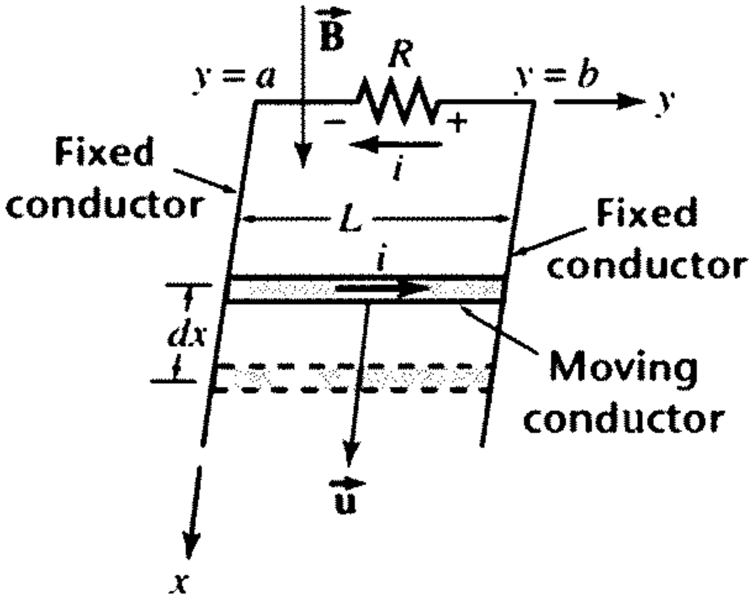


Figure 7.2 A sliding conductor

When the conductor moves a distance dx in a time interval dt, the work done by the external force is

$$dW = BLi\,dx = BLiu\,dt$$

$$dW = BLu\,dq$$

We now define the *electromotive force* or the *induced emf* as the amount of work done per unit positive charge by the external force:

$$\boxed{e = \frac{dW}{dq} = BLu} \quad \text{e=F.dl/dq = E.dl or } f_s\text{.dl in Griffith} \qquad (7.5)$$

Unit of e ?

In this case e is the *induced emf* between the two ends of the sliding conductor. It is also referred to as the *motional emf* because it is due to the motion (flux cutting action) of a conductor in a magnetic field.

General expression for motional emf

The general expression for the force that is required to keep a conductor in motion is

$$\vec{\mathbf{F}}_{ext} = -\int_c i\,\overrightarrow{d\ell}_c \times \vec{\mathbf{B}}$$

where $\overrightarrow{d\ell}_c$ is an element of length of the conductor in the direction of current i and c indicates the path of integration in the direction of the induced current in the conductor. The work done by the external force in moving the conductor a length $\overrightarrow{d\ell}$ in time dt is

$$dW = \vec{\mathbf{F}}_{ext} \cdot \overrightarrow{d\ell} = -i\,\overrightarrow{d\ell} \cdot \int_c \overrightarrow{d\ell}_c \times \vec{\mathbf{B}} \Rightarrow e = \frac{dW}{dq} = -\vec{\mathbf{u}} \cdot \int_c \overrightarrow{d\ell}_c \times \vec{\mathbf{B}}$$

As $\vec{\mathbf{u}}$ does not vary along the length of the conductor, we can also write the above equation as

$$e = -\int_c \vec{\mathbf{u}} \cdot (\overrightarrow{d\ell}_c \times \vec{\mathbf{B}}) = \int_c \vec{\mathbf{u}} \cdot (\vec{\mathbf{B}} \times \overrightarrow{d\ell}_c) \qquad \boxed{e = \int_c (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot \overrightarrow{d\ell}_c}$$

General expression for motional emf

$$\boxed{e = \int_c (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot d\vec{\ell}_c}$$

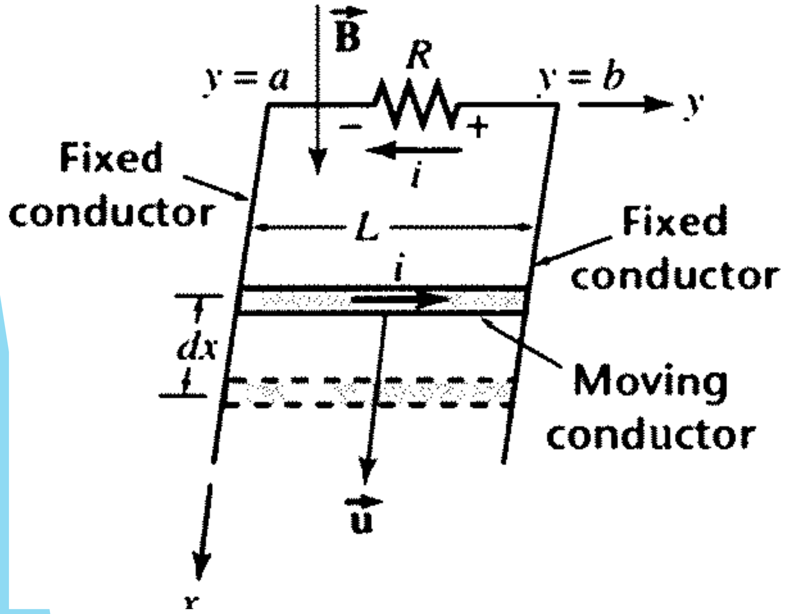
which reduces to (7.5) when $\vec{\mathbf{u}}$, $\vec{\mathbf{B}}$, and the length of the conductor are mutually perpendicular to each other. *Equation (7.6) is the one we will use to determine the motional emf in a conductor moving in a magnetic field.* → e=BLu

$$\mathcal{E} = \oint \mathbf{f}_s \cdot d\mathbf{l}.$$

We mention again that $\vec{\mathbf{u}} \times \mathbf{B}$ is the induced electric field intensity and its direction is the same as that of the induced current in the conductor. This fact enables us to establish the polarity of the motional emf across the two ends of a conductor. In a nutshell, *the induced current due to the motional emf is in the direction of the induced electric field.*

Faraday’s law of induction

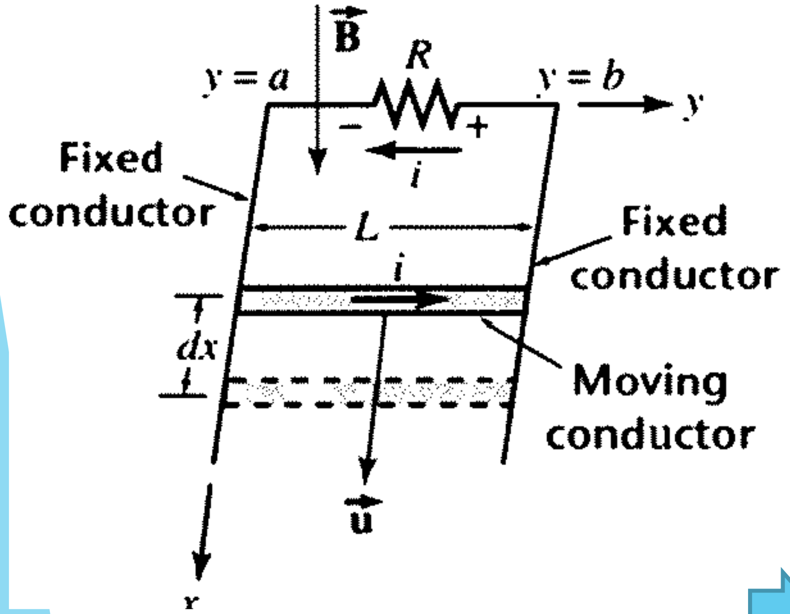
Concept of e can be realized in a diff. way



- a closed circuit/loop
- Due to motion along x change in cross sectional area ds = ?
- Change in flux through the closed loop = ?
- Rate of change of flux through the loop = ?

Faraday's law of induction

Concept of e can be realized in a diff. way



- a closed circuit/loop
- Due to motion along x change in cross sectional area

$$\vec{ds} = L\, dx\vec{a}_z$$

- Change in flux through the closed loop

$$d\Phi = \vec{B} \cdot \vec{ds} = -BL\, dx$$

- Rate of change of flux through the loop

$$\frac{d\Phi}{dt} = -BL\frac{dx}{dt} = -BLu$$

$$e = -\frac{d\Phi}{dt}$$

- Lenz's law?

It states that *the induced emf around a closed path is equal to the negative rate of change of the magnetic flux with respect to time passing through the area enclosed by the path.*

Induced emf

The process of inducing an emf in a coil is known as ***Electromagnetic induction***

How to achieve that?→ Any one of the following should be true

1. Flux through stationary coil is f(t)
2. Coil changes shape/position with t but B is uniform
3. Both 1 and 2 are true

$$\frac{d\Phi}{dt} = -BL\frac{dx}{dt} = -BLu$$

$$e = -\frac{d\Phi}{dt}$$

Thank You