Signals & Systems Tutorial - 4 EC23B1102
Madharnshetty Sathvika.

1) a) $\int_{0}^{\infty} e^{-st} e^{-(5+j\omega)t} dt = \int_{0}^{\infty} e^{-st} e^{-st} dt = \int_{0}^{\infty} e^{-(s+5)t} dt = \frac{-1}{s+5}[e^{-\infty} - e^{0}]$

If $s+5>0 \Rightarrow \sigma+j\omega+5>0 \Rightarrow \sigma>-5$.

b) $\int_{-\infty}^{0} e^{-5t} e^{-(\sigma+j\omega)t} dt = \int_{-\infty}^{0} e^{-5t} e^{-st} dt = \int_{-\infty}^{0} e^{-(s+5)t} dt = \frac{-1}{s+5}[e^{0} - e^{-(-\infty)}]$

If $s+5<0 \Rightarrow \sigma+j\omega+5<0 \Rightarrow \sigma<-5$

c) $\int_{-5}^{5} e^{-5t} e^{-(\sigma+j\omega)t} dt = \int_{-5}^{5} e^{-st} e^{-st} dt = \int_{-5}^{5} e^{-(s+5)t} dt = \frac{-1}{s+5}[e^{-5} - e^{+5}] \neq 0$.

$\therefore \sigma \in \mathbb{R}$

d) $\int_{-\infty}^{\infty} e^{-st} e^{-(\sigma+j\omega)t} dt = \int_{-\infty}^{-T} e^{-(s+5)t} dt + \int_{-T}^{\infty} e^{-(s+5)t} dt = \cancel{\frac{-1}{s+5}[e^{-(-T)} - e^{-(-\infty)}]}$

$-\frac{1}{s+5}[e^{-(-T)} - e^{-(-\infty)}]$

$\Downarrow$

$s+5<0 \Rightarrow \sigma<-5$

$s+5>0 \Rightarrow \sigma>-5$.

2) $x(t) = e^{-5t}u(t-1)$

$\int_{-\infty}^{\infty} e^{-5t} u(t-1) e^{-st} dt = \int_{1}^{\infty} e^{-5t} e^{-st} dt = \frac{-1}{s+5}[e^{-\infty} - e^{-1}]$

$\Rightarrow X(s) = \frac{e^{-1}}{s+5}$

$s+5>0 \Rightarrow \sigma>-5 \Rightarrow$ ROC > -5.

3) a) $\frac{1}{s+1} + \frac{1}{s+3} = \frac{s+3+s+1}{(s+1)(s+3)} = \frac{2s+4}{(s+1)(s+3)} = \frac{2(s+2)}{(s+1)(s+3)}$

~~No of~~ zero's ~~is~~ $\Rightarrow s+2=0 \Rightarrow s=-2$.

Poles $\Rightarrow s+1=0$ & $s+3=0 \Rightarrow s=-1$ & -3.

$\therefore$ No. of zeroes in finite s-plane = 1 i.e, '-2'

& No. of zeroes in infinite s-plane = 1.

b) $\frac{s+1}{s^2-1} \Rightarrow$ zeros $\Rightarrow s+1=0 \Rightarrow s=-1$

Poles $\Rightarrow s^2-1=0 \Rightarrow s=\pm 1$

$\therefore$ zero's in finite s-plane = 1

zero's in infinite s-palane = 1.

c) $\frac{s^3-1}{s^2+s+1} \Rightarrow$ ~~zeros = 1,~~ $\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$\Rightarrow$ Poles = $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ $\Rightarrow$ zero's $= 1, -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

4) $\frac{s-1}{(s+2)(s+3)(s^2+s+1)}$ Poles $= -2, -3, -\frac{1}{2} + j\frac{\sqrt{3}}{2}, -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$Re(s) > -\frac{1}{2}$ & $-2 < Re\{s\} < -\frac{1}{2}$ & $-3 < Re\{s\} < -2$

& $Re\{s\} < -3$.

$\therefore$ We may have four different signals.

5) a) $x(t) = 2^{-2t}u(t) + 4^{-4t}u(t)$

~~$x(t) = 2^{-2t}u(t) + 2^{-8t}u(t)$~~

~~$X(s) = \frac{\ln 2}{s+2} + \frac{\ln 2}{s+8}$~~

~~$X(s) = \ln 2\left[\frac{1}{s+2} + \frac{1}{s+8}\right]$~~

~~$X(s) = \ln 2\left[\frac{2s+10}{(s+2)(s+8)}\right]$~~

~~$X(s) = \frac{(2s+10)\ln 2}{(s+2)(s+8)}$~~ [∵ linearity property]

By linearity Property,

$X(s) = X_1(s) + X_2(s)$

$X_1(s) = \int_{-\infty}^{\infty} 2^{-2t}u(t)e^{-st}\,dt = \int_0^{\infty} 2^{-2t}e^{-st}\,dt$

$= \left[\frac{e^{-st}}{-s}2^{-2t}\right]_0^{\infty} + \frac{-1}{s^2}\int_0^{\infty} e^{-st} 2^{-2t}\ln(4)$

$= \left[0 - \frac{1}{-s}\right] + \frac{-1}{s^2}\left[0 - \ln(4)\right]$

$= \frac{1}{s} + \frac{1}{s^2}\ln(4) \quad \sigma > 0.$

$X_2(s) = \frac{1}{s} + \frac{1}{s^2}\ln(256) = \frac{1}{s} + \frac{8}{s^2}\ln(2).$

$\therefore X(s) = X_1(s) + X_2(s) = \frac{1}{s} + \frac{2}{s^2}\ln(2) + \frac{1}{s} + \frac{8}{s^2}\ln(2) = \frac{2}{s} + \frac{10}{s^2}\ln(2), \quad \sigma > 0.$

b) $x(t) = e^{-5t}[u(t) - u(t-5)] = e^{-5t}u(t) - e^{-5t}u(t-5).$

By linearity property, $X(s) = X_1(s) + X_2(s).$

$\Rightarrow X_1(s) \Rightarrow e^{-5t}u(t) \Rightarrow \frac{1}{s+5} \quad Re\{s\} > -5 \Rightarrow \sigma > -5.$

$X_2(s) \Rightarrow -e^{-5t}u(t-5) \Rightarrow -\left\{\frac{1}{s+5}\left(e^{-5s}\right)\right\} = \frac{e^{-5s}}{s+5}$ [∵ Time shifting]

$\therefore X(s) = X_1(s) + X_2(s) = \frac{1}{s+5} - \frac{e^{-5s}}{s+5} = \frac{1 - e^{-5s}}{s+5}.$

c) $x(t) = e^{-at}\sin\Omega_0 t\,u(t)$

$X(s) = \frac{\Omega_0}{(s+a)^2 + \Omega_0^2}$ [∵ time shifting in s-domain & $\sin\omega t \leftrightarrow \frac{\omega}{s^2+\omega^2}$]

d) $x(t) = t^2\cos\Omega_0 t\,u(t)$

W.K.T, $\mathcal{L}[t^n f(n)] = (-1)^n \frac{d^n F(s)}{ds^n}$ [∵ Frequency differentiation property]

$F(s) = \mathcal{L}[\cos\Omega_0 t\,u(t)] = \frac{s}{s^2+\Omega_0^2}$

$X(s) = \mathcal{L}[t^2(\cos\Omega_0 t)u(t)] = (-1)^2\frac{d^2}{ds^2}\left[\frac{s}{s^2+\Omega_0^2}\right]$

$= \frac{d}{ds}\left[\frac{(s^2+\Omega_0^2) - s(2s)}{(s^2+\Omega_0^2)^2}\right] = \frac{d}{ds}\left[\frac{\Omega_0^2 - s^2}{(s^2+\Omega_0^2)^2}\right] \Rightarrow$

$= \frac{(s^2+\Omega_0^2)^2(-2s) - (\Omega_0^2 - s^2)(2(s^2+\Omega_0^2))(2s)}{(s^2+\Omega_0^2)^4}$

$= \frac{2\cancel{(s^2+\Omega_0^2)}[-s(s^2+\Omega_0^2) - 2(\Omega_0^2 - s^2)s]}{(s^2+\Omega_0^2)^3}$

$= \frac{2s[-s^2 - \Omega_0^2 - 2\Omega_0^2 + 2s^2]}{(s^2+\Omega_0^2)^3}$

$$X(s) = \frac{2s[s^2 - 3\Omega_0^2]}{(s^2+\Omega_0^2)^3}$$

6) $x_1(t) = e^{-2t}u(t)$ & $x_2(t) = e^{-3t}u(t)$. & $y(t) = x_1(t-2) * x_2(-t+3)$

$X_1(t-2) = e^{-s(2)}\left(\frac{1}{s+2}\right) = \frac{e^{-2s}}{s+2}$ $\quad \sigma + j\omega + 2 > 0 \Rightarrow ROC > -2.$

$X_2(-t+3) = -\frac{1}{1}\cdot\left(\frac{+1}{s-3}\right) = \frac{-1}{s-3}$

$X_2(-t+3) = \frac{-e^{-3s}}{s-3} = \frac{e^{-3s}}{3-s}$ $\quad \Rightarrow Re\{s\} > -3$

$Y(s) = X_1(t-2) \times X_2(-t+3) = \left[\frac{e^{-2s}}{s+2}\right]\left[\frac{e^{-3s}}{3-s}\right]$

7) a) $\frac{s+5}{s^2+3s+2} = X(s)$

Initial value, $x(0^+) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{s(s+5)}{s^2+3s+2} = \lim_{s\to\infty} \frac{(1+5/s)}{1+3/s+2/s^2} = 1$

Final value, $x(\infty) = \lim_{s\to 0^+} sX(s) = \lim_{s\to 0} \frac{s(s+5)}{s^2+3s+2} = 0.$

b) $\frac{s^2+5s+7}{s^2+3s+2} = X(s)$

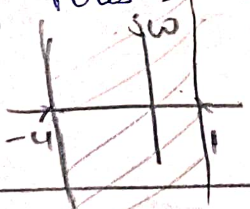
Initial value, $x(0^+) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{s(s^2+5s+7)}{s^2+3s+2} = \lim_{s\to\infty} \frac{s^3+5s^2+7s}{s^2+3s+2}$

$= \lim_{s\to\infty}\left(s + \frac{2s^2+5s}{s^2+3s+2}\right) = \infty.$

Final value, $x(\infty) = \lim_{s\to 0^+} sX(s) = \lim_{s\to 0} s\left(\frac{s^2+5s+7}{s^2+3s+2}\right) = 0.$

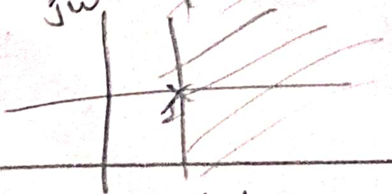
8) $X(s) = \frac{2}{(s+4)(s-1)}$ $\quad$ Poles $= -4, 1.$

a) $-4 < Re(s) < 1$



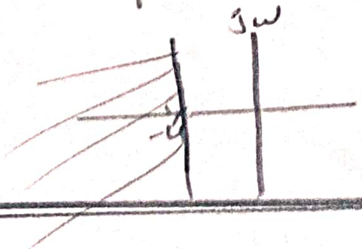
not causal because ROC lies left of 1.
Stable as it contains jω axis.

b) $Re(s) > 1.$



causal but unstable.

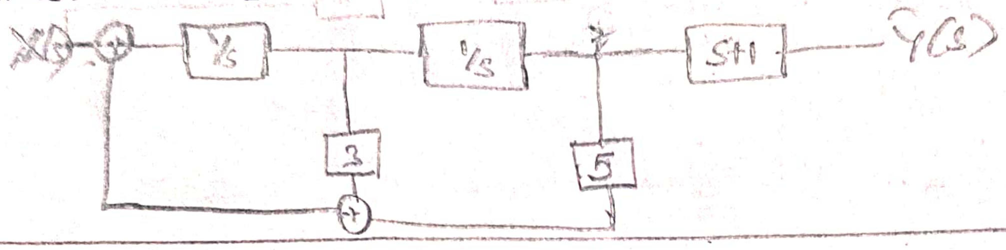
c) $Re(s) < -4$



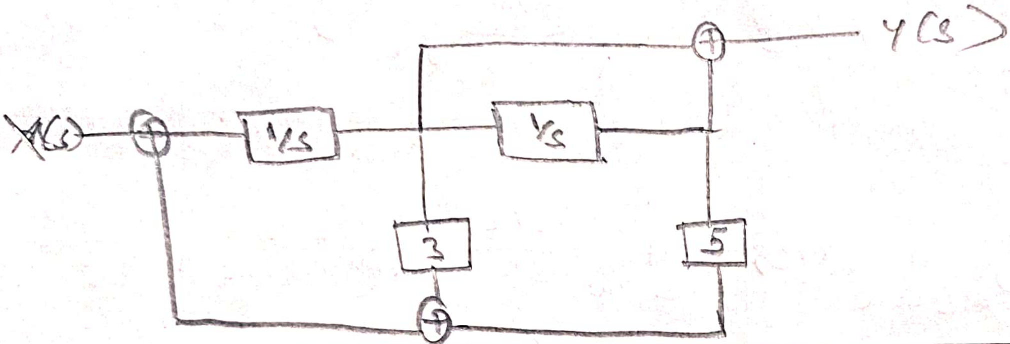
not causal and not stable.

9) $H(s) = \frac{s+1}{s^2+3s+5} = \frac{\frac{1}{s}+1/s^2}{1+3/s+5/s^2}$

Direct Form 1.

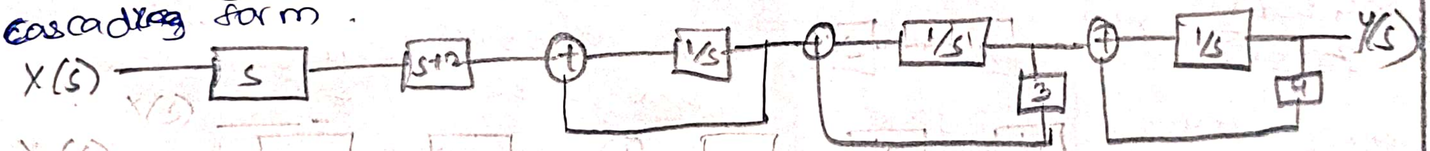


Direct Form 2



10) $H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+4} = \frac{-1/6}{s+1} + \frac{-1/2}{s+3} + \frac{8/3}{s+4}$

Cascading form.



Parallel form