Engineering Optics

Lecture 12

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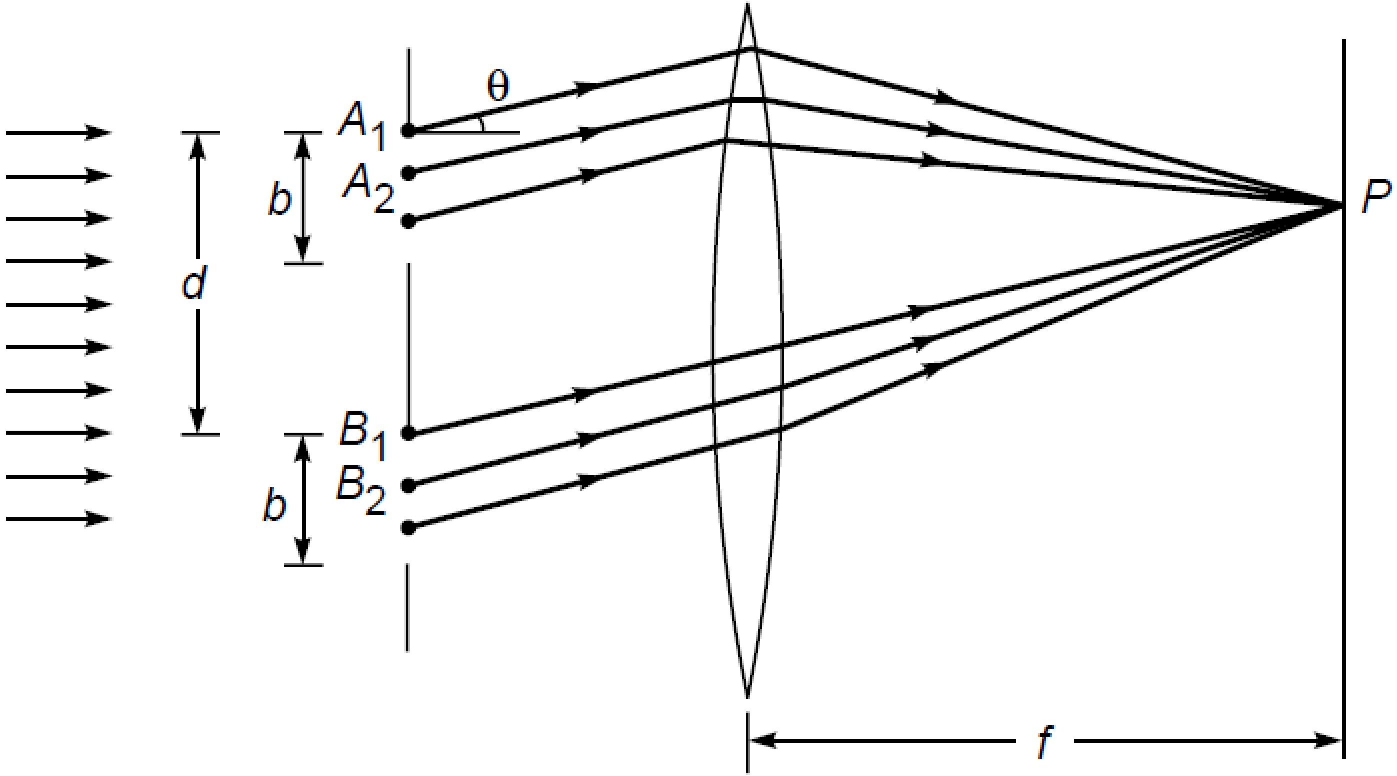
by

Debolina Misra
Department of Physics
IIITDM Kancheepuram, Chennai, India

Double slit diffraction



Fraunhofer diffraction of a plane wave incident normally on a double slit.

Distance between two consecutive points in either of the slits is Δ

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Similarly, the second slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

at point P, where

$$\Phi_1 = \frac{2\pi}{\lambda}d\sin\theta$$

Optics, Ghatak

Double slit diffraction continued

$$E = E_1 + E_2$$

$$= A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)]$$

$$E = 2A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{1}{2}\Phi_1\right)$$

where

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda}d\sin\theta$$

The intensity distribution will be of the form

$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$$\boxed{I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

Meaning?

Double slit diffraction continued

$$\boxed{I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

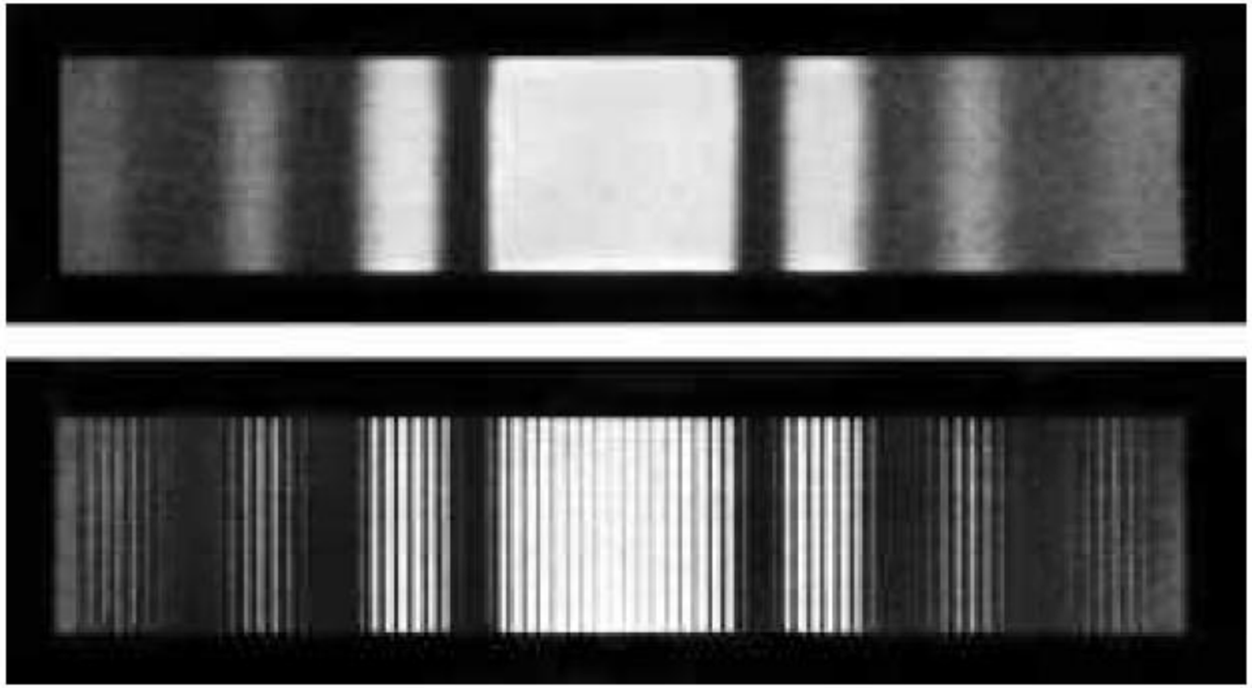
intensity distribution produced by one of the slits

Interference pattern produced by two point sources separated by a distance *d*

**if the slit widths are very small* → *β* small

Young's interference pattern

Diffraction pattern due to slits



Single- and double-slit Fraunhofer patterns. (*a*) Photographs taken with monochromatic light.

Minima

$$\boxed{I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma}$$

$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

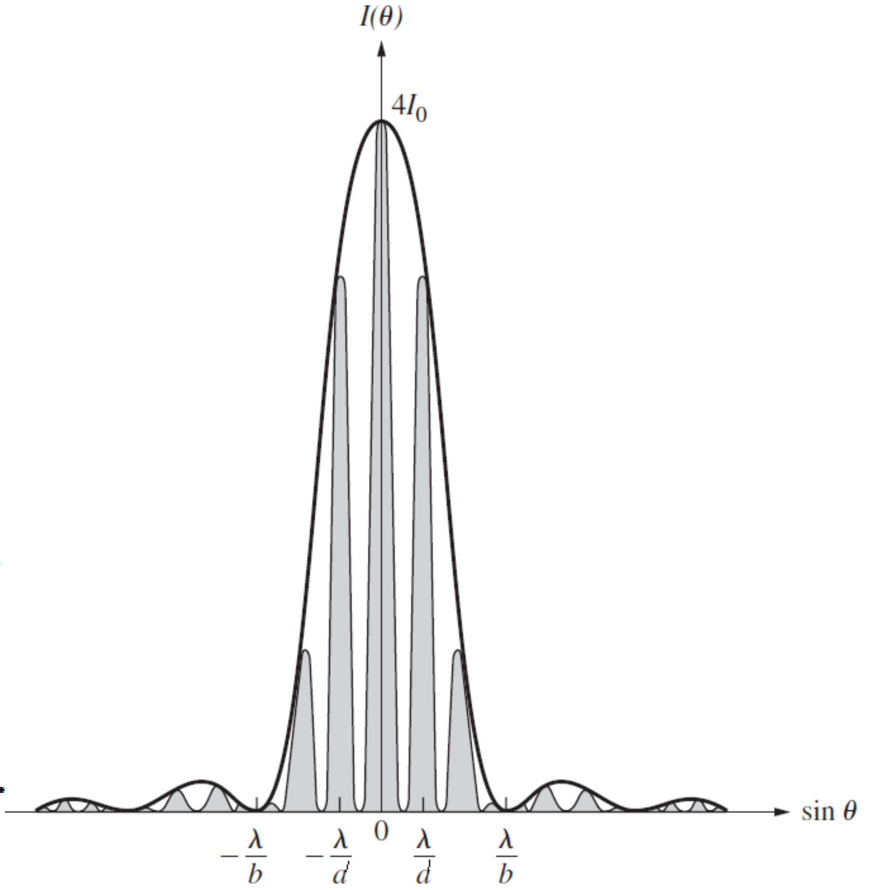
$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$

intensity is zero wherever

$$\beta = \pi, 2\pi, 3\pi, \ldots \quad b \sin\theta = m\lambda \quad \textbf{(1)} \qquad m = 1, 2, 3, \ldots$$

or when

$$\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \quad d \sin\theta = \left(n + \tfrac{1}{2}\right)\lambda \qquad n = 1, 2, 3, \ldots$$



The interference maxima occur when

$$\gamma = 0, \pi, 2\pi, \ldots$$

or when

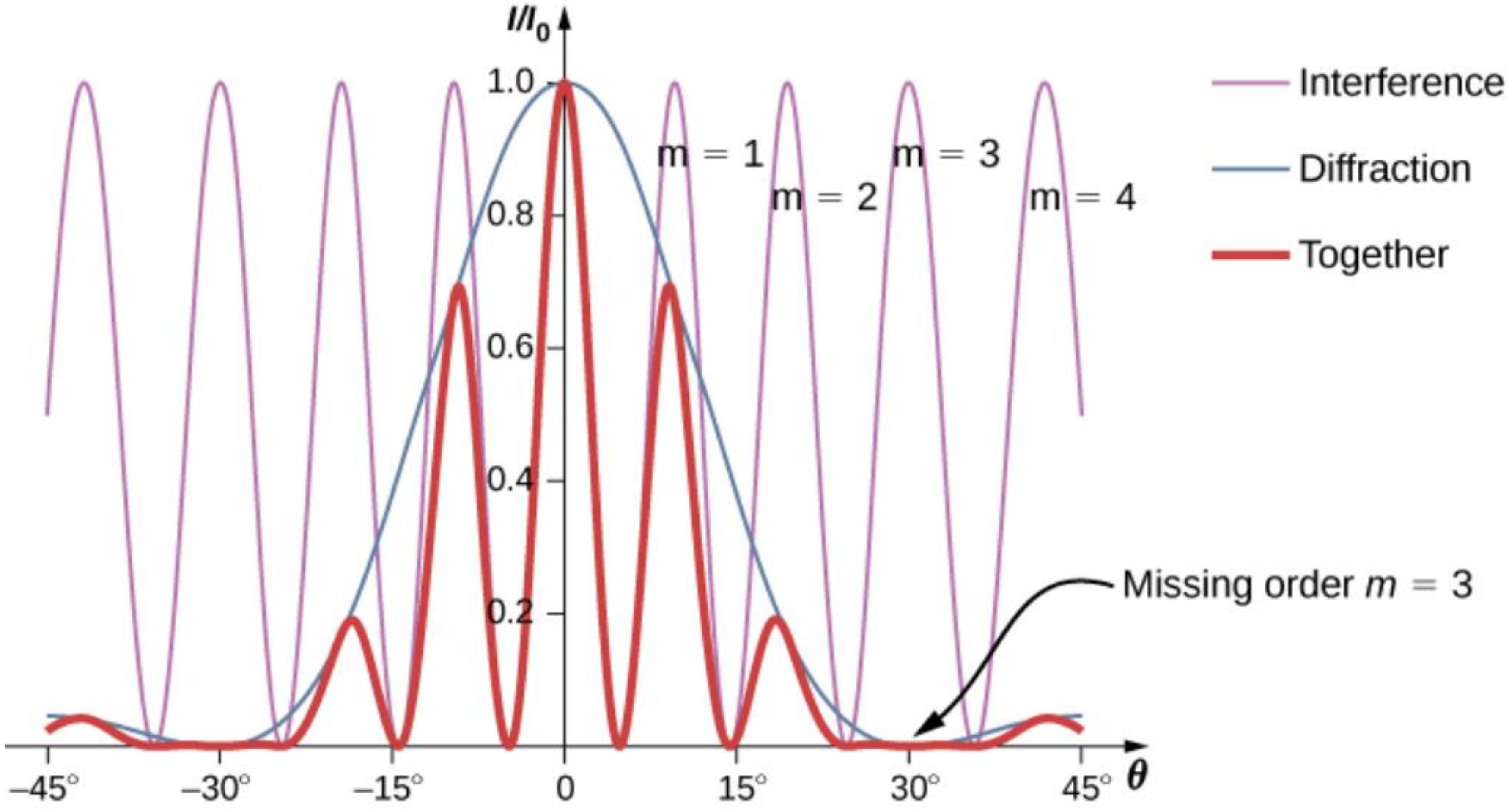
$$d \sin\theta = 0, \underbrace{\lambda}_{p\lambda}, 2\lambda, 3\lambda, \ldots \quad \textbf{(2)}$$

An interference maximum and a diffraction minimum (zero) may correspond to the same θ-value

(d/b = p/m) missing order

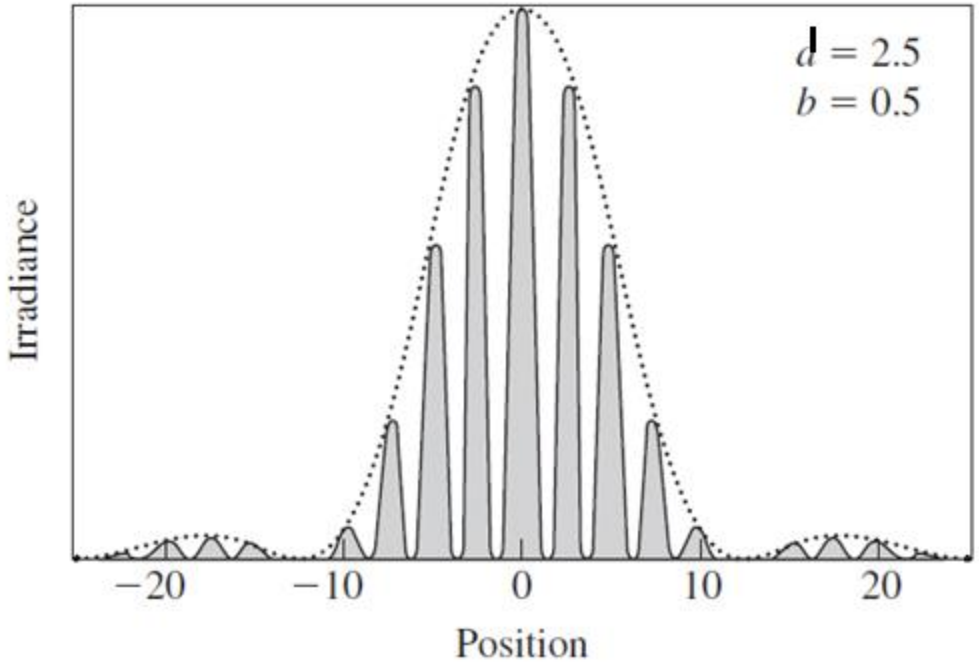
Optics, Ghatak; Hecht

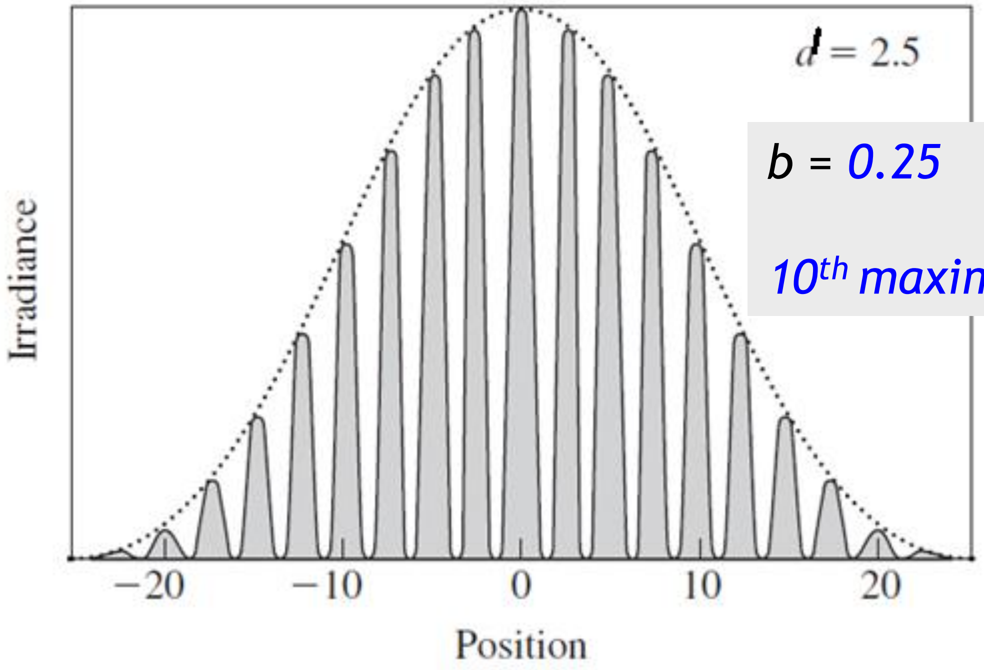
Double slit diffraction pattern



https://opentextbc.ca/universityphysicsv3openstax/chapter/double-slit-diffraction/

Missing orders



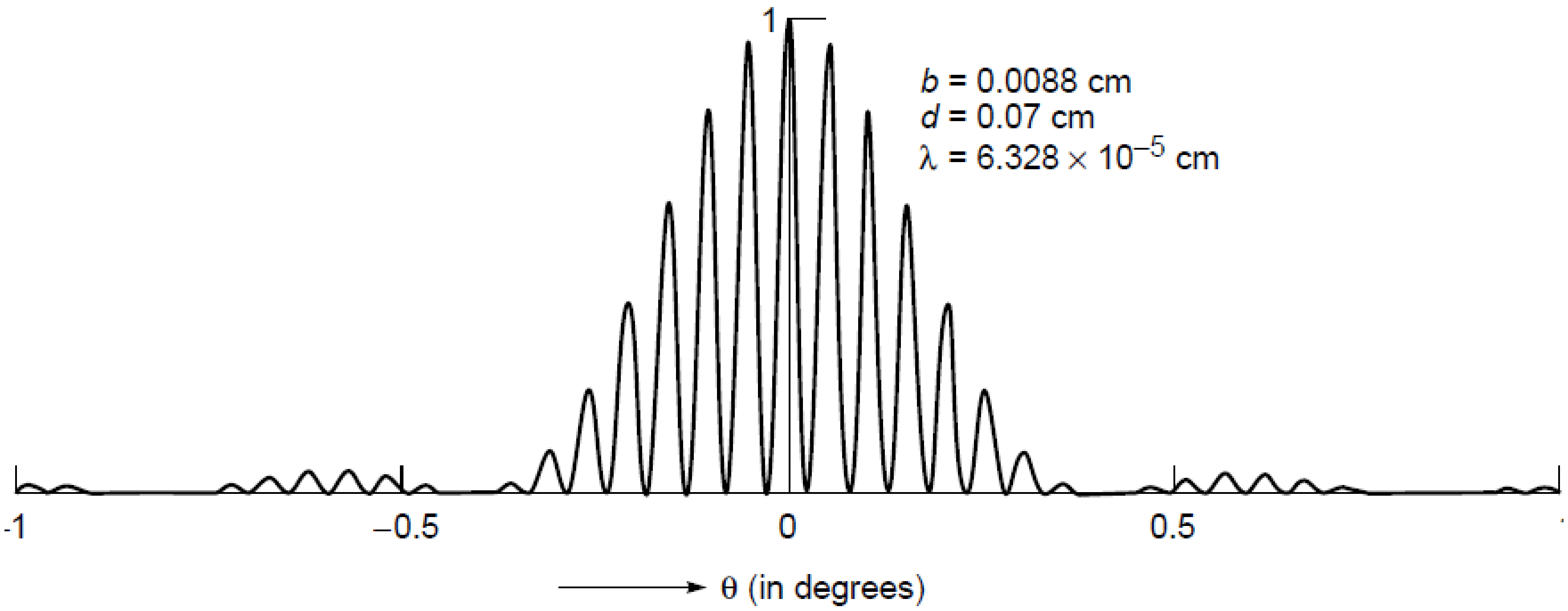


b = *0.25*

*10th maximum → **missing***

Optics, Hecht

Example 18.9 Consider the case when $b = 8.8 \times 10^{-3}$ cm, $d = 7.0 \times 10^{-2}$ cm, and $\lambda = 6.328 \times 10^{-5}$ cm (see Fig. 18.32). How many interference minima will occur between the two diffraction minima on either side of the central maximum?



Example 18.9 Consider the case when $b = 8.8 \times 10^{-3}$ cm, $d = 7.0 \times 10^{-2}$ cm, and $\lambda = 6.328 \times 10^{-5}$ cm (see Fig. 18.32). How many interference minima will occur between the two diffraction minima on either side of the central maximum?

Solution: The interference minima will occur when Eq. (46) is satisfied, i.e., when

$$\sin\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{d} = 0.904 \times 10^{-3}\left(n + \frac{1}{2}\right)$$

$$n = 0, 1, 2, \ldots$$

$$= 0.452 \times 10^{-3}, 1.356 \times 10^{-3}, 2.260 \times 10^{-3},$$

$$3.164 \times 10^{-3}, 4.068 \times 10^{-3}, 4.972 \times 10^{-3},$$

$$5.876 \times 10^{-3}, 6.780 \times 10^{-3}$$

Thus there will be 16 minima between the two first-order diffraction minima.

Thank You