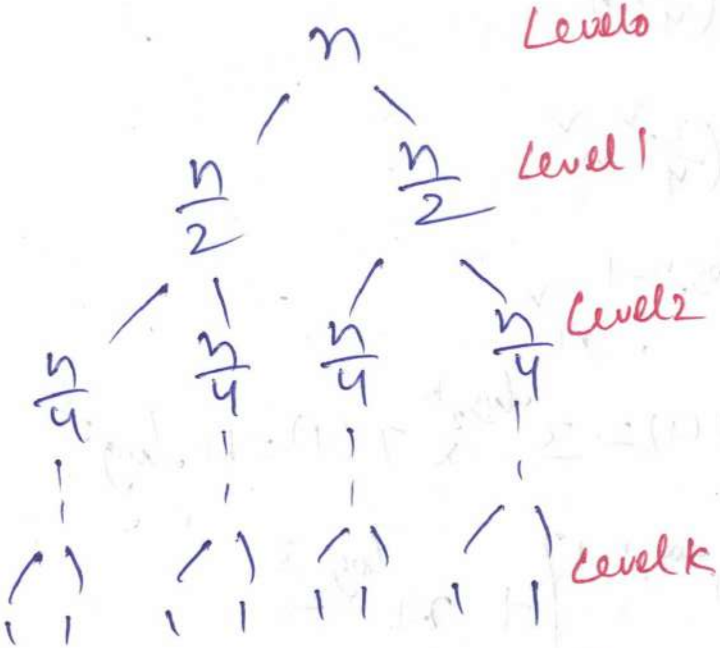
Recurrence Tree method

$T(n) = 2T(\frac{n}{2}) + n \rightarrow$ the cost of Combining partial solutions

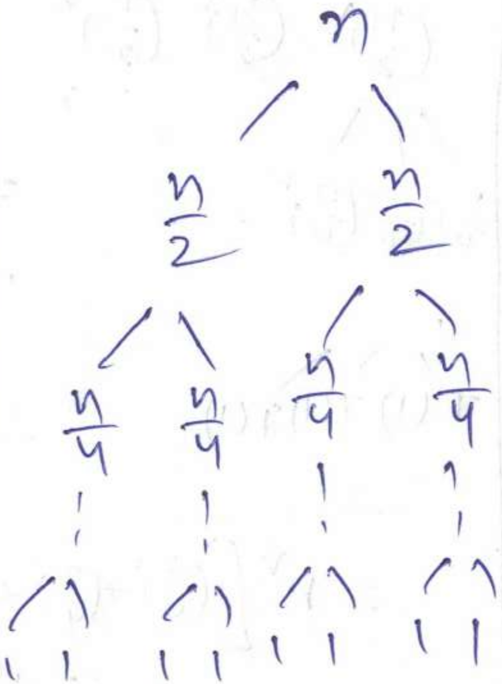
I/P Reduction Tree



$\frac{n}{2^k} = 1 \Rightarrow k = \log_2 n$

$\# \text{Levels} = \log_2 n + 1$

Computation Tree



$= n = n$

$= 2 \times \frac{n}{2} = n$

$= 4 \times \frac{n}{4} = n$

$= 1 \times n = n$

$T(n) = n \times (\log_2 n + 1)$

$n \log_2 n \le n\log_2 n + n \le 3\, n\log_2 n$

$T(n) = \Theta(n \log_2 n)$

Recurrence Tree Method

$T(n) = 3T(\frac{n}{2}) + n^2, \quad T(1) = 1$

I/P Reduction Tree

$n \Rightarrow 1 = 3^0$

$\frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2} \Rightarrow 3 = 3^1$

$\frac{n}{4} \quad \frac{n}{4} \quad \frac{n}{4}$

1

$\Rightarrow 3^k = 3^{\log_2 n}$ leaves

$= n^{\log_2 3}$ leaves

$\frac{n}{2^k} = 1, \; k = \log_2 n$

$\#\text{Levels} = k+1 = \log_2 n + 1$

Computation Tree

$n^2 \Rightarrow n^2 = \left(\frac{3}{4}\right)^0 n^2$

$\left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \Rightarrow \frac{3n^2}{4} = \left(\frac{3}{4}\right)^1 n^2$

$\left(\frac{n}{4}\right)^2 \left(\frac{n}{4}\right)^2 \left(\frac{n}{4}\right)^2 \; \cdots \Rightarrow \frac{9n^2}{16} = \left(\frac{3}{4}\right)^2 n^2$

$\Rightarrow \left(\frac{3}{4}\right)^{\log_2 n - 1} \cdot n^2$

$T(1) \; T(1) \; T(1) \Rightarrow \#\text{leaves} \cdot T(1) = 3^{\log_2 n} \times T(1) = n^{\log_2 3}$

$$= n^2 \left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \cdots + \left(\frac{3}{4}\right)^{\log_2 n - 1} \right] + n^{\log_2 3}$$

$$= n^2 \Bigg[\underbrace{\frac{1 - \left(\frac{3}{4}\right)^{\log_2 n}}{1 - \left(\frac{3}{4}\right)}}_{\text{Constant}} \Bigg] + n^{\log_2 3}$$

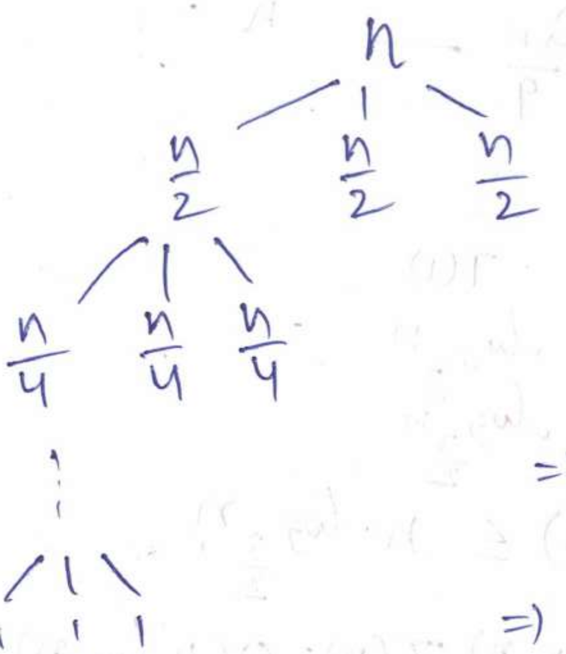
$n^2 \leq \qquad \leq 4 \cdot n^2$

$$= \Theta(n^2)$$

Recurrence Tree Method

$$T(n) = 3\,T\left(\frac{n}{2}\right) + n, \quad T(1) = 1$$

Computation Tree



$$\Rightarrow n = \left(\frac{3}{2}\right)^0 n$$

$$\Rightarrow \frac{3n}{2} = \left(\frac{3}{2}\right)^1 n$$

$$\Rightarrow \frac{9n}{4} = \left(\frac{3}{2}\right)^2 n$$

$$\Rightarrow \frac{27n}{8} = \left(\frac{3}{2}\right)^3 n$$

$$\Rightarrow = \left(\frac{3}{2}\right)^{\log_2 n - 1} \cdot n$$

$$\Rightarrow = 3^{\log_2 n} \times T(1)$$

$$= n^{\log_2 3} \times T(1)$$

$$\boxed{\#\text{leaves} = 3^{\log_2 n} = n^{\log_2 3}}$$

$$\Rightarrow \left(\frac{3}{2}\right)^0 n + \left(\frac{3}{2}\right)^1 n + \cdots + \left(\frac{3}{2}\right)^{\log_2 n - 1} \cdot n + n^{\log_2 3} \cdot T(1)$$

$$\Rightarrow n\left[\frac{\left(\frac{3}{2}\right)^{\log_2 n} - 1}{\frac{3}{2} - 1}\right] + \Theta\left(n^{\log_2 3}\right)$$

Constant

$$\Rightarrow n \cdot c \cdot \left[n^{\log_2 \frac{3}{2}} - 1\right] + \Theta\left(n^{\log_2 3}\right)$$

$$\Rightarrow n \cdot c \cdot \left[\frac{n^{\log_2 3}}{n^{\log_2 2}} - 1\right] + \Theta\left(n^{\log_2 3}\right)$$

$$\Rightarrow \cancel{n} \cdot c \cdot \left[\frac{n^{\log_2 3}}{\cancel{n}} - 1\right] + \Theta\left(n^{\log_2 3}\right)$$

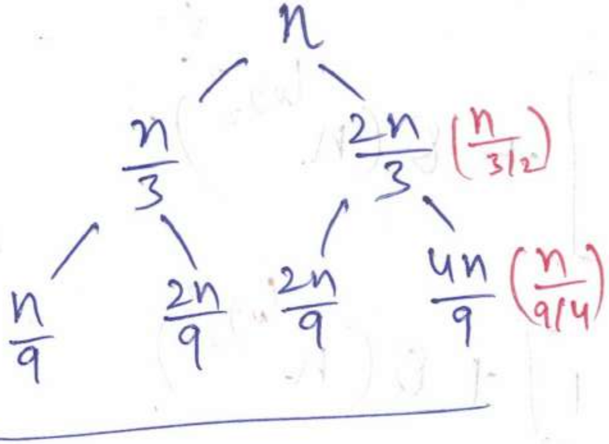
$$\Rightarrow \Theta\left(n^{\log_2 3}\right) + \Theta\left(n^{\log_2 3}\right)$$

$$\Rightarrow \underline{\underline{\Theta\left(n^{\log_2 3}\right)}}$$

Recurrence Tree Method

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n, \quad T(1) = 1$$

I/P Size Reduction Tree



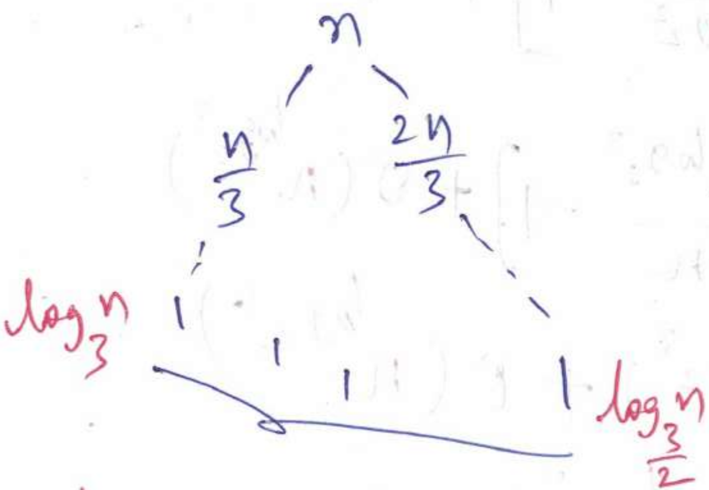
What is the value of k.

s.t.

$$\frac{n}{3^k} = 1 \Rightarrow k = \log_3 n$$

$$\frac{n}{\left(\frac{3}{2}\right)^{k'}} = 1 \Rightarrow k' = \log_{\frac{3}{2}} n$$

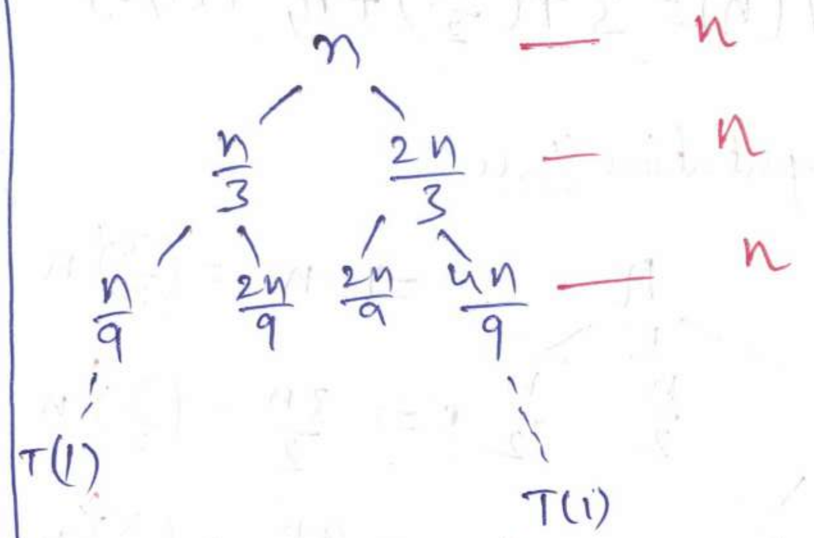
$$\log_3 n < \log_{\frac{3}{2}} n$$



leaves start at $\log_3 n$ and end at $\log_{\frac{3}{2}} n$

Leaves are distributed

Computation Tree



Min. height $= \log_3 n$

Max. height $= \log_{\frac{3}{2}} n$

$$n \log_3 n \le T(n) \le n \cdot \log_{\frac{3}{2}} n$$

$$T(n) = \Omega(n \log_3 n), \quad T(n) = O(n \log_{\frac{3}{2}} n)$$

$$\log_3 n = \Theta\left(\log_{\frac{3}{2}} n\right)$$

$$\Rightarrow T(n) = \Theta(n \log_3 n) = \Theta(n \log_{\frac{3}{2}} n)$$

$$= \Theta(n \log n).$$

Recurrence Tree Method

$T(n) = 5T\left(\frac{n}{2}\right) + n^2, \quad T(1) = 1$

$$\underbrace{\left(\frac{5}{4}\right)^0 n^2 + \left(\frac{5}{4}\right)^1 n^2 + \cdots + \left(\frac{5}{4}\right)^{\log_2 n - 1} n^2}_{\text{Cost from root until } \log_2 n - 1 \text{ level}} + \underbrace{n^{\log_2 5} \cdot T(1)}_{\text{Cost @ } \log_2 n \text{ level}} = \Theta\left(n^{\log_2 5}\right)$$

$T(n) = 9T\left(\frac{n}{4}\right) + n, \quad T(1) = 1$

$$\underbrace{n\left[\left(\frac{9}{4}\right)^0 + \left(\frac{9}{4}\right)^1 + \cdots + \left(\frac{9}{4}\right)^{\log_4 n - 1}\right]}_{\text{Cost from root to } \log_2 n - 1 \text{ level}} + \underbrace{n^{\log_4 9} \cdot T(1)}_{\substack{\text{Cost @ } \log_2 n \text{ level} \\ \text{@ sets of leaves}}} = \Theta\left(n^{\log_4 9}\right)$$