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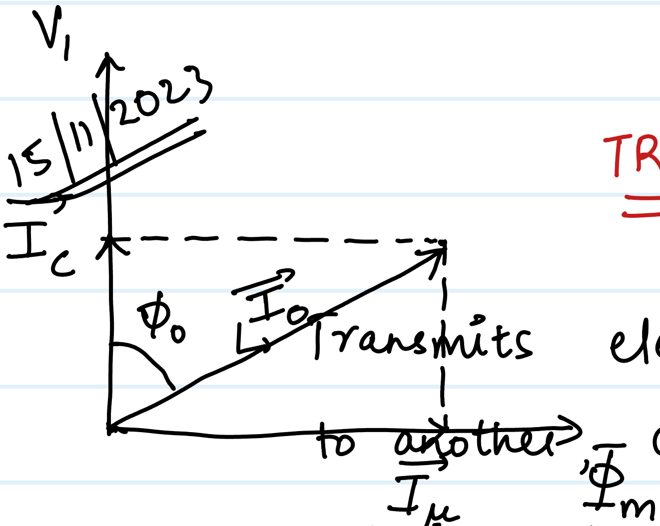



# TRANSFORMERS

↳ Transmits electrical energy from one circuit to another, at constant frequency, by the principle of Faraday's law of electromagnetic induction. Transformers are static devices.

↳ Changes Voltage levels. (e.g. 240 ⟶ 12 V in mobile chargers. This is a step-down transformer)

↳ Very high efficiency — 99.5% !!!

Transformers contain:

→ Winding in primary and secondary circuits made of copper / Aluminium conductor.

⟶ ELECTRICAL CIRCUITS

→ Magnetic core made of Iron

⟶ MAGNETIC CIRCUIT

Primary windings $\xrightarrow[\text{to}]{\text{connected}}$ supply
Secondary winding $\xrightarrow[\text{to}]{\text{connected}}$ external load

Are electrically isolated, but magnetically coupled!

## FARADAYS LAW OF ELECTROMAGNETIC INDUCTION

1. GENERATOR LAW : When a rotating conductor is placed in a uniform $\vec{B}$ field, it cuts the magnetic lines of flux, and emf is induced in it.

2. MOTORING LAW : When a current carrying conductor is placed in a uniform $\vec{B}$ field, it experiences a mechanical force.

BOTH LAWS PRODUCE DYNAMICALLY INDUCED EMF.

3. Transformer : emf induced in coil is directly proportional to the rate of change of flux linkage in it.

$$\boxed{e \propto -\frac{d\phi}{dt}}$$ [(-) due to Lenz law]

$\phi \rightarrow$ flux.

$\Rightarrow \boxed{\phi = f(t)}$

## MAGNETIC CIRCUIT:

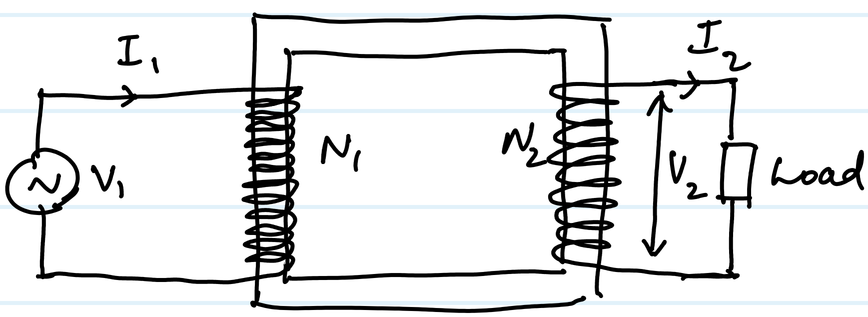
Core of transformer provides a continuous path for magnetic field. Its made of high grade silicon steel (or) sheet steel lamination, having low hysterisis loss.

- provides low reluctance path for carrying the flux.
- carries the windings required for electric power transfer.

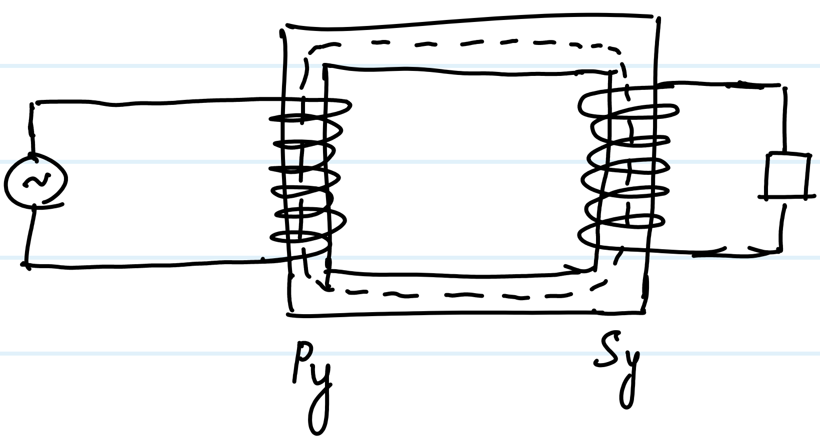
## ELECTRIC CIRCUIT:

- Primary & secondary windings
- input → primary winding
- output → secondary winding
- windings → copper.

$$\boxed{V_1 I_1 = V_2 I_2}$$



# PRINCIPLE of OPERATION




1. Py connected to supply
2. current flows through Py.
3. current in Py winding causes $\vec{B}$ field to be produced in the core, according to Ampere's Law.

if $i(t) = f(t)$,

$$\phi(t) = K f(t)$$

4. $\vec{B}$ flux $\Phi_m$ flows only through the magnetic core.
5. $\Phi_m$ links with the primary coil & it induces an emf, $E_1$ across it.
6. According to Faraday's Law: $\boxed{e_1 \propto -\frac{d\phi}{dt}}$

$$\Rightarrow \boxed{E_1 = -N_1 \frac{d\Phi}{dt}},$$

$N_1 \rightarrow$ n(turns) in Py
$E_2 \rightarrow$ emf in Py

7. further, $\Phi_m$ links with Secondary coil & hence it induces an emf $E_2$ across it.

$$\boxed{E_2 = -N_2 \frac{d\Phi}{dt}}$$

; $N_2 \rightarrow$ n(turns) in Sy
$E_2 \rightarrow$ emf in Sy

8. $E_1 \propto N_1$ , $E_2 \propto N_2$

$$\Rightarrow \boxed{E_2/E_1 = N_2/N_1}$$

TRANSFORMER RATIO

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# EMF EQUATION of TRANSFORMER

According to Faraday's law:

$$\boxed{E_1 \propto -\frac{d\Phi}{dt}} \Rightarrow \boxed{E_1 = -N_1 \frac{d\Phi}{dt}}$$

$$\boxed{\Phi(t) = \Phi_m \sin\omega t}$$

$$\Rightarrow \boxed{\frac{d\Phi(t)}{dt} = \Phi_m \omega \cos\omega t}$$

$$\Rightarrow E_1 = -N_1 \Phi_m \omega \cos\omega t$$

$$\Rightarrow |E_1|_{max} = +N_1 \Phi_m \omega = N_1 \Phi_m 2\pi f$$

$$\Rightarrow E_{rms} = \frac{E_{max}}{\sqrt{2}} = \frac{N_1 \Phi_m 2\pi f}{\sqrt{2}} = 4.44 f \Phi_m N_1$$

$$\therefore \boxed{E_{rms_1} = 4.44 f \Phi_m N_1 \ V};$$

where → $f$ : supply frequency (in Hz)

$\Phi_m$ : max flux (in Wb)

$N_1$ : n(turns) of Py

Similarly, induced emf in Sy is:

$$\boxed{E_{rms_2} = 4.44 f \Phi_m N_2 \ V}$$

$N_2$ : n(turns) of Sy.

$$\boxed{\frac{E_{rms\,②}}{E_{rms\,①}} = \frac{N_2}{N_1} = K}$$ ; where

$K$ : TRANSFORMER RATIO

also, Power is const.

$$\Rightarrow E_1 I_1 = E_2 I_2$$

$$\Rightarrow \boxed{\frac{E_2}{E_1} = \frac{I_1}{I_2}}$$

$\therefore$, $$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$

**K : TRANSFORMER RATIO**

$K>1$: $N_2 > N_1 \Rightarrow E_2 > E_1 \Rightarrow$ Transformer : STEP-UP TRFMR.

$K<1$: $N_1 > N_2 \Rightarrow E_1 > E_2 \Rightarrow$ Transformer : STEP-DOWN TRFMR

$K=1$: $N_1 = N_2 \Rightarrow E_1 = E_2 \Rightarrow$ Transformer : ISOLATION TRFMR.

ISOLATION → used to electrically isolate the load as the primary & secondary windings / circuits are magnetically coupled ONLY.

→ for safety purposes

**POWER TRFMR:** at power plants - handles in MW levels

11 KV || 400KV/600KV

**DISTRIBUTION TRFMR:**
- at consumer premises
- handes in KW levels
- 11 KV || 415V (AC two phase)

Transformers → 
- should have a good cooling medium
- increases efficiency.

## EXAMPLE :

$E_1 = 250V$ ; $E_2 = 3000V$ ; $f = 50Hz$ ; $\Phi_m = 1.2\ Wb/m^2$

emf/turn (e) = 8 V

(i) find $N_1$ & $N_2$ (ii) find $A_{core}$.

(i) $E_1 = N_1 \cdot e$

$$\Rightarrow N_1 = \frac{E_1}{e} = \frac{250}{8} = \frac{125}{4} = 31.25 \simeq 32.$$

$E_2 = N_2 \cdot e$

$$\Rightarrow N_2 = \frac{E_2}{e} = \frac{3000}{8} = \frac{1500}{4} = \frac{750}{2} = 375 \simeq 376.$$

(ii) $E_1 = 4.44 f \Phi_m N_1$

$$\Rightarrow E_1 = 4.44 f (B_m A_c) N_1 \Rightarrow \frac{E_1}{4.44 f B_m N_1} = A_c$$

$$A_c = \frac{250}{4.44 \times 50 \times 1.2 \times 32} = 0.029 m^2$$

$$\boxed{\therefore A_c = 0.029\ m^2}$$

**EXAMPLE**

$N_1 = 400$ ; $N_2 = 1000$ ; $A_c = 60\,cm^2$ ; $f = 50Hz$ ;
$E_1 = 520V$ $\quad = 60 \times 10^{-4}$

find (i) $\Phi_m/m^2$ (ii) $E_2$

(i) $E_1 = 4.44 f \Phi_m N_1$

$$\Rightarrow \frac{E_1}{4.44 f N_1} = \Phi_m$$

$$\Rightarrow \Phi_m = \frac{520}{4.44 \times 50 \times 400} = 5.85 \times 10^{-3}$$

$$\Phi_{dens} = \Phi_m / A_c = \frac{5.85 \times 10^{-3}}{6 \times 10^{-3}} = \frac{5.85}{6} \simeq 0.975\ Wb/m^2$$

$$\therefore \boxed{\Phi_{dens} = 0.975\ Wb/m^2}$$

(ii) $$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\Rightarrow E_2 = E_1 \cdot \left(\frac{N_2}{N_1}\right)$$

$$= 520 \times \left(\frac{10\cancel{00}}{4\cancel{00}}\right) = \frac{5200}{4} = 1300V$$

$$\Rightarrow \boxed{E_2 = 1300V}$$

## EXAMPLE

$P = 25\,KW$ ; $N_1 = 500$ ; $N_2 = 50$ ; $E_1 = 3000V$ ;

$f = 50Hz$

find (i) $I_1$ and $I_2$ (ii) $E_2$ and (iii) $\Phi_m$

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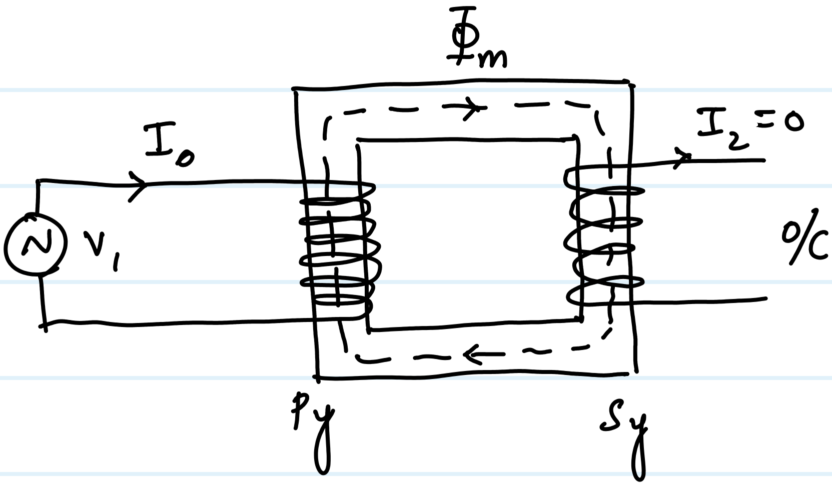
# TRANSFORMER on NO-LOAD




① Ideal Transformer : 100 % efficient → no losses in Trfmr

↳ NO leakage flux

↳ Not possible in reality.

(• winding loss ↓ ohmic /copper loss

• core loss (Iron loss) ↓ magnetic loss)

→ When supply is connected to primary, $I_o$ flows thru. it.

→ Secondary : open circuited (No Load connected)

$I_2 = 0 \Rightarrow V_2 I_2 = 0$

$\Rightarrow \boxed{P_{out} = 0}$

By $\boxed{\text{LOCOE}}$, $\boxed{P_{supplied} = P_{consumed}}$ (or)

$$\boxed{P_{in} = P_{consumed}}$$

but, $\boxed{P_{consumed} = P_{out} = 0}$ ||

Where does the $P_{in}$ get dissipated?

$P_{in} \longrightarrow$ magnetic loss in core

+ copper loss in primary winding.

No load Current $(I_0)$ = 2 to 6 % of Rated Current

$\Rightarrow$ Primary copper loss $(I_0^2 R)$ is negligible.

$\Rightarrow$ $\boxed{\text{No load input power } (W_1) = \text{Core loss of Trfmr}}$

No load current $\longrightarrow$ 2 components.

$I_\mu$ (magnetising current)

$\hookrightarrow$ produces flux

$I_c$ (core loss component)

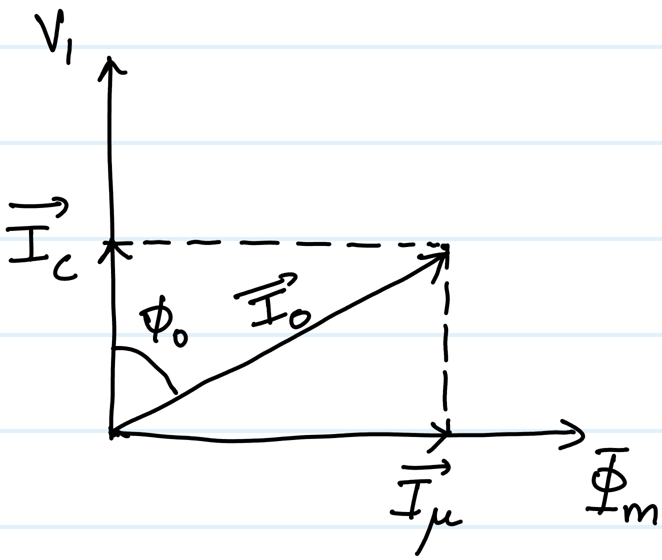
$$\Rightarrow \boxed{\vec{I_0} = \vec{I_\mu} + \vec{I_c}}$$

$$\Phi(t) = \Phi_m \sin \omega t ,$$

$$e = \frac{d\Phi(t)}{dt} = \Phi_m \omega \cos \omega t = \Phi_m \omega \sin(\omega t + \pi/2)$$

$\therefore$ $e$ (or) $V_1$ leads $\Phi$ by $\pi/2$ (or) $90^\circ$.

## Phasor Diagram




$$I_0 = \sqrt{I_c^2 + I_\mu^2}$$

$$\cos\phi_0 = \frac{I_c}{I_0}$$

$$\Rightarrow I_c = I_0 \cos\phi_0$$

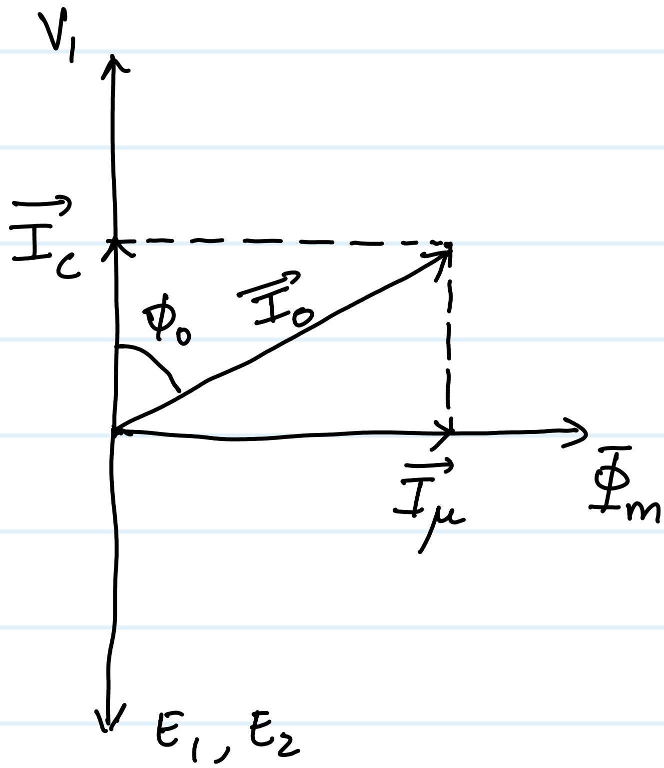
$$\Rightarrow I_\mu = I_0 \sin\phi_0$$

No-load power factor : $\cos\phi_0$ lagging.

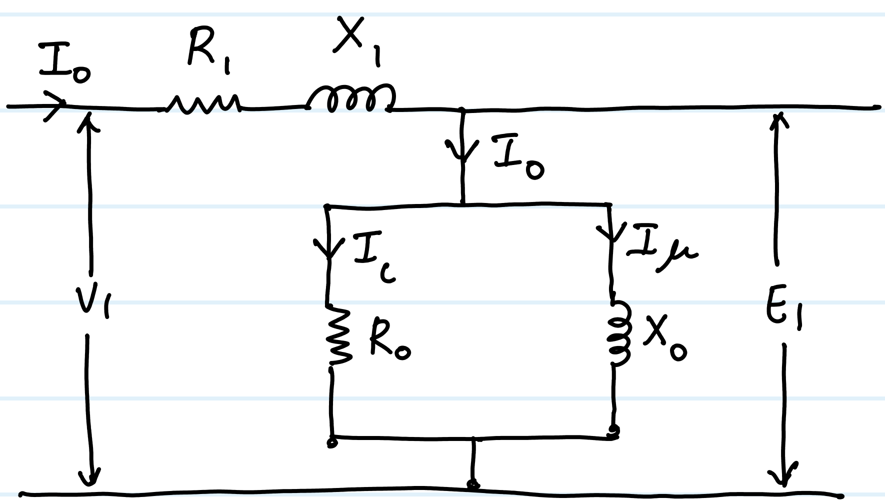
→ mostly inductive.

## What about $E_1$ & $E_2$?

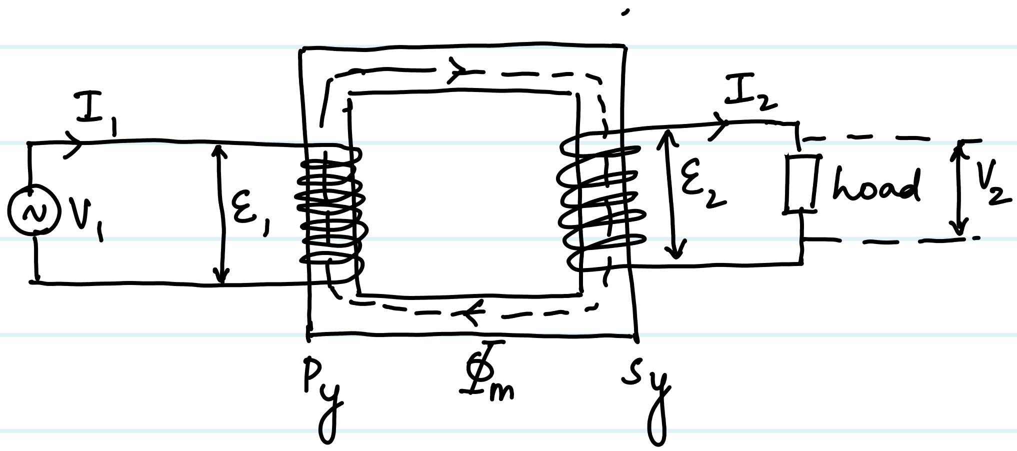
$E_1$ and $E_2$ are $180^\circ$ out of phase wrt $V_1$ according to Lenz law.




## EQUIVALENT CIRCUIT:



# TRANSFORMER on LOAD

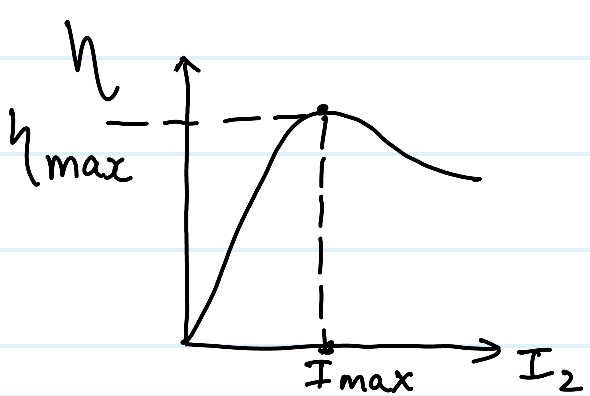



$I_2$ causes a back emf $E_1'$ at Py (coz of $\Phi_1'$) due to which $V_1 \downarrow$. but $P = $ const. $\therefore$, $I_1 \uparrow$ to $I_1'$.

$I_1'$ causes a flux $\Phi_2'$ which nullifies the effect of $\Phi_1'$, $\therefore$, net flux is just $\Phi_m$ in the core.

$I_2 \uparrow \mid P_{o/p} \uparrow \mid I_2 \uparrow \mid I_1 \uparrow \mid P_{o/p} \uparrow \mid \eta \uparrow$

$$\text{Efficiency } (\eta) = \frac{P_{out}}{P_{in}} = \frac{P_{o/p}}{P_{o/p} + \text{losses}} = \frac{P_{o/p}}{P_{o/p} + P_{core} + P_{cu}} \times 100\%$$



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21/11/2023

## ELECTRICAL MACHINES

- Electrical machines : Electromechanical energy conversion devices.

1. Electrical Energy is converted to mechanical Energy
   ↳ in Motors
2. Mechanical Energy is converted to Electrical Energy
   ↳ in Generators

## WORKING PRINCIPLES

- 2 electromagnetic phenomena take place:
  1. movement of conductor in a $\vec{B}$ field induces a voltage across its ends.
  2. current carrying conductor placed in a $\vec{B}$ field experiences a mechanical force.

① → GENERATOR ACTION , ② → MOTOR ACTION

① $|\mathcal{E}| = Blv$ ; $\vec{B}$ : mag. field; $l$ : length of condtr; $v$: vel. of cond-tr.

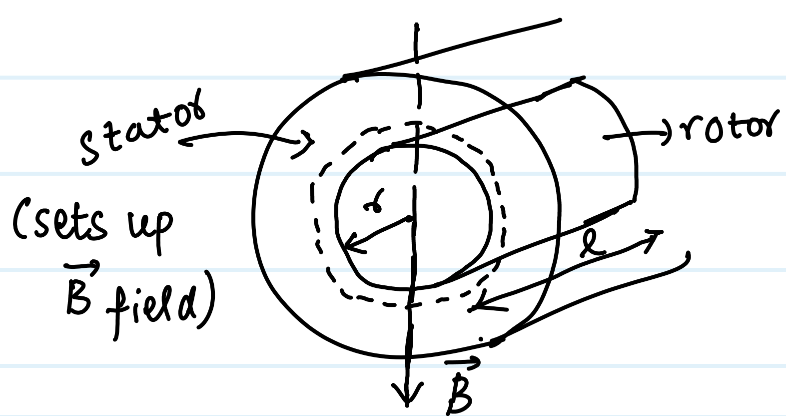
② $|\tau| = IlB$ ; $I$ : current ; $l$ : length of condtr; $B$ : mag field

## ROTATING MACHINES

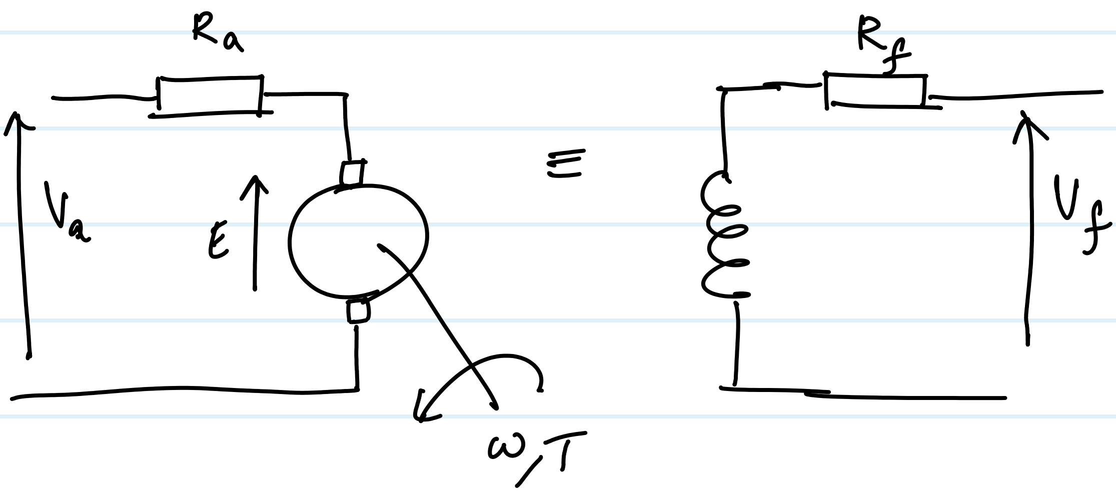
Machines → Electrical circuit
→ Magnetic circuit

Machines → fixed Stator : sets $\vec{B}$ field (usually)
→ rotating rotor : carries currents produced / supplied (usually)

for DC : Stator → $\vec{B}$ field . → field system.
Rotor → Electric circuit. → armature.
} → small gap b/w both (2-5mm)



emf of DC : $E = Nr\omega Bl = K\omega \Rightarrow \boxed{E \propto \omega}$




modern → Brushless DC Motor || permanent Magnet motor

Permanet magnets → no need of external excitation

↳ AUV's, EV's, Robotics, etc .............

22/11/2023

STUDY ELECTRICAL MACHINES – from Slides & Sadiku.

$\eta$ (permanent magnet motors) $\geq 90\%$

DC motor → universal motor (both DC & AC works)

# STUDY from SLIDES

## 3-PHASE INDUCTION MOTORS

↳ Workhorse of industry, 95% of industry uses it

↳ simple, rugged, cheap, easy maintenance, robust

parts : stator, rotor

formed by ↓ electrical circuit

rotor → rotatory member

principle: transformer action.

→ rotating trfmr.

{ Stator → primary
rotor → secondary

→ mutual induction

28/11/2023

induction motors → const. speed (depends on freq. of supply) → from no-load to full-load

{ stator → $\vec{E}$ field, rotor → $\vec{B}$ field

→ rotating transformer.

rotor → squirrel cage rotor
rotor → wound rotor (slip ring) → for variable speed appln. (usage of rheostat)

↓

- high starting torque (wrt squirrel cage)
- takes very low current (wrt squirrel cage)
- (initial + maintenance) cost is ↑↑ (wrt s.c)
- speed regulation is poor
- efficiency & p.f. is ↓↓ (wrt s.c)

## How is $\vec{B}$ field created?

elec. freq = $\omega_e = 2\pi f_e$ elec. rad/sec.

B rotates in space at $\omega_s = \frac{2}{P}\omega_e$ mech. rad/sec
(revolving $\vec{B}$ field)

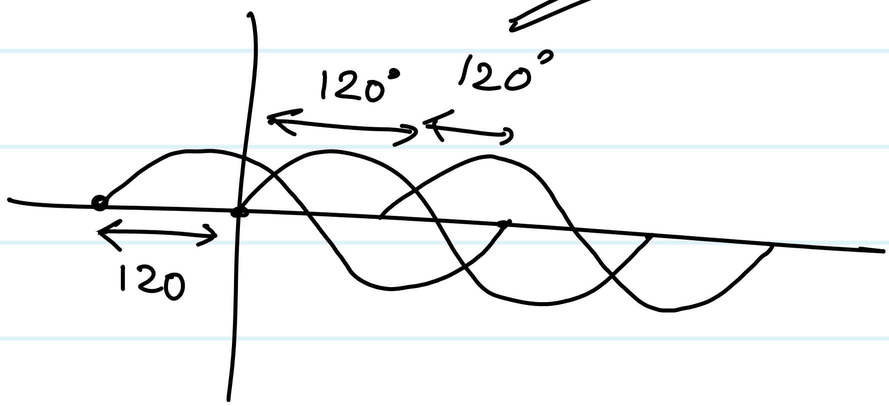
$\omega_s \rightarrow$ synchronous speed. Or $n_s = \frac{120f}{P}$ in rpm.

$P \rightarrow$ no. of poles....

if n (Poles) ↑↑, $\omega_s$ ↓↓. usually, $\omega_e \rightarrow$ const.
only $P \rightarrow$ changed.....

(heteropolar structure)

3 phase ≡ 3 phases separated by a phase angle of $360/3 = 120°$




time phase distribution of 3 phase AC supply is given to a 3-phase stator winding of induction motor, which is spatially distributed. A revolving $\vec{B}$ field is created, which runs at synchronous speed $(\omega_s / n_s)$.

$$\boxed{n_s = \frac{120f}{P}}$$

- the rotating $\vec{B}$ field cuts the stationary rotor/conductor. ∴ emf is induced in the rotor. Since the rotor conductors are permanently short circuited, it causes the electrical current flow. this current flow generates a magnetic field on the rotor conductor.

$$F = BIl = B_R * B_S ; \quad \tau_{ind} = K B_r \times B_s .$$

$B_R$ & $B_S \to$ magnetic flux densities due to Rotor & Stator

## SPEED of Induction Motor

$n_s = \dfrac{120f}{P}$ ; actual motor speed $= n_m$.

if $n_m = n_s$, rotor $\to$ stationary wrt $\vec{B}_s$ field.
∴, no induced current, no $\vec{B}_R$, no $\vec{\tau}_{ind}$, ∴, rotor speed falls below $n_s$.

When $n_m < n_s$, current is induced, $\vec{B}_R$ produced, ∴, torque is produced......

$\boxed{∴,\ n_m < n_s}$

Asynchronous motor

$s = \dfrac{n_s - n_m}{n_s}$

full load slip : 4-6 %

Slip $\to$ $\Delta(n_m$ & $n_s) \Rightarrow n_{slip} = n_{sync} - n_m$

$$\%\,\text{slip}\,(s) = \frac{n_s - n_m}{n_s} \times 100\,\%$$

Where $\boxed{n_s = \dfrac{120f}{P}}$

$s = 0 \to$ no motoring

$s = 1 \to$ stationary motor.

$\boxed{0 < s < 1}$

## frequency of rotor induced emf ($f_R$)

$$f_R = \frac{P \times n}{120}$$ ; $f_R$: rotor frequency

$P$: stator poles

$n$: slip speed (rpm)

$$f_R = \frac{P \times (n_s - n_m)}{120}$$

$$f_R = \frac{P \times S \times n_s}{120} = Sf_e$$

$\therefore$, $\boxed{f_R = sf_e}$

30/11/2023

$S=1$, $n_m = 0 \rightarrow$ motor $\equiv$ transformer

$S<1$, $n_m < n_s \rightarrow f_R = sf_e$ ($f_R$: rotor induced emf)

Mechanical Power $= \tau_m * \omega_m \equiv \tau_m * \frac{2\pi n_m}{60} = \frac{2\pi \tau_m n_m}{60}$

$\tau_m$: mech. torque in N-m

$\omega_m$: angular sp. in rad/s

$n_m$: rotor speed

w

$1\,HP \equiv 746\,W$

**EXAMPLE :** $V = 208\,V$, $P_o = 10\,HP$, $f_e = 60\,Hz$, $s = 5\%$

star connection (Y), $P = 4$

find: (i) synch. speed (ii) rotor speed (iii) rotor frequency

(iv) shaft torque {(ii), (iii) & (iv) at rated load}

(i) $$n_s = \frac{120 f_e}{P} = \frac{\overset{30}{\cancel{120}} \times 60}{\cancel{4}} = 1800\,rpm.$$

(ii) $$5 = \frac{n_s - n_m}{n_s} \times 100$$

$$\Rightarrow 0.05 = 1 - \frac{n_m}{n_s} \Rightarrow \frac{n_m}{n_s} = 0.95 \Rightarrow n_m = 0.95 \times 1800$$

$$= 1710\,rpm.$$

(iii) $$f_R = s f_e$$

$$\Rightarrow f_R = 0.05 \times 60$$

$$= 3\,Hz.$$

(iv) $$P = \tau\omega = \frac{\tau \cdot 2\pi n_m}{60} = 7460\,W \Rightarrow \tau = \frac{7460 \times 6\cancel{0}}{2\pi \times 171\cancel{0}} = 41.65\,N\text{-}m$$

**EXAMPLE :**

3 phase - cage rotor IM - 3 pole pairs - 50Hz supply - runs at 960 rpm - $\tau = 40$ Nm

find (i) $\omega_{field}$ (ii) slip (iii) $f_{\text{rotor currents}}$ (iv) $P_{mech}$

(i) $n_s = \frac{120f}{P} = \frac{120 \times 50}{2 \times 3} = 20 \times 50 = 1000$ Hz.

(ii) $\text{Slip} = \frac{n_s - n_m}{n_s} \times 100$

$$= \frac{1000 - 960}{1000} \times 100 = \frac{40}{1000} \times 100 = 4\ \%$$

(iii) $f_R = s f_e$

$$= \frac{4}{100} \times 50 = 2\ Hz$$

(iv) $P_m = \frac{2\pi n_m T_m}{60} = \frac{2\pi \times \cancel{960}^{16} \times 40}{\cancel{60}}$

$$= 640 \times 2\pi$$

$$= 1280\pi \simeq 1280 \times 3$$

$$= 3840 \simeq \underline{\underline{4\ KW}}$$

EXAMPLE :

emf in rotor → 1.5 Hz ($f_R$)

emf in stator → 50 Hz ($f_e$)

$P = 8$

$f_R = s f_e$

$\Rightarrow \frac{1.5}{50} = s \Rightarrow \boxed{s = 3\%}$

$$n_s = \frac{120 f}{P} = \frac{\cancel{120}^{\cancel{30}\,15} \times 50}{\cancel{8}\,\cancel{2}} = 750 \text{ rpm}$$

$$\frac{3}{100} = 1 - \frac{n_m}{750}$$

$$n_m = 750\,(0.97)$$

$$\boxed{n_m = 727.5 \text{ rpm.}}$$

**EXAMPLE :**

$$6 \text{ pole}, 3 \text{ phase}, \underset{\uparrow}{50\,HZ}^{f_e}, \tau = 150\ Nm,$$

$$120 \min^{-1} = \frac{120}{60} = 2HZ \rightarrow f_R, \quad P = ?$$

$$n_s = \frac{120f}{P} = \frac{\cancel{120}^{20} \times 50}{\cancel{6}} = 1000 \text{ rpm}$$

$$f_R = s f_e$$

$$\Rightarrow \ 2/50 = s \Rightarrow s = 4\%$$

$$s = \frac{n_s - n_m}{n_s} \times 100 \Rightarrow \frac{4}{100} = 1 - \frac{n_m}{1000}$$

$$\Rightarrow n_m = \frac{96}{\cancel{100}} \times 10\cancel{00}$$

$$= 960 \text{ rpm}.$$

$$P = \frac{2\pi N_m \tau_m}{60} = \frac{2\pi \times \cancel{960}^{16} \times 150}{\cancel{60}}$$

$$= 4800\pi = 15.07\ KW$$

$$\frac{15{,}070}{746} \simeq \underline{\underline{20.2}}\ HP$$

**EXAMPLE :** 3 phase, 50Hz, 400V IM -

6 poles, slip = 3 % .

find (i) $\text{Speed}_{\text{rotor}}$ (ii) $f_R$ (iii) $v_{\text{rotor } \vec{B} \text{ wrt stator}}$

(iv) $v_{\text{rotor } \vec{B} \text{ field wrt rotor}}$ (v) $v_{\text{rotor } \vec{B} \text{ wrt stator } \vec{B}}$