INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, DESIGN AND MANUFACTURING, KANCHEEPURAM

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

B.Tech. Second Semester End Examination - July 2021

Discrete Structures for Computer Science

Date: 19/07/2021 **Duration: 3 hrs** **Maximum marks: 40**

General instructions:
1. Write your **Roll number** and **Name** on each sheet.
2. Make your assumptions wherever necessary.
3. Submit all the answers as a single PDF file named as your **Roll number**.

Answer The Questions With Proper Justifications

SECTION A (Each question carries 1 mark. Answer all the questions)

A1) A box contains 6 red, 5 green, and 4 blue colored balls. How many balls you should take to be sure that you have at least 2 colors.

A2) Write the statement 'All Doctors are Engineers' using (i) universal quantifier only (ii) existential quantifier only.

A3) How many onto functions are there from set A of size 5 to set B of size 3.

A4) How many equivalence relations are there on a set of size 5.

A5) How many derangements are there on the set $\{1, 2, 3, 4\}$.

A6) Let $R1$ and $R2$ are partial order relations defined on a finite set. Prove or disprove; $R1 \cap R2$ a partial order.

A7) Write the definition of Weak induction and Strong induction using the first order logic.

A8) What is the chromatic number of a wheel graph on $n \geq 4$ vertices.

A9) Write the power set of $\{\emptyset, 1, \{1\}\}$.

A10) Let $A = \{1, 2, ..., n\}$. Given a function $f : A \to A$ is 1-1, does it follow that f is onto. Prove or Disprove.

SECTION B (Each question carries 3 marks. Answer any 10 questions)

B1) Let $A = \{1, 2, ..., n\}$. What is the binary relation R defined on A such that (i) R has maximum number of distinct equivalence classes (ii) R has least number of distinct equivalence classes.

B2) What should be the value of n (lower bound for n) such that in any group of n people, there exists either 4 mutual friends or 5 mutual enemies.

B3) Identify the following sets are finite / countably infinite / uncountable. (i) Wheel graphs and (ii) Trees on n vertices. n is a fixed integer.

B4) Coin exchange: Show that for any $n \geq n_0$, the change for n can be given using denominations 6 and 7. Prove using M.I. (By clearly mentioning base, hypothesis, induction step).

B5) What is the minimum and maximum number of edges in a bipartite graph on n vertices.

B6) Let $A = \{1, 2, 3, 4\}$. $R = \{(1, 1), (1, 2), (2, 3), (3, 4)\}$. Find reflexive, symmetric and transitive closure of R.

B7) Draw a simple graph with the degree sequence (3, 3, 3, 2, 2, 2, 1, 1, 1) and verify Euler's Planarity formula for that graph.

B8) Draw example graphs satisfying (i) G is planar whereas $\overline{G}$ (complement of G) is non-planar (ii) Both G and $\overline{G}$ are non-planar (iii) Both G and $\overline{G}$ are planar.

B9) Show that [3, 5] is uncountable.

B10) Write logical notation for each of the following by mentioning proper UOD and predicates. (i) There are exactly two engineers in an apartment (ii) If a person is both doctor and engineer, then he did PhD (iii) There are at most two doctors in an apartment.

B11) Present an example relation and a subset for each of the following. (i) Minimum and Maximum elements exist, however neither least nor greatest elements exist (ii) Upper and lower bounds exist, however neither greatest LB nor least UB exist

B12) (PIE) How many bit strings (binary) of length six do not contain three consecutive 0's.