Electrical Circuits for Engineers (EC1000)

Lecture 05
Capacitors & Inductors
(Chapter 6)

Learning Objectives

By using the information and exercises in this chapter student will be able to:

1. Fully understand the volt-amp characteristics of capacitors and inductors and their use in basic circuits.
2. Explain how capacitors behave when combined in parallel and in series.
3. Understand how inductors behave when combined in parallel and in series.

Capacitors & Inductors

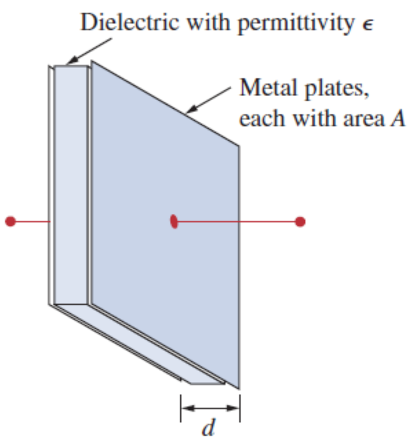
- In contrast to a resistor, which spends or dissipates energy irreversibly, an inductor or capacitor stores or releases energy (i.e., has a memory).

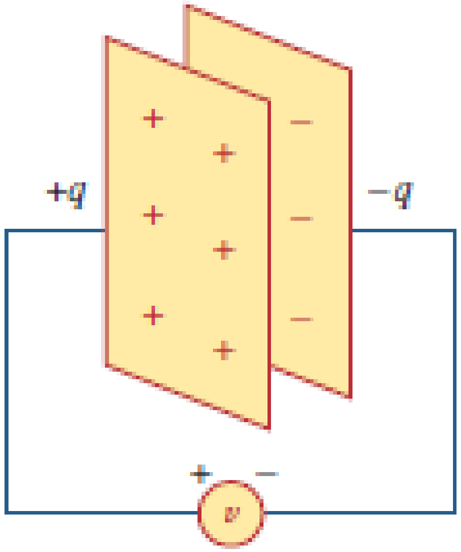
- Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time.

  - For this reason, capacitors and inductors are called *storage* elements.

1. Capacitor

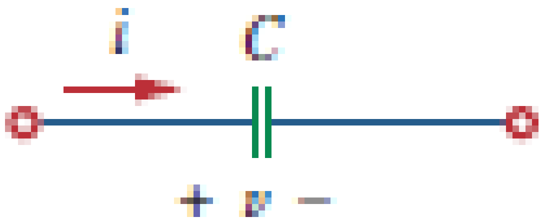
- A capacitor is a passive element designed to store energy in its electric field.
- A capacitor consists of two conducting plates separated by a dielectric.
- Capacitance is measured in Farads.
- Capacitors are used extensively in electronics, communications, and power systems. (i.e used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems).





$$q = C\,v$$

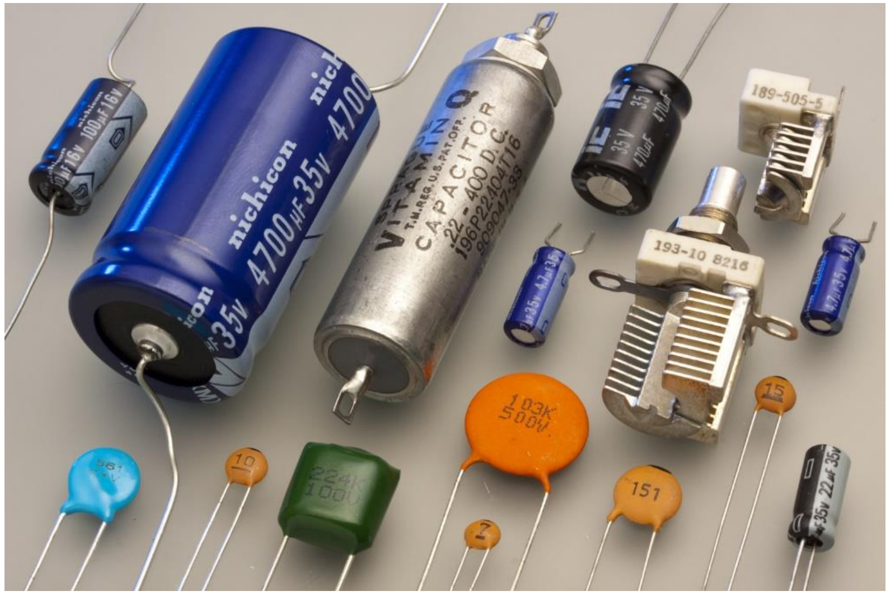
$$C = \frac{q}{v}$$



A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

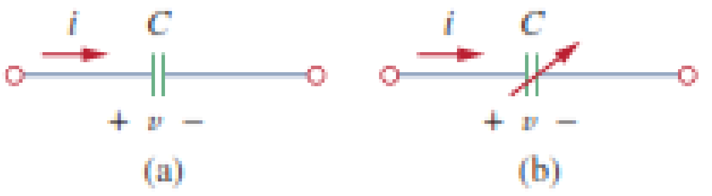
Different Capacitors



Capacitor

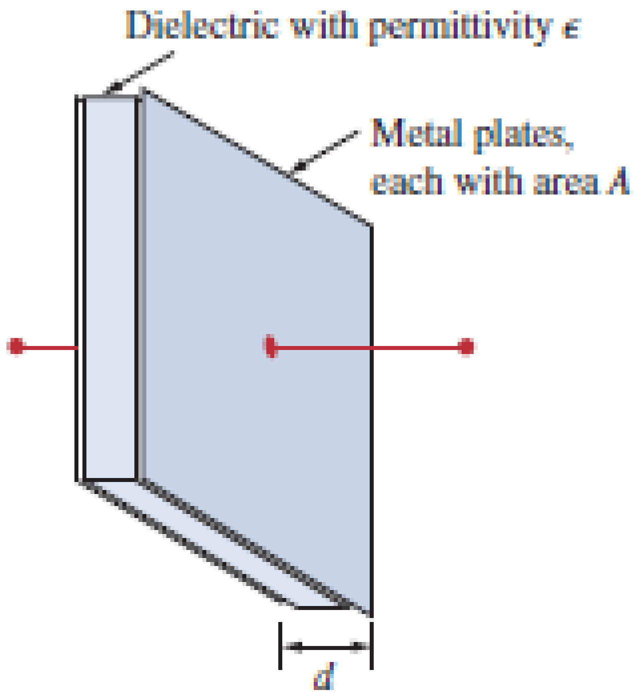
Capacitance of the capacitor dexpends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Figure, the capacitance is given by

Symbol



$$C = \frac{\in A}{d}$$

$$C = \frac{\in_o \in_r A}{d}$$



where

A is the surface area of each plate

d is the distance between the plates

Ɛ is the permittivity of the dielectric material.

$\in_o$ is the absolute permittivity of the vacuum 8.854×10^{-12} F/m

$\in_r$ is the relative permittivity (for air $\in_r$=1)

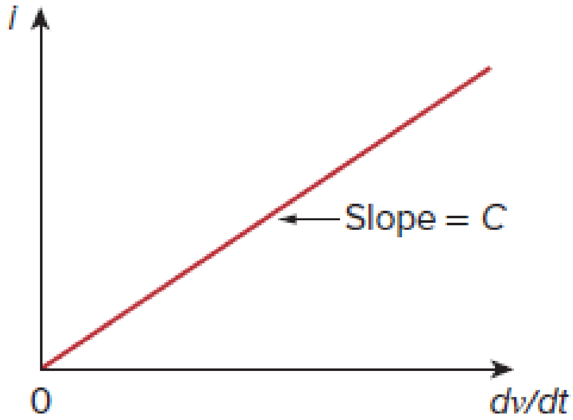
Capacitor

$$C = \frac{q}{v}$$

$$q = C\,v$$

$$\frac{dq}{dt} = C\,\frac{dv}{dt}$$

$$i = \frac{dq}{dt} = C\,\frac{dv}{dt}$$



$$p = vi = Cv\frac{dv}{dt}$$

$$\int p\,dt = C\int v\,dv$$

Energy stored in the capacitor (i.e. in terms of electro static field)

$$\boxed{w = \frac{1}{2}Cv^2}$$

Current through the capacitor $\boxed{i_c = C\,\frac{dv}{dt}}$

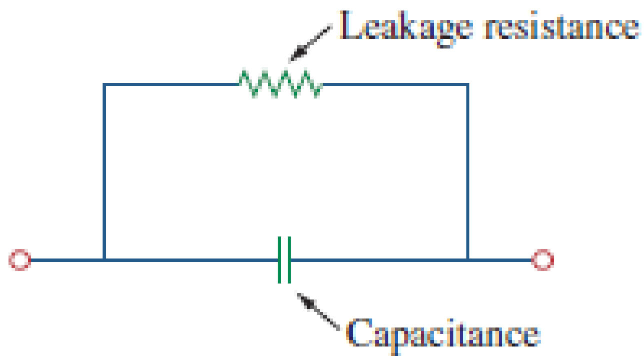
when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero.

voltage across the capacitor $\boxed{V_c = \frac{1}{C}\int I_c\,dt}$

Capacitor

- When the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero.
- A capacitor is an open circuit to dc.
- The voltage on a capacitor cannot change abruptly. Conversely, the current through a capacitor can change instantaneously.
- The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

A real, non-ideal capacitor has a parallel-model leakage resistance.



Example Problem

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

What is the voltage across a 4.5-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored? **Ans: 26.67 V,** 1.6 mJ

Example Problem

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t)$$

$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

If a 10-μF capacitor is connected to a voltage source with

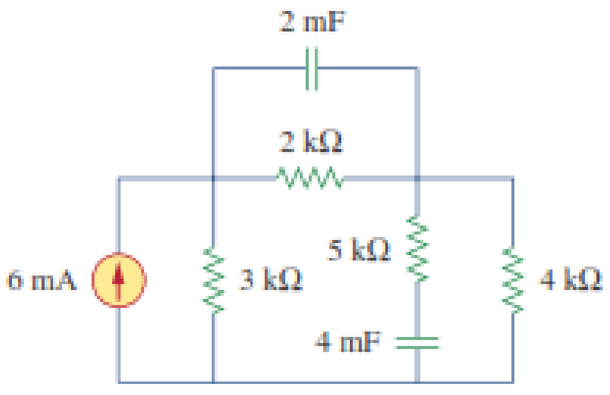
$$v(t) = 75 \sin 2000t \text{ V}$$

determine the current through the capacitor.

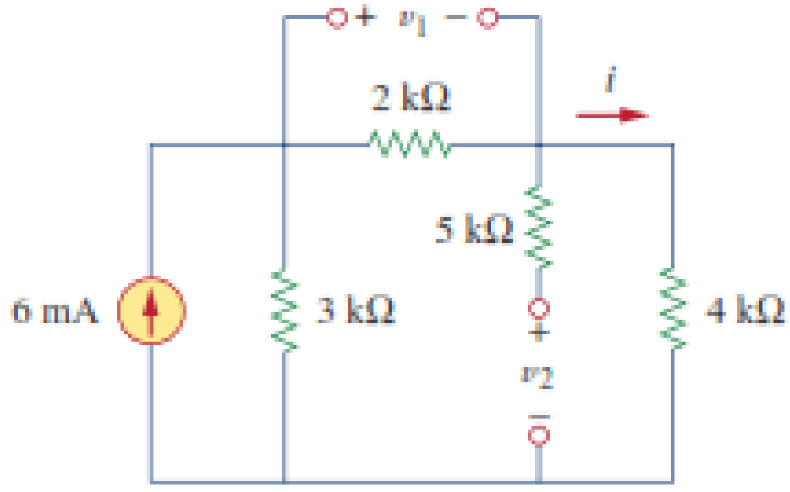
Answer: 1.5 cos 2000t A.

Example Problem

5. Obtain the energy stored in each capacitor in Fig. (a) under dc conditions.



(a)



(b)

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3+2+4}(6 \text{ mA}) = 2 \text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

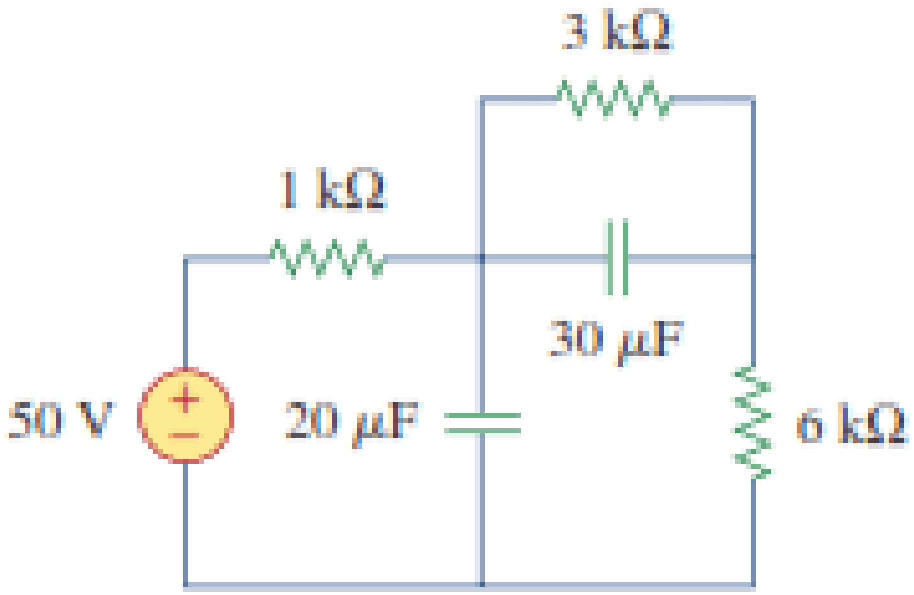
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

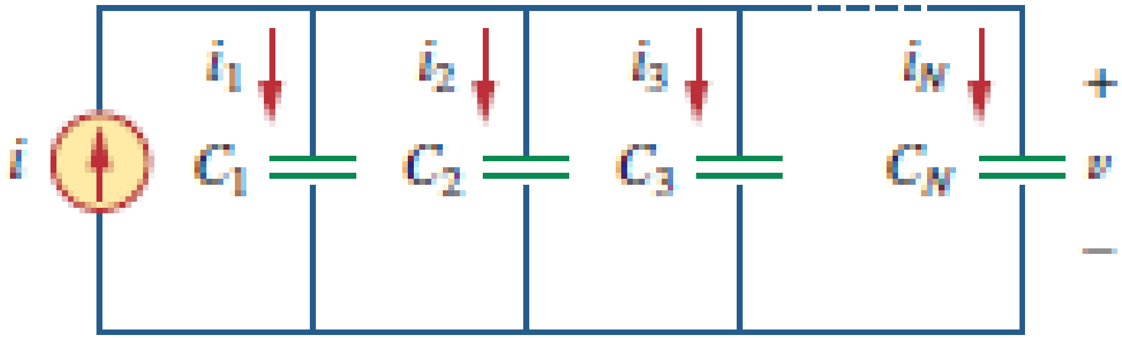
$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

Practical Problem

1. Under dc conditions, find the energy stored in the capacitors in the given Figure.
(**Ans:** 20.25 mJ, 3.375 mJ)



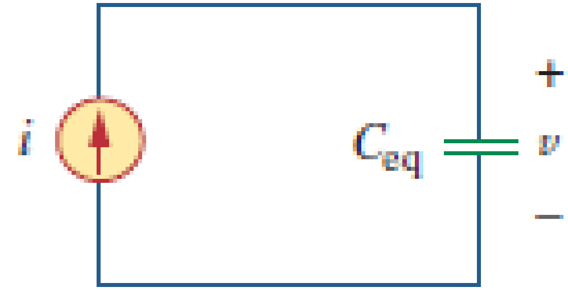
Parallel Capacitors



$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

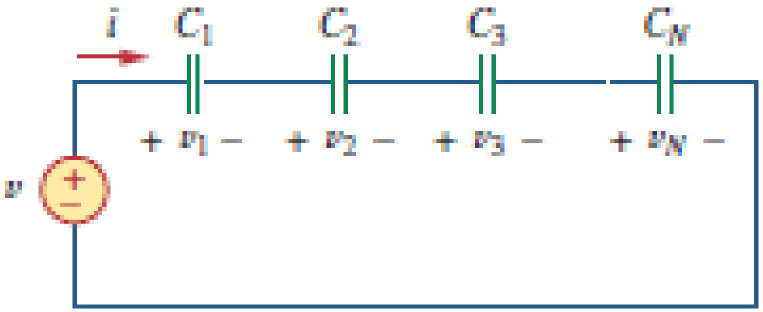
$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$



(b)

Capacitor

Series Capacitors

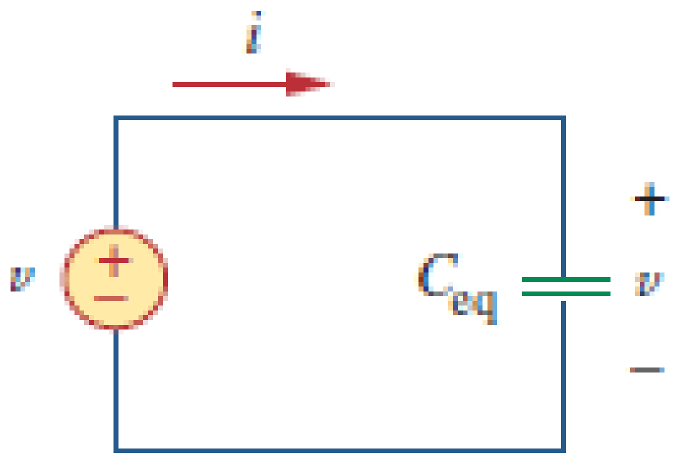


$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau$$

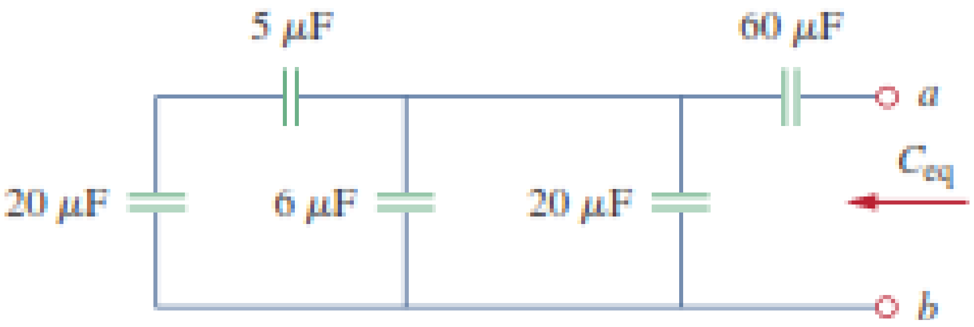
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$



$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

Example Problem

1. Find the equivalent capacitance seen between terminals *a and b of the* circuit in the given Figure.



Solution:

The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu\text{F}$$

This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

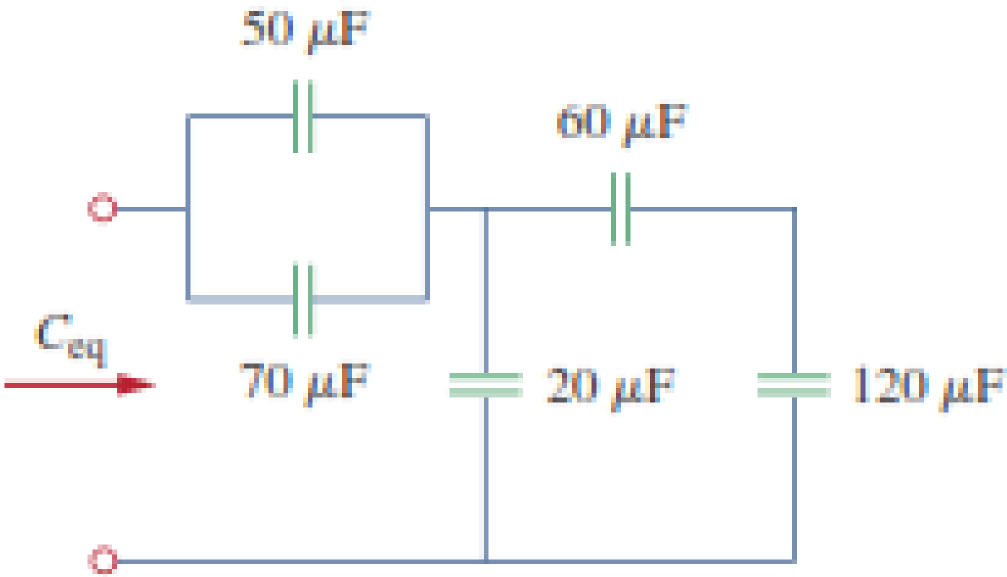
$$4 + 6 + 20 = 30\ \mu\text{F}$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$

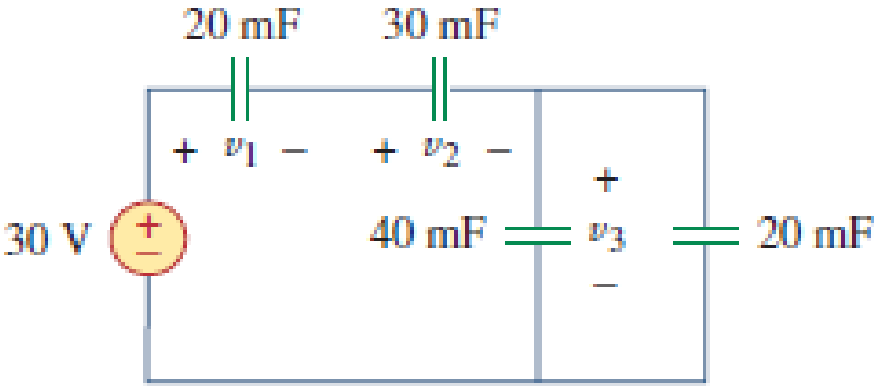
Practical Problem

Find the equivalent capacitance seen at the terminals of the circuit in the given figure below. (**Ans:** 40 μF)

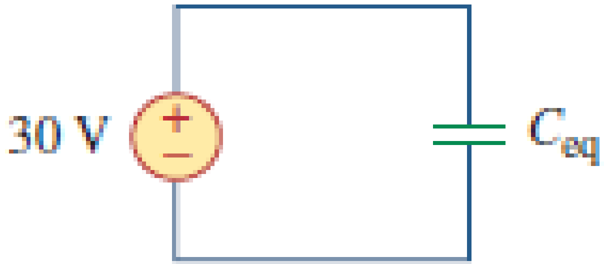


Example Problem

For the circuit in Figure, find the voltage across each capacitor



SOLUTION



$$C_{eq} = \frac{1}{\frac{1}{60}+\frac{1}{30}+\frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

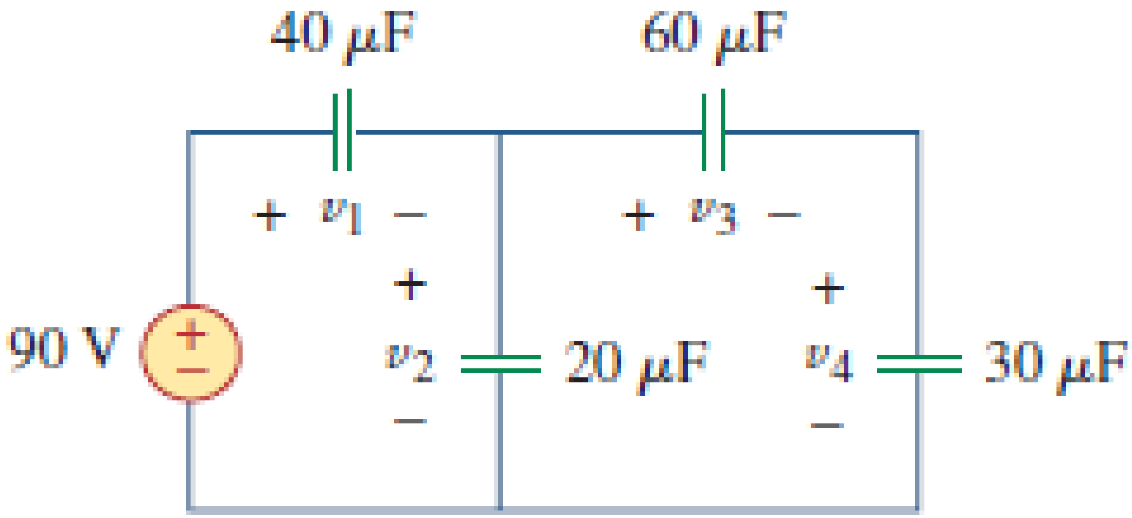
$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Practical Problem

1. Find the voltage across each of the capacitors in Figure.



Answer: v_1 **=45 V,** v_2 **=45 V,** v_3 **=15 V,** v_4 **=30 V**

Application of Capacitor & Inductor- Mobile Charger

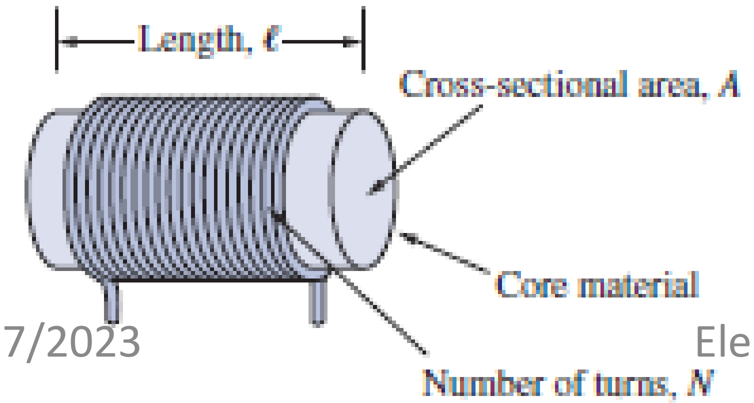
2. Inductor

- An inductor is a passive element designed to store energy in its magnetic field.
- Inductors find applications in electronic circuits, power supplies, transformers, and electric motors etc.
- An inductor consists of a coil of conducting wire.
- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it.
- Inductance is measured in Henrys (H).

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.

$$v = L\frac{di}{dt}$$

where L is the constant of proportionality called the *inductance* of the inductor.



$$L = \frac{N^2 \mu A}{\ell}$$

Inductor

Inductor

A selection of low-value inductors

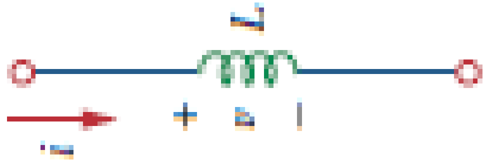
Type	Passive
Working principle	Electromagnetic induction
First production	Michael Faraday (1831)

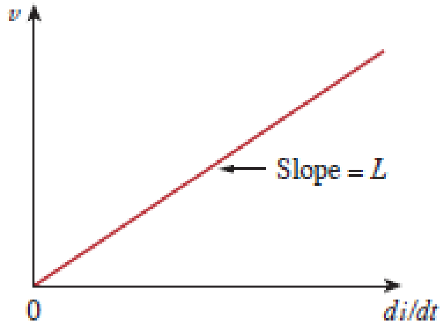
Electronic symbol

- Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems.
- Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms *coil* and *choke* are also used for inductors.

Inductor

Symbol





Joseph Henry, an American physicist, discovered inductance and constructed an electric motor by Michael Faraday.

$$\boxed{v = L\frac{di}{dt}}$$

voltage across the inductor

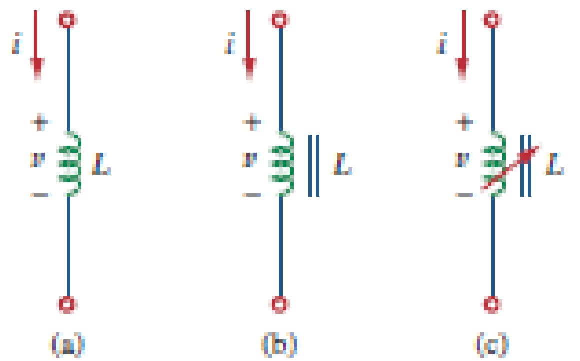


Figure 6.23
Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

$$i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

Current through the inductor

$$\boxed{w = \frac{1}{2}Li^2}$$

Energy stored in the inductor (Magnetic Field)

Important Properties

1. voltage across an inductor is zero when the current is constant. Thus, An inductor acts like a short circuit to dc.

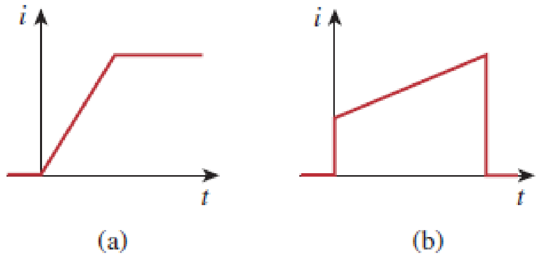
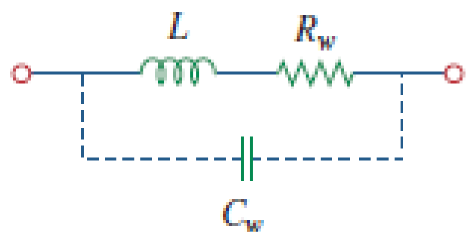


Figure 6.25
Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.

2. An important property of the inductor is its opposition to the change in current flowing through it.
The current through an inductor cannot change instantaneously.

3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.

4. A practical, non-ideal inductor has a significant resistive component, due to the fact that the inductor is made of a conducting material such as copper, which has some resistance.



Example Problems

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\, di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t})$$

$$= e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

2. Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:

Since $i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

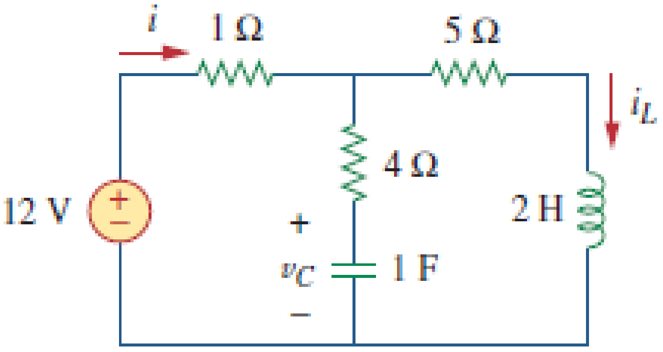
Practical Problems

1. The terminal voltage of a 2-H inductor is $v = 10(1 - t)$ V. Find the current flowing through it at $t = 4$ s and the energy stored in it at $t = 4$ s. Assume $i(0) = 2$ A.

Answer: -18 A, 320 J.

Example Problems

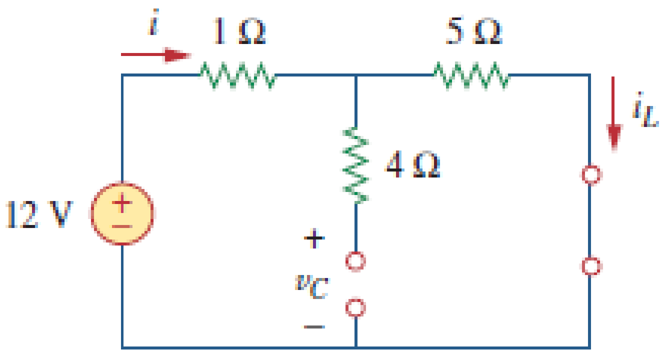
Consider the circuit in the given figure. Under dc conditions, find:
(a) i, V_c, and i_L (b) the energy stored in the capacitor and inductor.



(a)

Solution

Under dc conditions Capacitor acts like open circuited and Inductor acts like short circuited,



(b)

$$i = i_L = \frac{12}{1 + 5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

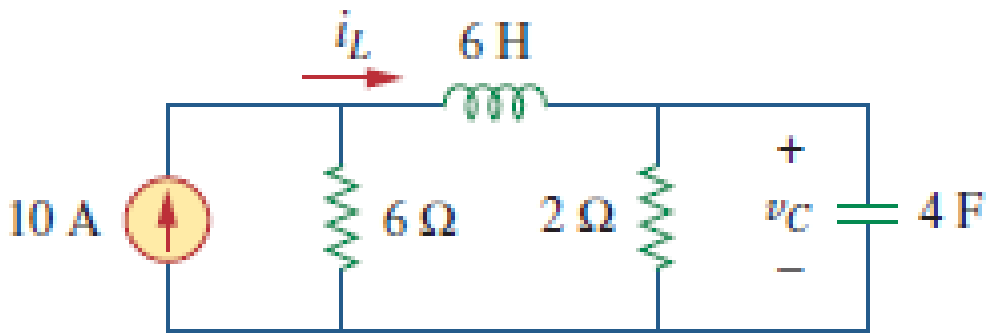
and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

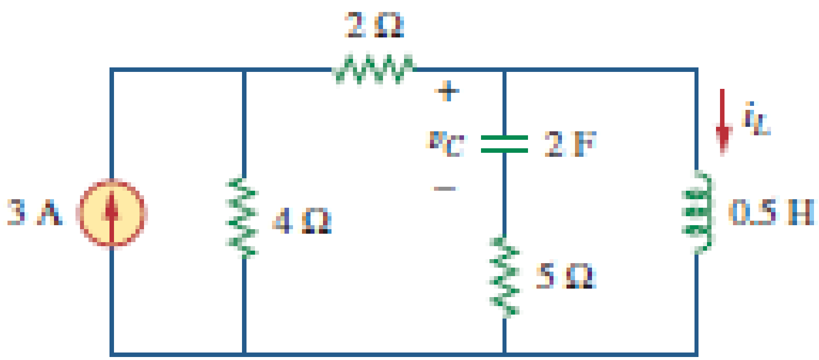
Practical Problems

1. Under steady-state dc conditions, find i_L and v_C and energy stored in the circuit in Figure.

Answer: 15 V, 7.5 A, 450 J, 168.75 J.

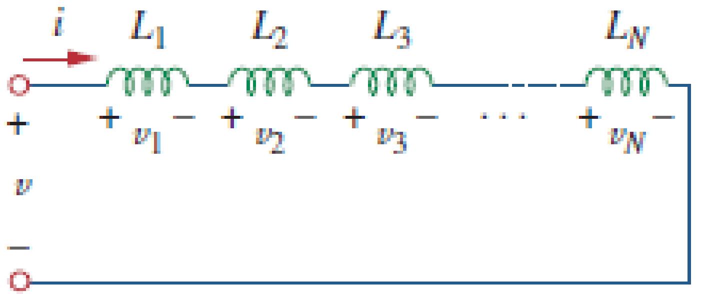


2. Under steady-state dc conditions, find i_L and v_C and energy stored in the circuit in Figure. (**Answer**: i_L=2A, v_C=0, W_L=1J, W_C=0)



Inductor

Inductors in series



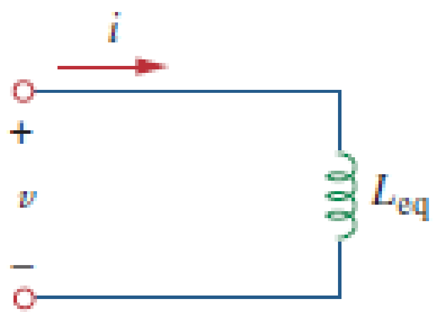
(a)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

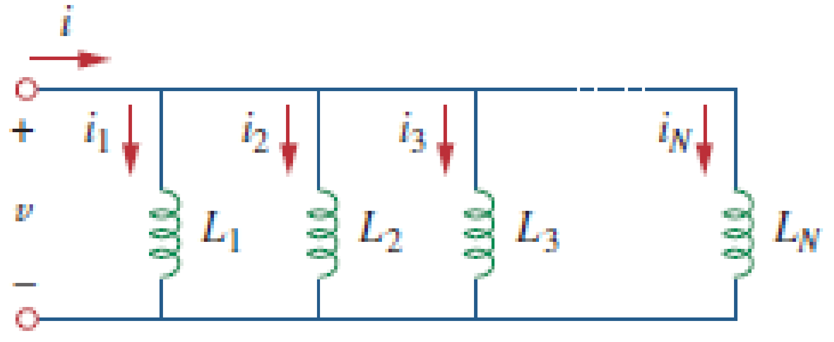
$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N}$$



(b)

Inductors in Parallel

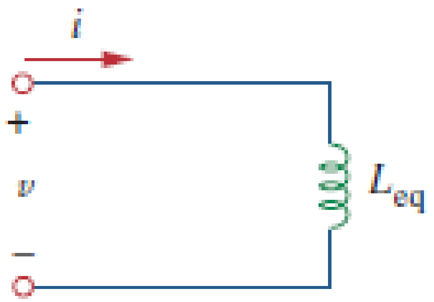


(a)

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt$$



(b)

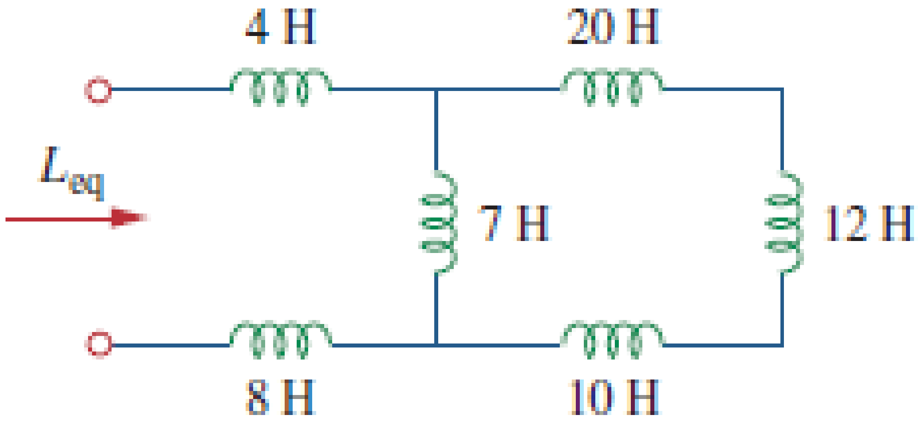
$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

If N=2

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Example Problems

1. Find the equivalent inductance of the circuit shown in Figure.



Solution:

The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

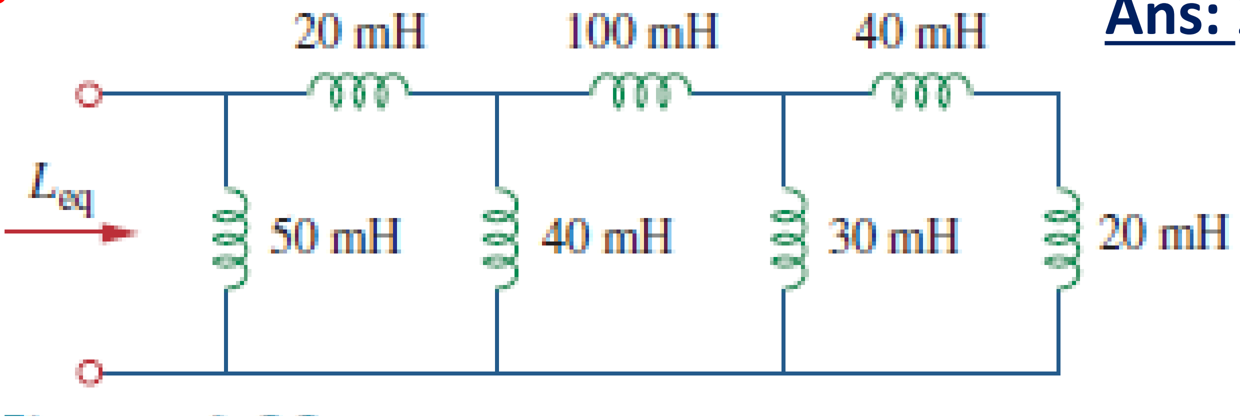
$$\frac{7 \times 42}{7 + 42} = 6 \text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

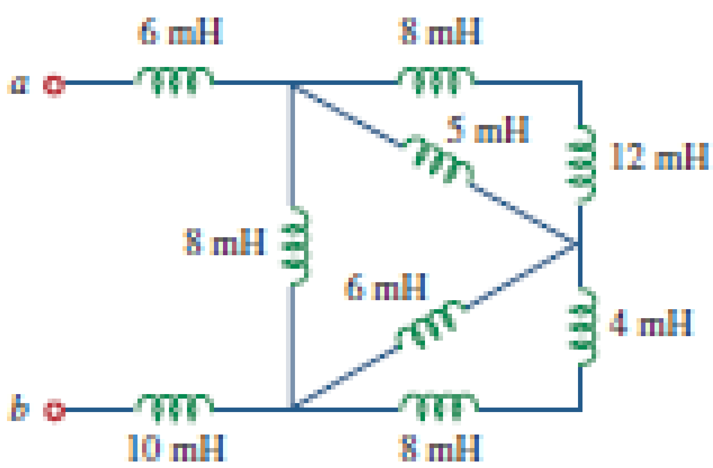
$$L_{eq} = 4 + 6 + 8 = 18 \text{ H}$$

Practical Problems

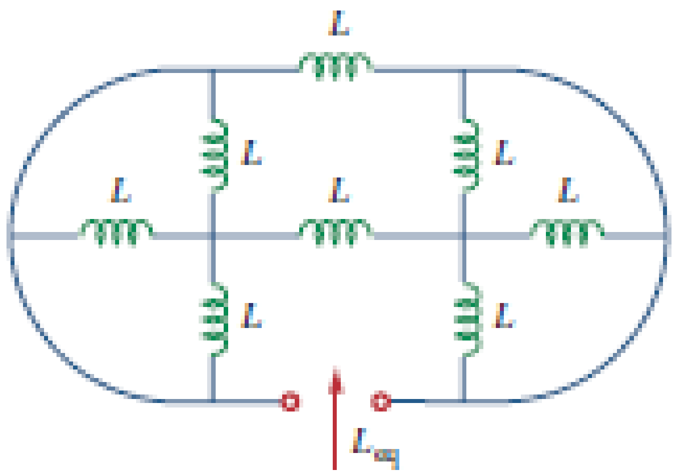
1. Find the equivalent inductance of the circuit shown in Figure.



Ans: 25 mH



Ans: 20 mH



Ans: 5/8*L

Important characteristics of the basic elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Summary

Capacitors and inductors possess the following three special properties that make them very useful in electric circuits:

1. The capacity to store energy makes them useful as temporary voltage or current sources. Thus, they can be used for generating a large amount of current or voltage for a *short period of time*.

2. Capacitors oppose any abrupt change in voltage, while inductors oppose any abrupt change in current. This property makes inductors useful for *spark or arc suppression* and for converting pulsating dc voltage into relatively *smooth dc* voltage.

3. Capacitors and inductors are frequency sensitive. This property makes them useful for *frequency discrimination*. Used as tuned circuit in Radio receivers

All the materials extracted from Fundamentals of Electric Circuits by Charles K. Alexander, Matthew N.O. Sadiku, 5th Edition, McGraw Hill, for the purpose of Teaching and Learning only.