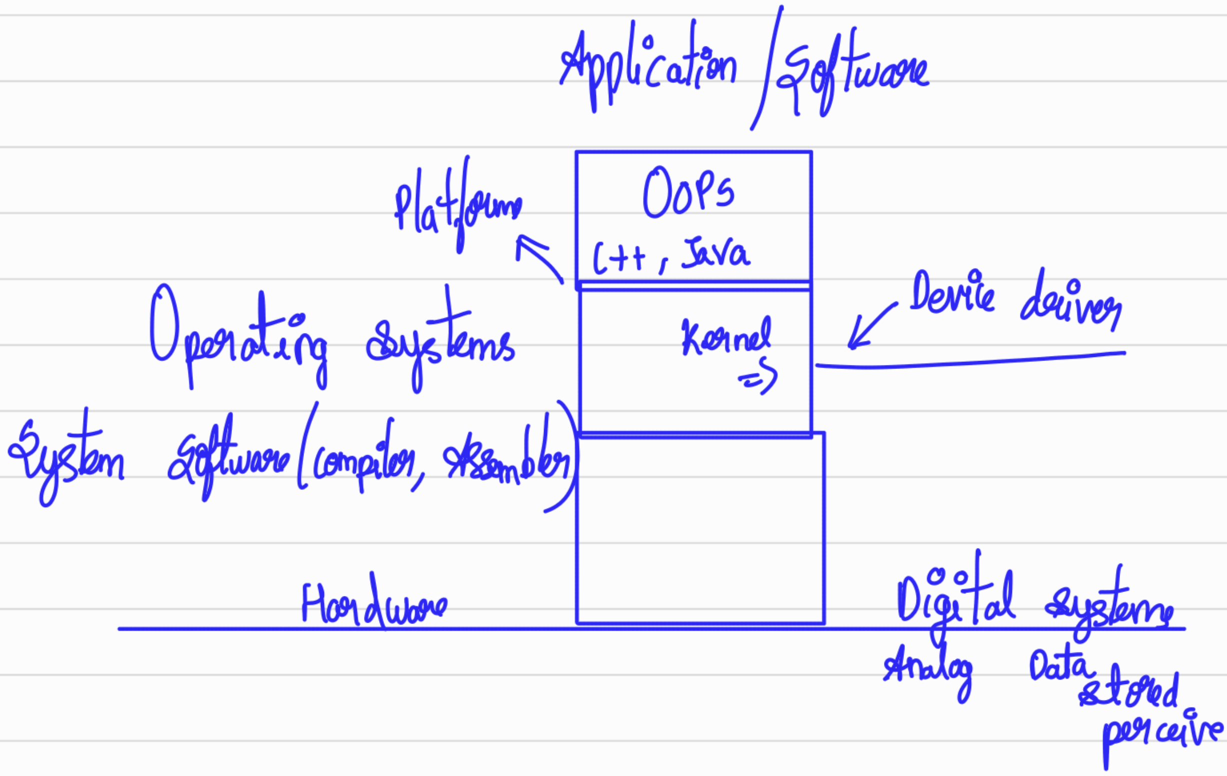
DSD



Architecture => Von neumann

Theory of computation - [decidability, computability)
Alan Turing

$(98)_{10}$

$$\begin{array}{r|l} 8 & 98 \\ \hline 8 & 12 - 2 \\ \hline & 1 - 4 \end{array}$$

$\Rightarrow (142)_8$

142

$2 \times 8^0 = 2$

$4 \times 8^1 = 32$

$1 \times 8^2 = 64$

$= 98$

A - 10
B - 11
C - 12
D - 13
E - 14
F - 15

$$\begin{array}{r|l} 4 & 43 \\ 4 & 10 - 3 \\ & 2 - 2 \end{array} \qquad (223)_4$$

$$(13)_{10} = (F)_{16}$$

$$(1000)_8 \quad \hookrightarrow \times 8^0 \quad \hookrightarrow \times 8^1 \quad \hookrightarrow \times 8^2 \quad \hookrightarrow \times 8^3 \quad = (512)_{10}$$

$$(1000)_4 \quad \hookrightarrow \times 4^3 \quad = 64$$

$$(11)_{10}$$

$$\begin{array}{r|l} 2 & 11 \\ 2 & 5 - 1 \\ 2 & 2 - 1 \\ & 1 - 0 \end{array} \qquad (1011)_2$$

$$10101$$

$$\begin{array}{ll} \hookrightarrow \times 2^0 & = 1 \\ \hookrightarrow \times 2^1 & = 0 \\ \hookrightarrow \times 2^2 & = 4 \\ \hookrightarrow \times 2^3 & = 0 \\ \hookrightarrow \times 2^4 & = 16 \\ \hline & \underline{21} \end{array}$$

$$\begin{array}{r|l} 2 & 112 \\ 2 & 56 - 0 \\ 2 & 28 - 0 \\ 2 & 14 - 0 \\ 2 & 7 - 0 \\ 2 & 3 - 1 \\ & 1 - 1 \end{array} \qquad (112)_{10} = (111000)_2$$

$(512)_{10}$

$$\begin{array}{r|l}
8 & 512 \\
8 & 64 - 0 \\
8 & 8 - 0 \\
 & 1 - 0
\end{array}$$

$(1000)_8$

$$\begin{array}{r|l}
4 & 512 \\
4 & 128 - 0 \\
4 & 32 - 0 \\
4 & 8 - 0 \\
 & 2 - 0
\end{array}$$

$(20000)_4$

$(1000000000)_2$

$(1023)_{10}$

$$\begin{array}{r|l}
8 & 1023 \\
8 & 127 - 7 \\
8 & 15 - 7 \\
 & 1 - 7
\end{array}$$

$(1777)_8$

$$\begin{array}{r|l}
16 & 1023 \\
16 & 63 - 15 \\
 & 3 - 15
\end{array}$$

$(3FF)_{16}$

$(110.11)_2 \to (\quad)_{10}$

110.11

$1 \times 2^{-2} = 0.25$

$1 \times 2^{-1} = 0.5$

$\overline{0.75}$

$0 \times 2^{0} = 0$

$1 \times 2^{1} = 2$

$1 \times 2^{2} = 4$

$\underline{(6.75)_{10}}$

16 4 1

10101 $\quad 001^{2^{-3}}$

21 $\quad = \frac{1}{8} = 0.125$

$(21.125)_{10}$

$(21.125)_{10}$

$0.125 \times 2 = 0.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.00$

.001

21 - 10101

=> 10101

$(41.75)_{10}$

41 - 101001

$0.75 \times 2 = 1.5$
$0.5 \times 2 = 1.00$

.11

101001.11

39.625

39- 100111

$0.625 \times 2 = 1.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.00$

.101

100111.101

$(1.3125)_{10}$

$0.3125 \times 2 = 0.625$
$0.625 \times 2 = 1.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.00$

1. 0101

14.4 $\quad$ $14 - 1110$

$0.4 \times 2 = 0.8$

$0.8 \times 2 = 1.6$

$0.6 \times 2 = 1.2$ $\Big\}$ recurring - take 6 points

$0.2 \times 2 = 0.4$

$0.4 \times 2 = 0.8$

$0.8 \times 2 = 1.6$

$\Rightarrow (14.4)_{10} \Rightarrow (1110.011001)$

$(20)_4 \rightarrow (8)_{10} \rightarrow (1000)_2$

$(1000)_2$

$4 = 2^{2}$

$10 | 00$

$2 \quad 0 \quad = (20)_4$

$8 = 2^{3}$

$1 | 000$

$1 \quad 0 \quad = (10)_8$

$(31)_8$ $\quad$ $011 | 001 \rightarrow (11001)_2$

$3 \quad 1$

31

$\rightarrow 1 \times 8^0 = 1$

$\rightarrow 3 \times 8 = 24$ $\quad (25)_{10} \rightarrow (11001)_2$

$(58)_{16} \rightarrow (0\overset{5}{101} | 1\overset{8}{000})_2$

$001 | 011 | 000$

$1 \quad 3 \quad 0 \rightarrow (130)_8$

$01 | 01 | 10 | 00$

$1 \quad 1 \quad 2 \quad 0$

$(1120)_4$

$(58)_{16}$

$8 \times 16^0 = 8$

$5 \times 16^1 = 80$

$(88)_{10}$

$$\begin{array}{r|l} 8 & 88 \\ 8 & 11 - 0 \\ & 1 - 2 \end{array}$$

$(130)_8$

$$\begin{array}{r|l} 4 & 88 \\ 4 & 22 - 0 \\ 4 & 5 - 2 \\ & 1 - 1 \end{array}$$

$(1120)_4$

$(131)_4$

01 | 11 | 01

$(001101)_2$

000 | 011 | 101
2 | 5

$(35)_8$

001 | 1101
1 D

$(1D)_{16}$

①

$$\begin{array}{r} 42 \\ -\ 39 \\ \hline 03 \end{array}$$

② - By Complement

9's complement

$39 = x$

↓ ↓

9 9

$99 - 39 \rightarrow 60 = y$

$99 - x = y$

10's complement   9's complement + 1

$60 + 1 = 61$

Using 9's complement

$$\begin{array}{r} 42 \\ 60 \\ \hline 1\,02 \\ \hookrightarrow 1 \\ \hline 03 \end{array}$$

using 10's complement

$$\begin{array}{r} 40 \\ 61 \\ \hline 1\,03 \end{array}$$

Binary addition and Subraction

$$\begin{array}{r} 1011 \\ +\ 0101 \\ \hline 10000 \end{array}$$

$$\begin{array}{r} 1011 \\ (-)\ 0101 \end{array} \rightarrow \begin{array}{r} 1010 \\ 1 \\ \hline 1011 \end{array} \begin{array}{l} (1\text{'s complement}) \\ \\ (2\text{'s complement}) \end{array}$$

$$\begin{array}{r} 1011 \\ +\ 1011 \\ \hline 1\,0110 \end{array}$$

Indicates positive

$$\begin{array}{r} 14 \\ (-)\ 9 \\ \hline 5 \end{array} \quad \begin{array}{r} 1110 \\ (-)\ 1001 \\ \hline \end{array} \rightarrow \begin{array}{r} 0110 \\ 1 \\ \hline 0111 \end{array}$$

$$\begin{array}{r} 1110 \\ +\ 0111 \\ \hline 1|0101 \\ \hline \end{array}$$

$\downarrow$ 5

$$\begin{array}{r} 9 \\ (-)\ 14 \\ \hline -5 \end{array} \quad \begin{array}{r} 1001 \\ (-)\ 1110 \end{array} \rightarrow \begin{array}{r} 0001 \\ 1 \\ \hline 0010 \end{array}$$

$$\begin{array}{r} 1001 \\ 0010 \\ \hline 0\,|1011 \end{array}$$

(-ve) ← 0

↳ If 0 comes repeat the process

$$\begin{array}{r} 0100 \\ 1 \\ \hline 0101 \end{array} \rightarrow 5$$

2's complement is the magnitude of negative integer

$$\begin{array}{r} 63 \\ -\ 48 \\ \hline 15 \\ \hline \end{array} \quad \begin{array}{r} \rightarrow 111111 \\ (-)\ 110000 \\ \hline \end{array} \longrightarrow \begin{array}{r} 001111 \\ 1 \\ \hline 010000 \end{array}$$

$$\begin{array}{r} 111111 \\ 010000 \\ \hline 1\,|001111 \end{array} \rightarrow 15$$

+ve ← 1

$$\begin{array}{r} 48 \\ (-)\ 63 \\ \hline -(15) \end{array} \quad \begin{array}{r} 110000 \\ (-)\ 111111 \end{array} \rightarrow \begin{array}{r} 000000 \\ 1 \\ \hline 000001 \end{array}$$

$$\begin{array}{r} 110000 \\ 000001 \\ \hline 0\,|110001 \end{array}$$

-ve ← 0

001110
1
001111
↳ magnitude 15

-14
-12
-26

14 - 1110

0001
1
0010

12 - 1100

0011
1
0100

0010
0100
00110

11001
1
11010
16 8 2 = 26

A 10110
B (-) 01110

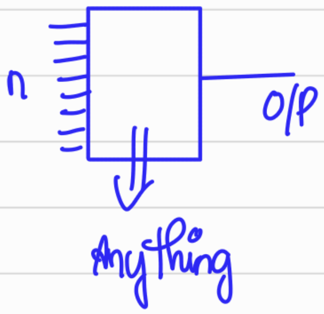
22
(-) 14
8

10001
1
10010

10110
10010
101000

+ve ← 1

Gate

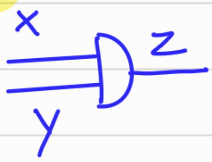


- AND
- OR
- NOT

AND

$X.Y$

XY



	X	Y	Z
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1

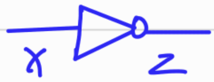
OR

$X+Y$

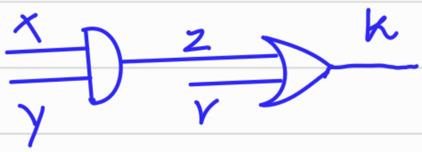
	X	Y	Z
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

NOT

$\overline{X}$

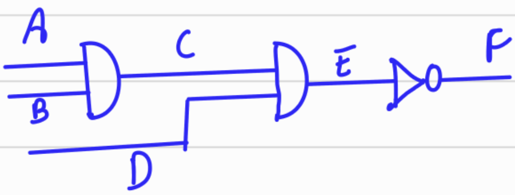


X	Z
0	1
1	0



$K = Z + V$

$= X.Y + V$

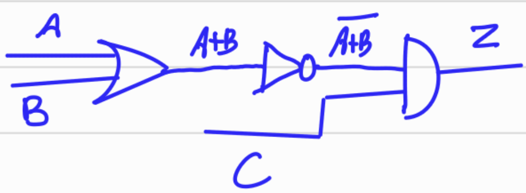


$F = \overline{E}$

$= \overline{C.D}$

$= \overline{A.B.C}$

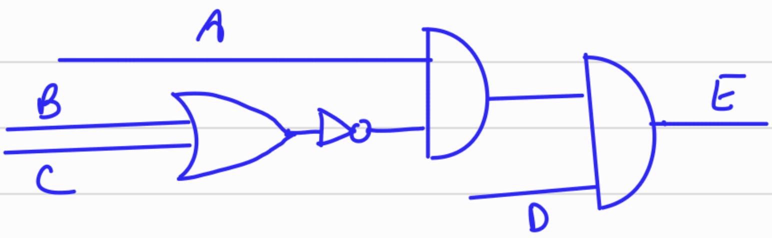
$Z = \overline{(A+B)}.C$



Order of precedence

NOT -> AND -> OR

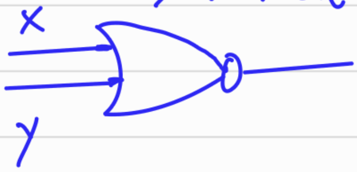
$E = A.\left(\overline{B+C}\right) D$



Universal gates

NOR

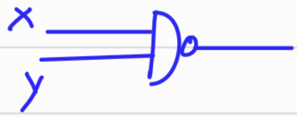
↳ NOT OR



x	y	z	$\bar{z}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

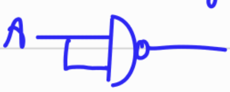
NAND

↳ NOT AND

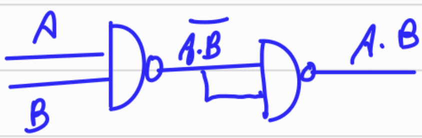


x	y	z	$\bar{z}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

NOT using universal gate



AND using universal gate



$F(A,B,C) = ABC + AB\bar{C} = AB$

A	B	C	ABC	$\bar{C}$	$AB\bar{C}$	F
0	0	0	0	1	0	0
0	0	1	0	0	0	0
0	1	0	0	1	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	0
1	0	1	0	0	0	0
1	1	0	0	1	1	1
1	1	1	1	0	0	1

	AND	OR	NOT
i)	$\cdot$	$+$	$-$
ii)	$I = 1$ $A \cdot I = A$	$I = 0$ $A + I = A$	$A + \bar{A} = 1$ $A \cdot \bar{A} = 0$
iii)	$A(BC) = (AB)C$	$(A+B)+C = A+(B+C)$	
iv)	$AB = BA$	$A+B = B+A$	
v)	$A \cdot A = A$	$A + A = A$	

a) $A.(A+B) = A.A + A.B = A + AB = A(1+B) = A$

a) $(A+B).(\bar{A}+B) = A(\bar{A}+B) + B(\bar{A}+B)$

$= A.\bar{A} + AB + B\bar{A} + B.B$

$= 0 + B\underbrace{(A+\bar{A})}_{1} + B$

$= B+B$

$= B$

a) $A.\bar{B} + AB + \bar{A}.\bar{B} = A\underbrace{(\bar{B}+B)}_{1} + \bar{A}\bar{B} = A + \bar{A}.\bar{B}$

$= (A+\bar{A})\bar{B} + AB = \bar{B} + AB$

a) $ABC + \bar{A}BC + AB\bar{C}$

$BC(A+\bar{A}) + AB\bar{C}$ (or) $AB(C+\bar{C}) + \bar{A}BC$

$BC + AB\bar{C}$ $\quad$ $AB + \bar{A}BC$

a) $AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$

$AB\bar{C} + A\bar{B}(C+\bar{C})$

$\Rightarrow AB\bar{C} + A\bar{B}$

$\Rightarrow A(B\bar{C} + \bar{B})$

a) $A+\bar{A}B$

$A\underbrace{(1+B)}_{1}+\bar{A}B$

$A+AB+\bar{A}B$

$A+B[A+\bar{A}]=A+B$

De Morgan's law

1. $\overline{A+B}=\bar{A}.\bar{B}$
2. $\overline{AB}=\bar{A}+\bar{B}$

a) $AB+\bar{A}B+A\bar{B}$

$B(A+\bar{A})+A\bar{B}$

$B+A\bar{B}$

$B(1+A)+A\bar{B}$

$B+AB+A\bar{B}$

$B+A(B+\bar{B})$

$B+A$ //.

2. $ABC+A\bar{B}C+AB\bar{C}+A\bar{B}\bar{C}$

$AB(C+\bar{C})+A\bar{B}(C+\bar{C})$

$AB+A\bar{B}$

$A(B+\bar{B})$

$=A$

(3) $ABC + AB + A\bar{C} + A\bar{B}\bar{C}$

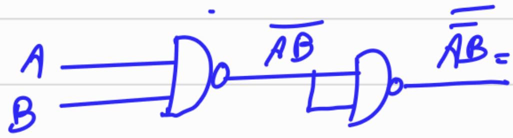
$AB(C+1) + A\bar{C}(1+\bar{B})$

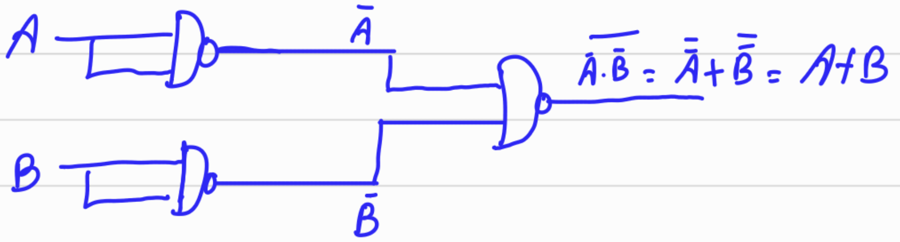
$AB + A\bar{C}$

$\Rightarrow A(B+\bar{C})$

NAND

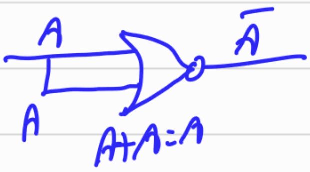
$\rightarrow$ NOT



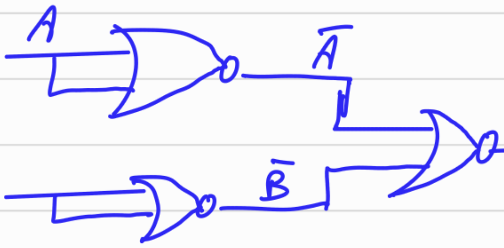


$\rightarrow$ OR gate using NAND

NOR



$\rightarrow$ NOT



$\overline{\bar{A}+\bar{B}} = \bar{\bar{A}}.\bar{\bar{B}} = AB$

$\Rightarrow$ AND using NOR

$AB + \bar{A}B + A\bar{B}$

A	B	F
		0
		1
		1
		1

Q) $AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC \rightarrow$ SOP canonical form

$AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC + ABC + ABC$ $\quad \Rightarrow A+A=A$

$AB(C+\bar{C}) + AC(\bar{B}+B) + BC(\bar{A}+A)$

$AB + AC + BC$

Term standard form (not unique)

Sum of Products (Unique) / Min term

$F(A,B) : AB + \bar{A}B + A\bar{B}$

A	B	AB	$\bar{A}B$	$A\bar{B}$	F
0	0	0	0	0	0
0	1	0	1	0	1
1	0	0	0	1	1
1	1	1	0	0	1

$\Rightarrow \bar{A}.B$
$\Rightarrow A\bar{B}$
$\Rightarrow AB$

In truth table see the 1 in F, in that row if the variable has zero put $\bar{\ }$, if the variable is 1, directly write it

$AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$
$110 \quad 101 \quad 011 \quad 111$

Match this code in truth table

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$AB + \bar{A}B + A\bar{B} \Rightarrow F(A,B)$
$11 \quad 01 \quad 10$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

$\Rightarrow A+B$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

$\rightarrow \bar{A}B$

$\rightarrow A\bar{B}$

$$\bar{A}B + A\bar{B}$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$\boxed{\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC}$$

$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$= \bar{A}(\bar{B}C + B\bar{C} + BC)$$

$$= \bar{A}(\bar{B}C + B) = \bar{A}(\bar{B}C + B(1+C))$$

$$= \bar{A}(\bar{B}C + B + BC)$$

$$\bar{A}(B+C)$$

$$A(\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC)$$

$$A(\bar{B}(\bar{C}+C) + B(\bar{C}+C))$$

$$A$$

$$\Rightarrow \bar{A}B + \bar{A}C + A$$

$$\Rightarrow \bar{A}B + \bar{A}C + A(1+B)$$

$$\Rightarrow \bar{A}B + \bar{A}C + A + AB$$

$$B + \bar{A}C + A(1+C)$$

$$B + \bar{A}C + A + AC$$

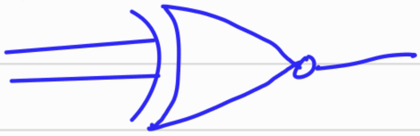
$$\Rightarrow A + B + C$$

XOR - 01, 10 -> 1

$A \oplus B$

XNOR 00, 11 -> 1

$A \odot B$



A -> 0
B -> 1

$A\bar{B} + \bar{A}B$

$\bar{A}\bar{B} + AB$

$$\overline{A \oplus B} = A \odot B$$

$$\overline{A \odot B} = A \oplus B$$

$$A \oplus \underbrace{B \oplus A}_{X}$$

$$X = \bar{A}B + A\bar{B}$$

$$A \oplus X$$

$$A\bar{X} + \bar{A}X$$

$$A(\overline{\bar{A}B + A\bar{B}}) + \bar{A}(\bar{A}B + A\bar{B})$$

$$A(\overline{\bar{A}B} \cdot \overline{A\bar{B}}) + \underbrace{\bar{A}\bar{A}B}_{B} + \underbrace{A\bar{A}\bar{B}}_{0}$$

$$A((\bar{\bar{A}} + \bar{B}) \cdot (\bar{A} + \bar{\bar{B}}))$$

$$A((A + \bar{B}) \cdot (\bar{A} + B))$$

$$A(\underbrace{A\bar{A}}_{0} + \bar{B}\bar{A} + BA + \underbrace{B\bar{B}}_{0})$$

$$A(\bar{B}\bar{A} + AB)$$

$$\underbrace{A\bar{A}}_{0}\bar{B} + AAB + B$$

$$\Rightarrow B$$

A	B	$A \oplus B$	$X \oplus A$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

X (over the $A \oplus B$ column); B (under the $X \oplus A$ column)

A B $\Rightarrow$ $C = A \oplus B$

2GB 2GB

$\Downarrow$ 2GB

Can be a backup

If A is lost, then $C \oplus B$

$A \oplus B \oplus B$

$\Rightarrow A$

$\Downarrow$

We got A

IEEE 754

Floating point number - 32 bit

64 bit

32 bit

s e m

sign exponent mantissa

1 8 23

$$\overset{12.5}{1100.1} = 1.1001 \times 2^{3 = e}$$

(Ex : $1024.32 = 1.02432 \times 10^3$)

B Bias $= 127$

$B + e = 127 + 3 = 130$

$(10000010)_2$

130 digits after point

s e m

0 10000010 10010000...

0.125 × 2 = 0.250
0.25 × 2 = 0.5
0.5 × 2 = 1.0

0.001

1.00×2^{-3}

B = 127
127 - 3
124

0 01111100 000000...
s e m

s e 101000...
0 01111110 m

(4) 1.101×2^{e}

0 1111110 = 126
e = 126 - 127
= -1

$1.101 \times 2^{-1} = (0.1101)_2 = 0.8125$

s e m

0 10000011 11 0000...

131

131 - 127 = 4

1.11×2^4

11100

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$= 2^2 + 2^3 + 2^4$

$= 4 + 8 + 16 = 28$

-24.625

s e m

1 10000011 1000 00010.....

$(24)_{10} = (11000)_2$

$0.625 \times 2 = 0.125$

$0.125 \times 2 = 0.250$

$0.25 \times 2 = 0.5$

$0.5 \times 2 = 1.0$

11000.0001

$1.1000\ 0001 \times 2^4$

B+e = 127+4

= 131

10000011

6 5 4 3 2 1

s e m

1 1000 0111 111 000

128
7
135

135-127
= 8

1.111×2^8

111100000
8 7 6 5 4 3 2 1 0

$2^5+2^6+2^7+2^8$

32 + 64 + 128 + 256

2
256
128
64
32
480

-480

① Signed Number 8bit
↳ subtraction will not work

1 7
sign ↳ mag

+1 0
-1 1

② 1's complement 4bit → magnitude representation

0 0111 0 0000 +0
1 1000 -7 1 1111 -1

③ 2's Complement

0 0111
1000
1
1 1001

(1) 28.875

28
11100

$0.875 \times 2 = 1.75$
$0.75 \times 2 = 1.5$
$0.5 \times 2 = 1$

11100.111
1.1100111×2^4

s	e	m
0	10000011	1100111

127+4 = 131

(2) -14.8

s	e	m
1	10000010	110110011

14
1110.110011
$0.8 \times 2 = 1.6$
$0.6 \times 2 = 1.2$
$0.2 \times 2 = 0.4$
$0.4 \times 2 = 0.8$
$0.8 \times 2 = 1.6$
$0.6 \times 2 = 1.2$

1.110110011×2^3

127+3
= 130

(4) 0.625

$0.625 \times 2 = 1.250$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.00$

101

s	e	m
0	01111110	0100...

127-1
= 126

0.101
$1.0100... \times 2^{-1}$

3 1 | 10000.11 0|000100.100

128
6
$e = 7$
$\times 2^7$

1.0001001×2^7
10001001.0
137

-137.00

① $A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + B$

② $\bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + ABC + \overline{ABC}$ $\Rightarrow$ Best Way $ABC + \overline{ABC} = 1$
If $\bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} = Y$.

③ $ABC + \bar{A}\bar{B}\bar{C} + AB\bar{C}$

④ $24 - 2$ (2^3 complement)

⑤ $A \odot B \odot C$ [T.T]

⑥ $(A+C)(A+B).A$

② $\bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + ABC + \bar{A} + \bar{B} + \bar{C}$

$\bar{A}\bar{C} + ABC + \bar{A} + \bar{B} + \bar{C}$

$\bar{C}(1+\bar{A}) + \bar{A} + \bar{B} + ABC$

$\bar{A} + \bar{B} + \bar{C} + ABC$

$\bar{A}(1+BC) + \bar{B} + \bar{C} + ABC = \bar{A} + BC(\bar{A}+A) + \bar{B} + \bar{C}$

$= \bar{A} + BC + \bar{B}(1+C) + \bar{C}$

$\bar{A} + BC + \bar{B} + \bar{B}C + \bar{C}$

$\bar{A} + \bar{B} + C(B + \bar{B}) + \bar{C}$

$= \bar{A} + \bar{B} + 1 \quad = 1 \mathbin{/\!/}.$

(6) $(A+C)(A+B).A$

$(AA + AC + BC + AB)A$

$AAA + AAC + ABC + AAB$

$A + AC + ABC + AB$

$A(1 + C + BC + B)$

$A(1 + C + B(1 + C))$

A

(5) $A \odot B \odot C$

A	B	C	$A \odot B$	$A \odot B \odot C$
0	0	0	1	0
0	0	1	1	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

3. $AB C + \bar{A}\bar{B}\bar{C} + AB\bar{C}$

$AB(C+\bar{C}) + \bar{A}\bar{B}\bar{C}$

$AB + \bar{A}\bar{B}\bar{C}$

① $A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + B$

$A\bar{B}(C+\bar{C}) + \bar{A}\bar{B}(C+\bar{C}) + B$

$A\bar{B} + \bar{A}\bar{B} + B$

$\bar{B}(A+\bar{A}) + B$

$\bar{B} + B$

$\Rightarrow 1$

④ 24 - 2

$$\begin{array}{r} 24 \\ -\ 2 \\ \hline 22 \end{array}$$

11000

00010 -

$$\begin{array}{r} 11101 \\ 1 \\ \hline 11110 \end{array}$$

$$\begin{array}{r} 11000 \\ 11110 \\ \hline 1\,|\,10110 \end{array}$$

+ve ← 1

16 4 2

16+4+2 = 22 //.

K-Maps

$F(A,B,C) = \bar{A}\bar{B}C + ABC + AB\bar{C}$ ⇒ canonical form

$\quad\quad\quad\quad\quad 001 \quad\quad 111 \quad\quad 110$

$F(A,B,C) = F(1,6,7)$

$F(A,B,C) = \Sigma(m_1, m_6, m_7)$

A	B	F(A,B)
0	0	0
0	1	0
1	0	1
1	1	1

K-Map

A \ B	0	1
0	0_0	0_1
1	1_2	1_3

A \ B	0	1
0	0_0	0_1
1	1_2	1_3

& combine 1 in either horizontal or vertical

& see which is common, If 0 then ‾

In this 1 in A common to both 1 1

$\therefore F(A,B) = A$

From truth table $A\bar{B} + AB = A(\bar{B} + B) = A$ //

Ex:

A \ B	0	1
0	1_0	0_1
1	1_2	0_3

$F(A,B) = \bar{B}$ (0 in B is common for both 1)

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

A \ BC	00	01	11	10
0	0_0	1_1	0_3	0_2
1	0_4	0_5	1_7	1_6

1 1 has common A in 1
and $\overset{11}{1}$ $\overset{10}{1}$ → 1 is common BC row, which is B

$\Rightarrow AB$

$\overset{01}{1} \Rightarrow \overset{001}{\bar{A}\bar{B}C}$

$\therefore \Rightarrow AB + \bar{A}\bar{B}C$

A \ BC	00	01	11	10
0	1_0	1_1	0_3	0_2
1	0_4	1_5	1_7	1_6

$\bar{A}\bar{B} + AB + \bar{B}C$

Only one time pairing with any one

A \ BC	00	01	11	10
0	0_0	1_1	1_3	0_2
1	0_4	1_5	1_7	0_6

C

A \ BC	00	01	11	10
0	1_0	1_1	1_3	0_2
1	0_4	1_5	1_7	1_6

$C + \bar{A}\bar{B} + AB$

A \ BC	00	01	11	10
0	0_0	0_1	1_3	1_2
1	1_4	1_5	1_7	1_6

$A + C$

A \ BC	00	01	11	10
0	0_0	0_1	0_3	0_2
1	1_4	0_5	0_7	1_6

$A\bar{C}$

A \ BC	00	01	11	10
0	1_0	0_1	1_3	1_2
1	1_4	0_5	0_7	1_6

A \ BC	00	10
0	1	1
1	1	0

$\Rightarrow \bar{C}$

$\Rightarrow \bar{C} + \bar{A}B$

A \ BC	00	01	11	10
0	1_0	1_1	0_3	0_2
1	1_4	0_5	1_7	1_6

$\bar{B}\bar{C} + AB + \bar{A}\bar{B}$

A \ BC	00	01	11	10
0	1_0	1_1	0_3	1_2
1	0_4	1_5	1_7	1_6

$\bar{A}\bar{B} + AC + B\bar{C}$

A \ BC	00	01	11	10
0	1_0	0_1	1_3	0_2
1	0_4	1_5	0_7	1_6

$\bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC + AB\bar{C}$

$\Sigma(m_0, m_2, m_3, m_4, m_6, m_7)$

$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

000   010   011   100   110   111

A \ BC	00	01	11	10
0	1_{0}	0_{1}	1_{3}	1_{2}
1	1_{4}	0_{5}	1_{7}	1_{6}

$B + \bar{C}$

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$\Sigma(m_0, m_2, m_3, m_4, m_5, m_6, m_7)$

A \ BC	00	01	11	10
0	1_{0}	0_{1}	1_{3}	1_{2}
1	1_{4}	1_{5}	1_{7}	1_{6}

$A + B + \bar{C}$

$\Sigma(m_0, m_2, m_3, m_4, m_5, m_7)$

A \ BC	00	01	11	10
0	1_{0}	0_{1}	1_{3}	1_{2}
1	1_{4}	1_{5}	1_{7}	0_{6}

$BC + A\bar{B} + \bar{A}\bar{C}$

$\Sigma(m_0, m_1, m_2, m_3, m_4, m_5, m_7, m_8, m_{12}, m_{13}, m_{15})$

AB \ CD	00	01	11	10
00	1_{0}	1_{1}	1_{3}	1_{2}
01	1_{4}	1_{5}	1_{7}	0_{6}
11	1_{12}	1_{13}	1_{15}	0_{14}
10	1_{8}	0_{9}	0_{11}	0_{10}

$\bar{A}\bar{B} + BD + \bar{C}\bar{D}$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

AB \ CD	00	01	11	10
00	1 (0)	1 (1)	1 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

AB \ CD	00	10
00	1	1
10	1	1

$\bar{B}\bar{D} + \bar{A}\bar{B} + BD$

AB \ CD	00	01	11	10
00	0 (0)	0 (1)	1 (3)	0 (2)
01	1 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	1 (9)	0 (11)	0 (10)

$BD + \bar{A}B\bar{C} + ABC$
$+ \bar{A}CD + A\bar{C}D$

Is this four needed?
No, instead of reducing the variable, it had a variable.

$\Rightarrow$

AB \ CD	00	01	11	10
00	0 (0)	0 (1)	1 (3)	0 (2)
01	1 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	1 (9)	0 (11)	0 (10)

$\bar{A}B\bar{C} + ABC$
$+ \bar{A}CD + A\bar{C}D$

Prime implicants
→ Essential PI
Redundant PI
(repitation)

AB \ CD	00	01	11	10
00	0 (0)	1 (1)	1 (3)	0 (2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	1 (8)	1 (9)	1 (11)	1 (10)

$A + B + D$

AB \ CD	00	01	11	10
00	1 (0)	1 (1)	1 (3)	0 (2)
01	1 (4)	0 (5)	0 (7)	0 (6)
11	1 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	1 (11)	0 (10)

$\Sigma(m_0, m_1, m_3, m_4, m_8, m_{11}, m_{12}, m_{13}, m_{15})$

Binary	Grey Code
00	00
01	01
10	11
11	10

000
001
011
010

110
111
101
100

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

AB \ CDE	000	001	011	010	110	111	101	100
00	1_{0}	1_{1}	1_{3}	1_{2}	0_{6}	0_{7}	0_{5}	0_{4}
01	0_{8}	1_{9}	1_{11}	0_{10}	0_{14}	0_{15}	0_{13}	0_{12}
11	0_{24}	1_{25}	1_{27}	0_{26}	0_{30}	0_{31}	0_{29}	0_{28}
10	0_{16}	1_{17}	1_{19}	0_{18}	0_{22}	1_{23}	1_{21}	$_{20}$

C

	001	011	111	101
10	1	1	1	1

$\bar{C}E + \bar{A}\bar{B}\bar{C} + A\bar{B}E$

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	1_{27}	26	30	1_{31}	29	28
10	16	17	1_{19}	18	22	1_{23}	21	20

	011	111
11	1	1
10	1	1

ADE //

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	1_{11}	1_{10}	14	1_{15}	1_{13}	12
11	24	25	1_{27}	1_{26}	30	1_{31}	1_{29}	28
10	16	17	19	18	22	23	21	20

X

	010	011	111	101
01	1	1	1	1
11	1	1	1	1

2 four's

B

X

AB \ CDE	000	001	011	010	110	111	101	100
00	1 0	1	3	1 2	1 6	7	5	1 4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	1 16	17	19	1 18	1 22	23	21	1 20

$\bar{B}\bar{E}$

AB \ CDE	000	001	011	010	110	111	101	100
00	1 0	1 1	1 3	2	6	1 7	5	4
01	8	1 9	11	10	14	15	1 13	12
11	24	25	27	26	30	31	29	28
10	16	1 17	1 19	18	22	23	21	20

$\bar{A}\bar{B}DE + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{B}\bar{C}E + \bar{A}B\bar{D}E$

	Binary $B_3\ B_2\ B_1\ B_0$		Gray code $e_3\ e_2\ e_1\ e_0$			
0	0000	->	0000	8	1000	1100
1	0001	->	0001	9	1001	1101
2	0010	->	0011	10	1010	1111
3	0011	->	0010	11	1011	1110
4	0100	->	0110	12	1100	1010
5	0101	->	0111	13	1101	1011
6	0110	->	0101	14	1110	1001
7	0111		0100	15	1111	1000

$G_3 = B_3$

$B_3B_2 \backslash B_1B_0$	00	01	11	10
00	0_0	0_1	1_3	1_2
01	1_4	1_5	0_7	0_6
11	1_{12}	1_{13}	0_{15}	0_{14}
10	0_8	0_9	1_{11}	1_{10}

$\rightarrow G_1$

$\bar{B}_2 B_1 + B_2 \bar{B}_1$
$= B_1 \oplus B_2$

$A \oplus B \oplus C = ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$

$B_2B_2 \backslash B_1B_0$	00	01	11	10
00	0_0	0_1	0_3	0_2
01	1_4	1_5	1_7	1_6
11	0_{12}	0_{13}	0_{15}	0_{14}
10	1_8	1_9	1_{11}	1_{10}

$= \bar{B}_3 B_2 + B_3 \bar{B}_2$

$G_2 = B_3 \oplus B_2$

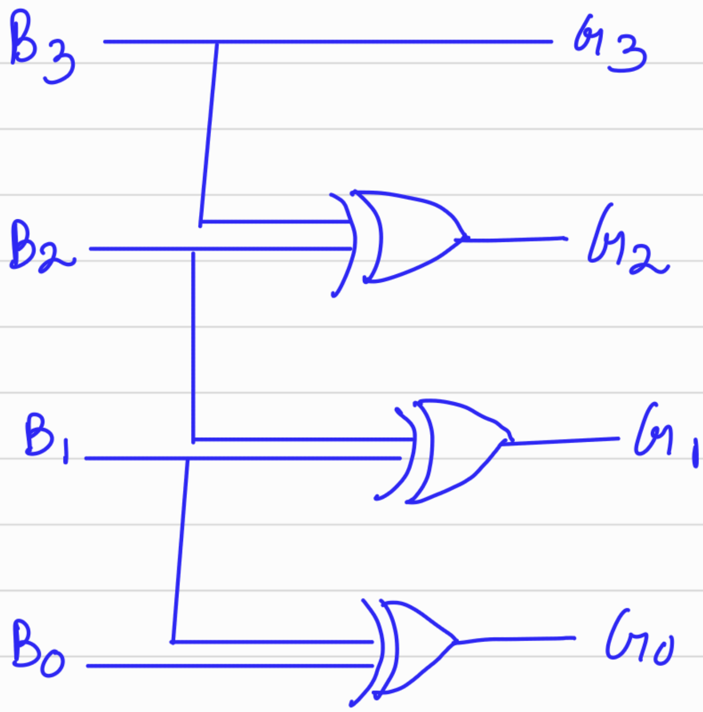
$B_3B_2 \backslash B_1B_0$	00	01	11	10
00	0_0	0_1	0_3	0_2
01	0_4	0_5	0_7	0_6
11	1_{12}	1_{13}	1_{15}	1_{14}
10	1_8	1_9	1_{11}	1_{10}

$G_3 = B_3$

$B_3B_2 \backslash B_1B_0$	00	01	11	10
00	0_0	1_1	0_3	1_2
01	0_4	1_5	0_7	1_6
11	0_{12}	1_{13}	0_{15}	1_{14}
10	0_8	1_9	0_{11}	1_{10}

$\bar{B}_1 B_0 + B_1 \bar{B}_0$
$G_0 = B_0 \oplus B_1$

$G_0 = B_0 \oplus B_1 \quad G_1 = B_1 \oplus B_2 \quad G_2 = B_2 \oplus B_3 \quad G_3 = B_3$


B3
G3
B2
G2
B1
G1
B0
G0

① $\Sigma(m_0, m_1, m_2, m_4, m_6, m_8, m_{12}, m_{13})$

AB \ CD	00	01	11	10
00	1 $_0$	1 $_1$	0 $_3$	1 $_2$
01	1 $_4$	0 $_5$	0 $_7$	1 $_6$
11	1 $_{12}$	1 $_{13}$	0 $_{15}$	0 $_{14}$
10	1 $_8$	0 $_9$	0 $_{11}$	0 $_{10}$

$\bar{C}\bar{D} + \bar{A}\bar{D} + AB\bar{C} + \bar{A}\bar{B}\bar{C}$

② $\Sigma(m_0\ m_4\ m_9\ m_{15}\ m_{16}\ m_{20}\ m_{25}, m_{31})$

AB \ CDE	000	001	011	010	110	111	101	100
00	1 $_0$	$_1$	$_3$	$_2$	$_6$	$_7$	$_5$	1 $_4$
01	$_8$	1 $_9$	$_{11}$	$_{10}$	$_{14}$	1 $_{15}$	$_{13}$	$_{12}$
11	$_{24}$	1 $_{25}$	$_{27}$	$_{26}$	$_{30}$	1 $_{31}$	$_{29}$	$_{28}$
10	1 $_{16}$	$_{17}$	$_{19}$	$_{18}$	$_{22}$	$_{23}$	$_{21}$	1 $_{20}$

$\bar{B}\bar{D}\bar{E} + B\bar{C}\bar{D}E + BCDE$

3 $\Sigma(m_0, m_2, m_4, m_6, m_{16}, m_{18}, m_{20}, m_{22})$

AB \ CDE	000	001	011	010	110	111	101	100
00	1 $_0$	$_1$	$_3$	1 $_2$	1 $_6$	$_7$	$_5$	1 $_4$
01	$_8$	$_9$	$_{11}$	$_{10}$	$_{14}$	$_{15}$	$_{13}$	$_{12}$
11	$_{24}$	$_{25}$	$_{27}$	$_{26}$	$_{30}$	$_{31}$	$_{29}$	$_{28}$
10	1 $_{16}$	$_{17}$	$_{19}$	1 $_{18}$	1 $_{22}$	$_{23}$	$_{21}$	1 $_{20}$

AB \ CDE	000	010	110	100
00	1	1	1	1
10	1	1	1	1

$\bar{B}\bar{E}$

4 $\Sigma(m_8, m_{10}, m_{12}, m_{14}, m_{26}, m_{30})$

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	1 8	9	11	1 10	1 14	15	13	1 12
11	24	25	27	1 26	1 30	31	29	28
10	16	17	19	18	22	23	21	20

AB \ CDE	000	010	110	100
01	1	1	1	1
11	0	1	1	0

$\bar{A}B\bar{E} + BD\bar{E}$

5) $\Sigma(m_0, m_1, m_3, m_4, m_5, m_7, m_{16}, m_{17}, m_{19}, m_{20}, m_{21}, m_{23})$

AB \ CDE	000	001	011	010	110	111	101	100
00	1 0	1 1	1 3	2	6	1 7	1 5	1 4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	1 16	1 17	1 19	18	22	1 23	1 21	1 20

AB \ CDE	000	001	101	100
00	1 0	1 1	1 5	1 4
10	1 16	1 17	1 21	1 20

AB \ CDE	001	011	111	101
00	1 0	1 1	1 5	1 4
10	1 16	1 17	1 21	1 20

$\bar{B}\bar{D} + \bar{B}E$

6 $\Sigma(m_8\ m_9\ m_{10}\ m_{11}\ m_{13}\ m_{14}\ m_{15}\ m_{27}\ m_{29})$

AB \ CDE	000	001	011	010	110	111	101	10
00	0	1	3	2	6	7	5	4
01	1 8	1 9	1 11	1 10	1 14	1 15	1 13	12
11	24	25	1 24	26	30	31	1 29	28
10	16	14	19	18	22	23	21	20

$\bar{A}B\bar{C} + B\bar{C}DE + BC\bar{D}E + \bar{A}BD$

7. $\Sigma\ m_1\ m_3\ m_5\ m_7\ m_9\ m_{10}\ m_{12}\ m_{17}\ m_{19}\ m_{21}\ m_{22}$
$m_{23}\ m_{25}\ m_{31})$

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1 1	1 3	2	6	1 7	1 5	4
01	8	1 9	11	1 10	14	15	13	1 12
11	24	1 25	24	26	30	1 31	29	28
10	16	1 14	1 19	18	1 22	1 23	1 21	20

AB \ CDE	001	011	111	101
00	1 0	1 1	1 5	1 4
10	1 16	1 14	1 21	1 20

$\bar{B}E + \bar{C}\bar{D}E$
$+ A\bar{B}CD + ACDE$
$+ \bar{A}B\bar{C}DE + \bar{A}BC\bar{D}\bar{E}$

Grey code → Binary

Binary $B_3\ B_2\ B_1\ B_0$		Grey code $G_3\ G_2\ G_1\ G_0$	Binary	Grey code
0000	←	0000 (0)	1000	1100 (12)
0001	←	0001 (1)	1001	1101 (13)
0010	←	0011 (3)	1010	1111 (15)
0011	←	0010 (2)	1011	1110 (14)
0100	←	0110 (6)	1100	1010 (10)
0101	←	0111 (7)	1101	1011 (11)
0110	←	0101 (5)	1110	1001 (9)
0111	←	0100 (4)	1111	1000 (8)

$B_3 = G_3$

→ B_2

$G_3G_2 \backslash G_1G_0$	00	01	11	10
00	0_0	0_1	0_3	0_2
01	1_4	1_5	1_7	1_6
11	0_{12}	0_{13}	0_{15}	0_{14}
10	1_8	1_9	1_{11}	1_{10}

$G_3\bar{G}_2 + \bar{G}_3 G_2$

$B_2 = G_3 \oplus G_2$

→ B_1

$G_3G_2 \backslash G_1G_0$	00	01	11	10
00	0_0	0_1	1_3	1_2
01	1_4	1_5	0_7	0_6
11	0_{12}	0_{13}	1_{15}	1_{14}
10	1_8	1_9	0_{11}	0_{10}

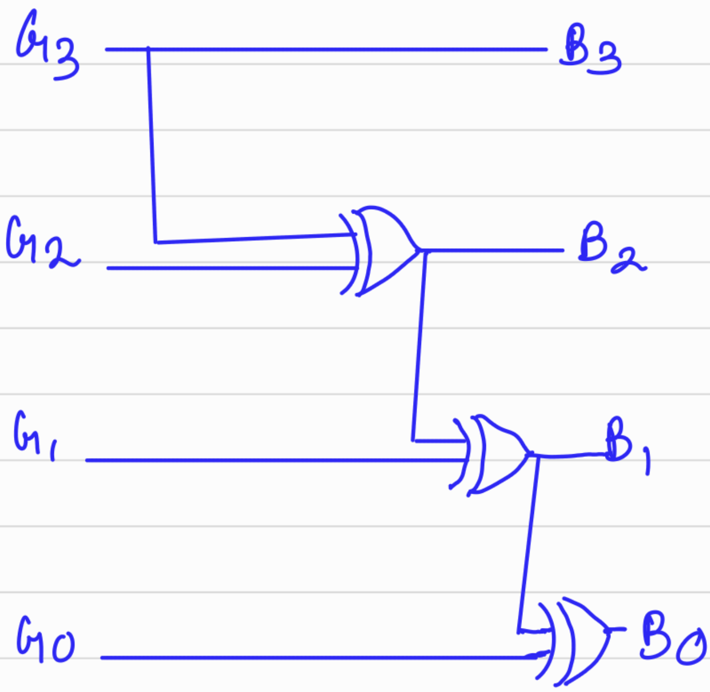
$\bar{G}_3\bar{G}_2 G_1 + \bar{G}_1 G_2 \bar{G}_1 + G_3 G_2 G_1 + G_3 \bar{G}_2 \bar{G}_1$

$= G_1 \oplus G_2 \oplus G_3$

$\rightarrow B_0$

$G_3G_2 \backslash G_1G_0$	00	01	11	10
00	0_0	1_1	0_3	1_2
01	1_4	0_5	1_7	0_6
11	0_{12}	1_{13}	0_{15}	1_{14}
10	1_8	0_9	1_{11}	0_{10}

$G_3 \oplus G_2 \oplus G_1 \oplus G_0$



Don't care

A	B	C	O/P
0	0	0	X
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

this can be considered as 0 and 1

$A \backslash BC$	00	01	11	10
0	X_0	1_1	0_5	1_4
1	1_4	1_5	0_7	0_6

$A \backslash BC$	00	01	11	10
0	1_0	1_1	0_5	1_4
1	1_4	1_5	0_7	0_6

$\bar{B} + \bar{A}\,\bar{C}$

A \ BC	00	01	11	10
0	X (0)	0 (1)	1 (5)	0 (4)
1	0 (4)	0 (5)	1 (7)	1 (6)

A \ BC	00	01	11	10
⇒ 0	0 (0)	0 (1)	1 (5)	0 (4)
1	0 (4)	0 (5)	1 (7)	1 (6)

↓
this time consider it as 0

Q) $\Sigma(m_0, m_2, m_3, m_4, m_6, m_8, m_9, m_?, m_{13}) + \Sigma(d_1, d_5, d_{15})$
⇓
$F(A,B,C,D)$

AB \ CD	00	01	11	10
00	1 (0)	X→1 (1)	1 (3)	1 (2)
01	1 (4)	X→1	(7)	1 (6)
11	1 (12)	1 (13)	X→0 (15)	(14)
10	1 (8)	1 (9)	(11)	(10)

$\bar{C} + \bar{A}\bar{B} + \bar{A}$

BCD - Binary Coded Decimal

1100   1101
12   13
0001 0010   0001 0011
1 2   1 3.

	B_3	B_2	B_1	B_0	D_4	D_3	D_2	D_1	D_0
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	1
2	0	0	1	0	0	0	0	1	0
3	0	0	1	1	0	0	0	1	1
4	0	1	0	0	0	0	1	0	0
5	0	1	0	1	0	0	1	0	1
6	0	1	1	0	0	0	1	1	0
7	0	1	1	1	0	0	1	1	1
8	1	0	0	0	0	1	0	0	0
9	1	0	0	1	0	1	0	0	1
10	1	0	1	0	1	0	0	0	0
11	1	0	1	1	1	0	0	0	1
12	1	1	0	0	1	0	0	1	0
13	1	1	0	1	1	0	0	1	1
14	1	1	1	0	1	0	1	0	0
15	1	1	1	1	1	0	1	0	1

$D_0 = B_0$

D_1

B_3B_2 \ B_1B_0	00	01	11	10
00	0_0	0_1	1_3	1_2
01	0_4	0_5	1_7	1_6
11	1_{12}	1_{13}	0_{15}	0_{14}
10	0_8	0_9	0_{11}	0_{10}

$D_1 = \bar{B}_1 B_2 B_3 + B_1 \bar{B}_3$

D_2

B_3B_2 \ B_1B_0	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	0 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	0 (10)

$D_2 = B_2\bar{B_3} + B_1 B_2$

D_2

B_3B_2 \ B_1B_0	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	0 (12)	0 (13)	0 (15)	0 (14)
10	1 (8)	1 (9)	0 (11)	0 (10)

$D_3 = \bar{B_1}\bar{B_2}B_3$

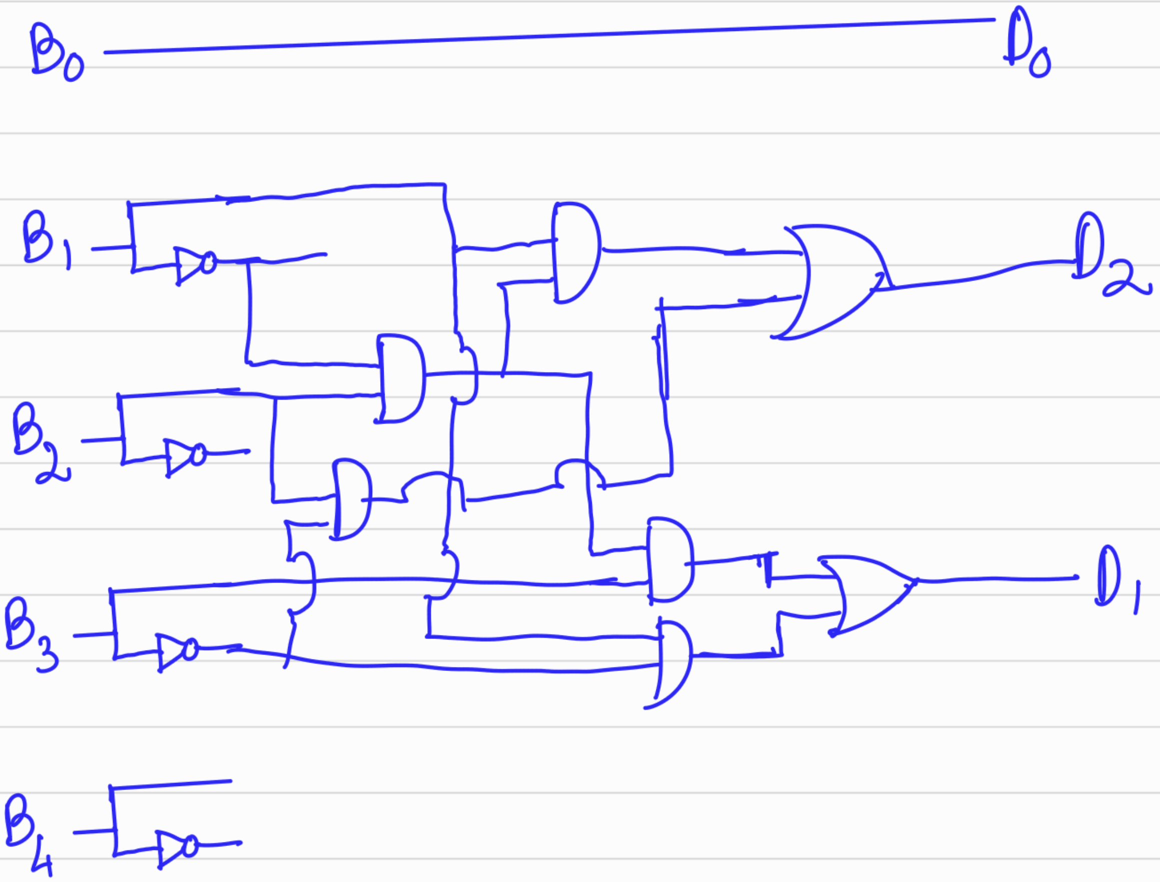
D_4

B_3B_2 \ B_1B_0	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	1 (11)	1 (10)

$D_4 = B_2B_3 + B_1B_2$

$D_0 = B_0$, $D_1 = \bar{B}_1 B_2 B_3 + B_1 \bar{B}_3$

$D_2 = B_2 \bar{B}_3 + B_1 B_2$, $D_3 = \bar{B}_1 \bar{B}_2 B_3$, $D_4 = B_2 B_3 + B_1 B_2$



Adder

$$\begin{array}{r} 1 \\ 1 \\ \hline 10 \end{array}$$

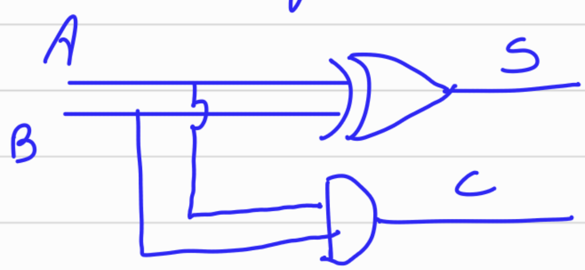
10 → Sum

Carry

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\begin{array}{r} 0 \\ 0 \\ \hline 00 \end{array} \quad \begin{array}{r} 0 \\ 1 \\ \hline 01 \end{array} \quad \begin{array}{r} 1 \\ 0 \\ \hline 01 \end{array} \quad \begin{array}{r} 1 \\ 1 \\ \hline 10 \end{array}$$

Half Adder.



$$\begin{array}{r} 1 \\ 11 \\ 11 \\ \hline 0 \end{array}$$

3 Inputs

⇒ But half adder can handle only 2 inputs

⇒ There is need for "Full adder" which can handle 3 inputs

x	y	z	s	c
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

S

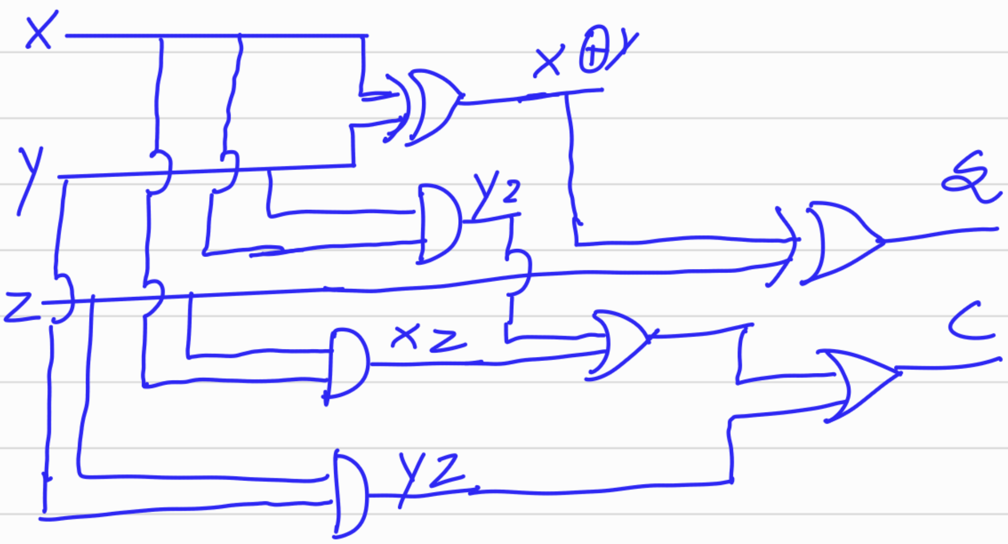
x \ yz	00	01	11	10
0	0_0	1_1	0_5	1_4
1	1_4	0_5	1_7	0_6

$S = x \oplus y \oplus z$

C

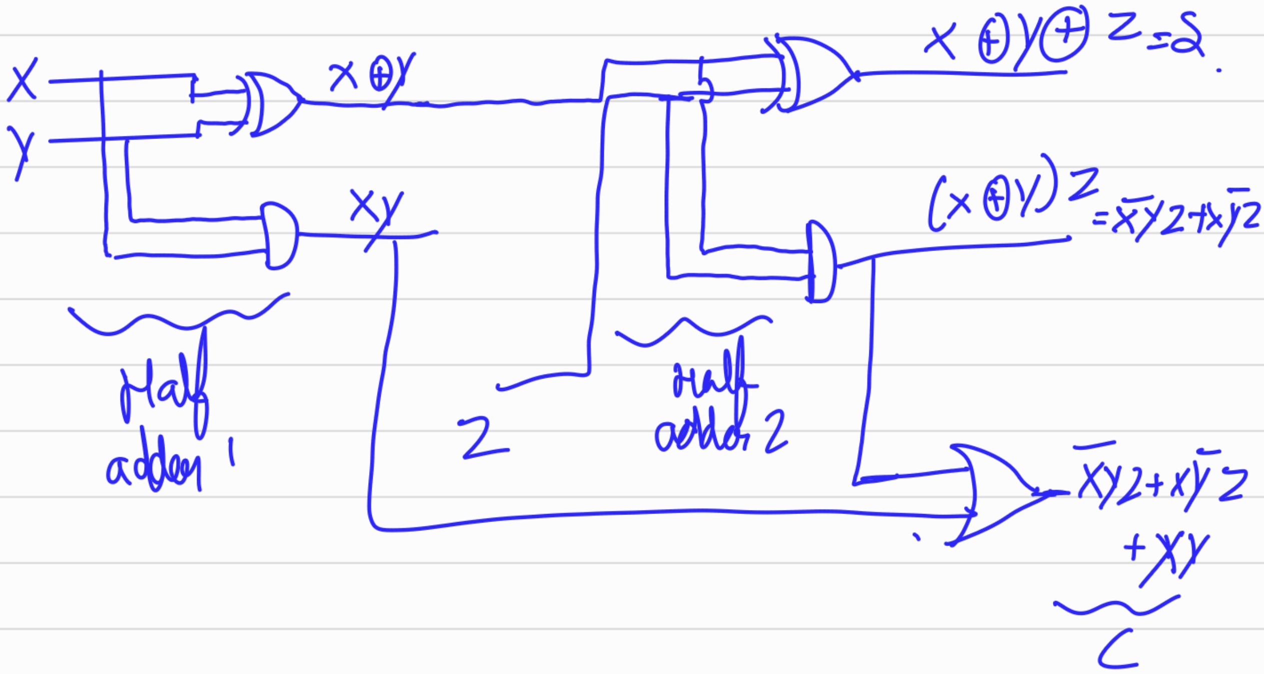
x \ yz	00	01	11	10
0	0_0	0_1	1_5	0_4
1	0_4	1_5	1_7	1_6

$C = xz + yz + xy$



Full adder using half adder

$S = x \oplus y \oplus z \quad C = xy + yz + xz$



$\bar{x}yz + x\bar{y}z + xy(z + \bar{z})$

$\bar{x}yz + x\bar{y}z + xyz + xy\bar{z} + xyz + xyz$

$yz + xy + xz$

Half subtractor

x	y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$0 - 0 = 0$

$0 - 1 \rightarrow$ B: $10 - 1 = 1$

$1 - 0 = 01$

$1 - 1 = 0$

$D = \bar{x}y + x\bar{y} = x \oplus y$

$B = \bar{x}y$

Full Subtractor

$x - y - z$

x	y	z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$x - y - z$

$0 - 0 - 1 = 0 - 1$

$B = 1 \Rightarrow 2 - 1 = 1$

$D = 1$

$1 - 0 - 0 = 1 - 0 = 1 \quad B = 0$

$0 - 1 - 0 = 2 - 1 - 0 = 1 - 0 = 1$

$B = 1 \quad D = 1$

$1 - 1 - 1 = 0 - 1 = 2 - 1 = 1$

$B = 1 \quad D = 1$

$D = x \oplus y \oplus z$

x \ yz	00	01	11	10
0	0	1 (1)	1 (3)	1 (2)
1	(4)	(5)	1 (7)	(6)

$B = \bar{x}z + \bar{x}y + yz$

4-bit adder.

```
  1 1 0 0
  1 0 1 0
+ 0 1 1 1
---------
1 0 0 0 1
```

→ 4 Full adder

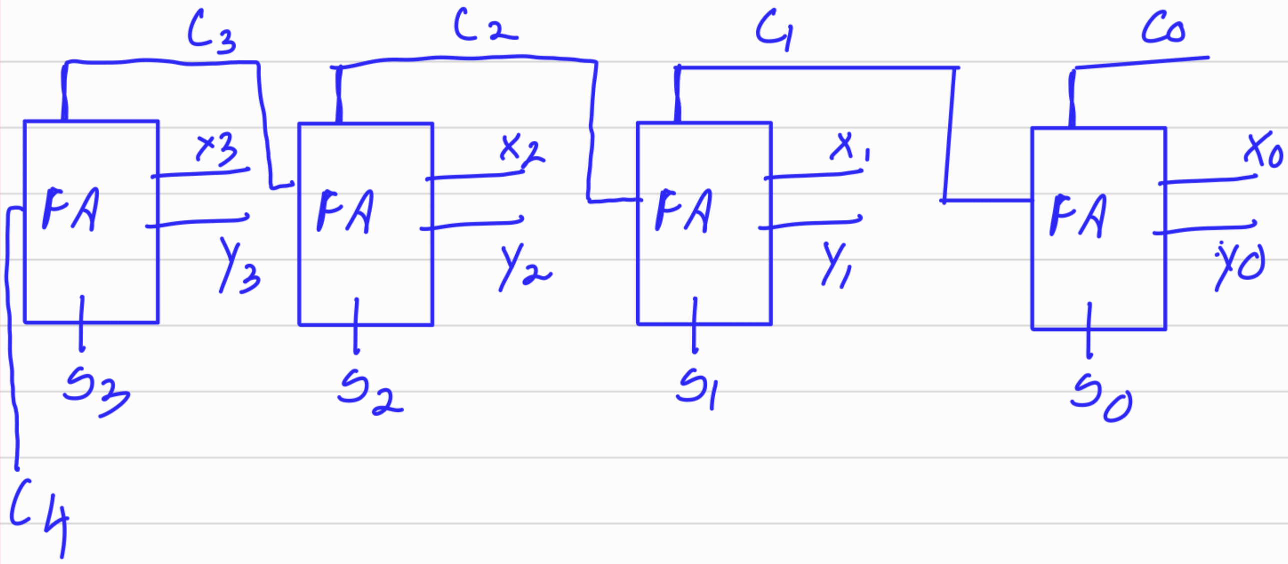
carry

this carry 1 which is given to the next one next one

this has carry 0

```
  1
  1 0 1 0 -
  1 0 1 0
  -------
1 0 1 0 0
```

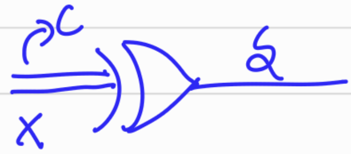
```
  0 0 0 1
  1 0 0 0
  -------
0 1 0 0 1
```



0, x - 4 bit Full adder
1, $\bar{x}$ - 4 bit Full Subtracter
↓
1's complement +1 = 2's complement → Subtraction

If 0 → x
If 1 → $\bar{x}$

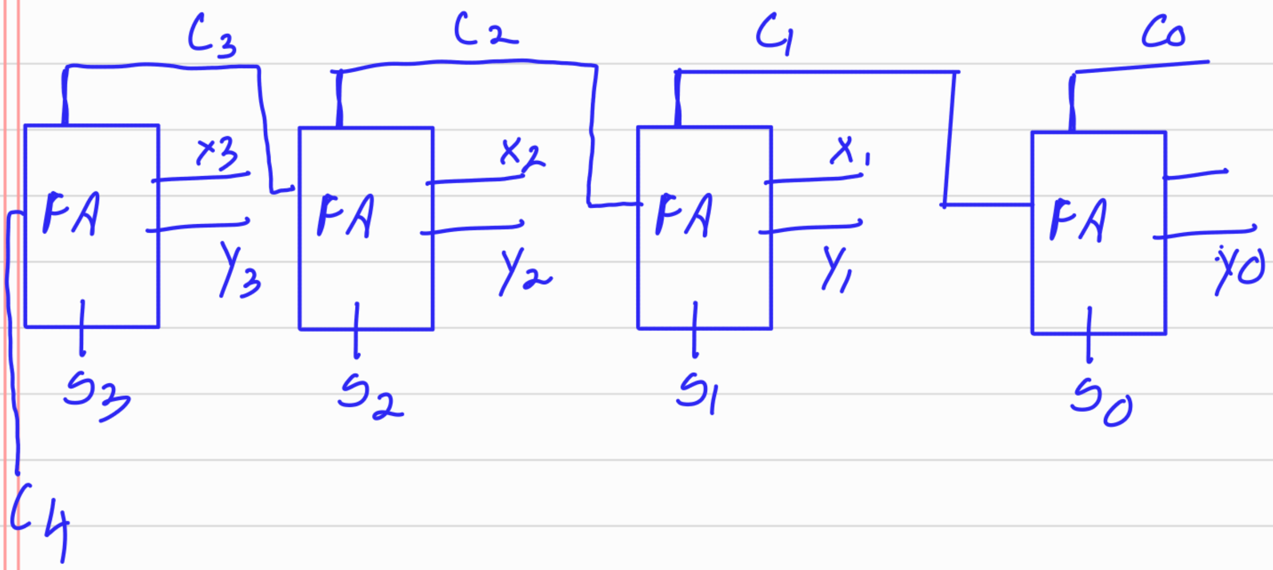
∴ If you 0 it will act as full adder
" " 1 " " " " " subtraction

⇒ 

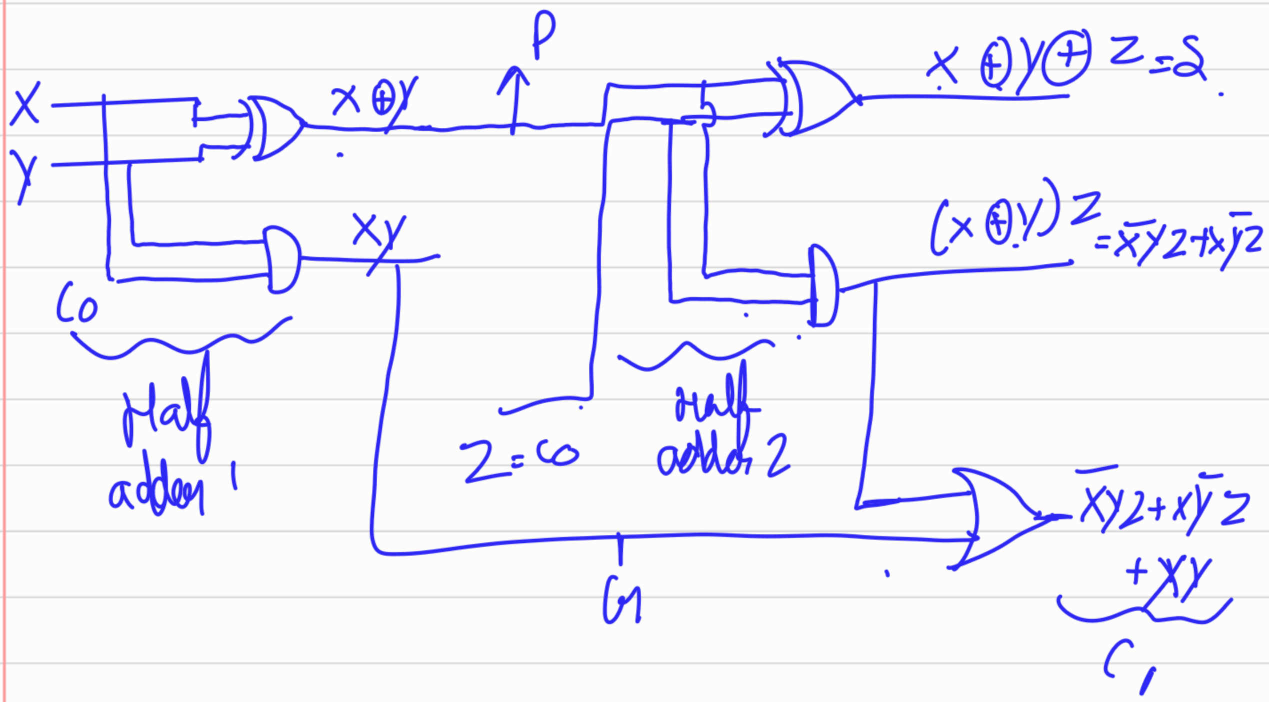
C=0 S=x
C=1 S=$\bar{x}$

-1

1111
0000
~~00001~~



4 bit look ahead carry



$P = x \oplus y \qquad G = xy \qquad S = P \oplus C_0$

$C = G + PC_0$

$xy + (x \oplus y)z$

For Full adder

$P_0 = X_0 \oplus Y_0 \quad G_0 = X_0 Y_0$

$C_1 = G_0 + P_0 C_0$
$C_2 = G_1 + P_1 C_1$
$C_3 = G_2 + P_2 C_2$

$C_2 = G_1 + P_1 C_1 = G_1 + P_1 (G_0 + P_0 C_0) = G_1 + P_1 G_0 + P_0 P_1 C_0$

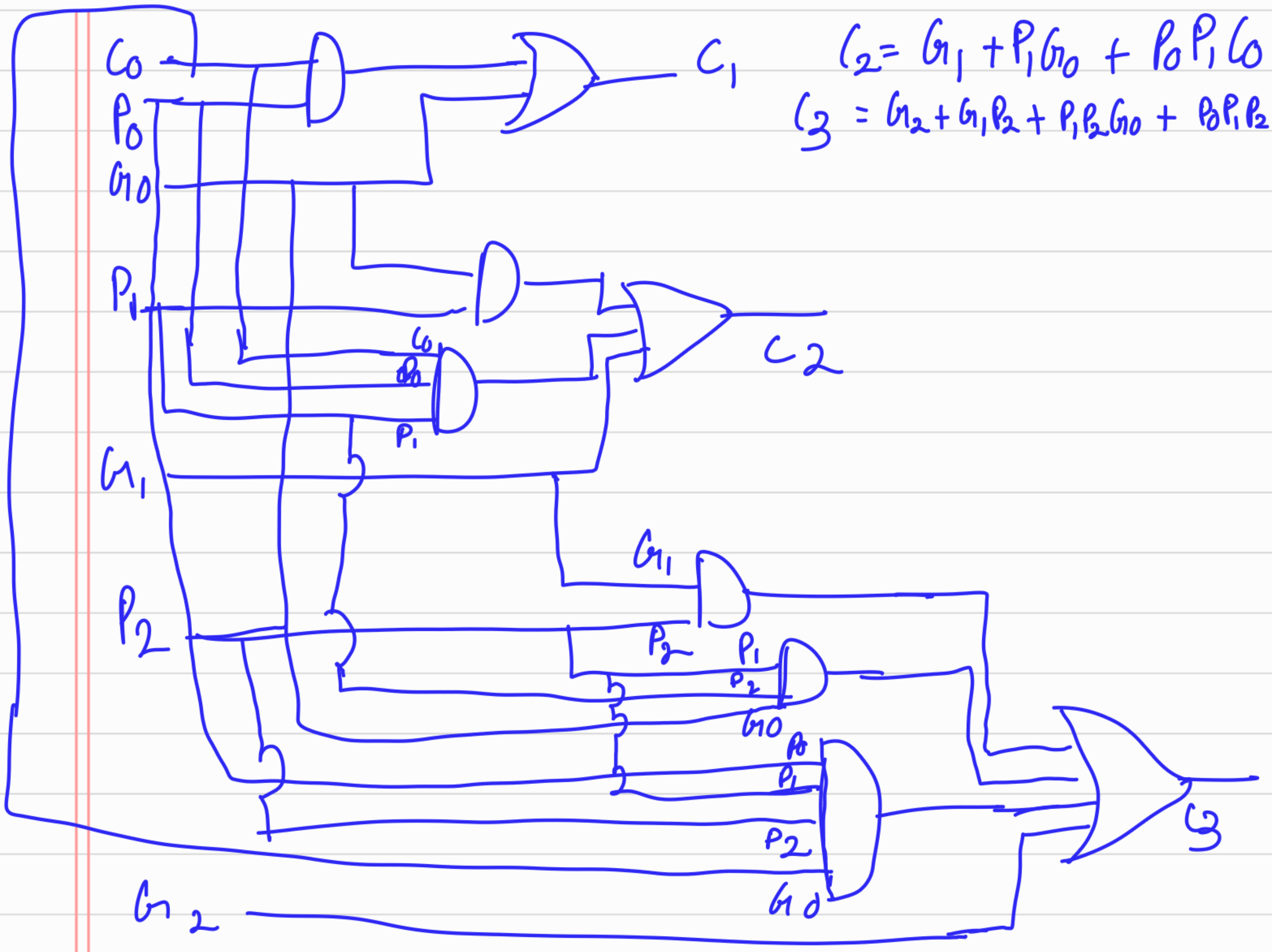
$C_3 = G_2 + P_2 C_2 = G_2 + P_2 (G_1 + P_1 (G_0 + P_0 C_0))$

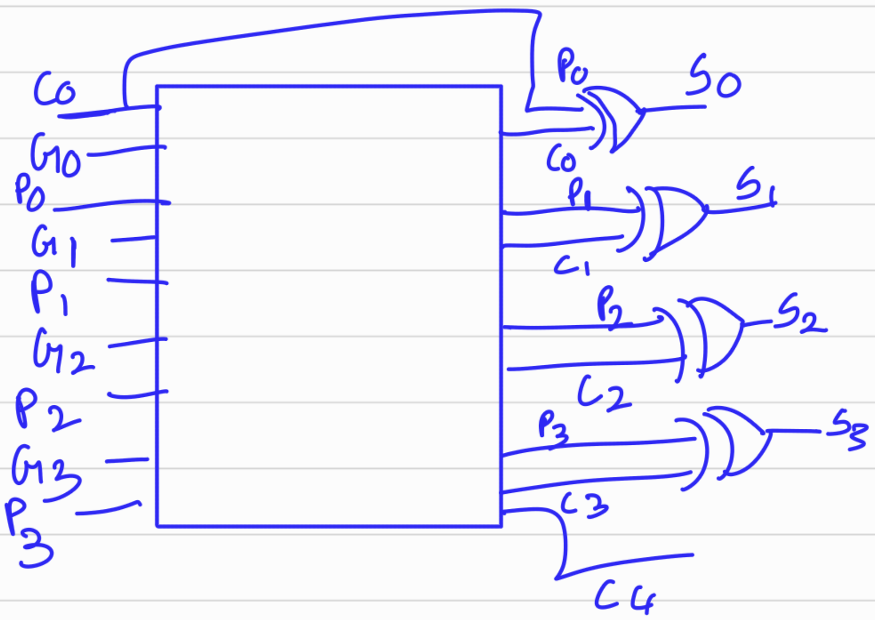
$= G_2 + P_2 (G_1 + P_1 G_0 + P_1 P_0 C_0)$

$= G_2 + G_1 P_2 + P_1 P_2 G_0 + P_0 P_1 P_2 C_0$

$C_2 = G_1 + P_1 G_0 + P_0 P_1 C_0$

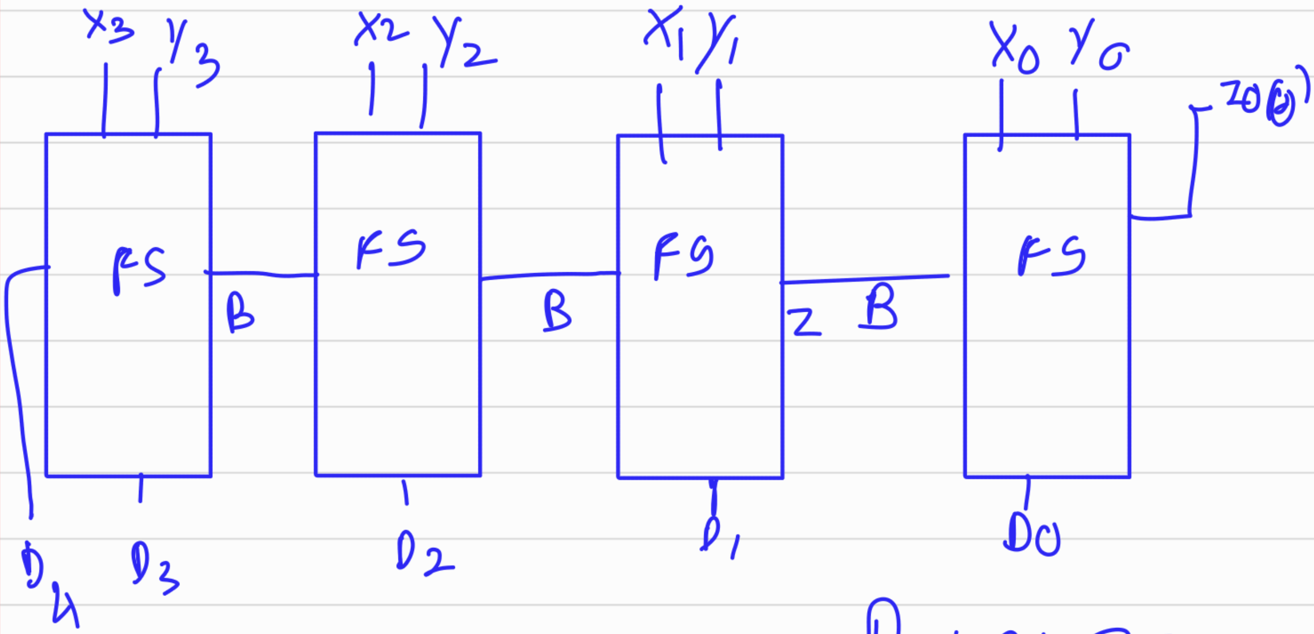
$C_3 = G_2 + G_1 P_2 + P_1 P_2 G_0 + P_0 P_1 P_2 C_0$





borrow for 0 in right

		1	0	1	0	
10		1	0	1	0	$-X$
-5	(-)	0	1	0	1	$-Y$
5		0	1	0	1	



$D = x \oplus y \oplus z$

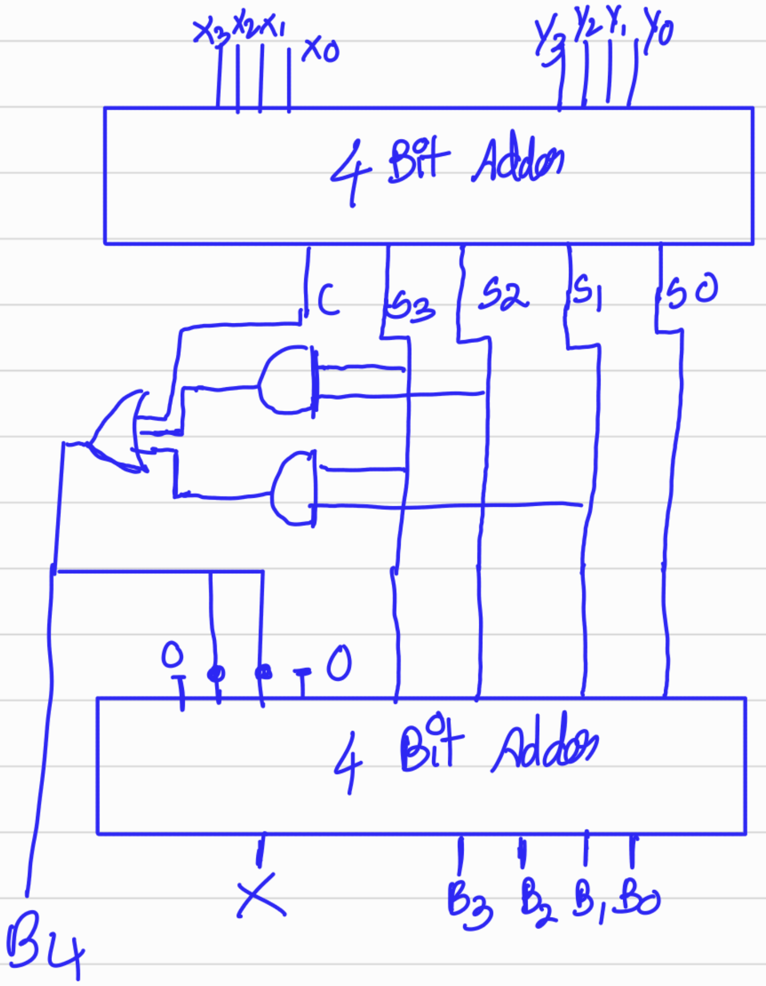
$B = \bar{x}y + \bar{x}z + zy$

BCD Converter

	$C\ S_3\ S_2\ S_1\ S_0$	B_4	B_3	B_2	B_1	B_0	
	0 0 0 0 0	0	0	0	0	0	
	0 0 0 0 1	0	0	0	0	1	
	⋮		⋮				
	0 1 0 0 1	0	1	0	0	1	
	0 1 0 1 0	1	0	0	0	0	(10)
		1	0	0	0	1	(17)
	⋮	1	0	0	1	0	(18)
		1	0	0	1	1	⋮
16	1 0 0 0 0	1	0	1	0	0	⋮
17	1 0 0 0 1	1	0	1	0	1	⋮
18	1 0 0 1 0	1	1	0	0	0	(24)

CS_3 \ $S_2S_1S_0$	000	001	011	010	110	111	101	100
00	0_0	0_1	0_3	0_2	0_6	0_7	0_5	0_4
01	0_8	0_9	1_{11}	1_{10}	1_{14}	1_{15}	1_{13}	1_{12}
11	1_{24}	1_{25}	1_{27}	1_{26}	1_{30}	1_{31}	1_{29}	1_{28}
10	1_{16}	1_{17}	1_{19}	1_{18}	1_{22}	1_{23}	1_{21}	1_{20}

$$C + S_3S_2 + S_3S_1$$



A_0	B_0	$A<B$	$A>B$	$A=B$
0	0	0	0	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

$A<B = \bar{A_0} B_0$

$A>B = A_0 \bar{B_0}$

$A=B = A_0 \odot B_0$

B_1	B_0	A_1	A_0	$A=B$	$A<B$	$A>B$
0	0	0	0	1	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	0	0	1
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	1	0	0

$$\begin{array}{ccccc} & A_3 & A_2 & A_1 & A_0 \\ A & 1 & 0 & 0 & 1 \\ B & 1 & 0 & 0 & 1 \\ & B_3 & B_2 & B_1 & B_0 \end{array}$$

$A = B \Rightarrow A \odot B = \overline{x} \quad x = A \oplus B$

$(A_3 \odot B_3)(A_2 \odot B_2)(A_1 \odot B_1)(A_0 \odot B_0)$

$\Rightarrow \overline{x_3}\,\overline{x_2}\,\overline{x_1}\,\overline{x_0}$

$A > B$

$A_3 \overline{B_3} + \overline{x_3} A_2 \overline{B_2} + \overline{x_3}\,\overline{x_2} A_1 \overline{B_1} + \overline{x_3}\,\overline{x_2}\,\overline{x_1} A_0 \overline{B_0}$

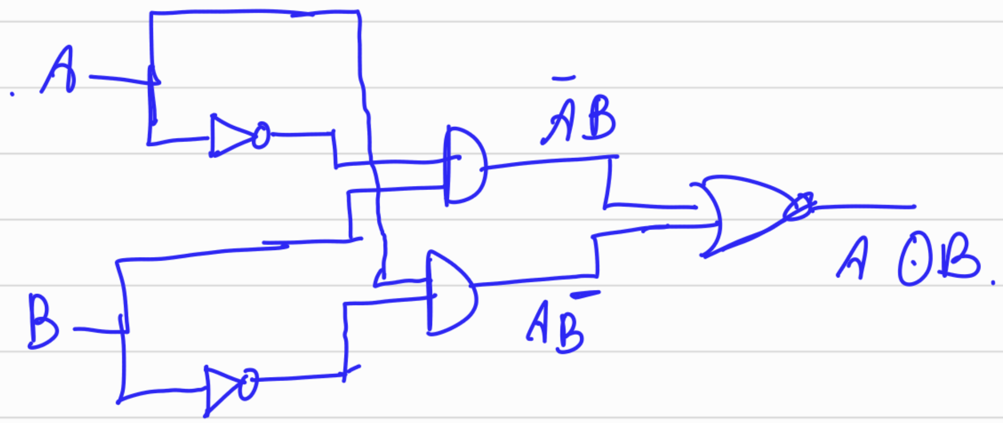
$A < B$

$\overline{A_3} B_3 + \overline{x_3}\,\overline{A_2} B_2 + \overline{x_3}\,\overline{x_2}\,\overline{A_1} B_1 + \overline{x_3}\,\overline{x_2}\,\overline{x_1}\,\overline{A_0} B_0$

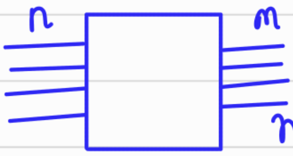
$A \odot B = \overline{A \oplus B}$

$\overline{A}B \quad A\overline{B}$

$\overline{\overline{A}B + A\overline{B}}$



Decoder



$m \leq 2^n$

high - 1 low - 0

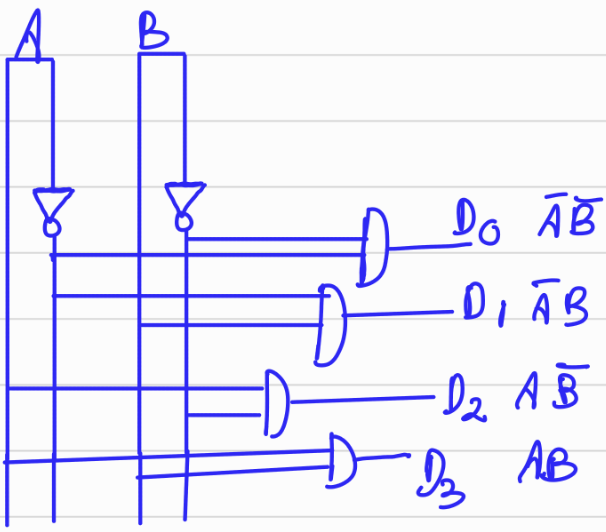
n : m decoder

2 : 4 decoder

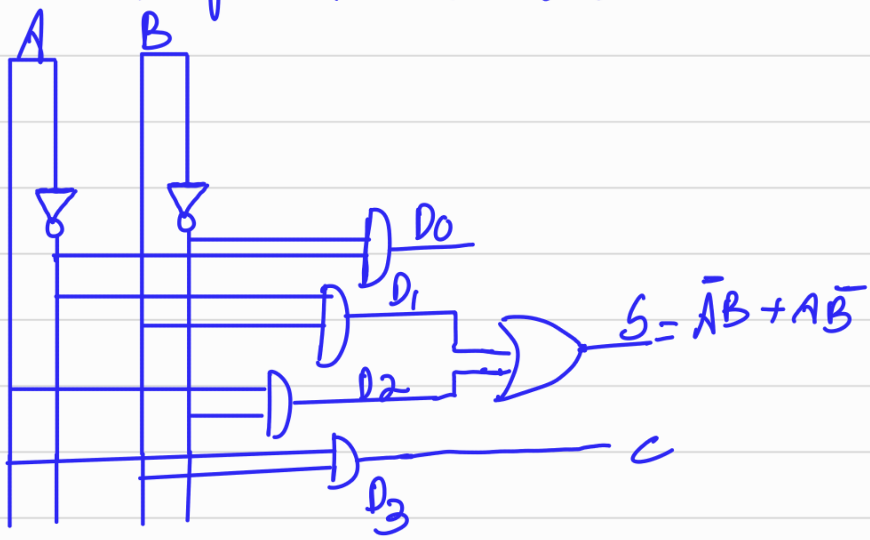
4 type of input 4 o/p

1 type of input -> 1 output

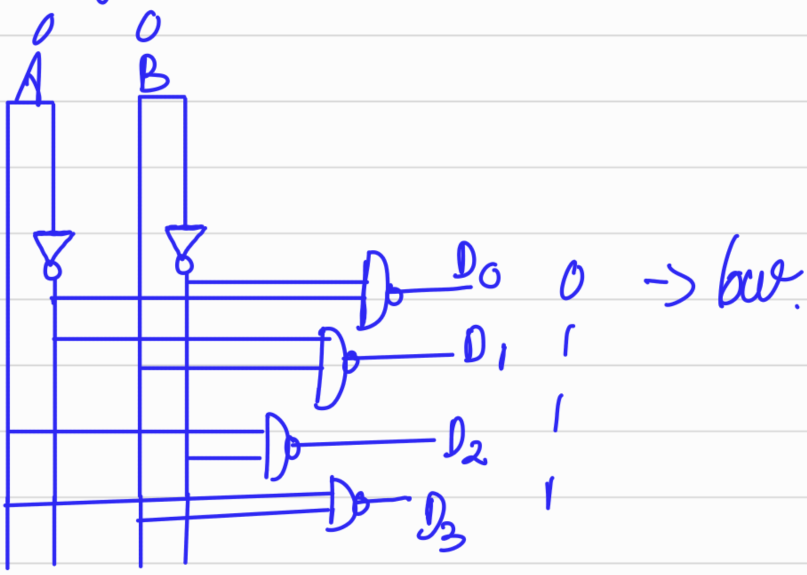
A	B	D_0	D_1	D_2	D_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



Half adder with decoder



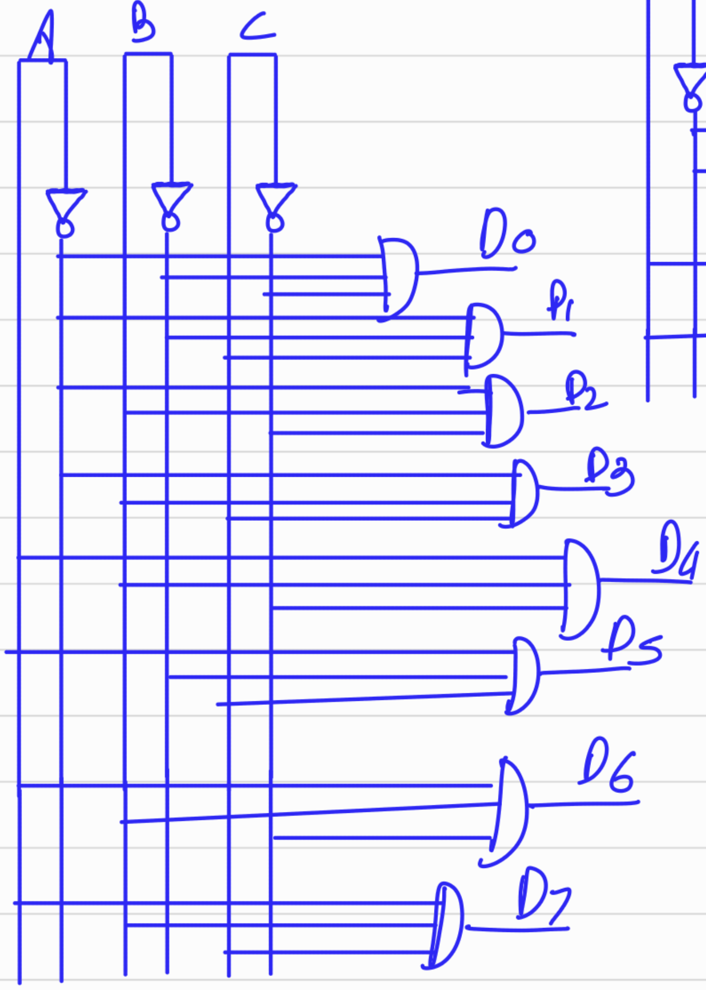
Using NAND Gate

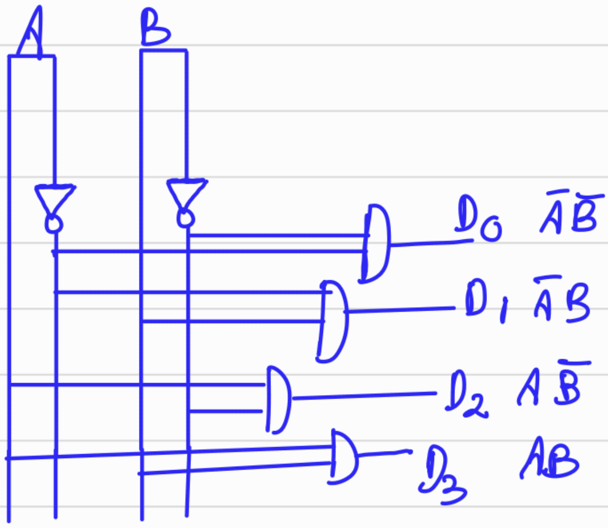


A_1A_0 \ B_1B_0	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	0	1
10	1	1	0	0

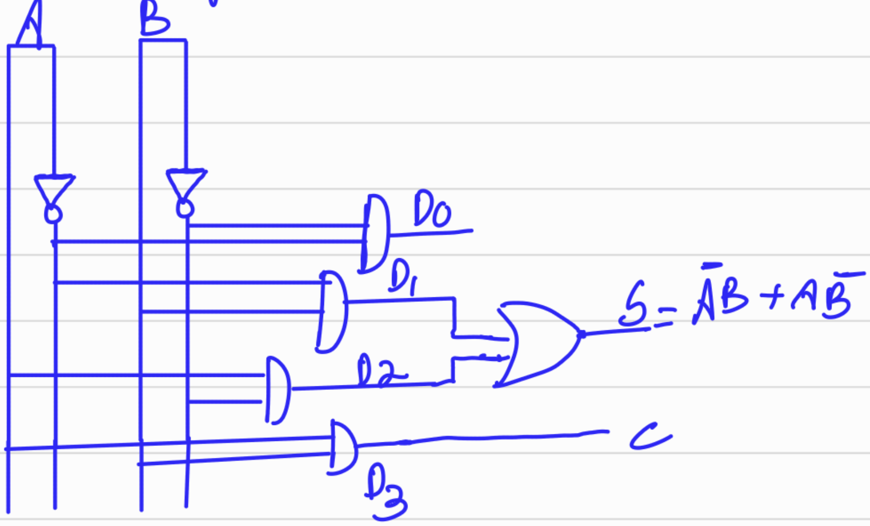
$A > B$

3:8 decoder

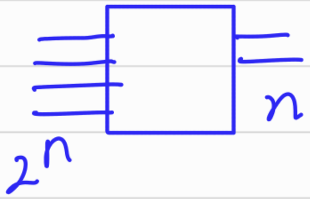


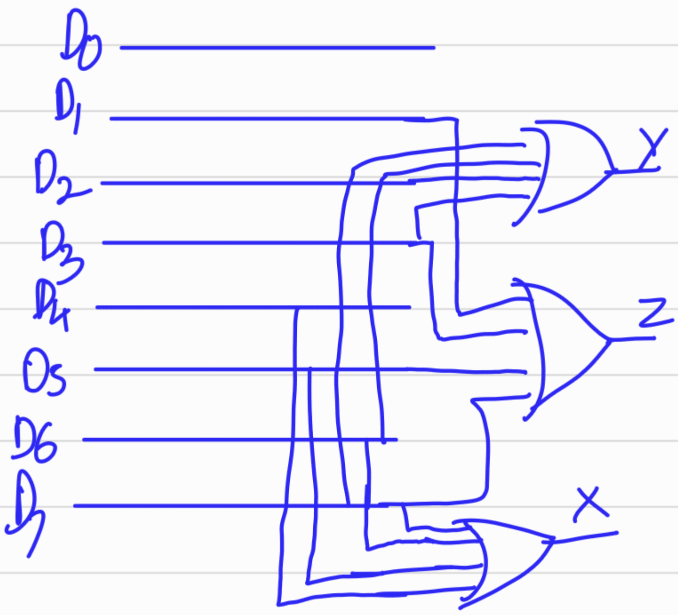


Half adder with decoder



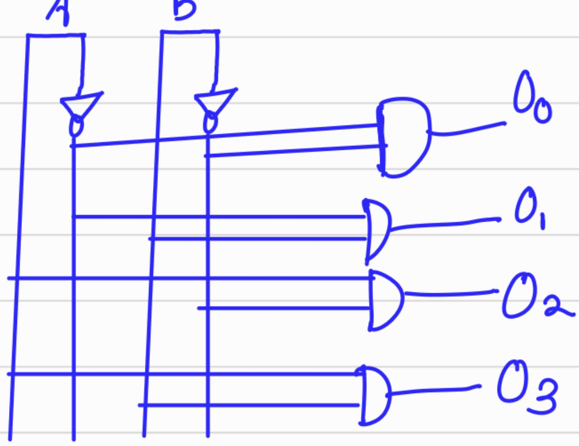
Encoder



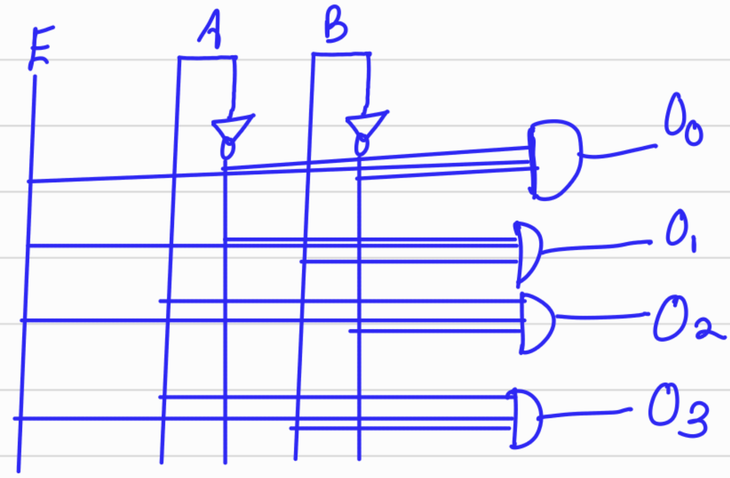


x	y	z	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

2:4 Decoder



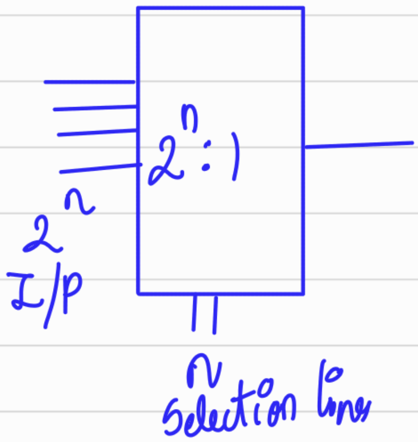
Enable bit

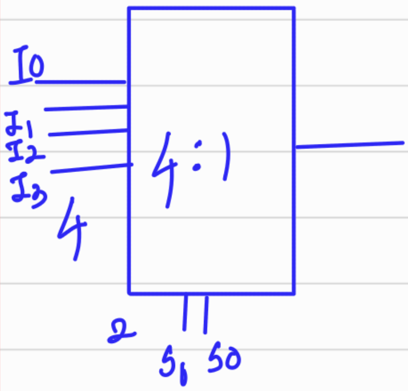


A	B	E	D_0	D_1	D_2	D_3
0	0	1	1			
0	1	1		1		
1	0	1			1	
1	1	1				1
X	X	0	0	0	0	0

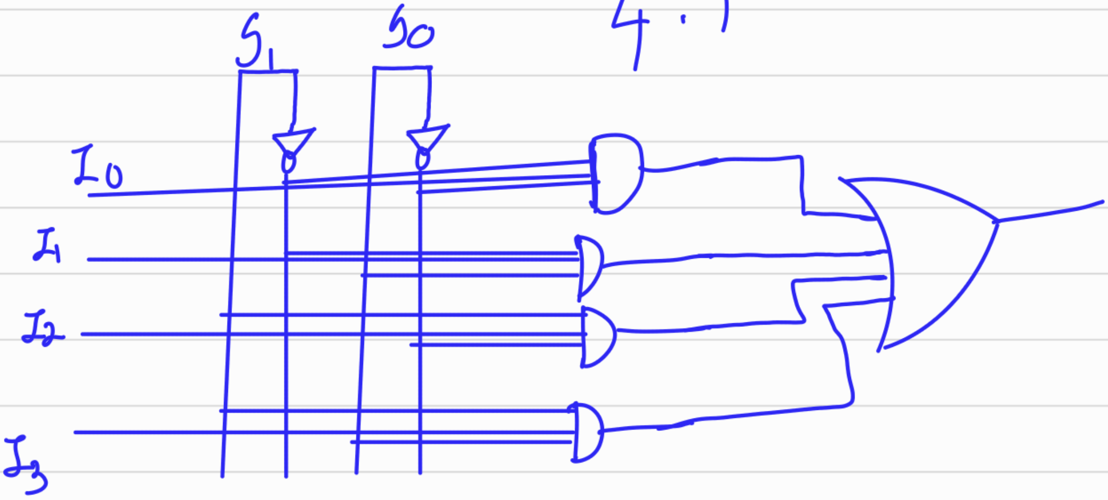
If Enable is 0
everything is 0 whatever may be A and B

Multiplexer

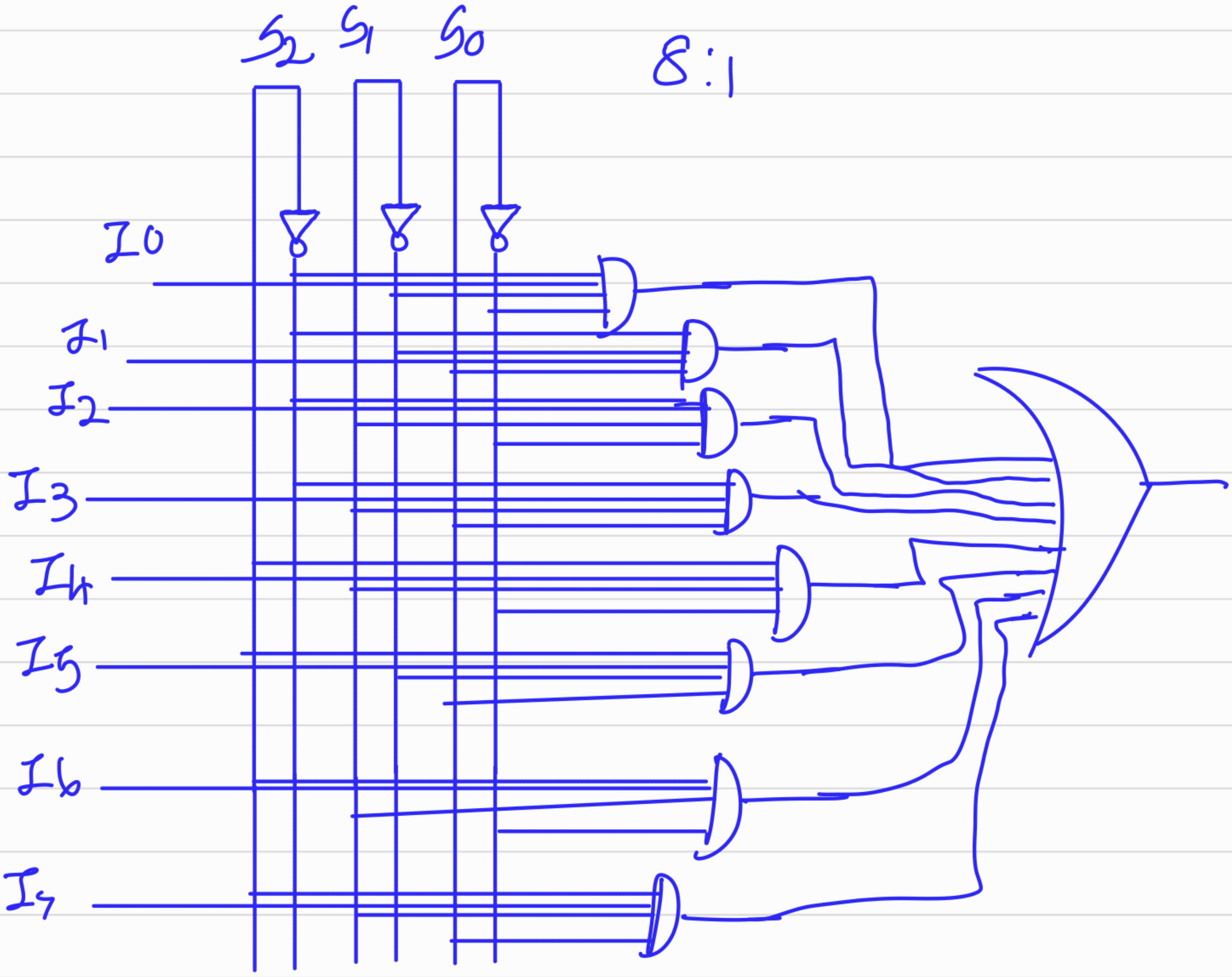



I0
I1
I2
I3
4
4:1
2
S1 S0

4:1

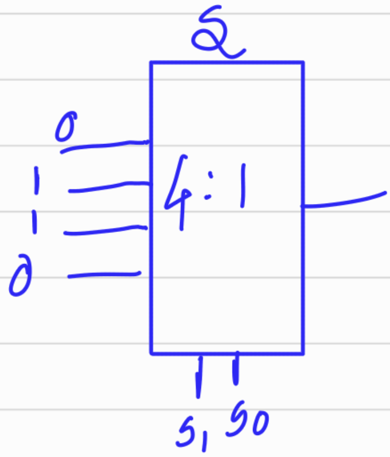

S1
S0
I0
I1
I2
I3

8:1

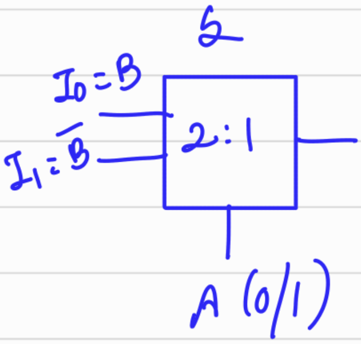

S2
S1
S0
I0
I1
I2
I3
I4
I5
I6
I7

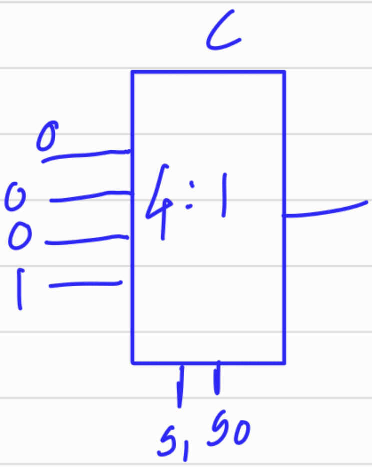
Half adder

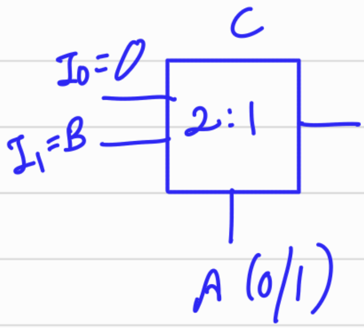
S_1 A	S_0 B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



$A=0,\ S=B$
$A=1,\ S=\overline{B}$







$F(A, B, C)$

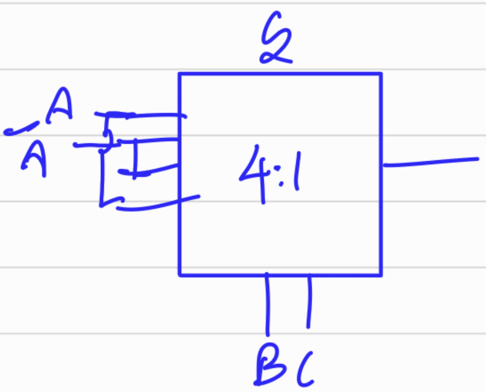
$2^{(3-1)} = 4$

4:1 Multiplexer

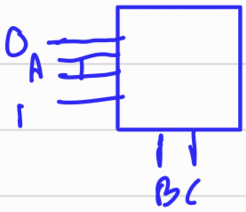
Full adder

A	B	C	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

	00	01	10	11
A 0	0 (0)	1 (1)	1 (2)	0 (3)
1	1 (4)	0 (5)	0 (6)	1 (7)
	A	$\bar{A}$	$\bar{A}$	A



A	00	01	10	11
0	0_0	0_1	0_2	1_3
1	0_4	1_5	1_6	1_7
	0	A	A	1



(A,B,C) 1,3,5,6

A	00	01	10	11
0	0_0	1_1	0_2	1_3
1	0_4	1_5	1_6	0_7
	0	1	A	$\bar{A}$

Priority encoder

↳ give prefence to the highest number

I_0	I_1	I_2	I_3	A	B	V
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1
0	0	0	0	x	x	0

$A = I_2\bar{I_3} + I_3$ $\qquad$ $V = I_0 + I_1 + I_2 + I_3$

$B = I_1\bar{I_2}\bar{I_3} + I_3$

Product of sum - Maxterms POS

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

POS - $(A+B)$ => Direct

SOP - $A\bar{B} + \bar{A}B + AB$

$= A\bar{B} + AB + \bar{A}B + AB$

$= A(B+\bar{B}) + B(\bar{A}+A)$

$= A+B$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

$(A+B)(\bar{A}+\bar{B})$

$= \underbrace{A\bar{A} + B\bar{B}}_{0} + A\bar{B} + \bar{A}B$

$= A\bar{B} + \bar{A}B$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

$F = A\bar{B} + \bar{A}B + AB$ SOP

$F = A + B$ POS

$\bar{F} \Rightarrow$ remaining min term $= \bar{A}\bar{B}$

$\overline{(\bar{F})} = \overline{\bar{A}.\bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A + B \Rightarrow$ POS

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

A \ BC	00	01	11	10
0	0_0	1_1	0_3	1_2
1	0_4	1_5	0_7	1_6

SOP . $\bar{B}C + B\bar{C}$
$= B \oplus C$

A \ BC	00	01	11	10
0	0_0	1_1	0_3	1_2
1	0_4	1_5	0_7	1_6

$F = (B+C)(\bar{B} + \bar{C})$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	0	1	1
10	1	1	1	1

$F(A,B,C) = \Pi(13)$

$(\bar{A} + \bar{B} + C + \bar{D}) \Rightarrow$ POS

$F(A,B,C) = \underbrace{\Pi(1,3,5) = \Sigma(0,2,4,6,7)}_{\text{Both are equal}}$

$$\overline{\prod(1,3,5)} = \prod(0,2,4,6,7)$$

$$\overline{\prod(0,2,4,6,7)} = \Sigma(0,2,4,6,7)$$

A \ BC	00	01	11	10
0	1_0	0_1	0_3	1_2
1	1_4	0_5	1_7	1_6

$$(A+\bar{C})(B+\bar{C})$$

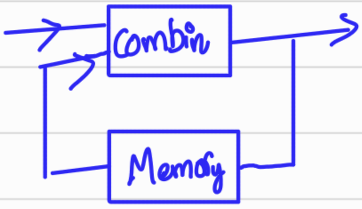
$\prod(1,3,5)$

001 $(A+B+\bar{C})$

011 $(A+\bar{B}+\bar{C})$

101 $(\bar{A}+B+\bar{C})$

Sequential Circuit

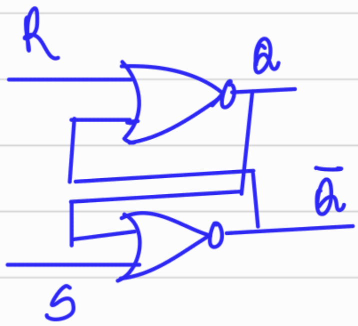


Flip Flop → Heart of sequential circuit

↳ store a single bit

Sequential Circuit
- asynchronous circuit
- Synchronous circuit → time → clock
  - discrete time events

RH latch



$Q(t)\ \bar{Q}(t)$ → Previous state

$Q(t+1)\ \bar{Q}(t+1)$ → next state

$Q(t)$ $\bar{Q}(t)$

0 1

Min 1½ cycle should be traced.

R	S	$Q(t+1)$	$\bar{Q}(t+1)$
0	1	1	0
1	0	0	1
0	0	0	1
1	1	0	0 Indeterminate

$Q(t)$ $\bar{Q}(t)$

1 0

R	S	$Q(t+1)$	$\bar{Q}(t+1)$
0	1	1	0 set
1	0	0	1 Reset
0	0	1	0 — Retain —
1	1	0	0 X

$Q(t)$ $\bar{Q}(t)$

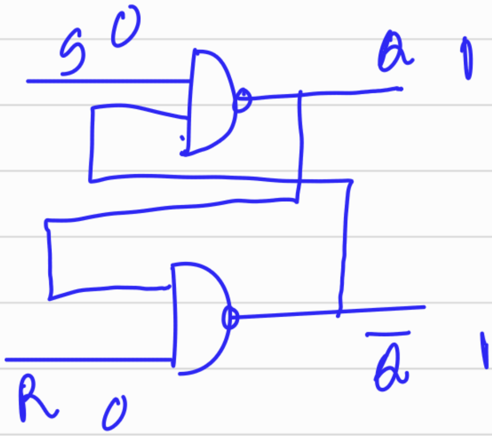
0 1

R	S	$Q(t+1)$	$\bar{Q}(t+1)$
0	1	1	0
1	0	0	1
0	0	0	1
1	1	0	0 X

0 1 1 -> Making 1 by giving 0 1
Set (whatever maybe the old value)

1 0 0 -> Making 0 by giving 1 0
Reset (whatever maybe the old value)

SR latch



$Q(t)$	$\bar{Q}(t)$
1	0

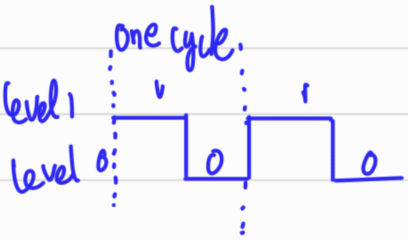
$Q(t)$	$\bar{Q}(t)$
0	1

S	R	$Q(t+1)$	$\bar{Q}(t+1)$	
0	1	1	0	Set
1	0	0	1	Reset
1	1	1	0	Retain
0	0	1	1	→ Indeterminate ←

S	R	$Q(t+1)$	$\bar{Q}(t+$
0	1	1	0
1	0	0	1
1	1	0	1
0	0	1	1

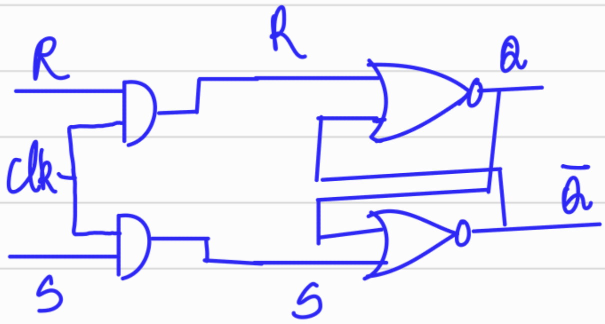
clock → generate 0 & 1 alternatively

Speed - How frequently it moves from 0 1 signal.



Pulse / level band signal

RS flip flop

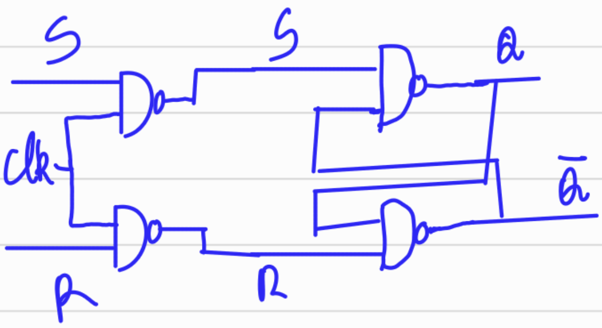


clk 0 -> 0 will be the output from AND gate -> Retain

clk 1 -> RS latche depending on R & S

R	S	Q(t+1)	$\bar{Q}$(t+1)
0	0	Q(t)	$\bar{Q}_t$
0	1	1	0
1	0	0	1
1	1	0	0 -> Indeterminate state

SR flip flop



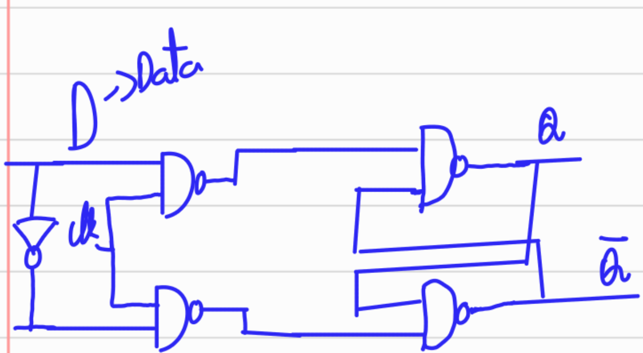
clk 0 -> 1 will be the output from NAND gate -> Retain

clk 1 -> SR latche depending on S & R

clk	S	R	Q(t+1)	$\bar{Q}$(t+1)
1	0	0	Q(t)	$\bar{Q}_t$
1	1	0	1	0
1	0	1	0	1
1	1	1	0	0 -> Indeterminate state
0	x	x	Q(t)	$\bar{Q}$(t) -> Retain

-> characteristic table

D flip flop

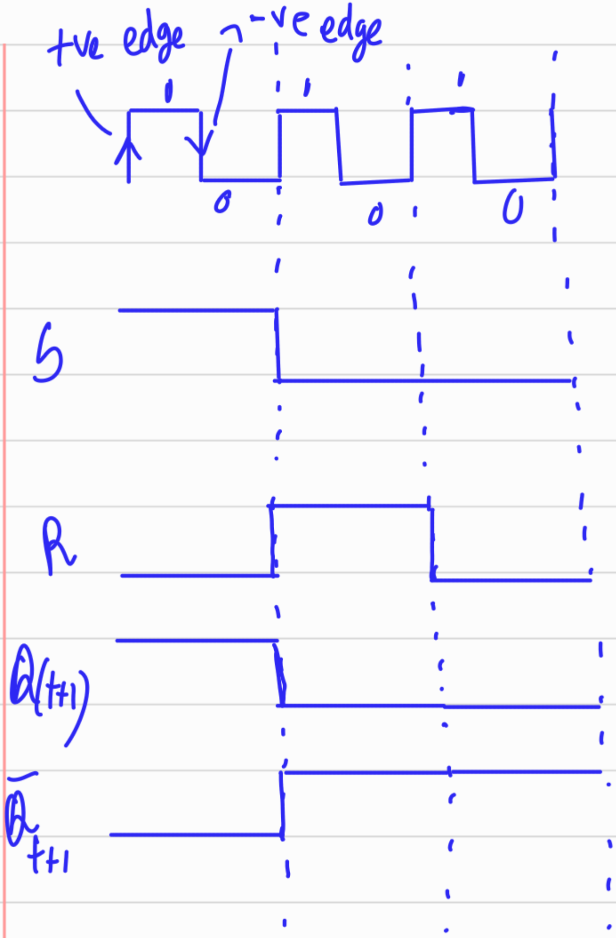


clk	D	Q(t+1)	$\bar{Q}$(t+1)
1	0	0	1
1	1	1	0
0	x	Q(t)	Q(t+1)

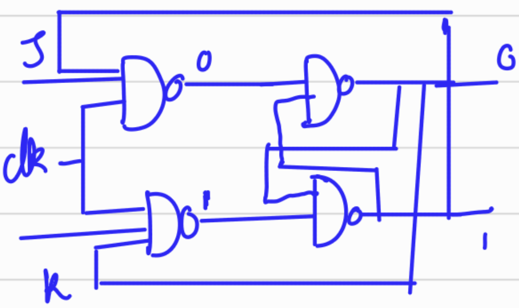
$Q(t+1) = D$

* Pulse based D flip flop cannot retain more than one cycle

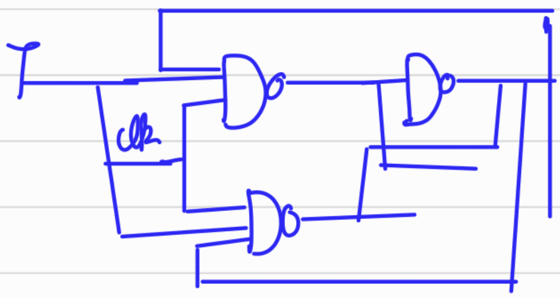
* No determinate cycle



JK flip flop



Toggle - changing the bit



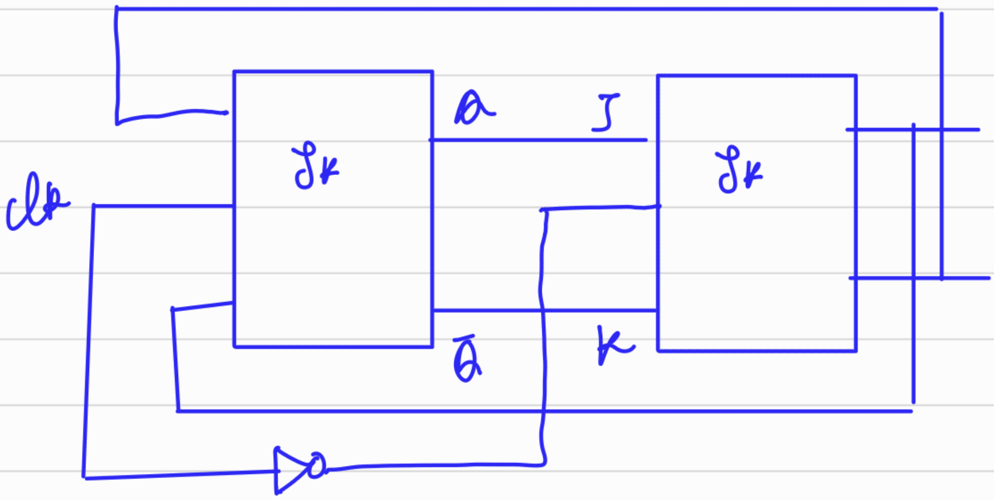
Q	J (S)	K (R)	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

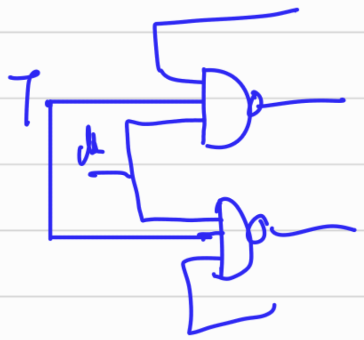
→ Characteristic table.

Q \ JK	00	01	11	10
0	0_0	0_1	1_3	1_2
1	1_4	0_5	0_7	1_6

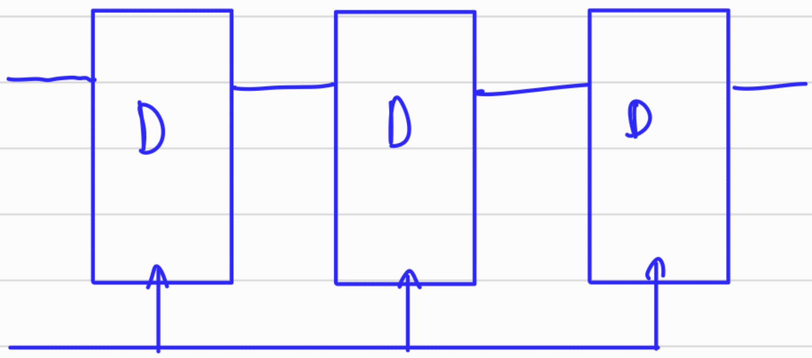
$$Q(t+1) = \bar{Q}J + Q\bar{K}$$

Master slave flip flop



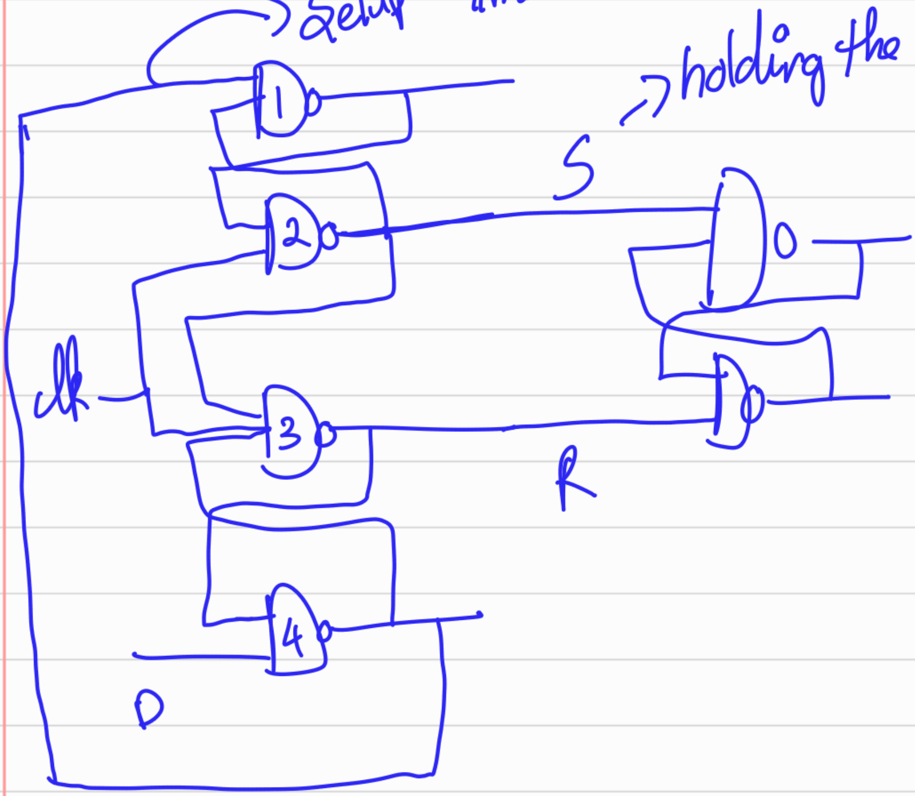


Edge triggered circuit



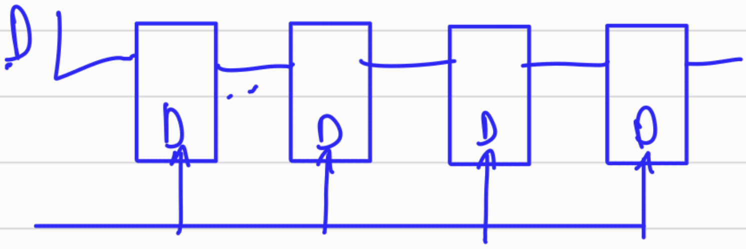
→ setup time

→ holding the input to get output
↳ hold time



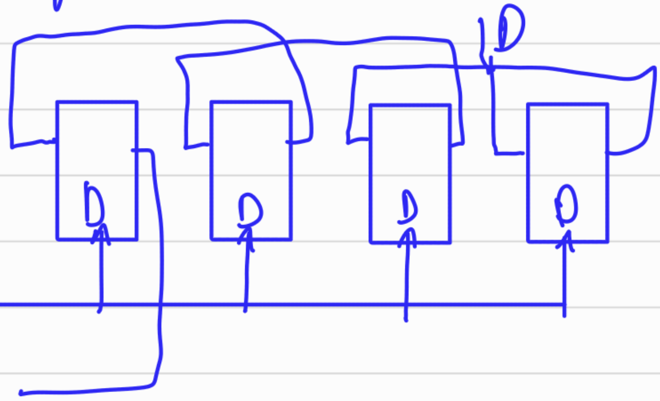
Shift Register

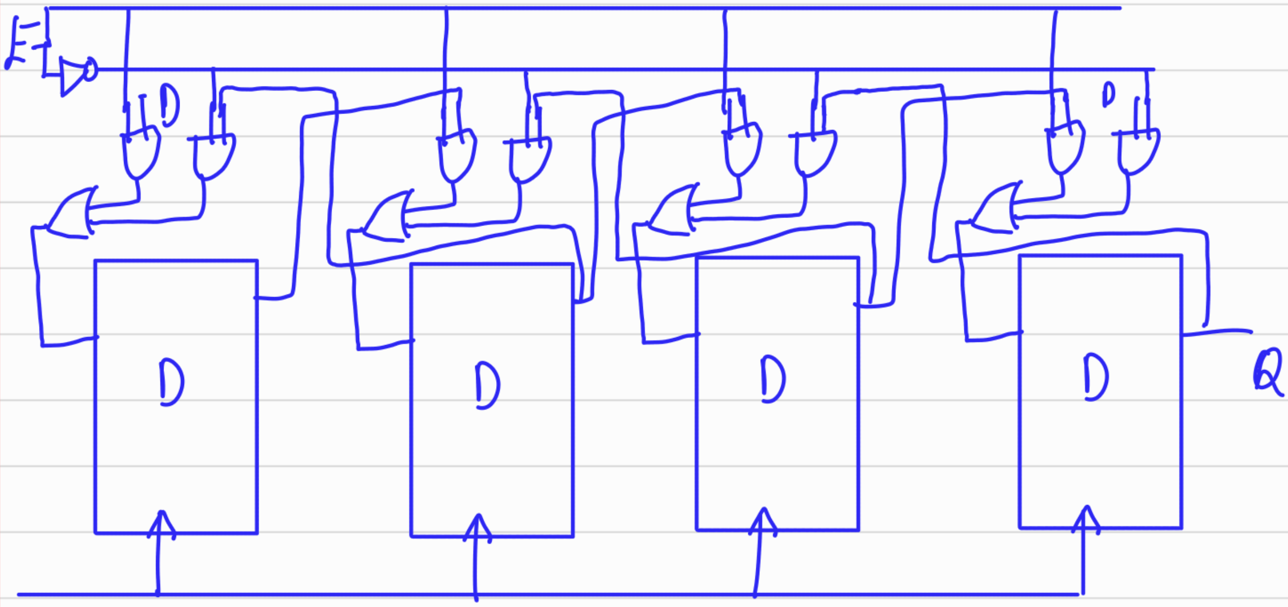
Edge triggered if D changes it will take 4 cycle

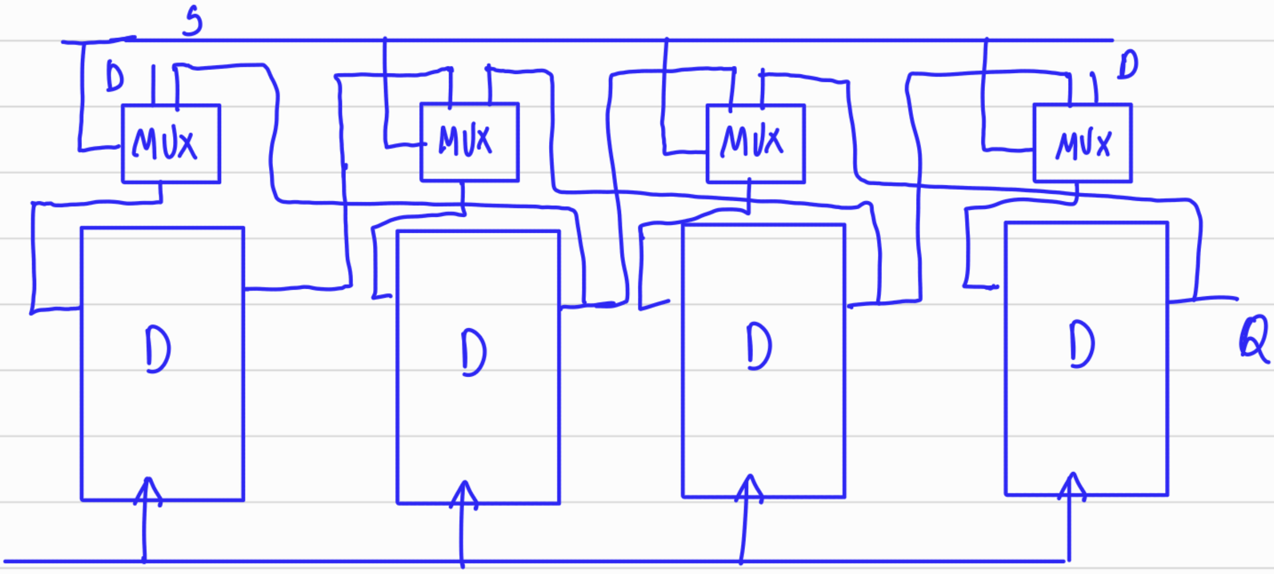


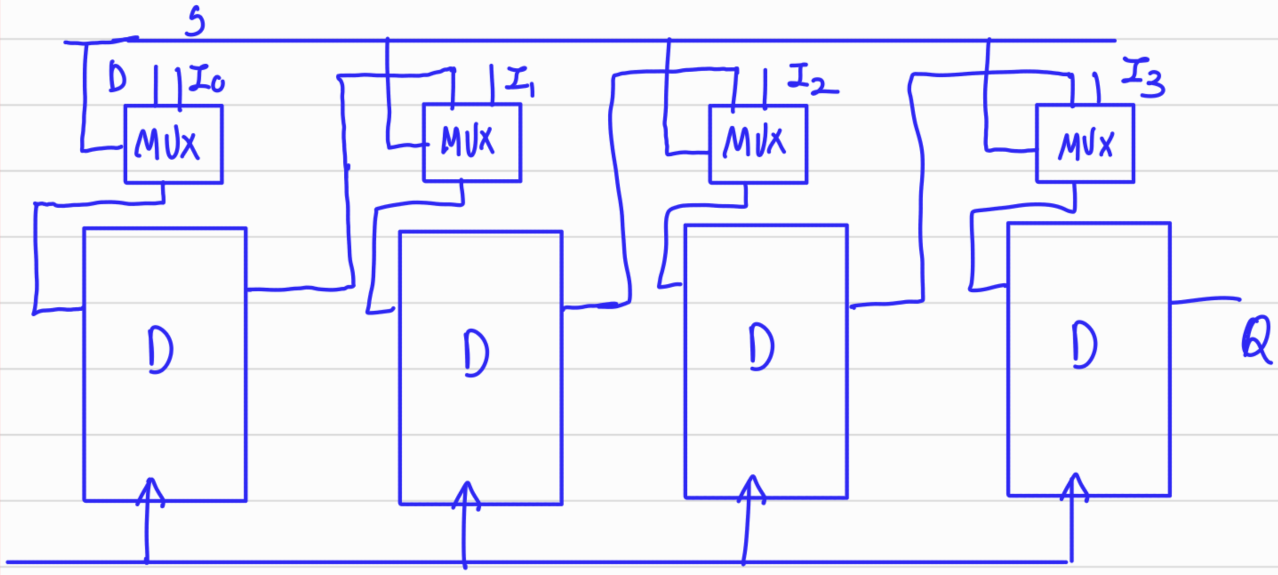
-> Right shift register

Left shift register

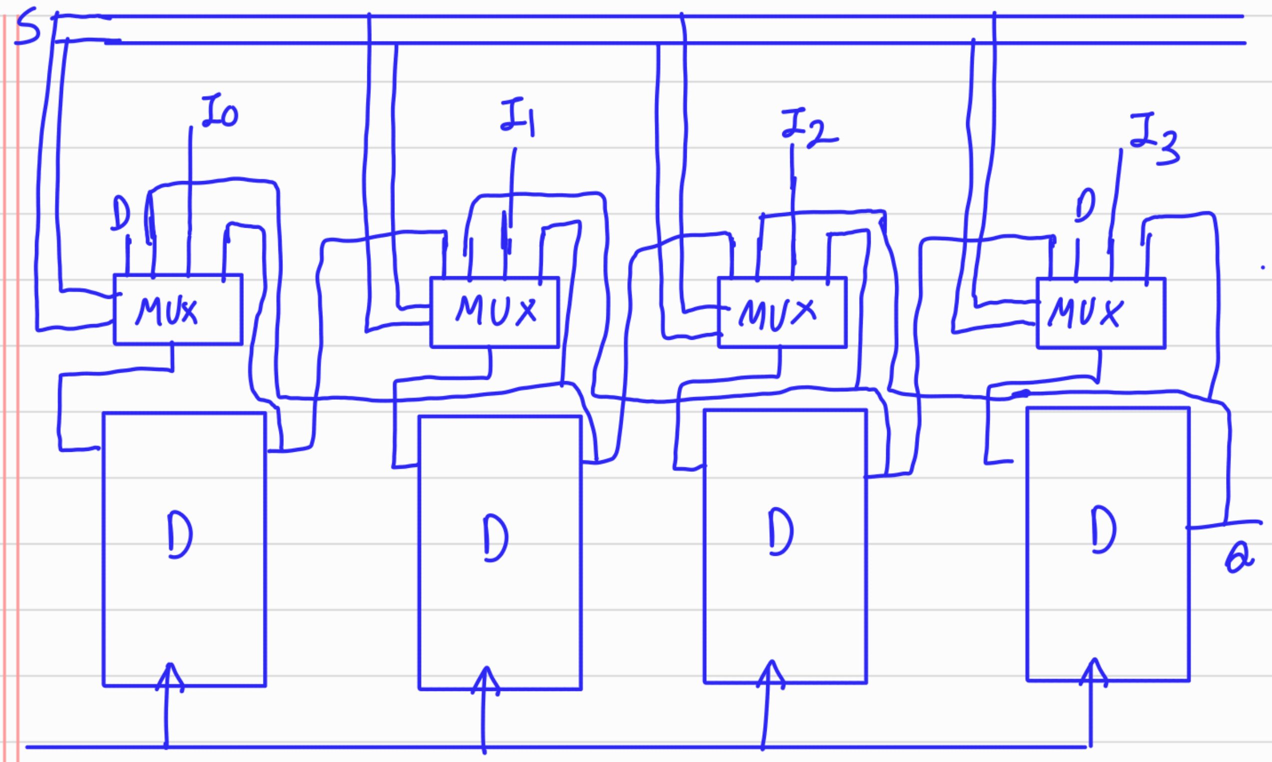








Parallel to serial converter
&
Right shift



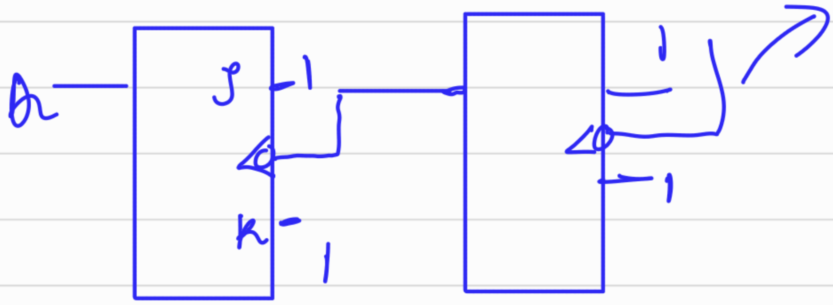
1) Right shift 00
2) Left shift 01
3) Parallel Input 10
4) Retain 11

Universal shift Register

Counter

Asynchronous Counter → Modulo 4 Counter

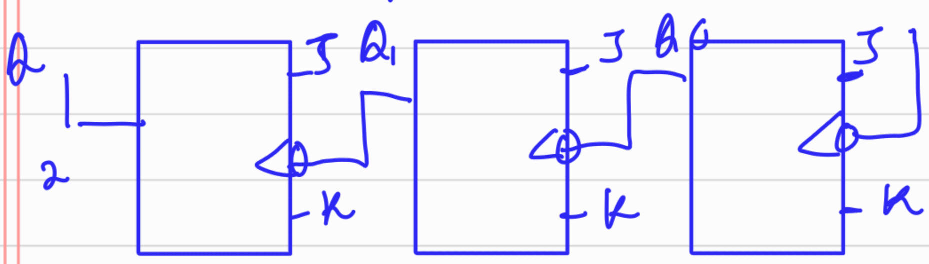
-ve edge clock



1 ↓ 0

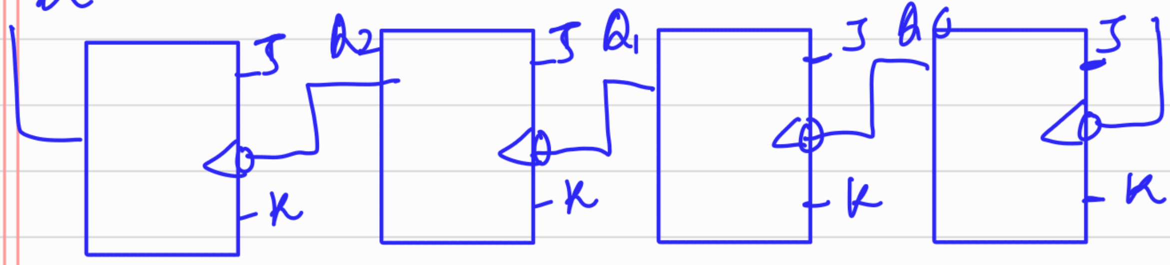
clk	Q_1	Q_0
0	0	0
1	0	1
2	0	0
3	1	1
4	0	0

Modulo 8 Asynchronous Counter



clk	Q_2	Q_1	Q_0
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8	0	0	0

Modulo 16 Asynchorous Counter

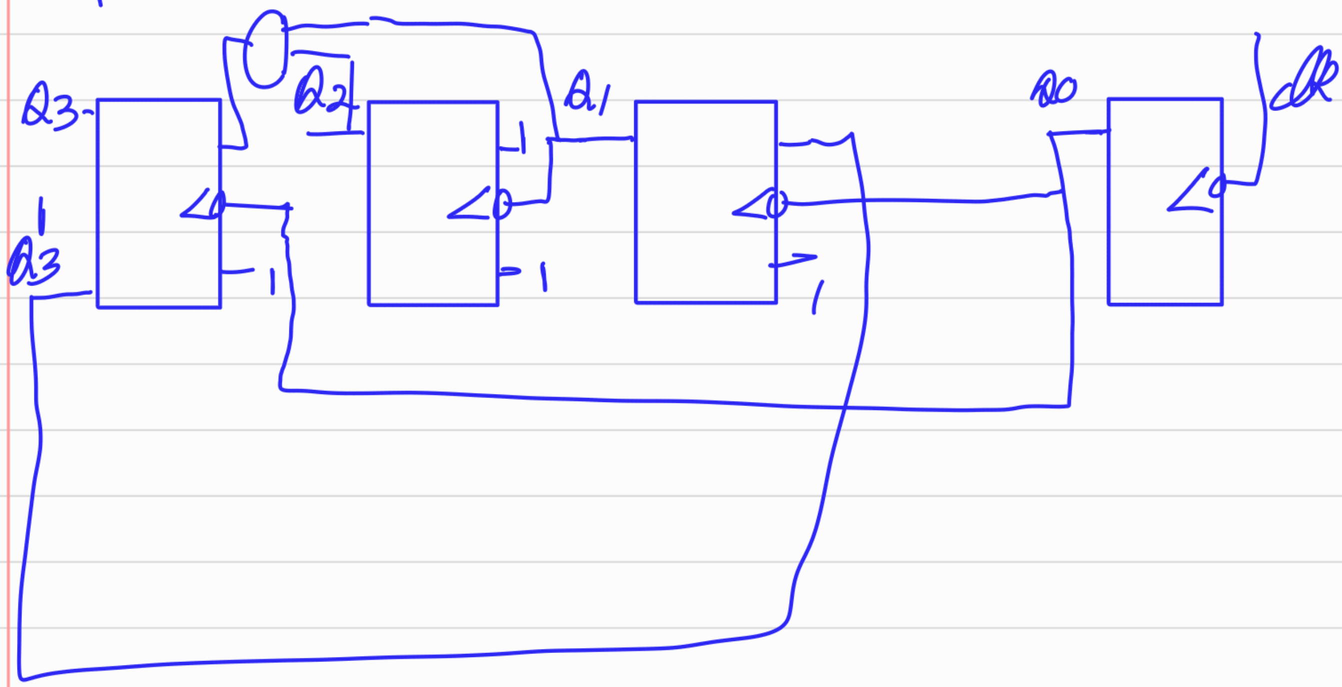


clk	Q_3	Q_2	Q_1	Q_0
0	0	0	0	0
1	0	0	0	1
2				
⋮				

7 segment display

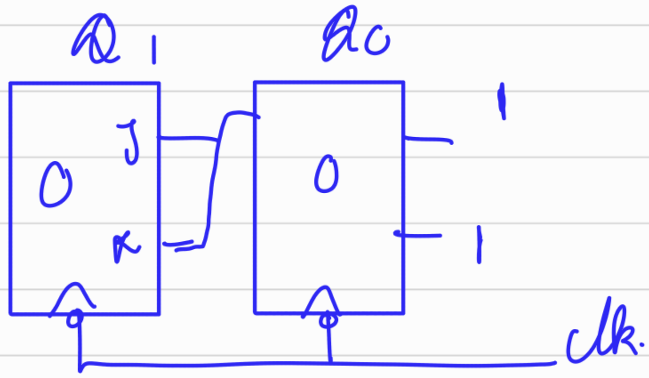
	a	b	c	d	e	f	g
0	1	1	1	1	1	1	0
1	0	0	0	1	1	0	0

Ripple counter -> Mod 10



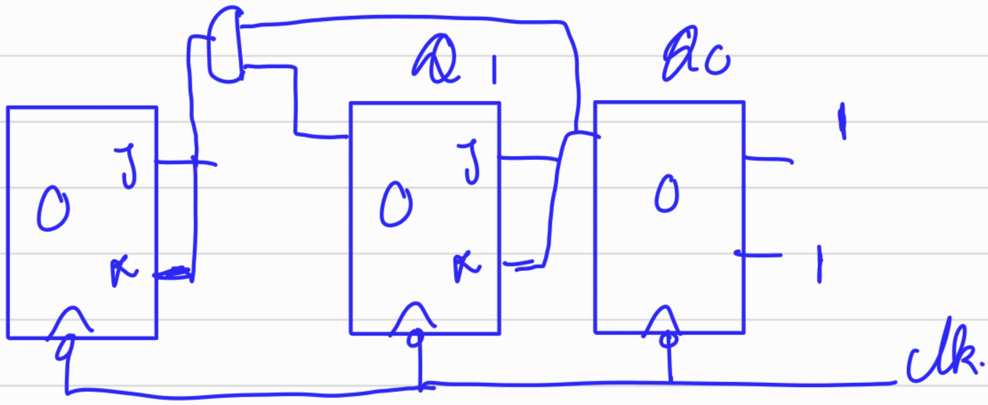
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
0	0	0	0

Same as before



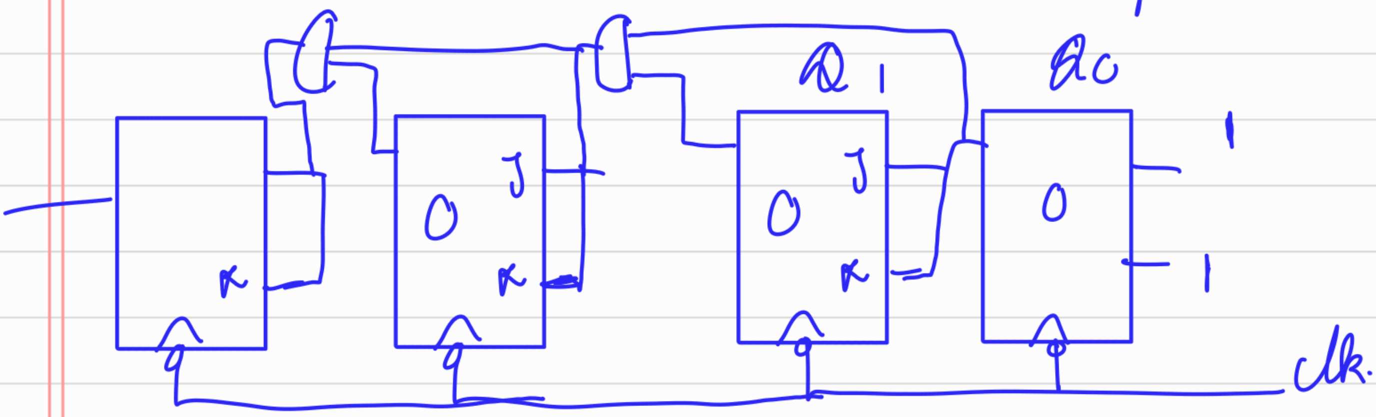
clk	Q_1	Q_0
0	0	0
1	0	1
2	1	0
3	1	1

Synchronous counter Mod 16

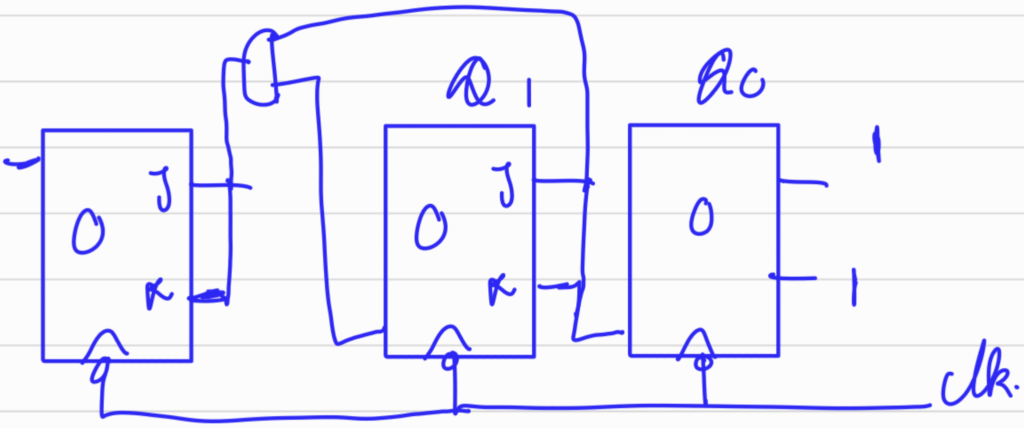


Q_2	Q_1	Q_0
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

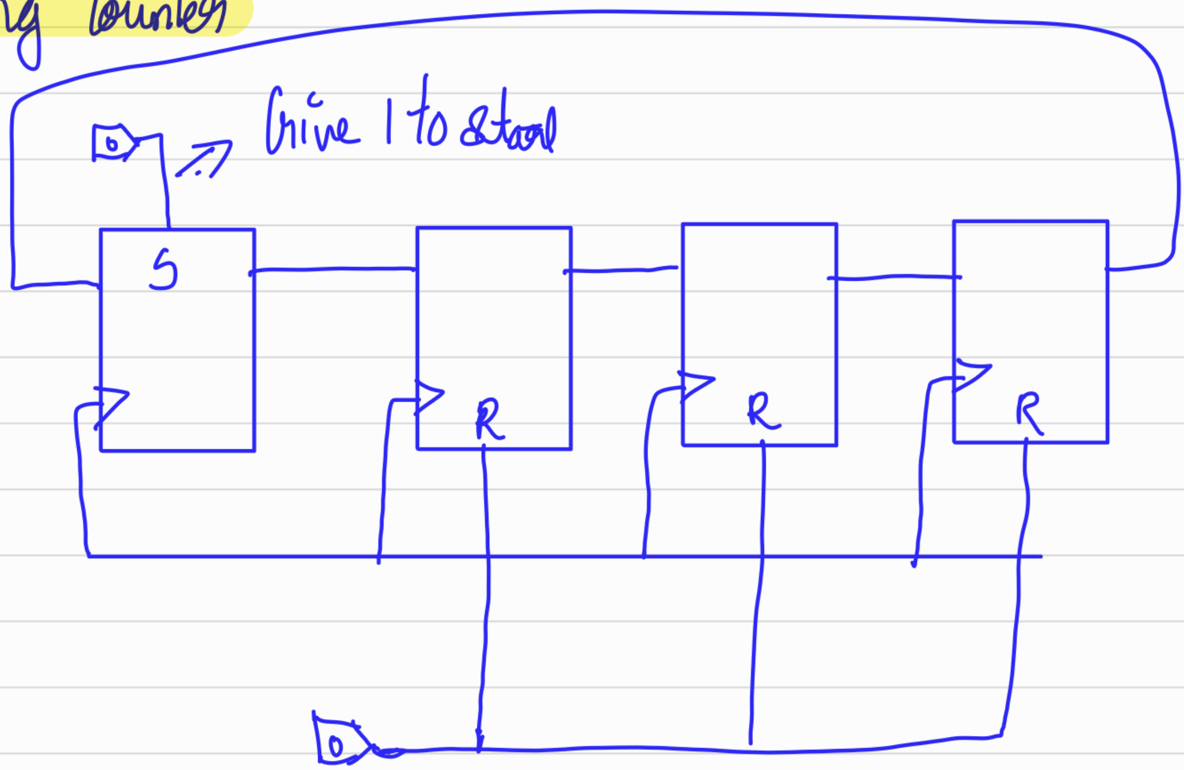
→ Up counter



Q_2	Q_1	Q_0
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0



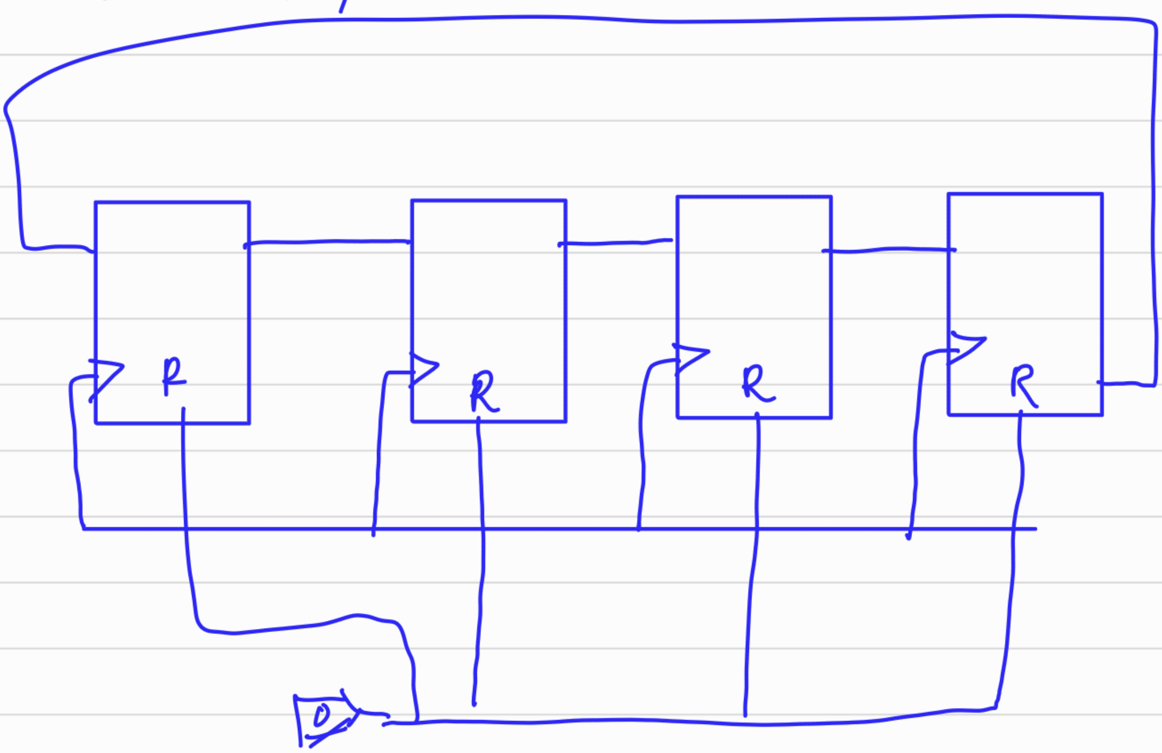
Ring Counter



No of state = 4 = No of flip flop

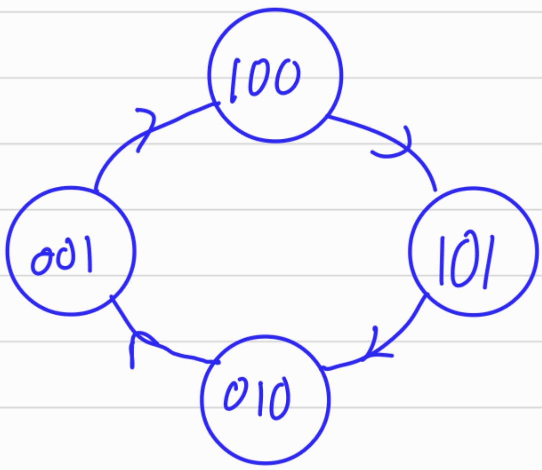
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1

Johnson Counter / twisted tail



No of state - 2(no of flip flop)

0	0	0	0
1	0	0	0
1	1	0	0
1	1	1	0
1	1	1	1
0	1	1	1
0	0	1	1
0	0	0	1
0	0	0	0



Excitation Table

Q_t	Q_{t+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

$D = Q_{t+1}$

Q_t	Q_{t+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

$T = Q_t \otimes Q_{t+1}$

Q_t	Q_{t+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

SR
00
01
0X

SR
00
10
X0

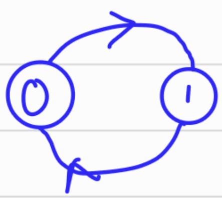
Q_t	Q_{t+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

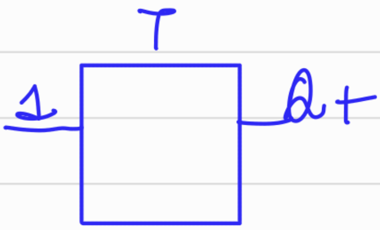
Q_t \ Q_{t+1}	0	1
0	0	1
1	X	X

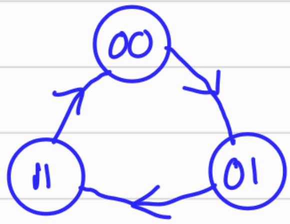
$J = Q_{t+1}$

Q_t \ Q_{t+1}	0	1
0	X	X
1	1	0

$K = \overline{Q_{t+1}}$







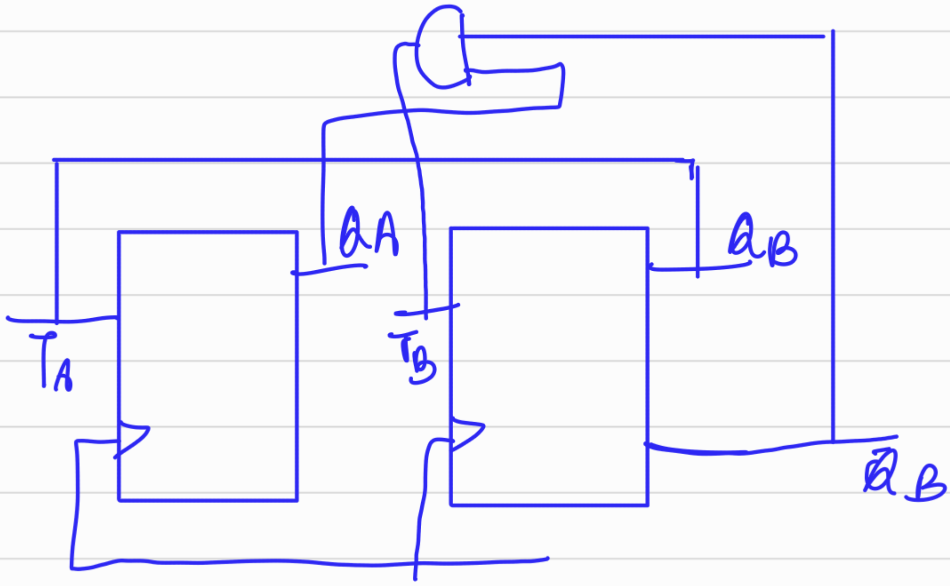
Q_A	Q_B	Q_{A+1}	Q_{B+1}	T_A	T_B
0	0	0	1	0	1
0	1	1	1	1	0
1	1	0	0	1	1

$Q_A \backslash Q_B$	0	1
0	0	1
1	X	1

$$T_A = Q_B$$

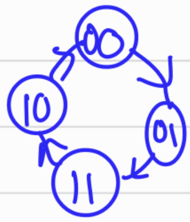
$Q_A \backslash Q_B$	0	1
0	1	0
1	X	1

$$T_B = Q_A + \bar{Q}_B$$



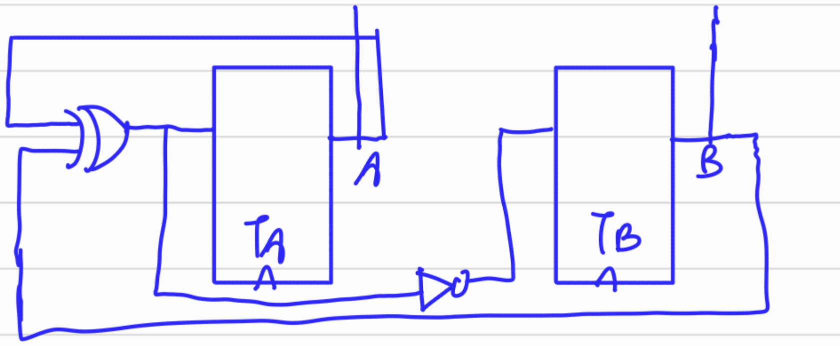
7476 - JK

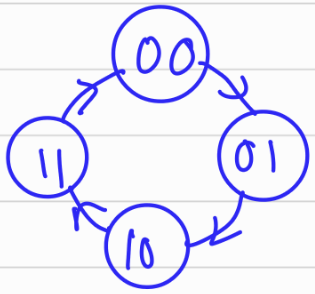
7474 - D



Prev A	Prev B	Next A	Next B	T_A	T_B
0	0	0	1	0	1
0	1	1	1	1	0
1	1	1	0	0	1
1	0	0	0	1	0

$T_A = A \oplus B$ $\qquad$ $T_B = A \odot B = \overline{A \oplus B}$





Prev A	Prev B	Next A	Next B	J_A	K_A	J_B	K_B
0	0	0	1	0	X	1	X
0	1	1	0	1	X	X	1
1	0	1	1	X	0	1	X
1	1	0	0	X	1	X	1

J_A

A \ B	0	1
0	0_0	1_1
1	X_2 (0)	X_3 (1)

J_B

A \ B	0	1
0	1_0	X_1 (1)
1	1_2	X_3 (1)

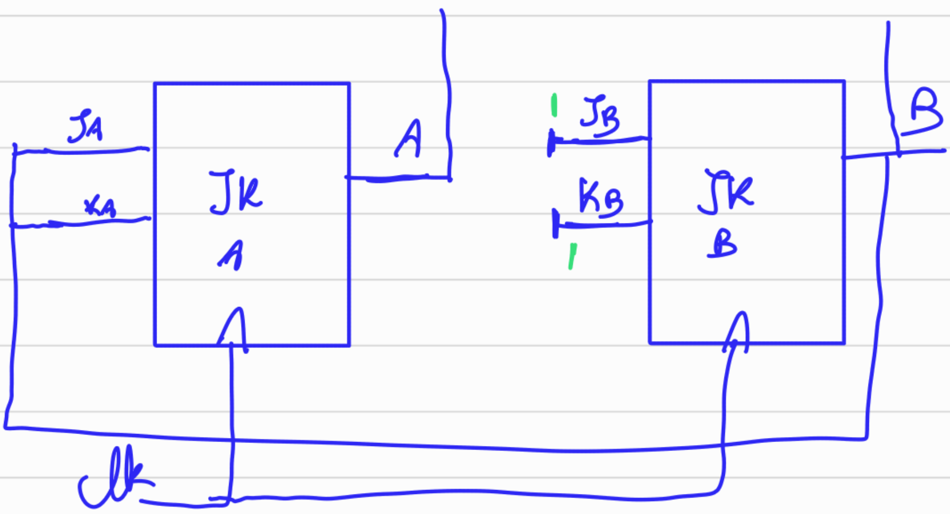
K_A

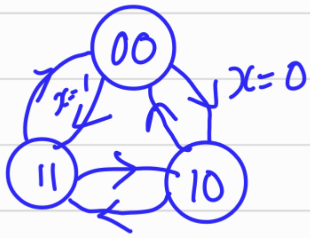
A \ B	0	1
0	X_0 (0)	X_1 (1)
1	0_2	1_3

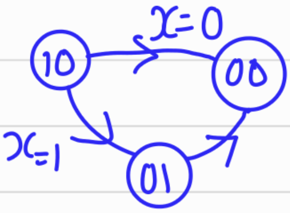
K_B

A \ B	0	1
0	X_0 (1)	1_1
1	X_2 (1)	1_3

$J_A = B$ $\quad J_B = 1$ $\quad K_A = B$ $\quad K_B = 1$







2,6,4,5

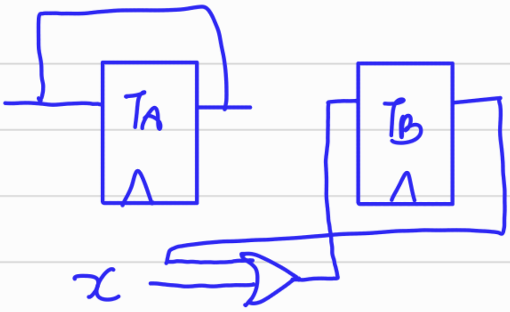
Prev x	Prev A	Prev B	Next A	Next B	T_A	T_B
0	1	0	0	0	1	0
1	1	0	0	1	1	1
0	0	1	0	0	0	1
1	0	1	0	0	0	0
0	0	0	X	X	X	X
0	1	1	X	X	X	X
1	0	0	X	X	X	X
1	1	1	X	X	X	X

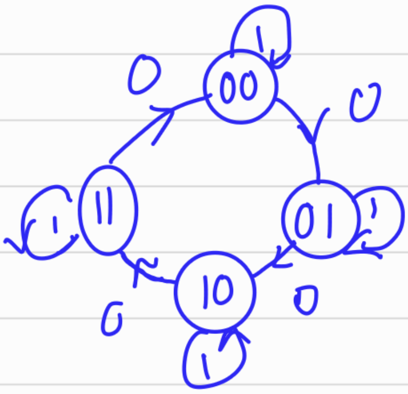
x \ AB	00	01	11	10
0	X (0)	0 (1)	X 1 (3)	1 (2)
1	X (4)	0 (5)	X 1 (7)	1 (6)

$T_A = A / B$

x \ AB	00	01	11	10
0	X (0)	1 (1)	X 1 (3)	0 (2)
1	X 1 (4)	1 (5)	X 1 (7)	1 (6)

$T_B = x + B / x + \bar{A}$





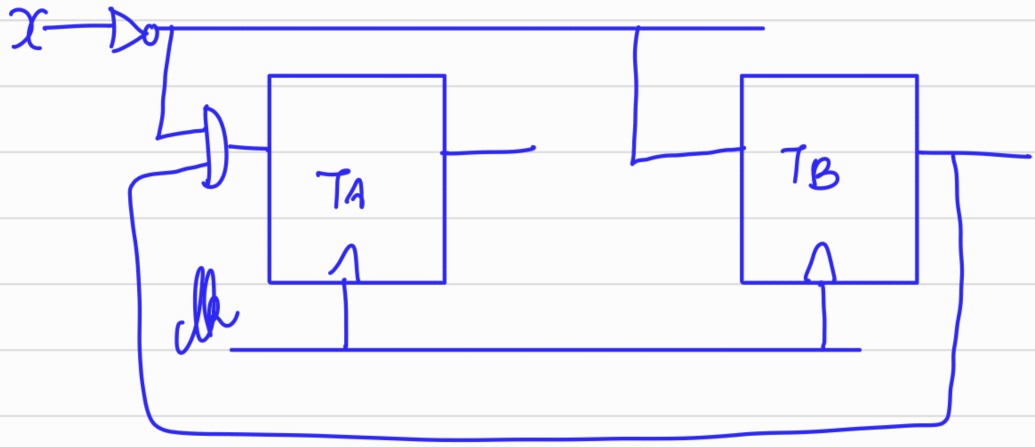
x	Prev A	B	Next A	B	T_A	T_B
0	0	0	0	1	0	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	0	0	1	1
1	0	0	0	0	0	0
1	0	1	0	1	0	0
1	1	0	1	0	0	0
1	1	1	1	1	0	0

$x \backslash AB$	00	01	11	10
0	0_0	1_1	1_3	0_2
1	0_4	0_5	0_7	0_6

$$T_A = \bar{x} B$$

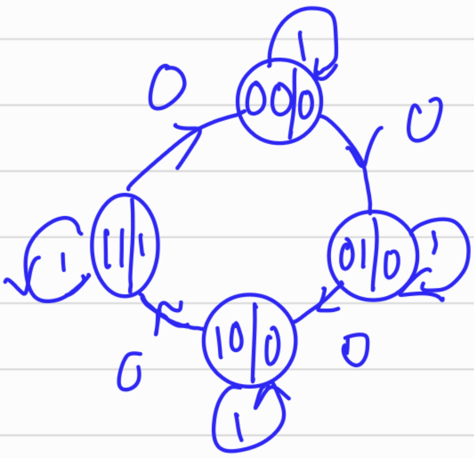
$x \backslash AB$	00	01	11	10
0	1_0	1_1	1_3	1_2
1	0_4	0_5	0_7	0_6

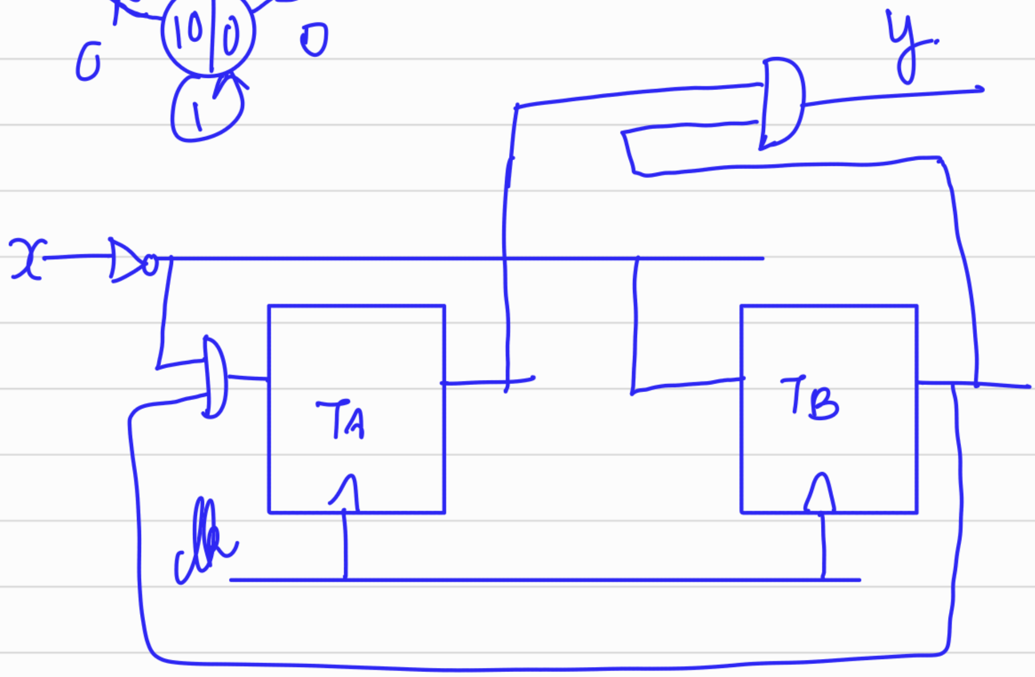
$$T_B = \bar{x}$$



Moore Machine

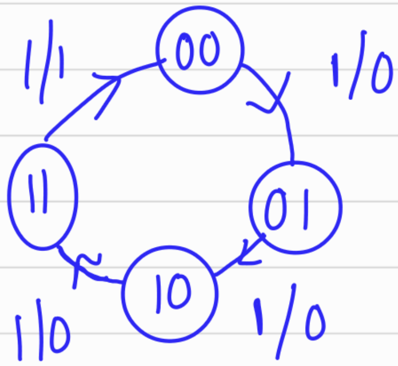
Present state ⇒ Output





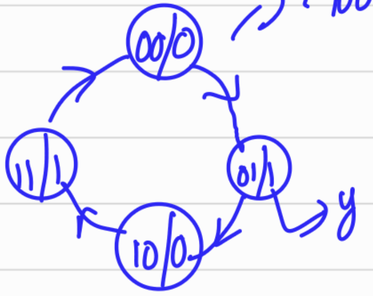
Mealy Machine

Present state + input ⇒ O/P



Jon Von Neuman

→ Moore machine



Alan Turing
∠ Alanzo church
↳ Lambda Calculus

Prev A	Prev B	Next A	Next B	TA	TB	y
0	0	0	1	0	1	0
0	1	1	0	1	1	1
1	0	1	1	0	1	0
1	1	0	0	1	1	1

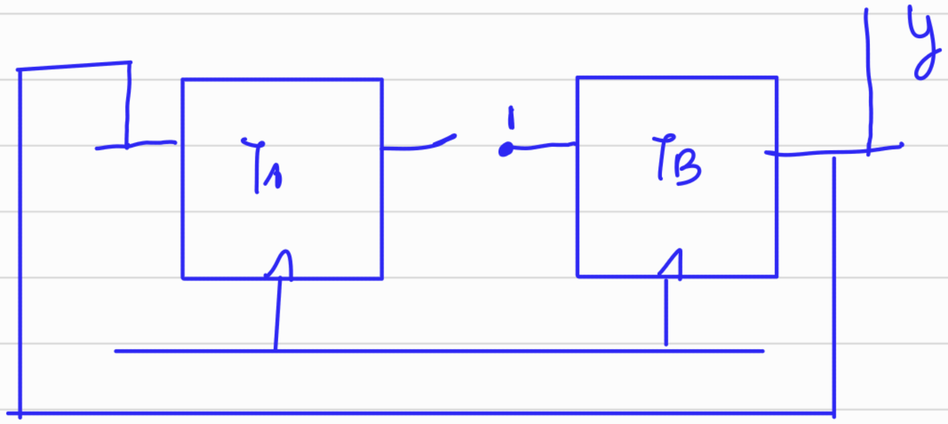
A \ B	0	1
0	0_0	1_1
1	0_2	1_3

$T_A = B$

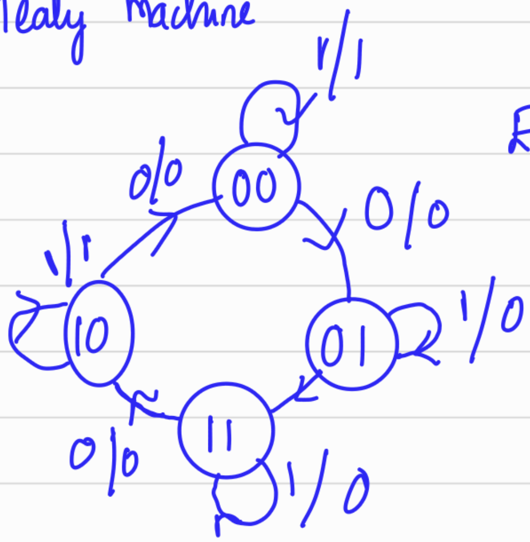
$T_B = 1$

A \ B	0	1
0	0_0	1_1
1	0_2	1_3

$y = B$



Mealy machine



Even + stay = 1

	Prev		Next				
x	A	B	A	B	T_A	T_B	y
0	0	0	0	1	0	1	0
0	0	1	1	1	1	0	0
0	1	0	0	0	1	0	0
0	1	1	1	0	0	1	0
1	0	0	0	0	0	0	1
1	0	1	0	1	0	0	0
1	1	0	1	0	0	0	1
1	1	1	1	1	0	0	0

$x \backslash AB$	00	01	11	10
0	0_0	1_1	0_3	1_2
1	0_4	0_5	0_7	0_6

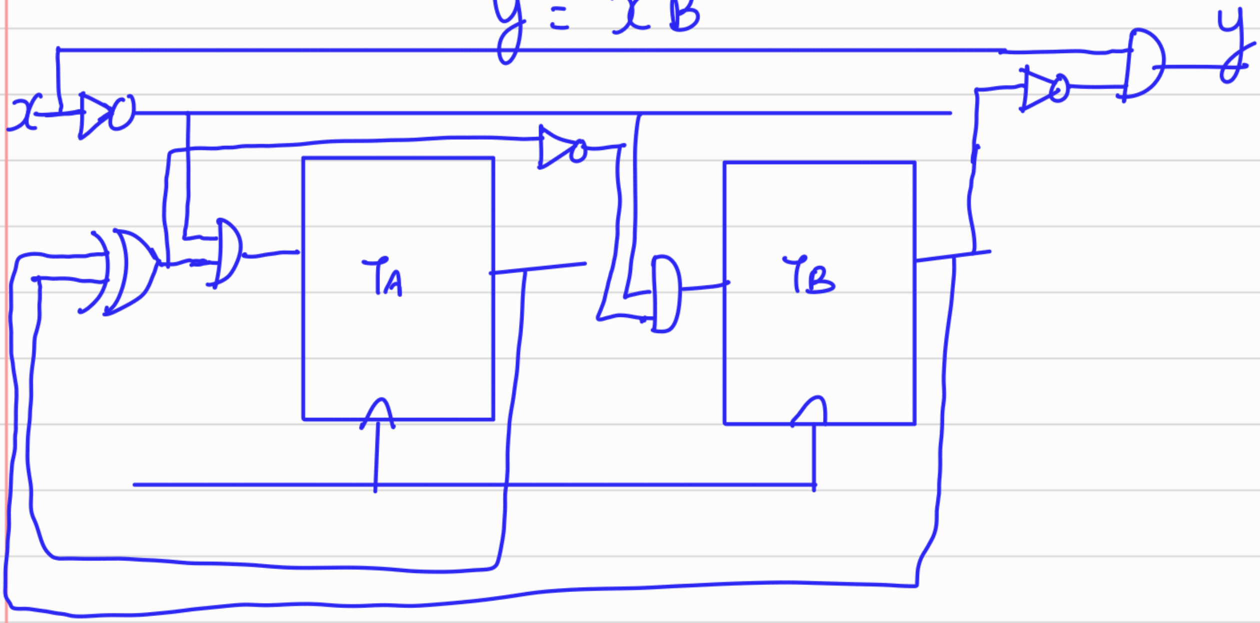
$x \backslash AB$	00	01	11	10
0	1_0	0_1	1_3	0_2
1	0_4	0_5	0_7	0_6

$$T_A = \bar{x}\bar{A}B + \bar{x}A\bar{B}$$
$$= \bar{x}(A \oplus B)$$

$$T_B = \bar{x}\bar{A}\bar{B} + \bar{x}AB$$
$$= \bar{x}(A \odot B)$$
$$= \bar{x}(\overline{A \oplus B})$$

$x \backslash AB$	00	01	11	10
0	0_0	0_1	0_3	0_2
1	1_4	0_5	0_7	1_6

$$y = x\bar{B}$$



Weighted

Non-weighted

* BCD 8421

1000

0×2^0 → +ve weight

0×2^1

0×2^2

1×2^3

* Grey

0011

→ 2 calculated by binary

Originally No weight

⇒ Non weighted

* BCD 2421

0111 - 7

1110 - 8

1111 - 9

* Excess 3

Excess 3

→ Add 3 to the given no

$$\begin{array}{r} 0001 \\ 0011 \\ \hline 0100 \end{array}$$

→ excess 3 equivalent code

Excess 3 in BCD

$$\begin{array}{cc} 0 & - & 9 \\ +3 & & +3 \\ \hline 3 & & 12 \end{array}$$

4bit 4bit

in Binary

0 - 15
+3   +3
3   18
↳ 5 bits

18 → BCD
13

1 → carry

0001+3   1000+3
0001+3   0011+3

1

0100   1011
0100   0110
1001   10001

0 ↳ carry 1 so add 3
carry 0 so subtract 3

0110   0100
6   4
↑   ↑
3   1

4 3
8 2

0100 +3 0011 +3
1000 +3 0010 +3

Excess3

0111 10110
1011 01101

10010 01011

carry 1 carry 3 subract 3

0010
0011
0101

1000

8
↑
5

5
4
2

carry 1

∴ 12

5

125

I/P BCD Excess 3

B_3	B_2	B_1	B_0	E_3	E_2	E_1	E_0	
0	0	0	0	0	0	1	1	- 3
0	0	0	1	0	1	0	0	
0	0	1	0	0	1	0	1	
0	0	1	1	0	1	1	0	
0	1	0	0	0	1	1	1	
0	1	0	1	1	0	0	0	
0	1	1	0	1	0	0	1	
0	1	1	1	1	0	1	0	
1	0	0	0	1	0	1	1	
1	0	0	1	1	1	0	0	- 12

$E_0 = \overline{B_0}$ $\quad$ $E_1 = B_1 \odot B_0$

B_3B_2 \ B_1B_0	00	01	11	10
00	0_0	0_1	0_3	0_2
01	0_4	1_5	1_7	1_6
11	X^1_{12}	X^1_{13}	X^1_{15}	X^1_{14}
10	1_8	1_9	X^1_{11}	X^1_{10}

$$E_3 = B_3 + B_2B_0 + B_2B_1$$
$$= B_3 + B_2(B_0 + B_1)$$

B_3B_2 \ B_1B_0	00	01	11	10
00	0_0	1_1	1_3	1_2
01	1_4	0_5	0_7	0_6
11	X^1_{12}	X^0_{13}	X^0_{15}	X^0_{14}
10	0_8	1_9	X^1_{11}	X^1_{10}

$$E_2 = B_2\overline{B_1}\,\overline{B_0} + \overline{B_2}\,\overline{B_1}B_0$$
$$+ \overline{B_2}B_1$$