Waves and Vibrations

Mid Semester Assessment | Date: 8.05.2023 | Time: 2 hours

IIITDM Kancheepuram | All questions are necessary | Maximum Marks (30)

1. Which of the following represent simple harmonic motion and why? (3)

(i) $x = A \sin \omega t + B \cos \omega t$

(ii) $x = A \sin \omega t + B \cos 2\omega t$

(iii) $x = A e^{i\omega t}$

(iv) $x = A \ln \omega t$

2. Consider a particle undergoing simple harmonic motion. The velocity of the particle at position x_1 is v_1 and velocity of the particle at position x_2 is v_2. Show that the ratio of time period and amplitude is (3)

$$\frac{T}{A} = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

3. A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works (2)

4. Prove that an overdamped oscillator can cross its equilibrium position at most once. (3)

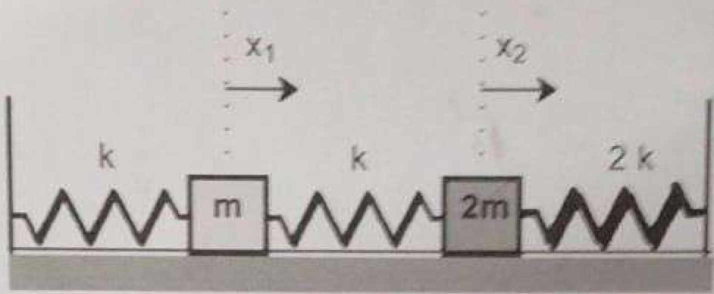
5. Consider a damped harmonic oscillator. Let us define T_1 as the time between adjacent zero crossings, $2T_1$ as its "period", and $\omega_1 = 2\pi/(2T_1)$ as its "angular frequency". If the amplitude of the damped oscillator decreases to 1/e of its initial value after n periods, show that the frequency of the oscillator must be approximately $[1 - (8\pi^2 n^2)^{-1}]$ times the frequency of the corresponding undamped oscillator. (3)

PTO

6. A 0.500 kg mass is attached to a spring of constant 150 N/m. A driving force $F(t) = (12.0\ N)\cos(\omega t)$ is applied to the mass, and the damping coefficient b is 6.00 Ns/m. What is the amplitude (in cm) of the steady-state motion if ω is equal to half of the natural frequency ω_0 of the system?

(3)

7. Consider the coupled oscillator given below. Write down the equation of motions for the normal modes of the system and determine their respective frequencies. (4)



8. Plot the variations in amplitude of a plane wave, spherical wave and cylindrical wave as these propagate away from their source. Also justify your answer mathematically and with proper reasoning. (3)

9. A plane wave having propagation vector $\mathbf{K} = 2\mathbf{a}_x + \mathbf{a}_y + 3\ \mathbf{a}_z\ mm^{-1}$ is propagating in the space with position vector $\mathbf{r} = t\mathbf{a}_x + 2t\mathbf{a}_y + \mathbf{a}_z$ m; where t stands for time. Determine the direction and speed of the wave. (3)

10. Write down the expressions for a solitary wave and a localized wave and differentiate between them physically. (3)

PTO