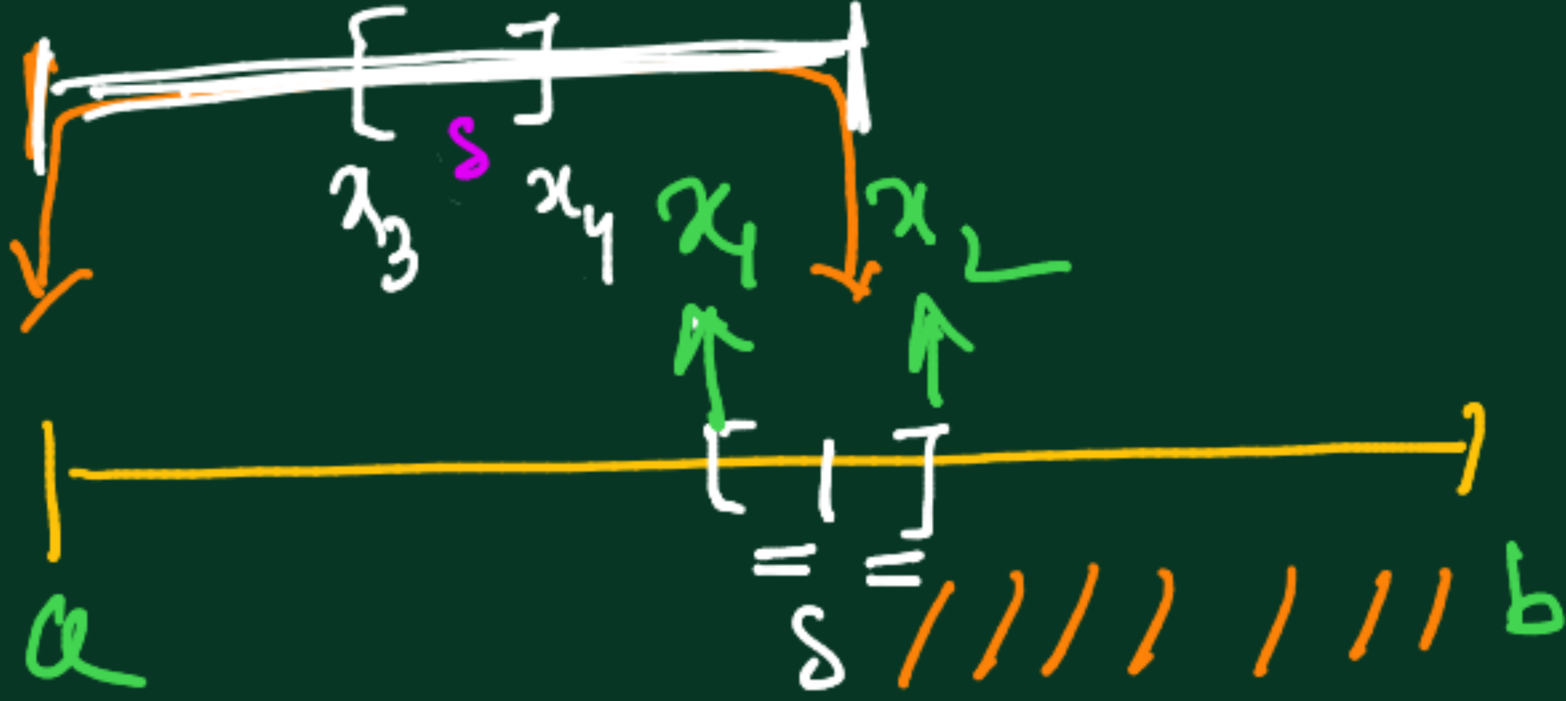
Dichotomous Search

$\boxed{f(x)}$

$$f'(x) = e^x + \sin x = 0$$

{ 50% per each iteration } & 2-function evaluat ?

25% per fun^n evaluation



$$x_1 = \boxed{a + \frac{L}{2}} - \frac{\delta}{2}$$

$$x_2 = b - \frac{L}{2} + \frac{\delta}{2} = a + \frac{L}{2} + \frac{\delta}{2}$$

$$f(x_1) \overset{<}{\underset{>}{\longleftrightarrow}} f(x_2)$$

$\downarrow$

$$f(x_3) \longleftrightarrow f(x_4)$$

$$\frac{L}{2} - \frac{\delta}{2}$$

$< 50\%$

For each function evaluation

$< 25\ \%.$

x

$\boxed{F_n = F_{n-1} + F_{n-2}}$ $\boxed{F_0 = F_1 = 1}$ F_5

1 1 2 3 5 8 13 $= F_6$

F_2 F_3 F_4

$\boxed{F_7 = 21}$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \boxed{\frac{1+\sqrt{5}}{2}} = 1.618$$

$$\lim_{n \to \infty} \frac{F_n}{F_{n+1}} = \boxed{0.618} = \lim_{n \to \infty} \frac{F_{n-1}}{F_n}$$

Q:- $\boxed{\lim_{n \to \infty} \frac{F_{n-2}}{F_n}}$

Fibonacci Search Method

Step-1

Choose lower bound a
& upper Bound b

Set $\boxed{L = b - a}$

Assume No of function evaluation : n

Step-2

Compute $L_k^* = \left(\dfrac{F_{n-k+1}}{F_{n+1}}\right) L$

Set $\boxed{x_1 = a + L_k^*}$ & $\boxed{x_2 = b - L_k^*}$

$\boxed{\text{Step-3}}$

Compute $f(x_1)$ & $f(x_2)$

$\boxed{\text{Step-4}}$

Is $\boxed{k = n}$? $\nearrow$ Yes - stop

$\searrow$ No then go to $\boxed{\text{Step-2}}$

Example:-

Minimize the function

$$\boxed{f(x) = x^2 + \frac{54}{x}}$$

$a = 0, \quad b = 5$

$\boxed{n = 3}$, $\boxed{k = 2}$

$$L_2^* = \left(\frac{F_{3-2+1}}{F_{3+1}}\right)L = \left(\frac{F_2}{F_4}\right)5$$

$$= \left(\frac{2}{5}\right)5 = 2$$

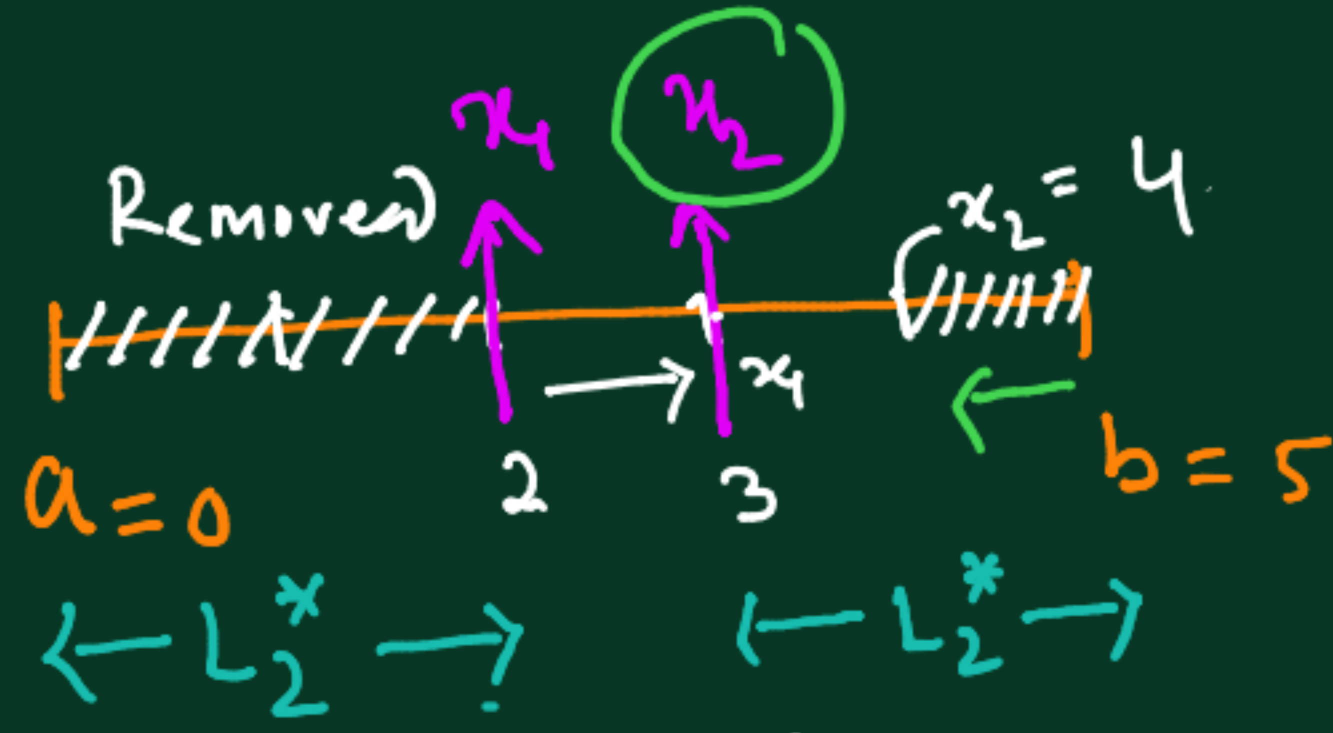
$$\left.\begin{aligned} x_1 &= a + L_2^* = 0+2=2 \\ x_2 &= b - L_2^* = 5-2=3 \end{aligned}\right\}$$

Step - 3

$$f(x_1) = x_1^2 + \frac{54}{x_1} = 4 + \frac{54}{2}$$

$$= \frac{62}{2} = 31$$

$$\boxed{f(x_2) = 27} \qquad f(x_1) > f(x_2)$$



$$L_2 = L - L_2^* = 5 - 2 = \boxed{3}$$

$f(x_1) > f(x_2)$

$a = x_1 = 2, \quad b = 5, \quad , \; x^* = 3.5$

Step-4

$k = 2 \neq 3 = n$ No, Go to Step-2

$\rightarrow k = k+1 = 3$

Step-2

$$L_3^* = \left(\frac{F_{n-k+1}}{F_{n+1}}\right) L = \left(\frac{F_1}{F_4}\right) L = \frac{1}{5} \cdot 5 = 1$$

Step-3

$x_1 = 3 \quad , \quad x_2 = 4$

$f(x_1) = 27 \quad , \quad f(x_2) = \boxed{29.5}$

$f(x_1) < f(x_2)$

$a = 2 \quad , \quad b = 4$

$\hookrightarrow \quad x^* = \frac{2+4}{2} = 3$

Step-4

$k = n = 3$

Stop

$$L_2 = L - \frac{F_{n-1}}{F_{n+1}} L = \left(\frac{F_{n+1} - F_{n-1}}{F_{n+1}}\right) L$$

L_2^*

$$F_{n+1} = \underline{F_n + F_{n-1}}$$

$$L_2 = \left(\frac{F_n}{F_{n+1}}\right) L$$

$$L_3 = L_2 - L_3^* = \left(\frac{F_n}{F_{n+1}}\right) L - \left(\frac{F_{n-2}}{F_{n+1}}\right) L$$

$$= \left(\frac{F_n - F_{n-2}}{F_{n+1}}\right) L$$

$$= \left(\frac{F_{n-1}}{F_{n+1}}\right) L$$

$$L_4 = \left(\frac{F_{n-2}}{F_{n+1}}\right) L$$

$$L_5 = \left(\frac{F_{n-3}}{F_{n+1}}\right) L$$

$$\vdots \qquad \ddots \qquad \ddots$$

$$\underline{\underline{L_n}} = L_{n-1} - L_n^* = \underline{\underline{\left(\frac{F_2}{F_{n+1}}\right) L}}$$

$$\boxed{> 25\%}$$

$$\boxed{\frac{2}{F_{n+1}} L < \epsilon \ \checkmark}$$