(Chapter-07) First order circuits

Solution for Practice Problems

The Source-Free RC Circuit

Q1.

$\tau = R_{th}C$

where R_{th} is the Thevenin equivalent at the capacitor terminals.

$$R_{th} = 120 \,||\, 80 + 12 = 60\ \Omega$$
$$\tau = 60 \times 200 \times 10^{-3} = \mathbf{12\ s.}$$

Q2.

For $t<0$, $v(0^-)=40$ V.

For $t>0$. we have a source-free RC circuit.

$$\tau = RC = 2x10^3 x10x10^{-6} = 0.02$$
$$v(t) = v(0)e^{-t/\tau} = \underline{40e^{-50t}\ \text{V}}$$

Q3.

(a) τ=RC = 1/200

For the resistor, V=iR= $56e^{-200t} = 8Re^{-200t}x10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ k\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200X7X10^3} = \underline{0.7143\mu F}$$

(b) τ =1/200= **5 ms**

(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2}x56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ ms}$$

Q4.

For t<0, $v(0^-) = \frac{3}{3+9}(36V) = \underline{9\ V}$

For t>0, we have a source-free RC circuit

$$\tau = RC = 3x10^3 x20x10^{-6} = 0.06s$$

$$v_o(t) = \mathbf{9e^{-16.667t}\ V}$$

Let the time be t_o.

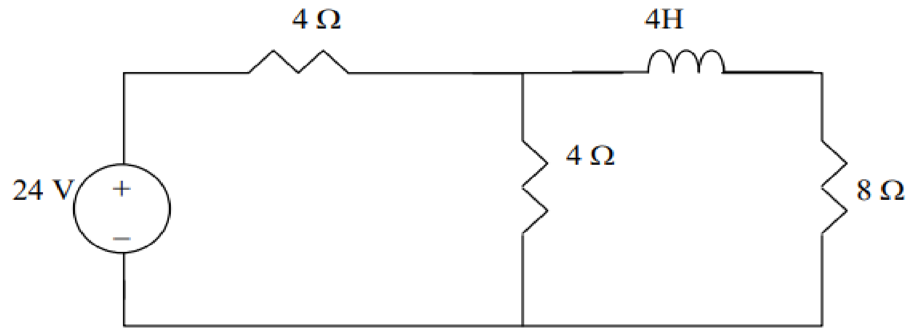
$3 = 9e^{-16.667to}$ or $e^{16.667to} = 9/3 = 3$

$$t_o = \ln(3)/16.667 = \mathbf{65.92\ ms.}$$

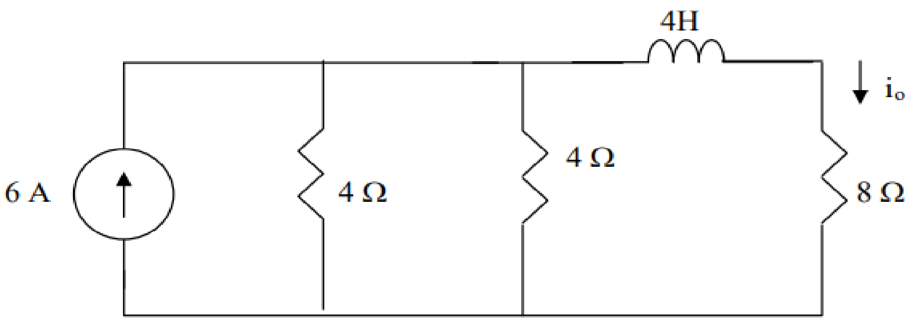
The Source-Free RC Circuit

Q1.

For t<0, we have the circuit shown below.



⇩



4||4= 4x4/8=2

$i_o(0^-) = [2/(2+8)]6 = 1.2$ A

For t >0, we have a source-free RL circuit.

$\tau = \frac{L}{R} = \frac{4}{4+8} = 1/3$ thus,

$$i_o(t) = \mathbf{1.2e^{-3t}\ A.}$$

Q2.

$$\tau = \frac{L_{eq}}{R_{eq}}$$

(a) $L_{eq} = L$ and $R_{eq} = R_2 + \frac{R_1 R_3}{R_1 + R_3} = \frac{R_2(R_1 + R_3) + R_1 R_3}{R_1 + R_3}$

$$\tau = \frac{\mathbf{L(R_1 + R_3)}}{\mathbf{R_2(R_1 + R_3) + R_1 R_3}}$$

(b) where $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$ and $R_{eq} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = \frac{R_3(R_1 + R_2) + R_1 R_2}{R_1 + R_2}$

$$\tau = \frac{\mathbf{L_1 L_2 (R_1 + R_2)}}{\mathbf{(L_1 + L_2)(R_3(R_1 + R_2) + R_1 R_2)}}$$

Q3.

(a) $\tau = \frac{1}{10^3} = \underline{1ms}$ $= \mathbf{1\ ms}$.

$v(t) = i(t)R = 80e^{-1000t}$ V $= R5e^{-1000t}x10^{-3}$ or $R = 80{,}000/5 = \mathbf{16\ k\Omega}$.

But $\tau = L/R = 1/10^3$ or $L = 16x10^3/10^3 = \mathbf{16\ H}$.

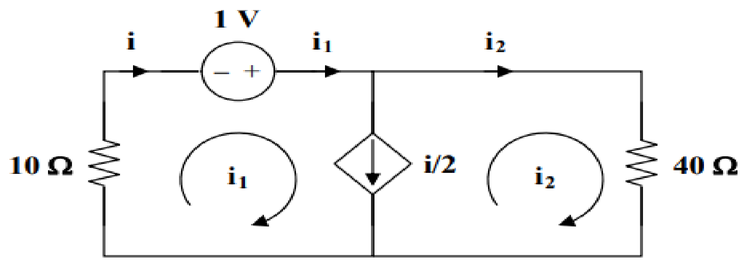
(b) The energy dissipated in the resistor is

$$w = \int_0^{0.0005} p dt = \int_0^{0.0005} 0.4e^{-2000t} dt = -\frac{0.4}{2000} e^{-2000t} \Big|_0^{0.0005}$$

$$= 200(1-e^{-1})x10^{-6} = \mathbf{126.42\ \mu J}.$$

(a) 16 kΩ, 16 H, 1 ms (b) 126.42 μJ

Q4.



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$10i_1 - 1 + 40i_2 = 0$

But $i = i_2 + i/2$ and $i = i_1$

i.e. $i_1 = 2i_2 = i$

$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$

$$R_{th} = \frac{1}{i} = 30\ \Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2\ s$$

$$i(t) = 6e^{-5t}u(t)\ A$$

Step Response of an RC Circuit

Q1.

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \mathbf{4\ V}$$

After t = 0,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20) e^{-t/8}$$

$$v(t) = 20 - 16 e^{-t/8}\ \text{V}$$

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12\ \text{V}$$

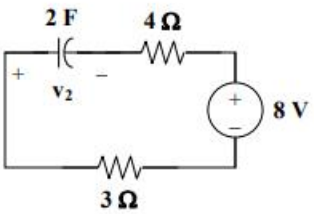
To get v_2, transform the current source as shown in Fig. (a).

$$v_2 = -8\ \text{V}$$

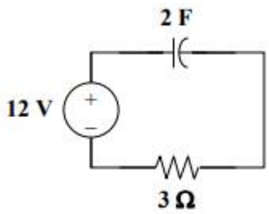
Thus,

$$v = 12 - 8 = \mathbf{4\ V}$$

After t = 0, the circuit becomes that shown in Fig. (b).



(a)



(b)

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12) e^{-t/6}$$

$$v(t) = \mathbf{12 - 8e^{-t/6}}\ \mathbf{V}$$

Q2.

(a)

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty) \right] e^{-t/\tau}$$

$$v_o(0) = 0, \qquad v_o(\infty) = \frac{4}{4+2}(12) = 8$$

$$\tau = R_{eq} C_{eq}, \qquad R_{eq} = 2 \,||\, 4 = \frac{4}{3}$$

$$\tau = \frac{4}{3}(3) = 4$$

$$v_o(t) = 8 - 8e^{-t/4}$$

$$v_o(t) = \mathbf{8(1 - e^{-0.25t})\ V}$$

(b) For this case, $v_o(\infty) = 0$ so that

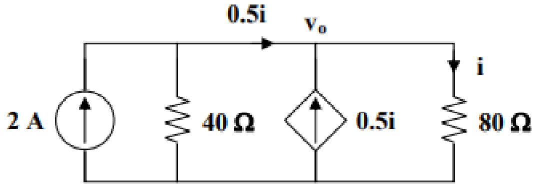
$$v_o(t) = v_o(0) e^{-t/\tau}$$

$$v_o(0) = \frac{4}{4+2}(12) = 8, \qquad \tau = RC = (4)(3) = 12$$

$$v_o(t) = \mathbf{8e^{-t/12}\ V}$$

Q3.

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



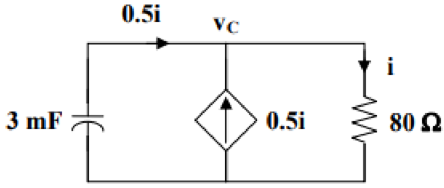
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

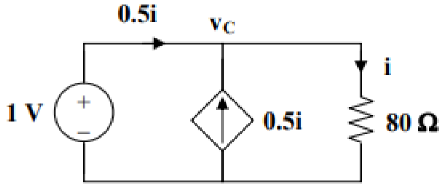
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64\text{ V}$$

$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \mathbf{800}\boldsymbol{e^{-t/480}u(t)\ mA}$$

Q4.

For $0 < t < 1$, $v(0) = 0$, $v(\infty) = (2)(4) = 8$

$$R_{eq} = 4 + 6 = 10, \qquad \tau = R_{eq}C = (10)(0.5) = 5$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(t) = 8\left(1 - e^{-t/5}\right) \text{ V}$$

For $t > 1$, $v(1) = 8\left(1 - e^{-0.2}\right) = 1.45$, $v(\infty) = 0$

$$v(t) = v(\infty) + \left[v(1) - v(\infty) \right] e^{-(t-1)/\tau}$$

$$v(t) = 1.45 e^{-(t-1)/5} \text{ V}$$

Thus,

$$v(t) = \begin{cases} \mathbf{8\left(1 - e^{-t/5}\right) V}, & \mathbf{0 < t < 1} \\ \mathbf{1.45 e^{-(t-1)/5} \ V}, & \mathbf{t > 1} \end{cases}$$

Step Response of an RL Circuit

Q1.

(a) Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \mathbf{1\ A}$$

After t = 0,

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + (4 \,||\, 12) = 7\ \Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{(4\,||\,12)}{4 + (4\,||\,12)}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right)e^{-2t}$$

$$i(t) = \mathbf{\frac{1}{7}\left(6 - e^{-2t}\right) A}$$

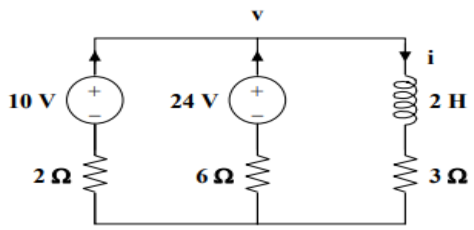
(b) Before t = 0, $i(t) = \frac{10}{2+3} = \mathbf{2\ A}$

After t = 0, $R_{eq} = 3 + (6 \,||\, 2) = 4.5$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at t = when the inductor becomes a short circuit,



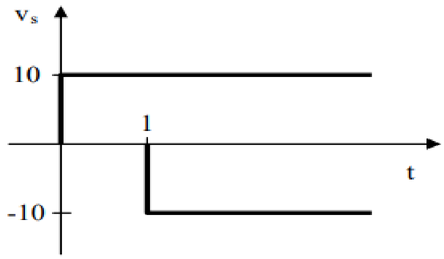
$$\frac{10 - v}{2} + \frac{24 - v}{6} = \frac{v}{3} \longrightarrow v = 9 \quad i(\infty) = \frac{v}{3} = 3 \text{ A and}$$

$$i(t) = 3 + (2 - 3)e^{-9t/4}$$

$$i(t) = \mathbf{3 - e^{-9t/4}\ \ A}$$

Q2.

Since $v_s = 10[u(t) - u(t-1)]$, this is the same as saying that a 10 V source is turned on at $t = 0$ and a -10 V source is turned on later at $t = 1$. This is shown in the figure below.



For $0 < t < 1$, $\quad i(0) = 0, \qquad i(\infty) = \frac{10}{5} = 2$

$$R_{th} = 5 \,||\, 20 = 4, \qquad \tau = \frac{L}{R_{th}} = \frac{2}{4} = \frac{1}{2}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$$
$$i(t) = 2(1 - e^{-2t})\text{ A}$$
$$i(1) = 2(1 - e^{-2}) = 1.729$$

For $t > 1$, $\qquad i(\infty) = 0 \qquad$ since $v_s = 0$

$$i(t) = i(1)\, e^{-(t-1)/\tau}$$
$$i(t) = 1.729\, e^{-2(t-1)}\text{ A}$$

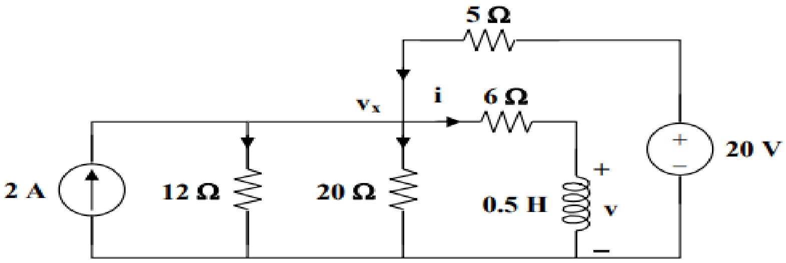
Thus,

$$i(t) = \begin{cases} \mathbf{2(1 - e^{-2t})\ A} & \mathbf{0 < t < 1} \\ \mathbf{1.729\, e^{-2(t-1)}\ A} & \mathbf{t > 1} \end{cases}$$

Q3.

$$R_{eq} = 6 + 20 \,||\, 5 = 10\,\Omega, \qquad \tau = \frac{L}{R} = 0.05$$
$$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$$

i(0) is found by applying nodal analysis to the following circuit.



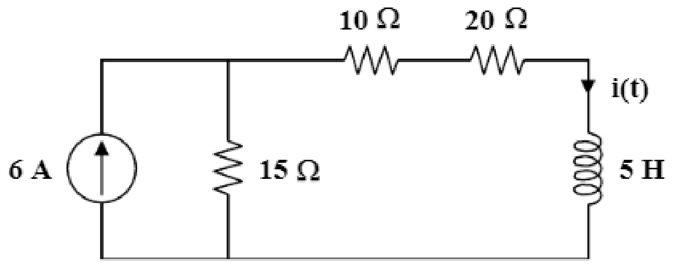
$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$
$$i(0) = \frac{v_x}{6} = 2\text{ A}$$

Since $20 \,||\, 5 = 4$,

$$i(\infty) = \frac{4}{4+6}(4) = 1.6$$
$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$
$$v(t) = L\frac{di}{dt} = \frac{1}{2}(0.4)(-20)e^{-20t}$$
$$v(t) = \mathbf{-4e^{-20t}\ V}$$

Q4.

For $0 < t < 2$, the given circuit is equivalent to that shown below.



Since switch S_1 is open at $t = 0^-$, $i(0^-) = 0$. Also, since i cannot jump, $i(0) = i(0^-) = 0$.

$$i(\infty) = \frac{90}{15+10+20} = 2 \text{ A}$$

$$R_{th} = 45\,\Omega, \quad \tau = \frac{L}{R_{th}} = \frac{5}{45} = \frac{1}{9}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 2 + (0-2)\,e^{-9t}$$

$$i(t) = 2\,(1 - e^{-9t}) \text{ A}$$

When switch S_2 is closed, the 20 ohm resistor is short-circuited.

$$i(2^+) = i(2^-) = 2\,(1 - e^{-18}) \cong 2$$

This will be the initial current

$$i(\infty) = \frac{90}{15+10} = 3.6 \text{ A}$$

$$R_{th} = 25\,\Omega, \quad \tau = \frac{5}{25} = \frac{1}{5}$$

$$i(t) = i(\infty) + \left[i(2^+) - i(\infty)\right] e^{-(t-2)/\tau}$$

$$i(t) = 3.6 + (2 - 3.6)\,e^{-5(t-2)}$$

$$i(t) = 3.6 - 1.6\,e^{-5(t-2)}$$

$$\text{Thus,}\quad i(t) = \begin{cases} \mathbf{0} & \mathbf{t < 0} \\ \mathbf{2\,(1-e^{-9t})\ A} & \mathbf{0 < t < 2} \\ \mathbf{3.6 - 1.6\,e^{-5(t-2)}\ A} & \mathbf{t > 2} \end{cases}$$

At $t = 1$, $\quad i(1) = 2\,(1 - e^{-9}) = \mathbf{1.9997\ A}$

At $t = 3$, $\quad i(3) = 3.6 - 1.6\,e^{-5} = \mathbf{3.589\ A}$