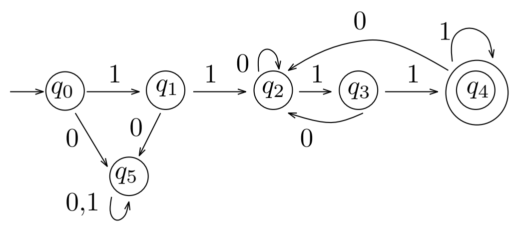
Assignment 1 Solution

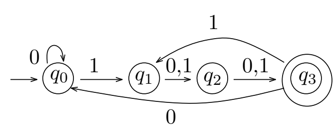
1. (3 marks) A Model is a representative of a system under study. Models give abstraction and many models (FA,PDA,TM) exist in computer science to model problems that arise in various CS domains, the popular domains are computer networks, computer organization, operating systems, big-data analysis, etc.,
   Case Study: High level Data link control protocol uses bit stuffing technique for each data transfer. i.e., bits 11 is stuffed as prefix and suffix with every frame. Frame contains data and CRC. For example, to transmit a frame 1101 with CRC bits 10, HDLC actually transmits 1111011011. Receiver is synchronized to this scheme so that when the data reaches the receiver the prefix and suffix are removed and the data part is taken for CRC verification. Model the above scenario using deterministic finite automaton. i.e., the set of all finite strings over $\{0, 1\}$ beginning with 11 and ending with 11.

   $L = \{\text{accepts all finite strings beginning with 11 and ending with 11}\}$
   $L = \{1111, 11011, 110101011, \ldots\}$

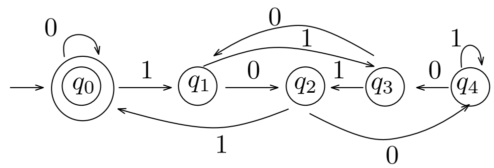
2. (3 marks) Consider a Cache memory system with set-associative mapping. The TAG fields follow a variable length scheme with the size ranging from 8 bits to 32 bits. The third bit from the right is a VALID bit which indicates that whether a block contains a valid data or not. If VALID=1 then the block is valid, otherwise it is invalid. The above scenario can be modelled using deterministic finite automata. i.e., the set of all finite strings over $\{0, 1\}$ such that the third bit from the right is 1.

   $L = \{\text{accepts all finite strings such that the third bit from the right is 1}\}$
   $L = \{11111111, 00000100, 11001101, \ldots\}$

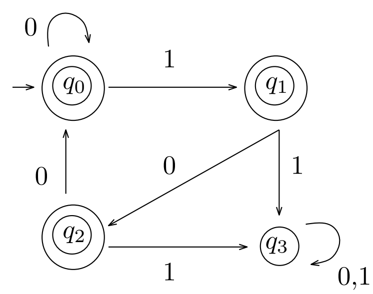
   

3. The following two DFA models find applications in Bio-informatics.

   - (3 marks) $L = \{x | x \text{ when interpreted as a binary string is a multiple of 5 }\}$. $x \in \{0, 1\}^*$.
     Some of the example strings are $L = \{101, 1111, 1010, 10100, \ldots\}$

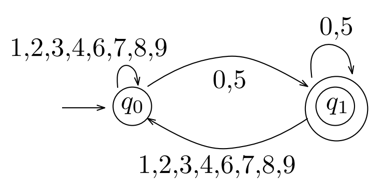


- (3 marks) $L = \{x|$ each block of 3 contains at least two 0s $\}$. $x \in \{0,1\}^*$.
  Some of the example strings are $L = \{000, 001, 100, 1001, 0100100, \ldots\}$



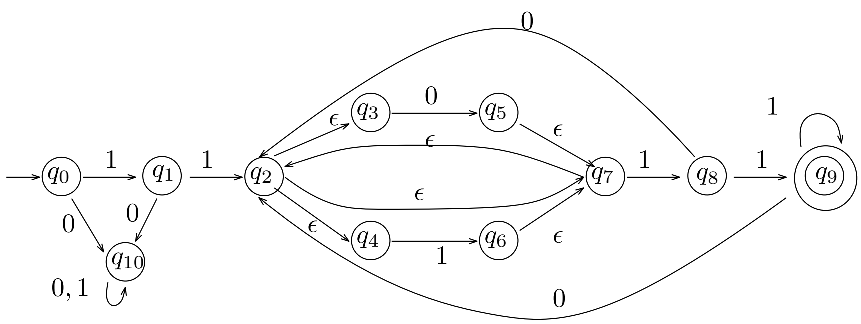
4. (3 marks) Construct a DFA if exists: $L = \{s \in (0+1) || n_0(s) n_1(s)| \leq 4\}$, $n_0(s)$ represents the number of 0s in the string s. If DFA does not exist, present an intuitive argument for non-existence of FA.
   Here, we need to count the number of 1's and the number of 0's. The difference between the number of 1's and 0's should be at most 4. But the problem is once we exceed the limit there is a possibility that we might cover up the difference in further part of the string.
   e.g., 01111111111111... Here, it needs infinite counting which is not possible in DFA and also L is not a regular language.

5. Let $\sum = \{0,1,2,3,4,5,6,7,8,9\}$. Consider the base 10 numbers formed by strings from $\sum^*$. Let $L_1 = \{x \in \sum^* \mid$ the numbers represented by x is exactly divisible by 5 $\}$.

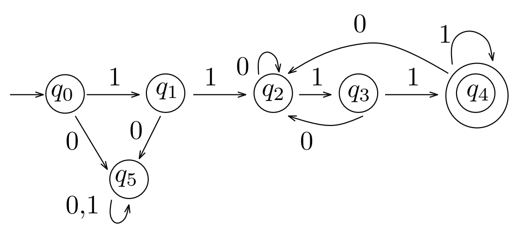


1. (1+1+3=5 marks) L : the set of all finite strings over $\{0, 1\}$ beginning with 11 and ending with 11. (i) Write the regular expression. (ii) Construct an equivalent NFA (with epsilon). (iii) Use the DFA constructed in Assignment 1 to obtain the Regular expression using Ardens Theorem.

   (i) The regular expression is $11(0 + 1)^*11$
   (ii) An equivalent NFA (with epsilon)



   (iii) Regular expression using Ardens Theorem.
   Arden's Theorem: If $R = RP + Q$ then $R = QP^*$



   $q_0 = \epsilon$, $q_1 = q_0 1$, $q_2 = q_1 1 + q_2 0 + q_3 0 + q_4 0$, $q_3 = q_2 1$, $q_4 = q_3 1 + q_4 1$.
   For start state we use ϵ and we omit all dead states.
   Now we substitute the value of q_0 in q_1.
   $q_0 = \epsilon$
   $q_1 = q_0 1 = \epsilon 1 = 1$.
   $q_3 = q_2 1$
   $q_4 = q_4 1 + q_3 1 = q_3 11^*$, by Arden's Theorem

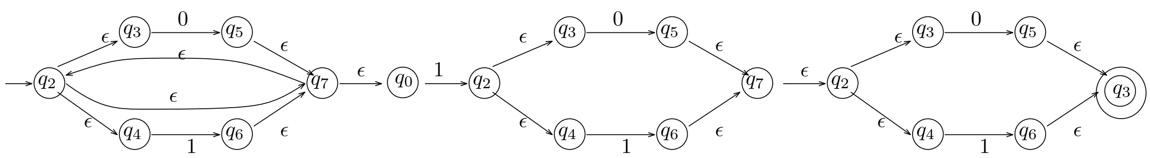
   $q_2 = q_0 11 + q_2 0 + q_3 0 + q_4 0$
   $= q_2 0 + (q_0 11 + q_3 0 + q_4 0)$
   $= q_2 0 + (\epsilon 11 + q_2 10 + q_3 11^* 0)$
   $= q_2 0 + (11 + q_2 10 + q_2 111^* 0)$
   $= q_2 (0 + 10 + 111^* 0) + 11$
   $= 11(0 + 10 + 111^* 0)^*$

   $q_3 = q_2 1 = 11(0 + 10 + 111^* 0)^* 1$

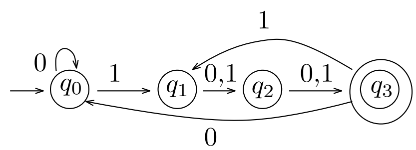
   $q_4 = q_3 11^* = 11(0 + 10 + 111^* 0)^* 111^*$
   The regular expression that we obtained in final state is our required solution. Here, the final state is q_4. Therefore, R.E is $11(0 + 10 + 111^* 0)^* 111^*$

2. (1+1+3=5 marks) L: the set of all finite strings over $\{0, 1\}$ such that the third bit from the right is 1. (1) Write the regular expression. (ii) Construct an equivalent NFA (with epsilon). (iii) Use the DFA constructed in Assignment 1 to obtain the Regular expression using Ardens Theorem.

   (i) The regular expression is $(0 + 1)^*1(0 + 1)(0 + 1)$
   (ii) An equivalent NFA (with epsilon)



   (iii) Regular expression using Ardens Theorem.
   Arden's Theorem: If $R = RP + Q$ then $R = QP^*$



   $q_0 = q_00 + q_30$, $q_1 = q_01 + q_31$, $q_2 = q_10 + q_11$, $q_3 = q_20 + q_21$.

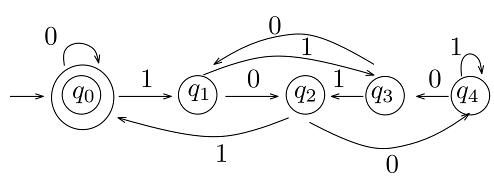
   $q_0 = q_00 + q_30 = q_300^*$, by Arden's Theorem

   $q_1 = q_01 + q_31 = q_300^*1 + q_31 = q_3(00^*1 + 1)$

   $q_2 = q_10 + q_11 = q_1(0 + 1) = q_3(00^*1 + 1)(0 + 1)$

   $q_3 = q_20 + q_21$
   $= q_2(0 + 1)$
   $= q_3(00^*1 + 1)(0 + 1)(0 + 1) + \epsilon$, consider Q is ϵ,
   $=(00^*1 + 1)(0 + 1)(0 + 1)$, by Arden's Theorem,
   Since the final state is q_3, R.E is $(00^*1 + 1)(0 + 1)(0 + 1)$

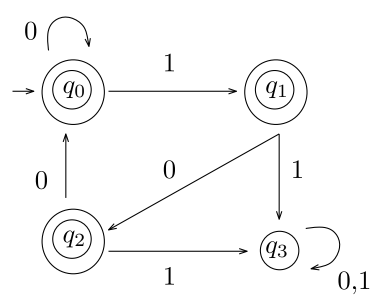
3. (3 marks) $L = \{x|x$ when interpreted as a binary string is a multiple of $5\}$. $x \in \{0, 1\}$. Use the DFA constructed in Assignment 1 to obtain the Regular expression using Ardens Theorem.



   Arden's Theorem: If $R = RP + Q$ then $R = QP^*$
   $q_0 = q_00 + q_21 + \epsilon$, $q_1 = q_01 + q_30$, $q_2 = q_10 + q_31$, $q_3 = q_40 + q_11$, $q_4 = q_41 + q_20$.

$q_0 = q_2 10^*$
$q_4 = q_2 01^*$
Substitute q_3 in q_1
$q_1 = q_0 1 + (q_4 0 + q_1 1)0 = q_0 1 + q_4 00 + q_1 10 = (q_0 1 + q_4 00)(10)^* = (q_2 10^* 1 + q_2 01^* 00)(10)^*$
Substitute q_1, q_3 in q_2
$q_2 = q_1 0 + q_3 1 = q_1 0 + (q_1 1 + q_4 0)1 = q_1 0 + q_1 11 + q_4 01 = q_1(0 + 11) + q_4 01$
$=((q_2 10^* 1 + q_2 01^* 00)(10)^*)(0 + 11) + q_2 01^* 01$
$=(q_2 10^* 1)(10)^*(0 + 11) + (01^* 00)(10)^*(0 + 11) + q_2 01^* 01$
$=q_2(10^* 1)(10)^*(0 + 11) + 01^* 01) + (01^* 00)(10)^*(0 + 11)$
$=(01^* 00)(10)^*(0 + 11)((10^* 1)(10)^*(0 + 11) + 01^* 01))^*$
Substitute q_2 in q_0
$q_0 = q_2 10^*$
$=((01^* 00)(10)^*(0 + 11)((10^* 1)(10)^*(0 + 11) + 01^* 01))^*)10^*$

4. (3 marks) $L = \{x|$ each block of 3 contains at least two 0s $\}$. $x \in \{0, 1\}^*$. Use the DFA constructed in Assignment 1 to obtain the Regular expression using Ardens Theorem



$q_0 = q_0 0 + q_2 0 + \epsilon$, $q_1 = q_0 1$, $q_2 = q_1 0$, $q_3 = q_1 1 + q_2 1 + q_3 0 + q_3 1$
$q_0 = q_2 00^*$
$q_2 = q_0 10 = q_2 00^* 10 + \epsilon = (00^* 10)^*$
$q_0 = (00^* 10)^* 00^*$
$q_1 = (00^* 10)^* 00^* 1$
$q_3 = q_3(0 + 1) + q_1 1 + q_2 1 = (q_1 1 + q_2 1)(0 + 1)^* = (((00^* 10)^* 00^* 1)1 + (00^* 10)^* 1)(0 + 1)^*$

DFA has multiple final states. Therefore, R.E is the language accepted by q_0, q_1, q_2

5. (5 marks) How many different DFAs are possible with $\sum = \{0, 1, 2\}$ and $Q = \{q_0, q_1, q_2\}$.

Start state can be chosen as any one among 3 in 3 ways.
Transition function is from $Q \times Z$ to Q, where Q is the set of states and Z is the set of input symbols. $|Q| = 3$, $|Z| = 3$. So, number of possible transition functions $= 3^{(3*3)}$=3^9.
Final state can be any subset of the set of states including empty set. With 3 states, we can have $2^3 = 8$ possible sub states.
Thus total number of DFAs possible is $=3 \times 3^9 \times 2^3$.
In general, For n states and m input alphabets the number of different DFA possible are $n \times n^{nm} \times 2^n$

6. (3 marks) Consider a DFA with write operation (after reading a symbol, FA can write on the input tape). What languages this DFA variant can accept which DFA cannot?