Indian Institute of Information Technology,
Design and Manufacturing Kancheepuram
MA1002 Linear Algebra

Date : 28/11/2024 | End Semester Examination
Time : 14.00 - 17.00 | Marks : 50

CS23I1027

? 1. Express the matrix A as a product of elementary matrices where (5)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

? 2. Prove that the set, W, of all $n \times n$ matrices having trace equal to zero is a subspace of $M_{n\times n}(F) = F^{n\times n}$. Find a basis for W. What is the dimension of W? (Note that trace is the sum of diagonal entries). (5)

? 3. Let

$$\begin{aligned} \alpha_1 &= (1,-1), \quad \beta_1 = (1,0) \\ \alpha_2 &= (2,-1), \quad \beta_2 = (0,1) \\ \alpha_3 &= (-3,2), \quad \beta_3 = (1,1) \end{aligned}$$

Does there exist a linear transformation T from R^2 to R^2 such that $T\alpha_i = \beta_i$ for $i = 1, 2$ and 3 ? Justify your answer. (5)

? 4. Let $T : R^{2\times 1} \longrightarrow R^{2\times 1}$ be a linear transforation defined as (5)
$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+3y \\ x+y \end{pmatrix}$. Let $\mathcal{B}_1 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ and $\mathcal{B}_2 = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ be two ordered bases of $R^{2\times 1}$. (a) Compute $[T]_{\mathcal{B}_1}$ and $[T]_{\mathcal{B}_2}$. (b) Find an invertible matrix Q such that $[T]_{\mathcal{B}_2} = Q^{-1}[T]_{\mathcal{B}_1}Q$. Justify your answer.

? 5. Let $T : R^{3\times 1} \longrightarrow R^{3\times 1}$ is given by (4)

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Find a basis of (a) the null space of T and (b) the range of T.

6. Find the eigen values and eigen spaces of $A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$. Prove or disprove that A is diagonlizable. (6)

7. State and prove rank-nullity-dimension theorem. (3)

8. Let $\mathcal{V}$ be a vector space and let W_1, W_2 be subspaces of $\mathcal{V}$. Suppose that $\dim \mathcal{V} = 10$, $\dim W_1 = 8$ and $\dim W_2 = 9$. What are the possible values of $\dim(W_1 \cap W_2)$? Justify your answer. (4)

9. Let $\lambda_1, \lambda_2, \lambda_3$ be distinct eigenvalues of the same matrix A with corresponding eigen vectors v_1, v_2, v_3. Prove or disprove that (i) the determinant of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{pmatrix}$ is zero and (ii) $\{v_1, v_2, v_3\}$ is a linearly independent set. (4)

10. Show that if $\mathcal{V}$ is a real inner product space and $x, y \in \mathcal{V}$, such that $\|x\| = \|y\|$, then $\langle x + y, x - y \rangle = 0$. Interpret this result in R^2. (3)

11. Use the Gram-Schmidt orthonormalisation process to find an orthonormal basis for the subspace of R^4 generated by (6)

$$\{(1, 1, 0, 1), (1, -2, 0, 0), (1, 0, -1, 2)\}.$$