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MA1000 Calculus Problem Set 2

- 1. For what value of r, if any, does the series $1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + \dots$ converge? Find the sum of the series when it converges.
- 2. Test the convergence of the following series:

(a)
$$\sum_{n=1}^{\infty} \frac{(3/2)^n}{n^5}$$
.

(b)
$$\sum_{n=1}^{\infty} \frac{n5^n}{(2n+3)\ln(n+1)}.$$

(c) (i)
$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p};$$
 (ii)
$$\sum_{n=3}^{\infty} \frac{1}{n \log n (\log \log n)^p}$$
 (p any number).

(d)
$$\sum \frac{1}{n^p} \cos\left(\frac{1}{n}\right)$$
 (p any number).

(e)
$$\frac{1 \cdot 2}{3^2 \cdot 4^2} + \frac{3 \cdot 4}{5^2 \cdot 6^2} + \frac{5 \cdot 6}{7^2 \cdot 8^2} + \dots$$

(f)
$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots$$

(g)
$$\sum \frac{1}{n^2 + n}$$
.

(h)
$$\sum \frac{1}{(1+\sqrt{n})^{3/2}}$$

(i)
$$\sum_{n=2}^{\infty} \frac{1}{(\log n)^p}$$
 (p any number).

(j) (i)
$$\sum \frac{(-1)^{n+1}}{n^2}$$
; (ii) $\sum \frac{(-1)^{n+1}}{n}$; (iii) $\sum \frac{(-1)^{n+1}}{\sqrt{n}}$.

(k)
$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1} x^n$$
 (x any number).

3. If $\sum a_n$ is a convergent series of positive terms, examine the convergence of the series $\sum \sqrt{a_n a_{n+1}}$.