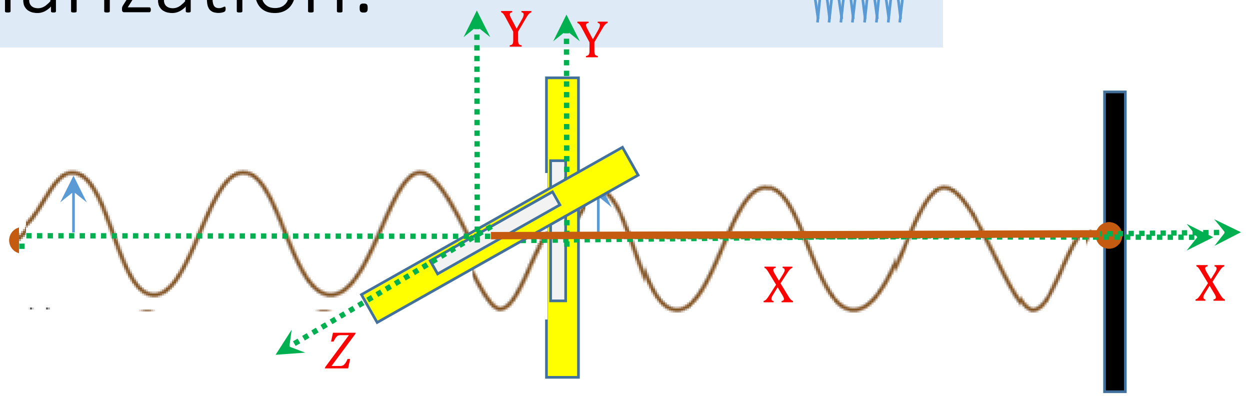
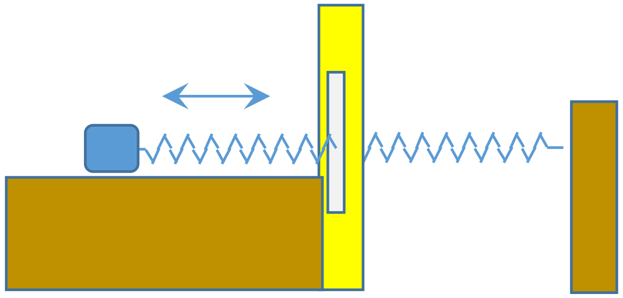
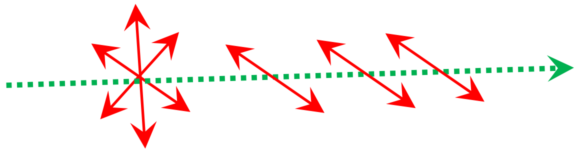
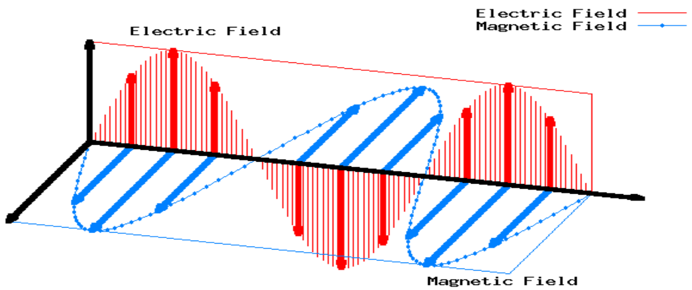
Polarization:







Transverse wave: direction of the wave and oscillation are mutually perpendicular.



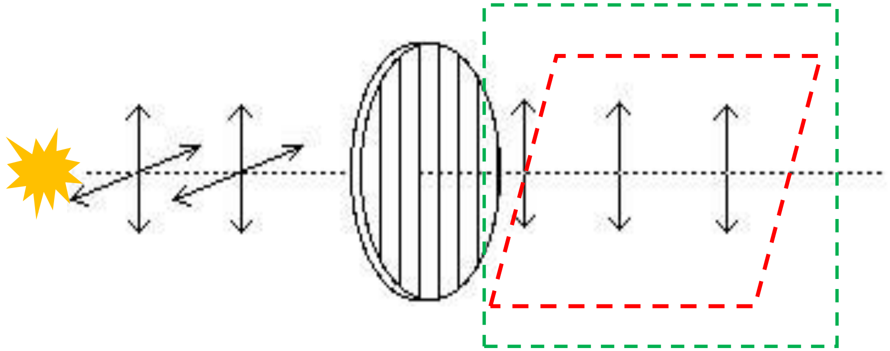
Oscillations may occur in any direction on the plane perpendicular to the direction of the wave ($\vec{k}$).– **Unpolarized wave.**

For EM wave, oscillation of $\vec{E}$ is considered to specify polarization.

If the direction of oscillation is specified – **Polarized wave**.

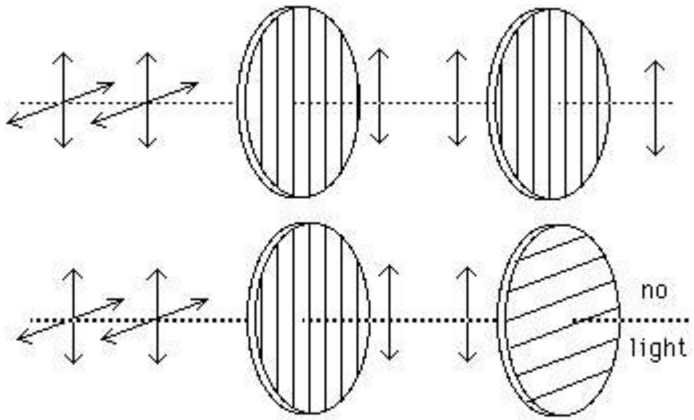
$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega} = \sqrt{\varepsilon\mu}\,\hat{n} \times \vec{E}$$

Polarization:



– **Plane of polarization --** No vibration occurs

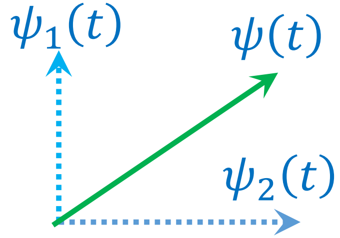
– **Plane polarized –** oscillation occurs along a straight line in a plane perpendicular to the direction of the propagation of the wave,



$$\psi(t) = \hat{e}A\cos\omega t,$$

which can be written as, $\psi(t) = \hat{\imath}A_1\cos\omega t + \hat{\jmath}A_2\cos\omega t \;= \psi_1(t) + \psi_2(t)$

$$A^2 = A_1^2 + A_2^2, \quad \hat{e} = \hat{\imath}A_1/A + \hat{\jmath}A_2/A$$

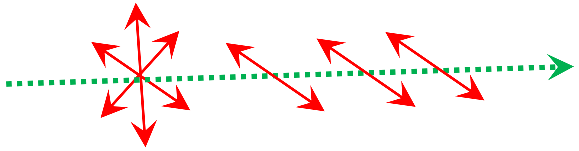


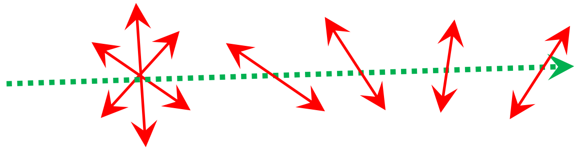
Plane polarized standing wave, $\psi(t) = [\hat{\imath}A_1 + \hat{\jmath}A_2]\sin(kz)\cos(\omega t)$

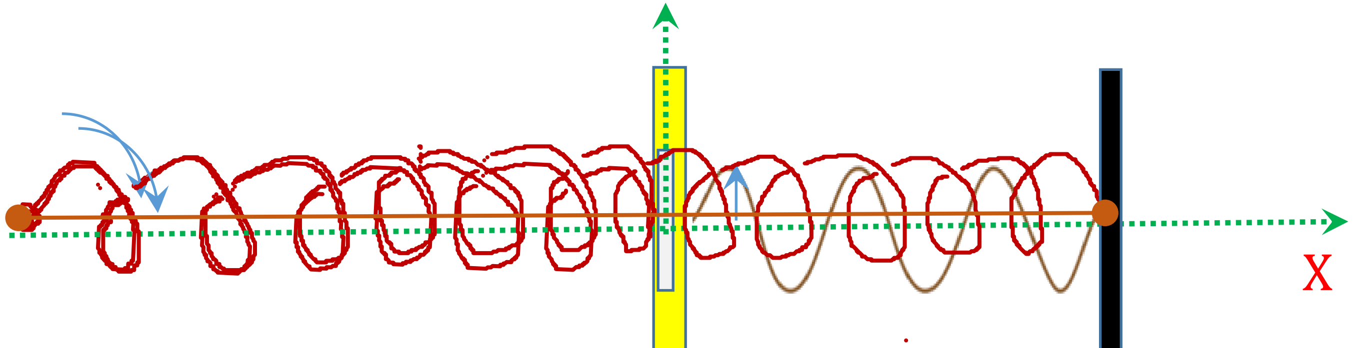
Plane polarized travelling wave, $\psi(t) = [\hat{\imath}A_1 + \hat{\jmath}A_2]\cos(kz - \omega t)$

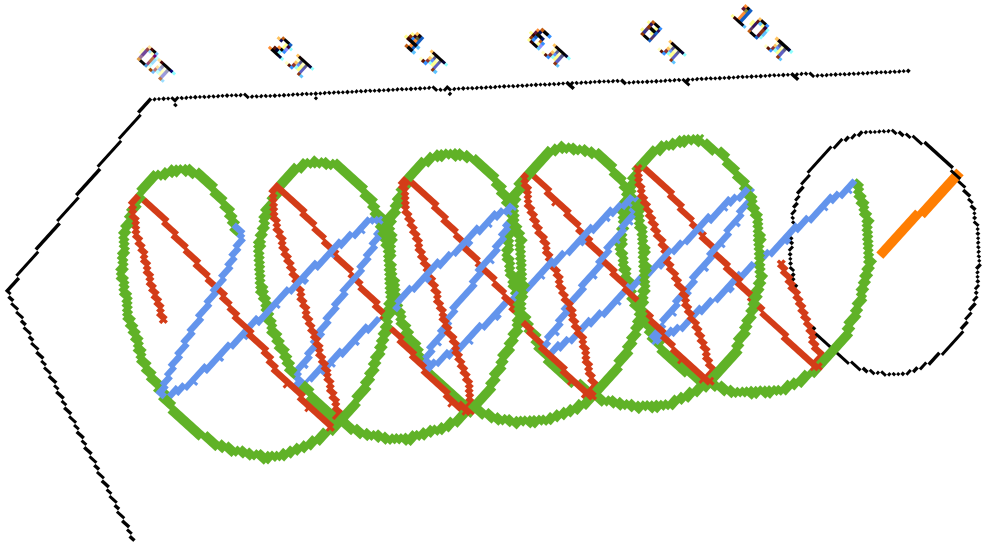
Polarization:

Circular Polarization: displacement is a motion on a circle

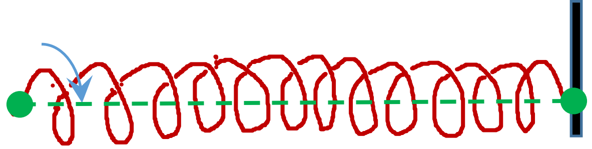








Polarization:



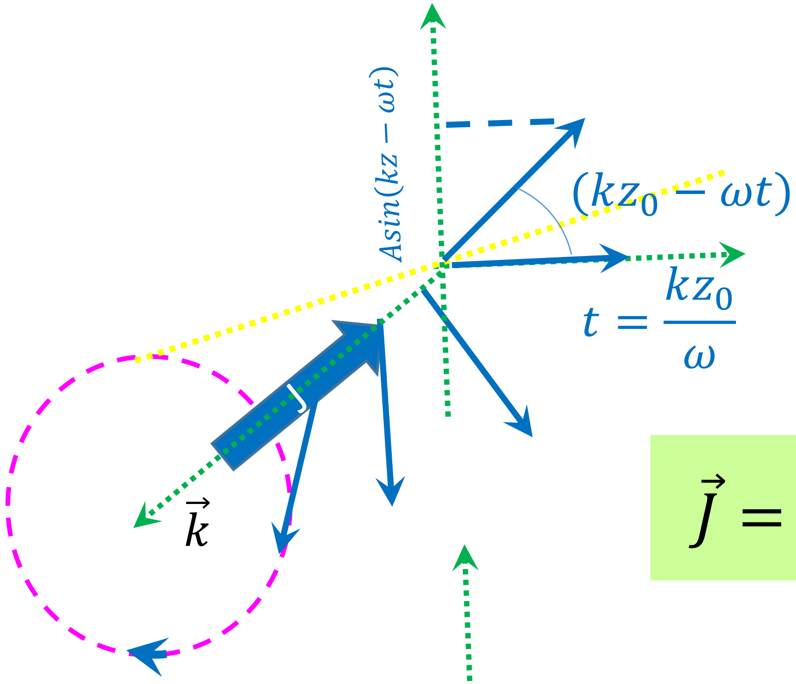
$$\psi(t) = \psi_1(t) + \psi_2(t)$$

If the components have equal amplitude and a relative phase difference $-\frac{\pi}{2} + 2m\pi, m = 0 \pm 1, \pm 2, \ldots\ldots,$

$$\psi(t) = \hat{\imath}A\cos\omega t + \hat{\jmath}A\cos(\omega t - \frac{\pi}{2}) = \hat{\imath}A\cos\omega t + \hat{\jmath}A\sin\omega t$$

Travelling : $\psi(t) = \hat{\imath}A\cos(kz - \omega t) + \hat{\jmath}A\sin(kz - \omega t)$

$\Rightarrow$ RCP wave



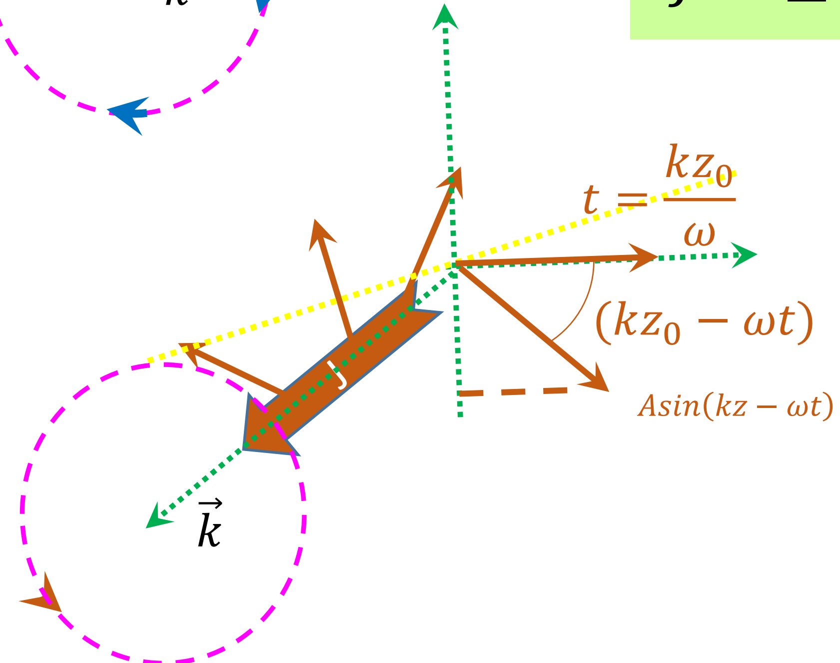
$$\vec{J} = \pm \frac{W}{\omega}\hat{z}$$

If the relative phase difference $\pi/2 + 2m\pi, m = 0 \pm 1, \pm 2, \ldots\ldots,,$

$$\psi(t) = \hat{\imath}A\cos\omega t + \hat{\jmath}A\cos(\omega t + \frac{\pi}{2}) = \hat{\imath}A\cos\omega t - \hat{\jmath}A\sin\omega t$$

Travelling: $\psi(t) = \hat{\imath}A\cos(kz - \omega t) - \hat{\jmath}\sin(kz - \omega t)$

$\Rightarrow$ LCP wave

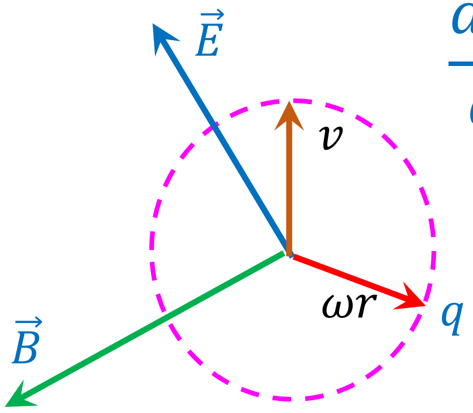


Polarization:

Circular Polarization: displacement is a motion on a circle

$$\frac{dW}{dt} = \omega\tau = \omega\frac{dJ}{dt}$$

$$\boxed{\vec{J} = \pm\frac{W}{\omega}\hat{z}}$$



$$\vec{P} = \frac{h\nu}{c} = \frac{W}{c}\hat{k}$$

$$W = (c^2P^2 + m^2c^4)^{1/2}$$

$$\vec{F} = q\vec{E} + \frac{q\vec{v}}{c}\times\vec{B}$$

considering $\vec{E} = \mathrm{iE_x}$ $\quad \vec{B} = jB_y \quad \vec{v} = iv_x + jv_y + kv_z$

$$\frac{dW}{dt} = \vec{v}.\vec{F} = q\dot{x}E_x$$

$$< \frac{dW}{dt} > = < q\dot{x}E_x >$$

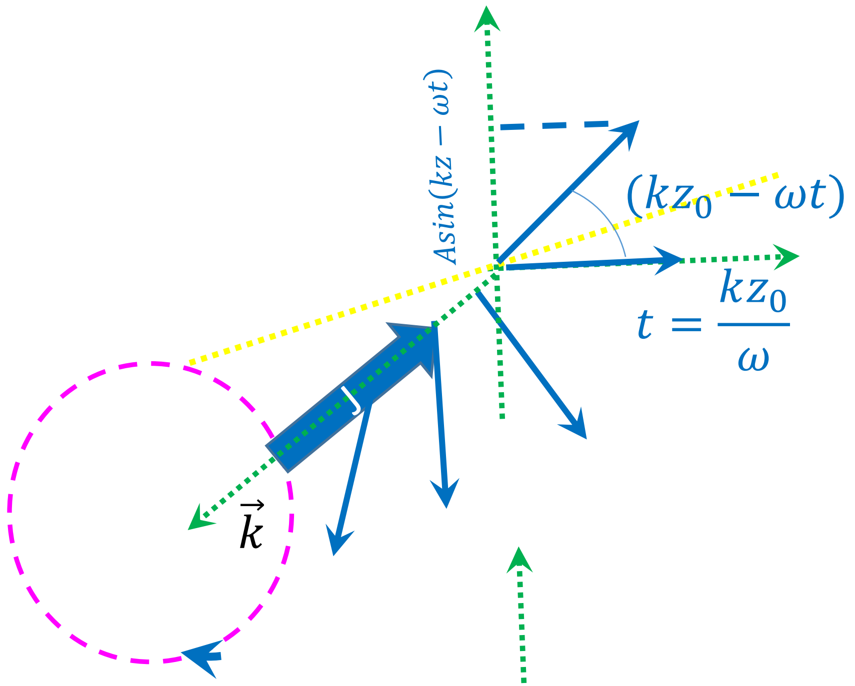
$$\omega\tau = \omega\vec{r}\times\vec{F} = q\omega\vec{r}\times\vec{E} + \omega\vec{r}\times\left(\underbrace{\frac{q\vec{v}}{c}}_{}\times\vec{B}\right) = \vec{v}.\vec{E}$$

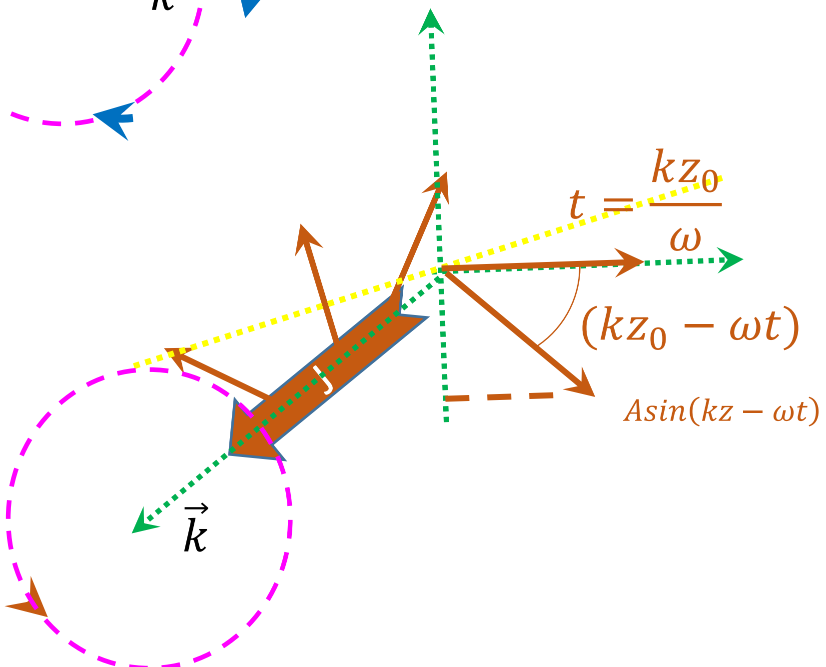
$\boxed{= -\vec{v}}$

$$<v_x> = <v_y \geq = <v_z> = 0$$

$$<\tau> = < \frac{dJ}{dt} > = \frac{k}{\omega} < q\vec{v}.\vec{E} > = \frac{k}{\omega} < \frac{dW}{dt} >$$

$$\boxed{\vec{J} = \pm\frac{W}{\omega}\hat{z}}$$





Polarization: **Elliptical Polarization:**

Let us consider superposition of two linearly polarized waves having different amplitude and a relative phase difference θ,

$$\psi_1(z,t) = a_1\cos(kz - \omega t) \qquad \psi_2(z,t) = a_2\cos(kz - \omega t + \theta)$$

If the polarizations of the waves are in the **same** direction, the resultant wave is written as,

$$\psi(z,t) = \hat{i}\,\psi_1(z,t) + \hat{i}\,\psi_2(z,t)$$

Plane polarized

$$\boxed{\boldsymbol{\psi}(\boldsymbol{z},\boldsymbol{t}) = \hat{i}\,\boldsymbol{A}\boldsymbol{sin}(\boldsymbol{kz} - \boldsymbol{\omega t} + \boldsymbol{\alpha})}$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(0 - \theta)$$

$$tan\alpha = \frac{a_1\sin 0 + a_2\sin\theta}{a_1\cos 0 + a_2\cos\theta}$$

If the polarizations of the waves are perpendicular, the resultant wave is given as,

$$\boldsymbol{\psi}(\boldsymbol{z},\boldsymbol{t}) = \hat{i}\,a_1\cos(kz - \omega t) + \hat{j}a_2\cos(kz - \omega t + \theta)$$

$\boldsymbol{\psi}$ will rotate, and change its magnitude, as well. In such cases the endpoint of $\boldsymbol{\psi}$ will trace out an ellipse, in a fixed-plane perpendicular to $\vec{k}$, as the wave sweeps by. We can see this better by actually writing an expression for the curve traversed by the tip of $\boldsymbol{\psi}$.

Polarization: **Elliptical Polarization:**

$$\psi_1(z,t) = a_1\cos(kz-\omega t)$$

$$\frac{\psi_1}{a_1} = \cos(kz-\omega t),$$

$$\psi_2(z,t) = a_2\cos(kz-\omega t+\theta)$$

$$\sin(kz-\omega t) = \sqrt{1-\left(\frac{\psi_1}{a_1}\right)^2}$$

$$\frac{\psi_2}{a_2} = \cos(kz-\omega t)\,cos\theta - \sin(kz-\omega t)\,sin\theta$$

$$\frac{\psi_2}{a_2} = \frac{\psi_1}{a_1}cos\theta - \sqrt{1-\left(\frac{\psi_1}{a_1}\right)^2}\;sin\theta$$

$$\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2\cos^2\theta - 2\left(\frac{\psi_1}{a_1}\right)\left(\frac{\psi_2}{a_2}\right)cos\theta = \sin^2\theta - \left(\frac{\psi_1}{a_1}\right)^2\sin^2\theta$$

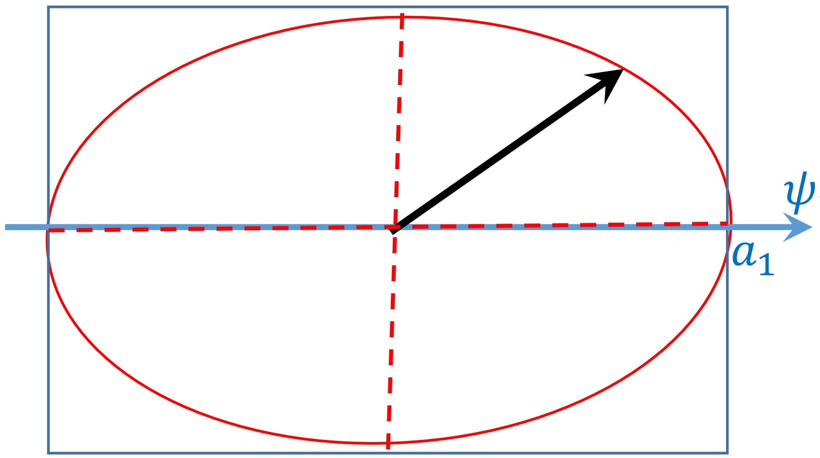
For $\theta = \pm\frac{(2n+1)\pi}{2},\ \alpha = 0$

$$\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2 = 1$$

An ellipse making an angle α with $\hat{i}$

$$\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2 - 2\left(\frac{\psi_1}{a_1}\right)\left(\frac{\psi_2}{a_2}\right)cos\theta = \sin^2\theta$$

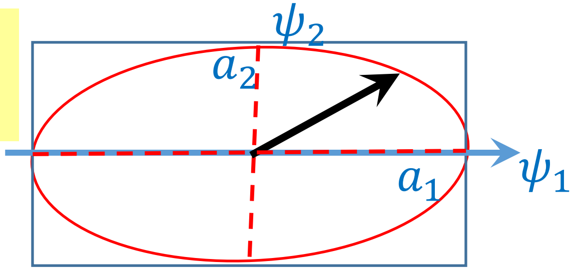
$$tan2\alpha = \frac{2a_1a_2}{a_1^2-a_2^2}cos\theta$$

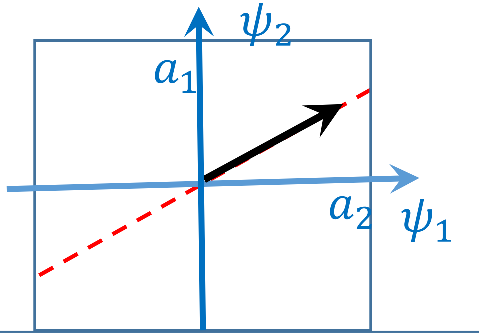


Polarization: **Elliptical Polarization:**

$$\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2 - 2\left(\frac{\psi_1}{a_1}\right)\left(\frac{\psi_2}{a_2}\right) cos\theta = \sin^2\theta$$

$$tan2\alpha = \frac{2a_1 a_2}{a_1^2 - a_2^2} cos\theta$$





For $\theta = \pm\frac{(2n+1)\pi}{2}, \ \alpha = 0$

$$\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2 = 1$$

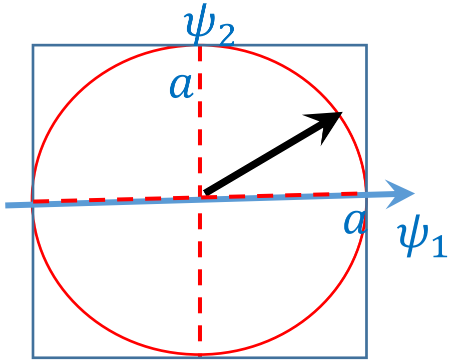
$$\boldsymbol{\psi(z,t)} = \hat{i}\, a_1\cos(kz-\omega t) + \hat{j}\, a_2\cos(kz-\omega t)$$

For $\theta = \pm n\pi, \ n = 2,4,6\ldots$

$$\psi_1 = \left(\frac{a_1}{a_2}\right)\psi_2$$

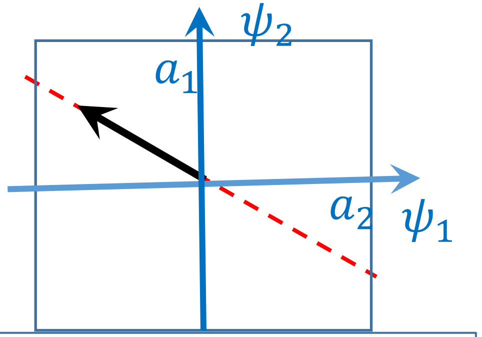
For $\theta = \pm\frac{(2n+1)\pi}{2}, \ \alpha = 0, \ a_1 = a_2 = a$

$$\psi_2^2 + \psi_1^2 = a^2$$



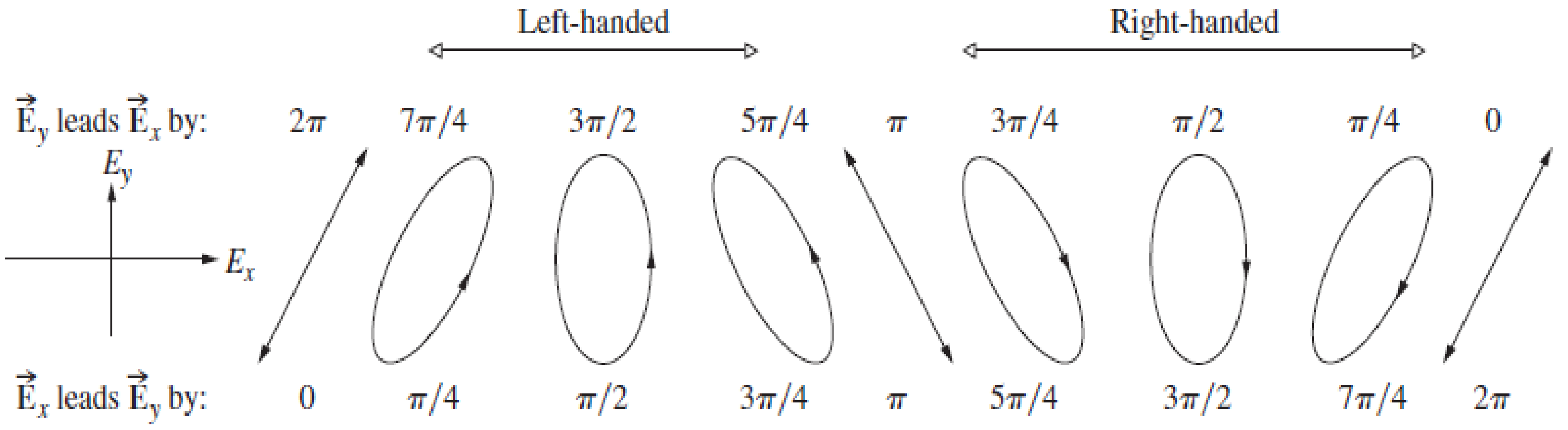
$$\psi_{RCP}(z,t) = a\,[\,\hat{i}\cos(kz-\omega t) + \hat{j}\cos(kz-\omega t)]$$

$$\psi_{LCP}(z,t) = a\,[\,\hat{i}\cos(kz-\omega t) - \hat{j}\cos(kz-\omega t)]$$



For $\theta = \pm n\pi, \ n = 1,3,\ldots$

$$\psi_1 = -\left(\frac{a_1}{a_2}\right)\psi_2$$

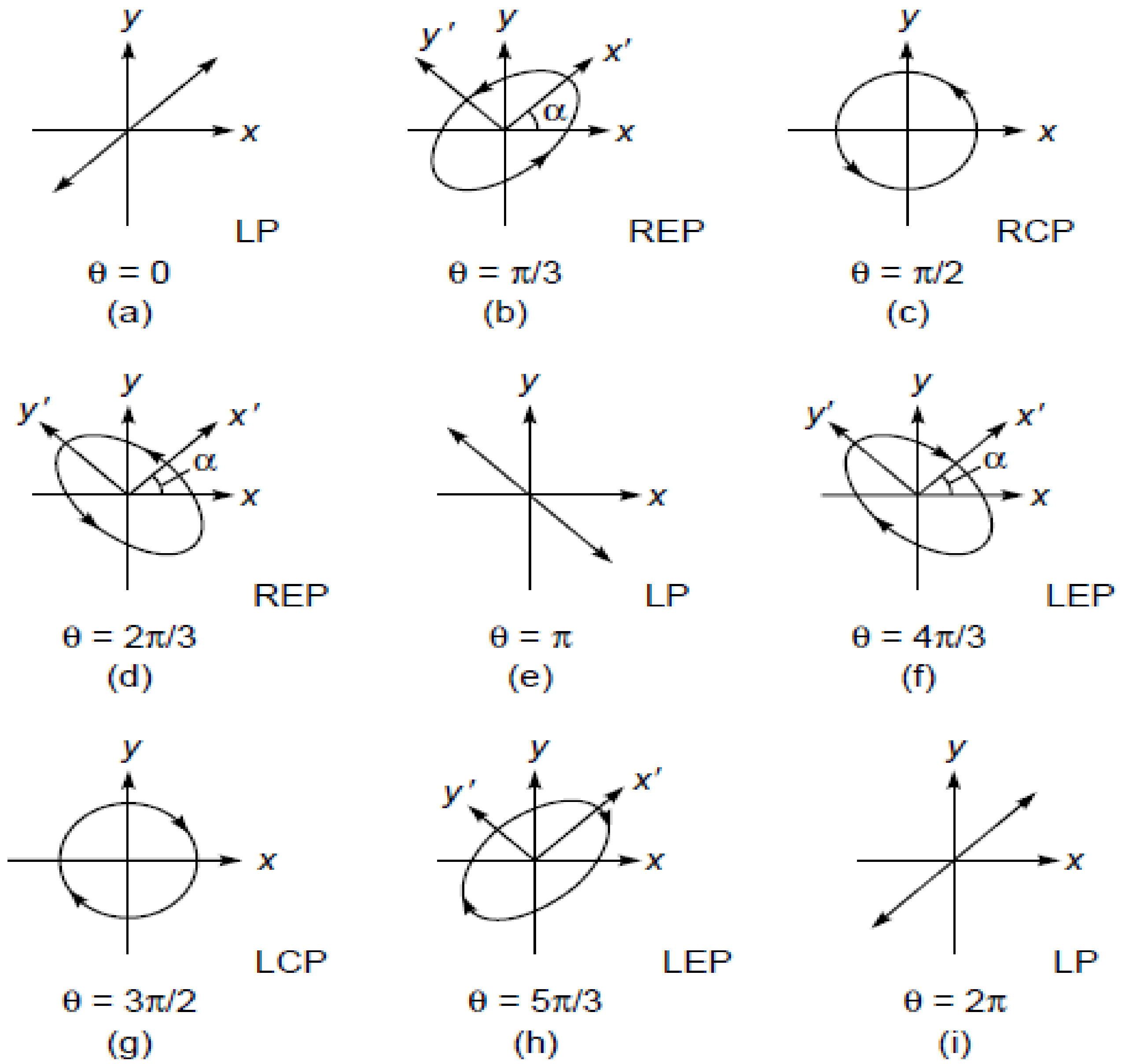


Polarization: $\left(\frac{\psi_2}{a_2}\right)^2 + \left(\frac{\psi_1}{a_1}\right)^2 - 2\left(\frac{\psi_1}{a_1}\right)\left(\frac{\psi_2}{a_2}\right)cos\theta = \sin^2\theta$

$$tan2\alpha = \frac{2a_1a_2}{a_1^2 - a_2^2}cos\theta$$

$$\boxed{\alpha = \pi/4}$$

$$\boxed{\psi_2^2 + \psi_1^2 - 2\psi_2\psi_1 cos\theta = a^2\sin^2\theta}$$



$$0 \Rightarrow (\psi_1 - \psi_2)\text{^}2 = 0 \Rightarrow \psi_2 = \psi_1$$

$$\pi/3 \Rightarrow \psi_2^2 + \psi_1^2 - 2\psi_2\psi_1(\frac{1}{2}) = \frac{3}{4}a^2$$

$$\pi/2 \Rightarrow \psi_2^2 + \psi_1^2 = a^2$$

$$2\pi/3 \Rightarrow \psi_2^2 + \psi_1^2 - 2\psi_2\psi_1(-\frac{1}{2}) = \frac{3}{4}a^2$$

$$\pi \Rightarrow (\psi_1 + \psi_2)\text{^}2 = 0 \Rightarrow \psi_2 = -\psi_1$$

$$4\pi/3 \Rightarrow \psi_2^2 + \psi_1^2 - 2\psi_2\psi_1(-\frac{1}{2}) = (-1)^2\frac{3}{4}a^2$$

$$3\pi/2 \Rightarrow \psi_2^2 + \psi_1^2 = (-1)^2a^2$$

$$5\pi/3 \Rightarrow \psi_2^2 + \psi_1^2 - 2\psi_2\psi_1(\frac{1}{2}) = (-1)^2\frac{3}{4}a^2$$

$$2\pi \Rightarrow (\psi_1 - \psi_2)\text{^}2 = 0 \Rightarrow \psi_2 = \psi_1$$