11.25/15

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CS22B2030

Design and Analysis of Algorithms - Quiz 2 - 04-Oct-2023 - 13.30-14.30

0. (0 marks) Name the Scientist whose name appears as a substring in our Institute name.

1. (1 mark) Name the two properties that any optimization problem must satisfy to become a candidate problem for the dynamic programming paradigm.

→ It should follow subproblems that mean subproblems should be saticfied

→ overlapping should not occur

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2. Consider the coin change problem; Input: Interger n, Denominations: 1, 3, 5, 7, the objective is to find the minimum number of coins required to give change for n using the given denominations. Answer the following;

- (i) (2 marks) Write the recursive subproblem for this problem along with base cases. Let $C[n]$ denote the minimum number of coins required to give change for n.

Recursive subproblem:

$c[n] = \min \{ 1+c[n-1], 1+c[n-3], 1+c[n-5], 1+c[n-7] \}$

1.75

Base cases:

$c[1] = 1 \quad c[5] = 1$

$c[3] = 1 \quad c[7] = 1$

c(2) = ?

General case: $c[d_1] = c[d_2] = \cdots = c[d_k] = 1$

$c[n] = \min \{ 1+c[n-d_1], 1+c[n-d_2], \ldots\ldots, 1+c[n-d_k] \}$

- (ii) (2 marks) Suppose, we wish to solve using brute force approach, what would be the lower bound and upper bound on the running time of your approach.

Given coins : 1, 3, 4, 7 (Denominations)

sorted order : $\begin{matrix} d_1 & d_2 & d_3 & d_4 \\ 1 & 3 & 4 & 7 \end{matrix}$

total coins : 4

$T(n) = T(n-d_1)+1$

$\leq T(n-1) \quad O(4^n)$ → upper bound

$T(n) \geq T(n-d_k)+1$

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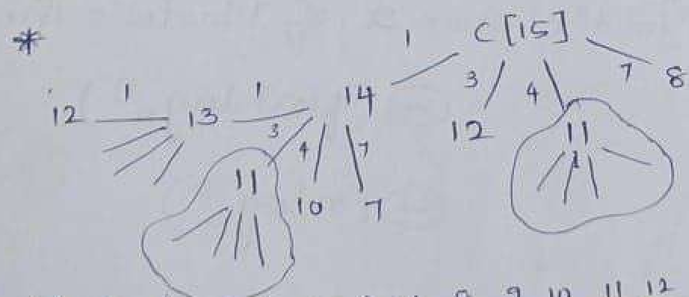
$\geq T(n-\cancel{7}) \quad \Omega(4^{n/\cancel{7}})$ → lower bound

- (iii) (1 mark) Justify the overlapping subproblem property for the case $C[15]$



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n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	1	2	1	2	1	2	1	2							

$c[15] = \min \{ 1+c[14], 1+c[12], 1+c[11], 1+c[8] \}$

$c[14] = \min \{ 1+c[13], 1+c[11], 1+c[10], 1+c[7] \}$

for finding c[14] we need c[11], c[13], c[10], c[7] and for c[15] we need c[14], c[12], c[11], c[8] we need subproblems to be solved for finding further by this we can justify overlapping subproblem property for C[15]

3. (1.5 marks) Consider the container loading problem with two containers. Suppose, the weights of the consignments are **distinct**, and the weights of containers are $W_1 = W_2 = 5$. Will greedy strategy with respect to 'min weight' yield optimum (maximum) number of consignments that can be placed onto the containers. Justify with a proof of correctness or a counter example.

W_1 (5) W_2 (5)

Let the distinct weights be : 1, 2, 3, 4 If we go with min weight greedy

1.5

W_1
1 → ✓
1+2 → ✓
1+2+3 → ✗
1+2 = (3)

W_2
3 → ✓ (3)
3+4 → ✗

greedy won't work here because (1+2)3, 3 are not best cases (1+4)5, (2,3) 5 are best cases

4. (3 marks) Solve using Master Theorem with proper justification.

- $T(n) = 15T(\frac{n}{3}) + n^2 \log n$

$f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 15}$

$= n^{(\log_3 5 + 1)}$

$\frac{n^2 \log n}{n^{(\log_3 5+1)}}$ ✗

$\frac{n^{(\log_3 5+1)}}{n^2 \log n}$ ✗

as $f(n) = n^2 \log n$ this won't goes to any of the 3 cases of the master's theorem as it won't satisfy e property. we can't solve this problem with Master's theorem Master's theorem is not applicable

1.5 0

- $T(n) = 5T(\frac{n}{5}) + 2^n$

$f(n) = 2^n$

$n^{\log_b a} = n^{\log_5 5}$

$= n$

$\frac{n}{2^n}$ $\frac{2^n}{n}$

$f(n) > n^{\log_b a}$

$f(n) = \Omega(2^n)$ ✗

it goes to the 3rd case of master's theorem

$= \Theta(f(n))$

$= \Theta(2^n)$

0.5

- $T(n) = 5T(\frac{n}{5}) + n$

$f(n) = n$

$n^{\log_b a} = n^{\log_5 5}$

$= n$

$f(n) = n$

$n^{\log_b a} = n$

$f(n) = n^{\log_b a}$

it goes to the case 2 of Master's theorem

$= \Theta(f(n) \log_b n)$

$= \Theta(n \log_5 n)$

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5. (2 marks) Is the following claim true for the recurrence $T(n) = aT(\frac{n}{b}) + f(n), a \geq 1, b > 1$; **CLAIM** If $af(\frac{n}{b}) \leq cf(n)), c < 1$ then $f(n) = \Omega(n^{\log_b a+\epsilon})$, for some $\epsilon > 0$. If not true, present a counter example.

Master's theorem:

Case 3: $T(n) = aT\left(\frac{n}{b}\right) + f(n) \quad a \geq 1,\ b > 1$

If Recursive property: $af\left(\frac{n}{b}\right) \leq cf(n)$, for some $c<1$

and $f(n) > n^{\log_b a}$ then only $f(n) = \Omega(n^{\log_b a+\epsilon})$ for some $\epsilon > 0$

counter example:

$T(n) = 3T\left(\frac{n}{2}\right) + \frac{1}{n}$ or

$T(n) = 3T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

6. (2.5 marks) Given an integer m, the objective is to find x and y such that $m = 4x + 7y$.

- (i) (1 mark) What is the value of m_0 such that for all $m \geq m_0$, $m = 4x + 7y$. Justify.

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$\boxed{m_0 = 24}$ $\Rightarrow c[x] = \min\{1 + c[x-4],\ 1 + c[x-7]\}$

$m_0 = 24$

$c[4] = 1 \quad c[7] = 1$

$24 = 4(6) + 7(0)$ $\quad 28 = 4(7) + 7(0)$

$25 = 4(1) + 7(3)$ $\quad 29 = 4(2) + 7(3)$ $\quad 32 = 4(8) + 7(0)$ $\quad 35 = 4(7) + 7(1)$

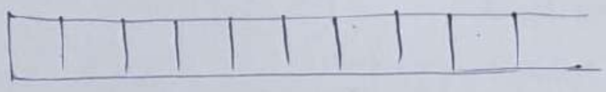
$26 = 4(3) + 7(2)$ $\quad 30 = 4(4) + 7(2)$ $\quad 33 = 4(3) + 7(3)$

$27 = 4(0) + 7(3)$ $\quad 31 = 4(6) + 7(3)$ $\quad 34 = 4(5) + 7(2)$

- (ii) (1.5 marks) What is your strategy to identify x and y so that $x + y$ is minimum. Justify.

1.5

Go with Array filling and D.P for finding minimum



after finding minimum If the number is direct multiple of x or y we get x and y directly If not

we use

$c[x] = \min\{1 + c[x-4],\ 1 + c[x-7]\}$

$c[4] = 1 \quad c[7] = 1$ $\quad$ we get x and y

(x, y)