KEY

Design and Analysis of Algorithms - End Semester - 20-Nov-2023 - 9.30-12.30

0. (0 marks) Name the algebraic structure that appears as a substring in our Institute name.

1 Light Dose

1. **(1.5 marks)** Given a sorted array of size $2^n, n > 0$; the asymptotic tight time complexity to search an element by the best known algorithm is....... Mention the algorithm and the asymptotic tight time complexity.

0.75 Binary Search

0.75 $\Theta(\log 2^n) = \Theta(n)$ in the Worst Case

2. **(1.5 marks)** With proper justification, explain what is the minimum number stacks required to simulate a queue. Similarly the minimum number of queues required to simulate a stack.

light dose - 13
Medium dose - 15
Strong dose - 12
40

3. **(1.5 marks)** Mention four applications of Breadth First Search

Any 4
1) Connectedness Testing
2) Cycle Testing
3) ODD Cycle / Even Cycle Testing
4) Treeness
5) Bipartiteness
6) 2-Colorability
7) No. of Connected Components

4. **(1.5 marks)** How do you test whether a given graph is bipartite.

0.5 [1) Run BFS

1 {
2) Chk for cross edges
3) If NO cross edges then no ODD cycles $\Rightarrow$ G is bipartite.
4) If $\exists$ cross edge declare G is NOT Bipartite

1

5. (1.5 marks) Define NP. Prove that MAX-CLIQUE problem is in NP.

NP: The Class of Non-deter Algorithms

0.5 A set of Computational problems whose Solution Can be obtained by designing Non-Determinstic Polynomial time Algorithms.

Max clique ∈ NP. ① Guess a subset in Non-Det fashion (Build a Guessing Tree)

② Verify the subset

1

6. (1.5 marks) For a problem P, there are five algorithms with run-times $n^n, 2^{n \log_2 n}, 4^{\log_2 n}, n^{O(1)}, \log_2 n$; arrange in increasing order of their time complexities.

1 $\log_2 n < n^{O(1)} < 2^{n \log n} = n^n$

0.5 $4^{\log n}$ Cannot be Compared with $n^{O(1)}$

$\log_2 n < 4^{\log_2 n} < 2^{n \log n} = n^n$

7. (1 mark) Mention the recursive subproblem for the Matrix Chain Multiplication problem

$$M[i,j] = \min_{i \le k < j} \{ M[i,k] + M[k+1,j] + P_{i-1} P_k P_j \}$$

8. (1.5 marks) Present two functions $f(n)$ and $g(n)$ such that $f(n) \neq O(g(n))$ and $f(n) \neq \Omega(g(n))$. Justify.

0.5 $f(n) = n$, $g(n) = n^{1+\sin n}$

0.5 if $\sin n = -1$ then $f(n) = n$, $g(n) = n^0 = 1$ — $g(n) = O(f(n))$

0.5 if $\sin n = 1$ then $f(n) = n$, $g(n) = n^2$ — $f(n) = O(g(n))$

9. (1.5 marks) What are the invariants maintained by Prim's and Kruskal's algorithms

0.75 Prim's: Connectedness

0.75 Kruskal's: Acyclicity

2 Medium Dose

1. **(2 marks) For the recurrence relation, $T(n) = aT(\frac{n}{b}) + f(n), a \geq 1, b > 1$, mention two good lower bounds with a proper justification.**

1 ← 1) # leaves is a good lower bound

$$\Omega(n^{\log_b a})$$

1 ← 2) The root of Computation tree $f(n)$ is also a LB

$$\Omega(f(n))$$

2. **(1+1.5=2.5 marks) Explain 3-way Merge sort along with time complexity analysis (present the asymptotic tight bounds)**

TO Merge two Sorted Arrays of size $n/3$ each

We need $n/3 + n/3 - 1 = \frac{2n}{3} - 1$ in W.C

1 Mark

TO Merge one Sorted Array of size $n/3$ and the other of size $\frac{2n}{3}$

We need $\frac{2n}{3} + n/3 - 1 = \frac{3n}{3} - 1$

Total; $\frac{2n}{3} - 1 + \frac{3n}{3} - 1 = \frac{5n}{3} - 2$

Recurrence;

$$T(n) = 3T(n/3) + \frac{5n}{3} - 2$$ in worst case

Explanation

1. Recursively Divide into 3 subproblems of size $n/3$ each } 0.5
2. Recursively Combine Sorted Arrays

Soln $T(n) = 3T(n/3) + \frac{5n}{3} - 2$

Case 2 of MT; $\Theta(n\log_3 n)$

1 mark

3. **(1+1.5=2.5 marks) Solve (i) $T(n) = 4T(\frac{n}{2}) + n^2$ (ii) $T(n) = 4T(\frac{n}{2}) + 2^n$**

0.5

(i) Case 2 of MT

0.5 $\Theta(n^2 \log_2 n)$

$f(n) = \Theta(n^{\log_b a})$

(ii) Case 3 of MT

0.5 $2^n = \Omega(n^{\log_2 4 + \epsilon})$, for some $\epsilon > 0$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, $\epsilon > 0$

0.5 Regularity condn chk is TRUE

$a f(\frac{n}{b}) < c f(n)$ for some $c < 1$

$4 \cdot 2^{n/2} < c \cdot 2^n$ $c = 3/4$ $n_0 \geq 10$

0.5 Soln: $T(n) = \Theta(2^n)$

4. **(2 marks) Given two sorted arrays of size n and m respectively, what are the minimum and maximum number of comparisons required (in the worst case) to merge them into a single array of size n + m. Present an example with justification for each meeting the number mentioned.**

MIN: min(n, m)

0.5

Example: $\underbrace{1\ 2\ 3\ 4}_{A_1}$ $\underbrace{5\ 6\ 7\ 8}_{A_2}$

0.5

Justify: We comp (1, 5) Move ptr_1

CMP (2, 5) "

CMP (3, 5) "

CMP (4, 5) "

4 Comp

STOP.

Max m+n-1

0.5

Example: $\underbrace{1\ 3\ 5\ 7}_{A_1}$ $\underbrace{2\ 4\ 6\ 8}_{A_2}$

0.5

CMP (1,2)

CMP (3,2)

CMP (3,4)

CMP (5,4)

CMP (5,6)

CMP (7,6)

CMP (7,8)

7 Comp

STOP

5. **(2 marks) Assume MIN-Vertex Cover is NP-Hard. Prove that MAX-CLIQUE is NP-Hard.**

$<G, K>$ of VC maps to $<G^C, n-K>$ of CLIQUE

If G has a VC of size K then, In G, the other (n-k) vertices form I Set

In G^c, (n-k) vertices form CLIQUE

Conversely, In G^c, Clique of size (n-k)

In G, ISet (n-k)

In G, the other 'k' vertices form VC.

6. **(2 marks) Given an unweighted graph G, present an algorithm to find a short cycle (a cycle with minimum number of edges) in G.**

For each edge $e = \{u, v\}$, Remove the edge $\{u, v\}$ and Run Spath (u) [Dijkstra] to obtain Spath from u to v.

Spath + the edge $\{u, v\}$ => Scycle containing $\{u, v\}$

Repeat for each edge and choose MIN

7. **(2 marks) Given a set S and an integer t, the objective is to find a subset $S' \subseteq S$ whose sum is t. Present a DP to solve this problem.**