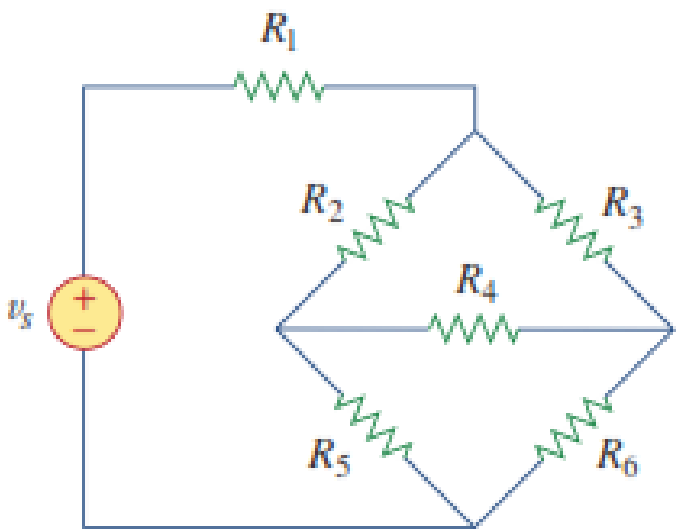
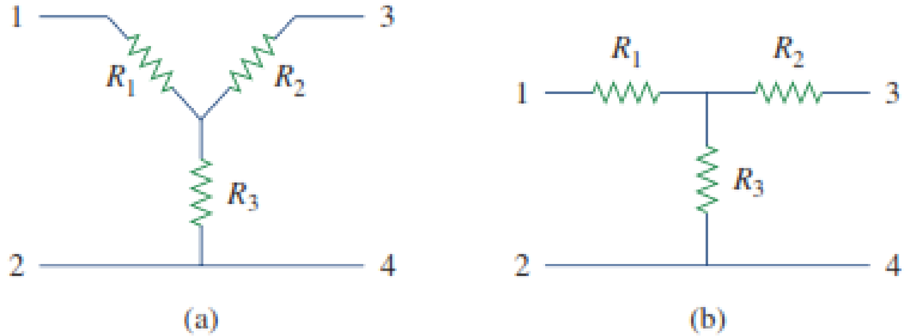
Electrical Circuits for Engineers (EC1000)

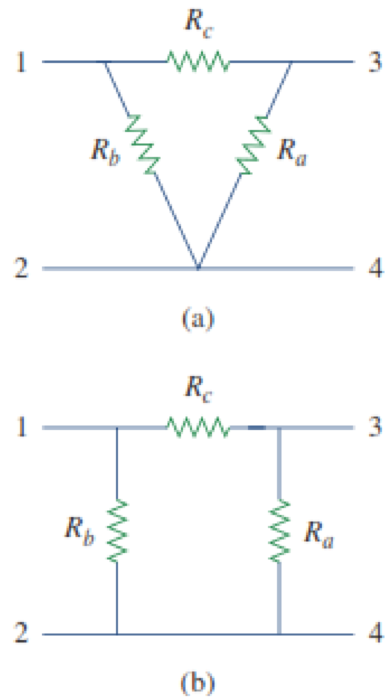
Lecture-2 (b)
Wye-Delta Transformations (Ch-2)

Wye-Delta Transformations (2.7)

- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series.
- In the given bridge network, How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel?
- It can be simplified by using three-terminal equivalent networks. (wye (Y) or tee (T) network)







How to apply wye-delta transformation in the analysis of that network?

(a) Δ (b) Π

Delta (Δ) to Y (Wye)Conversion

For terminals 1 and 2 in Figs. for example,

$$R_{12}(\mathrm{Y}) = R_1 + R_3 \tag{2.46}$$

$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.47a}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.47b}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.47c}$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.48}$$

Adding Eqs. (2.47b) and (2.48) gives

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}} \tag{2.49}$$

and subtracting Eq. (2.48) from Eq. (2.47b) yields

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}} \tag{2.50}$$

Subtracting Eq. (2.49) from Eq. (2.47a), we obtain

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}} \tag{2.51}$$

We do not need to memorize Eqs. (2.49) to (2.51). To transform a Δ network to Y, we create an extra node n as shown in Fig. 2.49 and follow this conversion rule:

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

One can follow this rule and obtain Eqs. (2.49) to (2.51) from Fig. 2.49.

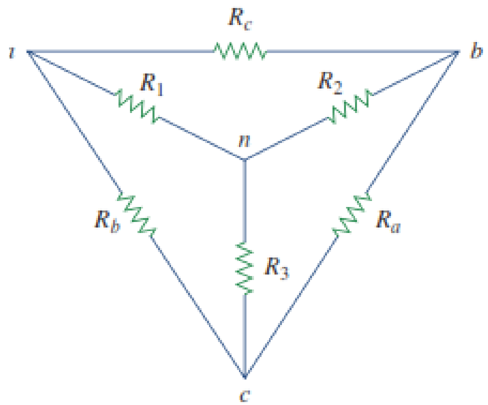


Figure 2.49 Superposition of Y and Δ networks as an aid in transforming one to the other.

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.49) to (2.51) that

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} = \frac{R_aR_bR_c}{R_a + R_b + R_c} \tag{2.52}$$

Dividing Eq. (2.52) by each of Eqs. (2.49) to (2.51) leads to the following equations:

$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} \tag{2.53}$$

$$\boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} \tag{2.54}$$

$$\boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}} \tag{2.55}$$

From Eqs. (2.53) to (2.55) and Fig. 2.49, the conversion rule for Y to Δ is as follows:

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta \tag{2.56}$$

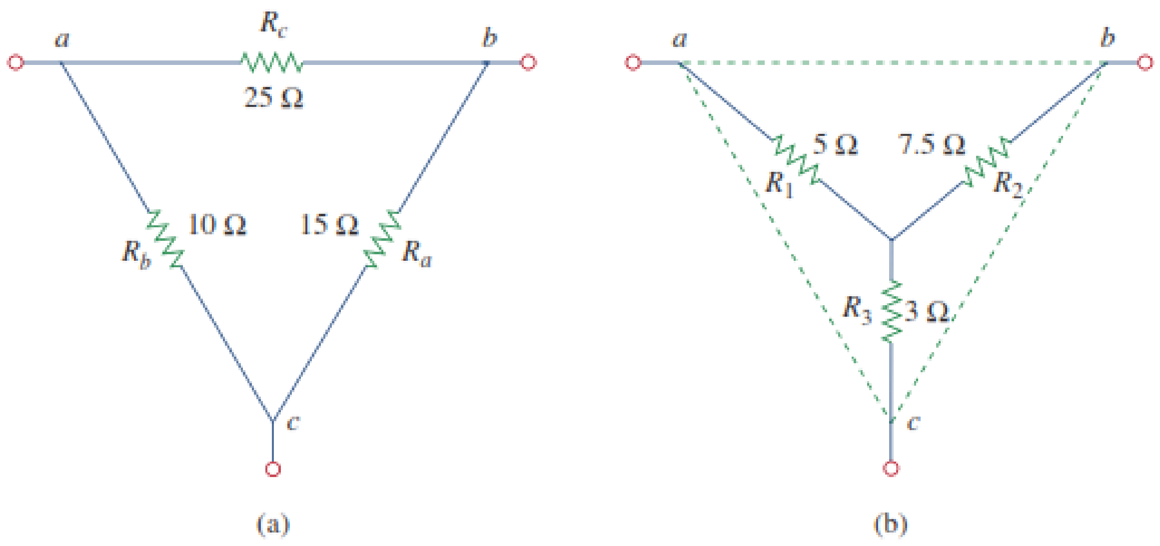
Under these conditions, conversion formulas become

$$\boxed{R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y} \tag{2.57}$$

One may wonder why R_Y is less than R_Δ. Well, we notice that the Y-connection is like a "series" connection while the Δ-connection is like a "parallel" connection.

Example

Convert the Δ network in Figure to an equivalent Y network.



Solution

Using Eqs. (2.49) to (2.51), we obtain

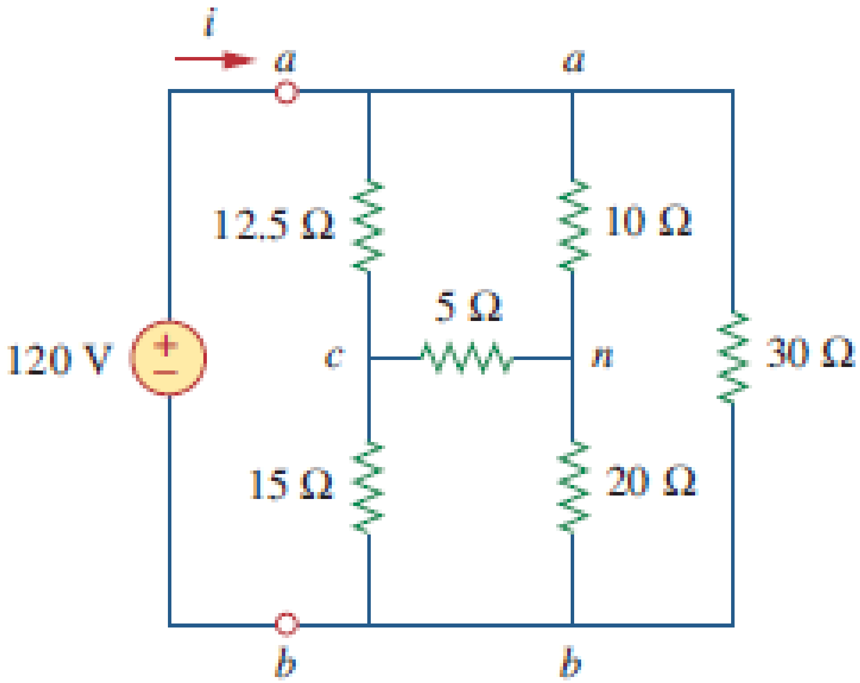
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

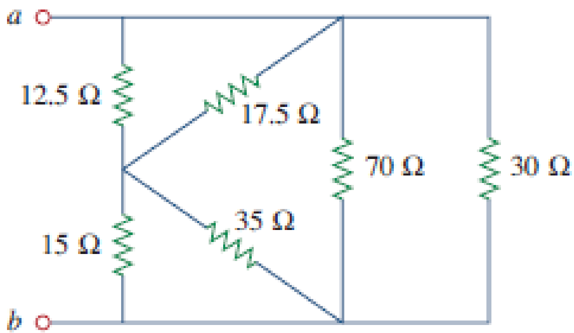
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

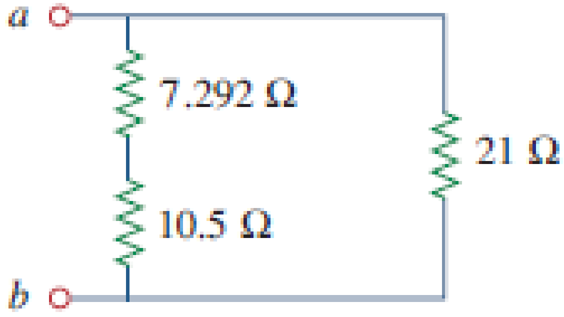
For the given network in Figure Find R_{ab} and *i.*



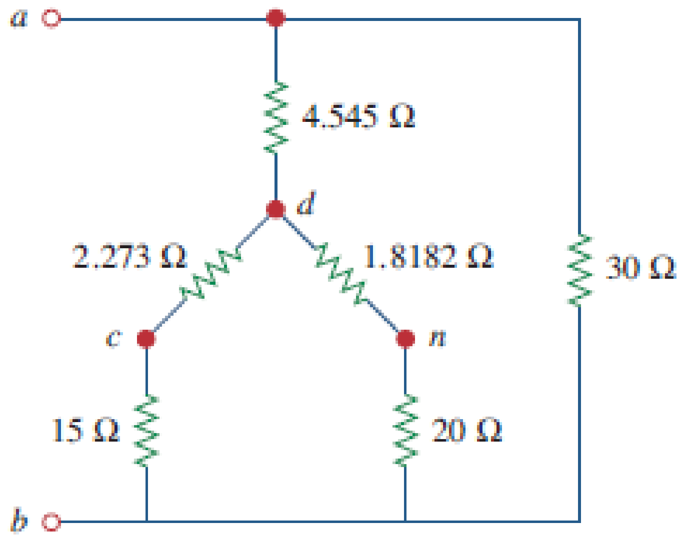
Hint



(a)



(b)

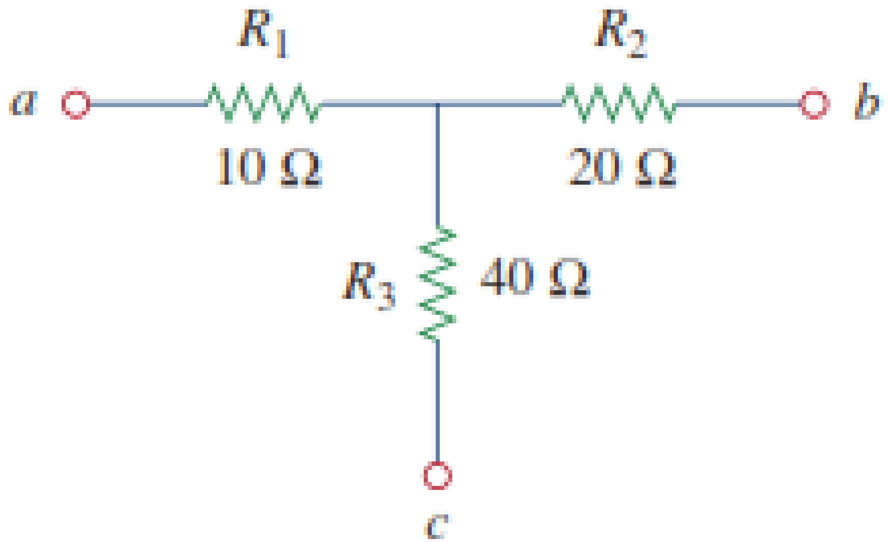


(c)

Answer: 9.632 Ohm, 12.458 A

Example

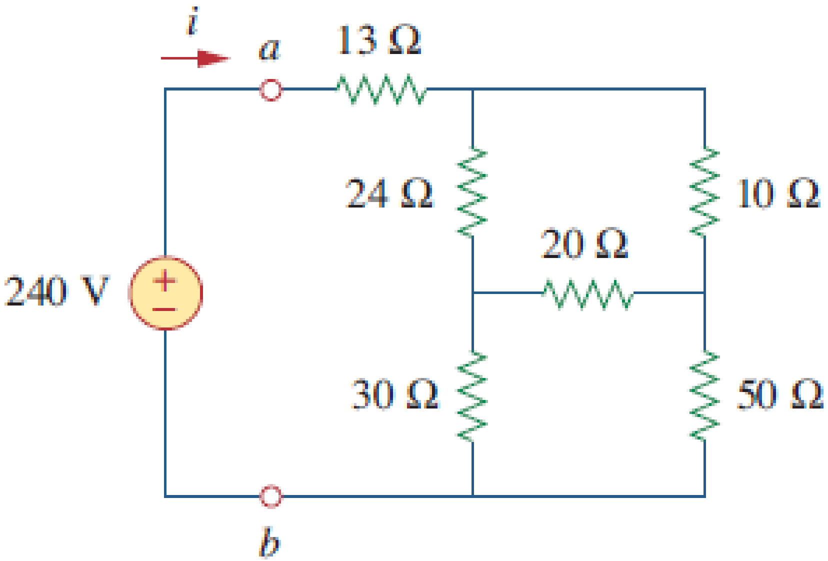
Transform the wye network in Fig. to a delta network



Answer: Ra = 140 Ohm, Rb=70 Ohm , Rc =35 Ohm.

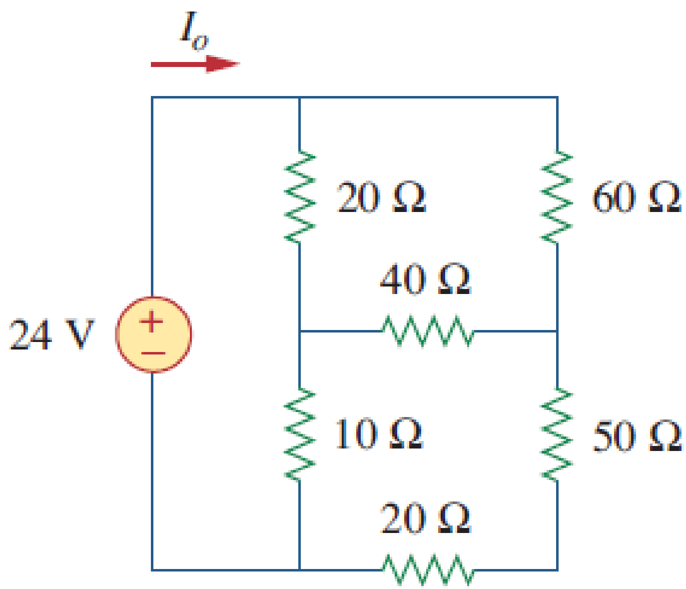
Practice Problem

1. For the given network in Figure Find R_{ab} and



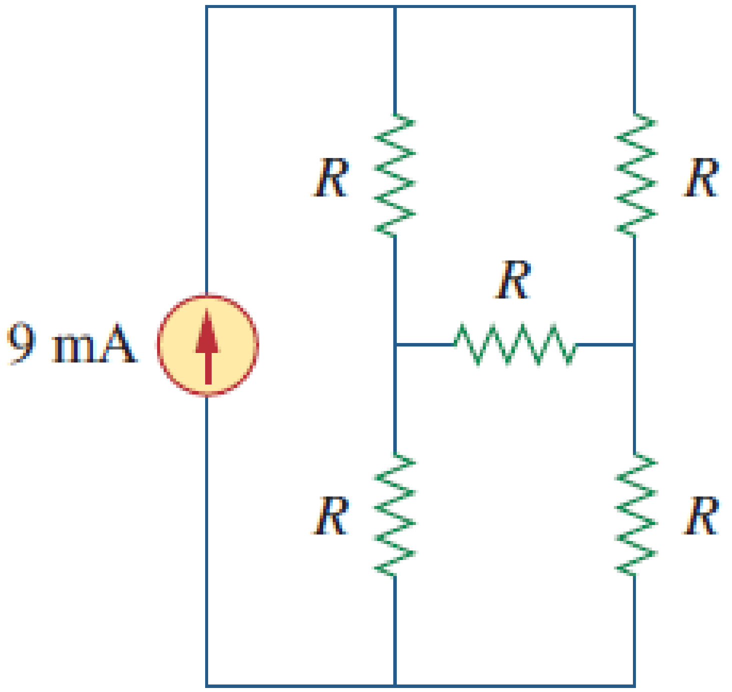
Answer: 40 Ohm, 6 A

2. For the given network in Figure Find i_0.



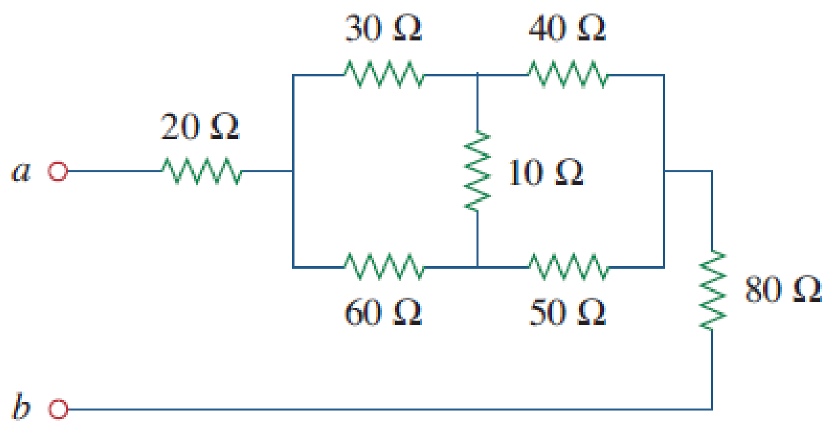
Answer: 24.0625 Ohm, 997.4 mA

3. What value of R in the circuit of Figure would cause the current source to deliver 800 mW to the resistors

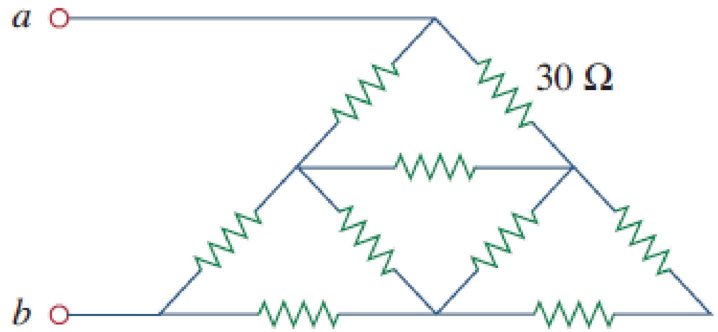


Answer: 9876 Ohm

4. For the given network in figure Find R_{ab}



(a)



(b)

Answer: (a) 142.32 Ohm, (b) 33.33 Ohm