

Engineering Optics

Lecture 34

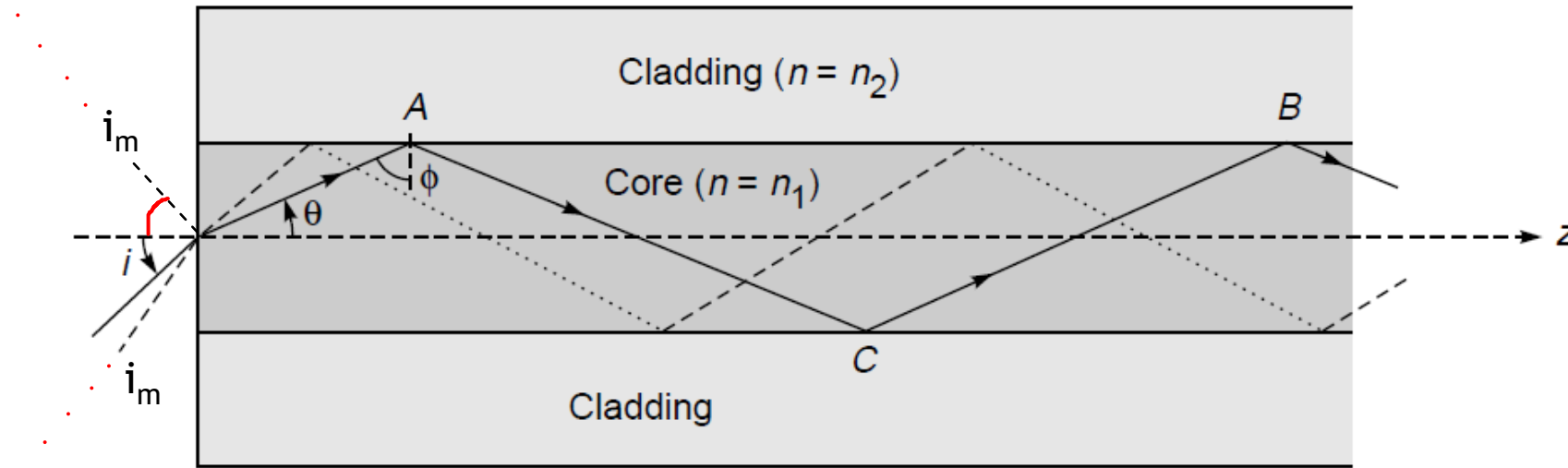
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by

Debolina Misra

Department of Physics
IIITDM Kancheepuram, Chennai, India

Numerical aperture



if light is incident on one end of the fiber, it will be guided through it provided $i < i_m$.

The quantity $\sin i_m$ is known as the **numerical aperture (NA)** of the fiber and is a measure of the light-gathering power of the fiber.

In almost all practical situations, $n_1^2 < n_2^2 + 1$.

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

Why glass for optical fiber?

- ▶ Easier to control w.r.t variation in temperature. There is a wide range of accessible temperatures where its viscosity is variable and can be well controlled unlike most materials, like water and metals which remain liquid until they are cooled down to their freezing temperatures and then suddenly become solid. Glass, on the other hand, does not solidify at a discrete freezing temperature but gradually becomes stiffer.
- ▶ The second most important property is that highly pure silica is characterized with extremely low-loss. Today, in most commercially available silica fibers 96% of the power gets transmitted after propagating through 1 km of optical fiber. This indeed represents a truly remarkable achievement.
- ▶ The third most remarkable property is the intrinsic strength of glass. Its strength is about 2,000,000 lb/in² so that a glass fiber of the type used in the telephone network and having a diameter (125 μm) can support a load of 40 lb.”

Materials for optical fiber

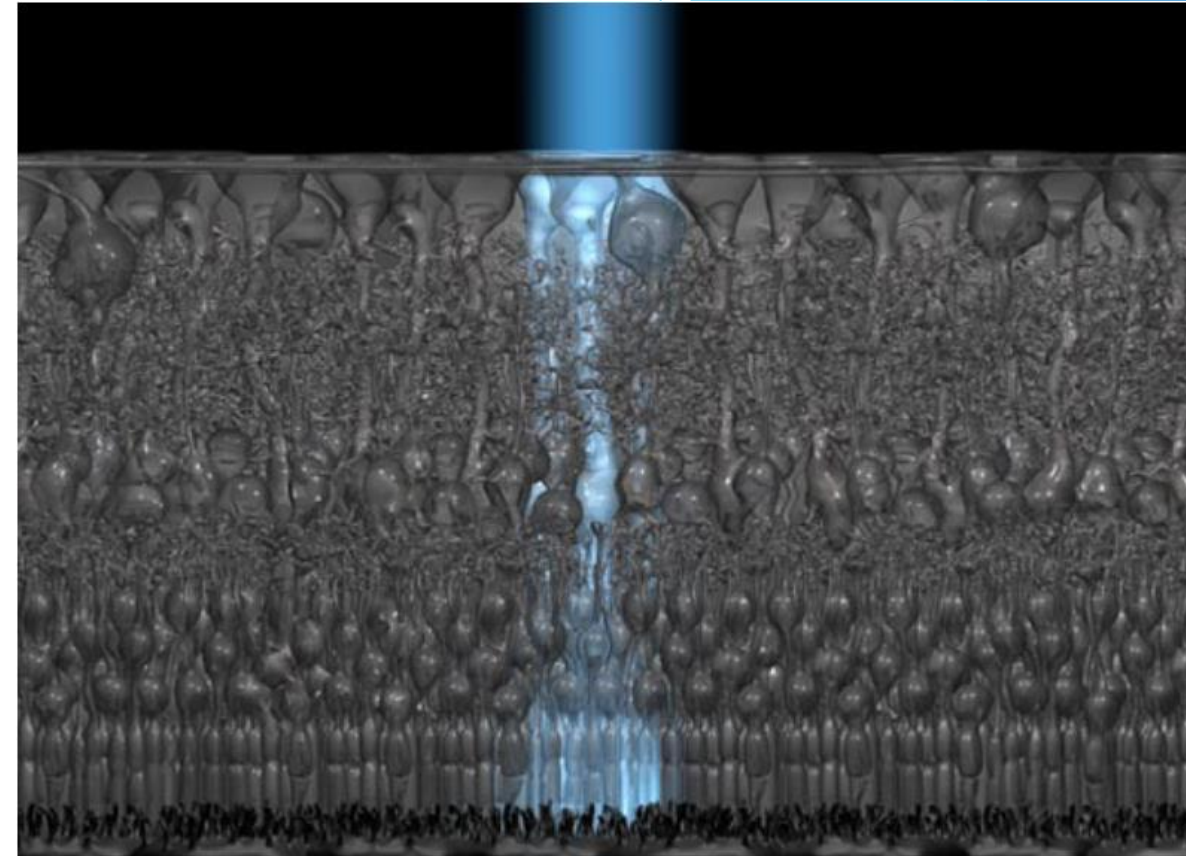
- ▶ Optical loss, mechanical strengths, and manufacturability

- ▶ Glass/plastic vs metals:

1. **Low loss:** metal wires → affected by other EM waves, glasses are not
2. **Safer:** glass → nonconducting → immune to EM and Radio Frequency interference → can safely be used around electrical transmission lines, as well as in high RF and magnetic fields.
 - ▶ A broken or damaged glass fiber → worst-case releases a few mJ of power. A break in a 1 kV electrical power cable will result in an arc discharge and huge energy → can be lethal
3. **Lighter:** metals → much heavier than glass fiber for similar power delivery capacity.
 - ▶ Conductor weight $\rightarrow 1/\text{voltage}^2$ (because power loss varies as V^2/R), so even at 400 volts, copper weighs over twice as much as fiber.
 - ▶ amazingly high mechanical strength against pulling and even bending
4. **Manufacturability** → silicon dioxide using Chemical Vapor Deposition technique—
layer by layer growth of optical fiber

Do you know?

- ▶ Eye → Retina → Müller cells → radial glial cells spanning the entire retinal thickness.
- ▶ Müller cells have an extended funnel shape, a *higher refractive index than their surrounding tissue*, and are *oriented along the direction of light propagation*.
- ▶ Transmission and reflection confocal microscopy showed that these cells provide a low-scattering passage for light from the retinal surface to the photoreceptor cells.
- ▶ individual Müller cells act as ?
- ▶ Finally the generated signals are transmitted to the brain through various nerves.



The inverted structure of the vertebrate retina. Light is coming from above and has to travel through several layers of light scattering tissue before reaching the sensitive photoreceptor cells at the back-side of the retina. The bright structures → represent Müller cells

Fermat's principle

According to the original statement of Fermat,

The actual path between two points taken by a beam of light is the one which is traversed in the least time.

The above statement is incomplete and slightly incorrect.
The correct form is:

The actual ray path between two points is the one for which the optical path length is stationary with respect to variations of the path.

This is expressed by Eq. (3), and in this formulation, the ray paths may correspond to maxima, minima, or stationary.

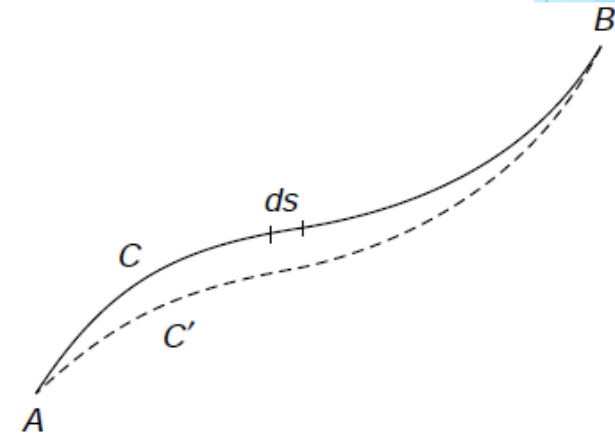
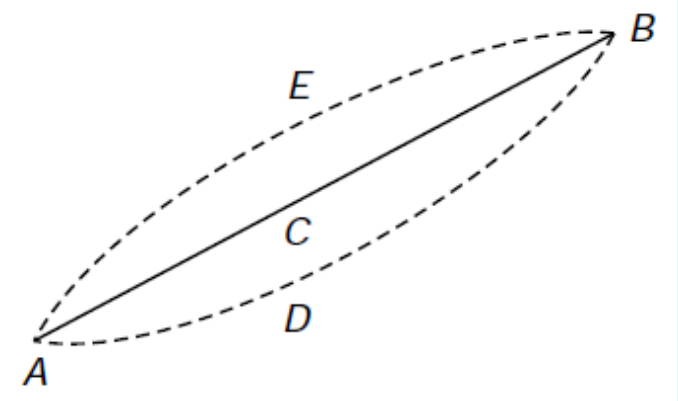
Fermat's principle

Let $n(x, y, z)$ represent the position-dependent refractive index. Then

$$\frac{ds}{c/n} = \frac{n ds}{c}$$

will represent the time taken to traverse the geometric path ds in a medium of refractive index n . Here, c represents the speed of light in free space. Thus, if τ represents the total time taken by the ray to traverse the path AB along the curve C (see Fig. 3.2), then

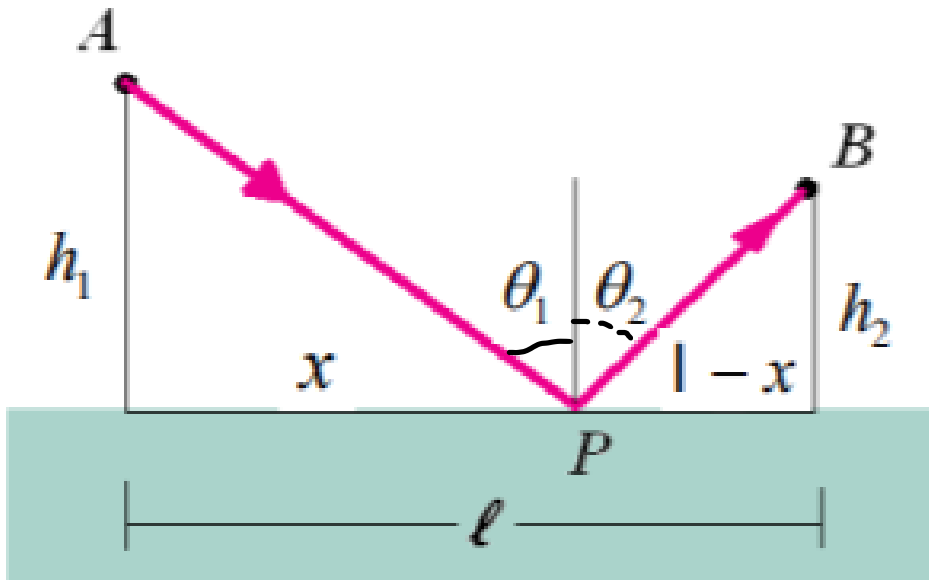
$$\tau = \frac{1}{c} \sum_i n_i ds_i = \frac{1}{c} \int_{A \rightarrow B} n ds \quad (1)$$



If path ACB represents the actual ray path, then the time taken in traversing path ACB will be an extremum in comparison to any nearby path $AC'B$.

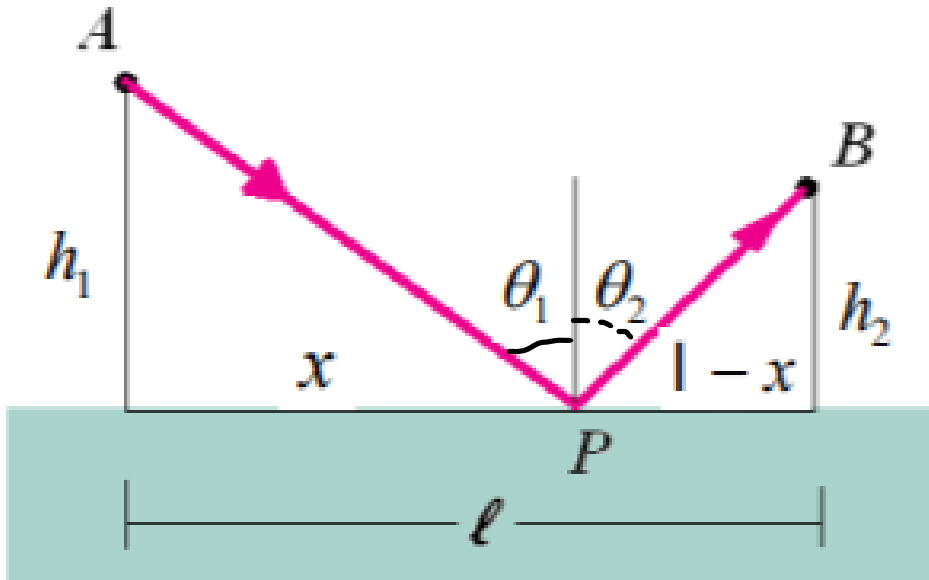
Fermat's principle

- ▶ Fermat's principle → determines the path of the rays
- ▶ According to this principle the ray will correspond to that path for which the time taken is an extremum in comparison to nearby paths, i.e., it is either a minimum or a maximum or stationary.



Fermat's principle

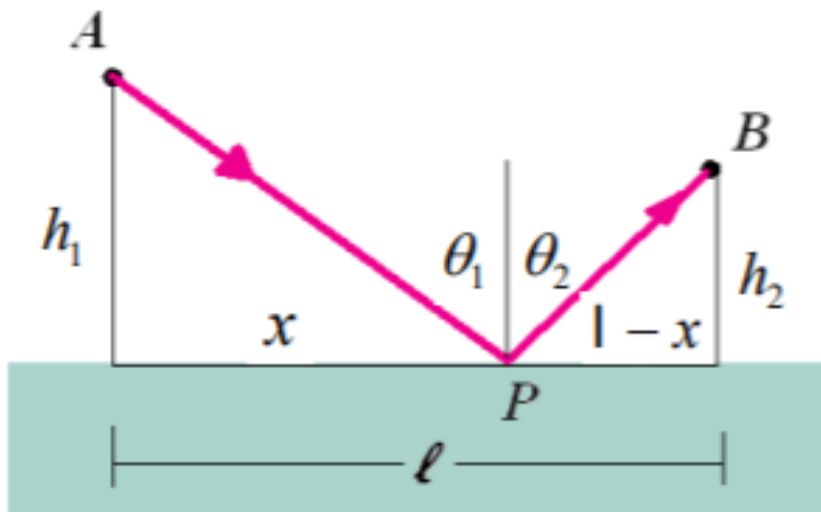
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$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c}$$

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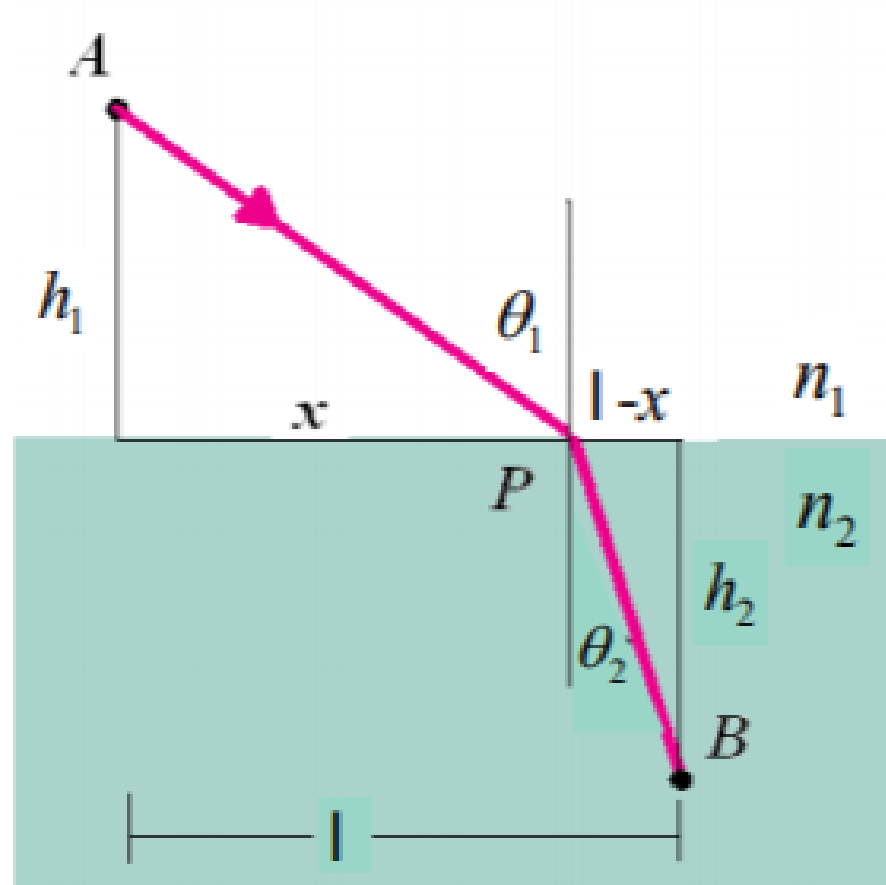
$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(l - x)^2 + h_2^2}}{c}$$

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2 + h_1^2}} + \frac{-(l - x)}{c\sqrt{(l - x)^2 + h_2^2}}$$

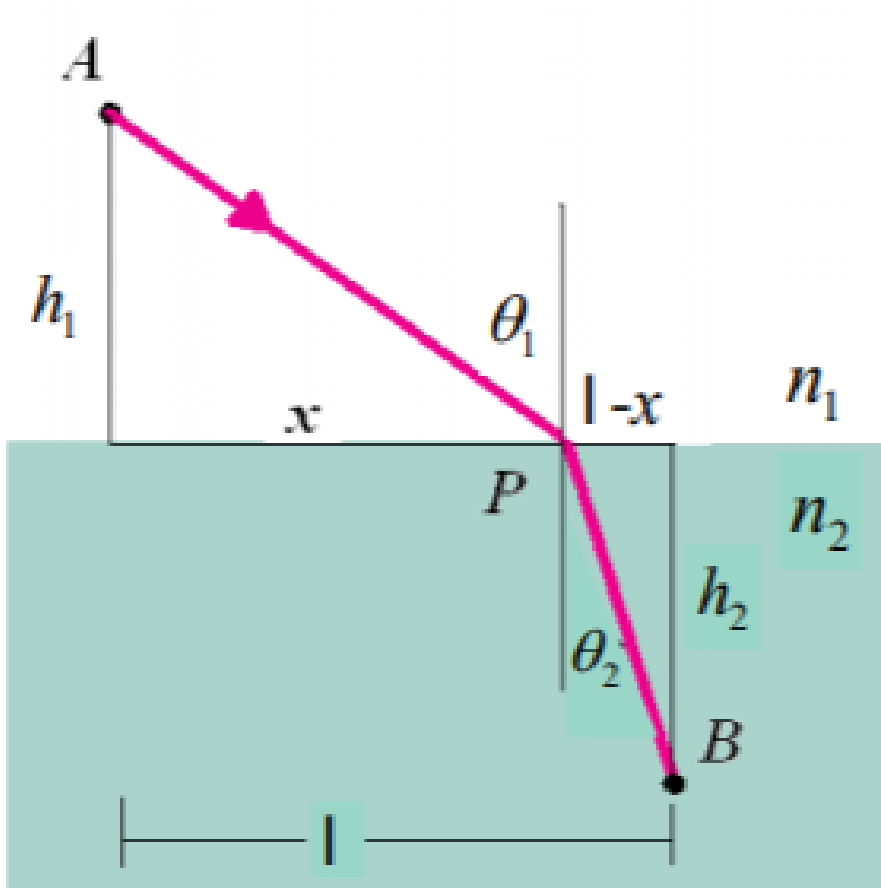
$$\frac{x}{\sqrt{x^2 + h_1^2}} = \frac{(l - x)}{\sqrt{(l - x)^2 + h_2^2}}$$

$$\sin \theta_1 = \sin \theta_2 \rightarrow \boxed{\theta_1 = \theta_2}$$

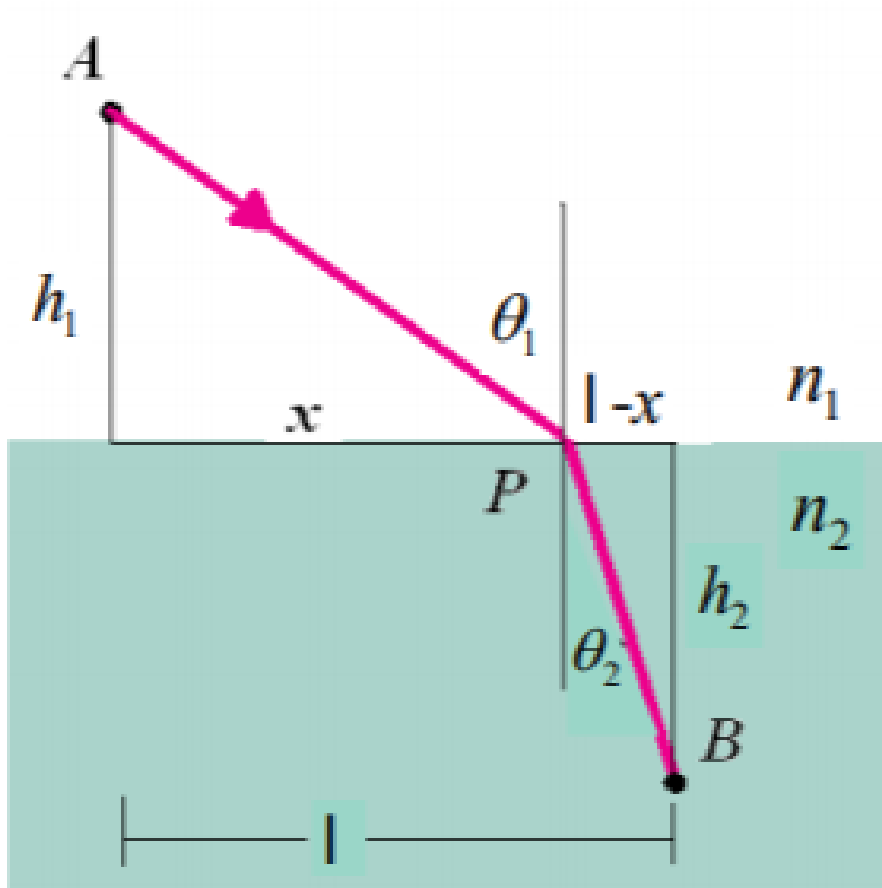
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Fermat's principle



Fermat's principle



$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(l-x)^2 + h_2^2}}{c/n_2}$$

$$0 = \frac{dt}{dx} = \frac{n_1 x}{c\sqrt{x^2 + h_1^2}} + \frac{-n_2(l-x)}{c\sqrt{(l-x)^2 + h_2^2}}$$

$$\frac{n_1 x}{\sqrt{x^2 + h_1^2}} = \frac{n_2(l-x)}{\sqrt{(l-x)^2 + h_2^2}}$$

$$\boxed{n_1 \sin \theta_1 = n_2 \sin \theta_2}$$

Thank You