23/08/23

Reference Books:

Hughes - Electrical and Electronic Technology

→ Edward Hughes,

→ Ian McKenzie Smith

→ John Hiley

→ Keith Brown

→ Tenth Edition, Pearson, 2010

(Charles Alexander)

→ Electrical Energy

→ Flexible

→ Efficiency

→ Clean

~~Electrical → W, kW, MW~~

~~Electronic → μW, mW~~

Electrical	Electronic
1) Fan, Light, Powerplant, etc	1) Video call, Voice call, etc
2) W, kW, MW	2) μW, mW
3) V, kV, MV	3) mV, V
4) A, kA	4) μA, mA
5) Power Frequency 50 Hz, 60 Hz	5) kHz, MHz, GHz

→ Sources of DC → Battery, Solar Cells, DC Generator, Fuel Cells

Sources of AC → AC Generator

Generation

↓

Transmition

↓

Distribution → Consumers

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Generated Power : 100 MW

Voltage : 10 kV

$$i = \frac{P}{V} = \frac{100 \times 10^6}{10 \times 10^3}$$

$$= 10\,kA \Rightarrow \boxed{I_v = 10\,kA}$$

$I \uparrow \Rightarrow$ Size $\uparrow \Rightarrow$ Volume $\uparrow \Rightarrow$ Cost $\uparrow$

distance $\uparrow \Rightarrow$ Resistance (Internal) $\uparrow \Rightarrow$ Voltage drop $\uparrow$

(i^2R loss) $\uparrow$

$\Downarrow$

Efficiency $\downarrow$

$P = \downarrow i \cdot V \uparrow$

V : 10 kV $\longrightarrow$ 100 kV (Transformer)

$\Rightarrow$ i : 10 kA $\longrightarrow$ 1 kA

Efficiency of Transformer : 99.5%

(i) Voltage
(ii) Current
(iii) Power
(iv) Work
(v) Resistance
(vi) Inductance
(vii) Capacitance

Primary Consumer $\rightarrow$ Bulk

Secondary Consumer $\rightarrow$ Domestic

CHAPTER-I

Electricity Fundamentals

Materials
- Conductors
- Semiconductors
- Insulator

Air gets Ionized $\longrightarrow$ 30 kV per cm

Electric Potential, $V = \dfrac{\text{Work done}}{\text{Charge}} = \dfrac{dW}{dq}$ [Volt (or) Joule/Coulomb]

EMF : Difference in potentials of two charged bodies is called Potential difference. It develops EMF.

$$i = \frac{dq}{dt}$$

$$\text{Energy} = \text{Work} = Vq = Vit$$

$$V = \frac{\text{Work}}{q}$$

$$P = \frac{dE}{dt} \quad \& \quad \text{Power} = \frac{\text{Work}}{t} = \frac{Vit}{t} = iV$$

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Law of Conservation of Energy

$$P_{supplied} = VI = P_{output}$$

Active Element $\longrightarrow$ Produces Energy

Passive Element $\longrightarrow$ Recieves Energy

Bilateral Elements → Resistors

Unilateral Elements → Diodes

$R = \rho \frac{l}{a}$ → Specific Resistance (or) Resistivity

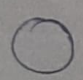
R → ohms (Ω)

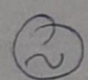
$G = \frac{1}{R}$

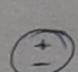
G → Siemens (S) (or) Mho ($\mho$)

For study, We assume sources are independent.

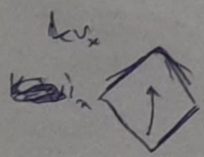
But in Reality, the sources are dependent.

 → Independent

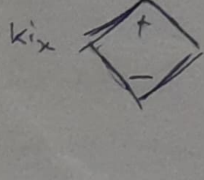
 → Independent AC source

 → Independent DC source

 → Independent Current source



→ Voltage Dependent



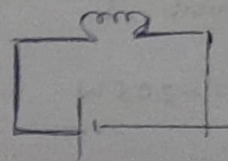
→ Current Dependent

Q) An energy source forces a constant circuit of 2 A for 10s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy. Calculate the voltage drop across the bulb.

Sol: Given:

$i = 2\,A$

$t = 10\,s$



$H = i^2Rt$

$= Vit$

$\Rightarrow 2300 = V \times 2 \times 10$

$\Rightarrow \boxed{V = 115\,V}$

(OR) $\Delta q = i\Delta t = 20\,C$

$v = \dfrac{\Delta w}{\Delta q} = 115\,V$

Q) How much energy does a ~~....~~ 100 W electric bulb consume in 2 hours?

Ans: $E = 2 \times 100 = 200$ Watt hrs

$= 200 \times 3600 = 720\,kJ$

$(W = P.t)$

Q) A 100 v lamp has a hot resistance of 250 Ω. Find the current taken by the lamp and its Power rating in watts. Calculate also the energy it will consume in 24 hours.

Sol: Given:

$P = iV$

$= i^2R$

$V = 100\,V$

$R = 250\,\Omega$

$P = V^2/R = \dfrac{100 \times 100}{250} = 40\,W$

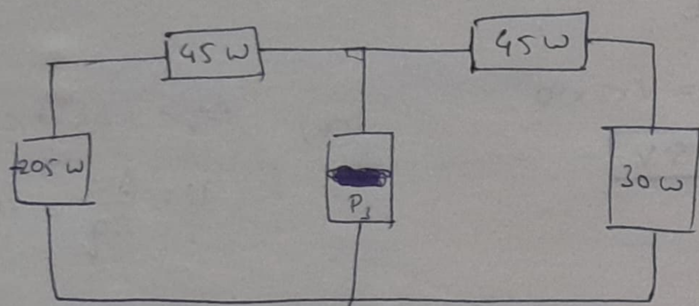
$W = Pt$

$= 40 \times 24 = 960$ Watthours

$i = V/R = \frac{100}{250} = 0.4$ A

Q) Figure shows a circuit with five elements. Calculate the Power P_3 recieved or Delivered by element 3.

$P_1 = -205$ W

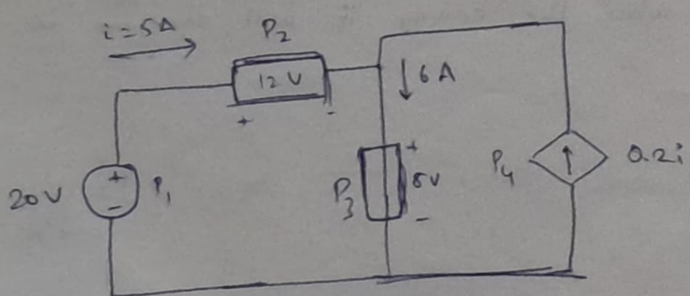


$\sum_{i=1}^{5} P_i = 0 \Rightarrow P_3 = 85$ W

Recieved

According to Law of Conservation of Energy

Q) Calculate the power absorbed or supplied by each element.



$\sum P = 0$

$P_1 + P_2 + P_3 + P_4 = 0$

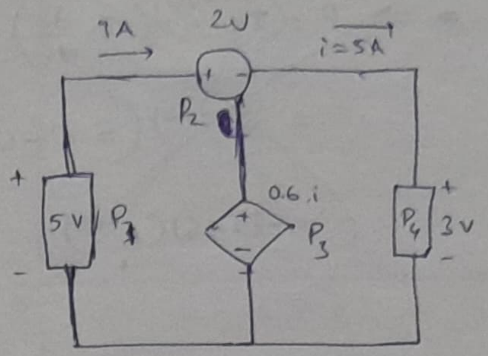
$P_3 = 48$ W

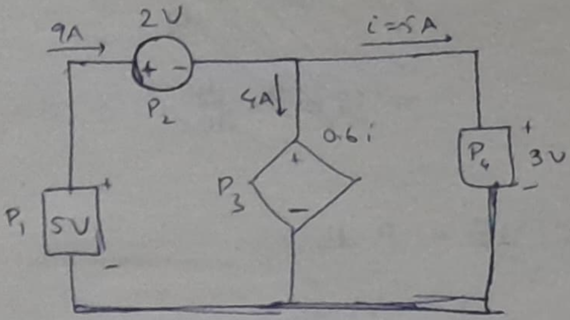
$P_1 = -100$ W [Supplied : -ve]

$P_2 = 60$ W

$P_4 = -8$ W

Q) Find the power absorbed or supplied by each component.





$P_1 = 45$ W

$P_4 = 15$ W

$P_2 = 18$ W

→ $P_3 = 9$ W

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→ <u>Practice</u> <u>Problems</u> :

Q3) $i = 85$ mA

$t = 12$ h

$q = it$

$= 85 \times 10^{-3} \times 12 \times 3600$

$= 3672$ C

$\boxed{q = 3.672 \text{ kC}}$

$E = (iV)t$ $\quad (P = E/t)$

$\Rightarrow E = 85 \times 10^{-3} \times 1.2 \times 12 \times 3600$

$= 4406.4$ J

$= 4.4064$ kJ

Q4) $i(t) = 3e^{-2t}$ A

$V(t) = 5\frac{di}{dt}$ V

(a) $dq = i \cdot dt$

$\Rightarrow \int_0^q dq = \int_0^2 3e^{-2t}\, dt$

$\Rightarrow q = \left[\frac{-3}{2} e^{-2t}\right]_0^2$

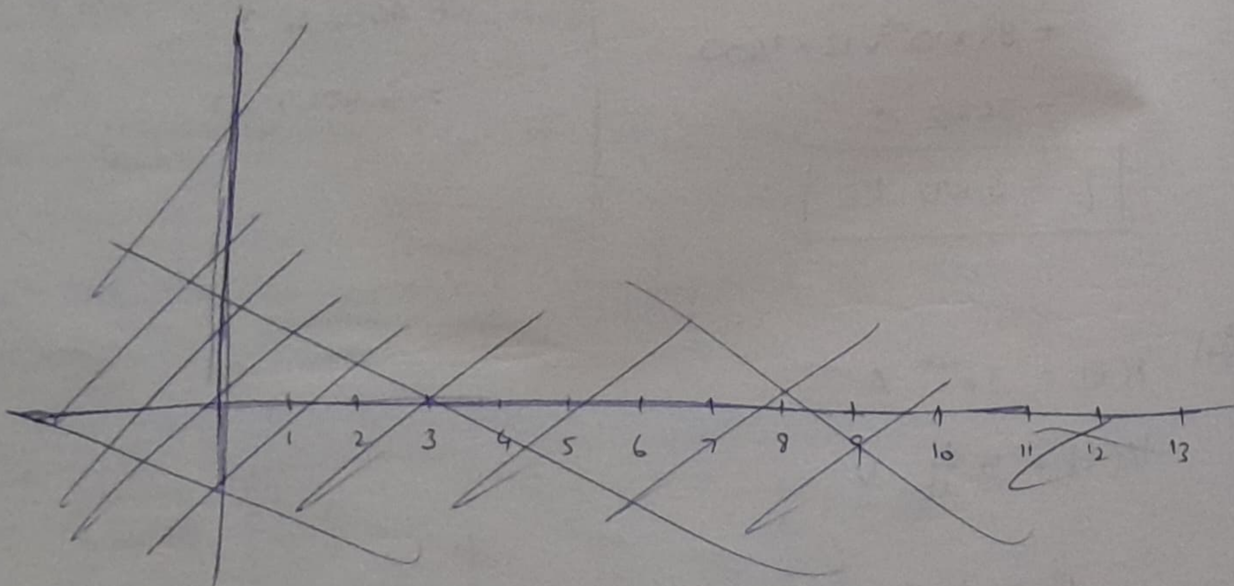
$= \left.\frac{3}{2} e^{-2t}\right]_2^0 = \left(\frac{3}{2} - \frac{3}{2}e^{-4}\right) = 1.4725$ C

$$= \int_0^3 15 \times e^{-2t} \, di.$$

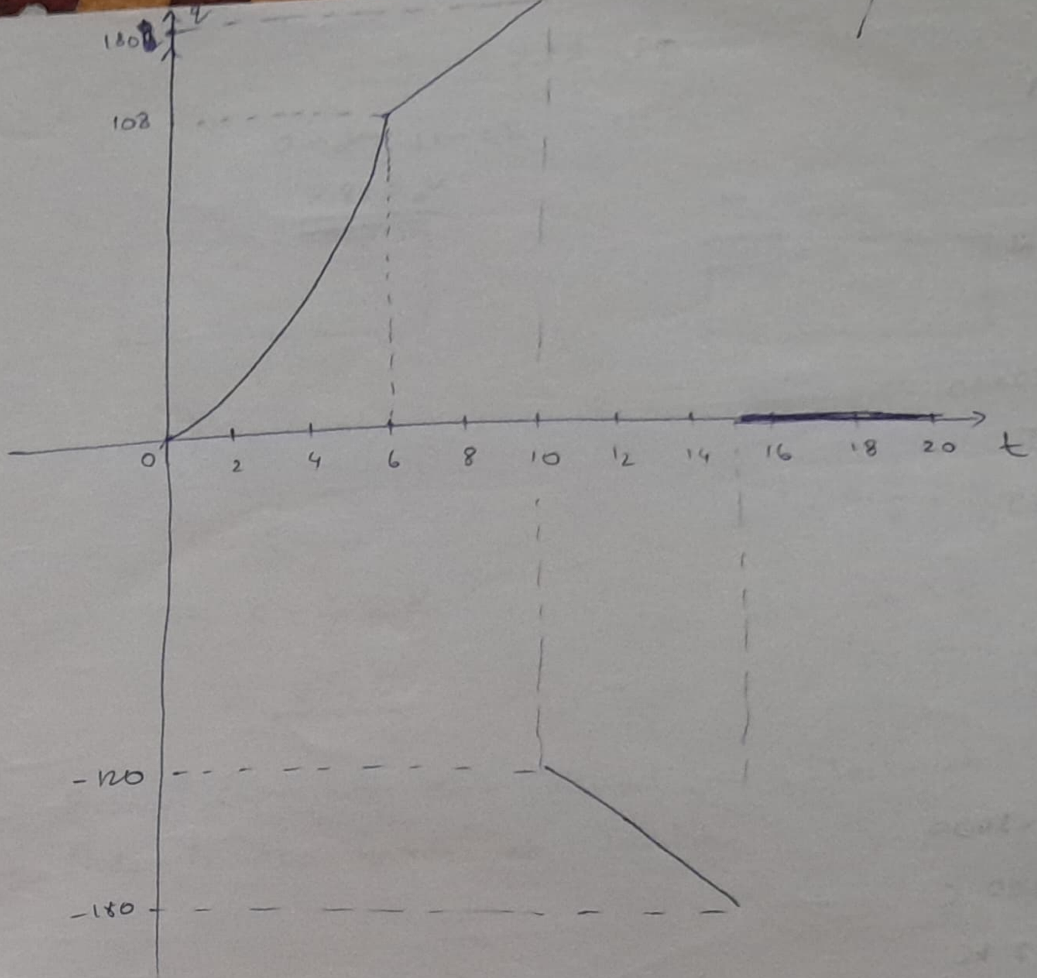
Q5)

$$i(t) = \begin{cases} 3t & , 0 \le t \le 6 \\ 18 & , 6 \le t < 10 \\ -12 & , 10 \le t < 15 \\ 0 & , 15 \le t \end{cases}$$

$\underline{\underline{0 < t < 20}}$

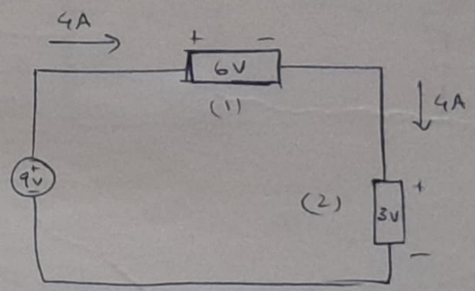


$$q(t) = \begin{cases} 3t^2 & , 0 \le t < 6 \\ 18t & , 6 \le t < 10 \\ -12t & , 10 \le t < 15 \\ 0 & , t \ge 15 \end{cases}$$



Q6)

(a)

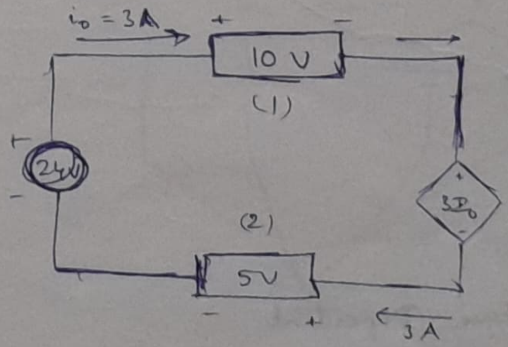


$P = iV$

(1) ⟶ $P = 4 \times 6 = 24$ W

(2) ⟶ $P = 4 \times 3 = 12$ W

(b)



$P = iV$

(1) $P = 3 \times 10 = 30$ V

(2) $P = 3 \times 5 = 15$ V

Q8) $V = 12\ V$

$i = 150 \times 10^{-3}\ A$

(a) $P = iV$

$= 1.8\ W$

(b) $E = P\,t$

$= 1.8 \times 20 \times 60$

$= 2160\ J$

$= 2.16\ kJ$

Q7) KCL :

$30 - 12 - V_0 = 0$

$\underline{\underline{V_0 = 18\ V}}$

Q9) $i = 3A$

$t = 4hr$

(a) $q = it$

$\Rightarrow q = 3 \times 4 \times 3600$

$= 43200\ C$

$= 43.2\ kC$

CHAPTER - 2

28/8/23

- Ohm's law
- Kirchoff's Law
- Network Topology
- Series and Parallel Connections

→ **Ohm's Law :**

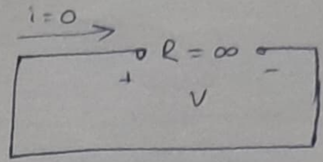
$V = RI$

Resistance : Temparature Dependent

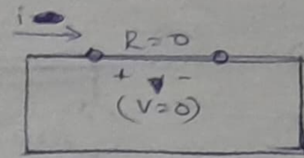
Resistor

- Linear Resistor
- Non-Linear Resistor

$P_R = I^2 R \longrightarrow$ Ohmic Loss



(Open Circuit)



(Short Circuit)

$i_{sc} = \frac{V}{R} = \frac{V}{0}$

$V = iR$

$\Rightarrow R = \frac{V}{i} = \frac{V}{0}$

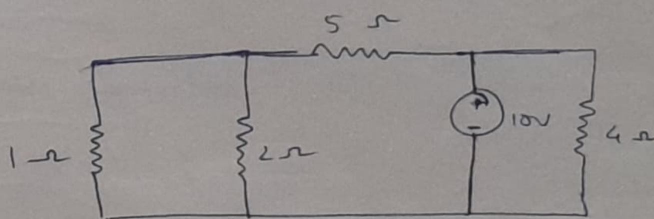
$\underline{R \rightarrow \infty}$

$i_{sc} \propto V$

$i_{sc} \longrightarrow \infty$

Branch Represents each element in Electrical Circuit.

Node is also known as Junction.



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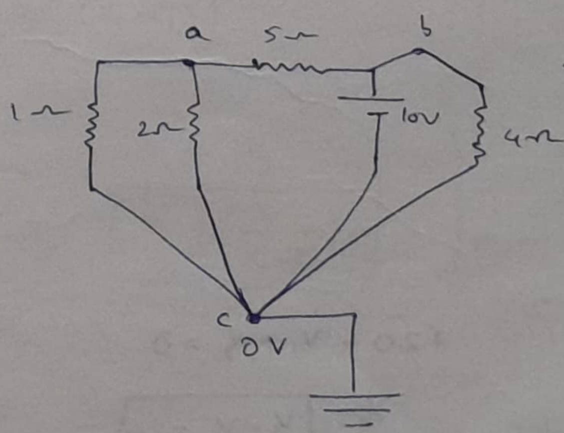


$\longrightarrow$ $\underline{3 \text{ nodes}}$

no. of branches = b

no. of nodes = n

no of Independent loops = l

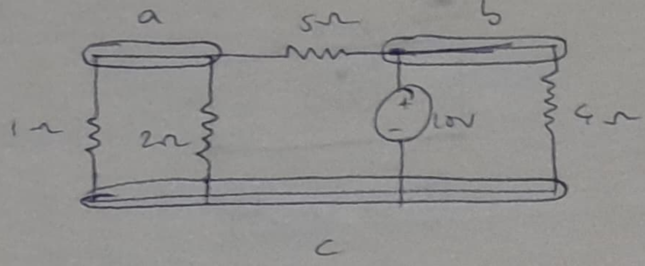
$\boxed{b = l + n - 1}$

$l = 3$

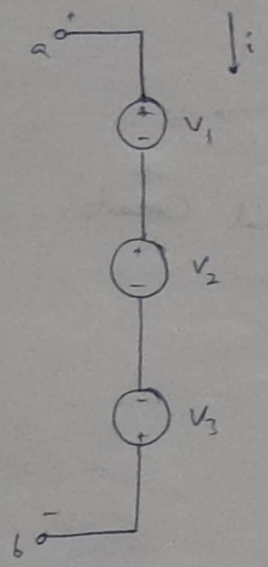
$n = 3$

$b = 3+3-1 = 5$ $\boxed{b=5}$

Ground → Common Node / Reference Node



Q)

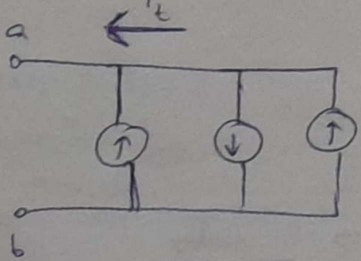


$V_a - V_1 - V_2 + V_3 = V_b$

$V_a - V_b = V_1 + V_2 - V_3$

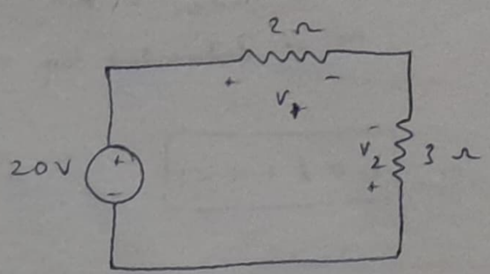
$\boxed{V_{ab} = V_1 + V_2 - V_3}$

Q)



$i_t = i_1 - i_2 + i_3$

Q) Find v_1 and v_2 using KVL



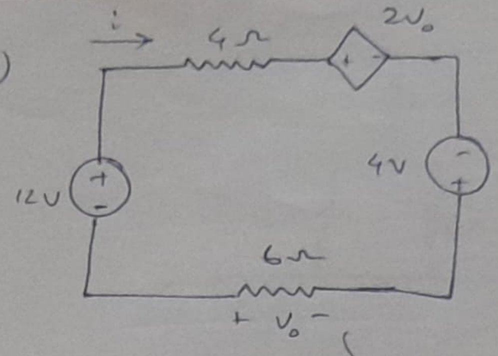
$+20 - v_1 + v_2 = 0$

$\boxed{v_1 - v_2 = 20}$

$\left.\begin{array}{l} v_1 = 2i \\ v_2 = 3i \end{array}\right\} \Rightarrow \boxed{i = 20\,A}$ ACW

$v_1 = 40\,V$
$v_2 = 60\,V$

Q)



$\boxed{6i = V_o}$

$-4i - 2V_o + 4 + 6i + 12 = 0$

$\Rightarrow 2i - 2V_o + 16 = 0$

$\Rightarrow 2i - 12i + 16 = 0$

$\Rightarrow 10i = 16$

$\boxed{i = 1.6} \Rightarrow \boxed{V_o = 9.6\,V}$

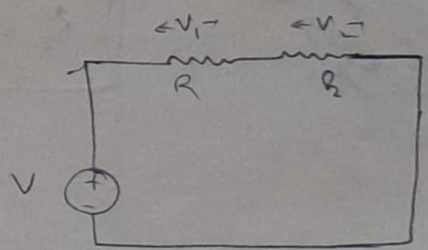
30/9/25:

→ Series Connection:

$V = V_1 + V_2$

$V_1 = iR_1$

$V_2 = iR_2$



$R_{eq.} = R_1 + R_2$

$[V = i(R_1 + R_2)]$

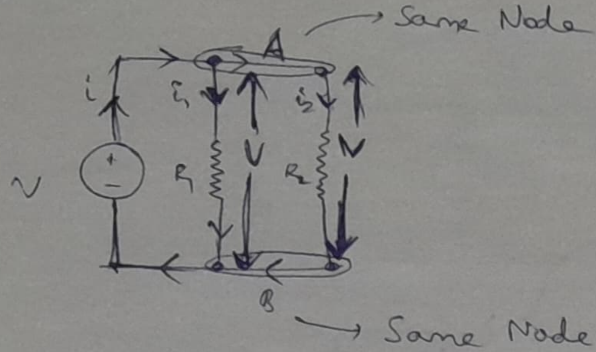
$V_1 = iR_1$

$\boxed{V_1 = \left(\frac{V}{R_1+R_2}\right) R_1}$

$V_2 = iR_2$

$\boxed{V_2 = \left(\frac{V}{R_1+R_2}\right) R_2}$

Parallel Connection :



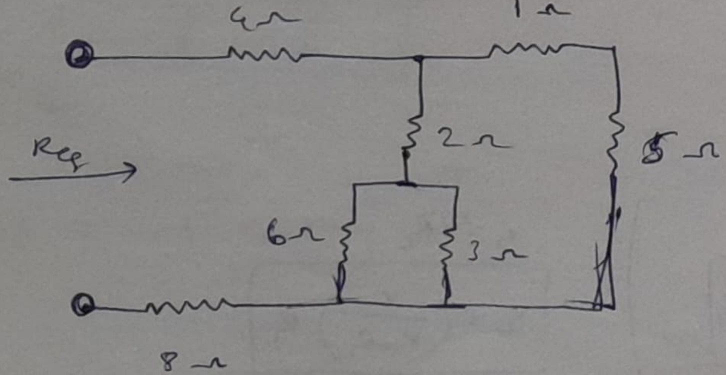
$$\boxed{i = i_1 + i_2}$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow \boxed{\frac{1}{R_{eq.}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

$$\boxed{R_{eq.} = \frac{R_1 \times R_2}{R_1 + R_2}}$$

$$\boxed{\begin{aligned} i_1 &= i\left(\frac{R_2}{R_1+R_2}\right) \\ i_2 &= i\left(\frac{R_1}{R_1+R_2}\right) \end{aligned}}$$

Q)

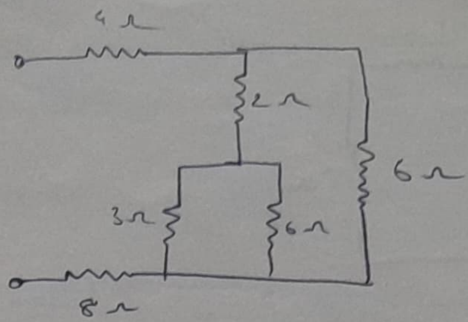


find $R_{eq.}$

A) Starts with Extreme Right side

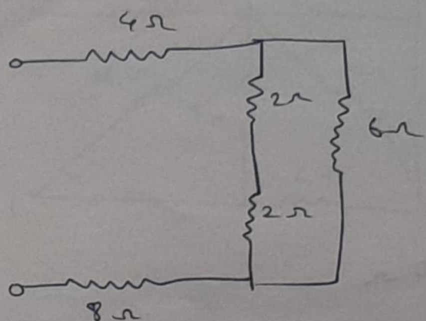
1 Ω series with 5 Ω

$\Rightarrow$ 6 Ω



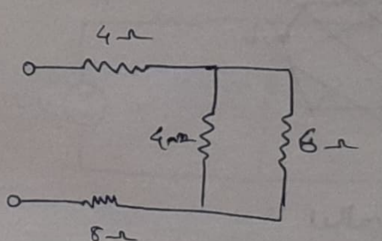
3 Ω parallel with 6 Ω

$\Rightarrow 2\ \Omega$



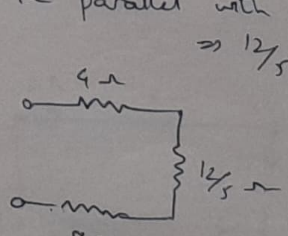
2 Ω series with 2 Ω

$\Rightarrow 4\ \Omega$



4 Ω parallel with 6 Ω

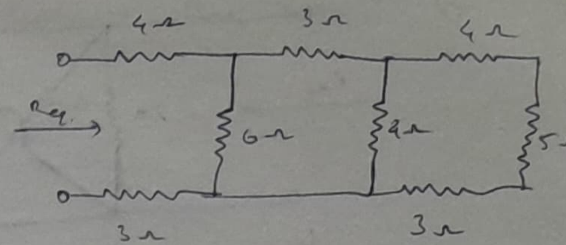
$\Rightarrow 12/5\ \Omega$



4 Ω, 8 Ω and 12/5 Ω in series

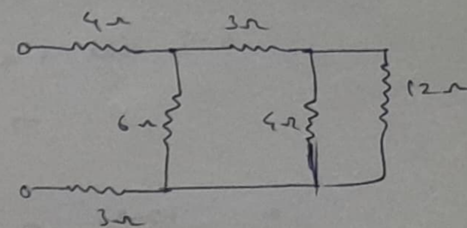
$\Rightarrow 14.4\ \Omega$

Q)



4 Ω, 5 Ω, 3 Ω in series

$\Rightarrow 12\ \Omega$



4 Ω and 12 Ω in parallel

$\Rightarrow 3\ \Omega$

3 Ω and 3 Ω in series

$\Rightarrow 6\ \Omega$

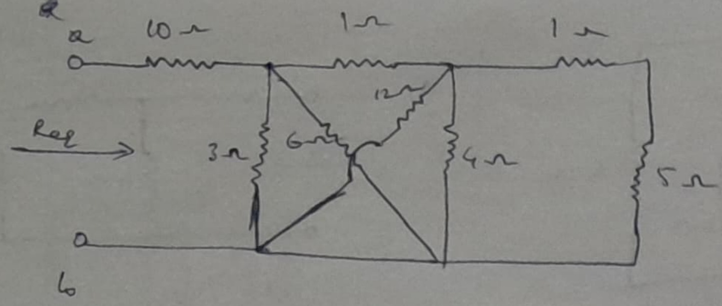
6 Ω and 6 Ω in parallel

$\Rightarrow 3\ \Omega$

4 Ω, 3 Ω, 3 Ω in series

$\Rightarrow 10\ \Omega$

Q



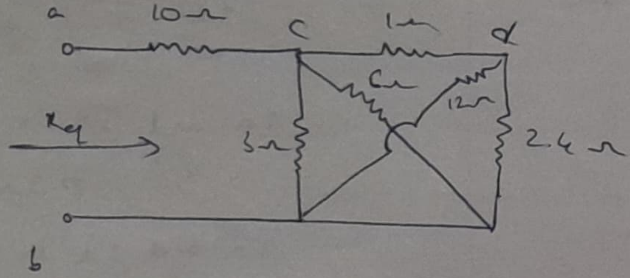
$\frac{2}{12} + \frac{3}{12}$

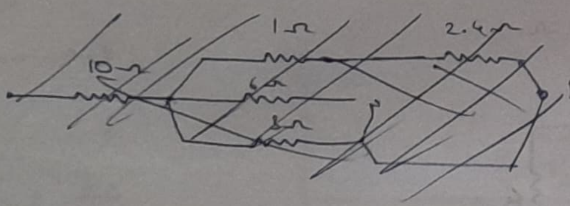
1 Ω and 5 Ω → series

⇒ 6 Ω

6 Ω and 4 Ω → parallel

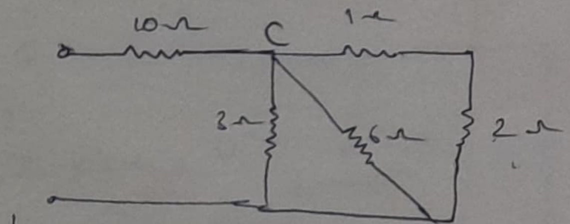
⇒ 2.4 Ω





12 Ω and 2.4 Ω in parallel

$\cancel{\frac{12}{5}}\ \frac{5}{12} + \frac{1}{12} = 2$



1 Ω and 2 Ω series ⇒ 3 Ω

3 Ω, 3 Ω, 6 Ω → parallel

⇒ 1.2

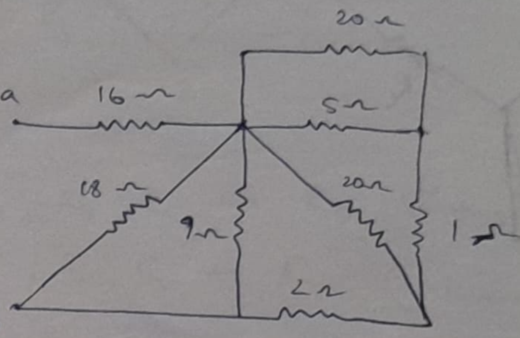
Note:

If the given Problem is relevant to Conductance 'g', The following Assumptions are made.

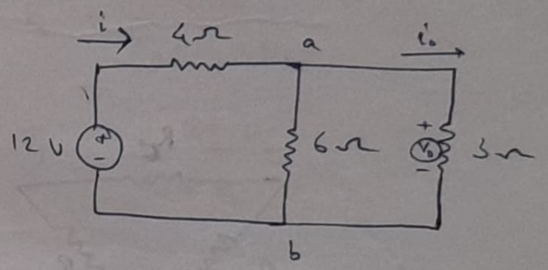
(i) Series : $\frac{1}{g_{eq.}} = \frac{1}{g_1} + \frac{1}{g_2}$

(ii) Parallel : $g_{eq} = g_1 + g_2$

Q3)



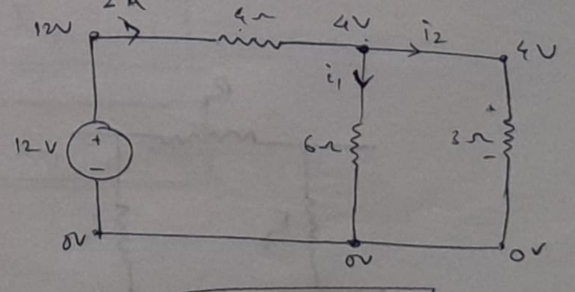
Q) Find i_o and V_o. Calculate Power Dissipated in the 3 Ω Resistor



$R_{eff} = 6$

$i = 12/6 = 2\text{ A}$

A)



$i_1 = i\left(\frac{R_2}{R_1+R_2}\right)$

$= 2 \times \frac{3}{9}$

$i_1 = \frac{6}{9} = \frac{2}{3}$

$\boxed{i_2 = \frac{4}{3}}$

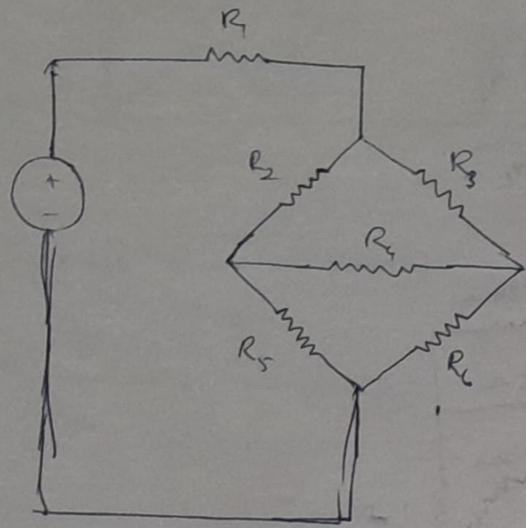
$\boxed{i_2 = i_0 = 1.33\text{A}}$

$\boxed{V_o = 4\text{V}}$

$P = i^2 R = \left(\frac{4}{3}\right)^2 \times 3 = \frac{16}{3}$

$\boxed{P = 5.33\text{ W}}$

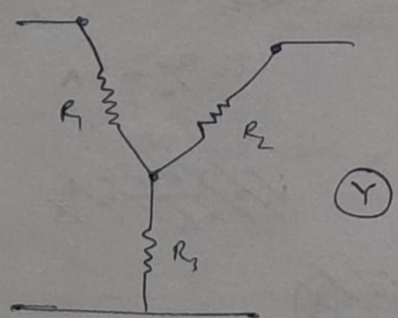
→ Wye - Delta Transformations :

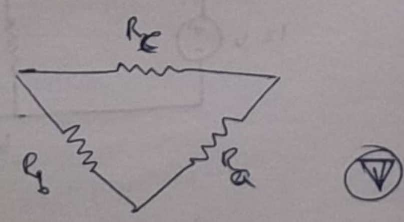


Y - T - Star

∇ - Mesh - π

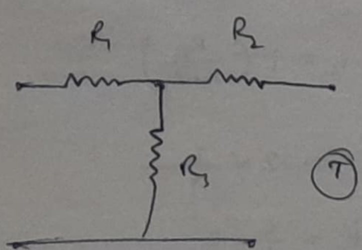
~~[illegible]~~ Representation :

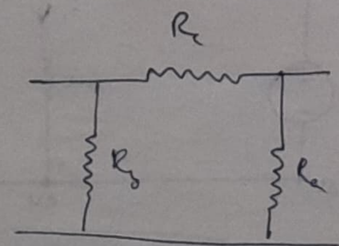




(OR)

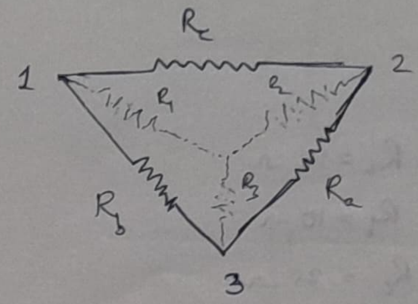
(OR)





Conversion :

① ∇ to Y Conversion



$R_a, R_b, R_c \longrightarrow R_\nabla$ (Known)

$R_1, R_2, R_3 \longrightarrow R_Y$ (Unknown)

$$R_1 = \frac{R_b \cdot R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

If $R_1 = R_2 = R_3 = R_Y$

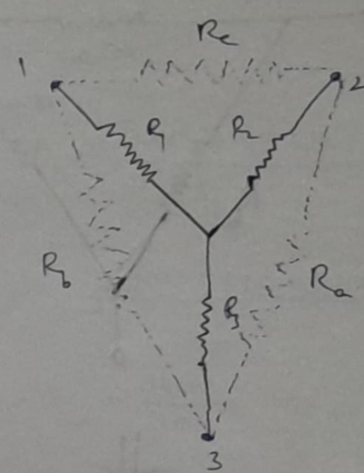
$R_a = R_b = R_c = R_\nabla$

$$\boxed{R_Y = \frac{R_\nabla}{3}}$$

$$\boxed{R_\nabla = 3R_Y}$$

Proof : HW

② Y to ∇ Conversion



$R_1, R_2, R_3 \longrightarrow R_Y$ (known)

$R_a, R_b, R_c \longrightarrow R_\nabla$ (Unknown)

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 \cdot R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

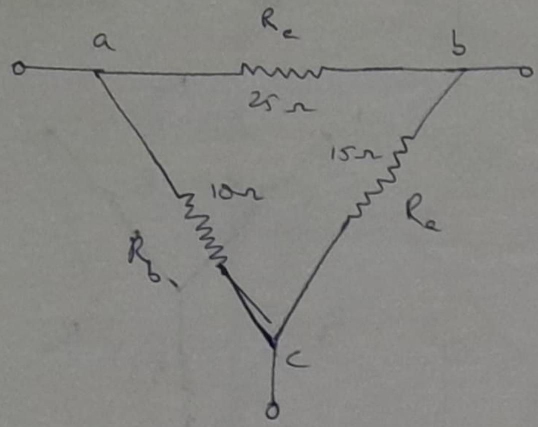
If $R_a = R_b = R_c = R_\nabla$

$R_1 = R_2 = R_3 = R_Y$

$$\boxed{R_\nabla = 3R_Y}$$

$$\boxed{R_Y = \frac{R_\nabla}{3}}$$

Q) Convert ∇ to Y

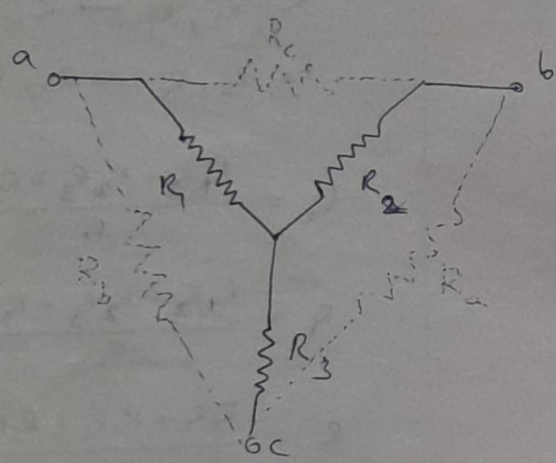


$R_a = 15\,\Omega$

$R_b = 10\,\Omega$

$R_c = 25\,\Omega$

$\Downarrow$



$$R_1 = \frac{R_b \cdot R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a \cdot R_c}{R_a + R_b + R_c}$$

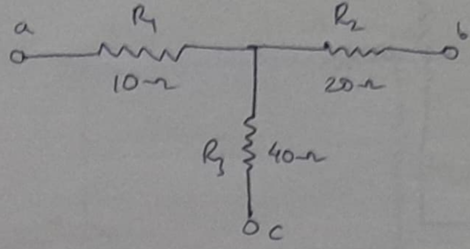
$$R_3 = \frac{R_a \cdot R_b}{R_a + R_b + R_c}$$

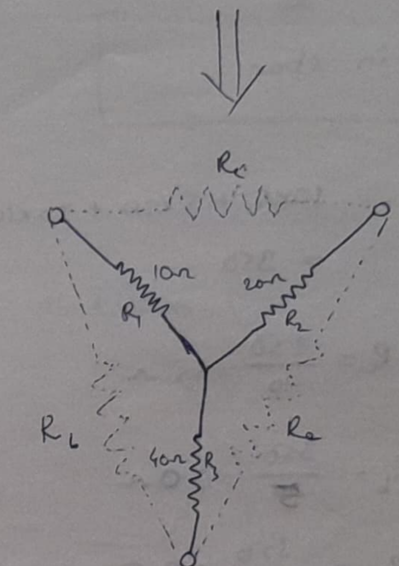
$$R_1 = \frac{25 \times 10}{50} = 5\,\Omega$$

$$R_2 = \frac{25 \times 15}{50} = \frac{7}{2}\,\Omega$$

$$R_3 = \frac{10 \times 15}{50} = 3\,\Omega$$

Q) Transform Y to ∇





$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

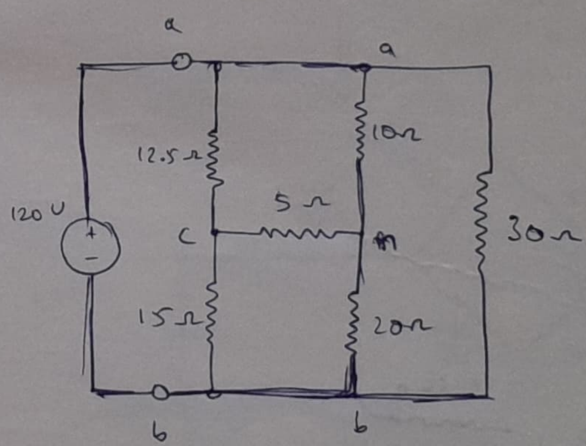
$$= \frac{400 + 800 + 200}{10}$$

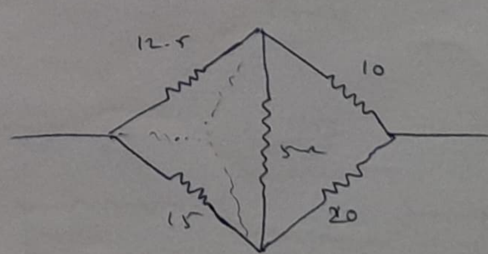
$$= \frac{1400}{10} = 140\ \Omega$$

$$R_b = \frac{1400}{20} = 70\ \Omega$$

$$R_c = \frac{1400}{40} = 35\ \Omega$$

Q)

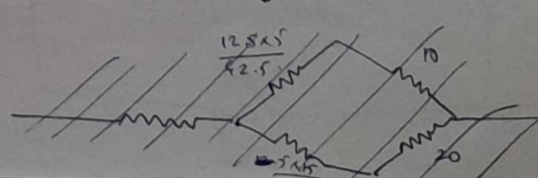


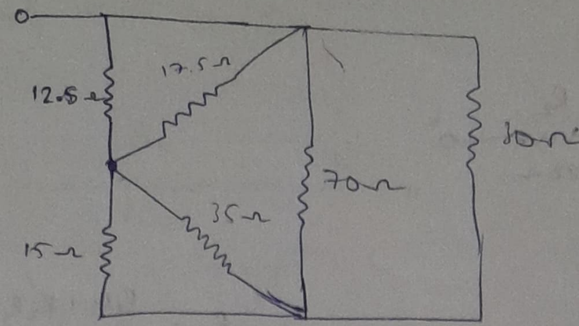


$$\frac{12.5 \times 5}{42.5}$$

$$\frac{12.5 \times 15}{42.5}$$

$$\frac{15 \times 5}{42.5}$$





Elements a, b, c are in star

$10\times5 + 5\times20 + 20\times10$

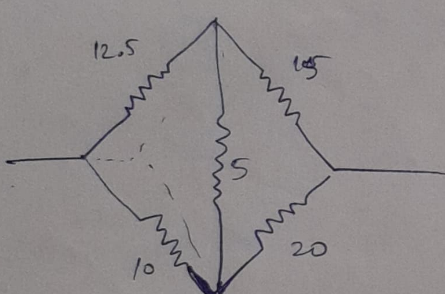
$= 350$

$R_c = \frac{350}{10} = 35\ \Omega$

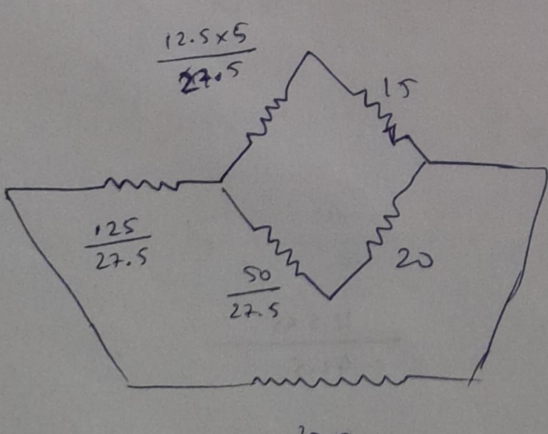
$R_b = \frac{350}{5} = 70\ \Omega$

$R_a = \frac{350}{20} = 17.5\ \Omega$

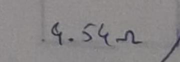
(OR)



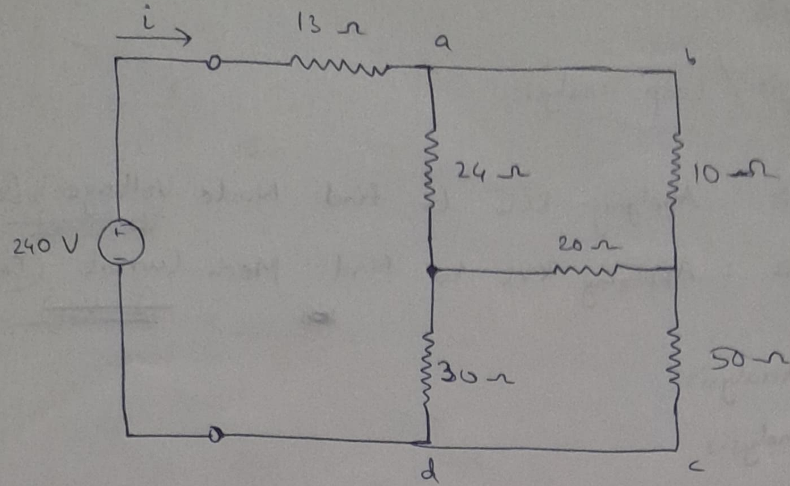
⇓



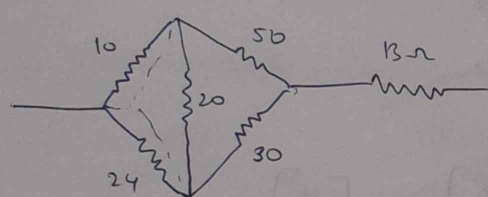
⇒



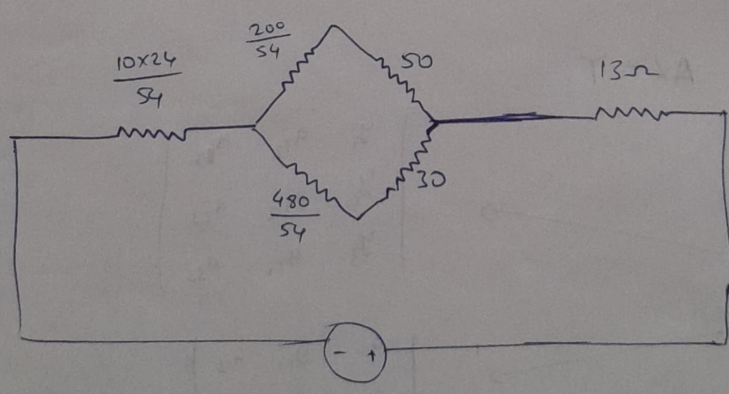
Q) For the Given network in figure. find R_{ab} and i



abcd $\longrightarrow$ ∇



$\Downarrow$



$\frac{100}{27} + 50$

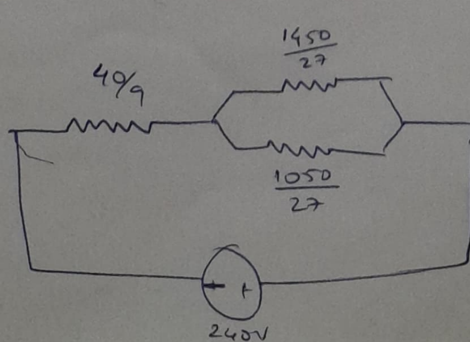
$50\left(1 + \frac{2}{27}\right)$

$\frac{50 \times 29}{27}$

$\frac{240}{27} + 30$

$30\left(1 + \frac{8}{27}\right)$

$\frac{30 \times 35}{27}$



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Chapter : 3 : Methods of Analysis

(1) Nodal Analysis

(2) Mesh Analysis / Loop analysis

Nodal Analysis : Applying KCL to find Node Voltages $([V]\cdot[G]=[I])$ ~~(RI = V)~~

Mesh Analysis : Applying KVL to find Mesh Current $([R]\cdot[I]=[V])$ ~~(R = VG)~~

(3) Super Node Analysis

(4) Super Mesh Analysis

Cramer's Rule:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = y_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = y_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = y_3$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$AX = Y$$

$|A|$

$$|A_1| \rightarrow \begin{vmatrix} y_1 & a_{12} & a_{13} \\ y_2 & a_{22} & a_{23} \\ y_3 & a_{32} & a_{33} \end{vmatrix}$$

$$|A_2| \rightarrow \begin{vmatrix} a_{11} & y_1 & a_{13} \\ a_{21} & y_2 & a_{23} \\ a_{31} & y_3 & a_{33} \end{vmatrix}$$

$|A_2|$

$$x_1 = \frac{|A_1|}{|A|}$$

$$x_2 = \frac{|A_2|}{|A|}$$

$$x_3 = \frac{|A_3|}{|A|}$$

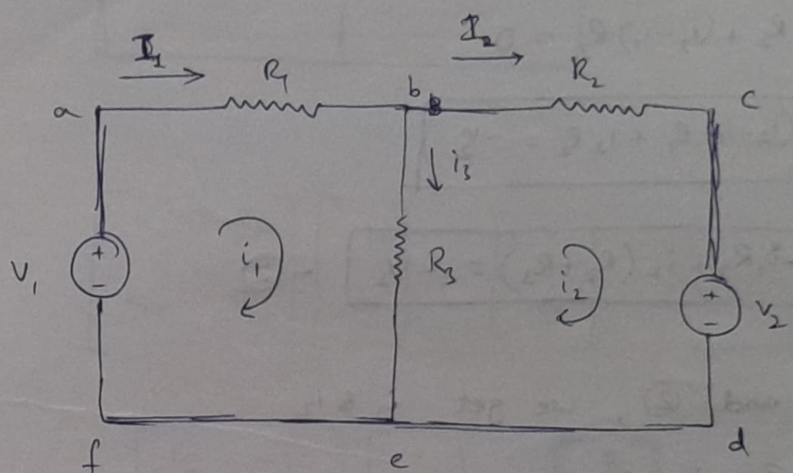
$A \cdot X = Y$

$R \cdot I = V$

$I = v\left(\frac{1}{R}\right)$

$I = V \cdot G$

→ Mesh Current Analysis :



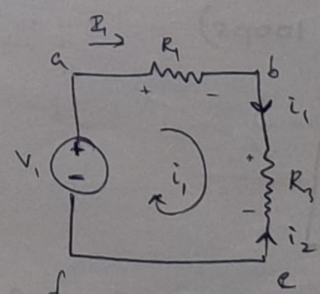
I_1, I_2 : Branch Currents

i_1, i_2 : Loop Currents

In the given Circuit, find i_1 and i_2.

Mess Analysis using KVL

Loop - I · Apply KVL :



$i_1 > i_2$ (Because loop of i_1)

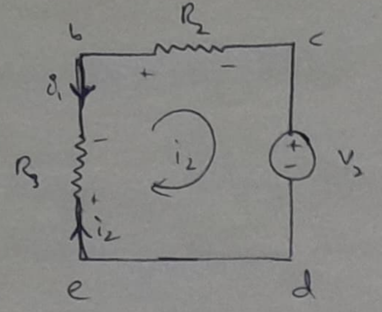
$V_1 - i_1R_1 - (i_1 - i_2)R_3 = 0$

$\Rightarrow V_1 = i_1R_1 + (i_1 - i_2)R_3$

$\boxed{V_1 = i_1R_1 + i_1R_3 - i_2R_3}$

$\boxed{V_1 = i_1(R_1 + R_3) - i_2R_3}$ — 1

Loop - 2 : Apply KVL :



$i_2 > i_1$ (Because of loop)

$V_2 + i_2 R_2 + (i_2 - i_1) R_3 = 0$

$\Rightarrow \boxed{(i_2 - i_1) R_3 + i_2 R_2 = -V_2}$

$\Rightarrow \boxed{-i_1 R_3 + i_2 (R_2 + R_3) = -V_2}$ — (2)

Solving (1) and (2), we get i_1 & i_2

$I_1 = i_1$

$I_2 = i_2$

$I_3 = (i_1 - i_2)$

Cramer Rule :

$[R] \cdot [I] = [V]$

Size $\Rightarrow$ 2x2 (2 loops)

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

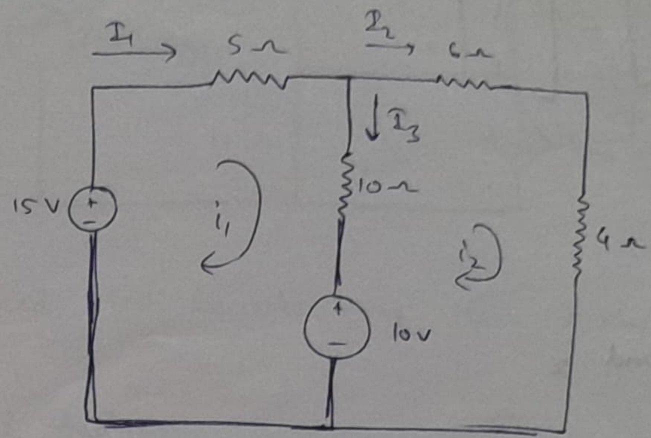
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} +V_1 \\ -V_2 \end{bmatrix}$$

R_{11} → Resistance in 1st loop

R_{12}, R_{21} → Resistence b/w 1st and 2nd loop

R_{22} → Resistance in 2nd loop

Q) Find I_1, I_2 and I_3



M-II:

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

M-I:

From Loop-I:

$$15 - 5i_1 - 10(i_1 - i_2) - 10 = 0 \Rightarrow -15i_1 + 10i_2 + 5 = 0$$

$$\Rightarrow \boxed{15i_1 - 10i_2 = 5} \quad -(1)$$

From loop-II:

$$-6i_2 - 4i_2 + 10 - 10(i_2 - i_1) = 0 \Rightarrow 10i_1 - 20i_2 + 10 = 0$$

$$\Rightarrow \boxed{20i_2 - 10i_1 = 10} \quad -(2)$$

$$30i_1 - 20i_2 = 10$$
$$-10i_1 + 20i_2 = 10$$

$$\boxed{i_1 = 1A} \Rightarrow \boxed{i_2 = 1A}$$

$$i_3 = i_1 - i_2 = 0A$$

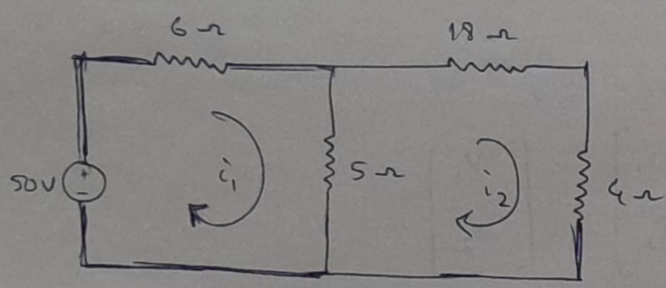
(OR)

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\Rightarrow i_1 = 1\,A$

$i_2 = 1\,A$

Q) Find Mesh Currents i_1 and i_2



$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11 & -5 \\ -5 & 27 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$\Rightarrow \boxed{11 i_1 - 5 i_2 = 50}$

$-5 i_1 + 27 i_2 = 0 \Rightarrow \boxed{27 i_2 = 5 i_1}$

$\cancel{i_2 = 0.9025}$

$\cancel{i_1 = 4.8735}$

(OR) :

Loop I : $50 - 6i_1 - 5(i_1 - i_2) = 0$

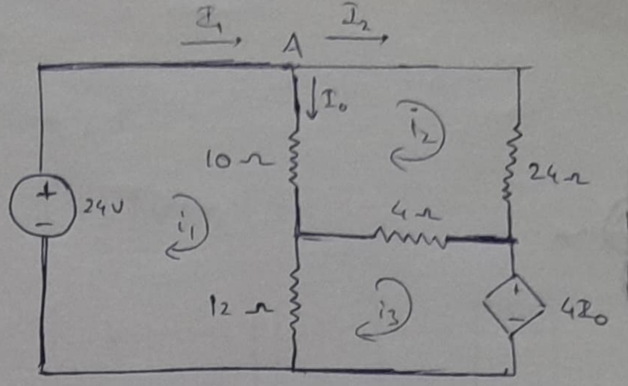
$\Rightarrow \boxed{50 = 11 i_1 - 5 i_2}$

Loop II : $-18 i_2 - 4 i_2 - 5(i_2 - i_1) = 0$

$\Rightarrow \boxed{5 i_1 = 27 i_2}$

$$\boxed{\begin{aligned} i_1 &= -4.8735 \\ i_2 &= 0.9025 \end{aligned}}$$

H.W Q)



Find Mesh Currents using Mesh Analysis

→ **Nodal Analysis:**

Assumption:

a (+) → Actual Node

b (−) → Reference Node (0V)

(i) Write the equations for 'n-1' nodes.

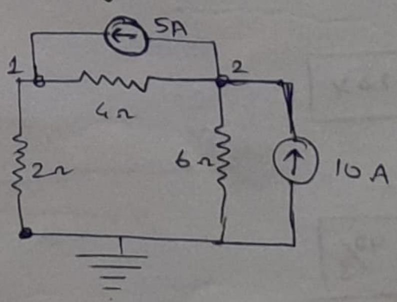
(ii) i → Always flows from higher Potential to lower Potential

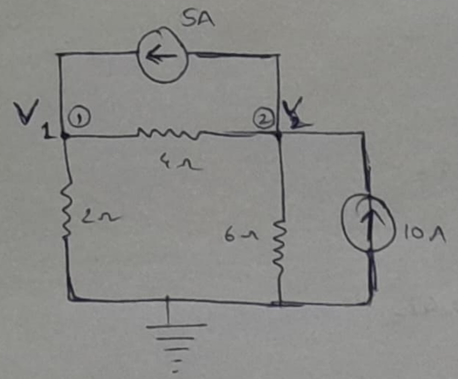
$$i = \frac{V_{high} - V_{low}}{R}$$

(iii) Apply KCL for each Node to obtain branch currents in terms of Node voltages using Ohm's Law.

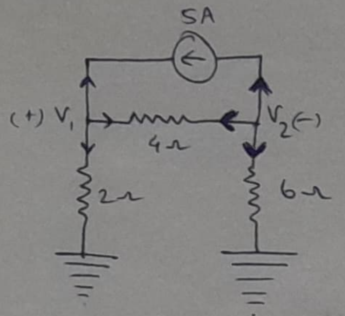
(iv) Find Node Voltage by solving Simultaneous equations

Q) Find Node Voltages at 1 and 2. for the given Circuit.





Step-I :



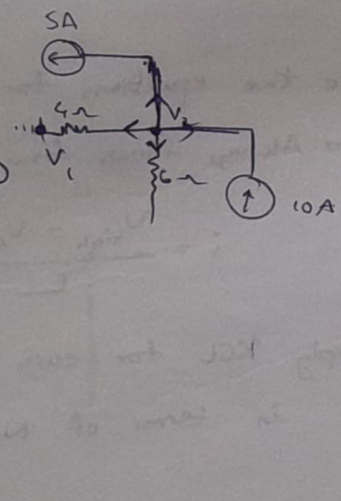
KCL : at V_1 Node

$$\frac{V_1}{2} + \frac{V_1 - V_2}{4} - 5 = 0$$

$$\Rightarrow 2V_1 + V_1 - V_2 - 20 = 0$$

$$\Rightarrow \boxed{3V_1 - V_2 = 20} \quad -①$$

KCL at Node V_2 :



$$\frac{V_2}{6} + 5 - 10 + \frac{V_2 - V_1}{4} = 0$$

$$\Rightarrow \frac{V_2}{6} + \frac{V_2 - V_1}{4} + 5 = 10$$

$$\Rightarrow \frac{V_2}{6} + \frac{V_2 - V_1}{4} = 5$$

$$\Rightarrow 2V_2 + 3V_2 - 3V_1 = 60$$

$$\Rightarrow \boxed{-3V_1 + 5V_2 = 60} \quad -②$$

① + ② ,

$$4V_2 = 80 \Rightarrow \boxed{V_2 = 20\,V}$$

$$3V_1 = 40$$

$$\Rightarrow \boxed{V_1 = 40/3}$$

(OR)

Cramer's Rule:

$$[G]\cdot[V] = [I]$$

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$|G| = 12$$

$$V_1 = \frac{\nabla V_1}{\nabla} \qquad \nabla V_1 = \begin{bmatrix} 20 & -1 \\ 60 & 5 \end{bmatrix}$$

$$V_2 = \frac{\Delta V_2}{\nabla}$$

$$|\nabla V_1| = 160$$

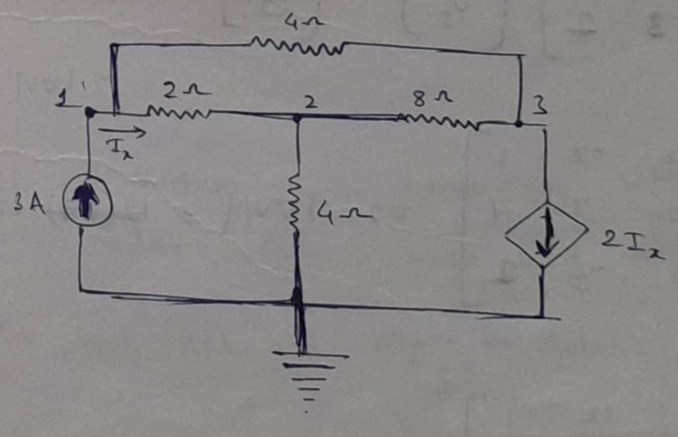
$$\nabla V_2 = \begin{bmatrix} 3 & 20 \\ -3 & 60 \end{bmatrix}$$

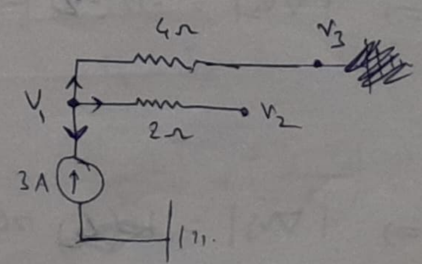
$$\Rightarrow V_1 = \frac{160}{12} = 13.33\text{ V}$$

$$V_2 = \frac{240}{12} = 20\text{ V}$$

$$|\nabla V_2| = 240$$

Q) Obtain Node Voltages



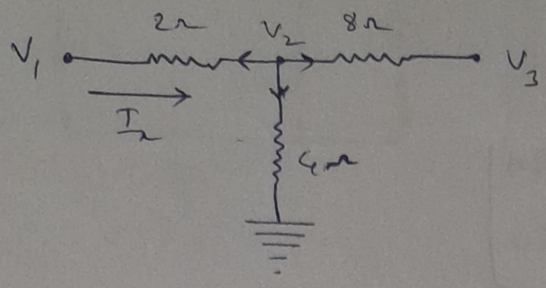


KCL for Node -I

$$\frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2} - 3 = 0$$

$$\Rightarrow V_1 + 2V_1 - 2V_2 - 12 = 0 - V_3$$

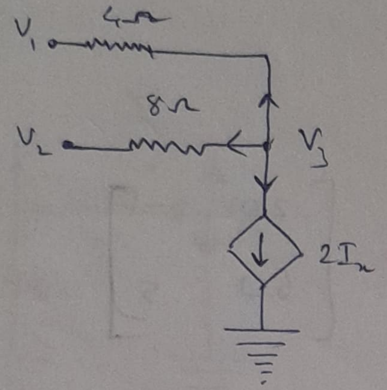
$$\Rightarrow \boxed{3V_1 - 2V_2 = 12 - V_3} \quad (1)$$



$$\frac{V_2}{4}+\frac{V_2-V_1}{2}+\frac{V_2-V_3}{8}=0$$

$$\Rightarrow 2V_2+4V_2-4V_1+V_2-V_3=0$$

$$\Rightarrow \boxed{-4V_1+7V_2-V_3=0}$$



$$\frac{V_3-V_1}{4}+\frac{V_3-V_2}{8}+2I_x=0$$

$$\boxed{I_x=\frac{V_1-V_2}{2}}$$

$$2V_3-2V_1+V_3-V_2+8V_1-8V_2=0$$

$$\Rightarrow \boxed{6V_1-9V_2+3V_3=0}$$

$$\Rightarrow \boxed{2V_1-3V_2+V_3=0}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow |\nabla V| = (3)(4) + 2(-2) + 0$$

$$\Rightarrow |\nabla V| = 10$$

$$\nabla V_1 = \begin{bmatrix} 12 & -2 & 1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{bmatrix} \Rightarrow |\nabla V_1| = 12 \times 4 = 48$$

$$\nabla V_2 = \begin{bmatrix} 3 & 12 & 1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow |\nabla V_2| = -12(-2) = 24$$

$$\nabla V_3 = \begin{bmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{bmatrix} \Rightarrow |\nabla V_3| = 12(-2) = -24$$

$$V_1 = \frac{48}{10} = 4.8V$$

$$V_2 = \frac{24}{10} = 2.4\ V$$

$$V_3 = \frac{-24}{10} = -2.4\ V$$

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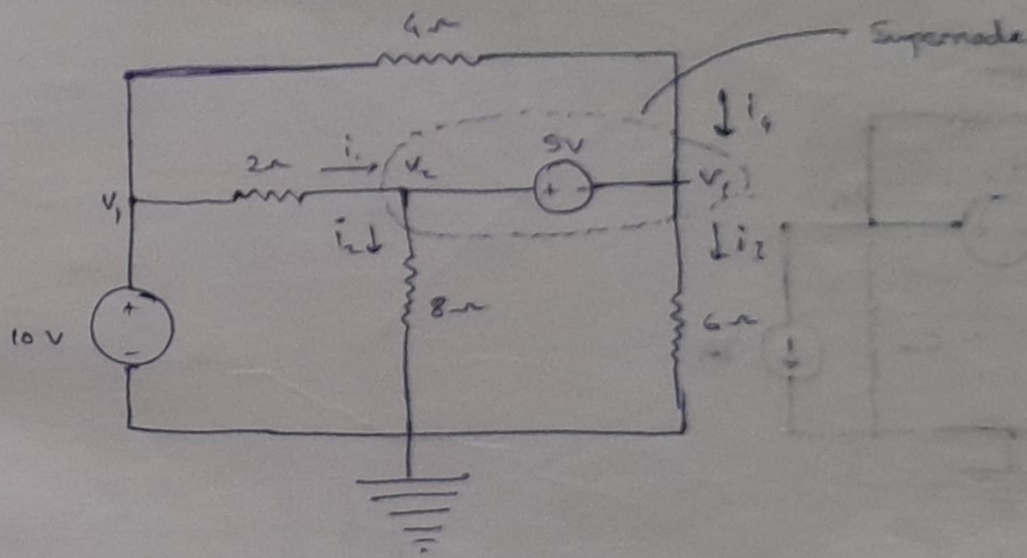
→ Supernode :



If a Voltage source connecting between 2 non-referencing nodes, form a generalised node (or) Supernode.

We apply KCL and KVL to determine the node voltages.

Apply KCL :

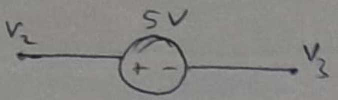
$$\boxed{i_1 + i_4 = i_2 + i_3}$$

$$\Rightarrow \left(\frac{V_1 - V_2}{2}\right) + \left(\frac{V_1 - V_3}{4}\right) = \frac{V_2}{8} + \frac{V_3}{6}$$

$$\Rightarrow 12V_1 - 12V_2 + 6V_1 - 6V_3 = 3V_2 + 4V_3$$

$$\Rightarrow 18V_1 - 15V_2 - 10V_3 = 0 \quad -(1)$$

Apply KVL for supernode :



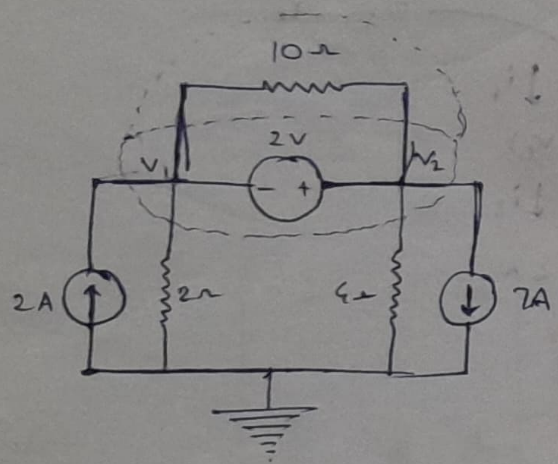
$\cancel{-V_2 + 5 + V_3}$

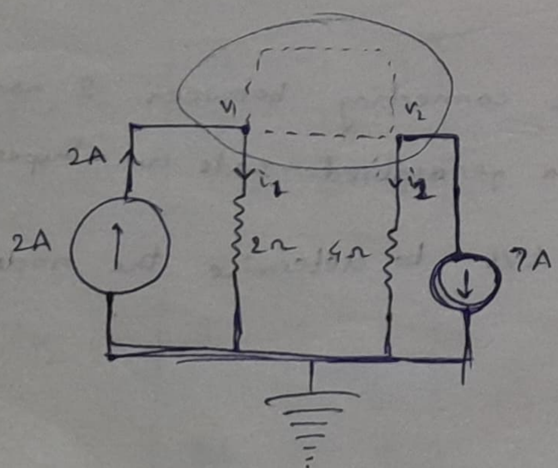
$$-V_2 + 5 + V_3 = 0$$

$$\Rightarrow V_2 - V_3 = 5 \quad -(2)$$

A supernode has no voltage of its own.

Q)





$$2 = i_1 + i_2 + 7$$

$$\Rightarrow 2 = \frac{V_1}{2} + \frac{V_2}{4} + 7$$

$$\Rightarrow \boxed{2V_1 + V_2 = -20} \quad -(1)$$

$$\boxed{V_1 + 2 = V_2} \quad -(2)$$

$\cancel{\Rightarrow 3V_1}$

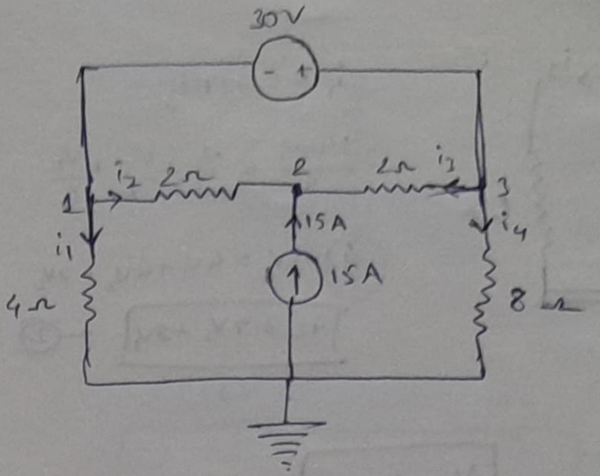
$$\Rightarrow 3V_1 = -22$$

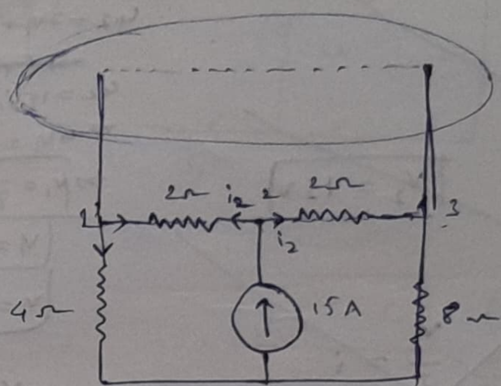
$$\Rightarrow \boxed{V_1 = -22/3}$$

$$V_2 = 2 - \frac{22}{3}$$

$$\boxed{V_2 = \frac{-16}{3}}$$

8)





$i_2 + i_3 = 15$

$\Rightarrow \frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} = 15$

$\cancel{V_3 - V_1 = 30}$

$\boxed{V_1 + 30 = V_3}$ — (2)

$\Rightarrow \boxed{2V_2 - V_1 - V_3 = 30}$ — (1)

$2V_2 - 2V_1 - 30 = 30$

$\Rightarrow \boxed{V_2 - V_1 = 30}$ — (3)

from (2) and (3),

$V_2 = V_3$

$i_2 - i_1 + i_4 + i_3$

$$15 = \frac{V_2 - V_1}{2} - \frac{V_1}{4} + \frac{V_3}{8} + \frac{V_2 - V_3}{2}$$

$$= \frac{4V_2 - 4V_1 - 2V_1 + V_3 + 4V_2 - 4V_3}{8}$$

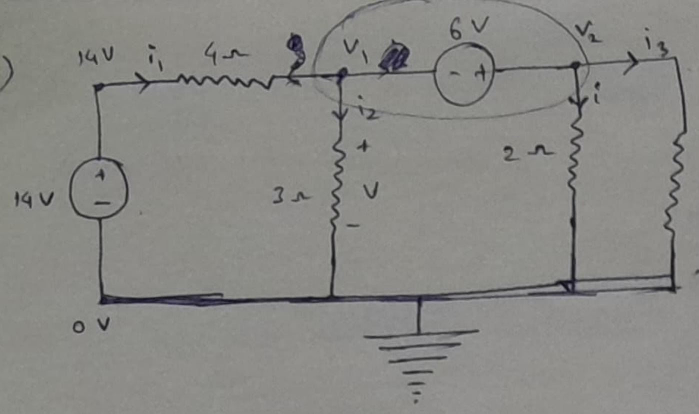
$$= \frac{-6V_1 + 8V_2 - 3V_3}{8}$$

$\Rightarrow -6V_1 + 8V_2 - 3V_3 = 120$

$\Rightarrow \boxed{-6V_1 + 5V_2 = 120}$ — (4) $[V_2 = V_3]$

from (3), (4) $\Rightarrow$ $\boxed{V_1 = 30}$ && $\boxed{V_2 = 60\text{ V}}$ $\boxed{V_3 = 60\text{ V}}$

9)



$i_1 = i_2 + i + i_3$

$$\frac{14-V_1}{4} = \frac{V_1}{3} + \frac{V_2}{2} + \frac{V_2}{6}$$

$$42 - 3V_1 = 4V_1 + 6V_2 + 2V_2$$

$$\boxed{42 = 7V_1 + 8V_2} \quad -②$$

$$\boxed{V_1 + 6 = V_2} \quad -①$$

~~$i_1 = i_2$~~

~~$\Rightarrow \frac{14-V_1}{4} = \frac{V_1}{3}$~~

~~$\Rightarrow 42 - 3V_1 = 4V_1$~~

~~$\Rightarrow \boxed{V_1 = 7V}$~~

~~$\boxed{V = 7V}$~~

~~$\boxed{V_2 = 13V}$~~

~~$\frac{V_2}{2} = i$~~

~~$\Rightarrow \boxed{i = 6.5A}$~~

$42 = 7V_1 + 8(V_1 + 6)$

$42 = 15V_1 + 48$

$\Rightarrow 15V_1 = -6$

$\Rightarrow \boxed{V_1 = \frac{-6}{15} V}$

$\boxed{V_1 = -0.4V}$

$\boxed{V_1 = -400 mV}$

$i = \frac{V_2}{2}$

$i = \frac{5.6}{2} = 2.8A$

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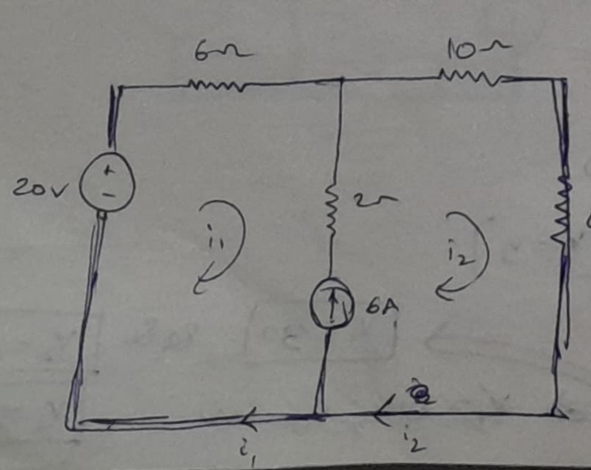
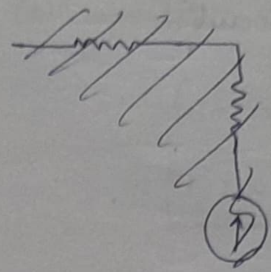
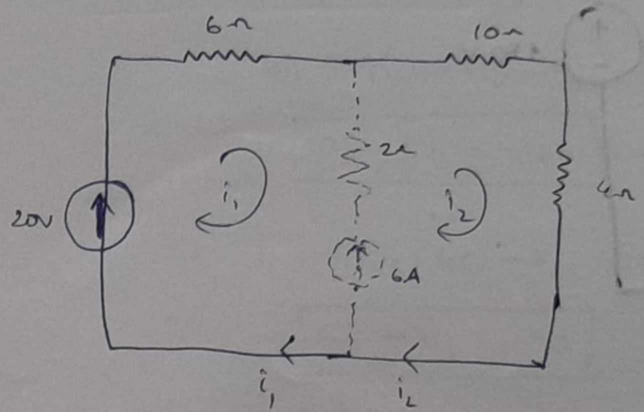
Supermesh : When a current source connected between two loops, it is encircled.

If a Resistance is connected in series with it, it also should be encircled.

Ex.







$$\boxed{i_2 = i_1 + 6}$$ (Super Mesh – KCL) — 1

Apply KVL for loop :

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

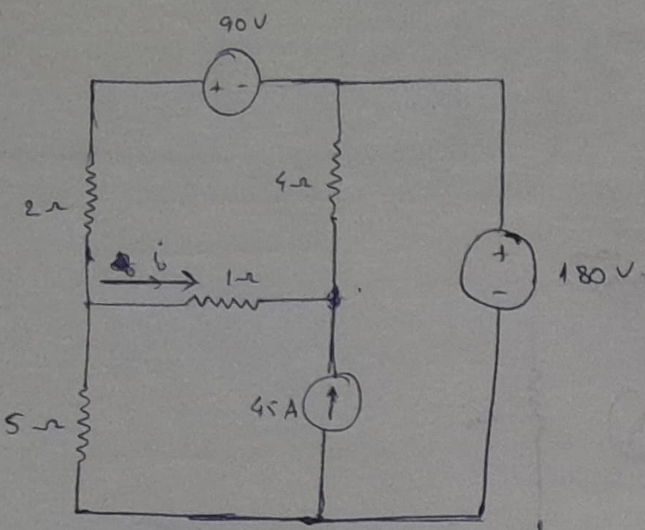
$$\Rightarrow \boxed{6i_1 + 14i_2 = 20} \quad \text{— 2}$$

Solving 1 and 2,

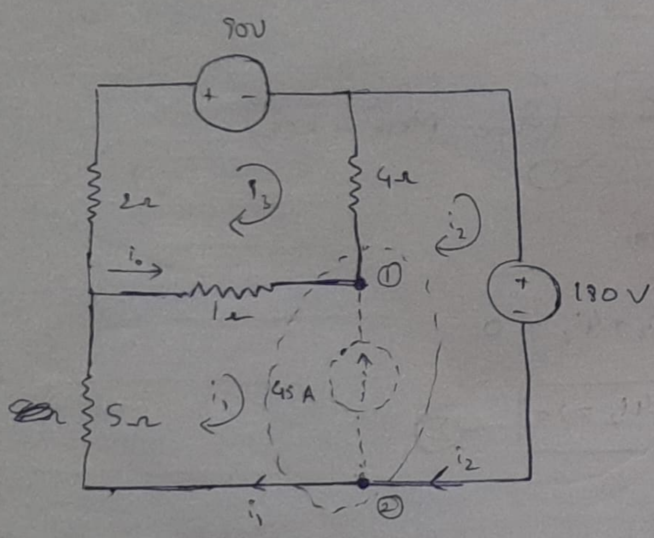
$$i_1 = -3.2\text{ A}$$
$$i_2 = 2.8\text{ A}$$

→ Supermesh does not have its own current.

Q) Use Mesh Analysis to determine i_o in circuit.



Sol:



45 A current source connected b/w 1 & 2. & Hence it forms ~~a~~ Supermesh.

$$\boxed{i_1 + 45 = i_2} \quad -(1)$$

By Applying KCL in Supermesh

Applying KVL in loops:

@ Loop - I and II :

$$-5i_1 + (i_1 - i_3) - 4(i_2 - i_3) - 180 = 0$$

$$\Rightarrow \boxed{6i_1 + 4i_2 - 5i_3 = -180} \quad -(2)$$

@ Loop - III :

$$-4(i_3-i_2) - (i_3-i_1) - 2i_3 - 90 = 0$$

$$\Rightarrow \boxed{i_1 + 4i_2 - 7i_3 = 90} \quad -③$$

Putting ① in ② & ③,

$$6i_1 + 4(i_1+45) - 5i_3 = -180$$

$$\Rightarrow \boxed{10i_1 - 5i_3 = -360} \quad -④$$

$$i_1 + 4(i_1+45) - 7i_3 = 90$$

$$\Rightarrow \boxed{5i_1 - 7i_3 = -90} \quad -⑤$$

$$\begin{bmatrix} 10 & -5 \\ 5 & -7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} -360 \\ -90 \end{bmatrix}$$

$|R| = -45$

$|R_1| = 2070$

$|R_3| = 900$

$i_1 = \frac{2070}{45} = -46\ A$

$i_3 = \frac{900}{45} = 20\ A$

$$R_1 = \begin{bmatrix} -360 & -5 \\ -90 & -7 \end{bmatrix}$$

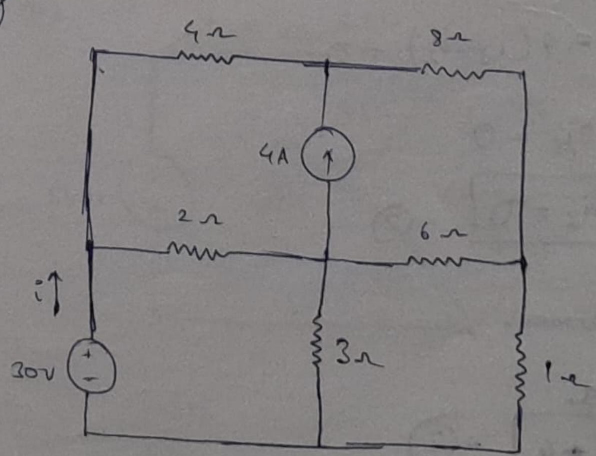
$|R_1| = 2520 - 450$
$= 2070$

$$R_3 = \begin{bmatrix} 10 & -360 \\ 5 & -90 \end{bmatrix}$$

$|R_3| = -900 + 1800$
$= 900$

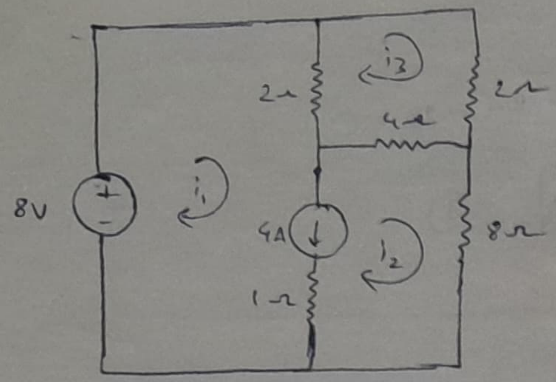
(H.W)

Q)

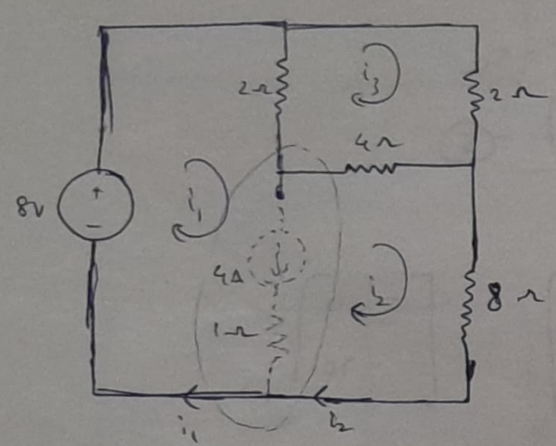


Find i

Q)



Sol:



Applying KVL for Loop ① and ②:

$$8 - 2(i_1 - i_3) - 4(i_2 - i_3) - 8i_2 = 0$$

$$\Rightarrow 8 - 2i_1 - 12i_2 + 6i_3 = 0$$

$$\Rightarrow \boxed{2i_1 + 12i_2 - 6i_3 = 8} \quad - ①$$

Applying KVL for Loop ③:

$$-2(i_3 - i_1) - 2i_3 - 4(i_3 - i_2) = 0$$

$$\Rightarrow 2i_1 + 4i_2 - 8i_3 = 0$$

$$\Rightarrow \boxed{i_1 + 2i_2 + 4i_3 = 0} \quad - ②$$

Applying KCL in Supermesh,

$\cancel{i_2 - i_1}$

$$\boxed{i_1 = i_2 + 4} \quad - ③$$

Sub ③ in ① and ② :

$3i_2 + 4 - 4i_3 = 0$

$\boxed{3i_2 - 4i_3 = -4}$ - ④

$\boxed{14i_2 - 6i_3 = 0}$

$\boxed{7i_2 = 3i_3}$ - ⑤

$3\left(\frac{3i_3}{7}\right) - 4i_3 = -4$

$\Rightarrow \frac{9i_3}{7} - 4i_3 = -4$

$\Rightarrow 9i_3 - 28i_3 = -28$

$\Rightarrow 19i_3 = 28$

$\boxed{i_3 = 28/19 = 1.473 \text{ A}}$

$i_2 = \left(\frac{3}{7}\right) i_3$

$\boxed{i_2 = 0.63158 \text{ A}}$

$\boxed{i_1 = i_2 + 4}$

$\boxed{i_1 = 4.6316 \text{ A}}$

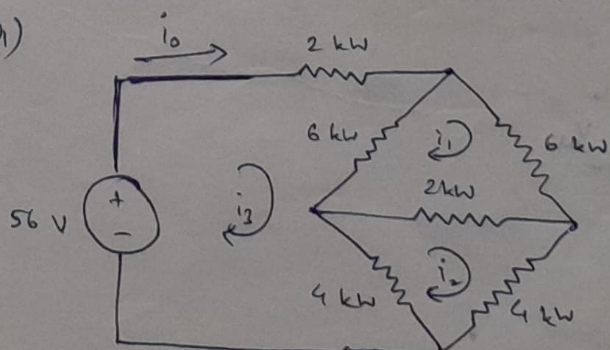
$$\begin{bmatrix} 2 & 12 & -6 \\ 1 & 2 & -4 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$$

$i_2 = \frac{12}{19}$

$i_1 = \frac{88}{19}$

→ Practice Problems :

Q1)



Mesh Analysis
Find i_o.

$-6i_1 - 2(i_1 - i_2) - 6(i_1 - i_3) = 0$ [KVL in Loop I]

$\Rightarrow 14i_1 - 2i_2 - 6i_3 = 0$

$\Rightarrow 7i_1 - i_2 - 3i_3 = 0$ — ①

$-4i_2 - 4(i_2 - i_3) - 2(i_2 - i_1) = 0$ [KVL in Loop 2]

$\Rightarrow 2i_1 - 10i_2 + 4i_3 = 0$

$\Rightarrow i_1 - 5i_2 + 2i_3 = 0$ — ②

$56 - 2i_3 - 6(i_3 - i_1) - 4(i_3 - i_2) = 0$

$\Rightarrow 56 + 6i_1 + 4i_2 - 12i_3 = 0$ — ③

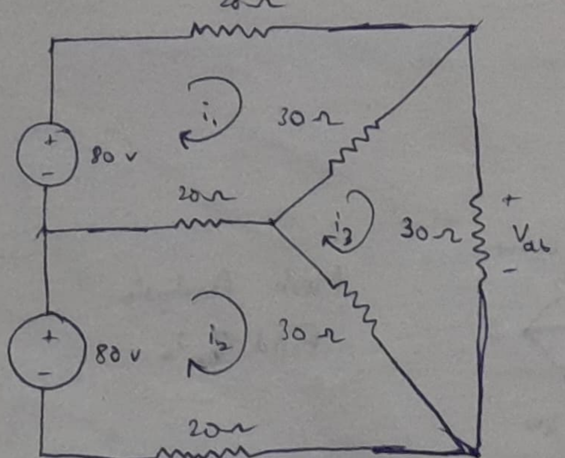
$$\begin{bmatrix} 7 & -1 & -3 \\ 1 & -5 & 2 \\ 6 & 4 & -12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -56 \end{bmatrix}$$

$i_1 = 4A$

$i_2 = 4A$

$i_3 = 6A = i_0$ ✓

9)



$$80 - 20i_1 - 30(i_1 - i_3) - 20(i_1 - i_2) = 0$$

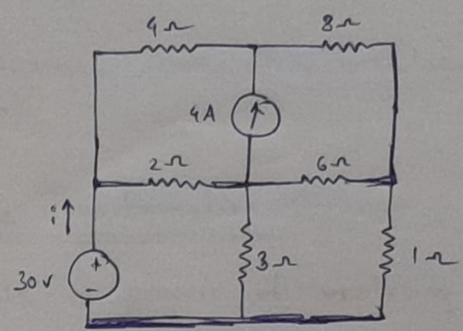
$$80 - 20(i_2 - i_1) - 30(i_2 - i_3) - 20(i_2) = 0$$

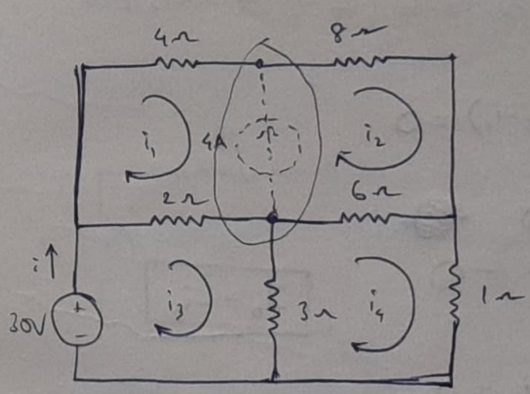
$$-30(i_3 - i_1) - 30(i_3 - i_2) - 30i_3 = 0$$

$$\boxed{V_{ab} = 30 i_3}$$

H.W

Sol :





Loop ③ :

$$30 - 2(i_3 - i_1) - 3(i_3 - i_4) = 0$$

$$\Rightarrow 30 = -2i_1 + 5i_3 - 3i_4 \quad - ①$$

Loop ④ :

$$-3(i_4 - i_3) - 6(i_4 - i_2) - i_4 = 0$$

$$\Rightarrow 6i_2 + 3i_3 - 10i_4 = 0 \quad - ②$$

$$\boxed{i_1 + 4 = i_2} \quad - ④$$

In loop ① and ② :

$$-2(i_1 - i_3) - 4(i_1) - 8(i_2) - 6(i_2 - i_4) = 0$$

$$\Rightarrow -6i_1 - 14i_2 + 2i_3 + 6i_4 = 0$$

$$\Rightarrow 3i_1 + 7i_2 - i_3 - 3i_4 = 0 \quad - ③$$

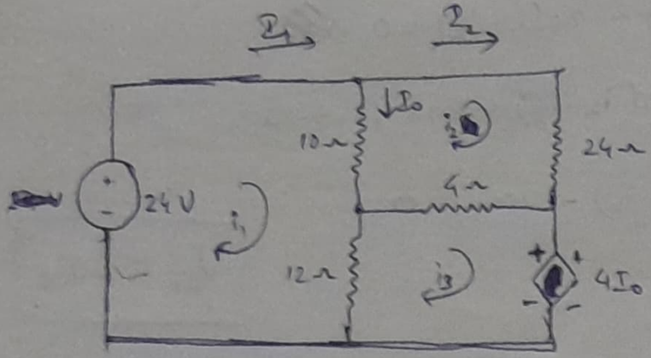
$$i_1 = \frac{-73}{132} = -0.553\ A$$

$$i_2 = \frac{455}{132} = 3.447\ A$$

$$i_3 = \frac{565}{66} = \underline{8.561\ A}$$

$$i_4 = \frac{51}{11} = 4.636\ A$$

HW
Sol :



Mesh Analysis

Loop ① :

$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$

$\Rightarrow 24 = 22i_1 - 10i_2 - 12i_3$

$\Rightarrow 12 = 11i_1 - 5i_2 - 6i_3$ — ①

Loop ② :

$-24i_2 - 4(i_2 - i_3) - 10(i_2 - i_1) = 0$

$\Rightarrow 10i_1 - 38i_2 + 4i_3 = 0$

$5i_1 - 19i_2 + 2i_3 = 0$ — ②

Loop ③ :

$\boxed{I_0 = -i_2}$

$-12(i_3 - i_1) - 4(i_3 - i_2) + 4i_2 = 0$

$\Rightarrow 12i_1 + 8i_2 - 16i_3 = 0$

$\Rightarrow 3i_1 + 2i_2 - 4i_3 = 0$

$i_1 = \frac{216}{65}$

$i_2 = \frac{6}{5}$

$i_3 = \frac{201}{65}$

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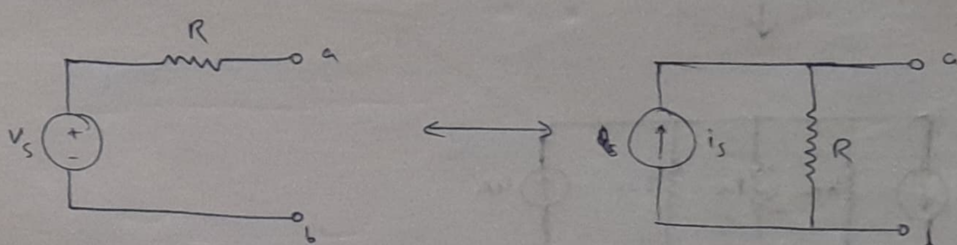
Chapter 4 :

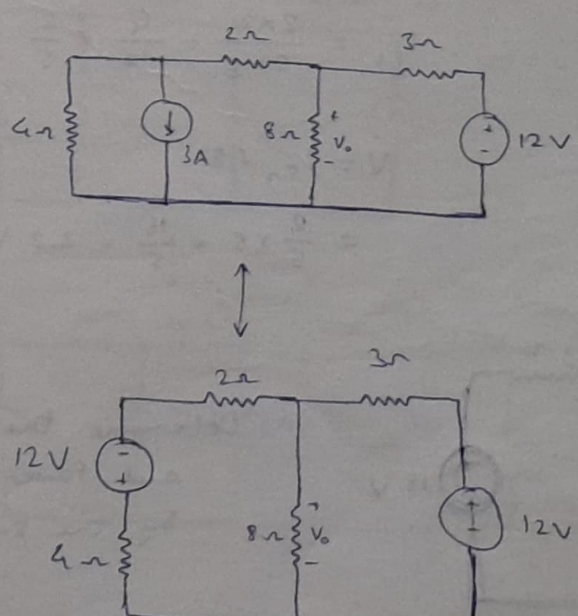
- → Source Transformation
- → Norton's Theorem
- → Thevenin's Theorem
- → Superposition Theorem
- → Maximum Power Transformation Theorem

Source Transformation :

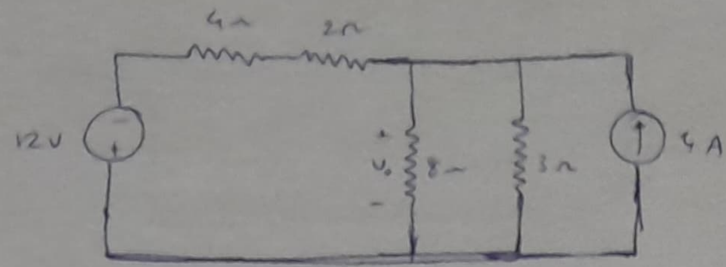
Is a process of replacing voltage source (V_s) in series with a Resistor R by a current source (i_s) in parallel with a Resistor R, or vice versa.



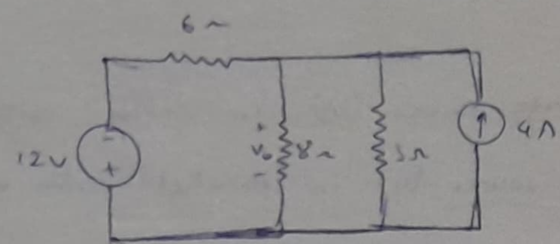
Q) Use Source Transformation to find V_o



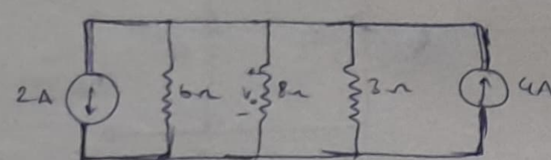
↕



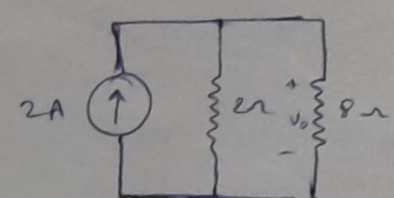
⇓



↕



⇓

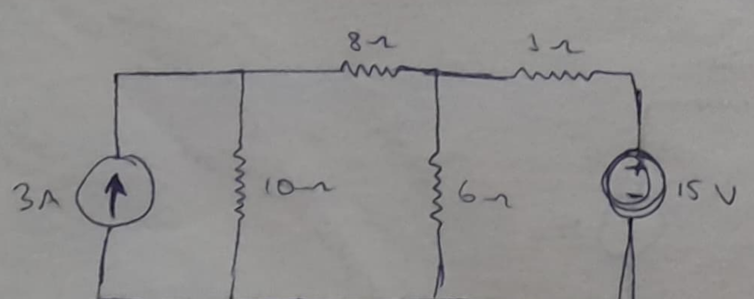


$$i_{8\Omega} = \frac{2\times 2}{2+8} = \frac{4}{10} = \frac{2}{5}$$

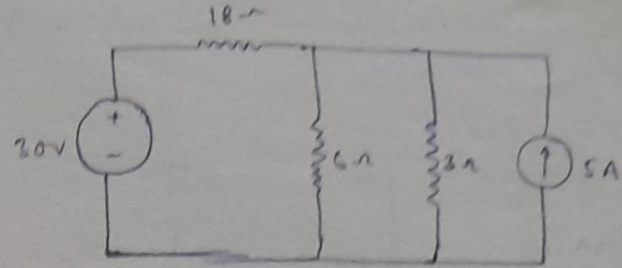
$$V = i_{8\Omega} \times 8$$

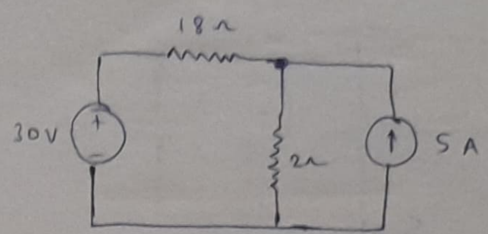
$$= \frac{2}{5} \times 8 = \frac{16}{5} = 3.2\ V$$

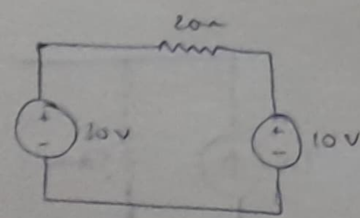
Q)



Determine the current and Power absorbed by the 8Ω resistor.



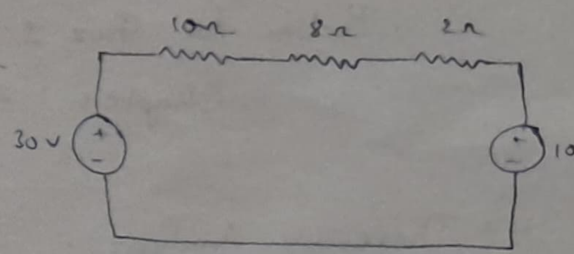




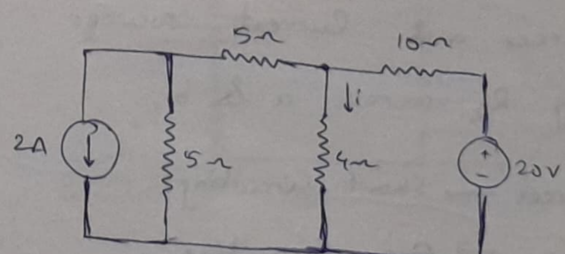
$i = \frac{20}{20} = 1\,A$

$\boxed{i = 1\,A}$

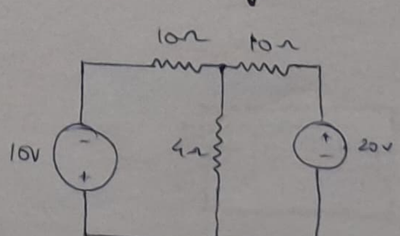
$P = i^2 R = 8\,W$

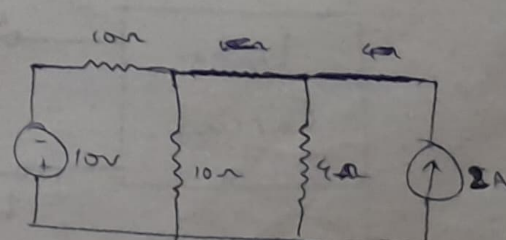


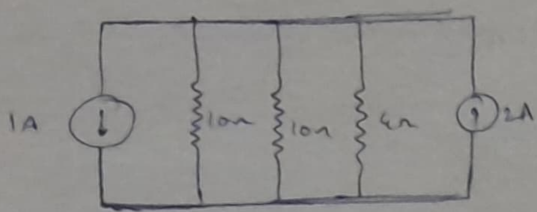
9)

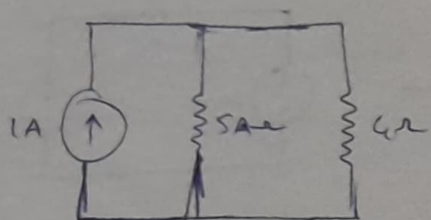


Find 'i'









$$i = \frac{5 \times 1}{9} = \frac{5}{9}\ A$$

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→ **LINEAR CIRCUITS :**

~~Portion for Quiz 1 : Ch 1 to 4~~

Portion for Quiz 1 :

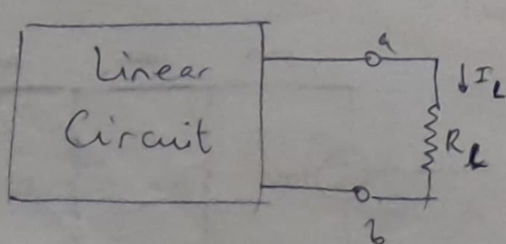
Chapters 1, 2, 3, 4

→ **Thevenin's Analysis :**

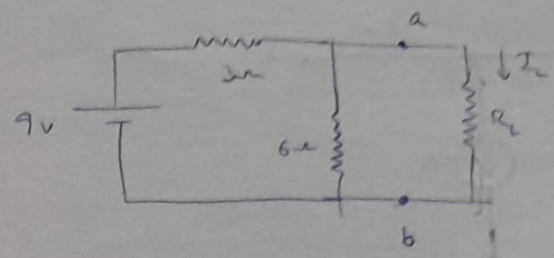
Turn off Voltage Sources and Current sources while calculating R_{Th} across a & b.

Voltage Sources → Short Circuiting

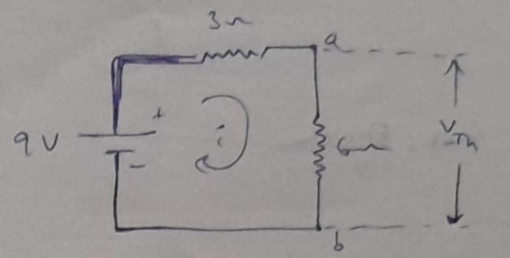
Current sources → Open Circuiting



Q) Find Thevenin's Voltage w.r.t R_L (Load Resistor)



Sol:

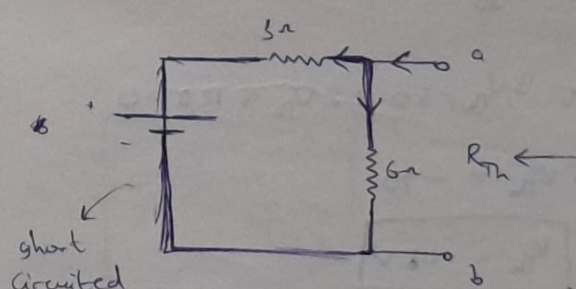


$$V_{Th} = V_{6\Omega}$$

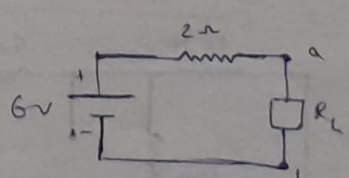
$$\Rightarrow V_{6\Omega} = \frac{9 \times 6}{6+3} = \frac{54}{9} = 6\,V$$

$$\boxed{\therefore V_{Th} = 6\,V}$$

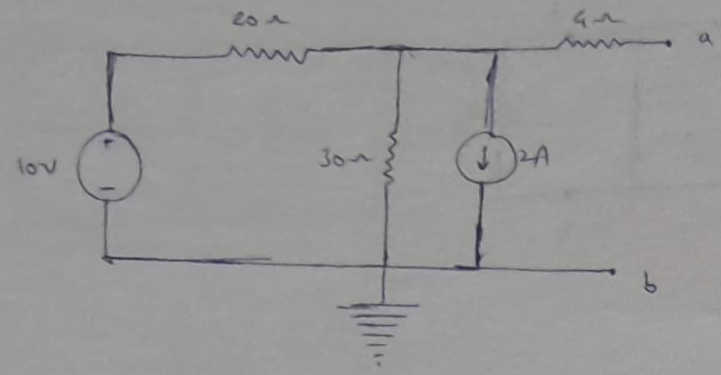
Finding R_{Th}:



$$R_{Th} = 3 \parallel 6 = 2\,\Omega$$

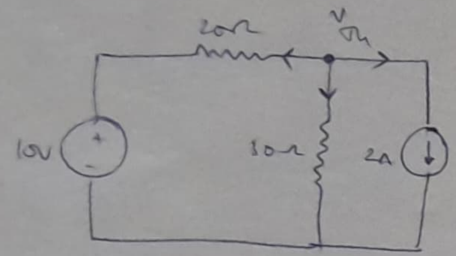


Q) Obtain Thevenin's Equivalent Circuit:



Sol:

No current flowing through 4Ω Resistor



Apply KCL @ V_{Th},

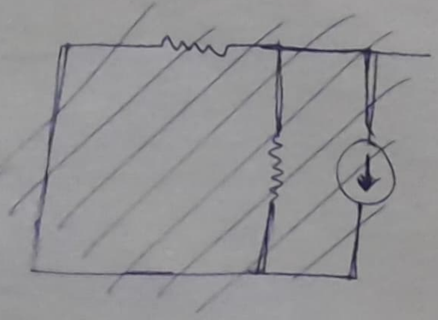
$$\frac{V_{Th}-10}{20}+\frac{V_{Th}}{30}+2=0$$

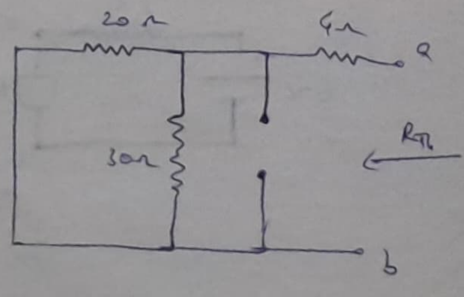
$$\Rightarrow 3V_{Th}-30+2V_{Th}+120=0$$

$$\Rightarrow 5V_{Th}=-90$$

$$\boxed{V_{Th}=-18\,V}$$

For R_{Th},

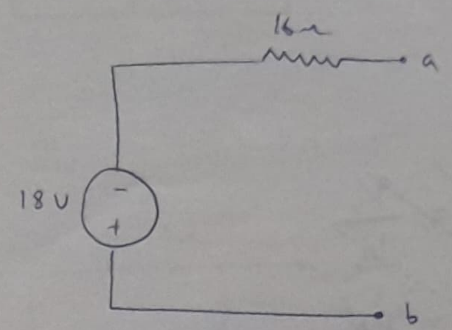




$R_{Th} = 20 \| 30 + 4$

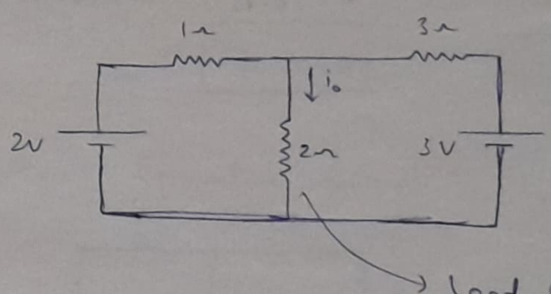
$= 12 + 4$

$\Rightarrow \boxed{R_{Th} = 16\,\Omega}$

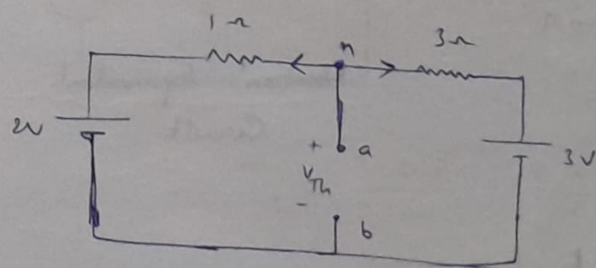


Thevenin's Equivalent Circuit

Q) Using Thevenin's Equivalent Circuit. Calculate i_o through $2\,\Omega$ Resistor.



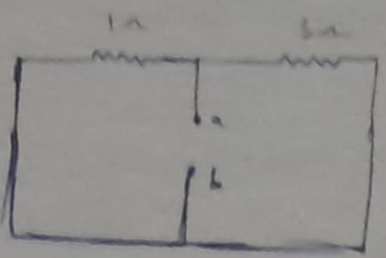
Sol :



$$\frac{V_{Th}-2}{1} + \frac{V_{Th}-3}{3} = 0 \qquad \text{(Applying KCL @ 'n' node)}$$

$\Rightarrow 3V_{Th} - 6 + V_{Th} - 3 = 0$

$\Rightarrow \boxed{V_{Th} = 2.25\ V}$

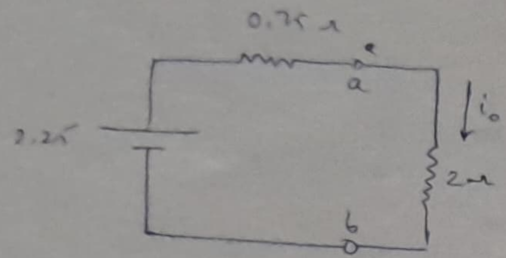


$1 + \frac{1}{3} \Rightarrow \frac{3}{4}$

$R_{Th} = 3/4\ \Omega = 0.75\ \Omega$

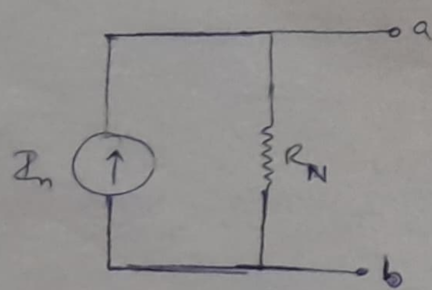
$\cancel{\therefore I_N = \frac{V_{Th}}{R_{Th}} = \frac{2.25}{0.75} = \frac{9/4}{3/4} = 3A}$

Thevenin's Equivalent Circuit :



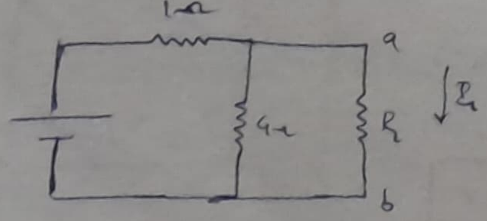
$i_o = \frac{2.25}{2.75} = 0.8182\ A$

→ <u>Norton's Theorem</u> :

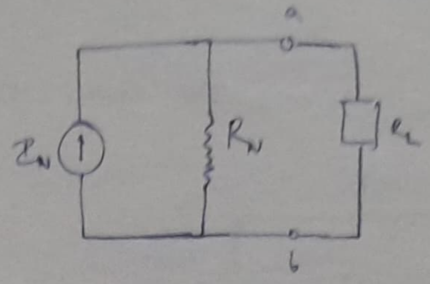


Norton's Equivalent Circuit

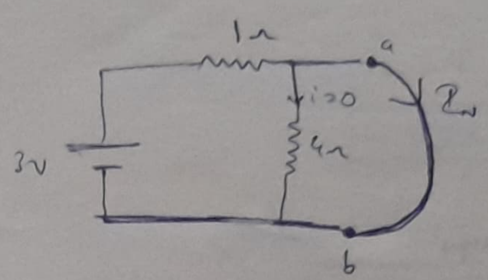
Q) Use Norton's Theorem to find current through R_L, $R_L = 0.7\ \Omega$



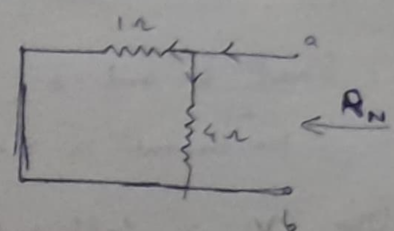
Sol



(i) I_N



(ii) R_N



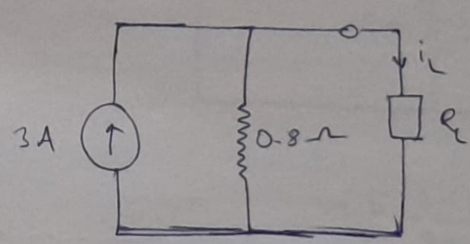
$I_n = \frac{3}{1} = 3A$

$\Rightarrow \boxed{I_N = 3A}$

$R_N = \frac{1\times 4}{1+4} \Rightarrow \boxed{R_N = 4/5\ \Omega}$

~~R_N~~

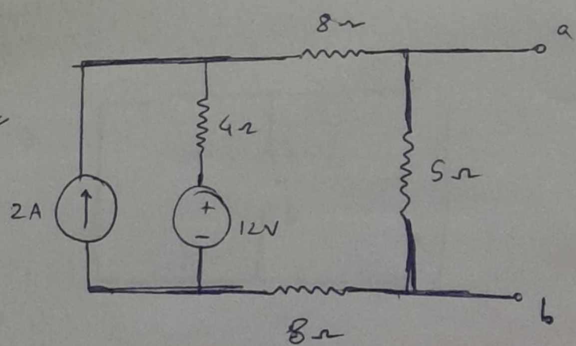
Norton's Equivalent Circuit:



$I_L = \frac{3\times 0.8}{0.8+0.7} = \frac{2.4}{1.5} = 1.6\,A$

$\boxed{\therefore I_L = 1.6\,A}$

Q)
H.W



Obtain Norton's Equivalent Circuit

i.e. find R_N and I_N

I_N : Short a and b

∴ No current in 5Ω Resistor

Use KVL in left and right loops

$i_1 = 2\,A$

$i_2\ (I_N) \rightarrow$ KVL

Ans : $i_2 = 1\,A = i_N$

$R_N = 4\,\Omega$

R_N : Short 12V voltage source

Open 2A current source

$5\Omega \parallel (8\Omega + 4\Omega + 8\Omega) \rightarrow R_N$

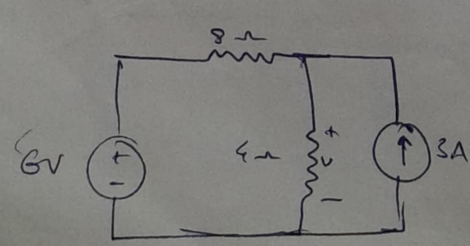
SQ) Solve using Thevenin's Theorem

→ Superposition Theorem :

Can be only used if Linear Circuit has 2 or more independent sources.

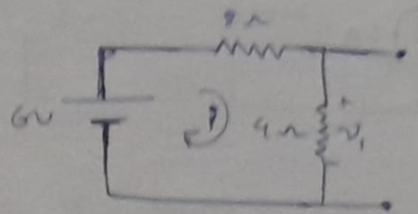
When 1 source is acting alone, the other sources should be shut off.

Q) Use Superposition Theorem to find V



Current Source → Open Circuit
Voltage Source → Close Circuit

(i) When 6V source is Acting :



$i = \frac{6}{12} = \frac{1}{2}$ A

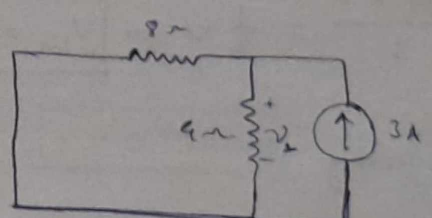
$\boxed{i = 0.5 A}$

$V_{4\Omega} = 0.5 \times 4 = 2V$

$\boxed{V_{4\Omega} = 2V}$ ⇒ $v_1 = 2V$

$\left[v_1 = \frac{6 \times 4}{12} = 2V \right]$

(ii) When 3A source is Acting :



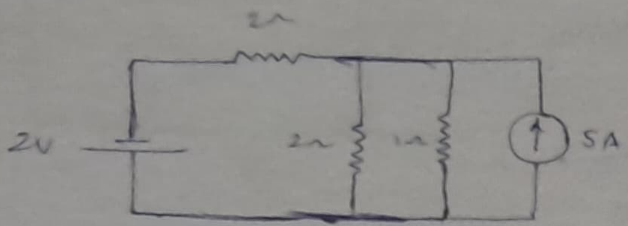
$i = \frac{3 \times 8}{8+4} = 2A$

$\boxed{i = 2A}$

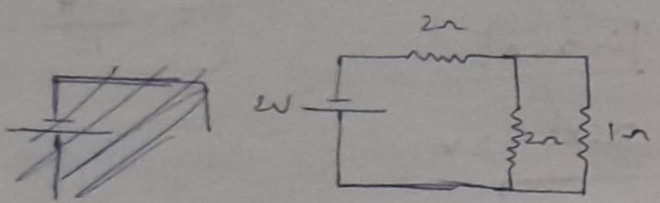
⇒ $\boxed{V_2 = 8V}$

$V_{2\Omega} = V_1 + V_2 = 2 + 8 = 10V$

Q) Calculate current through 1Ω Resistor in the circuit



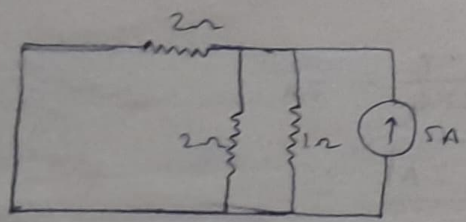
(i) When 2V source is acting



$$i = \frac{2}{\frac{2}{3}+2} = \frac{1}{\frac{1}{3}+1} = \frac{3}{4}\ A$$

$$V_{1\Omega} = \frac{3/4 \times 2}{3} = \frac{1}{2}V \Rightarrow \boxed{V_{1\Omega} = \frac{1}{2}V}$$

(ii) When 5A source is acting

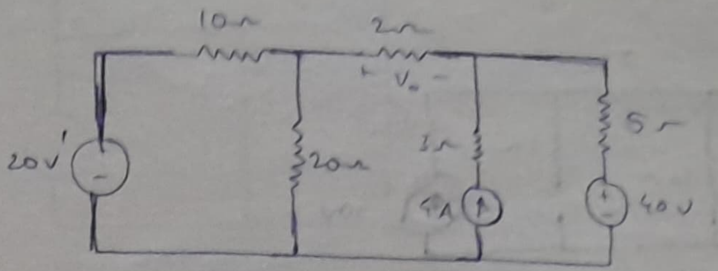


$$i = \frac{5 \times 1}{1+1} = \frac{5}{2}\ A$$

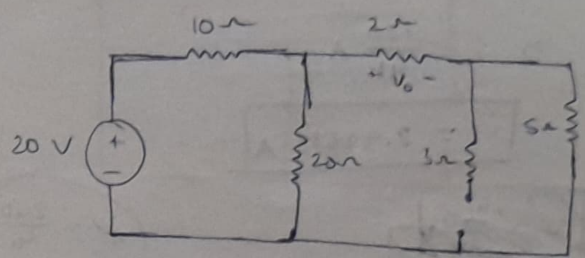
$$\boxed{V_{1\Omega} = \frac{5}{2}\ V}$$

$$\boxed{V_{1\Omega}\ (\text{Total}) = 2\Omega}$$

Q) Find V_0 across $2\,\Omega$



(i) When 20 V source is acting



$$R_{eff} = \frac{20\times7}{20+7} + 10$$

$$R_{eff} = \frac{140}{27} + \frac{270}{27} = \frac{410}{27}$$

$$i = \frac{20}{410}\times 27$$

$$\boxed{i = \frac{54}{41}\,A}$$

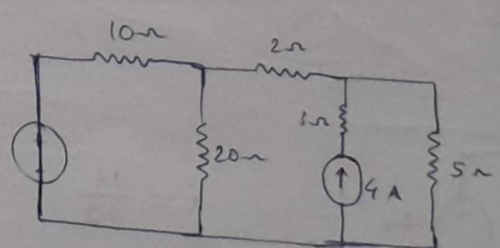
$\cancel{\frac{20\times7}{27} = \frac{140}{27}\,\Omega}$

$$i = \frac{\frac{54}{41}\times 20}{27}$$

$$= \frac{40}{41}\,A$$

$\boxed{i = 0.9756\,A}$ $\Rightarrow$ $\boxed{V = 1.9512\,V}$ $V = \frac{80}{41}$

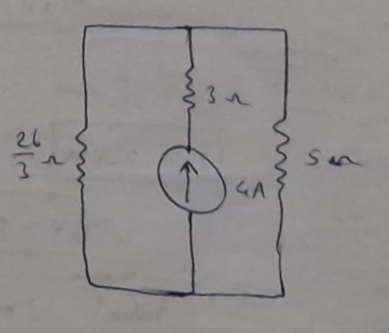
(ii) When 4A source is acting



$$\frac{10\times20}{30} = \frac{20}{3}$$

$$\frac{20}{3} + \frac{6}{3} = \frac{26}{3}$$

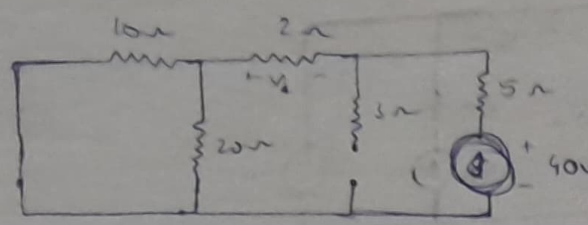
$$\frac{\frac{26}{3}\times5}{\frac{41}{3}} = \frac{26\times5}{41}$$



$$\frac{4\times5}{\frac{26}{3}+5} = \frac{60}{41}\,A \Rightarrow \boxed{i = 1.4634\,A}$$

$\boxed{V = 2.9268\,V}$ $V = \frac{120}{41}$

(iii) When 40V source is Acting



$\frac{20}{3}\Omega + 7\Omega$

$$i = \frac{40}{\frac{20}{3}+7}$$

$$\Rightarrow i = \frac{120}{41}\ A$$

$$\boxed{i = 2.92683\ A}$$

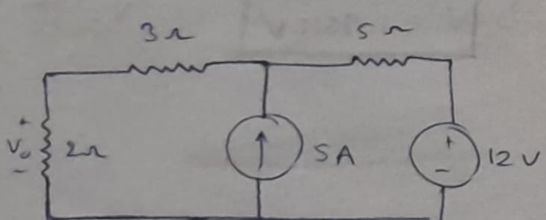
$$\cancel{V = \frac{240}{41}}$$

$$V = \frac{240}{41}$$

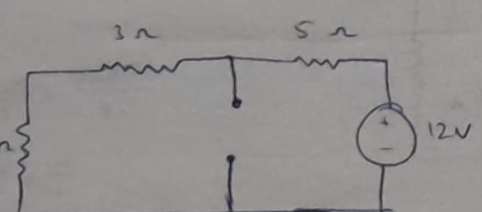
$$\boxed{V = 5.85\ V}$$

Total $\boxed{V = \frac{440}{41}\ V}$

9)



(i)

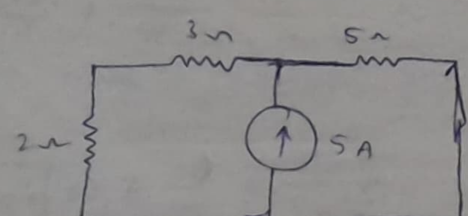


$$i = \frac{12}{10}$$

$$\boxed{i = 1.2\ A}$$

$$\Rightarrow \boxed{V = 2.4\ V}$$

(ii)

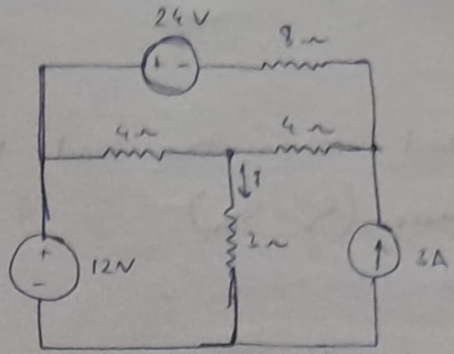


$$\boxed{i = 2.5\ A}$$

$$\Rightarrow \boxed{V = 5\ V}$$

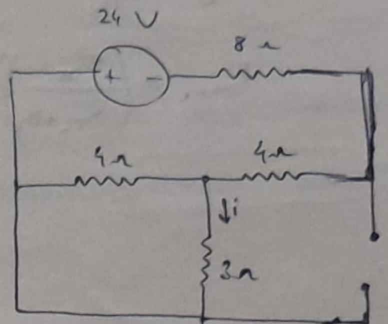
$\boxed{\text{Total } V = 7.4\ V}$

Q)
H.W



Find i in the circuit using Superposition principle and verify using Thevenin's Analysis.

(i)



$$R_{eff} = 12 + \frac{4 \times 3}{7}$$

$$= 12 \times \frac{8}{7}$$

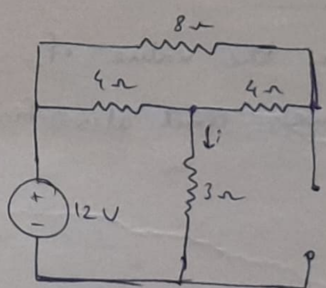
$$i = \frac{24}{12 \times \frac{8}{7}} = \frac{7}{4} \text{ A}$$

$$\boxed{i = 1.75 \text{ A}}$$

$$i_{3\Omega} = \frac{\frac{7}{4} \times 4}{7}$$

$$\boxed{i_{3\Omega} = 1 \text{ A}} \quad - (1)$$

(ii)



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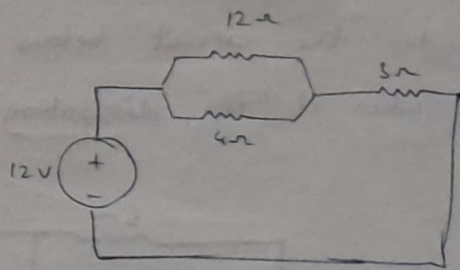
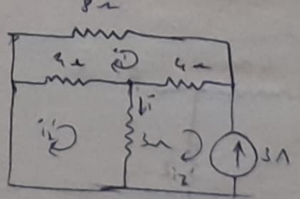


$$i = \frac{12}{6} = 2 \text{ A}$$

$$\boxed{i_{3\Omega} = 2 \text{ A}} \quad - (2)$$

(iii)



$$-3(i_3' - i_2') - 4(i_3' - i_1') - 3 = 0$$

$$\Rightarrow 4i_1' + 3i_2' - 7i_3' = 3 \quad - (1)$$

$$-4(i_2' - i_1') - 3(i_2' - i_3') = 0$$

$$\Rightarrow 4i_1' - 7i_2' + 3i_3' = 0 \quad - (2)$$

$$-8i_1' - 4(i_1' - i_3') - 4(i_1' - i_2') = 0$$

$$\Rightarrow -16i_1' + 4i_2' + 4i_3' = 0 \quad - (3)$$

$$i = i_2' - i_3'$$

$$= \frac{-3}{5} + \frac{9}{10}$$

$$\Rightarrow i = \frac{3}{10}$$

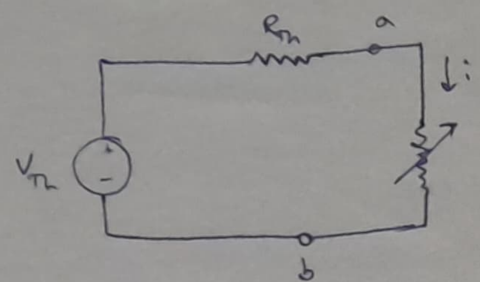
$$\boxed{i_{3\Omega} = \frac{3}{10} \text{ A}}$$

$$\text{Total } i = \frac{81}{20} \text{ A}$$

$$\boxed{i = 4.05 \text{ A}}$$

→ Maximum Power Transfer Theorem :

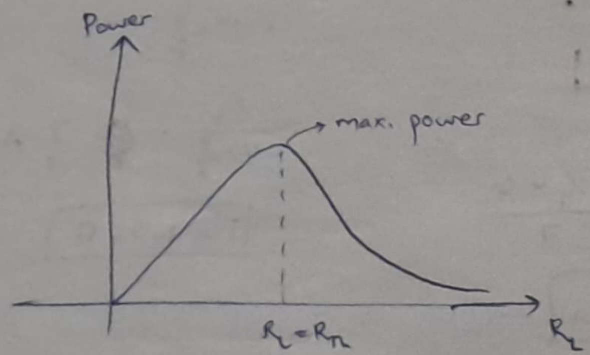
Max. power is transferred to load, when the load Resistance equals to Thevenin's Resistance ($R_L = R_{Th}$)



$R_{max.} = i^2 RL$

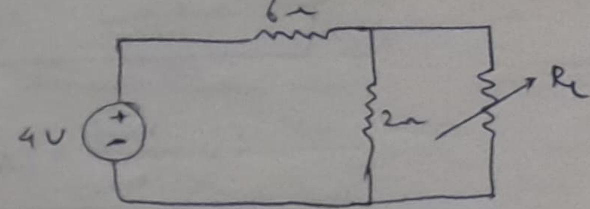
$R_L = R_{Th}$

$P_{max} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$

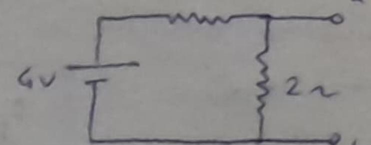


$P_{max.} = \frac{V_{Th}^2}{4 R_{Th}}$

Ex. Consider the circuit below. Determine the value of R_L when it is dissipating max. power and also find P_{max}.



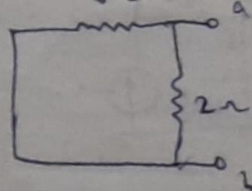
V_{Th}:



$V_{Th} = V_{2\Omega}$

$= \frac{4(2)}{8} = 1V$

R_{Th}:



$R_{Th} = \frac{6 \times 2}{8} = \frac{3}{2} = 1.5\,\Omega$

For max. power,

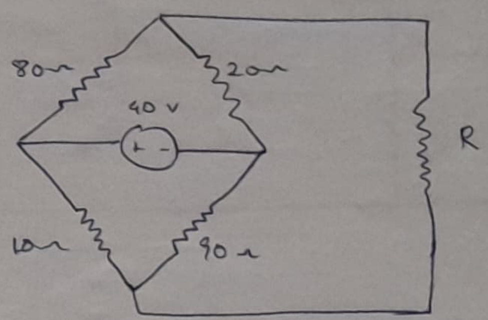
$R_{Th} = R_L = 1.5\ \Omega$

$P_{max.} = \dfrac{V_{Th}^2}{4 R_{Th}} = \dfrac{1^2}{4 \times 1.5} = \dfrac{1}{6}\ W$

Q) The variable Resistor R_L in given figure is adjusted until it absorbs the max. power from the circuit.

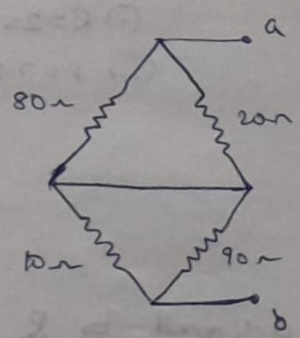
(i) Calculate the value of R for max. power.

(ii) Determine max. power absorbed by R



As R is adjusted, $\therefore R = R_{Th}$

For finding R_{Th}:



$20 \parallel 80 + 90 \parallel 10 \Rightarrow R_{Th}$

$\dfrac{20 \times 80}{100} + \dfrac{90 \times 10}{100} = R_{Th}$

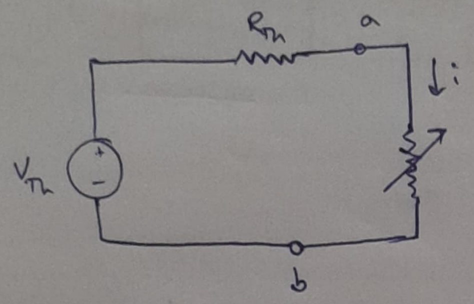
$16 + 9 = 25$

$\boxed{R_{Th} = 25\ \Omega}$

During max power absorption, $R = R_{Th} = 25\ \Omega$

→ Maximum Power Transfer Theorem :

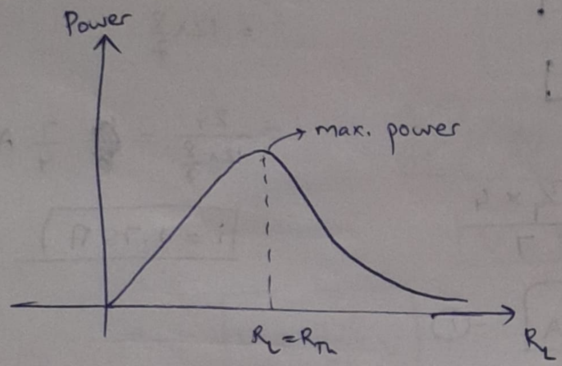
Max. power is transferred to load, when the load Resistance equals to Thevenin's Resistance ($R_L = R_{Th}$)



$$R_{max.} = i^2 RL$$

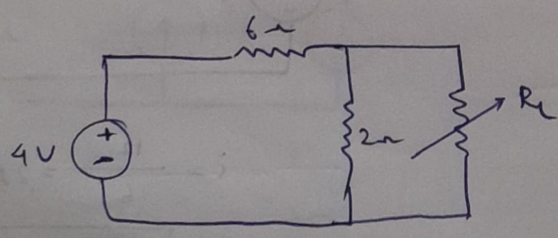
$$R_L = R_{Th}$$

$$P_{max} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

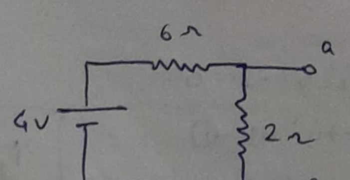


$$P_{max.} = \frac{V_{Th}^2}{4 R_{Th}}$$

Ex. Consider the circuit below. Determine the value of R_L when it is dissipating max. power and also find P_{max}



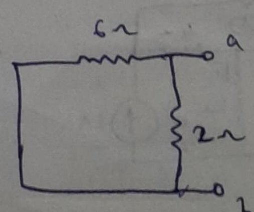
V_{Th} :



$$V_{Th} = V_{2\Omega}$$

$$= \frac{4(2)}{8} = 1\,V$$

R_{Th} :



$$R_{Th} = \frac{6\times 2}{8} = \frac{3}{2} = 1.5\,\Omega$$

For max. power,

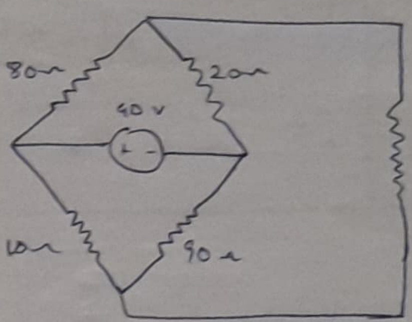
$R_{Th} = R_L = 1.5\ \Omega$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{1^2}{4 \times 1.5} = \frac{1}{6}\ W$$

Q) The variable Resistor R_L in given figure is adjusted until it absorbs the max. power from the circuit.

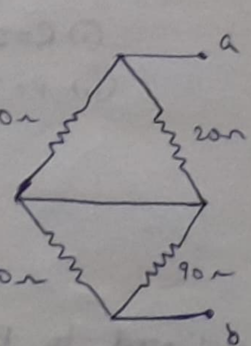
(i) Calculate the value of R for max power.

(ii) Determine max. power absorbed by R



As R is adjusted, ∴ $R = R_{Th}$

For finding R_{Th}:



$20 \parallel 80 + 90 \parallel 10 \Rightarrow R_{Th}$

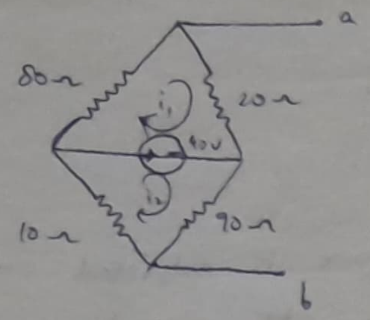
$$\frac{20 \times 80}{100} + \frac{90 \times 10}{100} = R_{Th}$$

$16 + 9 = 25$

$\boxed{R_{Th} = 25\ \Omega}$

During max power absorption, $R = R_{Th} = 25\ \Omega$

For finding V_{Th} :



for ①

$-20i_1 + 40 - 80i_1 = 0$

$\Rightarrow 40 = 100 i_1$

$\Rightarrow \boxed{i_1 = 0.4\ A}$

for ②:

$-90i_2 - 10i_2 - 40 = 0$

$\Rightarrow -100i_2 + 40 = 0$

$\Rightarrow \boxed{i_2 = -0.4\ A}$

$V_a - 20i_1 - 90i_2 = V_b$

$\Rightarrow V_a - V_b = 20i_1 + 90i_2$

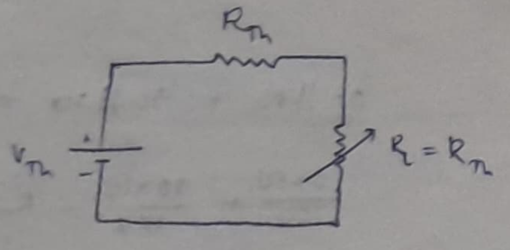
$= -70i_1$

$= -70 \times \frac{4}{10}$

$= -28\ V$

$\boxed{V_{Th} = -28\ V}$

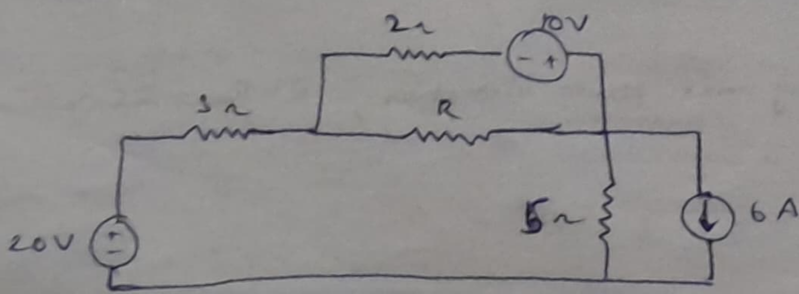
$$P_{max} = \frac{(V_{Th})^2}{4R_{Th}} = \frac{(28)^2}{4 \times 25} = \frac{(28)^2}{100} = 7.84\ W$$



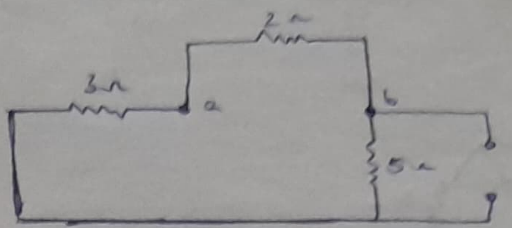
(i) $R = 25\ \Omega$

(ii) $P = 7.84\ W$

Q) Find Max. Power that can be delivered to R

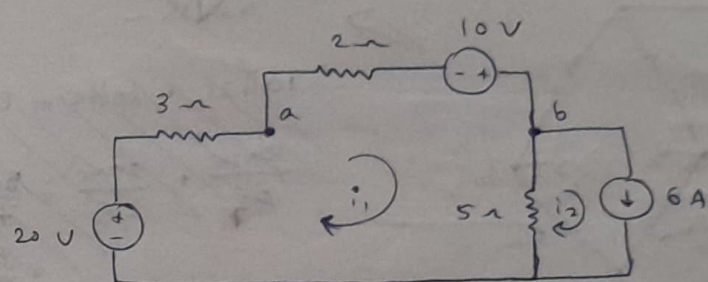


Finding R_{Th} :



$$R_{Th} = 8 \parallel 2$$
$$= \frac{8 \times 2}{10} = 1.6\ \Omega$$
$$\boxed{R_{Th} = 1.6\ \Omega}$$

Finding V_{Th} :



$$20 - 3i_1 - 2i_1 + 10 - 5(i_1 - i_2) = 0$$
$$\Rightarrow 30 - 10i_1 + 5i_2 = 0$$
$$\Rightarrow 10i_1 - 5i_2 = 30$$
$$\Rightarrow \boxed{2i_1 - i_2 = 6}$$

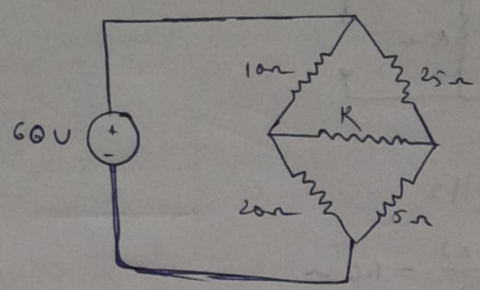
$i_2 = 6\ A$

$$\Rightarrow \quad 2i_1 - 6 = 6$$
$$\boxed{i_1 = 6\ A}$$

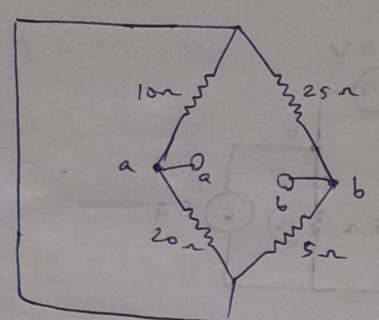
$$V_a - 2i_1 + 10 = V_b$$
$$\Rightarrow V_{Th} = 2i_1 - 10$$
$$= 12 - 10$$
$$= 2\ V$$
$$\boxed{V_{Th} = 2V}$$

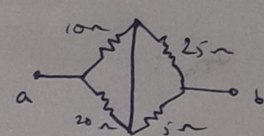
$$P_{max} = \frac{(V_{Th})^2}{4R_{Th}} = \frac{4}{4 \times 1.6} = \frac{5}{8}\ W \Rightarrow \boxed{P_{max} = 0.625\ W}$$
$$\boxed{P_{max} = 625\ mW}$$

Q) Determine max ~~?~~ power delivered to R.



For finding R_{Th} :





~~$10 \| 25 + 20 \| 5 = R_{Th}$~~

~~$\frac{10 \times 25}{35} + \frac{20 \times 5}{25} = \frac{50}{7} + 4 = \frac{78}{7}$~~

~~$R_{Th} = \frac{78}{7}\,\Omega$~~

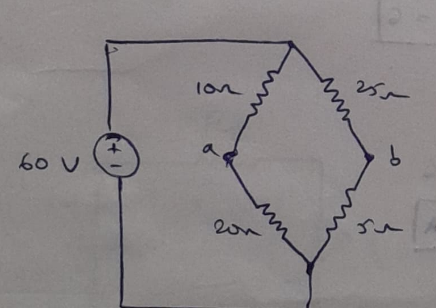
$\frac{35 \times 25}{60}$

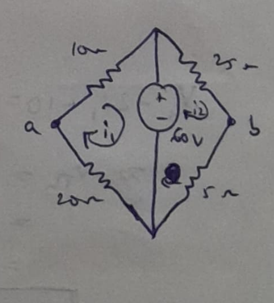
$10 \| 20 + 25 \| 5 = R_{Th}$

$R_{Th} = \frac{20}{3} + \frac{25}{6} = \frac{65}{6}$

$\Rightarrow \boxed{R_{Th} = 10.833\,\Omega}$

For finding V_{Th} :





$V_a - 10i_1 - 25i_2 = V_b$

$\Rightarrow V_a - V_b = V_{Th} = 10i_1 + 25i_2$

$= 15i_2$

$\Rightarrow V_{Th} = 30\,V$

$-10i_1 - 60 - 20i_1 = 0$

$-60 = 10i_1 + 20i_1$

$\boxed{i_1 = -2A}$

$-25i_2 - 5i_2 + 60 = 0$

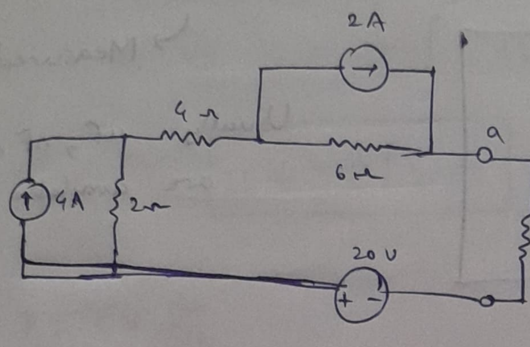
$\boxed{i_2 = 2A}$

$$P_{max} = \frac{(V_{Th})^2}{4R_{Th}} = \frac{900}{4 \times \frac{65}{6}} = \frac{270}{13}\ W$$

$$\therefore P_{max} = 20.769\ W$$

$$\Rightarrow \boxed{P_{max} = 20.77\ W}$$

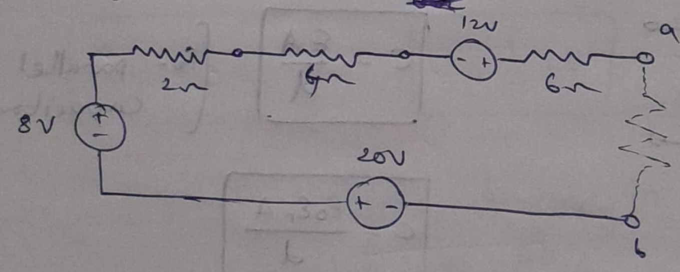
Q)



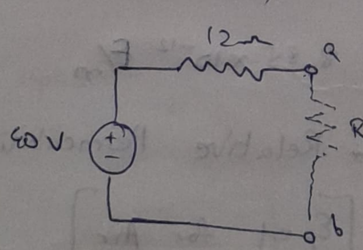
(i) Calculate ~~current in~~ R_L ~~= 10~~

(ii) Find R_L for max power deliverable

(iii) Determine that max power



$\Downarrow$



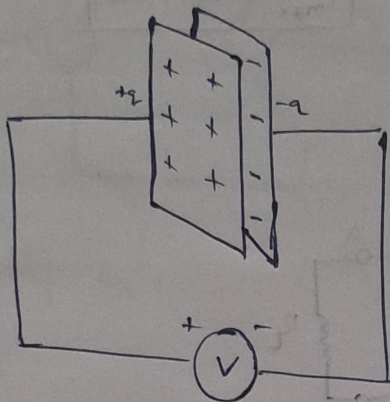
$\underline{R_{Th} = 12\ \Omega}$ & $\underline{V_{Th} = 40\ V}$

$$P = \frac{(V_{Th})^2}{4R_{Th}}$$

$$= \frac{40 \times 40}{4 \times 12} = \frac{400}{12} = 33.33\ W$$

Chapter - 6 : Capacitors and Inductors :

Capacitance and Inductor — Only store energy, No dissipation.

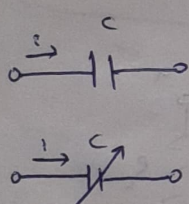


Capacitance
↳ Measured in Farads

Usually μF, nF and pF are available in market

$q \propto V$

$\Rightarrow \boxed{q = CV} \Rightarrow \boxed{C = \frac{q}{V}}$



$\boxed{C = \frac{\varepsilon A}{d}}$ [for parallel plate capacitors.]

$\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$

$q = CV$

$i = \frac{dq}{dt} = C\frac{dv}{dt}$

$\boxed{i = C \cdot \frac{dv}{dt}}$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\varepsilon_r \rightarrow$ Relative Permeability

[$\varepsilon_r = 1$ for Air]

No current flows in Capacitor as $\frac{dv}{dt} = 0$. Hence behaves like open circuited under DC conditions.

$$i_c = C \cdot \frac{dV}{dt}$$

$$\Rightarrow dV = \frac{i_c}{C} dt$$

$$\Rightarrow \int dV = \int \frac{i_c}{C} dt$$

$$\Rightarrow V_c = \frac{1}{C} \int i_c \, dt$$

$$\boxed{\therefore V_c = \frac{1}{C} \int i_c \cdot dt}$$

$$P = CV \cdot \frac{dV}{dt}$$

$$E = \int P \, dt$$

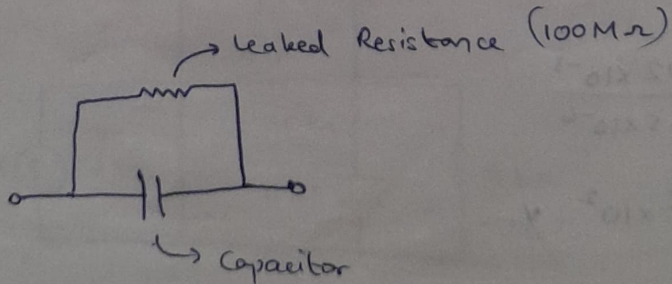
$$= C \int V \, dV$$

$$= \frac{1}{2} C V^2$$

$$\boxed{\therefore E = \frac{1}{2} C V^2} \Rightarrow \boxed{W = \frac{1}{2} C V^2}$$

Capacitor stores Energy in the form of Electrostatic field.

Real Capacitor:



Non-Linear Capacitor

Under DC, Capacitor behaves Open Circuited.

But Battery can Charge Capacitor.

Q) (1) Calculate charge stored on a 3 pF capacitor with 20V across it

A) $Q = cV$

$= 3 \times 10^{-12} \times 20$

$= 6 \times 10^{-11}$ C

$= 60$ pC

(2) Find the Energy Stored in the Capacitor.

A) $W = \frac{1}{2} cV^2$

$= \frac{1}{2} \times 3 \times 10^{-12} \times 20 \times 20$

~~$= 600 \times 10^{-12}$~~

$= 600 \times 10^{-12}$

$= 600$ pJ

$= 0.6$ nJ

Q) What is Voltage across 4.5 μF capacitor if the charge on one plate is 0.12 mC.? How much energy is stored?

$V = \frac{q}{C}$ $\qquad\qquad$ $q = CV$

$= \frac{0.12 \times 10^{-3}}{4.5 \times 10^{-6}}$

$= \frac{12}{45} \times 10^{2}$ V

$E = \frac{1}{2} cV^2$

$= \frac{1}{2} \times 4.5 \times 10^{-6} \times \frac{144}{(45)^2} \times 10^{4}$

$= \frac{1}{2} \times \frac{144}{45} \times 10^{-3} = \frac{72}{45} \times 10^{-3}$ J

Q) Voltage across 5 μF capacitor is

$$v(t) = 10 \cos(6000t) \ V$$

Calculate the current through it.

$$i_c = C \cdot \frac{dv}{dt}$$

$$= 5\times10^{-6} \times 10 \times 6000 \times (-\sin(6000t))$$

$$\Rightarrow i_c = -0.3 \sin(6000t) \ A$$

∴ Capacitor responds to AC sources only.

Q) If a 10 μF capacitor is connected to a voltage source

$$V(t) = 75 \sin(2000t) \ V$$

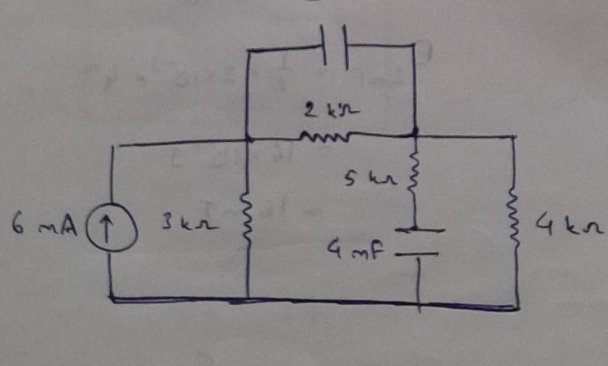
Determine the current

Ans :

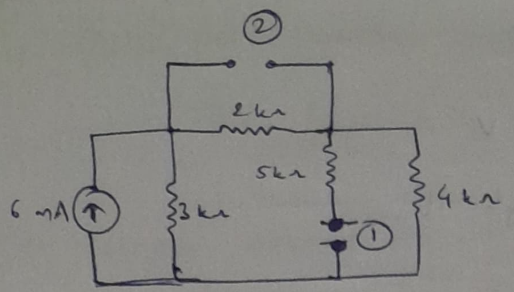
$$i_c = C \cdot \frac{dV}{dt}$$

$$\Rightarrow i_c = 10\times10^{-6} \times \cos(2000t) \times 75 \times 2000$$

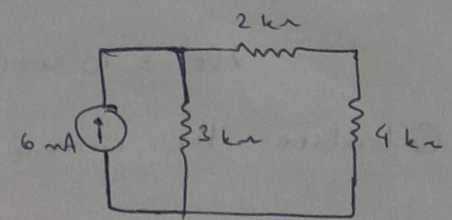
Q) Obtain the Energy stored in each Capacitor under DC condition



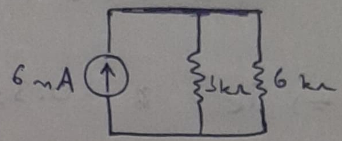
Under DC → Capacitor behaves as Open Circuit



$\Rightarrow$



$\Downarrow$



$V_{4mF} = V_{4k\Omega}$

$V_{2mF} = V_{2k\Omega}$

$\cancel{V_{4k\Omega} = 2A \times 4k\Omega = 8V}$

$\cancel{V_{2k\Omega} = 4 \times 2 =}$

$\cancel{i_{4k\Omega} = \frac{3 \times 6}{9} = 2\,mA}$

$\cancel{i_{2k\Omega} = 2\,mA}$

$V_{4k\Omega} = 2\,mA \times 4\,k\Omega$
$= 8\,V$

$V_{2k\Omega} = 2\,mA \times 2\,k\Omega$
$= 4\,V$

$i_{4k\Omega} = \frac{3 \times 6}{9} = 2\,mA$

$i_{2k\Omega} = 2\,mA$

$E = \frac{1}{2}CV^2 \Rightarrow E_{4mF} = \frac{1}{2} \times 4 \times 10^{-3} \times 8^2$

$= 128 \times 10^{-3}\,J$

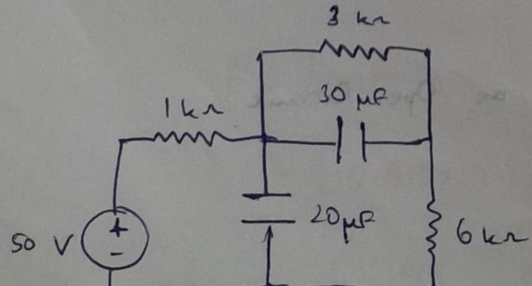
$= 128\,mJ$

$E_{2mF} = \frac{1}{2} \times 2 \times 10^{-3} \times 4^2$

$= 16 \times 10^{-3}\,J$

$= 16\,mJ$

Q)

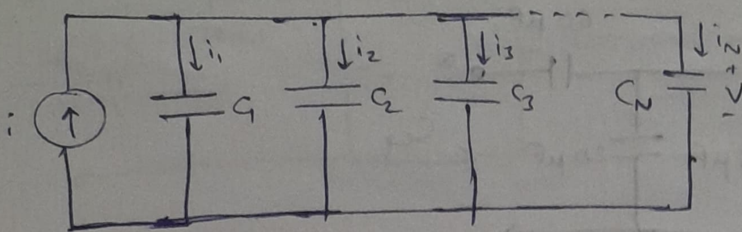


[Ans : 20.25 mJ , 3.375 mJ]

Find Energy Stored in the Capacitors under DC conditions in the given figure

27/09/23

→ Parallel Capacitors :

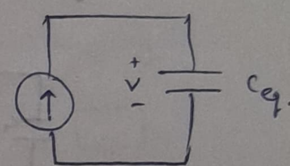


$$i = i_1 + i_2 + i_3 + \dots i_N$$

$$\Rightarrow i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$

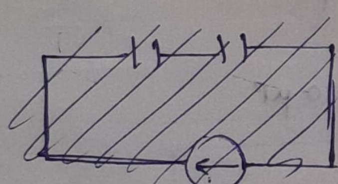
$$i = C_{eq} \cdot \frac{dv}{dt}$$

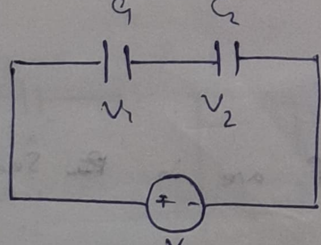
$$\Rightarrow \boxed{C_{eq} = C_1 + C_2 + C_3 + \dots C_N}$$



→ Capacitors are in Series :

$$\left[\begin{array}{c} W_c = \frac{1}{2} c v^2 \\ c = \frac{q}{v} \\ \Rightarrow v = \frac{1}{c} \int i \, dt \\ i = c \frac{dv}{dt} \\ P = cV \cdot \frac{dv}{dt} \end{array}\right]$$





$$V = V_1 + V_2$$

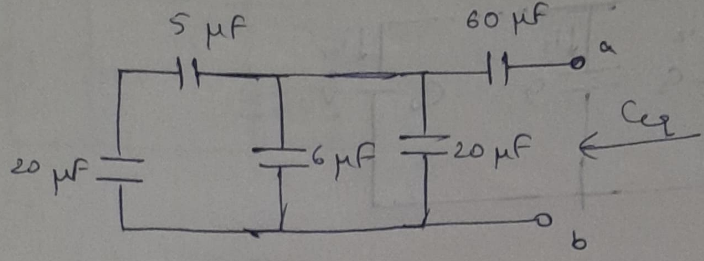
$$\Rightarrow \frac{q}{C_{eq}} = \frac{q_1}{C_1} + \frac{q_2}{C_2}$$

$$[q_1 = q_2 = q]$$

$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \frac{1}{C_N}}$$

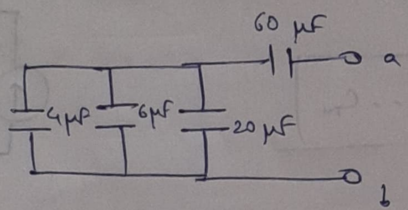
If Capacitors → Series behaves like Resistors → Parallel

Q)



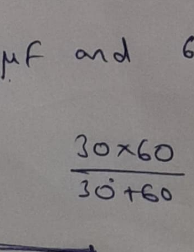
20 μF and 5 μF are in Series

$$\frac{20 \times 5}{20+5} = \frac{100}{25} = 4\ \mu F$$



4 μF, 6 μF and 20 μF → Parallel

$$= 4+6+20 = 30\ \mu F$$

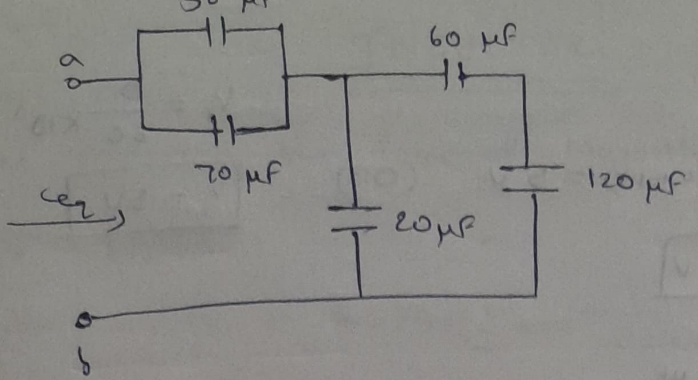


30 μF and 60 μF are in Series

$$\frac{30 \times 60}{30+60} = \frac{1800}{90} = 20\ \mu F$$

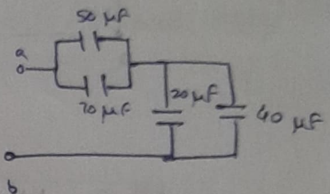
$$\boxed{C_{eff} = 20\ \mu F}$$

Q)

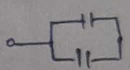


60 µF and 120 µF → Series

$$\frac{60\times120}{60+120} = \frac{7200}{180} = 40\ \mu F$$



⇓



40 µF and 20 µF → Parallel

$40 + 20 = 60\ \mu F$

50 µF and 70 µF → Parallel

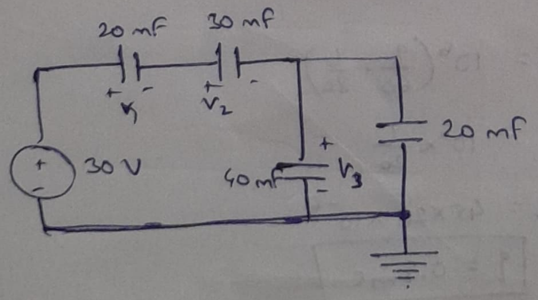
$50 + 70 = 120\ \mu F$

Now 60 µF and 120 µF → series

$$\frac{60\times120}{60+120} = 40\ \mu F$$

$$\boxed{\therefore C_{eff} = 40\ \mu F}$$

Q)



$$\frac{20\times30}{50} = \frac{600}{50} = \frac{60}{5}\ mF$$

20 mF and 40 mF → Parallel

⇒ $20+40 = 60$ mF

60 mF and $\frac{60}{5}$ mF → Series

⇒ 10 mF

$$\boxed{C_{eff} = 10\ mF}$$

$q = CV$

⇒ $q = 30\times10\times10^{-3}$

$= 0.3$ C

$$\boxed{\therefore q = 0.3\ C}$$

$V_1 = q/C_1 = \frac{0.3}{20}\times10^3$

$= 1.5\times10^1 = 15$ V

$V_2 = q/C_2 = \frac{0.3}{30}\times10^3 = 10$ V

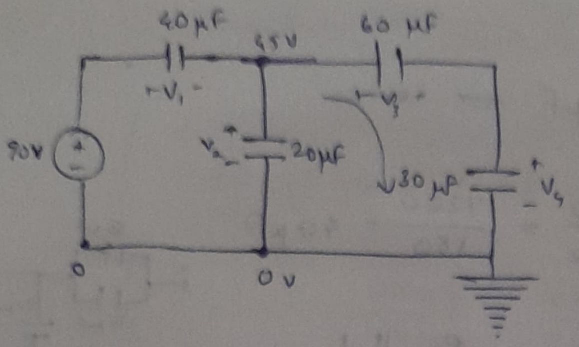
$V = 30$

$V = V_1 + V_2 + V_3$

$\Rightarrow V_3 = 30 - (10+15) = 5\,V$ (OR) $V_3 = \frac{0.3}{60} \times 10^3$

$\boxed{V_3 = 5\,V}$ $\boxed{V_3 = 5V}$

Q)



$C_{eff} = 20\,\mu F$

$q = CV$

$= 20 \times 90 \times 10^{-6}$

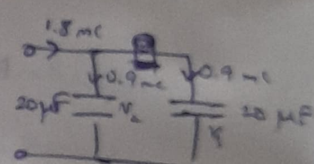
$\Rightarrow \boxed{q = 1.8 \times 10^{-3}\,C}$

$\boxed{q = 1.8\,mC}$

$V_1 = \frac{1.8 \times 10^{-3}}{40 \times 10^{-6}}$

$= \frac{180}{40} \times 10^{1}$

$= \frac{180}{4} = 45\,V$ $\boxed{\therefore V_1 = 45V}$



$V_2 = 45\,V$

$V_3 = 15\,V$

$V_4 = 30\,V$

Using KVL :

$$45 - \frac{q}{60} \times 10^6 - \frac{q}{30} \times 10^6 = 0$$

$$45 = 10^6\left(\frac{q}{60} + \frac{q}{30}\right)$$

$$= 10^6 \times \frac{q}{20}$$

$$q = 45 \times 20 \times 10^{-6}$$

$$\boxed{q = 0.9\,mC}$$

$$V_3 = \frac{0.9 \times 10^{-3}}{60 \times 10^{-6}} = 15\,V$$

$$V_4 = \frac{0.9 \times 10^{-3}}{30 \times 10^{-6}} = 30\,V$$

INDUCTORS :

→ Stores Energy in the form of Magnetic field.

~~Inductor consists~~

→ Inductor consists of a coil of conducting ~~coil~~ wire.

↳ Conductor.

↳ Measured in 'H'

(Length)

$\leftarrow l \rightarrow$

A (Cross sectional Area)

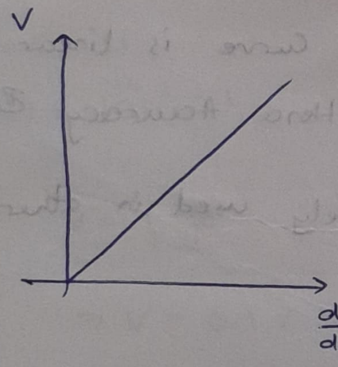
No. of turns of a coil = N

$$\boxed{V = L\frac{di}{dt}}$$

Constant of proportionality called the Inductance of Inductor.

$$\boxed{L = \frac{N^2 \mu A}{l}}$$

Inductance of any Inductor



(Linear Inductor)

$V \propto \frac{di}{dt}$

$\Rightarrow V = L\frac{di}{dt}$.

$\mu = \mu_0 \mu_r$

μ_r ↳ Relative Permeability

μ_0 ↓ Absolute Permeability

$\mu_0 = 4\pi \times 10^{-7}$ F/m

$\mu_r = 1$ for air

If current does not change with time,

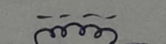
(V = ∞) Capacitor → Open Circuited

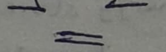
(V=0) Inductor → Short Circuited

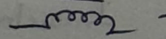
(Under DC situation, V = 0 for Inductor)

μH → Communication System

H → Power Systems

 → Inductor

 → Air Core Inductor

 → Iron core Inductor

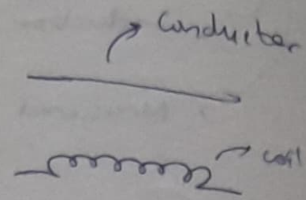
26/09/21

Core → Made up of Ferromagnetic Material

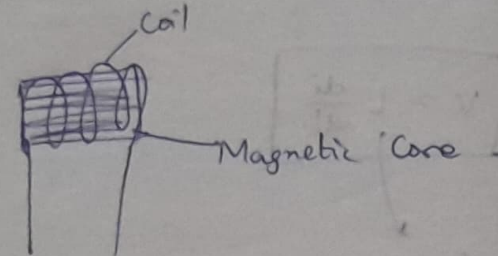
Coil → Made up of Conducting material

Core is wounded over by the coil.

This system → Inductor.



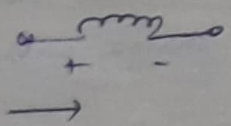
Inductor:

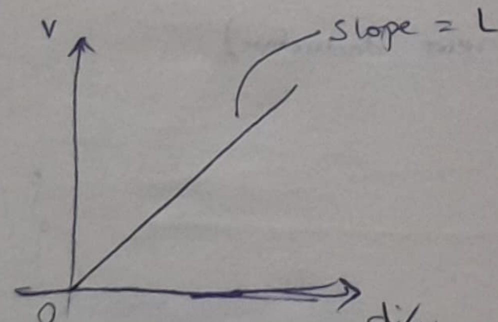


Air Core Inductors are used to make Ammetres and Voltmetres, so that BH Curve is linear and Saturation reduces and Hence Accuracy Increases.

But Air core Inductors are rarely used in other cases.

Symbol:





$$\boxed{V = L \cdot \frac{di}{dt}}$$

Voltage across the Inductor

$$\boxed{i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\, d\tau}$$

Current through the Inductor

$$\boxed{W = \frac{1}{2} L i^2}$$

Energy stored in the Inductor

$$v_L = L\frac{di}{dt}$$

$$i_L = \frac{1}{L}\int v\,dt$$

$$P = i_L \cdot v_L$$

$$\int P\,dt = W = \frac{1}{2}Li^2$$

Capacitors resist sudden change in Voltage.

Inductors resist sudden change in Current.

Resistors resist ~~sudden~~ flow of steady current.

Ideal Inductors do not dissipate power in form of heat. But it is otherwise in real Inductors.

Q) $L = 0.1\ H$

$i(t) = 10te^{-5t}\ A$

Sol: $V = L\frac{di}{dt}$

$\Rightarrow V = 0.1 \times 10\,(e^{-5t} - 5te^{-5t})$

$\cancel{4te^{-5t}(-4)}$

$\cancel{V = -4e^{-5t}\ \text{volts}}$

$\Rightarrow \boxed{V = e^{-5t}(1-5t)\ \text{volts}}$

$W = \frac{1}{2} \times 0.1 \times (10te^{-5t})^2$

$\boxed{W = 5t^2e^{-10t}\ J}$

Q) Find i through 5H Inductor

$$v(t) = \begin{cases} 10t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Find Energy stored at t = 5s. Assume $i(v) > 0$

$$V = L\frac{di}{dt}$$

$$\Rightarrow di = \frac{1}{L} V dt$$

$$\Rightarrow i = \int \frac{1}{L} V dt$$

$$= \frac{1}{L}\int_{-\infty}^{t} V\, dt$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v(t)\, dt$$

$$= \frac{1}{5}\left[\int_{-\infty}^{0} 0\, dt + \int_{0}^{5} 30t^2\, dt\right]$$

$$= \frac{1}{5}\int_{0}^{5} 30t^2\, dt$$

$$= \frac{1}{5}(10 \times 5^3) \Rightarrow i = 2 \times 125 \text{ A}$$

$\cancel{i = 15 \text{ A}}$ $\qquad$ $\boxed{i = 250 \text{ A}}$

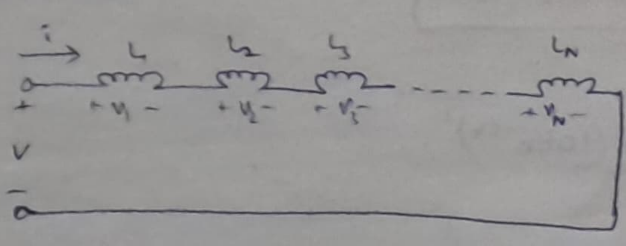
$$W = \frac{1}{2} \times 5 \times (250)^2 \qquad \left[\text{As } v(t) = 0 \ \forall t < 0\right]$$

$$= \frac{1}{2} \times 5 \times 250 \times 250$$

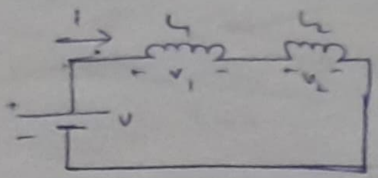
$$\boxed{W = 156.25 \text{ kJ}}$$

$$W\Big]_0^5 = \frac{1}{2}Li^2 - \frac{1}{2}L(0)^2$$

→ **Inductors in Series :**



Let us study only Two Inductors :



According to KVL :

$v = v_1 + v_2$

$\Rightarrow v = L_1 \cdot \frac{di}{dt} + L_2 \frac{di}{dt}$

Let $v = L_{eq} \frac{di}{dt}$

$\therefore \Rightarrow L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$

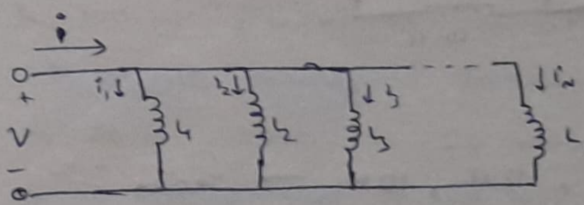
$$\boxed{\therefore L_{eq} = L_1 + L_2}$$

for N Inductors,

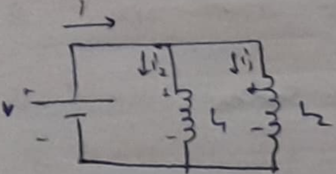
$$\boxed{L_{eq} = \sum_{i=1}^{N} L_i}$$

i.e. $L_{eq} = L_1 + L_2 + L_3 + \cdots L_N$

→ Inductors in Parallel :



Let us consider only two Inductors :



According to KCL :

$i = i_1 + i_2$

$\Rightarrow \frac{1}{L_1} \int_{-\infty}^{t} v\,dt + \frac{1}{L_2} \int_{-\infty}^{t} v\,dt = i$

Let $i = \frac{1}{L_{eq}} \int_{-\infty}^{t} v\,dt$

$\Rightarrow \frac{1}{L_{eq}} \int_{-\infty}^{t} v\,dt = \frac{1}{L_1} \int_{-\infty}^{t} v\,dt + \frac{1}{L_2} \int_{-\infty}^{t} v\,dt$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

For N Inductors,

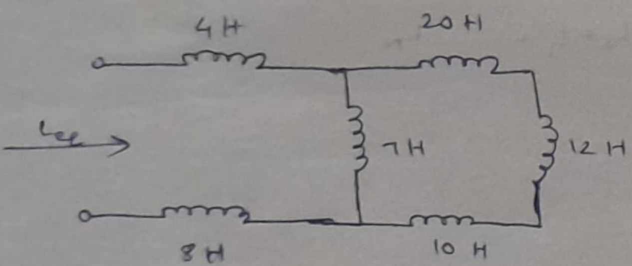
$$\boxed{L_{eq} = \left(\sum_{i=1}^{N} \frac{1}{L_i}\right)^{-1}}$$

i.e. $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots \frac{1}{L_N}$

Series combination of Resistors is similar to that of Inductors &

Parallel combination of Resistors is similar to that of Inductors

Q) Find L_{eq}



20 H, 12 H, 10 H → series

∴ 42 H

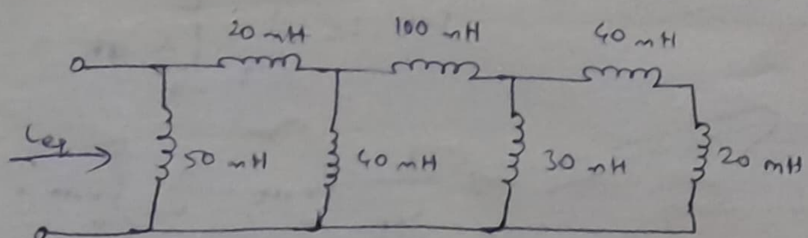
42 H and 7 H → Parallel

∴ 6 H

4 H, 6 H and 8 H → Series

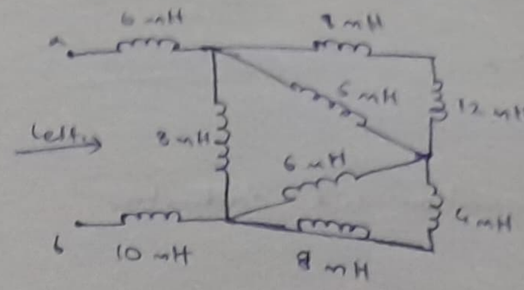
∴ L_{eff} = 18 H

Q)



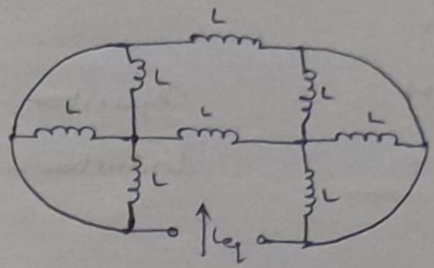
Ans : 25 mH

Q2)



Ans : 20 mH

Q3)



~~[illegible]~~

Ans : $\left(\frac{5}{8}\right)$ L

~~A3)~~

~~[illegible]~~

A1) 20 mH & 40 mH series ⇒ 60 mH

60 mH & 30 mH parallel ⇒ 20 mH

20 mH & 100 mH series ⇒ 120 mH

120 mH & 40 mH parallel ⇒ 30 mH

30 mH & 20 mH series ⇒ 50 mH

50 mH & 50 mH parallel ⇒ 25 mH

Q2

A2) 8 mH & 12 mH series ⇒ 20 mH (up)

4 mH & 8 mH series ⇒ 12 mH (down)

20 mH & 5 mH parallel ⇒ 4 mH

12 mH & 6 mH parallel ⇒ 4 mH

4 mH & 4 mH series ⇒ 8 mH

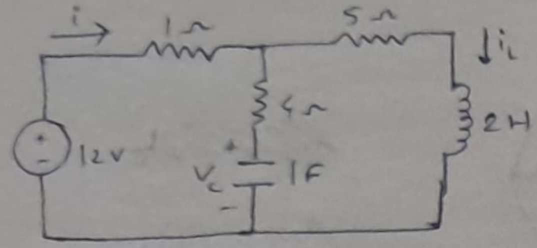
8 mH & 8 mH parallel ⇒ 4 mH

4 mH , 6 mH & 10 mH series ⇒ 20 mH

Q) Consider the circuit Under DC,

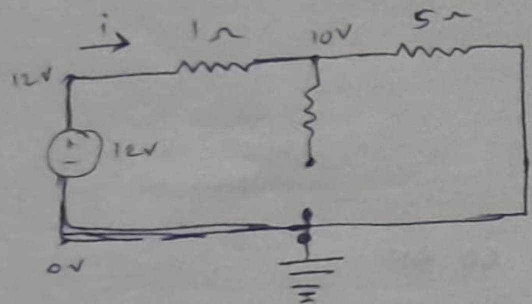
Find (a) i, V_c, and i_L

(b) Energy stored in Capacitor and Inductor



Capacitor – Open

Inductor – Close



$$i = \frac{12}{6} = 2\,A$$

$$V_c = 10\,V$$

$$i_L = i = 2\,A$$

$$E_{Cap.} = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 1 \times 10^2$$

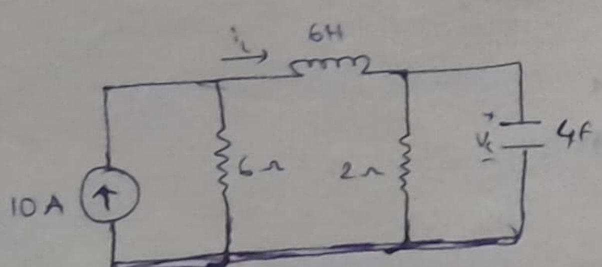
$$\Rightarrow E_{Cap.} = 50\,J$$

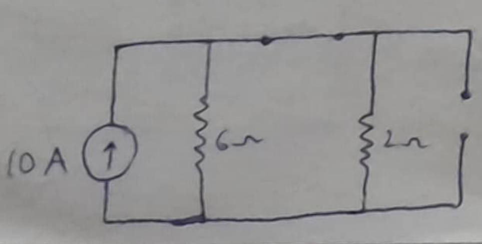
$$W_{ind.} = \frac{1}{2}Li^2$$

$$= \frac{1}{2} \times 2 \times 4$$

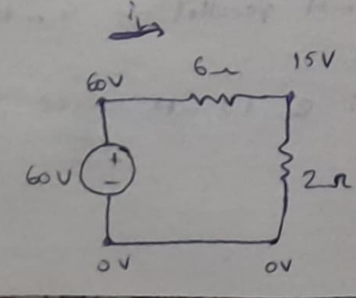
$$\Rightarrow E_{ind.} = 4\,J$$

Q)





$\Rightarrow$



$$i = \frac{60}{8} = \frac{30}{4} = \frac{15}{2}$$

$$= 7.5\,A$$

$$\boxed{I = 7.5\,A}$$

$$\boxed{V_c = 15\,V}$$

$V_c = 15V$

$$E = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 15 \times 15 \times 4$$

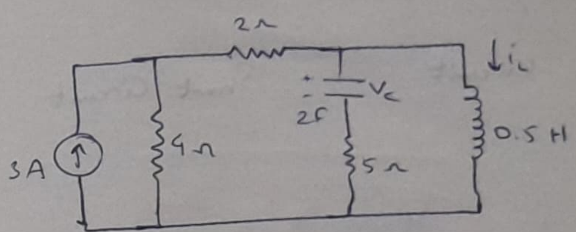
$$\boxed{E = 450 J}$$

$$E_{ind} = \frac{1}{2} Li^2$$

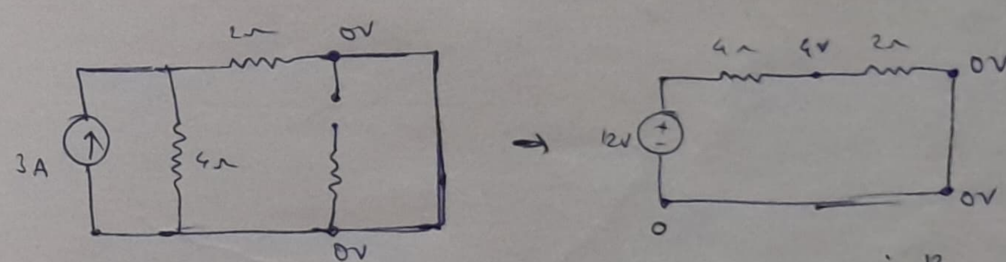
$$= \frac{1}{2} \times 6 \times \frac{15^2}{4}$$

$$\boxed{E_{ind} = 168.75 J}$$

9)



⇓



$$i = \frac{12}{6} = 2A$$

$\boxed{V_c = 0V} \Rightarrow \boxed{W_c = 0J}$ $\boxed{i = 2A}$

$$W_L = \frac{1}{2} \times \frac{1}{2} \times 4 = 1 J$$

$$\boxed{W_L = 1 J}$$

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
V - I	$V = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$V = L\frac{di}{dt}$
I - V	$i = V/R$	$i = C \cdot \frac{dV}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
P or W	$P = i^2R = V^2/R$	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}Li^2$
Series	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1+C_2}$	$L_{eq} = L_1 + L_2$
Parallel	$R_{eq} = \frac{R_1R_2}{R_1+R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1+L_2}$
At DC :	Same	Open Circuit	Short Circuit
Circuit variable that cannot change abruptly :	None	v	i

CHAPTER-7 : FIRST-ORDER CIRCUITS

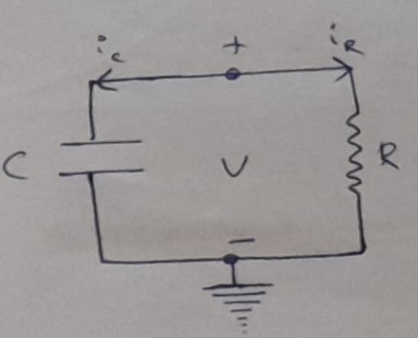
→ Excitation :

(i) No External Source : (Source Free Response)

Initial Energy Stored in Capacitor or Inductor.

(ii) External Source

→ Source Free RC Circuits



Response is Voltage across the capacitor, $v_c(t)$

Apply KCL at + Node :

$$i_R + i_c = 0$$

$$\Rightarrow \boxed{\frac{V_R}{R} + C \cdot \frac{dV}{dt} = 0}$$

Assume Initial Voltage across the Capacitor $V_0 = v(0)$ $[@ t=0]$

$$W_0 = \frac{1}{2} C V_0^2$$

$$\Rightarrow C \cdot \frac{dV}{dt} = \frac{-V_R}{R}$$

$$\Rightarrow \frac{dV}{dt} = \frac{-V_R}{RC}$$

$$\Rightarrow dV = \frac{-V_R}{RC} dt$$

$$\Rightarrow \int_{V_0}^{V} dV = \int_{0}^{t} \frac{-V_R}{RC} dt$$

$$\Rightarrow \int_{V_0}^{V} \frac{-1}{V}\, dV = \int_{0}^{t} \frac{1}{RC}\, dt$$

$$\Rightarrow \int_{V}^{V_0} \frac{1}{V}\, dV = \int_{0}^{t} \frac{1}{RC}\, dt$$

$$= \frac{t}{RC}$$

$$\Rightarrow \ln|V_0| - \ln|V| = \frac{t}{RC}$$

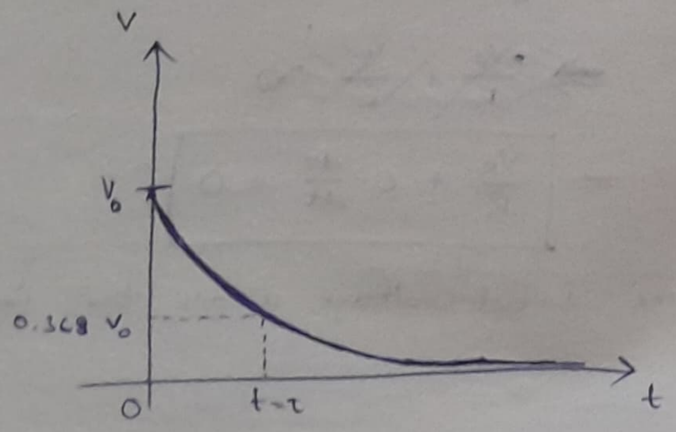
$$\Rightarrow \ln\left|\left(\frac{V_0}{V}\right)\right| = \frac{t}{RC}$$

$$\Rightarrow \ln\left(\frac{V}{V_0}\right) = \frac{-t}{RC}$$

$$\Rightarrow \frac{V}{V_0} = e^{-t/RC}$$

$$\Rightarrow \boxed{V = V_0 \cdot e^{-t/RC}}$$

$$\boxed{V(t) = V_0 \cdot e^{-t/RC}}$$



RC $\longrightarrow$ Time Constant (τ)

$RC = \tau$

$$\boxed{V_t = V_0 e^{-t/\tau}}$$

@ $t = \tau$,

$V(t) = V_0 \cdot e^{-t/t}$

$= V_0 \cdot e^{-1}$

~~$= 0.638\ V_0$~~

$= 0.368\ V_0$

$= (36.8\ \%)\ V_0$

v decays faster for small τ &
v decays slower for large τ.

If no. of Resistors are parallel with C,

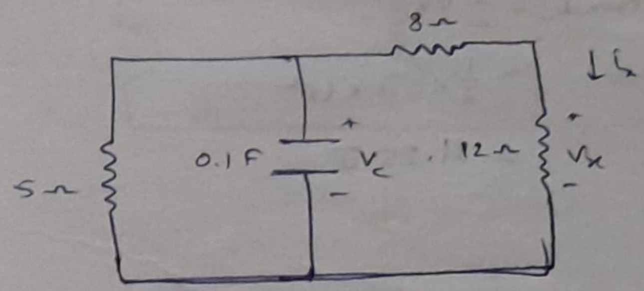
$$\boxed{\tau = R_{eq} \cdot C}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

$$P(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$$

$$W_R(\infty) \longrightarrow \frac{1}{2} C V_0^2$$

Q) Let $V_c(0) = 15\ V$



Find V_c, V_x and i_x for $t > 0$

8Ω and 12Ω are in Series ⇒ 20Ω

20Ω and 5Ω are in Parallel ⇒ $\frac{20 \times 5}{25} = 4\Omega$

$R_{eff} = 4\Omega$

$\tau = 4 \times 0.1 = 0.4\ s$

$$\boxed{V_c(t) = 15 \times e^{-t/0.4}}$$

$$\boxed{V_R(t) = 15 e^{-t/0.4}\ V}$$ (across 20Ω Resistor)

$$V_{12\Omega} = \frac{V \times 12}{20}$$

$$\Rightarrow V_x = 15 \times e^{-t/0.4} \times \frac{12}{20}$$

$$\boxed{V_x = 9e^{-t/0.4}\ V}$$

$$i_x = \frac{V_x(t)}{R} = \frac{9e^{-t/0.4}}{12}$$

$$= \frac{3}{4}e^{-t/0.4}$$

$$\boxed{i_x = \frac{3}{4}e^{-t/0.4}\ A}$$

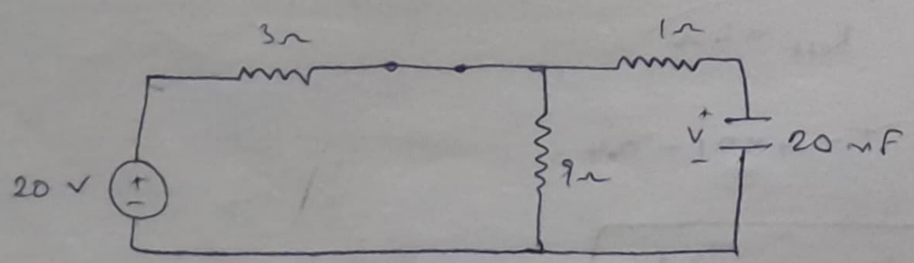
Initial Energy Stored $= \frac{1}{2}CV_0^2$

$= \frac{1}{2} \times 0.1 \times 15^2$

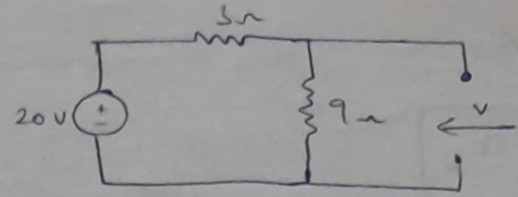
$= 11.25\ J$

Q) Switch has been closed for a long time, and opened at $t=0$. Find $v(t)$ for $t \geq 0$. Calculate Initial Energy stored in the capacitor.

(i) For $t<0$:



Capacitor → Open Circuited



$\boxed{V_{9\Omega} = 15\,V}$

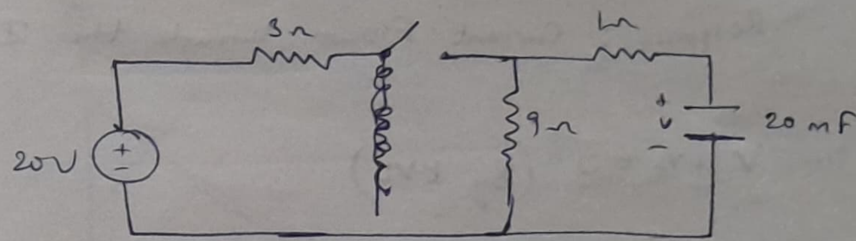
$V = \frac{20 \times 9}{3+9} = \frac{180}{12} = \frac{60}{4} = 15\,V$

$\boxed{V = 15\,V}$

$\Downarrow$ $\boxed{V_o = 15\,V}$

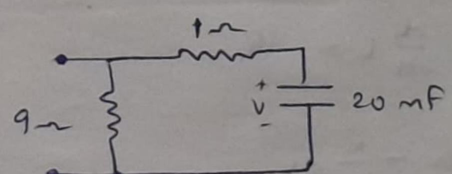
$V_{(t)} = 15$

(ii) At $t = 0$, Voltage cannot be changed inst. for cap.



$\Downarrow$

$V_c = V_o = 15\,V$



$W = \frac{1}{2} C V^2$

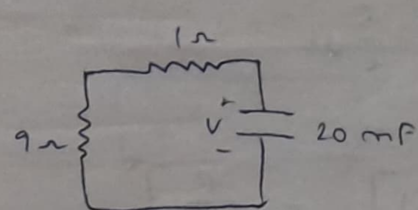
$= \frac{1}{2} \times 20 \times 10^{-3} \times 15^2$

$\boxed{W = 2.25\,J}$

τ

(iii) A' for $t > 0$,

Switch is open



$R_{eff} = 10\,\Omega$

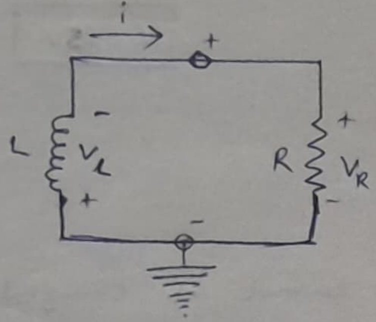
$\tau = 10 \times 20 \times 10^{-3}$

$\boxed{\tau = 0.2\,s}$

$\therefore V_0 = .15 e^{-t/0.2}$

$$\boxed{V_0 = .15 e^{-5t} \ V}$$

→ **Source free RL circuits :**



Response : Current flows through the Inductor

$V_L + V_R = 0$ (By KVL)

$\Rightarrow L \frac{di}{dt} + iR = 0$

$\Rightarrow \cancel{\frac{di}{dt}} \frac{di}{dt} L = -iR$

$\Rightarrow \frac{di}{i} = -\frac{R}{L} dt$

At $t = 0$, let Initial ~~current~~ Current flow through the Inductor, $i(0) = i_0$

Initial Energy Stored $= W_0 = \frac{1}{2} L i_0^2$

$$\int_{i_0}^{i} \frac{di}{i} = \int_0^t -\frac{R}{L} dt$$

$$\Rightarrow \ln(i) \Big]_{i_0}^{i} = -\frac{R}{L} t \Big]_0^t$$

$$\Rightarrow \ln\left(\frac{i}{i_0}\right) = -\frac{R}{L}t$$

$$\Rightarrow \frac{i}{i_0} = e^{-R/L\,t}$$

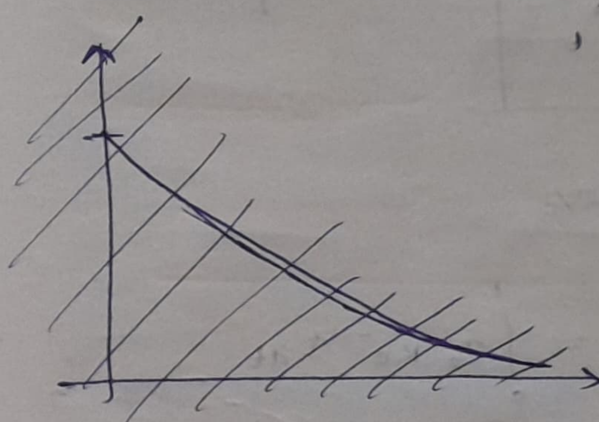
$$\Rightarrow i = i_0 \cdot e^{-R/L\,t}$$

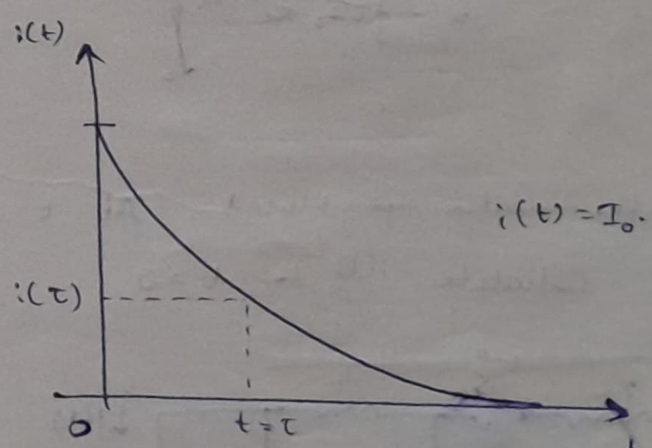
$$\boxed{i = i_0 \cdot e^{-\frac{Rt}{L}}}$$

$$i = i_0 \cdot e^{-t/(L/R)}$$

$$\boxed{i = i_0 \cdot e^{-t/\tau}} \qquad [\tau = L/R]$$

$[R \rightarrow R_{eq}]$





$$i(t) = I_0 \cdot e^{-t/\tau}$$

$$t = \tau \Rightarrow i_0 = I e^{-1}$$

$$\boxed{i_\tau = 0.368\, I_0}$$

$\tau \rightarrow$ large : Slow Decay

$\tau \rightarrow$ small : Fast Delay

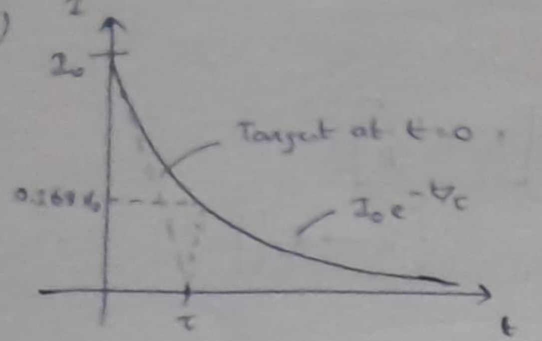
Comparison between RC and a RL Circuit

RL

(i) $i(t) = I_0 e^{-t/\tau}$

$\tau = \frac{L}{R}$

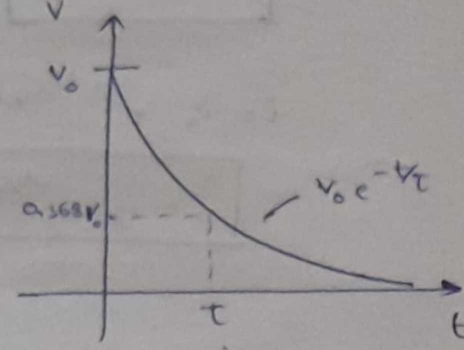
(ii)



RC

$V(t) = V_0 e^{-t/\tau}$

$\tau = RC$

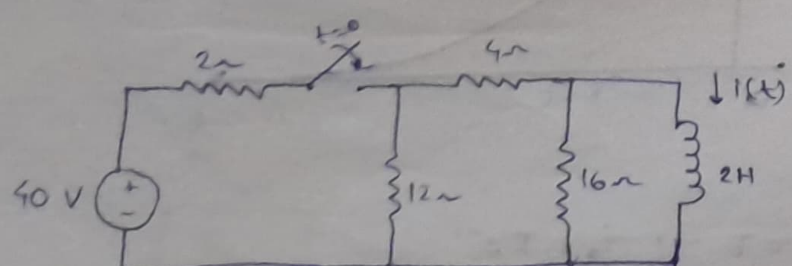


$P = V_R i = I_0^2 R e^{-2t/\tau}$

$W_R(t) = \int_0^t P\,dt = \int_0^t I_0^2 R e^{-2t/\tau}\,dt$

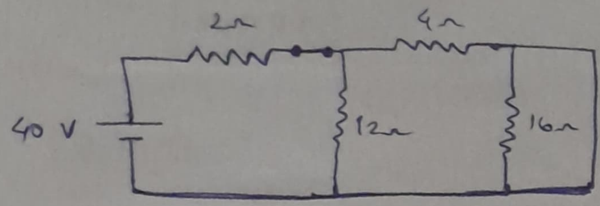
$= -\frac{1}{2}\tau I_0^2 R e^{-2t/\tau}\Big]_0^t \qquad \left[\tau = \frac{L}{R}\right]$

9) When $t < 0$, the switch is closed. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$

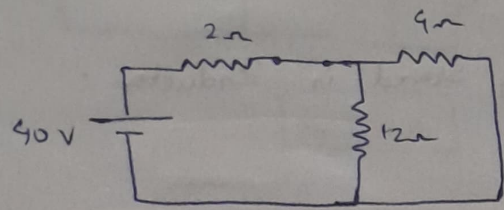


$\frac{48}{16}$

(a) $t < 0 \rightarrow$ Switch is closed



⇓



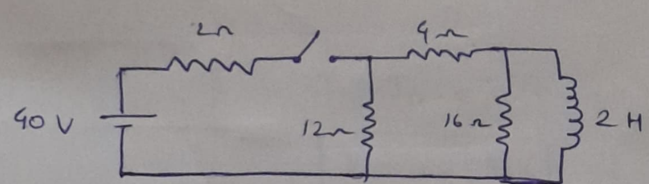
$$(4\Omega \parallel 12\Omega) + 2\Omega$$
$$= 3\Omega + 2\Omega$$
$$= 5\Omega$$

$$i_{total} = \frac{40}{5} = 8\text{ A}$$

$$i_{4\Omega} = \frac{12 \times 8}{12+4} = \frac{12 \times 8}{16}$$
$$= 6\text{A}$$

$$i_{4\Omega} = i_L = 6\text{ A}$$

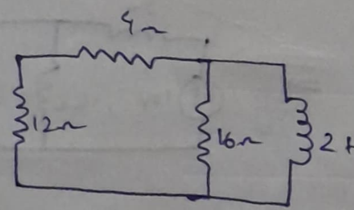
(b) $t = 0 \rightarrow$ Switch is opened



$$i_L = 6\text{A} = i_o$$

$$\boxed{i_o = 6\text{A}}$$

(c) $t > 0$



$\cancel{4\Omega \parallel 12\Omega}$

$$4\Omega + 12\Omega = 16\Omega$$
$$16\Omega \parallel 16\Omega = 8\Omega$$

$$\boxed{R_{eq} = 8\Omega}$$

$\tau = L/R = 2/8 = 1/4 \ s$

$i = i_0 \cdot e^{-t/\tau}$

$\Rightarrow i = 6 \cdot e^{-4t}$

$\boxed{i(t) = 6e^{-4t} A}$

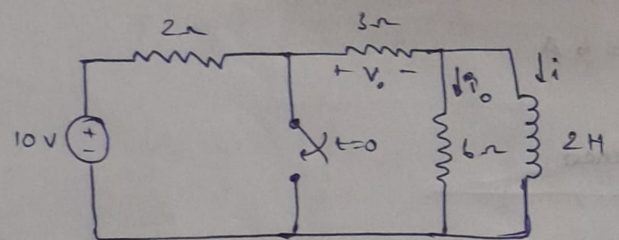
Calculate Initial Energy Stored in Inductor :

$W_0 = \frac{1}{2} L i_0^2$

$\Rightarrow W_0 = \frac{1}{2} \times 2 \times (6)^2$

$\boxed{W_0 = 36 \ J}$

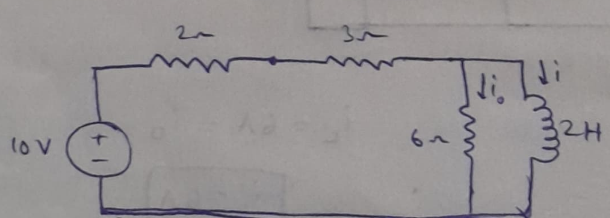
Q)



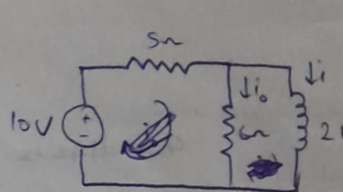
find i, i₀, v₀

Assuming Switch was open for a long time

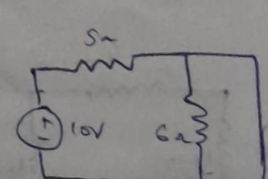
for t<0 :



⇓



⇒



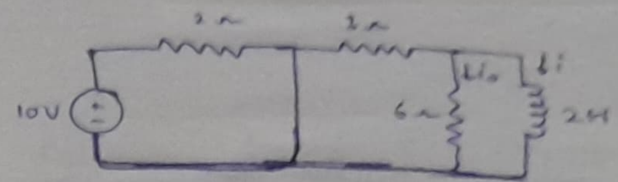
$\boxed{i = 2A}$

$\boxed{i_0 = 0A}$

$V_o(t) = 3(i(t)) = 3 \times 2 = 6\,V$ for $t<0$

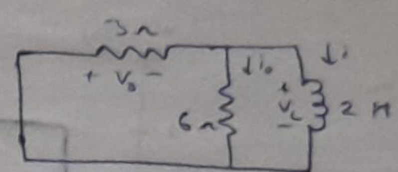
$\boxed{i_o = 2A}$ $\boxed{V_o = 6\,V}$

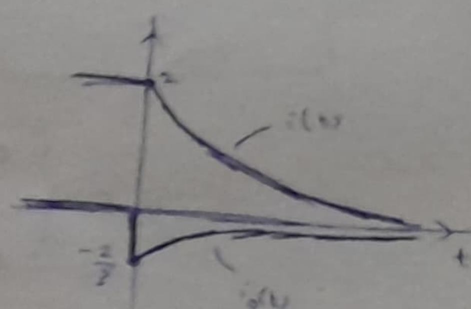
For $t = 0$:



$\boxed{i_o = 2\,A}$

For $t > 0$:





$R_{eq.} = 2\,\Omega$

$\tau = L/R = 2/2 = 1\,s$

$I = I_0 \cdot e^{-t/\tau}$

$\boxed{I = 2e^{-t}\,A}$

$W_0 = \frac{1}{2} L i_0^2$

$\Rightarrow W_0 = \frac{1}{2} \times 2 \times 2^2$

$\boxed{W_0 = 4\,J}$

$V_0(t) = -V_L = -L \cdot \frac{di}{dt}$

$= -2(-2e^{-t})$

$\boxed{V_0(t) = 4e^{-t}}$

$i_o(t) = \frac{V_L}{6}$

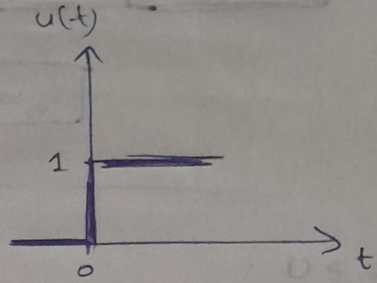
$\boxed{i_o(t) = -\frac{2}{3} e^{-t}\,A}$

→ Unit - Step function

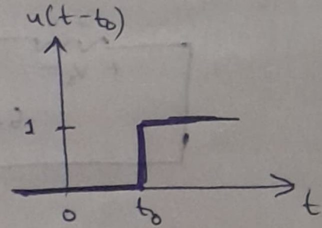
$\forall t < 0, \quad u(t) = 0$

$\forall t > 0, \quad u(t) = 1$

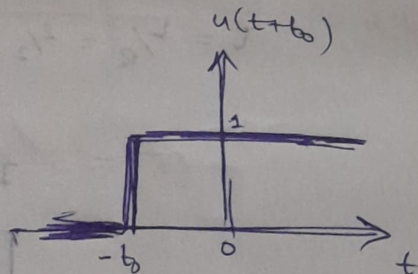
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

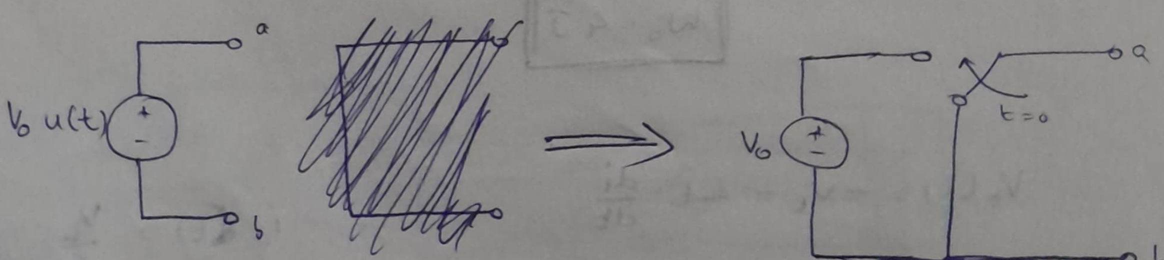


$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

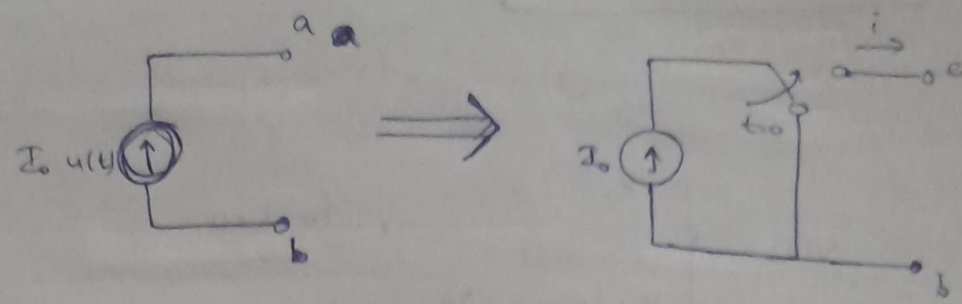


Represent an Abrupt source :

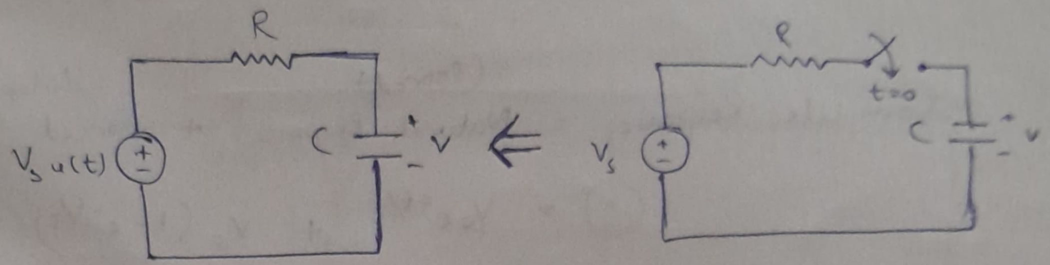
(i) Voltage Source



(ii) Current Source



→ **Step Response of an RC Circuit**



$$v(0^-) = v(0^+) = V_0$$

Applying KVL :

$$\cancel{C\frac{dv}{dt} + \frac{v - V_s u(t)}{RC}}$$

$$C\frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0$$

$$\frac{dv}{dt} = -\frac{v - V_s u(t)}{RC}$$

$[u(t) = 1]$

$$\frac{dv}{v - V_s} = -\frac{dt}{RC}$$

$$\Rightarrow \ln|v - V_s|\Big]_{V_0}^{v(t)} = -\frac{t}{RC}\Big]_0^t$$

$$\Rightarrow \ln(v(t) - V_s) - \ln(V_0 - V_s) = \frac{-t}{RC}$$

$$\Rightarrow \ln\left(\frac{v - V_s}{V_0 - V_s}\right) = \frac{-t}{RC}$$

$[\tau = RC]$

$$\Rightarrow \frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}$$

$$\boxed{V(t) = V_s + (V_0 - V_s)e^{-t/\tau}} \quad \text{for } t>0$$

$$V(t) = \begin{cases} V_0 & , \text{ for } t<0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & , \text{ for } t>0 \end{cases}$$

V_s : Final Value at $t \to \infty$

Complete Response = Natural Responce (Stored) + Forced Responce (Independent)

$= V_0 e^{-t/\tau} + V_s (1 - e^{-t/\tau})$

Complete Response = Transcient Response [Temporary] + Steady-State Response [Permenant]

$$V(t) = V_\infty + [V(0^+) - V(\infty)] e^{-t/\tau}$$

$$[V(0^+) = V(0^-) = V_0]$$

The complete Reason,

Independent Responce ⇒ Forced Responce → Steady state ~~Reason~~ Responce (Permenant)

Transient Responce → Temporary Response

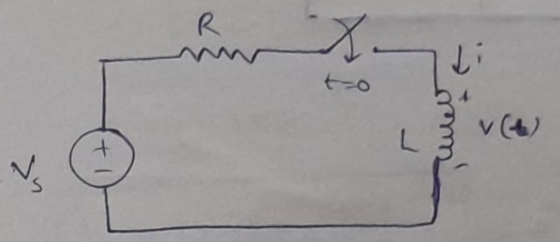
→ Step Response of RL Circuit :

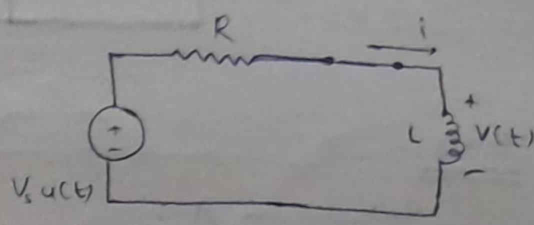
$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-t/\tau}\, u(t)$$

Initial Current : $i(0^-) = i(0^+) = i(0) = i_0$

Final Current : $i_\infty = V_s/R$

$$\tau = L/R$$





$$i(t) = i(\infty) + [i(0^+) - i(\infty)]\, e^{-t/\tau}$$

Natural Responce (Temporary) (Transcience)

Steady State Responce (Permanant)

Three steps to find out step responce of RL Circuit :

(i) Find Initial Inductor Current

$i(0)$ at $t = 0+$

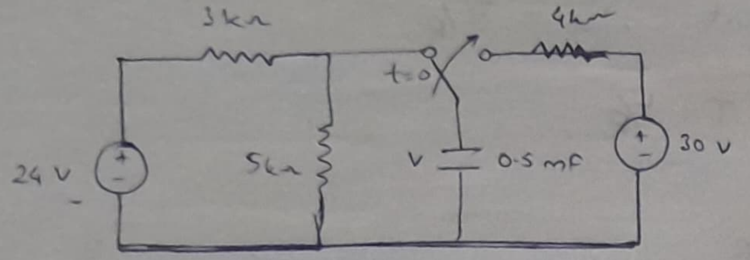
(ii) Find Final Inductor Current

$I_\infty = V_s/R$

(iii) Find Time Constant

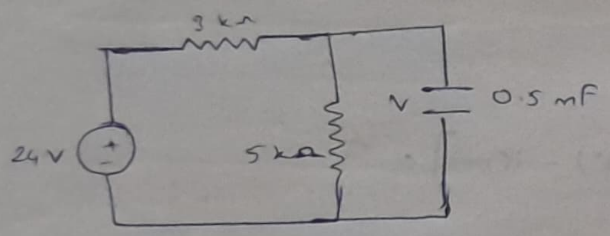
$\tau = L/R_{eq}$

① The switch in the figure has been in Position A for a long time. At $t=0$, the switch moves to B. Determine $v(t)$ for $t>0$ and calculate its value at $t=1s$ and $4s$.



$\underline{\underline{t<0}}$:

Switch is in A :



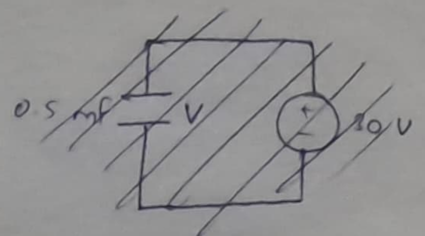
$$V_{cap} = V_{5k\Omega} = \frac{24 \times 5}{5+3} = 15\ V$$

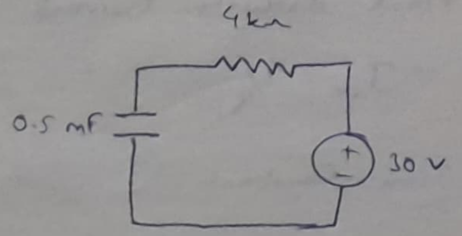
$$V_{cap.} = v(0^-) = 15V$$

$\underline{t=0}$:

$$v(0^-) = v(0^+) = v(0) = V_0 = 15\ V$$

$\underline{\underline{t>0}}$:





~~t=~~

$\tau = RC$

$\cancel{t = \frac{0.5\times10^{-3}}{4\times10^{-3}}}$

$\tau = RC = 4\times10^{-3}\times\frac{1}{2}\times10^{3} = 2\,s$

$\boxed{\tau = 2\,s}$

$\boxed{V_\infty = 30\,V}$

$V_t = v(\infty) + [v(0^+) - v(\infty)]\,e^{-t/\tau}$

$= 30 + (15 - 30)\,e^{-t/2}$

$= 30 - 15e^{-t/2}$

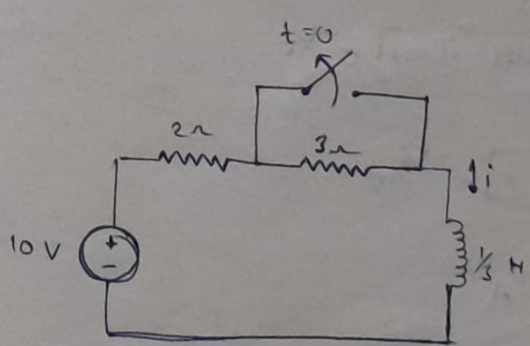
$\boxed{V_t = 15(2 - e^{-t/2})\ V}$

$v(1) = 15(2 - e^{-1/2}) = 20.9\ V$

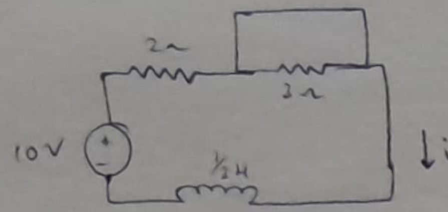
$v(4) = 15(2 - e^{-2}) = 27.97\ V$

Q) find $i(t)$

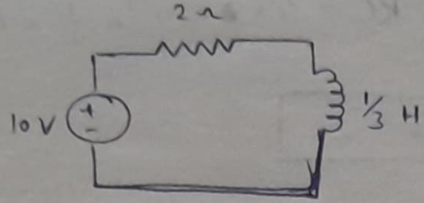
Assume switch is closed since long time



$\underline{t < 0}$:



$\Rightarrow$

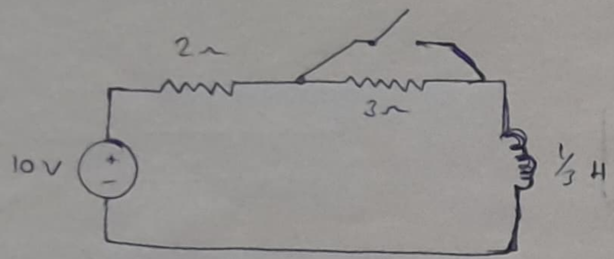


$$\boxed{i = 5A}$$

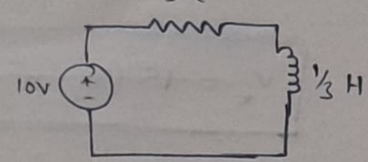
$\underline{t = 0}$

$$i(0^-) = i(0^+) = i(0) = i_0 = 5A$$

$\underline{t > 0}$



$\Rightarrow$



$$\tau = L/R_{eq} = \frac{1/3}{5} = \frac{1}{15} s$$
$$= 0.067 s$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$i(\infty) = \frac{10}{5} = 2A$$

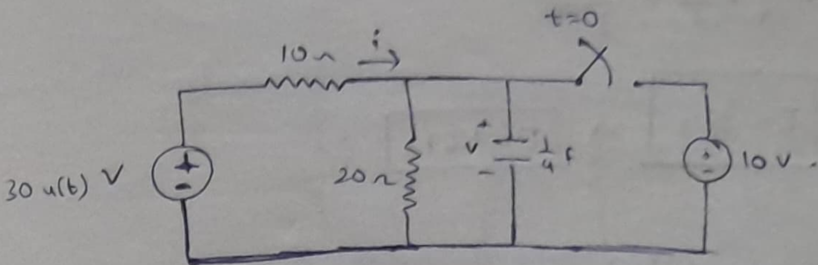
$$\Rightarrow i(t) = \frac{V_s}{R_{eq}} + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 2 + [5 - 2]e^{-t/(1/15)}$$

$$= 2 + 3e^{-15t} A$$

$$\boxed{\therefore i(t) = 2 + 3e^{-15t} A}$$

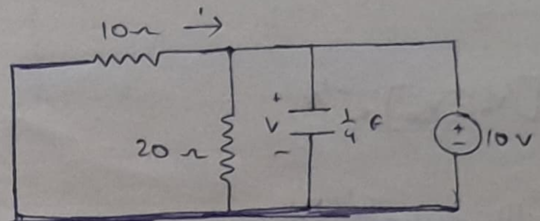
Q) The switch has been closed for a long time and is opened at $t=0$. Find i and v for all time.



for $t<0$:

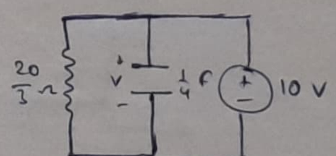
By Definition,

$$30\,u(t) = \begin{cases} 0, & t<0 \\ 30, & t>0 \end{cases}$$



$$\frac{10\times 20}{30}$$

$\Downarrow$



$$i = \frac{10\times 3}{20} \times \frac{20}{30} = 1$$

But $\boxed{i = -1\,A}$

$\boxed{V_c = 10V}$

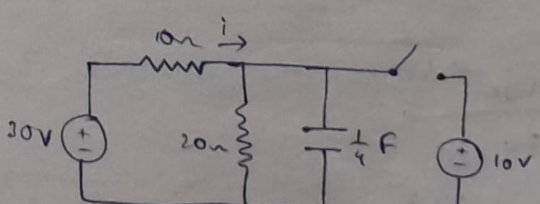
Under DC,
Capacitor - Open Circuit

$\boxed{V(0^-) = 10V}$

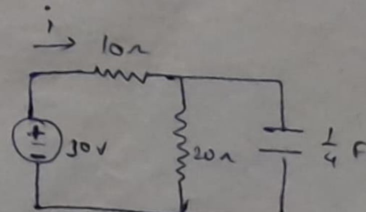
for $t=0$:

$$V(0^-) = V(0^+) = V(0) = V_0 = 10V$$

for $t>0$:



$\Rightarrow$



$$R_{eq} = \frac{20\times 10}{30} = \frac{20}{3}\,\Omega$$

$$\tau = RC = \frac{1}{4} \times \frac{20}{3} = \frac{5}{3}\ s$$

$$\boxed{\tau = 5/3\ s} \Rightarrow \boxed{\tau = 1.67\ s}$$

$$V_{cap.} = V_{20\Omega} = \frac{30 \times 20}{20 + 10} = 20\ V$$

$$\boxed{V(\infty) = 20\ V}$$

$$V(t) = V_\infty + [V_0 - V_\infty]\, e^{-t/\tau}$$

$$= 20 + (10 - 20)\, e^{-t/(5/3)}$$

$$= 20 - 10e^{-\frac{3t}{5}}$$

$$\boxed{V(t) = 20 - 10e^{-0.6t}}$$

$$i = C \frac{dV}{dt}$$

$$= \frac{1}{4} \times 10e^{-0.6t} \times \frac{+3}{5}$$

$$i_{20\Omega} = -1\ A$$

$$\boxed{i_{cap.} = \frac{+3}{2} e^{-0.6t}}$$

$$i_{20\Omega} = 1 \mp \frac{1}{2} e^{-0.6t}$$

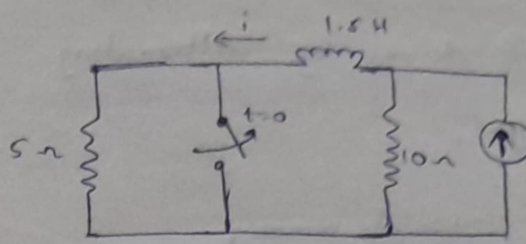
$$\frac{V_{20\Omega}}{20} = \frac{20 - 10e^{-0.6t}}{20}$$

$$= 1 - \frac{1}{2} e^{-0.6t}$$

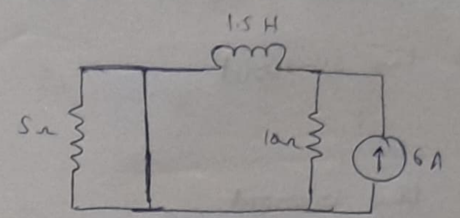
$$\therefore i = i_{20\Omega} + i_{cap} = 1 + e^{-0.6t}$$

Hw

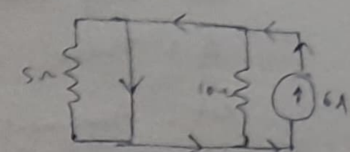
Q) The switch in the fig. has been closed for long time
Opens at $t=0$. Find $i(t)$ for $t>0$



for $t<0$:



⇒

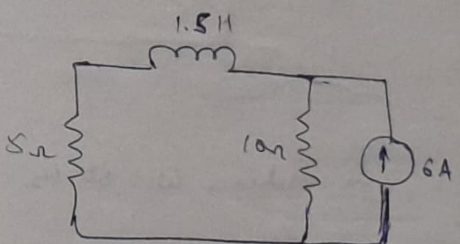


$\therefore i_0 = 6A$

$t=0$:

$$i(0^-) = i(0^+) = i(0) = 6A = i_0$$

$t>0$:



$R_{eq} = 5\Omega + 10\Omega = 15\Omega$

$\tau = L/R_{eq} = 1.5/15 = 0.1$

$i_\infty = i_{5\Omega}$ ~~[illegible]~~

By Source Transformation,

$$i_\infty = \frac{V_\infty}{R_{eq}} = \frac{60}{15} = 4A$$

$$\therefore i(t) = i_\infty + (i_0 - i_\infty)e^{-t/\tau}$$

$$= 4 + 2e^{-t/0.1}$$

$$\boxed{\therefore i(t) = 4 + 2e^{-10t} \text{ A}}$$

CHAPTER-9: AC Circuits

→ **Sinusoids and Phasor :**

AC Circuit : The Circuit driven by Alternating Currents and Voltages.

General expression for a sinusoidal wave,

$$V(t) = V_M \sin(\omega t + \phi)$$

V_m : Amplitude of the sinusoid

ϕ : Phase

ωt : Argument of the solenoid

ω : Angular frequency in radians/s

frequency (f) = no. of cycles/s

$= 1/T$

$$T = \frac{2\pi}{\omega}, \quad f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

In India, $\omega = 50$ Hz

$$\boxed{\omega = 2\pi f \text{ rad/s}}$$

$$\sin(\omega t \pm \pi) = -\sin(\omega t)$$

$$\cos(\omega t \pm \pi) = -\cos(\omega t)$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos(\omega t)$$

$$\cos(\omega t \pm \pi/2) = \mp \sin(\omega t)$$

For Calculations :

(i) Amplitudes must be positive

(ii) 'ω' should be same.

Q) Given a Sinusoid, $5 \sin(4\pi t - \pi/3^c)$

Calculate its Amplitude, Phase, Angular frequency, period and frequency.

sol) Amplitude $= 5$

Phase $= -\pi/3^c$

Angular frequency $= 4\pi$ rad/s

Time period $= \frac{2\pi}{4\pi} = \frac{1}{2}$ s

frequency $= \frac{1}{T} = \frac{1}{1/2} = 2$ Hz

Q) Find the phase angle between $i_1 = -4\sin(377t + 25°)$
$i_2 = 5\cos(377t - 40°)$

Does i_1 lag (or) lead?

$i_1 = -4\sin(377t + 25°)$

$= 4\cos(377t + 115°)$

$$\left[\begin{array}{l} 4\cos(377t + 115°) \\ = 4\cos(90° + 377t + 25°) \end{array}\right]$$

$\therefore i_1 = 4\cos(377t + 115°)$

$i_2 = 5\cos(377t - 40°)$

i_1 leads i_2 by $115° - (-40°) = \underline{\underline{155°}}$

Q) Calculate the phase angle between $v_1 = -10\cos(\omega t + 50°)$
$v_2 = 12\sin(\omega t - 10°)$

$v_1 = -10\cos(\omega t + 50°)$

$= 10\sin(\omega t - 40°)$

(OR)

$= 10\sin(\omega t + 320°)$

$= 10\sin(\omega t + 360 - 40)$

$= 10\sin(\omega t - 40)$

v_2 leads v_1 by $30°$

by $-10 - (-40)$

by $40 - 10 = 30°$

→ Phasor :

→ Complex Number

→ Vector Quantity. [Phase & Amplitude]

→ Representation :

① Rectangular form

$z = x + jy$ $\quad [j = \sqrt{-1}]$

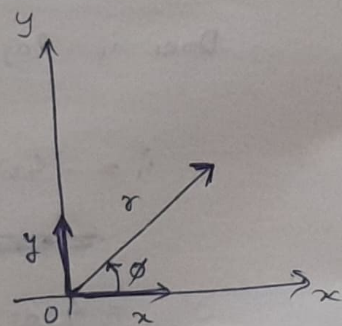
$= r(\cos\phi + j\sin\phi)$

$= re^{j\phi}$

$[e^{j\phi} = \cos\phi + j\sin\phi]$

$r = \sqrt{x^2+y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$



$$\left.\begin{array}{l} z_1 = a_1 + jb_1 \\ z_2 = a_2 + jb_2 \end{array}\right] \Rightarrow (z_1 + z_2) = (a_1 + a_2) + j(b_1 + b_2)$$

If adding (or) subtracting two phasors, they must be in rectangular form.

If Multiplying (or) Dividing two Phasors, they must be in Polar form.

$z = x + jy = r\angle\phi$

$z_1 = x_1 + jy_1 = r_1\angle\phi_1$

$z_2 = x_2 + jy_2 = r_2\angle\phi_2$

Note:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

$$\frac{1}{z} = \frac{1}{r} \angle -\phi$$

$$\sqrt{z} = \sqrt{r} \angle \phi/2$$

$$z^* = x - jy = r \angle -\phi = r e^{-j\phi}$$

$$e^{\pm j\phi} = \cos\phi \pm j \sin\phi$$

Q) Evaluate the following complex numbers

(a) $(40 \angle 50^\circ + 20 \angle -30^\circ)^{-1/2}$

$$z_1 = 40 \angle 50^\circ$$

$$z_2 = 20 \angle -30^\circ$$

$r_1 = 40$

$r_2 = 20$

$$z_1 = 25.7 + j(30.65)$$

$$z_2 = 17.32 + j(-10)$$

$$z_1 + z_2 = (25.7 + 17.32) + j(30.65 - 10)$$

$$= 43.03 + j\, 20.64$$

$$= 47.7 \angle 25.625$$

$$\sqrt{z_1 + z_2} = 6.91 \angle 12.81^\circ$$

(b) $\left[(5+2j)(-1+4j)\right] - 5\angle 60^\circ]$

$(5.385\angle 21.8)(4.12\angle 104.0)$

$= [-13+18j - 5\angle 60^\circ]$

(OR)

$= [-13+18j - (2.5+4.33j)]$

$= [-15.5 - j(13.67)]$

H.W

(C) $\dfrac{10+5j+3\angle 40^\circ}{-3+4j} + 10\angle 30^\circ + 5j$

Ans : $8.293 + 7.2j$

→ $v(t) = V_m \cos(\omega t + \phi)$ [Time Domain]

⇓

$V = V_m \angle \phi$ [Phasor Domain]

Only represented in cosine form (Alexander Book)

Represented in sine form in Indian ~~Auto~~ Books

Q) Transform following sinusoids into Phasor :

$i = 6\cos(50t - 40^\circ)$ A

$v = -4\sin(30t + 50^\circ)$ V

$i = 6\angle -40^\circ$ A

$v = -4\sin(30t + 50)$

$V = 4\angle 140^\circ$ V

$= 4\cos(90 + (30t + 50))$

$= 4\cos(30t + 140^\circ)$

Phasor Analysis applies only when frequency is constant.

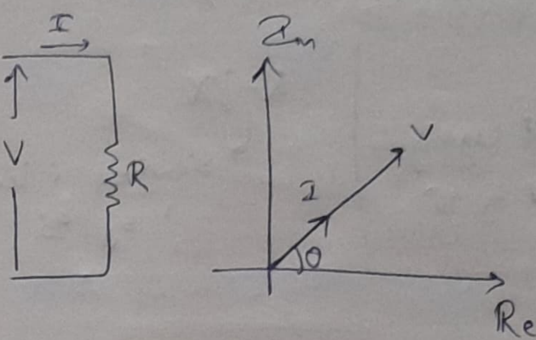
★ $v(t) \longleftrightarrow V = V\angle\phi$

$\frac{dv}{dt} \longleftrightarrow j\omega V$

$\int v\,dt \longleftrightarrow \frac{V}{j\omega}$

→ Phasor Relationship:

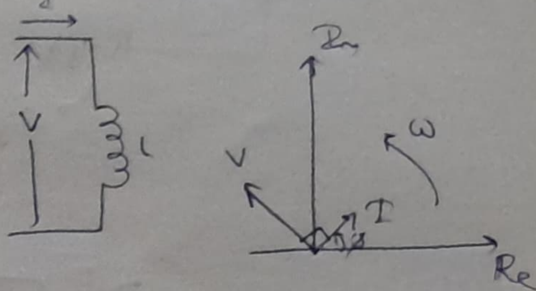
R



∴ I & V are in Phase

& $V = IR \Rightarrow I = \frac{V}{R}$

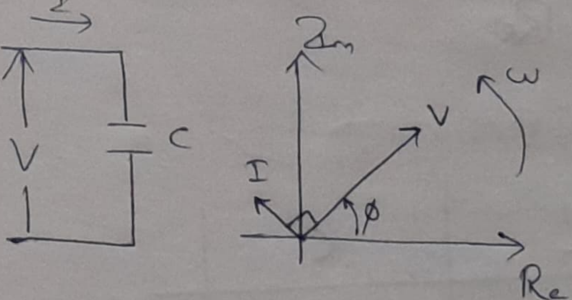
L



∴ Current lags behind Voltage by 90°.

& $v = L\frac{di}{dt} \Rightarrow V = Lj\omega I$

C



∴ Current leads Voltage by 90°

$i = C\frac{dv}{dt} \Rightarrow I = Cj\omega V$

→ Impedence: (Z)

↳ AC Resistance

$$Z = \frac{V}{I}$$

V → Phasor Quantity

I → Phasor Quantity

Phasor (Z) $= R \pm j\underbrace{X}_{\text{Reactance}}$ $\left[\begin{array}{l} \text{For } L, \; Z = R + jX \\ \text{For } C, \; Z = R - jX \end{array}\right]$

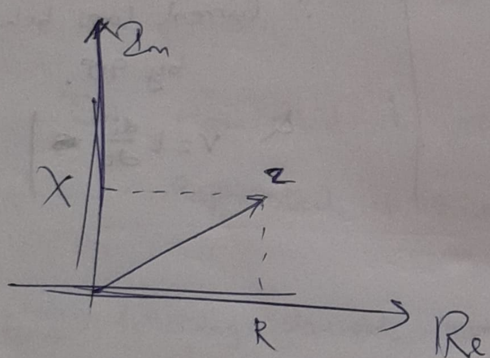
$R + jX$

↳ Signifies i lagging behind V by 90°

$R - jX$

↳ Signifies i leading V by 90°.

$$Z = \sqrt{R^2 + X^2}$$



R	L	C
$Z = R$	$Z = j\omega L$	$Z = \frac{1}{j\omega C}$ $\Rightarrow Z = \frac{-j}{\omega C}$

$$\boxed{Z_{eq} = R + j(X_L \pm X_C)}$$

Note: The parametres must be expressed in Phasor Domain

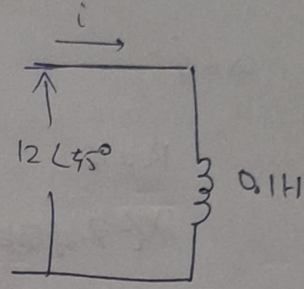
Q) $V = 12\cos(60t + 45^\circ)$

$L = 0.1\ H$

Find Steady State Current

Sol:

$V = 12\cos(60t + 45^\circ)$



$\Rightarrow V = 12\angle 45^\circ$

$Z = jL\omega = j \times 0.1 \times 60 = 6j\ \Omega$

$\therefore i = \dfrac{V}{Z} = \dfrac{12\angle 45^\circ}{6\angle 90^\circ} = 2\angle -45^\circ\ A$

$\Rightarrow i(t) = 2\cos(60t - 45^\circ)\ A$

Q) $V(t) = 6\cos(100t - 30^\circ)$

$C = 50\ \mu F$

Sol:

$V = 6\angle -30^\circ$

$X_c = \dfrac{-j}{C\omega} = \dfrac{-j}{100C} = -200j = Z$

$i = \dfrac{V}{Z} = \dfrac{6\angle -30^\circ}{200\angle -90^\circ} = 0.03\angle 60^\circ\ A$

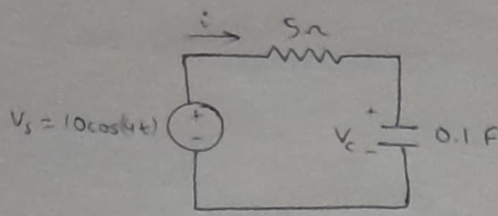
$\therefore i = 0.03\cos(100t + 60^\circ)\ A$

→ Reciprocal of Impedence (Z) is Y (Admittance)

$Y = \dfrac{1}{Z} = \dfrac{I}{V}$

Unit of Y : ℧ (mho), S (Siemens)

Q) Find $V(t)$ & $i(t)$



Sol :

$\omega = 4$,

$R = 5$

$X_c = \frac{-j}{\omega C} = \frac{-j}{4 \times 0.1} = -2.5j$
$\downarrow$
Z_c

$Z = 5 - 2.5j = 5.6\angle -26.565°$

$\Rightarrow I = \frac{V}{Z} = \frac{10\angle 0°}{5.6\angle -26.565°} = 1.786\angle 26.565°\ A$

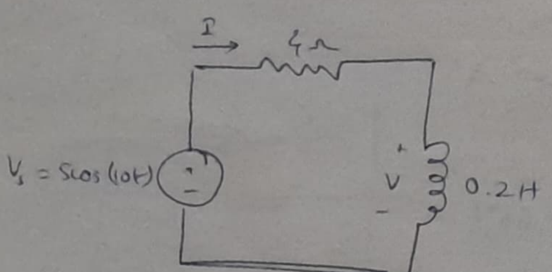
$\Rightarrow I(t) = 1.786\cos(4t + 26.565°)\ A$

& $V_c = I Z_c$

$= (1.786\angle 26.565°)(2.5\angle -90°)$

$\Rightarrow V_c = 4.464\angle -63.435°\ V$ $\Rightarrow V_c = 4.464\cos(4t - 63.435°)\ V$

Q) Determine $V(t)$ & $i(t)$



$X_L = j\omega L$
$= j \times 10 \times 0.2$
$= 2j$
$= 2\angle 90°$

$Z = 4 + 2j$

Sol :

$i(t) = \frac{V(t)}{Z} = \frac{5\angle 0°}{4.472\angle 26.565°}$

$= 1.12\angle -26.565°$

$\therefore i(t) = 1.12\cos(10t - 26.565°)$

$V(t) = I X_L$

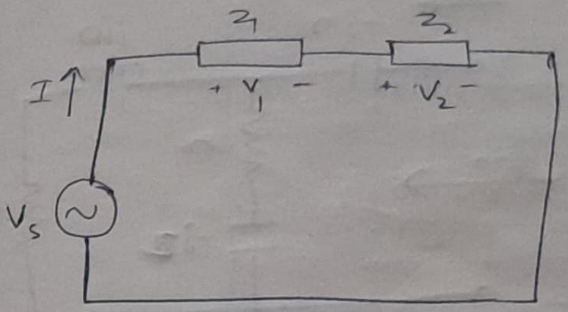
$= (1.12 \angle -26.565^\circ)(2 \angle 90^\circ)$

$= 2.24 \angle 63.435^\circ$ V

$\therefore V(t) = 2.24 \cos(10t + 62.435^\circ)$ V

→ Impedences Combination:

Series:



$V_s = V_1 + V_2$

$I Z_{eq} = I Z_1 + I Z_2$

$\Rightarrow \boxed{Z_{eq} = Z_1 + Z_2}$

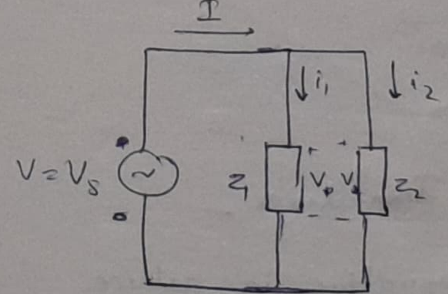
$\boxed{I = V/Z_{eq}}$

$I = \dfrac{V}{Z_1 + Z_2}$

By Voltage Division Techniques,

$$V_1 = \frac{Z_1 V}{Z_1 + Z_2} \quad \& \quad V_2 = \frac{Z_2 V}{Z_1 + Z_2}$$

Parallel:



$I = i_1 + i_2$

$\Rightarrow \dfrac{V}{Z_{eq}} = \dfrac{V}{Z_1} + \dfrac{V}{Z_2}$

$\Rightarrow \boxed{\dfrac{1}{Z_{eq}} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2}}$

$\boxed{Y_{eq} = Y_1 + Y_2}$

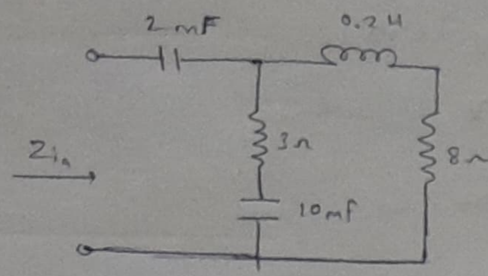
$V = I \cdot Z_{eq}$

$V = I\left(\dfrac{Z_1 Z_2}{Z_1 + Z_2}\right)$

From Current Division Techniques

$$i_1 = \frac{Z_2}{Z_1 + Z_2} I \quad \& \quad i_2 = \frac{Z_1}{Z_1 + Z_2} I$$

Q) Determine Input Impedence, $\omega = 50$ rad/s

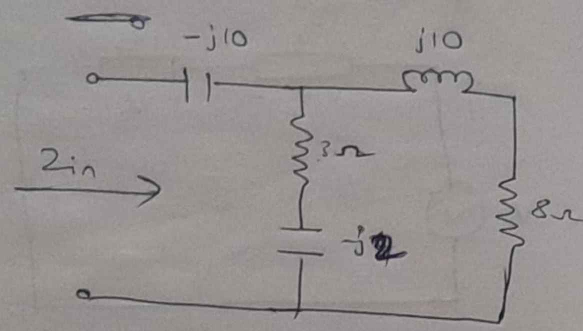


Sol :

$X_L = j \times 0.2 \times 50 = j10$

$X_{C_2} = \dfrac{-j}{2 \times 10^{-3} \times 50} = -j10$

$X_{C_{10}} = \dfrac{-j}{10 \times 10^{-3} \times 50} = -j2$



$(8 + j10) \,||\, (3 - j2) + (-j10) = Z_{eq}$

$\Rightarrow \quad Z_{eq} = -j10 + \dfrac{(12.8 \angle 51.34°)(3.6 \angle -33.69°)}{11 + j8}$

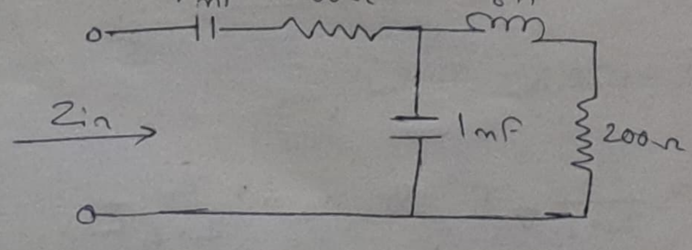
$= -j10 + (3.22 - 1.07j)$

$\therefore Z_{eq} = 3.22 - 11.07j$

Note : $-j$ Represents nature of total Impedence.

$\therefore$ Here, Total Impedence is Capacitive in nature

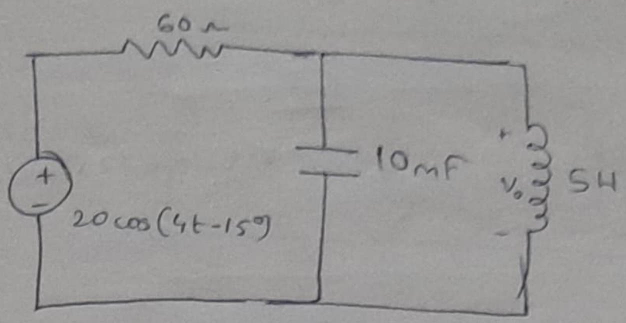
Q) Determine Input Impedence, $\omega = 10$ rad/s



Ans :

$Z_{eq} = 149.52 - j195$

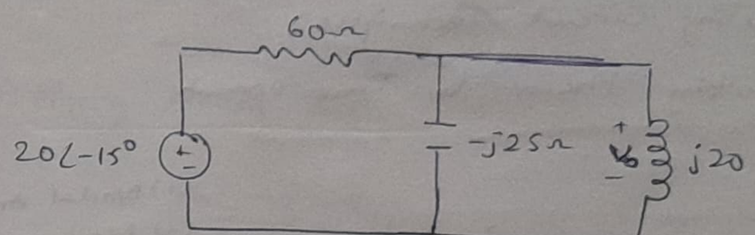
Q) Determine $V_o(t)$



Sol :

$$X_C = \frac{-j}{10\times10^{-3}\times4} = -25j\ \Omega \left[X_C = \frac{-j}{C\omega}\right]$$

$$X_L = j\omega L = j\times4\times5 = j20\ \Omega$$



$$V_{j20\Omega} = V_{-j25\Omega}$$

$$Z_{eq} = 60 + \frac{(-j25)(j20)}{-j5}$$

$$\Rightarrow Z_{eq} = 60 + j100$$

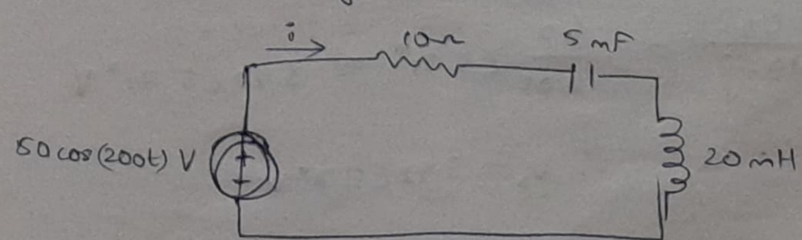
$$V_{100j} = V \times \frac{j100}{60+j100} = \frac{(20\angle-15^\circ)(100\angle90^\circ)}{(116.62\angle59.036^\circ)}$$

$$\Rightarrow V_{100j} = 16.488 + 4.7167j = V_o$$

$$\therefore V_o = 17.15\angle15.98^\circ\ V$$

$$\therefore V_o = 17.15\cos(4t + 15.93^\circ)V$$

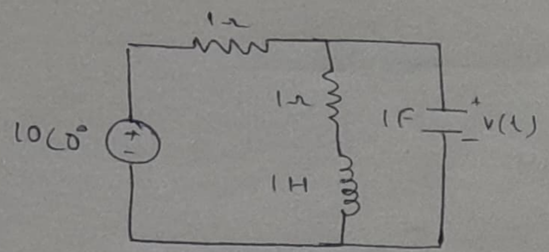
Q) Determine i(t) in given circuit



Sol :

$i(t) = 4.789\angle-16.7^\circ$ A

Q) Find $v(t)$



Sol:

$v(t) = 6.525\cos(t - 18.43°)$ V

CHAPTER-10

(SINUSOIDAL STEADY STATE ANALYSIS)

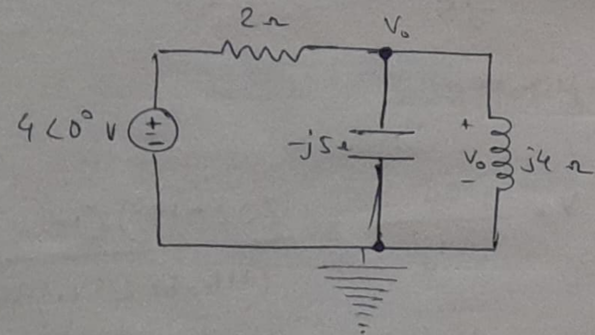
→ Steps while solving:

1. Transform the circuit into Phasor (or) Frequency domain
2. Solve the problem using Circuit Techniques
3. Transform the resulting Phasor to Time Domain

→ Nodal Analysis

(1) Nodal Analysis
(2) Mesh Analysis
(3) Superposition Theorem
(4) Thevenin's/Norton's Theorem

Q) Find V_o using Nodal Analysis



Sol: At top Node:

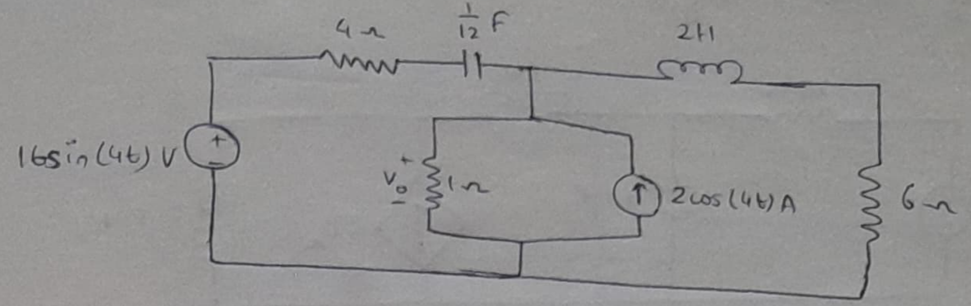
$$-\frac{V_o - 4\angle 0°}{2} = \frac{V_o}{-j5} + \frac{V_o}{j4} \Rightarrow \frac{V_o - 4\angle 0°}{2} = V_o j\left(\frac{1}{4} - \frac{1}{5}\right) = \frac{V_o j}{20}$$

$$\Rightarrow 40 = V_o(10+j)$$

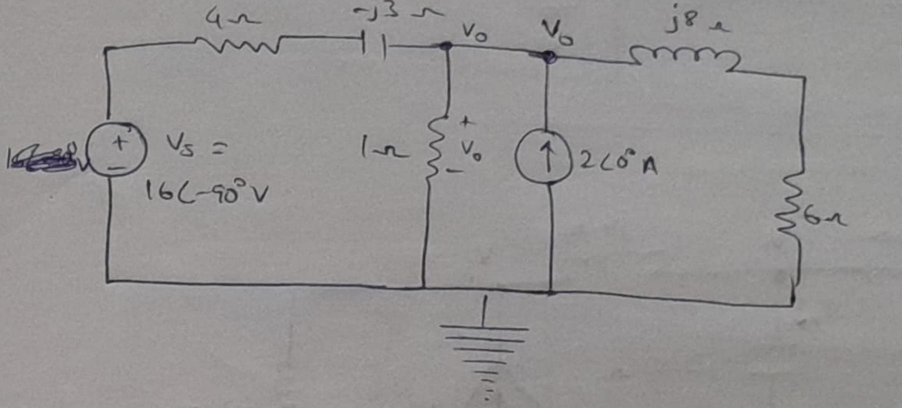
$$\Rightarrow V_o = \frac{40}{10-j} = \frac{40}{10.05}\angle 5.71° = 3.98\angle 5.71° \text{ V}$$

$$\therefore V_o = 3.98\angle 5.71° \text{ V}$$

Q) find $V_o(t)$



Sol :



$16\sin(4t)\ V = 16\angle -90^\circ\ V$

$\frac{1}{12}\ F = \frac{-j}{\frac{1}{12}\times 4} = -j3\ \Omega$

$2\cos(4t)\ A = 2\angle 0^\circ\ A$

$2H = j\omega L = 8j\ \Omega$

Applying Nodal Analysis @ Node V_o :

$$\frac{16\angle -90^\circ - V_o}{4-j3} + 2\angle 0^\circ = \frac{V_o}{1} + \frac{V_o - 0}{6+j8}$$

$$= V_o\left(\frac{53}{50} - \frac{2j}{25}\right)$$

$$\Rightarrow \frac{16\angle -90^\circ}{5\angle -36.87^\circ} + 2\angle 0^\circ = V_o\left(\frac{53-4j}{50} + \frac{1}{4-j3}\right)$$

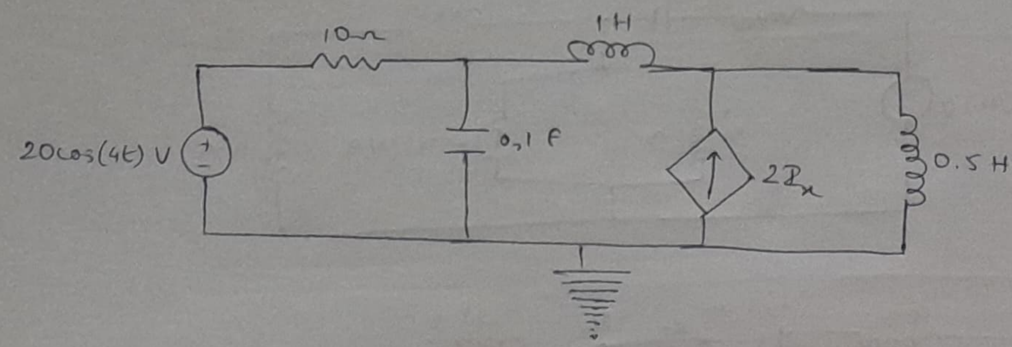
$$\Rightarrow \frac{98-64j}{25} = V_o\left(\frac{61+j2}{50}\right)$$

$$\Rightarrow V_o = \frac{196-128j}{61+j2}$$

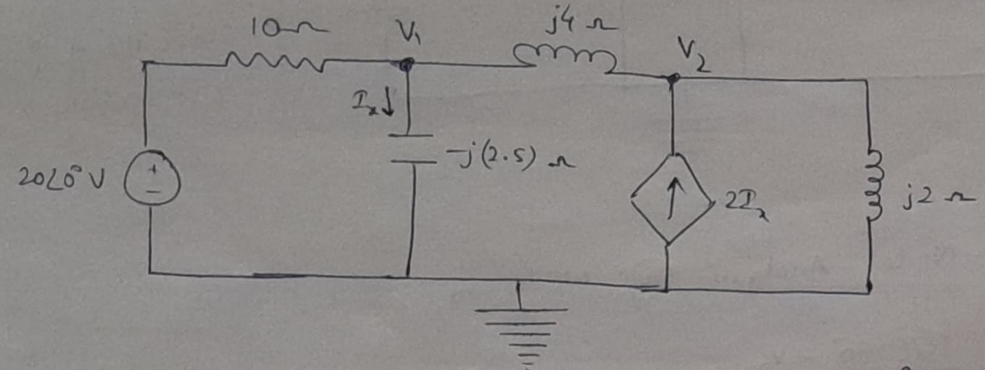
$$\boxed{\therefore V_o = 3.835\angle -35.025^\circ\ V}$$

$$\therefore V_o(t) = 3.835\cos(4t - 35.025^\circ)\ V$$

Q) Find I_x using Nodal Analysis :



Sol :



$20\cos(4t)\,V \rightarrow 20\angle 0^\circ\,V$

$1H \rightarrow j\times4\times1 = j4\,\Omega$

$0.5H \rightarrow j\times4\times0.5 = j2\,\Omega$

$0.1F \rightarrow -j/_{4\times0.1} = -j(2.5)$

Applying Nodal Analysis :

@ V_1 :

$$\frac{20\angle 0^\circ - V_1}{10} = \frac{V_1 - V_2}{j4} + \frac{V_1 - 0}{-j(2.5)} \quad -(1)$$

$$\left[\; I_x = \frac{V_1 - 0}{-j(2.5)} \qquad \therefore I_x = \frac{2jV_1}{5}\,A \;\right]$$

@ V_2 :

$$2I_x = \frac{V_2 - V_1}{j4} + \frac{V_2 - 0}{j2}$$

$$\Rightarrow \frac{4jV_1}{5} = \frac{V_2 - V_1}{j4} + \frac{V_2}{j2} \quad -(2)$$

By solving (1) and (2) :

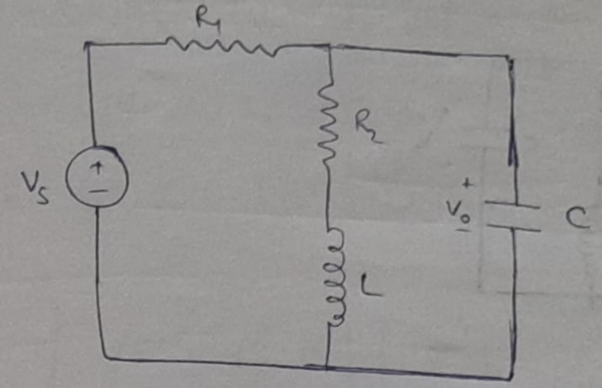
$V_1 = 18.97\angle 18.43^\circ\,V$

$= 18.97\cos(4t + 18.43^\circ)\,V$

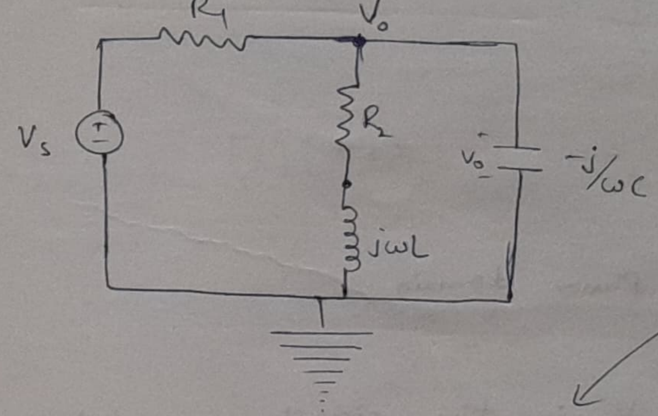
$V_2 = 13.91\angle 198.3^\circ\,V$

$= 13.91\cos(4t + 198.3^\circ)\,V$

Q) Find V_o/V_s Using Nodal Analysis in given Circuit :



<u>Sol</u> :



By Applying Nodal Analysis @ Node V_0

$L \longrightarrow j\omega L$

$C \longrightarrow -j/\omega C = 1/j\omega C$

$R_1 \longrightarrow R_1$

$R_2 \longrightarrow R_2$

$V_s \longrightarrow V_s$

$$\frac{V_s - V_o}{R_1} = \frac{V_o - 0}{1/j\omega C} + \frac{V_o - 0}{R_2 + j\omega L}$$

$$\Rightarrow \frac{V_s}{R_1} - \frac{V_o}{R_1} = (j\omega C)V_o + \frac{V_o}{R_2 + j\omega L}$$

$$\Rightarrow \frac{V_s}{R_1} = V_o\left(j\omega C + \frac{1}{R_1} + \frac{1}{R_2 + j\omega L}\right)$$

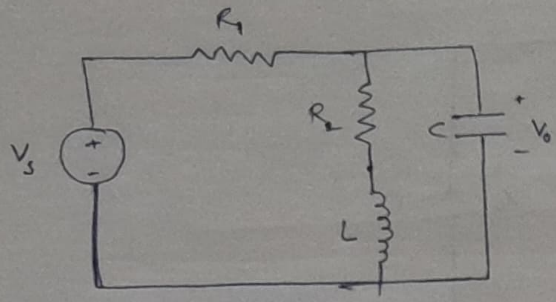
$$\Rightarrow \frac{V_s}{V_o} = \left(j\omega C R_1 + 1 + \frac{R_1}{R_2 + j\omega L}\right)$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{R_2 + j\omega L}{(R_2 + j\omega L)(1 + j\omega C R_1) + R_1} = \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L - \omega^2 L C R_1 + j\omega R_2 R_1 C}$$

$$\boxed{\therefore \frac{V_o}{V_s} = \frac{R_2 + j\omega L}{R_1 + R_2 - \omega^2 L C R_1 + j\omega(L + R_1 R_2 C)}}$$

Q) For the given circuit, find V_o/V_s using Nodal Analysis

[H.W]



Sol:

$$\frac{V_o}{V_s} = \frac{R_2 + j\omega L}{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1R_2C)}$$

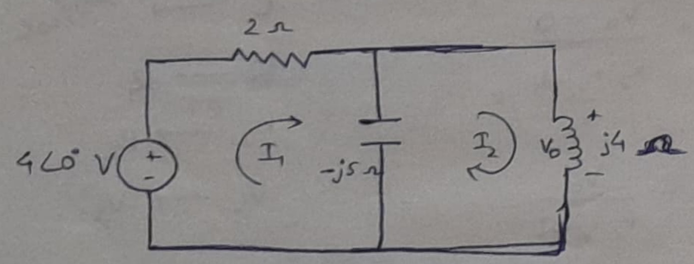
→ **Mesh Analysis** :

→ KVL for Loop

(To find loop current)

① Parameters should be in Phasor domain

Q) Use Mesh Analysis to find V_o in the circuit given below



$[V_o = I_2 \cdot j4]$

Apply KVL for loop 1 :

$$+4\angle 0° - 2I_1 - (-j5)[I_1 - I_2] = 0$$

$$\Rightarrow 4\angle 0° - 2I_1 + j5(I_1 - I_2) = 0$$

$$\Rightarrow -I_1(2 - j5) - j5I_2 = -4\angle 0°$$

$$\Rightarrow (2 - j5)I_1 + j5I_2 = 4\angle 0° \quad -①$$

Apply KVL for loop 2:

$$-(-j5)[I_2 - I_1] - I_2 \cdot (j4) = 0$$

$$\Rightarrow (j5 - j4) I_2 - j5 (I_1) = 0$$

$$\Rightarrow (j5) I_1 - (j5 - j4) I_2 = 0 \quad - (2)$$

$$\begin{bmatrix} j5 & j4 - j5 \\ 2 - j5 & j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \cancel{\begin{bmatrix} 0 \\ 4\angle 0^\circ \end{bmatrix}} \begin{bmatrix} 0 \\ 4\angle 0^\circ \end{bmatrix}$$

$$[4\angle 0^\circ = 4 + 0i]$$

$$[R][I] = [V]$$

$$\nabla = (j5)^2 - (j4 - j5)(2 - j5)$$

$$= \cancel{(j5)^2} - 2j4 + j4 \cdot j5 + 2j5 - \cancel{(j5)^2}$$

$$\Rightarrow \nabla = -2(j4) + j4 \cdot j5 + 2(j5)$$

$$= 20 - j2$$

$$\nabla I_2 = \begin{bmatrix} j5 & 0 \\ 2 - j5 & 4\angle 0^\circ \end{bmatrix}$$

$$= j5 \cdot 4\angle 0^\circ$$

$$= -20j$$

$$\left[I_1 = \frac{1}{5} I_2 \right]$$

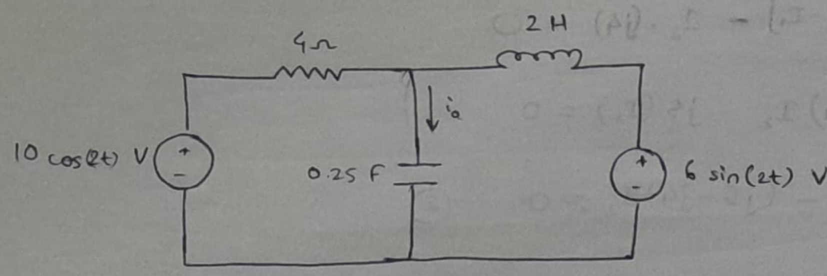
$$I_2 = \frac{\nabla I_2}{\nabla I} = \frac{-20j}{20 - j2}$$

$$I_2 = \frac{1}{0.1 + j}$$

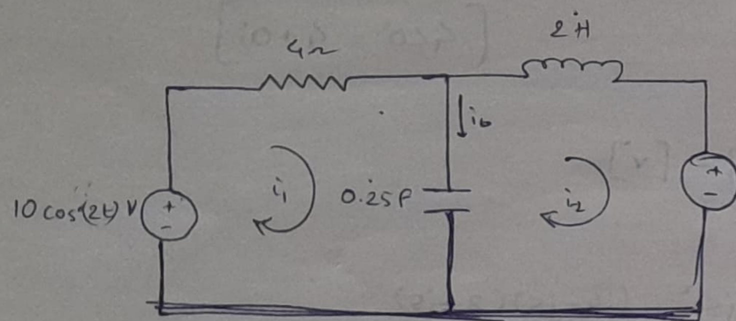
$$= 0.995 \angle -84.2^\circ$$

$$V_0 = j4 I_2 = \frac{j4}{0.1 + j} = \frac{j4}{1.00499\angle 84.29^\circ} = 3.98\angle 5.71^\circ \text{ V}$$

Q) Find I_o in circuit below



Assume i_1 in left loop and
i_2 in right loop



6 sin(2t) V $= 6\cos(90-2t)$
$= 6\cos(2t-90)$

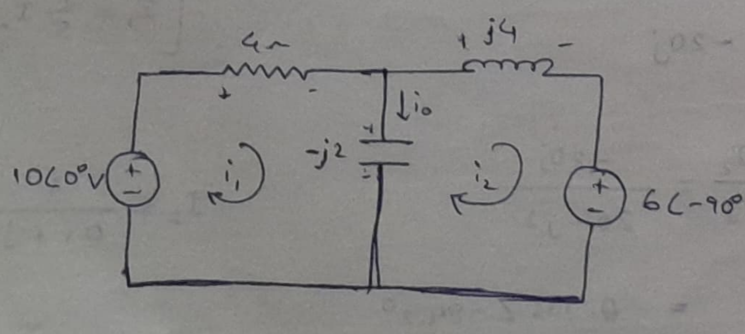
$V = 10\angle 0°$ V

$R = 4\Omega$

$0.25F \longrightarrow \dfrac{1}{j2 \times 0.25} = -j2$

$2H \longrightarrow j2 \cdot 2 = j4$

$6\sin(2t) \longrightarrow 6\angle -90°$



Applying KVL :

① $\rightarrow$ $+10\angle 0^\circ - 4i_1 - (-j2)(i_1 - i_2) = 0$

$\Rightarrow 10\angle 0^\circ - 4i_1 + j2(i_1 - i_2) = 0$

$\Rightarrow i_1(j2-4) - i_2(j2) = -10\angle 0^\circ$

$\Rightarrow i_1(4-j2) + i_2(j2) = 10\angle 0^\circ$ — ①

$\Rightarrow i_1(2-j) + i_2(j) = 5\angle 0^\circ$

② $\rightarrow$ $-j4 \cdot i_2 - 6\angle -90^\circ - (-j2)(i_2 - i_1) = 0$

$\Rightarrow -j4i_2 - 6\angle -90^\circ + j2(i_2 - i_1) = 0$

$\Rightarrow j4i_2 + 6\angle -90^\circ + j2(i_1 - i_2) = 0$

$\Rightarrow -(j2)i_1 - (j4-j2)i_2 = 6\angle -90^\circ$

$\Rightarrow -(j2)i_1 - (j2)i_2 = 6\angle -90^\circ$ — ②

$\Rightarrow -(j2)i_1 - (j2)i_2 = -6j \Rightarrow \boxed{i_1 + i_2 = 3}$

① + ② :

$i_1(4 - j2 - j2) = 10\angle 0^\circ + 6\angle -90^\circ$

$\Rightarrow i_1(4 - j4) = (10 + 0j) + (0 - 6j)$

$= 10 - 6j$

$= 11.662\angle -30.96^\circ$

$i_1 = \dfrac{11.662\angle -30.96^\circ}{4 - j4} = \dfrac{10-6j}{4-4j} = \dfrac{5-3j}{2-2j}$

$i_2 = 3 - i_1$

$= 3 - \dfrac{5-3j}{2-2j}$

$= \dfrac{6-6j-5+3j}{2-2j}$

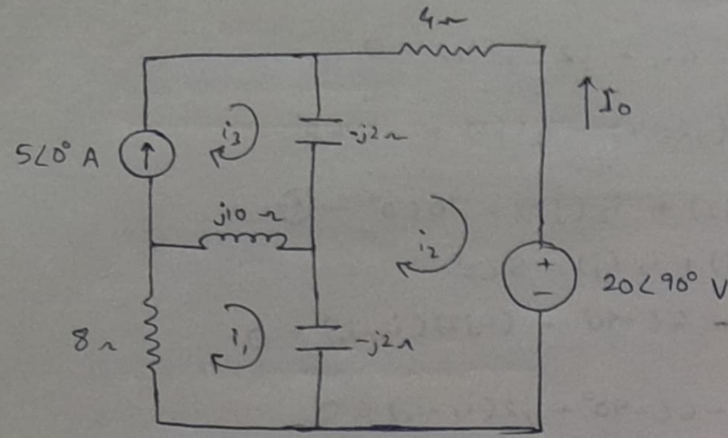
$\boxed{i_2 = \dfrac{1-3j}{2-2j}}$ $\boxed{i_1 = \dfrac{5-3j}{2-2j}}$

$I_0 = i_1 - i_2 = \dfrac{5-3j-1+3j}{2-2j} = \dfrac{4}{2-2j} = \dfrac{2}{1-j} = 1+j$

$\therefore i_0 = 1.414\angle 45^\circ$

$= 1.414\cos(2t + 45^\circ)$

Q) Determine I_0 using Mesh Analysis



[i_0 : $-6.12\angle 144.78^\circ$ A]

$\boxed{i_0 = -i_2}$

① → $-8i_1 - j10(i_1 - i_3) - (-j2)(i_1 - i_2) = 0$

⇒ $i_1(-8 - j10 + j2) - i_2(j2) + i_3(j10) = 0$

⇒ $-8i_1(1+j) - 2i_2(j) + 10i_3(j) = 0$

⇒ $4i_1(1+j) + i_2(j) - 5i_3(j) = 0$ — ①

② → $-(-j2)(i_2 - i_1) - (-j2)(i_2 - i_3) - 4i_2 - 20\angle 90^\circ = 0$

⇒ $j2(i_2 - i_1) + j2(i_2 - i_3) - 4i_2 = 20\angle 90^\circ$

⇒ $j2(2i_2 - i_3 - i_1) - 4i_2 = 20\angle 90^\circ$

⇒ $i_1(-j2) + i_2(j4 - 4) + i_3(-j2) = 20\angle 90^\circ$ — ②

③ → $-(-j2)(i_3 - i_2) - j10(i_3 - i_1) = 0$

⇒ $j2(i_3 - i_2) - j10(i_3 - i_1) = 0$

⇒ $j2(5 - i_2) - j10(5 - i_1) = 0$

⇒ $-j(40) - j10(i_1) - j2(i_2) = 0$

⇒ $10i_1 + 2i_2 = 40$ ⇒ $\boxed{5i_1 + i_2 = 20}$ — ③

Put ⑤ in ① :

$\Rightarrow 4\left(\frac{20-i_2}{5}\right)(1+j) + i_2(j) - 25j = 0$

$\Rightarrow \cancel{8}\ 4(20-i_2)(1+j) + 5i_2(j) - 125j = 0$

$\Rightarrow (80-4i_2)(1+j) + i_2(j5) - (125j) = 0$

$\Rightarrow 80 + 80j - 4i_2 - 4j(i_2) + (j5)i_2 - 125j = 0$

$\Rightarrow -45j - 4i_2 + i_2(j9) + 80 = 0$

$\Rightarrow i_2(-4+j9) - 45j = -80$

$\Rightarrow i_0(j9-4) + 45j = 80 \quad - ④$

$\Rightarrow \boxed{i_0 = \frac{80-45j}{-4+j9}}$

$i_0 = \frac{91.7818 \angle -29.3578}{9.8489 \angle 113.9625}$

$\boxed{i_0 = 9.32 \angle -143.32 \text{ A}}$

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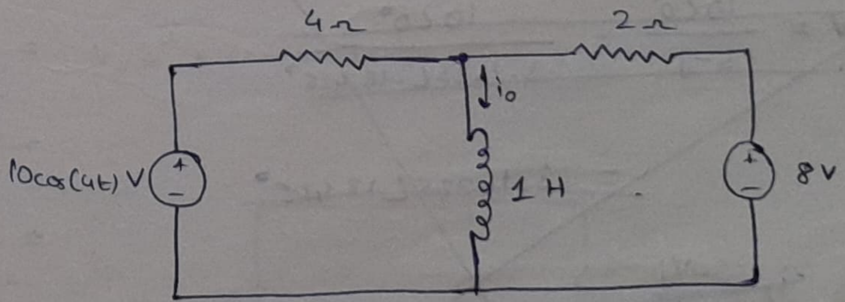
→ Superposition Theorem :

Applicable to Linear Circuits

Q) Determine i_0 using Superposition Theorem :



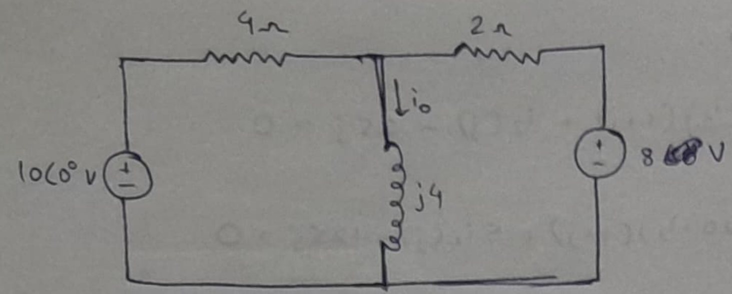
(i) Convert time domain to Phasor Domain

$1\text{ H} \longrightarrow j \times 1 \times 4 = j4$

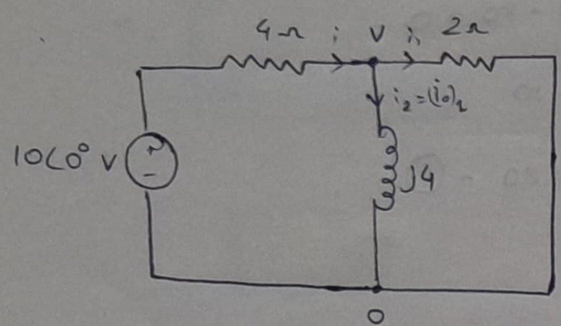
$8\text{ V} \longrightarrow 8\angle 0°$

$10\cos(4t) \longrightarrow 10\angle 0°$

8)



Finding $(i_o)_1$ by Shorting 8V source



$$\frac{10\angle 0^\circ - V}{4} = \frac{V-0}{2} + \frac{V-0}{j4}$$

$$\Rightarrow \frac{10\angle 0^\circ - V}{4} = \frac{V}{2} - \frac{Vj}{4}$$

$$\Rightarrow 10\angle 0^\circ - V = 2V - Vj$$

$$\Rightarrow 10\angle 0^\circ = V(3-j)$$

$$\cancel{\Rightarrow V = \frac{10\angle 0^\circ}{3-j} = \frac{10\angle 0^\circ}{3.1623\angle -18.435^\circ}}$$

$$\cancel{= 3.16225\angle 18.435^\circ}$$

$$\cancel{i_2 = \frac{-Vj}{4}}$$

$$\cancel{= -\frac{(3.16225\angle 18.435^\circ)(1\angle 90^\circ)}{4}}$$

$$\cancel{= -0.63245\angle 108.435^\circ}$$

$$V = \frac{10}{3-j}$$

$$i_2 = -\frac{\left(\frac{10}{3-j}\right)j}{4}$$

$$= \frac{10j}{4(j-3)}$$

$$= \frac{5j}{2j-6}$$

$$= \frac{-10+30j}{-4-36}$$

$$= \frac{-10+30j}{-40}$$

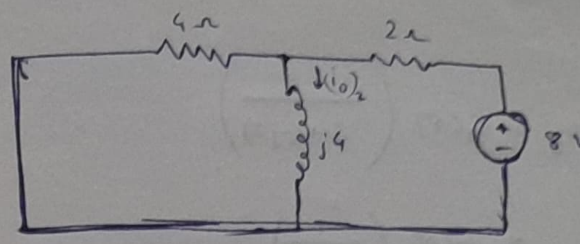
$$= \frac{10-30j}{40}$$

$$= \frac{1-3j}{4}$$

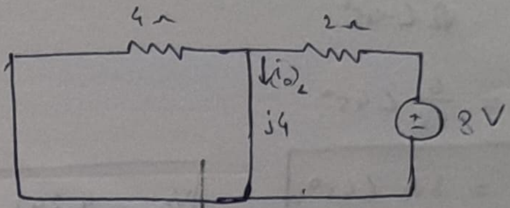
Correct Ans: ~~0.7915~~ $\cancel{\angle -71.56^\circ}$

$(i_o)_1 = 0.79057 \angle -71.56^\circ$ — ①

Finding $(i_o)_2$ by shorting $10\angle 0^\circ$ source



⇓



In DC,

Inductor is shorted

$$(i_o)_2 = \frac{8}{2} = 4 \text{ A}$$

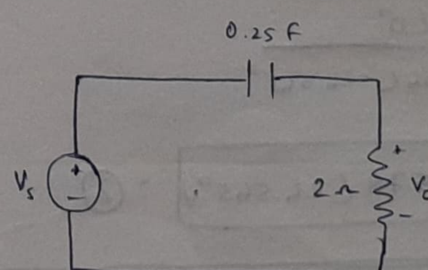
$$i_o = (i_o)_1 + (i_o)_2$$

$$\Rightarrow \boxed{i_o = 0.79057\angle -71.56^\circ + 4 \quad \text{A}}$$

$$\Rightarrow \boxed{i_o = 4 + 0.79\cos(4t - 71.56^\circ)\ \text{A}}$$

Q) Find v_o for the circuit

$v_s = 6\cos(2t) + 4\sin(4t)$ V



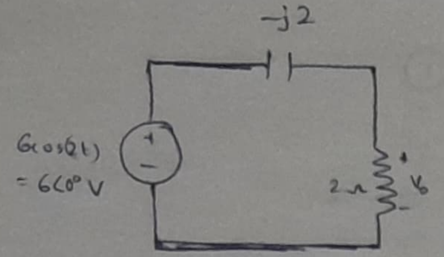
A:

(i) $v_s = 6\cos(2t)$ $[v_{o_1}]$

(ii) $v_s = 4\sin(4t)$ $[v_{o_2}]$

This problem can be solved only using Superposition Theorem (1 M)

(i)



$\frac{1}{j \times \frac{1}{4} \times 2} = \frac{2}{j}$

$= -2j$

$$V_{o_1} = (6\angle 0^\circ)\left(\frac{2}{2+(-j2)}\right)$$

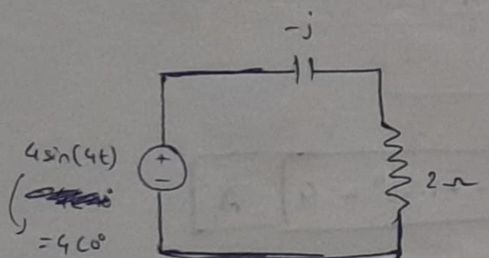
$$= 6\angle 0^\circ \left(\frac{1}{1-j}\right)$$

$$= \frac{6\angle 0^\circ}{\sqrt{2}\angle -45^\circ}$$

$$= \frac{6}{\sqrt{2}}\angle 45^\circ$$

$\boxed{V_{o_1} = 3\sqrt{2}\angle 45^\circ V} \simeq \boxed{V_{o_1} = 4.243\angle 45^\circ V}$ - ①

(ii)



$\frac{1}{j \times \frac{1}{4} \times 4} = -j$

sin → Convert into cos only when Angular frequencies are same.

Any AC load cannot be excited/supplied by 2 Different AC sources with 2 different frequency components in real time.

$$\cancel{V_{o_2} = (4\angle -90^\circ)\left(\frac{2}{2-j}\right)}$$

$$V_{o_2} = (4\angle 0^\circ)\left(\frac{2}{2-j}\right)$$

$$= \frac{8\angle 0^\circ}{2.236\angle -26.565^\circ}$$

$\boxed{V_{o_2} = 3.578\angle 26.565^\circ V}$ - ②

$$\boxed{V_o = 4.243\cos(2t + 45^\circ) + 3.578\sin(4t + 26.565^\circ) V}$$

Cannot be simplified further.

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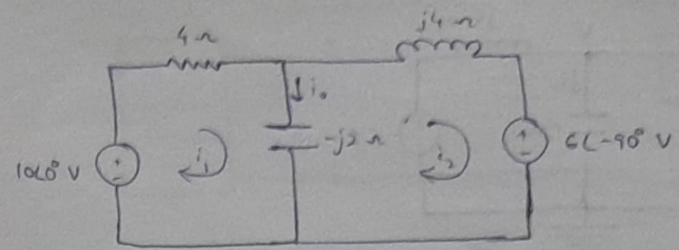
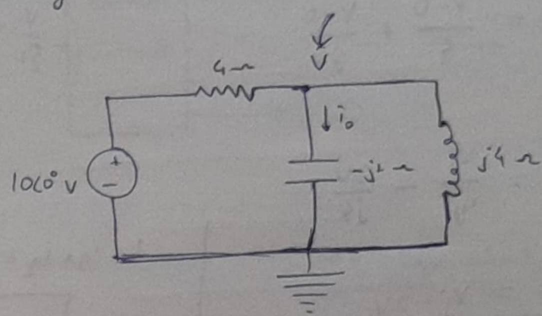
Q) Find i_o using Superposition Theorem:



A Shorting 6∠−90 Source first to find i_{o_1}



$$\frac{V-10}{4} + \frac{V}{-j2} + \frac{V}{j4} = 0$$

By Nodal Analysis,

$$-\frac{V-10\angle 0^\circ}{4} = \frac{V-0}{-j2} + \frac{V-0}{j4}$$

$$\Rightarrow \frac{V-10\angle 0^\circ}{4} = -\frac{Vj}{2} + \frac{Vj}{4}$$

$$\Rightarrow V-10\angle 0^\circ = -2Vj + Vj = -Vj$$

$$\Rightarrow 10\angle 0^\circ = V(1+j)$$

$$\Rightarrow V = \frac{1+j}{10\angle 0^\circ} = \frac{1+j}{10}$$

$$\boxed{V = \frac{1+j}{10}}$$

$$\boxed{V = \frac{10}{1+j}}$$

$$i_{o_1} = \frac{Vj}{2} = \frac{j}{2}\left(\frac{1+j}{10}\right)$$

$$= \frac{-1+j}{20} = \frac{\sqrt{2}\angle 135^\circ}{20\angle 0^\circ}$$

$$i_{o_1} = \frac{\sqrt{2}\angle 135^\circ}{20\angle 0^\circ}$$

$$\boxed{i_{o_1} = 0.0707\angle 135^\circ}$$

$$\boxed{i_{o_1} = \frac{\sqrt{2}}{20}\angle 135^\circ}$$

$$\boxed{i_{o_1} = -0.05 + 0.05j}$$

$$i_{o_1} = \frac{Vj}{2} = \frac{j}{2}\left(\frac{10}{1+j}\right)$$

$$= \frac{5j}{1+j}$$

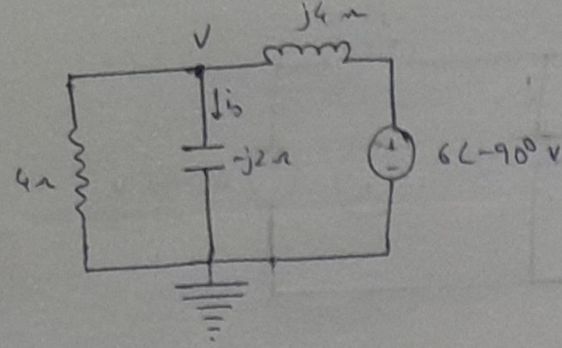
$$\boxed{i_{o_1} = \frac{5}{2}j(1-j)}$$

Generally $\omega = 2$ or 4 (In India)

$$\boxed{i_{o_1} = \frac{5}{2}(1+j)}$$

$$\boxed{i_{o_1} = 3.535\angle 45^\circ}$$

Shorting $10\angle 0^\circ$ source to find v_{o_2}, i_{o_2}.



$$\frac{6\angle -90^\circ - V}{j4} = \frac{V-0}{4} + \frac{V-0}{-j2}$$

$$\left(-\frac{V}{j2} = \frac{Vj}{2}\right)$$

$$\Rightarrow \frac{6\angle -90^\circ - V}{j4} = \frac{Vj}{4j} - \frac{2V}{j4}$$

$$\Rightarrow 6\angle -90^\circ - V = Vj - 2V$$

$$\Rightarrow 6\angle -90^\circ = V(j-1)$$

$$\Rightarrow V = \frac{6\angle -90^\circ}{j-1}$$

$$i_{o_2} = \frac{V-0}{-j2} = \frac{Vj}{2}$$

$$i_{o_2} = \frac{j}{2}\left(\frac{-6j}{j-1}\right)$$

$$= \frac{6}{2j-2}$$

$$i_{o_2} = \frac{3}{j-1}$$

$$\Rightarrow i_{o_2} = \frac{3\angle 0^\circ}{\sqrt{2}\angle 135^\circ}$$

$$i_{o_2} = \frac{3}{\sqrt{2}}\angle -135^\circ$$

$$\Rightarrow \boxed{i_{o_2} = \frac{3\sqrt{2}}{2}\angle -135^\circ}$$

$$i_o = i_{o_1} + i_{o_2} = \frac{5}{2}(1+j) - \frac{3}{2}(1+j)$$

$$\boxed{i_o = 1+j}$$

$$\boxed{\therefore i_o = \sqrt{2}\angle 45^\circ}$$

Correct Ans:

$$i_o(t) = \sqrt{2}\cos(2t+45^\circ)\,A$$

$$\cancel{i_o = i_{o_1} + i_{o_2} = -\frac{3}{2}(j+1) + \frac{5}{20}(1+j)}$$

$$\cancel{= \frac{-30j}{20} - \frac{30}{20} - \frac{1}{20} + \frac{j}{20}}$$

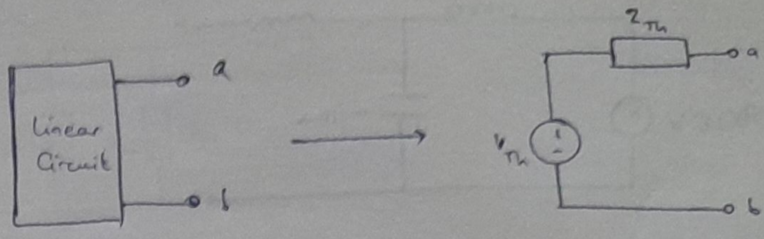
$$\cancel{= \frac{-29j}{20} - \frac{31}{20}}$$

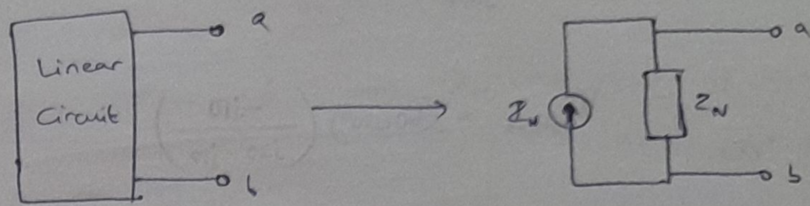
$$\cancel{= -1.55 - 1.45j}$$

$$\cancel{\Rightarrow i_o = 2.1225\angle -137^\circ}$$

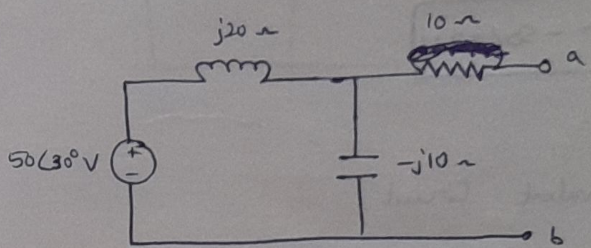
$$\cancel{\Rightarrow i_{o_2} = 2.12132\angle -135^\circ}$$

→ Thevenin's & Norton's Equivalent Circuit :



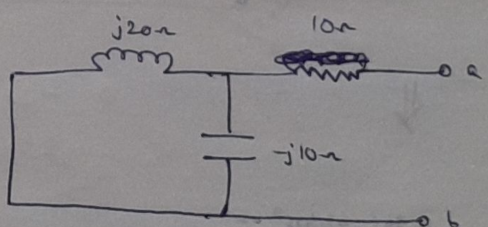


Q) find Thevenin's and Norton's Equivalent Circuit @ terminals a,b.



(i) Convert all quantities into Phasor Domain

For finding Z_{Th} :



$j20 \parallel -j10$

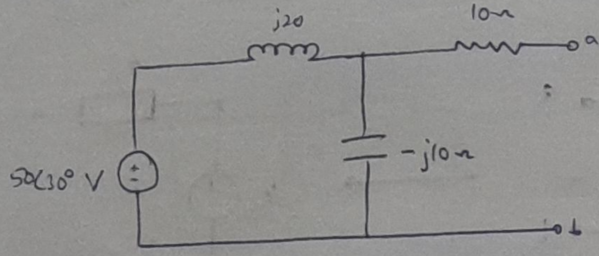
$$\Rightarrow \frac{j20 \times -j10}{10j}$$

$$\Rightarrow \frac{20}{j} \Rightarrow -20j \ \Omega$$

$-20j$ is Series with 10Ω = $\underline{10-20j}$ = $22.36\angle -63.435\ \Omega$

$$\boxed{Z_{Th} = 10-20j\ \Omega}$$

For finding V_{Th} :



$$V_{ab} = V_{-j10\Omega}$$

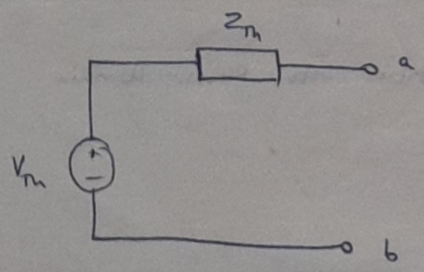
$$V_{-j10\Omega} = (50\angle 30^\circ)\left(\frac{-j10}{j20 - j10}\right)$$

$$= (50\angle 30^\circ)(-1) \qquad [-1 = -1\angle 0^\circ]$$

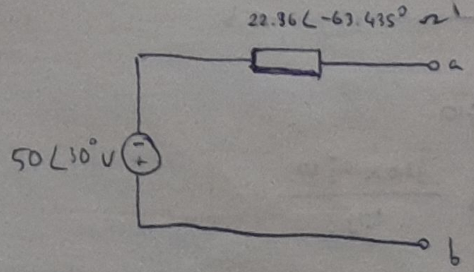
$$= -50\angle 30^\circ$$

$$\boxed{V_{Th} = -50\angle 30^\circ\ V}$$

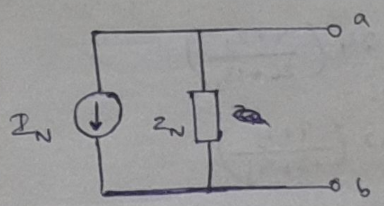
∴ Thevenin's Equivalent Circuit :



⇓



For finding Norton's Equivalent Circuit :

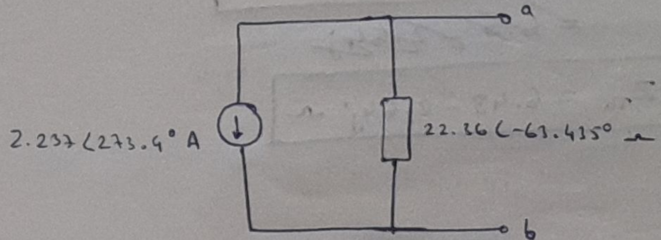


$Z_N = Z_{Th}$

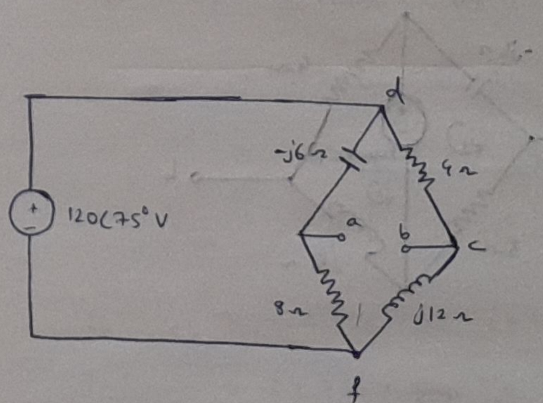
$$(I_N)^{-1} = \left(\frac{V_{Th}}{Z_{Th}}\right)^{-1} = \frac{22.361\angle -63.435^\circ}{50\angle 30^\circ} \text{ A}$$

$$\Rightarrow (I_N)^{-1} = 0.447\angle -93.435^\circ \text{ A}$$

$$= 2.237\angle 93.435^\circ \text{ A}$$

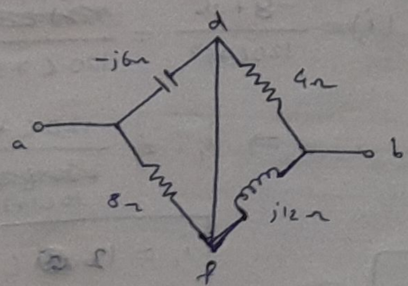


9)



Find Thevenin's Eq. ckt. and Norton's Eq. ckt.

~~V_{Th}~~: Z_{Th}:



$$Z_{Th} = -j6 \parallel 8\,\Omega + 4 \parallel j12\,\Omega$$

$$= \frac{-48j}{8-6j} + \frac{j48}{4+12j}$$

$$\Rightarrow Z_{Th} = j48\left(\frac{1}{4+12j} - \frac{1}{8-6j}\right)$$

$$= j48\left(\frac{8-6j-4-12j}{104+72j}\right)$$

32+

$$= 48\left(\frac{18+4j}{104+72j}\right)$$

$$\Rightarrow Z_{Th} = 48\left(\frac{9+2j}{52+36j}\right)$$

$$= 24\left(\frac{9+2j}{26+18j}\right)$$

$$= 12\left(\frac{9+2j}{13+9j}\right)$$

$$= \frac{108+24j}{13+9j}$$

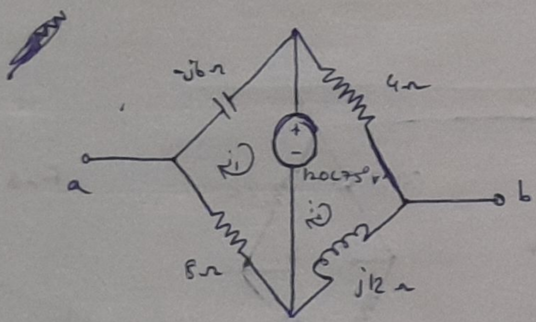
$$= \frac{110.6\angle 12.53^\circ}{15.8\angle 34.7^\circ}$$

$$\boxed{Z_{Th} = 7\angle -22.17^\circ\ \Omega}$$

$$\cancel{= 6.8 - 2.64j}$$

$$\boxed{Z_{Th} = 6.48 - 2.64j\ \Omega}$$

For finding V_{Th}:



$$-8i_1 + j6i_1 - 120\angle 75^\circ = 0 \quad (1)$$

$$120\angle 75^\circ = i_1(-8+j6)$$

$$(i_1)^{-1} = \frac{-8+j6}{120\angle 75^\circ} = \frac{10\angle 143.13^\circ}{120\angle 75^\circ}$$

$$\Rightarrow i_1 = \frac{120\angle 75^\circ}{10\angle 143.13^\circ} = 12\angle -68.13^\circ$$

$$\boxed{i_1 = 12\angle -68.13^\circ\ \text{A}}$$

$$\boxed{i_1 = 4.47 - 11.136j\ \text{A}}$$

$$120\angle 75^\circ - 4i_2 - j12i_2 = 0$$

$$\Rightarrow 120\angle 75^\circ = i_2(4+j12)$$

$$\Rightarrow i_2 = \frac{120\angle 75^\circ}{12.65\angle 71.565^\circ}$$

$$\boxed{i_2 = 9.4862\angle 3.435^\circ\ \text{A}}$$

$$\boxed{i_2 = 9.47 + 0.568j\ \text{A}}$$

Ans : $38 \angle 220° V$

$V_a + j6i_1 - 4i_2 = V_b$

$\Rightarrow V_{ba} = j6i_1 - 4i_2$

$= j6(4.47 - 11.136j) - 4(9.47 + 0.568j)$

$= 6(11.136 + 4.47j) - 4(9.47 + 0.568j)$

$= 66.816 + 26.82j - 37.88 - 2.272j$

$= 28.936 + 24.548j$

$\Rightarrow V_{ba} = 37.946 \angle 40.31°$

$V_{Th} = V_{ab} = -28.936 - 24.548j$

$= 37.946 \angle 220.31°$

$$\boxed{V_{Th} = 37.946 \angle 220.31°}$$

$\therefore I_N = \frac{V_{Th}}{Z_{Th}} = \frac{38 \angle 220°}{7 \angle -22.17°}$

$$\boxed{I_N = 5.4286 \angle 242.17°}$$

6/11/23

AC Power Analysis : (CHAPTER - 11)

(i) Instantaneous Power :

$$p(t) = v(t) \times i(t)$$

Power is absorbed by any electrical load at any instant of time.

Rate at which Power is being absorbed

Non-measurable Power by any Instrument.

$$v(t) = v_m \cos(\omega t + \theta_v)$$

$$i(t) = i_m \cos(\omega t + \theta_i)$$

$$\Rightarrow p(t) = V(t) \times i(t)$$

$$= V_m i_m \cos(\omega t + \theta_v) \cdot \cos(\omega t + \theta_i)$$

$$\Rightarrow p(t) = V_m i_m \left[\frac{1}{2}(\cos(\omega t + \theta_v - \omega t - \theta_i)) + \frac{1}{2}(\cos(\omega t + \theta_v + \omega t + \theta_i)) \right]$$

$$= V_m i_m \left[\frac{1}{2} \cos(\theta_v - \theta_i) + \frac{1}{2} \cos(2\omega t + \theta_v + \theta_i) \right]$$

$$= \underbrace{\frac{1}{2} V_m i_m \cos(\theta_v - \theta_i)}_{\substack{\text{Constant Power} \\ \text{(Time Independent)}}} + \underbrace{\frac{1}{2} V_m i_m \cos(2\omega t + \theta_v + \theta_i)}_{\substack{\text{Sinusoidal Power at } 2\omega t \approx 0 \\ \text{[Cannot be measured]}}}$$

(ii) Average Power :

$$\frac{\text{Area Under the curve}}{\text{Base}}$$

i.e. Average of Instantaneous Power over a cycle.

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt}$$

$$\Rightarrow P = \frac{1}{T}\int_0^T \frac{1}{2} V_m i_m \left(\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right) dt$$

$$= \frac{V_m \cdot i_m}{2T}\int_0^T \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\, dt$$

$$= \frac{V_m i_m}{2T}\left[\left[T\cos(\theta_v - \theta_i) + \frac{\sin(2\omega t + \theta_v + \theta_i)}{2\omega}\right]_0^T\right]$$

$$= \frac{V_m \cdot i_m}{2}\left[\cos(\theta_v - \theta_i) + \frac{\sin(2\omega T + \theta_v + \theta_i)}{2\omega T} - \frac{\sin(\theta_v + \theta_i)}{2\omega T}\right]$$

$$= \frac{1}{2} V_m i_m \left[\cos(\theta_v - \theta_i) + \frac{2\cos(\omega t + \theta_v + \theta_i)\cdot \sin(\omega T)}{2\omega T}\right]$$

$$= \frac{1}{2} V_m \cdot i_m \left[\cos(\theta_v - \theta_i) + \frac{\cos(\omega T + \theta_v + \theta_i)\cdot \sin(\omega T)}{\omega T}\right]$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

$$= \frac{1}{2} V_m \cdot i_m \left[\cos(\theta_v - \theta_i) + \frac{\cos(2\pi + \theta_v + \theta_i)\cdot \sin(2\pi)}{\omega T}\right]$$

$$\boxed{P_{avg.} = \frac{1}{2} V_m \cdot i_m \cos(\theta_v - \theta_i)}$$

$$\boxed{\sin(2\pi) = 0}$$

Not time dependent

Purely Resistive Load case : $\theta_v = \theta_i$

(In Phase)

$$\boxed{\therefore P = \frac{1}{2} V_m \cdot i_m}$$

Purely Reactive Load Case : $\theta_v - \theta_i = \pm 90^\circ$

$$\boxed{P = 0}$$

Current lags Voltage by 90° in Purely Inductive Load.

Voltage lags current by 90° in Purely Capacitor load.

$P=0$ means that the circuit absorbs no average Power.

Q) ~~Calcul~~ Calculate $p(t)$ and $P_{avg.}$ of Passive ~~tem~~ linear Network

$v(t) = 120\cos(377t + 45^\circ)$ V

$i(t) = 10\cos(377t - 10^\circ)$ A

$P = \frac{1}{2} V_m \cdot I_m (\cos(\theta_v - \theta_i))$ $\boxed{50 \text{ Hz}}$

$= \frac{1}{2} \times 10 \times 120 \times \cos(55^\circ)$

$= 600\cos(55^\circ)$

$= 344.146$ W

$\boxed{\therefore P_{avg.} = 344.146 \text{ W}}$

$p(t) = \frac{1}{2} \times V_m i_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m i_m \cos(2\omega t + \theta_v + \theta_i)$

$= 344.146 + 600\cos(2\omega t + 35^\circ)$ $[\omega = 377]$

$\boxed{p(t) = 344.146 + 600\cos(754t + 35^\circ) \text{ W}}$

Q). $v(t) = 330\cos(10t + 20^\circ)$ V

$i(t) = 33\sin(10t + 60^\circ)$ A

Calculate P and $p(t)$

$33\sin(10t+60^\circ)$

$= 33\cos(90 - (10t + 60^\circ))$

$= 33\cos(10t - 30^\circ)$

$P = \frac{1}{2} \times 330 \times 33 \times \cos(20 - \theta_i)$

$= \frac{1}{2} \times 330 \times 33 \times \cos(50^\circ)$

$\Rightarrow \boxed{P = 3.5 \text{ kW}}$

$$p(t) = 3.5\ kW + \frac{1}{2} \times 330 \times 33 \times \cos(20t - 10^\circ)$$

$$\boxed{\therefore p(t) = 3.5\ kW + 5.445 \cos(20t - 10^\circ)\ kW}$$

Q) $P_{avg} = ?$

$Z = (30 - j70)\ \Omega$

$V = 120\ V$

$V = 120\angle 0^\circ\ V$

$$i = \frac{V}{Z} = \frac{120\angle 0^\circ}{76.16\angle -66.8^\circ}\ A$$

$$\boxed{i = 1.5756\angle 66.8^\circ\ A}$$

$\therefore V = 120 \cos(\omega t)\ V$

$i = 1.5756 \cos(\omega t + 66.8^\circ)\ A$

$$P = \frac{1}{2} \times 120 \times 1.5756 \times \cos(-66.8^\circ)\ W$$

$$= 60 \times 1.5756 \times \cos(66.8^\circ)\ W$$

$$\Rightarrow \boxed{P_{avg} = 37.2417\ W}$$

Q) $\cancel{I =}$ $i = 33\angle 30^\circ\ A$

$Z = 40\angle -22^\circ\ \Omega$, $P_{avg} = ?$

$$\boxed{V = 1320\angle 8^\circ\ V}$$

$V = 1320 \cos(\omega t + 8^\circ)\ V$

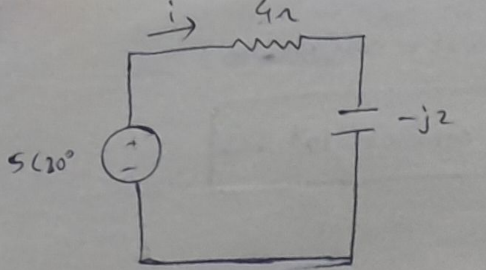
$i = 33 \cos(\omega t + 30^\circ)\ A$

$$P = \frac{1}{2} \times 1320 \times 33 \times \cos(-22^\circ)$$

$$\Rightarrow \boxed{P = 20.194\ kW}$$

$$\boxed{\therefore P_{avg} = 20.2\ kW}$$

9)



find P_{avg_s}, P_{avg_R}, $P_{avg_{Cap}}$

$$v(t) = 5\cos(\omega t + 30°)$$

$$z = 4 - j2$$

$$i(t) = \frac{5\angle 30°}{4.47\angle -26.565°}$$

$$\boxed{i(t) = 1.1186 \angle 56.565 \text{ A}}$$

$$P_{avg} \text{ (source)} = \frac{1}{2} \times 5 \times 1.1186 \times \cos(30° - 56.565°)$$

$$\Rightarrow \boxed{P_{avg.\,source} = 2.5 \text{ W}}$$

$$P_{avg.\,4\Omega} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$V_{4\Omega} = 5\angle 30° \left(\frac{4}{4-j2}\right)$$

$$\Rightarrow V_{4\Omega} = 4.472\angle 56.565°$$

(Capacitor does not consume any avg. Power)

$$\therefore P_{avg_{4\Omega}} = \frac{1}{2} \times 4.472 \times 1.1186 \times \cos(0°)$$

$$= 5 \text{ W}$$

$$\boxed{\therefore P_{avg_{4\Omega}} = 5 \text{ W}}$$

$$P_{avg.\,-j2} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$V_{-j2} = 5\angle 30° \times \left(\frac{-j2}{4-j2}\right)$$

$$\Rightarrow V_{-j2} = 2.236 \angle -33.435°$$

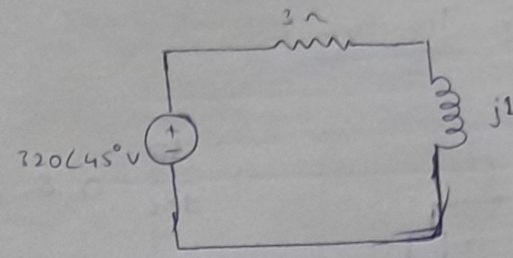
$$\therefore P_{avg_{-j2}} = \frac{1}{2} \times V_m \times I_m \times \cos(-33.435° - 56.565°)$$

$$= (\quad) \times \cos 90°$$

$$= 0$$

$$\boxed{\therefore P_{avg.\,Cap} = 0}$$

Q)



Find P_{avg} of source, Absorbed by Resistor & ~~Capacitor~~ Inductor

P_{avg} (Inductor) $= 0$ $\xrightarrow{\text{Reason}}$

& P_{avg} (source) $= P_{avg}$ (3 Ω)

$V_{j1} = 320\angle 45° \times \frac{j1}{3+j1}$

$\boxed{V_{j1} = 101.2\angle 116.565°}$

&

$\boxed{i_{j1} = 101.2\angle 26.565°}$

$\therefore P_{avg_{Ind.}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= \frac{1}{2} V_m I_m \cos 90°$

$= 0$

$i = \frac{V}{Z} = \frac{320\angle 45°}{3.1623\angle 18.435°}$

$\Rightarrow \boxed{i = 101.2\angle 26.565° \text{ A}}$

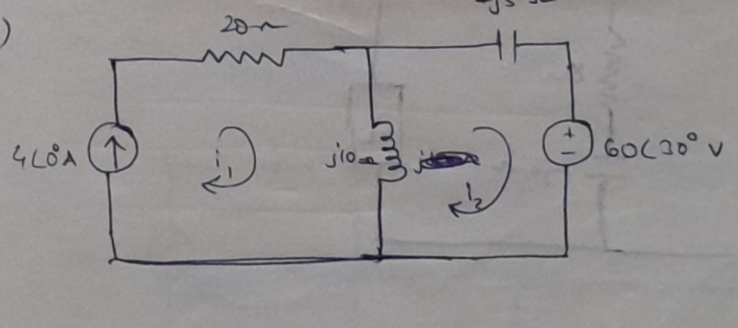
$P = \frac{1}{2} \times 320 \times 101.2 \times \cos(45° - 26.565°)$

$\therefore \boxed{P_{avg} = 15361.1 \text{ W}}$

$\boxed{P = 15.36 \text{ kW}} \rightarrow$ Source

H.W

Q)



Find Avg. Power generated by each source and Avg. Power absorbed by each passive element.

$\boxed{i_1 = 4\angle 0° \text{A}}$

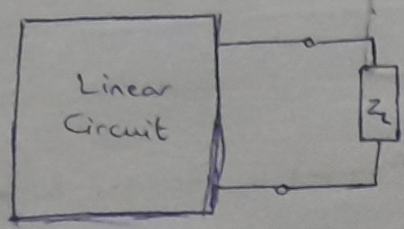
i_2

$-60\angle 30° - j10(i_2 - i_1) + j5(i_2) = 0$

$\Rightarrow 60\angle 30° = j10(i_2 - 4) - j5(i_2)$

$\Rightarrow 60\angle 30° + j40 = j5\, i_2$

→ Maximum Average Power Transform :



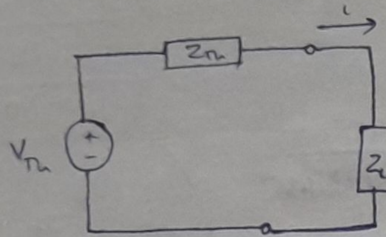
(a)

$$\frac{dP}{dR_L} = 0 \ \& \ \frac{dP}{dX_L} = 0$$

$$\Rightarrow Z_L = Z_{Th}^*$$

$$\begin{bmatrix} R_L = R_{Th} \\ X_L = X_{Th}^* \end{bmatrix}$$

$$Z_{Th} = R_{Th} + jX_{Th}$$
$$Z_L = R_L + jX_L$$



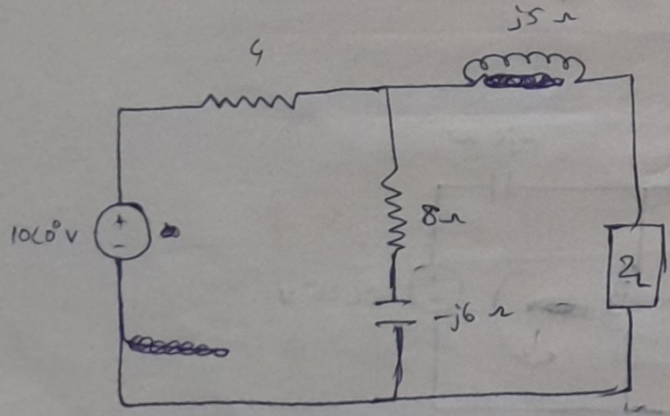
$$P_{max.} = \frac{|V_{Th}|^2}{8 R_{Th}}$$

$$X_L = -X_{Th}$$
$$R_L = R_{Th}$$

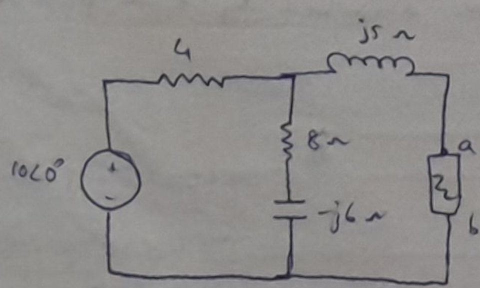
If load is purely Real,

$$R_L = |Z_{Th}|$$

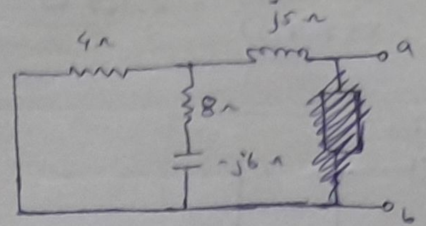
Q)



Z_{Th} :



Short sources



$$Z_{Th} = [(8-j6) \parallel 4\Omega] + j5$$

$$Z_{Th} = \frac{(8-j6)\times 4}{12-j6} + j5$$

$$Z_{Th} = \frac{(8-j6)(2)}{6-j3} + j5$$

$$= 4\left(\frac{4-j3}{6-j3}\right) + j5$$

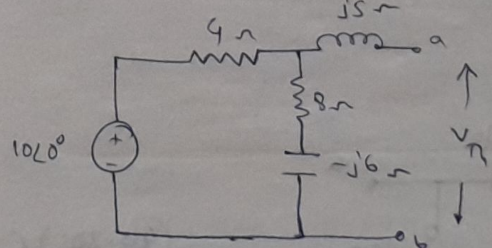
$$= \frac{16-j12+j30+15}{6-j3}$$

$$= \frac{31+j18}{6-j3} = \frac{35.847\angle 30.14}{6.71\angle -26.565}$$

$$= 5.34\angle 56.705$$

$$Z_{Th} = 2.93 + 4.46j \Rightarrow Z_L = Z_{Th}^* = 2.93 - 4.46j$$

for V_{Th}:



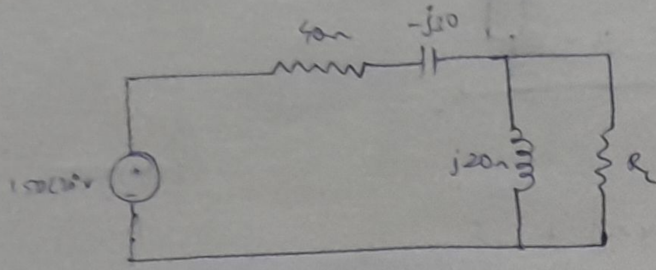
$$V_{Th} = \frac{8-j6}{4+8-j6}(10) = \frac{10\angle -36.87}{13.416\angle -26.565}(10)$$

$$= 7.454\angle -10.3\ V$$

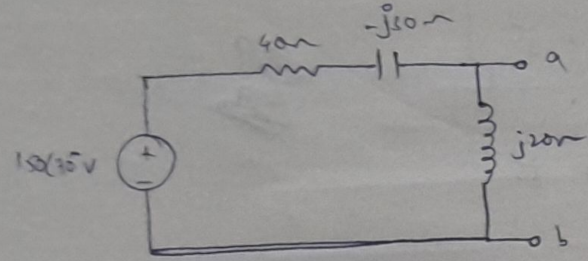
$$Z_L = Z_{Th}^* = (2.933 - j\,4.467)\ \Omega$$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368\ W$$

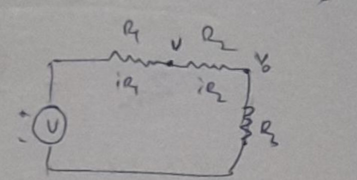
Q7)



For finding V_{Th} :



$$v_2 = \left(\frac{R_2 V}{R_1+R_2}\right)R_2$$



v_o

$v - iR_1 - iR_2 = v_o$

$$V_{Th} = V_{ab} = (150\angle 30^\circ)\left(\frac{j20}{40-j10}\right)$$

$Z_{eff} = 40 - j10$

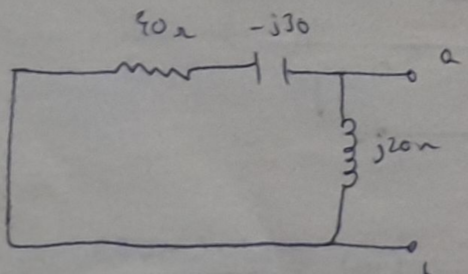
$$= (150\angle 30^\circ)\left(\frac{20\angle +90^\circ}{41.23\angle -14.03^\circ}\right) \Rightarrow$$

~~$V_{Th} = 60\angle 156.87$~~

$$\boxed{V_{Th} = 72.761\angle 134.036^\circ}$$

~~$R_{Th} =$~~

For R_{Th} :



$$Z_{Th} = \frac{(40-30j)(20j)}{(40-10j)}$$

$$= \frac{(50\angle -36.87^\circ)(20\angle 90^\circ)}{41.23\angle -14^\circ}$$

$\Rightarrow Z_{Th} = 9.411 + 22.3j$

$= 24.2\angle -67.12^\circ$

$R_{Th} = 9.411$

$Z_{Th}^* = Z_L = 9.411 - 22.3j$

$$P_{max} = \frac{(72.761)^2}{8 \times 9.411} = 70.32\ W \Rightarrow \therefore P_{max} = 70.32\ W$$

→ Effective (or) RMS value :

Any electrical Device is rated as RMS values.

$$I_{rms} = \frac{1}{\sqrt{T}}\sqrt{\int_0^T i^2(t)\,dt}$$

$$i(t) = i_m \cos(\omega t)$$

$$\Rightarrow I_{rms} = \frac{i_m}{\sqrt{2}} \quad \& \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

i_m → Max value

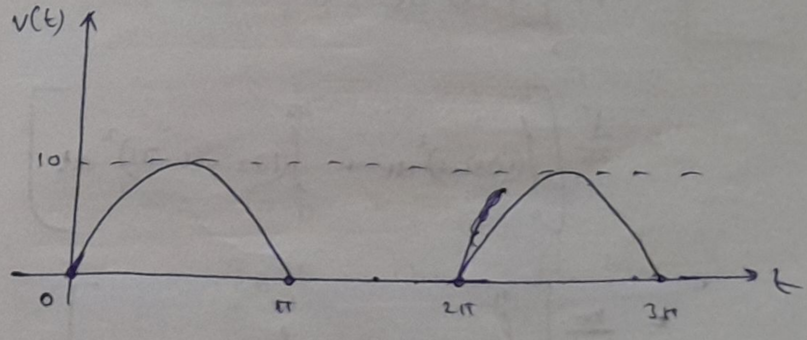
i_{rms} → rms value

$$P = \frac{1}{2} V_m i_m \cos(\theta_v - \theta_i)$$

$$\Rightarrow \boxed{P = V_{rms} \cdot i_{rms} \cdot \cos(\theta_v - \theta_i)}$$

Q) Half-wave Rectified sine wave

10 ohm Resistor, Find V_{RMS}



[T = 2π is the timeperiod]

Ans

$$v(t) = \begin{cases} 10\sin(t) & ; 0<t<\pi \\ 0 & ; \pi<t<2\pi \end{cases}$$

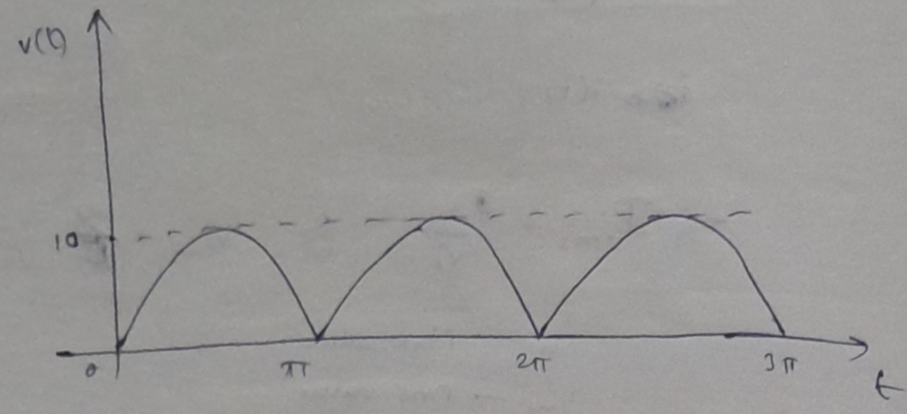
$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi}(10\sin t)^2\,dt + \int_{\pi}^{2\pi} 0\,dt\right]$$

$$= 25\ V$$

$$\Rightarrow V_{rms} = 5\ V$$

$$P = \frac{V_{rms}^2}{R} = \frac{5^2}{10} = \frac{25}{10} = 2.5$$

Q2) $R = 10\Omega$



Ans : Time period, $T = 2\pi$

$$v(t) = \begin{cases} 10\sin(t), & 0<t<\pi \\ 10\sin(t-\pi), & \pi<t<\pi \end{cases}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}(10\sin t)^2\,dt + \int_{\pi}^{2\pi}(10\sin(t-\pi))^2\,dt\right]$$

$$= \frac{100}{2\pi}\left[\int_0^{\pi}\sin^2 t\,dt + \int_{\pi}^{2\pi}\sin^2(\pi-t)\,dt\right]$$

$$= \frac{100}{2\pi}\left[\int_0^{\pi}\sin^2 t\,dt + \int_{\pi}^{2\pi}\sin^2 t\,dt\right]$$

$\cos 2\theta = 1-2\sin^2\theta$

$$= \frac{100}{2\pi}\left[\int_0^{2\pi}\sin^2 t\,dt\right]$$

$\int \sin^2 t\,dt$

$= \int \frac{1-\cos 2t}{2}\,dt$

$= \int \frac{1}{2} - \frac{\cos(2t)}{2}\,dt$

$= \frac{t}{2}$

$$= \frac{50}{\pi} \times \left[\frac{t}{2} - \frac{1}{4}\sin(2t)\right]_0^{2\pi}$$

$$= 50\pi \times (\pi - 0)$$

$$= 50$$

$$V_{rms}^2 = 50\ V^2$$

$$\boxed{V_{rms} = \sqrt{50}\ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{50}{10} = 5W$$

$$\boxed{P = 5\ W}$$

$\longleftrightarrow$ <u>Apparent Power</u> (S)

Rating of an electrical equipment.

$$P = V_{rms} \cdot i_{rms} \cdot \cos(\theta_v - \theta_i)$$

$$= Vi \cdot \cos(\theta_v - \theta_i)$$

$$\boxed{\therefore P = S\cos(\theta_v - \theta_i)}$$

Active Power $\left[S = V_{rms} \times i_{rms}\right]$

& $\left[\cos(\theta_v - \theta_i) = pf\right]$

[P : Avg. Power (or) Real Power (or) Useful Power.]

Power factor : $\cos(\theta_v - \theta_i)$

$\theta_v - \theta_i$ = Phase diff. b/w v and i.

Units of S : VA or volt-amperes.

Units of P : Watt

~~Power factor~~ $\cancel{= \frac{P}{S}}$

Power factor $= \frac{P}{S}$

Purely Resistive Load (R) $\Rightarrow$ $\theta_v - \theta_i = 0$, Pf = 1 [Unity Power factor] $\Rightarrow$ P/S = 1, All Power is being Consumed

Purely Reactive Load (L or C) $\Rightarrow$ $\theta_v - \theta_i = 90°$, Pf = 0 $\Rightarrow$ P/S = 0, No Real Power consumption ($\hookrightarrow$ P=0)

Resistive & Reactive Load (R and L(or)C) $\Rightarrow$
$\theta_v - \theta_i > 0 \Rightarrow$ lagging (Inductive Load) [P<S]
$\theta_v - \theta_i < 0 \Rightarrow$ leading (Capacitive load)

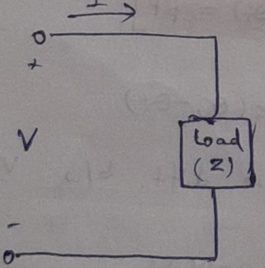
90% of Real loads are RL loads.
$\hookrightarrow$ Current lags behind Voltage

Power factor $\propto$ Efficiency $\propto$ Performance

$0 \leq$ Power factor ≤ 1

We use Capacitor invariably as a condensor in electrial loads in order to improve the Power factor.

→ <u>Complex Power</u>:



$V = V_m \angle \theta_v$
$i = i_m \angle \theta_i$

$Vi = Vi(\cos\theta + j\sin\theta)$
$= \underbrace{Vi\cos\theta}_{P} + j\underbrace{Vi\sin\theta}_{Q}$

$P = V_{rms} \cdot i_{rms}$

$S = Vi^*$
$= V_{rms} \cdot i_{rms} \cdot \angle(\theta_v - \theta_i)$

$S = P + Qj$

$\Rightarrow S = Vi\cos(\theta_v - \theta_i) + jVi\sin(\theta_v - \theta_i)$

$\boxed{S = P + jQ}$

Q : Reactive Power

$S = P + jQ$

S : Apparent Power (VA)

P : Real Power (W)

↳ $vi\cos\theta$

Q : Reactive Power (VAR)

↳ $vi\sin\theta = V_{rms} \cdot i_{rms} \cdot \sin(\theta_v - \theta_i)$

$Q = 0$ for Resistive Loads (Unity pf)

$Q < 0$ for Capacitive Loads (Leading pf)

$Q > 0$ for Inductive Loads (Lagging pf)

Reactive Power : The power which allows the work to happen (Power required to create magnetic field)

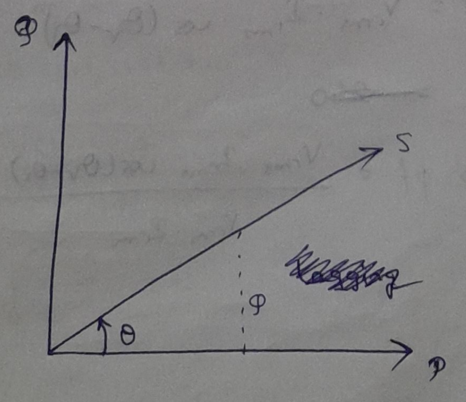
$$S = \underbrace{V_{rms} \cdot i_{rms} \cos(\theta_v - \theta_i)}_{P} + \underbrace{j\, V_{rms} \cdot i_{rms} \cdot \sin(\theta_v - \theta_i)}_{jQ}$$

Apparent Power , $S = |S|$

Real Power , $P = Re(S)$

Reactive Power , $Q = Im(S)$

$pf = P/S = \cos(\theta_v - \theta_i)$



(Lagging)

$\cos\theta = P/S$

$S = P + jQ = \sqrt{P^2 + Q^2}$

Purely Resistive Load (R) $\Rightarrow$ $\theta_v - \theta_i = 0$ $\Rightarrow$ $P/S = 1$, All Power is being Consumed

$Pf = 1$

[Unity Power factor]

Purely Reactive Load (L or C) $\Rightarrow$ $\theta_v - \theta_i = 90°$ $\Rightarrow$ $P/s = 0$, No Real Power consumption

$Pf = 0$ $\quad \hookrightarrow P = 0$

Resistive & Reactive Load (R and L or C) $\Rightarrow$

$\theta_v - \theta_i > 0 \Rightarrow$ lagging (Inductive Load)

[$P < S$]

$\theta_v - \theta_i < 0 \Rightarrow$ leading (Capacitive load)

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$\hookrightarrow$ Current lags behind Voltage

Power factor $\propto$ Efficiency $\propto$ Performance

$0 \leq$ Power factor ≤ 1

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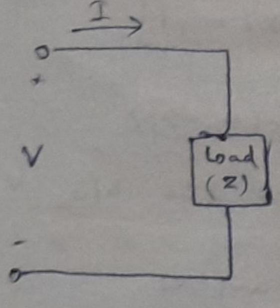
→ <u>Complex Power</u>:



$V = V_m \angle \theta_v$

$i = i_m \angle \theta_i$

$Vi = Vi(\cos\theta + j\sin\theta)$

$= \underbrace{Vi\cos\theta}_{P} + j\,\underbrace{Vi\sin\theta}_{Q}$

$P = V_{rms} \cdot i_{rms}$

$S = Vi^{*}$

$= V_{rms} \cdot i_{rms} \cdot \angle(\theta_v - \theta_i)$

$S = P + Qj$

$\Rightarrow S = Vi\cos(\theta_v - \theta_i)$ ~~j ... sin~~

$+ jVi\sin(\theta_v - \theta_i)$

$\boxed{S = P + jQ}$

Q : Reactive Power

$S = P + jQ$

S : Apparent Power (VA)

P : Real Power (W)

$\hookrightarrow$ $vi\cos\theta$

Q : Reactive Power (VAR)

$\hookrightarrow$ $vi\sin\theta = V_{rms} \cdot i_{rms} \cdot \sin(\theta_v - \theta_i)$

$Q = 0$ for Resistive Loads (Unity pf)

$Q < 0$ for Capacitive Loads (Leading pf)

$Q > 0$ for Inductive Loads (Lagging pf)

Reactive Power : The power which allows the work to happen (Power required to create magnetic field)

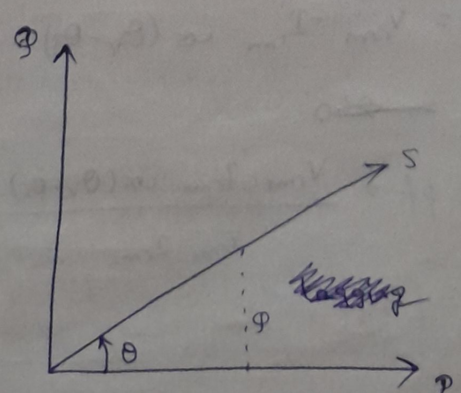
$$S = \underbrace{V_{rms} \cdot i_{rms} \cos(\theta_v - \theta_i)}_{P} + \underbrace{j V_{rms} \cdot i_{rms} \cdot \sin(\theta_v - \theta_i)}_{jQ}$$

Apparant Power , $S = |S|$

Real Power , $P = \mathrm{Re}(S)$

Reactive Power , $Q = \mathrm{Im}(S)$

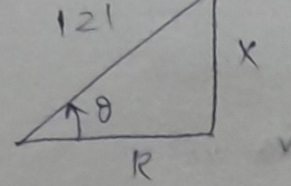
$pf = P/S = \cos(\theta_v - \theta_i)$



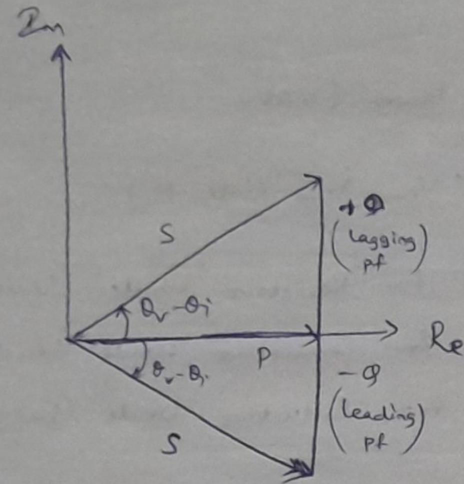
$\cos\theta = P/S$

$S = P + jQ = \sqrt{P^2 + Q^2}$

(Lagging)



$$\boxed{\theta = \tan^{-1}\left(\frac{X}{R}\right)}$$



$\cancel{\theta = \tan^{-1}\left(\frac{X}{R}\right)}$

Q)

$i(t) = 4\cos(100\pi t + 10^\circ)$ A

$v(t) = 120\cos(100\pi t - 20^\circ)$ V

Apparent Power $= V_{rms} \cdot I_{rms}$

$\Rightarrow S = \frac{4}{\sqrt{2}} \times \frac{120}{\sqrt{2}}$ VA

$\Rightarrow S = 240$ VA

Power factor $= \frac{P}{S}$

$P = V_{rms} \cdot I_{rms} \cdot \cos(\theta_v - \theta_i)$

$\cancel{240}$

$\Rightarrow pf = \frac{V_{rms} \cdot I_{rms} \cdot \cos(\theta_v - \theta_i)}{V_{rms} \cdot I_{rms}}$

$= \cos(\theta_v - \theta_i)$

$= \cos(-30^\circ)$

$= \sqrt{3}/2 \; (-)$ Negative

$= 0.866 \; (-)$

Capacitive $\Leftarrow$ [$\therefore$ Leading]

$\Rightarrow pf = \frac{R}{Z}$

$$Z = \frac{V}{I} = \frac{120\cos(100\pi t - 20^\circ)}{4\cos(100\pi t + 10^\circ)} = \frac{120\angle -20^\circ}{4\angle 10^\circ}\ \Omega$$

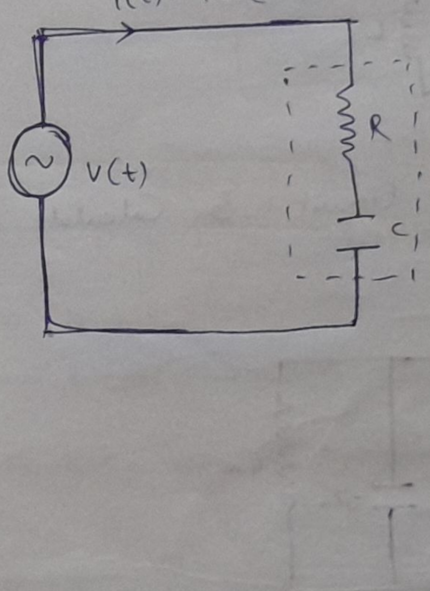
$$\Rightarrow \boxed{Z = 30\angle -30^\circ\ \Omega}$$

$$\Rightarrow \boxed{Z = 25.98 - j15\ \Omega}$$

$\cos(-30^\circ) = 0.866\ (-)$ [Leading]

$$\tan^{-1}\left(\frac{-15}{25.98}\right) = \tan^{-1}(-0.577)$$

$$= -30^\circ$$



$Z = R + jX_c\ ,\ X_c = -j15$

$= 25.98 - j15$

$X_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$\Rightarrow -15j = -\frac{j}{\omega C}$

$(\omega = 100\pi)$

$C = 212.2\ \mu F$

& $R = 25.98\ \Omega$

Q) Find pf and App. Power

$Z = 60 + j40\ \Omega$

$v(t) = 320\cos(377t + 10^\circ)\ V$

A:

$$i = \frac{v(t)}{Z} = \frac{320\angle 10^\circ}{72.111\angle 33.7^\circ} = 4.44\angle -23.7^\circ\ A$$

$$\boxed{\therefore i = 4.0630 - 1.78j}$$

$$S = V_{rms} I_{rms} = \frac{320}{\sqrt{2}} \times \frac{4.44}{\sqrt{2}} = 710.4\ VA$$

$$pf = \frac{V_{rms} \cdot I_{rms} \cdot \cos(\theta_v - \theta_i)}{V_{rms} \cdot I_{rms}} = \frac{P}{S}$$

$$= \cos(\theta_v - \theta_i)$$

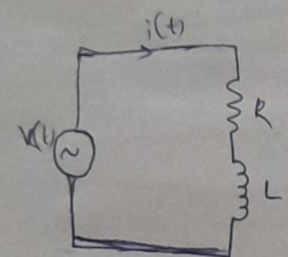
$$= \cos(10° + 23.69°)$$

$$= \cos(+33.69°)$$

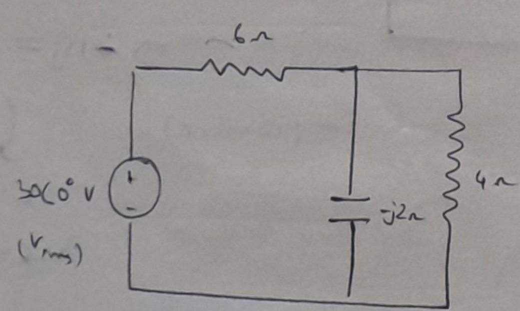
$$\Rightarrow pf = 0.832\ (+)$$

[Lagging]

↳ Inductive Load



Q) Determine pf of Entire Circuit : & Calculate Avg. Power



$$Z_{eq} = \frac{4 \times (-j2)}{4 - j2} + 6$$

$$= \frac{-j8}{4-j2} + 6$$

$$= \frac{-j8(4+j2)}{16+4} + 6$$

$$= \frac{-j2(4+j2)}{5} + 6$$

$$= \frac{4-j8}{5} + 6 = \frac{34-j8}{5}$$

$$\boxed{Z_{eq} = 6.8 - 1.6j\ \Omega} \Rightarrow Z_{eq} = 6.9857\angle -13.24$$

$$\theta = \tan^{-1}\left(\frac{-1.6}{6.8}\right)$$

$$\cancel{= \tan^{-1}(-0.2353)}$$

$$= \tan^{-1}(-0.235)$$

$$\Rightarrow \theta = 13.24\ (-) \Longrightarrow \text{Leading}$$

$$I = \frac{30\angle 0^\circ}{6.98\angle -13.24^\circ}$$

$$\Rightarrow \boxed{I_{rms} = 4.29\angle 13.24^\circ\ A}$$

$$\cos\theta = pf = \cos(13.24^\circ)$$

$$= 0.973$$

$$\therefore \boxed{pf = 0.973}\ [\text{Leading}]$$

$$S = (V_{rms})\cdot(I_{rms}) = \left(\frac{320}{\cancel{}}\right) \times \left(\frac{4.286}{\cancel{}}\right)$$

$$\boxed{S = 155\ VA}$$

$$P = 155 \times 0.973 = 125\ W$$

$$(OR)\ P = I^2 R_{eq} = (4.286)^2 \times 6.8 = \underline{125\ W}$$

Q) $v(t) = 60\cos(\omega t - 10^\circ)\ V$

$i(t) = 1.5\cos(\omega t + 50^\circ)\ A$

(a) Find Complex & Apparant Powers

(b) Find Real & Reactive Powers

(c) Power Factor & Load Impedence

(a) $S = V_{rms} \cdot I_{rms}$ (Apparant Power)

$$= \frac{60}{\sqrt{2}} \times \frac{1.5}{\sqrt{2}}$$

$$\boxed{S = 45\ VA}$$

$S = V_{rms} \cdot I_{rms} \angle \theta_v - \theta_i$ (Complex Power)

$$= 45\angle(-10^\circ - 50^\circ)$$

$$\Rightarrow \boxed{S = 45\angle -60^\circ\ (VA)} \Rightarrow \boxed{S = 22.5 - 38.97j}\ (VA)$$

(b) Real Power & Reactive Power

$$S = \underset{\downarrow \atop P}{22.5} \; \underset{\downarrow \atop Q}{-38.97j}$$

P : Real Power

Q : Reactive Power

$$\boxed{P = 22.5\ W}$$

$$\boxed{Q = -38.97\ VAR}$$

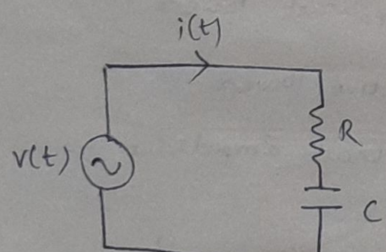
(c) $$pf = \frac{\cancel{V_{rms}} \cdot \cancel{I_{rms}} \cdot \cos(\theta_v - \theta_i)}{\cancel{V_{rms}} \cdot \cancel{I_{rms}}} = \cos(\theta_v - \theta_i)$$

$$pf = \cos(-10^\circ - 50^\circ)$$

$$= \cos(-60^\circ) \quad (-)$$

$$\boxed{pf = (-)0.5} \rightarrow \text{Leading}$$

↳ Capacitive Load



~~S = 22.5 − 38.97j~~

$$Z = R + jX_c$$

$$\Rightarrow Z_L = 20 - j34.64$$

$$Z = \frac{v(t)}{i(t)} = \frac{60\angle -10^\circ}{1.5\angle 50^\circ}$$

$$= 40\angle -60^\circ$$

$$= 20 - j34.64$$

$$X_c = -34.64j = \frac{1}{j\omega C}$$

$$\Rightarrow -34.64j = -\frac{j}{\omega C}$$

$$\Rightarrow C = \frac{1}{34.64 \times \omega}$$

$$\boxed{C = \frac{1}{34.64\omega}\ F}$$

H.W Q) $V_{rms} = 110\angle 85^\circ V$

$I_{rms} = 0.4\angle 15^\circ A$

Calculate (a), (b), (c)

Ans : (a) $44\angle 70^\circ VA$
$44 VA$
(b) $15.05 W$
$41.35 VAR$
(c) 0.342 (lagging)
$94.06 + j258.4 \Omega$

(a) $S = V_{rms} \cdot I_{rms}$ [Apparent Power]

$= \frac{110}{\sqrt{2}} \times \frac{0.4}{\sqrt{2}}$

$= 22 VA$

$\boxed{S = 22 VA}$

$S = V_{rms} \cdot I_{rms} \angle \theta_v - \theta_i$ [Complex Power]

$= \frac{110}{\sqrt{2}} \times \frac{0.4}{\sqrt{2}} \times \angle(85^\circ - 15^\circ)$

$= 22\angle 70^\circ VA = 7.524 + 20.67j$

$\boxed{S = 22\angle 70^\circ VA}$

(b) $S = 7.524 + 20.67j$

$\downarrow$ P $\quad \downarrow$ Q

P : Real Power
Q : Reactive Power

$\boxed{P = 7.524 W}$

$\boxed{Q = 20.67 VAR}$

(c) $pf = \cos(\theta_v - \theta_i) = \cos(85^\circ - 15^\circ) = \cos(70^\circ)$ (+)

15/11/23

TRANSFORMERS :

CHAPTER - 12

⇒ Transformer : Transfers energy from one circuit to another, at constant frequency (AC) by the principle of ~~a~~ Faraday's Laws. of EMI.

→ Static Device

Power Transfer is possible by changing the voltage level.

Efficiency : 99.5% (Very High)

$$\begin{array}{ccc} \text{Electrical Circuit} & \longleftrightarrow & \text{Magnetic Circuit} \\ \downarrow & & \downarrow \\ \text{Power} & & \text{Flux} \\ \downarrow & & \downarrow \\ \text{Winding / Coil} & & \text{Core} \end{array}$$

Primary Winding : External Supply
Secondary Winding : External load
} (connections)

Both Windings are magnetically coupled but electrically isolated.

⇒ Faraday's Law of EMI :

① ~~Motor~~ Generator Law :
When A rotating conductor is placed in a uniform magnetic field, It cuts the magnetic lines of flux and EMF is Introduced in it.

② Motoring Law :
When a current carrying conductor is placed in a

uniform magnetic field, then it experiences a mechanical force on it.

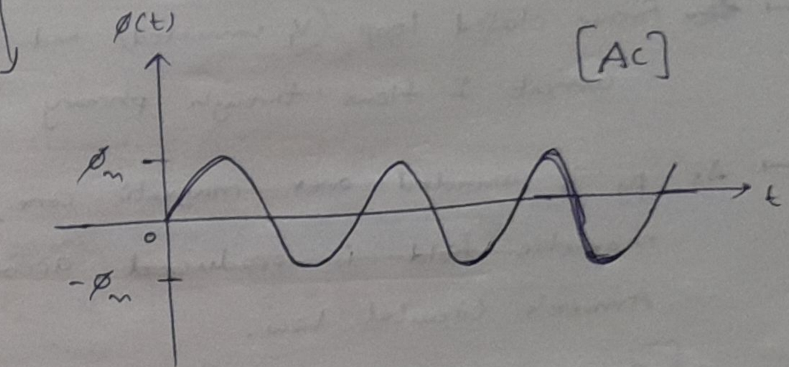
Both Laws produce dynamically induced EMFs.

⇒ Transformer Action:

An EMF induced in a coil is directly proportional to rate of change of flux linkage in it.

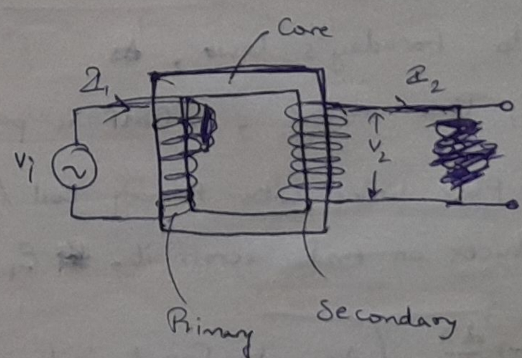
$$\boxed{EMF \propto -\frac{d\lambda}{dt}}$$

[Negative sign indicates Lenz's law]



Transformer responds only to Time varying magnetic flux i.e. AC circuits, not DC.

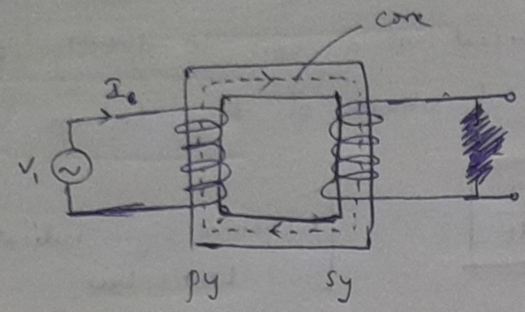
Core: Produces Magnetic field in the Transformer
↳ Made of Magnetic Material (High Grade Silicon steel)
↳ Minimum Energy Loss (Minimum Reluctance)



$P_1 = I_1 V_1$ $\qquad V_1 I_1 \longrightarrow V_2 I_2$

$P_2 = I_2 V_2$

→ Basic Principle :



I_e : Excitation Current

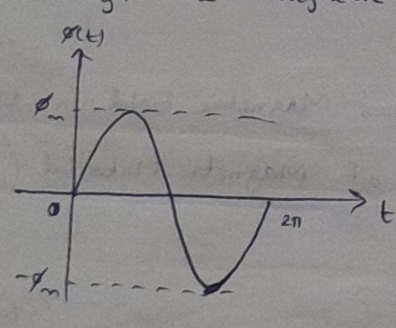
→ py is connected with supply.

→ ~~Ba~~ Forms closed loop (V_1 connected) and hence current I flows through primary winding

→ As py is wounded over magnetic core, magnetic field is produced according to Ampere's Circuital Law.

[Flux is confined to Magnetic Core]

→ ϕ_m flows through the magnetic core



(As current → AC & Sinusoidal)

$[\phi(t) = \phi_m \sin(\omega t)]$

→ According to Faraday's law,
As ϕ is time varying, ∴ EMF is produced
Magnetic ~~fie~~ Flux links with Primary Coil / winding and it induces an emf across it, $\mathcal{E}_1$

$$\mathcal{E}_1 \propto (-)\frac{d\lambda}{dt} \quad \text{[Acc. to Faraday's law]}$$

$$\boxed{E_1 = N_1 \frac{d\phi}{dt} \ (-)}$$

λ : Flux Linkage

ϕ : Flux

N_1 : No. of turns in the primary winding/coil

E_1 : EMF induced in the primary coil/windings

→ Further, the flux lines links with secondary coil/winding and Hence ~~Induce~~ EMF is induced in the secondary coil, E_2.

$$\boxed{E_2 = N_2 \frac{d\phi}{dt} \ (-)}$$

N_2 : No. of turns in the secondary coil/windings

$$\left.\begin{array}{l} E_1 \propto N_1 \\ E_2 \propto N_2 \end{array}\right] - \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1}}$$ — Transformation Ratio

Ex. 230/12 V ⟶ $N_1 > N_2$

(P) (S) $E_1 < E_2$

[Step - Down Transformer]

16/11/23

⇒ <u>EMF</u> <u>equation</u> <u>of</u> <u>a</u> <u>Transformer</u> :

According to Faraday's law of EMI,

Induced EMF in coil

$$E_1 \propto (-) \frac{d\lambda}{dt}$$

$$E_1 = -N_1 \frac{d\phi}{dt}$$

N_1 : No. of turns of Primary

E_1 : EMF induced in Primary

Ignore lenz law Temporarily,

$$E_1 = N_1 \cdot \frac{d\phi}{dt}$$

$$= N_1 \cdot \frac{d}{dt}\left[\phi_m \sin(\omega t)\right]$$

$$= N_1 \cdot \phi_m \cdot \omega \cdot \cos(\omega t)$$

$$\boxed{\therefore E_1 = N_1 \phi_m \omega \cos(\omega t)}$$

$$\boxed{E_{1(max)} = N_1 \phi_m \omega} \text{ Volts}$$

$$\therefore E_{1\,max} = N_1 \cdot \phi_m \cdot 2\pi f \text{ Volts}$$

$$E_{1\,rms} = \frac{E_{1\,max}}{\sqrt{2}} = \frac{N_1 \cdot \phi_m \cdot 2\pi f}{\sqrt{2}}$$

$$= \sqrt{2} N_1 \phi_m \pi f$$

$$= 4.443 \, N_1 \phi_m f$$

$$\boxed{\therefore E_1 = 4.443 \, N_1 \phi_m f \text{ Volts}} \quad \text{(RMS)}$$

where E_1 : EMF induced in primary coil (RMS)

f : Supply frequency in Hertz

ϕ_m : Max. flux in the core (Weber)

N_1 : No. of turns of Primary coil

Similarly,

$$\boxed{\therefore E_2 = 4.443 \, N_2 \phi_m f}$$

$$\therefore E_1 \propto N_1$$
$$E_2 \propto N_2$$

$$\Rightarrow \boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$$

Transformer Ratio

Input Power = Output Power

$$i_1 V_1 = i_2 V_2$$

$$\Rightarrow \boxed{\frac{V_2}{V_1} = \frac{I_2}{I_1} = k}$$

[Ohm's Law is Violated]

$$\boxed{\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = k}$$

if $N_2 > N_1 \Rightarrow E_2 > E_1$

↳ Step-Up Transformer

if $N_1 > N_2 \Rightarrow E_1 > E_2$

↳ Step-down Transformer

if $N_1 = N_2 \Rightarrow E_1 = E_2 \Rightarrow k = 1$

↳ 1:1 Ratio Transformer
[Isolation Transformer]

↳ Safety Purposes,
Isolates two electrically.

→ According to Application, Transformer is ~~cat~~ classified into two types :

① Power Transformer :

↳ Installed at Power Plant

↳ Power → MW

11 kV / 400-600 kV

∴ Step-Up Transformer

② Distribution Transformer :

↳ Installed at Locality (House / Local)
[Consumer Premesis]

↳ Power → kW

11 kV / 415 V Three Phase AC

∴ Step down Transformer

Q) $B_{max} = 1.2\ Wb/m^2$

$250/3000\ V \Rightarrow E_1 = 250\ V$
$E_2 = 3000\ V$

$f = 50\ Hz$

E per turn $= 8\ V$

(i) find N_1 & N_2

(ii) find Area of the core

Mineral Oil
↓
Coolant

A)

$$\boxed{\phi_{max} = B_{max} \cdot A_c}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

(i) $N_1 = \frac{250}{8} = 32$ turns [even]

$N_2 = \frac{3000}{8} = 375 \approx 376$ turns [even]

(ii) $E_1 = 4.443\ N_1 \phi_m f$

$\Rightarrow 250 = (\sqrt{2}\pi) \times 32 \times \phi_m \times 50$

$\Rightarrow \frac{5}{32\pi\sqrt{2}} = B_m \times A_c$

$\Rightarrow A_c = \frac{5}{32 \times 1.2 \pi\sqrt{2}} = 0.03\ m^2$

$$\boxed{\therefore A_c = 293.07\ cm^2}$$

Q) $N_1 = 400$ turns

$N_2 = 1000$ turns

$A_c = 60\ cm^2$

$f = 50\ Hz$

$E_1 = 520\ V$

Calculate (i) B_{max}

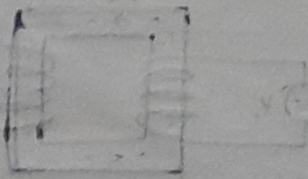
(ii) E_2

A) (ii) $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$\Rightarrow E_2 = \frac{N_2}{N_1} \times E_1$

$= \frac{1000}{400} \times 520$

$= \frac{5200}{4}$



$\Rightarrow \boxed{E_2 = 1300 \text{ V}}$

(i) $B_{max} = \frac{\phi_{max}}{A_c} = \frac{E_1}{\sqrt{2}\pi N_1 f A_c}$

As $E_1 = (\sqrt{2}\pi) N_1 \phi_m f$

$\Rightarrow \phi_m = \frac{E_1}{\sqrt{2}\pi N_1 f}$

$\therefore B_{max} = \frac{520 \times 10^4}{\sqrt{2}\pi \times 400 \times 50 \times 60}$ $[A_c = 60 \times 10^{-4} \text{ m}^2]$

$= \frac{520}{\sqrt{2}\pi \times 120}$

$= 0.975 \text{ W/m}^2$

$\boxed{\therefore B_{max} = 0.975 \text{ W/m}^2}$ ⟶ RMS Value

↳ But No Multiplication of $\sqrt{2}$

∵ Magnetic Parameter do not have RMS (or) max.

H.W

Q) $N_1 = 500$ turns

$N_2 = 50$ turns

$V_1 = 3000$ V

$f = 50$ Hz

$i_1 = ?$, $i_2 = ?$ (full load currents)

$E_2 = ?$, $\phi_m = ?$

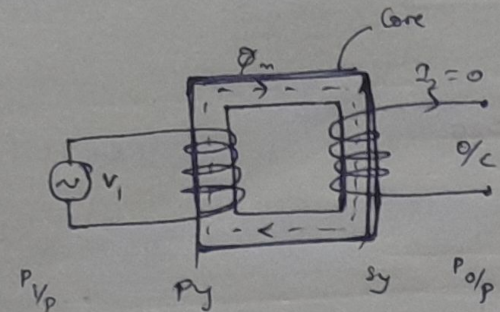
Neglect Leakage drop

25 kVA transformer

A) $V_{rms} \cdot I_{rms} = 25$ kVA $\Rightarrow V_1 I_1 = V_2 I_2 = 25$ kVA

20/11/23

→ Transformer on No-Load :



Ideal Transformer :

→ 100% Efficiency

No Losses → Winding loss (Copper loss) [Ohmic Loss]
→ Core Loss (Iron Loss) [Magnetic Loss]

→ No Leakage flux

→ It is not reliable in practice magnetic loss.

→ When supply is connected to py, I_0 flows through it.

→ Sy is open circuited [$\therefore I_2 = 0$]

No Load is connected

→ $P_{output} = 0$ $\left[\because P_2 = \underset{\overset{\uparrow}{0}}{I_2} V_2 = 0\right]$

($P_{o/p} = 0$)

→ $P_{i/p}$ = (magnetic loss in the core) + (copper loss in the py)

→ No Load current (I_0) is 2 to 6% of I_{rated}.
↳ very minimum

→ Py copper loss ($I_0^2 R_1$) is negligible

$\therefore$ No Load Input Power (W_0) = Magnetic loss in the core

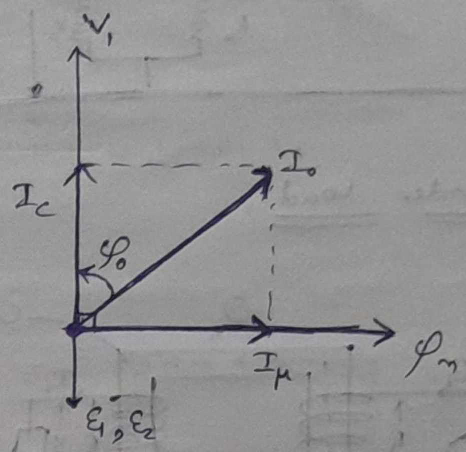
→ I_μ : Magnetising current
= Flux Producing Component

→ I_c : Core loss Current
= Core Loss Component

$\Rightarrow I_0 = I_\mu + I_c$

$$\boxed{\vec{I_0} = \vec{I_\mu} + \vec{I_c}}$$

→ Phasor Diagram :



$$I_0 = \sqrt{I_c^2 + I_\mu^2}$$

$$\cos(\varphi_0) = \frac{I_c}{I_0}$$

$$\Rightarrow I_c = I_0 \cos(\varphi_0)$$

$$\sin(\varphi_0) = \frac{I_\mu}{I_0}$$

$$\Rightarrow I_\mu = I_0 \sin(\varphi_0)$$

<u>Note</u> :

① $I_0 = \sqrt{I_c^2 + I_\mu^2}$

② $\cos(\varphi_0) = \frac{I_c}{I_0}$

③ $I_c = I_0 \cos(\varphi_0)$

④ $I_\mu = I_0 \sin(\varphi_0)$

No-Load Power factor ($\cos(\varphi_0)$) :

$\varphi_0 < 90^\circ$

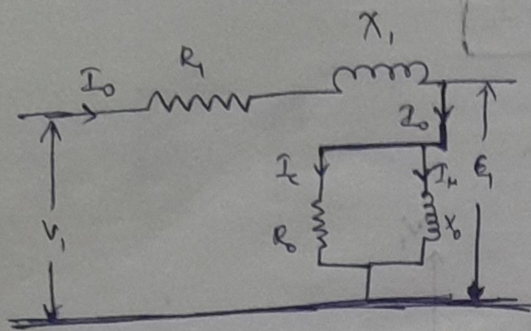
Assume $\varphi_0 = 80^\circ$,

$\cos(\phi_0) = 0.17$ lagging

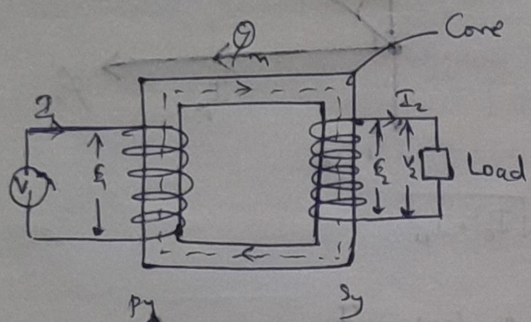
∴ Behaves Mostly Inductive

Very low pf.

Equivalent Circuit :



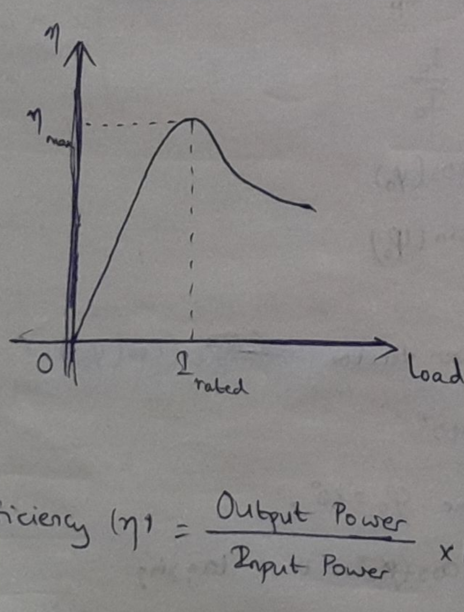
→ **Transformer under Load** :



→ On Load, I_2 flows

→ Flux ϕ_m is constant.

If $I_2 \uparrow \Rightarrow P_{o/p} \uparrow \Rightarrow I_1 \uparrow \ (\because I_2' \uparrow) \Rightarrow P_{I/p} \uparrow \Rightarrow \eta \uparrow$



$$\text{Efficiency } (\eta) = \frac{\text{Output Power}}{\text{Input Power}} \times 100\ \%$$

$$= \frac{P_{o/p} \times 100}{P_{o/p} + \text{Losses}}\ \% = \frac{P_{o/p} \times 100}{P_{o/p} + P_{core} + P_{Cu}}\ \%$$

P_{core} : Constant

P_{cu} : Variable [As $P_{cu} = i^2 R$, i is variable]

$$P_{core} = P_{cu} \Rightarrow \text{Efficiency} = \text{maximum}$$

$\therefore$ Variable loss = Constant loss

Transformer — Rated in kVA

Motors — Rated in W or hp

Reason:

21/11/23

→ Electrical Machines are electromechanical energy conversion devices.

(1) Electrical Energy to Mechanical Energy
↳ Electrical Motor

(2) Mechanical Energy to Electrical Energy
↳ ~~Ele~~ Generators

Electrical Machines

- Static Machine
  - Transformer
- Rotating Machine
  - DC Machines
    - DC Generator
    - DC Motor
  - AC Machines
    - Synchronous Machine
      - Synchronous Generator ← (Alternator) → (Power Plants)
      - Synchronous Motor
    - Asynchronous Machine
      - Induction Machines

P N [1 phase] → 230 V
(AC)

(N: Neutral)

R Y B N [3 phase]
(AC)

1 phase → Used to Transport 1 kW (Domestic)
3 phase → Used to Transport 3 kW (Industry)

Media for Energy Conversion – Magnetic field

∴ Electromagnetic conversion principle

Two electromagnetic conversion principles :

(1) Conductor moves in magnetic field

∴ Induced voltage in conductor

[Generator Action]

(2) Current carrying conductor placed in Magnetic field

∴ Conductor experiences mechanical force.

[Motor action]

Flemming's Right hand Rule : (OR) curling left hand

Induced EMF direction ⇓

Thumb : Magnetic field
Index : Direction of Rotation
Middle : Induced EMF ($\mathcal{E}$)

palm faces Magnetic field
Curl in direction of Rotation
Thumb : Induced EMF

$\boxed{\mathcal{E} \propto Blv}$ — Generator

$\boxed{\tau \propto Bli}$ — Motor

Q) H.W) Why Magnetic field ! Why not Electric field ?

$$\boxed{\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{l}}$$

Any Electrical Machine — { Electrical Circuit (Windings) ; Magnetic Circuit }

Any Electrical Machine — {
Stator (stationary / static member)
↳ Either circuit (Ex. Magnetic ckt)
Rotator (Rotating member)
↳ The other circuit (Ex. Electrical ckt) }

For DC machine :

Stator : Magnetic ckt (Field system)

Rotor : Electrical ckt (Armature)

$$Force = i(\vec{l} \times \vec{B})$$
$$|F| = ilB \text{ N}$$
$$\tau = \vec{r} \times \vec{F}$$
$$|\tau| = Fr \text{ N-m}$$
— <u>Motor</u>

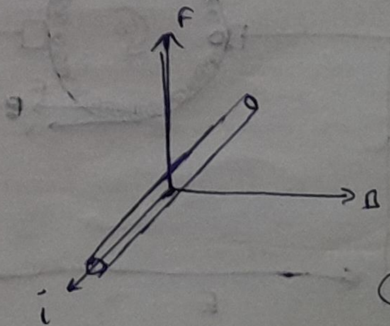
In DC machine,

Rotor is AKA Armature

& current supplied to Motor i.e. i_a is Armature current.

$$\boxed{F = i_a l B}$$

(1) Force on Conductor carrying I in field B

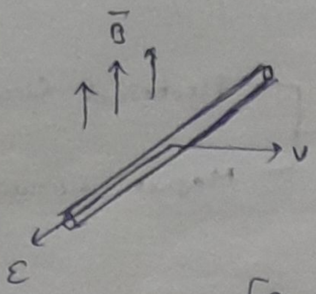


$$F = i(\vec{l} \times \vec{B})$$

l : Length of Conductor in Field.

(Flemming's left hand Rule)
[Motor Action]

(2) Voltage ($\mathcal{E}$) Induced in conductor travelling in field B.



$\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{l}$

~~$\mathcal{E} = v(\vec{l} \times \vec{B})$~~

$\mathcal{E}$: Voltage Induced along length l

[Flemming's Right hand Rule]
(Generating Action)

EMF induced in DC Motor:

$\mathcal{E} = NBvl$ $\quad \because \mathcal{E} = Ne$

e: induced in each conductor

$\mathcal{E}$: induced in all conductors 'N'.

$v = r\omega$

[ω: Angular velocity, rad/s]

$\mathcal{E} = N\omega r Bl$

$\therefore \mathcal{E} = k\omega$ $\quad \boxed{\mathcal{E} \propto \omega}$

Fig. 1 :

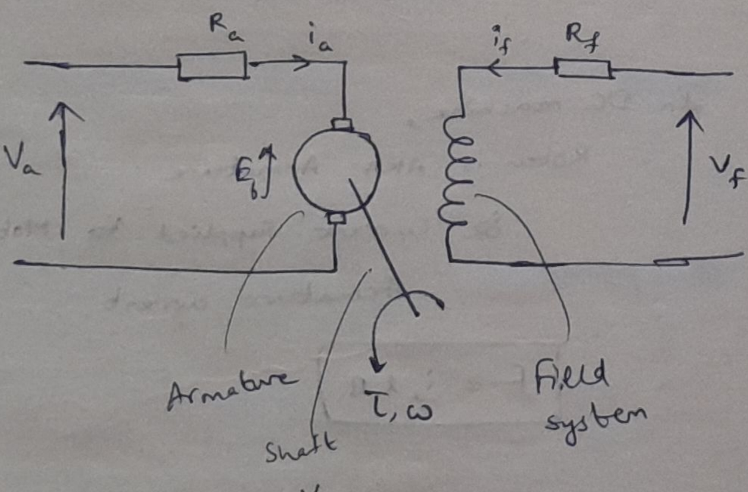
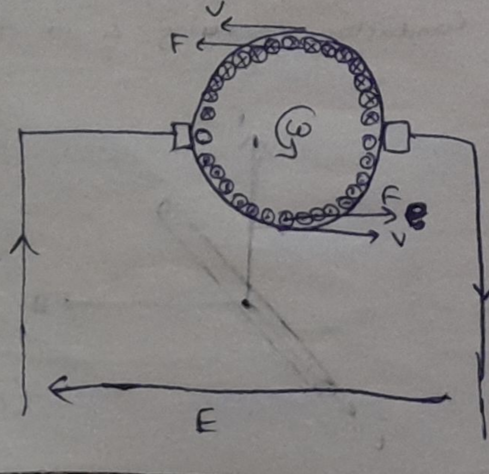


Fig. 2 :



DC machine is AKA Conduction Machine.
∵ Need current in both Armature & Field system.

∵ No connection b/w Primary & Secondary Windings in Transformer, works through Mutual Induction — AC Not DC.

DC Motor — Double Excitation Machine
∵ Should excite both Rotor & Stator.
(∴ Bulky & Needs maintenance)

↓

Not Popular ← Reason
(Outdated) ∴ Not used in Industry

Modified DC Motors : Permanant Magnet Motors
(OR)
Brushless Motor (BLDC Motors)
[Used in Robotics, Automobiles, etc]

Does not require external excitation.
∴ Field system removed → $\eta > 90\%$
↳ Energy loss ↓

Permanant Magnets
Ex. NeFeDb → ferrites
(Rare ~~Earth~~ Metals)
(OR)
Semarium Cobalt

~~DC M~~

DC Motor : Universal Motor

↳ Works with DC current and AC current

→ Three - Phase Induction Motor :

→ ~~Com~~. Most frequently encountered machines at households

→ Rugged, Cheap, Easy maintenance, Simple design, wide ranges of power rating, Constant speed motor.

→ Variable speed control is difficult

→ Parts of Induction Motor:

(i) Stator
(ii) Rotor

Stator - Electrical ckt

Rotor - Magnetic ckt

→ Works with the principle of Transformer Action (Mutual Induction)

∴ Essentially a Rotating Transformer

Stator : Primary coil
Rotor : secondary coil } - Similarity

Supply to Stator ⇒ ~~Moving~~ Rotor Rotates

No connection b/w Stator & Rotor

∴ Induction principle

∴ Induction Motor

→ According to Rotor's construction, ~~[illegible]~~ is classified into two types :

(a) Squirrel - Cage Rotor

(b) Wound Rotor (Slip - Ring Rotor)

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Induction Motors → Workhorse of Industry

Wound Rotor → Variable speed application

Squirrel cage Rotor → Constant speed application

Wound Rotor → high starting torque than Squirrel cage

i.e. Motor starts with load

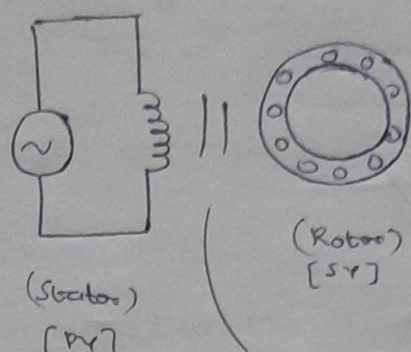
Ex. Lift, Train, Grinder

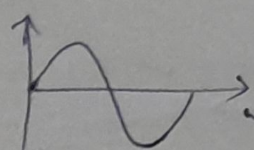
Adding external Rheostat

∴ Need frequent maintenance

& Efficiency is less than squirrel cage rotor.

→ How magnetic field is created:





Time varying magnetic field

⇓

i.e. Revolving Magnetic field

$\omega = 2\pi f$

$\Rightarrow \omega_e = 2\pi f_e$

f_e : Supply frequency ↳ (electrical frequency)

$\omega_s = \frac{2}{P}\omega_e$

ω_e ↳ electrical Angular frequency

ω_s ↳ mechanical Angular frequency

[P : No. of Poles]

ω_s : Synchronous speed

$\omega_s \cong n_s = \frac{120 f}{P}$ rpm

If $f_e = 50$ Hz,

$\therefore \omega = 2\pi f_e = 314$ rad/s

$P = 2 \Rightarrow n_s = 3000$ rpm $\Rightarrow \omega_s = 314$ rad/s

$P = 4 \Rightarrow n_s = 1500$ rpm $\Rightarrow \omega_s = 157$ rad/s

$P = 6 \Rightarrow n_s = 1000$ rpm $\Rightarrow \omega_s = 105$ rad/s

Max. speed available : 3000 rpm

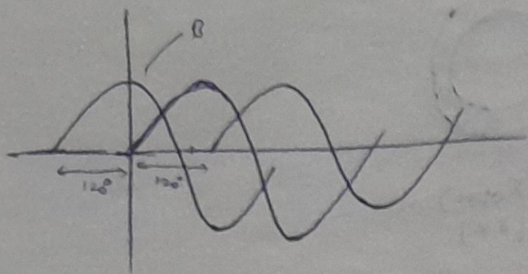
∵ Minimum no. of poles = 2

(N & S)

Magnetic Monopoles Don't exist.

~~Phase~~

Three Phase System:



Principle of Operation:

~~Three Phase~~

- A time Phase distribution of Three Phase AC Supply is given to a Three Phase Stator winding of Induction Motor which is spacially distributed.
- A Revolving magnetic field is created in the stator winding which runs at synchronous Speed. (n_s)
- This revolving magnetic field ~~cuts~~ is cut by stationary rotor conductor. And Hence EMF is induced in the Rotor Conductor.
- As the Rotor conductor are permanantly short circuited, a current flows through it.
- And Hence, a magnetic field is created on rotor conductors (Ampere Circuital Law)

$F = BIL$

$\Rightarrow F = B_s \times B_r$

$t \propto (F \times r)$

$\tau = k\, B_R \times B_s$

Torque is produced in the Motor due to interaction of those two magnetic fields.

→ At what speed Motor will run?

Speed of Rotating Magnetic Field of the stator $= \frac{120f}{P}$ (n_s)

n_m → Motor Speed

If $n_m = n_s$,

No Interaction between both Magnetic Fields.

∵ Both Appear Stationary to each other.

∴ Rotating Magnetic Field will not cut the Rotor.

∴ No Induced EMF in the Rotor.

∴ No Current produced in the Rotor.

∴ No Magnetic Field is created

∴ No Torque is Produced.

∴ Rotor speed falls below synchronous speed.

∴ Speed of Motor/Rotor should be less than synch. speed of the magnetic Field

∴ Relative speed exits.

~~i.e. $n_m < n_s$~~ → Always for Induction Motor.

i.e. $\underbrace{n_m < n_s}$ ⟹ Motoring Operation takes place

↳ Asynchronous Motor

Slip Speed (n_{slip})

$\Rightarrow n_{slip} = n_{sync} - n_m$

$$\boxed{\therefore n_{slip} = n_s - n_m}$$

$$\boxed{\% \text{ slip} = \frac{n_s - n_m}{n_s} \times 100\ \%}$$

$\therefore S\% = \frac{n_s - n_m}{n_s} \times 100\ \%$ & $n_s = \frac{120f}{P}$

n_m : Motor speed (rpm)

If Rotor Runs at Synchronous speed,

$S = 0\% \Rightarrow$ slip $= 0$

If Rotor is stationary,

$S = 100\% \Rightarrow$ slip $= 1$

$$\text{Slip} = \frac{n_s - n_m}{n_s}$$

S, Slip $\longrightarrow$ Ratio

$\therefore$ No Units

Slip varies between 0 and 1 for Induction Motor.

Full load Slip $\longrightarrow$ 4 to 6 %.

$\longrightarrow$ Frequency of an Induced EMF :

Because of slip,

frequency$_{\text{rotor}}$ $\neq$ frequency$_{\text{stator}}$ in motor.

$f_r \neq f_s$
↳ supply frequency

[In Transformer, $f_s = f_p$]

$$f_r = \frac{P \times n}{120}$$

$$= \frac{P(n_s - n_m)}{120}$$

$$= \frac{P \times s n_s}{120}$$

$$= s f_e \Longrightarrow \boxed{f_r = S f_e}$$

P : no. of stator Poles
S : slip
n : slip speed (rpm)
f_r : Rotor frequency (rpm)

(OR)

$$\boxed{f_r = S f_s}$$

If $S = 1 \Rightarrow f_r = f_s$

Induction Motor does not run,
It's a static device in this condition
i.e. Transformer.

30/11/22

→ ~~Tn~~ $n_{slip} = n_{sync} - n_m$

n_{slip} ↓ slip speed

n_{sync} ↓ synchronous speed ↳ speed of revolving magnetic field of stator

n_m ↳ Actual motor speed

$$n_{sync} = \frac{120 f_e}{P}$$

p : no. of poles
f_e : supply frequency (f_s)

$n_m < n_{sync}$

If $s = 1 \Rightarrow n_m = 0$ $\left[s = \frac{n_s - n_m}{n_s} \right]$

If $s < 1 \Rightarrow n_m < n_{sync}$

$f_r = s f_e$

f_e ↳ supply frequency

f_r ↳ frequency of motor induced EMF

→ $P_{mech} = T_m \times \omega_m$

P_{mech} ↳ Mechanical Power developed in Inductor Motor

ω_m ↳ Mechanical speed of motor (rad/s)

T_m → Torque (N-m)

$$P_{mech} = \frac{T_m (2\pi n_m)}{60}$$ $\left[\omega_m = \frac{2\pi n_m}{60} \right]$

$$\Rightarrow P_m = \frac{2\pi (T_m)(n_m)}{60} \text{ Watts}$$

n_m : motor speed
T_m : Motor Torque / Shaft Torque

→ 1 hp = 746 Watt

Usually Mech. power is measured in hp [horsepower]

☆ $n_m = n_{syn}(1 - s)$

n_m ↳ motor/rotor speed

Q) 208 V, 10 Hp, four pole, 60 Hz,
Y-connected Induction Motor,
Full Load slip of 5%

(i) $n_{sync} = ?$

(ii) $n_m = ?$ @ rated load

(iii) $f_r = ?$ @ rated load

(iv) $\tau_m = ?$ @ rated load

Sol:

$$P_m = \frac{2\pi (T_m) n_m}{60}$$

$$\Rightarrow 7460 \times 60 = 2\pi \times T_m \times n_m$$

$$\Rightarrow T_m \times n_m = \frac{7460 \times 60}{2\pi} \quad -(1) \quad \Rightarrow T_m = \frac{7460 \times 60}{2\pi \times 1800}$$

$$\boxed{\therefore T_m = 39.57653 \text{ N-m}}$$

$$f_r = \frac{P \times n_m}{120}$$

(iii)

slip = 0.05

$f_e = 60$

$f_r = s f_e$

$= 60 \times 0.05$

$\boxed{f_r = 3}$

(i)

$$\Rightarrow 60 = \frac{4 \times n_{sync}}{120}$$

$$\Rightarrow \boxed{n_{sync} = 1800 \text{ rpm}}$$

$$f_r = \frac{P \times n_m}{120}$$

$$s f_e = \frac{P \times s n_s}{120}$$

$$\Rightarrow \boxed{f_e = \frac{P \times n_s}{120}}$$

& (ii)

$$\% \text{ slip} = \left(\frac{n_{synch} - n_m}{n_{synch}}\right) \times 100 \%$$

$$\Rightarrow 5 = \frac{n_{sync} - n_m}{n_{sync}} \times 100$$

$$\Rightarrow \frac{1}{20} = \frac{n_{sync} - n_m}{n_{sync}}$$

$$\Rightarrow \boxed{\frac{n_m}{n_{synch}} = \frac{19}{20}}$$

$$\Downarrow$$

$$\cancel{n_{sync} = \frac{20}{19} \times 90}$$

$$n_m = \frac{19}{20} \times 1800$$

$$\boxed{\therefore n_m = 1710 \text{ rpm}}$$

Q) 3-Phase Cage Rotor,

3 Poles - pairs, $f_e = 50\,Hz$

Runs @ 960 rpm (n_m)

$\tau_m = 40\,Nm$

(i) ω of rotating field produced by stator winding

(ii) Slip

(iii) f_{rotor}

(iv) $Power_{mech.}$

Sol :

(i) $n_m = n_{sync}(1-s)$

$$n_{sync} = \frac{120 \times f_e}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\boxed{\therefore n_{sync} = 1000 \text{ rpm}}$$

(ii) $\Rightarrow \dfrac{n_m}{n_{sync}} = 1-s$

$$\Rightarrow \frac{960}{1000} = 1-s \Rightarrow s = \frac{40}{1000}$$

$$\boxed{\therefore s = 0.04} \Rightarrow \boxed{Slip = 0.04}$$

$\Downarrow$

$\boxed{slip\,\% = 4\%}$

(iii) $f_r = s f_e$

$$\Rightarrow f_{rotor} = 0.04 \times 50$$

$$\boxed{\therefore f_{rotor} = 2\cancel{0}\ Hz}$$

(iv)

$$P_{mech} = \frac{2\pi \times \tau_m \times n_m}{60} \Rightarrow P_{mech} = \frac{2\pi \times 40 \times 960}{60}$$

$$\Rightarrow \boxed{P_{mech} = 4021.24 \text{ Watts}}$$

$$\boxed{\therefore P_{mech} = 4.02\ kW}$$

Q) 8 pole Induction Motor, $f_s = 50$ Hz, $f_m = 1.5$ Hz

(i) n_m = ?

(ii) slip = ?

Sol:

$n_s = \frac{120 \times f_e}{P} \Rightarrow n_s = \frac{120 \times 50}{8}$

$\Rightarrow n_s = 750$ rpm

(ii) $f_r = s f_e$

$\Rightarrow 1.5 = (s) \times 50$

$\Rightarrow s = \frac{1.5}{50} = 0.03$

$\boxed{\therefore \text{slip} = 0.03} \Rightarrow \boxed{\text{slip } \% = 3\%}$

(i) $n_m = n_{sync}(1-s)$

$\Rightarrow n_m = 750(1-0.03)$

$\boxed{\therefore n_m = 727.5 \text{ rpm}}$

Q) 6 pole, 3 Phase, 50 Hz Induction Motor

Full Load Torque = 150 Nm

$f_m = 120$ cycle/min Find P_{shaft}

$= 2$ Hz

$[f_m = 2 \text{ cycles/sec}]$

1 Hz = 1 cycle/second

Hz = $\frac{x \text{ cycles}}{1 \text{ seconds}}$

Sol:

$P = \frac{2\pi T_m n_m}{60} = \frac{2\pi \times 150 \times n_m}{60} = 5\pi \times n_m$

$f_e = \frac{P \times n_s}{120} \Rightarrow n_s = \frac{120 \times f_e}{P}$

$\Rightarrow n_s = \frac{120 \times 50}{6} = 1000$

$\Rightarrow \boxed{n_s = 1000 \text{ rpm}}$

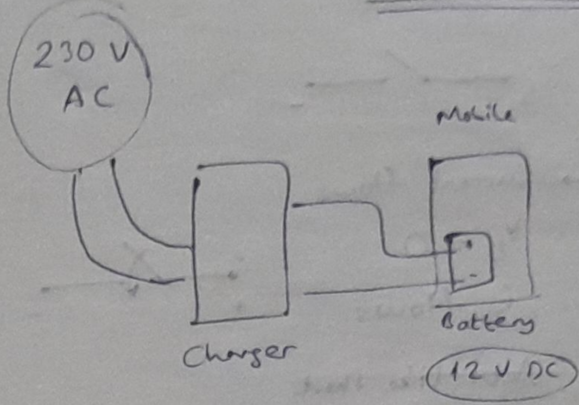
$n_m = n_{sync}\left(1 - \frac{f_r}{f_e}\right)$

$= 1000\left(1 - \frac{2}{50}\right)$

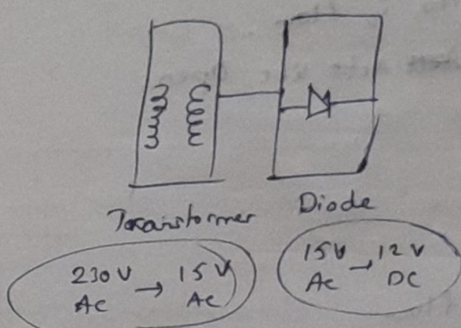
$\therefore n_m = 960 \Rightarrow P_{shaft} = 5\pi \times 960 = 15.08$ kW

2/11/23

Semiconductor Diodes



Charger :



Rectifier :

15 V AC → 12 V DC

Supply

Diode

→ Semiconductor Diode :

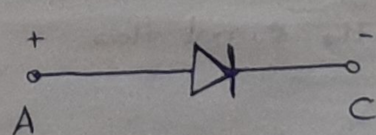
Basic Building block of electronic circuits.

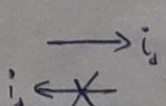
Diode → $\underbrace{\text{DI}}_{2}$ + $\underbrace{\text{ODE}}_{\text{electrode}}$

∴ Diode : 2 electrodes / 2 Terminals

[Anode & Cathode]

Symbol :

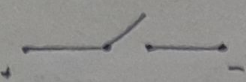




Only unidirectional current flow

Diode : Unilateral Device

↳ Electronic Switch

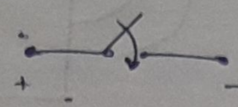


Diode is On ⟶ Current flows

$V_D = 0$

i_D flows

Acts like Short



Diode is Off ⟶ Current does not flow

V_D = Supply Voltage

No i_D flow.

~~Act~~ Acts like Open.

→ Conductor :

↳ Allows the current flow

Ex. Copper / Aluminium, etc

Insulator :

↳ Isolates the current flow, Reduces to minimum current flow

Ex. Polythene, PVC, ~~etc~~ Porsalein, etc

Property holds till Threshhold Temp

After Breakdown Temp., Material acts like Conductor.

Semiconductor :

Ex. GaAs, Ge, Si, etc

Properties lie b/w Conductor & Insulator.

Able to control the current flow.

→ Pure semiconductors :

- no. of e^- = no. of holes
- Can't Control Electric Current ∴ Not Useful

- e^- → -ve charge carriers

holes → +ve charge carriers [holes are absence of e^-]

- Intrinsic Semiconductor ⟷ Pure Semiconductors

→ Extrinsic Semiconductors :

↳ Add some Impurity, so that electrical current flow can be controlled.

Process : Doping

∴ Electrical charecteristics are Improved.

Doped Semiconductor materials

n-type

Majority carriers ; e^-

p-type

Majority carriers ; holes

Gap b/w Valency Band and Conduction band - Band Gap (eV)

Doped Semiconductors have greater conductivity than pure conductors.

Band Gap for Germanium Ge Semiconductor : 0.7 eV

Band Gap for Silicon Semiconductor : 1.12 eV

GaAs → LED, laser

Si, Ge → Rectifiers

Si more preferable than Ge

∵ Band Gap is more.

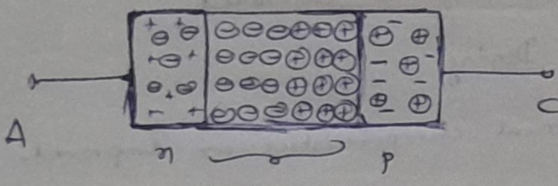
∴ Sustains high Temp, upto 200°C.

(Wherear Ge. sustains upto 100°C only)

Si – Naturally Available in Earth (Sand) [∴ Cheap]

[SiO_2 (silica)]

→ p-n junction ~~p type~~ :



Depletion Region

↘ forms an electric Potential across depletion region

↓

Barrier Potential

$= 0.7$ V for Si

~~= 1.12 V for GaAs~~

$= 0.3$ V for Ge

$= 1.2$ V for GeAs

Charecteristics :

(i) No bias : No ext ~~ext~~ Voltage

$(V_{AC} = 0)$

(ii) Forward Biasing :

With voltage $V_{AC} < 0.7$: (for Si)

$\therefore i_d = 0$, No current flow

$i_d \approx 0$ $\because$ Minority Carriers $[i_d > 0]$

With Voltage $V_{AC} > 0.7$ (for Si)

$\therefore$ Current flows

$\therefore$ Diode starts Conducting

$(i_d \propto V_{AC})$

Anode : Connected with +ve of supply

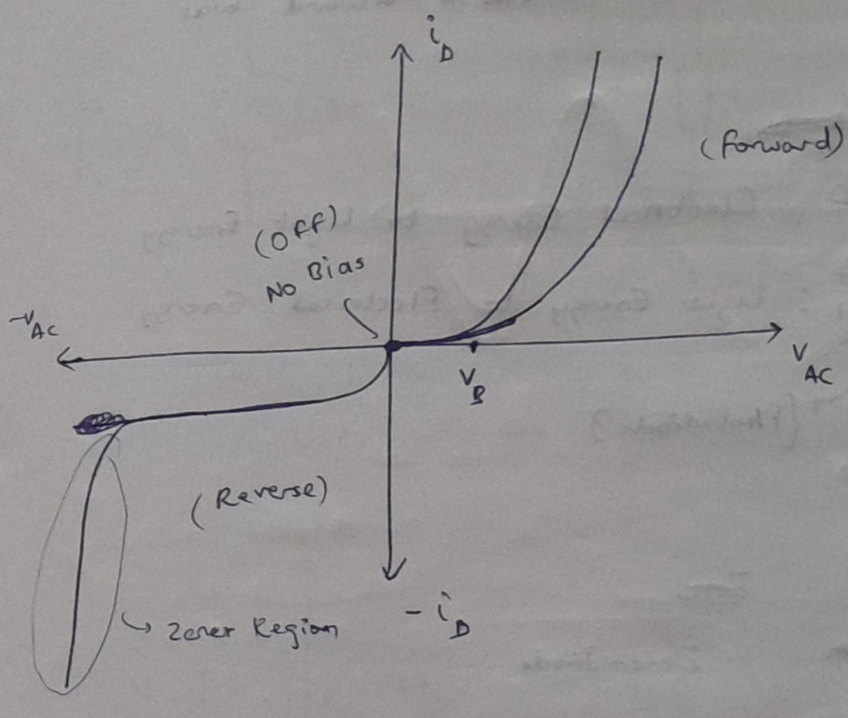
Cathode : Connected with -ve of supply.

(iii) Reverse Biasing:

Anode : Connected with -ve of supply

Cathode: Connected with +ve of supply

Diode is said to be off state.



For Both Operating characteristics,

Junction Temp. should be within Threshold value

If it exceeds, Breakdown Voltage

(Diode damages)

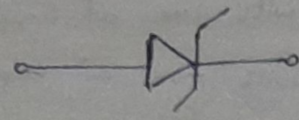
In order to safely operate the diode,

We use heat sink.

→ Zener Region:

Zener Voltage : Voltage Remains const. during Reverse Bias

Zener Diode ⟶ Voltage Regulator



During Reverse Bias, Diode can be easily damaged
∵ Temp. Rise

But Zener Diode won't get damaged.
(Red colour)

p-n junction diode : Rectifier
(Black Colour) ↳ Works in forward bias

~~LED Dio~~

LED : Electrical Energy to Light Energy

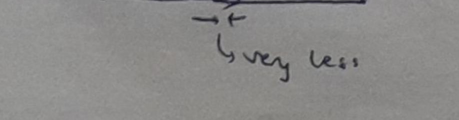
Solar Cell : Light Energy to Electrical Energy
↳ [Photodiode]

~~Ze~~

~~Ze.~~ Zener Diode :

↳ Indicates Cathode
(Curve)

→ Schottky Diode :



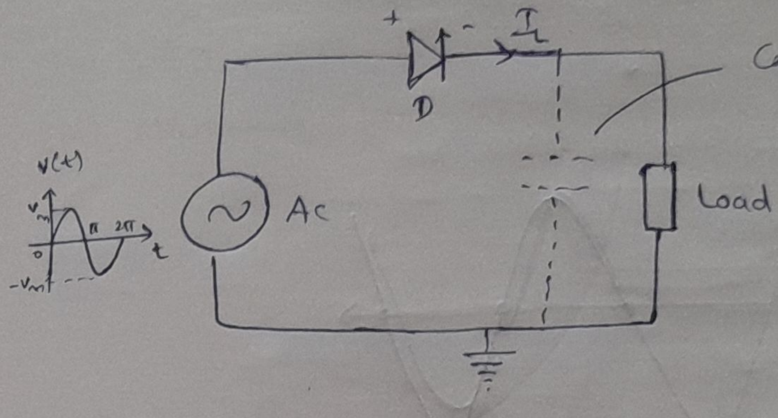
↳ very less

Use : High Switching speed applications

→ Application of Diodes :

~~Naturally~~, Used as Rectifiers
to convert sinusoidal voltages (AC), into Unidirectional (DC) voltages.

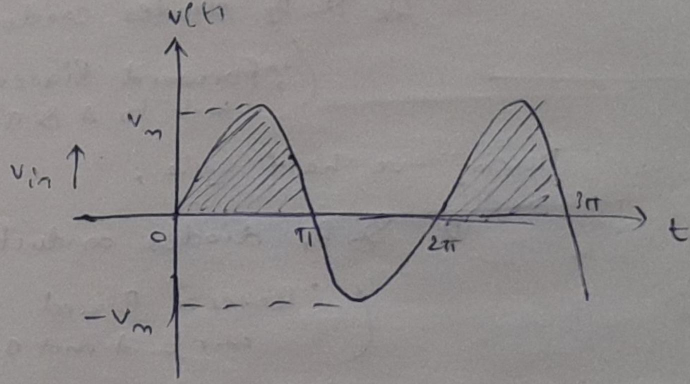
(i) Half-Wave Rectifiers



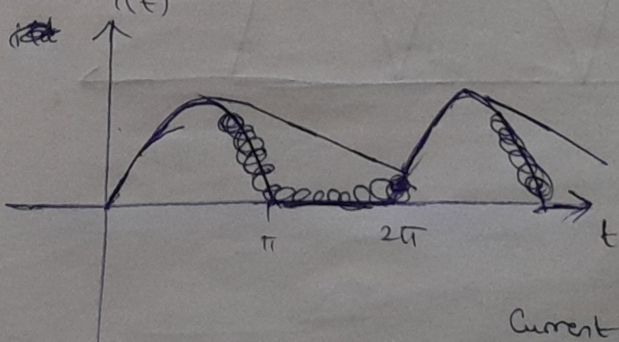
Capacitor — Acts as a filter

[Makes current Continuous]

Half of the AC cycle is rectified & Uses only one Diode



Forward Bias — + is connected to +ve

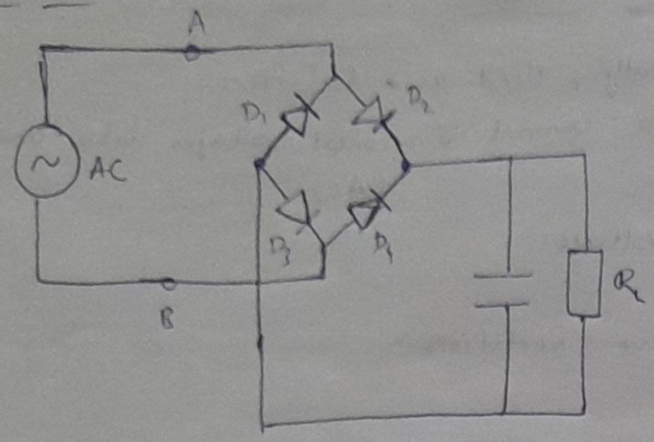


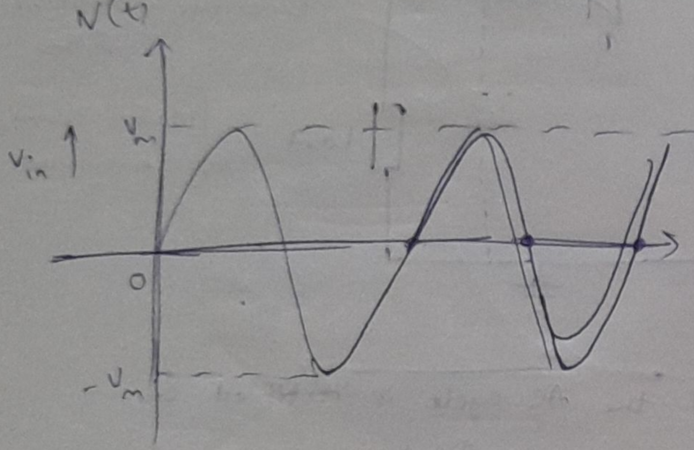
(Without filter)

Current flow is not continuous

∴ To make continuous, Capacitor is added.

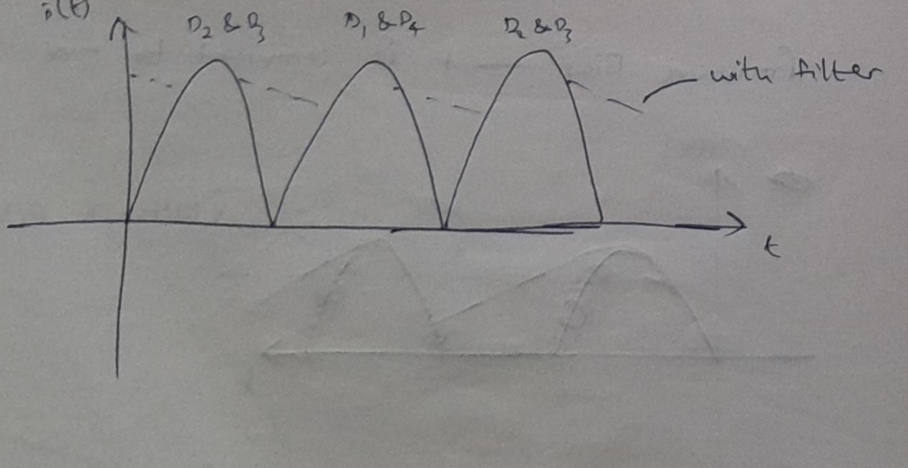
→ Full Wave :





During +ve half cycle,
D_2 & D_3 diodes conduct
(∵ Forward Biased w.r.t to A & B)

During -ve half cycle,
D_1 & D_4 diodes conduct
(∵ Reverse Biased w.r.t A and B)



→ **Applications of Diodes**

(i) Rectifier Circuits,

(ii) Power Supplies,

(iii) Wave shaping Circuits
(Clippers & Clampers)

(iv) Time Delay Circuits
(Running Light) &

(v) LEDs

(vi) Microwave communications

6/11/23

Operational Amplifiers : CHAPTER - 5

(Op-Amp) → very High Gain Amplifier (10^4 to 10^6)

→ Gain = $\frac{\text{Output voltage}}{\text{Input voltage}}$

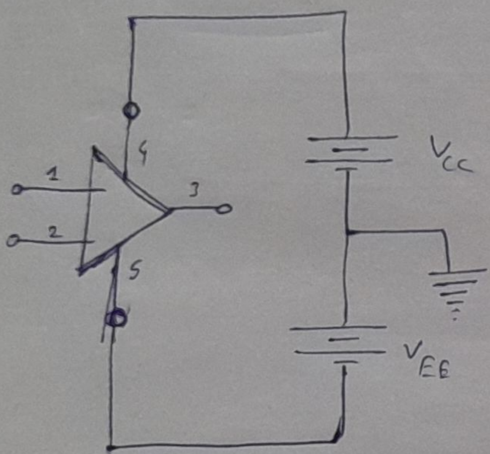
→ Single Output, ~~Double Input~~ Two Terminal Input Device

→ Input Impedence – ↑↑

Output Impedence – ↓↓

→ Assume Op-Amp to be black box

→



Terminal 1 : Inverting Input

Terminal 2 : Non-Inverting Input

Terminal 3 : Output

Terminal 4 : Positive supply V_{CC}

Terminal 5 : Negative supply V_{EE}

→ Ideal Gain:

$$V_3 = A(V_2 - V_1)$$

Ideal Input Impedence = ∞
Ideal Output Impedence = 0
Differential Gain (A) = ∞
Bandwith Gain is const. from DC to high frequency

Amplifier with ∞ gain — No Use &

— Practically Impossible

V_3 : Output Voltage (V)

A : Gain

V_2 : Non-Inverting Input (V)

V_1 : Inverting Input (V)

Ideal Op-Amp → Zero common-mode gain ($A_{cm} = 0$)

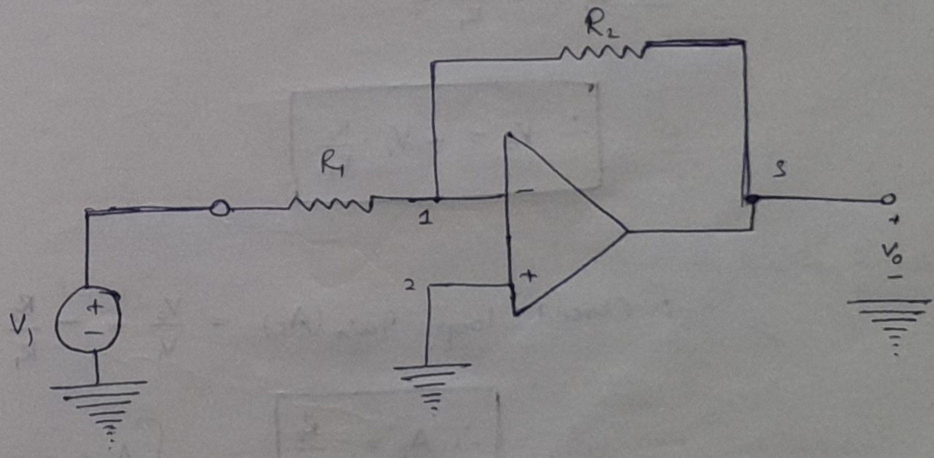
→ Complete common-mode rejection

→ Infinite Bandwith

→ Types of Op-Amp :

- Inverting Op-Amp
- Non-Inverting Op-Amp

(1) Inverting Amplifier :



Source is applied to Inverting (terminal) input.

Non-Inverting (terminal) input is grounded.

In order to limit the gain, Resistance R_2 is connected b/w Non-Inverting terminal & Output Terminal. R_2 : Negative feedback & R_2 AKA Feedback Resistance.

→ Relationship b/w V_0 and V_1 :

$$\cancel{+V_1 - I_1(R_1 + R_2) = V_0}$$

$$\cancel{\Rightarrow I_1 = \frac{V_1 - V_0}{R_1 + R_2}}$$

Current flowing through inverting Terminal from source $= I_1$

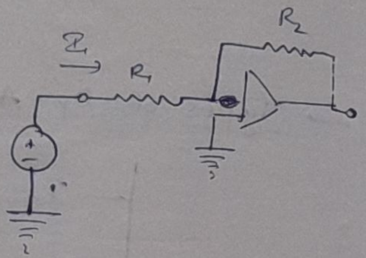
If Ideal Op-Amp, Infinite Input Impedance

⇓

No Current to -ve terminal

$V_1 = I_1 R_1$

$$\Rightarrow \boxed{I_1 = \frac{V_1}{R_1}}$$



$V_0 = I_2 R_2$

But Negative Feedback

$\Rightarrow \quad V_0 = -I_1 R_2$

$\therefore V_0 = -\frac{V_1}{R_1} R_2$

$$\boxed{\therefore V_0 = -V_1 \cdot \frac{R_2}{R_1}}$$

$\therefore$ Closed loop gain $(A_c) = \frac{V_0}{V_1} = -\frac{R_2}{R_1}$

$$\boxed{\therefore A_c = -\frac{R_2}{R_1}}$$

[$A_c < \infty$ and is finite]

Voltage @ Inverting Terminal = 0

(Virtual Ground) $\because V_b = 0 \therefore V_a \approx 0$

$$I_1 = \frac{V_1 - V_{virtual}}{R_1} = \frac{V_1 - 0}{R_1} = \frac{V_1}{R_1}$$

$$I_2 = \frac{V_{virtual} - V_0}{R_2} = \frac{0 - V_0}{R_2} = -\frac{V_0}{R_2}$$

& $I_2 = I_1$

Negative sign Indicates Inverting the signal

i.e. 180° phase diff. b/w Input & Output Voltages

Ex. Input : Sine wave

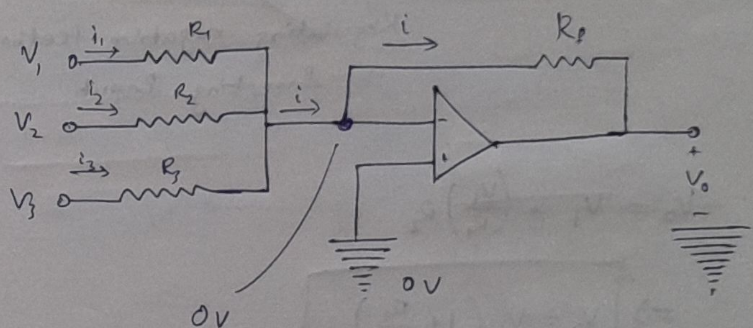
Output : Cos wave

→ **Application of Inverted Configuration:**

① (Weighted summer).

Output Voltage = Weighted sum of all Input

i.e. $V_o = V_1 + V_2 + V_3 + \ldots$



$$V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} \cdot V_3\right)$$

↳ Weighted sum of Inputs

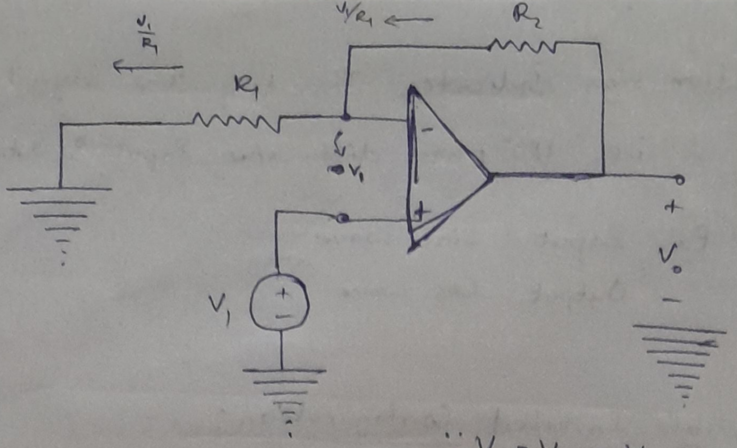
Hence :

$$V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \cdots \frac{R_f}{R_n} V_n\right)$$

[R_f : Resistance b/w Inverted Input & Output Terminal]

② **Non-Inverting Op-Amp :**

Polarity / Phase of Output is same as Input.



$\because V_a = V_1 \Rightarrow V_b = V_1$

Source is Applied to Non-Inverting Terminal.
Inverted (Terminal) Input is grounded through R_1

$\underbrace{R_1 \text{ \& } R_2}$ forms Voltage Divider Circuit.

↳ Act as a Voltage Divider,
Regulating negative feedback to the Inverting Input

$$V_0 = V_1 + \left(\frac{V_1}{R_1}\right) R_2$$

$$\Rightarrow \boxed{V_0 = V_1\left(1 + \frac{R_2}{R_1}\right)}$$

As Negative feedback, $I_1 = -\frac{V_1}{R_1}$

$$\Downarrow$$

$$I_1 = \frac{V_1}{R_1}$$

Closed Loop Gain $(A_c) = \frac{V_0}{V_1}$

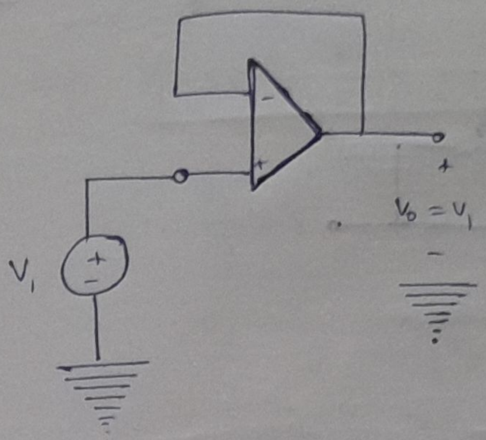
$$= 1 + \frac{R_2}{R_1}$$

$$\boxed{\therefore A_c = 1 + \frac{R_2}{R_1}}$$

$A_c < \infty$ & is finite

→ Application of Non-Inverting Op-Amp :

(Buffer / Voltage Follower)

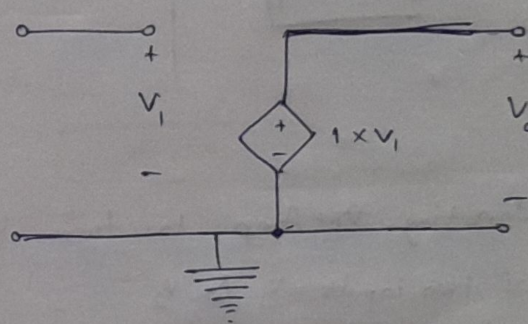


Unity Gain-Buffer

(OR)

Follower Amplifier

Equivalent Circuit :



Gain = 1

Input Voltage = Output Voltage

This Op-Amp is used after Amplification of another circuit.

It acts as a Buffer ∵ Output Voltage always follows Input Voltage

[Transformer : Electrical Buffer Amplifier]

~~Buffer Amplification~~

Buffer ⟶ Impedence Matching Device

[In order to extract Max possible Power by Load Resistance = Source Resistance]

Q) Use circuit below to design an Inverted Amplifier,

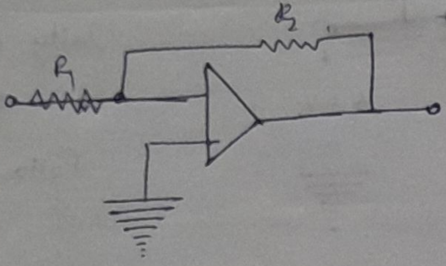
Gain = −10

Input R = 100 kΩ

$\begin{bmatrix} R_1 = 100\,k\Omega \\ R_2 = 1\,M\Omega \end{bmatrix}$

Find R_1 & R_2

Sol :



$$A = \text{Gain} = -\frac{R_2}{R_1}$$

$$\Rightarrow 10 = \frac{R_2}{R_1}$$

$$\boxed{R_2 = 10R_1}$$

$R_1 = 100\ k\Omega$

$\Rightarrow R_2 = 10 \times 100\ k\Omega$

$= 1000\ k\Omega$

$\boxed{\therefore R_2 = 1\ M\Omega}$

Feedback Resistance

Q) Design an Inverting Op-Amp to form Weighted-sum V_o of two inputs V_1 & V_2

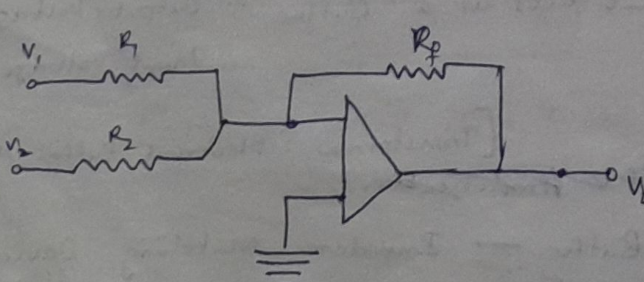
$$V_o = -(V_1 + 5V_2)$$

Max Output Voltage = 10 V

Max Current in Feedback Resistor = 1 mA

Design R_1, R_2 & R_f

Sol :



$$V_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right)$$

$$V_0 = -(v_1 + 5v_2)$$

$$\Rightarrow \quad v_1 + 5v_2 = \left(\frac{R_f}{R_1}\right) v_1 + \left(\frac{R_f}{R_2}\right) v_2 .$$

$$\therefore \frac{R_f}{R_1} = 1 \Rightarrow \boxed{R_f = R_1}$$

$$\frac{R_f}{R_2} = 5 \Rightarrow \boxed{R_f = 5R_2}$$

$$\Rightarrow \boxed{R_1 = 5R_2}$$

$$\frac{10}{R_f} \leq 1\,mA \Rightarrow \boxed{R_f \geq 10\,k\Omega}$$

Assume $R_f = 10\,k\Omega$

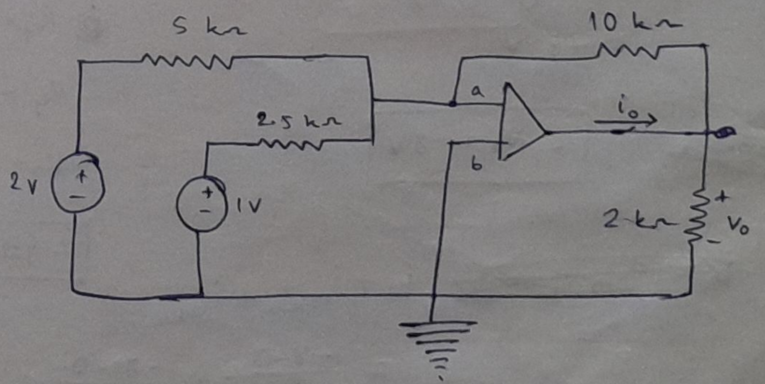
$$\boxed{\therefore R_1 = 10\,k\Omega}$$

$$\boxed{\therefore R_2 = 2\,k\Omega} \quad \left[\because \frac{10}{5} = 2\right]$$

$$\therefore (R_f, R_1, R_2) = (10, 10, 2)\,k\Omega$$

H.W

Q1)

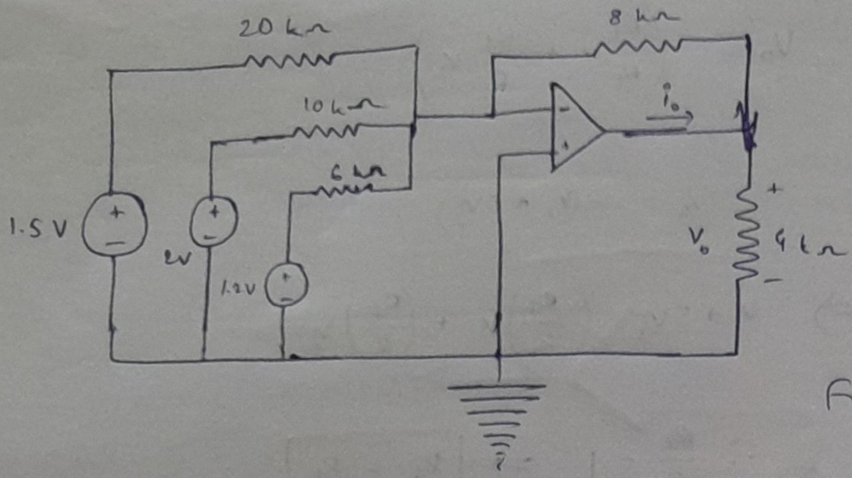


Find i_o & V_o

End-sem:
(Pattern)
10 Q
↓
1Q - 5M

H.W

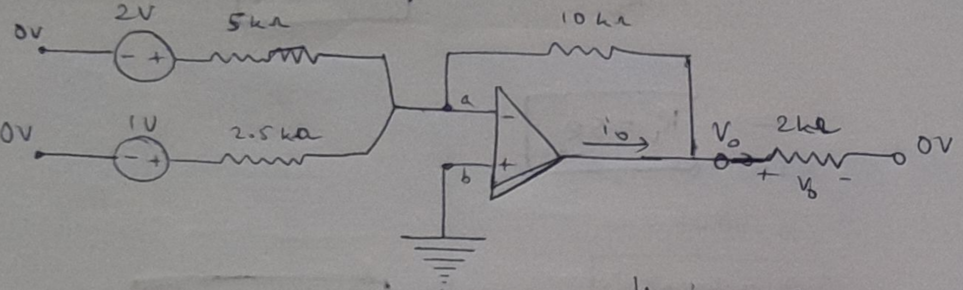
Q2)



Find i_o & V_o

Sol

Q2)



$V_b = 0$ ∵ Grounded

~~$V_a = 0$ ∵ $Z = \infty$ for Inverting Terminal~~

$V_a = 0$ ∵ Terminal is Virtual Ground

$v_1 = 2V$

$v_2 = 1V$

$$V_o = -\left[\left(\frac{R_f}{R_1}\right)V_1 + \left(\frac{R_f}{R_2}\right)V_2\right]$$

$$\Rightarrow V_o = -\left[\left(\frac{10\times10^3}{5\times10^3}\right)(2) + \left(\frac{10\times10^3}{2.5\times10^3}\right)(1)\right]$$

$$= -(4+4)$$

$$= -8\ V$$

$$\boxed{\therefore V_o = -8V}$$

$R_{eq} = \frac{5\times 5/2}{15/2}$

$\therefore R_{eq} = 5/3$

$\text{Gain} = \frac{-R_f}{R_{inq}} = \frac{-10}{5/3}$

$\boxed{\therefore \text{Gain} = -6}$

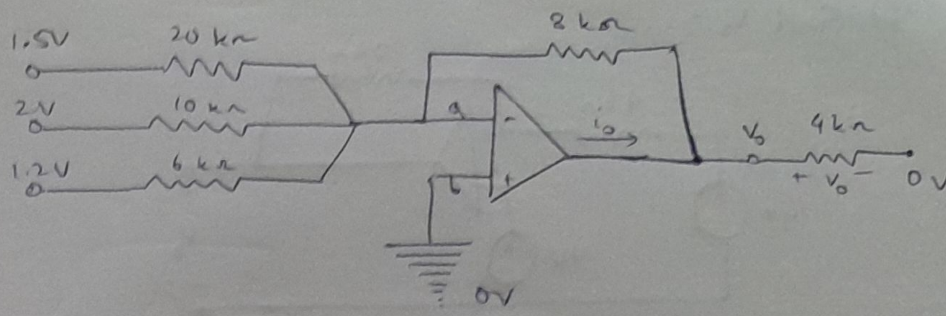
$$i_o = i_{10k\Omega} + i_{2k\Omega}$$

$$= \frac{-8-0}{10k\Omega} + \frac{-8-0}{4k\Omega}$$

$$= -0.8\ mA - 4\ mA$$

$$\boxed{\therefore i_o = -4.8\ mA}$$

Sol:
Q2)



$V_b = 0$ ∵ Grounded

$V_a = 0$ ∵ Infinite Impedance to Inverting Terminal

$V_1 = 1.5\ V$ & $R_1 = 20\ k\Omega$

$V_2 = 2\ V$ & $R_2 = 10\ k\Omega$

$V_3 = 1.2\ V$ & $R_3 = 6\ k\Omega$

$$V_o = -\left[\left(\frac{R_f}{R_1}\right)V_1 + \left(\frac{R_f}{R_2}\right)V_2 + \left(\frac{R_f}{R_3}\right)V_3\right]$$

$$\Rightarrow V_o = -\left[\frac{8}{20}\left(\frac{3}{2}\right) + \frac{8}{10}(2) + \frac{8}{6}\left(\frac{6}{5}\right)\right]$$

$$= -\left[\frac{3}{5} + \frac{8}{5} + \frac{8}{5}\right]$$

$$= -\frac{19}{5}\ V$$

$$\boxed{\therefore V_o = -3.8\ V}$$

$$i_o = i_{8k\Omega} + i_{4k\Omega}$$

$$= \frac{V_o - 0}{8\ k\Omega} + \frac{V_o - 0}{4\ k\Omega}$$

$$= -3.8\left(\frac{1}{8} + \frac{1}{4}\right)\ mA$$

$$= -\frac{19}{5} \times \frac{3}{8}\ mA$$

$$= -\frac{57}{40}\ mA$$

$$\boxed{\therefore i_o = -1.425\ mA}$$

(OR)

$$\frac{1}{R_{eq}} = \frac{2}{3} + \frac{1}{2} + \frac{5}{6}$$

$$= \frac{4+3+5}{6}$$

$$R_{eq} = \frac{6}{12} = \frac{1}{2}\ k\Omega$$

$$V_{eq} = R_{eq}\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

$$= \frac{1}{2}\left(\frac{3}{40} + \frac{2}{10} + \frac{1}{5}\right)$$

$$= \frac{19}{80}\ V$$

$$\boxed{R_{eq} = \tfrac{1}{2}\ k\Omega}$$

$$\boxed{V_{eq} = 19/80\ V}$$

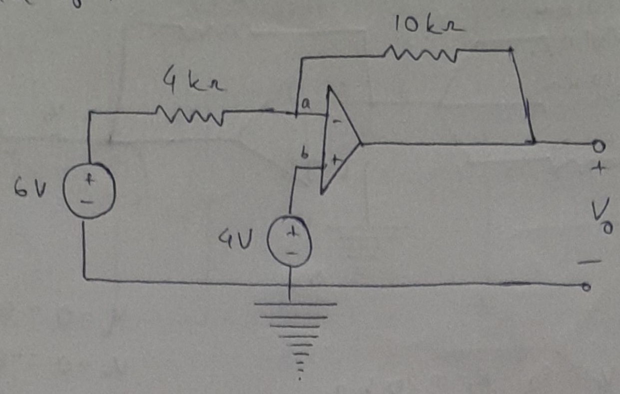
$$V_o = -\frac{8}{(1/2)} \times \frac{19}{80}$$

$$\boxed{V_o = -\frac{19}{5}\ V}$$

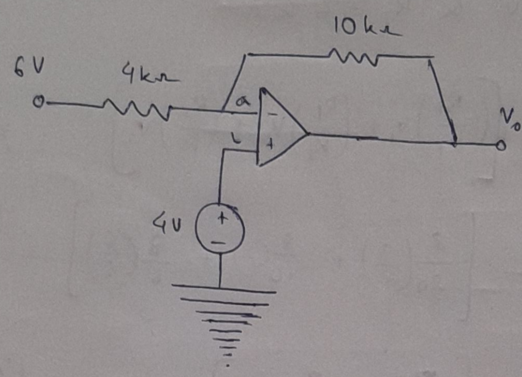
$$\text{Gain} = -\frac{R_f}{R_{eq\,in}} = -\frac{8}{1/2}$$

$$\boxed{\therefore \text{Gain} = -16}$$

Q) Find V_o :



Sol :



$V_{o_2} = V_2\left(1+\frac{R_f}{R_1}\right)$

$\Rightarrow V_{o_2} = 4\left(1+\frac{10}{4}\right)$

$= 14V$

$\boxed{\therefore V_{o_2} = 14V}$

$A_{c_2} = \frac{V_o}{V_2} = \frac{14}{4} = \frac{7}{2}$

$V_{o_1} = -V_1\left(\frac{R_f}{R_1}\right)$

$= -6 \times \frac{10}{4}$

$= -15V$

$\boxed{\therefore V_{o_1} = -15V}$

$A_{c_1} = \frac{V_o}{V_1} = \frac{-15}{6} = \frac{-5}{2}$

$V_o = (14-15)V$

$\boxed{\therefore V_o = -1V}$

$V_a = V_b$

Whatever V_b = , Same V_a is.

$V_a = V_b = 4$

$\text{Gain}_2 = A_{c_2} = 1+\frac{R_2}{R_1}$

$A_{c_2} = 1+\frac{10}{4} = \frac{7}{2}$

$\boxed{A_{c_2} = 7/2}$

$\text{Gain}_1 = A_{c_1} = -\frac{R_2}{R_1}$

$\boxed{A_{c_1} = \frac{-5}{2}}$

$\boxed{\therefore A_c = 1}$