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MA1000 Calculus
Assignment 1

January 20, 2021

Submit by: January 26, 2021 | Marks: 10

1. Find the limit of the sequence $a_n = \left(1 + \frac{3}{4n}\right)^{\frac{8}{3}n}$.
2. If α is a rational number, find $\lim\limits_{n\to\infty} \sin(n!\alpha\pi)$.
3. Let $a_0 = 1$ and $a_1 = 1$. For $n \geq 2$, let $a_n = a_{n-1} + a_{n-2}$. Then the sequence $\{a_n\}$ is called the Fibonacci sequence. Find $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n}$.
4. Find the limit of the sequence $\{a_n\}$, where $a_1 = \sqrt{2}$, $\quad a_n = \sqrt{2 + \sqrt{a_{n-1}}}$ for $n \geq 2$.
5. Find the limit of the sequence $\{a_n\}$ if $a_n = \frac{1}{n^2+1} + \frac{1}{n^2+2} + \cdots + \frac{1}{n^2+n}$.
6. If $a_n \to a$ and $b_n \to b$ and if $a < b$, show that the sequence $\{s_n\}$, where $s_n = \max\{a_n, b_n\}$, converges to b.
7. Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p} \cos(\frac{1}{n})$ converges for $p > 1$ and diverges for $0 < p \leq 1$.
8. If $\sum a_n$ converges and $a_n \geq 0$, does $\sum a_n^2$ converge? If yes, prove.
9. If $\sum a_n$ converges and $a_n \geq 0$, does $\sum \sqrt{a_n a_{n+1}}$ converge? If yes, then prove.
10. Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \cdots = 9$.
11. For what values of r, if any, does the infinite series $1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + \cdots =$ converge? Find the sum of the series when it converges.
12. Are there any values of x for which $\sum \frac{1}{nx}$ converges? Give reason.
13. Show by an example that $\sum a_n b_n$ may diverge even if $\sum a_n$ and $\sum b_n$ both converge.
14. Decide whether the following series converge or diverge.
    (a) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$.
    (b) $\sum_{n=2}^{\infty} \frac{1}{\ln(\ln n)}$.
    (c) $\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$.
    (d) $\sum_{n=1}^{\infty} \left[\frac{(n+1)^{n+1}}{n+1} - \frac{n+1}{n}\right]^{-n}$.

(e) $\sum_{n=1}^{\infty} \frac{n5^n}{(2n+3)\ln(n+1)}$.

15. Find the radius and interval of convergence of the power series below. For what values of x does the series converge (i) absolutely, (ii) conditionally?

(a) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{\sqrt{n+3}}$

(c) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$

16. Find the Taylor series of $f(x) = \cos(2x + \frac{\pi}{2})$ about $x = \frac{\pi}{4}$.

17. Find the Maclaurin series of $f(x) = \frac{1}{(1-x)^3}$.

18. Using substitution, find the Taylor series about $x = 0$ (i.e., the Maclaurin series) of the following functions.

(a) $\tan^{-1}(3x^4)$;

(b) $\frac{1}{1+\frac{3}{4}x^3}$.

19. Use the idea of power series multiplication to find the Taylor series about $x = 0$ (i.e., the Maclaurin series) of the functions

(a) $x^2 \cos(x^2)$,

(b) $x \ln(1 + 2x)$,

(c) $\cos^2 x$.

20. Use the identity $\sin^2 x = \frac{1-\cos 2x}{2}$ to obtain the Maclaurin series for $\sin^2 x$. Then differentitate this series to obtain the Maclaurin seies for $2 \sin x \cos x$. Check that this is the series for $\sin 2x$.