Engineering Electromagnetics

Lecture 17

09/10/2023

by

Debolina Misra

Dept. of Physics
IIITDM Kancheepuram, Chennai, India

For continuous charge distribution

$$W = \frac{1}{2}\int \rho V \, d\tau$$

OR

$$W = \frac{1}{2}\int \sigma V \, da$$

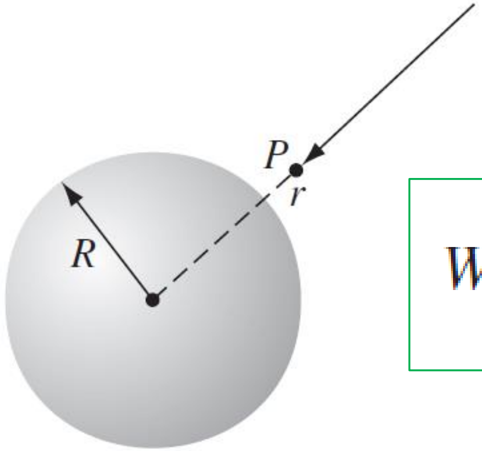
$$\rho = \epsilon_0 \nabla \cdot \mathbf{E}$$

Evaluate the integral

$$W = \frac{\epsilon_0}{2}\int E^2 \, d\tau$$

- λ → line charge density?

1. Find the energy of a uniformly charged spherical shell of total charge *q* and radius *R*.



$$W = \frac{1}{2}\int \sigma V \, da$$

q= total charge.

V for spherical shell =$\frac{q}{4\pi\varepsilon_0 R}$

Hence W = $\frac{q^2}{8\pi\varepsilon_0 R}$

From Griffith Book

Problem-2

A metallic sphere of radius 10 cm has a surface charge density of 10 nC/m^2. Calculate the electric energy stored in the system.

Solution

The potential on the surface of the sphere is

$$V = \int_s \frac{\rho_s \, ds}{4\pi \epsilon_0 R} = 9 \times 10^9 \times 10 \times 10^{-9} \times 0.1 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$
$$= 113.1 \text{ V}$$

where Q_t is the total charge on the sphere. For uniform charge distribution, the total charge is

$$Q_t = 4\pi R^2 \rho_s = 4\pi (0.1)^2 \, 10 \times 10^{-9} = 1.257 \text{ nC}$$

Thus,

$$W = 0.5 \times 1.257 \times 10^{-9} \times 113.1 = 71.08 \times 10^{-9} \text{ joules (J)}$$

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Superposition for Energy??

Superposition for Energy??

(iii) The superposition principle. Because electrostatic energy is *quadratic* in the fields, $\boxed{\text{it does } \textit{not} \text{ obey a superposition principle.}}$ The energy of a compound system is *not* the sum of the energies of its parts considered separately—there are also "cross terms":

$$\begin{aligned} W_{\text{tot}} &= \frac{\epsilon_0}{2}\int E^2\,d\tau = \frac{\epsilon_0}{2}\int (\mathbf{E}_1 + \mathbf{E}_2)^2\,d\tau \\ &= \frac{\epsilon_0}{2}\int \left(E_1^2 + E_2^2 + 2\mathbf{E}_1\cdot\mathbf{E}_2\right)\,d\tau \\ &= W_1 + W_2 + \epsilon_0\int \mathbf{E}_1\cdot\mathbf{E}_2\,d\tau. \end{aligned} \tag{2.47}$$

For example, if you double the charge everywhere, you *quadruple* the total energy.

From Griffith Book

Electric Dipole

We define an *electric dipole* as a pair of equal charges of opposite signs that are very close together.

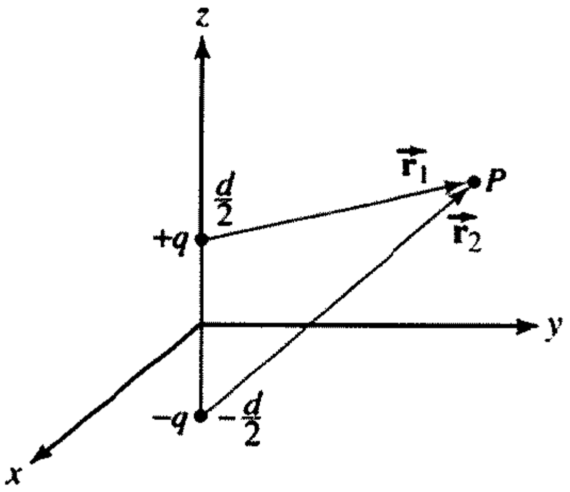

Figure 3.24 An electric dipole

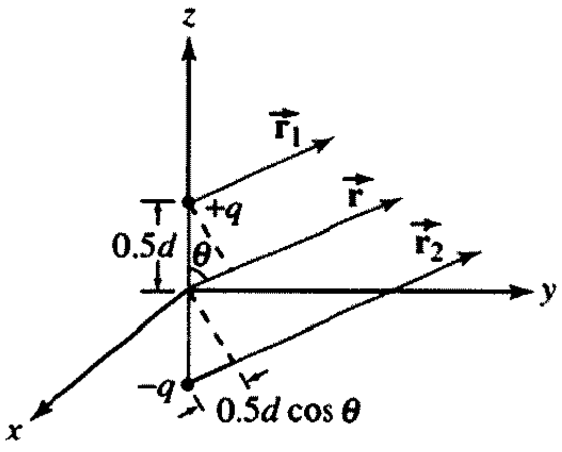

Figure 3.25 Distance approximations when P is far away from the dipole ($r \gg d$)

Potential due to Dipole

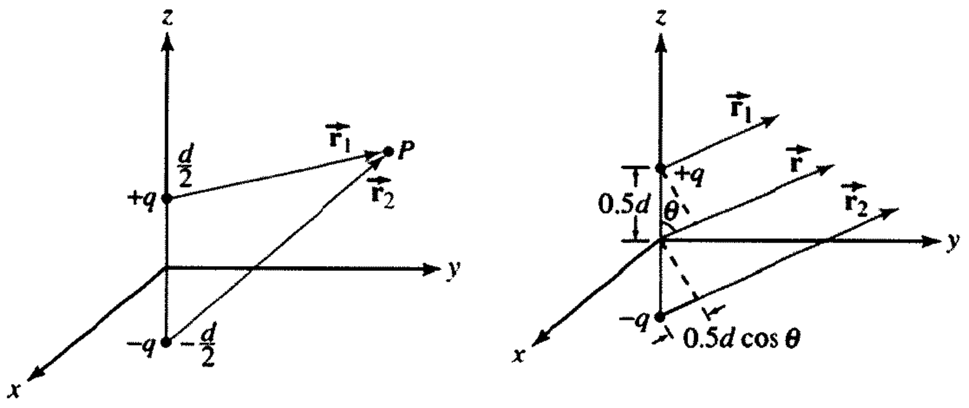

Figure 3.24 An electric dipole

Figure 3.25 Distance approximations when P is far away from the dipole ($r \gg d$)

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$r_1 \approx r - 0.5d\cos\theta, \quad r_2 \approx r + 0.5d\cos\theta$$

and

$$r_1 r_2 = r^2 - (0.5d\cos\theta)^2 \approx r^2$$

The potential at P can now be written as

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{d\cos\theta}{r^2}\right]$$

$$\boxed{\vec{\mathbf{p}} = qd\vec{\mathbf{a}}_z}$$

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{a}}_r}{4\pi\epsilon_0 r^2}}$$

Note that the potential at a point falls off as the square of the distance for a dipole, whereas it is inversely proportional to distance for a single-point charge.

Field due to dipole

$$\vec{\mathbf{E}} = \frac{p}{4\pi\epsilon_0 r^3}[2\cos\theta\,\vec{\mathbf{a}}_r + \sin\theta\,\vec{\mathbf{a}}_\theta]$$

However,

$$\begin{aligned} 2\cos\theta\,\vec{\mathbf{a}}_r + \sin\theta\,\vec{\mathbf{a}}_\theta &= 3\cos\theta\,\vec{\mathbf{a}}_r - (\cos\theta\,\vec{\mathbf{a}}_r - \sin\theta\,\vec{\mathbf{a}}_\theta) \\ &= 3\cos\theta\,\vec{\mathbf{a}}_r - \vec{\mathbf{a}}_z \end{aligned}$$

Thus, we can write the electric field intensity at point P as

$$\boxed{\vec{\mathbf{E}} = \frac{3(\vec{\mathbf{p}}\cdot\vec{\mathbf{r}})\,\vec{\mathbf{r}} - r^2\vec{\mathbf{p}}}{4\pi\epsilon_0 r^5}}$$

An **electric dipole** *is defined as two charges of equal strength but of opposite polarity but separated by a small distance.* Associated with each dipole is a vector called the **dipole moment**. If q is the magnitude of each charge and $\vec{\mathbf{d}}$ is the distance vector from the negative to the positive charge, then the dipole moment is $\vec{\mathbf{p}} = q\vec{\mathbf{d}}$.

Problem-2

An electron and a proton separated by a distance of 10^{-11} meter are symmetrically arranged along the z axis with $z = 0$ as its bisecting plane. Determine the potential and $\vec{\mathbf{E}}$ field at $P(3, 4, 12)$.

Solution

The position vector: $\vec{\mathbf{r}} = 3\vec{\mathbf{a}}_x + 4\vec{\mathbf{a}}_y + 12\vec{\mathbf{a}}_z \quad r = 13$ m
The dipole moment: $\vec{\mathbf{p}} = 1.6 \times 10^{-19} \times 10^{-11}\vec{\mathbf{a}}_z = 1.6 \times 10^{-30}\vec{\mathbf{a}}_z$
From (3.38), the potential at point P is

$$V = \frac{\vec{\mathbf{p}} \cdot \vec{\mathbf{r}}}{4\pi\epsilon_0 r^3} = \frac{9 \times 10^9 \times 1.6 \times 10^{-30} \times 12}{13^3} = 7.865 \times 10^{-23}\text{V}$$

The electric field intensity at point P, from (3.42), is

$$\vec{\mathbf{E}} = \frac{9 \times 10^9}{13^5}(1.6 \times 10^{-30})[3 \times 12(3\vec{\mathbf{a}}_x + 4\vec{\mathbf{a}}_y + 12\vec{\mathbf{a}}_z) - 13^2\vec{\mathbf{a}}_z]$$
$$= [4.189\vec{\mathbf{a}}_x + 5.585\vec{\mathbf{a}}_y + 10.2\vec{\mathbf{a}}_z] \times 10^{-24} \text{ V/m}$$

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Conductors

- Conductors: examples?
- Insulators: examples?

In an **insulator,** such as glass or rubber, each electron is on a short leash, attached to a particular atom. In a metallic **conductor,** by contrast, one or more electrons per atom are free to roam. (In liquid conductors such as salt water, it is ions that do the moving.) A *perfect* conductor would contain an *unlimited* supply of free charges. In real life there are no perfect conductors, but metals come pretty close, for most purposes.

From Griffith Book

Conductors

Let us first raise a question: Can excess charge reside inside a conductor? The answer, of course, is emphatically no because of the force of mutual repulsion among the charges. They will continue to "fly away" due to the repulsive forces until their mutual repulsion is balanced by surface barrier forces. In other words, the excess charge will disappear from inside and redistribute itself on the surface of an isolated conductor. How long does this process take? Again, a quantitative answer is given in Chapter 4; however, the time is extremely small, of the order of 10^{-14} second for a good conductor like copper. This implies that under steady-state (equilibrium) conditions, the net volume charge density within the conductor is zero. That is,

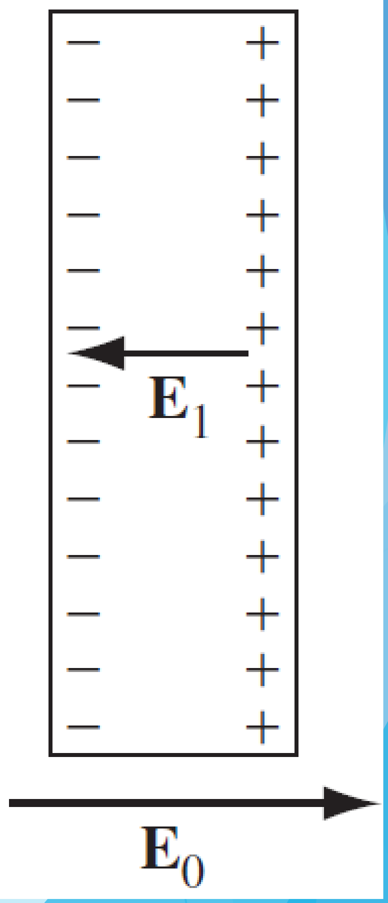
$\rho_v = 0$ inside the conductor.

From Bhagguru Book

Conductor in a field

Now let us assume that we place an isolated conductor in an electric field, as shown in Figure 3.27. The externally applied electric field exerts a force on the free electrons and causes them to move in a direction opposite to the $\vec{\mathbf{E}}$ field. One side of the conductor becomes negatively charged, and the other side becomes positively charged. We refer to such a separation of charges as *induced charges* because they are caused without any direct contact with the conductor. The effect of these induced charges is to produce an electric field within the conductor which is finally equal and opposite to the externally applied electric field. In other words, the net electric field inside the conductor is zero when the steady state is reached. Thus,

$\vec{\mathbf{E}} = 0$ inside a conductor in the equilibrium state.



From Bhagguru Book

Conductors

(ii) $\rho = 0$ inside a conductor.

(iii) Any net charge resides on the surface. That's the only place left.

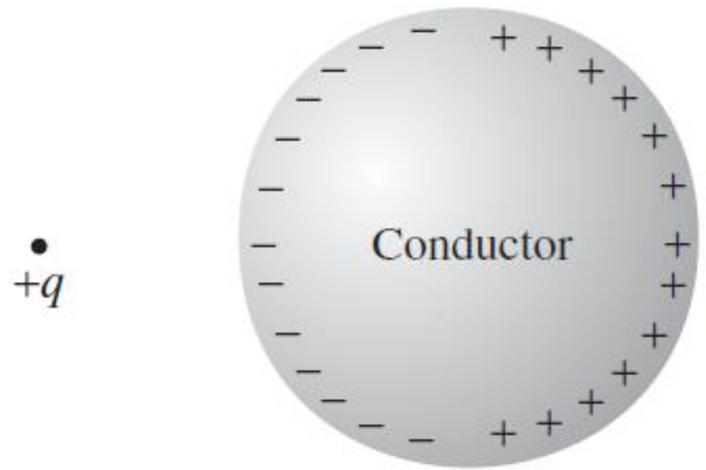
(iv) A conductor is an equipotential.

(v) $\mathbf{E}$ is perpendicular to the surface, just outside a conductor.

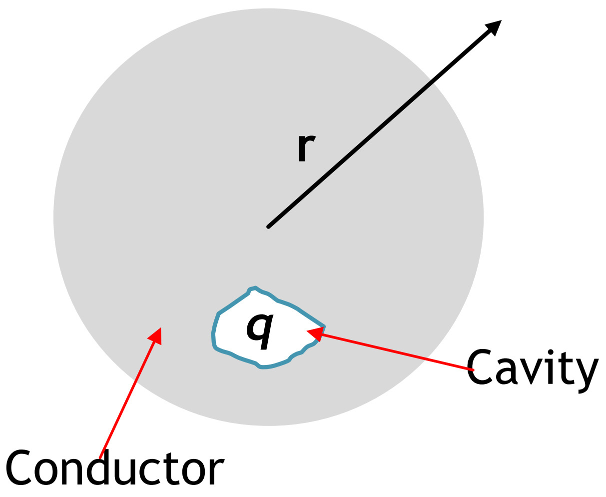
From Griffith Book

Induced Charges

If you hold a charge $+q$ near an uncharged conductor (Fig. 2.44), the two will attract one another. The reason for this is that q will pull minus charges over to the near side and repel plus charges to the far side. (Another way to think of it is that the charge moves around in such a way as to kill off the field of q for points inside the conductor, where the total field must be zero.) Since the negative induced charge is closer to q, there is a net force of attraction. (In Chapter 3 we shall calculate this force explicitly, for the case of a spherical conductor.)



Q. An uncharged spherical conductor has a cavity carrying a charge q. What is the field (i) within the cavity? (ii) outside the cavity (iii) outside the sphere?



Thank You