

KEY

Design and Analysis of Algorithms - End Semester - 20-Nov-2023 - 9.30-12.30

0. (0 marks) Name the algebraic structure that appears as a substring in our Institute name.

1 Light Dose

1. (1.5 marks) Given a sorted array of size $2^n, n > 0$; the asymptotic tight time complexity to search an element by the best known algorithm is..... Mention the algorithm and the asymptotic tight time complexity.

0.75 Binary Search

0.75 $\Theta(2^n) = \Theta(n)$ in the worst case

2. (1.5 marks) With proper justification, explain what is the minimum number stacks required to simulate a queue. Similarly the minimum number of queues required to simulate a stack.

light dose - 13
Medium dose - 15
strong dose - 12
40

3. (1.5 marks) Mention four applications of Breadth First Search

1) Connectedness Testing

7) No. of Connected Components

Any 4 2) Cycle Testing

3) ODD Cycle / Even Cycle Test

4) Treeness

5) Bipartiteness

6) 2-colorability

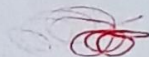
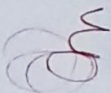
4. (1.5 marks) How do you test whether a given graph is bipartite.

0.5 [① Run BFS

② Chk for cross edges

③ If NO cross edges then NO ODD cycles
 $\Rightarrow G$ is bipartite.

④ If \exists cross edge declare G is NOT Bipartite.



5. (1.5 marks) Define NP. Prove that MAX-CLIQUE problem is in NP.

0.5 NP: The class of Non-deter Algorithms
A set of Computational problems whose solution
can be obtained by designing Non-deterministic Polynomial time
Algorithms.

1 Max clique NP: ① Guess a subset in Non-det fashion (Build a Guessing Tree)
② Verify the Subset

6. (1.5 marks) For a problem P , there are five algorithms with run-times $n^n, 2^{n \log_2 n}, 4^{\log_2 n}, n^{O(1)}, \log_2 n$; arrange in increasing order of their time complexities.

① $\leftarrow \log_2 n < n^{O(1)} < 2^{n \log_2 n} = n^n$

0.5 $\leftarrow 4^{\log_2 n}$ Cannot be compared with $n^{O(1)}$
 $\log_2 n < 4^{\log_2 n} < 2^{n \log_2 n} = n^n$

7. (1 mark) Mention the recursive subproblem for the Matrix Chain Multiplication problem

$$M[i, j] = \min_{i \leq k < j} \{ M[i, k] + M[k+1, j] + P_{i-1} P_k P_j \}$$

8. (1.5 marks) Present two functions $f(n)$ and $g(n)$ such that $f(n) \neq O(g(n))$ and $f(n) \neq \Omega(g(n))$. Justify.

0.5 $\begin{cases} f(n) = n \\ g(n) = n^{1+\sin n} \end{cases}$ if $\sin n = -1$ then $f(n) = n$ $g(n) = n^0 = 1$ $g(n) = O(f(n))$
if $\sin n = 1$ then $f(n) = n$ $g(n) = n^2$ $f(n) = O(g(n))$

9. (1.5 marks) What are the invariants maintained by Prim's and Kruskal's algorithms

0.75 Prim's: Connectedness

0.75 Kruskal's: Acyclicity

2 Medium Dose

1. (2 marks) For the recurrence relation, $T(n) = aT(\frac{n}{b}) + f(n)$, $a \geq 1, b > 1$, mention two good lower bounds with a proper justification.

① \leftarrow 1) # leaves is a good lower bound
 $\Omega(n^{\log_b a})$

① \leftarrow 2) The root of computation tree $f(n)$ is also a LB
 $\Omega(f(n))$

2. (1+1.5=2.5 marks) Explain 3-way Merge sort along with time complexity analysis (present the asymptotic tight bounds)

To Merge two Sorted Arrays of size $n/3$ each
 We need $n/3 + n/3 - 1 = \frac{2n}{3} - 1$ in W.C

To Merge one Sorted Array of size $n/3$ and the other of size $\frac{2n}{3}$
 We need $\frac{2n}{3} + n/3 - 1 = \frac{3n}{3} - 1$

Total; $\frac{2n}{3} - 1 + \frac{3n}{3} - 1 = \frac{5n}{3} - 2$

Recurrence;

$T(n) = 3T(n/3) + \frac{5n}{3} - 2$ in worst case

Explanation

- ① Recursively Divide into 3 subproblems of size $n/3$ each } 0.5
 ② Recursively Combine Sorted Arrays

Soln $T(n) = 3T(n/3) + \frac{5n}{3} - 2$

Case 2 of HT; $\Theta(n \log_3 n)$

1 mark

3. (1+1.5=2.5 marks) Solve (i) $T(n) = 4T(\frac{n}{2}) + n^2$ (ii) $T(n) = 4T(\frac{n}{2}) + 2^n$

(i) Case 2 of MT (ii) Case 3 of MT

for some $\epsilon > 0$

$2^n = \Omega(n^{\log_2 4 + \epsilon})$

$f(n) = \Omega(n^{\log_2 4 + \epsilon})$

Regularity condn chk is TRUE

$a f(n) < c f(n)$ for some $c < 1$

$4 \cdot 2^{n/2} < c \cdot 2^n$ $c = 3/4$ $n_0 \geq 10$

Soln: $T(n) = \Theta(2^n)$

4. (2 marks) Given two sorted arrays of size n and m respectively, what are the minimum and maximum number of comparisons required (in the worst case) to merge them into a single array of size $n + m$. Present an example with justification for each meeting the number mentioned.

MIN: $\min(n, m)$

Example: $A_1: 1 \ 2 \ 3 \ 4$ $A_2: 5 \ 6 \ 7 \ 8$

0.5

0.5

Justify: We Comp (1, 5) Move ptr,

Comp (2, 5) "

Comp (3, 5) "

Comp (4, 5) "

4 Comp

Stop.

Max $m+n-1$

0.5

0.5

Example: $A_1: 1 \ 3 \ 5 \ 7$ $A_2: 2 \ 4 \ 6 \ 8$

Comp (1, 2)

Comp (3, 2)

Comp (3, 4)

Comp (5, 4)

Comp (5, 6)

Comp (7, 6)

Comp (7, 8)

STOP

7 Comp

5. (2 marks) Assume MIN-Vertex Cover is NP-Hard. Prove that MAX-CLIQUE is NP-Hard.

$\langle G, k \rangle \in \text{VC}$ maps to $\langle G^c, n-k \rangle \in \text{CLIQUE}$

If G has a VC of size k then, In G , the other $(n-k)$ vertices form ISet

In G^c , $(n-k)$ vertices form CLIQUE

Conversely, In G^c , Clique of size $(n-k)$

In G , ISet $(n-k)$

In G , the other ' k ' vertices form VC.

6. (2 marks) Given an unweighted graph G , present an algorithm to find a short cycle (a cycle with minimum number of edges) in G .

For each edge $e = \{u, v\}$, Remove the edge $\{u, v\}$
and Run $\text{spath}(u)$ [Dijkstra]
to obtain spath from u to v .

$\text{spath} + \text{the edge } \{u, v\} \Rightarrow \text{Scycle containing } \{u, v\}$

Repeat for each edge and choose MIN

7. (2 marks) Given a set S and an integer t , the objective is to find a subset $S' \subseteq S$ whose sum is t . Present a DP to solve this problem.