Engineering Optics

Lecture 21

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by

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What is 'Population'?

- Imagine a chamber filled with a gas in equilibrium at some temperature T
- **If T is relatively low, most of the atoms will be in their ground states**, but a few will "rise" into an excited state
- Maxwell-Boltzmann distribution (N: number of atoms/volume)

$$N_i = N_0 \, exp^{-E_i/k_BT}$$

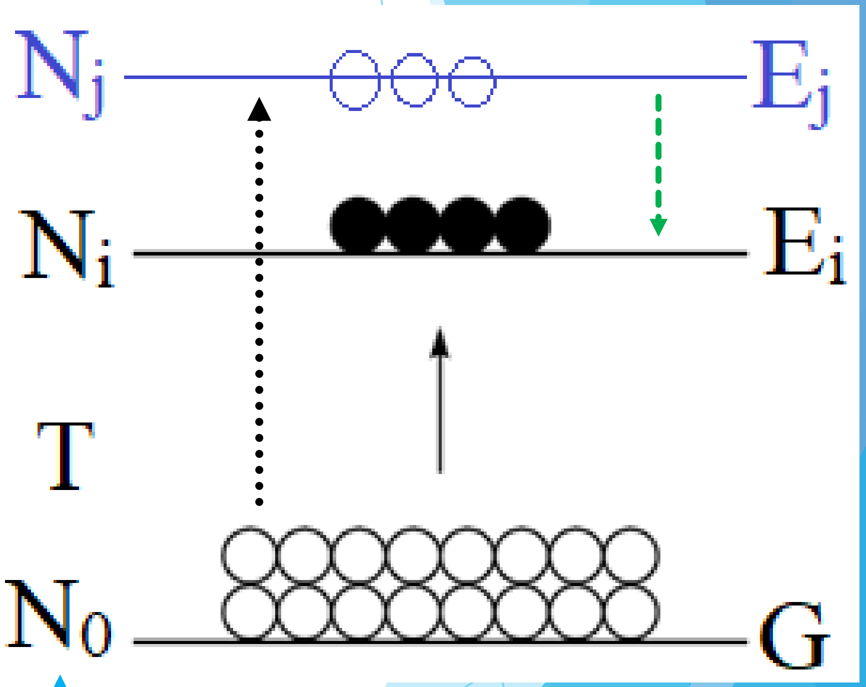
- higher E → fewer atoms there will be in that state

$$N_j = N_0 \, exp^{-E_j/k_BT}$$

- Where E_j > E_i
- ratio of the populations occupying these two states

$$\frac{N_j}{N_i} = \frac{exp^{-E_j/k_BT}}{exp^{-E_i/k_BT}}$$

relative population, $N_j = N_i \, exp^{-(E_j-E_i)/k_BT}$



constant for a given T

A transition from j^{th} to i^{th} state is also possible!

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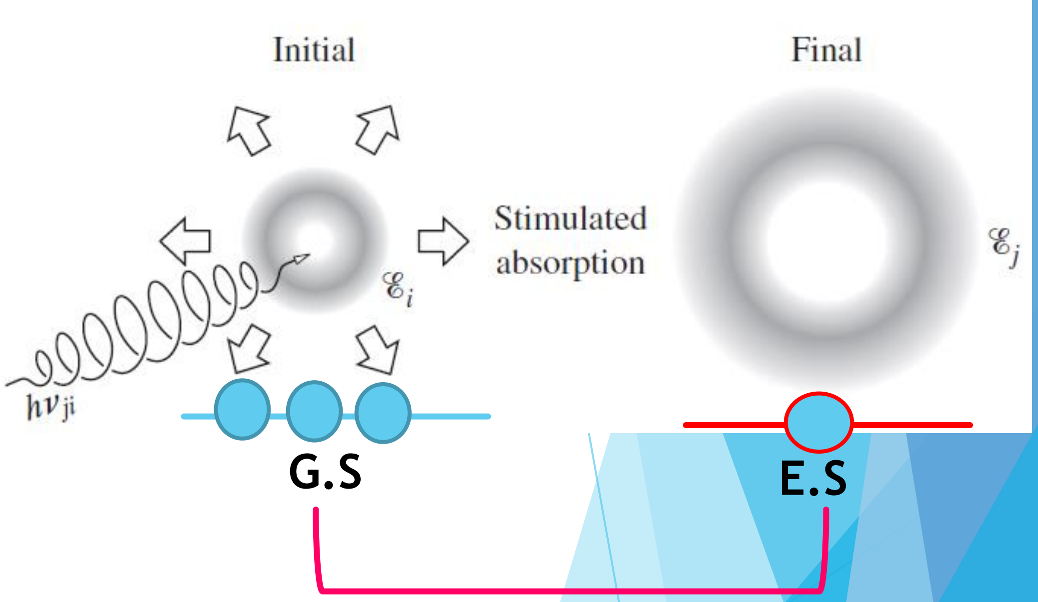
The Einstein A and B Coefficients: Stimulated absorption

stimulated absorption, whereupon the transition rate is

$$\left(\frac{dN_i}{dt}\right)_{\text{ab}} = -B_{ij}N_i u_\nu$$

Here B_{ij} is a constant of proportionality, the *Einstein absorption coefficient*, and the minus arises because N_i is decreasing.



u_ν: spectral energy density = energy per unit volume per unit frequency interval ($J.s/m^3$)

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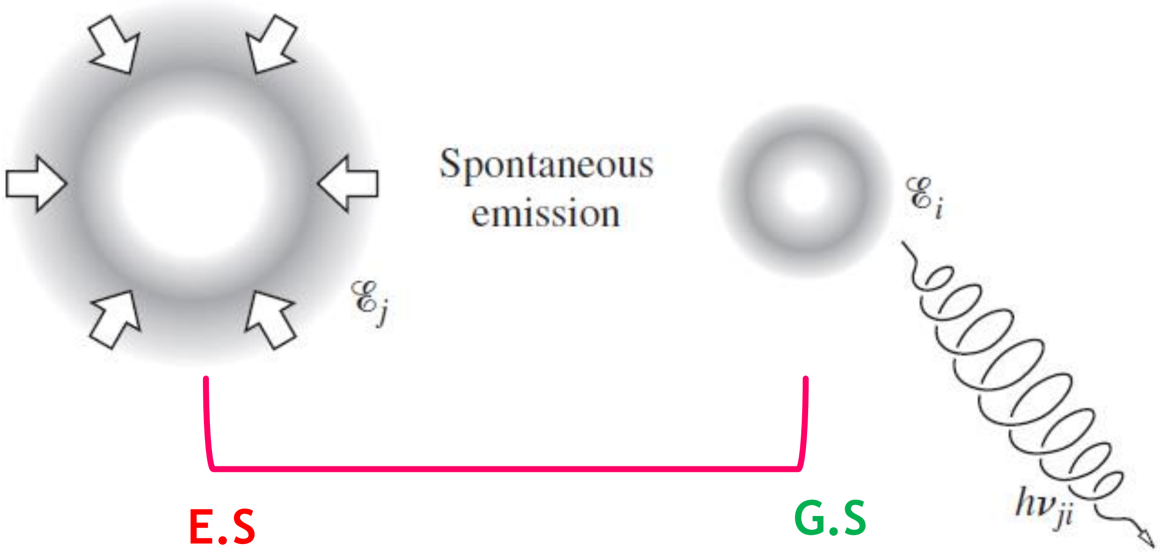
The Einstein A and B Coefficients: Spontaneous emission

Such an excess-energy configuration is usually (though not always) exceedingly short-lived, and in 10 ns or so, without the intercession of any external influence, the atom will emit its overload of energy as a photon. As it does, it reverts to a stable state in a process called *spontaneous emission*



[spontaneous emission] $$\left(\frac{dN_j}{dt}\right)_{\text{sp}} = -A_{ji}N_j$$

This is the rate of decrease of the higher-energy population, N_j, due to spontaneous emission. And A_{ji} is the *Einstein spontaneous emission coefficient* associated with a drop from energy level-j to level-i.

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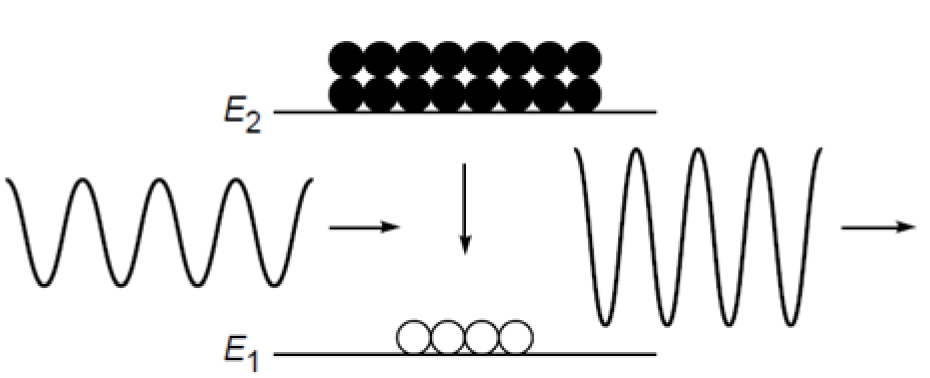
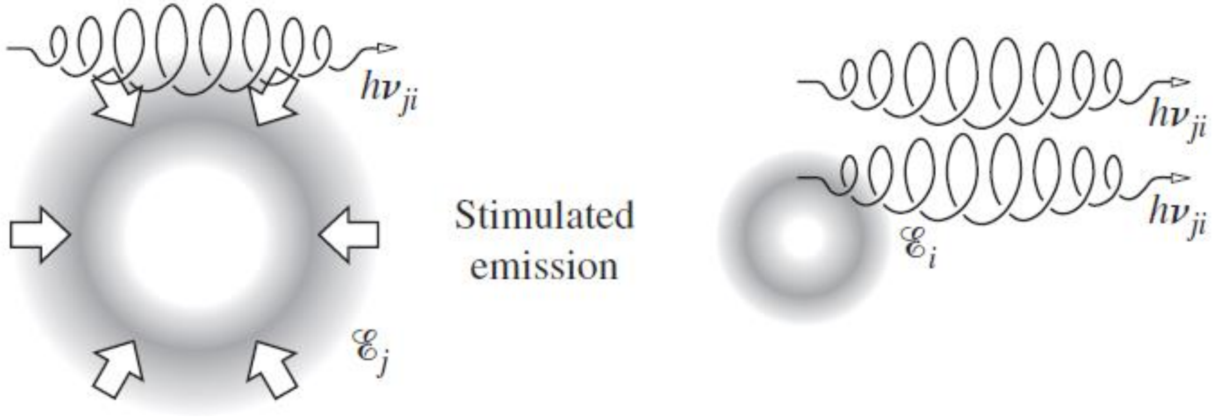
The Einstein A and B Coefficients: Stimulated emission





[stimulated emission] $$\left(\frac{dN_j}{dt}\right)_{\text{st}} = -B_{ji}N_j u_\nu$$

The constant B_{ji} is the *Einstein stimulated emission coefficient*.

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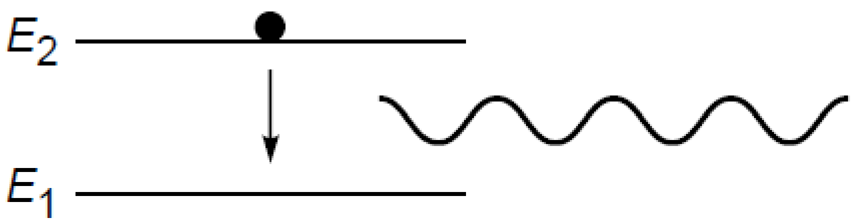
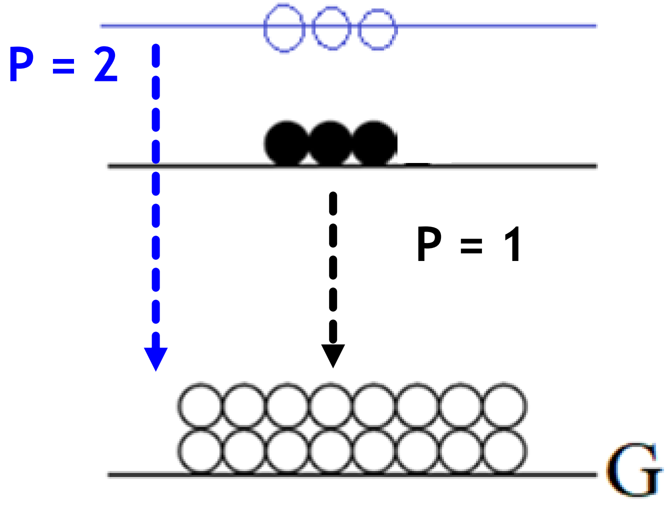
Probability and life time of a state

Keep in mind that the transition rate, the number of atoms making transitions per second, divided by the number of atoms, is the probability of a transition occurring per second, $\mathscr{P}$. Consequently, the probability per second of spontaneous emission is $\mathscr{P}_{sp} = A_{ji}$.





Q: most preferred state?

For a single excited atom making a spontaneous transition to a lower state, the inverse of the transition probability per second is the **mean life** or **lifetime** of the excited state τ. Thus (operating under conditions that exclude any other mechanism but spontaneous emission), if N atoms are in that excited state, the total rate of transitions, that is, the number of emitted photons per second, is $N\mathscr{P}_{sp} = NA_{ji} = N/\tau$. A low-transition probability means a long lifetime.

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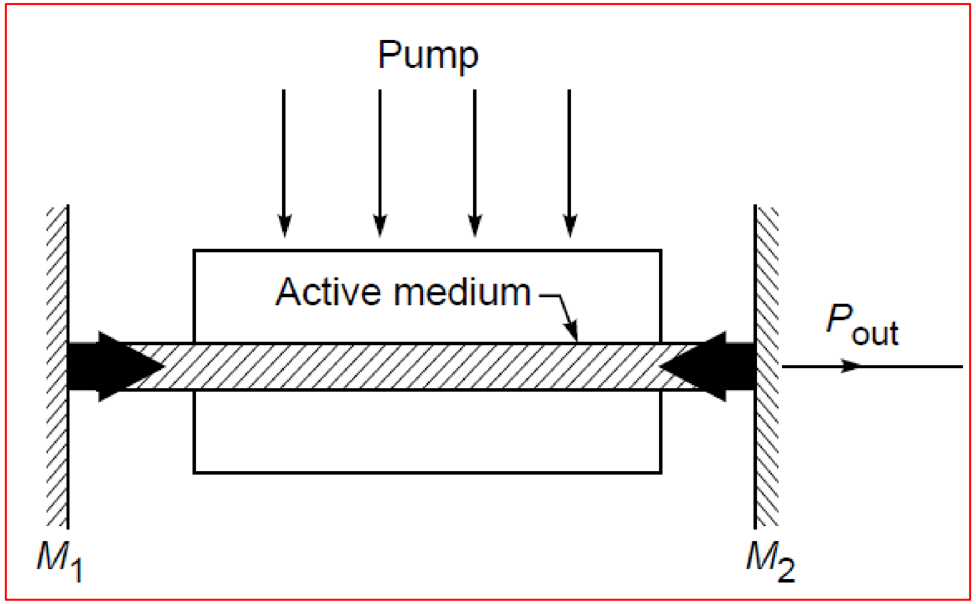
Main Components of the Laser

- 1. Active medium
- 2. Pumping source
- 3. Optical resonator



Points to note: *Some of the energy is coupled back to the system → oscillator.*

LOSER → light oscillation by stimulated emission of radiation.

Since it would have been difficult to obtain a research grant for LOSERs, it was decided to retain the name LASER.

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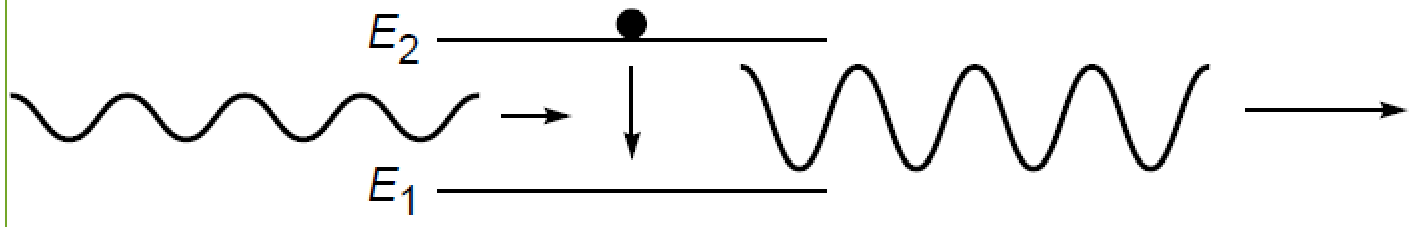
Problem-1

- **A 10-mW laser is emitting at a mean wavelength of 500 nm. Determine the rate of occurrence of stimulated emission.**

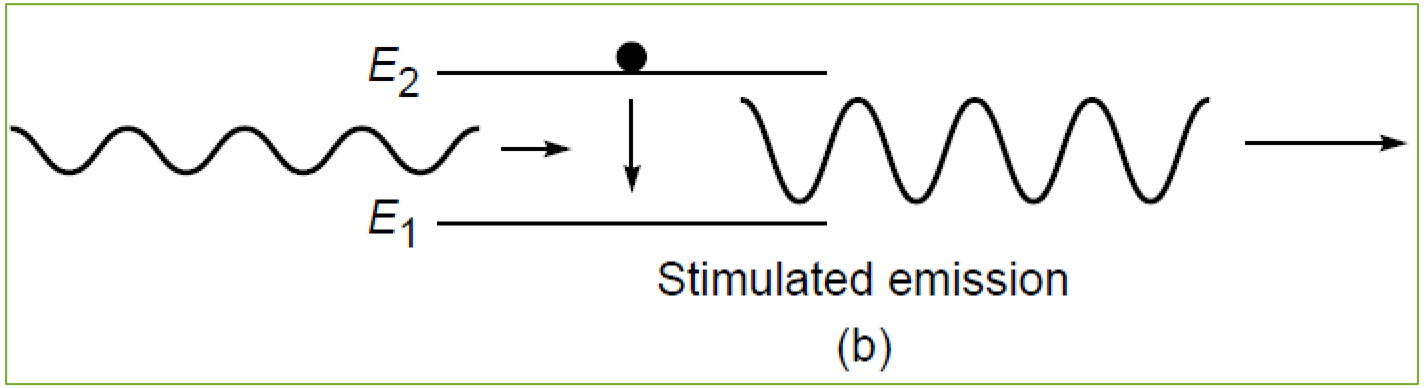


Stimulated emission

(b)

Problem-1

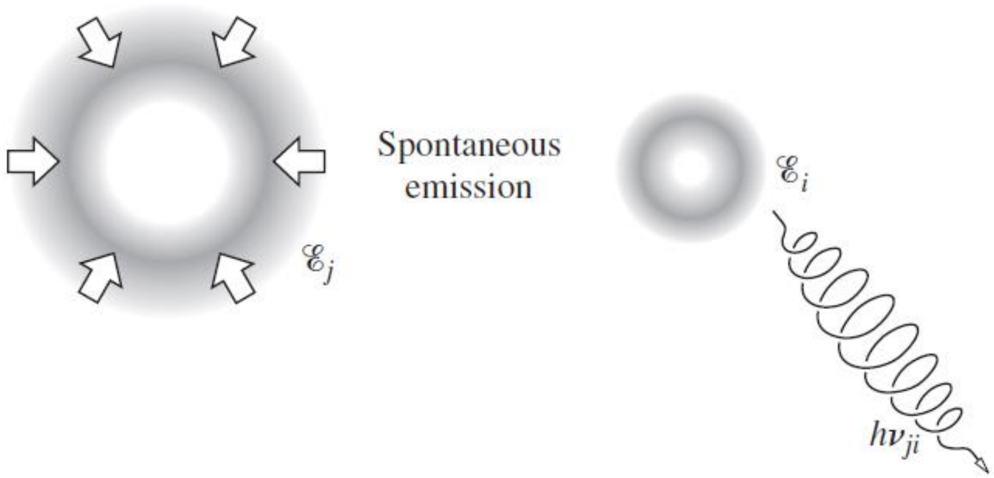
- A 10-mW laser is emitting at a mean wavelength of 500 nm. Determine the rate of occurrence of stimulated emission.

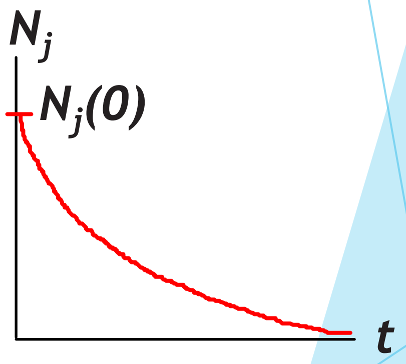


- Rate of occurrence of stimulated emission = how many photons are emitted/sec ?
- Power P = Total energy/time = 10 mW = 0.01 J/Sec
- Energy of each photon E = $h\nu$ = hc/λ = 3.973×10^{-19} J
- No. of photons = P/E = 2.52×10^{16}

Problem-2

Q: Suppose a sample exists where there are N_j excited electrons per unit volume in energy level-j just above the ground state level-i. Show that the population of energy level-j falls exponentially as electrons leave via spontaneous emission. What can be said about the lifetime of level-j?





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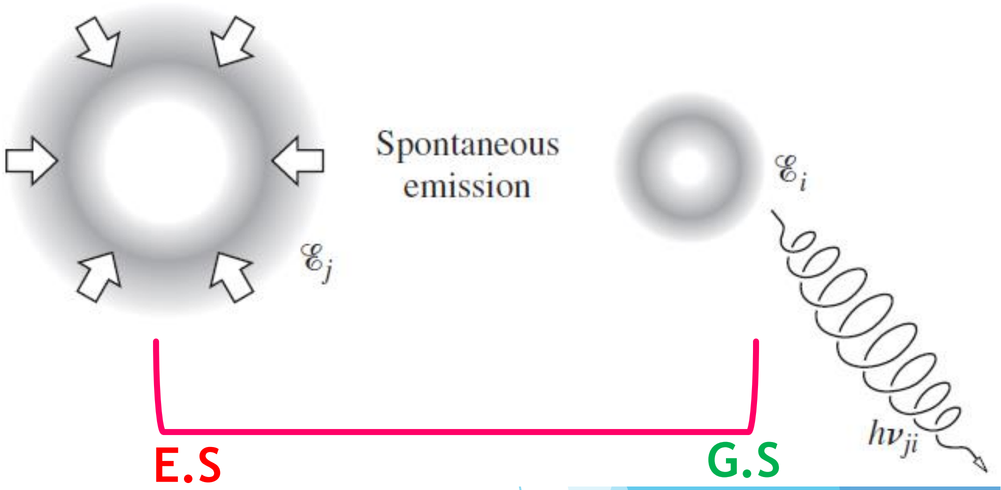
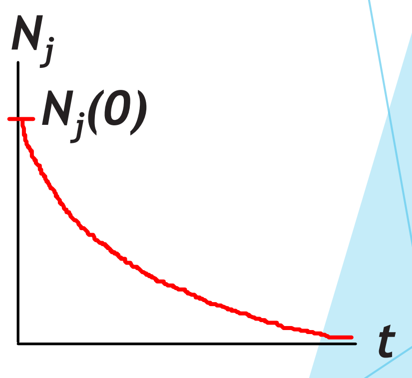


[spontaneous emission] $$\left(\frac{dN_j}{dt}\right)_{\text{sp}} = -A_{ji}N_j$$

This is the rate of decrease of the higher-energy population, N_j, due to spontaneous emission. And A_{ji} is the *Einstein spontaneous emission coefficient* associated with a drop from energy level-j to level-i.

$$\frac{dN_j}{N_j} = -A_{ji}\,dt \Rightarrow \int \frac{dN_j}{N_j} = \int -A_{ji}\,dt + C$$

Say at $t = 0$, $N_j = N_j(0)$ $\longrightarrow$ $\boldsymbol{N_j = N_j(0)e^{-A_{ji}t}}$



Probability of transition occurrence/Sec = P = A_{ji}

→ Lifetime $\tau = 1/P = 1/A_{ji}$

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Problem-3

Q: Determine the mks units of $u(\nu)$, A, and B

[spontaneous emission] $$\left(\frac{dN_j}{dt}\right)_{sp} = -A_{ji}N_j$$

A_{ij} :

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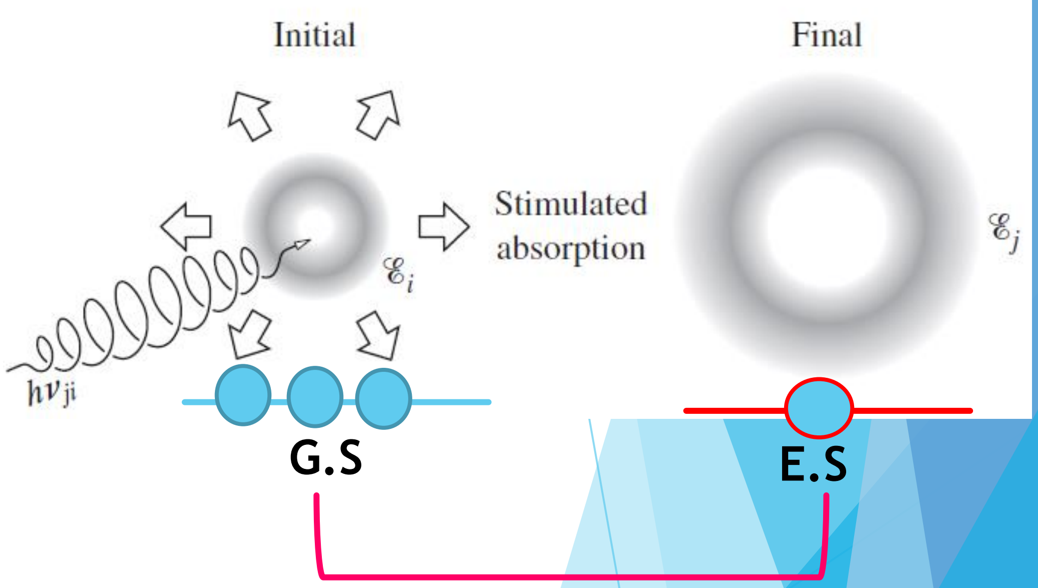

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Problem-3

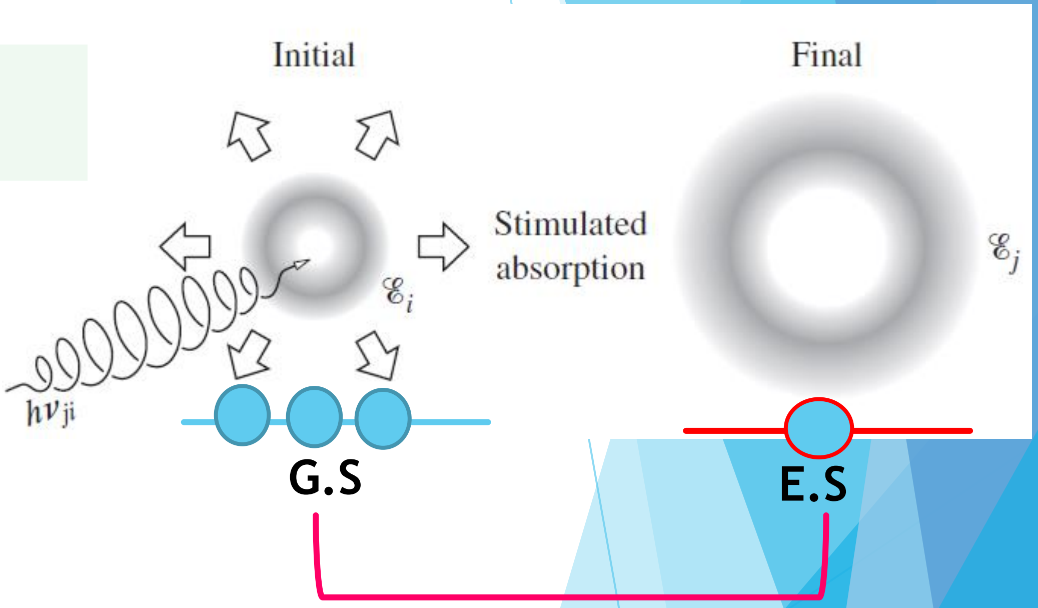
Q: Determine the mks units of $u(v)$, A, and B

Ans: u_v: spectral energy density = energy per unit volume per unit frequency interval **(J.s/m³)**

[spontaneous emission] $$\left(\frac{dN_j}{dt}\right)_{sp} = -A_{ji}N_j$$

A_{ij} : S^{-1}

This is the rate of decrease of the higher-energy population, N_j, due to spontaneous emission. And A_{ji} is the *Einstein spontaneous emission coefficient* associated with a drop from energy level-j to level-i.



stimulated absorption, whereupon the transition rate is

$$\left(\frac{dN_i}{dt}\right)_{ab} = -B_{ij}N_iu_\nu \longrightarrow B_{ij} = \left(\frac{dN_i}{dt}\right)/(N_iu_v)$$

$B_{ij}: m^3.J^{-1}.s^{-2}$

Here B_{ij} is a constant of proportionality, the *Einstein absorption coefficient*, and the minus arises because N_i is decreasing.

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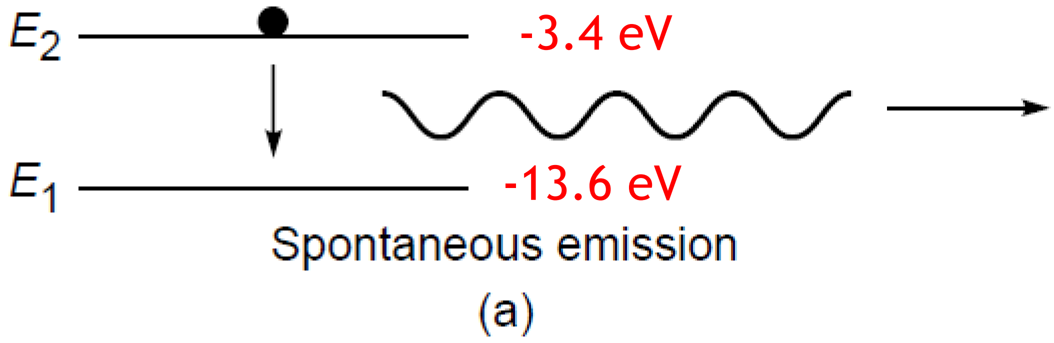
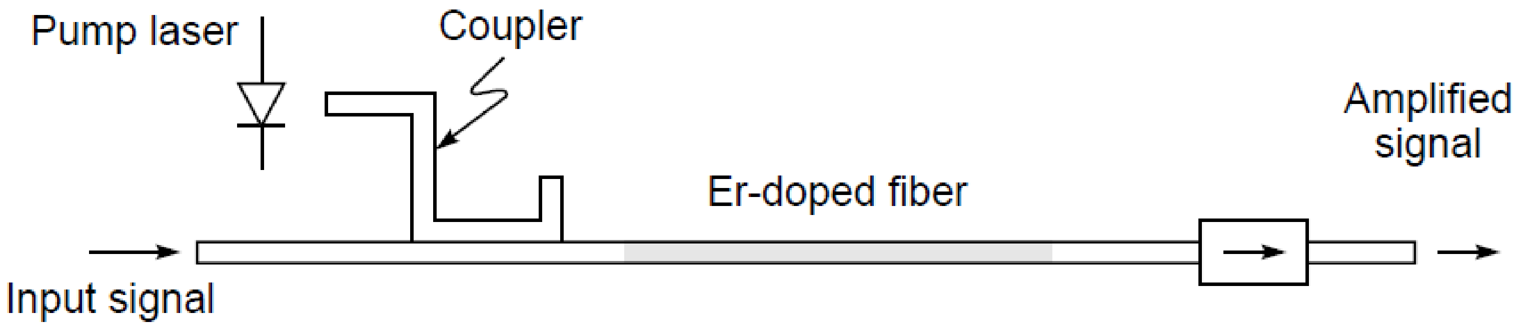
Problem 4

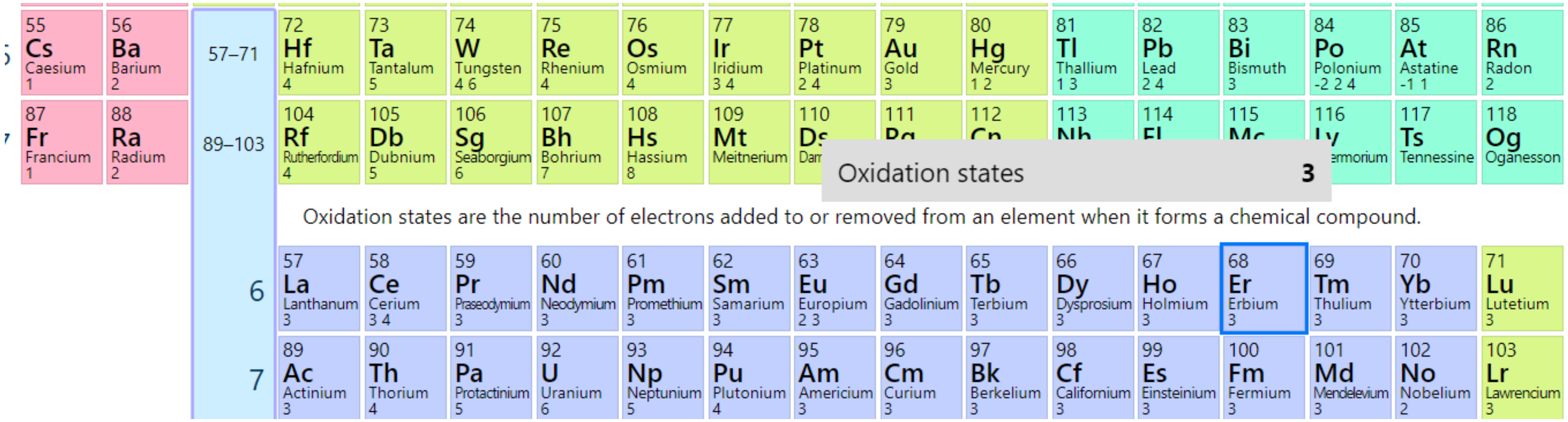
- Q: For the 2P⟶1S transition in the hydrogen atom, calculate frequency of emitted wave.

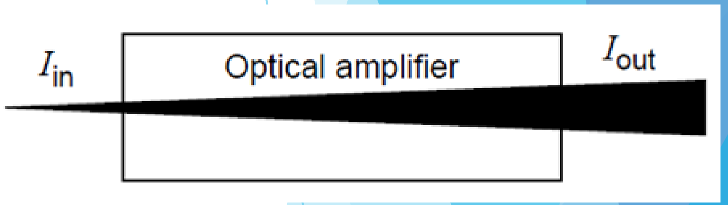


- Ans: $E_2 - E_1 = -3.4 + 13.6 = 10.2$ eV $= h\nu$
- h = Planck's constant → ν = ?

Understanding Optical Amplification







- understand optical amplification → EDFA
- **Erbium doped fiber amplifier**
- The EDFA: consists of a silica optical fiber the core doped with erbium oxide (Er_2O_3)
- light is guided through the optical fiber
- Valence state of Er in silica?

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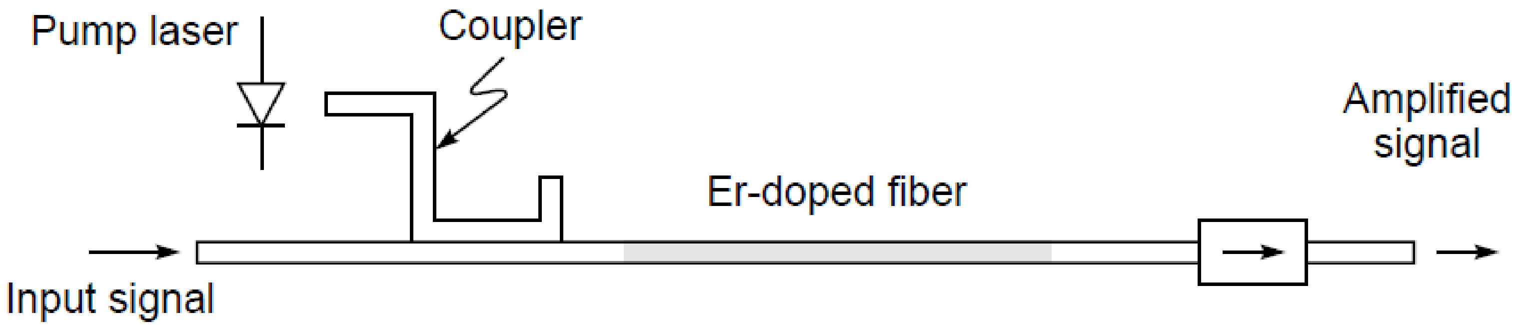
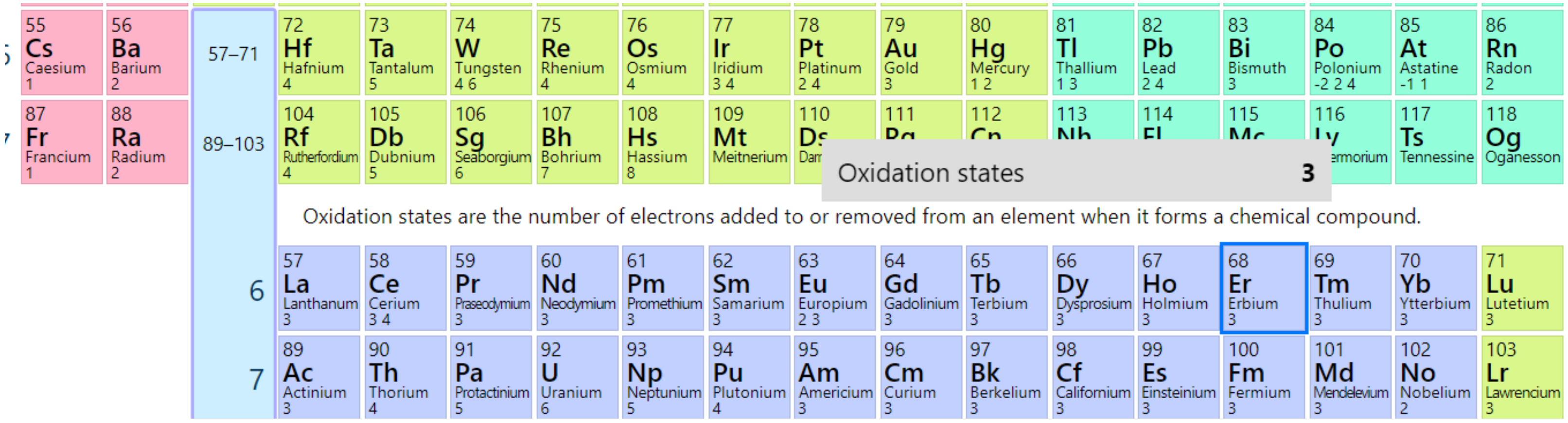
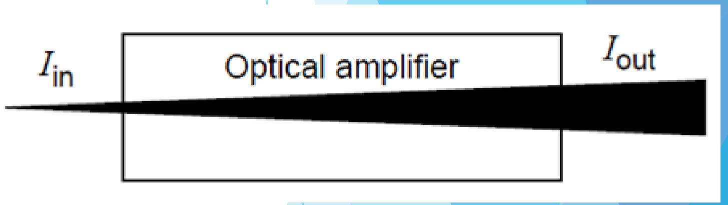
Understanding Optical Amplification







- understand optical amplification → EDFA
- **Erbium doped fiber amplifier**
- The EDFA: consists of a silica optical fiber the core doped with erbium oxide (Er_2O_3)
- light is guided through the optical fiber
- three discrete energy levels of Er^{3+} ion in silica host glass.

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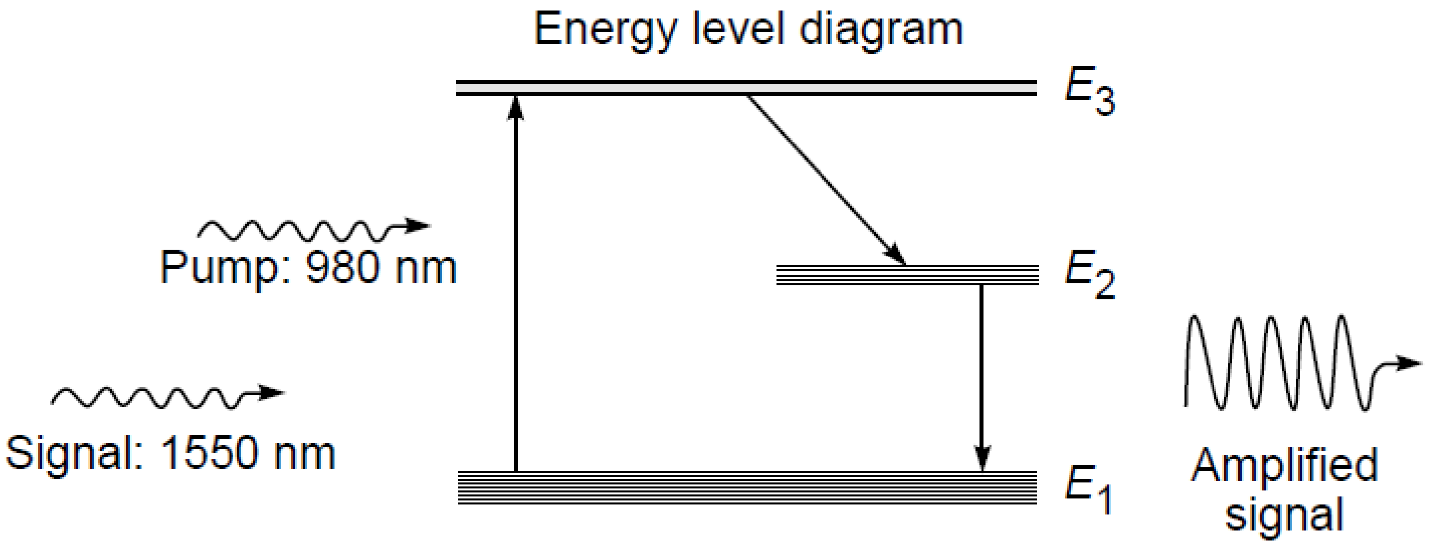
EDFA



Absorption of 980 nm pump

↓

Population inversion between E_2 and E_1

↓

Amplification of signal (at 1550 nm)

Underlying principle of optical amplification

OR

of *light amplification through stimulated emission of radiation*

→ *LASER*

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Thank You