1. Show that the series $\sum_{n=1}^{\infty} \dfrac{1}{n^p} \cos\left(\dfrac{1}{n}\right)$ converges for $p > 1$ and diverges for $0 < p \leq 1$.

2. If $\sum a_n$ converges and $a_n \geq 0$, does $\sum a_n^2$ converge? If yes, prove.

3. If $\sum a_n$ converges and $a_n \geq 0$, does $\sum \sqrt{a_n a_{n+1}}$ converge? If yes, then prove.

4. Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \cdots = 9$.

5. For what values of r, if any, does the infinite series $1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + \cdots =$ converge? Find the sum of the series when it converges.

6. Show by an example that $\sum a_n b_n$ may diverge even if $\sum a_n$ and $\sum b_n$ both converge.

7. Decide whether the following series converge or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\ln n}{n^2}$.

   (b) $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{\ln(\ln n)}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \dfrac{2^n}{3 + 4^n}$.

   (d) $\displaystyle\sum_{n=1}^{\infty} \left[\dfrac{(n+1)^{n+1}}{n+1} - \dfrac{n+1}{n} \right]^{-n}$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n 5^n}{(2n+3)\ln(n+1)}$.

8. Find the radius and interval of convergence of the power series below. For what values of x does the series converge (i) absolutely, (ii) conditionally?

   (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{(x-1)^n}{n^3 3^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \dfrac{x^{n+1}}{\sqrt{n+3}}$