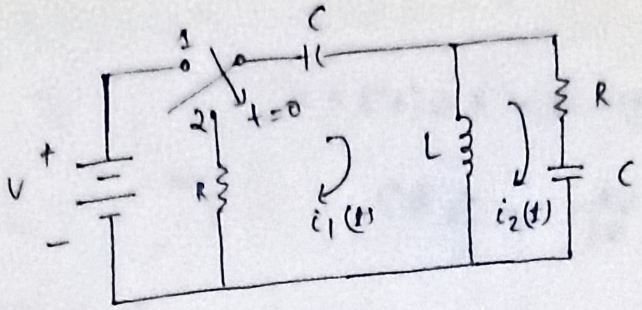
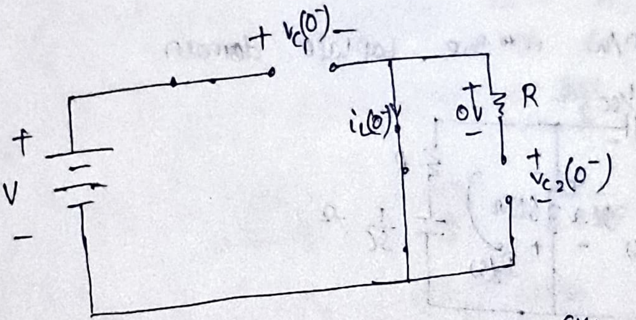
Q11



Det. $i_1(0^+)$

(a) 0 (b) $-\frac{V}{R}$ ~~(c)~~ $-\frac{V}{2R}$ (d) $\frac{V}{2R}$.

(A)



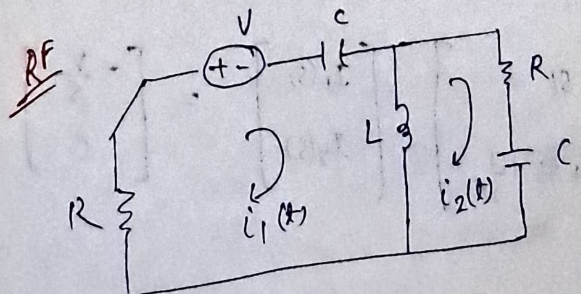
At $t=0^-$: S.S ! A res. CKt.

BY KVL $\Rightarrow$ $V - V_{C_1}(0^-) = 0 \Rightarrow V_{C_1}(0^-) = V = V_C(0^+)$.

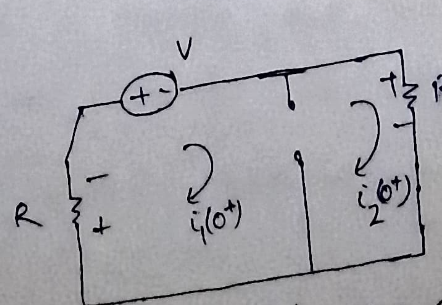
$i_L(0^-) = 0A = i_L(0^+)$

$V_{C_2}(0^-) = 0 = V_{C_2}(0^+)$

RF



for $t \geq 0$:



At $t = 0^+$: Res. CKt ! NW is in TR. state

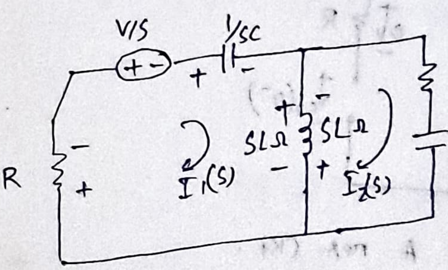
$i_1(0^+) = i_2(0^+)$

BY KVL $\Rightarrow$ $-R \cdot i_1(0^+) - V - R\, i_1(0^+) = 0$

$\Rightarrow i_1(0^+) = \frac{-V}{2R} = i_2(0^+)$.

Q// Let $I_1(s)$ and $I_2(s)$ are the L.T of $I_1(t)$ and $I_2(t)$ respectively then det. the expressions for $I_1(s)$ and $I_2(s)$

(A) Transform the RF n/w into the Laplace domain



$V(s) = Z(s) \cdot I(s)$

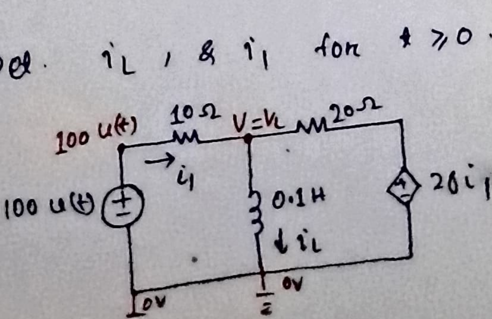
BY KVL in S-D $\Rightarrow$

$-RI_1(S) - \frac{V}{S} - \frac{1}{SC} \cdot I(s) - SL\left(I_1(s) - I_2(s)\right) = 0$

$\Rightarrow -R\, I_2(s) - \frac{1}{SC} \cdot I_2(s) - SL \cdot \left(I_2(s) - I_1(s)\right) = 0$.

$$\begin{bmatrix} R + SL + \frac{1}{SC} & -SL \\ -SL & R + SL + \frac{1}{SC} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\frac{V}{S} \\ 0 \end{bmatrix}$$

Q// Det. i_L, & i_1 for $t \geq 0$.



$$i_L(0^-) = 0A = i_L(0^+)$$

$$i_1 = \frac{100\,u(t) - v_L}{10} \qquad v_L = 0.1\frac{di_L}{dt}$$

$$\Rightarrow i_1 = 10\,u(t) - \frac{1}{100}\frac{di_L}{dt}.$$

NODAL $\quad -i_1 + i_L + \frac{v_L - 20 i_1}{20} = 0$

$$\Rightarrow -2i_1 + i_L + \frac{1}{200}\frac{di_L}{dt} = 0$$

$$\Rightarrow \frac{di_L}{dt} + 40 i_L = 800\,u(t) \longrightarrow \text{A state eqn.}$$

$$s I_L(s) + I_L(0^+) + 40\, i_L(s) = \frac{800}{s}$$

$$\Rightarrow I_L(s) = \frac{800}{s(s+40)} = \frac{20}{s} - \frac{20}{s+40}$$

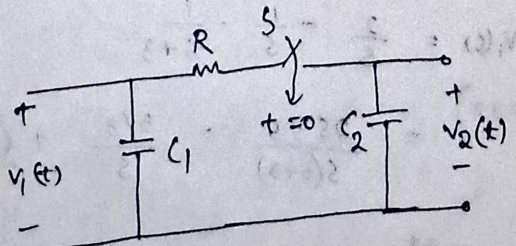
$$\Rightarrow i_L(t) = 20\left(1 - e^{-40t}\right) u(t)$$

$$\tau = \frac{1}{40}\text{ sec.}$$

$$i_1 = 10\,u(t) - \frac{1}{100}\frac{di_L}{dt} = \left(10 - 8e^{-40t}\right) u(t).$$

13.11.10.

Q// Determine the S.S voltages across the capacitors.



$C_1 = 1F$

$C_2 = \frac{1}{2}F$

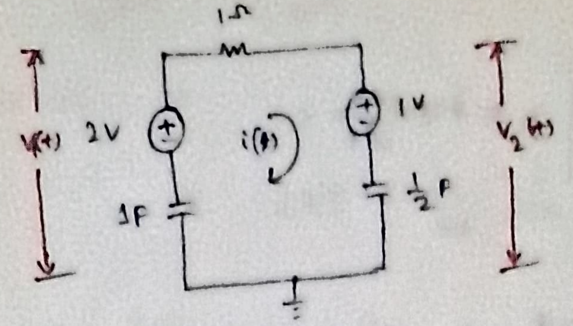
$v_1(0) = 2V$

$v_2(0) = 1V$

$R = 1\Omega$,

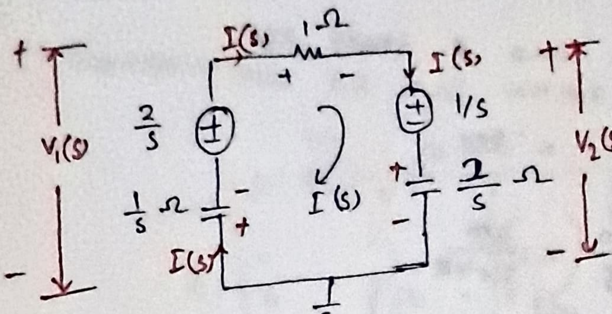
(A) It is a source free RC ckt, but the energy in S.S is non-zero, provided the capacitors are said to be ideal.

$$v_1(\infty) = v_2(\infty) = \frac{v_1 C_1 + v_2 C_2}{C_1 + C_2} \qquad \begin{matrix} v_1 = v_1(0) \\ v_2 = v_2(0) \end{matrix} \qquad = \frac{2 + \frac{1}{2}}{1 + \frac{1}{2}} = \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3}.$$



for $t \geq 0$:

Transform the above ntw into the Laplace domain



(S-D)

$V(s) = Z(s) \cdot I(s)$

By KVL in S-D ⇒

$-\frac{1}{s} \cdot I(s) + \frac{2}{s} - 1 \cdot I(s) - \frac{1}{s} - \frac{2}{s} \cdot I(s) = 0$

$\Rightarrow I(s) = \frac{1}{s+3} \Rightarrow i(t) = e^{-3t}$ A for $0 \leq t \leq \infty$

Check: $i(0^+) = 1$ A.

$i(\infty) = 0$ A

$\tau = \frac{1}{3}$ sec $= R\, C_{eq} = \frac{R\, C_1 C_2}{C_1 + C_2}$

By KVL in S-D ⇒ $v_1(s) - \frac{2}{s} + \frac{1}{s} \cdot I(s) = 0$

$\Rightarrow V_1(s) = \frac{2}{s} - \frac{1}{s} \cdot \frac{1}{s+3}$

$= \frac{2s+5}{s(s+3)} = \frac{5/3}{s} + \frac{\left(\frac{1}{3}\right)}{s+3}$

$v_1(t) = \frac{5}{3} + \frac{1}{3} e^{-3t}$ V for $0 \leq t \leq \infty$.

Check: $v_1(0) = \frac{5}{3} + \frac{1}{3} = 2$V.

$v_1(\infty) = \frac{5}{3}$ V.

Similarly, $V_2(s) - \frac{1}{s} - \frac{2}{s} I(s) = 0$

$$\Rightarrow \quad V_2(s) = \frac{1}{s} + \frac{2}{s} \cdot \frac{1}{s+3}$$

$$= \frac{s+5}{s(s+3)} = \frac{5/3}{s} - \frac{2/3}{s+3}.$$

$$v_2(t) = \frac{5}{3} - \frac{2}{3} e^{-3t} \text{ V} \quad \text{for } 0 \le t \le \infty.$$

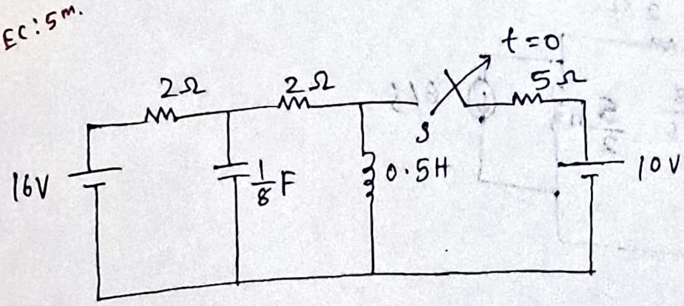
Check!

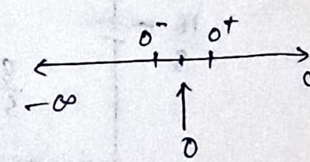
$$v_2(0) = \frac{5}{3} - \frac{2}{3} = 1 \text{ V}.$$

$$v_2(\infty) = \frac{5}{3} \text{ V}.$$

G-2001 : EC : 5m.

Q//





Det. (a) $e_1(0^+)$ (b) $i_L(0^+)$ (c) $e_1(t)$ for $t \geq 0$ (d) $i_L(t)$ for $t \geq 0$.

(A) Evaluation of initial condn :- (must for any procedure)

$i_L(0^+) = 6\text{A}$

$e_1(0^+) = 8\text{V}.$

Evaluation of final condn :- (Required only for short cut)

$i_L(\infty) = 4\text{A}$

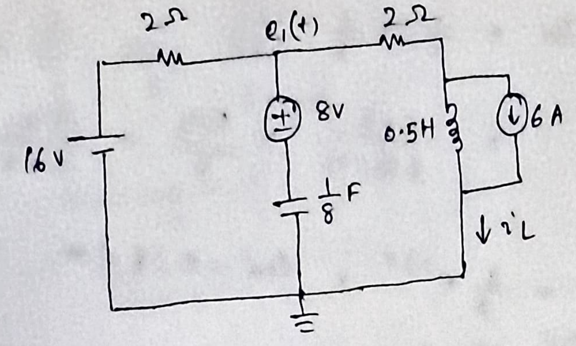
$e_1(\infty) = 8\text{V}$

Evaluation of first derivative initial condn. (Required only for short cut)

$$\left.\frac{di_L(t)}{dt}\right|_{t=0^+} = -8 \frac{\text{A}}{\text{sec}}$$

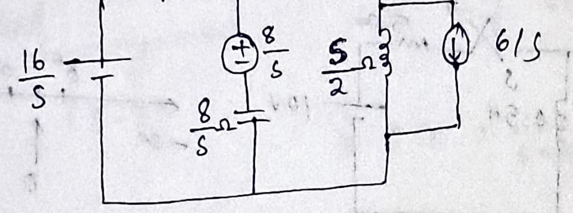
$$\left.\frac{de_1(t)}{dt}\right|_{t=0^+} = -16 \frac{\text{V}}{\text{sec}}$$

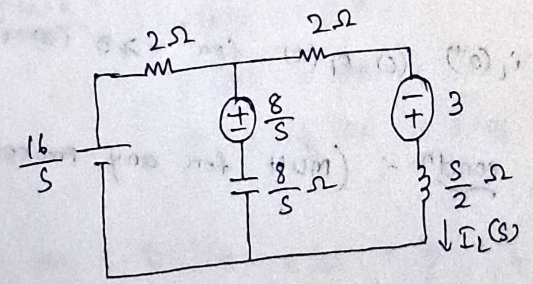
Evaluation of $i_L(t)$ and $e_1(t)$ for $t \geq 0$ by LTA :



(for $t \geq 0$:)

Transform the above n/w into the Laplace domain





(S - D)

Nodal in S-D $\Rightarrow$

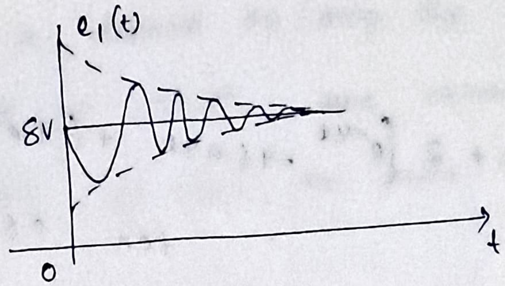
$$\frac{E_1(s) - \frac{16}{s}}{2} + \frac{E_1(s) - \frac{8}{s}}{8/s} + \frac{E_1(s) + 3}{2 + \frac{s}{2}} = 0$$

$$\Rightarrow E_1(s) = \frac{8}{s}\left(\frac{s^2 + 6s + 32}{s^2 + 8s + 32}\right)$$

$$= \frac{8}{s}\left(1 - \frac{2s}{(s+4)^2 + 4^2}\right)$$

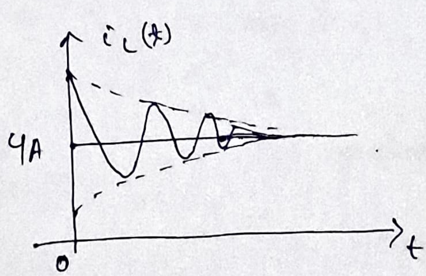
$$= \frac{8}{s} - \frac{4 \cdot 4}{(s+4)^2 + 4^2}$$

$\Rightarrow e_1(t) = 8 - 4e^{-4t} \sin 4t \; V$ for $t \geq 0$.



$$I_L(s) = \frac{E_1(s) + 3}{2 + \frac{s}{2}}$$

$\Rightarrow I_L(t) = 4 + 2e^{-4t} \cos 4t \; A$ for $t \geq 0$.



$\tau = \frac{1}{4}$ sec $= \frac{1}{\xi \omega_n}$ | $\omega_n = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{\frac{1}{2} \cdot \frac{1}{8}}} = 4$ rad/sec.

$\Rightarrow \xi = 1 \Rightarrow$ A critically damped sys / NW.

$e_1(t) = 8 - 4e^{-4t} \sin 4t \; V$ for $t \geq 0$.

$\sin \omega_n t$.

Evaluation of $i_L(t)$ and $e_1(t)$ for $t \geq 0$ directly from choices by applying the short cut Procedure :-

$e_1(t)$: (i) $e_1(0^+) = 8V$.

(ii) $e_1(\infty) = 8V$.

(iii) $\frac{de_1(t)}{dt} = 0 - 4\left[e^{-4t} 4\cos 4t + (-4e^{-4t}).\sin 4t\right]$ V/s for $0 \leq t \leq \infty$

$\Rightarrow \left.\frac{de_1(t)}{dt}\right|_{t=0^+} = -4 \times 4 = -16$ V/s.

$\underline{i_L(t)}$:

(i) $i_L(0^+) = 6\,A$

(ii) $i_L(\infty) = 4A.$

(iii) $\dfrac{di_L(t)}{dt} = 0 + 2\left[e^{-4t} \cdot -4\sin 4t + (-4e^{-4t}) \cdot \cos 4t\right]$ A/s

for $0 \leq t \leq \infty$

$\Rightarrow \left.\dfrac{di_L(t)}{dt}\right|_{t=0^+} = 2 \times -4 = -8\ A/s.$

H.W
Q.//

Repeat the above problem procedure for all the R, L, C ckts of case ②.

The phaser representation :-

It is defined to only for cosinusoidal signals.

All the sinusoidal signals are converted in to the cosinusoids by substracting 90° from the phase.

→ $v(t) = V_m \cos(\omega t + \phi)$

$= \text{Real part}\left[V_m e^{j(\omega t + \phi)}\right]$

$= R.P\left[V_m e^{j\phi} \cdot e^{j\omega t}\right]$

$= R.P\left[V \cdot e^{j\omega t}\right]$

$V = V_m e^{j\phi}$ → The exponential form of the phaser ╲ Div & Mul.

$= V_m \angle\phi$ → The polar " " " " ╱

$= V_m(\cos\phi + j\sin\phi)$ → The rect. form of the phaser → for Add & Sub.

→ $i(t) = I_m \cos(\omega t + \theta)$

$I = I_m \angle\theta$

$= I_m e^{j\theta}$

$= I_m(\cos\theta + j\sin\theta)$

eg. $v(t) = 10\cos(2t + 30°)$

$V = 10\angle 30°$

$= 10 e^{j30}$

$= 10(\cos 30° + j\sin 30°)$

$= 10\left(\frac{\sqrt{3}}{2} + j\,\frac{1}{2}\right)$

eg

$$i(t) = 10 \sin(2t + 30^\circ)$$

$$= 10 \cos(2t + 30^\circ - 90^\circ)$$

$$= 10 \cos(2t - 60^\circ)$$

$$= 10 \angle -60^\circ$$

$$= 10\, e^{-j60^\circ}$$

$$= 10\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

→ $\boxed{\frac{V}{I} = Z}$ Ω $\Rightarrow V = Z \cdot I$

$\Rightarrow I = \frac{V}{Z}$

→ $\boxed{\frac{I}{V} = Y}$ $\mho$ $\Rightarrow I = Y \cdot V$

$\Rightarrow V = \frac{I}{Y}$

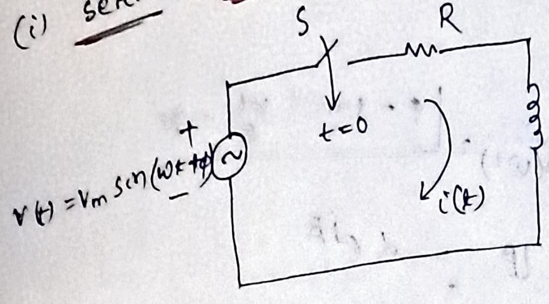
→ $\boxed{Y = \frac{1}{Z}}$ $\mho$

Time domain		Phaser Domain
$i(t)$	⟷	I
$v(t)$	⟷	V
R	⟷	$R\ \Omega$
L	⟷	$j\omega L\ \Omega$
C	⟷	$\frac{1}{j\omega C}\ \Omega$

Note - The analysis of A.C is generally carried in the phaser domain i.e the KCL, KVL, NODAL, MESH, ohm's law and the source transformation are written only in the phaser domain

A.C. Transients.

(i) series R-L ckt :-



$\rightarrow$ C.S = C.F + P.I

= Natural resp. + Forced resp.

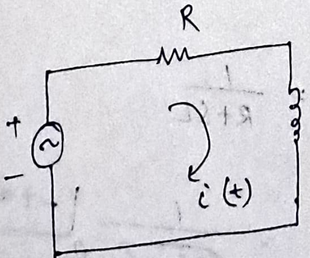
= Resp. upto 5τ + resp. after 5τ

= Trans resp + S.S. resp.

$i(t) = i_{tr}(t) + i_{ss}(t) = Ke^{-t/\tau} + i_{ss}(t)$

$= Ke^{-R/L\, t} + i_{ss}(t)$

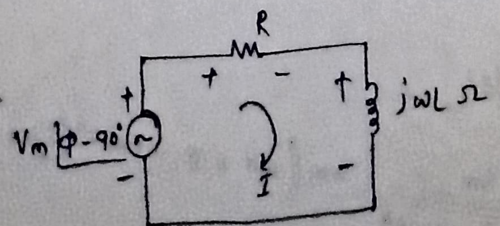
$i_{ss}(t)$



N/w is in S.S.

Method ① : By phasor approach :

Transform the above n/w into the phasor domain



N/w is in P.D

$V = Z \cdot I$

By KVL in P-D $\Rightarrow$

$$V_m \angle \phi - 90^\circ - R \cdot I - j\omega L \cdot I = 0$$

$$\Rightarrow I = \frac{V_m \angle \theta - 90^\circ}{R + j\omega L} = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ$$

$$= \alpha \angle \beta = \alpha e^{j\beta}.$$

$$\rightarrow i(t) = R.P.\left[I \cdot e^{j\omega t}\right] A.$$

$$\Rightarrow i(t) = \alpha \cos(\omega t + \beta)$$

$$= \alpha \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ\right)$$

$$= \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) = i_{ss}(t)$$

Method ② : BY LTA

$$v(t) = V_m \sin(\omega t + \phi)$$

Desired resp. L.T

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{R + sL}$$

Excitation L.T.

$$\Rightarrow H(j\omega) = \frac{1}{R + j\omega L} = \frac{1}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\rightarrow I(s) = H(s) \cdot V(s).$$

$$\Rightarrow i(t) = \frac{1}{\sqrt{R^2 + (\omega L)^2}} \cdot V_m \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) A = i_{ss}(t)$$

$$\rightarrow i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= K e^{-R/L\, t} + \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$i(0^-) = 0A = i(0^+)$$

$$\Rightarrow K = \frac{-V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right) \quad << 1$$

$$V_L(t) = L\frac{di(t)}{dt} \quad \Big| \text{ By ohm's law.}$$

→ suppose $\phi - \tan^{-1}\frac{\omega L}{R} = 0 \Rightarrow K = 0 \Rightarrow i_{tr}(t) = 0$

$\Rightarrow i(t) = i_{ss}(t)$ A tr. free resp.

So, the condition for the transient free response at $t=0$ is

$$\boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$$

i.e. $(\omega t + \phi)\Big|_{t=0} = \tan^{-1}\frac{\omega L}{R}$

Note-! i.e if the total phase of the excitation at the time of switching is equal to $\tan^{-1}\frac{\omega L}{R}$ then no transients will result in the system at the time of switching for the sinusoidal excitation.

→ If the switch is closed at $t = t_0$ then the condn for the transient free response at $t = t_0$ is

$$(\omega t + \phi)\Big|_{t=t_0} = \tan^{-1}\frac{\omega L}{R}$$

$$\Rightarrow \boxed{\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R}}$$

→ If the excitation is $V(t) = V_m \cos(\omega t + \phi)$ then sine is replaced by cosine in the steady state response and hence K is a function of cosine

→ suppose $\phi - \tan^{-1}\frac{\omega L}{R} = \frac{\pi}{2}$ ⇒ $K = 0$ ⇒ $i_{tr}(t) = 0$

→ $i(t) = i_{ss}(t)$ — transient free resp.

* $\boxed{\phi = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2}}$ → at $t = 0$

* $\boxed{\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2}}$ → at $t = t_0$

→ In the above cases, if L is replaced by 'C' then

$\tau = L/R$ is Replaced by $\tau = RC$.

→ The above condⁿ are the same even if the elements are connected in parallel, with current excitation i.e. for the parallel R-L and R-C ckts.

Note. → The transient free condⁿ is not possible for the n/ws with both the energy storing elements i.e. for the RLC n/wk.

Reason →

$S_1, S_2 = \alpha \pm j\beta$.

→ $i(t) = i_{tr}(t) + i_{ss}(t)$

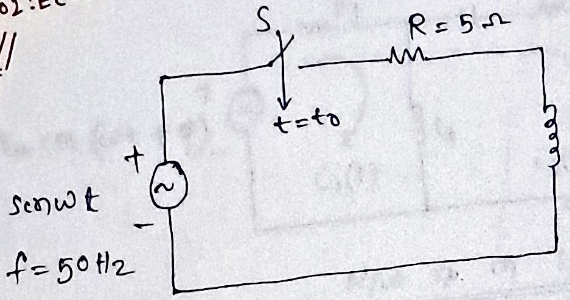
$= e^{\alpha t}(K_1 \cos\beta t + K_2 \sin\beta t) + i_{ss}(t)$

By applying the initial condⁿ $i_L(0^+)$ and $V_C(0^+)$, K_1 is a funcⁿ of sine and K_2 is a funcⁿ of cosine (vice versa), hence no time will satisfy both K_1 and K_2 simultaneously to zero, so the transient term is always present in the complete response

Obs : So, the transient free time (t_0) value in R-L and R-C ckts with a.c excitation will depends on the source freq. (ω), its initial phase (π), on the ckt constants (R, L, C values) and the nature of the excitation (sine or cosine), but not on the max. value of the excitation. (V_m or I_m)

G-2002:EC:2M

Q//



The value of t_0 which result a transient free response.

(A) $\omega t \big|_{t=t_0} = \tan^{-1} \frac{\omega L}{R}$

$\Rightarrow \omega t_0 = \tan^{-1} \frac{\omega L}{R}$

$\Rightarrow 2\pi \times 50 \times t_0 = \tan^{-1} \left(\frac{2\pi \times 50 \times 0.01}{5} \right)$

$= 32.14^\circ \times \frac{\pi}{180}$

$\Rightarrow t_0 = 1.78\,ms.$

Q// In the above case if the excitation is $\cos \omega t$ then,

$\omega t \big|_{t=t_0} = \tan^{-1} \frac{\omega L}{R} + \pi/2$

$\Rightarrow \omega t_0 = \tan^{-1} \frac{\omega L}{R} + \frac{\pi}{2}$

$\Rightarrow 2\pi \times 50 \times t_0 = \tan^{-1} \left(\frac{2\pi \times 50 \times 0.01}{5} \right) + \frac{\pi}{2}$

$= 32.14^\circ \times \frac{\pi}{180} + \frac{\pi}{2}$

$\Rightarrow t_0 = 6.78\,ms.$

Q// In the above case if the excitation is $\sin(\omega t - 10°)$

(A) $$\omega t - 10 \Big|_{t=t_0} = \tan^{-1}\frac{\omega L}{R}$$

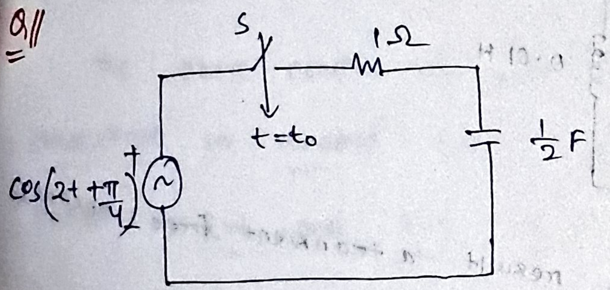
$$\Rightarrow \omega t_0 = \tan^{-1}\frac{\omega L}{R} + 10°$$

$$\rightarrow 2\pi \times 50 \times t_0 = \tan^{-1}\left(\frac{2\pi \times 50 \times 0.01}{5}\right) + 10°$$

$$= 42.14° \times \frac{\pi}{180°}$$

$$\Rightarrow t_0 = 2.34\ ms.$$

Q//



(A) $$\omega t_0 + \phi = \tan^{-1}\omega CR + \frac{\pi}{2}$$

$$\Rightarrow 2t_0 + \frac{\pi}{4} = \tan^{-1}\left(2 \cdot \tfrac{1}{2} \cdot 1\right) + \frac{\pi}{2}$$

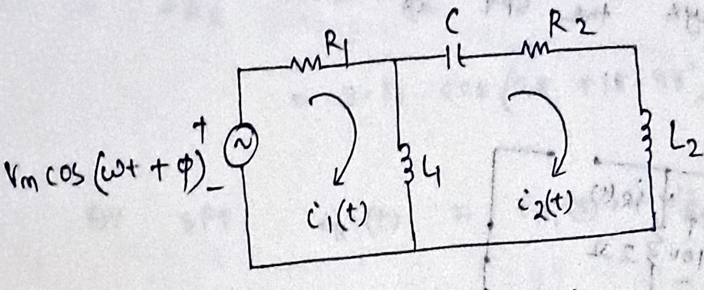
$$\Rightarrow 2t_0 + \frac{\pi}{4} = \frac{\pi}{4} + \frac{\pi}{2}$$

$$\rightarrow 2t_0 = \frac{\pi}{2} \Rightarrow t_0 = \frac{\pi}{4} = \frac{3.14}{4} = 0.785\ sec.$$

IV/ A.C. Analysis.

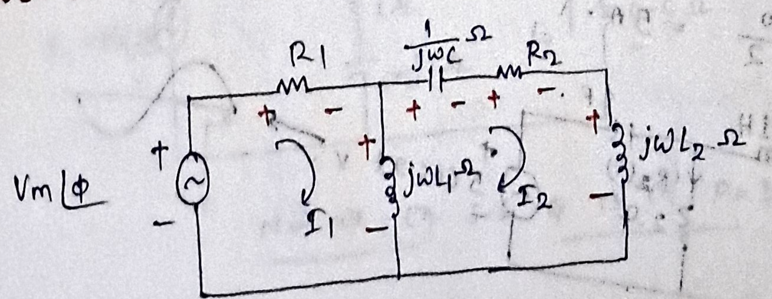
Sinusoidal steady state Analysis →

Steady state indicates the absence of transients, it is achieved practically after '5 τ' constants of the switching action and the analysis in steady state is generally carried out by using phasors.



N/w is in S.S.

Transform the above n/w into the phasor domain →



N/w is in P.D.

$V = Z \cdot I.$

By KVL in P-D ⇒

$$V_m \angle\phi - R_1 \cdot I_1 - j\omega L_1 (I_1 - I_2) = 0$$

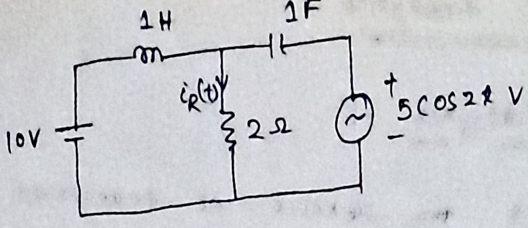
$$-\frac{1}{j\omega C} \cdot I_2 - R_2 \cdot I_2 - j\omega L_2 \cdot I_2 - j\omega L_1 \cdot (I_2 - I_1) = 0$$

→ $I_1 = \frac{\Delta_1}{\Delta}$ | $i_1(t) = R.P\left[I_1 \cdot e^{j\omega t}\right]$ A.

$I_2 = \frac{\Delta_2}{\Delta}$ | $i_2(t) = R.P\left[I_2 \cdot e^{j\omega t}\right]$ A.

-EE - 5m

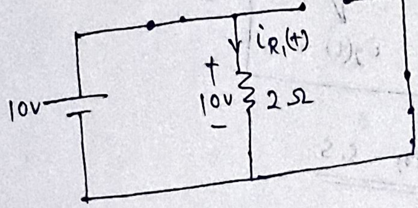
Q4



Det. the current $i_R(t)$, provided the n/w is in the S.S

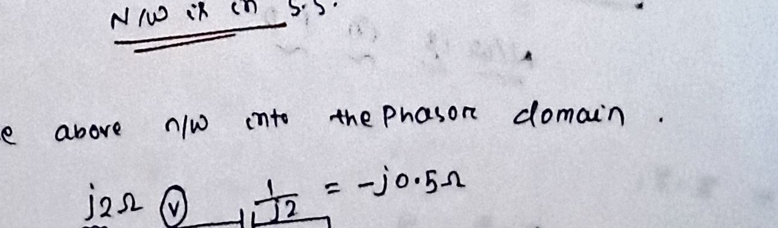
(A)

Note: Since two diff. frequencies are operating on the n/w simultaneously, always the SPT is used to evaluate the responses.



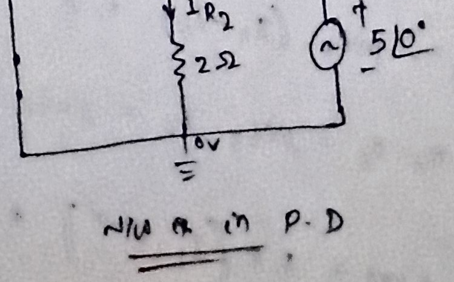
N/w is in S.S : A res. ckt.

$$i_{R_1}(t) = \frac{10}{2} = 5\,A.$$



N/w is in S.S.

Transform the above n/w into the phasor domain.



N/w is in P.D

NODAL in P-D ⇒

$$\frac{V}{j2} + \frac{V}{2} + \frac{V - 5\angle 0^\circ}{-j0.5} = 0$$

$$\Rightarrow \quad V = 6.32 \angle 18.44^\circ$$

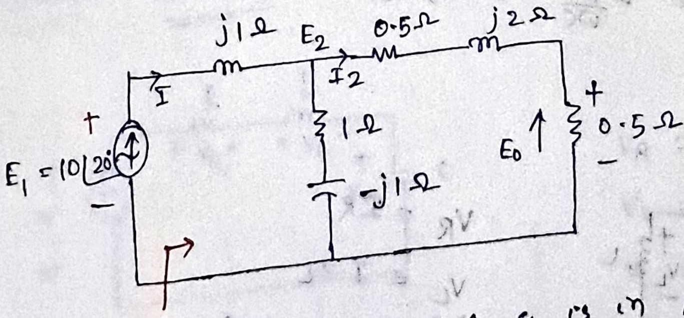
$$I_{R_2} = \frac{V}{2} = 3.16 \angle 18.44^\circ = 3.16\, e^{j18.44^\circ}.$$

$$i_{R_2} = R.P\left[I_{R_2} \cdot e^{j2t}\right] A$$

$$= 3.16 \cos(2t + 18.44^\circ)\ A.$$

BY SPT $\quad i_R(t) = i_{R_1}(t) + i_{R_2}(t)$.

Q11



NW is in S.S & is in P-D

Det. I_1, I_2, E_0, E_2.

$$Z = j1 + (1 - j1) \parallel (1 + j2)$$

$$=$$

$$I = \frac{E_1}{Z} \quad \Big| \text{ By ohm's Law.}$$

$$I_1 = \frac{I \cdot (1 + j2)}{1 - j1 + 1 + j2}$$

$$I_2 = \frac{I \cdot (1 - j1)}{1 - j1 + 1 + j2}$$

$E_2 = (1 - j1) \cdot I_1$

$E_0 = 0.5\, I_2$ | By ohm's law.

The Phasor diagrams. →

→ Phasor diagram is a pictorial respresentation of all the phase voltages and their respective currents in a n/w.

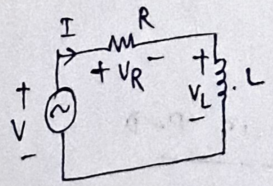
** → The Phasor is a rotating vector, which rotates in the anticlockwise direction with angular frequency 'ω' in the time domain.

→ $Z_R = R\,\Omega = R\angle 0^\circ\,\Omega$

$Z_L = j\omega L\,\Omega = jX_L\,\Omega = X_L\angle 90^\circ\,\Omega$ | where $X_L = \omega L$.

$Z_C = \frac{1}{j\omega C}\,\Omega = -\frac{j}{\omega C} = -jX_C = X_C\angle -90^\circ\,\Omega$ | where $X_C = \frac{1}{\omega C}$

① Series RL-ckt :

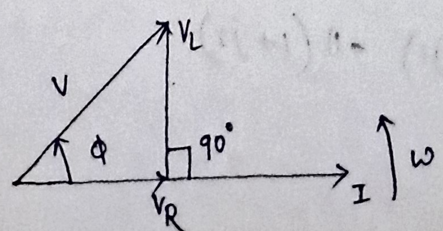


$V_R = IR$

$V_L = I Z_L$

$= I X_L \angle 90^\circ$

Now i in S.S

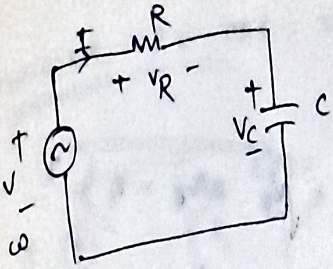


$V = \sqrt{V_R^2 + V_L^2}$

$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$ = imp. angle = Adm. angle.

$\cos\phi$ = P.f = $\frac{V_R}{V}$ (lag)

(2)

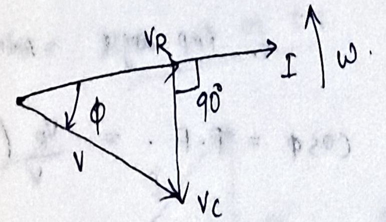


Nw ckt in S.S

$V_R = IR$

$V_C = I Z_C$

$= I X_C \angle -90^\circ$



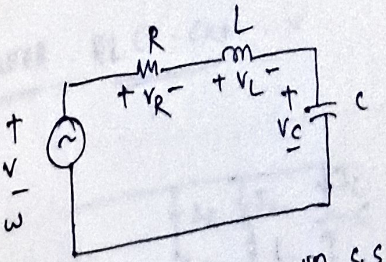
$V = \sqrt{V_R^2 + V_C^2}.$

$\phi = \tan^{-1} \frac{V_C}{V_R}$ = imp. angle.

= Adm. angle.

$\cos\phi$ = p.f $= \left(\frac{V_R}{V}\right)$ lead.

(3) Series RLC - ckt.



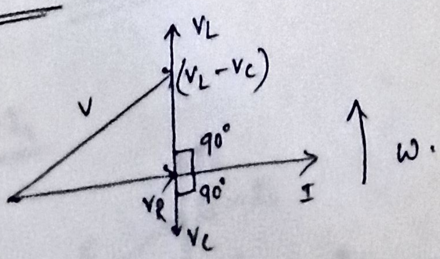
Nw ckt in S.S.

$V_R = IR$

$V_L = I X_L \angle 90^\circ$

$V_C = I X_C \angle -90^\circ$

(i) $V_L > V_C$

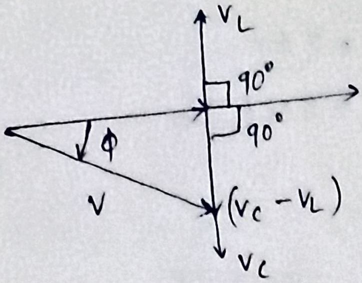


$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$ = Impedance angle = Adm. angle.

$\cos\phi$ = P.F = $\frac{V_R}{V}$ (Lag ~~lead~~)

(ii) $V_C > V_L$.



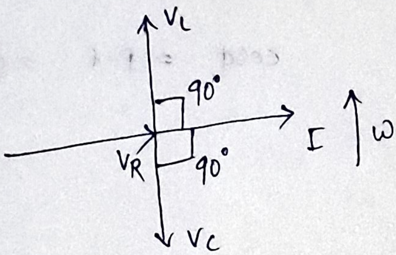
$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$$

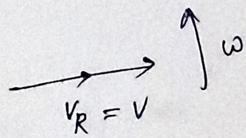
= imp angle = Adm. angle

$$\cos\phi = P.F. = \frac{V_R}{V} \text{ (lead)}$$

(iii) $V_L = V_C$

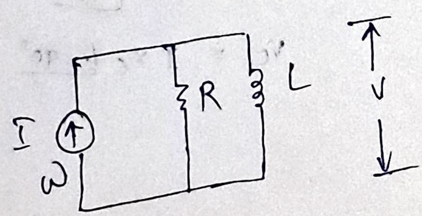


$\equiv$

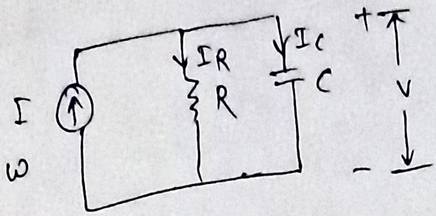


$\phi = 0° \Rightarrow$ UPF.

(4) Parallel RL - ckt:



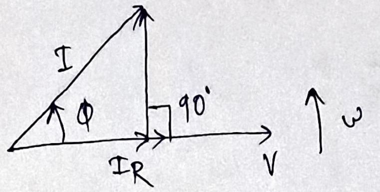
(5) Parallel RC - ckt. →



$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{Z_C} = \frac{V}{X_C} \angle 90^\circ$$

Now it is in S.S.

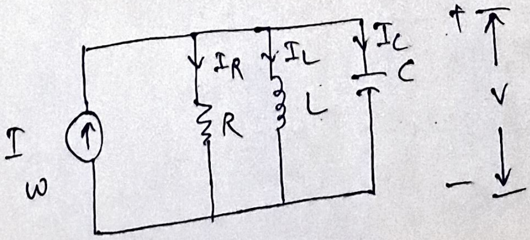


$$I = \sqrt{I_R^2 + I_C^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right) = \text{imp. angle} = \text{Adm. angle}$$

$$\cos\phi = \text{P.f.} = \left(\frac{I_R}{I}\right) \text{ (lead)}$$

(6) Parallel RLC - ckt. →



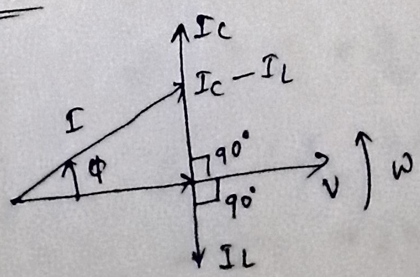
$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

Now it is in S.S.

(i) $I_C > I_L$

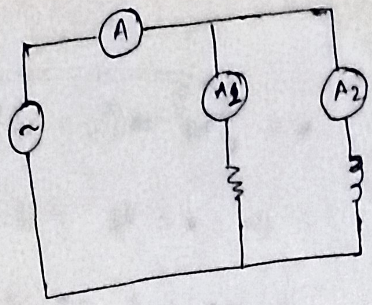


$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right) = \text{imp. angle} = \text{Adm. angle}$$

$$\cos\phi = \text{P.f} = \frac{I_R}{I} \text{ (lead)}$$

Q//



A_1 leads 8 A

A_2 leads 6 A

Then A leads.

<u>Now ckt in S.S.</u>

By KCL in P-D ⇒

$$I = I_R + I_L = \frac{V}{Z_R} + \frac{V}{Z_L}$$

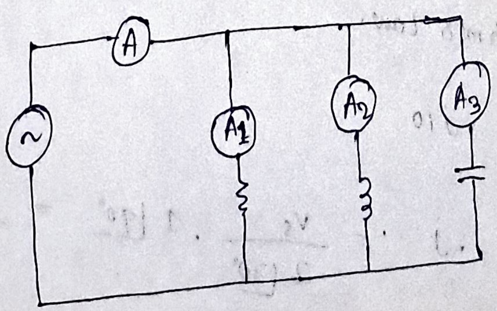
$$= \frac{V}{R} - j\frac{V}{X_L}$$

$$= (8 - j6)\,A.$$

$$I = \sqrt{8^2 + 6^2} = 10A$$

$$\text{p.f} = \frac{I_R}{I} = \frac{8}{10} = 0.8 \text{ (lag)}.$$

Q//



A_1 leads 6 A

A_2 leads 12 A

A_3 leads 20 A

Then A leads.

By KCL in P-D ⇒

$$I = I_R + I_L + I_C$$

$$= \frac{V}{Z_R} + \frac{V}{Z_L} + \frac{V}{Z_C}$$

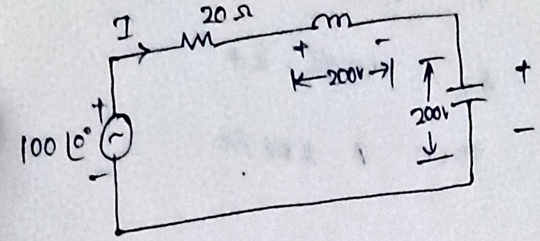
$$= \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C}$$

$$= 6 - j12 + j20$$

$$= (6 + j8)\,A.$$

$$I = \sqrt{6^2 + 8^2} = 10A \qquad \text{p.f} = \frac{I_R}{I} = \frac{6}{10} = 0.6 \text{ (lead)}$$

Q) Det I.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

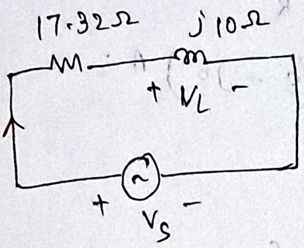
$$\Rightarrow V = V_R = I \cdot R$$

$$\Rightarrow 100 = I \cdot 20 \Rightarrow I = 5A.$$

Now it in S.S.

$$\therefore p.f = \frac{V_R}{V} = \frac{V_R}{V_R} = 1.$$

Q) The angle betn V_L & V_S is ________



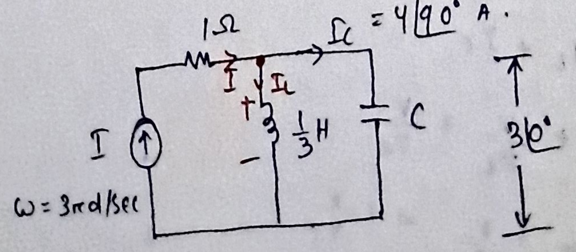
Now it in S.S.

(A). $V_L = I \cdot Z_L$ | By Ohm's Law.

$$= \frac{V_S}{10\sqrt{3} + j10} \cdot j10$$

$$= \frac{V_S}{\sqrt{3} + j} \cdot j = \frac{V_S}{2\angle 30^\circ} \cdot 1\angle 90^\circ = \frac{V_S}{2}\angle 60^\circ.$$

Q) Det I.



Now it in S.S

$$I_L = \frac{V}{Z_L} = \frac{V}{j\omega L} = \frac{3\angle 0^\circ}{j3\frac{1}{3}} = \frac{3\angle 0^\circ}{1\angle 90^\circ} = 3\angle -90^\circ \text{ A}$$

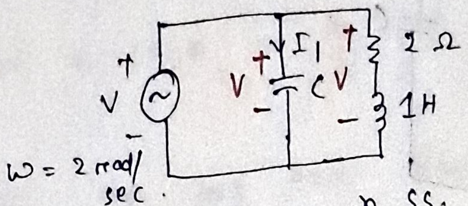
By KCL in P-D $\Rightarrow$ $I = I_L + I_C$

$= 3\angle -90^\circ + 4\angle 90^\circ$

$= -j3 + j4.$

$= j1 = 1\angle 90^\circ$ A

Q// The phasor I_1 leads I_2 by ______

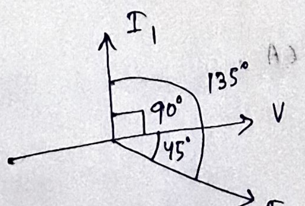


Nω is in S.S.

Soln

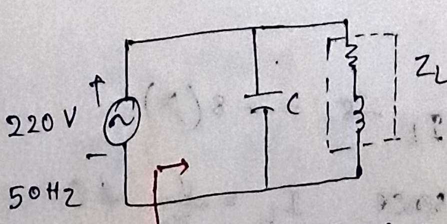
$I_1 = I_C = \frac{V}{Z_C} = \frac{V}{X_C}\angle 90^\circ$

$I_2 = \frac{V}{2 + j\omega L} = \frac{V}{2 + j2} = \frac{V}{2\sqrt{2}}\angle -45^\circ$



$\uparrow$ ω = 2 rad/sec.

Q//



$Z_L = 30\angle 40^\circ\ \Omega$

y Nω is in S.S

what value of 'C' will result a unity p.f at the a.c source

(A)

$$y = \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$= \frac{1}{\left(\frac{1}{j\omega C}\right)} + \frac{1}{30\angle 40^\circ}$$

$$= j\omega C + \frac{1}{30}\angle -40^\circ$$

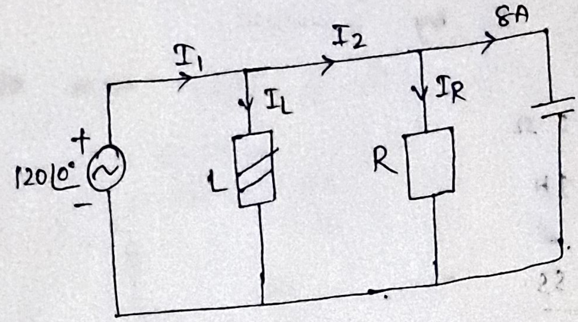
$$Y = j\omega C + \frac{1}{30}\left(\cos 40^\circ - j\sin 40^\circ\right)$$

j- term = 0 ⇒

$$\Rightarrow \omega C = \frac{\sin 40^\circ}{30}$$

$$\Rightarrow C = \frac{\sin 40^\circ}{2\pi \times 50 \times 30} = 68.1\,\mu F$$

Q4



Now is on S.S

If $|I_1| = |I_2| = 10A$.

then I_L & I_R are

$$I_2 = \sqrt{I_R^2 + I_C^2}$$

$$\Rightarrow 10 = \sqrt{I_R^2 + 8^2} \quad = I_R = 6A$$

$$I = I_1 = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$10 = \sqrt{6^2 + (I_L - 8)^2}$$

$$\Rightarrow (I_L - 8)^2 = 64$$

$$\Rightarrow I_L - 8 = \pm 8$$

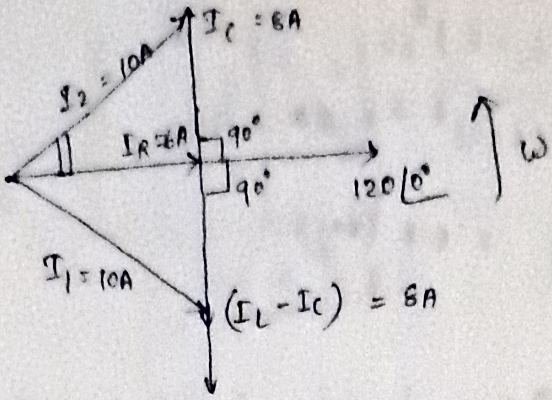
$I_L - 8 = -8$ (✗)

since $I_L = \frac{V}{Z_L} \neq 0$

so, $I_L - 8 = 8$

$\Rightarrow I_L = 16A$,

$I_L > I_C$

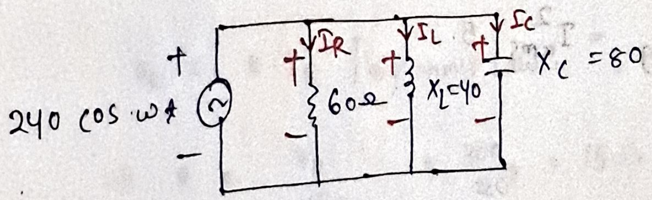


→ I_2 leads $120\angle 0°$ by $\tan^{-1}\frac{8}{6}$

→ I_1 lags " " "

→ $p.f = \cos\phi = \frac{I_R}{I} = \frac{I_R}{I} = \frac{6}{10} = 0.6$ (lag)

Q12 Det. the avg. power dissipated and the p.f of the ckt.



NW in S.S.

$V_R(t) = 240 \cos \omega t$ = A periodic voltage.

$\Rightarrow P_{avg} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{240}{\sqrt{2}}\right)^2}{60} = 480\ W$

$V = 240\angle 0°$

$|I_R| = \left|\frac{V}{R}\right| = \frac{240}{60} = 4A.$

$|I_L| = \left|\frac{V}{Z_L}\right| = \frac{V}{X_L} = \frac{240}{40} = 6A.$

$|I_C| = \left|\frac{V}{Z_C}\right| = \frac{V}{X_C} = \frac{240}{80} = 3A$

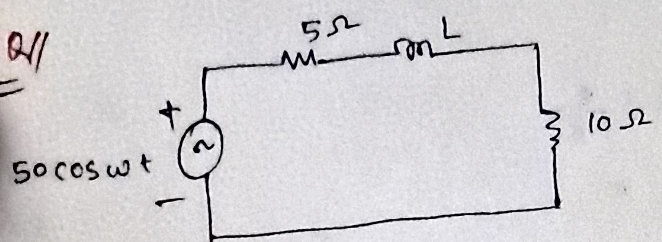
Since $I_L > I_C \Rightarrow$ inductive nature of the ckt.

$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$= \sqrt{4^2 + (6-3)^2} = 5\,A$$

$$P.f = \frac{I_R}{I} = \frac{4}{5} = 0.8 \text{ (lag)}$$

Q11



Now it in S.S.

If the power dissipated in 5Ω resistor is 10 watt, then the p.f. of the ckt.

(A)

$$P_{5\Omega} = 10W = P_{avg} = I_{rms}^2 \cdot 5$$

$$\Rightarrow I_{rms} = \sqrt{2}\,A.$$

$$P_{del} = P_{abs}.$$

$$\Rightarrow V_{rms} \cdot I_{rms} \cos\phi = I_{rms}^2 (5 + 10)$$

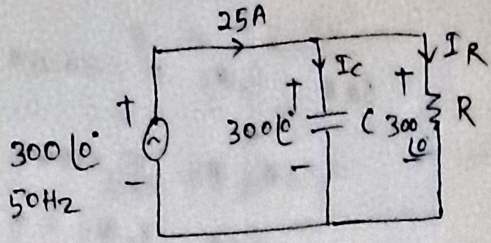
$$\Rightarrow \frac{50}{\sqrt{2}} \cdot \sqrt{2} \cdot \cos\phi = (\sqrt{2})^2 \cdot 15.$$

$$\Rightarrow p.f = \cos\phi = \frac{30}{50} = 0.6 \text{ (lag)}$$

~~Q11~~

A 159.23 μF capacitor is in parallel with a resistor 'R'. ~~draws~~ draws a current of 25 A from a 300 V, 50 Hz supply. Det. the freq. ~~of~~ at which the ckt draws the same 25 A from a 360 V supply.

(A)



Now is in S.S

$$|I_C| = \left|\frac{V}{Z_C}\right| = \left|\frac{V}{\frac{1}{j\omega C}}\right| = V\omega C.$$

$$= 300 \times 2\pi \times 50 \times 159.23 \times 10^{-6}.$$

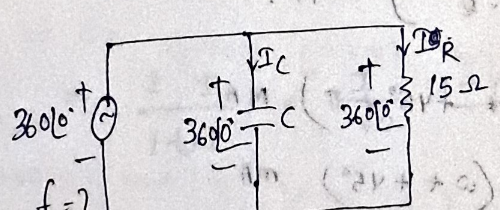
$$= 15A.$$

$$\Rightarrow \quad I_R = \sqrt{I^2 - I_C^2}$$

$$= \sqrt{25^2 - 15^2} = 20A!$$

$V_R = R \cdot I_R$ | By ohm's Law

$$\rightarrow R = \frac{V_R}{I_R} = \frac{300}{20} = 15\Omega$$



The new $I_R' = \frac{V_R}{R} = \frac{360}{R} = \frac{360}{15} = 24A.$

So, the required $I_C = \sqrt{25^2 - 24^2}$

$$\Rightarrow V\omega C = 7$$

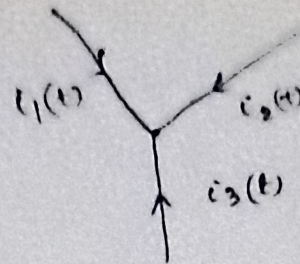
$$\Rightarrow 360 \times 2\pi \times f \times 159.23 \times 10^{-6} = 7$$

$$\Rightarrow f = 19.4\,Hz.$$

Obs: $I_C = \frac{V}{Z_C}$ $\quad Z_C = \frac{1}{j\omega C}\ \Omega$

$$f \downarrow \Rightarrow Z_C \uparrow \Rightarrow I_C \downarrow$$

Q.



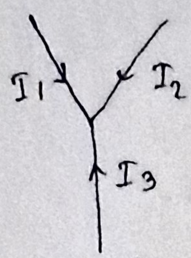
let $i_1(t) = -\sin\omega t$ mA

&

$i_2(t) = \cos\omega t$ mA

then $i_3(t) = ?$

(A)



P-D

By KCL $\Rightarrow$

$-I_1 - I_2 - I_3 = 0$

$\Rightarrow I_3 = -(I_1 + I_2)$

$i_1(t) = \cos(\omega t + 90^\circ)$ mA

$I_1 = 1\angle 90^\circ = j1$

$I_2 = 1\angle 0^\circ = 1 + j0$

$I_3 = -1 - j1 = -a - jb = \sqrt{a^2+b^2}\ \angle \pi + \tan^{-1} b/a$

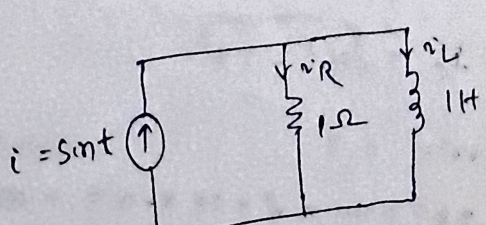
$= \sqrt{2}\ \angle \pi + 45^\circ = \sqrt{2}\ e^{j(\pi + 45^\circ)}$

$i_3(t) = R.P\left[I_3 \cdot e^{j\omega t}\right]$ mA.

$= \sqrt{2}\cos(\omega t + 45^\circ + \pi)$ mA

$= -\sqrt{2}\cos(\omega t + 45^\circ)$ mA.

Q. Det. the steady state current i_R & i_L.



Now in S.S

$$I = 1\angle{-90^\circ}$$

$$Z_R = 1\,\Omega$$

$$Z_L = j\omega L\,\Omega = j1\,\Omega.$$

$$I_R = \frac{I \cdot j1}{1+j1} = \frac{I \cdot 1\angle{90^\circ}}{\sqrt{2}\angle{45^\circ}} = \frac{I}{\sqrt{2}}\angle{45^\circ}$$

$$\Rightarrow i_R = \frac{1}{\sqrt{2}} \cdot 1 \cdot \sin(t + 45^\circ)\ A.$$

Another method :-

By inverse phaser approach :- $I_R = \frac{I \times j1}{1+j1}$

$$I_R = \frac{I}{\sqrt{2}}\angle{45^\circ} = \frac{1\angle{-90^\circ}}{\sqrt{2}}\angle{45^\circ} = \frac{1}{\sqrt{2}}\angle{45^\circ - 90^\circ}$$

$$= \frac{1}{\sqrt{2}} e^{j(45^\circ - 90^\circ)}$$

$$\Rightarrow i_R = R.P\left[I_R \cdot e^{j1.t}\right]$$

$$\Rightarrow i_R = \frac{1}{\sqrt{2}} \cos(t + 45^\circ - 90^\circ)$$

$$= \frac{1}{\sqrt{2}} \sin(t + 45^\circ)\ A$$

$$I_L = \frac{I \cdot 1}{1+j1} = \frac{I}{\sqrt{2}}\angle{-45^\circ}$$

$$\Rightarrow i_L = \frac{1}{\sqrt{2}} \cdot 1 \cdot \sin(t - 45^\circ)\ A$$

Check:

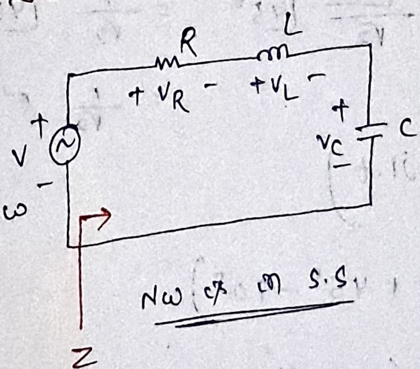
BY KCL $\Rightarrow$ $i_R + i_L = \sin t = i$ ✓

The Resonance.

Resonance in the electric ckts is because of the presence of both the energy storing elements called the inductor and the capacitor. At a fixed freq. called 'wo', the 'L' & 'C' element will exchange the energy freely as a func^n of time, which results the sinusoidal oscillations either across inductor or across capacitor.

Series RLC ckt → Series Resonance.

① The series RLC ckt



Nw is in S.S

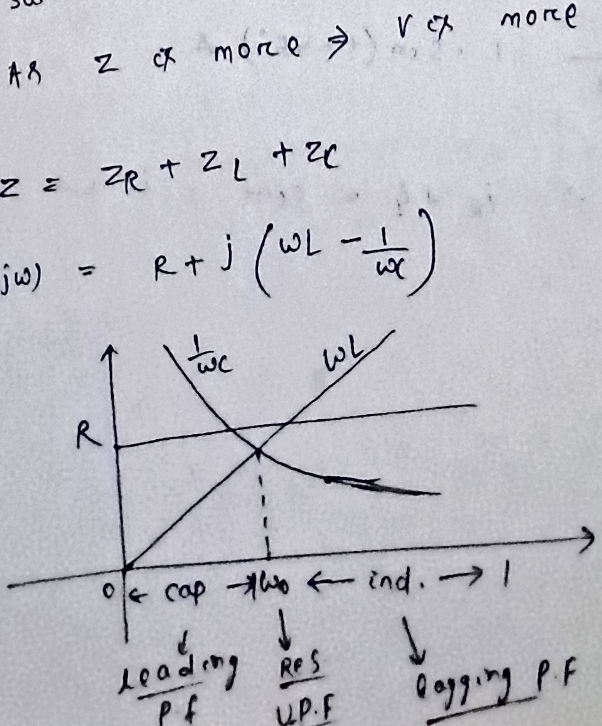
Important → $V = Z \cdot I$

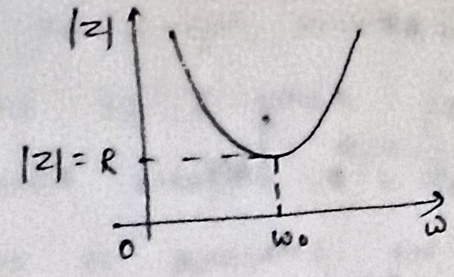
in series the current flowing through all the elements are the same.

As Z is more ⇒ V is more ⇒ that nature of the ckt

→ $Z = Z_R + Z_L + Z_C$

$$Z(j\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$





$\rightarrow \omega = \omega_0 \Rightarrow \frac{1}{\omega_0 C} = \omega_0 L$

$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec.

$\Rightarrow Z(j\omega_0) = R.$

Since $I = \frac{V}{Z}$.

$\rightarrow \boxed{I = \frac{V}{R}}$ = The response at resonance.

$\rightarrow$ $\underline{\underline{\omega = \omega_0}}$

$V_R = IR = \frac{V}{R} \cdot R \Rightarrow V_R = V$

$V_L = I \cdot Z_L = \frac{V}{R} \cdot j\omega_0 L \Rightarrow \boxed{V_L = QV \angle 90^\circ}$

where $\boxed{Q = \frac{\omega_0 L}{R}}$.

$V_C = I \cdot Z_C = \frac{V}{R} \cdot \frac{1}{j\omega_0 C} \Rightarrow \boxed{V_C = QV \angle -90^\circ}$

Where $\boxed{Q = \frac{1}{\omega_0 CR}}$

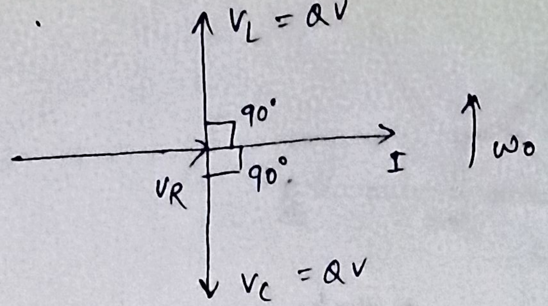
Check :

By KVL $\Rightarrow V_R + V_L + V_C = V + jQV - jQV$

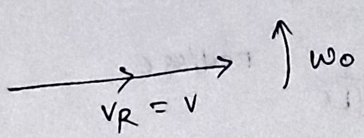
$= V.$

Obs! Here $\underline{V_L + V_C = 0} \Rightarrow$ LC - combination is like a $\underline{S.C.}$

The Phasor dia.



$\equiv$



$\Rightarrow \phi = 0° \quad \rightarrow$ UPf.

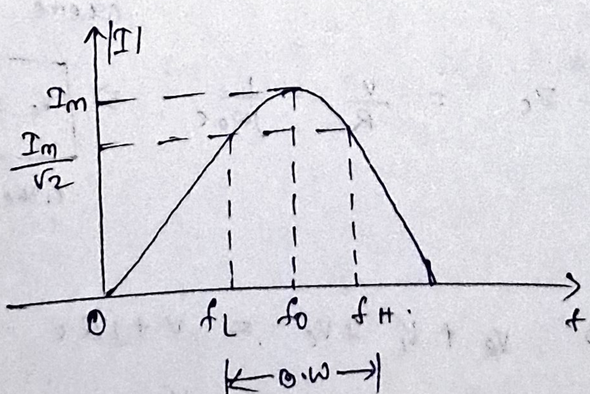
→ The freq. resp.

$$|I| = \left|\frac{V}{Z}\right| = \frac{V}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$\omega = 0 \quad \Rightarrow |I| = 0$

$\omega = \omega_0 \quad \Rightarrow |I| = \frac{V}{R} = I_m$.

$\omega = \infty \quad \Rightarrow |I| = 0$.



$$\boxed{B.W = f_H - f_L = f_0/Q}$$ where $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$

$$\boxed{\sqrt{f_H \cdot f_L} = f_0}$$

→ At resonance the impedance reaches a minimum value, and hence the ckt acts as max. current $\left(I = \frac{V}{Z}\right)$, so called ACCEPTOR ckt.

→ Since the voltages across 'L' & 'C' elements are magnified by Q. times. (This amplification is due to the stored energies in L & C elements), the series RLC ckt at Resonance are called as the voltage magnification ckts.

The oscillations phenomenon →

Let $v(t) = V_m \cos \omega_0 t$

$\omega = \omega_0 \Rightarrow Z = R$

$\Rightarrow i(t) = \frac{V_m}{R} \cos \omega_0 t = i_R = i_L = i_C$

→ $V_C = \frac{1}{C} \int i_C \cdot dt$

$= \frac{V_m}{R\omega_0 C} \sin \omega_0 t$

→ $P_{avg} = I_{rms}^2 \cdot R$

$= \left(\frac{V_m}{\frac{R}{\sqrt{2}}}\right)^2 \cdot R = \frac{V_m^2}{2R}$ (W)

→ The total energy stored in the ckt at resonance is

$w(t) = w_L(t) + w_C(t)$

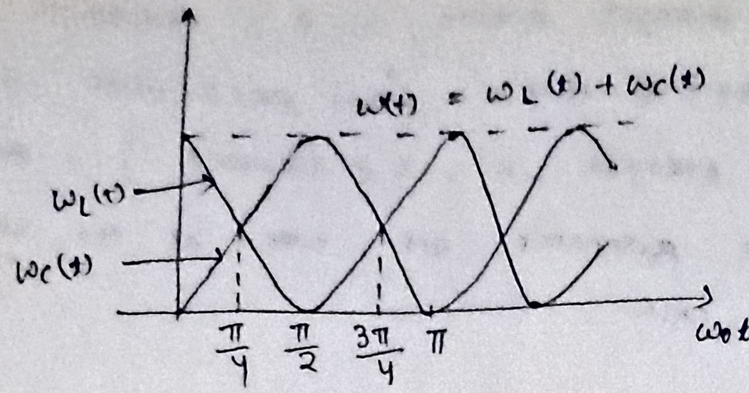
$= \frac{1}{2} L \cdot i_L^2(t) + \frac{1}{2} C \cdot v_C^2(t)$

$= \frac{1}{2} L \cdot \frac{V_m^2}{R^2} \cos^2 \omega_0 t + \frac{1}{2} C \cdot \frac{V^2}{R^2 \omega_0^2 C^2} \cdot \sin^2 \omega_0 t$

$\left(\because \omega_0 = \frac{1}{\sqrt{LC}} \right.$

$\left. \Rightarrow \omega_0^2 L = \frac{1}{C} \Rightarrow \omega_0^2 C = \frac{1}{L} \right)$

$w(t) = \frac{1}{2} \cdot \frac{V_m^2}{R^2}$ (J)



→ $Q = \omega_0 \times \dfrac{\text{max. energy stored in 'L' or 'C' at resonance}}{\text{Avg. power dissipated at resonance}}$.

$$= \omega_0 \times \frac{\frac{1}{2} L \frac{V_m^2}{R^2}}{\frac{1}{2}\frac{V_m^2}{R}} = \frac{\omega_0 L}{R} = \frac{\omega_0^2 L}{\omega_0 R} = \frac{1}{\omega_0 CR}.$$

$$\Rightarrow \boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

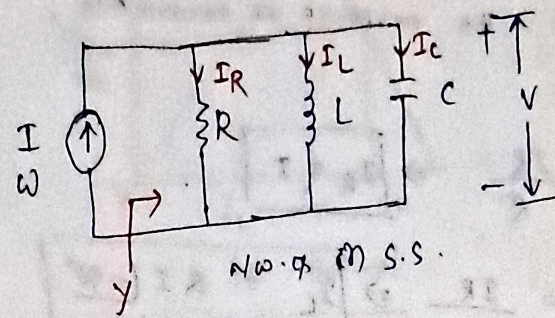
→ So, the Q-factor is a function of ckt constants. As Q is more than the ckt is said to be more selective and hence the oscillations produced are high quality in nature.

→ As $R \to 0 \Rightarrow Z \to 0 \Rightarrow \phi \to \infty \Rightarrow B.W \to 0$

$\Rightarrow f_H - f_L \to 0 \Rightarrow f_H = f_L = f_0$ x(not possible)

which is impossible to achieve, since the violation of KVL in the ckt i.e the voltage source gets short ckted.

(2) The parallel RLC ckt → (parallel Resonance)



Nw. is in S.S.

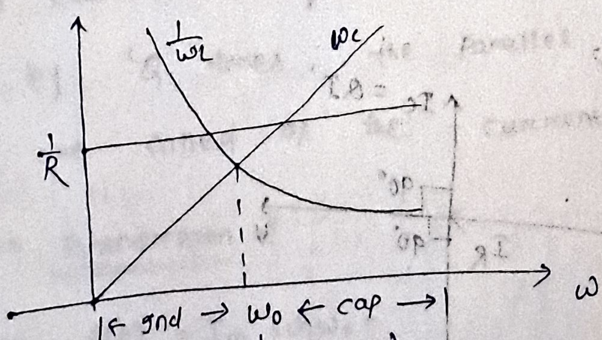
important → $I = Y.V$

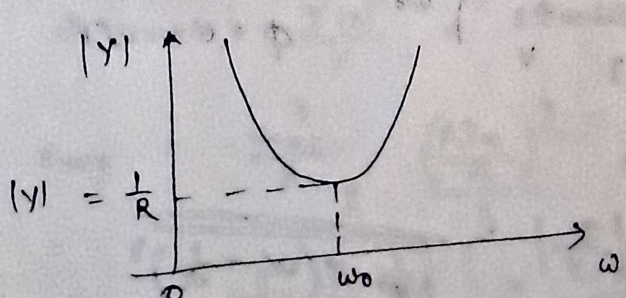
In parallel the voltage across all the elements are the same.

AS Y is more ⇒ I is more ⇒ that nature of the ckt.

→ $Y = Y_R + Y_L + Y_C$

⇒ $Y(j\omega) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$





→ $\omega = \omega_0$ ⇒ $\frac{1}{\omega_0 L} = \omega_0 C$ ⇒ $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec.

⇒ $Y(j\omega_0) = \frac{1}{R}$

Since $V = I/Y = I/(\frac{1}{R})$

$\boxed{V = IR}$ = The response at resonance.

$\omega = \omega_0$

$I_R = \frac{V}{R} = \frac{IR}{R} \Rightarrow \boxed{I_R = I}$

$I_L = \frac{V}{Z_L} = \frac{IR}{j\omega L} \Rightarrow \boxed{I_L = QI\angle -90^\circ}$

where $Q = \frac{R}{\omega_0 L}$.

$I_C = \frac{V}{Z_C} = \frac{IR}{\frac{1}{j\omega C}} \Rightarrow \boxed{I_C = QI\angle 90^\circ}$

where $Q = \omega_0 CR$.

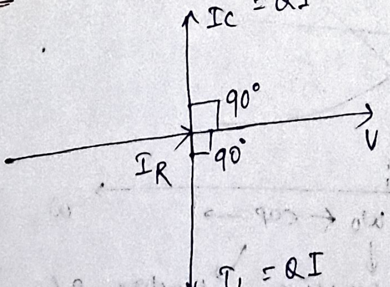
Check:

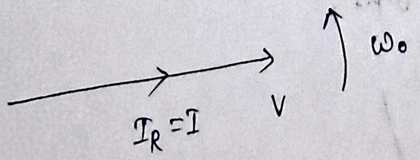
BY KCL $\Rightarrow I_R + I_L + I_C = I - jQI + jQI$

$= I$

Obs: Here $I_L + I_C = 0 \Rightarrow$ LC-combination ∝ like an O.C.

The phasor dia. →





$\phi = 0^\circ$ = UPF

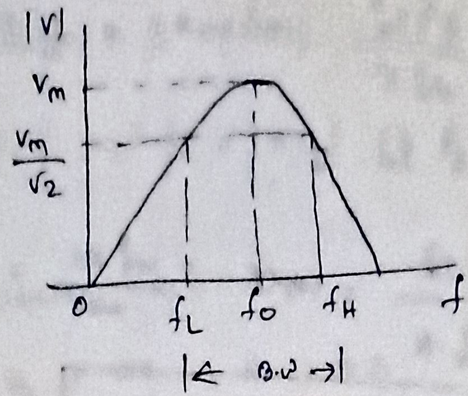
The freq. resp. →

$$V = \left|\frac{I}{Y}\right| = \frac{I}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

$\omega = 0 \Rightarrow |V| = 0$

$\omega = \omega_0 \Rightarrow |V| = IR = V_m$.

$\omega = \infty \rightarrow |V| = 0$



$$\boxed{B.W = f_H - f_L = \frac{f_o}{Q}}$$ where $$\boxed{Q = \frac{R}{\omega_0 L} = \omega_0 C R}$$

$$\boxed{\sqrt{f_H \cdot f_L} = f_o}$$

→ At resonance admittance reaches a min. value and hence the ckt ~~accepts~~ accepts minimum current. $(I = Y \cdot V)$, so called the rejector ckt.

→ Since the currents through 'L' and 'C' elements are magnified by 'Q' times, the parallel RLC ckt at resonance are called as the current magnification ckts

The oscillation phenomenon :-

Let $i(t) = I_m \sin \omega_0 t$

$\omega = \omega_0 \Rightarrow Y = \frac{1}{R}$

$\Rightarrow v(t) = \frac{I(t)}{Y} = R I_m \sin \omega_0 t = V_R(t) = V_L(t) = V_C(t)$

→ $P_{avg} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{R I_m}{\sqrt{2}}\right)^2}{R} = \frac{1}{2} I_m^2 \cdot R\ (\omega)$

→ $i_L = \frac{1}{L} \int V_L \cdot dt = \frac{-R I_m}{\omega_0 L} \cos \omega_0 t$

→ $\omega(t) = \omega_C(t) + \omega_L(t)$
$= \frac{1}{2} C\, V_C^2(t) + \frac{1}{2} L \cdot i_L^2(t)$

$$= \frac{1}{2} L \cdot \frac{R^2 I_m^2}{\omega_0^2 L^2} \cos^2 \omega_0 t + \frac{1}{2} C R^2 I_m^2 \sin^2 \omega_0 t.$$

$$W(t) = \frac{1}{2} C R^2 I_m^2 \ J.$$

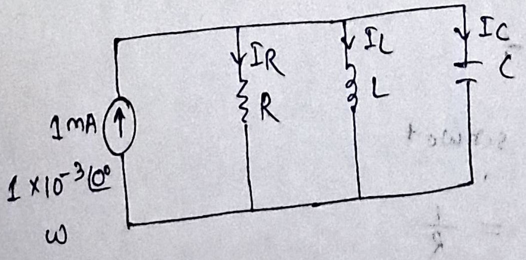
$$\rightarrow Q = \omega_0 \times \frac{\frac{1}{2} C R^2 I_m^2}{\frac{1}{2} I_m^2 \cdot R} = \omega_0 C R = \frac{\omega_0^2 RC}{\omega_0} = \frac{R}{\omega_0 L}.$$

$$\boxed{Q = \omega_0 C R = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}}$$

$\rightarrow$ As $R \to \infty \Rightarrow Y = \frac{1}{R} \to 0 \Rightarrow Z \to \infty \Rightarrow Q \to \infty$

$\Rightarrow$ B.W $\to 0 \Rightarrow f_H - f_L \to 0 \Rightarrow f_L = f_L = f_0$ (X)

$\rightarrow$ So, the ckt operation approaches to a single frequency called the resonant frequency, which is impossible to achieve, since the current source gets open ckted i.e. the violation of KCL of the ckt



(a) $|I_R| < 1$ mA

(b) $|I_R + I_L| < 1$ mA

(c) $|I_R + I_C| > 1$ mA

(d) $|I_L + I_C| > 1$ mA.

At resonance which one of the statements is true.

(A) $I_R = I$, $\quad \omega = \omega_0$

$I_L = QI\angle{-90^\circ} = -jQI$

$I_C = QI\angle{90^\circ} = jQI$

$I_L + I_C = 0$

$\rightarrow \quad |I_R + I_L| = |I - jQI| = I\sqrt{1+Q^2} > I$

$\rightarrow \quad |I_R + I_C| = |I + jQI| = I\sqrt{1+Q^2} > I$

Q/ In a series RLC ckt.

$V_s = 100$, $R = 10\,\Omega$ $X_L = 20$ & $X_C = 20$,

the voltage across the capacitor is.

(A) $X_L = X_C \Rightarrow \omega = \omega_0 \Rightarrow V_C = Q V_s \angle -90^\circ$

$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 2$

$Q = \frac{1}{\omega_0 C R} = \frac{X_C}{R} = 2$.

$\Rightarrow V_C = 200 \angle -90^\circ = -j200\,V$.

Q/ In a series RLC ckt the Q factor is 100. If all the component are doubled then.

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2R}\sqrt{\frac{2L}{2C}} = \frac{1}{2R}\sqrt{\frac{L}{C}} = \frac{1}{2}Q$

Q/ In the above case if the ckt a parallel.

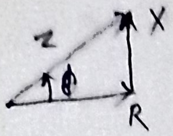
$Q = R\sqrt{\frac{C}{L}} = 2R\sqrt{\frac{2C}{2L}} = 2Q$

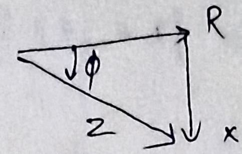
Q/ In a series RLC ckt, the P.f at $f = f_L$ is

$|I| = \frac{I_m}{\sqrt{2}} \quad \Big| \; I_m = \frac{V}{R}$

$\Rightarrow \left|\frac{V}{Z}\right| = \frac{V}{\sqrt{2}R} \quad \Rightarrow |Z| = \sqrt{2}R.$

$z = R \pm jX = \frac{V}{I}$





$z = R + jX$ $\qquad$ $z = R - jX$

$P.f = \cos\phi = \left(\frac{R}{z}\right)$

$\rightarrow f = f_L \Rightarrow P.f = \frac{R}{\sqrt{2}\, R} = 0.707$ (lead)

$\rightarrow f = f_H \Rightarrow P.f = 0.707$ (lag)

$\rightarrow f = f_0 \Rightarrow Z = R \Rightarrow P.f = \frac{R}{z} = \frac{R}{R} = 1$

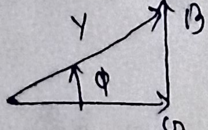
Q/1 In the above case if the ckt is a parallel RLC then

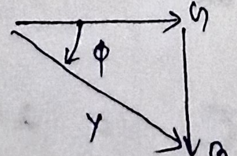
$f = f_L \Rightarrow |V| = \frac{V_m}{\sqrt{2}} \Big| V_m = IR$

$\rightarrow \left|\frac{I}{Y}\right| = \frac{IR}{\sqrt{2}} = \frac{I}{\sqrt{2}\, G}$

$\Rightarrow |Y| = \sqrt{2}\, G.$

$Y = G \pm jB = \frac{I}{V}.$





$P.f = \cos\phi = \left(\frac{G}{Y}\right)$

$\rightarrow f = f_L \Rightarrow P.f = \frac{G}{\sqrt{2}\, G} = 0.707$ (lag)

Q1 $f = f_H \Rightarrow P.f = 0.707$ lead.

$f = f_0 \Rightarrow Y = \frac{1}{R} = G \Rightarrow P.f = \frac{G}{Y} = \frac{G}{G} = 1$

Q/ 2

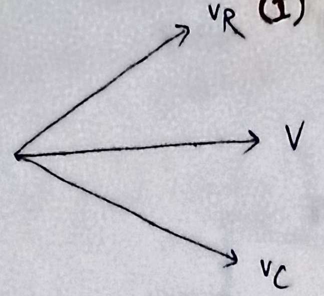


fig.

(a) $f = 0$

(b) $f = f_0$

(c) $f < f_0$

(d) $f > f_0$

In a ~~Series~~ RLC ckt the phasor dia. at a certain freq. is shown in fig., the operating freq. is

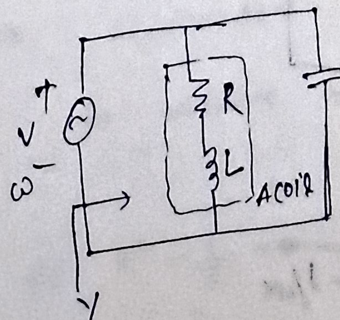
(A) $V_R = IR$ since current leads the voltage, the nature of the ckt is capacitive and hence the operating freq. is $f < f_0$.

Q/ At resonance, the parallel ckt of the fig. constructed by an iron cored coil and the capacitor will behaves like

(a) an open ckt (b) a S.C (c) pure Resistor of 'R'.

(d) A pure resistor of value much higher than 'R'.

(A)



$Y = Y_L + Y_C = \frac{1}{Z_L} + \frac{1}{Z_C}$

$= \frac{1}{R+j\omega L} + \frac{1}{\frac{1}{j\omega C}}$

$\Rightarrow Y = \frac{R - j\omega L}{R^2 + (\omega L)^2} + j\omega C.$

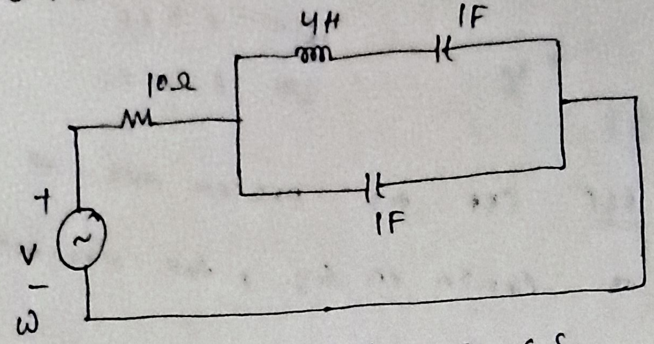
i term $= 0 \Rightarrow \omega = \omega_0$.

$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{C}{R} R^2}$ rad/sec.

$\Rightarrow Y = \frac{R}{R^2 + (\omega_0 L)^2}$ ℧

$$z = \frac{1}{Y} = R + \frac{(\omega_0 L)^2}{R} >> R.$$

Q) Det. ω_0



NW is in S.S

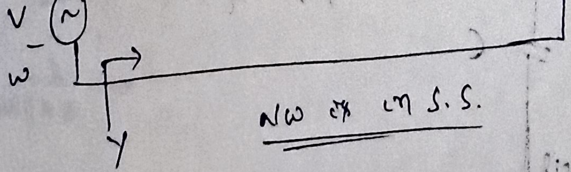
$$z = 10 + \left(j4\omega + \frac{1}{j\omega}\right) \parallel \left(\frac{1}{j\omega}\right)$$

$$= 10 + \frac{4 - \frac{1}{\omega^2}}{j\left(4\omega - \frac{2}{\omega}\right)} = \frac{10 - j\left(4 - \frac{1}{\omega^2}\right)}{4\omega - \frac{2}{\omega}}$$

j term is zero ⇒

$$4 - \frac{1}{\omega^2} = 0 \Rightarrow \omega_0 = \frac{1}{2} \text{ rad/sec}.$$

Q)



Det. ω_0. What happens when $L = CR^2$

NW is in S.S.

(A)

$$Y = \frac{1}{R + j\omega L} + \frac{1}{R - j/\omega C}$$

$$= \frac{R - j\omega L}{R^2 + (\omega L)^2} + \frac{R + j/\omega C}{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

j term = 0 ⇒

$$\frac{\omega_0 L}{R^2 + (\omega_0 L)^2} = \frac{\frac{1}{\omega_0 C}}{R^2 + \left(\frac{1}{\omega_0 C}\right)^2}$$

$$\Rightarrow \omega_0 L R^2 + \frac{\omega_0 L}{(\omega_0 C)^2} = \frac{R^2}{\omega_0 C} + \frac{(\omega_0 L)^2}{\omega_0 C}$$

$$\Rightarrow R^2\left(\omega_0 L - \frac{1}{\omega_0 C}\right) - \frac{\omega_0 L}{\omega_0 C}\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0.$$

$$\Rightarrow \left(R^2 - \frac{L}{C}\right)\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

case-(i) if $R^2 - \frac{L}{C} \neq 0$ then $\omega_0 L - \frac{1}{\omega_0 C} = 0$ must for

j term to be zero

i.e. the ckt will resonant at only one freq. called

$\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec. then

$$Y = \frac{R}{R^2+(\omega_0 L)^2} + \frac{R}{R^2+\left(\frac{1}{\omega_0 C}\right)^2}\Bigg|_{\omega_0 = \frac{1}{\sqrt{LC}}} \mho.$$

$$\Rightarrow \boxed{Y = \frac{2R}{R^2+\frac{L}{C}}} \mho$$

case-(ii) If $R^2 - \frac{L}{C} = 0$

$\Rightarrow R^2 = \frac{L}{C} \Rightarrow L = CR^2$ then j term is zero

and it is independent of frequency. So, the ckt will

resonant for infinite no. of frequencies.

case-(iii) If $R^2 - \frac{L}{C} = 0$ and $\omega_0 L - \frac{1}{\omega_0 C} = 0$

then $R^2 = \frac{L}{C}$ & $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec.

i.e. out of infinite no. of resonant frequencies we are

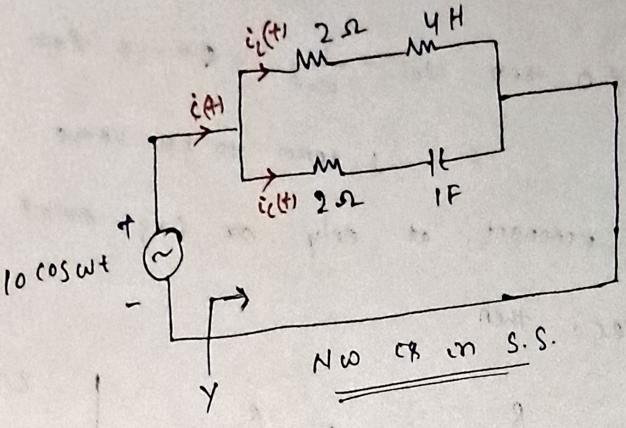
selecting one frequency called 'ω_0'

$$\Rightarrow Y = \frac{R}{R^2+(\omega_0 L)^2} + \frac{R}{R^2+\left(\frac{1}{\omega_0 C}\right)^2}\Bigg|_{\substack{\omega_0 = \frac{1}{\sqrt{LC}} \\ R^2 = L/C}} \mho$$

$$\boxed{Y = \frac{2R}{R^2 + \frac{L}{C}}} \quad \mho$$

$R^2 = L/C \Rightarrow Y = \frac{2R}{2R^2} = \frac{1}{R} \Rightarrow Z = R.$

Q) Det. the avg power dissipated in the ckt at resonance.



Now ckt in S.S.

A) $\frac{L}{C} = R^2 \Rightarrow$ ckt will resonant for all the frequencies. i.e. we are out of infinite no. of resonant frequencies selecting one frequency called 'ω_0' $= \frac{1}{\sqrt{LC}} = \frac{1}{2}$ rad/sec

$\Rightarrow Z = R = 2\,\Omega$

$$I = \frac{V}{2} = \frac{10\angle 0^\circ}{2} = 5\angle 0^\circ$$

$\Rightarrow i(t) = 5 \cos \frac{t}{2}$ A

$Z_L = j\omega_0 L = j2\,\Omega$

$Z_C = \frac{1}{j\omega_0 C} = -j2\,\Omega$

$$I_L = \frac{I\cdot(2-j2)}{2+j2+2-j2} = \frac{I}{\sqrt{2}}\angle -45^\circ$$

$\Rightarrow i_L(t) = \frac{5}{\sqrt{2}} \cos\left(t/2 - 45^\circ\right)$ A.

$$I_C = \frac{I\,(2+j2)}{2-j2+2+j2}$$

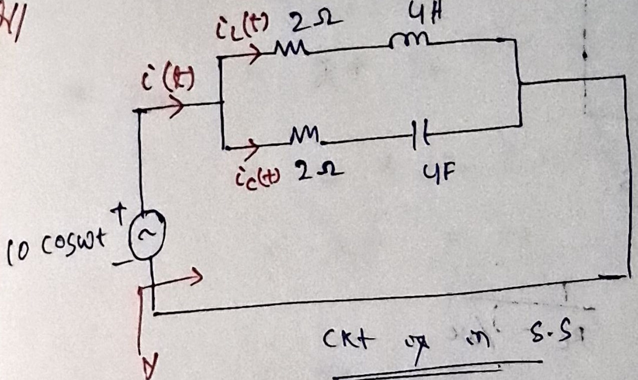
$$= \frac{I}{\sqrt{2}}\angle 45^\circ$$

$$\Rightarrow i_C = \frac{5}{\sqrt{2}}\cos(t+45^\circ)\ A.$$

$$P_{avg} = I_{L\,rms}^2 R + I_{C\,rms}^2 \cdot R$$

$$= \left(\frac{\frac{5}{\sqrt{2}}}{\sqrt{2}}\right)^2 \cdot 2 + \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2 \cdot 2 = 25\,W.$$

Q/



Ckt in S.S

since $L/C = R^2$, ckt will resonant at only one freq.

called $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot 4}} = \frac{1}{4}$ rad/sec.

$$Y = \frac{2R}{R^2 + \frac{L}{C}}\ \mho = \frac{4}{5}\ \mho$$

$$Z = \frac{5}{4}\ \Omega.$$

$$I = \frac{V}{Z} = \frac{10\angle 0^\circ}{5/4} = 8\angle 0^\circ$$

$$\Rightarrow i(t) = 8\cos\frac{t}{4}\ A.$$

$$Z_L = j\omega_0 L = j1\,\Omega.$$

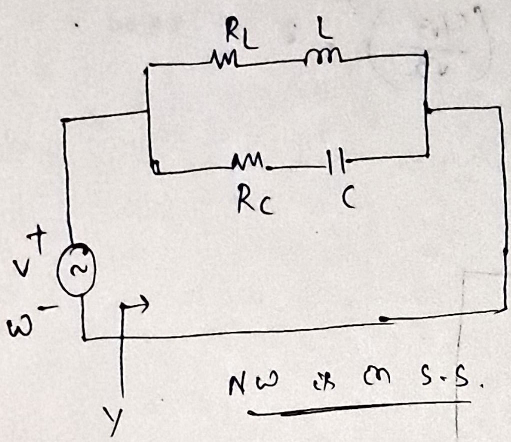
$$Z_C = \frac{1}{j\omega_0 C} = -j1\,\Omega$$

$$I_L = \frac{I\,(2-j1)}{2+j1+2-j1}$$

$$I_C = \frac{I \cdot (2+j1)}{2+j1+2-j1}$$

$$\Rightarrow P_{avg} = I_{L\,rms}^2 \cdot 2 + I_{C\,rms}^2 \cdot 2 \ (W)$$

Q11



NW is in S.S.

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - j/\omega C}$$

$$= \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{R_C + j/\omega C}{R_C^2 + \left(\frac{1}{\omega C}\right)^2}$$

j term $= 0$ $\Rightarrow$ $\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$ rad/sec.

(i) If $R_L = R_C = 0$ $\Rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec.

(ii) If $R_L = R_C = R$ $\Rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec.

(iii) If $R_C = 0$ $\Rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{C}{L} R_L^2}$ rad/sec.

(iv) If $R_L = 0$ $\Rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}\sqrt{1 - \frac{C}{L} R_C^2}}$ rad/sec.

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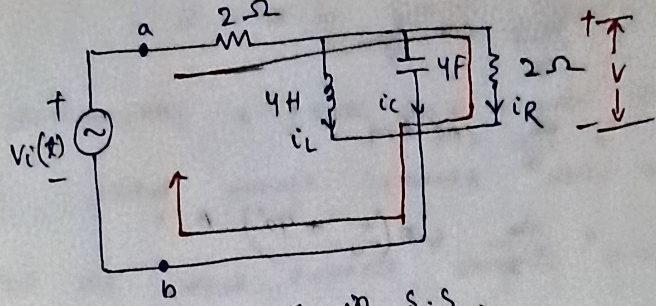
8//



NW is in S.S.

Det. (a) $\omega = \omega_0$ at which Z_{ab} is max.

(b) $Z_{ab}\Big|_{\omega=\omega_0} = ?$

(c) if $V_i(t) = V_m \sin \omega_0 t$ then i_R, i_L & i_C are.

(A) (a) $Z_{ab} = 2 + Z_L \parallel Z_C \parallel 2$

$= 2 + jX_L \parallel -jX_C \parallel 2$

$= 2 + \dfrac{X_L X_C}{j(X_L - X_C)} \parallel 2.$

$Z_{ab} = 2 + \dfrac{2 X_L X_C}{X_L X_C + j2(X_L - X_C)}$

$= 2 + \dfrac{2X_L X_C\left(X_L X_C - j2(X_L - X_C)\right)}{(X_L X_C)^2 + 4(X_L - X_C)^2}$

j-term $= 0$

$\Rightarrow -2(X_L - X_C) = 0$

$\Rightarrow X_L = X_C$

$\Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C} \Rightarrow \omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{4 \cdot 4}} = \dfrac{1}{4}$ rad/sec.

(b) $Z_{ab}\Big|_{\omega=\omega_0} = 2 + 2 = 4\,\Omega$

or

$X_L = X_C$

(c) $V_i(t) = V_m \sin \omega_0 t = V_m \sin \dfrac{t}{4}$ V. ; $Z = 4\Omega$

$i(t) = \dfrac{V_i(t)}{2} = \dfrac{V_m}{4} \sin t/4 = i_R.$

$$V = 2 \cdot i_R = \frac{V_m}{2} \cdot \sin\frac{t}{4} = V_C = V_L$$

$$i_C = C\frac{dv_C}{dt} = \frac{V_m}{2}\cos t/4$$

$$= \frac{V_m}{2}\sin\left(\frac{t}{4} + 90^\circ\right) A$$

$$\rightarrow \quad i_L = \frac{1}{L}\int V_L \, dt$$

$$= -\frac{V_m}{2}\cos\frac{t}{4}$$

$$= \frac{V_m}{2}\sin\left(\frac{t}{4} - 90^\circ\right) A$$

Obs:

Here $i_L + i_C = 0 \Rightarrow$ LC - combination Like an O.C.