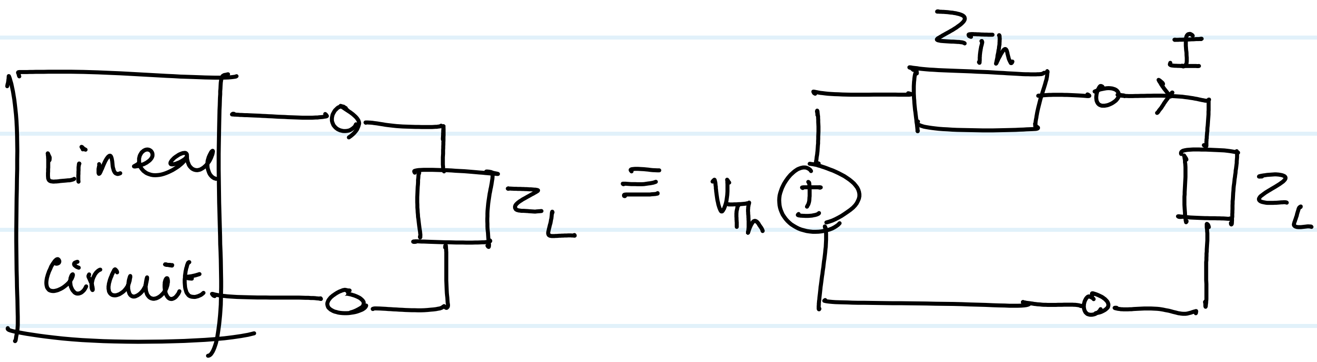
08/11/2023

MAXIMUM AVERAGE POWER TRANSFER



$$Z_{Th} = R_{Th} + jX_{Th}$$

$$Z_L = R_L + jX_L$$

Power is max if $R_L = R_{Th}$

equivalently, $\boxed{Z_L = \overline{Z}_{Th}}$

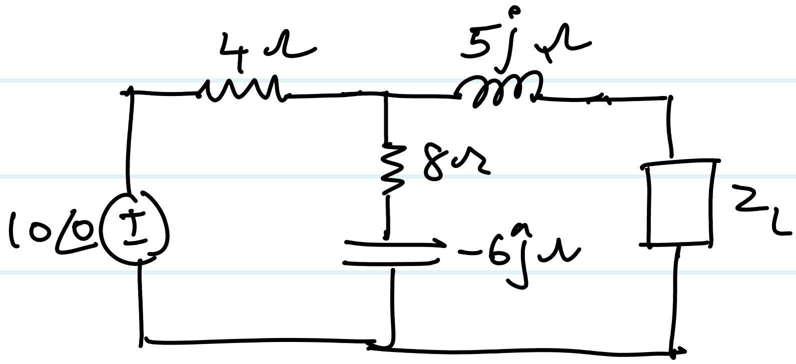
$$\underbrace{\frac{dP}{dR_L} = 0 \qquad \frac{dP}{dX_L} = 0}_{\text{gives this.}} \qquad \boxed{Z_L = \overline{Z}_{Th}}$$

$$P_{max} = \frac{V_{Th}^2}{8R_{Th}}$$

$$R_L = \sqrt{R_{Th}^2 + \cancel{X_{Th}^2}^{\,0}} = |Z_L|$$

EXAMPLE

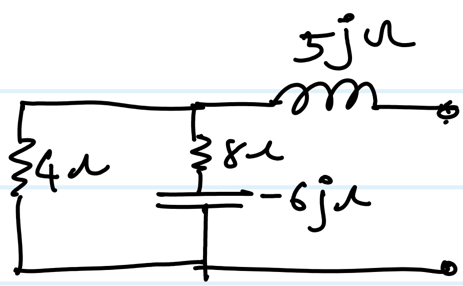
find load impedence (Z_L) that gets max Power. Calculate P_{max}



$$P_{max} \Rightarrow \boxed{Z_L = \overline{Z_{Th}}}$$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}.$$

find Z_{Th}:



$$Z_{Th} = \left((4\Omega) \,\|\, (8-6j)\right) + 5j$$

$$= \frac{4(8-6j)}{12-6j} + 5j$$

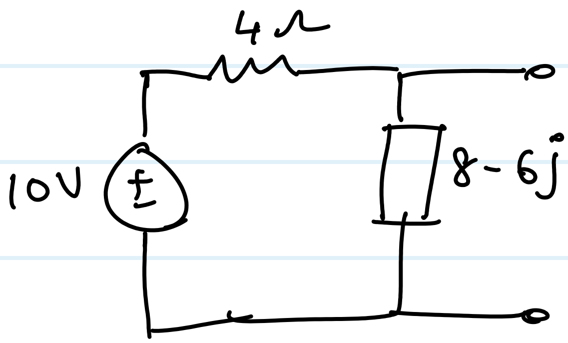
$$= \frac{44}{15} - \frac{8}{15}j + 5j$$

$$Z_{Th} = \frac{44}{15} + \frac{67}{15}j$$

$$Z_L = \overline{Z_{Th}} = \frac{44}{15} - \frac{67}{15}j$$

$$P_{max} = \frac{V_{Th}^2}{8R_{Th}}$$

find V_{Th}:



$$10 = 4i + (8-6j)i$$

$$10 = i(12-6j)$$

$$\frac{10}{12-6j} = i$$

$$i = \left(\frac{2}{3} + \frac{1}{3}j\right) A$$

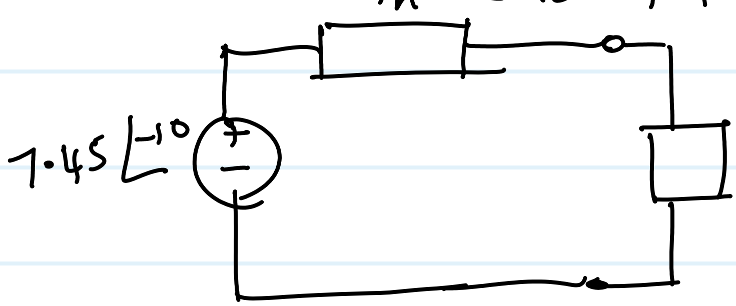
$$\left(\frac{2}{3} + \frac{1}{3}j\right)(8-6j)$$

$$V_{Th} = \frac{22}{3} - \frac{4}{3}j$$

$$= 7.33 - 1.33j = 7.45\angle{-10}.$$

R_{Th}

$$Z_{Th} = 2.93 + 4.47j$$

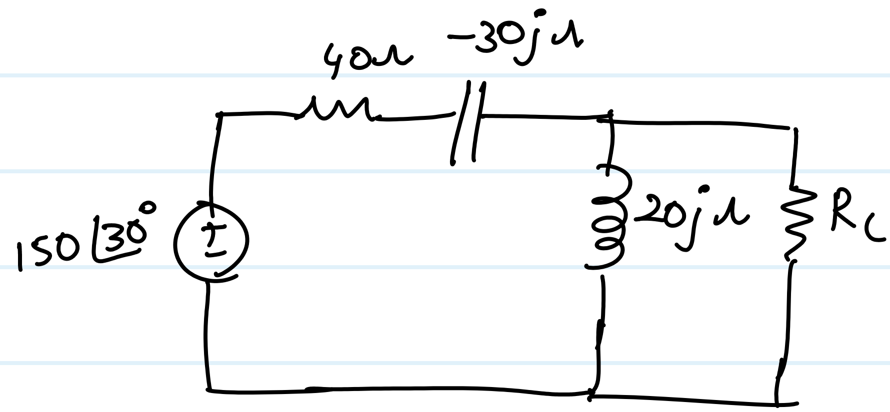


R_{Th}

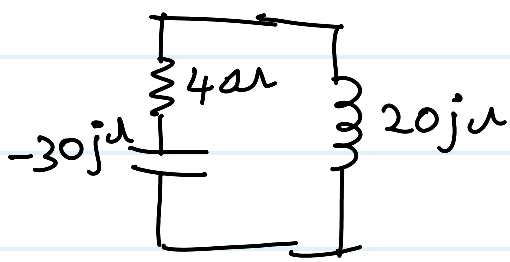
$$Z_c = 2.93 - 4.47j$$

$$P_{max} = \frac{(7.45)^2}{8(2.93)} = 2.368\,W$$

EXAMPLE :



Z_{Th} :



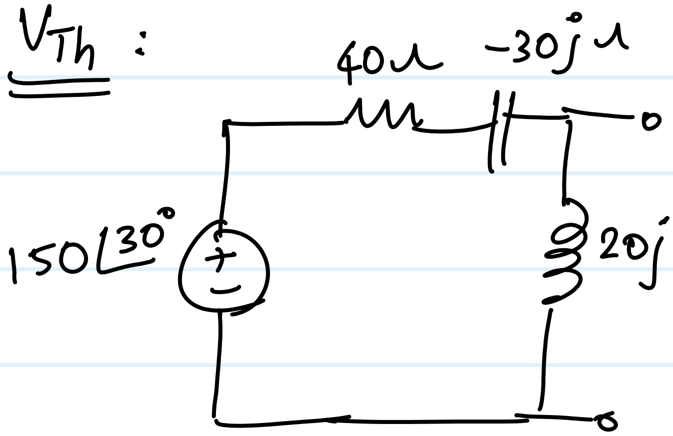
$$Z_{Th} = (40-30j) \parallel 20j$$

$$= \frac{(40-30j)(20j)}{40-10j}$$

$$= \frac{160}{17} + \frac{380}{17} i$$

$$R_L = |Z_{Th}| = 24.25$$

V_{Th} :



$$150\angle 30 = (40-30j+20j)i$$

$$\frac{150\angle 30}{40-10j} = i$$

$$= 2.615 + 2.53j$$

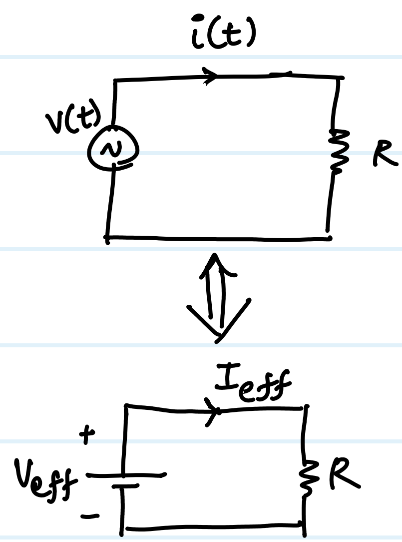
$$V_{Th} = -50.577 + 52.3j$$
$$= 72.76 \angle 134.$$

$$P_{max} = \frac{V_{Th}^2}{8R_{Th}} \leftarrow \underline{\underline{Re(Z_{Th})}}$$

$$\frac{(72.76)^2}{8 \times \left(\frac{160}{17}\right)} = \underline{\underline{70.311\ V}}$$

Effective (or) RMS Value

$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt = I_{RMS}^2 R$$



$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = I_{RMS}}$$

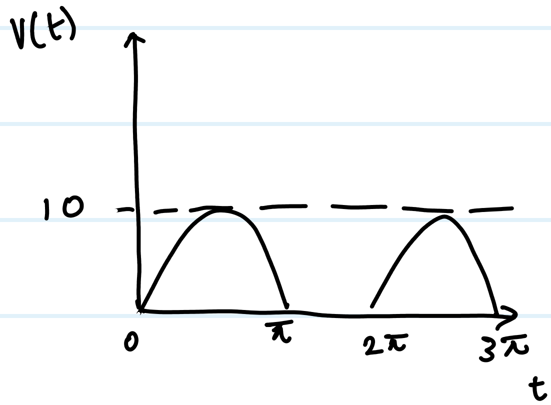
$$i(t) = I_m \cos wt$$
$$i^2(t) = I_m^2 \cos^2 wt$$
$$i^2(t)\,dt = I_m^2 \left(\frac{1+\cos 2wt}{2}\right) dt$$
$$\int i^2(t)\,dt = I_m^2 \left(\int_0^T \frac{dt}{2} + \int_0^T \frac{\cos 2wt}{4w}\, d(2wt)\right)$$
$$\int i^2(t)\,dt = I_m^2 \left[\frac{T}{2} + \cancel{\left[\frac{\sin 2wt}{4w}\right]_0^T}^{\,0}\right] = \frac{I_m^2 T}{2}$$
$$\frac{\int i^2(t)\,dt}{T} = \frac{I_m^2}{2} \Rightarrow \sqrt{\frac{\int i^2(t)\,dt}{T}} = \frac{I_m}{\sqrt{2}} = I_{RMS} \,//$$

Avg. Power using RMS :

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{RMS} I_{RMS} \cos(\theta_v - \theta_i)$$

EXAMPLE : find RMS , $R = 10\Omega$



$$v(t) = \begin{cases} 10\sin t & 0<t<\pi \\ 0 & \pi<t<2\pi \end{cases}$$

$$V_{RMS}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{\pi} 100\sin^2 t\,dt + \cancel{\int_{\pi}^{2\pi} 0^2\,dt}^{\,0}\right]$$

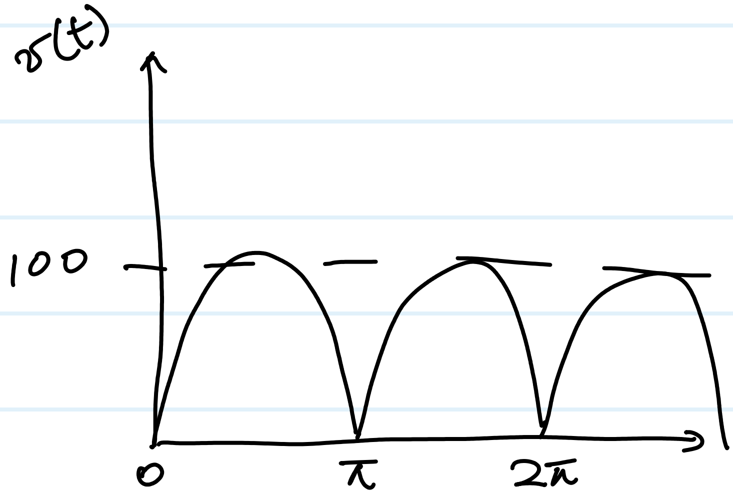
$$= \frac{100}{T}\int_0^{\pi}\left(\frac{1-\cos 2t}{2}\right)dt$$

$$= \frac{100}{2T}\left[\pi\right] = \frac{100}{4\pi}\pi$$

$$= 25$$

$$\boxed{V_{RMS} = \sqrt{25} = 5\,V}$$

$$\boxed{P = \frac{V_{RMS}^2}{R} = \frac{25}{10} = 2.5\,W}$$



find RMS

find Pavg

$R = 6\Omega$

find $P_{6\Omega}$

$0 < t < \pi$: $v(t) = 10\sin t$ V

$\pi < t < 2\pi$: $v(t) = 10\sin t$ V

$$v_{RMS} = \sqrt{\frac{\int_0^T v^2(t)\,dt}{T}} = \sqrt{\frac{\int_0^{\pi} 100\sin^2 t\,dt + \int_{\pi}^{2\pi} 100\sin^2 t\,dt}{T}}$$

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APPARENT POWER (S)

↳ "S" is the product of rms. values of Voltage and current. Its the "Rating" of equipments.

↳ $P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \underbrace{V_{RMS} I_{RMS}}_{\text{"S"}} \underbrace{\cos(\theta_v - \theta_i)}_{\text{power factor ("pf")}}$

⇒ $P_{avg} = S \cdot pf$

pf : cosine of phase diff. b/w voltage & current
(or) cosine of $\angle^{le}$ of load impedence.

↳ S → has units as Volt-Ampere (VA)

→ we specifically say VA & not W to distinguish S from P_{avg} (P_{avg}/P → active power – useful/real power)

↳ $pf = P/S$; P → active power , S → Apparent power

Purely Resistive (only R)	$\theta_v - \theta_i = 0$, $P_f = 1$	$P/S = 1$, all P_{avg} consumed
Purely reactive (only L or C)	$\theta_v - \theta_i = \pm 90°$, $pf = 0$	$P = 0$, no P_{avg} consumed
Resistive & Reactive (R and L or C)	• $\theta_v - \theta_i > 0$ • $\theta_v - \theta_i < 0$	• Lagging - inductive • Leading - capacitive

Lagging / Leading
↳ wrt current.

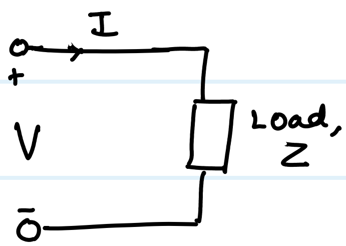
* bill = actual power + pf. penalty *

to make $pf \to 1$, L & C are used together.

Capacitors are used as condensers in order to improve pf.

COMPLEX POWER (S)

↳ "S" is the product of the voltage and the complex conjugate of the current



$V = V_m \angle \theta_v$ $\quad I = I_m \angle \theta_i$

$I^* = I_m \angle -\theta_i$

$$S = \frac{1}{2} V I^* = V_{RMS} I_{RMS} \angle \theta_v - \theta_i$$

$$= V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) + (V_{RMS} I_{RMS} \sin(\theta_v - \theta_i)) j$$

reactive power
↓
Allows work to happen (to create $\vec{B}$ field)

measured in VAR ← $S = P + Qj$

P → active power

Q → reactive power (exchanged b/w source & reactive load)

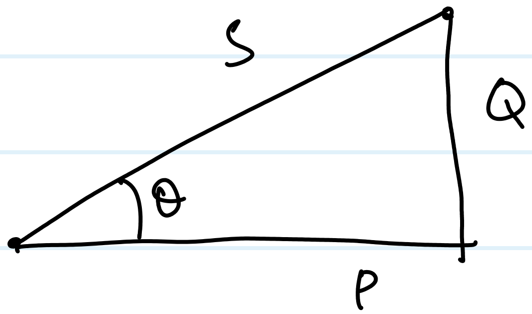
active → actual work done by source

$Q = 0 \rightarrow$ resistive, $pf = 1$

$Q < 0 \rightarrow$ capacitive, leading pf

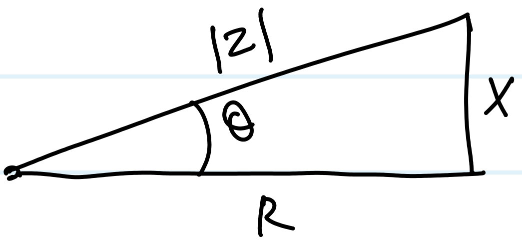
$Q > 0 \rightarrow$ inductive, lagging pf

Q is reqd., but in low levels.. else, more power will be drawn by reactive load...



$S = P + Qj$; $|S| = \sqrt{P^2+Q^2}$

$\cos\theta = \frac{P}{S}$; $\sin\theta = \frac{Q}{S}$



$Z = R + jX = \sqrt{R^2+X^2} = |Z|$

$\cos\theta = \frac{R}{|Z|}$

$\theta = \tan^{-1}(X/R)$

$\cos\theta = \cos\left[\tan^{-1}(X/R)\right]$

Apparent Power $\rightarrow$ $V_{RMS} \cdot I_{RMS} = \sqrt{P^2+Q^2}$ (S)

Real Power $\rightarrow$ $S\cos(\theta_v - \theta_i)$ $(Re(S)) \rightarrow (P)$

Reactive Power $\rightarrow$ $S\sin(\theta_v - \theta_i)$ $(Im(S))$

Power factor $\rightarrow$ $\cos(\theta_v - \theta_i)$ $(P/S \rightarrow pf)$

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EXAMPLE :

$$i(t) = 4\cos(100\pi t + 10^\circ)\,A$$
$$v(t) = 120\cos(100\pi t - 20^\circ)\,V$$

find $\longrightarrow$ Apparent Power (S), power factor (P/S)

$$S = V_{RMS} I_{RMS} = \frac{V_0 I_0}{2} = \frac{4\times 120}{2} = 240 \text{ VA}$$

$$pf = \cos(\theta_v - \theta_i) = \cos(-20-10) = +\cos 30 = \frac{\sqrt{3}}{2}$$
$$= 0.867$$

$\theta_i > \theta_v \Rightarrow$ pf : leading $\rightarrow$ capacitive

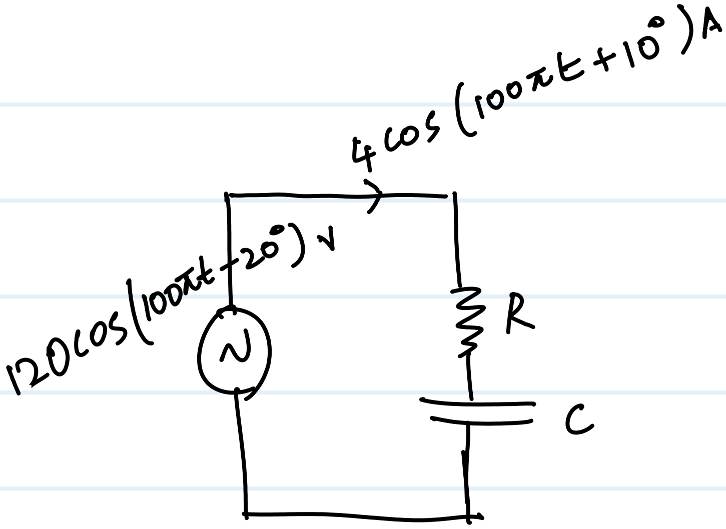
$$pf = R/Z = \frac{Re(Z)}{Z} = \frac{\cancel{30}\cos 30\angle 0}{\cancel{30}\angle -30} \qquad Z = \frac{V(t)}{i(t)}$$

$$= \cos 30 \angle 30 \leftarrow \text{leading} \qquad = \frac{V_m \angle \theta_v}{I_m \angle \theta_i}$$

$$= 0.867\angle 30 \qquad = Z_m \angle \theta_v - \theta_i$$

$$= 30\angle -30$$

$$30\angle -30$$
$$\downarrow$$
$$30\cos(-30) + 30\sin(-30)$$



$$R = \mathrm{Re}(Z)$$
$$= 30 \cos 30$$
$$= 30 \times 0.867 = 25.98\,\Omega$$

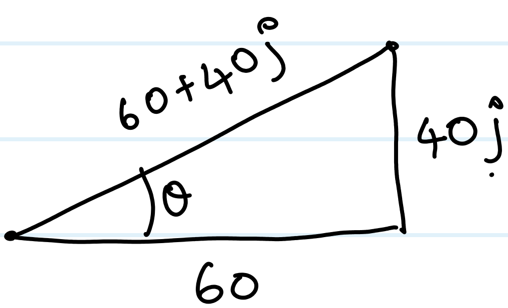
$$I_m(Z) = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$-15j = \frac{-j}{\omega C} \Rightarrow \omega C = \frac{1}{15}$$

$$\Rightarrow C = \frac{1}{15 \times 100\pi}$$
$$= \frac{1}{1500\pi}$$
$$= 2.12 \times 10^{-6}\,F$$
$$= 212\,\mu F$$

EXAMPLE :

$$Z = (60 + 40j)\,\Omega\ ; \quad v(t) = 320 \cos(377t + 10^\circ)\,V$$



$$\cos\theta = \frac{60}{|60 + 40j|} = 0.832$$

$$Z = R + jX \quad , \quad X \rightarrow +ve \Rightarrow \theta_v - \theta_i > 0$$

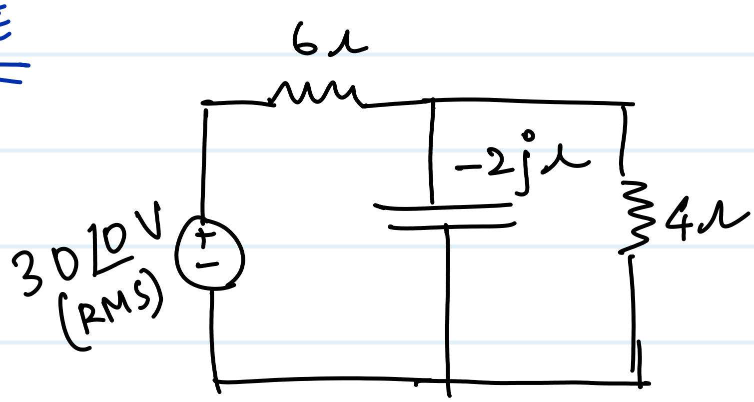
$$\Rightarrow \boxed{\theta_v > \theta_i} \rightarrow \text{lagging (inductive)}$$

$$\frac{1}{2} \times \left(\frac{320\angle 10 \cdot 320\angle 10}{72.111\angle 33.7} \right) = \frac{320^2}{2 \times 72.111} \angle -13.7 = \frac{1420}{2} \angle -13.7 \quad \text{Check}$$

$$= 710 \angle -13.7\ \text{VA}$$

find pf, P_{avg}.

EXAMPLE

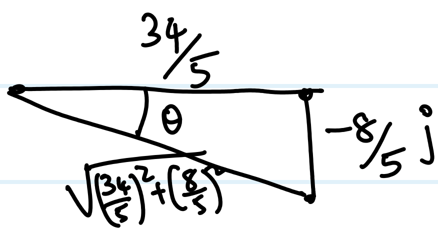


$$\frac{-8j}{4-2j} = \frac{4}{5} - \frac{8}{5}j$$

$$\frac{4}{5} + 6 = \frac{34}{5}$$

$$Z = 6.98\angle{-13.24}\ \Omega$$

$$Z_{eq} = \left(\frac{34}{5} - \frac{8}{5}j\right)\Omega$$



$$\cos\theta = \frac{(34/5)}{6.98} = \frac{6.8}{6.98} = 0.9734$$

(leading)

capacitive

$$\text{Average } P = V_{RMS} \cdot I_{RMS} \cos(\theta_v - \theta_i)$$

$$= 30 \times 4.29 \times 0.9734$$

$$= 125.5$$

$$i_{RMS} = \frac{V_{RMS}}{Z}$$

$$= \frac{30\angle 0}{6.98\angle{-13.24}}$$

$$= 4.29\angle 13.24.$$

EXAMPLE

$$v(t) = 60\cos(\omega t - 10)\,V$$
$$i(t) = 1.5\cos(\omega t + 50^\circ)\,V$$

(a) complex & apparent power, (b) real & reactive power

(c) pf & impedence.

(a) $V = 60\angle -10$; $V_{RMS} = 60/\sqrt{2}\,\angle -10$ & $I = 1.5\angle 50$;

$$I_{RMS} = \frac{1.5\angle 50}{\sqrt{2}}$$

$$\text{Complex } P = V_{RMS} \cdot \overline{I_{RMS}}$$
$$= \frac{60}{\sqrt{2}}\angle -10 \cdot \frac{1.5}{\sqrt{2}}\angle -50 = 45\angle -60^\circ\ VA$$

Apparent : $V_{RMS} \cdot I_{RMS} = \frac{60}{\sqrt{2}} \times \frac{1.5}{\sqrt{2}} = 45\ VA.$

(b) Real $=$ Re(complex P) $= 45\cos(-60) = 22.5\ VA$

Reactive $=$ Im(complex P) $= 45\sin(-60) = -45 \times 0.867$

$= -38.97\ VAR$

(leading)

↓

capacitive

(c) $pf = \cos(\theta_v - \theta_i) = \cos(-60) = 0.5$ (leading)

$$Z = \frac{60\angle -10}{1.5\angle 50} = \underline{\underline{40\angle -60\ \Omega}}$$

EXAMPLE

$$V_{RMS} = 110\angle 85^\circ \text{ V}, \quad I_{RMS} = 0.4\angle 15^\circ \text{ A}$$

(a) complex & apparent powers. (b) real and reactive powers (c) the power factor & load impedence.