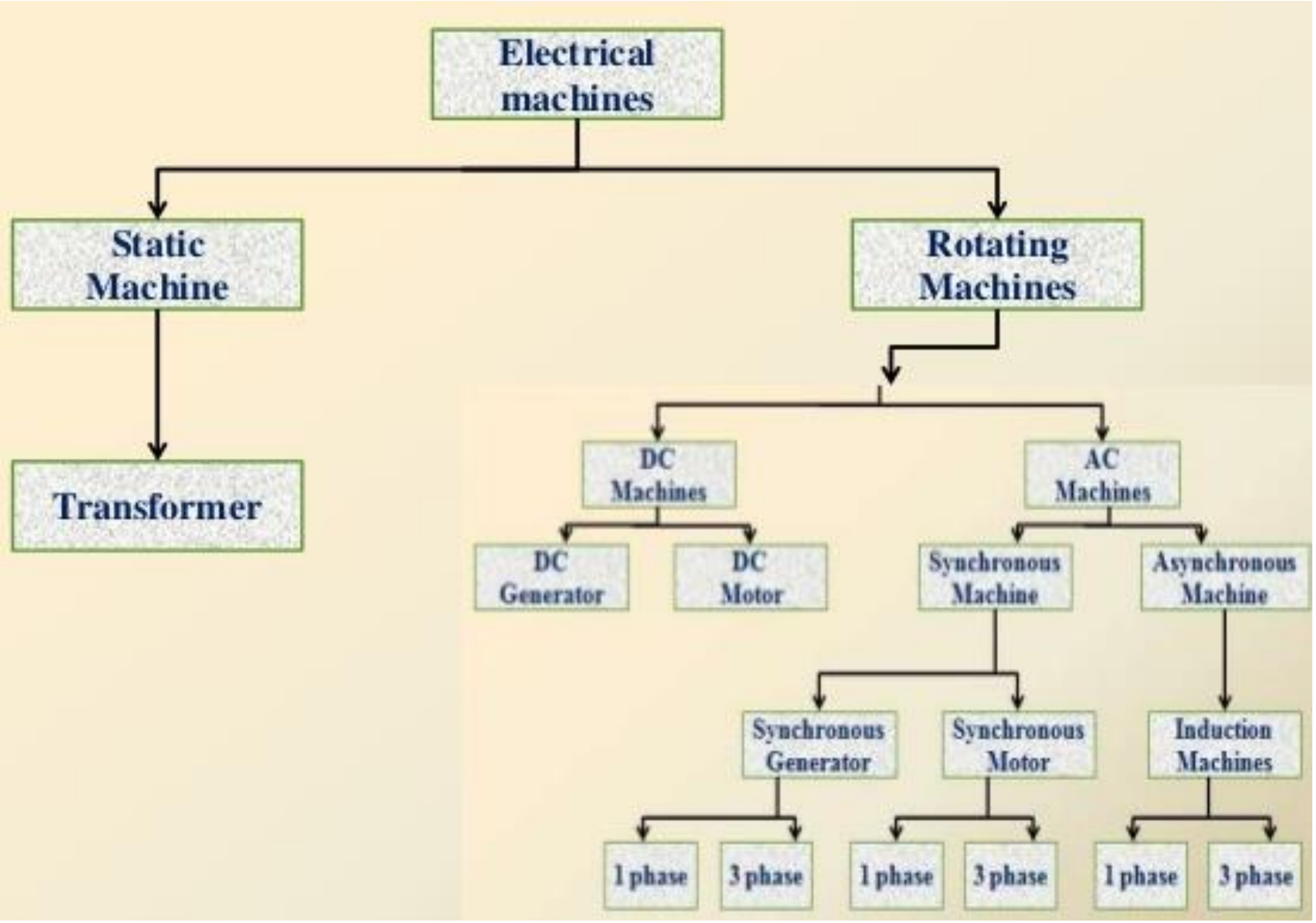
Electrical Circuits for Engineers (EC1000)

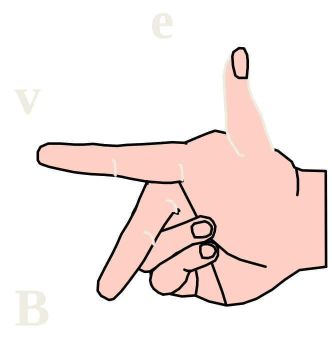
Lecture-12
Electrical Machines

Types of Electrical Machines

Electrical Machines are Electro Mechanical Energy Conversion devices, which converters Electrical energy into mechanical energy and vice-versa

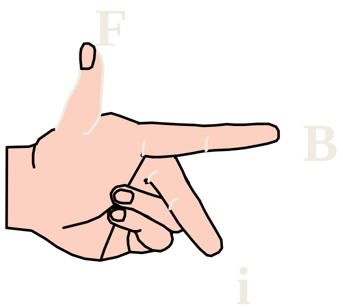


How they work??



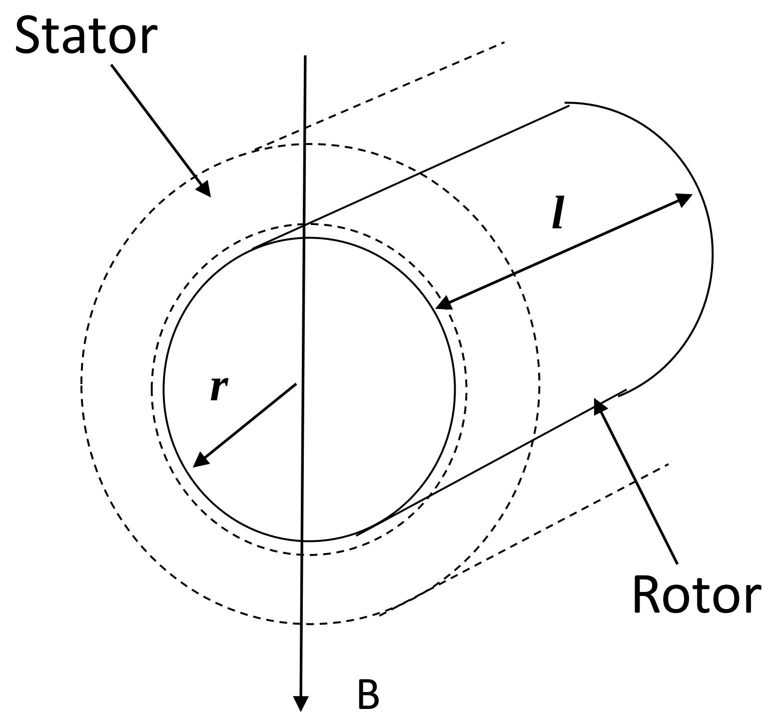
Two electromagnetic conversion phenomena:

- **Induced voltage:** when a conductor moves in a magnetic field, voltage is induced in the conductor. (**Generator action**)
- **Force and developed torque:** when a current-carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. (**Motor action**)

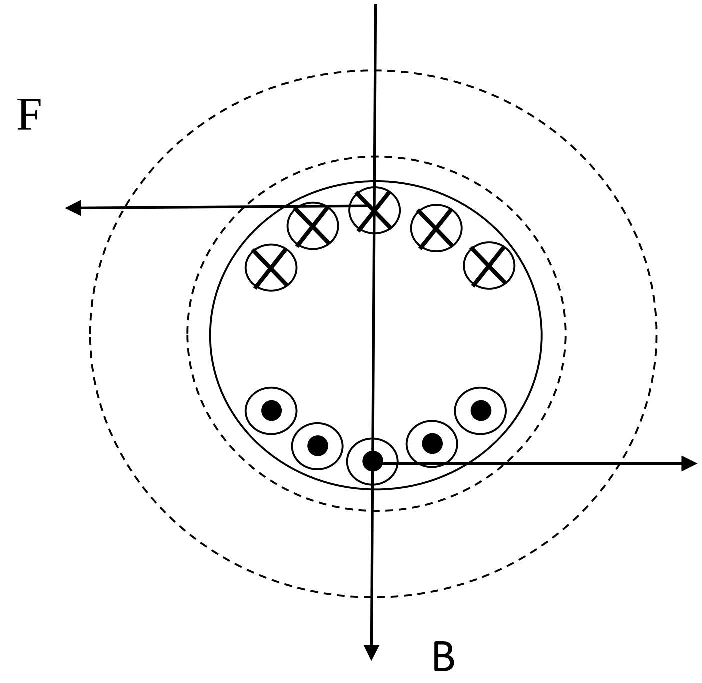


Rotating Electrical Machines

Fixed stator: This (normally) sets up magnetic field.
Rotating rotor: This (normally) carries currents (either supplied from a power source or (induced).

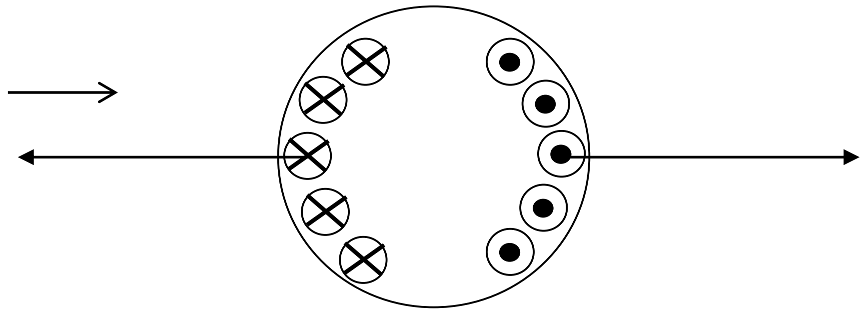


Stator sets up B field



Current (I) flows in rotor.
Force on conductor: $F=IBl$
Torque: $T=Fr$

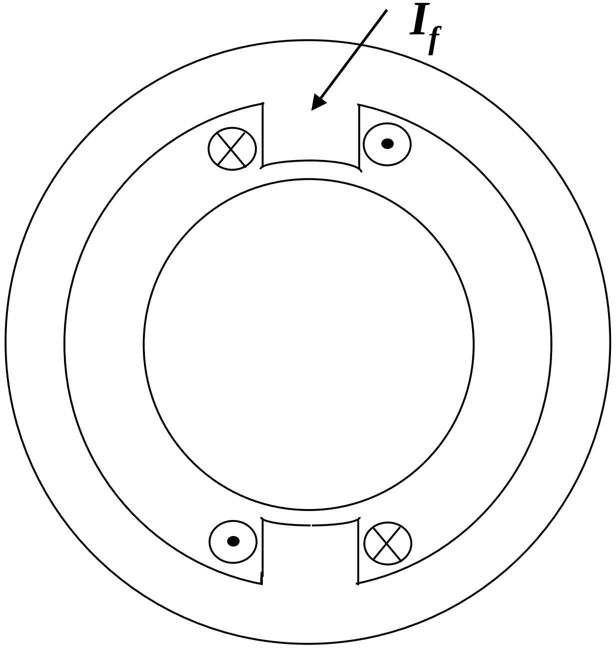
Rotor rotates (anticlockwise)
1/4 revolution later we have

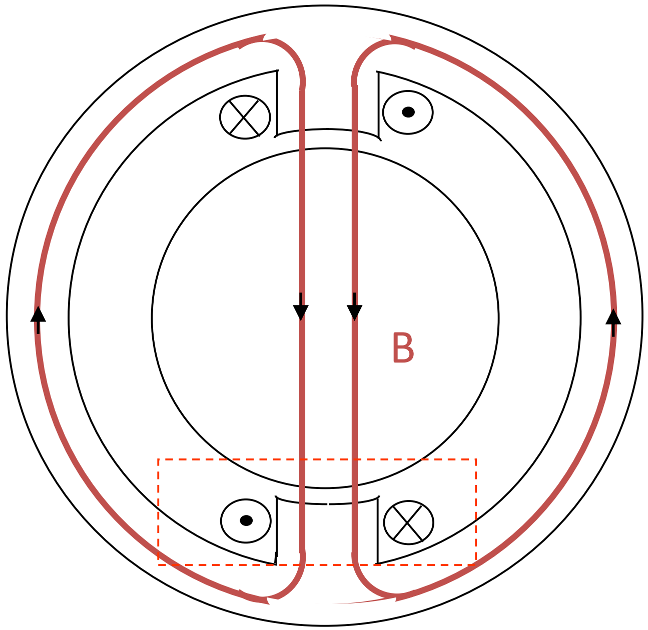


For a particular machine:

- How B is produced?
- How do we get currents in rotor to produce continuous torque?

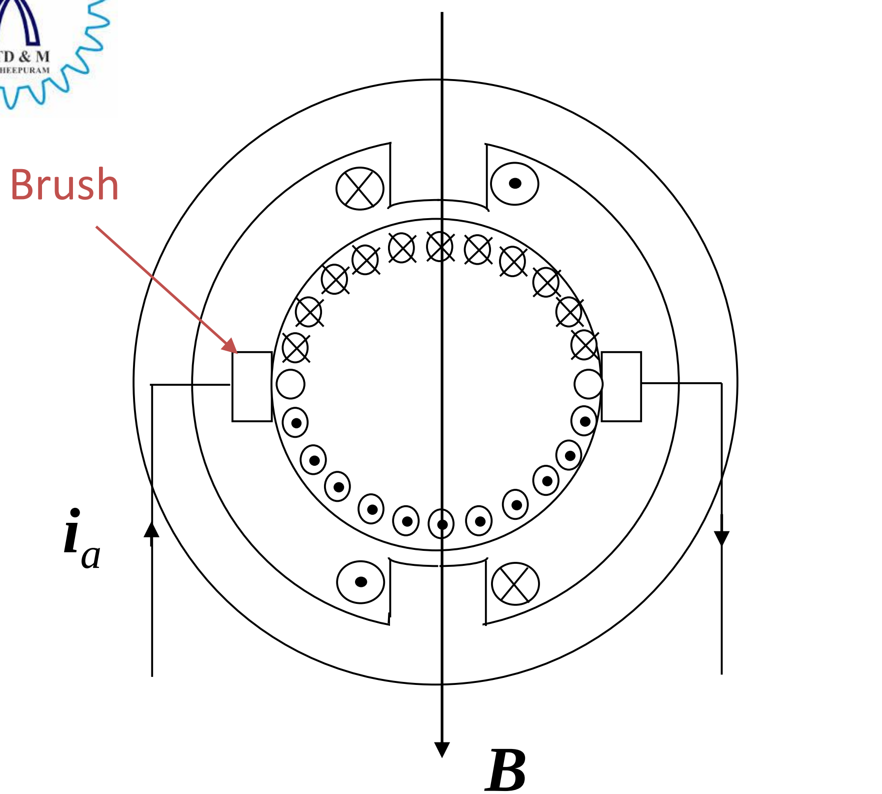
Principle of Operation of DC Motors

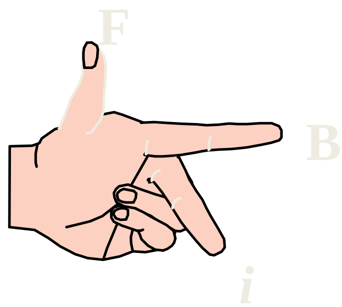


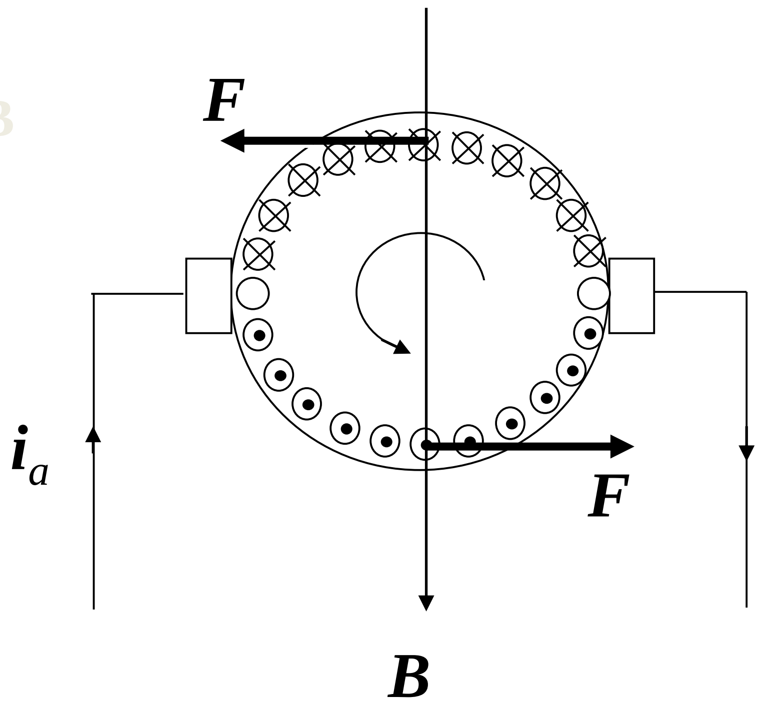


Field winding (2 coils in series as shown) carries field current I_f

I_f sets up magnetic field shape as shown





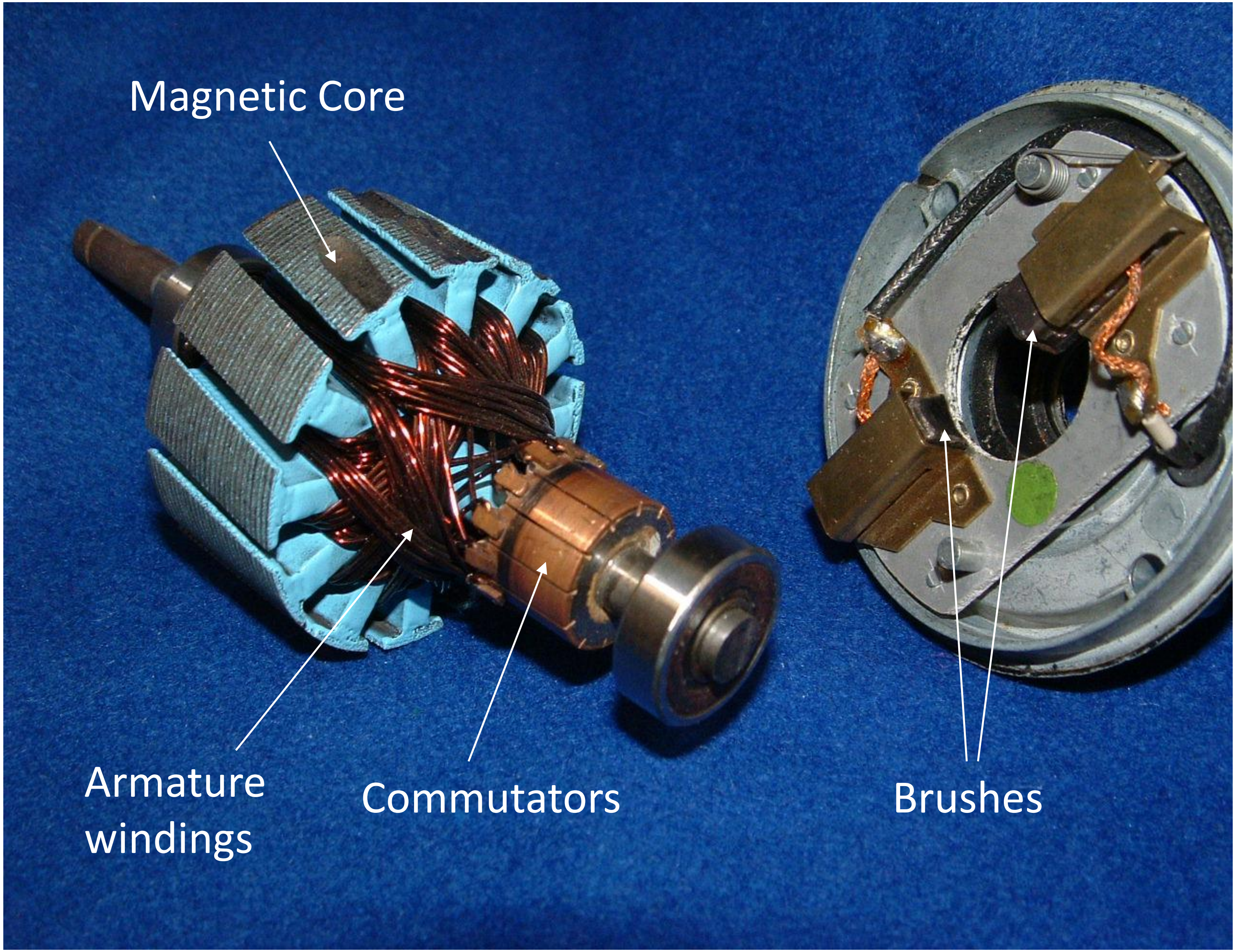


Rotor carries "armature" winding. *Armature current i_a fed to through commutator and brushes.*

This feeds i_a to different windings as rotor rotates. Result is that the current distribution *in space* is fixed.

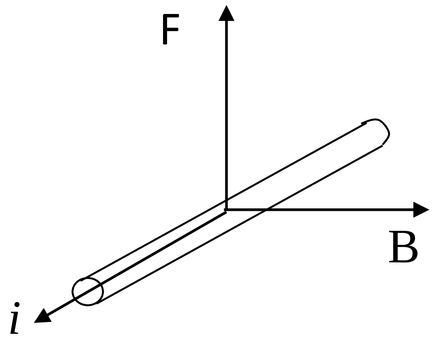
Force = $i_a B.l$ always in direction shown since current distribution is fixed in space.
Torque = Force x radius
Torque = $i_a B.l\ r$

Permanent Magnet DC Motor

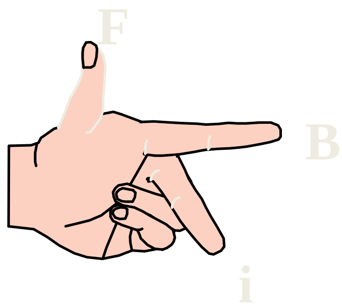


Contd.,

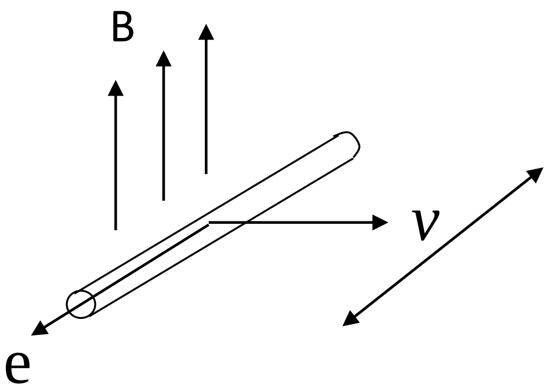
1. Force on conductor carrying I in field B



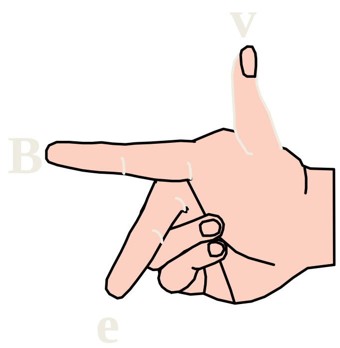
$F = I\,B\,l$
l = length of conductor in field



2. Voltage (emf) induced in conductor traveling in field B



$e = v\,B\,\boldsymbol{l}$
e = voltage induced along length $\boldsymbol{l}$

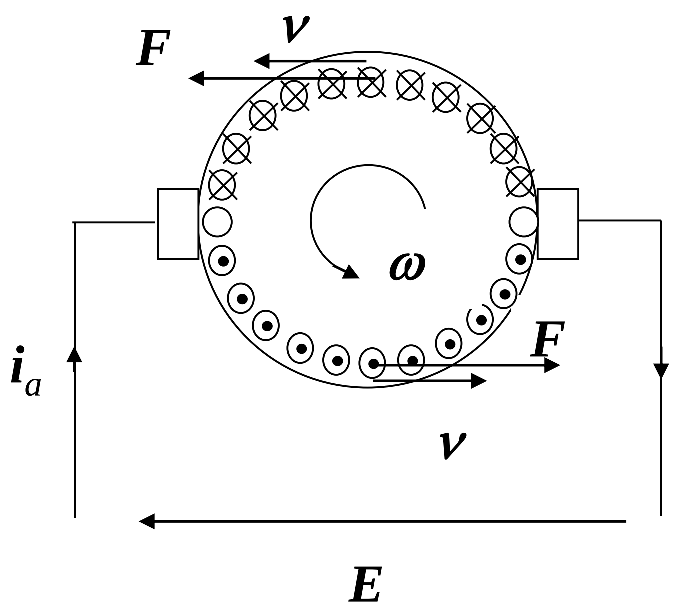


EMF induced in DC- Motor

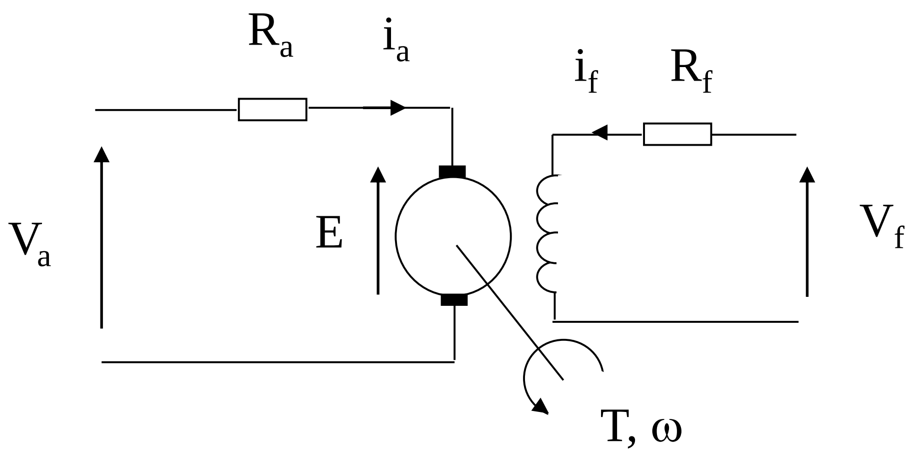
- e induced in each conductor
- N conductors are all in series
- Therefore total "back-emf " induced in armature winding is E = Ne
- $E = N\boldsymbol{\nu} B \boldsymbol{l}$

  $\boldsymbol{\nu} = \omega r$ (ω = angular velocity, rad/s)

  $E = N\omega r B\boldsymbol{l} = k\omega$
- **E is proportional to ω**



Equivalent circuit of a separately excited DC motor



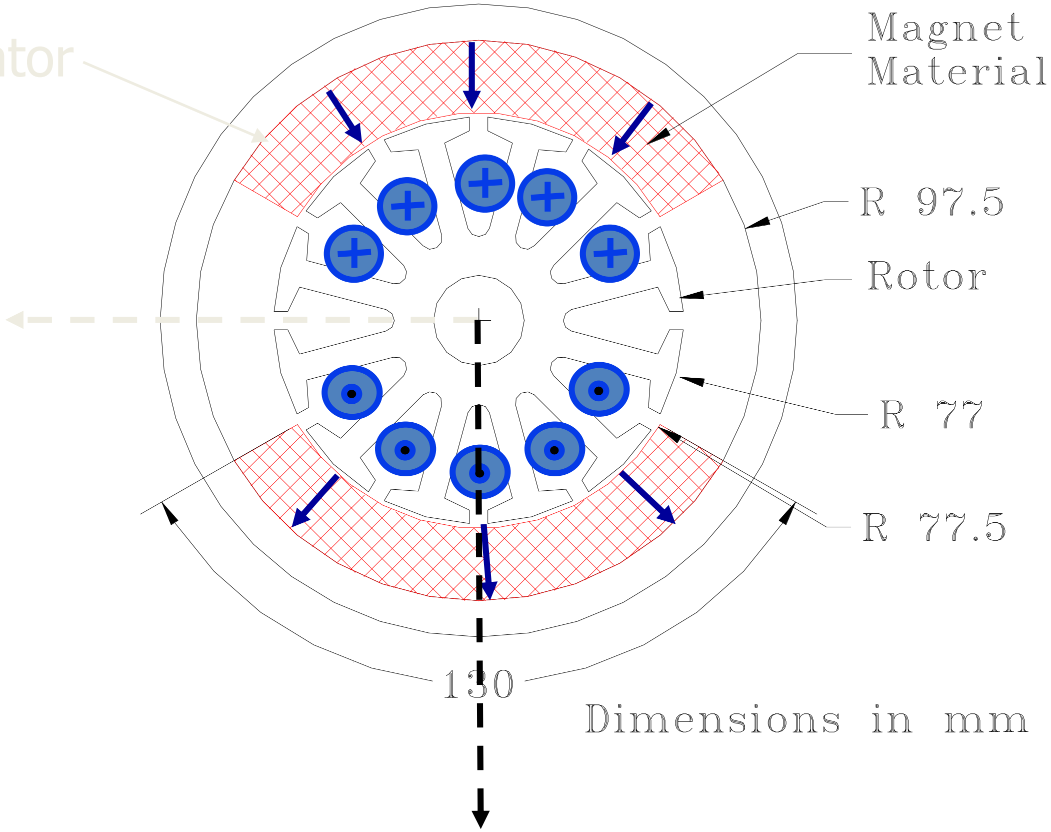
Cross section of a permanent magnet DC motor

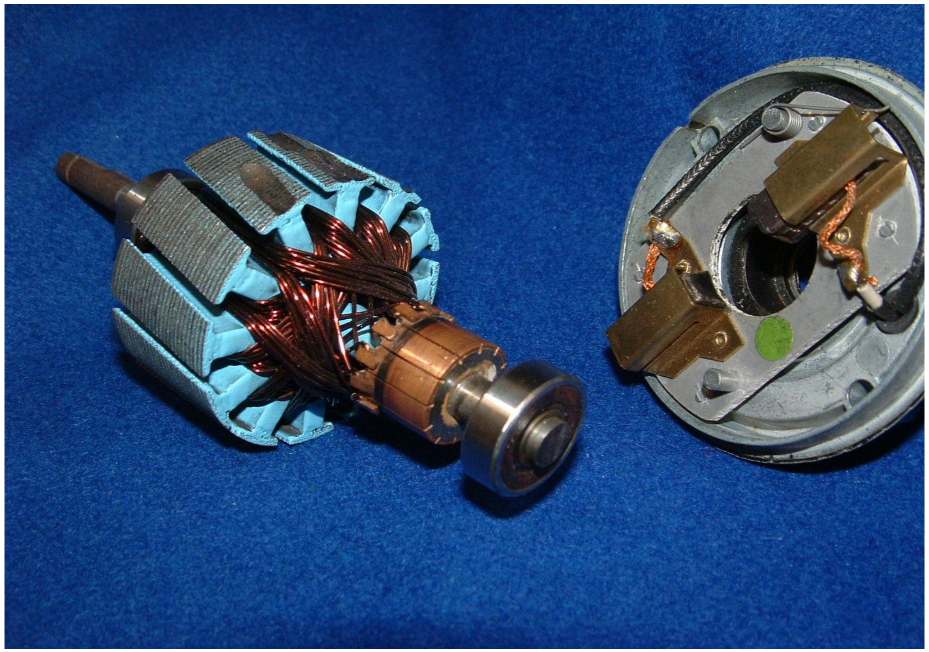
Stator

Direction of B

Winding Axis of Rotor

Note it is perpendicular to the magnet flux



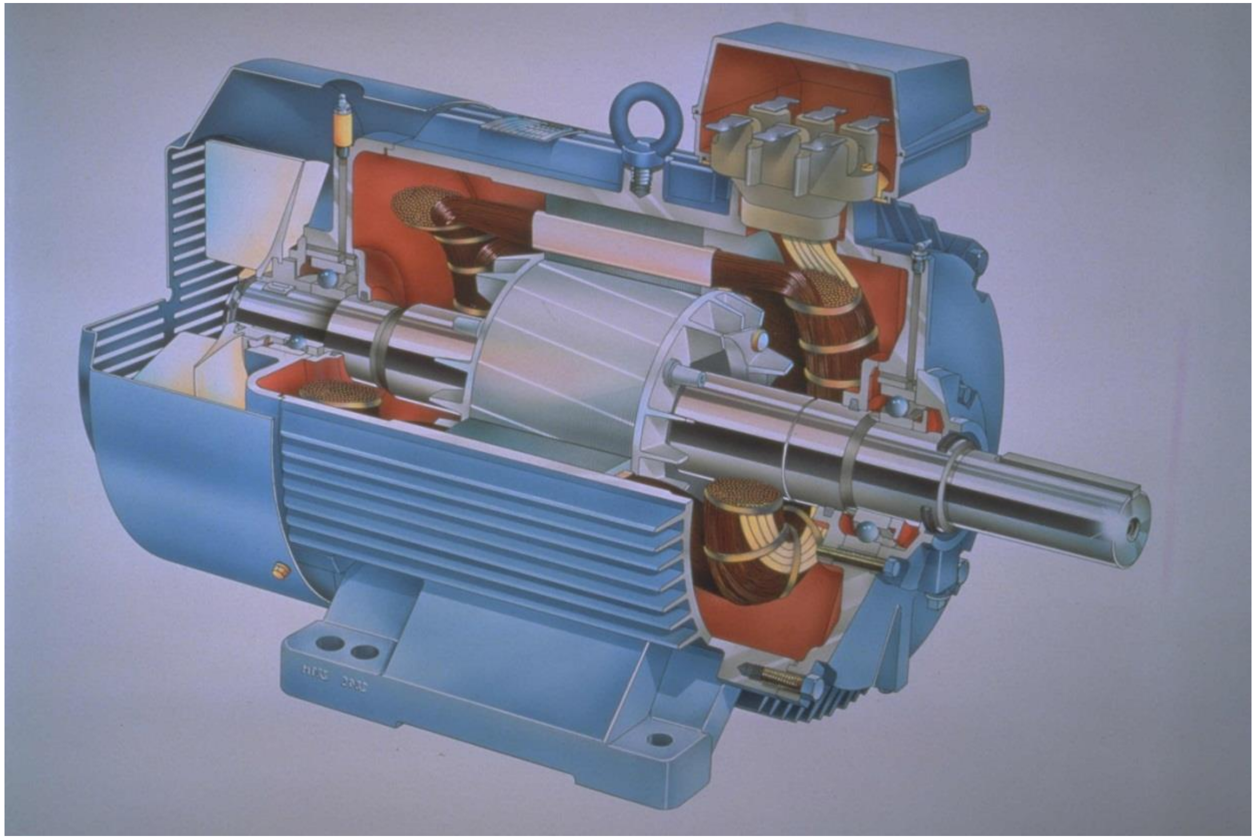


Rotor with Commutator and End Frame with Brushes of Simple Permanent Magnet Machine

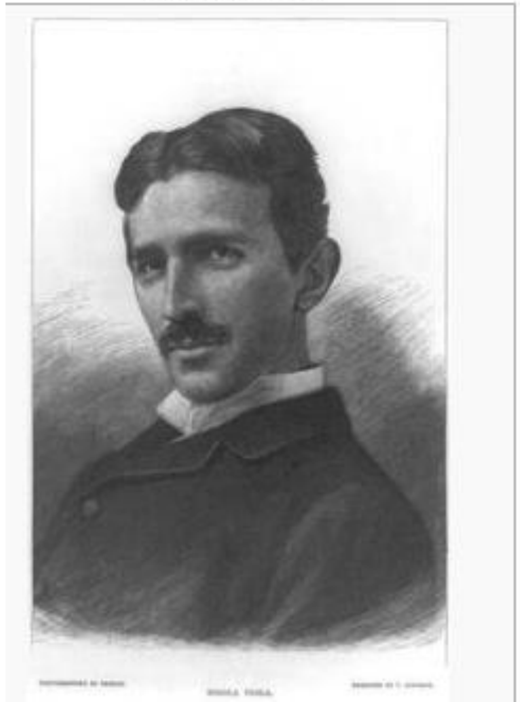
Characteristics of DC Machine

- I_f sets field B. This normally constant
- Torque = $k\ i_f\ i_a$. If i_a supplied form separate supply then i_a controls torque. (If we want more torque then we inject more i_a)
- Carbon brushes required – sparking is common and brushes wear out – regular maintenance
- Quite expensive due to commutator and armature winding construction.
- Very common for small motors (tape recorders, CDs, drills, printers, photocopiers etc. In these cases, B is often supplied by a permanent magnet.
- Can put armature and field winding in series and run it directly off single phase household mains. This called a "universal motor". Used in lawn mowers, washing machines, cheap tools, hairdryers etc.

Three-Phase Induction Motors



Nikola Tesla



Introduction

Three-phase induction motors are the most common and frequently encountered machines in industry (***<u>Workhorse of Industry</u>***)

simple design, rugged, low-price, easy maintenance

wide range of power ratings: fractional horsepower to 10 MW

run essentially as constant speed from no-load to full load

Its speed depends on the frequency of the power source

- not easy to have variable speed control
- requires a variable-frequency power-electronic drive for optimal speed control

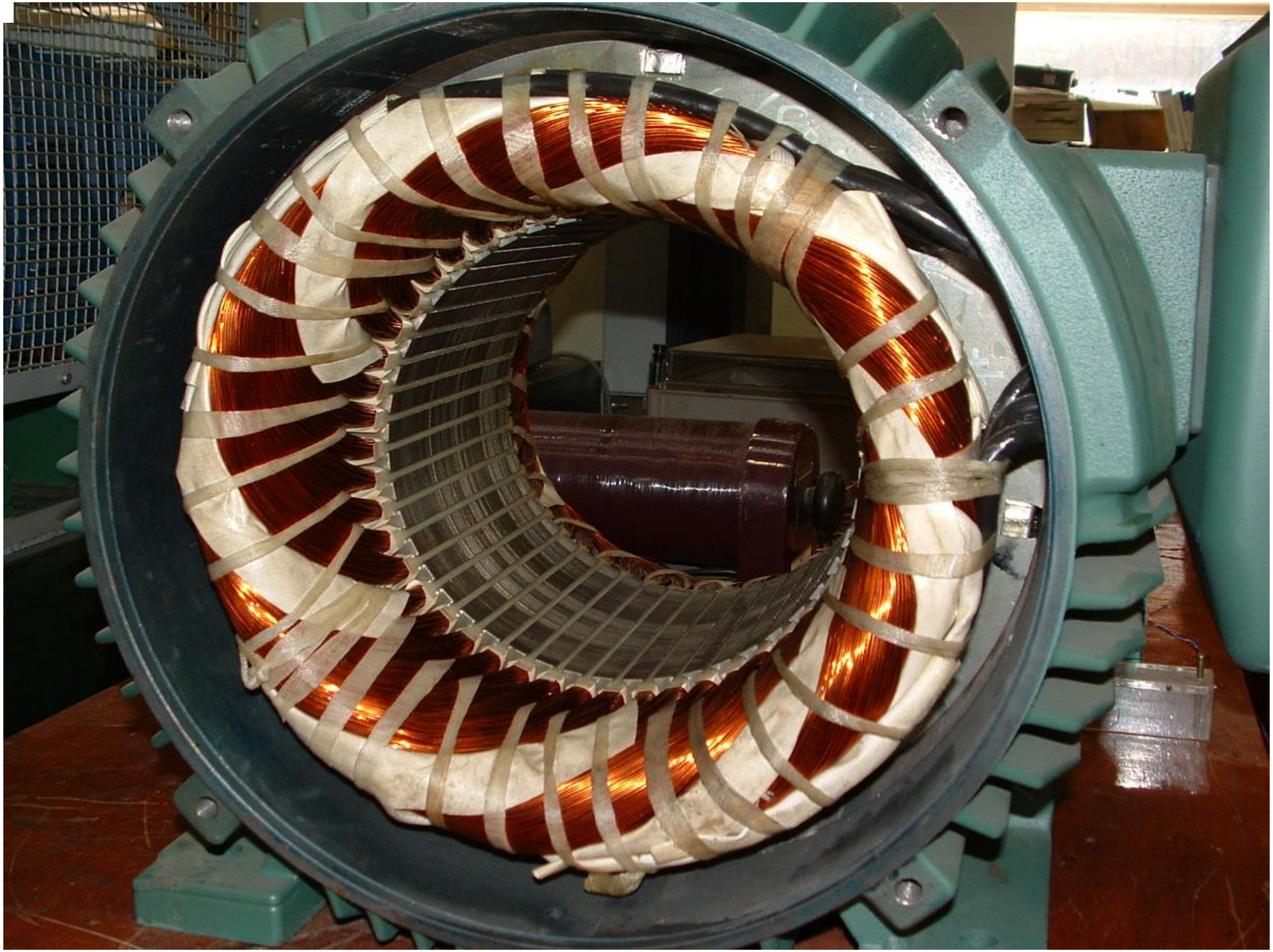
Construction

An induction motor has two main parts

-a stationary stator

- consisting of a steel frame that supports a hollow, cylindrical core
- core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding
- Three-phase stator winding is either Star or Delta connected and coils are made of Copper conductors.

3-Phase Stator Winding



Rotor Construction

-a revolving rotor

composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding

one of two types of rotor windings

conventional 3-phase windings made of insulated wire (wound-rotor)

» similar to the winding on the stator

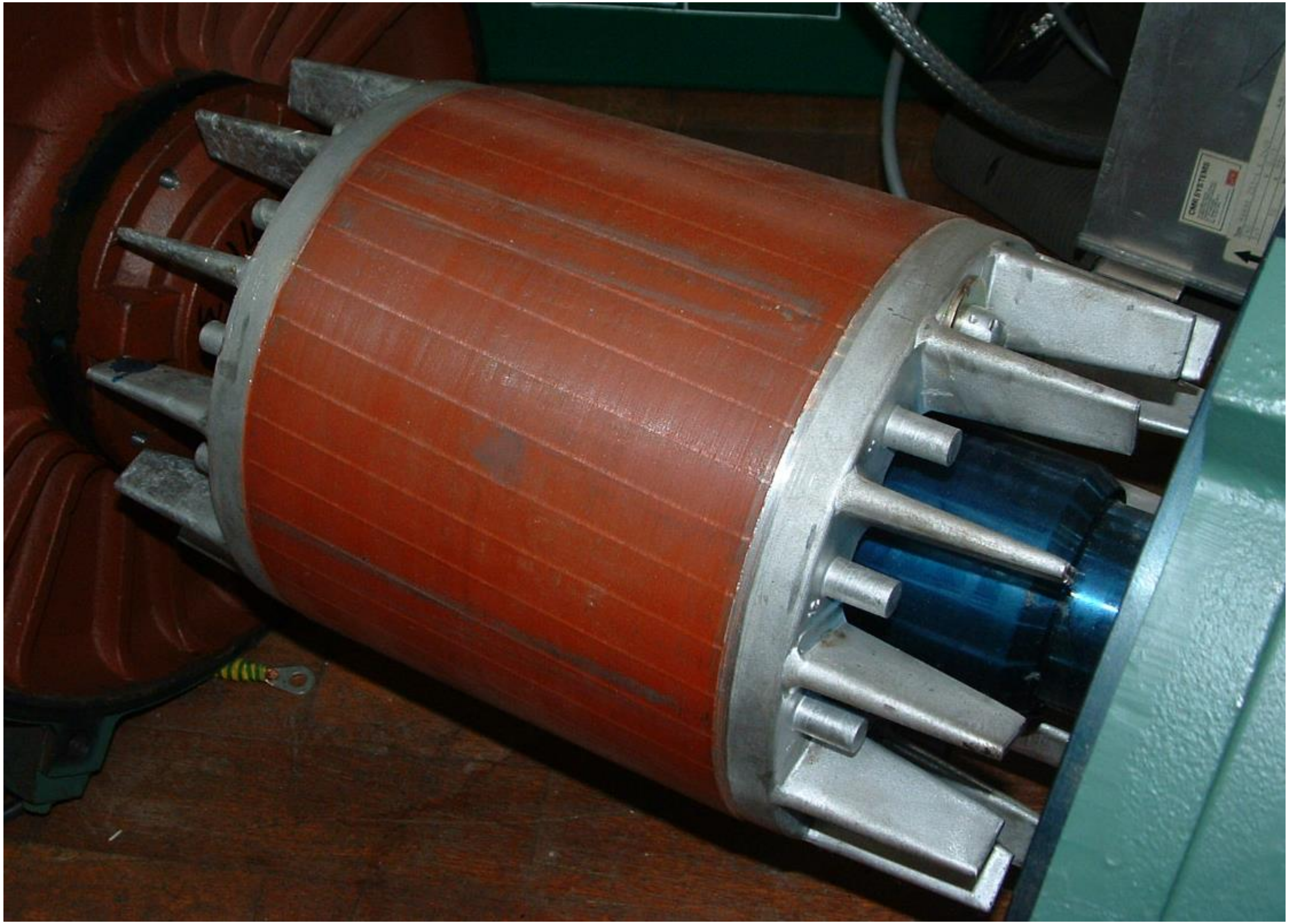
aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

Two basic design types depending on the rotor design

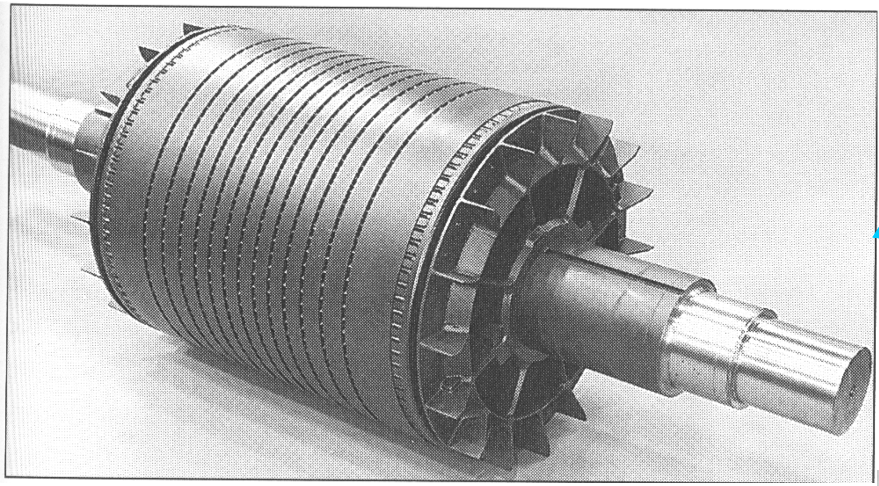
squirrel-cage: conducting bars laid into slots and shorted at both ends by shorting rings.

wound-rotor (Slip-Ring Rotor): complete set of three-phase windings exactly as the stator. Usually Y-connected, the ends of the three rotor wires are connected to 3 slip rings on the rotor shaft. In this way, the rotor circuit is accessible.

Rotor Showing Single Bars Short Circuited by 'End Rings'

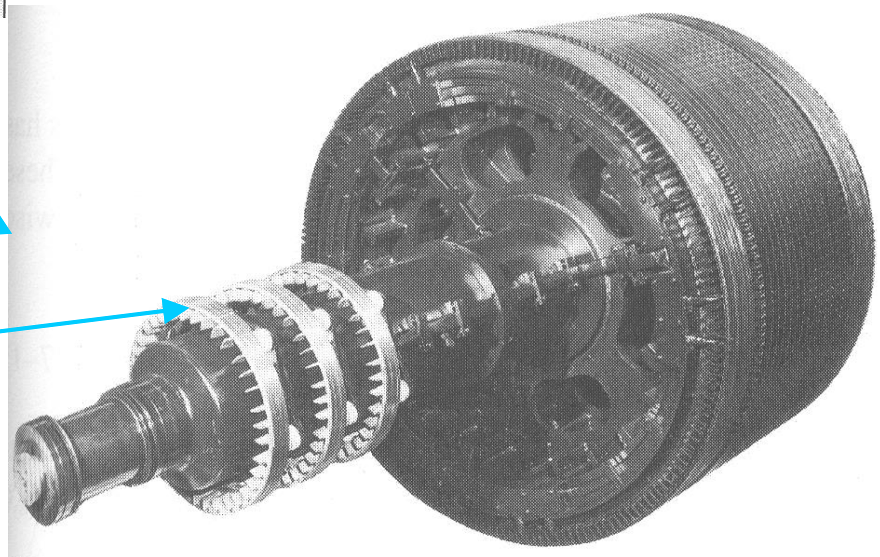


Rotor Construction



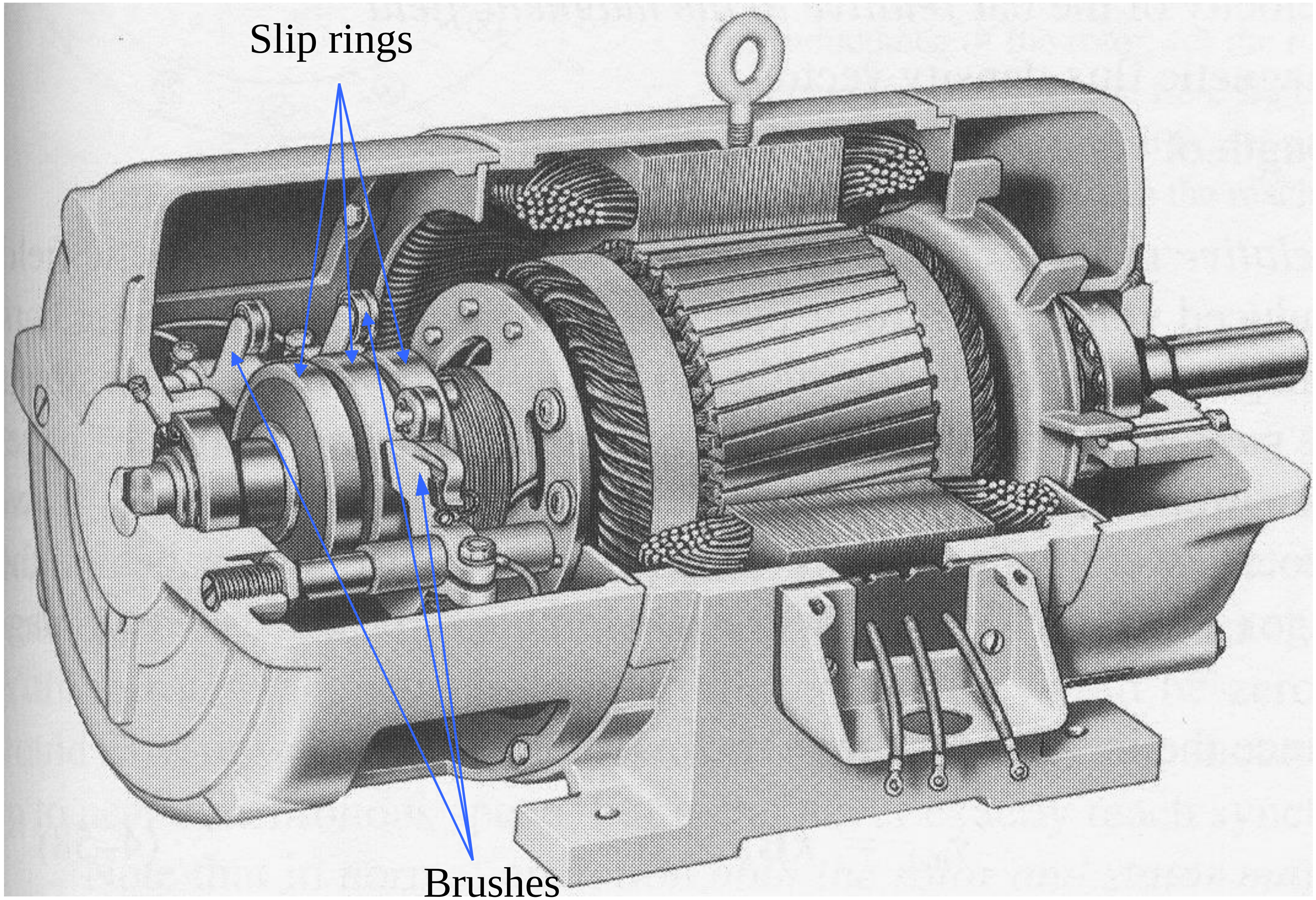
Squirrel cage rotor

Wound rotor



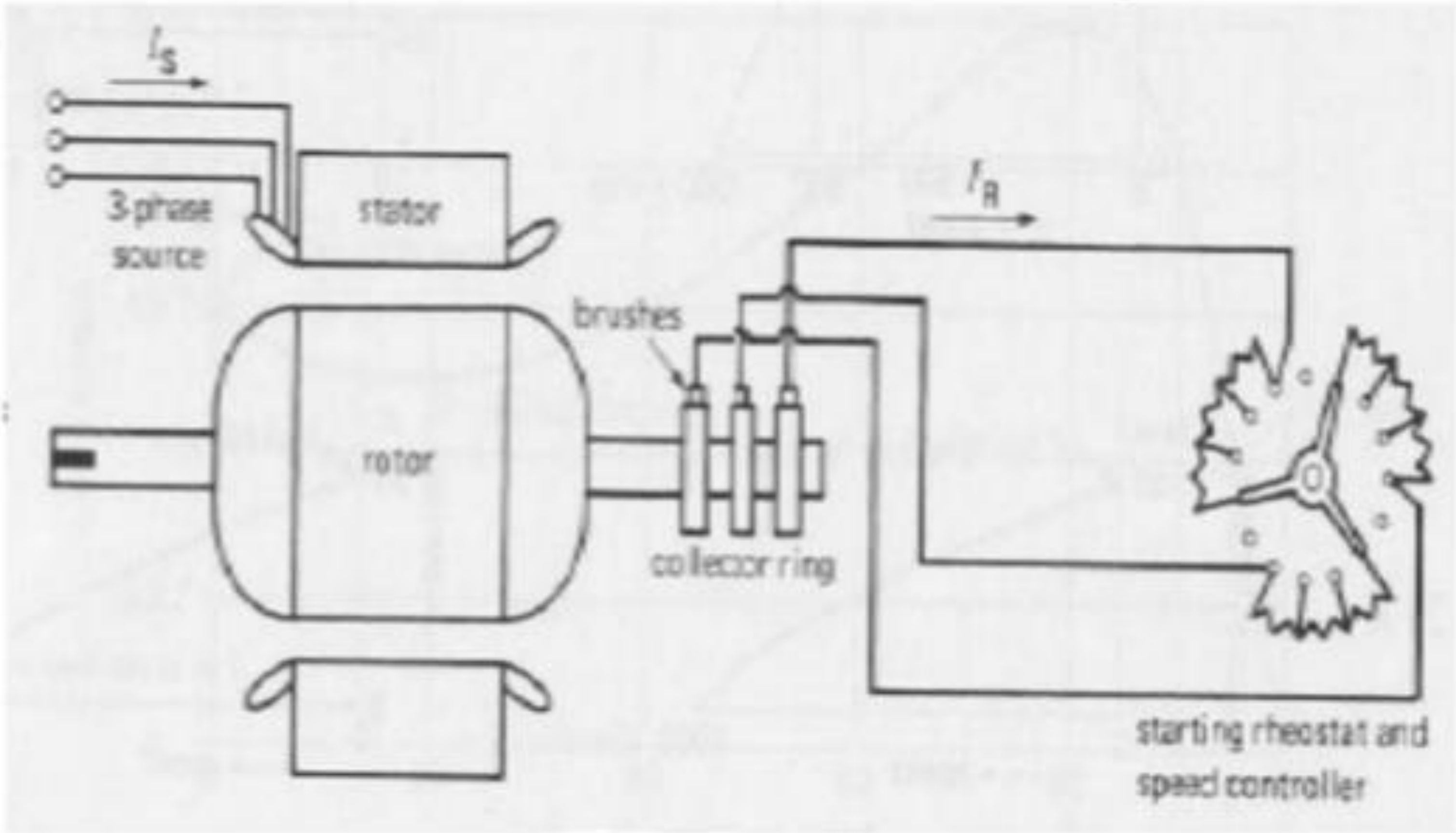
Notice the
slip rings

Slip Ring Rotor (Wound Rotor)



Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings

Slip Ring Induction Motor



Advantages of Slip Ring Induction Motor

- *The main advantage of a slip ring induction motor is that its speed can be controlled easily.*

- *It has a high starting torque when compared to squirrel cage induction motor. Approximately 200 - 250% of its full-load torque.*

- *A squirrel cage induction motor takes 600% to 700% of the full load current, but a slip ring induction motor takes a very low starting current approximately 250% to 350% of the full load current*

Advantages of Slip Ring Induction Motor

- *Initial and maintenance cost is more compared to squirrel cage motor because presence of slip rings, brushes, short circuiting devices etc.*
- *Speed regulation is poor when operated with external resistances in rotor circuit.*
- *Efficiency and power factor of slip ring motor is lower compared to squirrel cage induction motor.*
- *Sensitivity to fluctuations in supply voltage*

How Magnetic Field B is Produced

Electrical frequency is $\omega_e = 2\pi f_e$ elec. rads/s

B rotates in space at $\omega_s = \frac{2}{P}\omega_e$ mech. rads/s

ω_s is called the synchronous speed. Or $n_s = 120f/P$ in rpm.

The greater no. of poles, the slower the synchronous speed

f_e	$\omega_e = 2\pi f_e$	P	ω_s (rad/s)	n_s (rpm)
50	314	2	314	3000
50	314	4	157	1500
50	314	6	105	1000
50	314	8	78	750
50	314	10	63	600

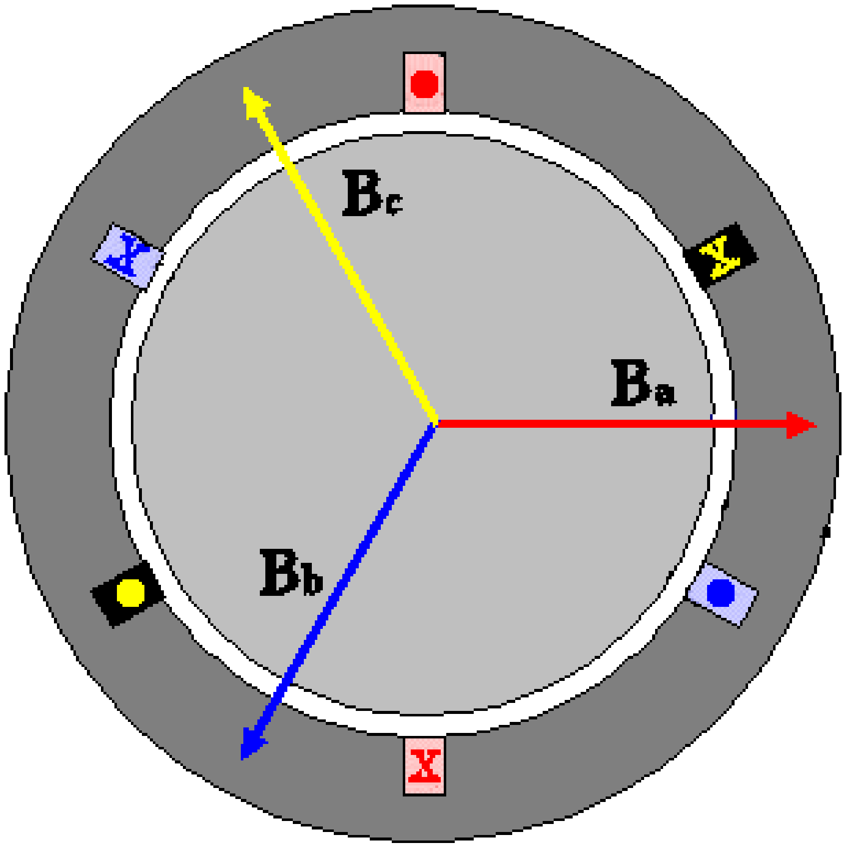
Rotating Magnetic Field

Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source

A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120 f_e}{P} \quad rpm$$

Where f_e is the supply frequency and P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

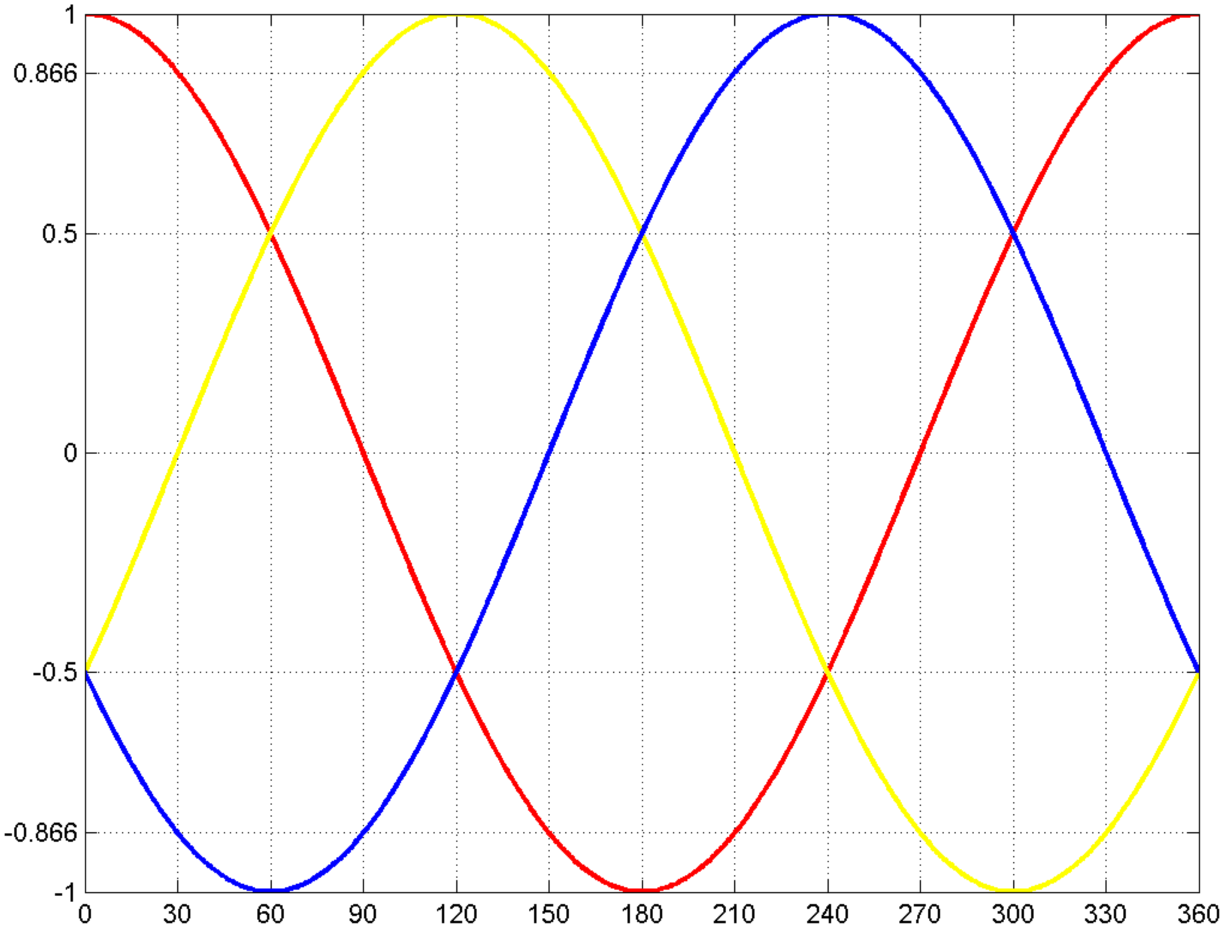


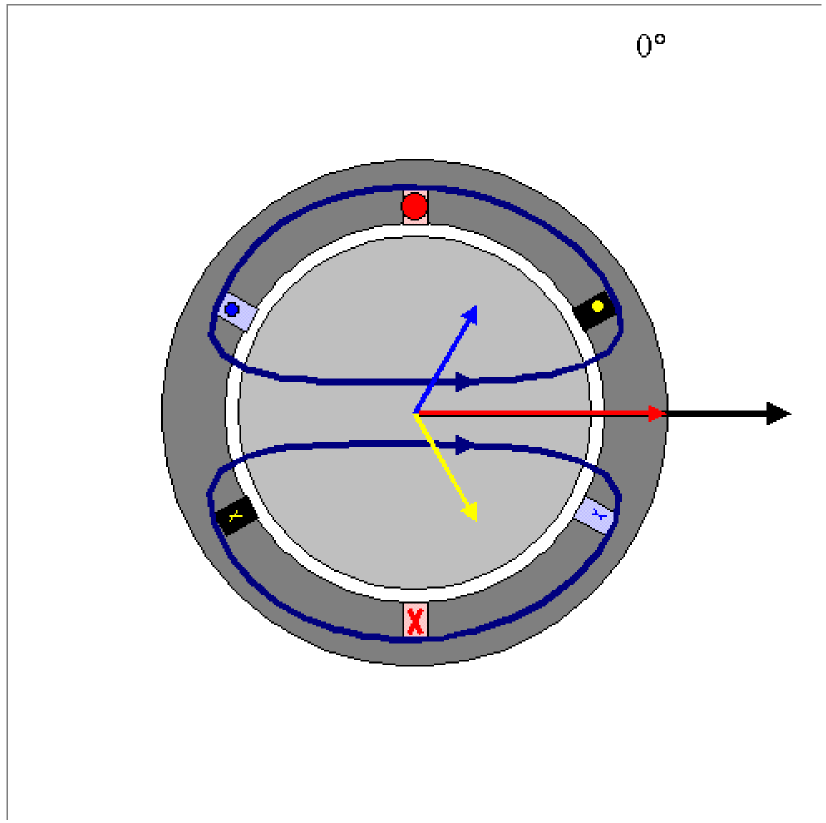
1 rpm = 2π radians/minute = $2\pi/60$ radian/second (rad-s^{-1})

Therefore 1 rad-s^{-1} = $60/2\pi \approx 10$ rpm

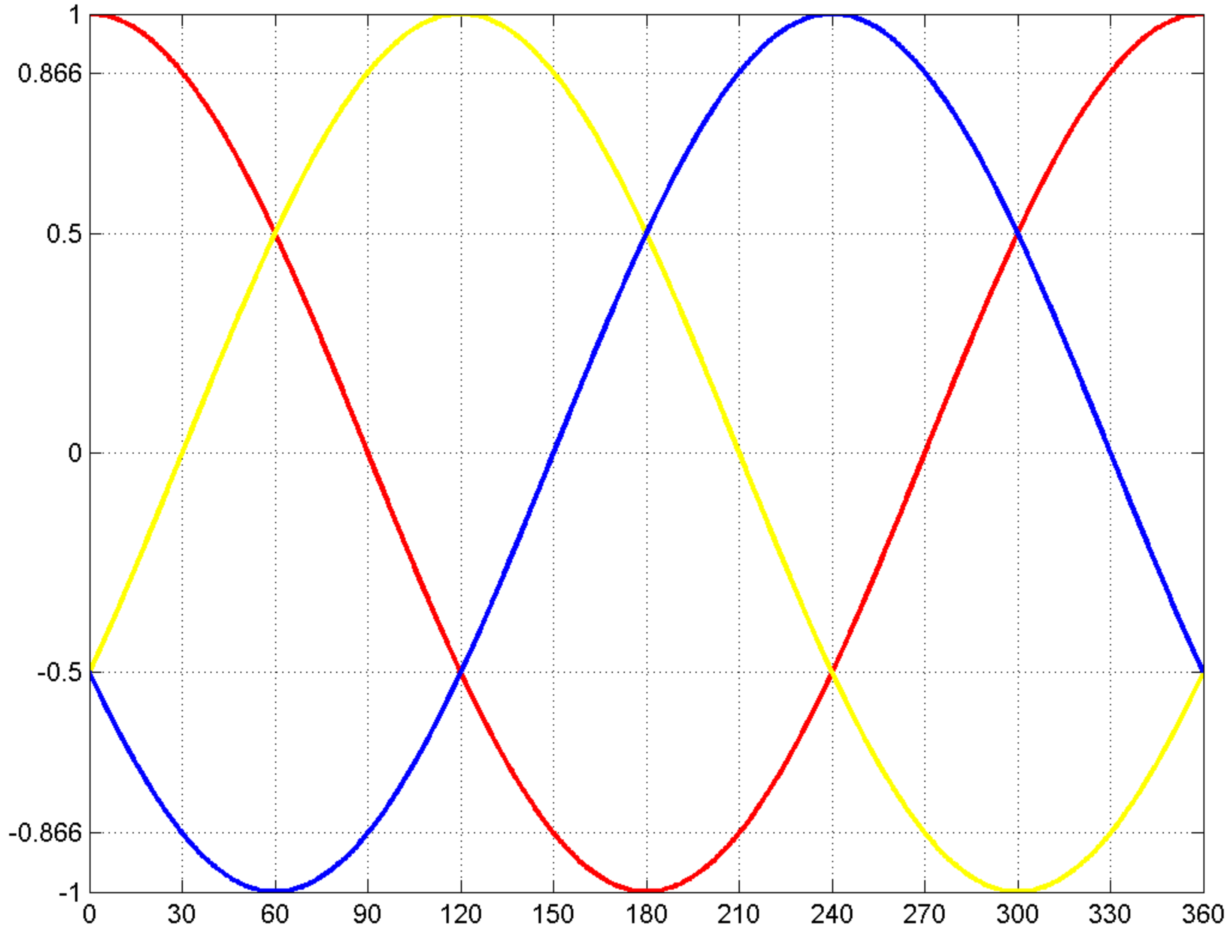
Stator windings of an Induction Machine (IM) can only be wound in one way. P is fixed for an individual machine. An IM can either be a 2-pole machine, or a 4-pole machine or ….etc

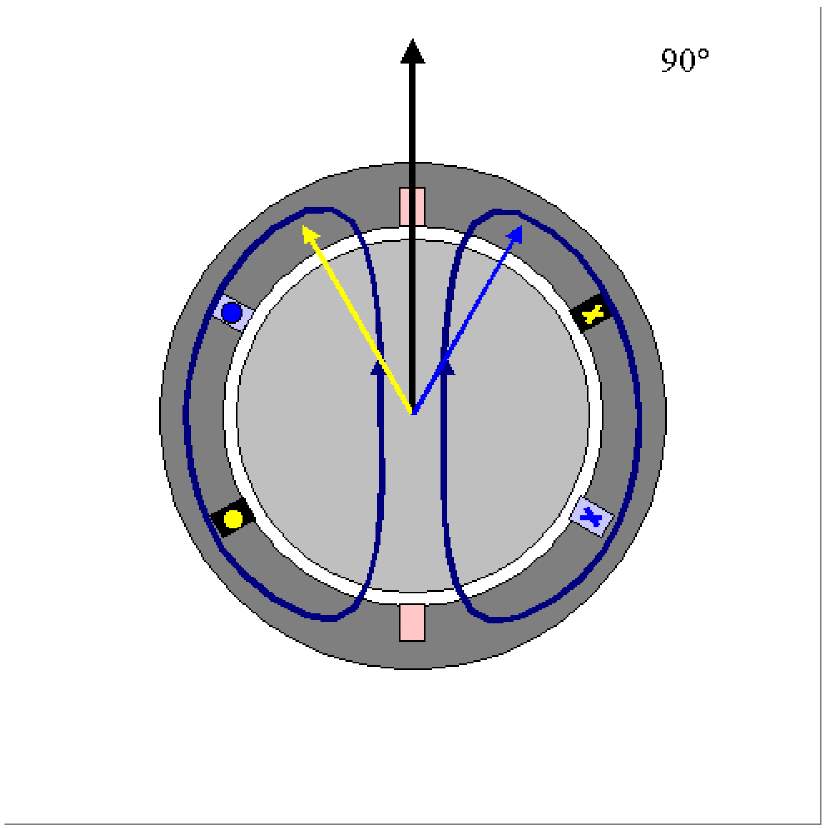
Rotating Magnetic Field





Rotating Magnetic Field





Rotating Magnetic Field

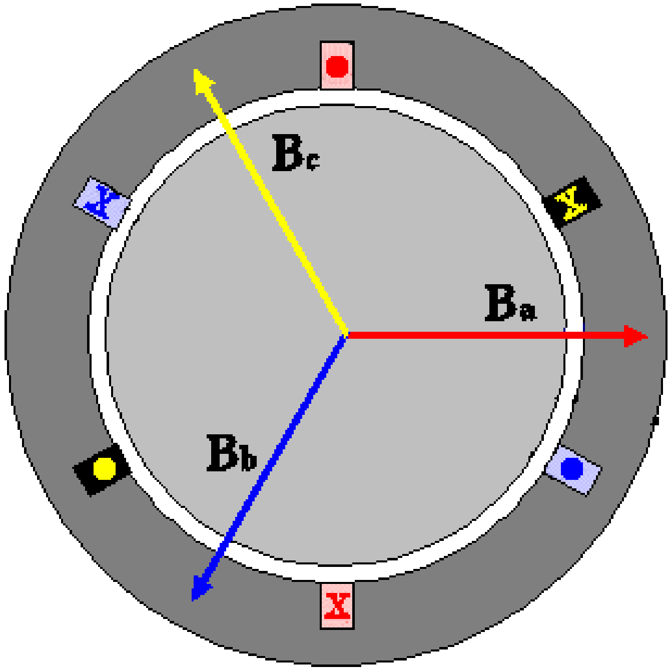
$$B_{net}(t) = B_a(t) + B_b(t) + B_c(t)$$

$$= B_M \sin(\omega t)\angle 0° + B_M \sin(\omega t - 120°)\angle 120° + B_M \sin(\omega t - 240)\angle 240°$$

$$= B_M \sin(\omega t)\hat{\mathbf{x}}$$

$$-[0.5B_M \sin(\omega t - 120°)]\hat{\mathbf{x}} - [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)]\hat{\mathbf{y}}$$

$$-[0.5B_M \sin(\omega t - 240°)]\hat{\mathbf{x}} + [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)]\hat{\mathbf{y}}$$

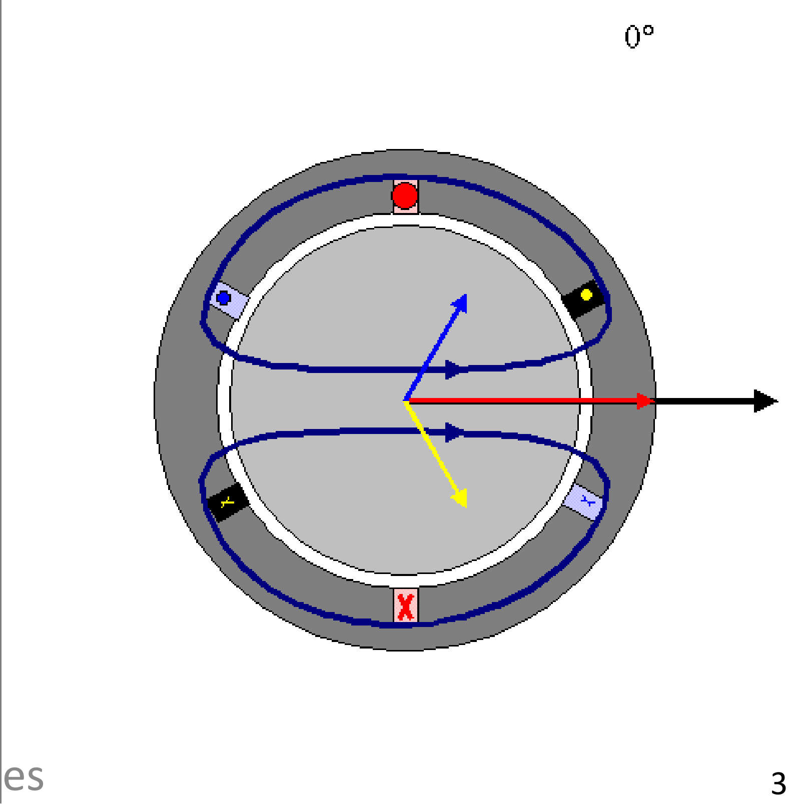


Rotating Magnetic Field

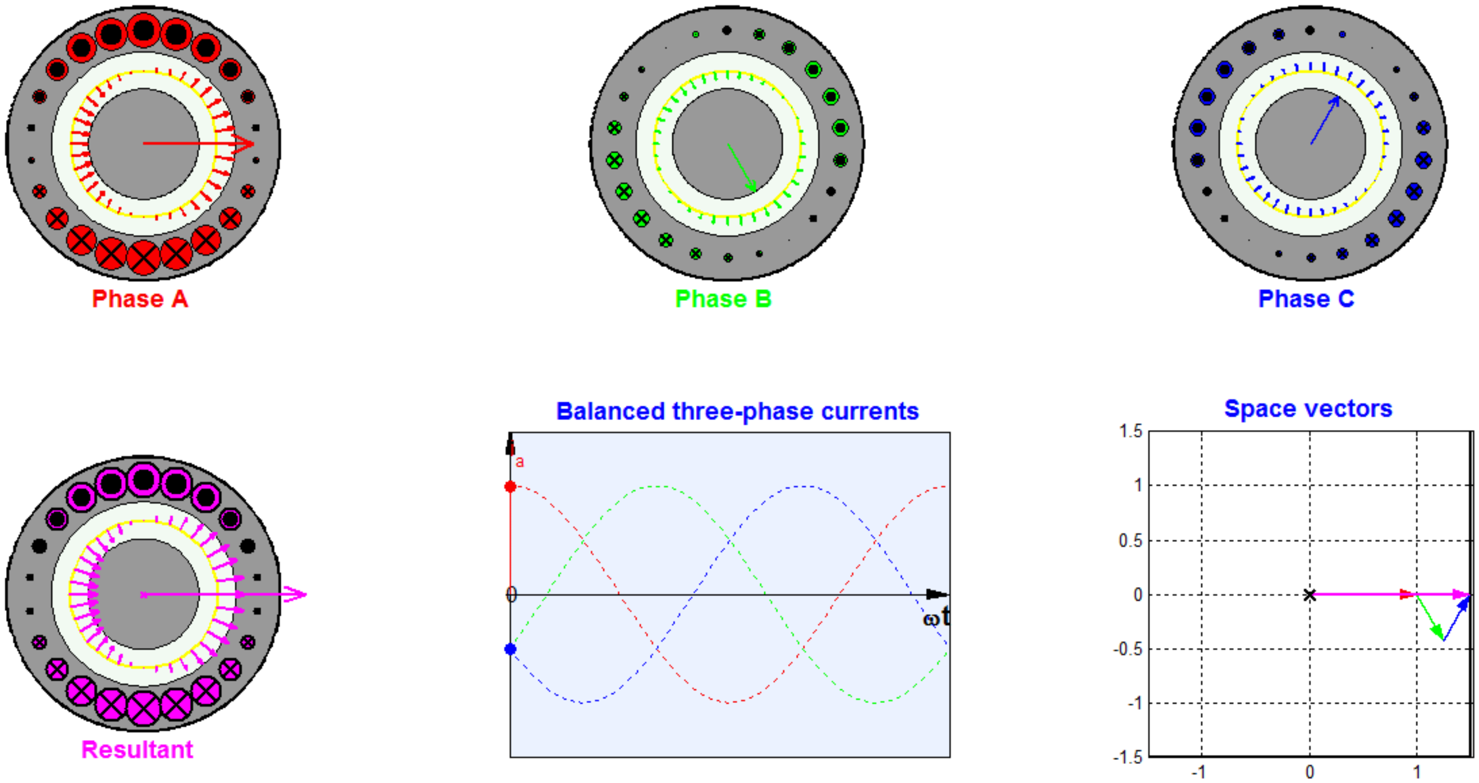
$$B_{net}(t) = [B_M \sin(\omega t) + \frac{1}{4} B_M \sin(\omega t) + \frac{\sqrt{3}}{4} B_M \cos(\omega t) + \frac{1}{4} B_M \sin(\omega t) - \frac{\sqrt{3}}{4} B_M \cos(\omega t)]\hat{\mathbf{x}}$$

$$+[-\frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t) + \frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t)]\hat{\mathbf{y}}$$

$$= [1.5 B_M \sin(\omega t)]\hat{\mathbf{x}} - [1.5 B_M \cos(\omega t)]\hat{\mathbf{y}}$$



Rotating Magnetic Field



Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

- Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

At what speed will the IM run?

Can the IM run at the synchronous speed, why?

- If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
- When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous **speed**
- **The difference between the motor speed and the synchronous speed is called the *Slip***

$$\boxed{n_{slip} = n_{sync} - n_m}$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}}}$$

Where s is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Induction Motors and Transformers

Both IM and transformer works on the principle of induced voltage

- **Transformer**: voltage applied to the primary windings produce an induced voltage in the secondary windings
- **Induction motor**: voltage applied to the stator windings produce an induced voltage in the rotor windings
- The difference is that, in the case of the induction motor, the secondary windings can move
- Due to the rotation of the rotor (the secondary winding of the IM), the induced voltage in it does not have the same frequency of the stator (the primary) voltage

Frequency

The frequency of the voltage induced in the rotor is given by

$$f_r = \frac{P \times n}{120}$$

Where f_r = the rotor frequency (Hz)
P = number of stator poles
n = slip speed (rpm)

$$f_r = \frac{P \times (n_s - n_m)}{120}$$

$$= \frac{P \times sn_s}{120} = sf_e$$

Frequency

What would be the frequency of the rotor's induced voltage at any speed n_m?

$$\boxed{f_r = s\ f_e}$$

When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency
On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero

Torque

- While the input to the induction motor is electrical power, its output is mechanical power and for that we should know some terms and quantities related to mechanical power
- Any mechanical load applied to the motor shaft will introduce a Torque on the motor shaft. This torque is related to the motor output power and the rotor speed

$$\boxed{\tau_{load} = \frac{P_{out}}{\omega_m} \quad N.m}$$

and

$$\boxed{\omega_m = \frac{2\pi n_m}{60} \quad rad / s}$$

Horse power

- Another unit used to measure mechanical power is the Horse power (HP)
- It is used to refer to the mechanical output power of the motor
- Since we, as an electrical engineers, deal with watts as a unit to measure electrical power, there is a relation between horse power and watts

$$\boxed{hp = 746\ watts}$$

Example

1. A 208-V, 10 HP, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

   1. What is the synchronous speed of this motor?
   2. What is the rotor speed of this motor at rated load?
   3. What is the rotor frequency of this motor at rated load?
   4. What is the shaft torque of this motor at rated load?

Solution

1.

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

2.

3.

$$n_m = (1 - s) n_s$$

4.

$$= (1 - 0.05) \times 1800 = 1710\ rpm$$

$$f_r = s f_e = 0.05 \times 60 = 3 Hz$$

$$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$

$$= \frac{10\, hp \times 746\, watt / hp}{1710 \times 2\pi \times (1/60)} = 41.7\ N.m$$

Example

2. A 3-phase cage rotor induction motor with 3 pole-pairs is fed from 50Hz supply and runs at 960 rpm generating a torque of 40 Nm. Determine:

(i) The angular velocity (in rpm) of the rotating field produced by the stator winding,

(ii) The slip at which the machine is operating,

(iii) The frequency of the rotor currents flows,

(iv) The mechanical power produced by the machine.

(**Ans:** 1000 rpm, 4%, 2 Hz, 4.02 kW)

Solution

1) Given.

No. of pole pair = 3 [6 poles]

frequency [f] = 50 Hz

Motor speed (A. n_r) = 960 rpm

Torque (T_m) = 40 Nm.

To find

(i) Speed of Revolving mag. field of of stator (n_s) $= \frac{120f}{P}$

$= \frac{120 \times 50}{6} = 1000$ rpm

(ii) Slip (S) $= \frac{n_s - n_r}{n_s} \times 100$ %

$= \frac{1000 - 960}{1000} \times 100 = 0.04$

$\boxed{S = 4\%}$

(iii) frequency of rotor current (f_r) = Sf

$= 0.04 \times 50$

$\boxed{f_r = 2 \text{ Hz}}$

(iv) $P_{mech} = \frac{2\pi n_r T_m}{60}$ watts

$= \frac{2 \times \pi \times 960 \times 40}{60}$

$\boxed{P_{mech} = 4.02 \text{ KW}}$

Example

3. If the e.m.f. in the stator of an 8-pole induction motor has a frequency of 50 Hz and that in the rotor of 1.5 Hz , at what speed is the motor running and what is the slip.

(i) n_r (ii) slip (s)

We know that, $f_r = s f$

Hence $s = \frac{f_r}{f} = \frac{1.5}{50} = 0.03$

$$\boxed{s = 3\%}$$

(ii) $0.03 = \frac{n_s - n_r}{n_s}$ $\left[\therefore n_s = \frac{120 \times 50}{8} = 750 \text{ rpm} \right]$

$0.03 = \frac{750 - n_r}{750}$; $\boxed{n_r = 727.5 \text{ rpm}}$

Example

4. A 6-pole, 3-pha, 50 Hz induction motor develops a full load torque of 150 Nm when the rotor current makes 120 complete cycles per minute. Determine the shaft power output.

Given:

$P = 6$

$f = 50\ Hz$

$T_m = 150\ Nm$

$f_r = 2\ Hz\ (120\ \text{cycles}/\text{min})$

Find P_{shaft}

$\rightarrow P_{shaft} = \frac{2\pi N_r T_m}{60} \times 60$

$n_s = \frac{120 f}{P} = \frac{120 \times 50}{6} = 1000\ rpm$

$S = \frac{f_r}{f} = \frac{2}{50} = 0.04$

$n_r = \text{rotor speed} = [n_s - s \times n_s]$

$= [1000 - 1000 \times 0.04]$

$\boxed{n_r = 960\ rpm}$

$\therefore P_{shaft} = \frac{2 \times \pi \times 960 \times 150}{60}$

$\boxed{P_{shaft} = 15.08\ KW}$

Practice Problems

A 3-phase 50 Hz, 400 V induction motor has 6 poles and operates with a slip of 3% at a certain load. Determine;

(i) the speed of the rotor.

(ii) the frequency of rotor current.

(iii) the speed of the rotor magnetic field with respect to stator.

(iv) the speed of the rotor magnetic field with respect to the rotor.

(v) the speed of the rotor magnetic field with respect to the stator magnetic field.