Engineering Optics

Lecture 31

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by

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Michelson's interferometer

The wavelength λ in Michelson interferometer is given by following equation.

$$\lambda = \frac{2d_0}{N}$$

Here, d_0 is the distance moved by the mirror, and N is the number of fringes.

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2, . . . , we will have disappearance of the fringe pattern; and if it is equal to 1, 2, 3, . . . , then the interference pattern will appear.

Fabry-Perot etalon

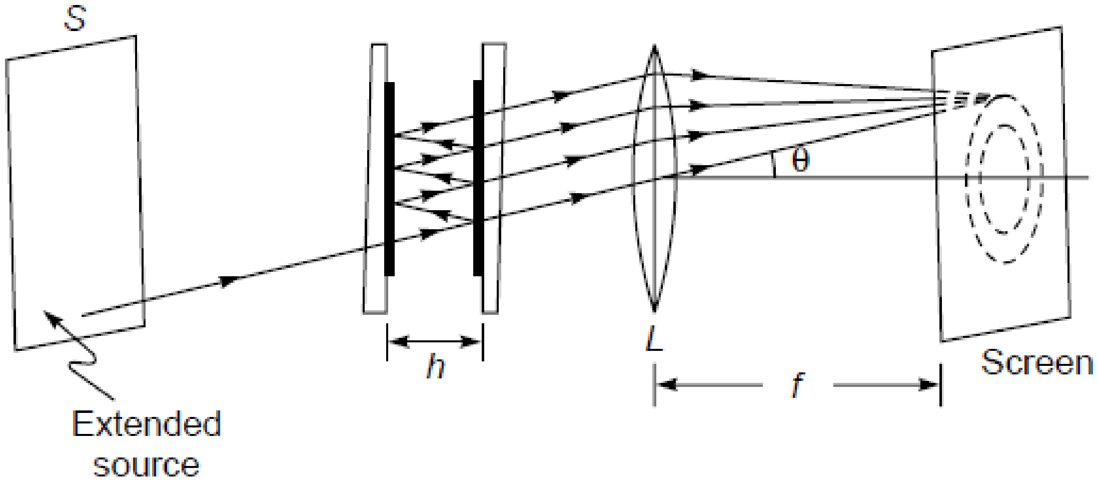


Fig. 16.3 The Fabry–Perot etalon.

It is immediately seen that the reflectivity and the transmittivity of the Fabry–Perot etalon add to unity. Further,

$$T = 1$$

when

$$\delta = 2m\pi \qquad m = 1, 2, 3, \ldots \tag{6}$$

In a typical experiment, light from a broad source is collimated by a lens and is passed through the Fabry–Perot etalon as shown in Fig. 16.3. Thus, if we consider light of a specific wavelength λ_0, the incident light will be completely transmitted (i.e., $T = 1$) if the angle of incidence is such that

$$\delta = \frac{4\pi}{\lambda_0} n_2 h \cos\theta_2 = 2m\pi \tag{9}$$

or

$$\cos\theta_2 = \frac{m\lambda_0}{2n_2 h} \tag{10}$$

Problem:1

In the Michelson interferometer experiment, calculate the various values of θ' (corresponding to bright rings) for $d = 5 \times 10^{-3}$ cm. Show that if d is decreased to 4.997×10^{-3} cm, the fringe corresponding to $m = 200$ disappears. What will be the corresponding values of θ'? Assume $\lambda = 5 \times 10^{-5}$cm.

Answer:

The condition of bright rings in Michelson interferometer experiment is given by following equation.

$$2d\cos\theta' = \left(m+\frac{1}{2}\right)\lambda$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\lambda}{2d}\right)$$

$$\Rightarrow$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\left(5\times10^{-5}\text{ cm}\right)}{2\left(5\times10^{-3}\text{ cm}\right)}\right)$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{200}\right)$$

For $m = 197{,}198{,}199$ The corresponding θ'=9.07°, 7.02°,4.05°

For $d = 4.997 \times 10^{-3}$ cm

$$\theta' = cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)5\times 10^{-5}}{2\times 4.997\times 10^{-3}}\right)$$

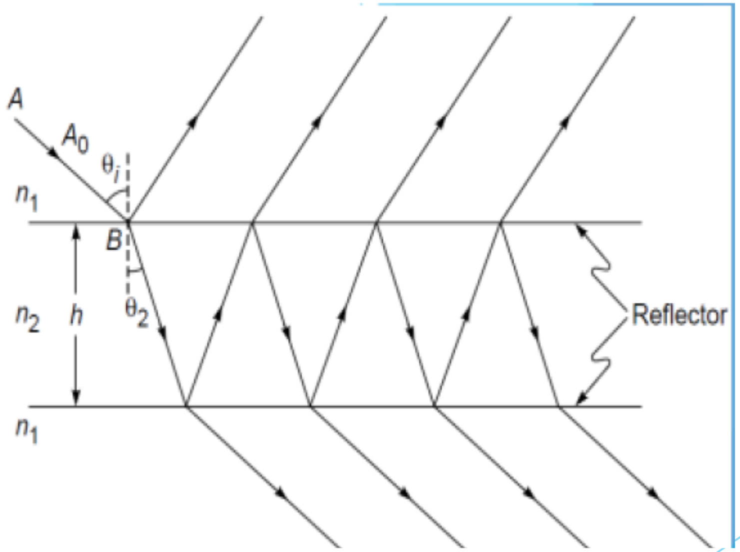
$$\theta' = cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{198.8}\right)$$

$\Rightarrow m = 200$ will disappear

Problem: 2

The following device shown in the figure contains two media of refractive indices $n_1 = 1 \, and \, n_2 = 1.5.$

a) calculate the reflection coefficient (r)?
b) calculate transmittivity (τ)?
c) calculate Finesse F?
d) calculate the reflectivity of the device if $\delta = \frac{\pi}{2}$?



Answer:

a) reflection coefficient (r):

$$r = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

$$r = \frac{(1 - 1.5)^2}{(1 + 1.5)^2} = \frac{0.5^2}{2.5^2}$$

r =0.04

b) transmittivity (τ)

For that particular interface:
Reflectivity $R = r^2$
Transmittivity $\tau = 1 - R$

$$R = 0.04^2$$

$$R = 0.0016$$

$$\tau = 1 - 0.0016$$

$$\tau = 0.9984$$

c) Finesse (F)

$$F = \frac{4R}{(1-R)^2}$$

$$R = 0.0016$$

$$F = \frac{4 \times 0.0016}{(1 - 0.0016)^2}$$

$$F = 0.0064$$

d) reflectivity of the device ($\mathcal{R}$)

$$\mathcal{R} = \frac{F \sin^2 \delta/2}{1 + F \sin^2 \delta/2}$$

$$\mathcal{R} = \frac{0.0064 \times \sin^2 \frac{\pi}{4}}{1 + 0.0064 \times \sin^2 \frac{\pi}{4}}$$

$$\mathcal{R} = \frac{0.0032}{1 + 0.0032}$$

$$\mathcal{R} \approx 0.0032$$

Problem:3

In Fabry-Perot interferometer a bright fringe occur when initial separation between two mirrors h $= 10\ c.m$ $m = 100000$. At h $= 12\ c.m$ bright fringe occur at $m = 200000$. Then calculate the wavelength of the light?

Answer:

Wavelength λ :

$$\boxed{\lambda = \frac{2(d_2 - d_1)}{m_2 - m_1}}$$

$$\lambda = \frac{2(12 - 10)}{200000 - 100000}$$

$$\lambda = 4 \times 10^{-5}\ c.m$$

Problem:4

Consider now two wavelengths 6000 and 5999.9 Å incident on a Fabry–Perot etalon with $n_2 = 1, h = 1\ cm, and\ F\ =\ 200$. Calculate the angle corresponding of the first three bright rings corresponding to each wavelength.

Answer:

$$\cos\theta_2 = \frac{m\lambda_0}{2n_2h}$$

$$cos\theta_2 = \frac{m \times 6000 \times 10^{-8}}{2 \times 1 \times 1} = \frac{m}{33333}$$

$$\theta_2 = \cos^{-1}\frac{m}{33333}$$

For $m = 33333{,}33332{,}33331\ we\ get\ \theta_2 = 0°, 0.44°, 0.63°$

Answer:

$$\cos\theta_2 = \frac{m\lambda_0}{2n_2h}$$

$$cos\theta_2 = \frac{m\times5999.9\times10^{-8}}{2\times1\times1} = \frac{m}{33333.8}$$

$$\theta_2 = \cos^{-1}\frac{m}{33333.8}$$

For $m = 33333{,}33332{,}33331\ we\ get\ \theta_2 = 0.40°, 0.60°, 0.74°$

Problem:5

A Fabry-Perot interferrometer would resolve two lines with $\Delta\lambda = 0.1$ Å at 6000 Å. Calculate the resolving power? Calculate the value of reflectivity if F=80?

Answer:

$$\text{Resolving power} = \left|\frac{\lambda_0}{\Delta\lambda_0}\right|$$

$$= \frac{6000\text{Å}}{0.1\text{Å}}$$

$$= 6 \times 10^4$$

$$F = \frac{4R}{(1-R)^2}$$

$$80 = \frac{4R}{(1-R)^2}$$

$$80R^2 - 164R + 80 = 0$$

Possible values of $\mathrm{R} = 1.25, 0.8$

$\mathrm{R} = 0.8$

Thank You