24/1/24

~~Interference~~

Waves

→ Variation (Disturbance) of Physical Quantity that propagates through space.

i.e Oscillation in space and time

$$y(x, t) = A\sin(kx - \omega t)$$

→ Superposition Theorem :

When two or more waves overlap, the resultant displacement at any instant is the sum of the displacements of each of the individual waves

Interference : Superposition due to Primary wavefronts

Diffraction : Superposition due to Secondary wavefronts.

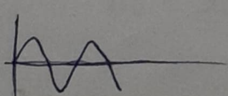
Interference

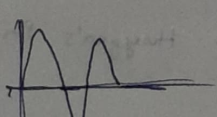
Constructive

$l_2 - l_1 = m\lambda$

$m \in W$

(Path difference of any two waves being Integral multiple of wavelength)

- Maxima
- Bright Fringes
- 

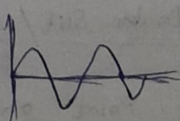


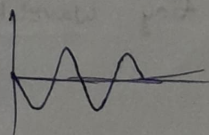
Destructive

$l_1 - l_2 = (m + \frac{1}{2})\lambda$

$m \in W$

(Path difference of any two waves being half-Integral multiple of wavelength)

- Minima
- Dark Fringes
- 



Optics - Ajay G
└ UG Mect

$E_1 = E_0 \sin\theta_1$

$E_2 = E_0 \sin\theta_2$

$E_0 \sin\theta_3 = \bar{E}_1 + \bar{E}_2 = E_3$

→ **Interference Requirement:**

- Need two (or) more waves
- Must have same frequency
- * Must be coherent

i.e. $\underbrace{\text{phase difference}}_{\text{between the two waves}}$ should not change with time.

Note: For Sound,

$l_1 - l_2 = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots \longrightarrow$ No Sound

$l_1 - l_2 = \lambda, 2\lambda, \ldots \longrightarrow$ Max Sound

[Two speakers @ Certain Distance]

For light,

All Sources of light : Incoherent Sources (Fluctuating @ 10^{14} Hz)

∴ We need two waves from the same source taking two different paths

- Two slits
- Reflection ~~Thin/slits~~ (Thin films)
- Diffraction

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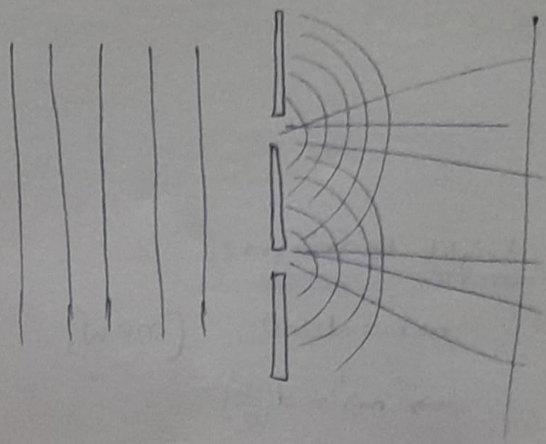
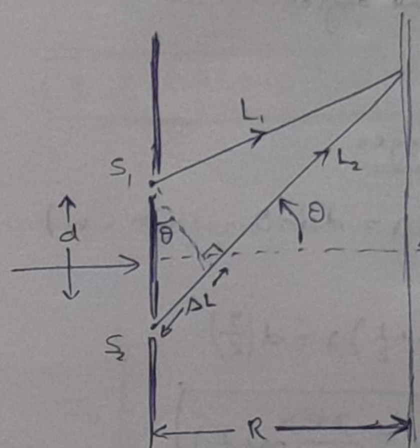
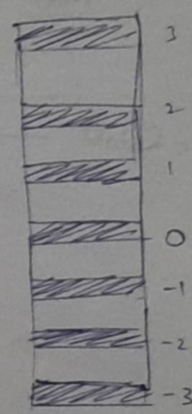
Young's Double Slit/Huygens:

Every Point on a wavefront acts as a source of tiny wavelets that move forward — Huygen's Principle

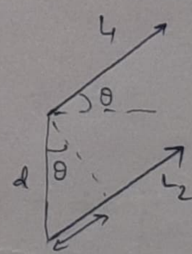


→ Consider two rays travelling @ an angle θ:





for Infinitely distant screen:



$\tan\theta = \frac{y}{R}$

~~$\Delta L = L_2 - L_1 = d\sin\theta$~~

$\Delta L = L_2 - L_1 = d\sin\theta$

$y = R\tan\theta$

$\Delta L = d\sin\theta = m\lambda$

$m = 0, \pm 1, \pm 2, \dots$ } Constructive Interference

$\Delta L = d\sin\theta = (m + \frac{1}{2})\lambda$

$m = 0, \pm 1, \pm 2, \dots$ } Destruction Interference

Central Bright Fringe ⇒ Order = 0

For small Angles,

$y = R\tan\theta$

$\Downarrow$

$y = R\sin\theta \longrightarrow$ **Bright Fringes:**

$m\lambda = d\sin\theta \quad [m \in W]$

$\Rightarrow m\lambda = d\left(\frac{y}{R}\right)$

$\Rightarrow \boxed{y = \frac{\lambda R}{d} m}$

* **Note**: Only for small angles

$y = R\sin\theta \longrightarrow$ **Dark Fringes:**

$(m+\frac{1}{2})\lambda = d\sin\theta \quad [m \in W]$

$\Rightarrow (m+\frac{1}{2})\lambda = d\left(\frac{y}{R}\right)$

$\Rightarrow \boxed{y = \frac{\lambda R}{d}(m+\frac{1}{2})}$

Q) $R = 1.2\ m$

$d = 0.03\ mm$

$m = 2,\ y = 4.5\ cm$

(a) $\lambda = ?$

Sol: $\lambda = \frac{yd}{mR}$

$\Rightarrow \lambda = \frac{4.5\times10^{-2} \times 0.03\times10^{-3}}{2\times1.2}$

$= 562.5\times10^{-9}\ m$

$\boxed{\therefore \lambda = 562.5\ nm}$

(b) $\beta = ?$ ⟶ Fringe Width: distance b/w two adjacent Maximas or minimas

$$\beta = m\left(\frac{\lambda R}{d}\right) - (m-1)\left(\frac{\lambda R}{d}\right)$$

$$= \frac{\lambda R}{d}$$

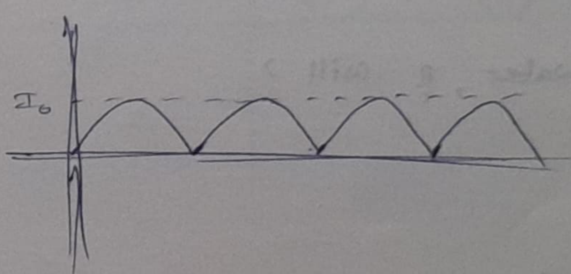
$$\Rightarrow \cancel{\beta = \frac{\lambda}{m}} \quad \boxed{\beta = \frac{\lambda R}{d}}$$

$$\therefore \beta = \frac{562.5 \times 10^{-9} \times 1.2}{0.03 \times 10^{-3}}$$

$$= 225 \times 10^{2} \times 10^{-6}$$

$$\boxed{\beta = 2.25 \times 10^{-2}\ m} \rightarrow \boxed{\beta = 0.0225\ m}$$

(c) Intensity Pattern:



Q) $m = 2$

$\theta = 5.4°$, $d/\lambda = ?$

Sol: $(m + \frac{1}{2})\lambda = d\sin\theta$

$$\Rightarrow 2.5 \times \lambda = d\sin(5.4°)$$

$$\Rightarrow \frac{d}{\lambda} = \frac{2.5}{\sin(5.4°)}$$

$$\cancel{\frac{d}{\lambda} = 26.565}$$

$$\boxed{d/\lambda = 16}$$

Q) $\beta = 0.0240\, m = y$

$\lambda = 475\, nm$

$\sin\theta \approx \tan\theta$

$\lambda = 611\, nm,\ y = ?$

$$\beta = \frac{\lambda R}{d}$$

$$\Rightarrow \frac{\beta_1}{\lambda_1} = \frac{\beta_2}{\lambda_2}$$

$$\Rightarrow \frac{0.024}{475} = \frac{y}{611} \Rightarrow y = \frac{0.024}{475} \times 611$$

$$\therefore y = 0.03087$$

Q) If placed under water, β will ?

Decrease

→ Intensity Pattern :

$$E = E_1 + E_2$$

$$\left[\Delta\phi = \frac{2\pi}{\lambda}\Delta x\right]$$

$$E = E_p \sin(\omega t) + E_p \sin(\omega t + \phi)$$

$$= E_p\left[\sin(\omega t) + \sin(\omega t + \phi)\right]$$

$$\therefore E = 2E_p \cos(\phi/2) \sin(\omega t + \phi/2)$$

$d\sin\theta = \phi/2\pi$ → Path diff

Path diff $= \phi/2\pi$

$$r_2 - r_1 = d\sin\theta = \frac{\lambda\phi}{2\pi} = \frac{\phi}{k}$$

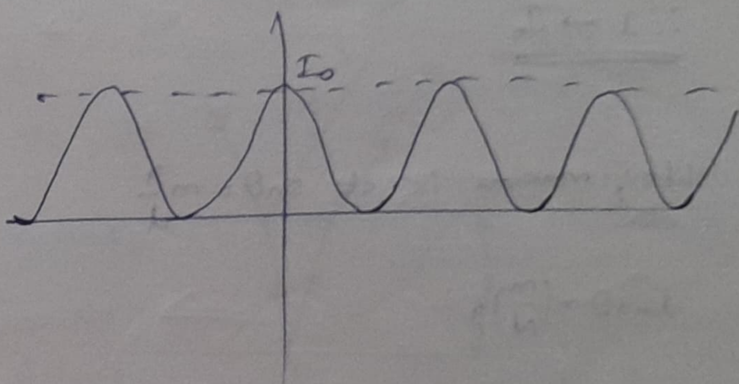
$$\Rightarrow \phi = kd\sin\theta$$

$$I = I_0\cos^2(\phi/2) = I_0\cos^2\left(\frac{kd\sin\theta}{2}\right)$$

$\sin\theta \approx \frac{y}{R}$ I

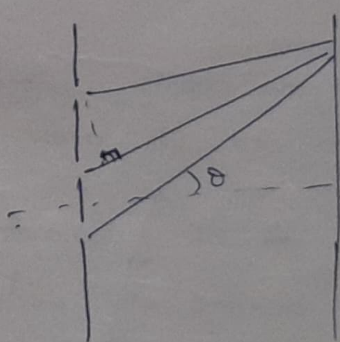
$$\Rightarrow I_0\cos^2\left(\frac{kdy}{2R}\right)$$

$$\Rightarrow I_0\cos^2\left(\frac{\pi dy}{\lambda R}\right)$$



~~Q) 3 Fr~~

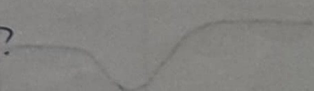
Q) 3 Fringes, What Interference



Constructive Interference

Ray 1 $\rightarrow I_0$

Combined Intensity = ?



Sol: $E_0 \longrightarrow$ ray 1, 2, 3

$$\therefore E_0 + E_0 + E_0 = 3E_0$$

$$I_0 \propto E^2$$

$$I = 9I_0$$

Q) When ray 1 and 2 — Destructive

Intensity - Minimum ?

$$E_0 - E_0 + E_0 = E_0$$

$$\therefore I \rightarrow I_0$$

For many slits, maxima is at $\sin\theta = m\frac{\lambda}{d}$

$$d\sin\theta = \left(\frac{m}{N}\right)\lambda$$

$\hookrightarrow$ for N Slits

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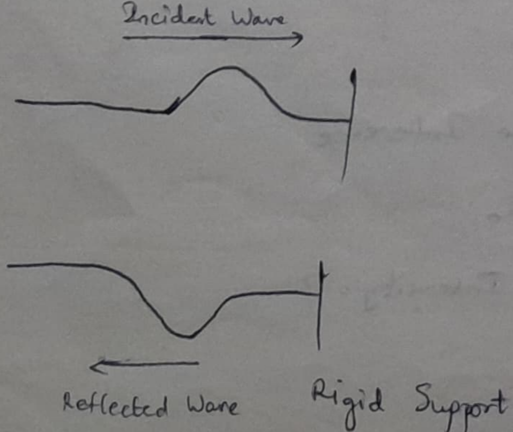
→ Interference of two types:

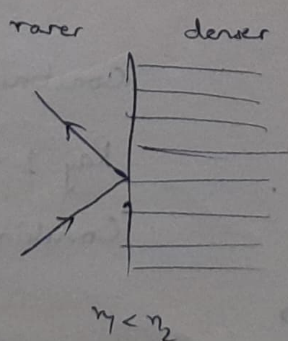
(1) Division of Wavefront

(2) Division of Amplitude

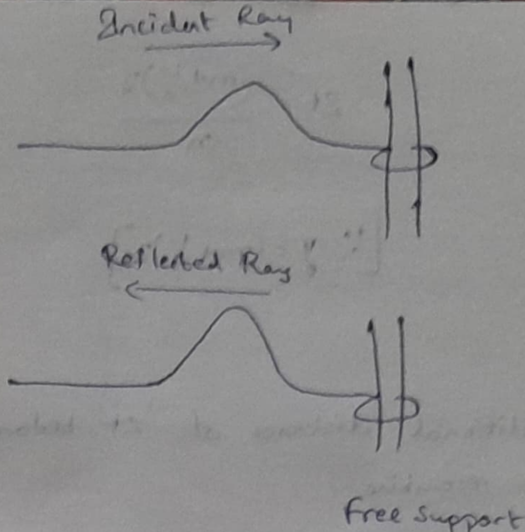
→ Phase change due to Reflection

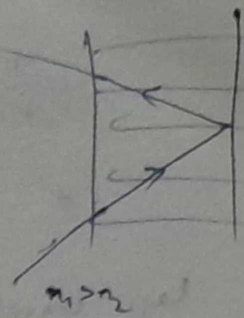
①



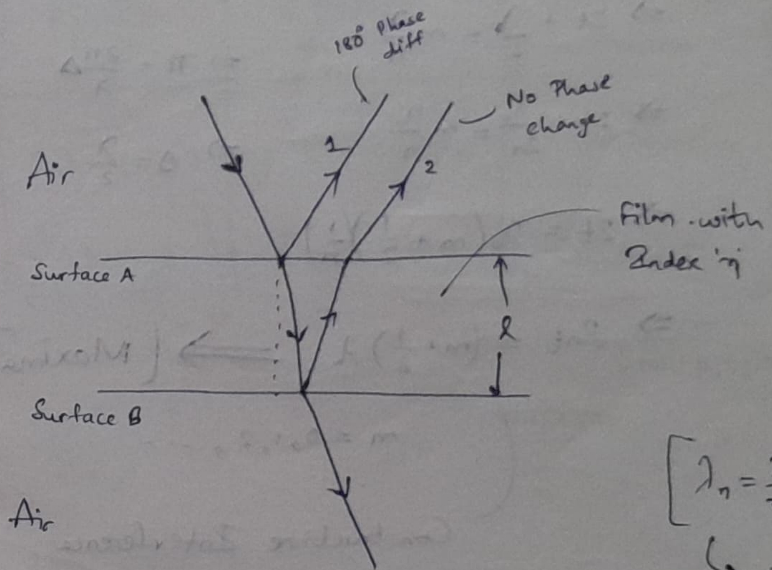


(2)





→ <u>Interference in Thin films</u> :



$$\boxed{\lambda_n = \frac{\lambda_0}{n}}$$

↳ $\Delta x = \frac{\lambda_0 m}{n}$

- Interference effects are observed in soap bubbles, oil in water, etc are examples of this.
- Interference is due to interaction of the waves reflected from both surfaces of the film.
- Ray 1 and Ray 2 give Interference

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$$\Rightarrow \pi = \frac{2\pi}{\lambda}(\Delta x)$$

$$\therefore \Delta x = \frac{\lambda}{2}$$

$\cancel{\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)}$ $\therefore\ 2t = \frac{(m+\frac{1}{2})\lambda}{n}$

$\cancel{= \frac{2\pi}{\lambda} \cdot \frac{m\lambda_0}{n}}$ $\left[\because\ y = (m+\tfrac{1}{2})\lambda_0\right]$

ϕ

Rays travel additional distance of '2t' before the ~~rays~~ waves recombine

$$2t + \pi = m\frac{\lambda}{n}$$

$$\phi = \frac{2\pi}{\lambda}\Delta$$

$$\Rightarrow 2t + \frac{\lambda_0}{2} = m\frac{\lambda_0}{n}$$

$$\Rightarrow \pi = \frac{2\pi}{\lambda}\Delta$$

$$\Rightarrow 2t + \frac{\lambda_0}{2n} = m\frac{\lambda_0}{n}$$

$$\Rightarrow \Delta = \frac{\lambda}{2}$$

$$\Rightarrow 2t = \lambda_0\left(m + \frac{1}{2}\right)\left(\frac{1}{n}\right)$$

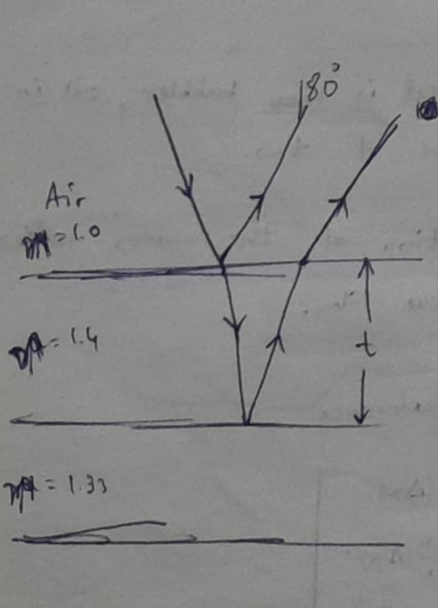
$$\Rightarrow 2nt = \left(m + \frac{1}{2}\right)\lambda \Longrightarrow \boxed{\text{Maxima}}$$

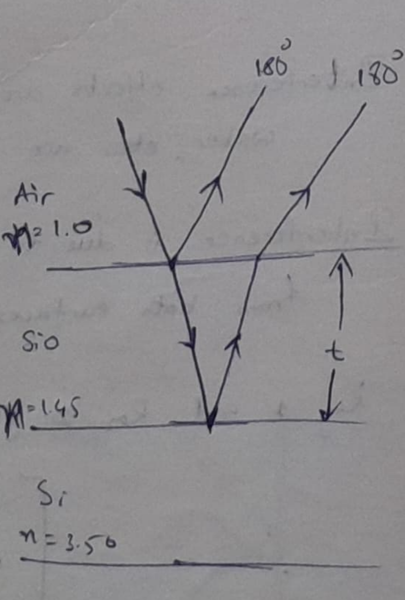
$m = 0, 1, 2, \ldots$

Constructive Interference

1/2/24

Q)





$2nt = (m+\frac{1}{2})\lambda$

$2nt = m\lambda_0$

→ Problem Solving for Thin Films,

- Identify the thin film causing the Interference
- Determine refractive Index
  - (to find which is rare and dense)
  - (to find Δ, ϕ)
- Determine Phase reversal (0 (or) 1 (or) 2)

Equation	1 Phase Reversal	0 (or) 2 Phase Reversal
$2nt = (m+\frac{1}{2})\lambda$	Constructive	Destructive
$2nt = m\lambda$	Destructive	Constructive

λ_{vacuum}

($m : 0, 1, 2, \ldots$)

(OR)

$2t = (m+\frac{1}{2})\lambda_{film}$

$2t = m\lambda_{film}$

Q) Soap film - refractive Index 'n'
& Air on both sides

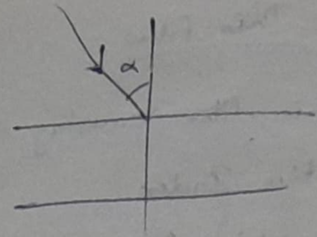
One Region - Yellow - Destructive, Removed blue ($\lambda_{vac.} = 469$ nm)

Second Region - Magenta - Destructive, Removed Green ($\lambda_{vac.} = 555$ nm)

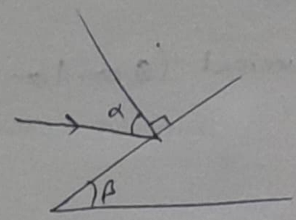
Thickness 't'

$t_{magenta} / t_{yellow} = ?$

2/2/24



$$2nt\cos\alpha = \left(m+\frac{1}{2}\right)\lambda$$



$$2nt\cos(\alpha+\beta) = \left(m+\frac{1}{2}\right)\lambda$$

Sol: Light falls Normal to surface $\therefore 2t + \underset{\text{due to reflection}}{\text{Addition}} \pm m\lambda_{film}$

$$\left[\Delta l = \left(m+\frac{1}{2}\right)\lambda \quad m: 0, 1, 2, \ldots\right]$$

$$\therefore 2t + \frac{m\lambda_{film}}{2} = m\lambda_{film}$$

$$\Rightarrow 2t = \frac{m\lambda_{film}}{2}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\lambda_{film_1}}{\lambda_{film_2}}$$

$$\left[\lambda_{film} = \frac{\lambda_{vacuum}}{n}\right]$$

$$\left[t = \frac{\lambda_{vacuum}}{2n}\right]$$

$$\frac{t_{magenta}}{t_{yellow}} = \frac{\lambda_{film(magenta)}}{\lambda_{film(yellow)}} = \frac{555}{469}$$

Q) Orange light ($\lambda_{vac.} = 611$ nm)

Soap film, $n = 1.33$, Air on both sides

Light strikes normal to surface

$t_{min} = ?$ for Constructive Interference

Sol: ~~2t~~ $2nt + \frac{m\lambda_{film}}{2}$

$$\left(\Delta l = m\lambda \quad m: 0, 1, 2, \ldots\right)$$

$$2t + \frac{1}{2}\lambda_{film} = \lambda_{film},\ 2\lambda_{film}, \ldots$$

$$\Rightarrow \left[2t = \left(m + \frac{1}{2}\right)\lambda_{film} \right] \quad m: 0, 1, 2, \ldots \qquad \left[\lambda_{film} = \frac{\lambda_{vacuum}}{n} \right]$$

$$2t = \left(0 + \frac{1}{2}\right)\lambda_{film}$$

$$\Rightarrow 2t = \frac{1}{2} \times \frac{611}{(1.33)}$$

$$\boxed{\therefore t = 115\ nm} \longrightarrow t = 115 \times 10^{-9}\ m = 1.15 \times 10^{-7}\ m$$

$$\boxed{\therefore t = 115 \times 10^{-3}\ \mu m}$$

Q) yellow ($\lambda_{vac.} = 580\ nm$) and Violet ($\lambda_{vac.} = 410\ nm$)

Falls Perpendicular

$n_{gasoline} = 1.40$

$n_{water} = 1.33$

$t_{min} = ?$ for (a) yellow, (b) violet } Destructive

Sol.

$$2t + \frac{m\lambda_{film}}{2} = \frac{m\lambda_{film}}{2},\ \frac{3m\lambda_{film}}{2},\ \frac{5m\lambda_{film}}{2}$$

$$\Rightarrow 2t = 0,\ m\lambda_{film},\ 2m\lambda_{film}, \ldots$$

$$\left[2t = \lambda_{film} \Rightarrow t = \frac{\lambda_{vac}}{2n} \right] \qquad \left[2t = \lambda_{film},\ t = \frac{\lambda_{vacuum}}{2n} \right]$$

(a) $t = \frac{1}{2} \times \frac{580}{1.40} = 210\ nm$

(b) $t = \frac{1}{2}\ \frac{410}{1.40} = 150\ nm$

6/3/24

Q) bubble – Thin, Soapy film

- looks blue @ normal incidence

$t = ?$

$n_{film} = 1.35$

$\lambda_{blue} = 400\,nm$

Sol:
$2t = \lambda_{film}$

$$2t = \frac{(m+\frac{1}{2})\lambda_{blue}}{n_{film}} = \frac{\lambda_{blue}}{2n_{film}}$$

$\Rightarrow 2t = \frac{\lambda_{vac}}{2n}$

$\therefore t = \frac{400}{2\times1.35} \times \frac{1}{2}$

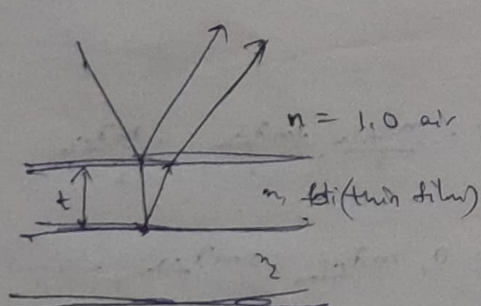
$\boxed{\therefore t = 74\,nm}$

Q) Blue light $\lambda_0 = 500\,nm$

incident on glass ($n_1 = 1.5$)

cover slip ($t = 167\,nm$)

floating on top of water ($n_2 = 1.3$)



$\delta_1 = 1$

$\delta_2 - \delta_1 = \pi$

if $n_2 > n_1 \Rightarrow \delta_2 - \delta_1 = 0/2\pi$

→ Diffraction:

→ A single slit placed between a distant light source and a screen produces a diffraction pattern.

- Sharp Edges create diffraction pattern.
  (Sharpness is comparable to wavelength of light)

- Intensity keeps decreasing maxima by maxima.
  - Consits broad, Intense central band.
  - Less Intensity secondary band - Secondary Maxima
  - Series of Dark bands - Minima
  - Cannot be explained by Geometric Optics.

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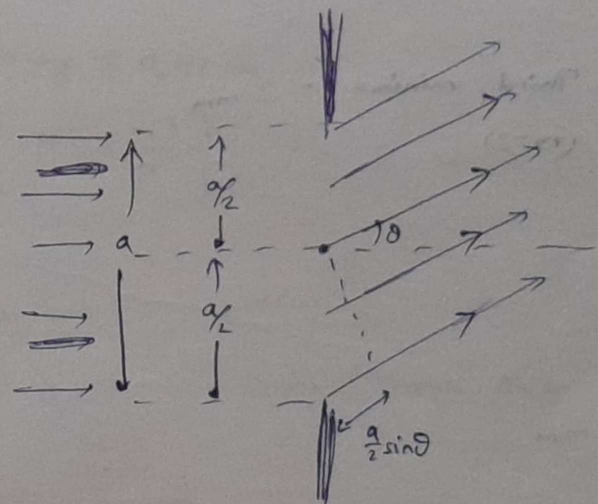
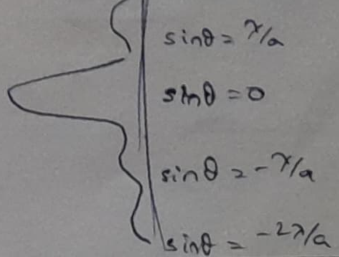


$$\sin\theta_{dark} = \frac{m\lambda}{a} \qquad (m = \pm 1, \pm 2, \ldots)$$

[Destructive Interference]



Q) $\lambda = 500\ nm$

$a = 50\ \mu m$

$D = 50\ cm$

$$\Delta\theta = \lambda/a$$

$$\Delta y = L\,\Delta\theta$$

$$= \frac{(50\times10^{-2})(500\times10^{-9})}{50\times10^{-6}}$$

$$= 5\ mm$$

Q) $D = 50\ cm$

$\lambda = 680\ nm$

dist. b/w first & Third minima $= 3\ mm$
(m=1) (m=3)

$a = ?$

Sol :

$$\frac{3\lambda}{a} - \frac{\lambda}{a} = 3\ mm$$

$$\Rightarrow \frac{2\lambda}{a} = 3\ mm$$

$$\Rightarrow a = \frac{2\lambda}{3\times10^{-3}}$$

$$\therefore a = 4.53\times10^{-4}\ m$$

$$\boxed{\therefore a = 0.453\ mm}$$

Q) $\lambda = 610\ nm$

$a = 0.2\ mm$

$D = 1.5\ m$

Width of Central Maxima = ?

Sol:

$\cancel{\frac{2\lambda}{a} = \frac{2 \times 610 \times 10^{-9}}{0.2 \times 10^{-3}}}$

$\cancel{= 0.0061\ m}$

$\cancel{= 6.1\ mm}$

$\cancel{y = x\frac{m\lambda}{D}}$

$\Rightarrow 2y \approx 0.92\ cm$

$\left[y = x\frac{m\lambda}{D} \right]$

Q) $\lambda = 687\ nm$

$a = 0.75\ mm$

$D = ?$

$\frac{2\lambda}{a} = 1.7\ mm$

Sol:

$\cancel{\sin\theta_{dark} = \frac{2\lambda}{a}}$

$\cancel{\Rightarrow}$

$y = x\frac{m\lambda}{D}$

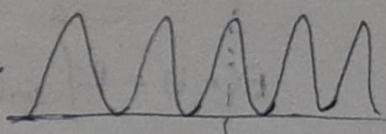
$\Rightarrow x = \frac{yD}{m\lambda}$

$\Rightarrow x \approx 0.93\ cm$

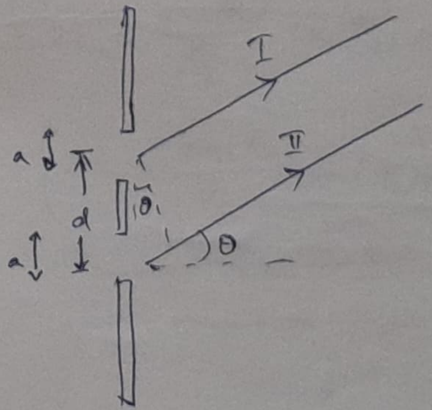
8/2/24

→ **Double Slit Diffraction:**

$a < \lambda$ → Young's Double slit →

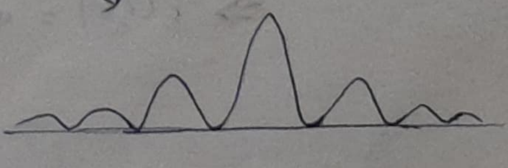


[Interference]

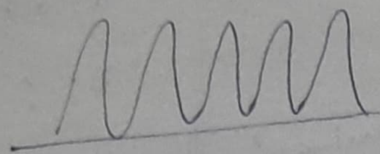


Single slit (d=0)

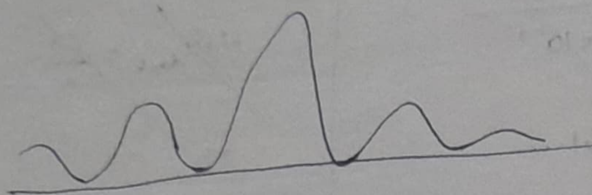
[Diffraction]



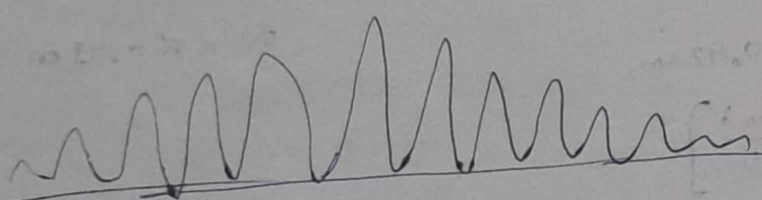
$a < \lambda$ — Double Slit Interference



$a > \lambda$ — Single Slit

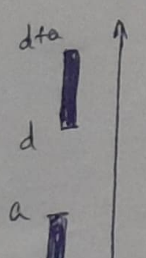


$a > \lambda$ — Double Slit



$$E(r,t) = E_m \int_0^a \sin(kr - \omega t + kz\sin\theta)\, dz + E_m \int_d^{d+a} \sin(kr + \omega t + kz\sin\theta)\, dz$$

$[\phi = ka\sin\theta]$



Solve
(H.W)

$$\Rightarrow E(r,t) = a\frac{\sin(\phi/2)}{(\phi/2)} \sin(kr - \omega t + \delta + \phi/2)$$

$$+ a\frac{\sin(\phi/2)}{\phi/2} \sin(kr - \omega t + \delta + \phi/2 + \delta)$$

$$\int_0^a \sin(kr - \omega t + kz\sin\theta)\, dz$$

$$= \left. \frac{-\cos(kr - \omega t + kz\sin\theta)}{k\sin\theta} \right]_0^a$$

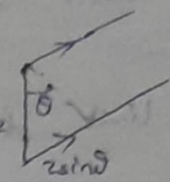
$$= \frac{1}{k\sin\theta}\left[\cos(kr-\omega t) - \cos(kr - \omega t + ak\sin\theta)\right]$$

How $kz\sin\theta = \phi$?

Sol:

$\sin(kr - \omega t + \phi)$

$$\phi = \frac{2\pi}{\lambda}\Delta$$



$$\therefore \phi = \frac{2\pi}{\lambda} z\sin\theta$$

$$\left[\frac{2\pi}{\lambda} = k\right]$$

$$\Rightarrow \phi = kz\sin\theta$$

9/2/24

$$E(r,t) = E_m a \frac{\sin(\phi/2)}{\phi/2}\left[\sin(A) + \sin(A+\delta)\right]$$

$$\left[\begin{array}{l} A = kr - \omega t + \phi/2 \\ \phi = ka\sin\theta \\ \delta = kd\sin\theta \end{array}\right]$$

$$\left(\begin{array}{l} \frac{0}{0} \\ \text{let } x \to 0 \\ = \frac{x}{x} \\ = 1 \end{array}\right)$$

$$= 2E_m a \frac{\sin(\phi/2)}{\phi/2}\cos(\phi/2)\sin(A + \delta/2)$$

$$= \underbrace{\left[2E_m a \frac{\sin(\phi/2)}{\phi/2}\cos(\phi/2)\right]}_{\text{Amplitude}} \sin(kr - \omega t + \phi/2 + \delta/2)$$

→ Intensity:

$\cancel{I_0 = 2}$

$$I = I_0 \left(\frac{\sin(\phi/2)}{\phi/2}\right)^2 \left(\cos^2\left(\frac{\delta}{2}\right)\right)$$

$\phi = ka\sin\theta$

$\delta = kd\sin\theta$

limit $a \Rightarrow 0$	$I = I_0\left(\cos\frac{\delta}{2}\right)^2$	Young's Double Slits
limit $d \Rightarrow 0$	$I = I_0\left(\frac{\sin(\phi/2)}{(\phi/2)}\right)^2$	Single Slit

Note :

$$\boxed{\phi/\delta = a/d}$$

(A) Amplitude : $2E_m a\left(\frac{\sin \phi/2}{\phi/2}\right)\cos(\delta/2)$

$$A^2 = 4E_m^2 a^2\left(\frac{\sin\phi/2}{\phi/2}\right)^2\cos^2(\delta/2)$$

$$I = I_0\left(\frac{\sin(\phi/2)}{\phi/2}\right)^2\cos^2(\delta/2)$$

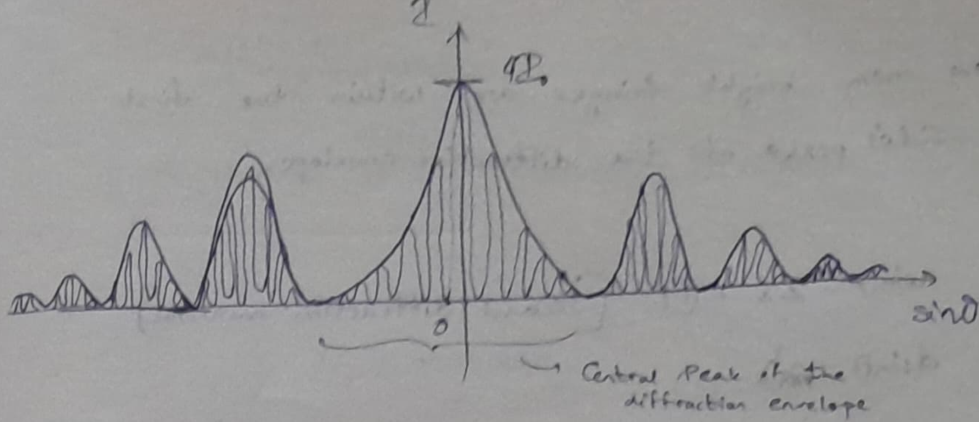
Proof :

$$\lim_{a\to 0} I = \lim_{a\to 0} I_0\left(\frac{\sin(\phi/2)}{(\phi/2)}\right)^2\cos^2(\delta/2)$$

$$= \lim_{a\to 0} I_0\left(\frac{\sin\left(\frac{ka\sin\theta}{2}\right)}{\left(\frac{ka\sin\theta}{2}\right)}\right)^2\cos^2(\delta/2)$$

$$= I_0\cos^2(\delta/2)$$

$$I = \underbrace{I_0\left(\frac{\sin(\phi/2)}{(\phi/2)}\right)^2}_{\text{Diffraction Envelope}}\underbrace{\cos^2(\delta/2)}_{\text{Two slit Interference}}$$



(Diffraction Envelope)

13/2/24

Q) $\lambda = 405$ nm

$d = 19.44\ \mu m$ (centre to centre - slit seperation)

$a = 4.050$ mm (slit width)

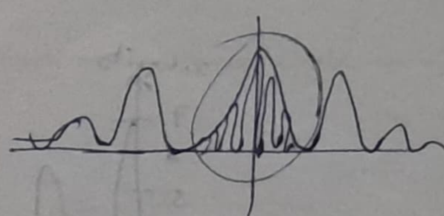
Q.) How many bright fringes are within the central Peak of the diffraction envelope ?

Sol :

$m = \pm 1 \longrightarrow a\sin\theta = m\lambda$

[Central Peak limit] [Diffraction minima]

$\Rightarrow a\sin\theta = \lambda \quad -①$



$d\sin\theta = m\lambda \quad -②$

$\Rightarrow m = d/a$ [from ②/①]

$\Rightarrow m = \dfrac{19.44 \times 10^{-6}}{4.05 \times 10^{-3}} = 4.8$

$\boxed{\therefore m = 4.8} \longrightarrow 2m = 9.6$

$[9.6] = 9$

$\Rightarrow 9$ fringes

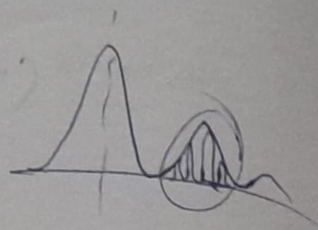
$\therefore$ 9 Fringes

i.e. $0, \pm 1, \pm 2, \pm 3, \pm 4$

Q) How many bright fringes are within the first side's peak of the diffraction envelope.

Sol: $a\sin\theta = 2\lambda$ — ① [Second diffraction minima]

$d\sin\theta = m'\lambda$ — ②



$\Rightarrow m' = \frac{2d}{a} = 9.6$

$\boxed{\therefore m' = 9.6}$

$m' = 5, 6, 7, 8, 9$ lie within the side's peak

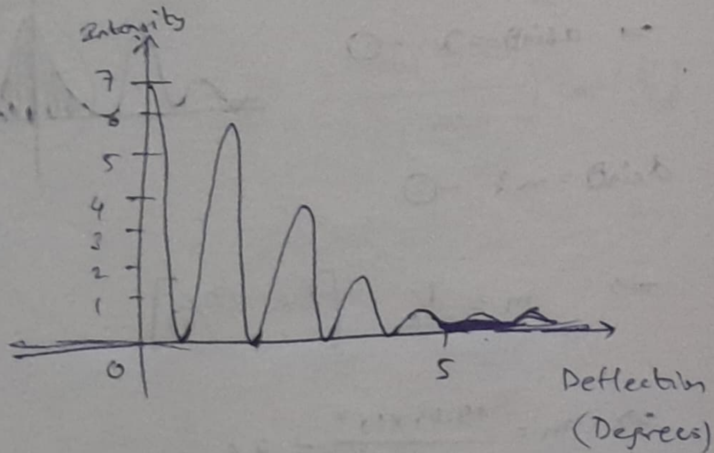
$\therefore$ 5 fringes

Q) $\lambda = 440$ nm

double slit

Q(a) Slit width = ?

Q(b) Slit seperation = ?



Sol: $a\sin\theta = m\lambda$

$\left.\begin{array}{l} m = 4 \\ \lambda = 440\text{nm} \end{array}\right\} \Rightarrow a = \frac{m\lambda}{\sin(5°)} = \frac{4 \times 440 \times 10^{-9}}{\sin(5°)}$

$$I = I_0\left(\frac{\sin(\phi/2)}{\phi/2}\right)^2\cos^2\left(\frac{\delta}{2}\right)$$

First Minima of Diffraction $- \theta \approx 5^\circ$

$$\Rightarrow \cancel{\frac{5\pi}{180}}$$

$$\phi/2 = \pi a\sin\theta/\lambda$$

$$\Rightarrow \phi/2 = \pi$$

$$\therefore a = \lambda/\sin\theta \Rightarrow a = 5.048\ \mu m$$

First Maxima of Interference $- \theta \approx 1.25^\circ$

$$\delta/2 = \pi$$

$$\Rightarrow d = \lambda/\sin\theta \Rightarrow d = 20.2\ \mu m$$

14/2/24

Diffraction Grating :

- An arrangement of many slits
- Assumptions :

  (1) The slits are Narow (Each one produces single wave)

  (2) The screen is very far

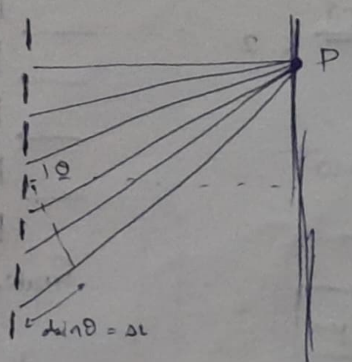
- For two adjacent slits :

$$\Delta L = d\sin\theta$$

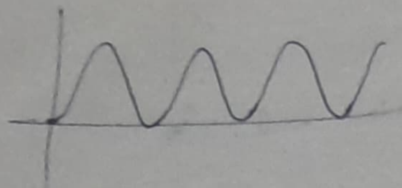
Constructive :

$$d\sin\theta = m\lambda$$

$$[m \in W]$$



No '0' Intensity in this case



2 slits

Ex. $\lambda = 630$ nm

$h = 0.15$ m

↳ Diffraction slits are seperated by

$W = \cancel{0}2$ m

↳ distance of screen from the grating

Find 'd',

↳ distance b/w slits in the grating

Sol:

$d\sin\theta = m\lambda$ $\left[\sin\theta = \frac{y}{D} = \frac{h}{W}\right]$

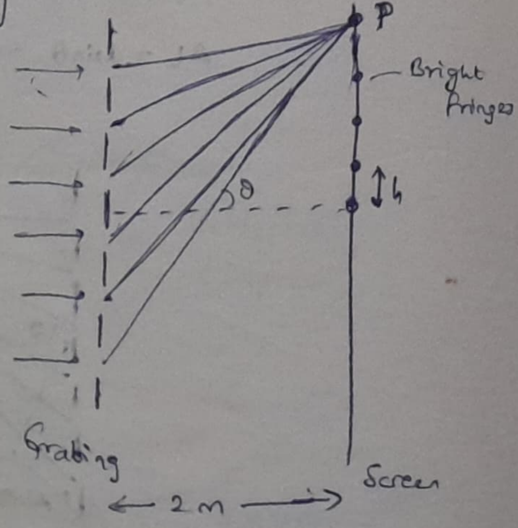
$\Rightarrow d = \frac{m\lambda}{\sin\theta}$

$\Rightarrow d = \frac{630\times10^{-9}}{0.15} \times 2$

$= 8400\times10^{-9}$

$= 8.4\ \mu m$

$\boxed{\therefore d = 8.4\ \mu m}$



(OR)

$\tan\theta = \frac{h}{w}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{h}{w}\right)$ rad

Convert into degrees

$\pi^c = 180^\circ$

$1^c = \frac{180^\circ}{\pi}$

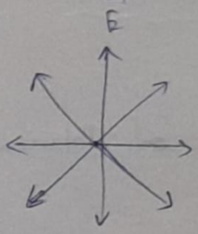
$\tan^{-1}\left(\frac{h}{w}\right)^c = \frac{180}{\pi} \times \tan^{-1}\left(\frac{h}{w}\right)^\circ$

$d\sin\theta = m\lambda \Rightarrow d\sin\theta = \lambda$

$\Rightarrow d = \lambda / \sin\left(\tan^{-1}\left(\frac{h}{w}\right)^\circ\right)$

20/2/24

Polarisation



Unpolarised Light
(Daily life)

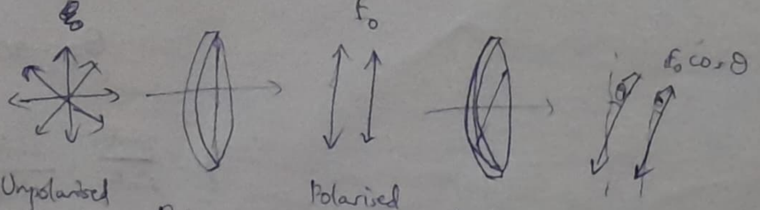
Linearly
Polarised Light
(Lasers)

Planar Wavefronts
(Plane Polarised Light)

Polarisation can be obtained by

(1) Absorption
(2) Reflection
(3) Selection

(1) Absorption:



Unpolarised light   Polarizer   Polarised Light   Analyser

$I = I_0 \cos^2\theta$: Malus' Law

I_0 → Before Analyser

I → After Analyser

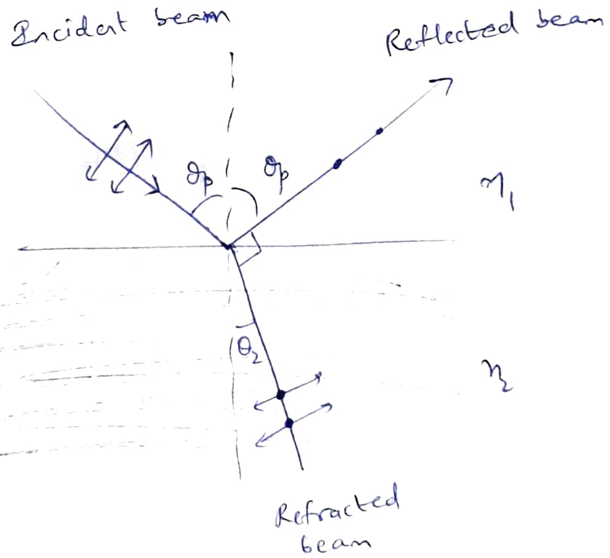
(2) <u>Reflection</u> :

Angle : 0 or 90°,
After passing through a medium and gets reflected ~~afte~~ from a surface,

Reflected beam is unpolarised.

Angle – b/w 0° and 90° – Some degree of Polarisation

Particular Angle : Completely Polarised



$$\theta_p + 90° + \theta_2 = 180°$$

$$\theta_2 = 90° - \theta_p$$

$$\eta = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2} = \frac{\sin\theta_p}{\cos\theta_p}$$

$$\therefore \eta = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$$

Q) Light - Intensity I_0

Polarised parallel to Transmission axes of a Polariser

& is incident on an Analyser

(a) Transmission axis of Analyser makes 45° with axis of Polariser

Sol: $I_0 = I_0$

$$I = \frac{I_0}{2}$$

(b) Angle for which $\frac{I}{I_0} = \frac{1}{3}$

$$I = I_0 \cos^2\theta$$

$$\Rightarrow \frac{I_0}{3} = I_0 \cos^2\theta$$

$$\Rightarrow \cos^2\theta = \frac{1}{3}$$

$$\cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \theta$$

13/3/24

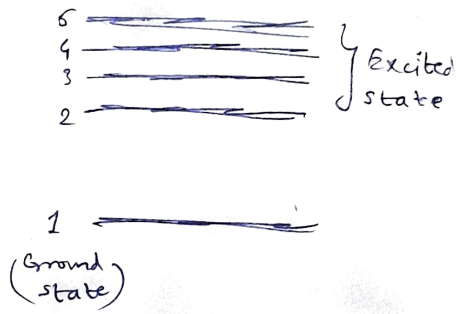
LASERS

→ **LASER :**

L : Light
A : Amplification by
S : Stimulated
E : Emmission of
R : Radiation

→ LASER : Photons are in Phase

↳ Emits Coherent sources



Population:
no. of atoms per unit vol. that occupy a given energy state. (N)

$$N \propto e^{-E/kT}$$

[k : Boltzmann's constant]

Atoms tend to be at lowest ~~energy~~ possible energy level.

$$N = N_0 \cdot e^{-E/kT}$$

At temp. above 0K :

- Atoms have some thermal energy.
- Distribute themselves among available energy levels according to their energy.

At T = 0 K :

All atoms are in the ground state.

Note :

Population at each energy level decreases with increase in energy level.

→ Boltzmann's Equation :

$\underline{\underline{N_1 = ke^{-E_1/kT}}}$ & $\underline{\underline{N_2 = ke^{-E_2/kT}}}$

$$\frac{N_1}{N_2} = e^{-(E_2 - E_1)/kT}$$

$\boxed{\therefore N_2 = N_1 \cdot e^{-\Delta E/kT}}$; $\Delta E = E_2 - E_1$

↳ Relative Population

Depends on two factors - (i) Energy Diff. $(E_2 - E_1)$

(ii) Temperature (T)

At lower temp.s , All atoms - Ground ~~floor~~ State

At higher temp.s , ~~All~~ Some atoms - Higher state

Limiting Cases :

$E_2 - E_1 \to 0 \implies N_2 - N_1 = 0 \Rightarrow \underline{\underline{N_1 = N_2}}$

$T \to \infty \implies N_2 = N_1$

$\therefore$ Material in Thermal equilibrium → Population of higher states cannot exceed the population of lower states

For Lasing Action - More than 2 energy levels

14/3/24

Simple Absorption : When energy is available $n=1$ to $n=2$

Excitation : Induced / Stimulated absorption

De-excitation : Spontaneous emission

$$h\nu = E_2 - E_1$$

Monotwisted
Lifetime of excitated state – μs or ms

Lasing Action ↑ Lifetime ↑

Properties of laser, also depend on material & setup (design)

Lasers – mW

High-Power lasers – kW

→ Stimulated Emmission :

(a) Absorption :

Rate of Absorption = $B_{12} \cdot \rho(\nu) \cdot N_1$ – (1)

$[A + h\nu = A^*]$

N_1 : Population at lower level N_1

$\rho(\nu)$: Energy density of incident light

B_{12} : Proportionality constant ~~for~~
(Einstein coefficient for induced Absorption)

Indicates probability of an induced transition from level 1-2.

(b) Spontaneous emmision :

Rate of Spt. emmision = $A_{21} \cdot N_1$

A_{21} : Eienstein coeff. for spt. emission and is a function of frequency and properties of material.

$A_{21} = 1/\tau_{sp}$ – (2) [τ_{sp} : Lifetime of spt. emission]

Important features:

(1) No oustide control
[No stimulus]

(2) Probabilistic in nature

(3) Incoherent

(4) Non- Monochromatic
[Will be monochromatic, if there is a film]
(Still Incoherent)

(5) Lack of Directionality

$$I_{total} = NI$$

19/3/24

* ⓒ Stimulated Emission:

→ Most Important for Lasers

→ When no. of e^- in higher energy is more.

$$A^* + h\nu = A = 2h\nu$$

$$R_{st} = B_{21} \cdot \rho(\nu) . N_2 \quad -(7)$$

B_{21} — Einstein coefficient for Stimulated Emmission.

Important features:

(1) Controlled from Outside

(2) Same freq, same phase & same phase of Polarisation

(3) Coherent

(4) Monochromatic

(5) Directional

Note: All these are properties of Laser as well

→ Light Amplification :

- Multiplication of Photons
- All in same phase & travel in same direction

$$\boxed{I_{total} = N^2 I}$$

Intensity of Stimulated Photons

→ Steady State Condition :

Rate of Absorption = Rate of Spontaneous emission + Rate of stimulated emmission

i.e. Thermal Equilibrium

→ Einstein Relations

@ Thermal eq, Rate of Absorption = Rate of Emmission

$$B_{12}\,\rho(\nu)\cdot N_1 = A_{21} N_2 + B_{21}\,\rho(\nu)\,N_2$$

$$\Rightarrow \rho(\nu)\cdot\left[B_{21}N_1 - B_{12}N_2\right] = A_{21}N_2$$

$$\boxed{\therefore\ \rho(\nu) = \frac{A_{21}\cdot N_2}{B_{21}\cdot N_1 - B_{12}\cdot N_2}}$$

Can be known from Plank's Law

$$\rho(\nu) = \frac{A_{21}/B_{12}}{N_1/N_2 - B_{21}/B_{12}}$$

As $\frac{N_1}{N_2} = e^{(E_2-E_1)/KT} = e^{h\nu/KT}$

$$\therefore \rho(\nu) = \frac{A_{21}}{B_{12}} \left[\frac{1}{e^{h\nu/kT} - \frac{B_{21}}{B_{12}}} \right]$$

Must be Identical to Black body Radiation

& be consistent ~~to~~ with Planck's Law of

Radiation Law.

→ Conditions for large Stimulated Emmissions:

Key to Laser Action - Existence of Stimulated Emmission

$$\frac{B_{21}}{A_{21}} = \frac{B_{12}}{A_{12}} = \frac{e^3}{8\pi h \nu^3 \mu^3}$$

$$R_1 = \frac{1}{e^{h\nu/kT} - 1}$$

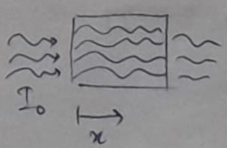
$$R_2 = \frac{\text{Stimulated Transitions}}{\text{Absorption Transitions}} = \frac{B_{21}\, \rho(\nu) \cdot N_2}{B_{12} \cdot \rho(\nu) \cdot N_1}$$

$$\text{As } B_{21} = B_{12} \Rightarrow R_2 = \frac{N_2}{N_1}$$

Lifetime in excited state ↑ ⇒ Better Material for Laser

$$I = I_0 \cdot e^{-\alpha x}$$

↳ Intensity inside the material



$[N_2 > N_1]$

Gain Coefficient - γ $[\gamma \geq 0]$

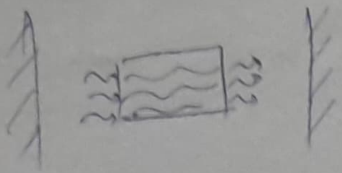
$$I = I_0 e^{\gamma x}$$

$[N_2 > N_1]$

↳ α → -ve

$N_1 > N_2 \Rightarrow \alpha > 0$ Always

[γ : Gain coeff. per unit length]



$$\gamma = (N_2 - N_1)\frac{B_{12} h\nu}{V}$$ → Conditions of Amplification

$\gamma > 0$ when $N_2 > N_1$

~~Population inversion~~ :

16/4/24

$Ar^+ \Rightarrow$ Electronic Transitions

$CO_2 \Rightarrow$ Vibrational levels

$CO_2 \Rightarrow O = C = O$

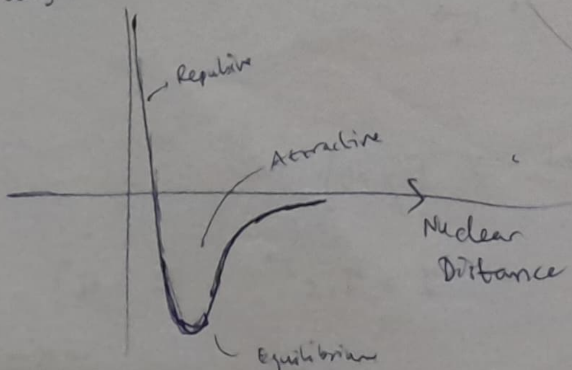
$E_{rotation} < E_{transitional} < E_{electronic}$

17/4/24

Excimer Laser — UV Laser

i.e High Energy

P.E diagram:



No Cavity Mirrors required.

$AB^* \longrightarrow A + B$: Lasing Action

(Meta Stable) (Ground State)

UV Lasers used mostly in Photolithography.

Ground State – Monomers (A + B)

12/4/24

Chemical Lasers :

NOT IN SYLLABUS

Dye Lasers

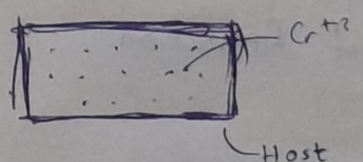
Dye : Fluorescent

Singlet & Triplet — According to Spin, states
$(2s+1, s=0)$ $(2s+1, s=1)$

Singlet $\xrightarrow{\text{Transition}}$ Singlet

Triplet $\xrightarrow{\text{Transition}}$ Triplet

Single $\xrightarrow[\text{Not Possible}]{\text{Transition}}$ Triplet



Lasing Action : Dye

Solid State Lasers – singular Wavelength

Gas Laser – 3-4 to wavelengths

Dye Lasers – 320 nm ⟶ 400 nm
(UV) (FIR)

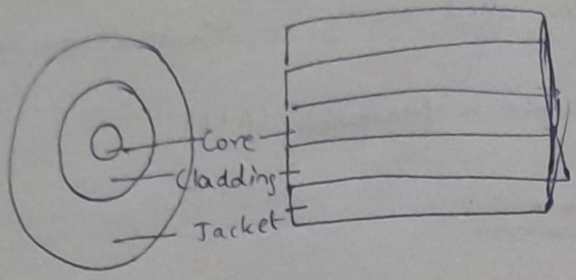
Note :

Hazardous to Human health

Pumps High Energy - Less Visibility
but Coherent.

23/4/24

FIBRE OPTICS



Fiber Cross section

Optical Fibre Material — Glass/plastic

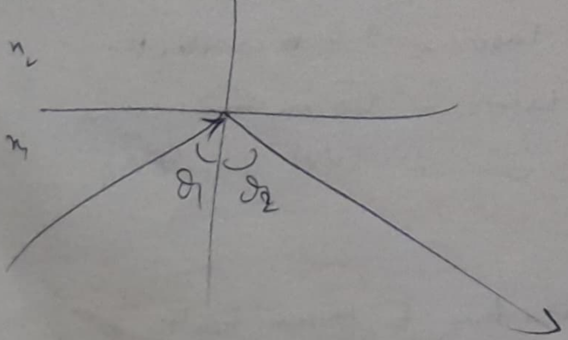
Imp:

TIR : Snell's Law

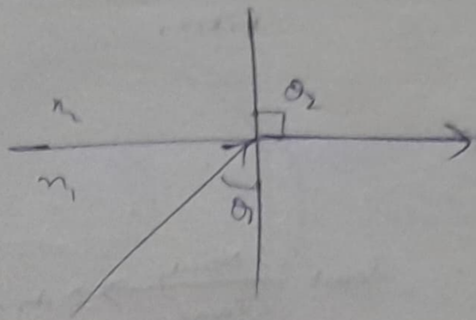
$n_1 \sin\theta_1 = n_2 \sin\theta_2$ [Relation b/w Core & Cladding]

↳ Angle of Refraction

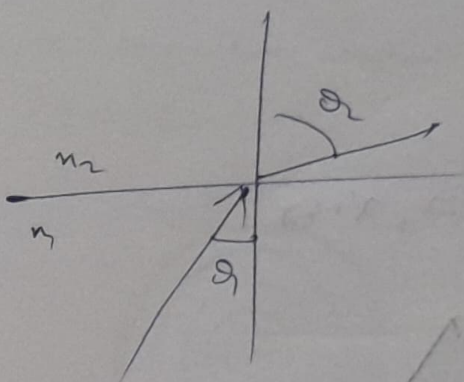
(i) Angle of Incidence > Critical Angle



(ii) Angle of Incidence = Critical Angle



(iii) Angle of Incidence < Critical Angle
↳ Min. angle for TIR



→ Numerical Apperture :

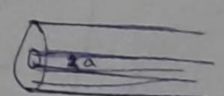
Solid Angle within which if light falls,
TIR occurs

$$NA = \sqrt{n_1^2 - n_2^2} = \sin\theta_a$$

$$V = 2\pi \cdot \frac{NA}{\lambda} a$$

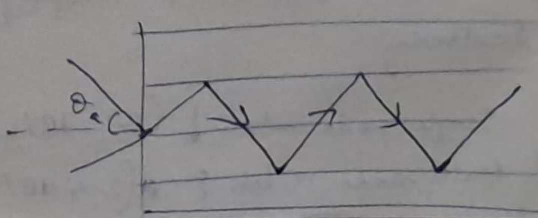
a : Core Radius

Angle of Acceptance = $2\sqrt{n_1^2 - n_2^2}$



24/4/24

Refractive Index changes step-by-step : Step-Index Fibre

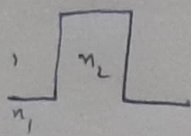


Graded Index Fibre : Less Dispersion in multi mode compared to the step-Index fibre.

Advantage

Single mode : Input $\xrightarrow[\text{Same}]{\text{Nearly}}$ Output

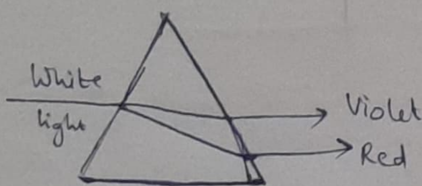
[slight diff. in diametre]



$n_2 > n_1$ for TIR

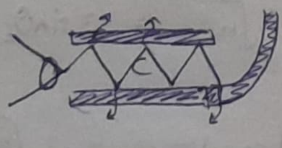
→ Dispersion :

$\lambda \longrightarrow \lambda' + 2\lambda, \lambda' + 4\lambda$



→ Losses :

↳ White Coupling of Signals



Escaping light through Cladding (leakage)
most of the time because of Bending

After long distance, Signal becomes weak
∴ Signal needs Amplifying

Scattering

Single mode – loss ↓ [0.4 dB/km] (Dispersion)
Multi mode – loss ↑ [2-4 dB/km]

- Bending ~~because~~ losses because light falls beyond critical Angle and light goes to cladding. ∴ loss.

→

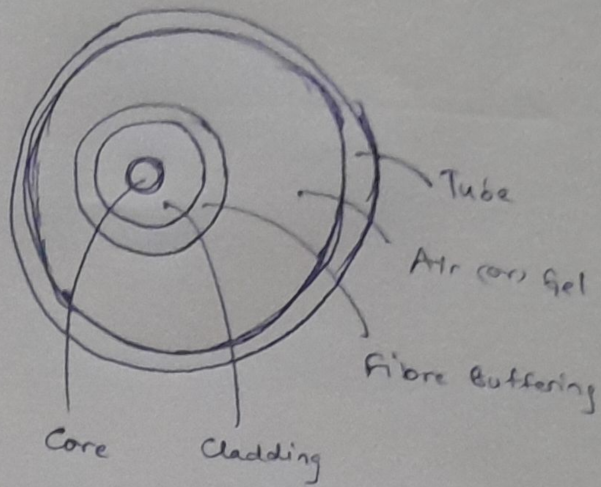


Fig.(a) Loose - Tube Construction

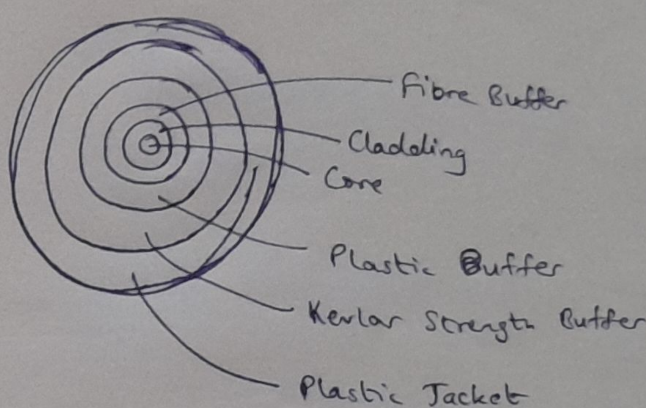


Fig. (b) Tight-buffer Construction