Engineering Optics

Lecture 4

by

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Harmonic waves

1-D differential wave equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Simplest waveform: Sine or Cosine → ***Sinusoidal / harmonic waves***

$$\boxed{\psi(x, t)\big|_{t=0} = \psi(x) = A \sin kx = f(x)}$$

Any wave → superposition of harmonic waves

k **: propagation number** → a +*ve* constant

$|\psi(x)|_{max}$ = → *maximum disturbance* → ***amplitude***

Argument of Sine function → *'phase (φ)'*

Reference: Optics *by* Hecht

Harmonic waves : *wavelength*

$$\boxed{\psi(x, t)\big|_{t=0} = \psi(x) = A \sin kx = f(x)}$$

To transform it to a wave travelling with a speed *v*

$$\boxed{\psi(x, t) = A \sin k(x - vt) = f(x - vt)}$$

Fix ‘*x*’ or ‘*t*’ → sinusoidal disturbance → periodic wave in both space and time

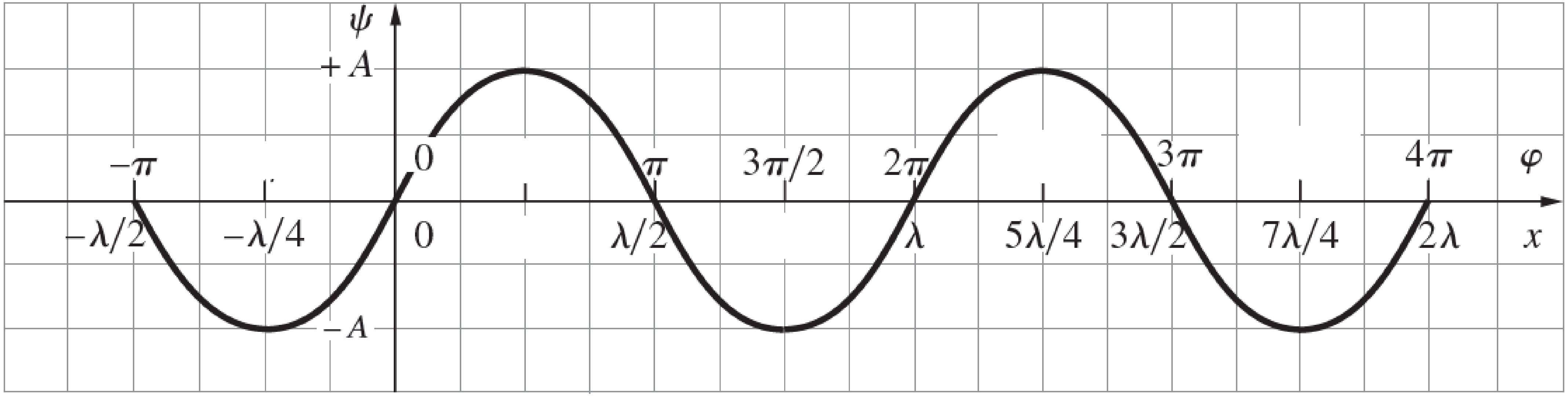
A change in x by λ = change in φ by 2π

$$\sin k(x - vt) = \sin k[(x \pm \lambda) - vt] = \sin [k(x - vt) \pm 2\pi]$$

$$|k\lambda| = 2\pi \qquad k = 2\pi/\lambda$$

Reference: Optics *by* Hecht

Harmonic waves continued



- *Spatial period* → wavelength ***'λ'*** → *meaning?* $\psi(x, t) = \psi(x \pm \lambda, t)$
- Units?

Spatial frequency: wave number (κ) = 1/λ

Reference: Optics *by* Hecht

Harmonic waves: *Frequency*

Temporal period: τ

$$\psi(x, t) = \psi(x, t \pm \tau)$$

$$\sin k(x - vt) = \sin k[x - v(t \pm \tau)]$$

$$\sin k(x - vt) = \sin [k(x - vt) \pm 2\pi]$$

$$kv\tau = 2\pi$$

$$\frac{2\pi}{\lambda} v\tau = 2\pi$$

$$\tau = \lambda / v$$

Temporal frequency: $\nu \equiv 1/\tau$

Hence, $v = \nu\lambda$

units: cycles/second or Hertz

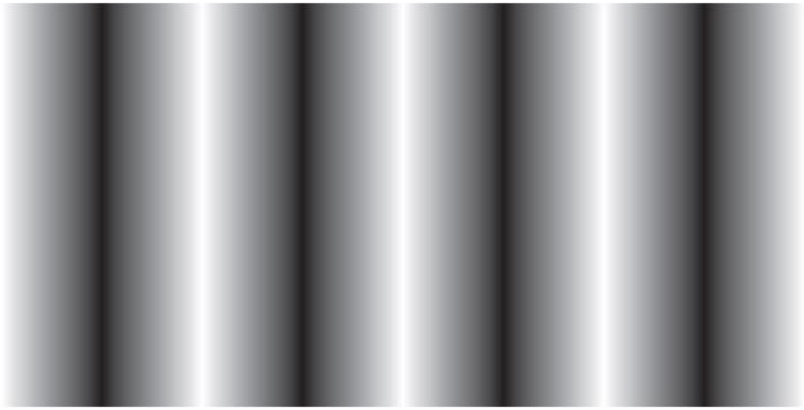
Monochromatic (monoenergetic) waves
(ideal, not reality)

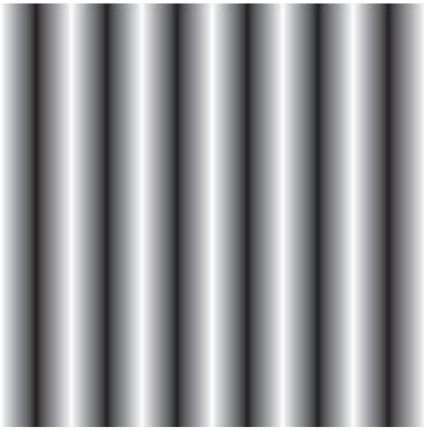
angular (temporal) frequency: $\omega = 2\pi\nu$

Reference: Optics *by* Hecht

Harmonic waves: *Frequency*

optical information → spread out in space → periodically like a harmonic wave





(a) _ and (b) _ spatial frequency (high/low)

Single κ (λ) → monochromatic.

Superposition of various such waves (each with unique λ)→ images

Reference: Optics *by* Hecht

Phase and Phase velocity

Phase

- Consider a sinusoidal wave:

$$\psi = A \sin k(x - vt)$$

[k(x-vt) = kx-kvt = kx - (2π/λ)(vλ)t = kx - (2πv)t = kx - ωt]

$$\boxed{\psi(x, t) = A \sin(kx - \omega t)}$$

Phase $\boxed{\varphi = (kx - \omega t)}$

$$\text{At } t = x = 0,\ \psi(x, t)\Big|_{\substack{x=0\\t=0}} = \psi(0, 0) = 0$$

$$\psi(x, t) = A \sin(kx - \omega t + \varepsilon)$$

ε is the **initial phase**.

Initial phase → contribution from the generator.

Phase velocity

$$\varphi(x, t) = (kx - \omega t + \varepsilon)$$

Rate-of change of phase with time: $\left|\left(\frac{\partial \varphi}{\partial t}\right)_x\right| = \omega$ (1)

Rate of change of phase with distance: $\left|\left(\frac{\partial \varphi}{\partial x}\right)_t\right| = k$ (2)

(1)/(2) → $\frac{\omega}{k} = v$ → *phase velocity*

Superposition principle

$$\frac{\partial^2 \psi_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2}$$

$$\frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2} + \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2}{\partial x^2}(\psi_1 + \psi_2) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(\psi_1 + \psi_2)$$

$$\boxed{\psi = \psi_1 + \psi_2}$$

Problems

1. Draw $\psi_1 = 1.0 \sin kx$
   $\psi_2 = 0.9 \sin kx$

   and $\psi = \psi_1 + \psi_2$

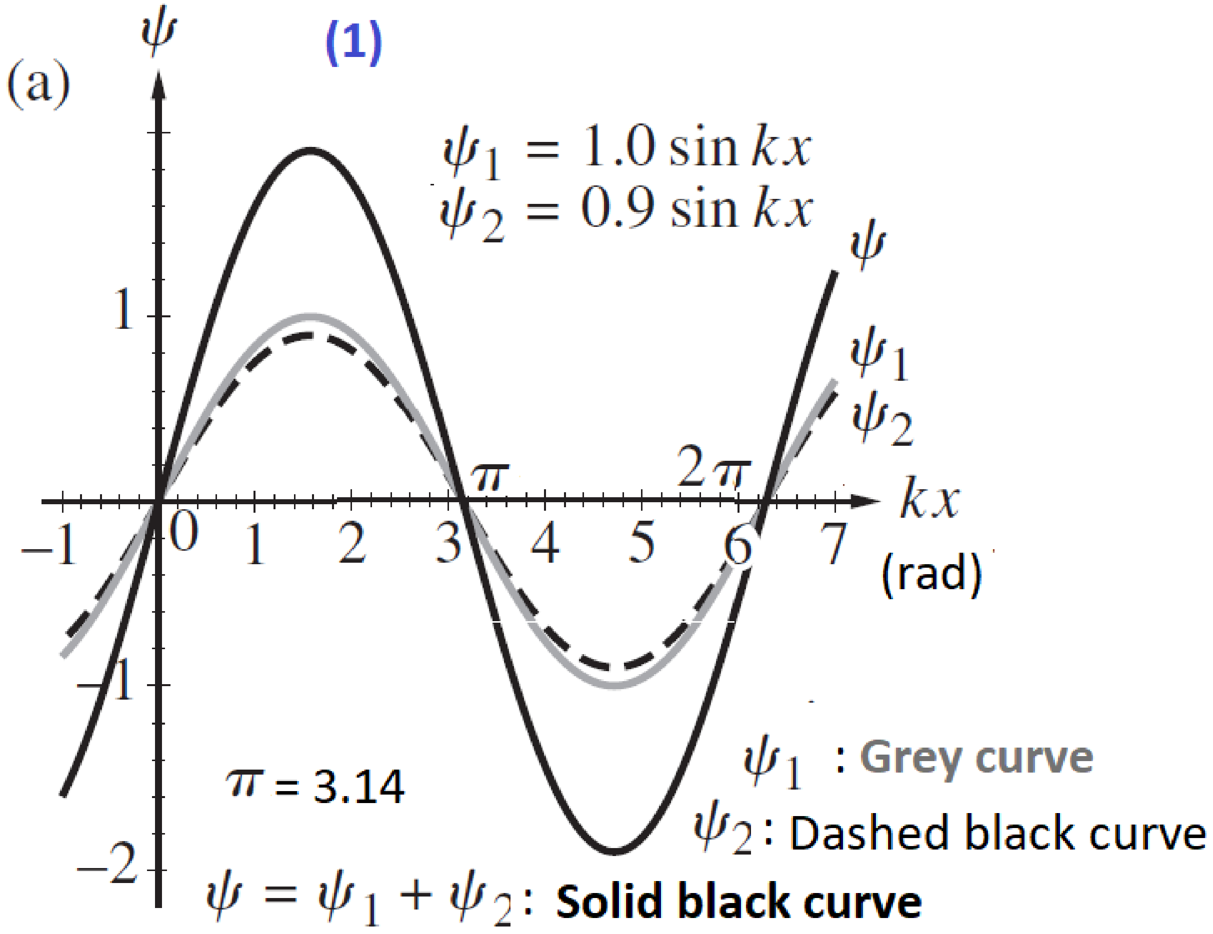
2. Draw $\psi_1 = 1.0 \sin kx$
   $\psi_2 = 0.9 \sin (kx - \pi/3)$

   and $\psi = \psi_1 + \psi_2$

3. Draw $\psi_1 = 1.0 \sin kx$
   $\psi_2 = 0.9 \sin (kx - \pi)$

   and $\psi = \psi_1 + \psi_2$

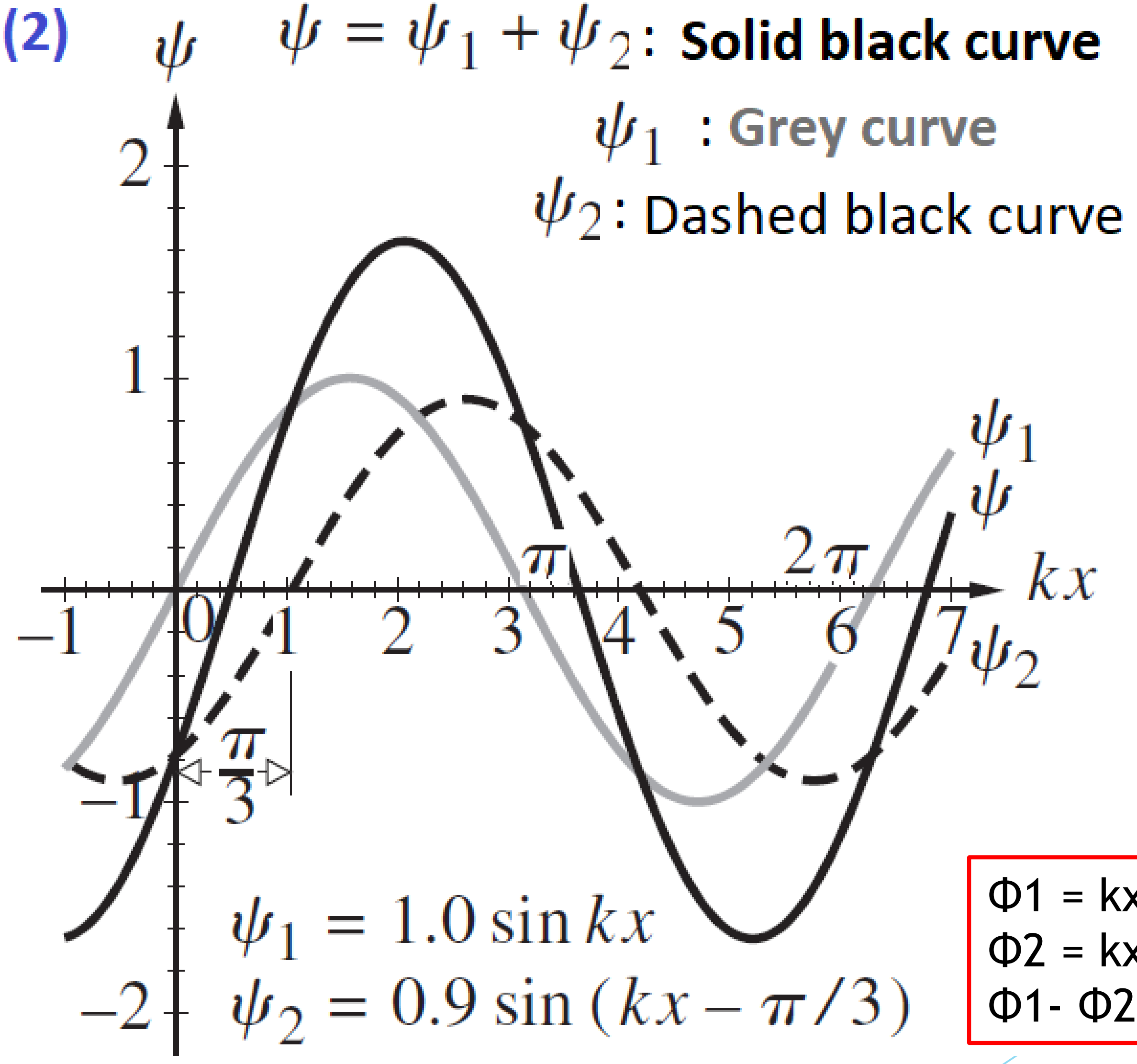
In-phase



***Constructive intereference**

Phase difference

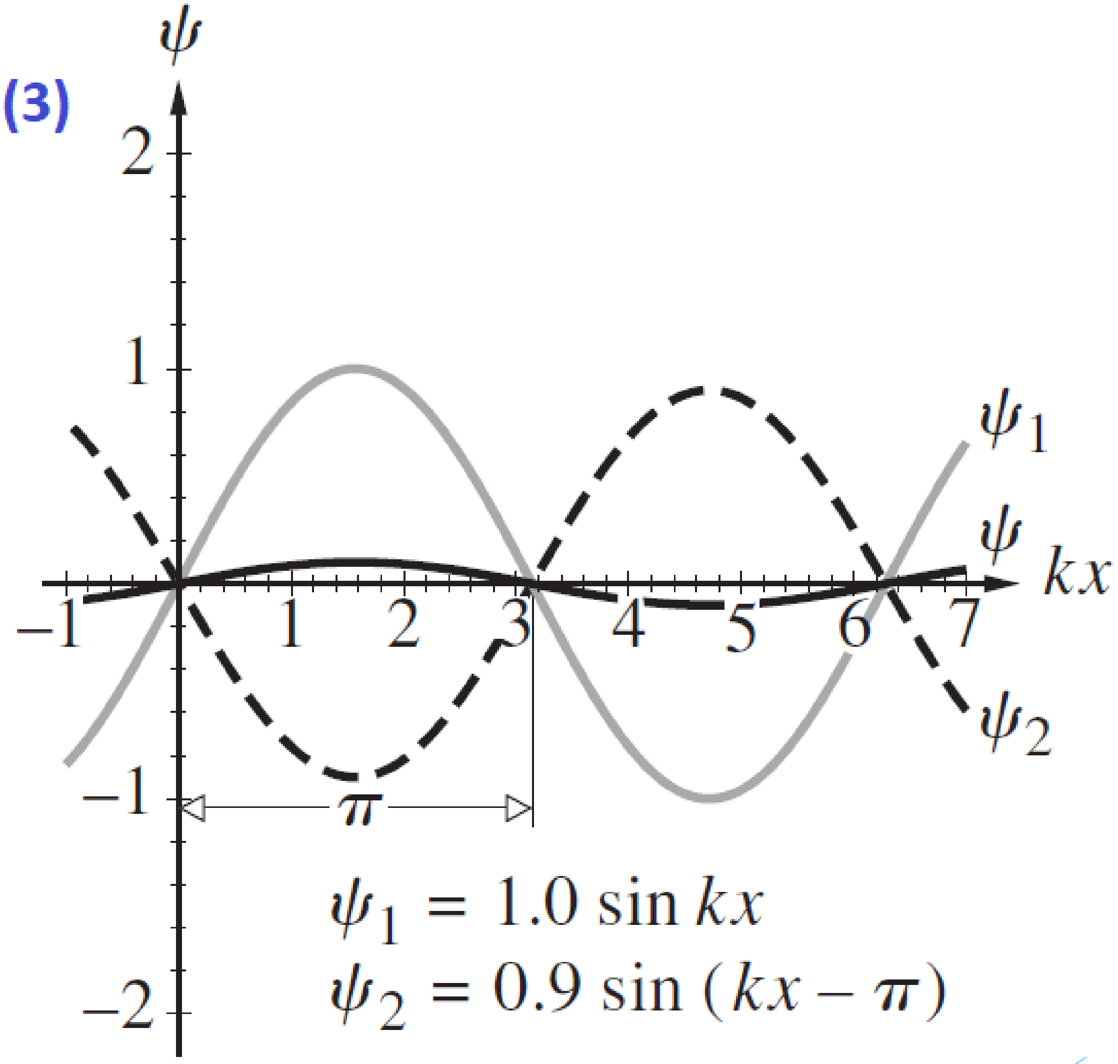
(2)



$\Phi 1$ = kx
$\Phi 2$ = kx $-\pi/3$
$\Phi 1 - \Phi 2 = \pi/3$

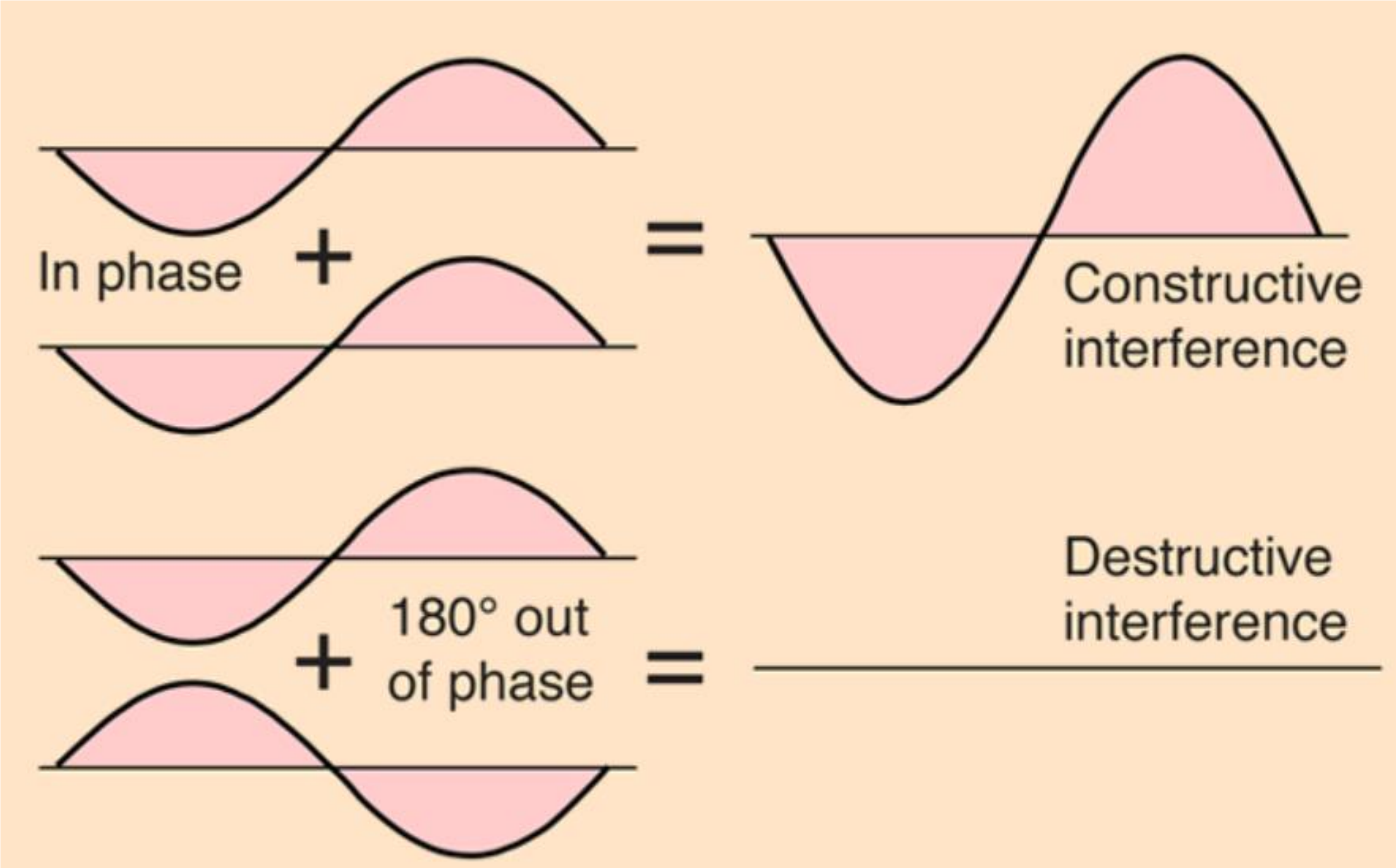
Out-of-phase

(3)



*Destructive intereference

Relative phase → Interference



Reference: http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/interf.html

Thank You