EC1001: Digital Circuits
Solution: Assignment-2_Chapter2

1. Reduce the following Boolean expressions to a minimum number of literals:

   (a) $A'C' + ABC + AC'$
   (b) $(x'y' + z)' + z + xy + wz$
   (c) $A'B(D' + C'D) + B(A + A'CD)$
   (d) $(A' + C)(A' + C')(A + B + C'D)$
   (e) $A'BD' + ABC'D' + ABCD'$

Solution:

(a) $A'C' + ABC + AC' = C'(A'+A) + ABC = C' + ABC = (C' + AB)(C'+C) = C' + AB$

(b) $(x'y' + z)' + z + xy + wz = (x'y')'z' + z + xy + wz = [(x + y)z' + z] + xy + wz$
$= (z + z')(z + x + y) + xy + wz = z + wz + x + xy + y$
$= z(1 + w) + x(1 + y) + y = x + y + z$

(c) $A'B(D' + C'D) + B(A + A'CD) = B(A'D' + A'C'D + A + A'CD) = B(A'D' + A + A'D(C + C')$
$= B(A + A'(D' + D)) = B(A + A') = B$

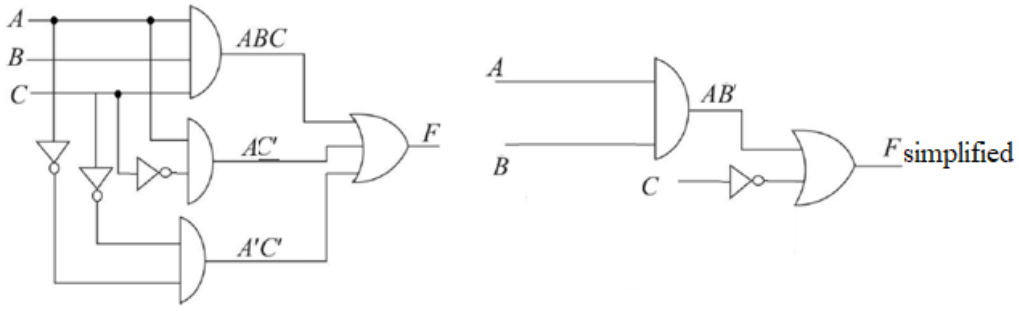
(d) $(A' + C)(A' + C')(A + B + C'D) = (A' + CC')(A + B + C'D)$
$= A'(A + B + C'D) = AA' + A'B + A'C'D = A'(B + C'D)$

(e) $A'BD' + ABC'D' + ABCD' = BD'(A' + AC' + AC)$
$= BD'$

2. Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 1 (a), (b), (c).

Solution:

(a) $A'C' + ABC + AC' = C' + AB = F$



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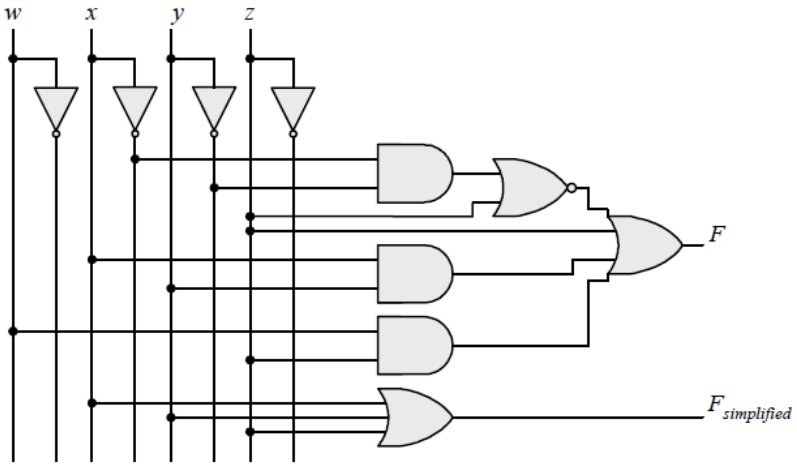
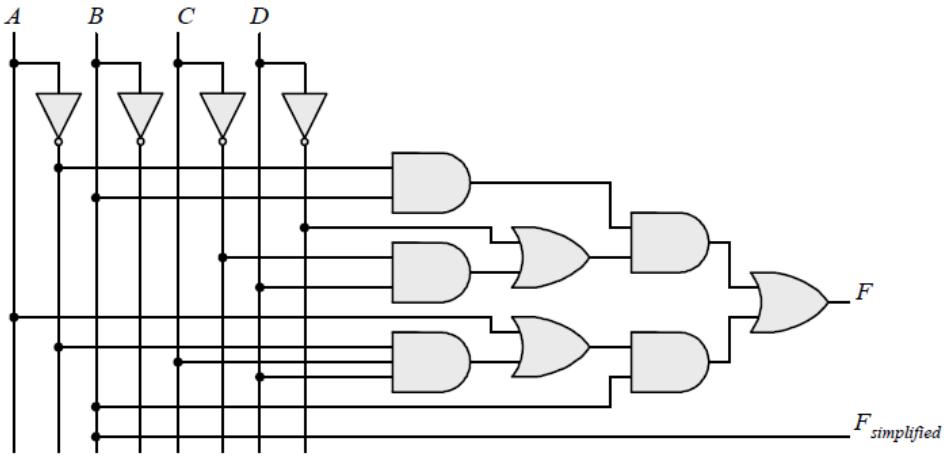
(b)



(c)



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3. Find the complement of the following expressions:
   (a) $x'y' + xy$ (b) $ac + ab' + a'bc'$
   (c) $z + z'(v'w + xy)$

Solution:

(a)
$$F = x'y' + xy$$
$$F' = (x'y' + xy)' = (x'y')'(xy)' = (x + y)(x' + y') = xy' + x'y$$

(b)
$$F = ac + ab' + a'bc'$$
$$F' = (ac + ab' + a'bc')'$$
$$= (ac)'(ab')'(a'bc')'$$
$$= (a' + c')(a' + b)(a + b' + c)$$
$$= (a' + a'b + a'c' + bc')(a + b' + c)$$
$$= (a'(1 + b + c') + bc')(a + b' + c)$$
$$= (a' + bc')(a + b' + c)$$
$$= (a' + bc')(a + (bc')')$$
$$= (a' + x)(a + x') \quad \text{assume: } bc' = x$$
$$= a'x' + ax$$
$$= (a \text{ xnor } x) = (a \text{ xnor } bc')$$

(c)
$$F' = [z + z'(v'w + xy)]' = z'[z'(v'w + xy)]'$$
$$= z'[z'v'w + xyz']' = z'[(z'v'w)'(xyz')']$$
$$= z'[(z + v + w') + (x' + y' + z)]$$
$$= z'z + z'v + z'w' + z'x' + z'y' + z'z$$
$$= z'(v + w' + x' + y')$$

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4. Express each function in sum-of-minterms and product-of-maxterms, and obtain the truth table.

(a) $F = (a' + c' + d')(ab + cd)$ (b) $F = (b + c'd')(a + bc')$

(c) $F = a'b'c' + acd + ab'd' + b'cd$

Solution:

(a) $F = (a' + c' + d')(ab + cd) = a'cd + abc' + abd'$

$= a'b'cd + a'bcd + abc'd' + abc'd + abc'd' + abcd'$

$= \Sigma(3,7,12,13,14)$

$F' = \Sigma(0,1,2,4,5,6,8,9,10,11,15)$

$F = \Pi(0,1,2,4,5,6,8,9,10,11,15)$

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

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(b) $F = (b + c'd')(a + bc') = ab + bc' + ac'd' + bc'd'$

$= abc'd' + abc'd + abcd' + abcd + a'bc'd' + a'bc'd$

$+ abc'd' + abc'd + ab'c'd' + abc'd' + abc'd' + a'bc'd'$

$= \Sigma(4,5,8,12,13,14,15)$

$F' = \Sigma(0,1,2,3,6,7,9,10,11)$

$F = \Pi(0,1,2,3,6,7,9,10,11)$

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

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(c) $F = a'b'c' + acd + ab'd' + b'cd$

$= a'b'c'd' + a'b'c'd + ab'cd + abcd + ab'c'd' + ab'cd' + a'b'cd + ab'cd$

$= \Sigma(0,1,3,8,10,11,15)$

$F' = \Sigma(2,4,5,6,7,9,12,13,14)$

$F = \Pi(2,4,5,6,7,9,12,13,14)$

a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

5. Convert each of the following expressions into sum of products and product of sums:

(a) $(u + x'w)(x + u'v)$
(b) $x' + z\,(x + y')\,(y + z')$

Solution:

(a) $(u + x'w)(x + u'v) = ux + x'wu'v$ → (SOP form)

$= (u + x')(u + w)(x + u')(x + v)$ → (POS form)

(b) $x' + z\,(x + y')\,(y + z') = x' + (xz + zy')\,(y + z') = x' + xyz$

$= x' + yz$ → SOP form

$= (x' + y)(x' + z)$ → POS form

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6. Find the complement of $F = AC + BD$; then show that $FF' = 0$ and $+F' = 1$.

Solution:

$$
\begin{aligned}
F &= AC + BD \\
F' &= (AC + BD)' = (AC)'(BD)' = (A' + C')(B' + D') \\
F.F' &= (AC + BD)(AC)'(BD)' \\
&= (AC)\,(AC)' + (BD)\,(BD)' \\
&= ((AC) + (AC)')' + ((BD) + (BD)')' \\
&= (1)' + (1)' \qquad \text{because,} \quad (AC) + (AC)' = 1 \text{ and } \quad (BD) + (BD)' = 1 \\
&= 0 \\
F + F' &= (AC + BD) + (A' + C')(B' + D') \\
&= ((AC)'(BD)')' + (AC)'(BD)' \\
&= X' + X \qquad \text{Assume: } (AC)'\,(BD)' = X \\
&= (X.X')' = (0)' \qquad \text{because, } X.X' = 0 \\
&= 1
\end{aligned}
$$

7. Simplify the following Boolean functions T_1 and T_2 to a minimum number of literals:

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

Solution:

$$T_1 = A'B'C' + A'B'C + A'BC' = A'B' + A'BC' = A'(B' + BC') = A'(B' + C')$$

$$
\begin{aligned}
T_2 &= A'BC + AB'C' + AB'C + ABC' + ABC = A'BC + AB'(C' + C) + AB(C' + C) \\
&= A'BC + AB' + AB = A'BC + A(B' + B) = A'BC + A = (A' + A)(BC + A) \\
&= A + BC
\end{aligned}
$$

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8. Represent each of the following statements by a boolean Expressions.
   (a) The company safe (S) should be unlocked only when Mr. Jones (J) is in the office or Mr. Evans (E) is in the office, and only when the company (C) is open for business, and only security guard (G) is present.
   (b) The elevator door should open if the elevator is stopped, it is level with the floor, and the timer has not expired, or if the elevator is stopped, it is level with the floor, and a button is pressed. [Note: Assume the variables are, D: Elevator door opens, S: Elevator is stopped, F: Elevator is level with floor, T: Timer has expired, B: Button is pressed]

   Solution:

   (a) $S = (J + E).CG$

   (b) $D = SFT' + SFB$

9. For the Boolean function

   $F = xy'z' + x'y'z' + xy + x'y$

   (a) Obtain the truth table of F.
   (b) Draw the logic diagram, using the original Boolean expression.
   (c) Use Boolean algebra to simplify the function to a minimum number of literals.
   (d) Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part (a).
   (e) Draw the logic diagram from the simplified expression, and compare the total number of gates with the diagram of part (b).

   Solution:

$$F = xy'z' + x'y'z' + xy + x'y = xy'z' + x'y'z' + xyz + xyz' + x'yz + x'yz' = \Sigma(0, 2, 3, 4, 6, 7)$$

(a)

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

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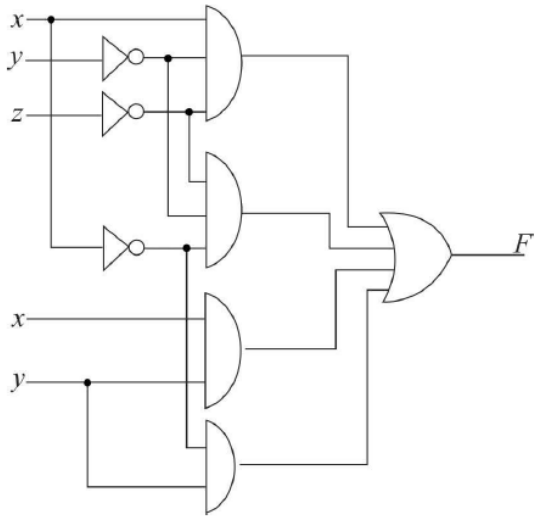
(b)



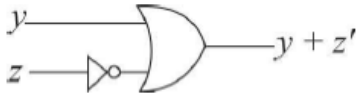
(c) $F = xy'z' + x'y'z' + xy + x'y = y'z' + y = y + z'$

(d)

x	y	z	(a)	(d)
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

Hence, $(a) = (d)$.

(e)



10. Determine whether the following Boolean equations are true or false.
    (a) $y'z' + yz' + x'z = x' + xz'$
    (b) $x'y' + xz' + yz = y'z' + xy + x'z'$

Solution:

(a) True
(b) False