Engineering Electromagnetics

Lecture 18

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by

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Conductors

(ii) $\rho = 0$ inside a conductor. This follows from Gauss's law: $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$. If $\mathbf{E}$ is zero, so also is ρ. There is still charge around, but exactly as much plus as minus, so the *net* charge density in the interior is zero.

(iii) Any net charge resides on the surface. That's the only place left.

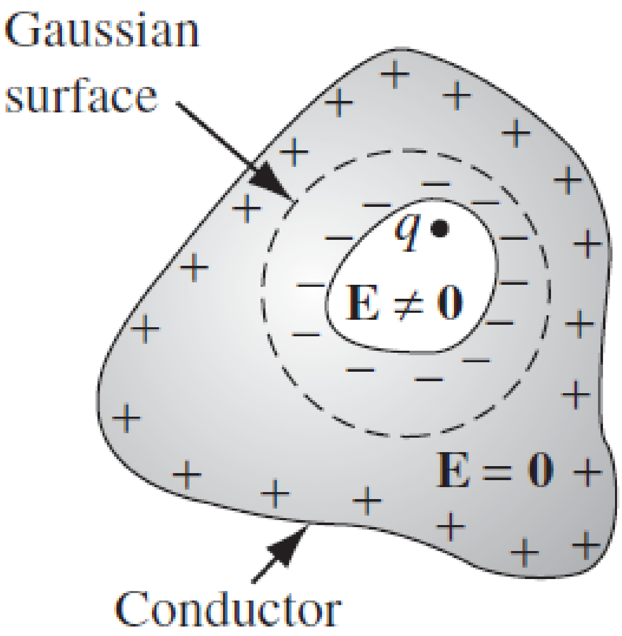
(iv) A conductor is an equipotential.

(v) E is perpendicular to the surface, just outside a conductor.

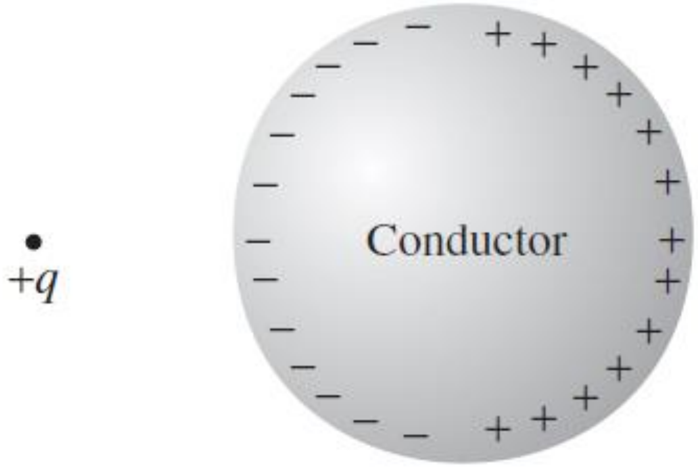
From Griffith Book

Cavity in a conductor

within that cavity you put some charge, then the field *in the cavity* will *not* be zero.



if we surround the cavity with a Gaussian surface, all points of which are in the conductor (Fig. 2.45), $\oint \mathbf{E} \cdot d\mathbf{a} = 0$, and hence (by Gauss's law) the net enclosed charge must be zero. But $Q_{\text{enc}} = q + q_{\text{induced}}$, so $q_{\text{induced}} = -q$. Then if the conductor as a whole is electrically neutral, there must be a charge $+q$ on its outer surface.



Charge is uniformly distributed within a spherical region of radius a. An isolated conducting spherical shell with inner radius b and outer radius c is placed concentrically, as shown in Figure 3.28. Determine the electric field intensity everywhere in the region.

Solution We can divide the space into four regions, as indicated in the figure.

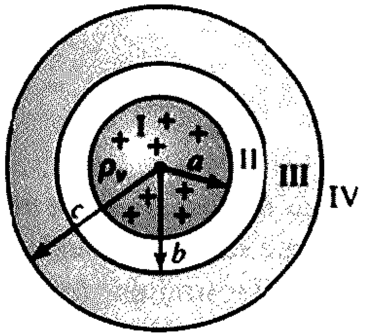


Figure 3.28 A spherical charge distribution enclosed by a conducting shell

a) Region I: For any radius $r < a$, the total charge enclosed is

$$Q = \frac{4\pi}{3} r^3 \rho_v$$

Owing to the uniform charge distribution, the $\vec{\mathbf{E}}$ field must be not only in the radial direction but also constant on a spherical (Gaussian) surface. Thus,

$$\oint_s \vec{\mathbf{E}} \cdot \vec{d\mathbf{s}} = 4\pi r^2 E_r$$

Hence, $\vec{\mathbf{E}} = \dfrac{r}{3\epsilon_0} \rho_v \vec{\mathbf{a}}_r$ for $0 < r < a$

b) Region II: $a \le r < b$. The total charge enclosed is

$$Q = \frac{4\pi}{3} a^3 \rho_v$$

and from Gauss's law,

$$\vec{\mathbf{E}} = \frac{a^3}{3\epsilon_0 r^2} \rho_v \vec{\mathbf{a}}_r \quad \text{for} \quad a \le r \le b$$

c) Region III: $b \le r \le c$. Since $\vec{\mathbf{E}}$ within a conductor must be zero, the surface at $r = b$ must possess a negative charge with magnitude equal to the total charge enclosed. If ρ_{sb} is the surface charge density, then the charge on the surface must be $-4\pi b^2 \rho_{sb}$. Thus,

$$\rho_{sb} = -\frac{a^3}{3b^2} \rho_v$$

d) Region IV: $r \geq c$: If the inner side of an isolated conducting shell acquires a negative charge, the outer side at $r = c$ must acquire an equal amount of positive charge. If ρ_{sc} is the surface charge density on the outer surface, then

$$\rho_{sc} = \frac{a^3}{3c^2}\rho_v$$

The electric field intensity in this region is

$$\vec{\mathbf{E}} = \frac{a^3}{3\epsilon_0 r^2}\rho_v\,\vec{\mathbf{a}}_r \quad \text{for} \quad r \geq c$$

• • •

Dielectrics

- Conductors: “unlimited” supply of free charges
- In dielectrics: all charges are attached to specific atoms or molecules—they’re on a tight leash, and all they can do is move a bit within the atom or molecule.

Induced Dipoles

- What happens to a neutral atom when it is placed in an electric field **E**?
- Atom as a whole is electrically neutral, nucleus + negatively charged electron cloud surrounding it.
- External field: the nucleus is pushed in the direction of the field, and the electrons the opposite way.
- Result? → (i) If the field is large enough, it can pull the atom apart completely, “ionizing” it (the substance then becomes a conductor).
- (ii) With less extreme fields → two opposing forces → (i) E pulling the electrons and nucleus apart (ii) their mutual attraction drawing them back together→ a balance (equilibrium)

⇩

atom is polarized

(plus and minus charges shifted from each other)

- The atom now has a tiny dipole moment **p**, in the same direction of **E**.

$$\mathbf{p} = \alpha \mathbf{E}$$

α → atomic polarizability

(proportionality constant, depends on structure of atom)

Induced Dipoles

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- (ii) With less extreme fields → two opposing forces → (i) E pulling the electrons and nucleus apart (ii) their mutual attraction drawing them back together→ a balance (equilibrium) → **atom is polarized** (plus and minus charges shifted from each other)
- The atom now has a tiny dipole moment **p** =q**d** is along **E**. **p** = α **E** (α → atomic polarizability)

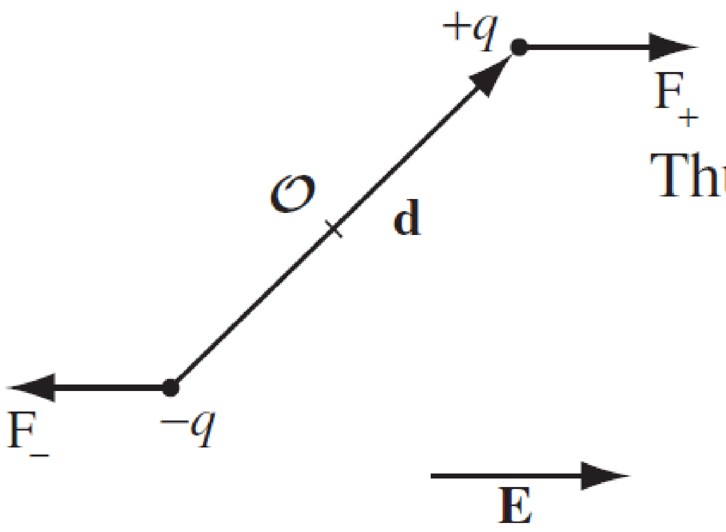
state where the center of a positive charge coincides with that of a negative charge.

separation between charge pairs

Griffith book

Polar and non-polar molecules

The neutral atom discussed in Sect. 4.1.2 had no dipole moment to start with—**p** was *induced* by the applied field. Some molecules have built-in, permanent dipole moments. In the water molecule, for example, the electrons tend to cluster around the oxygen atom (Fig. 4.4), and since the molecule is bent at 105°, this leaves a negative charge at the vertex and a net positive charge on the opposite side. (The dipole moment of water is unusually large: 6.1×10^{-30} C·m; in fact, this is what accounts for its effectiveness as a solvent.) What happens when such molecules (called **polar molecules**) are placed in an electric field?



Thus a dipole $\mathbf{p} = q\mathbf{d}$ in a uniform field $\mathbf{E}$ experiences a torque

$$\boxed{\mathbf{N} = \mathbf{p} \times \mathbf{E}.}$$

Griffith book

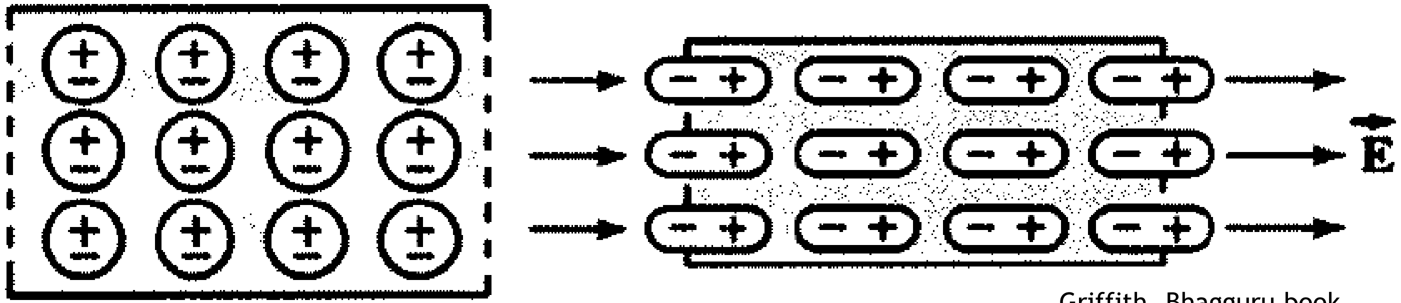
Polarization

What happens to a piece of dielectric material when it is placed in an electric field? If the substance consists of neutral atoms (or nonpolar molecules), the field will induce in each a tiny dipole moment, pointing in the same direction as the field.[3] If the material is made up of polar molecules, each permanent dipole will experience a torque, tending to line it up along the field direction.

Notice that these two mechanisms produce the same basic result: *a lot of little dipoles pointing along the direction of the field*—the material becomes **polarized**. A convenient measure of this effect is

$$\boxed{\mathbf{P} \equiv \textit{dipole moment per unit volume},}$$

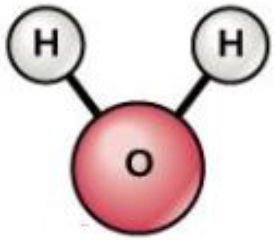
which is called the **polarization**.



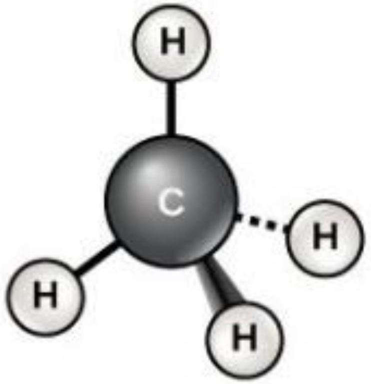
Griffith, Bhagguru book

Polar/non-polar ?

(i) polar

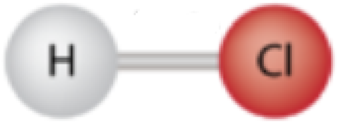


(ii) non-polar

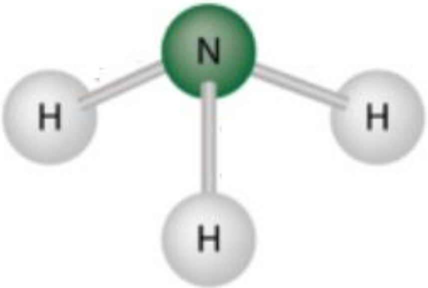


(iii) non-polar

(iv)



(v)



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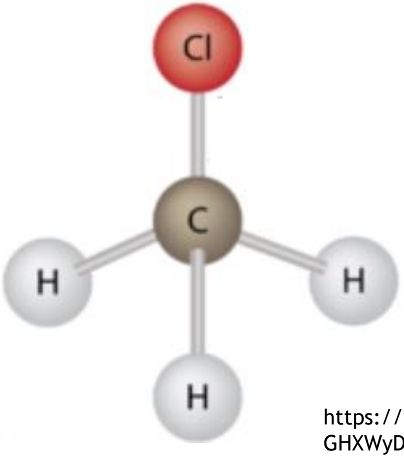
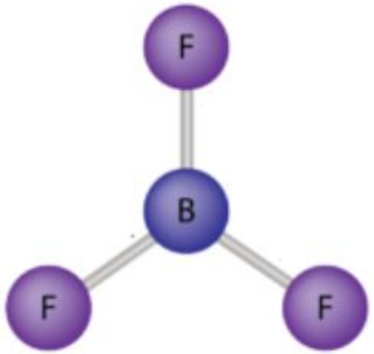
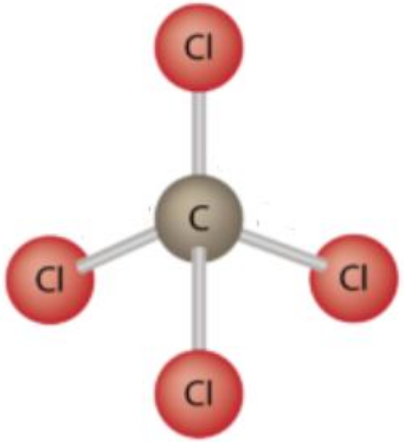


(vii)



(viii)



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Bound charge

Suppose we have a piece of polarized material—that is, an object containing a lot of microscopic dipoles lined up. The dipole moment per unit volume **P** is given. *Question:* What is the field produced by this object (not the field that may have *caused* the polarization, but the field the polarization *itself* causes)? Well, we know what the field of an individual dipole looks like, so why not chop the material up into infinitesimal dipoles and integrate to get the total? As usual, it's easier to work with the potential. For a single dipole **p** (Eq. 3.99),

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}, \leftarrow \text{For a single dipole}$$

where $\boldsymbol{\mathscr{r}}$ is the vector from the dipole to the point at which we are evaluating the potential

For unit volume?

Griffith book

Bound charge

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where $\boldsymbol{\mathscr{r}}$ is the vector from the dipole to the point at which we are evaluating the potential (Fig. 4.8). In the present context, we have a dipole moment $\mathbf{p} = \mathbf{P}\, d\tau'$ in each volume element $d\tau'$, so the total potential is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\mathbf{P}(\mathbf{r}') \cdot \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\, d\tau'. \qquad \nabla' \left(\frac{1}{\mathscr{r}} \right) = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

Griffith book

Bound charge

$$V = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \mathbf{P} \cdot \nabla' \left(\frac{1}{\mathscr{r}} \right) d\tau'.$$

Integrating by parts, using product rule number 5 (in the front cover), gives

$$V = \frac{1}{4\pi\epsilon_0} \left[\int_{\mathcal{V}} \nabla' \cdot \left(\frac{\mathbf{P}}{\mathscr{r}} \right) d\tau' - \int_{\mathcal{V}} \frac{1}{\mathscr{r}} (\nabla' \cdot \mathbf{P})\, d\tau' \right],$$

or, invoking the divergence theorem,

$$V = \frac{1}{4\pi\epsilon_0} \oint_{\mathcal{S}} \frac{1}{\mathscr{r}} \mathbf{P} \cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{1}{\mathscr{r}} (\nabla' \cdot \mathbf{P})\, d\tau'.$$

The first term looks like the potential of a surface charge

$$\boxed{\sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}}}$$

Griffith book

Bound charge

(where $\hat{\mathbf{n}}$ is the normal unit vector), while the second term looks like the potential of a volume charge

$$\boxed{\rho_b \equiv -\nabla \cdot \mathbf{P}.}$$

With these definitions, Eq. 4.10 becomes

$$\boxed{V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b}{\mathscr{r}}\, da' + \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho_b}{\mathscr{r}}\, d\tau'}.$$

What this means is that the potential (and hence also the field) of a polarized object is the same as that produced by a volume charge density $\rho_b = -\nabla \cdot \mathbf{P}$ plus a surface charge density $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$. Instead of integrating the contributions of all the infinitesimal dipoles, as in Eq. 4.9, we could first find those **bound charges**, and then calculate the fields *they* produce, in the same way we calculate the field of any other volume and surface charges (for example, using Gauss's law).

Griffith book

Thank You