Engineering Optics

Lecture 28

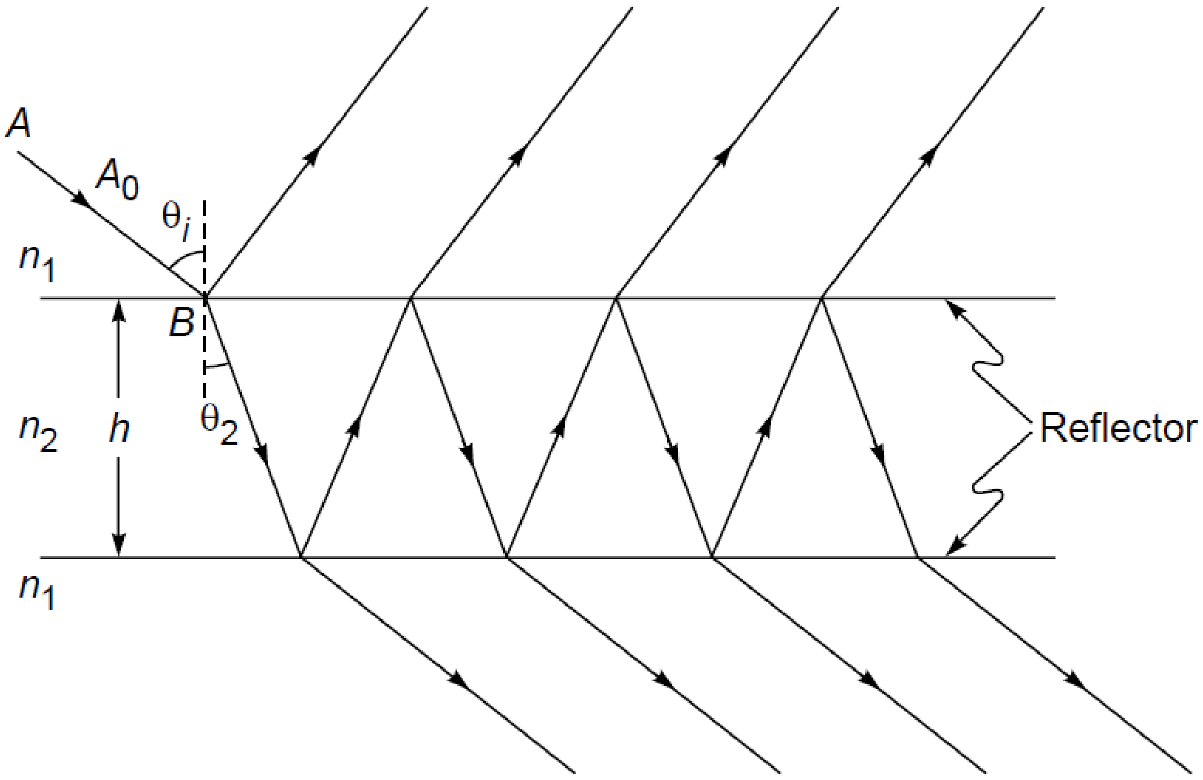
13/06/2022

by

Debolina Misra

Assistant Professor in Physics
IIITDM Kancheepuram, Chennai, India

Multiple reflections from a plane parallel film



Q: Do R and τ give any hint about single/multiple reflection?
What else do you need for multiple reflection?

http://www1.udel.edu/chem/sneal/sln_tchng/CHEM620/CHEM620/Chi_4._Light_at_Interfaces.html

(1) We've 2 media (say air-glass/ air-diamond/ water-glass etc.).
2 different media → hence 2 different r.i. (n_1, n_2)

(2) For a particular interface/pair (air-glass/water-glass etc.) reflection coefficient

$$r = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

For upper and lower surfaces $r_1 = -r_2$

(3) For that particular interface:
Reflectivity $R = r^2$
Transmittivity $\tau = 1 - R$

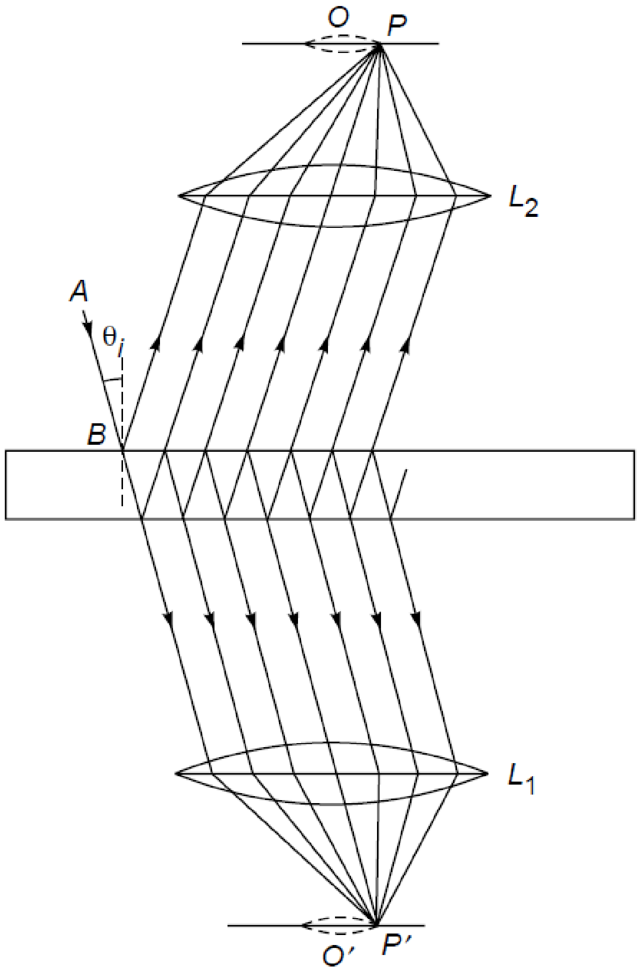
(4) Generally T → here τ to avoid confusion between transmittivity for a given interface and transmittivity of a device having multiple reflection and transmission

Optics, Ghatak

Multiple reflections from a plane parallel film

Any ray parallel to AB will focus at the same point P. If ray AB is rotated about the normal at B, then point P will rotate on the circumference of a circle centered at point O; this circle will be bright or dark depending on the value of θ_i .

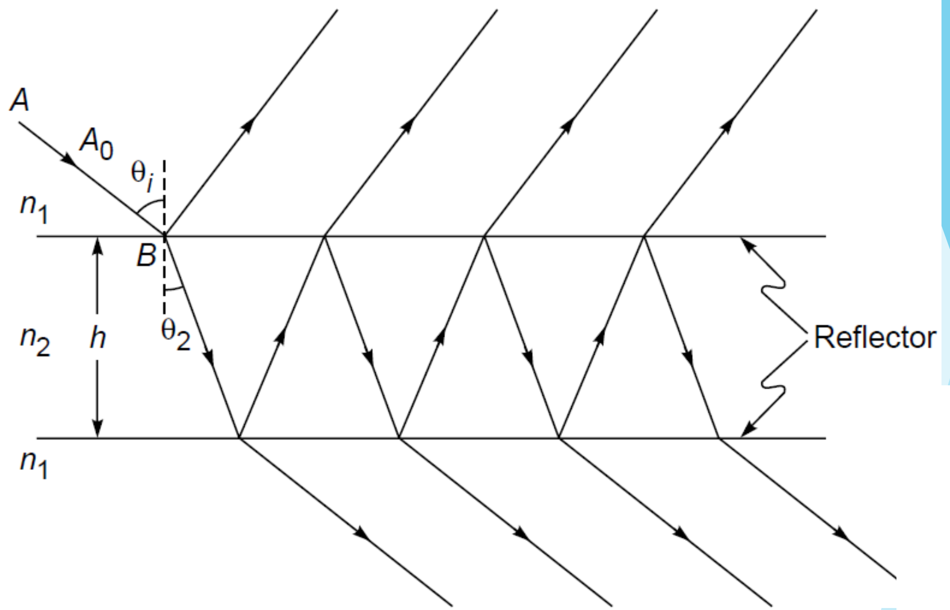
Rays incident at different angles will focus at different distances from point O, and one will obtain concentric bright and dark rings for an extended source.



Optics, Ghatak

Multiple reflections from a plane parallel film

$$A_r = A_0[r_1 + t_1 t_2 r_2 e^{i\delta}(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots)]$$
$$= A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$



$$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{4\pi n_2 h \cos\theta_2}{\lambda_0}$$

represents the phase difference (between two successive waves emanating from the plate) due to the additional path traversed by the beam in the film

So, we already had all the info (R, T for a particular interface)

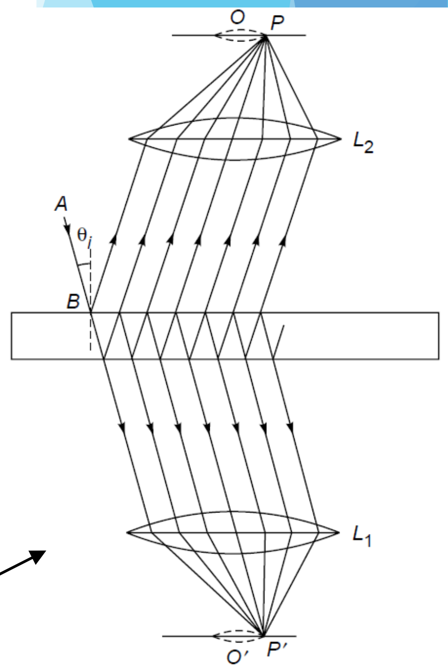
What do we need now?

1. *Resultant amplitude of reflected beam*

   (after successive/multiple reflections)

2. *Resultant amplitude of transmitted beam*

Why?



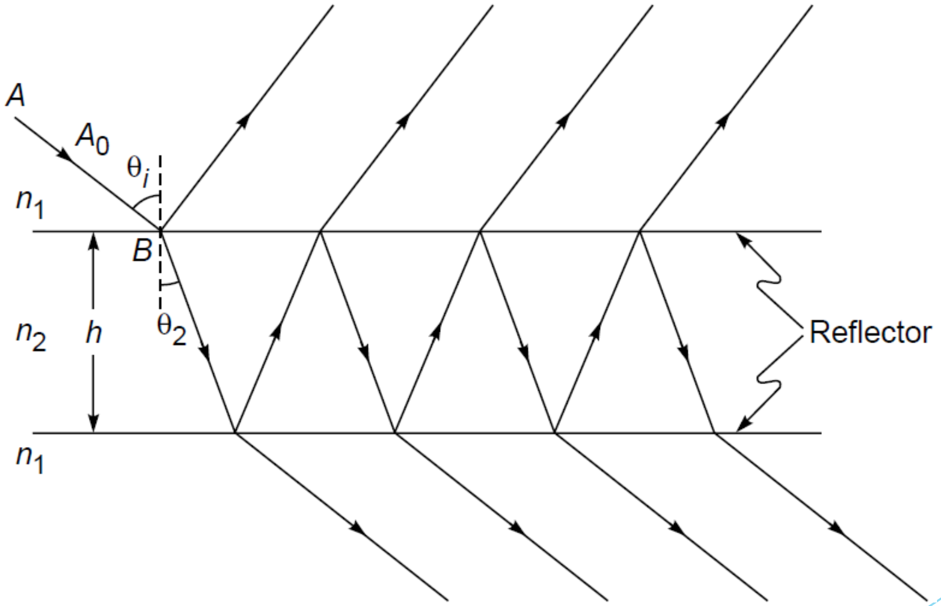
Optics, Ghatak

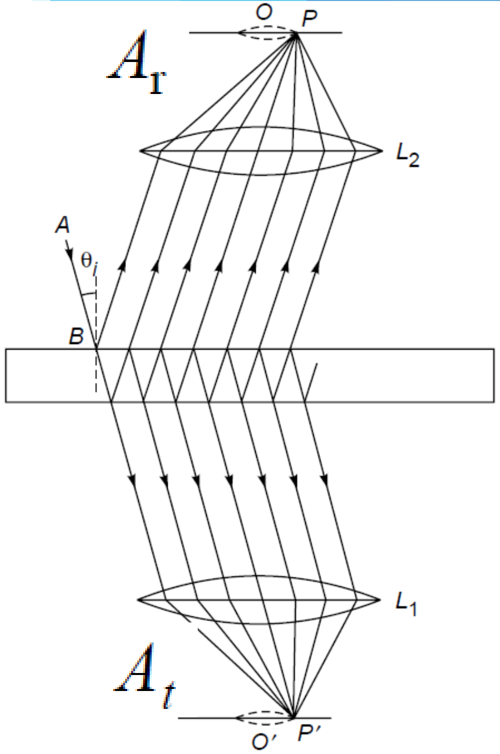
Resultant amplitude of reflected wave

$$A_r = A_0[r_1 + t_1 t_2 r_2 e^{i\delta}(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots)]$$

$$= A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$

$$\boxed{\frac{A_r}{A_0} = r_1\left[1 - \frac{(1-R)e^{i\delta}}{1 - Re^{i\delta}}\right]}$$





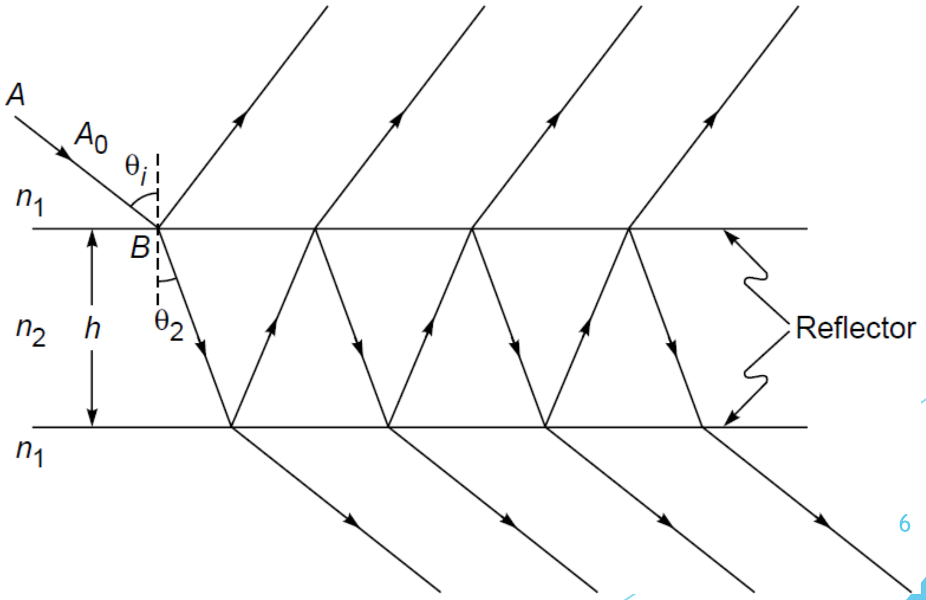
Optics, Ghatak

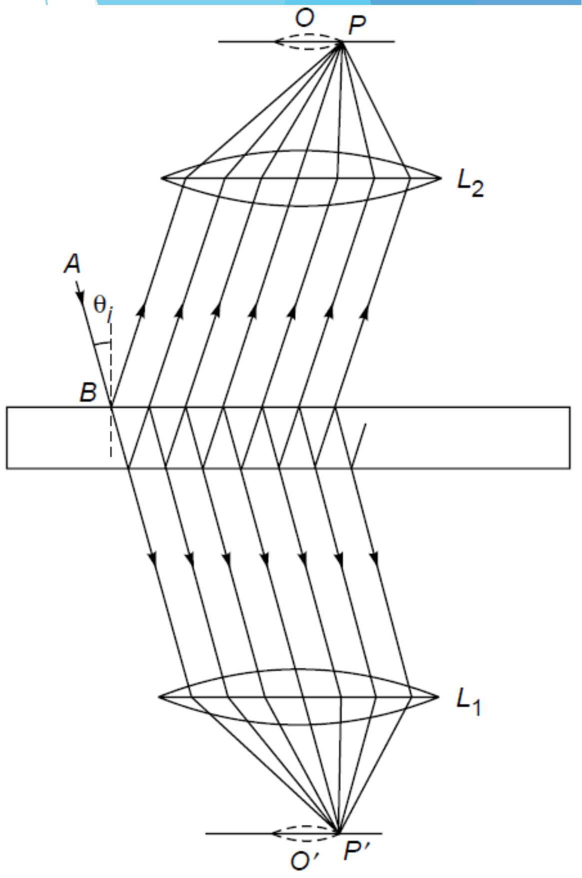
Resultant amplitude of transmitted wave

resultant amplitude of the transmitted wave

$$A_t = A_0 t_1 t_2 \left(1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \cdots\right)$$

$$= A_0 \frac{t_1 t_2}{1 - r_2^2 e^{i\delta}} = A_0 \frac{1 - R}{1 - R e^{i\delta}}$$





Optics, Ghatak

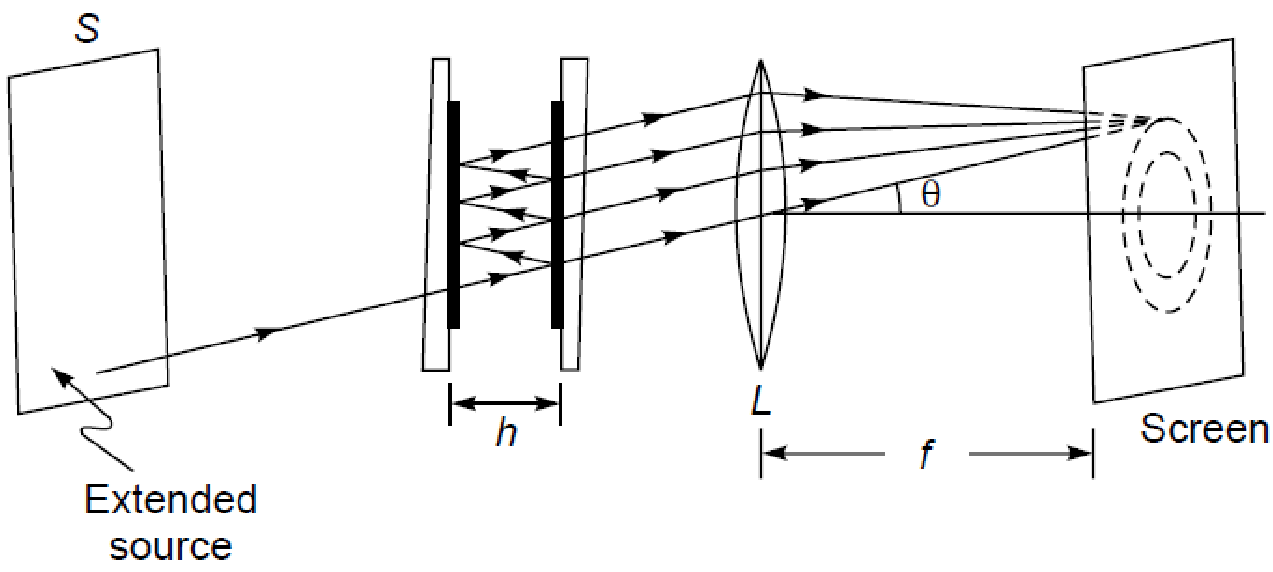
Fabry–Perot interferometer

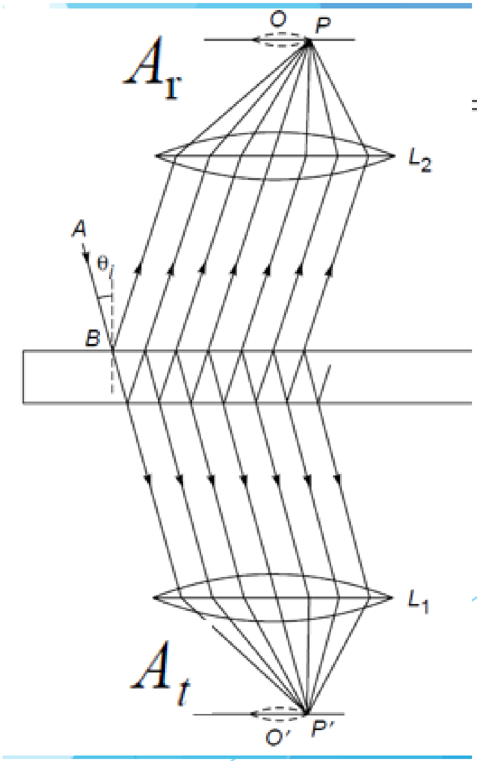
Reflectivity
of the instrument

$$\mathcal{R} = \left|\frac{A_r}{A_0}\right|^2 = R\left|\frac{1-e^{i\delta}}{1-Re^{i\delta}}\right|^2$$

$$= R\,\frac{(1-\cos\delta)^2+\sin^2\delta}{(1-R\cos\delta)^2+R^2\sin^2\delta}$$

$$= \frac{4R\sin^2\delta/2}{(1-R)^2+4R\sin^2\delta/2}$$





Transmittivity
of the instrument
(not τ → property
of the 2 media
forming interface)

$$T = \left|\frac{A_t}{A_0}\right|^2 = \frac{(1-R)^2}{(1-R\cos\delta)^2+R^2\sin^2\delta}$$

Optics, Ghatak

Measuring the wavelength of light

- Let the initial separation between the mirrors is d_1.
- vary the distance between the mirrors to d_2
- Count the number of fringes (say maxima) appearing or disappearing at the center ($\theta \approx 0$)
- then λ can be determined as follows:

$$2d_1 = m_1 \lambda,$$

$$m_2 - m_1 = \text{Number of maxima counted}$$

$$2d_2 = m_2 \lambda,$$

$$\boxed{\lambda = \frac{2(d_2 - d_1)}{m_2 - m_1}}$$

https://www.niser.ac.in/sps/sites/default/files/basic_page/Fabry-Perot%20Interferometer.pdf

Mach–Zehnder interferometer

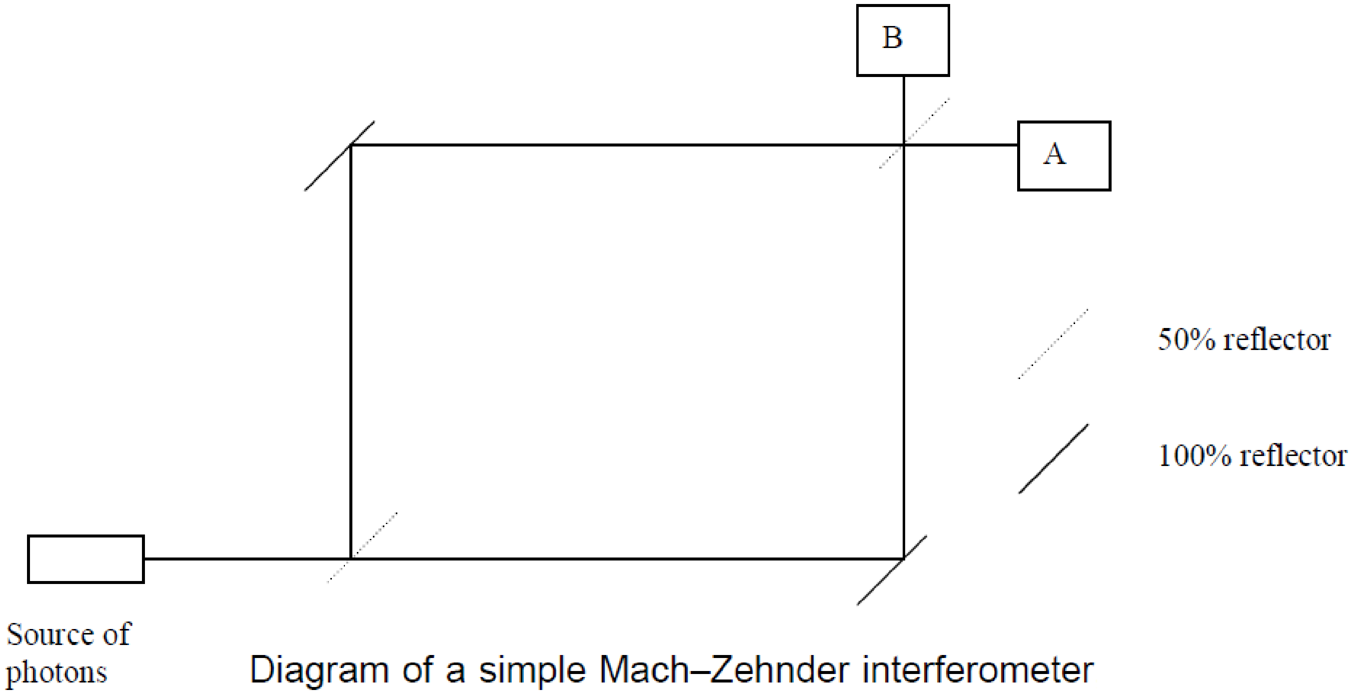


Diagram of a simple Mach–Zehnder interferometer

- used to determine the relative phase shift variations between two collimated beams derived by splitting light from a single source.
- caused by a sample or a change in length of one of the paths
- named after the physicists Ludwig Mach and Ludwig Zehnder
- In contrast to the well-known Michelson interferometer, each of the well-separated light paths is traversed only once

wikipedia

So, which path shows constructive interference, the path towards A or B?

the entire situation is symmetrical with respect to the two detectors and should one path allow constructive interference, so will the other
But is that allowed?

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Mach–Zehnder interferometer

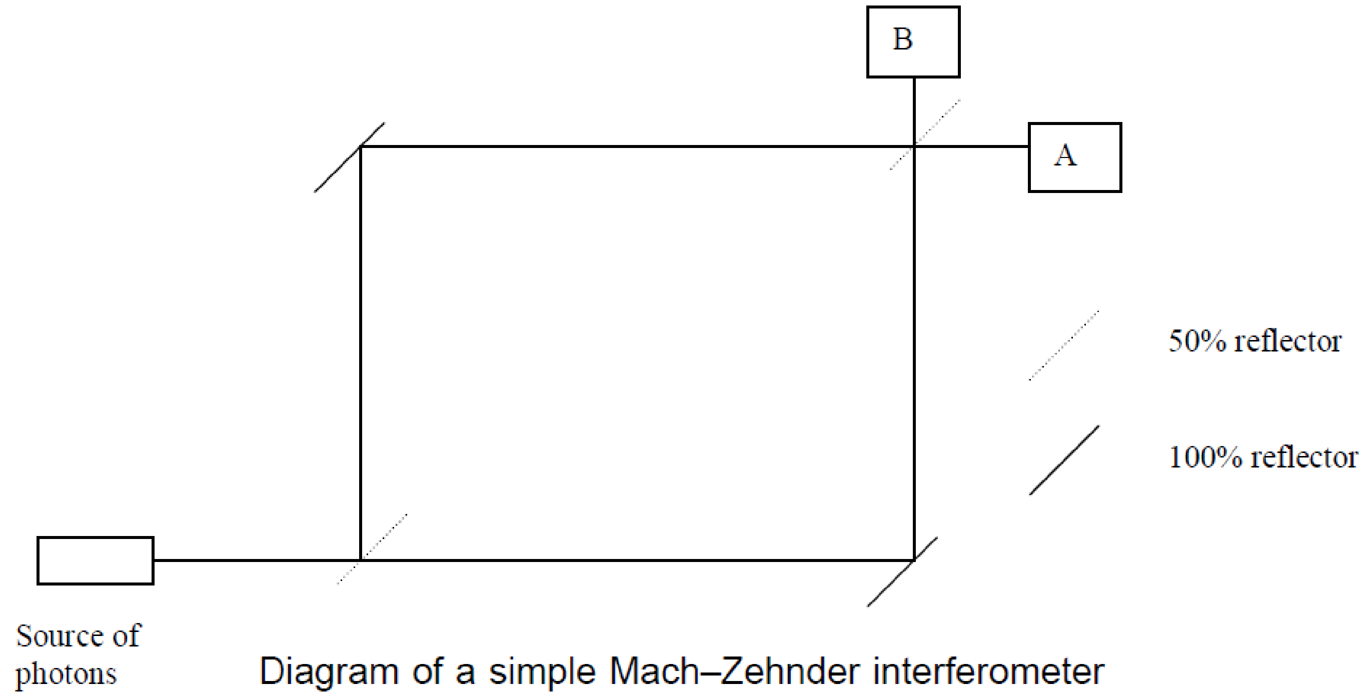


Diagram of a simple Mach–Zehnder interferometer

- look at the phase shift on reflection
- A standard piece of physics lore is that on transmission a wave picks up no phase shift, but on reflection it picks up a phase shift of π. So now let's investigate.
- We shall break the problem into two parts: (1) first the path from the source to the second beamsplitter, and then
- (2)the final stretch from the second beamsplitter to the detectors A and B.

- On the lower path, the beam undergoes -- transmission and -- reflection before the second beamsplitter. Phase shift?
- On the upper path there are -- reflection—a total phase shift of --

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Mach–Zehnder interferometer

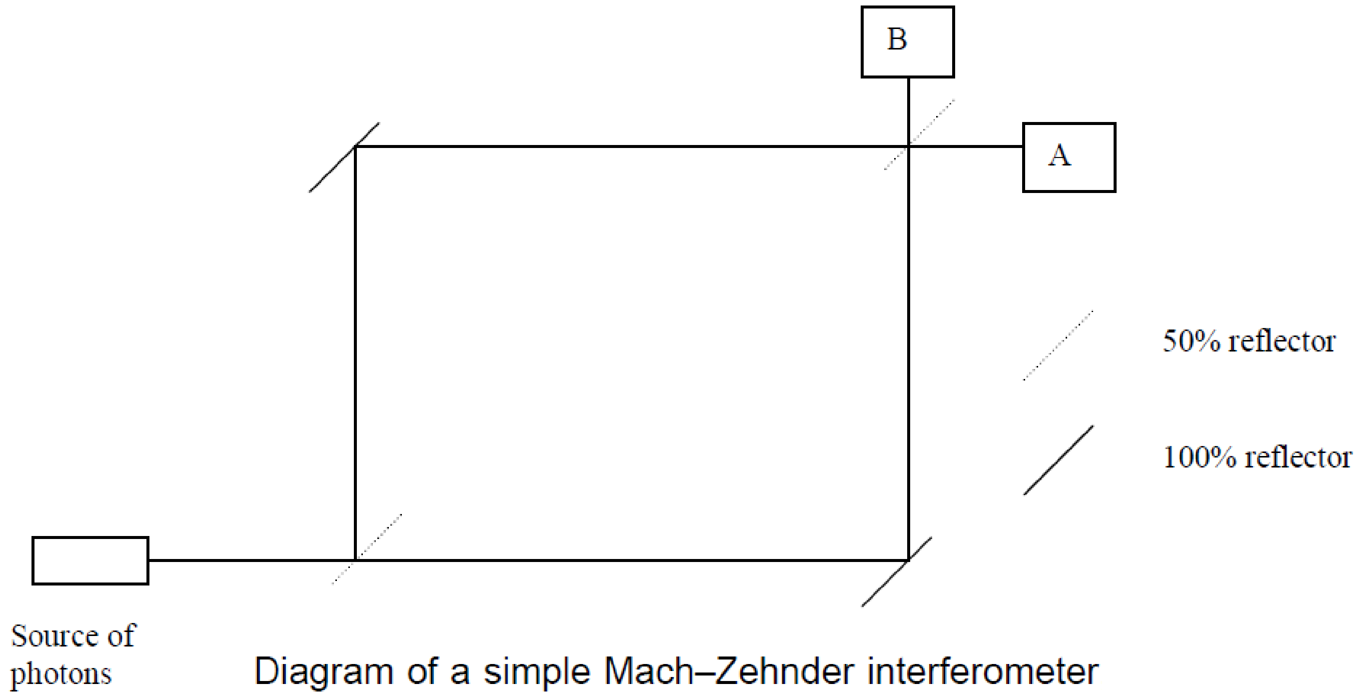


Diagram of a simple Mach–Zehnder interferometer

- Now if we continue on to **detector A**:
- the lower path makes one more reflection and the upper path one transmission.
- So now each path has a phase shift of 2π and they will **interfere constructively**.
- look at the path to **detector B**:
- the lower path makes one more transmission, picking up a total phase shift of π. The upper path makes a further reflection, so its total phase shift is 3 π.

- The puzzle/dilemma: difference is 2 π and **again we expect constructive interference.**
- So that is the problem. Energy conservation???

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

1 photon interference??

- 1905 Einstein → hypothesis that light is made of quanta
- Light follows many paths simultaneously → merge→ interfere→ pattern depends on the optical path difference between those various paths
- **Can we say the same for a photon? How can a photon follow different paths simultaneously?**
- **1 incident photon → 1 (in A) + 1 (in B) → is that possible?**
- Q: does a photon interfere with itself? → *intriguing puzzle*
- **Where did we go wrong?**

Thank You