Tutorial - 4

1. For each of the following integrals, specify the values of the parameters σ which ensures that the integral converges:

a) $\int_0^{\infty} e^{-5t}\, e^{-(\sigma+j\omega)t} dt$

b) $\int_{-\infty}^{0} e^{-5t}\, e^{-(\sigma+j\omega)t} dt$

c) $\int_{-5}^{5} e^{-5t}\, e^{-(\sigma+j\omega)t} dt$

d) $\int_{-\infty}^{\infty} e^{-5t}\, e^{-(\sigma+j\omega)t} dt$

2. Consider the signal $x(t) = e^{-5t}u(t-1)$, evaluate $X(s)$ and specify its region of convergence.
3. For each of the following algebraic expressions for the Laplace transform of a signal, determine the number of zeros located in the finite s-plane and the number of zeroes located at infinity:
   a) $\frac{1}{s+1} + \frac{1}{s+3}$
   b) $\frac{s+1}{s^2-1}$
   c) $\frac{s^3-1}{s^2+s+1}$
4. How many signals have a Laplace transform that may be expressed as $\frac{s-1}{(s+2)(s+3)(s^2+s+1)}$ in its region of convergence?
5. Find the Laplace transform of the following signals using properties
   a. $x(t) = 2^{-2t}u(t) + 4^{-4t}u(t)$
   b. $x(t) = e^{-5t}[u(t) - u(t-5)]$
   c. $x(t) = e^{-at} sin\Omega_0 t u(t)$
   d. $x(t) = t^2\, Cos\Omega_0 t u(t)$
6. Given $x_1(t) = e^{-2t}u(t)$ and $x_2(t) = e^{-3t}u(t)$. Determine Y(s) where, $y(t) = x_1(t-2) * x_2(-t+3)$.
7. Find the initial and final values, for the following transforms
   a. $\frac{s+5}{s^2+3s+2}$ b. $\frac{s^2+5s+7}{s^2+3s+2}$
8. Find the causality and stability of the system $X(s) = \frac{2}{(s+4)(s-1)}$ for the following ROC's.
   a. $-4 < Re(s) < 1$
   b. $Re(s) > 1$
   c. $Re(s) < -4$
9. Realize the transfer function of the system given in direct form I and direct form II.

$$H(s) = \frac{s+1}{s^2+3s+5}$$

10. Realize the transfer function of the system given in cascade form and parallel form.

$$H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)}$$