Problem:1

Calculate the ratio of population densities of upper and lower laser levels. Assume that the material is in thermal equilibrium. It is given that the wave length separation between energy levels is $1\mu m$ at a temperature 295 K

Answer:

- Maxwell-Boltzmann distribution (N: number of atoms/volume)

$$N_i = N_0\, exp^{-E_i/k_BT}$$

- higher E → fewer atoms there will be in that state

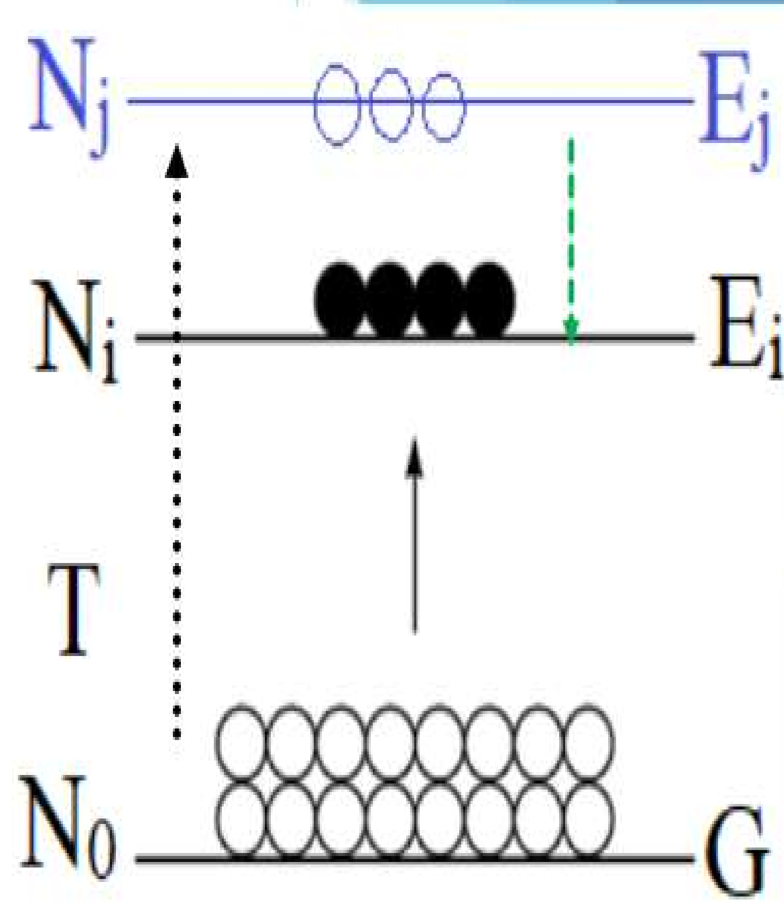
$$N_j = N_0\, exp^{-E_j/k_BT}$$

- Where $E_j > E_i$

relative population,

$$\frac{N_j}{N_i} = \frac{exp^{-E_j/k_BT}}{exp^{-E_i/k_BT}}$$

$$= exp^{-(E_j-E_i)/k_BT} = exp^{-\Delta E/k_BT} = exp^{-h\nu_{ji}/k_BT}$$



constant for a given T

Here $\upsilon_{ji} = c/\lambda_{ji}$

Then
$$\frac{N_j}{N_i} = e^{-\frac{hc}{\lambda_{ji} k_B T}}$$

Where
$$hc = 1.98 \times 10^{-25} J\ m \qquad h = 6.626 \times 10^{-34} J\ s$$

$$\lambda_{ji} = 10^{-6}\ m, \text{ T=295 K}$$

$$k_B = 1.38 \times 10^{-23} J/K$$

$$\frac{N_j}{N_i} = e^{-\frac{1.98\times10^{-25}}{10^{-6}\times1.38\times10^{-23}\times295}} = 6.21 \times 10^{-22}$$

Problem:2

A Nd:YAG laser rod is composed of Nd ions doped at a 1% concentration into an yttrium aluminum garnet host. That corresponds to a Nd^{+3} ion density in the laser rod of about $1.38 \times 1026\, m^{-3}$. Suppose all of these ions are pumped to their upper $^{4}F_{3/2}$ levels essentially all at once. From there they cascade downward, emitting radiation at 1060 nm. Determine the energy radiated per cubic meter of rod.

Answer:

Let's first determine the energy of each photon. Then if we assume all the Nd ions radiate, we can find the total energy emitted. At 1060 nm the photon energy is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\,\mathrm{J \cdot s})(2.998 \times 10^{8}\,\mathrm{m/s})}{1060 \times 10^{-9}\,\mathrm{m}}$$

$$= 1.874 \times 10^{-19}\,\mathrm{J}$$

Now if there are 1.38×10^{26} ions/m^3, each radiating a 1.874×10^{-19} J photon, the total amount of energy emitted per cubic meter is

$$E_T = (1.874 \times 10^{-19}\ \mathrm{J})(1.38 \times 10^{26}\ \mathrm{m}^{-3})$$

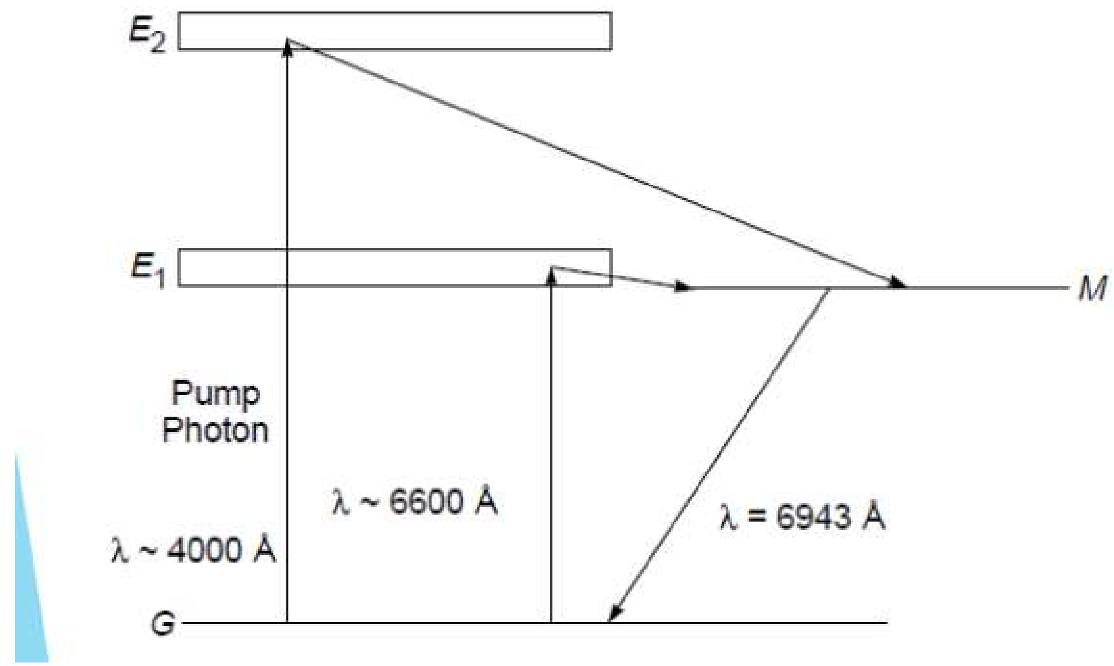
$$E_T = 25.9 \times 10^{6}\ \mathrm{J/m^3}$$

Problem:3

In Ruby laser a an output of wavelength 694.3nm is obtained. Calculate the relative population at a temperature 300K between E_1 and ground state

We know that

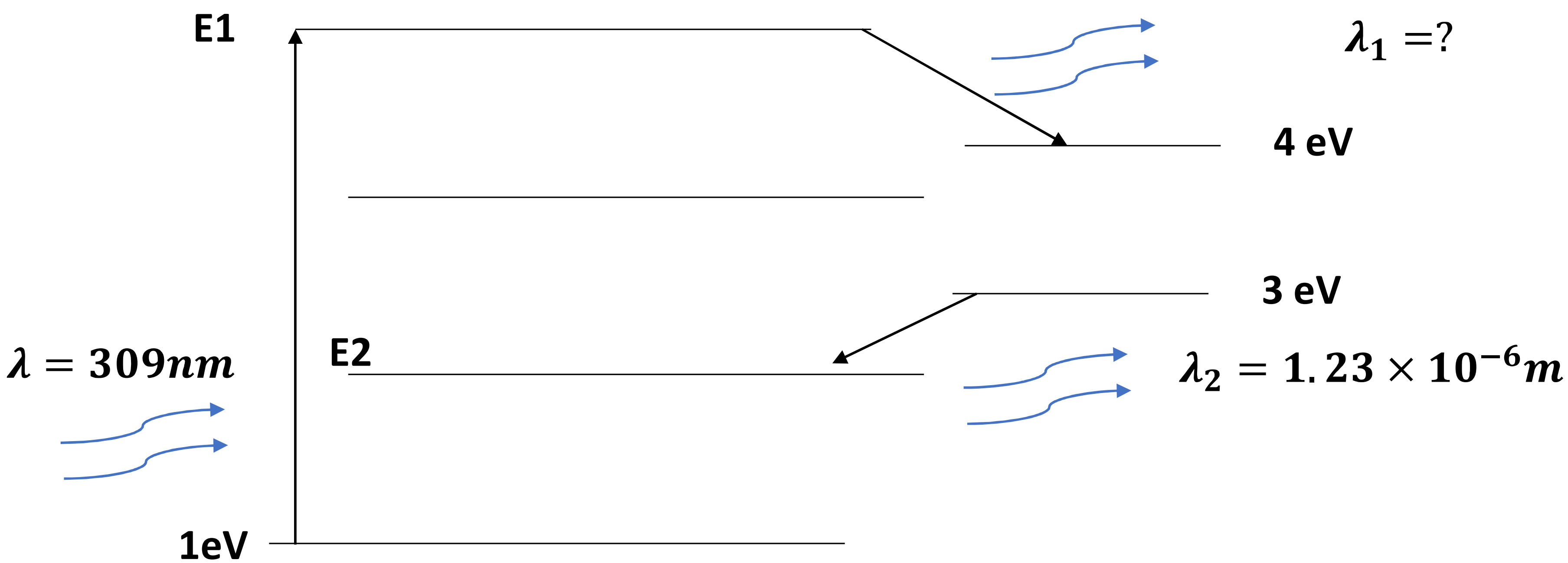
$$\frac{N_j}{N_i} = e^{-\frac{hc}{\lambda_{ji} k_B T}}$$



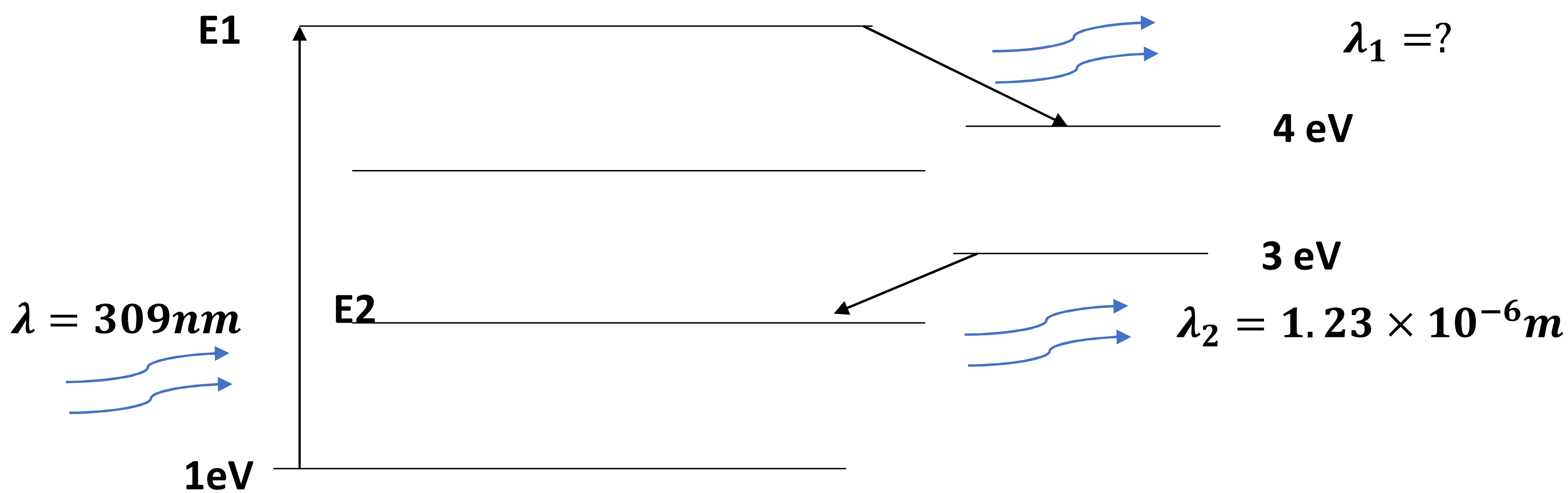
$$\frac{N_j}{N_i} = e^{-\frac{1.98\times10^{-25}}{694.3\times10^{-9}\times1.38\times10^{-23}\times300}} = 1.21\times10^{-30}$$

Problem:4

From the given energy spectra of the 4 level laser find out the unknown energy levels E_1, E_2 and λ_1.



Answer:



E_1 calculation

$$E_1 - 1eV = \text{hc}/\boldsymbol{\lambda}(\boldsymbol{eV})$$

$$\frac{\text{hc}}{\boldsymbol{\lambda}}(\boldsymbol{eV}) = \frac{1.98644582 \times 10^{-25}}{309 \times 10^{-9} \times 1.62 \times 10^{-19}} = 4eV$$

$$E_1 = 5eV$$

$\boldsymbol{\lambda_1}$ calculation

$$5eV - 4eV = \mathrm{hc}/\lambda_1\ (eV)$$

$$1.62\times10^{-19} = \mathrm{hc}/\lambda_1$$

$$\lambda_1 = 1.98644582\times10^{-25}/1.62\times10^{-19}$$

$$\lambda_1 = 1.23\times10^{-6}m$$

$$\Rightarrow E_2 = 2eV$$