Engineering Electromagnetics

Lecture 32

20/11/2023

by

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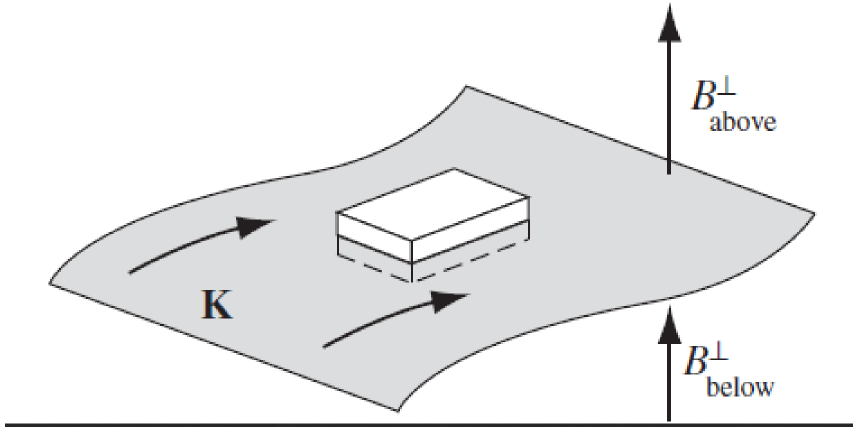
Boundary condition

Just as the electric field suffers a discontinuity at a surface *charge*, so the magnetic field is discontinuous at a surface *current*. Only this time it is the *tangential* component that changes. For if we apply Eq. 5.50, in integral form,

$$\oint \mathbf{B} \cdot d\mathbf{a} = 0,$$

$$B^{\perp}_{\text{above}} = B^{\perp}_{\text{below}}$$



$$\oint \mathbf{B} \cdot d\mathbf{l} = \left(B^{\parallel}_{\text{above}} - B^{\parallel}_{\text{below}}\right) l = \mu_0 I_{\text{enc}} = \mu_0 K l,$$

$$B^{\parallel}_{\text{above}} - B^{\parallel}_{\text{below}} = \mu_0 K.$$

From Griffith book

Static E and B

In the study of static fields we concluded that

(a) static electric fields are created by charges

(b) static magnetic fields are produced by charges in motion or steady currents.

(c) the static electric field is a conservative field because it has no curl

(d) the static magnetic field is continuous because its divergence is zero

(e) the static electric field can exist even when there is no static magnetic field and vice versa

2 new concepts

- **E** produced by **B**(t) → Expt. By Faraday
- **B** produced by **E**(t) → Theoretical concept by Maxwell
- Oersted → Current carrying wire → deflects a needle
- Faraday professed → Mag. Field can also produce *I*
- Worked for 10 year → 1831 → toroid → 2 separate windings → Galvanometer and battery. Deflection in Galv during closed and disconnected circuits.

electro-motive force (emf)

- Now we know:
- Time-varying B→ produces an ***electro-motive force (emf)*** → that produces a current in a suitable closed circuit
- Emf (e or ε) is a voltage generated
- When?
- i. either B changes
- ii. circuit is in motion
- iii. both

emf

- $emf = -\frac{d\Phi}{dt}$
- What kind of closed path? → not necessarily only conductor→ R, C
- Flux through any surface with closed perimeter
- Change in flux Φ → $\frac{d\Phi}{dt}$
- $\frac{d\Phi}{dt} \neq 0$ when (i) B is f(t)
- (ii) conductor is in motion.
- Why? $\Phi = \int B.dS$ → either B or S (effective area) need to change with t
- ‘-’ve sign? → change in Φ is resisted by emf
- $\boxed{emf = -\frac{d\Phi}{dt} \text{ Lenz's law}}$
- If N turns? $emf = -N\frac{d\Phi}{dt}$

Motional electric field

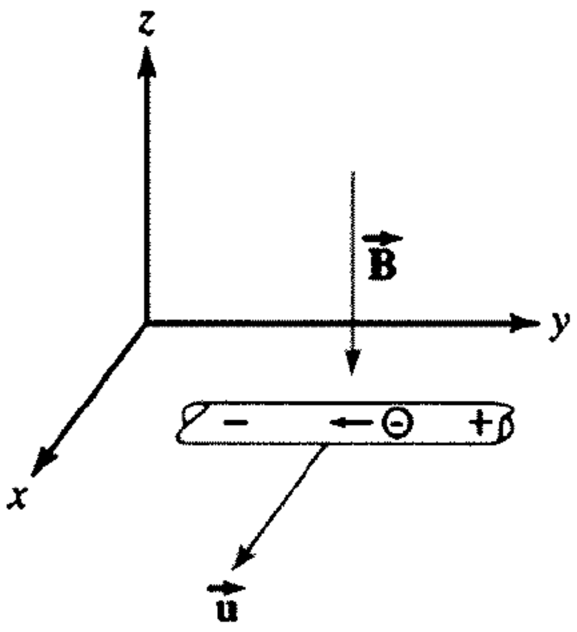


Figure 7.1 A conductor moving in a uniform magnetic field

uniform flux density $\vec{\mathbf{B}}$ such that $\vec{\mathbf{B}} = -B\vec{\mathbf{a}}_z$. The magnetic force acting on each of the free electrons in the conductor is

$$\begin{aligned}\vec{\mathbf{F}} &= q_e\vec{\mathbf{u}} \times \vec{\mathbf{B}} \\ &= q_e uB\vec{\mathbf{a}}_y\end{aligned} \tag{7.1}$$

$\vec{\mathbf{E}} = \vec{\mathbf{u}} \times \vec{\mathbf{B}} = uB\vec{\mathbf{a}}_y$ *motional electric field.*

The induced electric field is tangential to the surface of the conductor.

field.

When both **v** and **B**(t) exist?

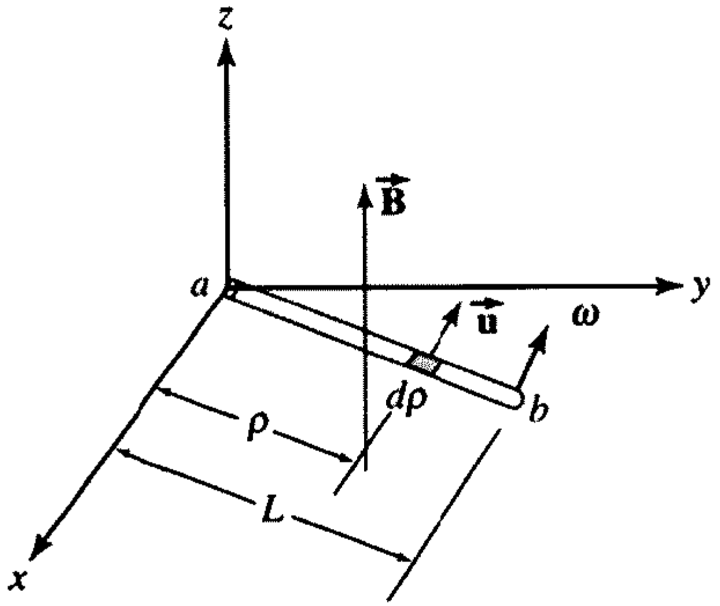
$\frac{d\Phi}{dt} \neq 0$ when (i) B is f(t)
(ii) conductor is in motion

We now define the *electromotive force* or the *induced emf* as the amount of work done per unit positive charge by the external force:

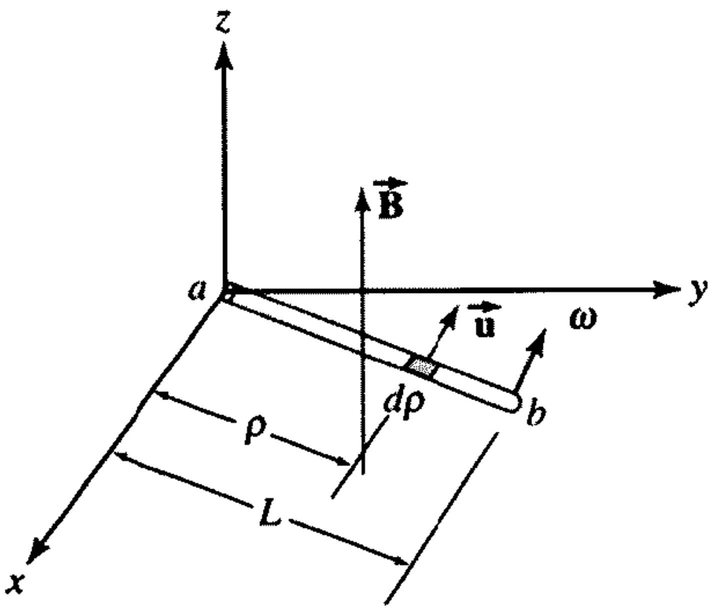
$$e = \frac{dW}{dq} = BLu \tag{7.5}$$

Problem-1

A copper strip of length L pivoted at one end is rotating freely with an angular velocity ω in a uniform magnetic field, as shown in Figure 7.3. What is the induced emf between the two ends of the strip?



Solution-1



The velocity at any radius ρ of the strip is

$$\vec{\mathbf{u}} = \rho\omega\vec{\mathbf{a}}_\phi$$

The induced electric field intensity is

$$\vec{\mathbf{E}} = \vec{\mathbf{u}} \times \vec{\mathbf{B}} = \rho\omega B(\vec{\mathbf{a}}_\phi \times \vec{\mathbf{a}}_z) = \rho\omega B\vec{\mathbf{a}}_\rho$$

$$e_{ba} = \omega B \int_0^L \rho \, d\rho$$

$$= \tfrac{1}{2} B\omega L^2$$

W=F.dl/q = E.dl or f_s.dl in Griffith

Thank You