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Indian Institute of Information Technology, Design and Manufacturing, Kancheepuram
End Semester Examination – January-May, 2024

Course Code: MA1001
Batches: All
Date of Examination: 01.05.2024
Duration: 3 Hours

Course Title: Differential Equations
Category: Core

Maximum Marks: 50

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1. (a) State Picard's theorem for first order ordinary differential equations. (2)
   (b) Find all solutions of the initial value problem $y' = 3y^{2/3}, \quad y(0) = 0$. (1)
   (c) Does part (b) contradict Picard's theorem? If not, why? (1)
2. Solve: $(y \log y - 2xy)dx + (x + y)dy = 0$. (3)
3. Solve by reducing the equation $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ into a linear equation. (3)
4. Find the general solution of the following differential equation in **three** different ways: $x^2 y'' + xy' - y = 0$. (6)
5. Derive the formula of the method of variation of parameters for a particular solution of the equation $y'' + P(x)y' + Q(x)y = R(x)$. (4)
6. If p is not zero or a positive integer, show that $(1+x)^p = \sum_{n=0}^{\infty} \frac{p(p-1)(p-2)\ldots(p-n+1)}{n!} x^n$ for $|x| < 1$. [Here $p(p-1)(p-2)\ldots(p-n+1) = 1$ when $n = 0$.] (4)
7. Find *the* Frobenius series solution of the differential equation $xy'' + y' + xy = 0$. (5)
8. (a) Find the Laplace transform of $f(x) = \int_0^x \frac{1 - e^{-u}}{u} du$. (2)
   (b) Find the inverse Laplace transform of $F(p) = \log\left(\frac{p^2 + 1}{p(p+1)}\right)$. (2)
9. Prove: If $L[f(x)] = F(p)$, then $\int_0^\infty \frac{f(x)}{x} dx = \int_0^\infty F(p) dp$. Hence compute $\int_0^\infty \frac{\sin x}{x} dx$. (3)
10. State and prove the convolution theorem on Laplace transforms. (4)
11. Find the Fourier series of the function $f(x) = x, \quad -\pi \le x < \pi$. (4)
12. Prove: $\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots = \frac{\pi}{4}$ for each $0 < x < \pi$. (3)
13. Prove: $\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \ldots = \frac{\pi}{4}\left(\frac{\pi}{2} - x\right)$ for each $0 < x < \pi$. (3)