Engineering Optics

Lecture 36

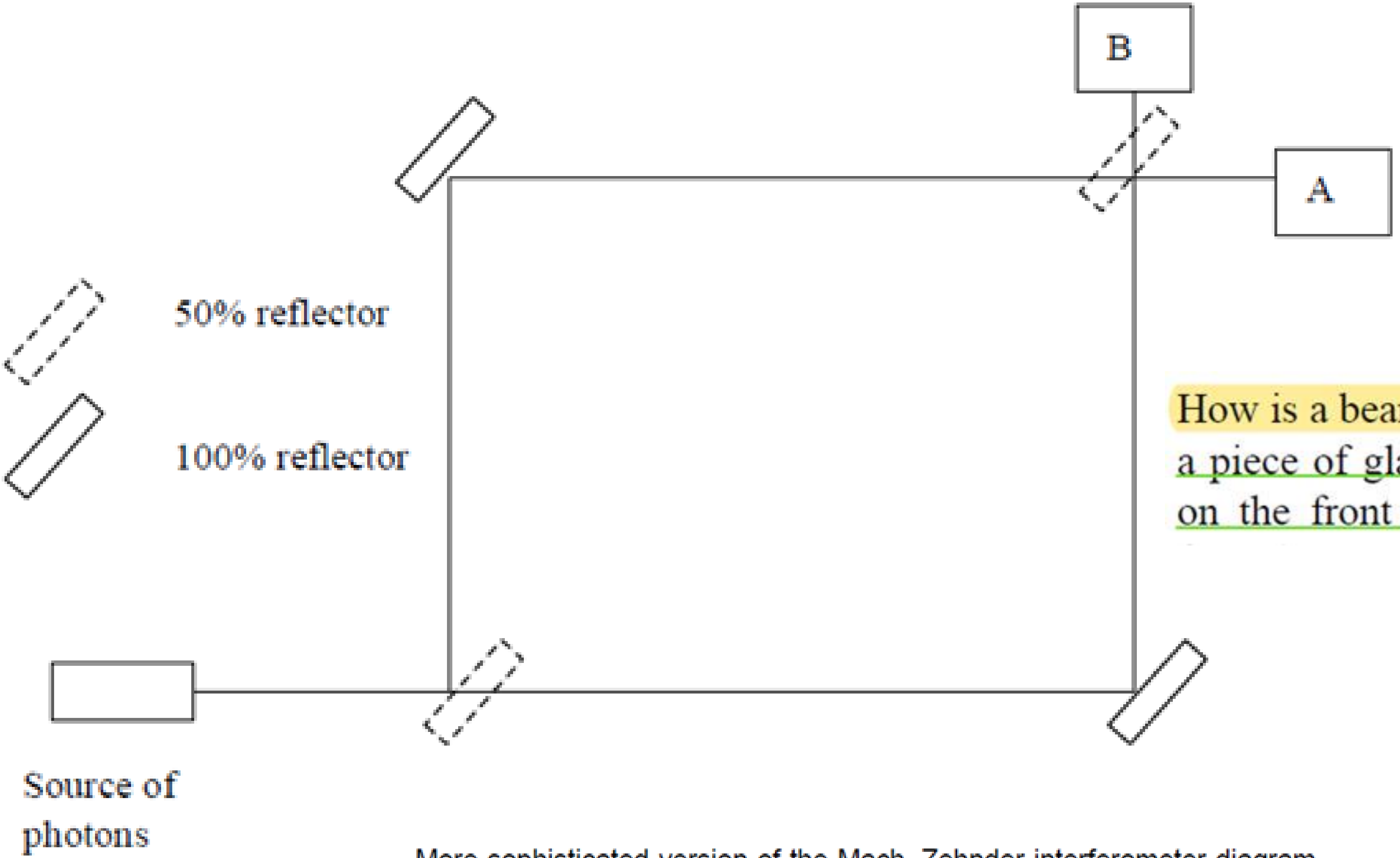
09/06/2023

by

Debolina Misra

Dept. of Physics
IIITDM Kancheepuram, Chennai, India

Resolution of the problem

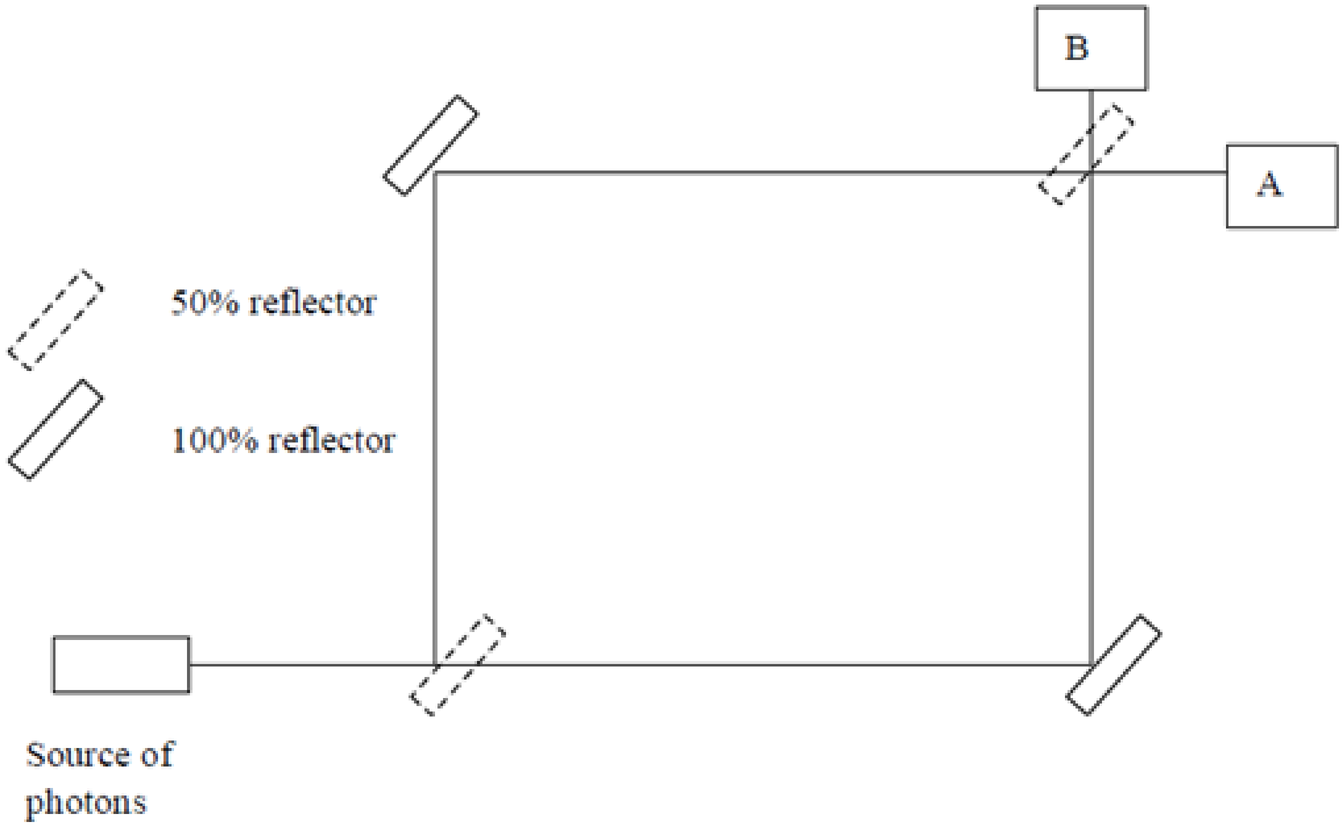


How is a beamsplitter actually made? Usually it is a piece of glass with a dielectric or metal coating on the front surface.

More sophisticated version of the Mach–Zehnder interferometer diagram.
Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Resolution of the problem



More sophisticated version of the Mach–Zehnder interferometer diagram.
Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

Are we calculating the *phase* accurately?

However, the key to the problem lies in what happens to a photon approaching the beamsplitter from behind. There it first enters the glass (ignoring the small chance of reflection off the air–glass interface) and has a 50% chance of reflecting off the dielectric coating whilst within the glass. Here is the crux of the matter—*that* reflection does not induce a phase change. Given that, let us once again examine the phase shifts on the two paths.

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Resolution of the problem

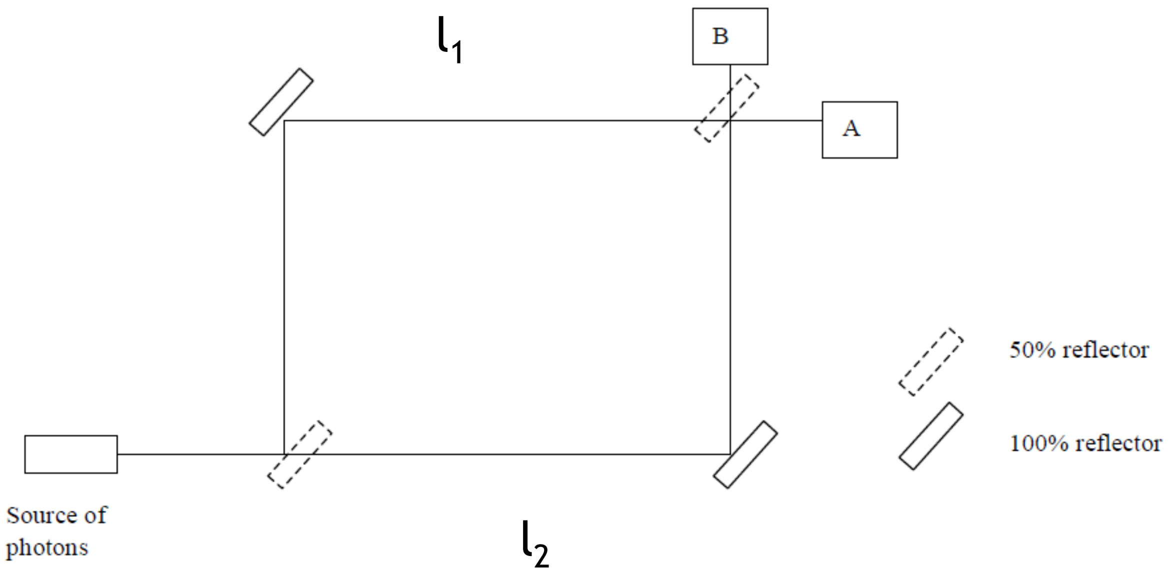


Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

l_1= total length from source to detector for upper path

l_2= total length from source to detector for lower path

Extra phase when the light passes through the glass of the beamsplitter $2\pi t/\lambda$

t: optical path length through the BS
Depends on length (thickness) and r.i.

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Resolution of the problem: detector A

To detector A

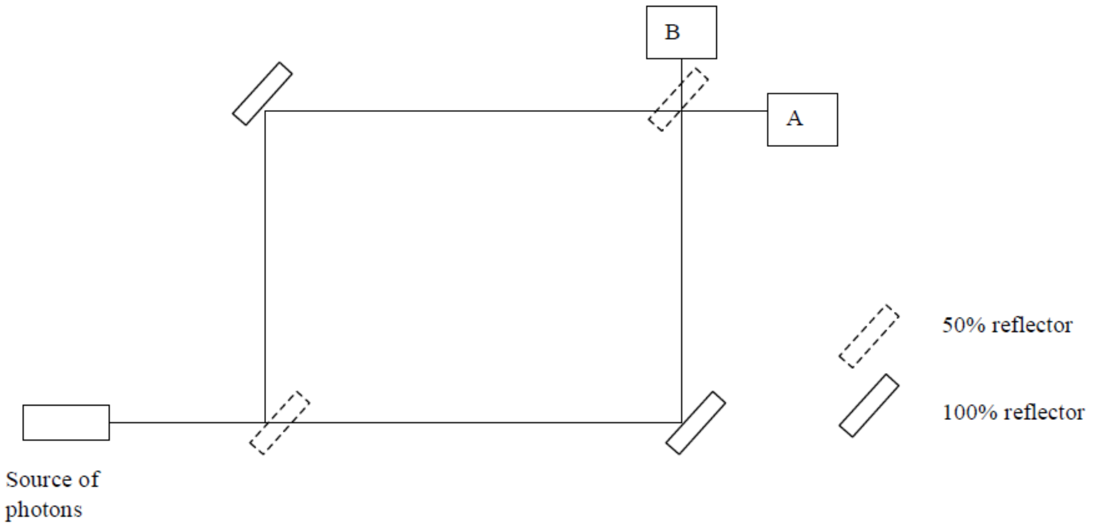


Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

The upper path picks up the following phase shifts on the way to detector A: π at the first reflection, π at the second (100%) reflection, nothing at the transmission, $2\pi l_1/\lambda$ for the distance travelled, and $2\pi t/\lambda$ for the extra phase picked up in traversing the glass substrates where the wavelength is reduced. This gives a total of

$$2\pi + 2\pi\left(\frac{l_1 + t}{\lambda}\right).$$

The lower path, also on its way to A, picks up a phase shift of π off the 100% reflector, π at the second beamsplitter, a phase shift of $2\pi l_2/\lambda$ for the distance travelled, and an extra phase shift of $2\pi t/\lambda$ from passing through the glass substrate at the first beamsplitter.

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Resolution of the problem: detector A

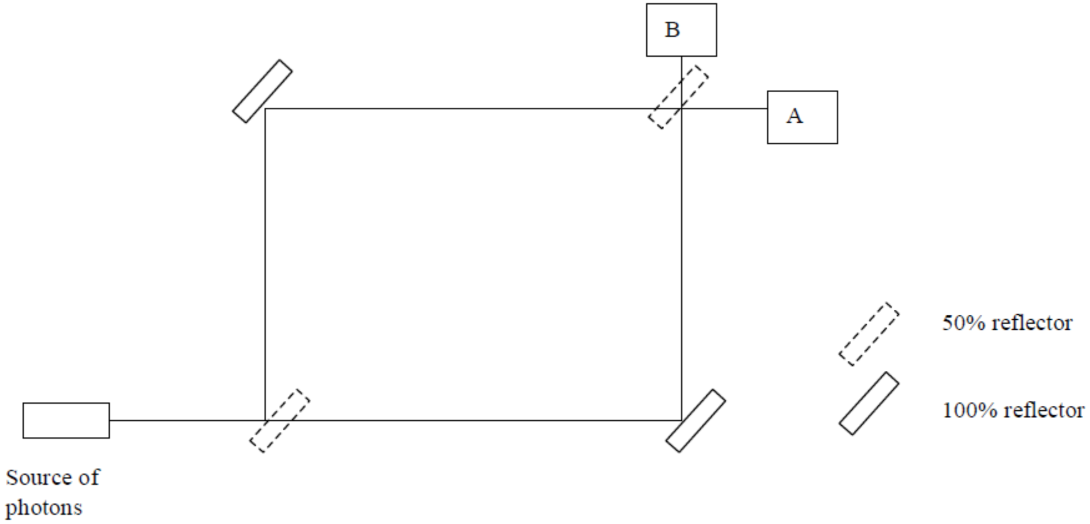


Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

The phase difference between the two paths is

$$2\pi + 2\pi\left(\frac{l_1 + t}{\lambda}\right) - 2\pi - 2\pi\left(\frac{l_2 + t}{\lambda}\right)$$
$$= 2\pi\left(\frac{l_1 - l_2}{\lambda}\right) = \delta$$

where δ is the phase shift due to the difference in the path lengths.

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Resolution of the problem : detector B

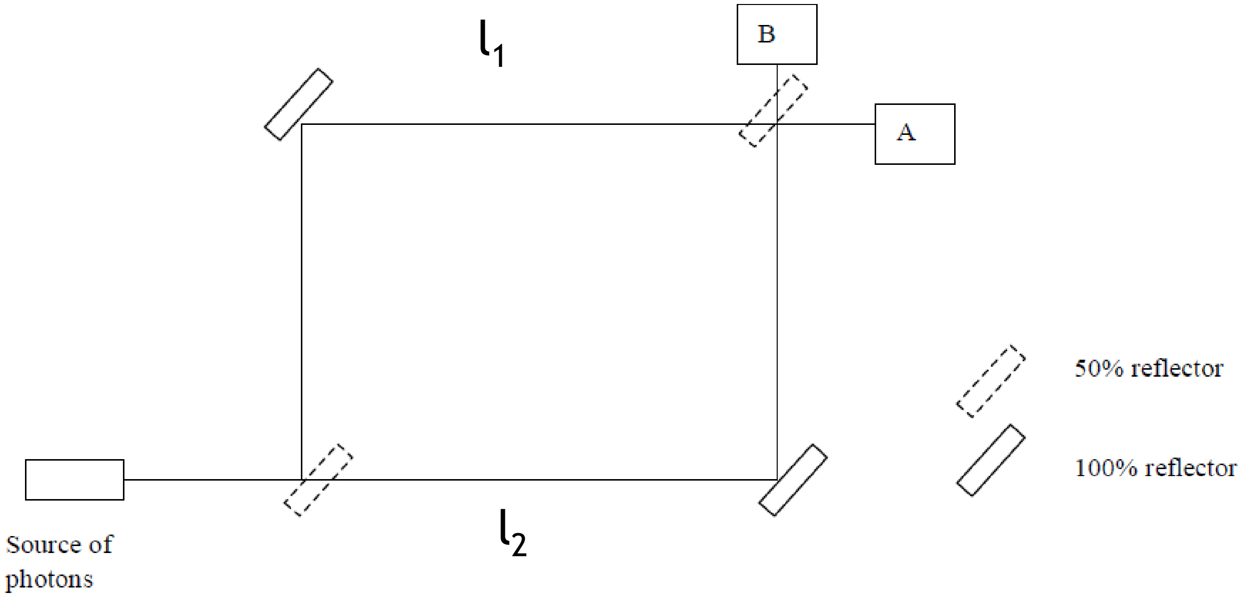


Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

Similarly, we can calculate the phase difference between the two paths on their way to detector B. We obtain

$$2\pi + 2\pi\left(\frac{l_1 + 2t}{\lambda}\right) - \pi - 2\pi\left(\frac{l_2 + 2t}{\lambda}\right)$$
$$= \pi + 2\pi\left(\frac{l_1 - l_2}{\lambda}\right) = \pi + \delta.$$

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Resolution of the problem : detector B

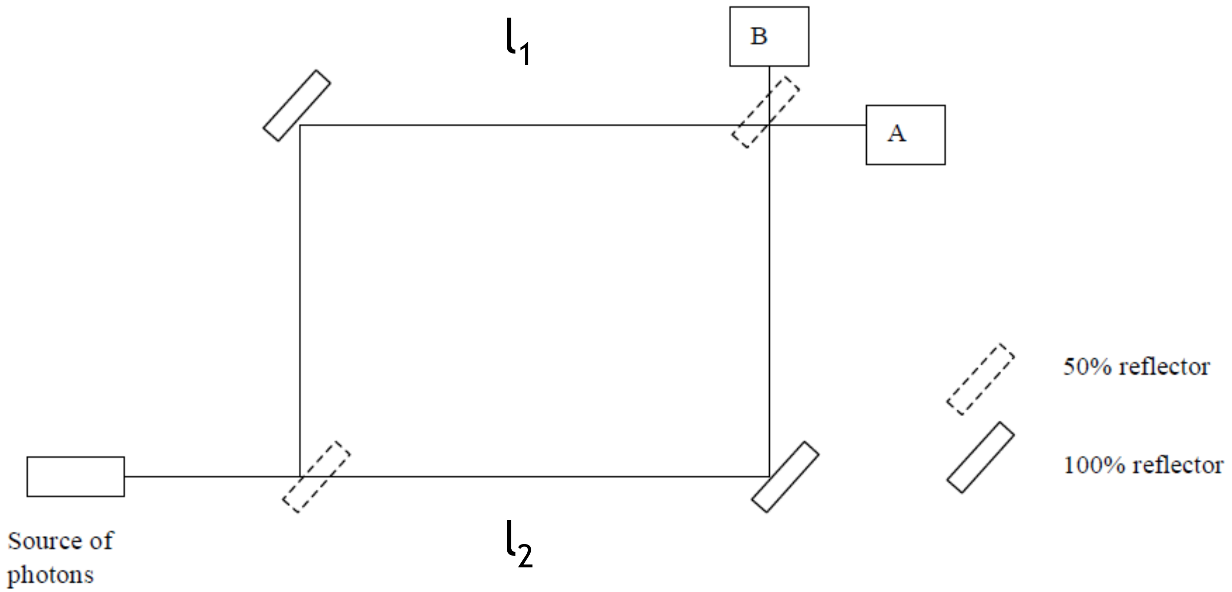


Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

Detector A: phase diff = δ
Detector B: phase diff. = π+δ

When δ = 0
Detector A→ Constructive interference
Detector B→ Destructive Interference

Change δ → pattern will vary
OR
Probability of detecting a photon at either detector: 0 to 1

Puzzle solved

All of the physics is contained in this analysis. In practice, the beamsplitters may be of different thicknesses but this will simply add a fixed phase difference, as will placing the second beamsplitter the other way around.

K P Zetie, S F Adams and R M Tocknell, Physics Department, Westminster School, London SW1 3PB, UK

Problem:1

For a sodium lamp, the distance traversed by the mirror between two successive disappearances is 0.289 mm for Michelson interferometer. Calculate the difference in the wavelengths of the D_1 and D_2 lines. Assume λ= 5890 Å

Answer:

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2, . . . , we will have disappearance of the fringe pattern; and if it is equal to 1, 2, 3, . . . , then the interference pattern will appear.

Answer:

When the mirror moves through a distance 0.289 mm, the additional path introduced is 0.578 mm. Thus

$$\frac{0.578}{\lambda} - \frac{0.578}{\lambda + \Delta\lambda} = 1$$

$$\Delta\lambda \simeq \frac{\lambda^2}{0.578} = \frac{(5890 \times 10^{-7})^2}{0.578}\ \text{mm}$$

$$\simeq 6\ \text{Å}$$

Assume $\Delta\lambda \times \lambda \ll \lambda^2$

Problem:2

In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength.

Answer:

The wavelength λ in Michelson interferometer is given by following equation.

$$\lambda = \frac{2d_0}{N}$$

Here, d_0 is the distance moved by the mirror, and N is the number of fringes.

$$\lambda = \frac{2(0.08\,\text{mm})\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}{250}$$

$$= 6.4 \times 10^{-5}\,\text{cm}\left(\frac{10^{8}\,\text{A}^\circ}{1\,\text{cm}}\right)$$

$$= 6400\,\text{A}^\circ$$

Problem:3

The Michelson interferometer experiment is performed with a source which consists of two wavelengths of 4882 and 4886 Å. Through what distance does the mirror have to be moved between two positions of the disappearance of the fringes?

Answer:

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1$$ For appearance of fringe

Then

$$d = \frac{1}{2}\left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\right)$$

$$
\begin{aligned}
d &= \frac{1}{2}\left(\frac{(4882\,\text{A}^\circ)(4886\,\text{A}^\circ)}{4886\,\text{A}^\circ - 4882\,\text{A}^\circ}\right) \\
&= 2981681.5\,\text{A}^\circ\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right) \\
&= 0.0298\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right) \\
&= 0.298\,\text{mm}
\end{aligned}
$$

Problem:4

Consider a monochromatic beam of wavelength 6000 Å incident (from an extended source) on a Fabry–Perot etalon with $n_2 = 1$, $h = 1$ cm, and $F = 200$. Concentric rings are observed on the focal plane of a lens of focal length 20 cm . Calculate the reflectivity of each mirror.

Answer:

$$F = \frac{4R}{(1-R)^2} \quad \Rightarrow \quad 200 = \frac{4R}{(1-R)^2}$$

$$50 = \frac{R}{(1+R^2-2R)}$$

$$50 + 50R^2 - 100R = R$$

$$50R^2 - 101R + 50 = 0$$

By solving above quadratic equation,

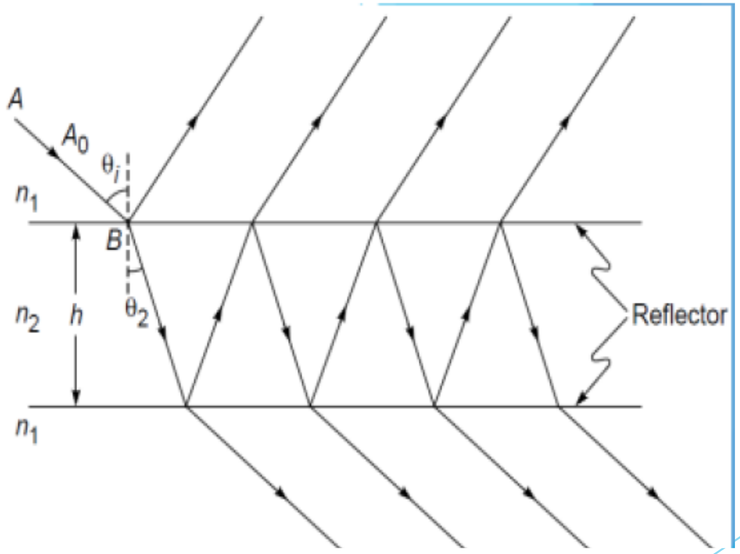
$$R = 1.15 \text{ or } 0.87$$

R=0.87 (Since R will be less than 1)

<u>Problem: 2</u>

The following device shown in the figure contains two media of refractive indices $n_1 = 1\ and\ n_2 = 1.5$.

a) calculate the reflection coefficient (r)
b) calculate transmittivity (τ)
c) calculate Finesse F
d) calculate the reflectivity of the device if $\delta = \frac{\pi}{2}$



Answer:

a) reflection coefficient (r):

$$r = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

$$r = \frac{(1 - 1.5)^2}{(1 + 1.5)^2} = \frac{0.5^2}{2.5^2}$$

r =0.04

b) transmittivity (τ)

For that particular interface:
Reflectivity $R = r^2$
Transmittivity $\tau = 1 - R$

$$R = 0.04^2$$

$$R = 0.0016$$

$$\tau = 1 - 0.0016$$

$$\tau = 0.9984$$

c) Finesse (F)

$$F = \frac{4R}{(1-R)^2}$$

$$R = 0.0016$$

$$F = \frac{4 \times 0.0016}{(1-0.0016)^2}$$

$$F = 0.0064$$

d) reflectivity of the device ($\mathcal{R}$)

$$\mathcal{R} = \frac{F \sin^2 \delta/2}{1 + F \sin^2 \delta/2}$$

$$\mathcal{R} = \frac{0.0064 \times \sin^2 \frac{\pi}{4}}{1 + 0.0064 \times \sin^2 \frac{\pi}{4}}$$

$$\mathcal{R} = \frac{0.0032}{1 + 0.0032}$$

$$\mathcal{R} \approx 0.0032$$

Thank You