FSA :

Powerful ??

< $\#_1 \% 5 = 0$

Minimum FA

M_1 , M_2 — $M_1 \cap M_2$

I/p : String x

One has to simulate the behavior of

$\langle M_1, x \rangle$ and

$\langle M_2, x \rangle$

$\hat{\delta}(q_0^1, x) \rightsquigarrow \bigcirc$

$\hat{\delta}(q_0^2, x) \rightsquigarrow \bigcirc$

q_0^1 : start state of M_1

q_0^2 : " of M_2

$\Rightarrow x \in L(M_1)$

$x \wedge L(M_2)$

'Product Automaton' — $M_1 \times M_2$

$x \in L(M_1)$

x

x

$\Rightarrow x \in L(M_1) \wedge x \in L(M_2) \Rightarrow x \in L(\underline{M_1 \cap M_2})$

$M_1 \times M_2 \qquad \delta(M_1 \times M_2)$

$\delta((q_0^1, q_0^2), x)$

$= (\delta(q_0^1, x), \delta(q_0^2, x))$

$(q_{nf}^1, \underline{q_f^2}) \quad (q_f^1\, q_f^2) \quad (\underline{q_f^1}, q_{nf}^2) \qquad (q_f^1, q_f^2) \Rightarrow x \in L(M_1 \cap M_2)$

$x \in L(M_1 \cup M_2)$ final st Non-final $\in F_1$ $\in F_2$

$(q_f^1, q_{nf}^2) \Rightarrow x \in L(M_1 \setminus M_2)$

$(q_{nf}^1, q_f^2) \Rightarrow \in \mid M_2 \setminus M_1)$

$M_1 \times M_2 = (Q_1 \times Q_2, \Sigma, \delta, (q_0^1, q_0^2), F)$

$\delta((q, q'), a) = (\delta^1(q, a), \delta^2(q', a))$

$a \in \Sigma$

$\hat{\delta}((q_0^1, q_0^2), x) = \underline{\ ?\ }$

$\underline{F}$ — $F(M_1 \cap M_2)$ interms of F_1, F_2 — $M_1 \cap M_2$ — $F = (F_1 \times F_2)$

— $M_1 \cup M_2$ — $F = \{ (q_f^1, q_{nf}^2)$ or (q_1^f, q_2^f)

(q_{nf}^1, q_f^2) or $\}$

$F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$

— M

$M_1 \setminus M_2$ — $F(M_1 \setminus M_2)$ — $\underleftrightarrow{(q_f^1, q_{nf}^2)}$

$$F(M_1 \setminus M_2) = (F_1 \times F_2^c)$$
$$= (F_1 \times \underline{(Q_2 \setminus F_2)})$$

Non final st of M_2

$M_2 \setminus M_1$ — $F(M_2 \setminus M_1)$ — $(F_1^c \times F_2)$

$$= \underleftrightarrow{((Q_1 \setminus F_1) \times F_2)}$$

$\hat{\delta}((q_0^1, q_0^2), x)$ — $(q, q') \in$ $\Rightarrow$ $x \in L(M_2 \setminus M_1)$

$\in$ $\Leftarrow$ $x \in L(M_1 \wedge M_2)$.

M^c

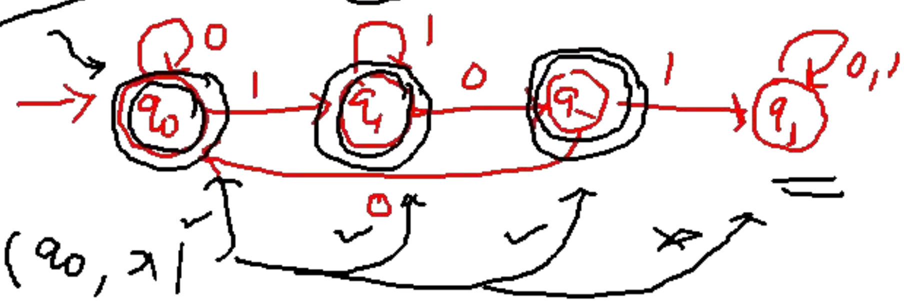
$M_1^c = (Q, \Sigma, \delta, q_0, F_1^c)$

$F_1^c = F(M_1^c) = \boxed{Q \setminus F_1}$

$L = \{ x \mid x$ does not contain 101 as a substring $\}$.

$M_1 - L_1 = \{ x \mid x$ contains 101 as a subst $\}$

$M_1^c - L_1^c$ - does NOT contain 101



$\hat{\delta}(q_0, x)$

M_1^c, $\underset{L}{M_1 \cup M_2}$, $M_1 \cap M_2$, $M_1 \setminus M_2$, $M_2 \setminus M_1$, =

Given a $\underline{\underline{Q}}$ L_1, L_2, L_3

$fn(\underline{\underline{L_1}}, \underline{\underline{L_2}}, \underline{\underline{L_3}}, \dots, L_K)$

$L_1 = \{ x \mid$ x begins w [L_1] ^
ends " [L_2] ^
not contain 1010 as a subst $\}$ ^

L_3^c

$L_2 = \{ x \mid x$ contains $\underset{P_1}{\underline{101}}$ and does not contain $\underset{P_2}{\underline{11}}$ $\}$

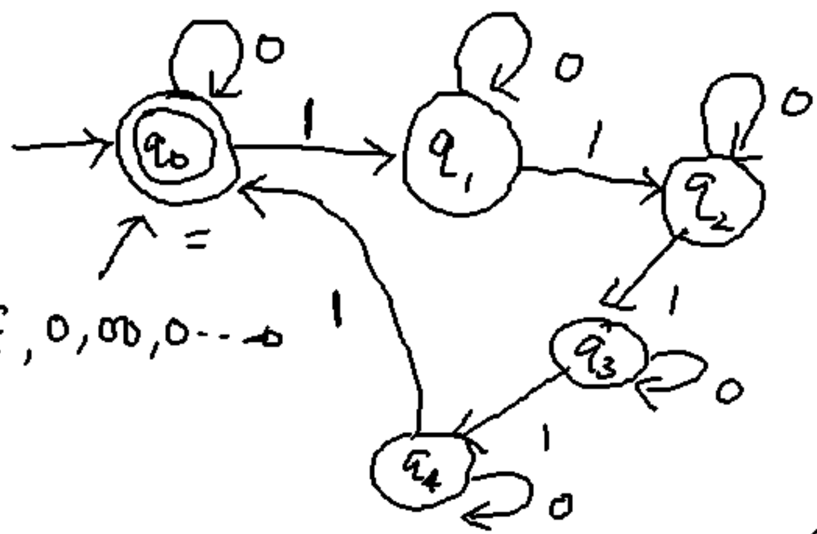
\# 1's

$L = \{ x \mid$ No. of 1's in x is div by 5 $\}$

$L = \{ 00, 0, 0\cdots0, \underline{0110111}, 111110\cdots0110\cdots0111,$

$L^c = \{01, 011, 0110, 110\cdots01, \ldots$

Does $\exists$ FA



Focus is on '#1's'

There should not be any self-loop on '1' @ any st

$0\cdots0 \mid 0\cdots0 \mid 0\cdots0 \mid 0\cdots0 \mid 0\cdots0 \mid 1\cdots1$

$\cdots 1 \cdots 1 \cdots 1$ q_0

$\delta : Q \times \Sigma \to Q$

$\delta(\overset{\in}{q}, \overset{\in}{a}) \to q' \in Q$

The transition is always unique

Certain

$M = (Q, \Sigma, \delta, q_0, F)$

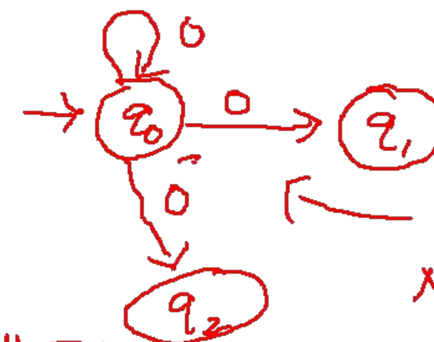
$\delta : Q \times \Sigma \to Q$

Deterministic Finite Auto (DFA)

$\delta : Q \times \Sigma \to$? Any subset of Q

$\delta(q_0, 0) = \{q_0, q_1, q_2\}$

$M = (Q, \Sigma, \delta, q_0, F)$



$\delta : Q \times \Sigma \to$ Any subset of Q

uncertainty

Non-Det FSA (NFA)

FA

DFA

$\delta : Q \times \Sigma \to Q$

$q_0, 0$
$q_0, 1$
$\vdots$

q_0
q_1
q_2
$\vdots$

NFA

$\delta : Q \times \Sigma \to 2^Q$

Any subset of Q

Powerset of Q

$q_0, 0$
$q_0, 1 \to \{q_0\}$
$q_1, 0 \to \{q_0, q_1, q_2\}$
$(q_1, 1)$
$\vdots$

ϕ
$\{q_0\}$
$\vdots$
$\{q_0, q_1\}$
$\vdots$
$\{q_0, q_1, q_2\}$
$\vdots$
$\{q_0, q_1, \ldots, q_{n-1}\}$

ϕ

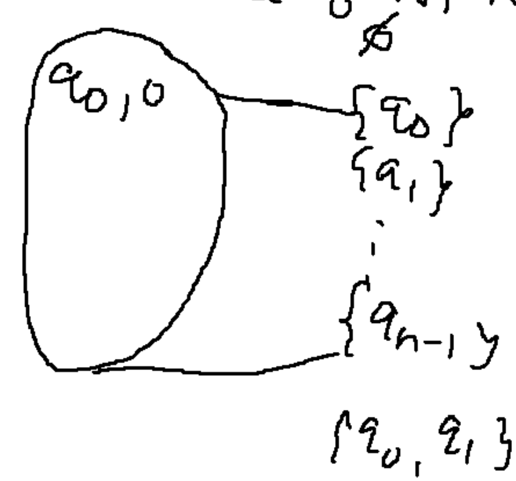
$q_1, 0$: q_0, q_1, $q\_$

$q_1, 0 \to \{q_0\, q_1 \ldots\}$

DFA NFA

$Q \times \Sigma \to Q$

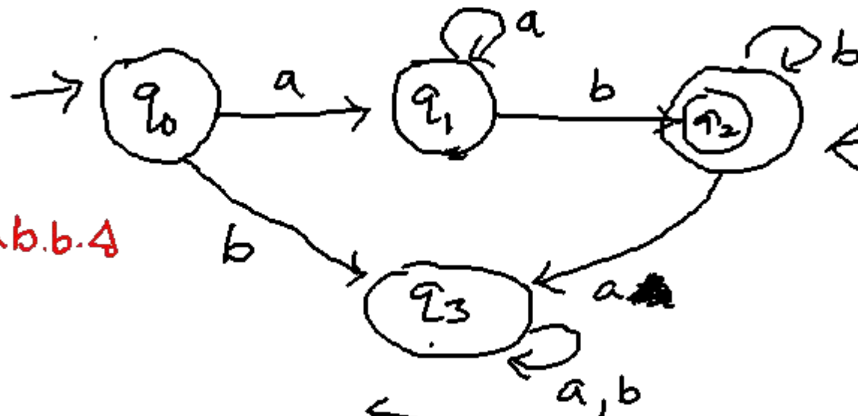
① DFA is a special case of NFA



③ Does NFA increase the computing power of FA.

④ Are there computations for which NFA exist but DFA do not exist.

② NFA is a generalization of DFA



a..ab.b.a

Σ i

$M = Q, \Sigma = \{a,b\}, \delta, q_0, F = \{q_2\}$

$L(M) = ?$

$\nexists\ \delta(q,a) \to \{q', q'', \dots\}$, $\forall q, \forall a$

$\Rightarrow$ DFA

"reverse loding"

$abaabb \notin L(M)$

$a \dots ab \in L(M)$

$\boxed{a} \dots abb \dots \boxed{b} \in L(M)$

?

$b \dots \notin$

$a \dots ab \dots a \boxed{a} \dots \in L(M)$

$q_3\ q_3\ q_3 - q_3$

Reject — b ___
— a..ab..a a
— a—a

$L = \{\underline{ab}, \underline{aab}, aaabb, aaab, aaaab, aaaabb, aaaabbb, aaaabbbbbbb, \dots$

$L = \{ a^{\underline{n}} b^{\underline{m}} \mid n, m \geq 1 \}$

DFA $\Rightarrow$ NFA

$\vdash$ FA

$\underline{\underline{a^3 b^2}}$

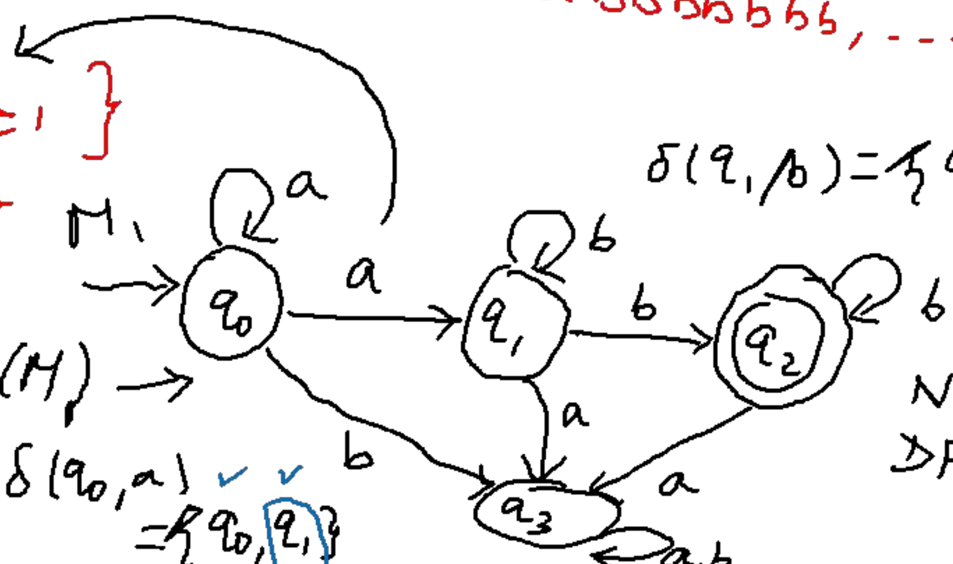
$\underline{q_0 q_0 q_0 q_3 q_3} \notin$

$q_0 q_0 q_1 q_1 q_2 \in$

$q_0 q_0 q_1 q_2 q_2 \in$

M_1

$L(M)$



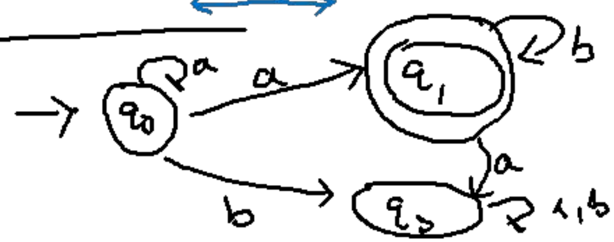
$\delta(q_0, a) = \{q_0, \boxed{q_1}\}$

$\delta(q_1, b) = \{q_1, q_2\}$

NFA ✓

DFA X

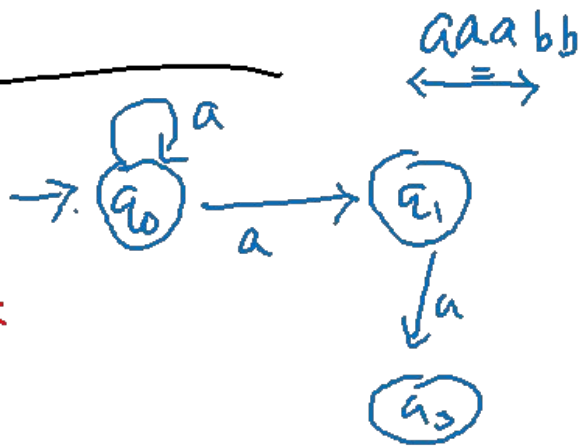
M_2

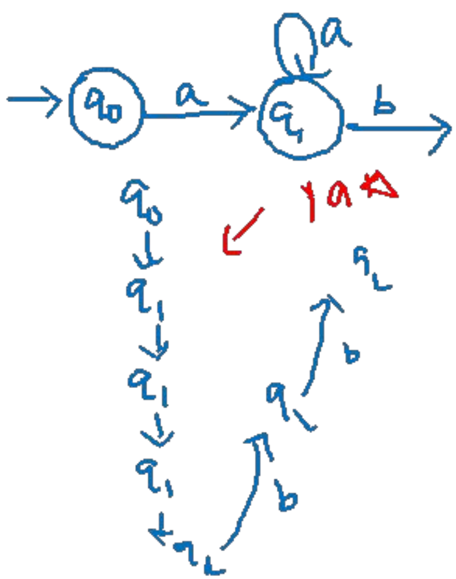


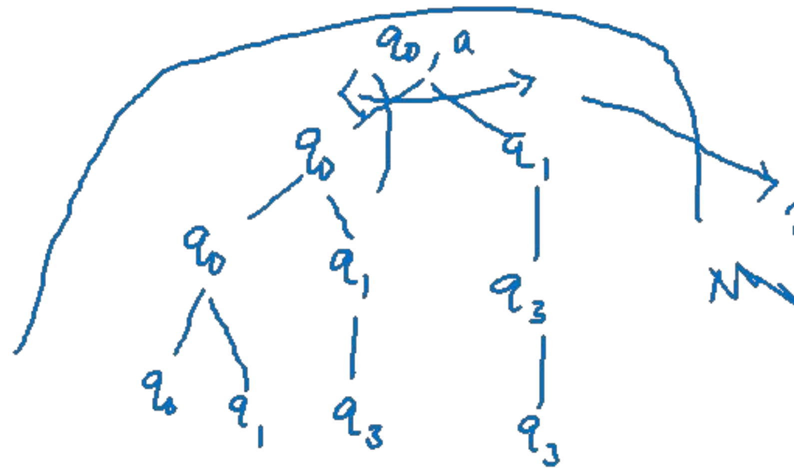
$L = \{a^n b^m \mid n, m \geq 1\}$ → ∃ NFA M_1 → Equivalent DFA M

? Is it true that for every NFA there is an equivalent DFA

DFA







For a DFA — Comp Gh is a Path | uncertain Non-Det

NFA — " Tree

G → MST

↳ Spath

100%.

DFA → Class P ~ Det poly-time Algo

NFA → Class NP ~ Non-Det Poly -

? P = NP ST ✓

DFA ⟷ NFA ⟷

Alh

AI AN Path ✓ AI Alas | → Approximate Algo

Gate PEMJ AA !

> 10m

??

FA Can Model Any Conj

DFA

$\delta(q_0, a) = q' \in Q$

$\hat{\delta}(q_0, x = a_1 a_2 a_3 \ldots a_n) = ((q_0, a_1), a_2), a_3, \ldots$

$a_i \in \Sigma$

q^1

q^2

$q^3 \in Q$

Tree

NFA

q_{nf} $\quad q_f \in F$ $\quad q_f \in F$

↳ ∃ unique path

$q_0 \rightsquigarrow$ some $q \in F$

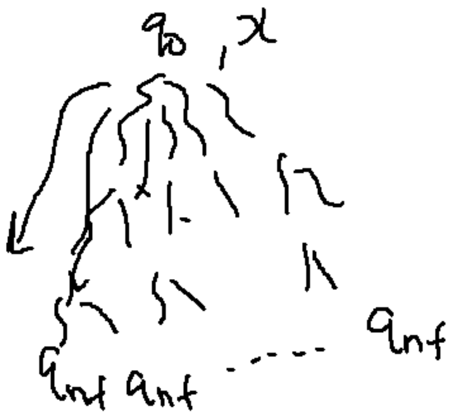
$L(M) = \{ x \mid \hat{\delta}(q_0, x) = q_f \in F \}$

DFA

NFA

$L(M) = \{ x \mid \hat{\delta}(q_0, x) = q_f \in F \}$

There exist a path in Comp



If Every Comp path in the Comp tree is leading to a Non-final st then $x \notin L(M)$.

If $\hat{\delta}(q_0, x) = q_{nf}$ for each path in the Tree then $x \notin L(M)$

$\exists \geq 1$ path/branch leading to q_f $\rightarrow x \in L(M)$.
$\longleftrightarrow$

Are we guessing

How do we discover this branch $\leq$ Det approach

NFA ~ power of guessing [cannot go wrong]

If $x \in L(M)$,

NFA is ALL Powerful

Hypothetical

If $x \in L(M)$ then NFA will indeed stop @ ◎

NFA will 'Some how' discover
'Guess'
$q_0 \cdots\cdots q_f$

Can simultanously simulate x @ q_0

Parallel explore all paths

DFA $Q \times \Sigma \to Q$

NFA $Q \times \Sigma \to 2^Q$

? $L(NFA) \equiv L(DFA)$

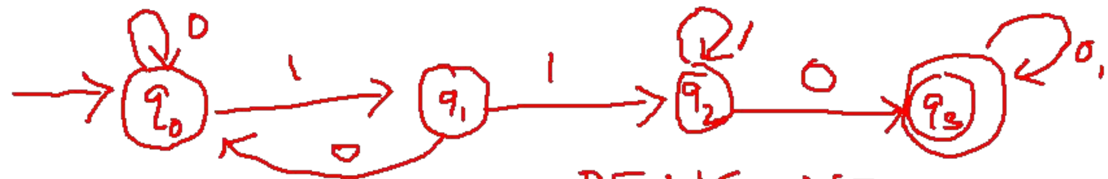
$q, a \to \{q_0, q_1, \dots, q_k\}$

✓ ✓ ✗

$L = \{x \mid x \text{ contains } 110 \text{ as a substring}\}$

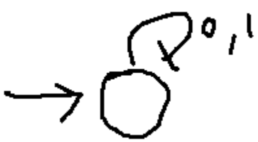
DFA | NFA

NFA but not DFA

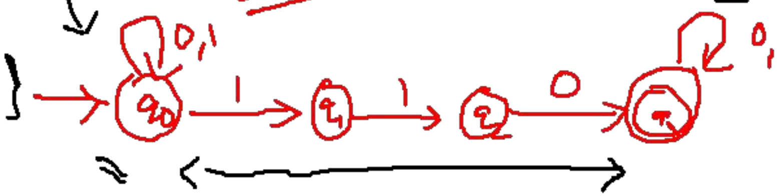


DFA ✓ NFA ✓

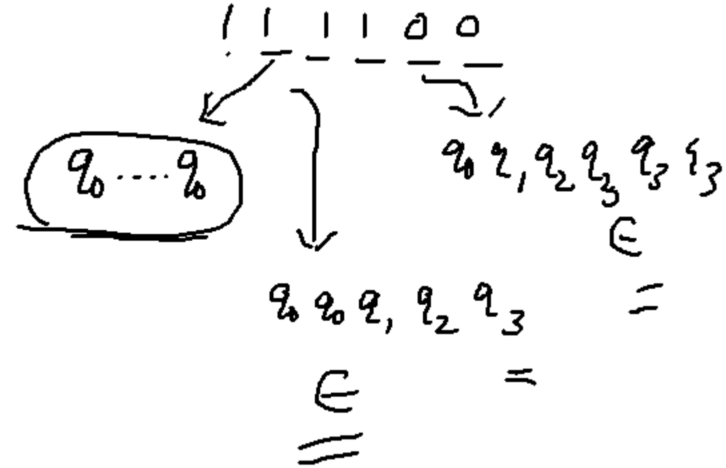
Prefix $\{0,1\}^*$ 110 Suffix $\{0,1\}^*$



$\delta(q_0, 1) = \{q_0, q_1\}$



$\underline{\{0,1\}^*}\ 110\ \underline{\{0,1\}^*}$



$\exists$ path $q_0 \cdots q_3$

$\forall x \in L(M)$, $\exists$ path $q_0 \cdots\cdots q_3$

① NFA | DFA

② Containing a pattern

DFA

NFA

⋮

$\longleftrightarrow$

Also an NFA

Contain 100

1010

$\delta(q_1, 0|1) = q_1$ ×

Invalid st

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

$1010 \in L(M)$

$\in$ Contain 110 as a Substr.

Also an NFA
Contain 100

1 0 1 0

$\delta(q_1, 0|1) = q_1$ ×

Invalid ste

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

$1010 \in L(M)$

$\in$ Contain 110 as a Substr.

Also an NFA
Contain 1[illegible]0

1 0 1 0

$\delta(q_1, 0|1) = q_1$ ✗

Invalid ste

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

$1010 \in L(M)$

$\in$ Contain 110 as a substr.

Also an NFA

Contain 1 0 0

1 0 1 0

$\delta(q_1, 0|1) = q_1$ ×

Invalid st

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

$1010 \in L(M)$

$\in$ Contain 110 as a Substr.

Also an NFA
Contain 100

1010

$\delta(q_1, 0|1) = q_1$ ×

Invalid st

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

$1010 \in L(M)$

$\in$ Contain 110 as a substr.

Also an NFA

Contain 100

1010

$\delta(q_1, 0|1) = q_1$ ✗

$q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

Invalid st

$1010 \in L(M)$

$\in$ Contain 110 as a substr.

Valid st $\exists$ path $q_0 \cdots q_f$

Invalid st $\forall$ path $q_0 \cdots \underline{q_{nf}}$

$L = \{ x \mid x \in \{0,1\}^*$ s.t the third symbol from the right is '1' $\}$

$L = \{ \boxed{1}01,\ 000101,\ 10100110,\ 000\underline{1}00, \cdots \}$

$L^c = \{ 000,\ 011,\ 10111011, \cdots \}$ X

✓ Does $\exists$ FA ? (NFA | DFA)

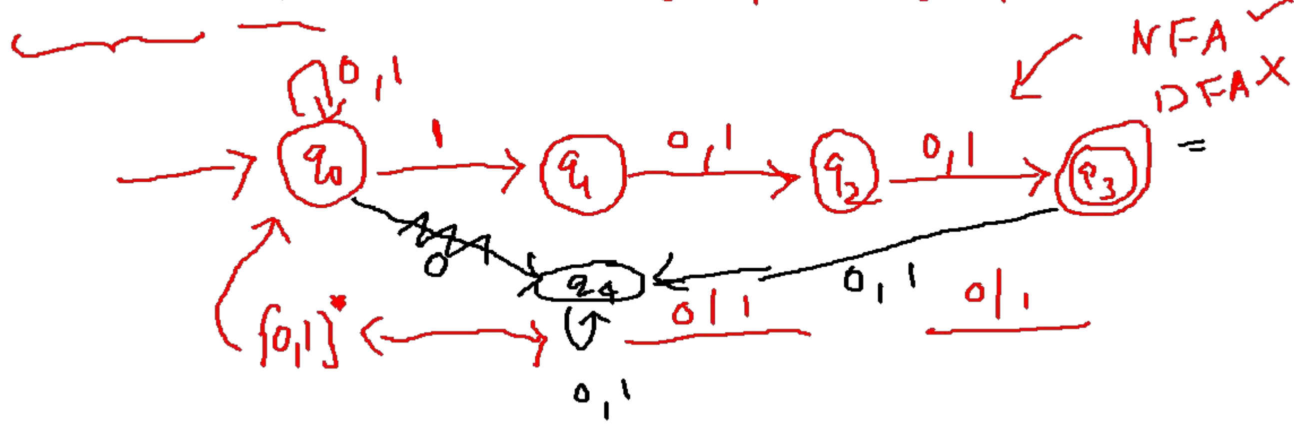
$\underbrace{\{0,1\}^*}\ 1\ \overbrace{\underline{0}\ \underline{0}}$

$\underline{0}\ \underline{1}$

$\underline{1}\ \underline{0}$

$\underline{1}\ \underline{1}$

$\delta(q_0, 1) = \{q_0, q_1\}$



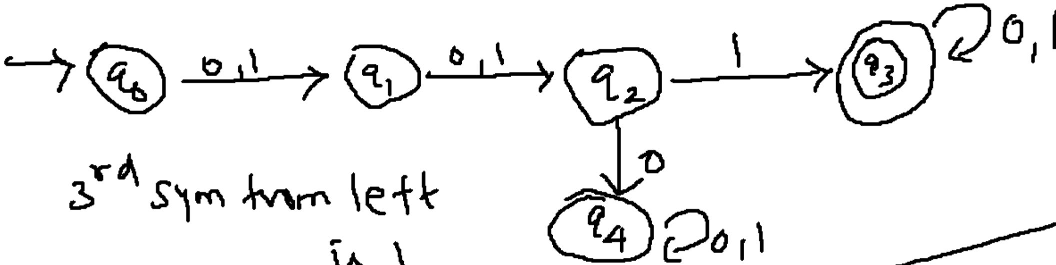
$L = \{ x \mid$ Third symbol is 1 from the beg is 1 $\}$

101, 001, 000, 011 ____

$=1$ ✗

0|1 0|1 1 $\underline{\{0,1\}^*}$

M_1'



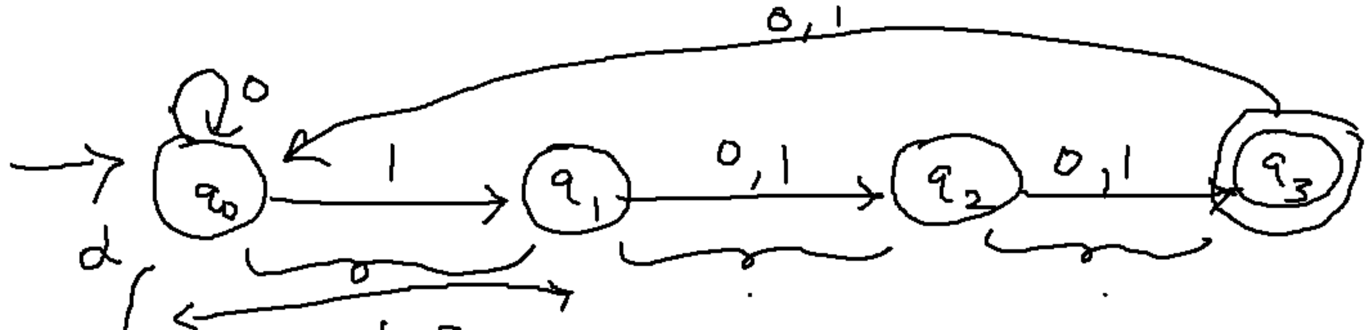
3^{rd} sym from left is 1

~~DFA~~ | NFA

M

3^{rd} sym from Right is 1

NFA | ?? DFA

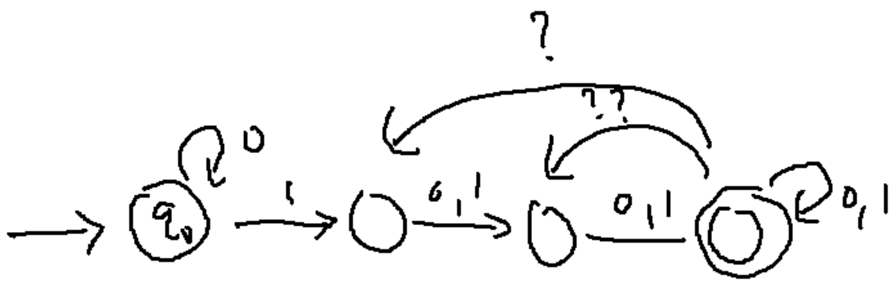


DFA

3^{rd} sym from Right is 1

11100 0

→ valid st but rejected by the Automaton.

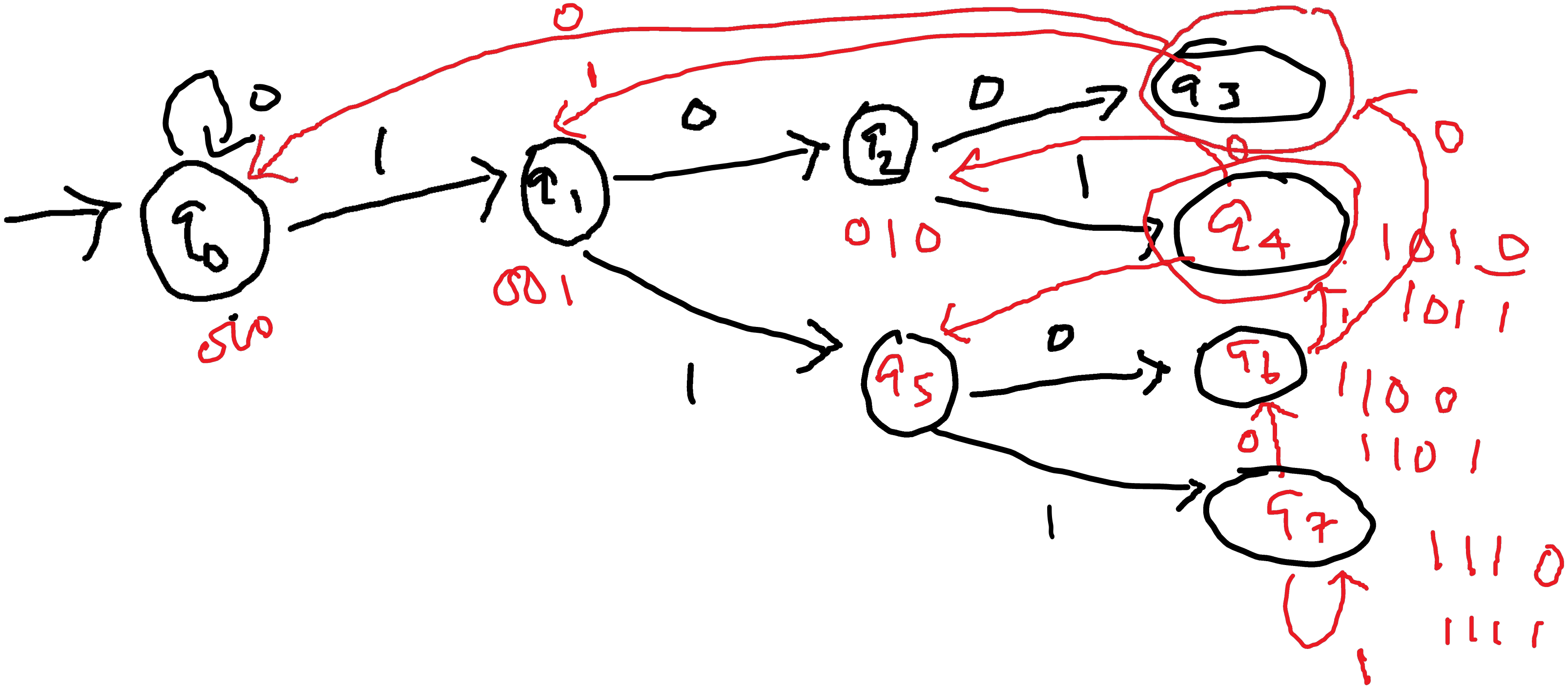


110

11001

1100

100000 Accept



q0
q1
q2
q3
q4
q5
q6
q7
0
1
010
001
000
1010
1011
1100
1101
1110
1111