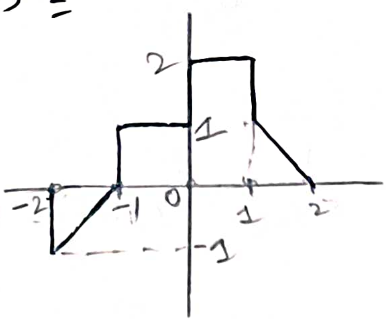
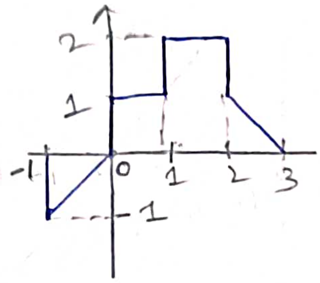
Tutorial - 1

Madhamshetty Sathvika

1) $x(t) =$

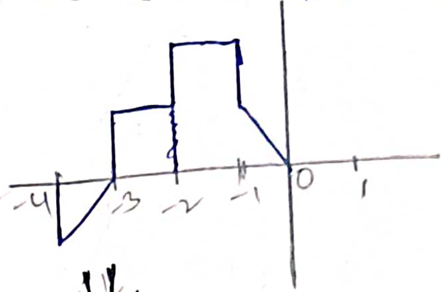


a) $x(t-1)$

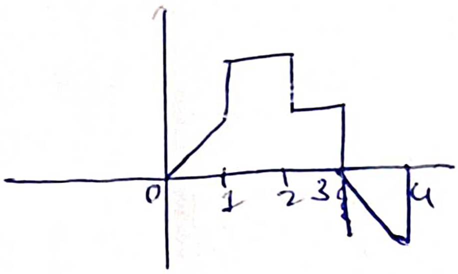


b) $x(2-t)$

$x(t+2)$

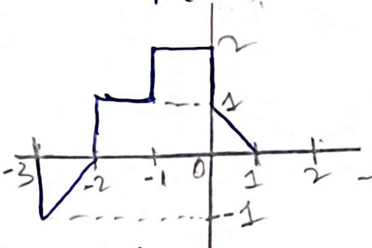


$x(2-t)$

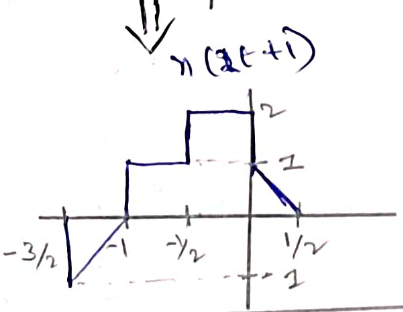


c) $x(2t+1)$

$x(t+1)$

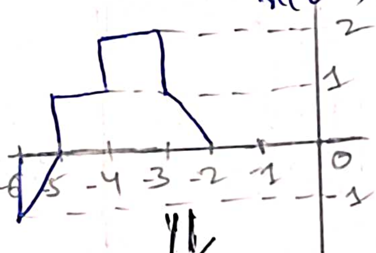


$x(2t+1)$

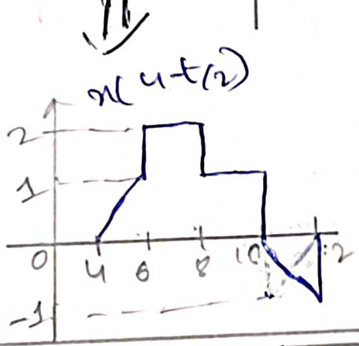


d) $x(4-t/2)$

$x(t+4)$



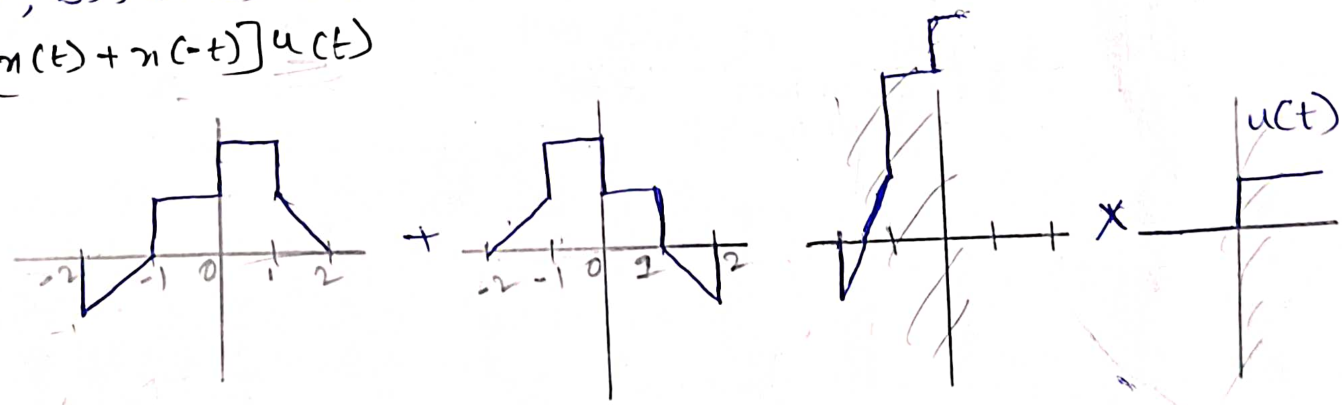
$x(4-t/2)$



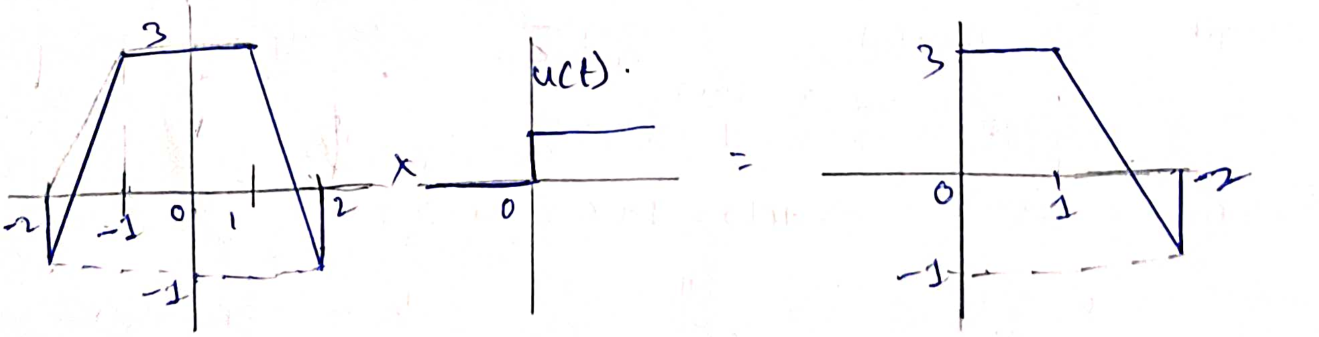
2) Graph is same as 1st question. i.e, $x(t)$ graph.

a), b), c) d) are same as 1st question.

e) $[x(t)+x(-t)]u(t)$

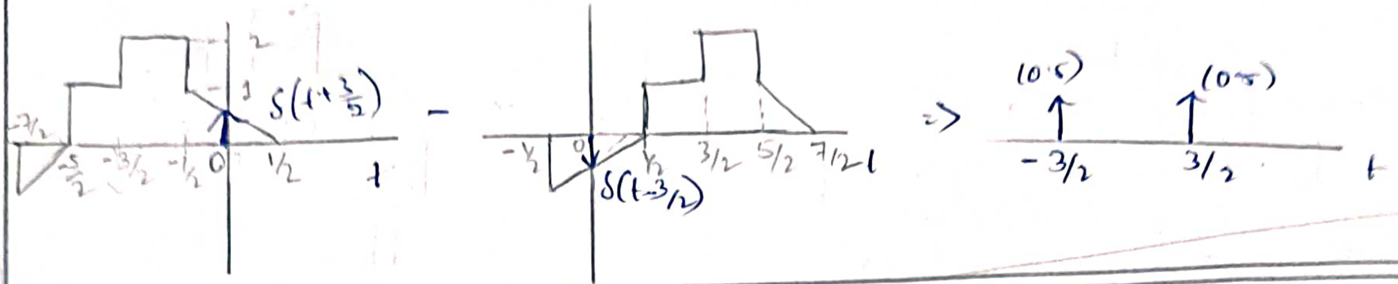


$u(t)$

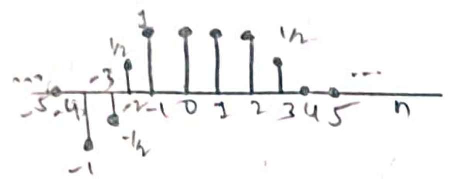


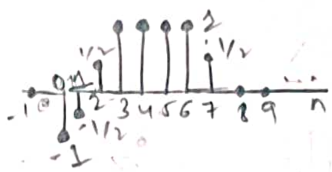
$u(t)$.

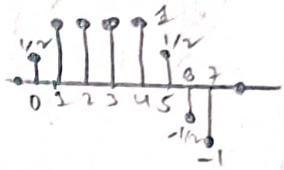
f) $x(t)[\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})]$



3) $x[n]$ a) $x[n-4]$ b) $x[3-n]$

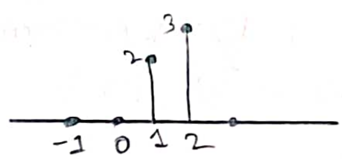






c) $x[3n]$ d) $x[3n+1]$

4) $x[n]$ a) $x[n-2]$ b) $x[2n]$



c) $x[-n]$ d) $x[-n+2]$

5) a) Digital b) Digital c) ~~Digital~~ Analog d) Analog.

6) a) $x(t) = 3\cos(4t+\frac{\pi}{3}) \Rightarrow \omega_0 = 4 \Rightarrow T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \pi/2$ sec. (Periodic)

b) $x(t) = [\cos(2t-\frac{\pi}{3})]^2 \Rightarrow x(t) = \frac{1+\cos 2(2t-\frac{\pi}{3})}{2} = \frac{1}{2}+\frac{1}{2}\cos(4t+\frac{2\pi}{3})$

$\omega_0 = 4$ and $T_0 = \frac{2\pi}{\omega_0} \Rightarrow \frac{2\pi}{4} = \pi/2$ sec (Periodic)

c) $x(t) = \sin 10\pi t \Rightarrow \omega_0 = 10\pi \Rightarrow T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10\pi} = \frac{1}{5}$ sec. (Periodic).

d) $x(t) = \sin(2t) + \cos(3\pi t) = \sin(\frac{\pi}{2} - 2t) + \cos(3\pi t) = \cos(2t - \pi/2) + \cos(3\pi t)$.

~~$\omega_{0_1} = HCF($~~ $\omega_{0_1} = 2$, $\omega_{0_2} = 3\pi$ $\quad \frac{\omega_{0_1}}{\omega_{0_2}} = \frac{2}{3\pi}$ which is not the ratio of integers

∴ $x(t)$ is aperiodic.

7) a) $x[n] = \sin\left(\left(\frac{6\pi}{7}\right)n + 1\right)$

$\omega_0 = \frac{6\pi}{7} \Rightarrow T_0 = \frac{r2\pi}{\omega_0} = \frac{2\pi}{6\pi/7} = \frac{r7}{3}$

as it is discrete siginal we need to have T_0 as integer so we multiply with '3' [∵ $r=3$].

∴ $T_0 = 7$. (It is periodic)

b) $x[n] = \cos(\frac{n}{8} - \pi)$

$\omega_0 = \frac{1}{8} \Rightarrow T_0 = \frac{r2\pi}{1/8} = r16\pi$.

for any integer 'r', 'T_0' doesnot become integere.

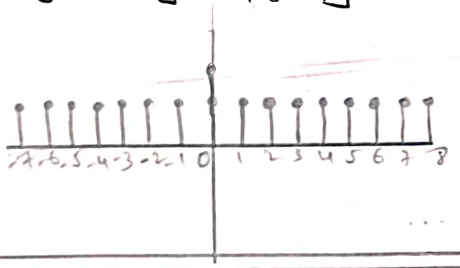
∴ $x[n]$ is aperiodic signal

c) $\cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n) = x[n] \Rightarrow x[n] = \frac{1}{2}\left(\cos(\frac{3\pi}{4}n) + \cos(\frac{\pi n}{4})\right)$

$\omega_0 = \frac{HCF(3,1) \times \pi}{LCM(4,4)} = \frac{\pi}{4}$. $\Rightarrow T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/4} = 8$.

∴ $x[n]$ is periodic signal.

d) $x[n] = u[n] + u[-n]$



By graph we can tell that $x[n]$ is ~~aperiodic~~ not periodic.

8) a) $x(t) = 2\cos(10t+1) - \sin(4t - 1)$

$T_1 = \frac{2\pi}{\omega_{0_1}} = \frac{2\pi}{10} = \frac{\pi}{5}$ $\qquad T_2 = \frac{2\pi}{\omega_{0_2}} = \frac{2\pi}{4} = \frac{\pi}{2}$

∴ $LCM(\frac{\pi}{5}, \frac{\pi}{2}) = \pi$

∴ $T_0 = \pi$.

b) $x(t) = \cos(\frac{2\pi}{7}t) + \cos(\frac{3\pi}{4}t) + \cos(\frac{6\pi}{5}t)$

$\omega_0 = \frac{HCF(2,3,6) \times \pi}{LCM(7,4,5)} = \frac{1 \times \pi}{140}$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/140} = 280$

c) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

Period of '1' is 1.

Period of $e^{j4\pi n/4} = r\left(\frac{2\pi}{4\pi/7}\right) = 7$ (∵ $r=2$)

Period of $e^{j2\pi n/5} = r(2\pi/(2\pi/5)) = 5$ (∵ $r=1$)

$T_0 = LCM(1,7,5) = 35$

∴ $T_0 = 35$.

9) a) $x(t) = e^{-5t}$ $\quad x(-t) = e^{-5(-t)} = e^{5t}$

$\therefore x(t)$ is neither even nor odd signal.

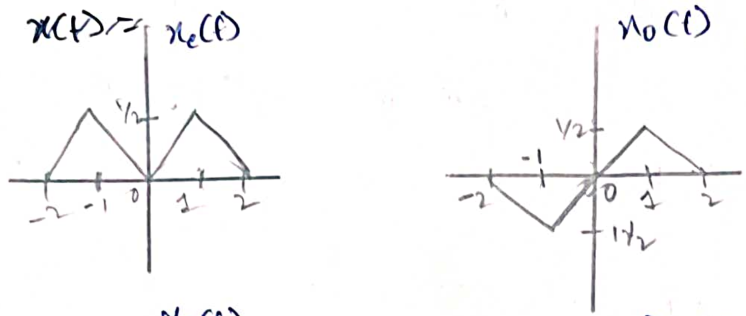
b) $x(t) = \sin 2t$ $\quad x(-t) = \sin 2(-t) = -\sin 2t = -x(t)$

$\therefore x(t)$ is odd signal.

c) $x(t) = \cos 5t$. $\quad x(-t) = \cos 5(-t) = \cos 5t = x(t)$

$\therefore x(t)$ is even signal.

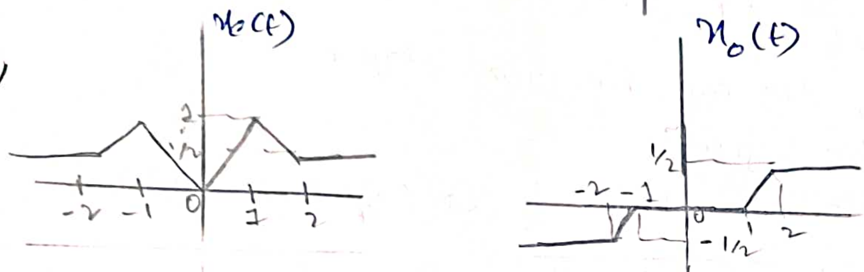
d) (i)



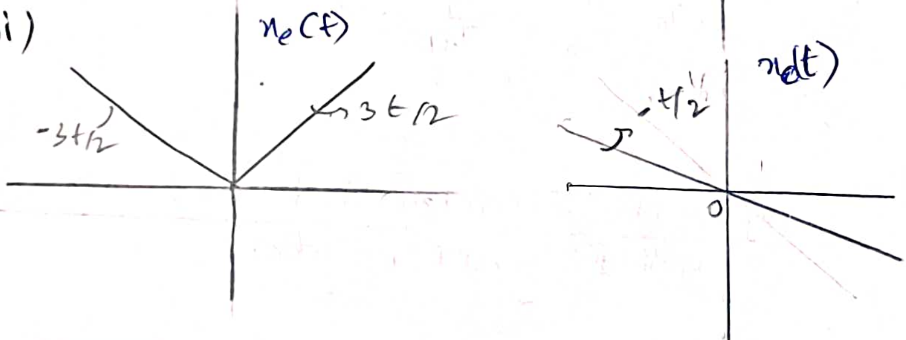
$x_e(t) = \dfrac{x(t)+x(-t)}{2}$

$x_o(t) = \dfrac{x(t)-x(-t)}{2}$

(ii)



(iii)



10) a) $x(t) = e^{-2t}u(t)$

$E_\infty = \int_0^\infty |e^{-2t}|^2 dt = -\frac{1}{4}\left[e^{-4t}\right]_0^\infty = \frac{1}{4}$ $\Rightarrow P_\infty = 0$ [$\because E_\infty$ = finite]

b) $x(t) = \cos(t)$.

$E_\infty = \int_{-\infty}^{\infty} \cos^2(t)\, dt = \infty$.

$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2(t)dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}\left(\frac{1+\cos 2t}{2}\right)dt = \frac{1}{2}$.

c) $x(t) = e^{j(2t+\pi/4)}$ $\quad \Rightarrow |x(t)| = 1$.

$\Rightarrow E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} dt = \infty$.

$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} 2T = 1$.

d) $x(t) = (\frac{1}{2})^n u[n]$ $\quad |x[t]|^2 = (\frac{1}{4})^n u[n]$ $\quad \Rightarrow E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} (\frac{1}{4})^n$

$= \frac{1}{1-1/4} = \frac{4}{3}$.

$E_\infty = \frac{4}{3}$.

$P_\infty = 0$ [$\because E_\infty$ = finite]

e) $x(t) = e^{j\left(\frac{n\pi}{2}+\frac{\pi}{8}\right)} \Rightarrow |x(t)|^2 = 1 \Rightarrow E_\infty = \int_{-\infty}^{\infty} 1\,dt = \infty$.

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1\,dt = 1$.

11) a) $y(t) = 2t^2 x(t)$

~~$x_1(t) \rightarrow y_1(t) = 2t^2 x_1(t)$ — $x_2(t) \rightarrow y_2(t) = t^2 x_2(t-t)$~~

$x_1(t) \Rightarrow y_1(t) = 2t^2 x_1(t)$

$x_2(t) = x_1(t-t_0)$

$y_2(t) = 2t^2 x_2(t) = 2t^2(x_1(t-t_0)) \rightarrow ①$

$y_1(t-t_0) = 2(t-t_0)^2 x_1(t-t_0) \rightarrow ②$

$\therefore$ Eq ① $\neq$ Eq ②.

$\therefore$ system is time varient.

$x_1(t) \rightarrow y_1(t) = 2t^2 x_1(t)$ $\quad x_2(t) \rightarrow y_2(t) = 2t^2 x_2(t)$

$x_3(t) = a x_1(t) + b x_2(t)$

$y_3(t) = 2t^2[x_3(t)] \Rightarrow y_3(t) = 2t^2(a x_1(t) + b x_2(t))$

$\Rightarrow y_3(t) = a y_1(t) + b y_2(t)$.

$\therefore$ The system is linear.

y(t) doesn't have any memory unit. since it is only dependent on 't'.

As the system is memoryless it is causal.

$y(t) = 2t^2 x(t) \Rightarrow x(t) = \frac{y(t)}{2t^2}$

$\therefore$ y(t) is invertible.

b) $y(t) = 3e^{3x(t)}$

$y_1(t) = 3e^{3x_1(t)}$. $\quad y_2(t) = 3e^{3x_2(t)}$.

$x_3(t) = a x_1(t) + b x_2(t)$

$y_3(t) = 3e^{3x_3(t)} = 3e^{3(a x_1(t) + b x_2(t))} = 3\left[e^{3a x_1(t)} \cdot e^{3b x_2(t)}\right]$

$= \frac{1}{3} \cdot e^{a} \cdot y_1(t) \cdot e^{b} \cdot y_2(t)$

$\therefore$ y(t) is non linear.

$x_1(t) \Rightarrow y_1(t) = 3e^{3x_1(t)}$ $\quad x_2(t) = x_1(t-t_0)$.

$y_2(t) = 3e^{3x_2(t)} = 3e^{3x_1(t-t_0)} \rightarrow ①$

$y_1(t-t_0) = 3e^{3x_1(t-t_0)} \rightarrow ②$

$\therefore$ Eq ① = Eq ② $\Rightarrow$ system is time invarient.

y(t) doesn't have any memory unit & y(t) is causal.

$y(t) = 3e^{3x(t)} \Rightarrow x(t) = \frac{1}{3}\ln\left(\frac{y(t)}{3}\right)$ $\quad \therefore$ y(t) is invertible.

c) $y(t) = x(t) + t\,x(t-1)$

$y_1(t) = x_1(t) + t\,x_1(t-1)$ $\qquad$ $y_2(t) = x_2(t) + t\,x_2(t-1)$.

$x_3 = a x_1 + b x_2$

$y_3(t) = x_3(t) + t\,x_3(t-1) = a x_1(t) + b x_2(t) + t\,[a x_1(t-1) + b x_2(t-1)]$

$y_3(t) = a[x_1(t) + t\,x_1(t-1)] + b[x_2(t) + t\,x_2(t-1)]$

$y_3(t) = a\,y_1(t) + b\,y_2(t)$

$\therefore$ $y(t)$ is linear.

$x_1 \Rightarrow y_1(t) = x_1(t) + t\,x(t-1)$ $\qquad$ $x_2 = x_1(t-t_0)$.

$x_2 \Rightarrow y_2(t) = x_2(t) + t\,x_2(t-1) = x_1(t-t_0) + t\,x_1(t-t_0-1) \rightarrow (1)$

$y_1(t-t_0) = x_1(t-t_0) + (t-t_0)(x_1(t-t_0-1)) \rightarrow (2)$

Eq (1) $\neq$ Eq (2).

$\therefore$ $y(t)$ is time varient

It has a memory unit '$t-1$',

It is causal system.

$y(t)$ is non-invertible function.

d) $y(t) = \sin[x(t)]$

$y_1(t) = \sin x_1(t)$. $\qquad$ $y_2(t) = \sin x_2(t)$.

$x_3 = a x_1 + b x_2$

$y_3(t) = \sin x_3(t) = \sin[a x_1(t) + b x_2(t)]$.

$= \sin a x_1(t) \cos b x_2(t) + \cos a x_1(t) + \sin b x_2(t)$

$\therefore$ $y(t)$ is non linear

$x_1(t) \Rightarrow y_1(t) = \sin x_1(t)$ $\qquad$ $x_2 = x_1(t-t_0)$

$x_2(t) \Rightarrow y_2(t) = \sin x_2(t) = \sin x_1(t-t_0) \rightarrow (1)$

$y_1(t-t_0) = \sin x_1(t-t_0) \rightarrow (2)$

Eq (1) = Eq (2)

$\therefore$ $y(t)$ is time invarient.

It doesn't have memory unit.

It is causal system.

$y(t) = \sin x(t) \Rightarrow x(t) = \sin^{-1}(y(t))$

for same value of $y(t)$ there are different $x(t)$ values.

$\therefore$ $y(t)$ is non-invertible function.

e) $y[n] = x^2[n] - x[n-1]x[n+1]$

$y_1[n] = x_1^2[n] - x_1[n-1]x_1[n+1]$ $\quad$ $y_2[n] = x_2^2[n] - x_2[n-1]x_2[n+1]$

$x_3 = ax_1 + bx_2$.

$y_3[n] = x_3^2[n] - x_3[n-1]x_3[n+1]$

$= (ax_1[n] + bx_2[n])^2 - [ax_1[n-1] + bx_2[n-1]][ax_1[n+1] + bx_2[n+1]]$

$\therefore$ $y[n]$ is non linear.

$x_1[n] \Rightarrow y_1[n] = x_1^2[n] - x_1[n-1]x_1[n+1]$

$x_2[n] = x_1[n-n_0] \Rightarrow x_2[n] \Rightarrow y_2[n] = x_2^2[n] - x_2[n-1]x_2[n+1]$

$\Rightarrow y_2[n] = x_1^2[n-n_0] - x_1[n-n_0-1]x_1[n-n_0+1] \rightarrow (1)$

$y_1[n-n_0] = x_1^2[n-n_0] - x_1[n-n_0-1]x_1[n-n_0+1] \rightarrow (2)$

Eq (1) = Eq (2)

$\therefore$ system is time invarient.

It has memory unit $n-1$ & $n+1$.

It is non-cawsal.

and the system is non-invertible.