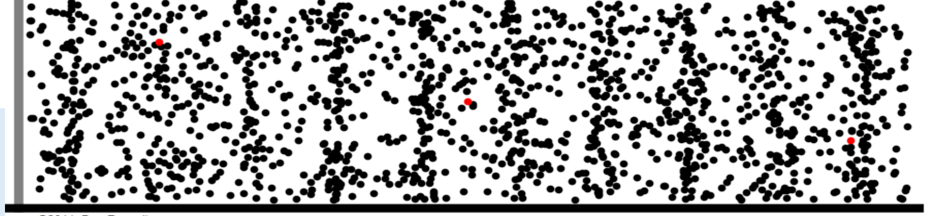
Energy associated with a wave:



A wave propagates through a medium, the particles execute SHM about their equilibrium positions, and associated with this motion is a certain amount of energy. As the wave propagates through, the energy gets transported from one end of the medium to the other. We consider the time variation of the displacement of a particle, which can be

$$\psi(x,t) = A\sin(\omega t + \delta)$$

Instantaneous velocity, $v = \omega A\cos(\omega t + \delta)$

Total energy of a SH oscillator, $\mathrm{E} = \mathrm{K.E.} + \mathrm{P.E.} = \mathrm{K.E.}_{\mathrm{max}}$

The wave carries energy, $\mathrm{E} = \frac{1}{2}mv^2|_{max}$

$$\mathrm{E} = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \delta)\Big|_{max}$$

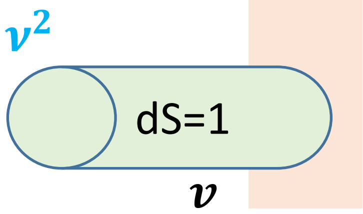
$$\mathrm{E} = \frac{1}{2}m\omega^2A^2$$

Sound wave through a gas of particle density n

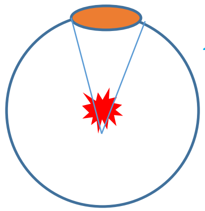
Energy density $\epsilon = \frac{1}{2}mnA^2\omega^2 = \frac{1}{2}\rho A^2\omega^2$

$$= \frac{1}{2}\rho A^2(2\pi\nu)^2 = 2\pi^2\rho A^2\nu^2$$

Speed of wave v,



Intensity (energy/unit area), $\mathrm{I} = \epsilon.Vol = 2\pi^2\rho A^2\nu^2 v$



$$I = \frac{W}{4\pi r^2}$$

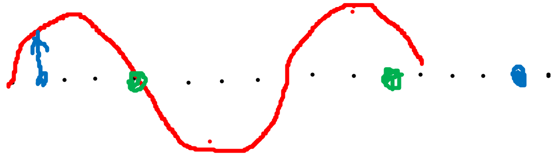
$$\frac{W}{4\pi r^2} = 2\pi^2\rho A^2\nu^2 v$$

$$A = \frac{1}{r}\sqrt{\frac{W}{8\pi^2 v\nu^2}} = \frac{A_0}{r}$$

$$\psi_s(x,t) = \frac{A_0}{r}\sin(kr - \omega t + \delta)$$

Phase and phase velocity:

A sinusoidal wave propagates through a medium is represented by,



$$\psi(x,t) = A\sin(kx - \omega t + \delta)$$

Phase $\Rightarrow \delta$, Phase angle $\Rightarrow \phi = (kx - \omega t + \delta)$

$$\boxed{\frac{\partial\phi}{\partial t} = -\omega} \qquad \boxed{\frac{\partial\phi}{\partial x} = k} \qquad \left.\frac{\partial x}{\partial t}\right|_{\phi} = \left.\frac{\partial\phi}{\partial t}\right|_{x} \Big/ \left.\frac{\partial\phi}{\partial x}\right|_{t}$$

The LHS is the speed of propagation of the condition of constant phase (velocity of the wave):

Phase velocity: $v_p = \mp\frac{\omega}{k}$

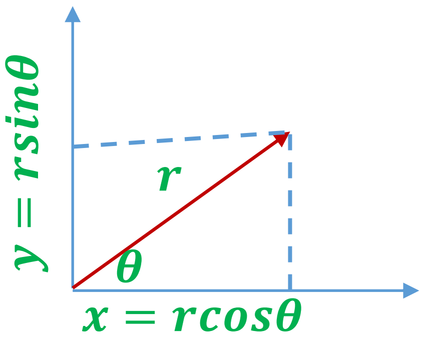
$$\psi(x,t) = A\sin[k(x - v_p t + \delta)$$

$$\boxed{\tilde{z} = x + iy}$$

$$\boxed{\tilde{z} = r(\cos\theta + i\sin\theta)}$$

$$\boxed{\tilde{z} = re^{i\theta}}$$

$$\boxed{r = \sqrt{x^2 + y^2}} \quad \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$



$$\boxed{Re(\tilde{z}) = r\cos\theta} \quad \boxed{Im(\tilde{z}) = r\sin\theta}$$

$$\psi(x,t) = A\sin(kx \mp \omega t + \delta) = Im\{Ae^{i(kx \mp \omega t + \delta)}\}$$

$$\psi(x,t) = A\cos(kx \mp \omega t + \delta) = Re\{Ae^{i(kx \mp \omega t + \delta)}\}$$

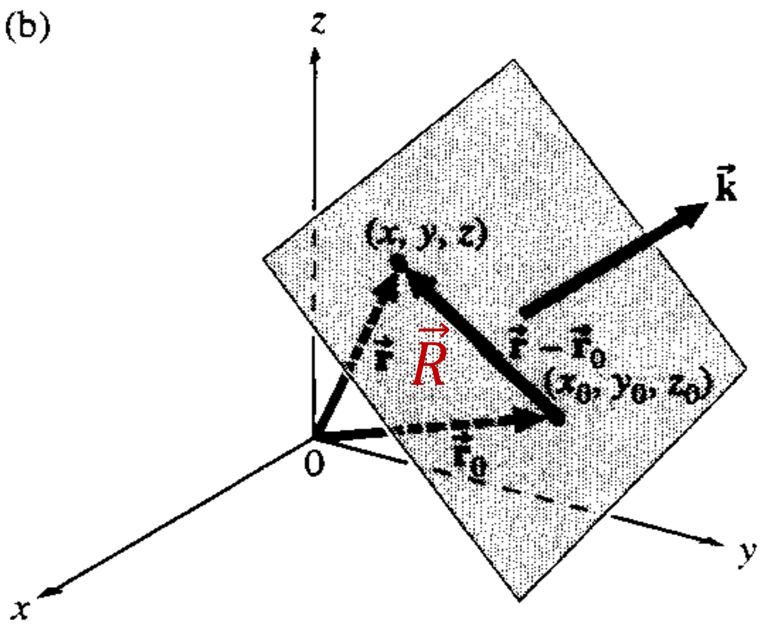
$$\psi(x,t) = Ae^{i(kx \mp \omega t + \delta)} = Ae^{ikx}e^{i(\mp\omega t + \delta)}$$

Plane wave:

At any instant of time, the surfaces joining all points of equal phase are known as **wavefronts**

If at a given time, all the surfaces on which **a disturbance has constant phase** form a set of planes, each generally perpendicular to the propagation direction then the wave is known as a plane wave.

To derive the expression for **a plane** that is perpendicular to a given vector $\vec{k}$ and that passes through some point (x_0, y_0, z_0), we write,



$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad \vec{k} = k_x\hat{\imath} + k_y\hat{\jmath} + k_z\hat{k}$$

$$\vec{R} = \vec{r} - \vec{r_0} = (x - x_0)\hat{\imath} + (y - y_0)\hat{\jmath} + (z - z_0)\hat{k}$$

$$\vec{k} \cdot \vec{R} = 0 \Rightarrow \text{for } \vec{R} \text{ perpendicular to } \vec{k}$$

$$(x - x_0)k_x + (y - y_0)k_y + (z - z_0)k_z = 0$$

$$x_0k_x + y_0k_y + z_0k_z = xk_x + yk_y + zk_z$$

If we assume, for the fixed point $\vec{r_0}(x_0, y_0, z_0)$, $\vec{k} \cdot \vec{r_0} = constant$ (say)

$$\boxed{\vec{k} \cdot \vec{r} = a, \quad \text{constant}}$$

The plane is the locus of all points each of whose position vectors have a constant projection on $\vec{k}$.

$$\psi(\vec{r}) = A\sin(\vec{k} \cdot \vec{r})$$

$$\psi(\vec{r}) = A\cos(\vec{k} \cdot \vec{r})$$

$$\psi(\vec{r}) = Ae^{i\vec{k} \cdot \vec{r}}$$

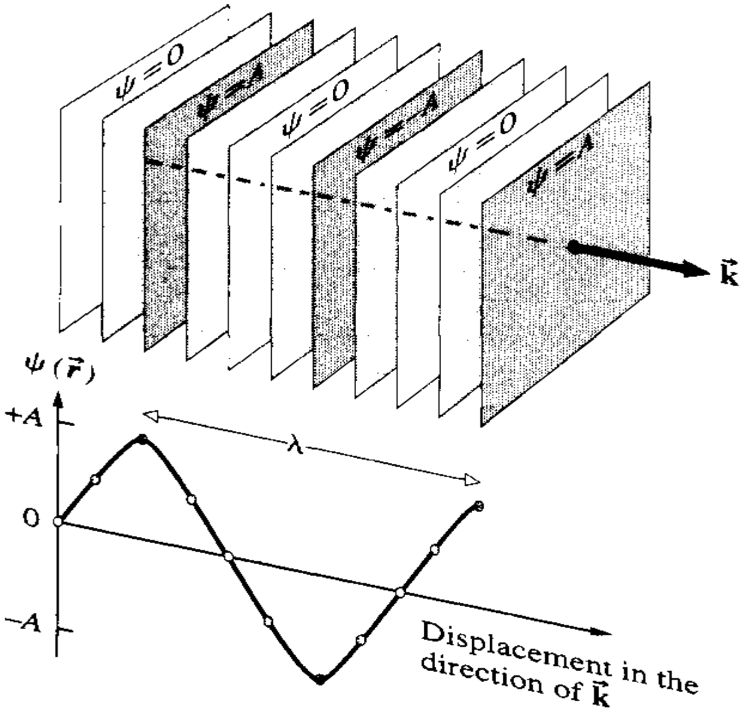
Plane wave:

$$\psi(\vec{r}) = A sin(\vec{k}\cdot\vec{r}) \qquad \psi(\vec{r}) = Ae^{i\vec{k}\cdot\vec{r}}$$

The spatially repetitive nature of these harmonic functions can be expressed by

$$\psi(\vec{r}) = \psi\left(\vec{r} + \lambda\left(\vec{k}/_{k}\right)\right) \qquad Ae^{i\vec{k}\cdot\vec{r}} = Ae^{i\vec{k}\cdot\vec{r}}e^{i\lambda k}$$

$$e^{i\lambda k} = 1 = e^{i2\pi} \qquad \vec{k} = \frac{2\pi}{\lambda} \Rightarrow wave\ number$$



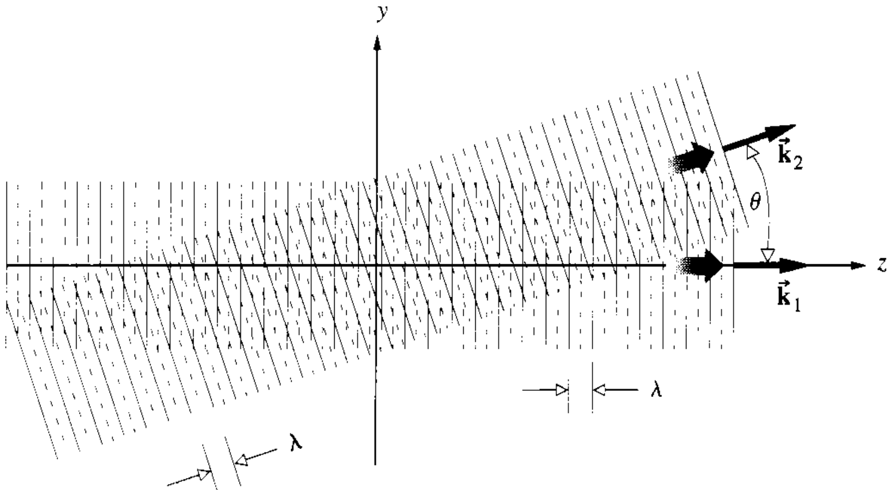
The disturbance on a wavefront is constant, so that after a time ***dt*,** if the front moves along k a distance $\boldsymbol{dr_k}$(r_k is the projection of $\vec{r}$ along $\vec{k}$ **),** we must have

$$\psi(\vec{r},t) = \psi(r_k + dr_k, t + dt) = \psi(r_k, t)$$

$$\psi(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r}\mp\omega t)} = Ae^{i(kr_k + kdr_k \mp \omega t \mp \omega dt)} = Ae^{i(kr_k\mp}$$

$$kdr_k \mp \omega dt = 0 \qquad \boxed{\frac{dr_k}{dt} = \pm\frac{\omega}{\text{k}} = \pm v_p}$$

$$r_1 = \hat{k}z \Rightarrow \overrightarrow{k_1}\cdot\vec{r} = kz$$
$$r_2 = \hat{j}y + \hat{k}z \Rightarrow \overrightarrow{k_2}\cdot\vec{r} = k(ysin\theta + zcos\theta)$$



$$\psi_1(\vec{r},t) = A_1 sin(kz - \omega t) = A_1 e^{i(kz-\omega t)}$$

$$\psi_1(\vec{r},t) = A_1 sin(k\{ysin\theta + zcos\theta\} - \omega t) = A_1 e^{i(k(xsin\theta + zcos\theta)-\omega t)}$$

$$\lambda = \frac{2\pi}{k_1} = \frac{2\pi}{k_2} = \frac{2\pi}{k}$$