

Case (1) TOMORROW'S DISCUSSION  
 Type 4: Non-homogeneous but reducible to homogeneous.

$$\frac{dy}{dx} = \frac{ax+by+c}{dx+ey+f} \quad \text{where } \frac{a}{d} \neq \frac{b}{e} \quad \text{or } ae \neq b \cdot d \quad \text{--- (1)}$$

then transformation

$$x = \underset{\substack{\downarrow \\ \text{new } x}}{X} + \underset{\substack{\downarrow \\ \text{unknown}}}{h} \quad \text{and} \quad y = \underset{\substack{\downarrow \\ \text{new } y}}{Y} + \underset{\substack{\downarrow \\ \text{unknown constant}}}{k} \quad \text{--- (2)}$$

$$dx = dX \quad \text{and} \quad dy = dY \quad ; \quad \frac{dy}{dx} = \frac{dY}{dX} \quad \text{--- (2)}$$

From (1) and (2)

$$\frac{dY}{dX} = \frac{dY}{dX} = \frac{ax+by+c}{dx+ey+f} = \frac{a(X+h)+b(Y+k)+c}{d(X+h)+e(Y+k)+f}$$

$$\frac{dY}{dX} = \frac{ax+by+c}{dx+ey+f} = \frac{aX+bY+\cancel{ah+bk+c}}{dX+eY+\cancel{dh+ek+f}} = 0 \quad \text{solve for unknown } h \text{ and } k.$$

Now

$$\frac{dY}{dX} = \frac{aX+bY}{dX+eY} \quad \left. \vphantom{\frac{dY}{dX}} \right\} \rightarrow \text{homogeneous.}$$

$$Z = \frac{Y}{X} \rightarrow \begin{matrix} Z(X) \\ \downarrow \\ Y(X) \end{matrix}$$

Case (2) if  $ae = b \cdot d$

$$\text{or } \frac{a}{d} = \frac{b}{e} = k \quad (\text{let it be})$$

$$\text{then } \frac{dy}{dx} = \frac{k(dx+ey)+c}{dx+ey+f} = \frac{kz+c}{z+f}$$

$$\text{let } z = dx+ey \Rightarrow \frac{dz}{dx} = d + e \cdot \frac{dy}{dx}$$

NOW SEPERABLE.