

1. Give a list of (at least two) properties of matrix multiplication that differ from those of the multiplication of real numbers.
2. Find inverses of all 3×3 elementary matrices.
3. Find elementary matrices E_1, E_2, E_3 such that

$$E_1 E_2 E_3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note: Your answers may vary depending on the elementary operations you choose.

4. Let

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}.$$

For which triples (y_1, y_2, y_3) does the system $AX = Y$ have a solution?

5. Find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$$

using (i) the (usual) adjoint method and (ii) the row-reduced echelon method. Also, verify that the matrix you got from both methods is the same and satisfies the definition of matrix inverse.