Engineering Optics

Lecture 37

12/06/2022

by

Debolina Misra

Dept. of Physics
IIITDM Kancheepuram, Chennai, India

Sagnac Interferometer

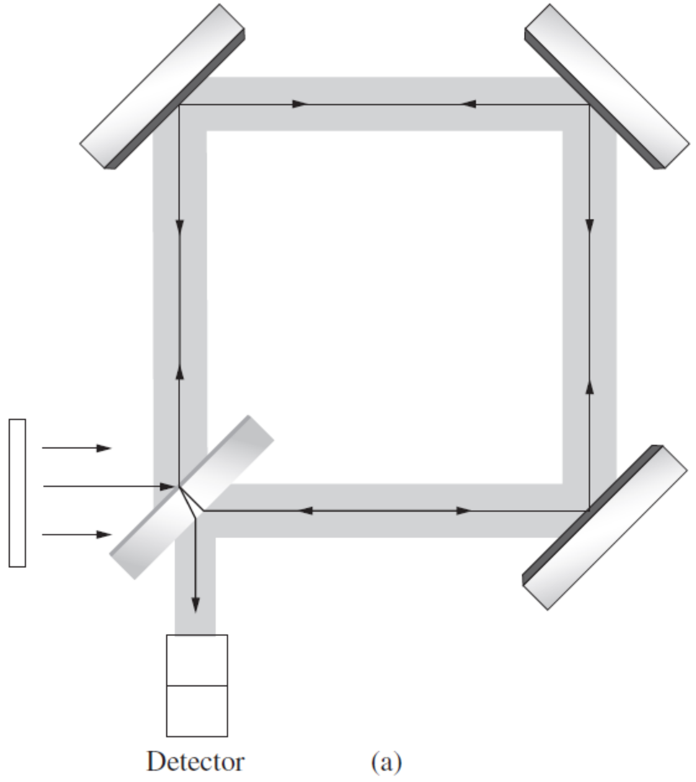
- amplitude-splitting device
- very easy to align and quite stable
- Application of the device →

- main feature of the device:
- there are two identical but oppositely directed paths taken by the beams
- both form closed loops before they are united to produce interference

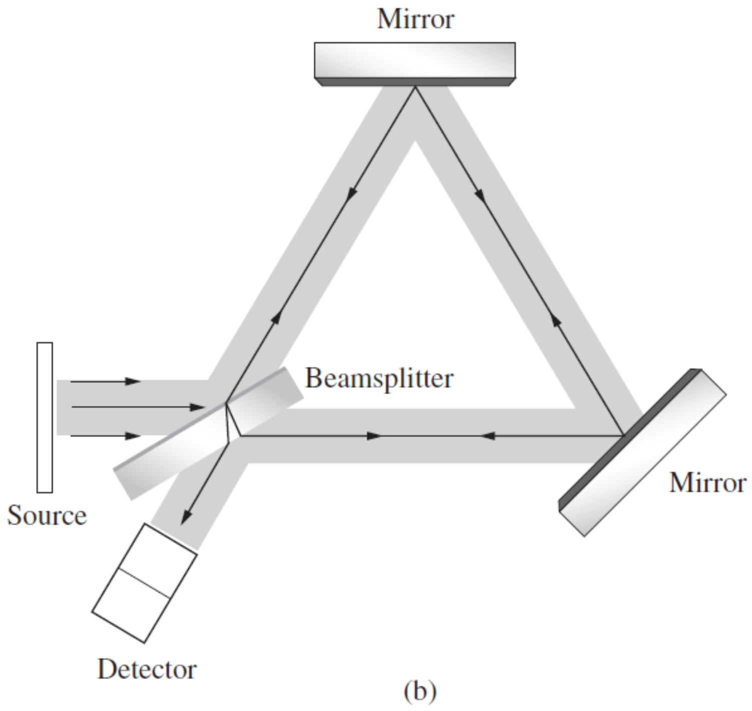
A deliberate slight shift in the orientation of one of the mirrors will produce a path length difference and a resulting fringe pattern.

Since the beams are superimposed and therefore inseparable, the interferometer cannot be put to any of the conventional uses.

These in general depend on the possibility of imposing variations on only one of the constituent beams.



(a) A Sagnac Interferometer.



(b) Another variation of the Sagnac Interferometer.

Optics, Hecht

Problem:1

For a sodium lamp, the distance traversed by the mirror between two successive disappearances is 0.289 mm for Michelson interferometer. Calculate the difference in the wavelengths of the D_1 and D_2 lines. Assume λ= 5890 Å

Answer:

$$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2}$$

is 1/2, 3/2, 5/2, . . . , we will have disappearance of the fringe pattern; and if it is equal to 1, 2, 3, . . . , then the interference pattern will appear.

Answer:

When the mirror moves through a distance 0.289 mm, the additional path introduced is 0.578 mm. Thus

$$\frac{0.578}{\lambda} - \frac{0.578}{\lambda + \Delta\lambda} = 1$$

$$\Delta\lambda \simeq \frac{\lambda^2}{0.578} = \frac{(5890 \times 10^{-7})^2}{0.578} \text{ mm}$$

$$\simeq 6 \text{ Å}$$

Assume $\Delta\lambda \times \lambda \ll \lambda^2$

Problem:2

In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength.

Answer:

The wavelength λ in Michelson interferometer is given by following equation.

$$\lambda = \frac{2d_0}{N}$$

Here, d_0 is the distance moved by the mirror, and N is the number of fringes.

$$\lambda = \frac{2(0.08\,\text{mm})\left(\frac{1\,\text{cm}}{10\,\text{mm}}\right)}{250}$$

$$= 6.4 \times 10^{-5}\,\text{cm}\left(\frac{10^{8}\,\text{A}^{\circ}}{1\,\text{cm}}\right)$$

$$= 6400\,\text{A}^{\circ}$$

Problem:3

Consider a monochromatic beam of wavelength 6000 Å incident (from an extended source) on a Fabry–Perot etalon with $n_2 = 1$, $h = 1$ cm, and $F = 200$. Concentric rings are observed on the focal plane of a lens of focal length 20 cm . Calculate the reflectivity of each mirror.

Answer:

$$F = \frac{4R}{(1-R)^2} \quad \Rightarrow \quad 200 = \frac{4R}{(1-R)^2}$$

$$50 = \frac{R}{(1+R^2-2R)}$$

$$50 + 50R^2 - 100R = R$$

$$50R^2 - 101R + 50 = 0$$

By solving above quadratic equation,

$$R = 1.15 \text{ or } 0.87$$

R=0.87 (Since R will be less than 1)

Problem:4

The Michelson interferometer experiment is performed with a source which consists of two wavelengths of 4882 and 4886 Å. Through what distance does the mirror have to be moved between two positions of the disappearance of the fringes?

Answer:

$\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1$ For appearance of fringe

Then

$$d = \frac{1}{2}\left(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\right)$$

$$d = \frac{1}{2}\left(\frac{(4882\,\text{A}^\circ)(4886\,\text{A}^\circ)}{4886\,\text{A}^\circ - 4882\,\text{A}^\circ}\right)$$

$$= 2981681.5\,\text{A}^\circ\left(\frac{1\,\text{cm}}{10^8\,\text{A}^\circ}\right)$$

$$= 0.0298\,\text{cm}\left(\frac{10\,\text{mm}}{1\,\text{cm}}\right)$$

$$= 0.298\,\text{mm}$$

Problem:5

In the Michelson interferometer experiment, calculate the various values of θ' (corresponding to bright rings) for $d = 5 \times 10^{-3}$ cm. Show that if d is decreased to 4.997×10^{-3} cm, fringe due to m=200 disappears. What will be the corresponding values of θ'? Assume $\lambda = 5 \times 10^{-5}$cm.

Answer:

The condition of bright rings in Michelson interferometer experiment is given by following equation.

$$2d\cos\theta' = \left(m+\frac{1}{2}\right)\lambda$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\lambda}{2d}\right)$$

$$\Rightarrow$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)\left(5\times10^{-5}\text{ cm}\right)}{2\left(5\times10^{-3}\text{ cm}\right)}\right)$$

$$\theta' = \cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{200}\right)$$

For $m = 197{,}198{,}199$ The corresponding θ'=9.07°, 7.02°,4.05°

For $d = 4.997 \times 10^{-3}$ cm

$$\theta' = cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)5\times 10^{-5}}{2\times 4.997\times 10^{-3}}\right)$$

$$\theta' = cos^{-1}\left(\frac{\left(m+\frac{1}{2}\right)}{198.8}\right)$$

$\Rightarrow m = 200$ will disappear

Problem: 6

In Fabry-Perot interferometer a bright fringe occur when initial separation between two mirrors $h = 10\ c.m$ $m = 100000$. At $h = 12\ c.m$ bright fringe occur at $m = 200000$. Then calculate the wavelength of the light

Answer:

Wavelength λ :

$$\boxed{\lambda = \frac{2(d_2-d_1)}{m_2-m_1}}$$

$$\lambda = \frac{2(12-10)}{200000-100000}$$

$$\lambda = 4 \times 10^{-5}\ c.m$$

Problem: 7

Consider now two wavelengths 6000 and 5999.9 Å incident on a Fabry–Perot etalon with $n_2 = 1, h = 1\ cm, and\ F\ =\ 200$. Calculate the angle corresponding of the first three bright rings corresponding to each wavelength.

Answer:

$$\cos\theta_2 = \frac{m\lambda_0}{2n_2h}$$

$$cos\theta_2 = \frac{m\times 6000\times 10^{-8}}{2\times 1\times 1} = \frac{m}{33333}$$

$$\theta_2 = \cos^{-1}\frac{m}{33333}$$

For $m = 33333, 33332, 33331\ we\ get\ \theta_2 = 0°, 0.44°, 0.63°$

Answer:

$$\cos \theta_2 = \frac{m\lambda_0}{2n_2h}$$

$$cos\theta_2 = \frac{m \times 5999.9 \times 10^{-8}}{2 \times 1 \times 1} = \frac{m}{33333.8}$$

$$\theta_2 = \cos^{-1}\frac{m}{33333.8}$$

For $m = 33333{,}33332{,}33331\ we\ get\ \theta_2 = 0.40°, 0.60°, 0.74°$

Problem: 8

A Fabry-Perot interferrometer would resolve two lines with $\Delta\lambda = 0.1$ Å at 6000 Å. Calculate the resolving power . Calculate the value of reflectivity if F=80 .

Answer:

$$\text{Resolving power} = \left| \frac{\lambda_0}{\Delta\lambda_0} \right|$$

$$= \frac{6000\text{Å}}{0.1\text{Å}}$$

$$= 6 \times 10^4$$

$$F = \frac{4R}{(1-R)^2}$$

$$80 = \frac{4R}{(1-R)^2}$$

$$80R^2 - 164R + 80 = 0$$

Possible values of $\mathrm{R} = 1.25, 0.8$

$$\mathrm{R} = 0.8$$

Thank You