15/11/2023

MAGNETISM

- charges in motion $\Longrightarrow$ Magnetism.
- magnetic materials on an atomic scale $\rightarrow$ e^- orbiting around nucleus + spinning about their axis
- for macroscopic purposes $\rightarrow$ the current loops $\equiv$ magnetic dipoles
- due to random orientation $\longrightarrow$ dipole moments cancel off.
- if $\vec{B}$ field applied $\rightarrow$ dipoles align $\rightarrow$ medium magnetically polarized / magnetized.
- electric polarization ($\vec{P}$) : $\vec{P}$ $\|$lel to $\vec{E}$.
- magnetization depends on materials:
  - ↳ paramagnetic : $\vec{P}$ $\|$lel to $\vec{B}$
  - ↳ diamagnetic : $-\vec{P}$ $\|$lel to $\vec{B}$
- ferromagnetic : $\vec{P}$ $\|$lel to $\vec{B}$, but if $\vec{B}$ becomes 0, $\vec{P}$ is retained in ferromagnets.
- Anti-ferromagnetic : neighbouring dipoles are opposite.
- ferrimagnetic : $\vec{P}$ is reduced (neighbouring dipoles are oppositely oriented, but not equal in magnitude, so effect is not nullified, but reduced)

Measure of Magnetization = M

where M = magnetic moment per unit volume.

M → magnetization

$\vec{M}$ (in magnetism) $\equiv$ $\vec{P}$ (in electrostatics)

$$\frac{d\mu_m}{d\tau} = \vec{M}$$

$$\frac{d\mu_E}{\partial\tau} = \vec{P}$$

$$\Rightarrow d\mu_m = \vec{M}\, d\tau$$

$$\Rightarrow \boxed{\mu_E = \int_V \vec{P} \cdot d\tau}$$

$$\vec{E} = -\vec{\nabla} V.$$

$$\Rightarrow \boxed{\mu_m = \int_V \vec{M}\, d\tau}$$

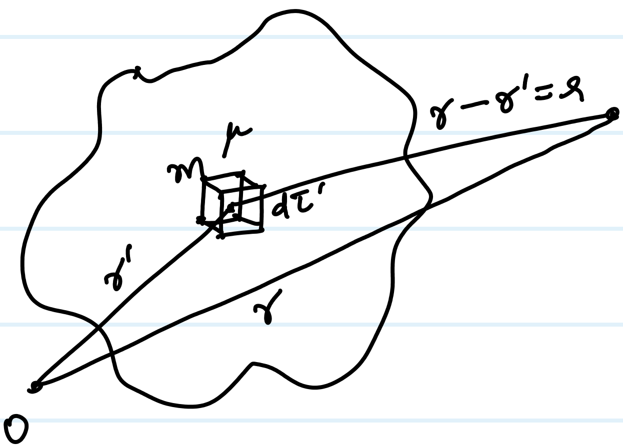
$$\boxed{V_E = K \cdot \frac{\vec{\mu}_E \cdot \hat{r}}{r^2}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \cdot \frac{\vec{\mu}_m \times \hat{r}}{r^2}}$$

$$\vec{A}_M \equiv V_E$$

BOUND CURRENTS



$$d\vec{A}(r) = \frac{\mu_0}{4\pi}\,\frac{d\vec{\mu}_m \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$d\vec{A}(r) = \frac{\mu_0}{4\pi}\cdot\frac{(\vec{M}\times\hat{\mathscr{r}})\,d\tau}{\mathscr{r}^2}$$

$$\vec{A}(r) = \frac{\mu_0}{4\pi}\int \frac{\vec{M}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau$$

$$= \frac{\mu_0}{4\pi}\int \vec{M}\times\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right)d\tau$$

$$= \frac{\mu_0}{4\pi}\int\left(\frac{1}{\mathscr{r}}(\vec{\nabla}\times\vec{M}) - \vec{\nabla}\times\left(\frac{\vec{M}}{\mathscr{r}}\right)\right)d\tau'$$

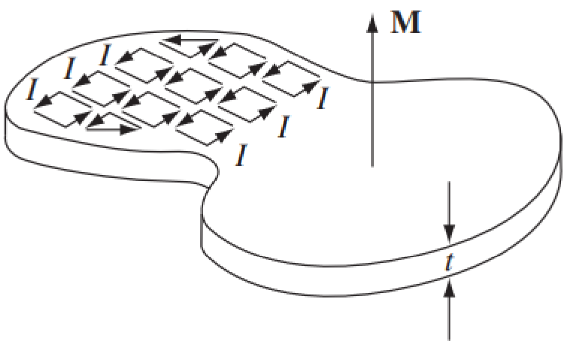
$$= \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}(\vec{\nabla}\times\vec{M})\,d\tau' - \frac{\mu_0}{4\pi}\int\vec{\nabla}\times\left(\frac{\vec{M}}{\mathscr{r}}\right)d\tau'$$

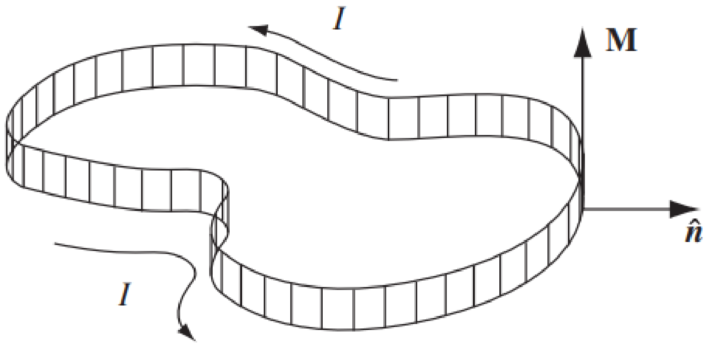
$$= \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}(\vec{\nabla}\times\vec{M})\,d\tau' + \frac{\mu_0}{4\pi}\oint\frac{1}{\mathscr{r}}(\vec{M}\times d\vec{a}')$$

$$\boxed{\begin{aligned} J_b &= \vec{\nabla}\times\vec{M} \\ K_b &= \vec{M}\times\hat{n}\end{aligned}}$$

PHYSICAL INTERPRETATION of BOUND CURRENTS

In the last section, we found that the field of a magnetized object is identical to the field that would be produced by a certain distribution of "bound" currents, $\mathbf{J}_b$ and $\mathbf{K}_b$. I want to show you how these bound currents arise physically. This will be a heuristic argument—the rigorous derivation has already been given. Figure 6.15 depicts a thin slab of uniformly magnetized material, with the dipoles represented by tiny current loops. Notice that all the "internal" currents cancel: every time there is one going to the right, a contiguous one is going to the left. However, at the edge there is *no adjacent loop to do the canceling*. The whole thing, then, is equivalent to a single ribbon of current I flowing around the boundary (Fig. 6.16).





TOTAL CURRENT

$$J = J_b + J_f$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{B})\cdot d\vec{s} = \mu_0 \int_S \vec{J}\cdot d\vec{s}$$

$$\Rightarrow \vec{J} = \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})$$

$$\therefore \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = J_f + \vec{\nabla}\times\vec{M}$$

$$\Rightarrow \vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{B}\right) - \vec{\nabla}\times\vec{M} = J_f$$

$$\Rightarrow J_f = \vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)$$

$\therefore$ $J_f = \vec{\nabla}\times\vec{H}$; where

$$\boxed{\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}}$$

$$\vec{\nabla}\times\vec{H} = J_f$$

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{H})\cdot d\vec{s} = \int_S \vec{J}_f\cdot d\vec{s}$$

$$\Rightarrow \int_l \vec{H}\cdot d\vec{l} = I_{f_{enc}}$$

$$\therefore \boxed{\oint \vec{H}\cdot d\vec{l} = I_{f_{enc}}}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

$$\Rightarrow \vec{B} = \mu_0(\vec{H} + \vec{M})$$
$$= \mu_0(\vec{H} + \chi_m \vec{H})$$
$$= \mu_0(1+\chi_m)\vec{H}$$
$$= \mu_0 \mu_r \vec{H}$$
$$\Rightarrow \boxed{\vec{B} = \mu \vec{H}}$$

for a linear medium, $\boxed{\vec{M} = \chi_m \vec{H}}$

PROBLEM:

$l = 0.25\,m$, $N = 400$, $I = 15\,A$

find $\vec{B}$, $\vec{H}$ (magnetic field strength).

$$\vec{B} = \mu_0 n I$$
$$= 4\pi \times 10^{-7} \times 400 \times 4 \times 15$$
$$= 4\pi \times 400 \times 4 \times 15 \times 10^{-7}$$
$$= 4\pi \times 24000 \times 10^{-7}$$
$$= 96\pi \times 10^{-4}$$
$$\vec{B} = 0.0301\ T$$

$$\vec{B} = \mu_0 \vec{H} \Rightarrow \frac{\vec{B}}{4\pi \times 10^{-7}} = \vec{H} = \frac{0.0301}{4\pi} \times 10^{7}$$

$$\vec{H} = 23952$$

PROBLEM

$$\vec{B} = 0.63\,T, \quad \vec{H} = 5\times10^{5}\,A/m$$

(a) μ (b) χ_m (c) Type of magnetism

(a) $\vec{B} = \mu\vec{H}$

$$\Rightarrow \mu = \frac{B}{H} = \frac{0.63}{5}\times10^{-5} = 1.26\times10^{-6}$$

(b) $$\mu_r = \frac{\mu}{\mu_0} = \frac{1.26\times10^{-6}}{4\pi\times10^{-7}} = \frac{1.26}{4\pi}\times10$$

$$= \frac{12.6}{4\pi}$$

$$= 1.0026$$

$$1+\chi_m = \mu_r \Rightarrow \chi_m = \mu_r - 1 = 1.0026 - 1$$

$$= 0.0026$$

(c) paramagnetism ($\chi_m > 0$)

$\chi_m < 0 \rightarrow$ diamagnetism

$\chi_m >> 0 \rightarrow$ ferromagnetism

PROBLEM

find $\vec{H}$

$\forall s$

when: $s<R$

$$\int_l \vec{H}\cdot d\vec{l} = I_{enc} = \int J_{enc}\cdot ds = J_{enc}\int ds$$

$$\Rightarrow H\cdot 2\pi \cancel{s} = \frac{I}{\cancel{\pi} R^2}\cdot \cancel{\pi} s^{\cancel{2}}$$

$$\Rightarrow \boxed{H = \frac{Is}{2\pi R^2}\hat{\phi}}$$

When $\underline{s \geq R}$:

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} = \frac{I}{\cancel{\pi R^2}} \cdot \cancel{\pi R^2} = I$$

$$\Rightarrow \vec{H} \cdot 2\pi s = I$$

$$\Rightarrow \boxed{\vec{H} = \frac{I}{2\pi s} \hat{\phi}}$$

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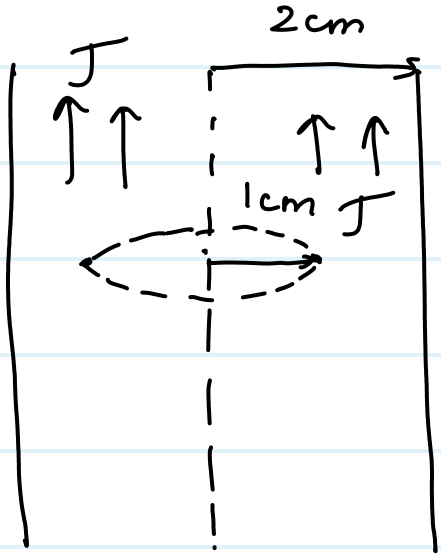
PROBLEM - 1



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$= \mu_0 \int J \cdot ds$$

$$\Rightarrow \vec{B} \cdot 2\pi s = \mu_0 \cdot J \cdot \pi s^2$$

$$\Rightarrow \vec{B} \cdot 2\pi \cancel{s} = \mu_0 \frac{I}{\cancel{\pi} R^2} \cancel{\pi} s^{\cancel{2}}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I s}{2\pi R^2} \hat{\phi}$$

$$\therefore \vec{B} = \frac{\mu_0 \times 10 \times 1 \times 10^{-2}}{2\pi \times (2 \times 10^{-2})^2}$$

PROBLEM-2

∞-solenoid, n turns/unit length, radius R and current I.

$$\phi = \int_S \vec{B}\cdot\vec{ds} = \oint_l \vec{A}\cdot\vec{dl}$$

$$\Rightarrow \mu_0 n I \cdot \cancel{\pi} s^{\cancel{2}} = A \cdot 2\cancel{\pi}\cancel{s}$$

$$\Rightarrow \vec{A} = \frac{\mu_0 n I}{2} s\,\hat{\phi}$$

$(s = R)$

$$\Rightarrow \boxed{\vec{A} = \frac{\mu_0 n I}{2} R\,\hat{\phi}}$$

PROBLEM-3

$l = 0.25\,m$, $N = 400$, $I = 15\,A$, $\chi_m = 3.13\times10^{-4}$

$$\vec{B} = \mu_0(\vec{H} + M)$$

$$\vec{B} = \mu n I$$
$$= \mu_0 \mu_r n I$$
$$= \mu_0 (1+\chi_m) n I$$

$$n = \frac{400}{0.25} = \underline{\underline{1600}}$$

$$\Rightarrow \vec{B} = \mu_0 (1 + 3.13\times10^{-4}) \times 400 \times 15 \times 4$$

$$\Rightarrow \vec{B} = 6000\mu_0 (1.000313) \times 4$$

$$= 24000\mu_0 (1.000313)$$

$$\vec{H} = \frac{\vec{B}}{\mu_0 \mu_r}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0 (1.000313)}$$

$$= 24,000.$$

$$\frac{1}{\mu_0}\vec{B} = \vec{H} + \vec{M}$$

$$\Rightarrow \vec{M} = \frac{1}{\mu_0}\vec{B} - \vec{H}$$

$$= 24000(0.000313)$$

$$= 7.512 \text{ A/m}$$

PROBLEM

∞-ly long cylinder, radius b

$\vec{M} = b s \hat{z}$. $I_{free} = 0$.

find H, J_b, K_b, B_{in}, B_{out}

$$\oint \vec{H}\cdot d\vec{l} = I_{enc_f} = 0$$

$$\Rightarrow \boxed{\vec{H} = 0}$$

$$J_b = \vec{\nabla} \times \vec{M}$$

$$= \vec{\nabla} \times (b\rho \hat{z})$$

$$\frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & b\rho \end{vmatrix}$$

$$\frac{1}{\rho}\left(\frac{\partial}{\partial \phi}\hat{\rho} - \frac{\partial}{\partial \rho}\rho\hat{\phi}\right) b\rho$$

$$-\frac{\partial \rho b}{\partial \rho}\hat{\phi} = -b\hat{\phi}$$

$$K_b = \vec{M} \times \hat{n} = \vec{M} \times \hat{\rho}$$

$$= br\, \hat{z} \times \hat{\rho}$$

$$K_b = br\, \hat{\phi}$$

$\underline{\vec{B}_{in}}$:

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) \qquad = \mu_0 \vec{M}$$

$$= \mu_0 (0 + b\rho\, \hat{z})$$

$$\boxed{\vec{B} = \mu_0 b \rho\, \hat{z}}$$

$\underline{\vec{B}_{out}}$:

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$= \mu_0 (0 + 0)$$

$$\Rightarrow \boxed{\vec{B} = \vec{0}}$$

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BOUNDARY CONDITIONS

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\Rightarrow \int_V (\vec{\nabla}\cdot\vec{B})\,d\tau = 0$$

$$\Rightarrow \int_S \vec{B}\cdot\vec{ds} = 0 \longrightarrow B_1{\perp} - B_2{\perp} = 0$$

$$\Rightarrow \boxed{B_1{\perp} = B_2{\perp}}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{B})\cdot\vec{ds} = \mu_0\int_S \vec{J}\cdot\vec{ds} = \mu_0 I_{enc}$$

$$\Rightarrow \int_l \vec{B}\cdot\vec{dL} = \mu_0 I_{enc}$$

$$\Rightarrow (B_1^{\parallel el} - B_2^{\parallel el})\cdot l = \mu_0 I_{enc}$$

$$\Rightarrow B_1^{\parallel el} - B_2^{\parallel el} = \mu_0\frac{I_{enc}}{l} = \mu_0 K$$

$$\boxed{K = \frac{I_{enc}}{l}}$$

$$\therefore\ \boxed{B_{net}^{\parallel el} = \mu_0 K}$$

* NEW *

$\vec{E} \rightarrow$ due to $\vec{B}(t)$: expt. by Faraday.

$\vec{B} \rightarrow$ due to $\vec{E}(t)$: Maxwell.

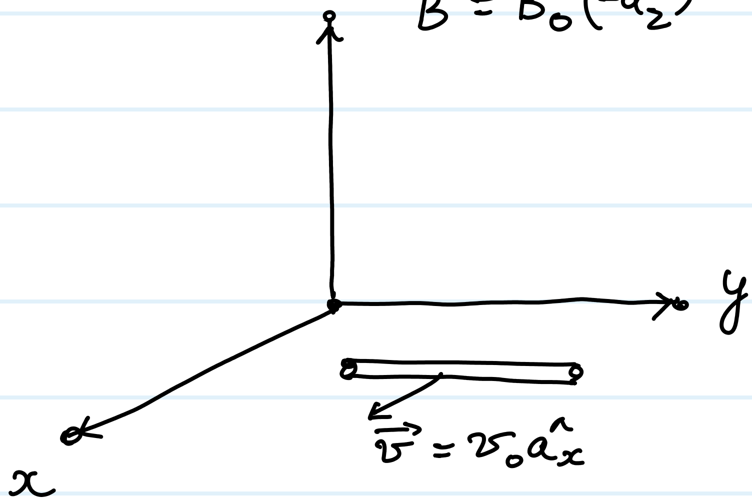
$\vec{B}(t) \rightarrow$ produces emf $\rightarrow$ causes a current in a closed circuit

emf $\rightarrow$ e or $\mathcal{E}$ $\rightarrow$ voltage generated.

When :
- $B = f(t)$
- $v_{circuit} \neq 0$
- both!!!

<u>Motional $\vec{E}$</u>

$$\vec{B} = B_0(-\vec{a}_z)$$



$$\vec{v} = v_0 \hat{a}_x$$

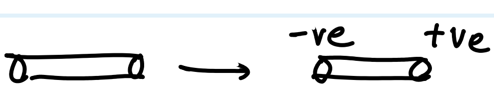
$$\vec{F}_q = q(\vec{v} \times \vec{B})$$

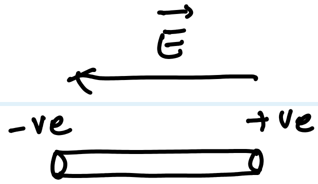
$$= q\left(v_0 \vec{a}_x \times B_0(-\vec{a}_z)\right)$$

$$\vec{F}_q = q v_0 B_0 \hat{y}$$

$$\vec{F}_{-q} = q v_0 B_0 (-\hat{y})$$

$\therefore$





$\vec{E}$ produced.

$$\phi = BA$$

$$-\frac{d\phi}{dt} = B\cdot\frac{dA}{dt} = -B\cdot\left(\frac{l\cdot x}{dt}\right)$$

$$= Bl\cdot v$$

$$\Rightarrow e = -B_0 l v_0$$

$$\Rightarrow \boxed{e/l = -B_0 v_0}$$

$$\vec{E} = -B_0 v_0 (-\vec{a_y})$$

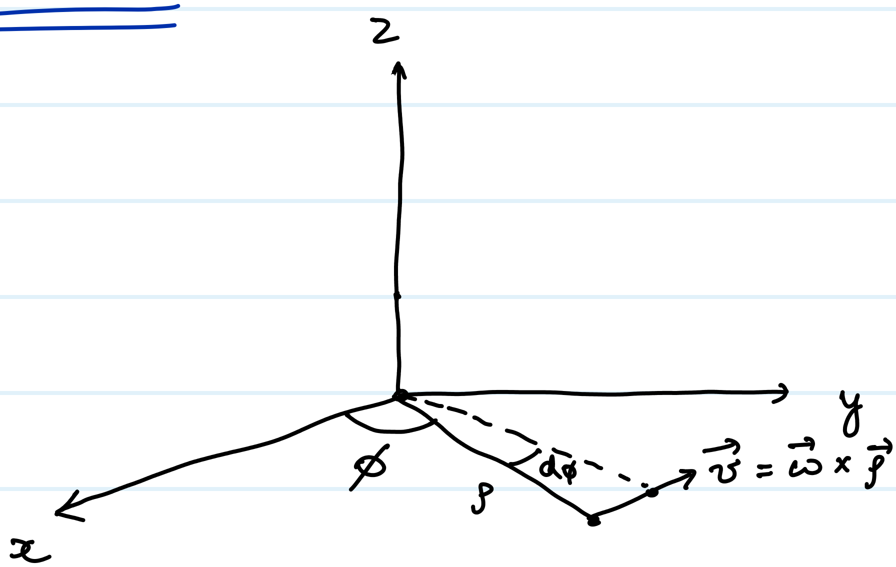
$$= B_0 v_0 \vec{a_y}$$

$$\boxed{\vec{E} = \vec{v}\times\vec{B}}$$

$$e = \frac{dW}{dq} = \frac{F\cdot dl}{q} = \frac{q\vec{E}\cdot\vec{dl}}{q} = \vec{E}\cdot\vec{dl}$$

$$\Rightarrow e = \int \vec{E}\cdot\vec{dl} = \int(\vec{v}\times\vec{B})\cdot\vec{dl}$$

PROBLEM-1

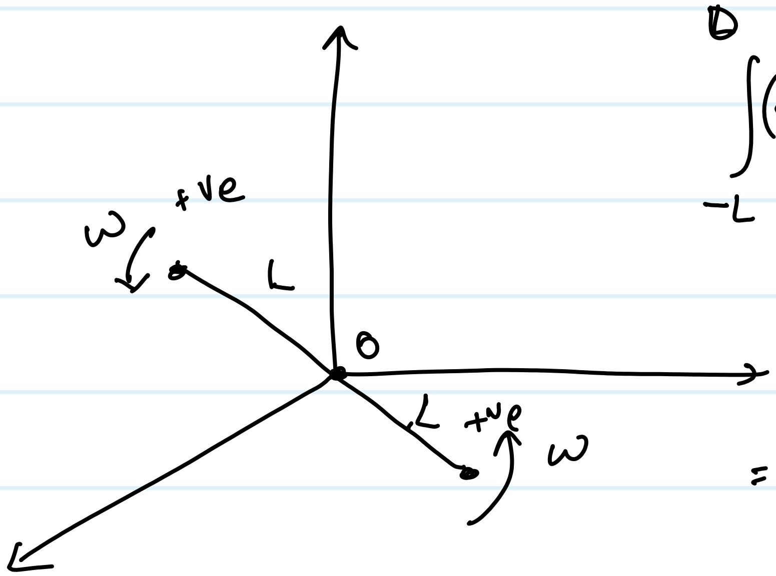


$$e = \int(\vec{v}\times\vec{B})\,d\rho$$

$$= \int_0^R (B\omega\rho)\,d\rho \;\hat{\rho}$$

$$= \left.\frac{B\omega\rho^2}{2}\right|_0^R$$

$$= BR^2\omega/_2 = \frac{Bl^2\omega}{2}$$

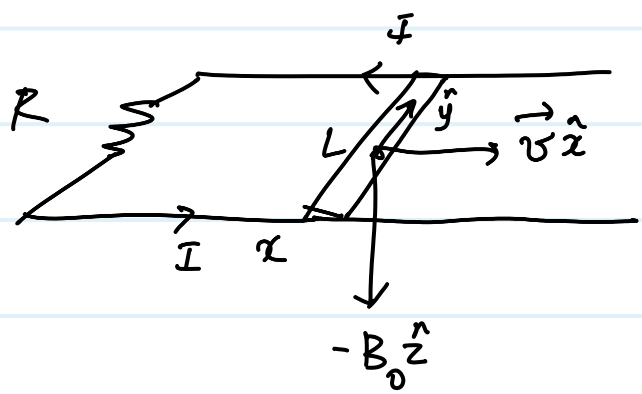


$$\int_{-L}^{0} (\vec{v}\times\vec{B})\cdot\vec{dL} + \int_{0}^{L} (\vec{v}\times\vec{B})\cdot\vec{dL}$$

$$= -\int_{-L}^{0} (Br\omega)\cdot dr + \int_{0}^{L} Br\omega\, dl$$

$$= -\left(\frac{Br^2\omega}{2}\right)_{-L}^{0} + \left(\frac{Br^2\omega}{2}\right)_{0}^{L}$$

$$= \underline{\underline{0}}$$



$$\phi = \vec{B}\cdot\vec{A}$$

$$\phi = -B_0 A$$

$$e = -\frac{d\phi}{dt} = B_0\cdot\frac{dA}{dl} = B_0 L v$$

$$dW = +ilB\cdot dx$$

$$=$$

$$i = \frac{e}{R} = \frac{B_0 L v}{R}$$

$$F = i(\vec{L}\times\vec{B})$$

$$= \frac{B_0 L v}{R}\left(L\hat{y}\times B_0(-\hat{z})\right)$$

$$F = \frac{B_0^2 L^2 v}{R}(-\hat{x})$$

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$$\mathcal{E} = \int \vec{E} \cdot \vec{dl} = \int (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} \quad -①$$

$$\mathcal{E} = -\frac{d\phi}{dt} = -\int \frac{\partial B}{\partial t} \cdot ds$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}_f$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = -\frac{\partial \rho_f}{\partial t}$$

$\Rightarrow 0 = -\frac{\partial \rho_f}{\partial t}$?? $\longrightarrow$ NOT POSSIBLE.

So, $\vec{\nabla} \times \vec{H} = \vec{J}_f + \vec{a}$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot (\vec{J}_f + \vec{a})$$

$$\Rightarrow 0 = \vec{\nabla} \cdot \vec{J}_f + \vec{\nabla} \cdot \vec{a} \Rightarrow \boxed{\vec{\nabla} \cdot \vec{a} = \frac{\partial \rho_f}{\partial t}}$$

$$\Rightarrow \vec{\nabla} \cdot \vec{a} = \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot \frac{\partial D}{\partial t} \Rightarrow \boxed{\vec{a} = \frac{\partial D}{\partial t}}$$

MAXWELLS

$$\vec{\nabla}\cdot\vec{D} = \rho \quad ; \quad \vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t} \;;$$

$$\vec{\nabla}\times\vec{H} = J + \frac{\partial D}{dt}$$

$$0 = \underbrace{\sigma\vec{E}}_{J_c} + \frac{\partial\vec{D}}{dt} \qquad \frac{\partial\vec{D}}{\partial t} = \varepsilon_0\varepsilon_r\frac{\partial\vec{E}}{\partial t}$$

PROBLEM : $\sigma = 0.11$, $\varepsilon_r = 1.2$, at $t = 5$:

find (i) J_c, (ii) J_D $\Big\} \longrightarrow$ if $\vec{E} = \cos 0.1t$

$$J_c = \sigma E = 0.11\cos 0.1t = 0.11\cos 0.5$$

$$J_D = -\frac{1.2}{2}\varepsilon_0\sin 0.5 = -0.6\varepsilon_0\sin 0.5$$

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if $A \to$ const & $\phi = f(t) \Rightarrow B = g(t)$

$$v = e = N\frac{d\Phi}{dt} \;, \quad \lambda = N\Phi \;; \quad \lambda \to n \text{ (flux linkage)}$$

$\frac{\text{flux}}{\text{current}} = \text{const}(L)$

$$\frac{\lambda}{i} = L \Rightarrow \lambda = Li \;; \quad L \to \text{self inductance.}$$

$$N\Phi = Li \qquad \boxed{\phi \propto i}$$

$$\Rightarrow NBA = Li$$

$$\Rightarrow N\mu_0 nIA = Li$$

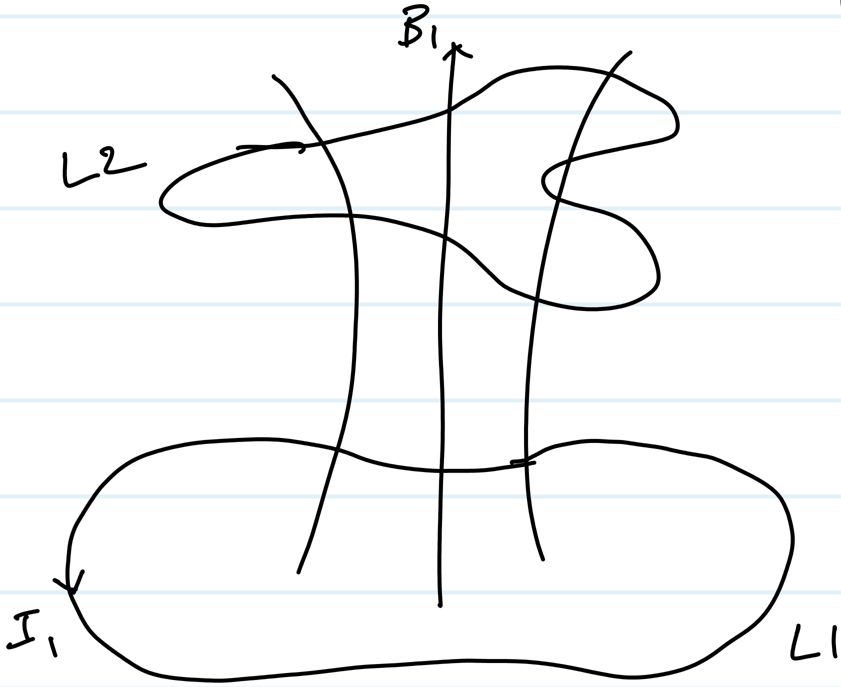
$$\Rightarrow \frac{\mu_0 N^2 A}{l} = L$$

$$\Rightarrow \boxed{L = \frac{\mu_0 N^2 \pi r^2}{l}}$$

$N_1 = N_2$, $l_1 = 2l_2$, $r_1 = 3/2\, r_2$

$$\frac{L_1}{L_2} = \frac{\cancel{\mu_0 N_2^2 \pi} (3/2\, r_2)^2 / 2\cancel{l_2}}{\cancel{\mu_0 N_2^2 \pi}\, r_2^2 / \cancel{l_2}}$$

$$= \frac{9}{4} \times \frac{1}{2} = \frac{9}{8} \;//$$



$$\phi_2 = \int \vec{B_1} \cdot dS_2 = M_{21} I_1$$

$$\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{I\vec{dl} \times \hat{r}}{r^2}$$

$$W = -\mathcal{E}\, q$$
$$\Rightarrow \frac{dW}{dt} = -\mathcal{E}\frac{dq}{dt}$$
$$\frac{dW}{dt} = -\mathcal{E}\, i$$

→ work done against emf.

$$\mathcal{E} = -\frac{d\phi}{dt} = -L\frac{di}{dt}$$

$$\Rightarrow \frac{dW}{dt} = \left(L\frac{di}{dt}\right) i \Rightarrow dW = L i\, di$$
$$\Rightarrow \boxed{W = \tfrac{1}{2} L i_0^2}$$

$$W = \frac{\mu_0}{2}\int H^2 d\tau = \frac{\varepsilon_0}{2}\int E^2 d\tau$$
$$= \frac{1}{2\mu_0}\int B^2 d\tau$$

$\therefore \boxed{L/l = 0.0064 \text{ H/m}}$

$r = 20$ cm, $n = 200$.

$$L = \frac{\mu_0 N^2 A}{l}$$

$$L = \frac{\mu_0 n \cdot N \cdot A}{l}$$

$1600\pi\mu_0$
$= 1600 \times 3.14 \times 4\pi \times 10^{-7}$
$= 1600 \times 9.8 \times 4 \times 10^{-7}$
$= 6400 \times 9.8 \times 10^{-7}$
$= 64000 \times 10^{-7}$
$= 0.0064$

$$\frac{20}{100} = \frac{400}{10000}$$

$$\frac{L}{l} = \mu_0 n^2 A$$
$$= \mu_0 \times \cancel{40000} \times \pi \frac{400}{\cancel{40000}} = \underline{\mu_0 n \cdot n l A} = \mu_0 n^2 A l$$

$\vec{H} = H_0 \sin\theta \, \vec{a_y}$, $\theta = \omega t - \beta z$

find (a) disp. current density (b) $\vec{E}$ field intensity

$\vec{\nabla} \times \vec{H} = J_C + J_D$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_0 \sin(\omega t - \beta z) & 0 \end{vmatrix} = 0 + \frac{\partial D}{\partial t}$$

$$\left(\frac{\partial}{\partial z} \vec{a_x} - \cancel{\frac{\partial}{\partial x} \vec{a_z}}^{\,0} \right) H_0 \sin(\omega t - \beta z) = \frac{\partial D}{\partial t}$$

$$\Rightarrow \quad +H_0 \beta \cos(\omega t - \beta z) = \frac{\partial D}{\partial t}$$

$$\Rightarrow \frac{+H_0 \beta \sin(\omega t - \beta z)}{\omega} \vec{a_z} = \vec{D}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{H_0 \beta}{\omega \varepsilon_0} \sin(\omega t - \beta z) \, \vec{a_z} \; //$$

01/12/2023

POYNTING THEOREM

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$$

$$\vec{F} \cdot \vec{u} = P = q(\vec{E} + \vec{u} \times \vec{B}) \cdot \vec{u} = q(\vec{u} \cdot \vec{E})$$

$$dP = dq(\vec{u} \cdot \vec{E})$$

$$dP = \rho_v d\tau (\vec{u} \cdot \vec{E})$$

$$\frac{dP}{d\tau} = \rho_v (\vec{u} \cdot \vec{E})$$

$$= (\rho_v \vec{u}) \cdot \vec{E} = \vec{J} \cdot \vec{E}$$

$$\Rightarrow \frac{dP}{d\tau} = \vec{J} \cdot \vec{E} = \mathbb{P} \quad (\mathbb{P} \rightarrow \text{power dens.})$$

* DERIVE POYNTING THEOREM LATER *

EXAMPLE : $\vec{E}(t) = E\cos(\omega t - kz)\,\vec{a}_x$ V/m

(a) $\vec{\nabla}\times\vec{E} = -\dfrac{\partial B}{\partial t}$

$$\vec{\nabla}\times\vec{E}$$

$$= \vec{\nabla}\times\left(E\cos(\omega t - kz)\,\vec{a}_x\right)$$

$$= E\cdot\vec{\nabla}\times\left(\cos(\omega t - kz)\,\vec{a}_x\right)$$

$$E\cdot\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(\omega t - kz) & 0 & 0 \end{vmatrix}$$

$$= E\cos(\omega t - kz)\left(\frac{\partial}{\partial z}\vec{a}_y - \cancel{\frac{\partial}{\partial y}\vec{a}_z}^{\,0}\right)$$

$$= +E\frac{\partial}{\partial z}\cos(\omega t - kz)\,\vec{a}_y$$

$$+\frac{\partial B}{\partial t} = +kE(\sin \omega t - kz)\,\hat{y}$$

$$B = \int kE\sin(\omega t - kz)\,dt$$

$$= \frac{-kE}{\omega}\cos(\omega t - kz)\,\vec{a}_y$$

$$H = \frac{-kE}{\mu\omega}\cos(\omega t - kz)\,\vec{a}_y$$

(b) $\vec{S} = \vec{E}\times\vec{H} = \vec{a}_x\times\vec{a}_y = \vec{a}_z$

(c) $S = \dfrac{-kE}{\mu\omega}\cos(\omega t - kz)\cdot E\cos(\omega t - kz)\,\hat{z} = \dfrac{-kE^2}{\mu\omega}\underline{\underline{\cos^2(\omega t - kz)\,\hat{z}}} = S$

$$\frac{\dfrac{-KE^2}{\mu\omega}\displaystyle\int_0^T \cos^2(\omega t - kz)\,dt}{T}$$

$$= \frac{-KE^2}{\mu\omega T}\left[\frac{1}{4}\int_0^T \cos(2\omega t - 2kz)\,dt + \int_0^T \frac{1}{2}\,dt\right]$$

$$= \frac{-KE^2}{\mu\omega T}\left[\overset{0}{\cancel{\frac{1}{4}\sin(2\omega t - 2kz)\Big|_0^T}} + \frac{T}{2}\right]$$

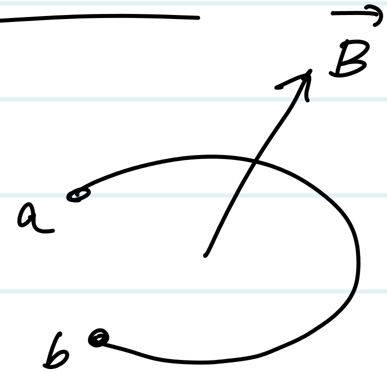
$$= \frac{-KE^2}{\mu\omega \cancel{T}}\left[\frac{\cancel{T}}{2}\right] = \frac{-KE^2}{2\mu\omega}$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\therefore \boxed{|P_{avg}| = \frac{KE^2}{2\mu\omega}}$$

$\vec{B}$ and $\vec{e}$

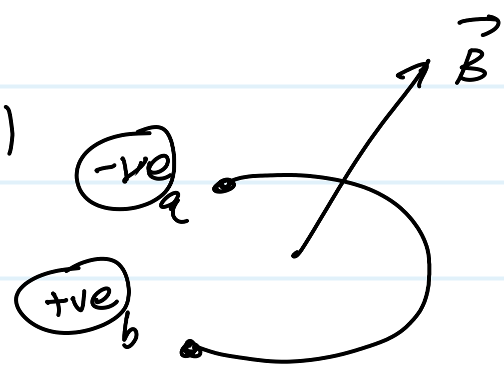
(i)



B is const.

$e = 0$

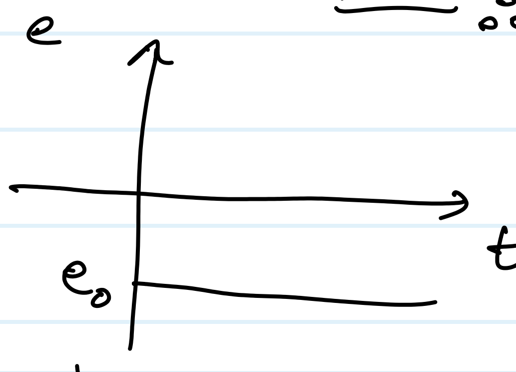
(ii)



B ↑

$$-e = \frac{d\phi}{dt} = A\boxed{\frac{dB}{dt}} \rightarrow \text{const. (+ve)}$$

$\therefore \boxed{e \rightarrow -ve}$



$\vec{B}\downarrow\downarrow$

$e = -\frac{dB}{dt}A$ → $-ve$

$\therefore\ \underline{\underline{e = +ve}}$

(iii)