[1] Find ordinary and singular (regular and irregular singular) points of the following differential equations.

(a) $x^2(x^2-1)^2y'' - x(1-x)y' + 2y = 0$

(b) $x^4y'' + \sin(x)y = 0$

(c) $x^2y'' + \sin(x)y' + \cos(x)y = 0$

[2] Find the roots of the indicial equation for following differential equations about $x = 0$.

(a) $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$

(b) $4x^2y'' - 4xe^xy' + 3\cos(x)y = 0$

[3] Show that indicial equation has only one root for $x^2y'' + xy' + x^2y = 0$ and corresponding one solution is

$$\sum_{r=0}^{\infty} \frac{(-1)^r}{(r!)^2} \left(\frac{x}{2}\right)^{2r}$$

[4] Show that $x = 0$ is a irregular singular point of the following differential equation

$$y'' + \frac{1}{x^2}y' - \frac{1}{x^3}y = 0$$

Also find the solution of the differential equation.

[5] Find the solution of the following differential equations about point $x = 0$.

(a) $x^2y'' + xy' + (x^2 - n^2)y = 0$, where m is not an integer

(b) $(1-x^2)y'' - 2xy' + 2y = 0$

[6] Prove the following relations for Bessel function.

(i)$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$

(ii) $J_{-n}(x) = (-1)^n J_n(x)$, where n is integer.

[7] Find the first three terms in the expansion of the following function in the terms of the Legendre polynomials

$$f(x) = \begin{cases} 0 & \text{if} \quad -1 < x < 0 \\ \text{x} & \text{if} \quad 0 < x < 1 \end{cases}$$

[8] Let y be a polynomial solution of the differential equation

$$(1-x^2)y'' - 2xy' + 12y = 0.$$

If $y(1) = 2$, then find the value of the integral $\int_{-1}^{1} y^2 dx$.

9 Let P_n be a solution of the Legendre differential equation: $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ then show that $P_n(1) = \frac{1}{2}n(n+1)$

10 Let the Legendre equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ have n^{th} degree polynomial solution $y_n(x)$ such that $y_n(1) = 3$. If $\int_{-1}^{1} \left(y_n^2(x) + y_{n-1}^2(x)\right) dx = \frac{144}{15}$. Find the value of n.