

Engineering Optics

Lecture 21

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by

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What is ‘Population’?

- ▶ Imagine a chamber filled with a gas in equilibrium at some temperature T
- ▶ If T is relatively low, most of the atoms will be in their ground states, but a few will “rise” into an excited state
- ▶ Maxwell-Boltzmann distribution (N : number of atoms/volume)

$$N_i = N_0 \exp^{-E_i/k_B T}$$

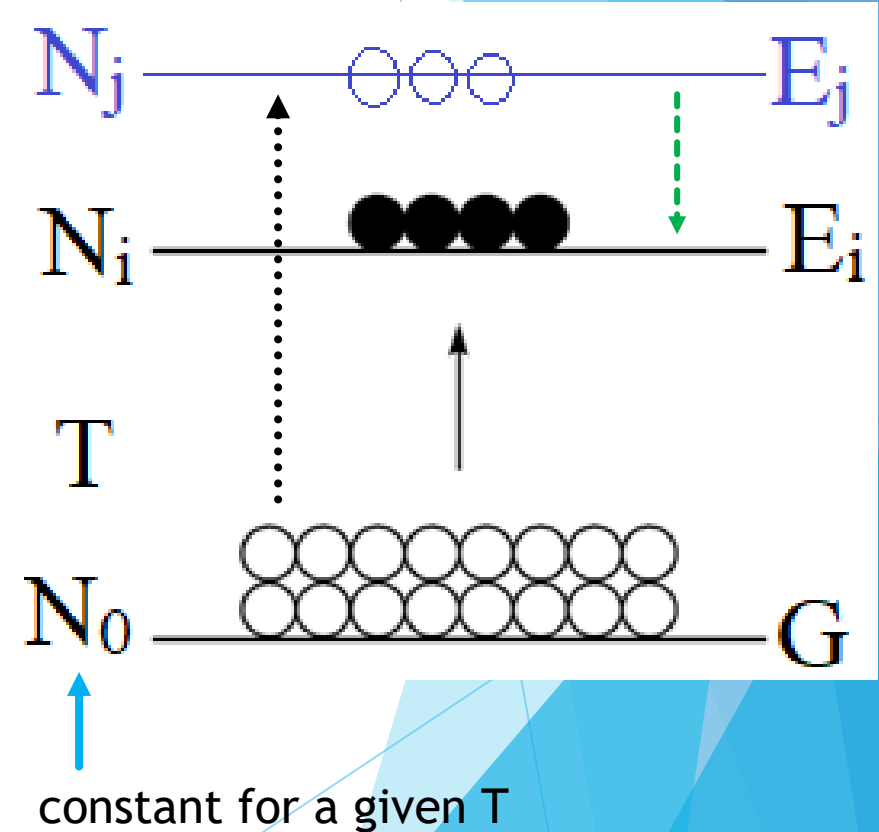
- ▶ higher $E \rightarrow$ fewer atoms there will be in that state

$$N_j = N_0 \exp^{-E_j/k_B T}$$

- ▶ Where $E_j > E_i$
- ▶ ratio of the populations occupying these two states

$$\frac{N_j}{N_i} = \frac{\exp^{-E_j/k_B T}}{\exp^{-E_i/k_B T}}$$

relative population, $N_j = N_i \exp^{-(E_j - E_i)/k_B T}$



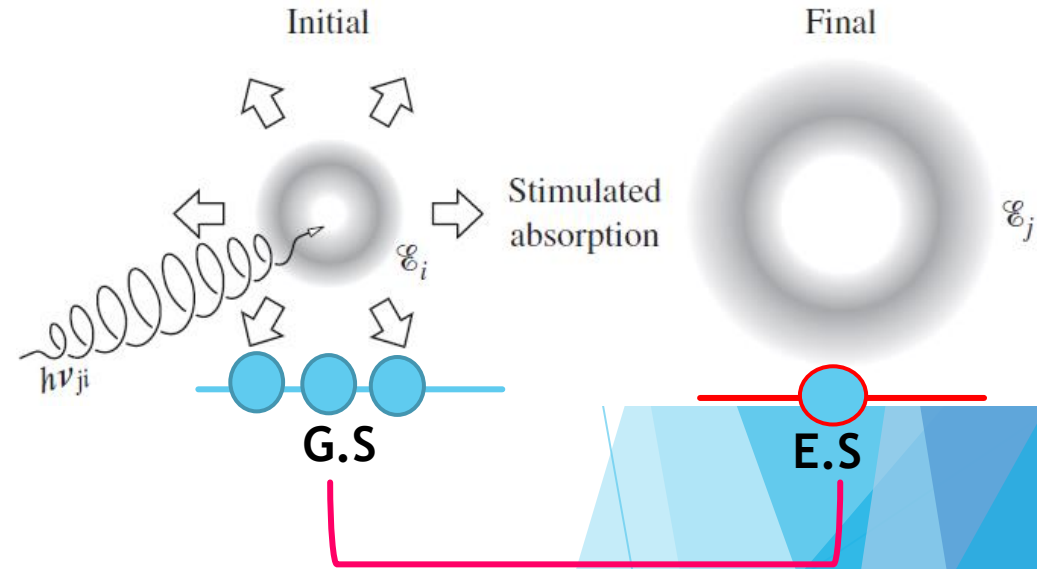
A transition from j^{th} to i^{th} state is also possible!

The Einstein A and B Coefficients: Stimulated absorption

stimulated absorption, whereupon the transition rate is

$$\left(\frac{dN_i}{dt}\right)_{\text{ab}} = -B_{ij}N_i u_\nu$$

Here B_{ij} is a constant of proportionality, the *Einstein absorption coefficient*, and the minus arises because N_i is decreasing.



u_ν : spectral energy density =
energy per unit volume per
unit frequency interval
(J.s/m³)

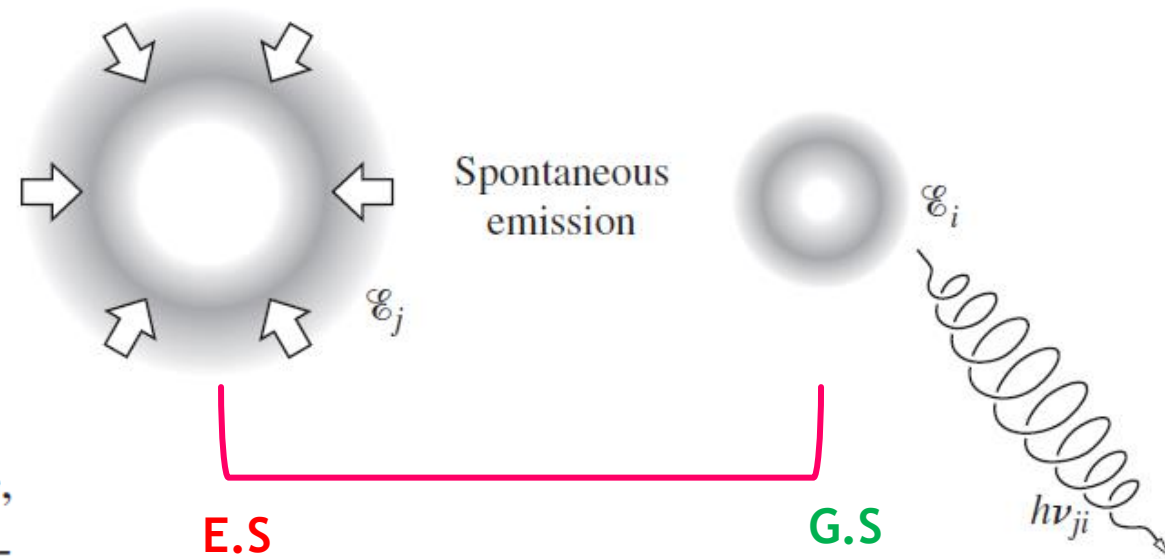
The Einstein A and B Coefficients: Spontaneous emission

Such an excess-energy configuration is usually (though not always) exceedingly short-lived, and in 10 ns or so, without the intercession of any external influence, the atom will emit its overload of energy as a photon. As it does, it reverts to a stable state in a process called *spontaneous emission*

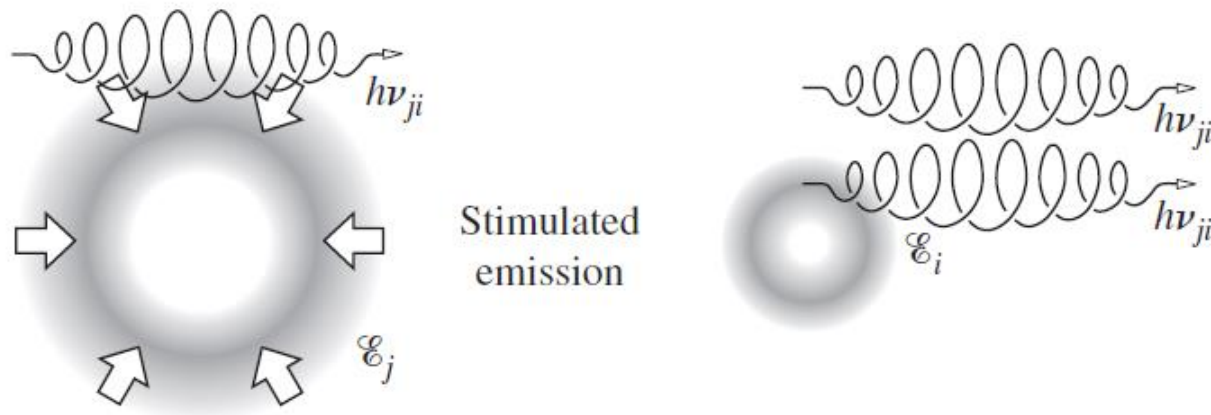
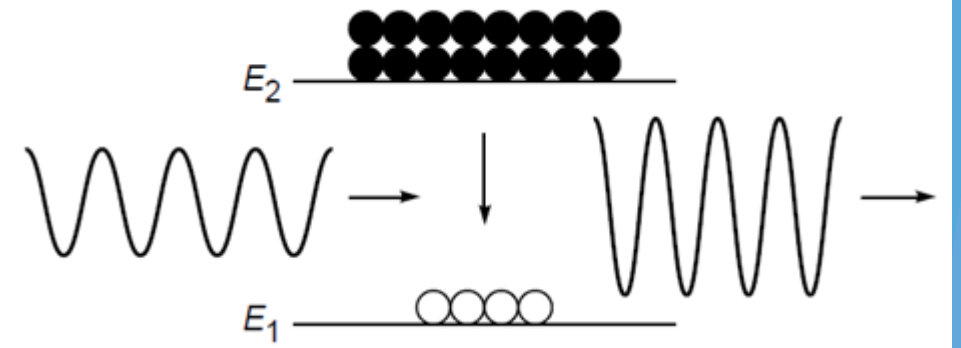
[spontaneous emission]

$$\left(\frac{dN_j}{dt}\right)_{\text{sp}} = -A_{ji}N_j$$

This is the rate of decrease of the higher-energy population, N_j , due to spontaneous emission. And A_{ji} is the Einstein spontaneous emission coefficient associated with a drop from energy level- j to level- i .



The Einstein A and B Coefficients: Stimulated emission

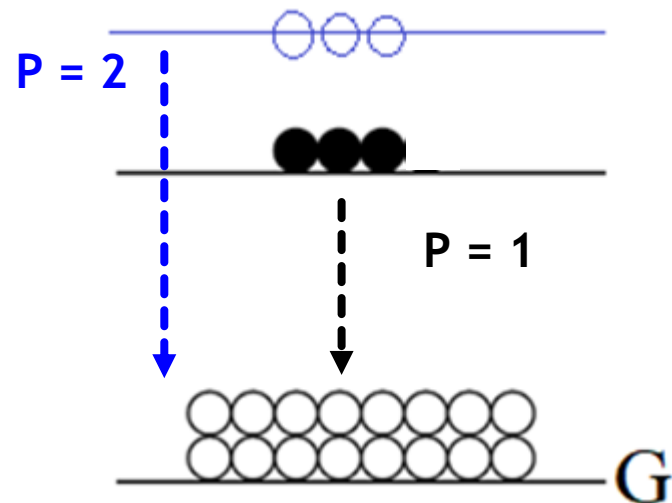
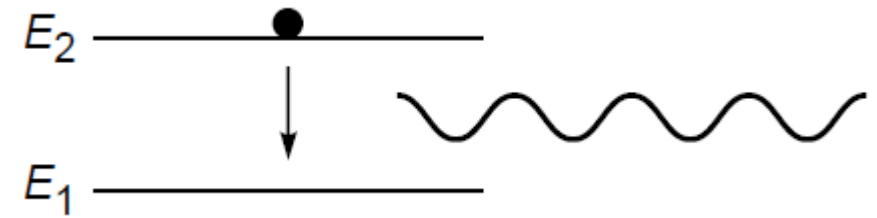


[stimulated emission]
$$\left(\frac{dN_j}{dt} \right)_{\text{st}} = -B_{ji} N_j u_\nu$$

The constant B_{ji} is the *Einstein stimulated emission coefficient*.

Probability and life time of a state

Keep in mind that the transition rate, the number of atoms making transitions per second, divided by the number of atoms, is the probability of a transition occurring per second, \mathcal{P} . Consequently, the probability per second of spontaneous emission is $\mathcal{P}_{\text{sp}} = A_{ji}$.

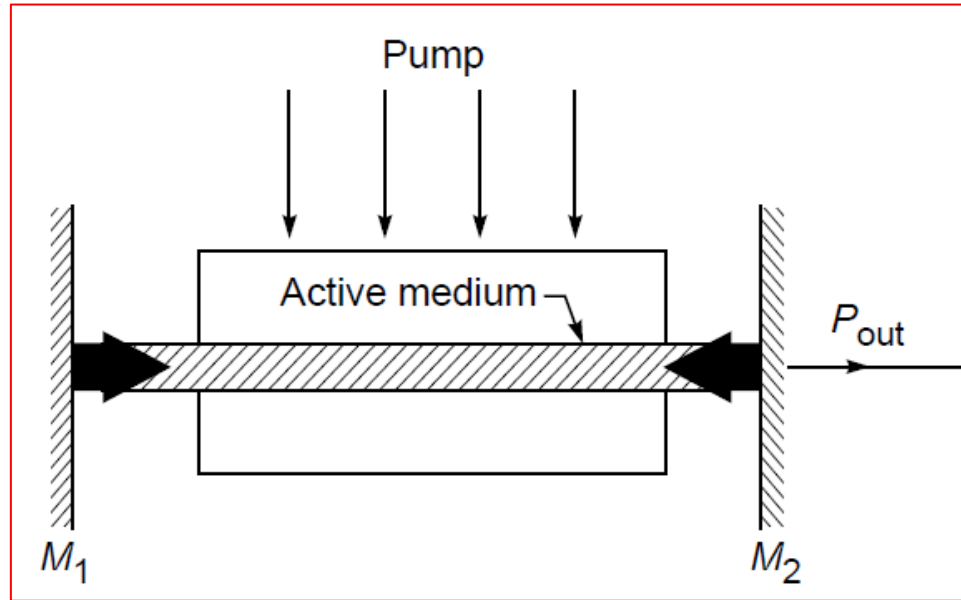


Q: most preferred state?

For a single excited atom making a spontaneous transition to a lower state, the inverse of the transition probability per second is the **mean life or lifetime** of the excited state τ . Thus (operating under conditions that exclude any other mechanism but spontaneous emission), if N atoms are in that excited state, the total rate of transitions, that is, the number of emitted photons per second, is $N\mathcal{P}_{\text{sp}} = NA_{ji} = N/\tau$. A low-transition probability means a long lifetime.

Main Components of the Laser

- ▶ 1. Active medium
- ▶ 2. Pumping source
- ▶ 3. Optical resonator



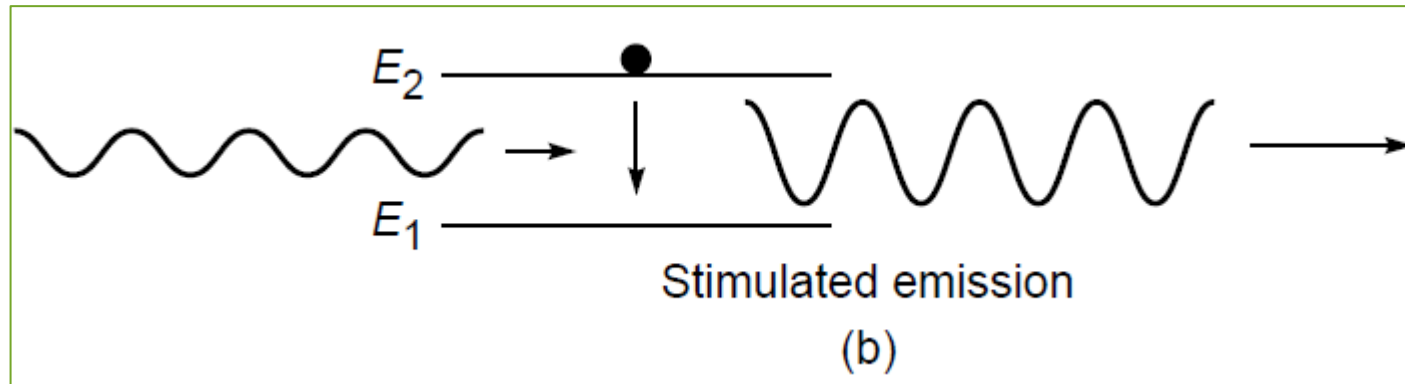
Points to note: Some of the energy is coupled back to the system \rightarrow oscillator.

LOSER \rightarrow light oscillation by stimulated emission of radiation.

Since it would have been difficult to obtain a research grant for LOSERs, it was decided to retain the name LASER.

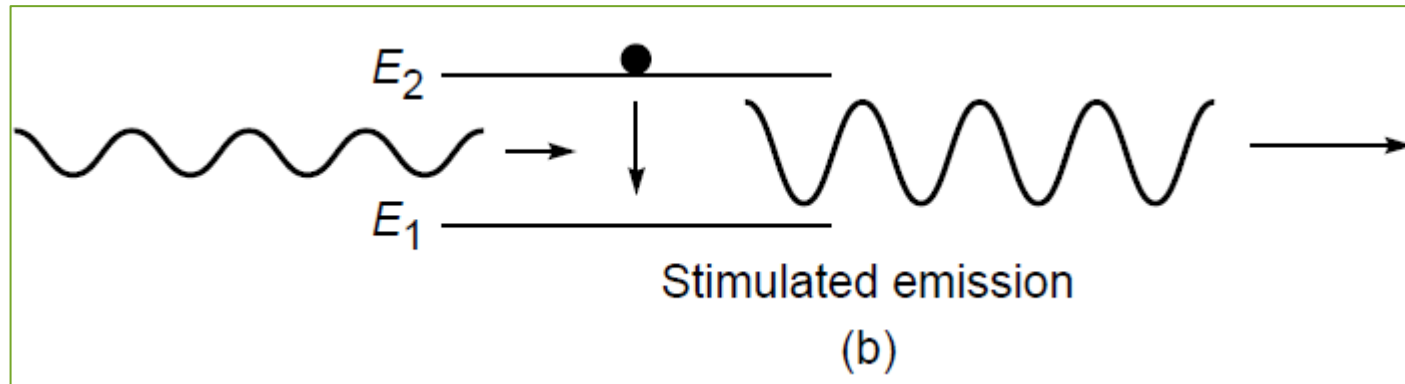
Problem-1

- ▶ A 10-mW laser is emitting at a mean wavelength of 500 nm. Determine the rate of occurrence of stimulated emission.



Problem-1

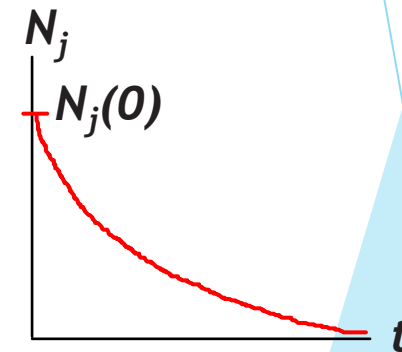
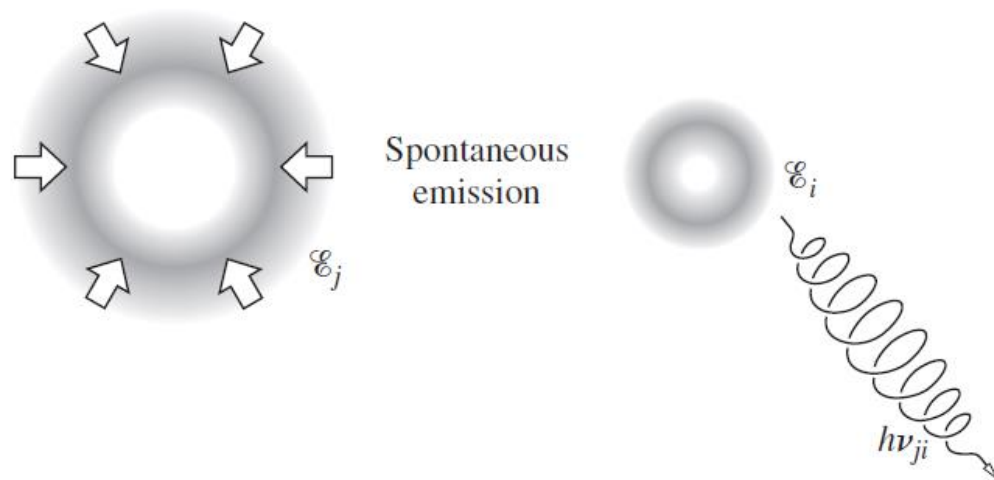
- ▶ A 10-mW laser is emitting at a mean wavelength of 500 nm. Determine the rate of occurrence of stimulated emission.



- ▶ Rate of occurrence of stimulated emission = how many photons are emitted/sec ?
- ▶ Power P = Total energy/time = 10 mW = 0.01 J/Sec
- ▶ Energy of each photon $E = h\nu = hc/\lambda = 3.973 \times 10^{-19} \text{ J}$
- ▶ No. of photons = $P/E = 2.52 \times 10^{16}$

Problem-2

Q: Suppose a sample exists where there are N_j excited electrons per unit volume in energy level-j just above the ground state level-i. Show that the population of energy level-j falls exponentially as electrons leave via spontaneous emission. What can be said about the lifetime of level-j?



Problem-2

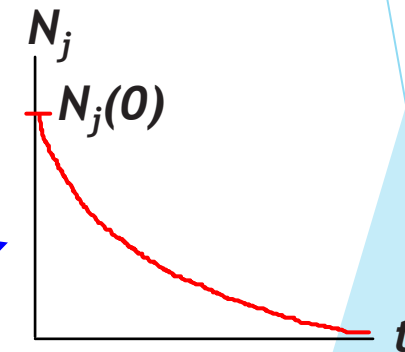
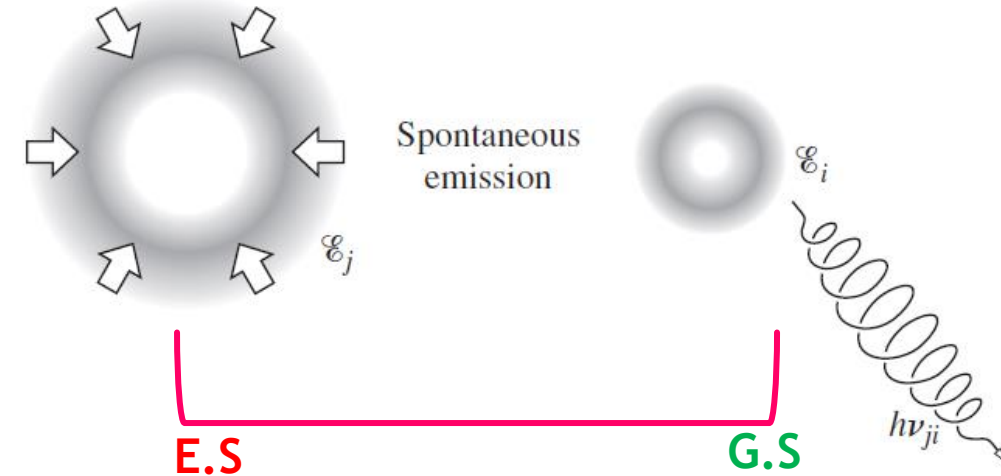
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[spontaneous emission]
$$\left(\frac{dN_j}{dt}\right)_{\text{sp}} = -A_{ji}N_j$$

This is the rate of decrease of the higher-energy population, N_j , due to spontaneous emission. And A_{ji} is the Einstein spontaneous emission coefficient associated with a drop from energy level-j to level-i.

$$\frac{dN_j}{N_j} = -A_{ji} dt \Rightarrow \int \frac{dN_j}{N_j} = \int -A_{ji} dt + C$$

Say at $t = 0$, $N_j = N_j(0) \rightarrow N_j = N_j(0)e^{-A_{ji}t}$



Probability of transition occurrence/Sec = $P = A_{ji}$

\rightarrow Lifetime $\tau = 1/P = 1/A_{ji}$

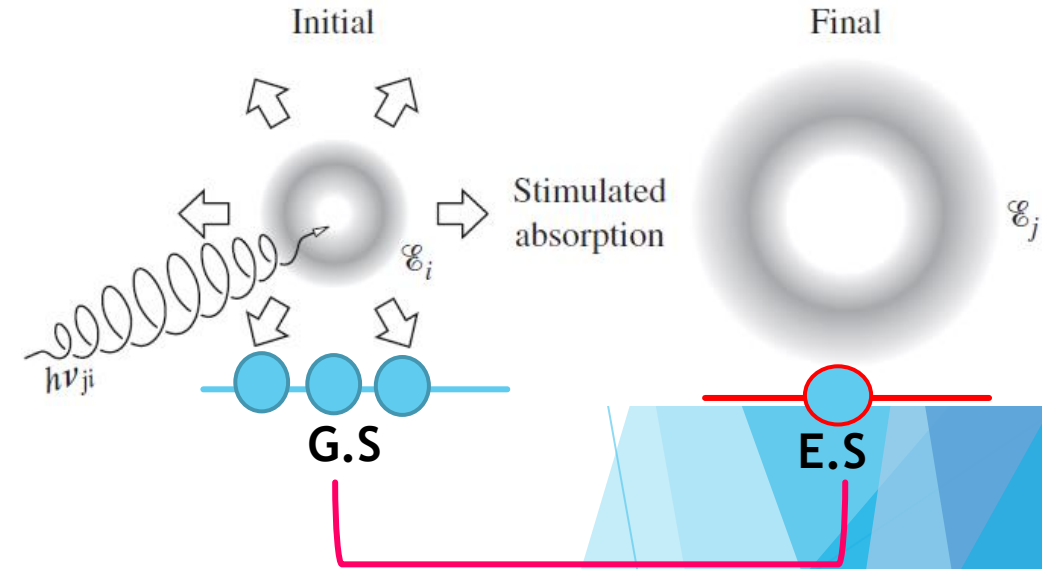
Problem-3

Q: Determine the mks units of $u(\nu)$, A , and B

[spontaneous emission] $\left(\frac{dN_j}{dt}\right)_{\text{sp}} = -A_{ji}N_j$

A_{ij} :

This is the rate of decrease of the higher-energy population, N_j , due to spontaneous emission. And A_{ji} is the *Einstein spontaneous emission coefficient* associated with a drop from energy level- j to level- i .



Problem-3

Q: Determine the mks units of $u(\nu)$, A , and B

Ans: u_ν : spectral energy density = energy per unit volume per unit frequency interval (**J.s/m³**)

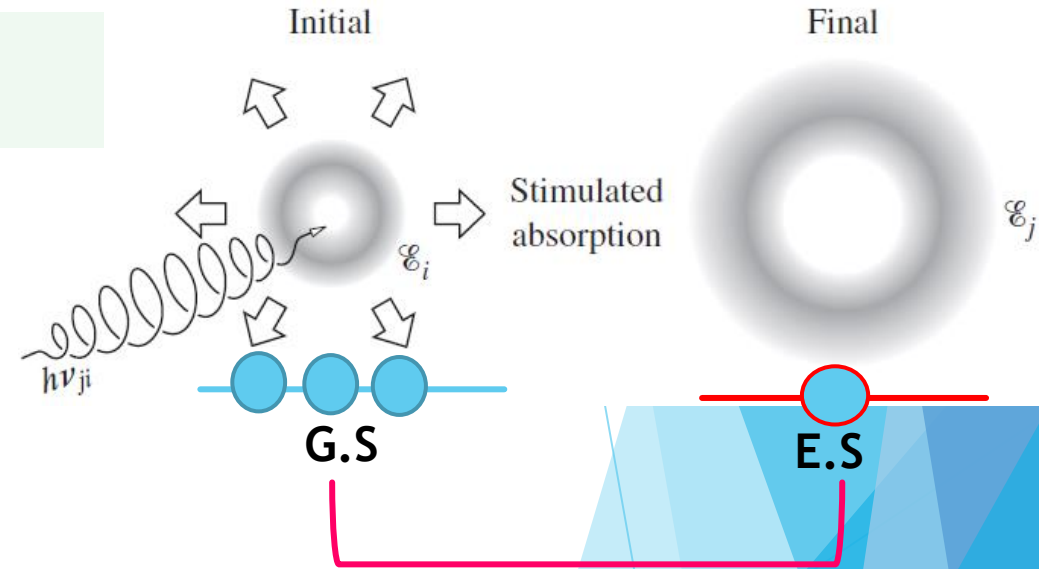
[spontaneous emission] $\left(\frac{dN_j}{dt}\right)_{sp} = -A_{ji}N_j$ **$A_{ij} : S^{-1}$**

This is the rate of decrease of the higher-energy population, N_j , due to spontaneous emission. And A_{ji} is the *Einstein spontaneous emission coefficient* associated with a drop from energy level- j to level- i .

stimulated absorption, whereupon the transition rate is

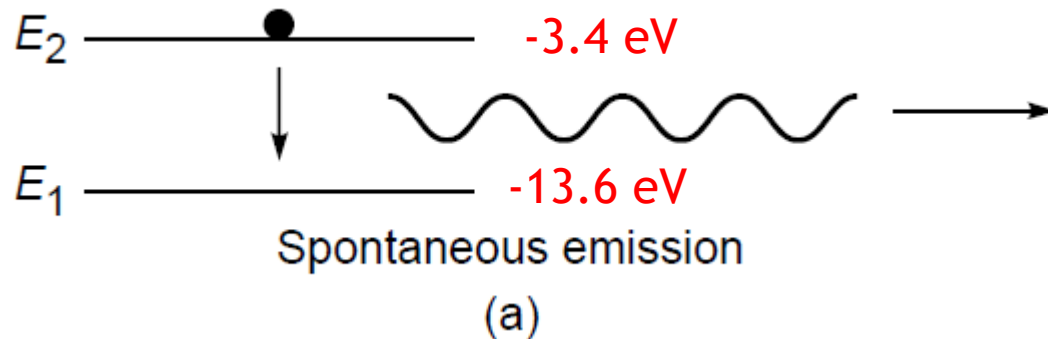
$$\left(\frac{dN_i}{dt}\right)_{ab} = -B_{ij}N_iu_\nu \longrightarrow B_{ij} = \left(\frac{dN_i}{dt}\right)/(N_iu_\nu)$$
 $B_{ij} : m^3 \cdot J^{-1} \cdot s^{-2}$

Here B_{ij} is a constant of proportionality, the *Einstein absorption coefficient*, and the minus arises because N_i is decreasing.



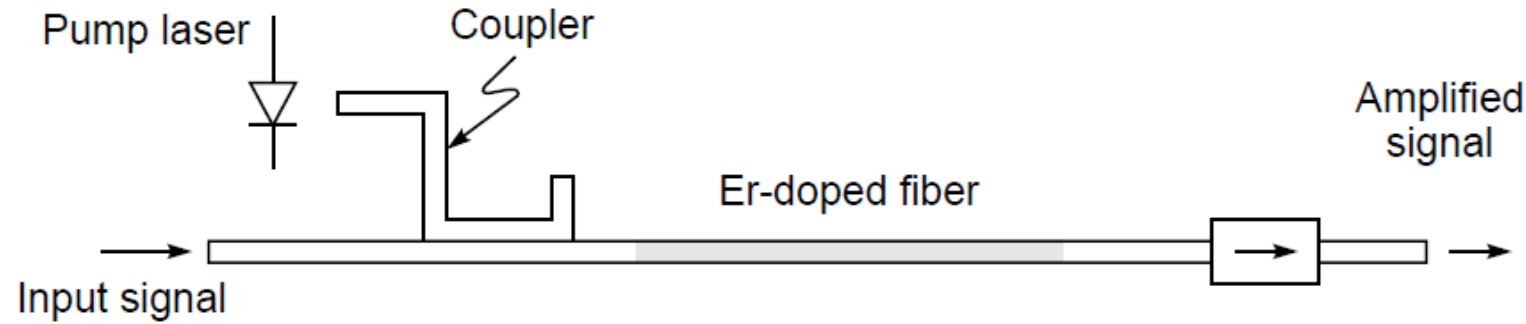
Problem 4

- ▶ Q: For the $2P \rightarrow 1S$ transition in the hydrogen atom, calculate frequency of emitted wave.



- ▶ Ans: $E_2 - E_1 = -3.4 + 13.6 = 10.2 \text{ eV} = h\nu$
- ▶ $h = \text{Planck's constant} \rightarrow \nu = ?$

Understanding Optical Amplification

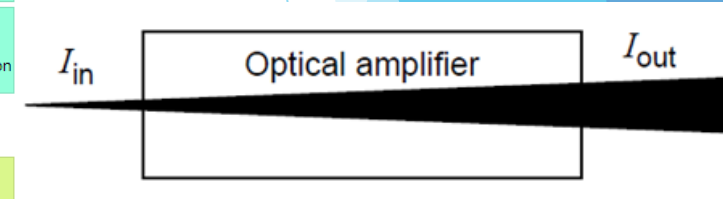


55 Cs Caesium 1	56 Ba Barium 2	57-71	72 Hf Hafnium 4	73 Ta Tantalum 5	74 W Tungsten 4 6	75 Re Rhenium 4	76 Os Osmium 4	77 Ir Iridium 3 4	78 Pt Platinum 2 4	79 Au Gold 3	80 Hg Mercury 1 2	81 Tl Thallium 1 3	82 Pb Lead 2 4	83 Bi Bismuth 3	84 Po Polonium -2 2 4	85 At Astatine -1 1	86 Rn Radon 2
87 Fr Francium 1	88 Ra Radium 2	89-103	104 Rf Rutherfordium 4	105 Db Dubnium 5	106 Sg Seaborgium 6	107 Bh Bohrium 7	108 Hs Hassium 8	109 Mt Meitnerium 3 4	110 Ds Darmstadtium 3 4	111 Rg Roentgenium 3	112 Cn Copernicium 3	113 Nh Nihonium 3	114 Fl Flerovium 3	115 Mc Moscovium 3	116 Lv Livermorium 3	117 Ts Tennessine 3	118 Og Oganesson 3
		6	57 La Lanthanum 3	58 Ce Cerium 3 4	59 Pr Praseodymium 3	60 Nd Neodymium 3	61 Pm Promethium 3	62 Sm Samarium 3	63 Eu Europium 2 3	64 Gd Gadolinium 3	65 Tb Terbium 3	66 Dy Dysprosium 3	67 Ho Holmium 3	68 Er Erbium 3	69 Tm Thulium 3	70 Yb Ytterbium 3	71 Lu Lutetium 3
		7	89 Ac Actinium 3	90 Th Thorium 4	91 Pa Protactinium 5	92 U Uranium 6	93 Np Neptunium 5	94 Pu Plutonium 4	95 Am Americium 3	96 Cm Curium 3	97 Bk Berkelium 3	98 Cf Californium 3	99 Es Einsteinium 3	100 Fm Fermium 3	101 Md Mendelevium 3	102 No Nobelium 2	103 Lr Lawrencium 3

Oxidation states

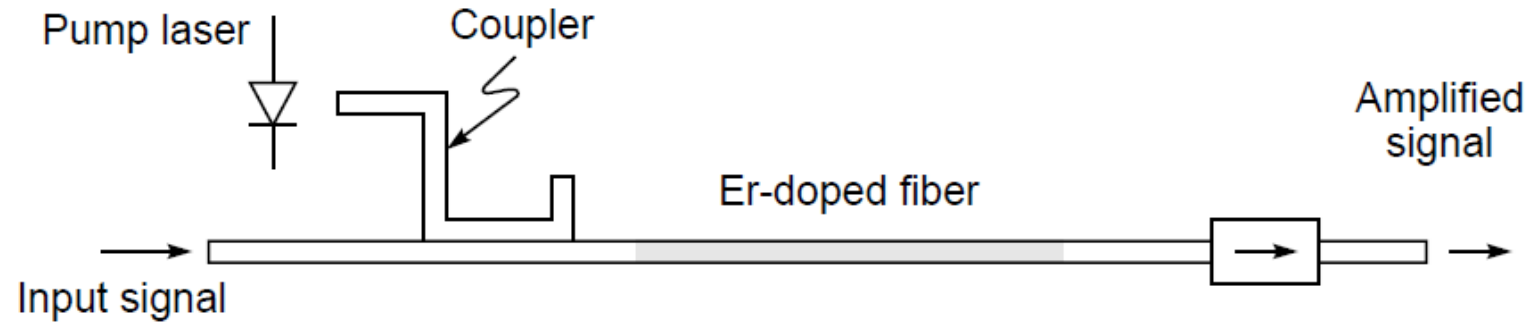
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Oxidation states are the number of electrons added to or removed from an element when it forms a chemical compound.

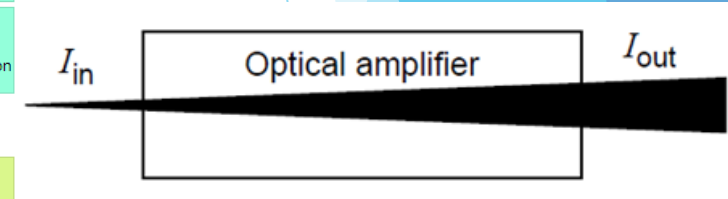


- ▶ understand optical amplification → EDFA
- ▶ **Erbium doped fiber amplifier**
- ▶ The EDFA: consists of a silica optical fiber the core doped with erbium oxide (Er_2O_3)
- ▶ light is guided through the optical fiber
- ▶ **Valence state of Er in silica?**

Understanding Optical Amplification

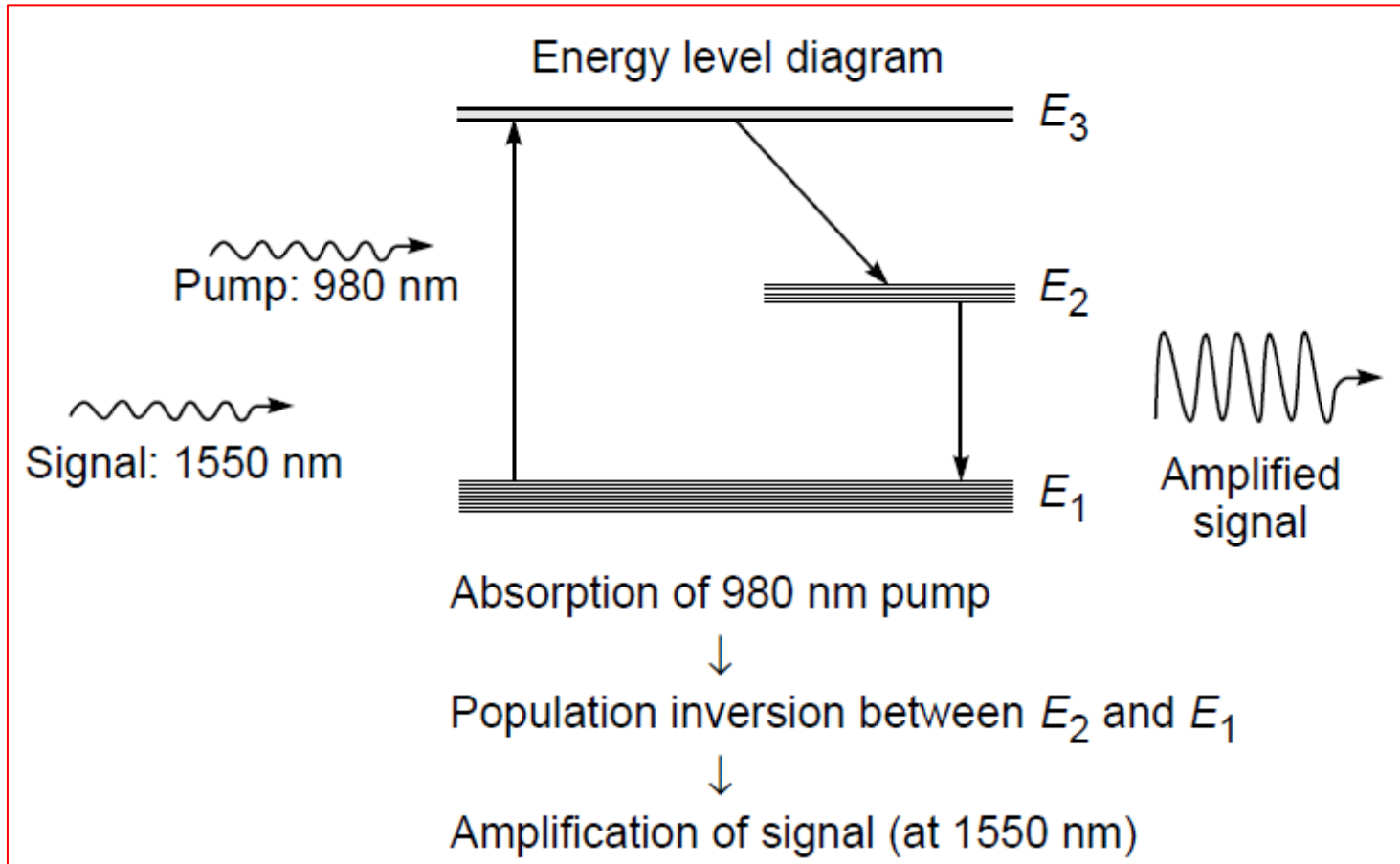


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- ▶ understand optical amplification → EDFA
- ▶ **Erbium doped fiber amplifier**
- ▶ The EDFA: consists of a silica optical fiber the core doped with erbium oxide (Er_2O_3)
- ▶ light is guided through the optical fiber
- ▶ three discrete energy levels of Er^{3+} ion in silica host glass.

EDFA



Underlying principle of optical amplification
OR
of *light amplification through stimulated emission of radiation*
→ LASER

Thank You