IIITDM KANCHEEPURAM

MA1001 Differential Equations

Problem Set 7

[1] Find the Laplace transform of the following functions.

$(a)\ f(x) = 4e^{-2x} + \sin(3x) - 4\cos(5x) + 12x^3 - 5$ $(b)\ f(x) = \sin(5x)\cos(3x)$

$(c)\ f(x) = \cosh^2(4x)$ $(d)\ f(x) = (x+2)^2 e^x$

$(e)\ f(x) = \cos(ax)\sinh(bx)$ $(f)\ f(x) = e^{-2x}\cos^2(x)$

$(g)\ f(x) = \begin{cases} \sin(x-\pi/3) & \text{if} \quad x > \pi/3 \\ 0 & \text{if} \quad x < \pi/3 \end{cases}$ $(h)\ f(x) = \sin(ax) - ax\cos(ax) + \dfrac{\sin(x)}{x}$

$(i)\ f(x) = xe^{ax}\sin(bx)$ $(j)\ f(x) = \displaystyle\int_0^x \frac{1-e^{-u}}{u}du$

$(k)\ f(x) = \displaystyle\int_0^x \frac{\sin t}{t}dt$

[2] Dose the Laplace transform of following function exist?

$(i)\ \dfrac{1}{x+2}$ $(ii)\ e^{x^2-x}$ $(iii)\ \sin(x^2)$

[3] Find $L[\sin\sqrt{t}]$. Also obtain $L[\dfrac{\cos\sqrt{t}}{\sqrt{t}}]$.

[4] Prove that $L[\dfrac{\cos(at)-\cos(bt)}{t}] = \dfrac{1}{2}\log\left(\dfrac{p^2+b^2}{p^2+a^2}\right)$ and deduce that

$$\int_0^\infty \frac{\cos(6t)-\cos(4t)}{t}dt = \log(2/3).$$

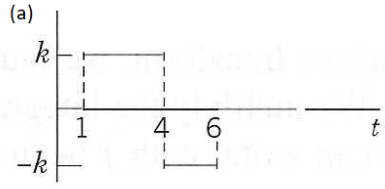
[5] Prove that $L[\dfrac{\sin^2(x)}{x}] = \dfrac{1}{4}\log\left(\dfrac{p^2+4}{p^2}\right)$ and deduce that

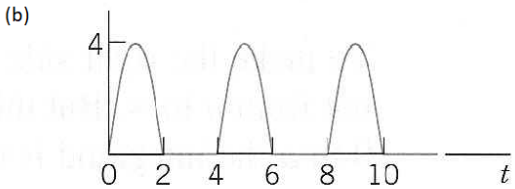
(i) $\displaystyle\int_0^\infty e^{-x}\frac{\sin^2(x)}{x}dx = 0.25\log(5)$, (ii) $\displaystyle\int_0^\infty \frac{\sin^2(x)}{x^2}dx = \pi/2$

[6] Evaluate the following integral with the help of Laplace Transform.

$$(i)\ \int_0^\infty x^3 e^{-x}\sin(x)dx, \quad (ii)\ \int_0^\infty \frac{e^{-x}\ \sin(x)}{x}dx, \quad (iii)\ \int_0^\infty xe^{-2x}\cos(x)dx.$$

[7] Find the Laplace Transform of following representation.





[8] Find inverse Laplace transform of the following functions.

(a) $F(p) = \dfrac{1}{p^4} + \dfrac{3p}{p^2+16} + \dfrac{5}{p^2+4}$ (b) $F(p) = \dfrac{6}{2p-3} - \dfrac{3+4p}{9p^2-16} + \dfrac{8-6p}{16p^2+9}$

(c) $F(p) = \dfrac{p^2-1}{(p^2+1)^2}$ (d) $F(p) = \dfrac{p}{(p+3)^{7/2}}$

(e) $F(p) = \dfrac{p}{(p+1)^5}$ (f) $F(p) = \dfrac{1}{\sqrt{(2p+3)}}$

(g) $F(p) = \dfrac{1}{\sqrt{(p^2-4p+20)}}$ (h) $F(p) = \log\left(\dfrac{p^2+a^2}{p^2+b^2}\right)$

(i) $F(p) = \dfrac{1}{p}\log\left(\dfrac{p+2}{p+1}\right)$ (j) $F(p) = \dfrac{1}{(p+2)(p^2+4)}$

(k) $F(p) = \dfrac{p}{(p^2+a^2)^3}$ (l) $F(p) = \cot^{-1}(p+1)$

(m) $F(p) = \dfrac{1}{p^3(p^2+1)}$ (l) $F(p) = \dfrac{1}{(p+1)(p^2+1)}$

9 Solve the following differential equation with the help of Laplace transform.

(a) $y'' + y = x\cos(2x), \quad y(0) = y'(0) = 0$

(b) $y'' + 2y' + y = x, \quad y(0) = -3, \quad y(1) = -1$

(c) $xy'' + 2y' + xy = 0, \quad y(0) = 1, \quad y(\pi) = 0$

(d) $xy'' + (x-1)y' - y = 0, \quad y(0) = 5, \quad y(\infty) = 0$

10 Solve the following equations using Laplace transforms.

(a) $y' + 4y + 5\int_0^x y dx = e^{-x}, \quad y(0) = 0.$

(b) $y(x) = x^3 + \displaystyle\int_0^x \sin(x-t)y(t)dt.$