MECHANICAL PROPERTIES

- **Stress and strain: What are they and why are they used instead of <u>force and deformation</u>?**

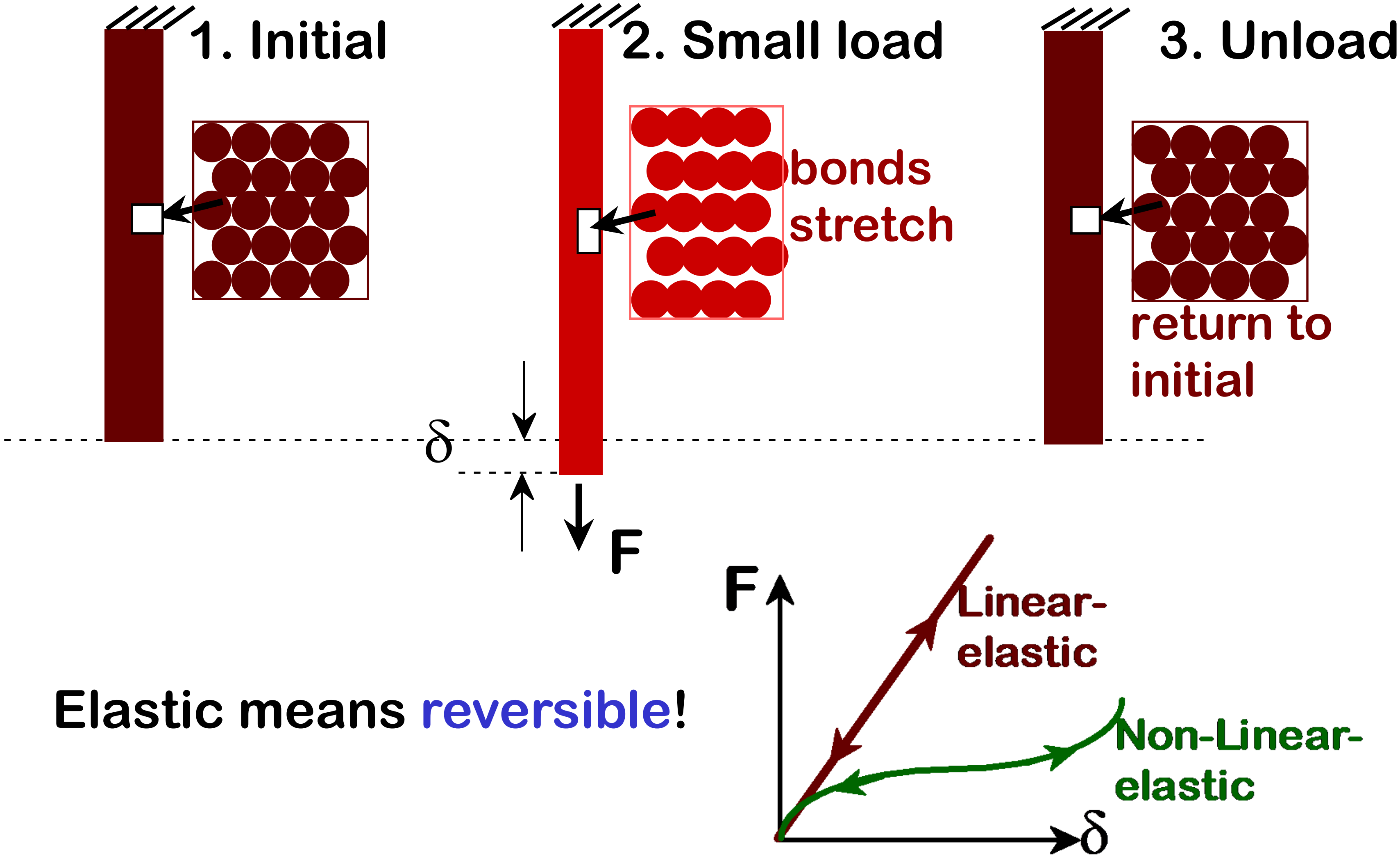
- **Elastic behavior:**

- **Plastic behavior:**

- **Youngs Modulus, Yield Strength, and Tensile Strength**

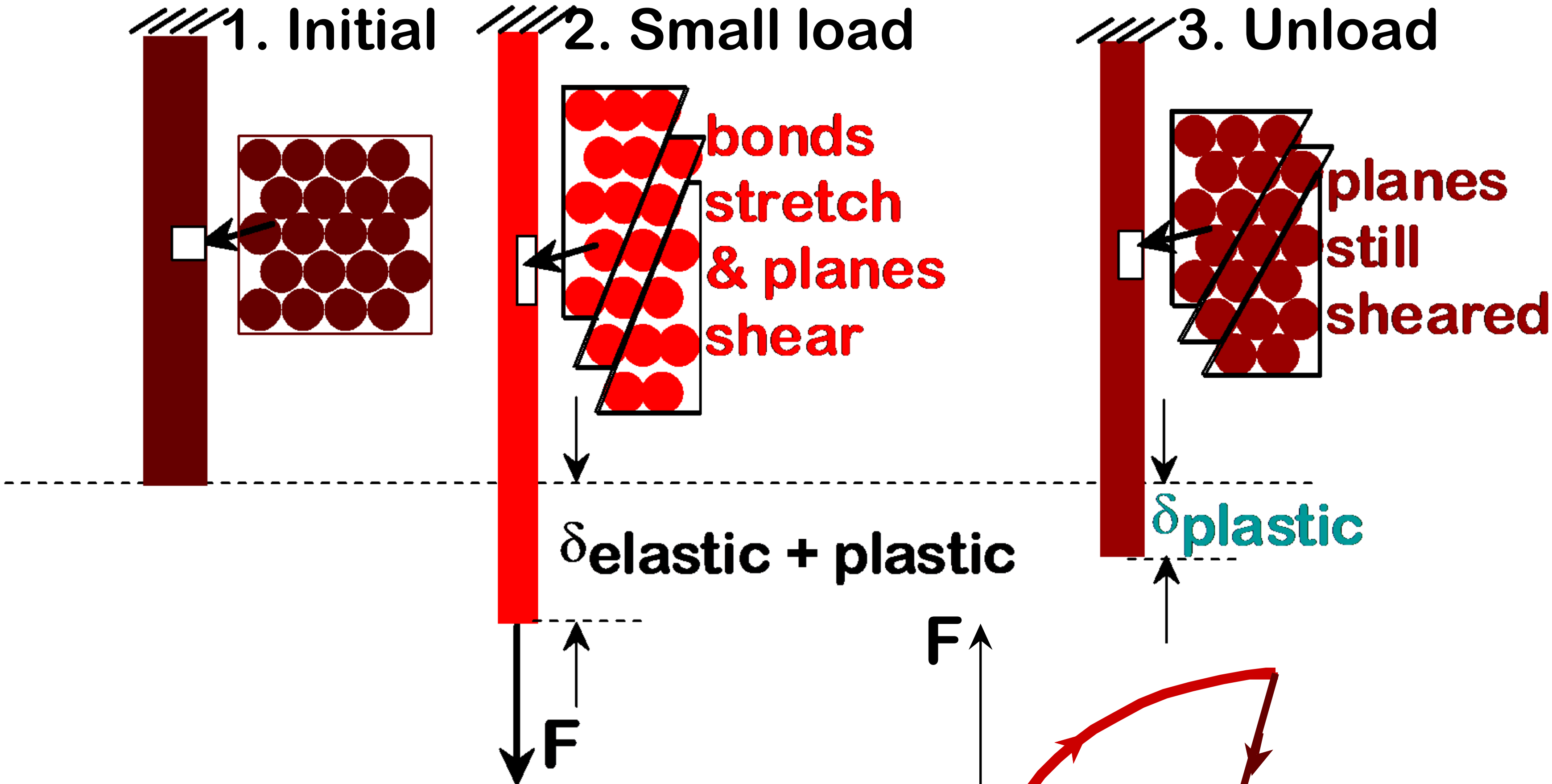
- **Toughness and ductility:**

ELASTIC DEFORMATION

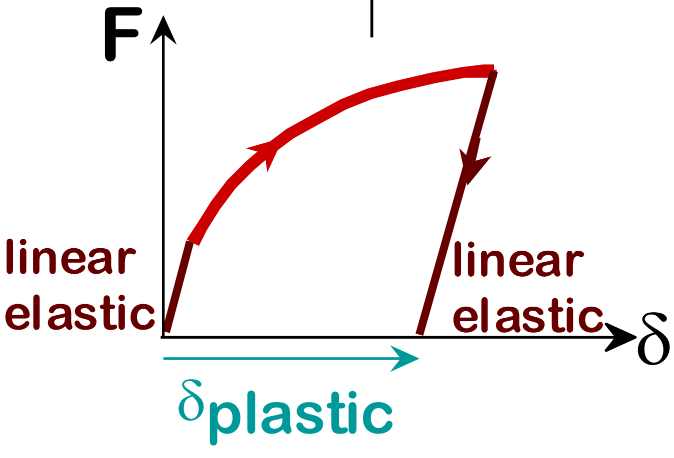


Elastic means reversible!

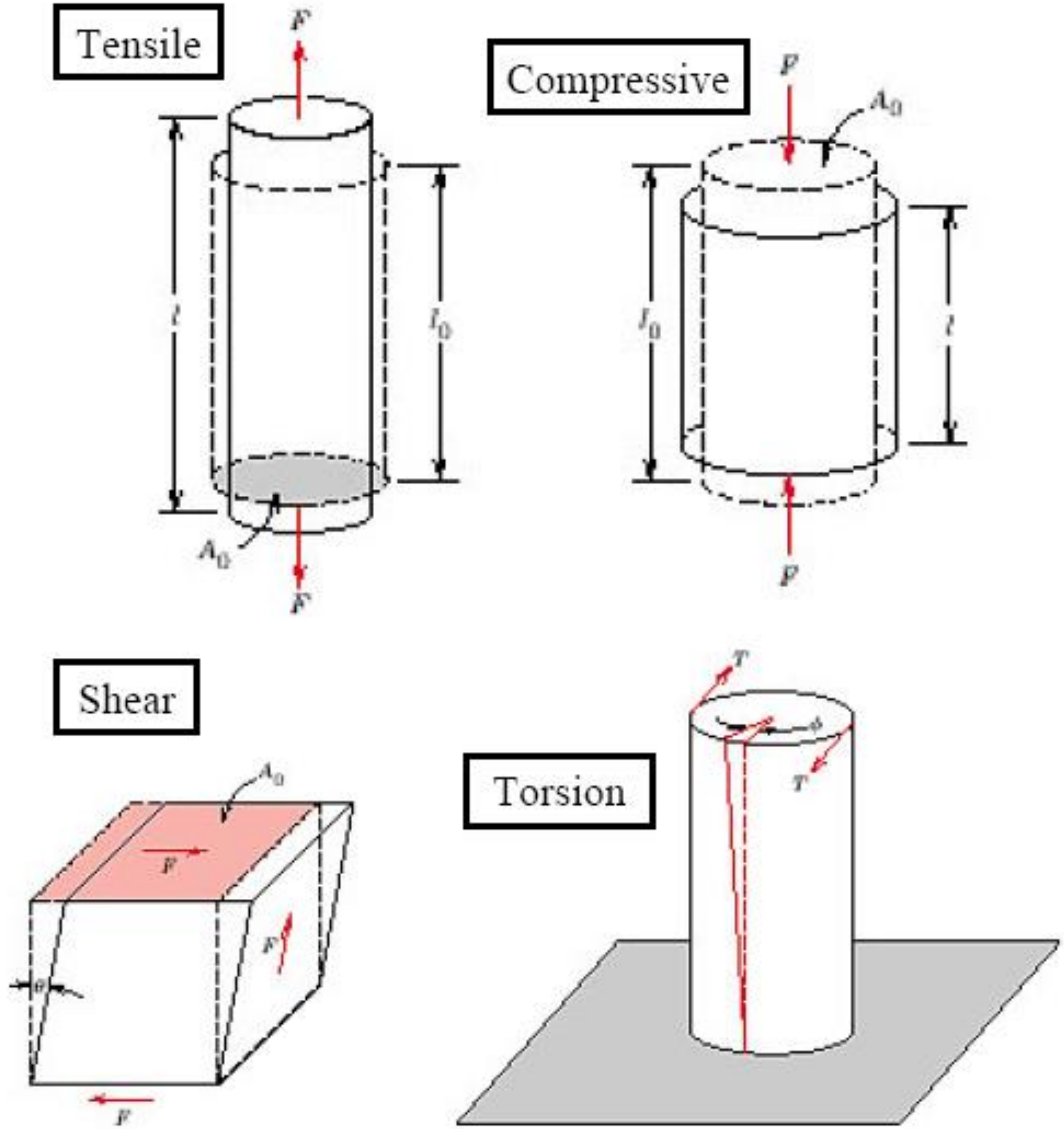
PLASTIC DEFORMATION (METALS)



Plastic means permanent!

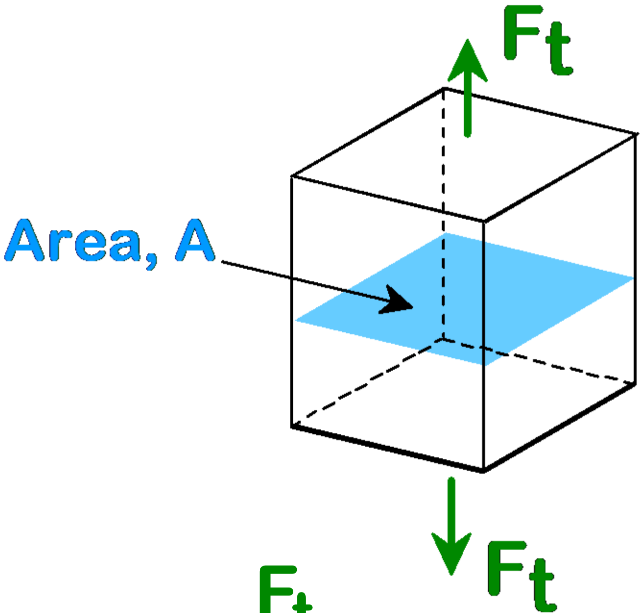


Types of Loading



ENGINEERING STRESS

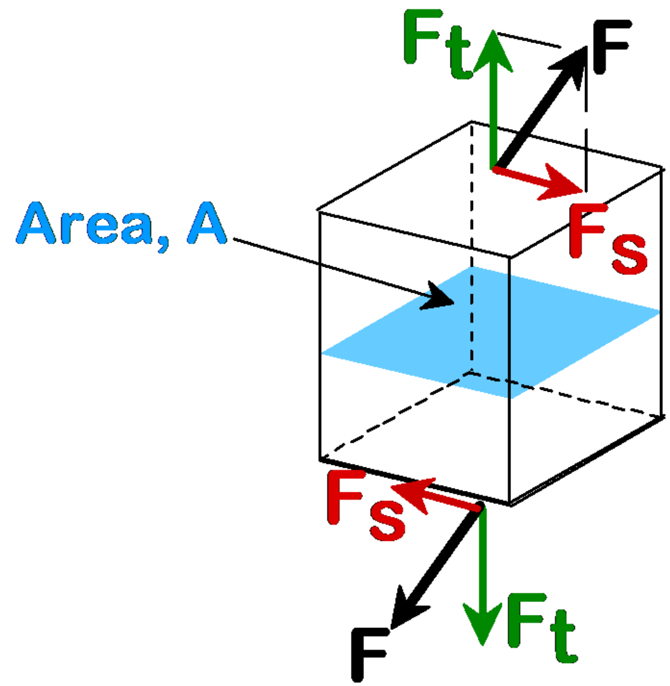
- **Tensile stress, σ:**



$$\sigma = \frac{F_t}{A_o}$$

original area
before loading

- **Shear stress, τ:**



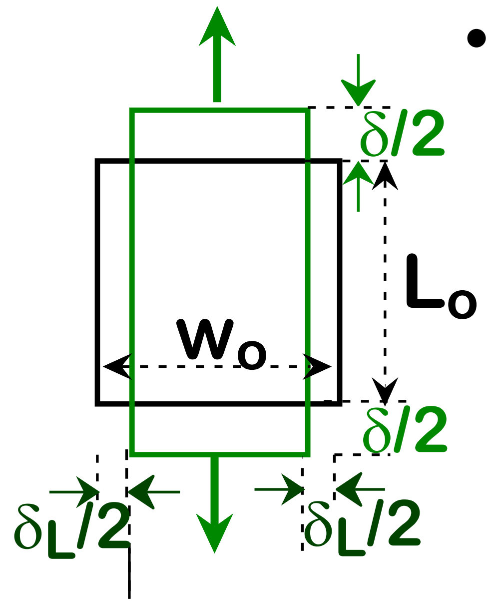
$$\tau = \frac{F_s}{A_o}$$

Stress unit?

ENGINEERING STRAIN

- **Tensile strain:**

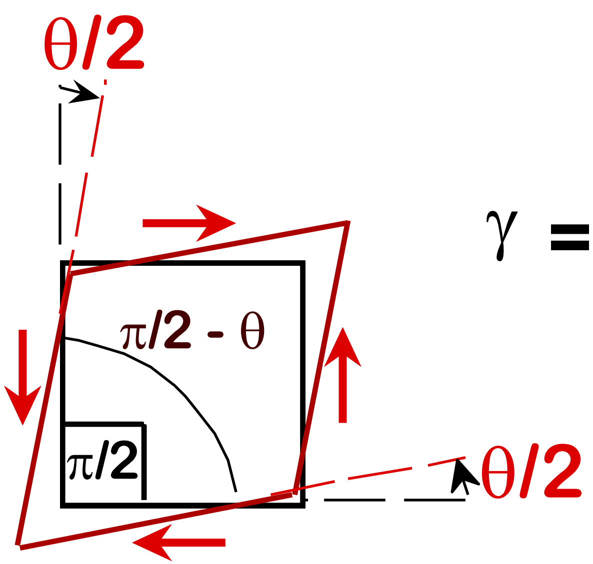
$$\varepsilon = \frac{\delta}{L_o}$$



- **Lateral strain:**

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

- **Shear strain:**

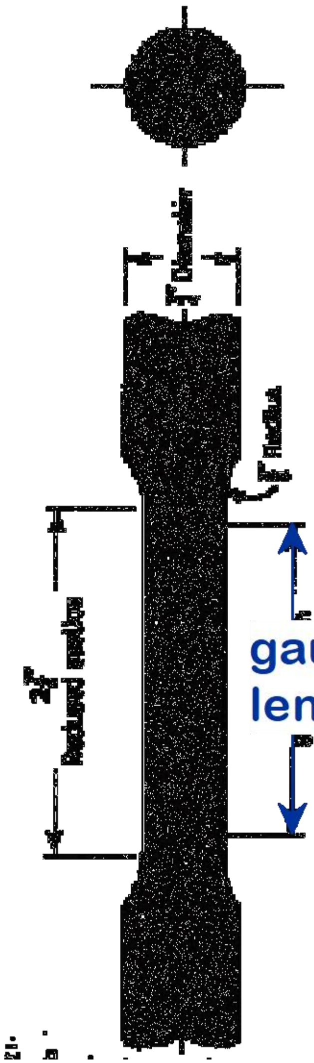


$$\gamma = \tan\theta$$

Strain unit?

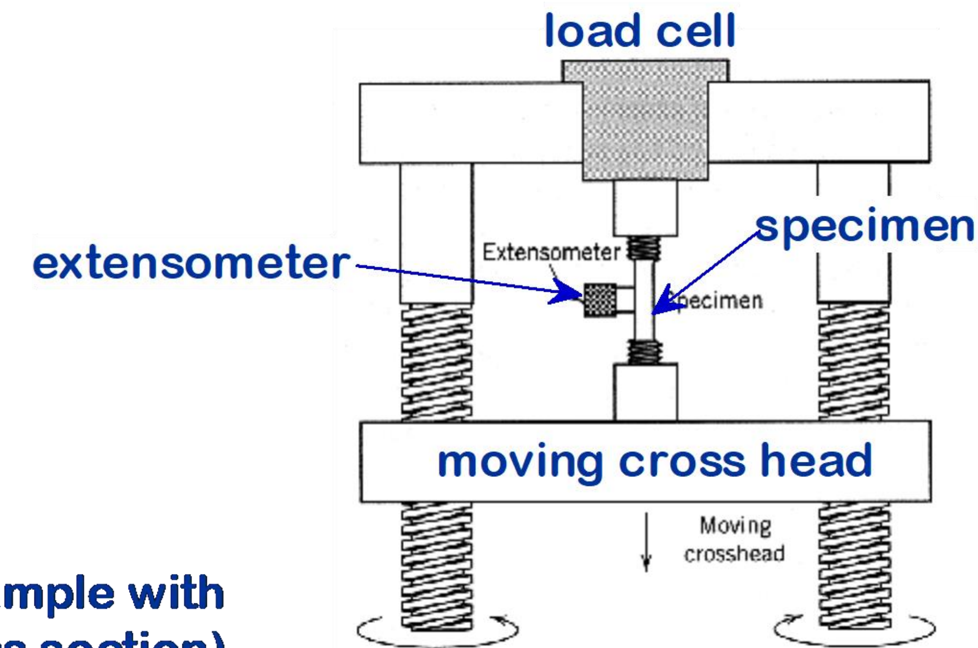
STRESS-STRAIN TESTING

- Typical tensile specimen



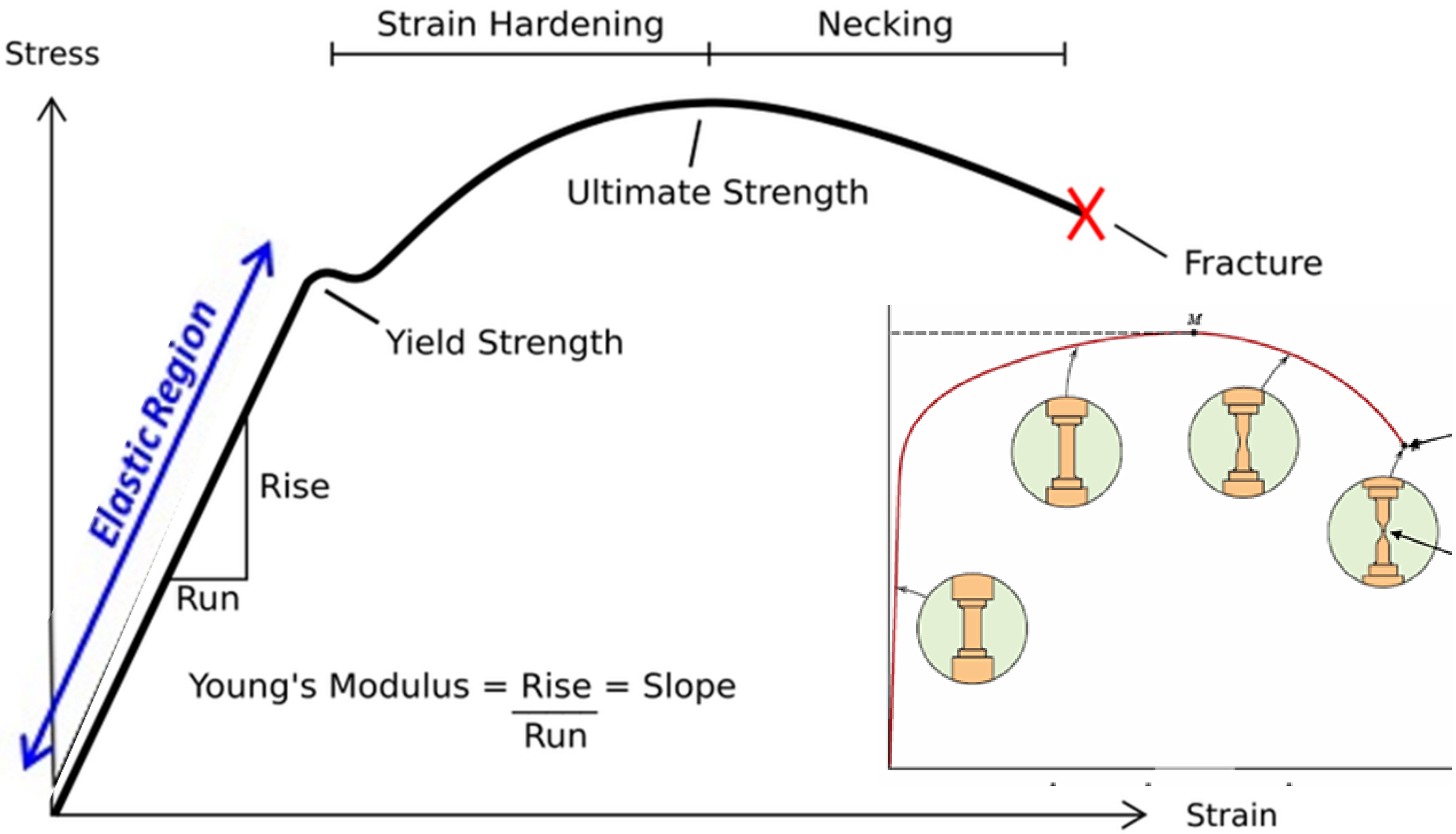
gauge length = (portion of sample with reduced cross section)

- Typical tensile test machine

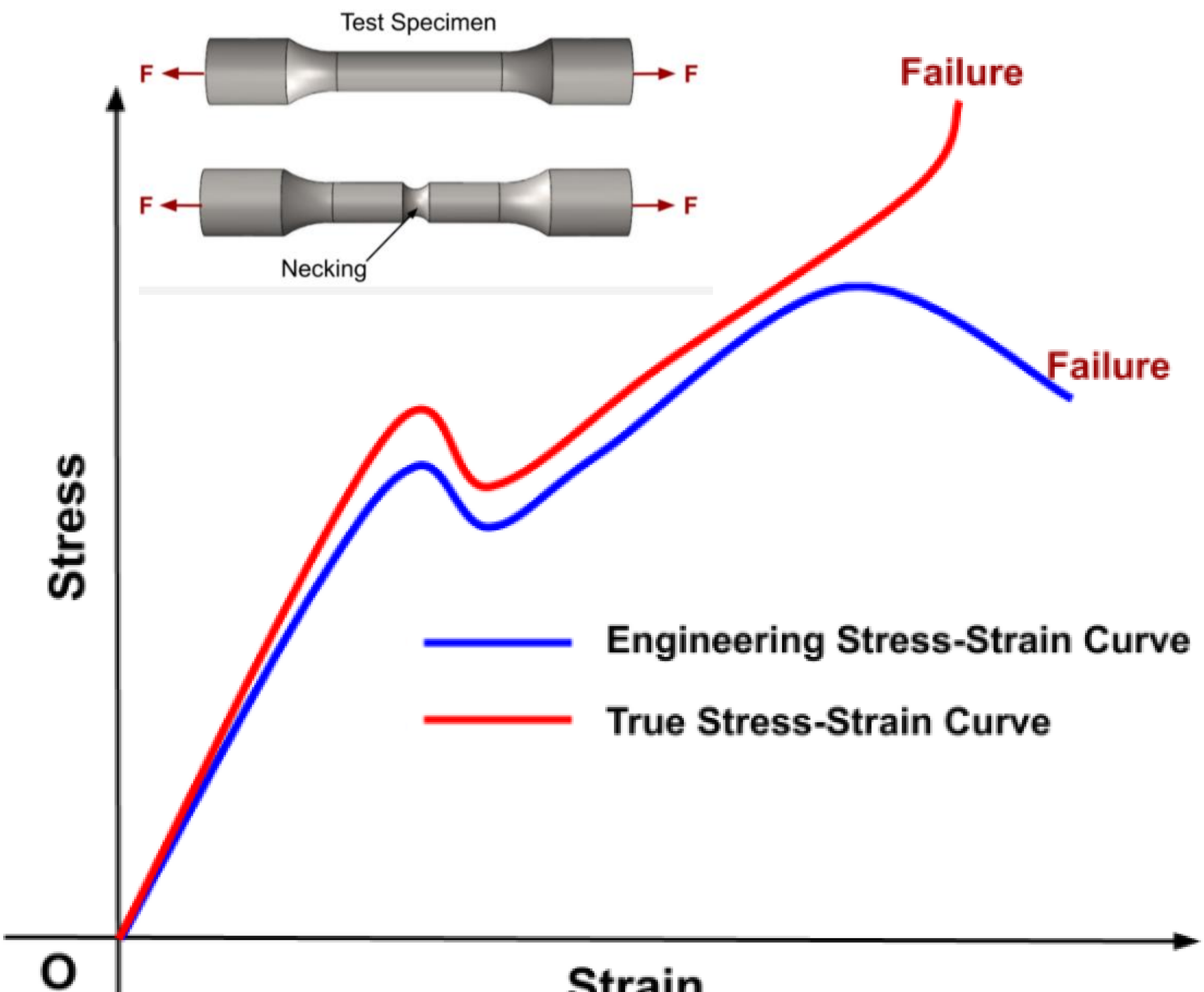


Adapted from Fig. 6.3, *Callister 6e.* (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

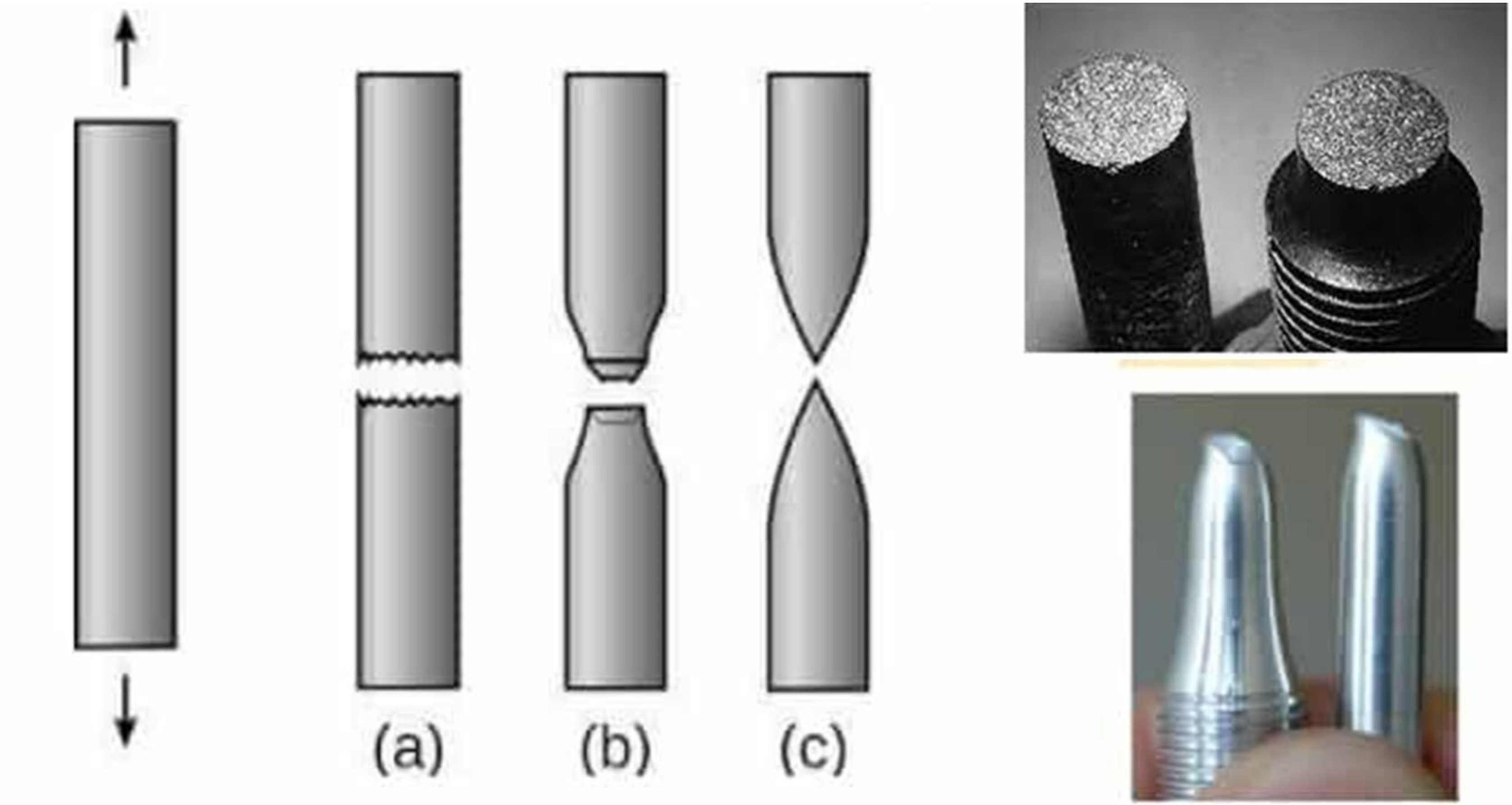
Typical Stress-Strain Curve



Engineering Strain vs True Strain

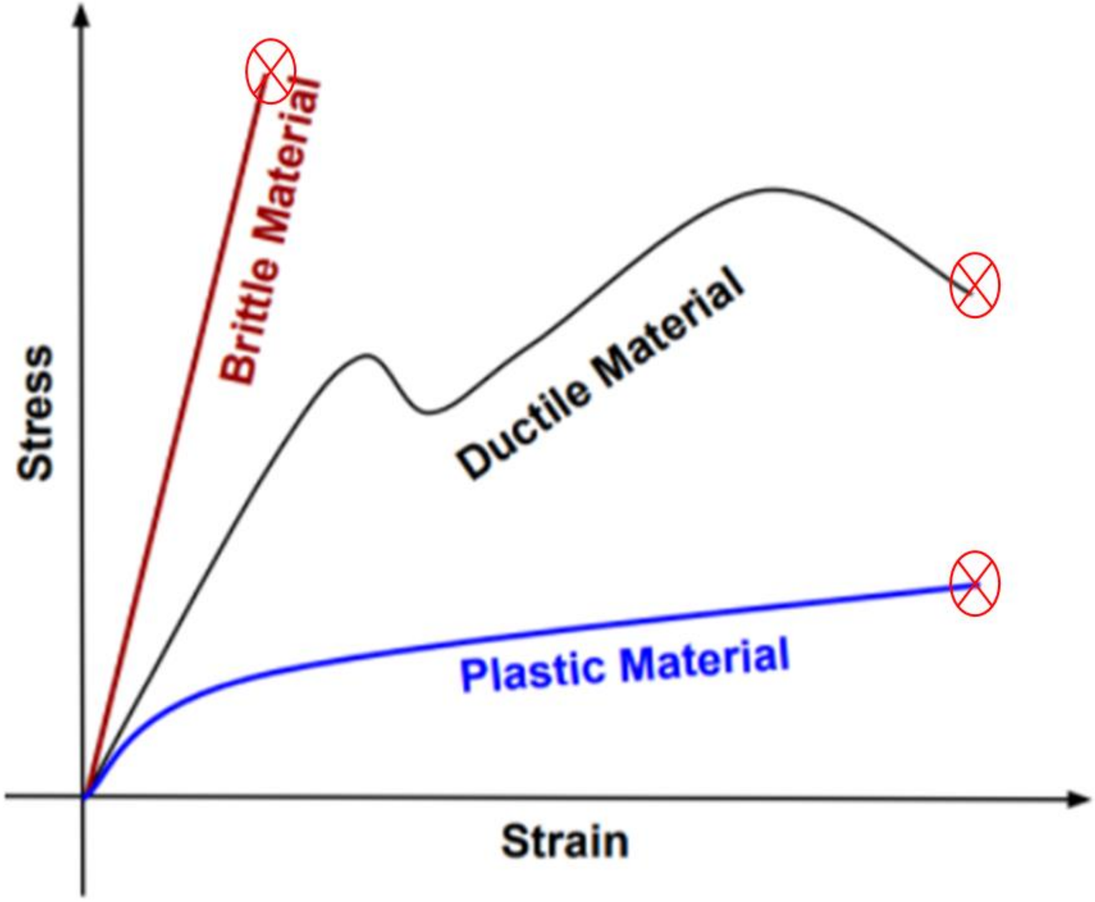


Ductile Vs Brittle Failure



(a) Brittle fracture (b) Ductile fracture (c) Completely ductile fracture

Stress Strain Curve of Brittle Ductile Plastic materials



Linear Elastic Properties

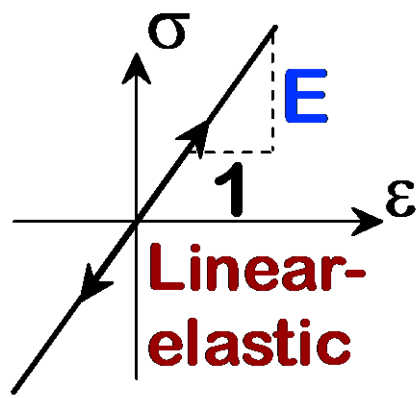
- **Modulus of Elasticity, E**:
  (also known as Young's modulus)
- **Hooke's Law**:

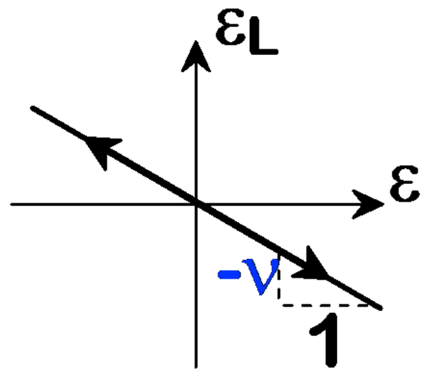
$$\sigma = \mathbf{E}\,\varepsilon$$

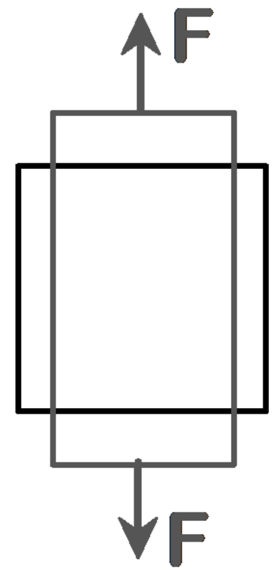
- **Poisson's ratio, ν**:

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$

metals: $\nu \sim 0.33$
ceramics: ~0.25
polymers: ~0.40







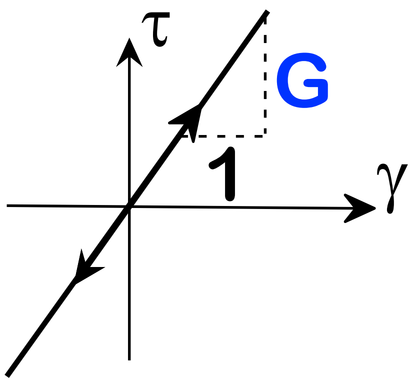
simple
tension
test

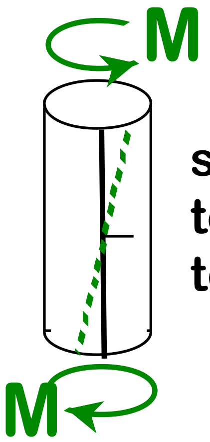
Unit of E?

Other elastic properties

- Elastic **Shear modulus, G:**

$$\tau = G\,\gamma$$

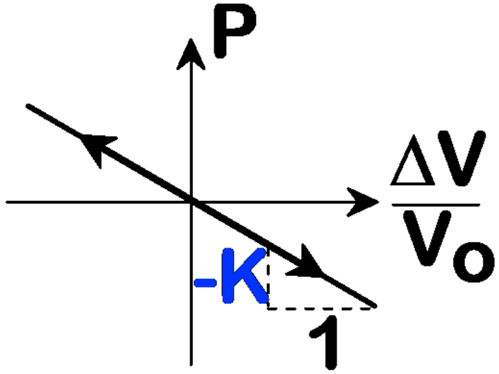


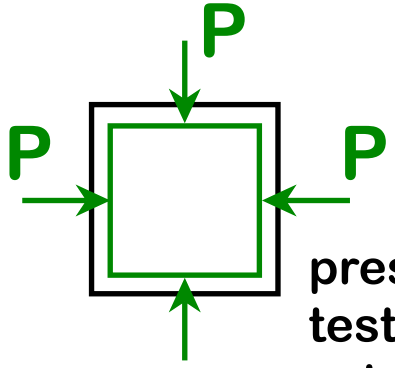


simple torsion test

- Elastic **Bulk modulus, K:**

$$P = -K\frac{\Delta V}{V_o}$$



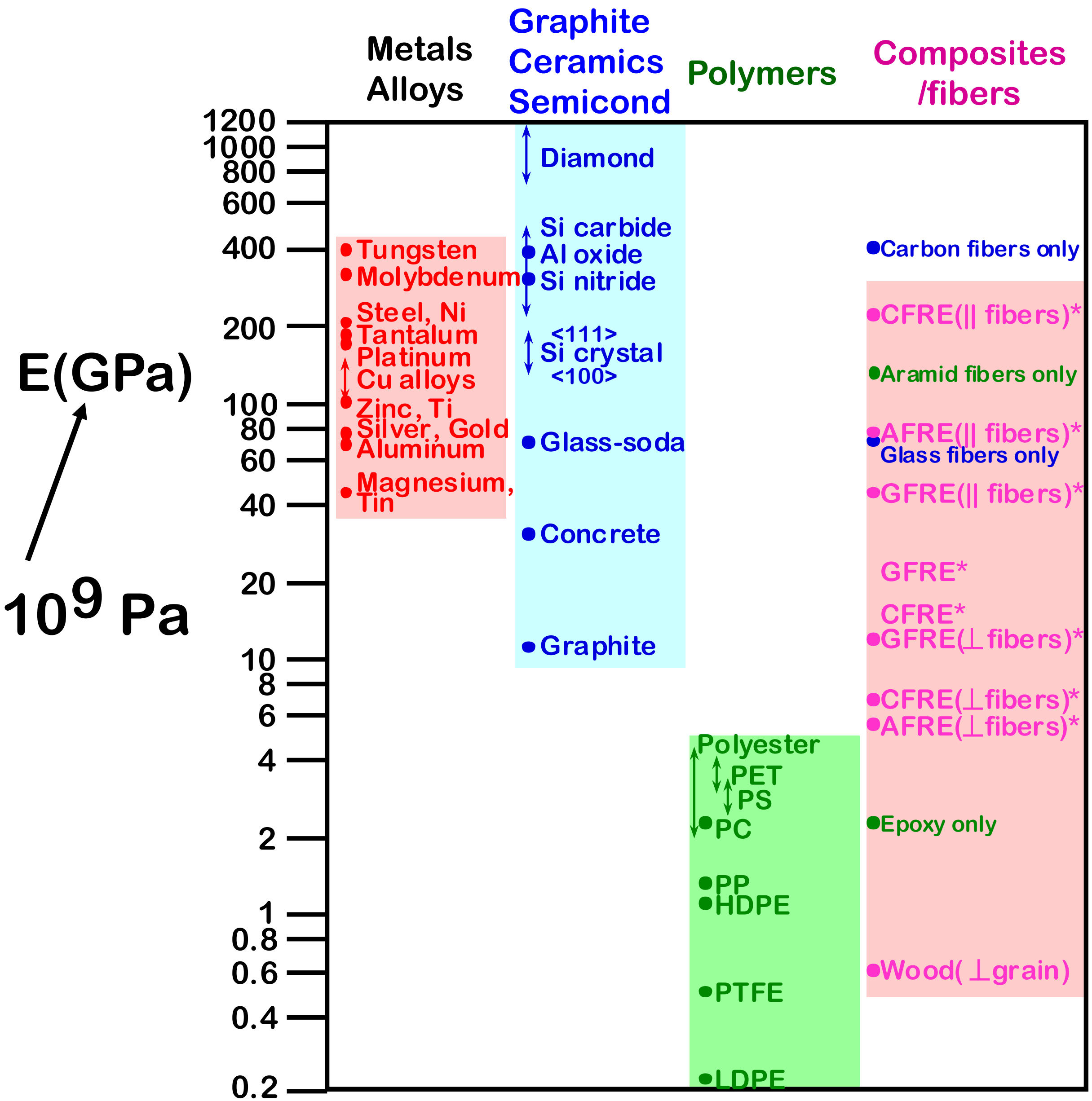


pressure test: Init. vol $=V_o$. Vol chg. $=\Delta V$

- Special relations for isotropic materials:

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Comparison of Young's Modulus, E:



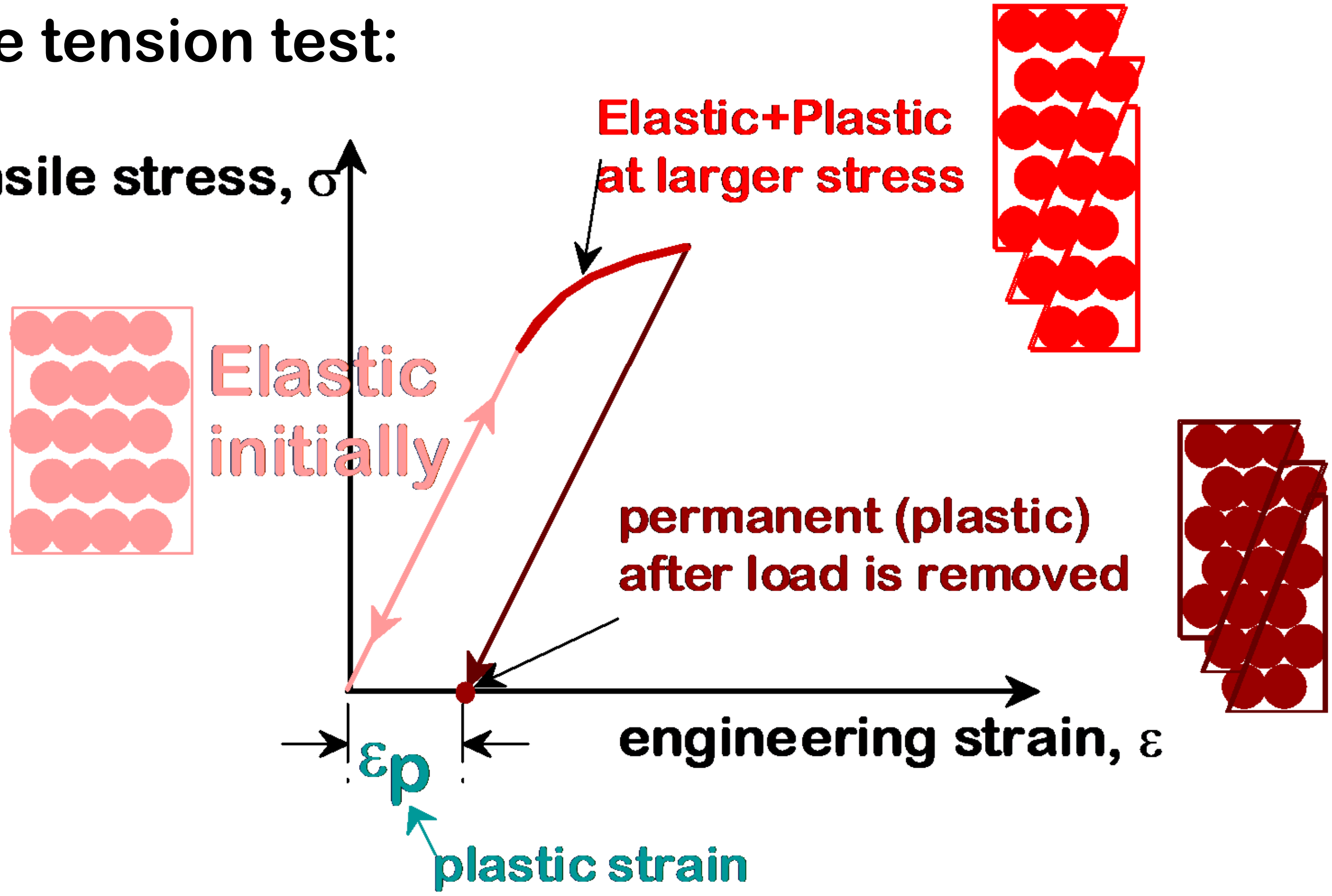
$E_{ceramics}$

$> E_{metals}$

$>> E_{polymers}$

PLASTIC (PERMANENT) DEFORMATION

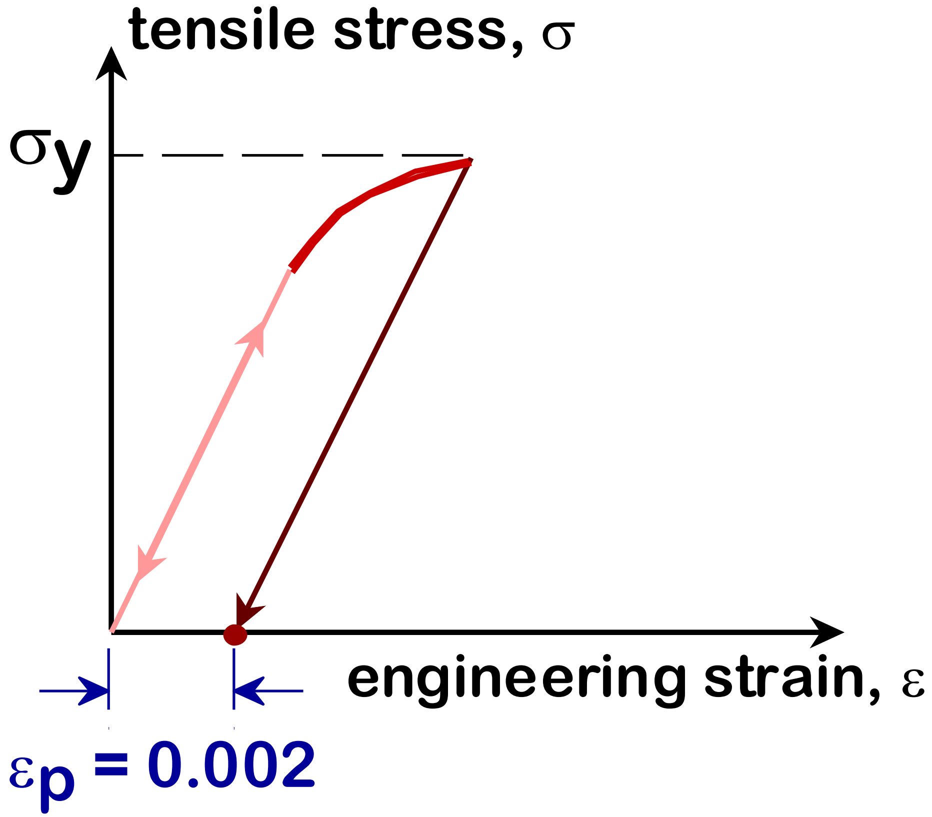
- **Simple tension test:**



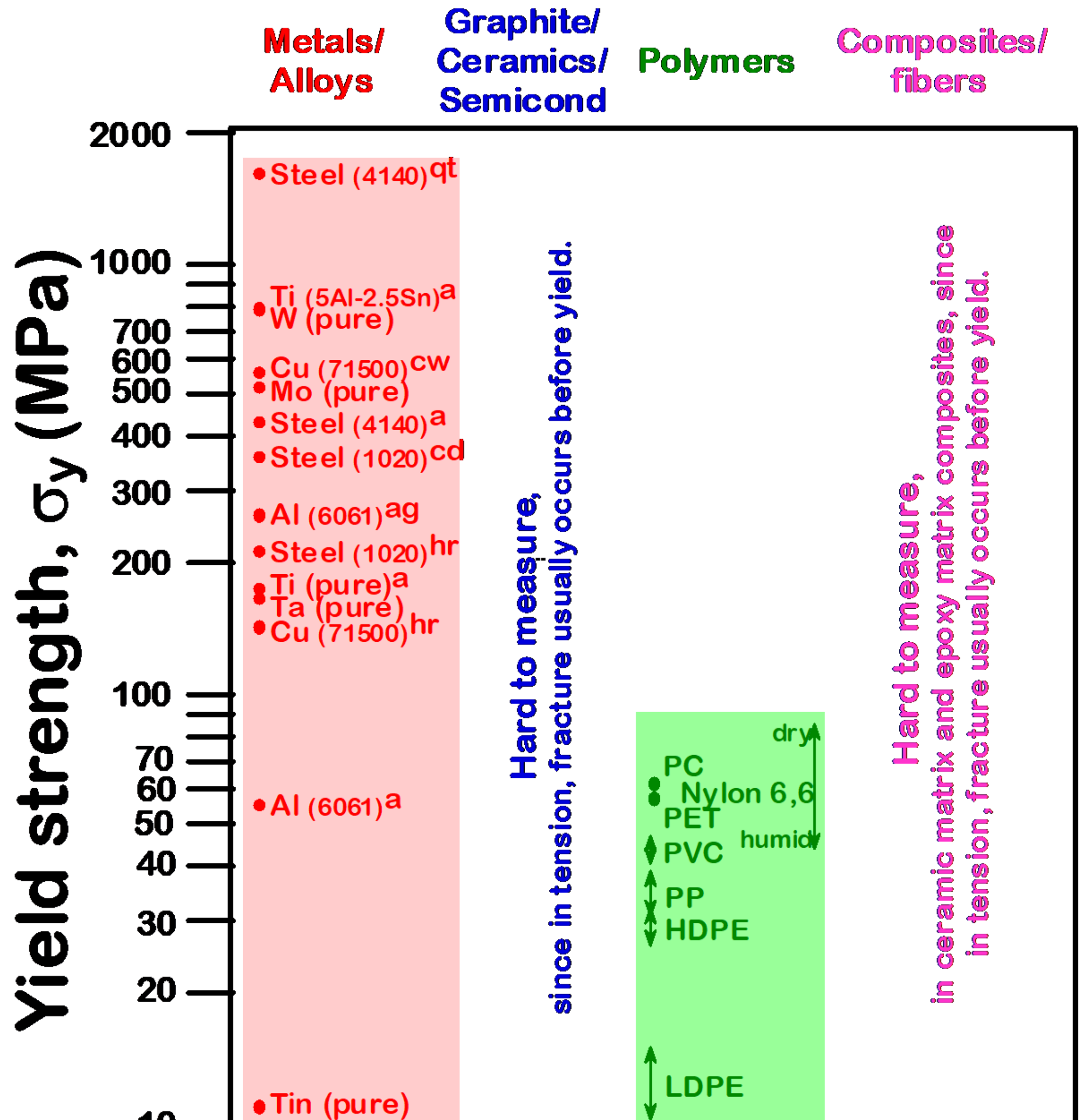
Yield Strength

- Stress at which *noticeable* plastic deformation has occurred.

when $\varepsilon_p = 0.002$



Yield Strength: Comparison



σ_y(ceramics)
>> σ_y(metals)
>> σ_y(polymers)

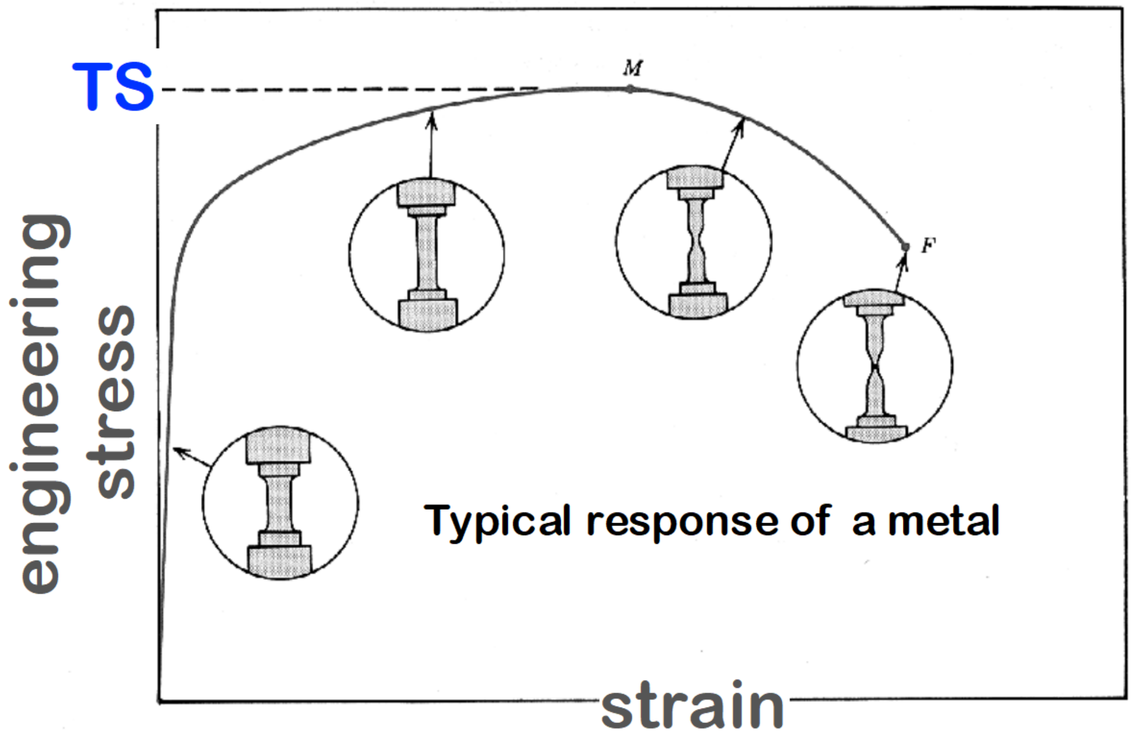
Room T values

Based on data in Table B4, *Callister 6e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength, TS

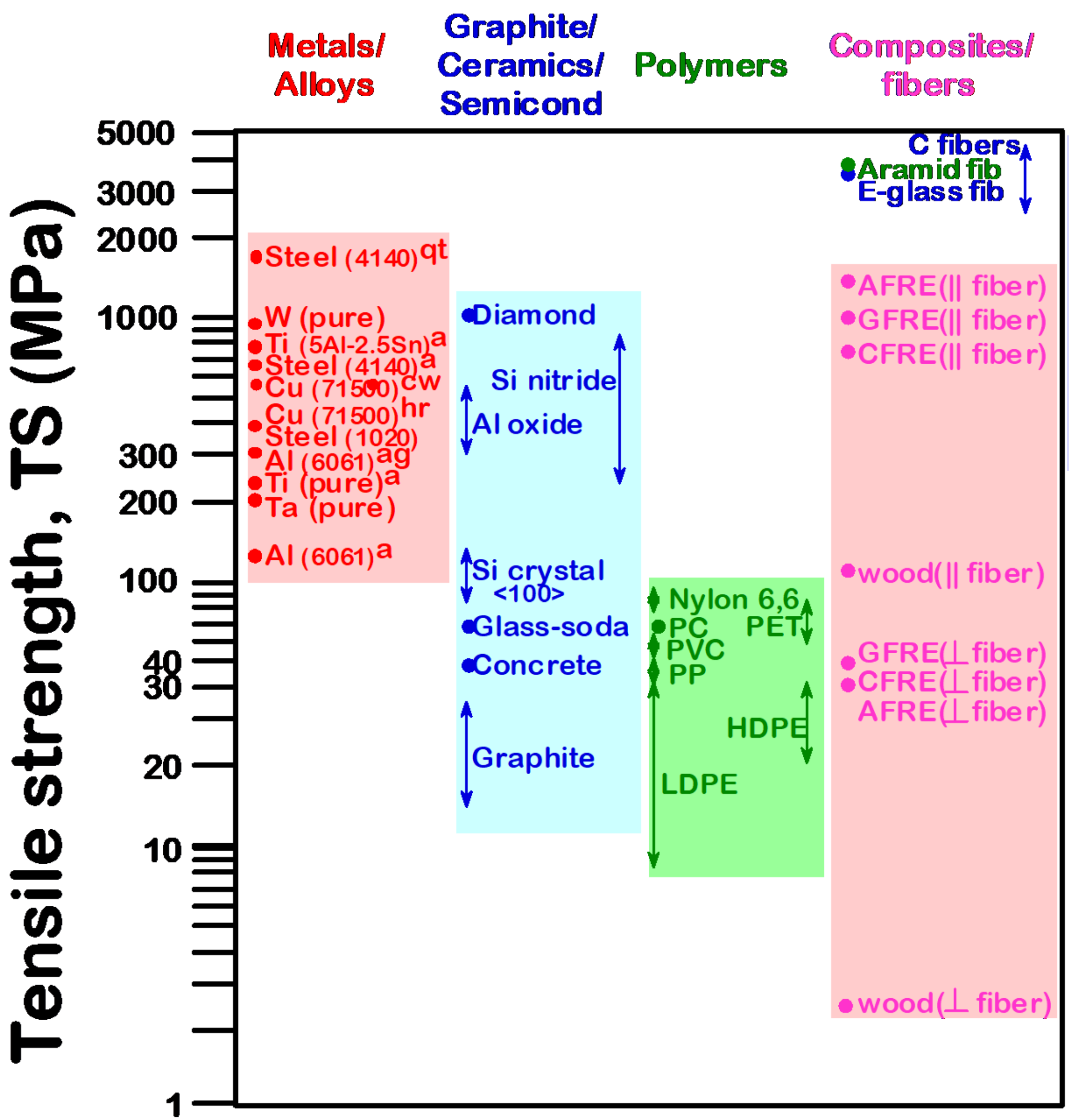
- Maximum possible engineering stress in tension.



Adapted from Fig. 6.11, *Callister 6e.*

- **Metals:** occurs when noticeable necking starts.
- **Ceramics:** occurs when crack propagation starts.
- **Polymers:** occurs when polymer backbones are aligned and about to break.

Tensile Strength: Comparison



$TS_{(ceram)}$
$\sim TS_{(met)}$
$\sim TS_{(comp)}$
$\gg TS_{(poly)}$

Room T values

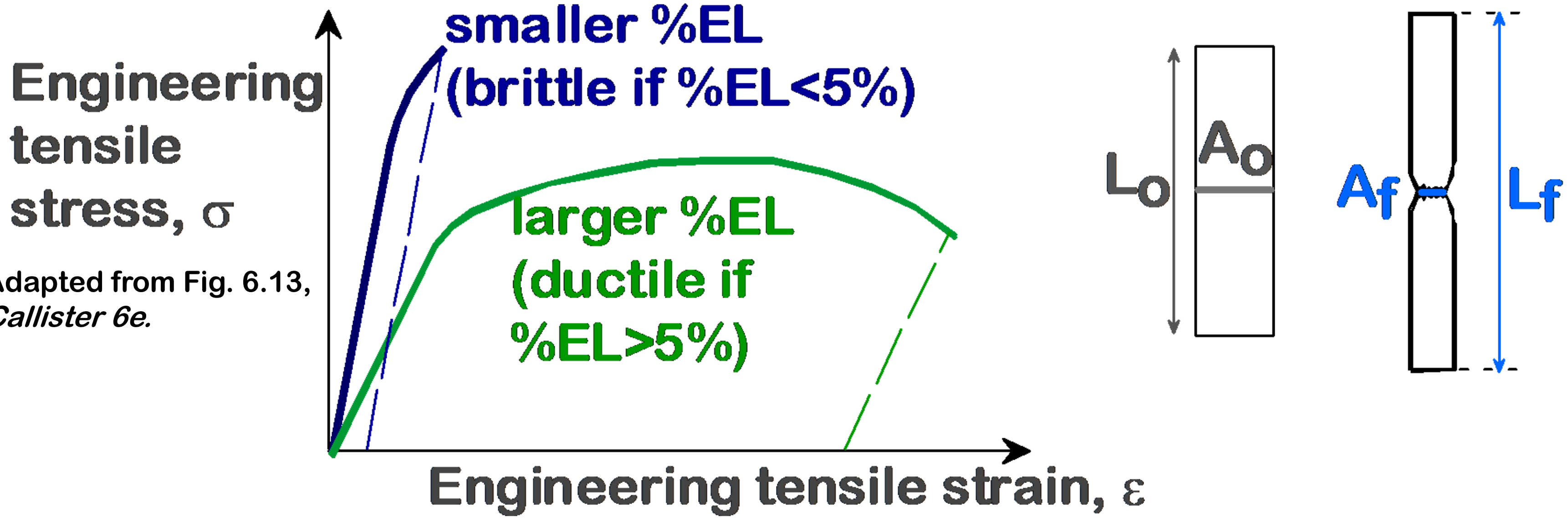
Based on data in Table B4, *Callister 6e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

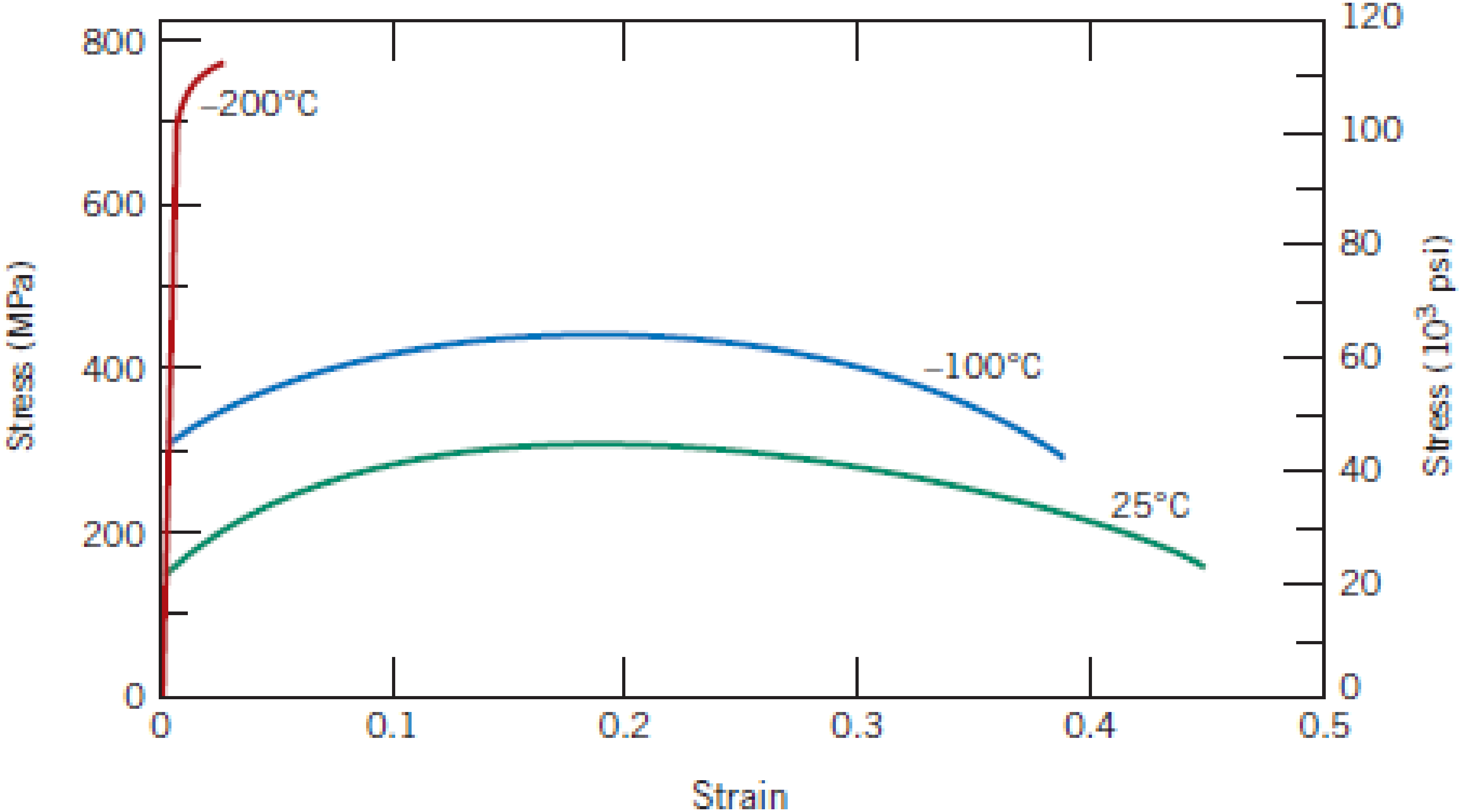
Ductility, %EL

- **Plastic tensile strain at failure:** $$\%EL = \frac{L_f - L_o}{L_o} x100$$



Adapted from Fig. 6.13, *Callister 6e.*

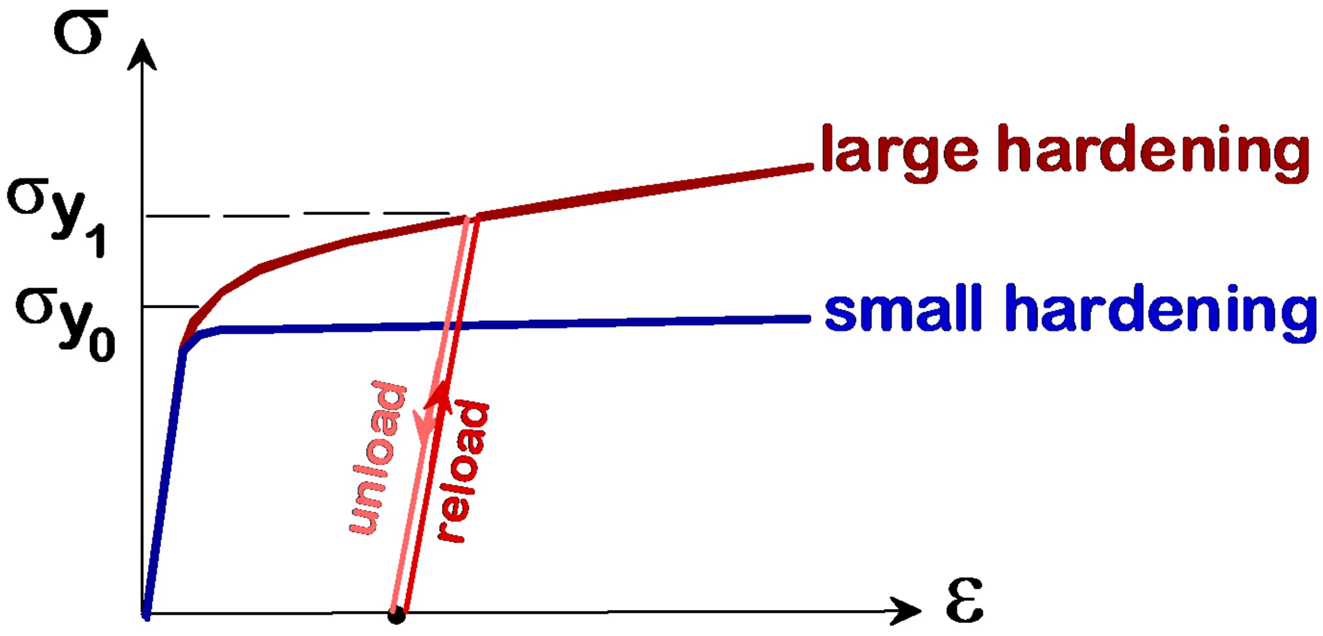
- **Another ductility measure:** $$\%AR = \frac{A_o - A_f}{A_o} x100$$



Engineering stress–strain behavior for iron at three temperatures.

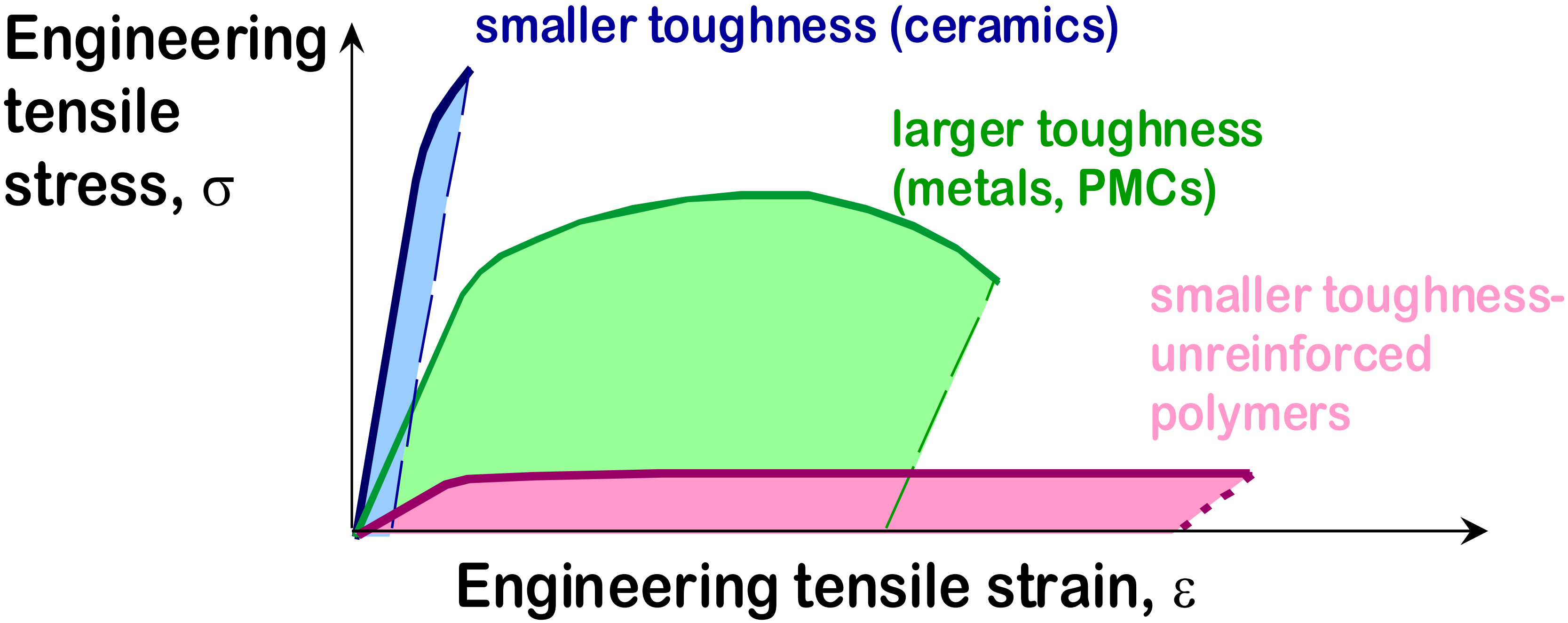
STRAIN HARDENING

- An increase in σy due to plastic deformation.



Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.



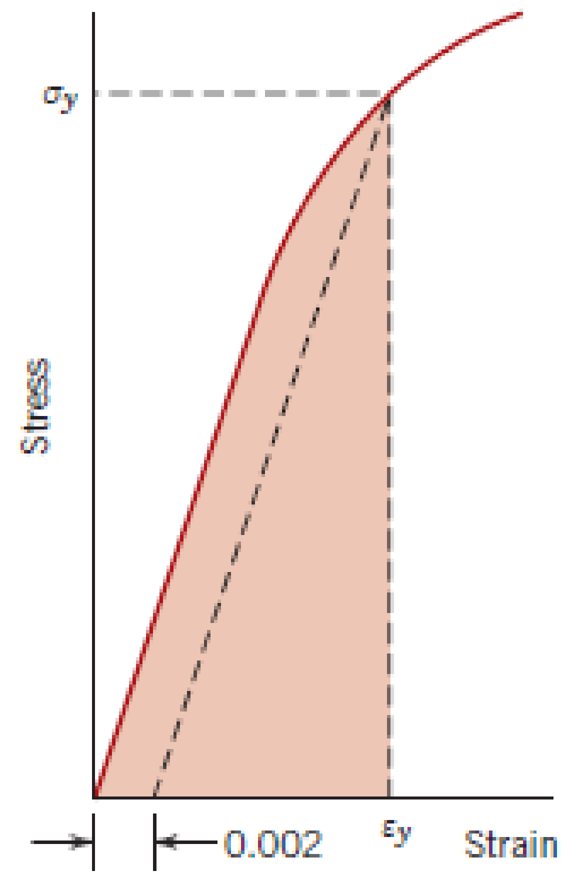
Resilience

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered. The associated property is the *modulus of resilience*, U_r, which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielc

$$U_r = \frac{1}{2}\sigma_y \varepsilon_y$$

ε_y is the strain at yielding.

$$U_r = \frac{1}{2}\sigma_y \varepsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E}$$



Problem 1

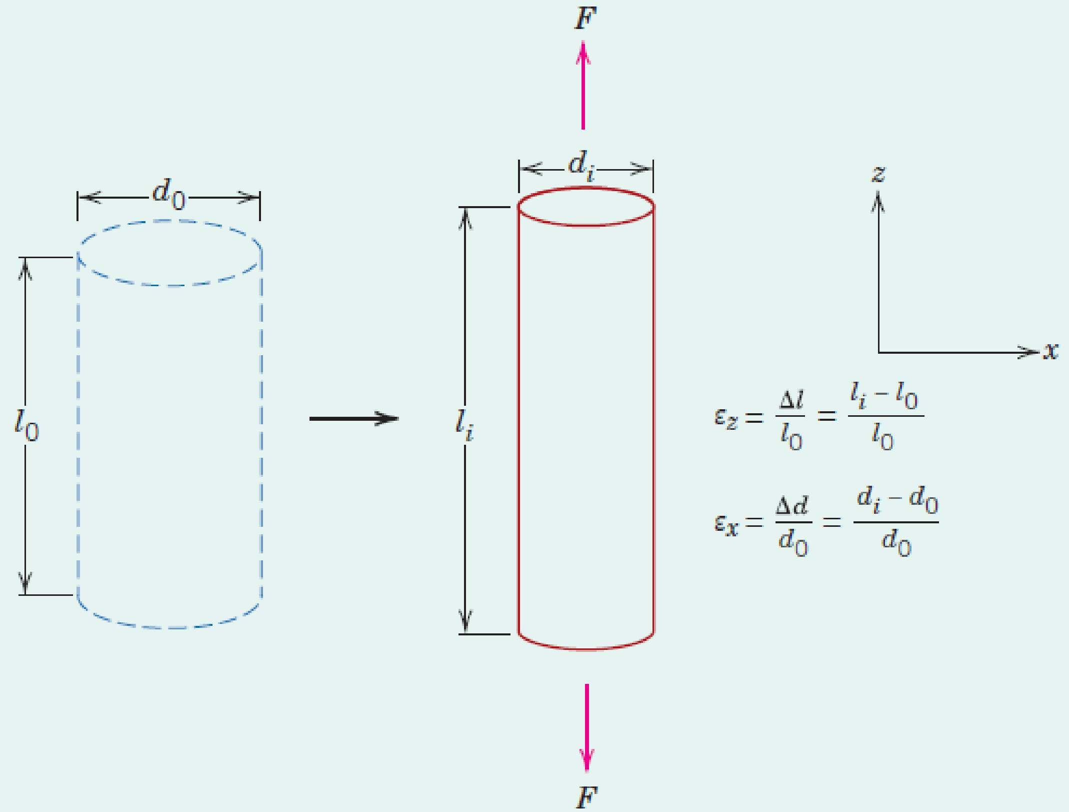
Elongation (Elastic) Computation

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

	Modulus of Elasticity		Shear Modulus		
Metal Alloy	*GPa*	*10^6 psi*	*GPa*	*10^6 psi*	*Poisson's Ratio*
Aluminum	69	10	25	3.6	0.33
Brass	97	14	37	5.4	0.34
Copper	110	16	46	6.7	0.34
Magnesium	45	6.5	17	2.5	0.29
Nickel	207	30	76	11.0	0.31
Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

Computation of Load to Produce Specified Diameter Change

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5×10^{-3}-mm (10^{-4}-in.) change in diameter if the deformation is entirely elastic.



	Modulus of Elasticity		*Shear Modulus*		
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Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

True Stress

$$\sigma_T = \frac{F}{A_i}$$

True Strain

$$\varepsilon_T = \ln\frac{l_i}{l_0}$$

No Volume Change/
Incompressible

$$A_i l_i = A_0 l_0$$

Relation between True & Engineering stress and strain

$$\sigma_T = \sigma(1 + \varepsilon)$$

$$\varepsilon_T = \ln(1 + \varepsilon)$$

Relation between True stress and True strain

$$\sigma_T = K\varepsilon_T^n$$

n is often termed the *strain-hardening exponent*

		K	
Material	***n***	***MPa***	***psi***
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered @ 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

Ductility and True-Stress-at-Fracture Computations

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile-tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine

(a) The ductility in terms of percentage reduction in area
(b) The true stress at fracture

Calculation of Strain-Hardening Exponent

Compute the strain-hardening exponent n in Equation for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

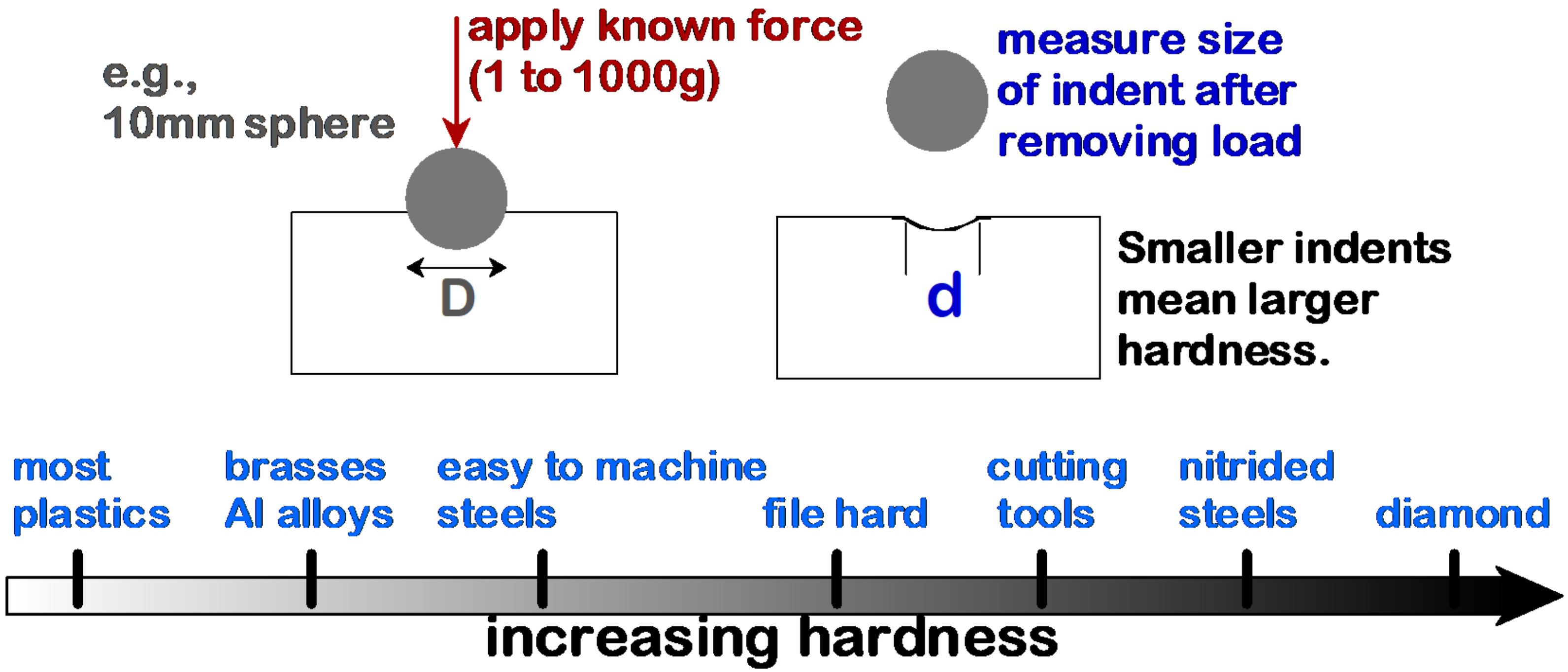
$$\sigma_T = K\varepsilon_T^n$$

HARDNESS

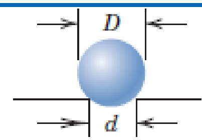
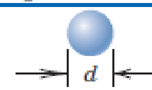
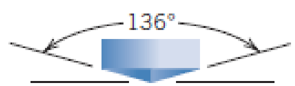
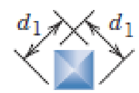
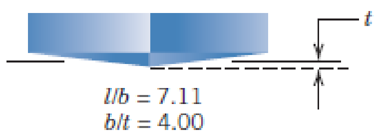
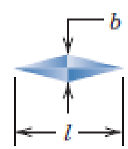
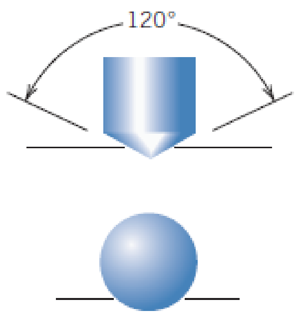
- Measure of a material's resistance to localized plastic deformation (e.g., a small dent or a scratch).
- Large hardness means:

  --resistance to plastic deformation or cracking in compression.

  --better wear properties.

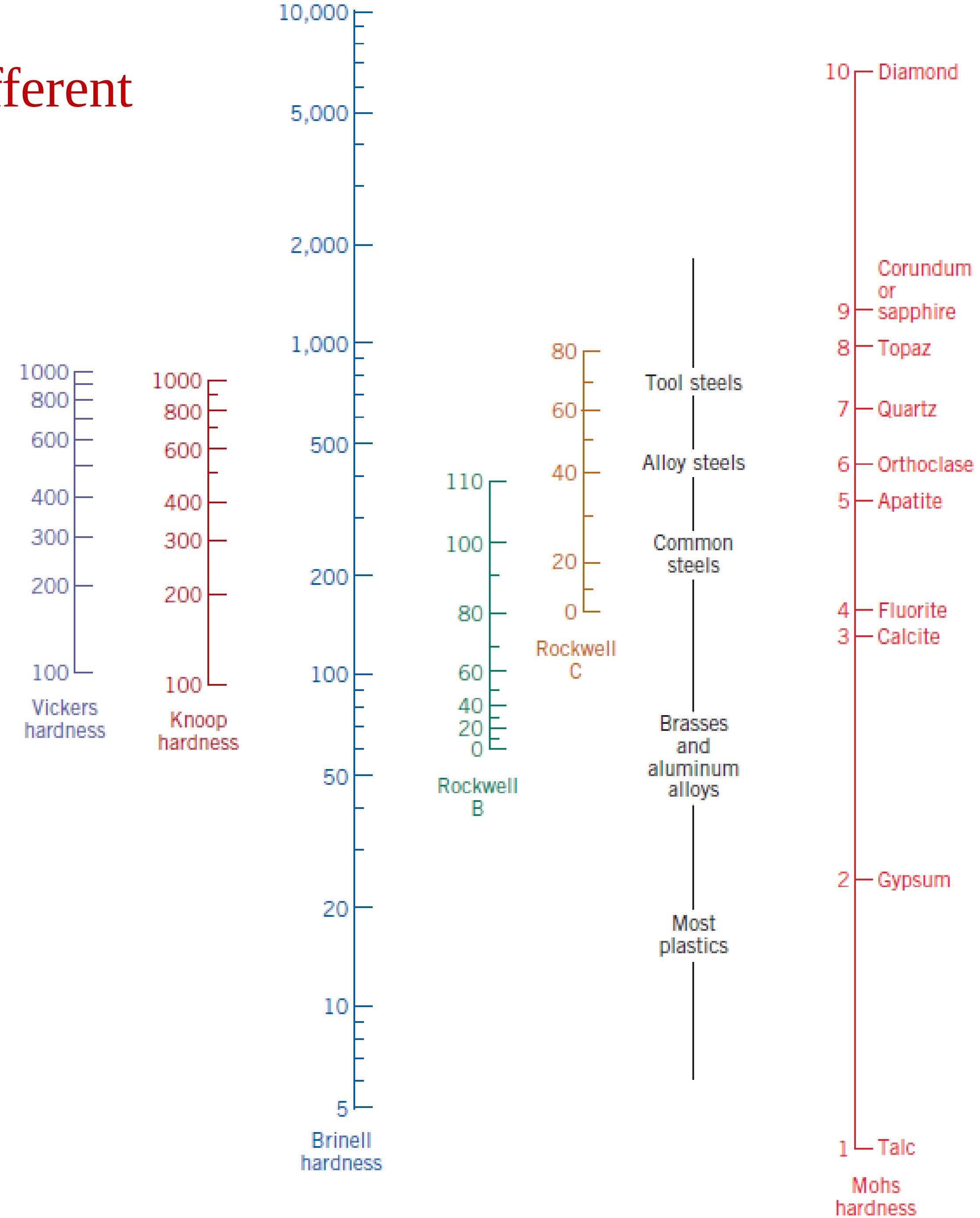


Various Hardness Tests

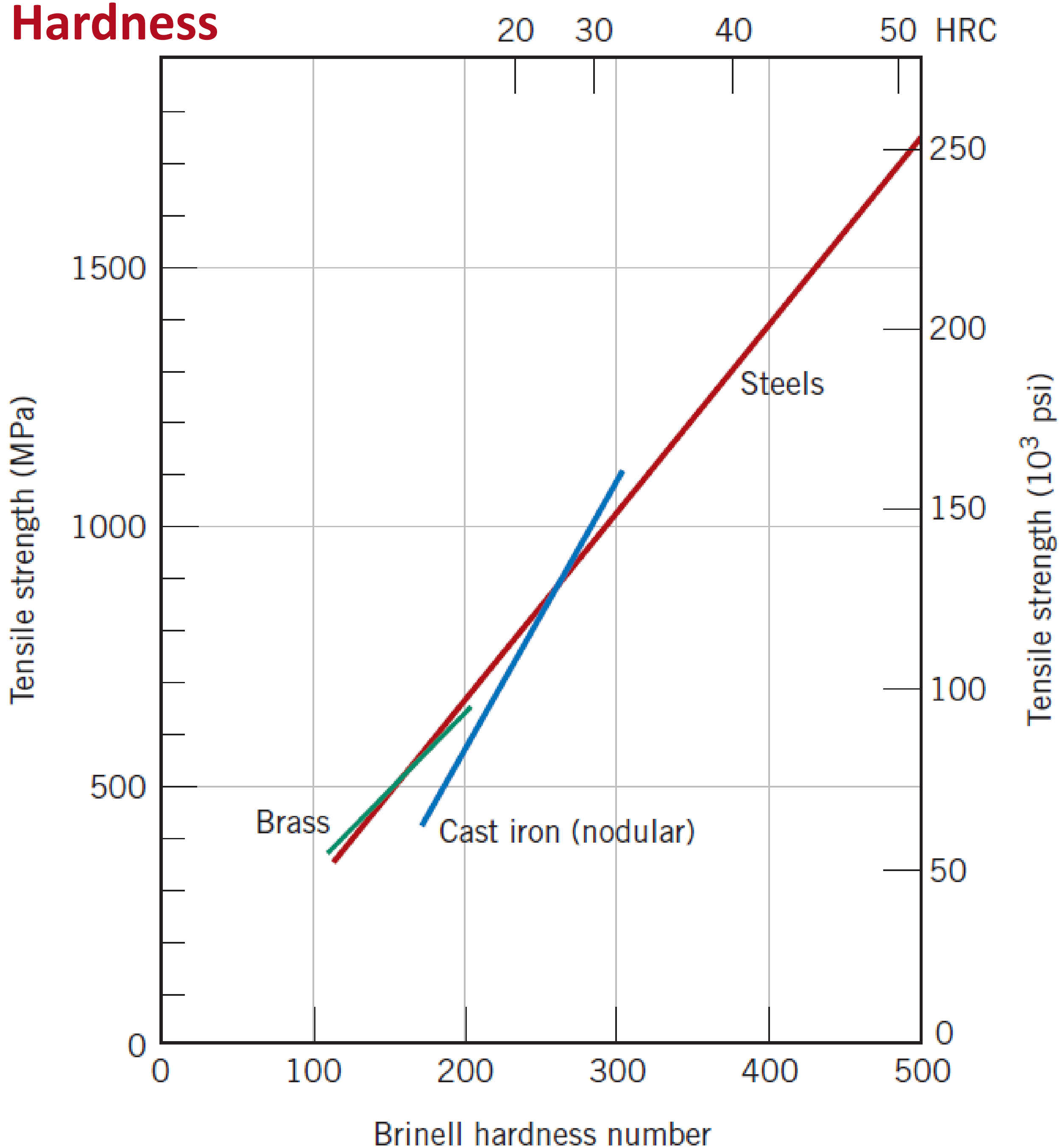
Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$HK = 14.2P/l^2$
Rockwell and superficial Rockwell	Diamond cone: $\frac{1}{16}$-, $\frac{1}{8}$-, $\frac{1}{4}$-, $\frac{1}{2}$-in. diameter steel spheres			60 kg, 100 kg, 150 kg: Rockwell; 15 kg, 30 kg, 45 kg: Superficial Rockwell	

[a] For the hardness formulas given, P (the applied load) is in kg, and D, d, d_1, and l are all in millimeters.

Comparison of Different hardness scales



Correlation between Hardness and Tensile Strength



For Most Steels,

$$TS(\text{MPa}) = 3.45 \times \text{HB}$$

$$TS(\text{psi}) = 500 \times \text{HB}$$

VARIABILITY OF MATERIAL PROPERTIES

Average and Standard Deviation Computations

The following tensile strengths were measured for four specimens of the same steel alloy:

Sample Number	*Tensile Strength (MPa)*
1	520
2	512
3	515
4	522

(a) Compute the average tensile strength.
(b) Determine the standard deviation.

Average value

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Standard deviation s

$$s = \left[\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1} \right]^{1/2}$$

n is the number of observations or measurements and x_i is the value of a discrete measurement.

Average and Standard Deviation Computations

The following tensile strengths were measured for four specimens of the same steel alloy:

Sample Number	*Tensile Strength (MPa)*
1	520
2	512
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(a) Compute the average tensile strength.
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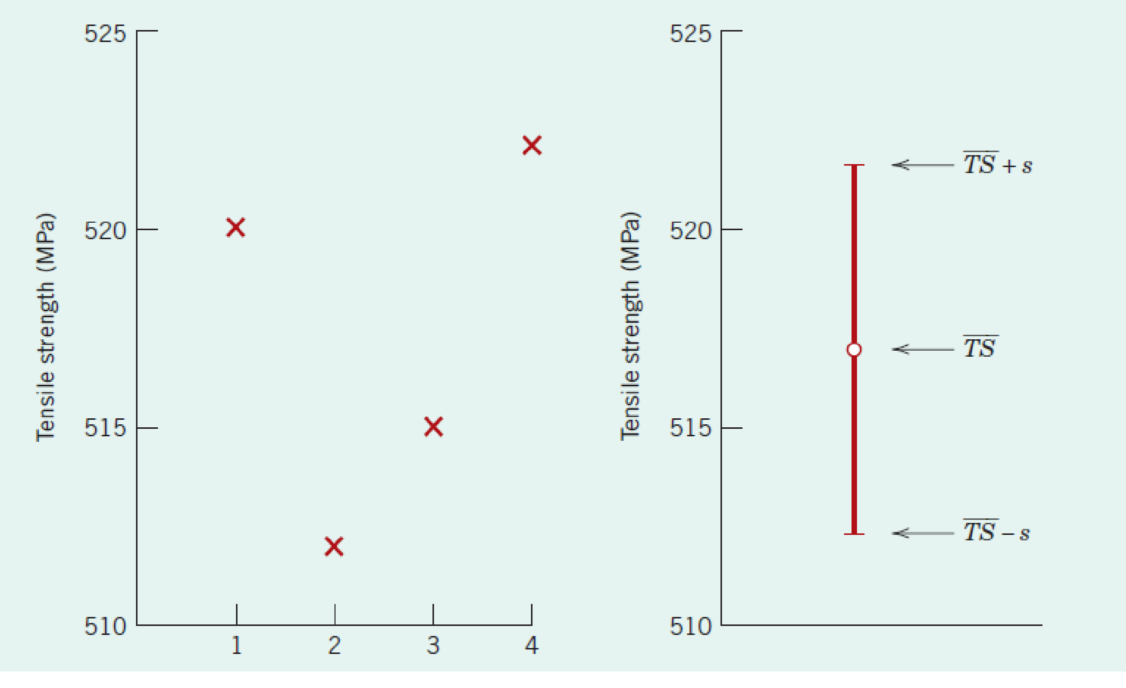


DESIGN OR SAFETY FACTORS

- **Design uncertainties mean we do not push the limit.**
- **Factor of safety, N**

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is between 1.2 and 4

Specification of Support-Post Diameter

A tensile-testing apparatus is to be constructed that must withstand a maximum load of 220,000 N (50,000 lb_f). The design calls for two cylindrical support posts, each of which is to support half of the maximum load. Furthermore, plain-carbon (1045) steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 MPa (45,000 psi) and 565 MPa (82,000 psi), respectively. Specify a suitable diameter for these support posts.

Materials Specification for a Pressurized Cylindrical Tube

(a) Consider a thin-walled cylindrical tube having a radius of 50 mm and wall thickness 2 mm that is to be used to transport pressurized gas. If inside and outside tube pressures are 20 and 0.5 atm (2.027 and 0.057 MPa), respectively, which of the metals and alloys listed in Table 6.8 are suitable candidates? Assume a factor of safety of 4.0.

For a thin-walled cylinder, the circumferential (or "hoop") stress (σ) depends on pressure difference (Δp), cylinder radius (r_i), and tube wall thickness (t) as follows:

$$\sigma = \frac{r_i \Delta p}{t}$$

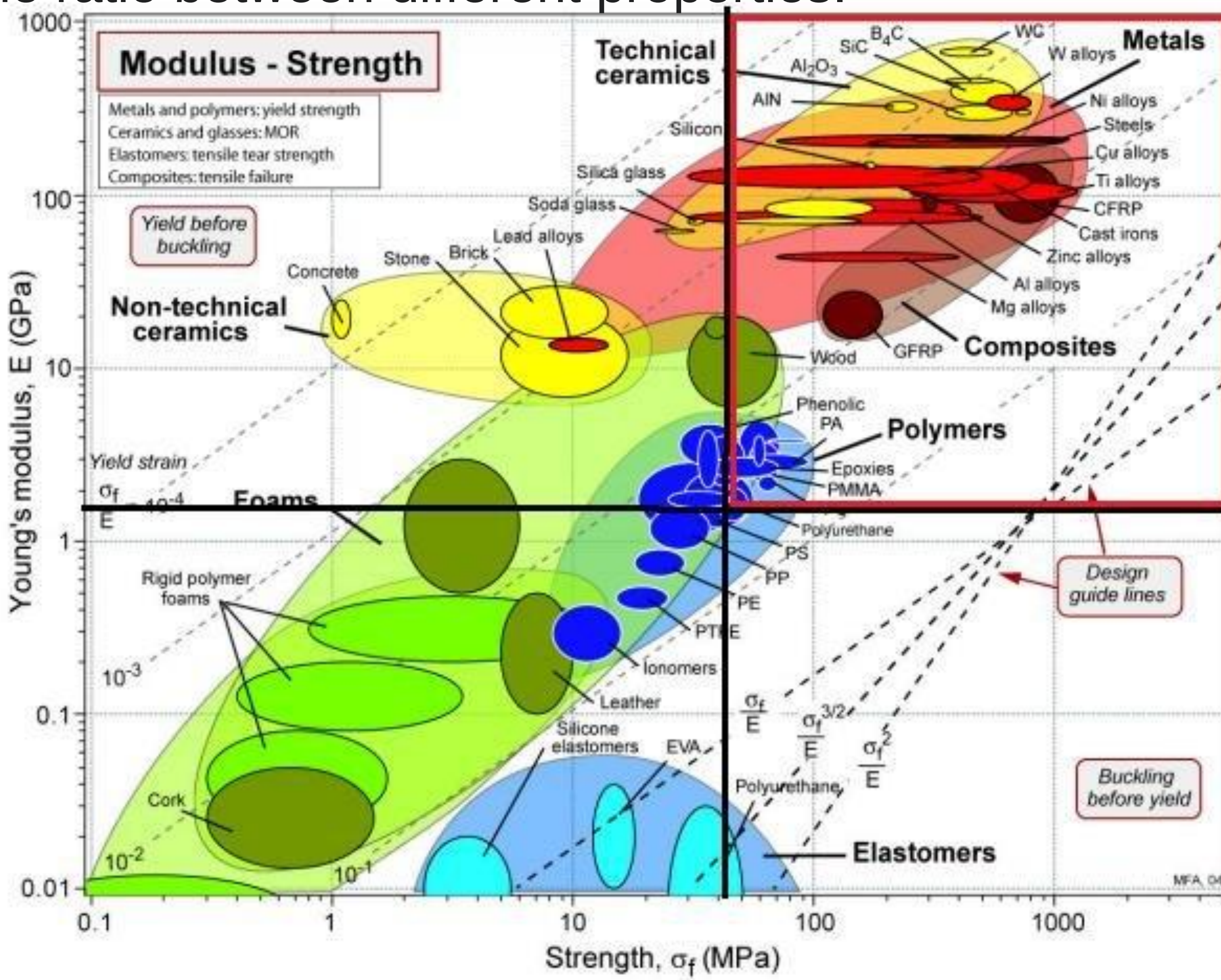
These parameters are noted on the schematic sketch of a cylinder presented in Figure

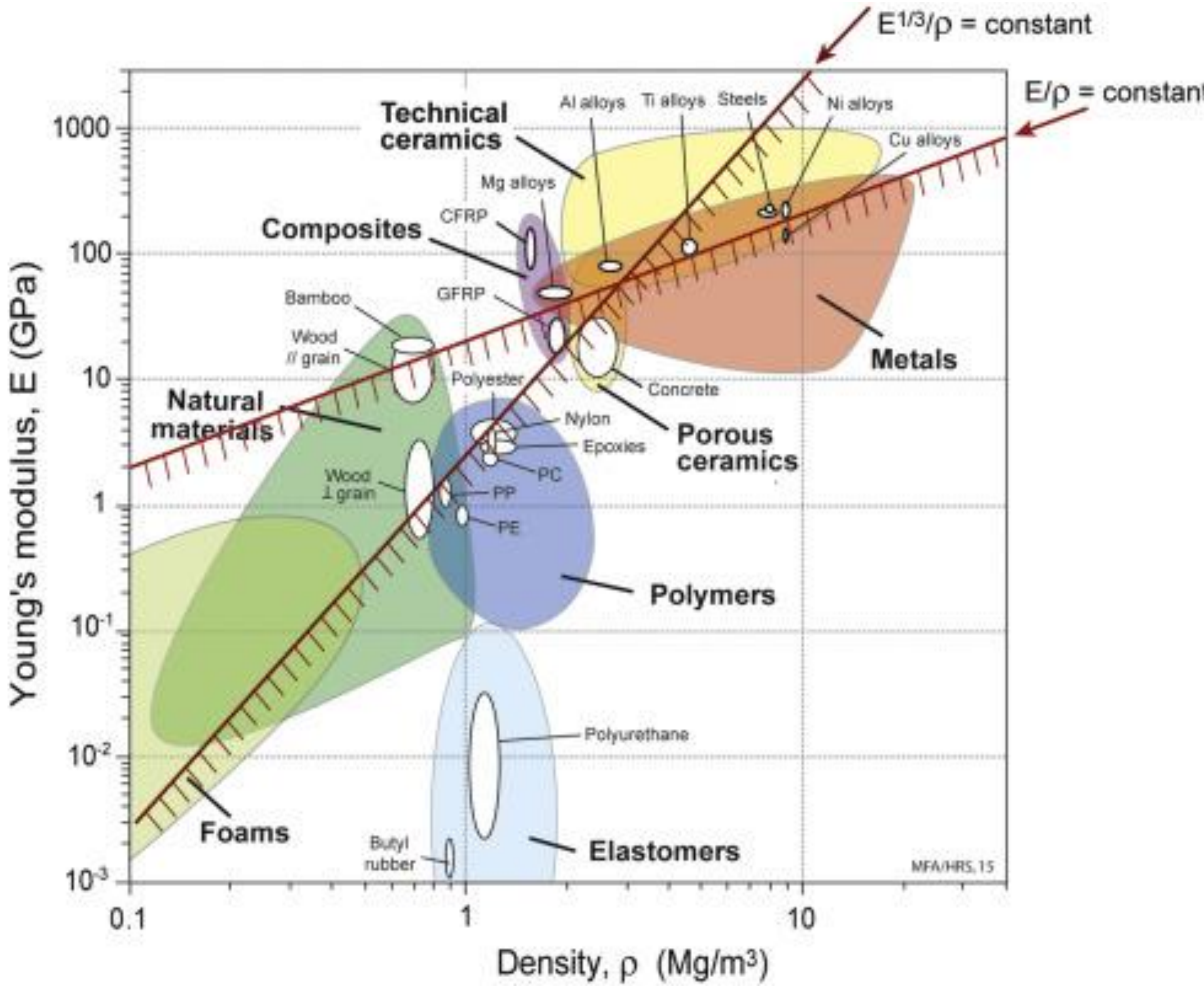
(b) Determine which of the alloys that satisfy the criterion of part (**a**) can be used to produce a tube with the lowest cost.

Alloy	*Yield Strength, σ_y (MPa)*	*Density, ρ (g/cm^3)*	*Unit mass cost, $\bar{c}$ (\$US/kg)*
Steel	325	7.8	1.25
Aluminum	125	2.7	3.50
Copper	225	8.9	6.25
Brass	275	8.5	7.50
Magnesium	175	1.8	14.00
Titanium	700	4.5	40.00

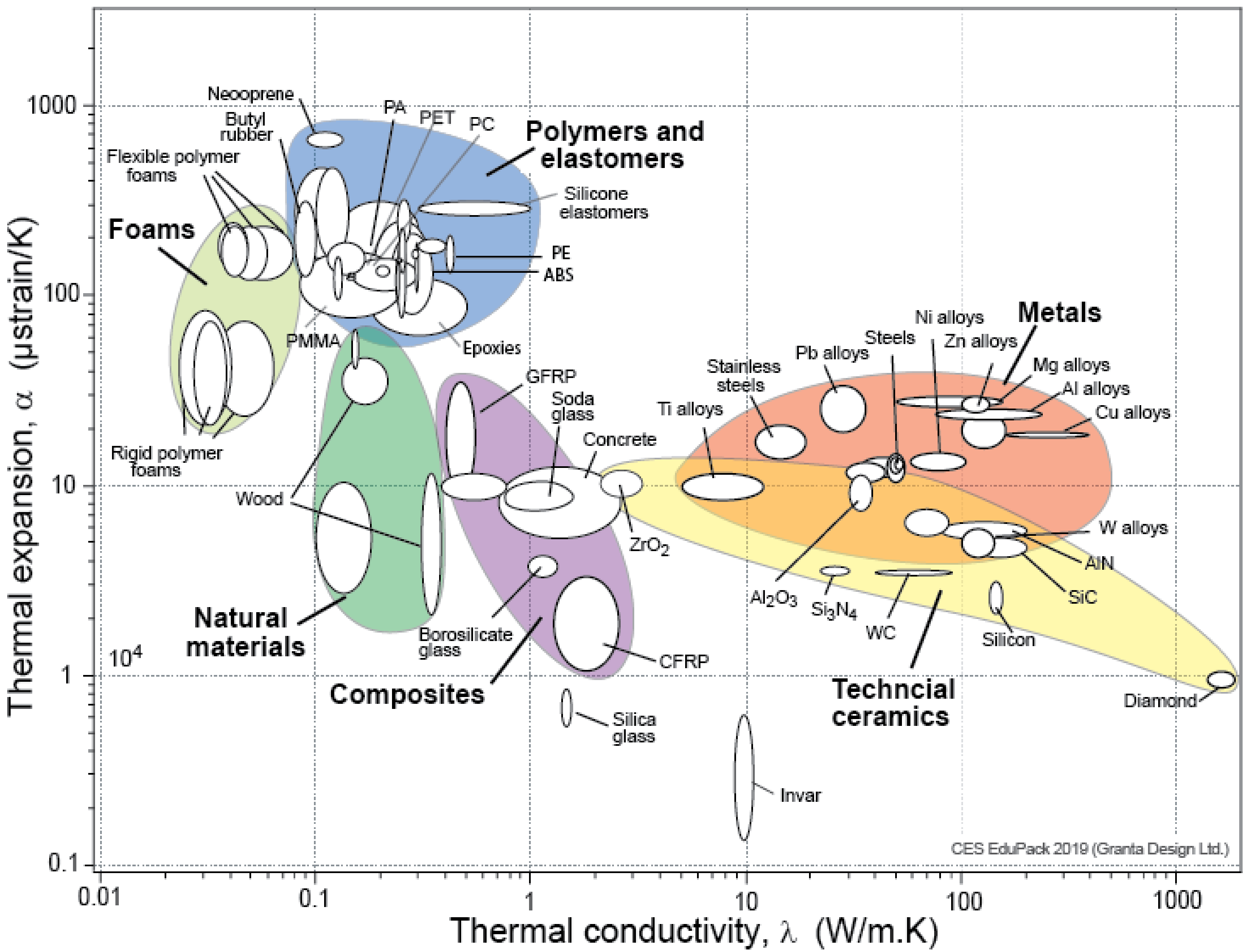
An Ashby plot –

is a scatter plot which displays classes of materials. Useful to compare the ratio between different properties.

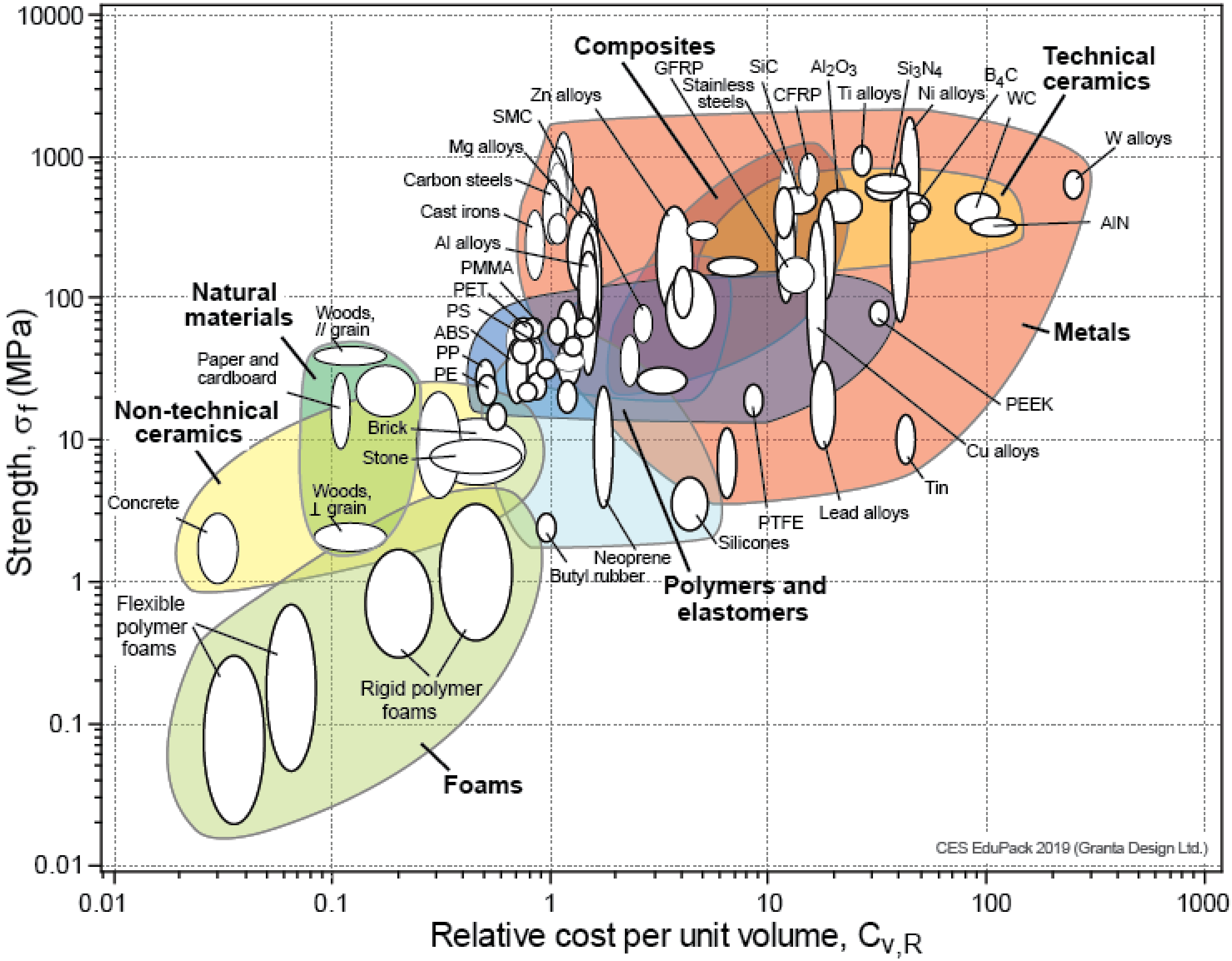




E^1/3/ρ = constant
E/ρ = constant
Technical ceramics
Al alloys
Ti alloys
Steels
Ni alloys
Cu alloys
Mg alloys
CFRP
Composites
GFRP
Bamboo
Wood // grain
Natural materials
Polyester
Nylon
Epoxies
PC
PP
PE
Wood ⊥ grain
Concrete
Porous ceramics
Metals
Polymers
Polyurethane
Foams
Butyl rubber
Elastomers
MFA/HRS, 15
Young's modulus, E (GPa)
Density, ρ (Mg/m3)
1000
100
10
1
10-1
10-2
10-3
0.1
1
10



Polymers and elastomers
Neoprene
Butyl rubber
PA
PET
PC
Flexible polymer foams
Silicone elastomers
Foams
PE
ABS
PMMA
Epoxies
Rigid polymer foams
Wood
GFRP
Soda glass
Concrete
ZrO2
Natural materials
Borosilicate glass
Composites
CFRP
Silica glass
Invar
Metals
Ni alloys
Zn alloys
Steels
Pb alloys
Stainless steels
Mg alloys
Al alloys
Cu alloys
Ti alloys
W alloys
AlN
SiC
Al2O3
Si3N4
WC
Silicon
Techncial ceramics
Diamond
10^4
CES EduPack 2019 (Granta Design Ltd.)
Thermal expansion, α (μstrain/K)
Thermal conductivity, λ (W/m.K)
0.1
1
10
100
1000
0.01

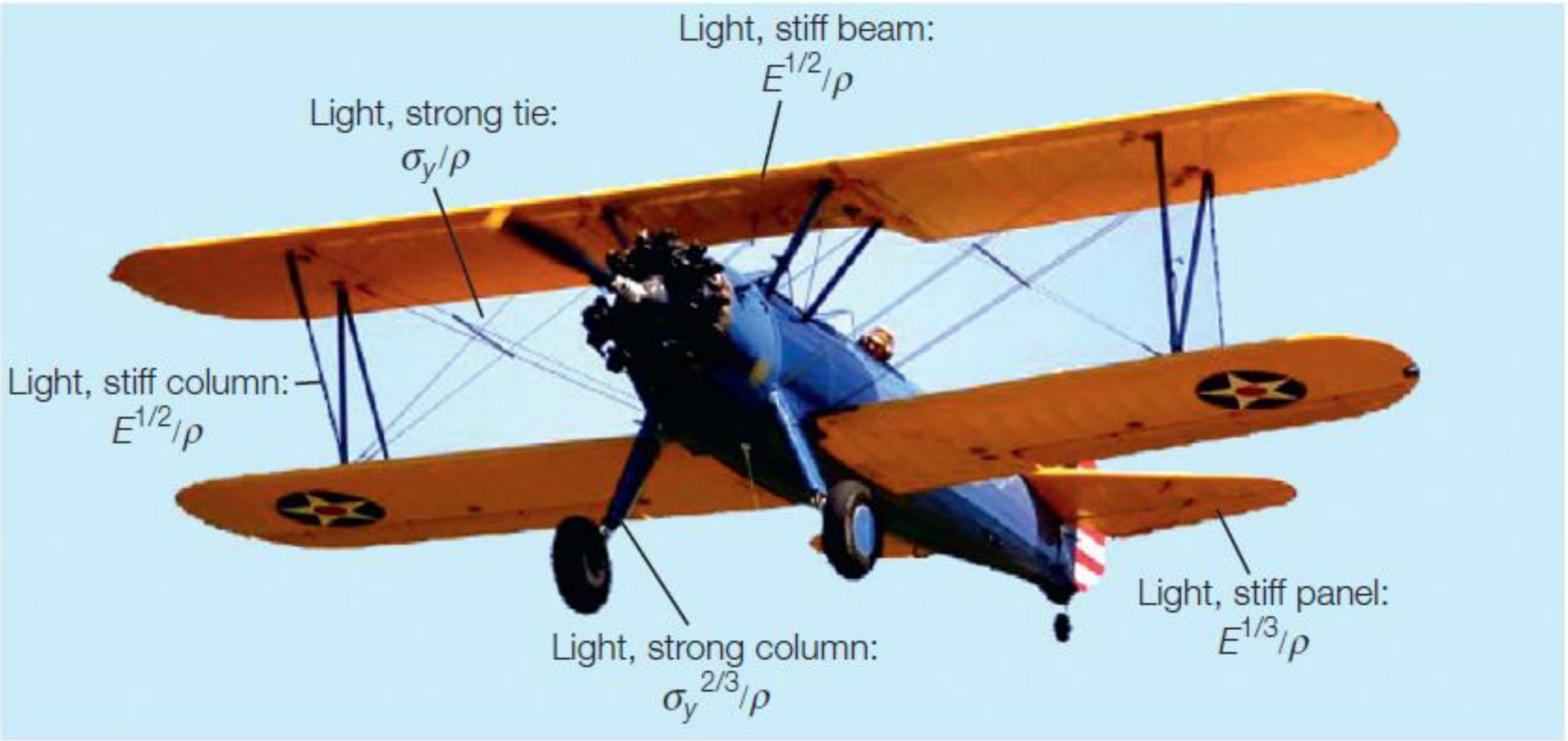


Composites
Technical ceramics
Natural materials
Non-technical ceramics
Metals
Polymers and elastomers
Foams
GFRP
SiC
Al_2O_3
Si_3N_4
B_4C
WC
Zn alloys
Stainless steels
CFRP
Ti alloys
Ni alloys
SMC
Mg alloys
Carbon steels
Cast irons
Al alloys
PMMA
PET
PS
ABS
PP
PE
W alloys
AlN
PEEK
Cu alloys
Tin
Lead alloys
PTFE
Silicones
Neoprene
Butyl rubber
Woods, // grain
Paper and cardboard
Brick
Stone
Woods, ⊥ grain
Concrete
Flexible polymer foams
Rigid polymer foams
CES EduPack 2019 (Granta Design Ltd.)
Strength, σ_f (MPa)
Relative cost per unit volume, $C_{v,R}$
10000
1000
100
10
1
0.1
0.01
0.01
0.1
1
10
100
1000

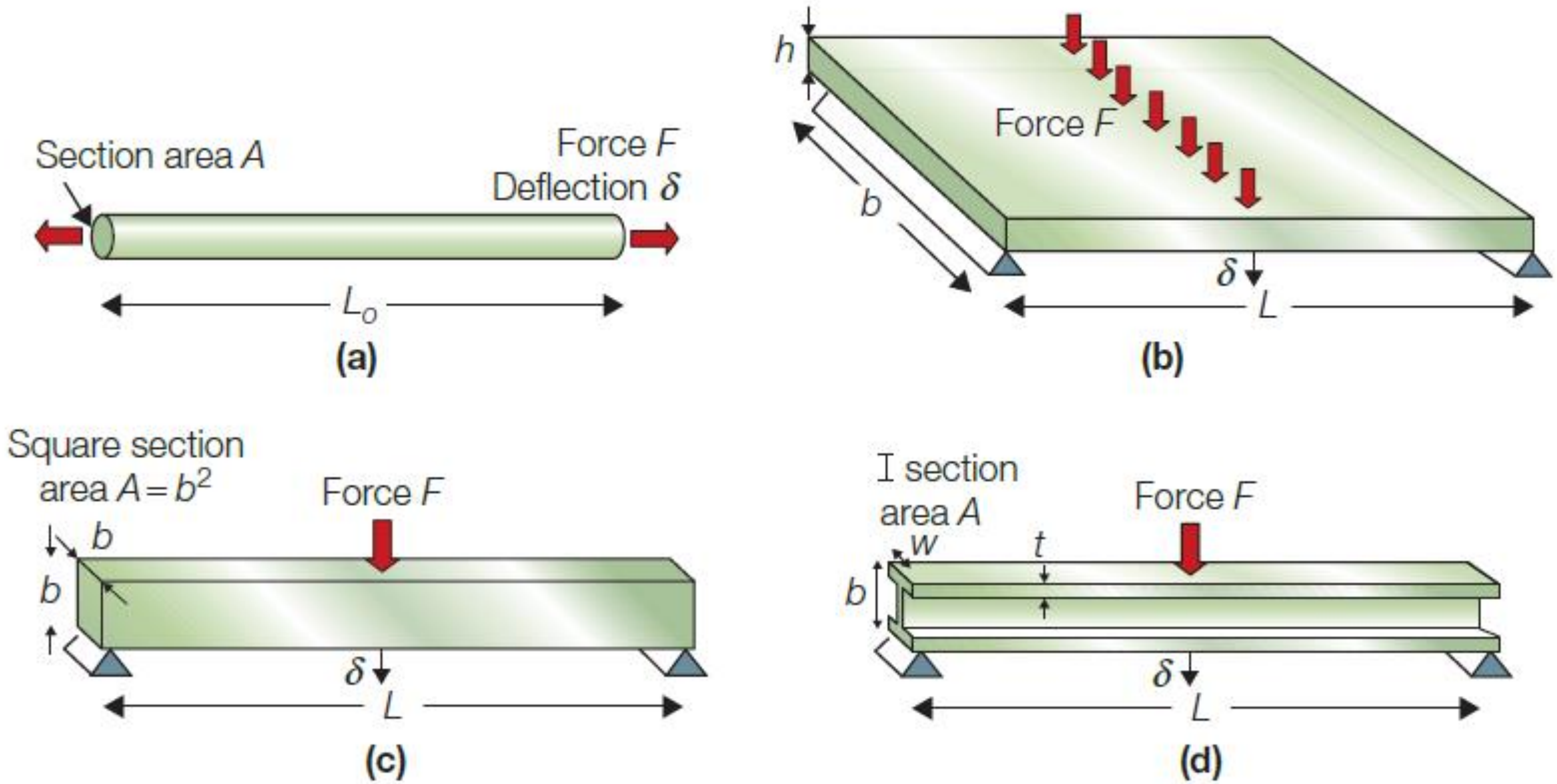
Table 5.5 Examples of Material Indices

Function, Objective, and Constraints	Index
Tie, minimum weight, stiffness prescribed	$\dfrac{E}{\rho}$
Beam, minimum weight, stiffness prescribed	$\dfrac{E^{1/2}}{\rho}$
Beam, minimum weight, strength prescribed	$\dfrac{\sigma_y^{2/3}}{\rho}$
Beam, minimum cost, stiffness prescribed	$\dfrac{E^{1/2}}{C_m\rho}$
Beam, minimum cost, strength prescribed	$\dfrac{\sigma_y^{2/3}}{C_m\rho}$
Column, minimum cost, buckling load prescribed	$\dfrac{E^{1/2}}{C_m\rho}$
Spring, minimum weight for given energy storage	$\dfrac{\sigma_y^{2}}{E\rho}$
Thermal insulation, minimum cost, heat flux prescribed	$\dfrac{1}{\lambda C_p\rho}$
Electromagnet, maximum field, temperature rise prescribed	$\dfrac{C_p\rho}{\rho_e}$

ρ = density; E = Young's modulus; σ_y = elastic limit; C_m = cost/kg; λ = thermal conductivity; ρ_e = electrical resistivity; C_p = specific heat


Light, stiff beam:
$E^{1/2}/\rho$
Light, strong tie:
σ_y/ρ
Light, stiff column:
$E^{1/2}/\rho$
Light, strong column:
$\sigma_y^{2/3}/\rho$
Light, stiff panel:
$E^{1/3}/\rho$

Cheap strong tie:
$\sigma_y / C_m\rho$
Cheap stiff panel:
$E^{1/3} / C_m\rho$
Cheap strong beam:
$\sigma_y^{2/3} / C_m$
Cheap stiff column:
$E^{1/2} / C_m\rho$



(a) a tie, a tensile component;
(b) a panel, loaded in bending;
(c) and (d) beams, loaded in bending.

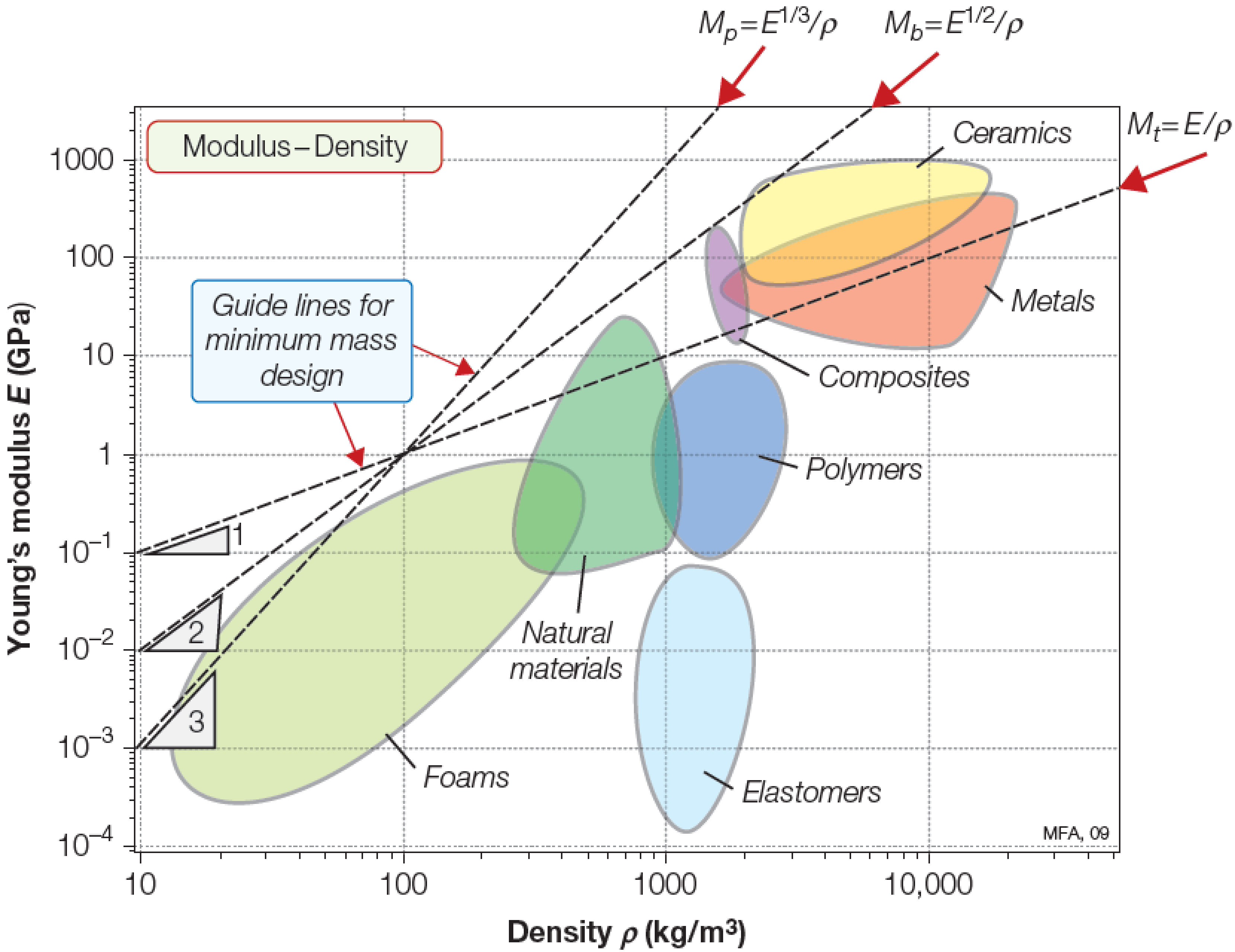
Objective Function for Light-Strong Stiff Tie rod:

$$m = A\,L\,\rho \qquad\qquad \frac{F^*}{A} \le \sigma_f$$

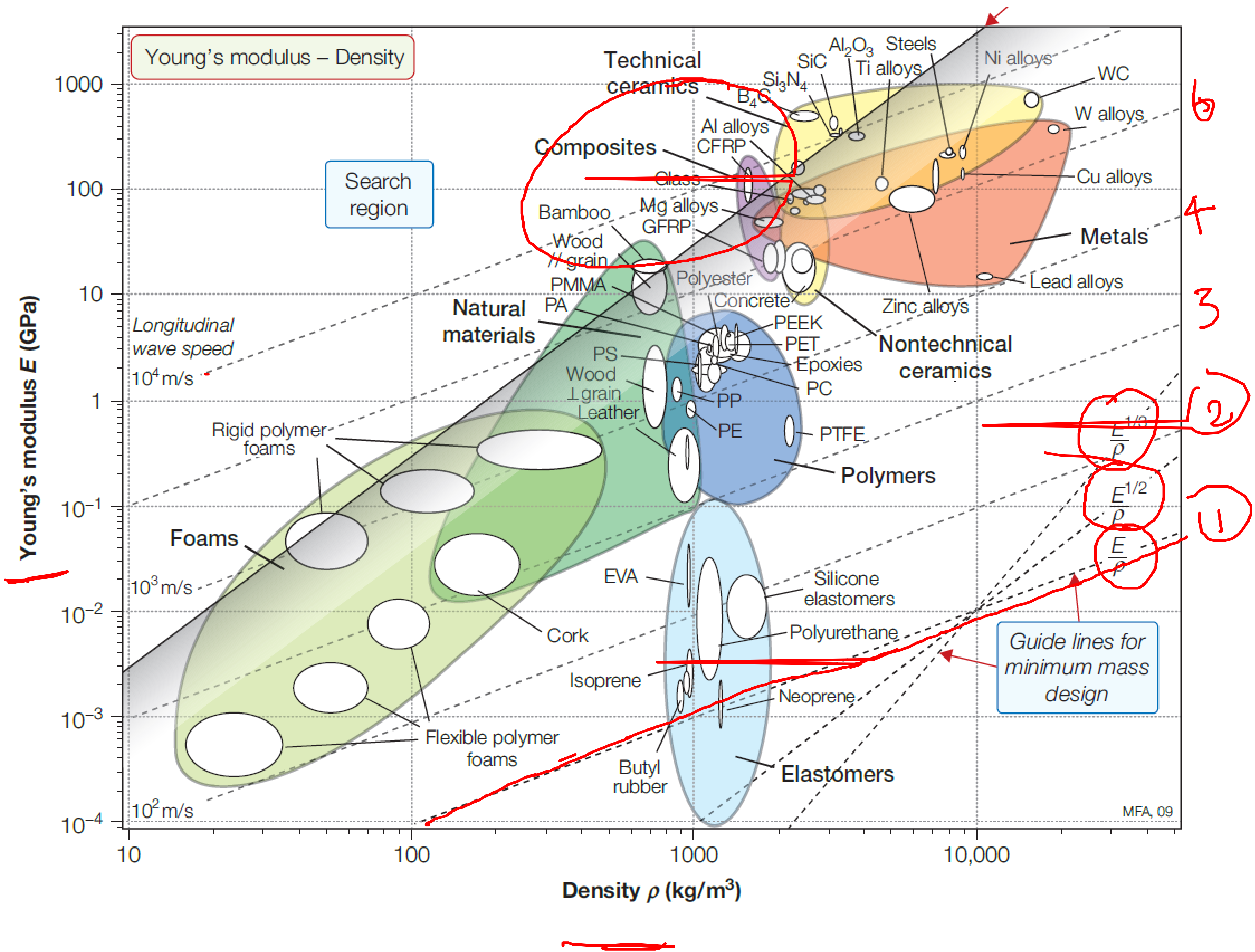
$$m \ge \underbrace{(F^*)}_{\text{Functional constraint}}\underbrace{(L)}_{\text{Geometric constraint}}\underbrace{\left(\frac{\rho}{\sigma_f}\right)}_{\text{Material properties}}$$

Material Index:

$$M_{t1} = \frac{\sigma_f}{\rho}$$



$M_p = E^{1/3}/\rho$
$M_b = E^{1/2}/\rho$
$M_t = E/\rho$
Modulus – Density
Guide lines for minimum mass design
Ceramics
Metals
Composites
Polymers
Natural materials
Foams
Elastomers
1
2
3
MFA, 09
1000
100
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Young's modulus E (GPa)
10
100
1000
10,000
Density ρ (kg/m³)



Young's modulus – Density
Search region
Technical ceramics
Composites
Natural materials
Foams
Metals
Nontechnical ceramics
Polymers
Elastomers
Al$_2$O$_3$
Steels
SiC
Ti alloys
Si$_3$N$_4$
Ni alloys
WC
B$_4$C
W alloys
Al alloys
CFRP
Cu alloys
Glass
Mg alloys
GFRP
Bamboo
Wood // grain
Polyester
Concrete
PMMA
PA
PEEK
PET
Epoxies
PC
PS
Wood ⊥grain
Leather
PP
PE
PTFE
Zinc alloys
Lead alloys
Rigid polymer foams
Longitudinal wave speed
10^4 m/s
10^3 m/s
10^2 m/s
EVA
Silicone elastomers
Polyurethane
Cork
Isoprene
Neoprene
Butyl rubber
Flexible polymer foams
$\frac{E^{1/3}}{\rho}$
$\frac{E^{1/2}}{\rho}$
$\frac{E}{\rho}$
Guide lines for minimum mass design
MFA, 09
Young's modulus E (GPa)
Density ρ (kg/m^3)
1000
100
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10
100
1000
10,000
6
4
3
2
1

Table 5.2 Design Requirements for the Light, Strong Tie

Function	Tie rod
Constraints	Length L is specified (geometric constraint) Tie must support axial tensile load F^* without failing (functional constraint)
Objective	Minimize the mass m of the tie
Free variables	Cross-section area A Choice of material