Engineering Electromagnetics

Lecture 29

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by

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Magnetism

- Magnetic phenomena → electric charges in motion
- magnetic material on an atomic scale → tiny currents: electrons orbiting around nuclei and spinning about their axes.
- For macroscopic purposes, these current loops are so small that we may treat them as magnetic dipoles.
- Ordinarily, they cancel each other out because of the random orientation of the atoms.
- When a magnetic field is applied → alignment of magnetic dipoles→ medium magnetically polarized/magnetized.
- electric polarization always in the same direction as E
- materials acquire a magnetization parallel to B (paramagnets) / opposite to B (diamagnets).
- Ferromagnets retain magnetization after field is removed
- Antiferrro/Ferri-magnet??

Magnetization

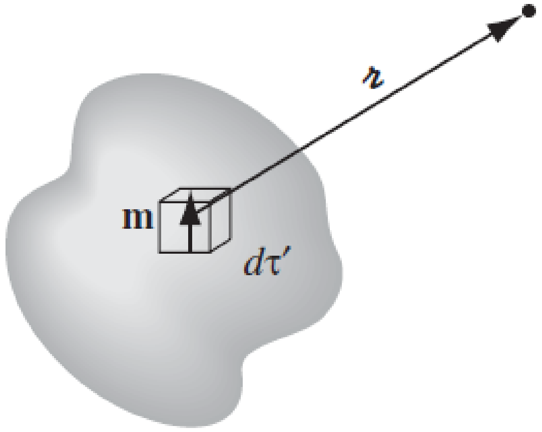
In the presence of a magnetic field, matter becomes *magnetized*; that is, upon microscopic examination, it will be found to contain many tiny dipoles, with a net alignment along some direction.

$$\mathbf{M} \equiv \textit{magnetic dipole moment per unit volume.}$$

M is called the **magnetization**; it plays a role analogous to the polarization **P** in electrostatics.

From Griffith book

Bound currents



Suppose we have a piece of magnetized material; the magnetic dipole moment per unit volume, **M**, is given. What field does this object produce? Well, the vector potential of a single dipole **m** is given by Eq. 5.85:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}. \tag{6.10}$$

In the magnetized object, each volume element $d\tau'$ carries a dipole moment $\mathbf{M}\,d\tau'$, so the total vector potential is (Fig. 6.11)

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{M}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,d\tau'. \tag{6.11}$$

That *does* it, in principle. But, as in the electrical case (Sect. 4.2.1), the integral can be cast in a more illuminating form by exploiting the identity

$$\nabla'\frac{1}{\mathscr{r}} = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.$$

With this,

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\left[\mathbf{M}(\mathbf{r}')\times\left(\nabla'\frac{1}{\mathscr{r}}\right)\right]d\tau'.$$

Integrating by parts, using product rule 7, gives

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left\{\int\frac{1}{\mathscr{r}}[\nabla'\times\mathbf{M}(\mathbf{r}')]\,d\tau' - \int\nabla'\times\left[\frac{\mathbf{M}(\mathbf{r}')}{\mathscr{r}}\right]d\tau'\right\}.$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{\mathscr{r}} [\nabla' \times \mathbf{M}(\mathbf{r}')] \, d\tau' + \frac{\mu_0}{4\pi} \oint \frac{1}{\mathscr{r}} [\mathbf{M}(\mathbf{r}') \times d\mathbf{a}'].$$

The first term looks just like the potential of a volume current,

$$\boxed{\mathbf{J}_b = \nabla \times \mathbf{M},}$$

while the second looks like the potential of a surface current,

$$\boxed{\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}},}$$

where $\hat{\mathbf{n}}$ is the normal unit vector. With these definitions,

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}} \, d\tau' + \frac{\mu_0}{4\pi} \oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}} \, da'.$$

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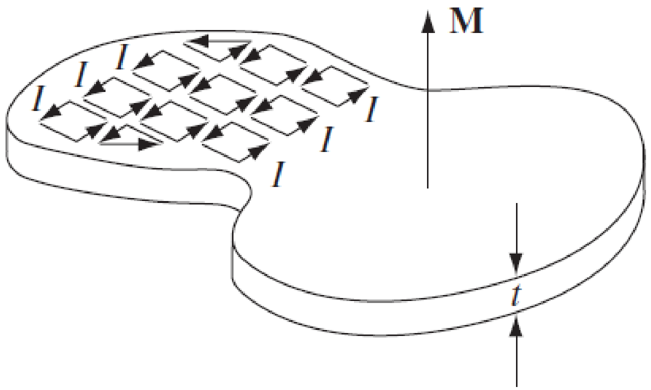
What this means is that the potential (and hence also the field) of a magnetized object is the same as would be produced by a volume current $\mathbf{J}_b = \nabla \times \mathbf{M}$ throughout the material, plus a surface current $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$, on the boundary.

$$\rho_b = -\nabla \cdot \mathbf{P}$$
$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}.$$

From Griffith book

Physical Interpretation of Bound Currents

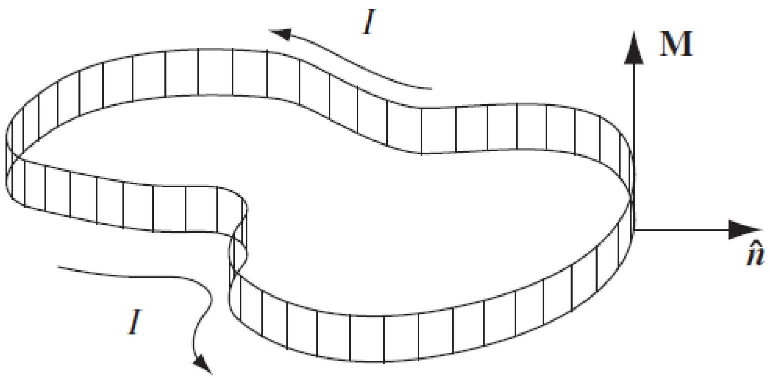
- the field of a magnetized object is identical to the field that would be produced by a certain distribution of "bound" currents, J_b and K_b.
- How these bound currents arise physically?



- a thin slab of uniformly magnetized material, with the dipoles represented by tiny current loops.

From Griffith book

- Notice that all the “internal” currents cancel: every time there is one going to the right, a contiguous one is going to the left. However, at the edge there is no adjacent loop to do the canceling. The whole thing, then, is equivalent to a single ribbon of current I flowing around the boundary.



From Griffith book

Thank You