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CS 2009 Theory of Computation - Mid-Sem - 06-Mar-2024 - 11.30-13.00 - 25 marks

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0. What is your source of preparation (Class notes, Scribe, Text book, Internet). If text book, mention the name...

1 Light Dose

1. (2 marks) Mention a regular expression for each of the following regular languages defined over $\{a, b\}$.
   1. Strings beginning with 'a' ending with 'b'. $a(a+b)^*b$ ✓
   2. Strings containing 'aba' as a substring. $(a+b)^*(aba)(a+b)^*$ ✓
   3. Strings having even number of b's. $(a^*(bb)^*a^*)^*$ ✓
   4. Strings having even number of a's and b's. $((aa+bb)+(ab+ba)(aa+bb)^*(ab+ba))^*$ ✓

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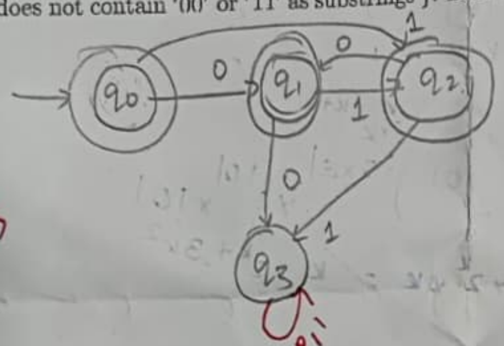
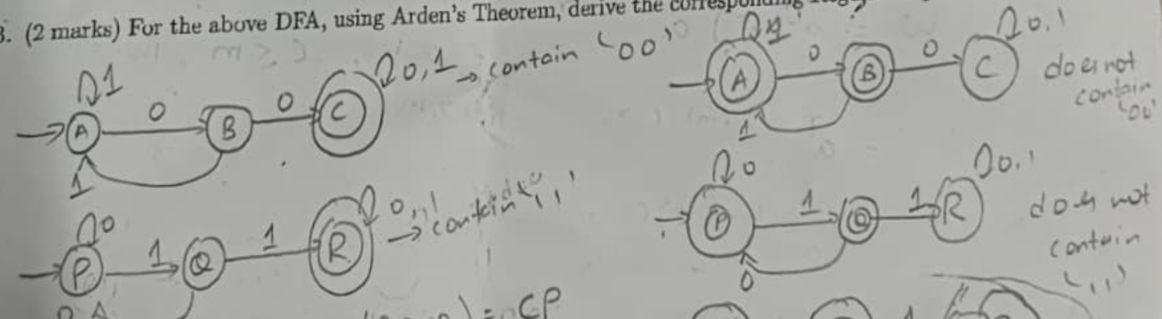
2. (2 marks) Draw a Deterministic Finite Automaton (DFA) for the language $L = \{x \mid x \in \{0,1\}^*$ and x does not contain '00' or '11' as substrings $\}$. DFA must be complete and well-defined for all transitions.



1.75

3. (2 marks) For the above DFA, using Arden's Theorem, derive the corresponding Regular Expression.

0



δ(AP,0) = BP
δ(AP,1) = AQ
δ(BP,0) = CP
δ(BP,1) = AQ
δ(CP,0) = CP
δ(CP,1) = CQ
δ(AQ,0) = BP
δ(AQ,1) = AR
δ(BQ,0) = CP
~~δ(BQ,1)~~ = AR
δ(CQ,0) = CP
δ(CQ,1) = CR
δ(AR,0) = BR
δ(AR,1) = AR
δ(BR,0) = CR
δ(BR,1) = AR
δ(CR,0) = CR
δ(CR,1) = CR

'BQ' does not have any incoming lines so outline the part is rejected

simplifying we get the the DFA BP & (AQ) and ...

4. (2 marks) Define the Pumping Lemma for Regular sets using First Order Logic. Also, its contrapositive. Be precise and ensure all variables used in the expression are well defined.

Pumping lemma:- $\exists n \forall w (|w| \geq n \rightarrow \exists u \exists v \exists z (\cancel{|w| \geq n}\ w = uvz \ |uv| \leq n \ |v| \geq 1 \wedge \forall i (i \geq 0 \rightarrow (uv^iz \in L))))$ Regular

Contra positive :- $\forall n \exists w (|w| \geq n \wedge \forall u \forall v \forall z (|w| \geq n \ |uv| \leq n \ |v| \geq 1 \rightarrow \exists i (i \geq 0 \wedge (uv^iz \notin L))))$ L is not regular

1.5 $w = uv^iz$, $w \in L$.

5. (2 marks) Let $\Sigma = \{0, 1, 2\}$, $Q = \{q_0, \cdots, q_{k-1}\}$. How many different DFAs exist on Q over Σ. Similarly, count the number of NFAs.

No. of DFA's $= |Q|^{|Q \times \Sigma|} \times 2^{|Q|} \times |Q|$.

$|Q| = k$, $|Q \times \Sigma| = k \times 3 = 3k$

No. of DFA's $= k \times k^{3k} \times 2^k = 2^k k^{3k+1}$

2 No. of NFA's $= (2^{|Q|})^{|Q \times \Sigma|} \times 2^{|Q|} \times |Q|$

$= (2^k)^{3k} \times 2^k \times k = k \times 2^{k + 3k^2}$.

$(k \times 3 \times 3) \times k$ $k^{3 \times k}$

2 Medium Dose

1. (3 marks) Let $\Sigma = \{a\}$. Prove that $L = \{a^{m!} \mid m \geq 1\}$ is non-regular using pumping lemma.

$\forall m$ $w = a^r (a^c)^i a^{m! - c - r}$, $c \geq 1$, $c \leq m$, $|w| = m!$

$|w| \geq m$.

$= a^{r + ic + m! - c - r}$

$= a^{m! + (i-1)c}$

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put $\underline{i = 2}$.

$m! + (i-1)c = m! + c$

$m! < m! + 1 \leq m! + c \leq m! + m < (m+1)!$

$m! + c \neq k!$

where $k > m$.

so L is non-regular.

2. (3 marks) Draw a DFA with minimum number of states for the language defined over $\Sigma = \{1\}$. Intuitively, argue that the DFA constructed is represented by the regular expression $(1111 + 111)^*$. Intuitively, argue that the DFA constructed is minimum.

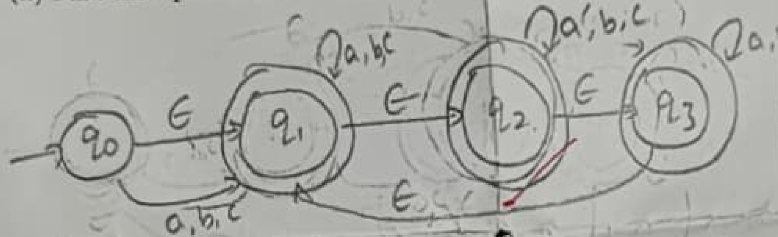
$(\epsilon, 1^3, 1^4, 1^6, 1^7, 1^8, 1^9 \ldots)$ everthing after 1^6 should be accepted

1

If we take five states (or) less than that we have connect q_4, 1 to (q_0, q_1, q_2, q_4). If we connect to q_0 the '111111' won't be accepted, similarly with q_2, if we connect q_4, 1 with q_2 then 1^8 will be accepted. If we connect q_4, 1 with q_3 then 1^7 won't be accepted. For 4 states 1^5 should not be accepted so If we connect to q_3, 1 to q_4 then 1^8 won't be accepted so the given DFA is minimum

2.75

3. (3 marks) For the language represented by $(a^*b^*c^*)^*$ on $\Sigma = \{a, b, c\}$, draw NFA with epsilon (FA should have at least 4 arcs on Epsilon). (i) Compute Epsilon Closure for each state in the drawn FA M (ii) Find the equivalent NFA (iii) Find the equivalent DFA



i) $E(q_0) = \{q_0, q_1, q_2, q_3\}$

$E(q_1) = \{q_1, q_2, q_3\}$

$E(q_2) = \{q_1, q_2, q_3\}$

$E(q_3) = \{q_3, q_1\} q_2$

ii) $\delta(q_0, a)$

$= E(\delta(E(q_0)), a))$

$= E(\delta(q_0, q_1, q_2, q_3), a))$

$= \{q_1, q_2, q_3\}$

$\delta(q_0, b)$

$= \cancel{E(\delta(q_0, q}$ $\{q_1, q_2, q_3\}$

$\delta(q_0, c) = \{q_1, q_2, q_3\}$

$\delta(q_1, a) = \{q_1, q_2, q_3\}$

$\delta(q_1, c)$

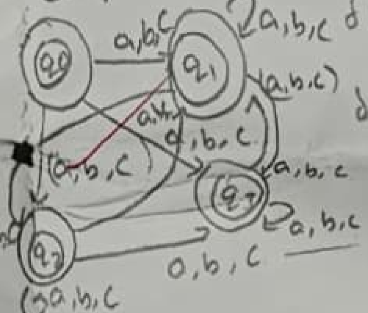
$\delta(q_2, c) = \{q_1, q_2, q_3\}$

$\delta(q_3, a) = \delta\{q_1, q_3\}$

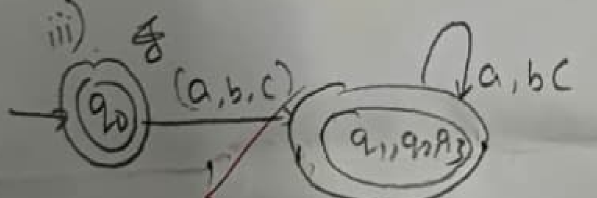
$\delta(q_3, b) = \{q_1, q_3\}$

$\delta(q_3, c) = \{q_1, q_3\}$

diagram



iii) δ



$\delta(q_0, a/b/c) = \{q_1, q_2, q_3\}$

$\delta(\{q_1, q_2, q_3\}, a/b/c) = \{q_1, q_2, q_3\}$

3 Strong Dose

1. (2 marks) Is $L = \{s = xwx \mid x, w$ are some non-empty strings over $\{a, b\}$ (I.e., $x, w \in (a+b)(a+b)^*$) $\}$. Is L regular. Present a neat justification (DFA or Pumping Lemma). Note: each string s in L can be decomposed into x, w, x where the first part and the last part are same. Ex: $abaa (x = a, w = ba)$, $bbaaabb (x = bb, w = aaa)$.

2. (4 marks) Let L be a language over $\{a, b\}$ with the property that all strings in L are of odd length. Draw two different DFAs. Also, find two different regular expressions (direct method or Arden's theorem).

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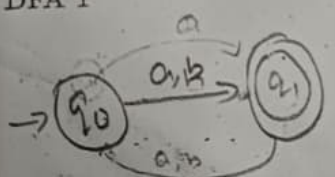
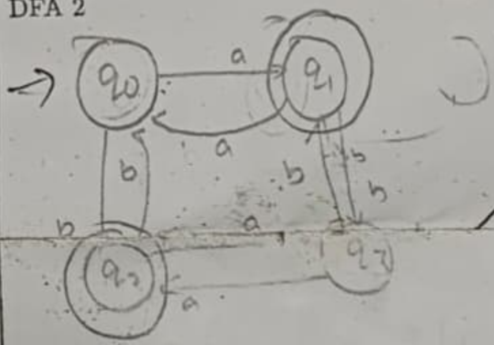
DFA 1	DFA 2
	
RE 1 $(a+b)(aa+bb+ab+ba)^* + (aa+bb+ab+ba)^*(a+b)$	**RE 2** $a(aa+bb+ab+ba)^* + (aa+bb+ab+ba)^*a + b(aa+ab+ba+bb)^*$

SPACE for ROUGH WORK

$(ab)(a)$