Engineering Optics

Lecture 7

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by

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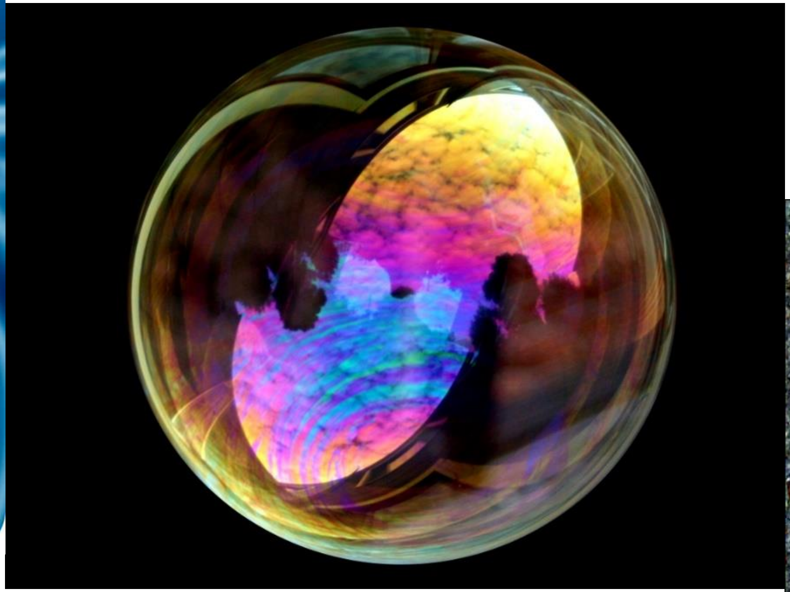
Interference

https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0/section/11.5/primary/lesson/wave-interference-ms-ps

Superposition of waves→ resultant wave



Soap bubble

https://simple.wikipedia.org/wiki/Interference#/media/File:Soap_bubble_sky.jpg

Oil on water

https://simple.wikipedia.org/wiki/Interference#/media/File:Soap_bubble_sky.jpg

Coherence: constant phase relationship

- Whenever the phase difference is constant, a stationary interference pattern is produced.
- The positions of the maxima and minima → depend on the phase difference
- Two sources which vibrate with a fixed phase difference between them are said to be **coherent**.'

Optics, Hecht

Constantly changing phase

- Changing phase difference → sometimes in phase, sometimes out of phase,
- No stationary interference can be observed,
- sources are said to be **incoherent**

Few points to note

- The wave theory for EM nature of light provides a natural basis from which to proceed.
- As we have seen, it obeys the important Superposition Principle.
- The resultant electric-field intensity **E**, at a point in space where two or more lightwaves overlap, is equal to the vector sum of the individual constituent disturbances.
- Optical interference corresponds to the interaction of two or more lightwaves yielding a resultant irradiance that deviates from the sum of the component irradiances.
- After being superimposed, the individual waves separate and continue on, completely unaffected by their previous encounter.

Interference

$$\psi(x, t) = A \sin(kx - \omega t + \varepsilon)$$

Light vector : $\vec{\mathbf{E}} \rightarrow E\ \mathrm{Sin}(\mathbf{k.r} - \omega t + \varepsilon)$

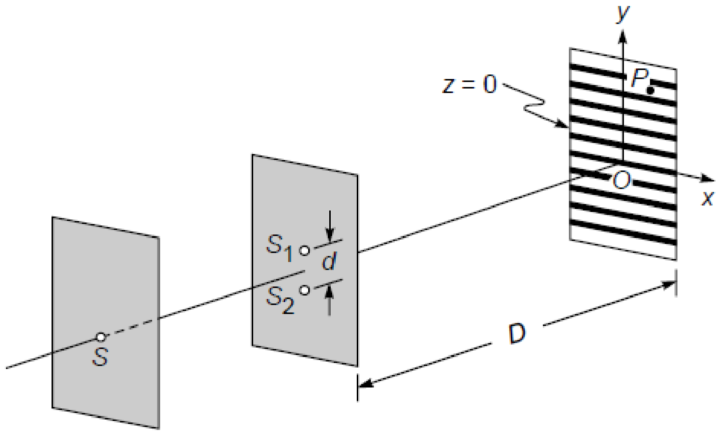


Fig. 14.6 Young's arrangement to produce interference pattern.

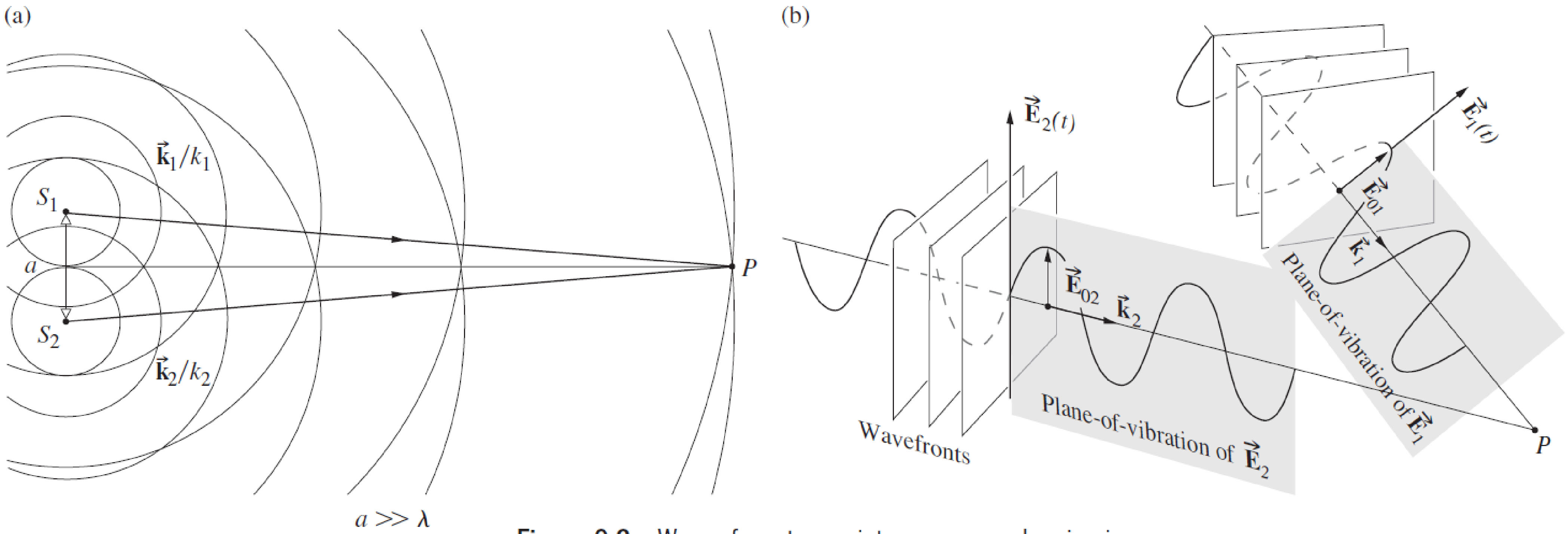


Figure 9.2 Waves from two point sources overlapping in space.

Optics, Hecht

Light vectors and interference

Wave 1

- $\mathbf{E_1} = E_1 \, \mathrm{Sin}(\mathbf{k_1.r} - \omega t + \varepsilon_1)$

- $\mathbf{E_2} = E_2 \, \mathrm{Sin}(\mathbf{k_2.r} - \omega t + \varepsilon_2) \longrightarrow$ Wave 2

- **Intensity→ Irradiance = $<E^2>_{\text{Time T}}$**

Resultant wave

Most important term → interefeerence

- $\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} \Rightarrow \quad \mathbf{E.E} = (\mathbf{E_1} + \mathbf{E_2}) \cdot (\mathbf{E_1} + \mathbf{E_2}) = \mathbf{E_1.\,E_1} + \mathbf{E_2.\,E_2} + 2\,\mathbf{E_1.\,E_2}$

Superposition principle

Because we're more interested in intensity

Interference equation

1st term : $E_1.E_1 = E_1^2 \sin^2(k_1.r-\omega t+\varepsilon)$

$$\frac{1}{T}\int_t^{t+T} E_1.E_1\, dt = \frac{1}{2}E_1^2$$

$$\therefore \boxed{I_1 = \frac{1}{2}E_1^2} \text{ — (1)}$$

Similarly $\boxed{I_2 = \frac{1}{2}E_2^2} \rightarrow$ 2nd term — (2)

3rd term : $2\vec{E_1}.\vec{E_2}$

$$= 2.\ E_1 \sin(\bar{k_1}.\bar{r}-\omega t+\varepsilon_1).\ E_2 \sin(\bar{k_2}.\bar{r}-\omega t+\varepsilon_2)$$

$$= 2E_1E_2\left\{\sin(\bar{k_1}.\bar{r}+\varepsilon_1-\omega t).\sin(\bar{k_2}.\bar{r}+\varepsilon_2-\omega t)\right\}$$

$$= 2E_1E_2\left\{\sin(\bar{k_1}.\bar{r}+\varepsilon_1)\cos\omega t - \cos(\bar{k_1}.\bar{r}+\varepsilon_1)\sin\omega t\right\}\times$$
$$\left\{\sin(\bar{k_2}.\bar{r}+\varepsilon_2)\cos\omega t - \cos(\bar{k_2}.\bar{r}+\varepsilon_1)\sin\omega t\right\} \text{ — (3)}$$

Interference equation

Take the time average $\Rightarrow \frac{1}{T}\int_t^{t+T} \sin^2\omega t \, dt \ (\text{or } \cos^2\omega t \, dt) = \frac{1}{2}$

$\langle \sin^2 \omega t \rangle_T$

$\langle \sin\omega t \cos\omega t \rangle_T = 0$

equation (3)

$$\langle 2 \vec{E}_1 \cdot \vec{E}_2 \rangle_T = 2 E_1 E_2 \Big[\frac{1}{2} \sin(\vec{k}_1 \cdot \bar{r} + \varepsilon_1) \sin(\bar{k}_2 \cdot \bar{r} + \varepsilon_2) + \frac{1}{2} \cos(\bar{k}_1 \cdot \bar{r} + \varepsilon_1) \cos(\bar{k}_2 \cdot \bar{r} + \varepsilon_2)$$

$$I_{12} = E_1 E_2 \cos(\bar{k}_1 \cdot \bar{r} + \varepsilon_1 - \bar{k}_2 \cdot \bar{r} - \varepsilon_2)$$

$$= \sqrt{2I_1} \cdot \sqrt{2I_2} \cos\delta \quad \text{where } \delta = \bar{k}_1 \cdot \bar{r} - \bar{k}_2 \cdot \bar{r} + \varepsilon_1 - \varepsilon_2$$

$$\boxed{I_{12} = 2\sqrt{I_1 I_2} \cos\delta}$$

$$\therefore \boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta}$$

Phase difference and interference

total irradiance is $$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

when $\cos\delta = 1, \;\; \delta = 0, \pm 2\pi, \pm 4\pi, \ldots$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

total constructive interference

disturbances are *in-phase.*

At $\delta = \pi/2, \cos\delta = 0,$

$$I = I_1 + I_2$$

When $0 < \cos\delta < 1$

$$I_1 + I_2 < I < I_{\max}$$

constructive interference

minimum irradiance

when $\cos\delta = -1, \;\; \delta = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

total destructive interference

Ranges

$0 > \cos\delta > -1$

$$I_1 + I_2 > I > I_{\min}$$

destructive interference

Interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

What will happen if

$$I_1 = I_2 = I_0.$$

$$I = 2I_0(1 + \cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

$$I_{\min} = 0$$

$$I_{\max} = 4I_0$$

Problem:1

A propagating wave at time $t = 0$ can be expressed in SI units as

$$\psi(y, 0) = 0.030 \cos\left(\frac{\pi y}{2.0}\right).$$

The disturbance moves in the negative y-direction with a phase velocity of 2.0 m/s. Write an expression for the wave at a time of 6.0s. What is the Time period?

Answer:

Given that:

$$\psi(y,0) = 0.030\cos\left(\frac{\pi y}{2.0}\right) \qquad (1)$$

A wave Equation can be written in the form (here displacement is in terms of *y*)

$$\psi(y,t) = A cos k(y + vt)] \qquad (2)$$

Time period τ?

temporal velocity: $v = \nu\lambda$

$$\Rightarrow v = \frac{\lambda}{\tau} \qquad \Rightarrow \tau = \frac{\lambda}{v}$$

Given that $\lambda = 4.0\ m$ and $v = 2.0\ m/s$ $\quad \Rightarrow \tau = \dfrac{4.0\ m}{2.0\ m/s} = 2.0\ s$

Eq 3 can be written as:

$$\psi(y,t) = 0.030cos2\pi\left(\frac{y}{4} + \frac{t}{2}\right) \qquad (4)$$

At $t = 6$ $\qquad \psi(y,6) = 0.030cos2\pi\left(\dfrac{y}{4} + 3\right)$

Problem:2

Consider a point P such that $S_2P - S_1P = \frac{\lambda}{3}$. Find the ratio of the intensity at point P to that at a maximum.

Answer:

Assume that $S_1P = acos\omega t$ and $S_2P = \text{acos}(\omega t - \frac{2\pi}{3})$

Let $I_1 = I_2$ be the intensities of two waves with $\delta = 2\pi/3$

Then the resultant intensity is,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Then the resultant intensity is,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(2\pi/3)$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(2\pi/3)$$

$$I = I_1 \quad (\, since\ I_1 = I_2\,)$$

Maximum occurs when $\delta = 0, \pm 2\pi$ ...

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Then $\quad I_{max} = 4I_1$

The intensity is therefore one-fourth of the intensity at the maxima

Conditions for Interference

- Same frequency
- Clearest pattern → amplitudes are almost same
- White lights from 2 sources → red with red, green with green etc.
- Sources → same initial phase? → not necessary
- Can have a phase difference (δ) → δ should not change with time
- If δ between S1 and S2 = constant → ***Coherent sources***

Interference *by* division of wavefronts

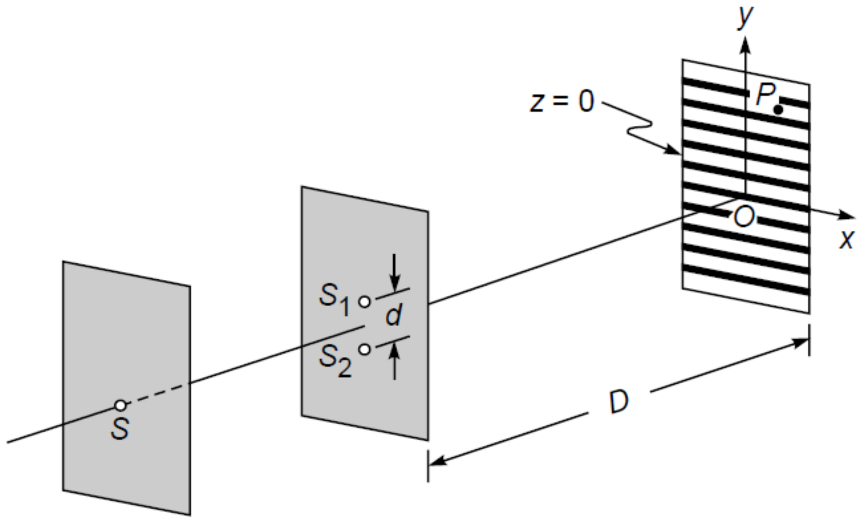
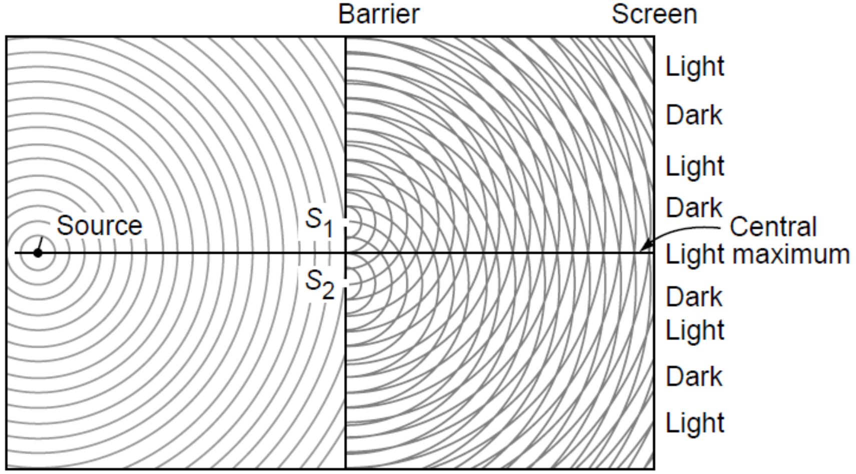
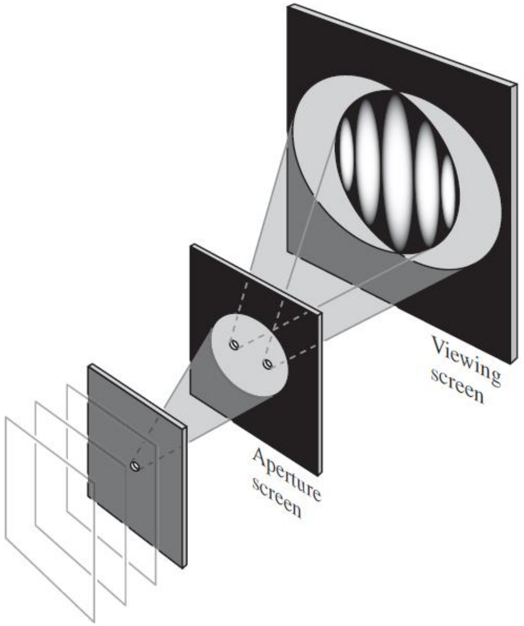


Fig. 14.6 Young's arrangement to produce interference pattern.



$(r_1 - r_2) = m\,\lambda$		**Maxima**
	$=(m+\frac{1}{2})\lambda$	**Minima**



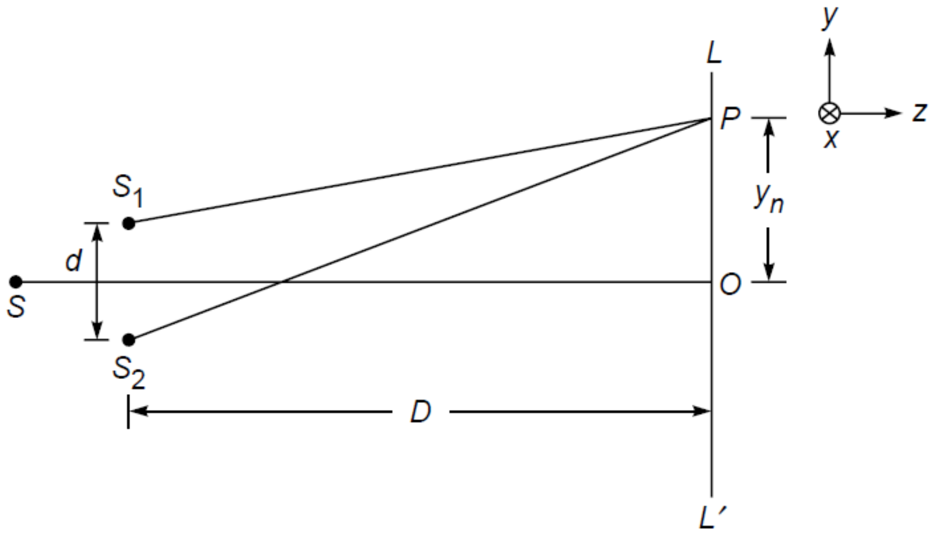


Fig. 14.8 Arrangement for producing Young's interference pattern.

Optics, by A. Ghatak

Young's double slit experiment

For an arbitrary point P (on line LL') to correspond to a maximum, we must have

$$S_2P - S_1P = n\lambda \qquad n = 0, 1, 2, \ldots$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y_n - \frac{d}{2}\right)^2\right] = 2y_nd$$

$$\boxed{y_n = \frac{n\lambda D}{d}}$$

$S_1S_2 = d$ and $OP = y_n$

Thus

$$S_2P - S_1P = \frac{2y_nd}{S_2P + S_1P}$$

If y_n, $d << D$,
$S_2P + S_1P \approx 2D$

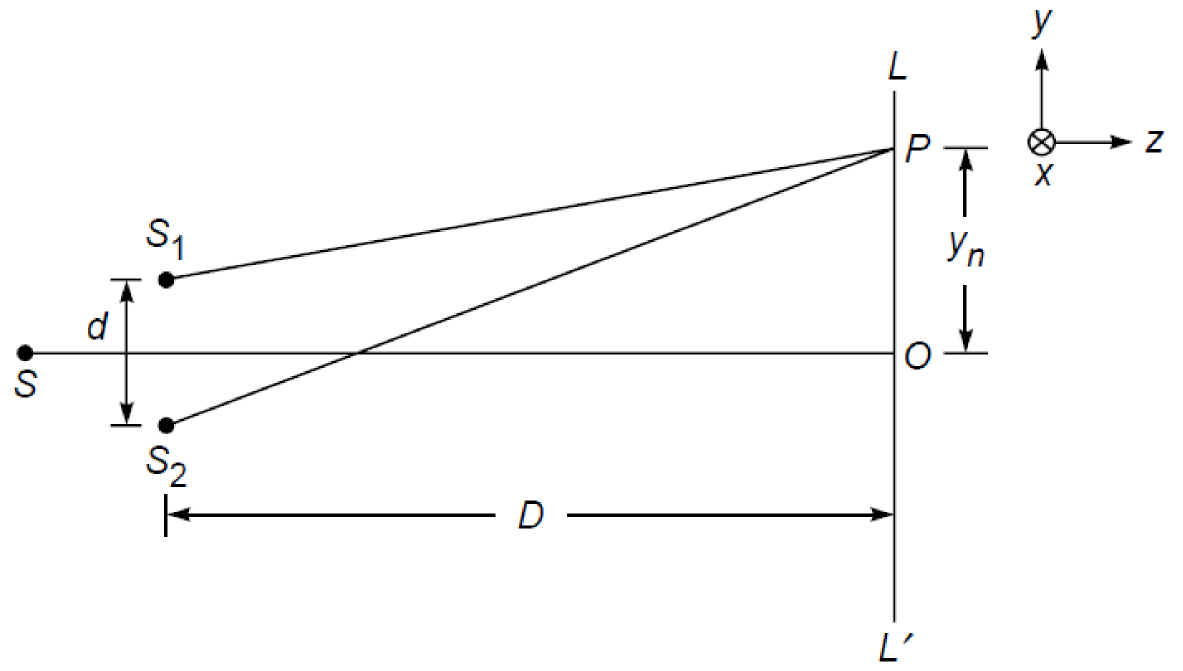


Fig. 14.8 Arrangement for producing Young's interference pattern.

distance between two consecutive bright fringes

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

fringe width $\beta = \frac{\lambda D}{d}$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\delta$$

$$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

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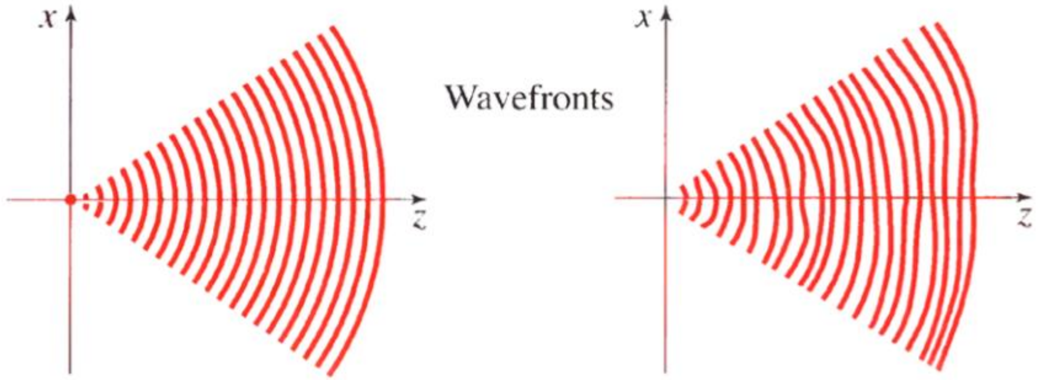
Coherence

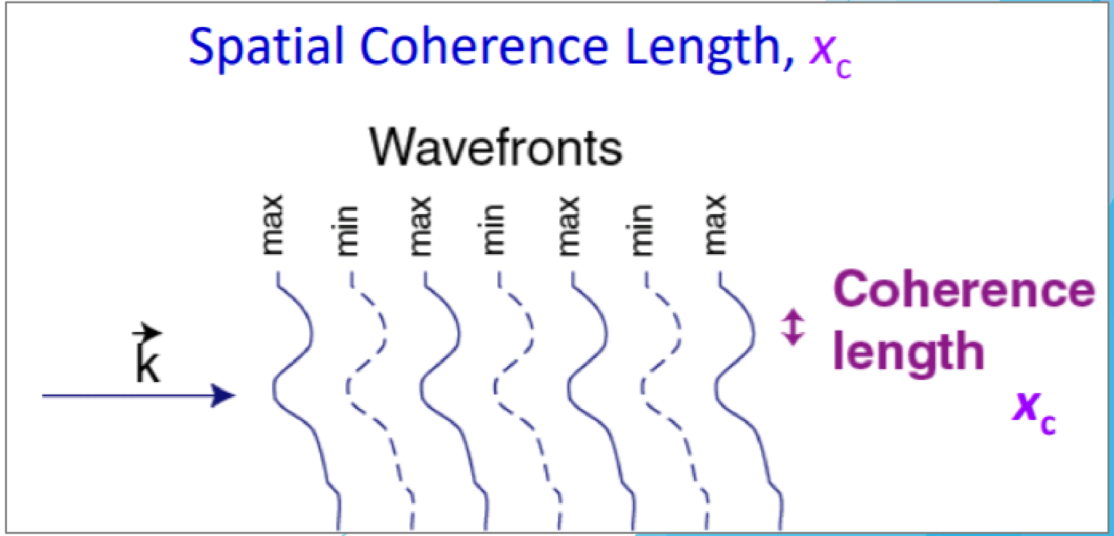
Coherence

Incoherence

A measure of the phase correlation at different temporal and spatial points on a wave.

Spatial Coherence: at different points (transverse to k)→ how uniform the phase of a wavefront is

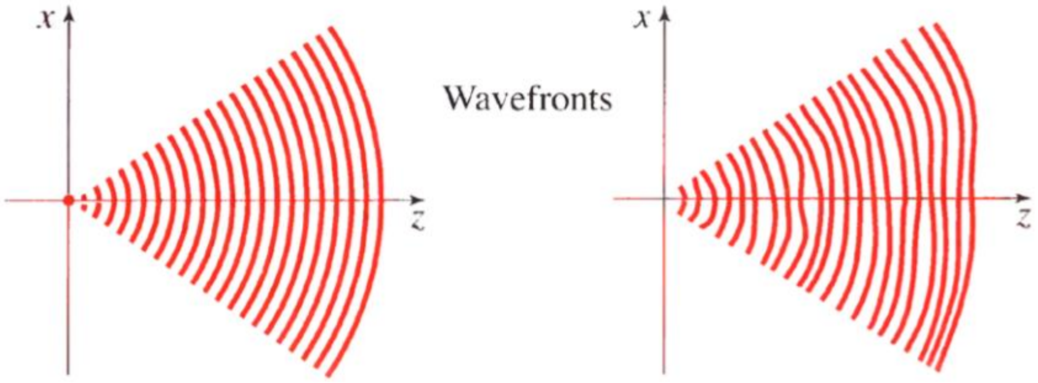




Pictures taken from web

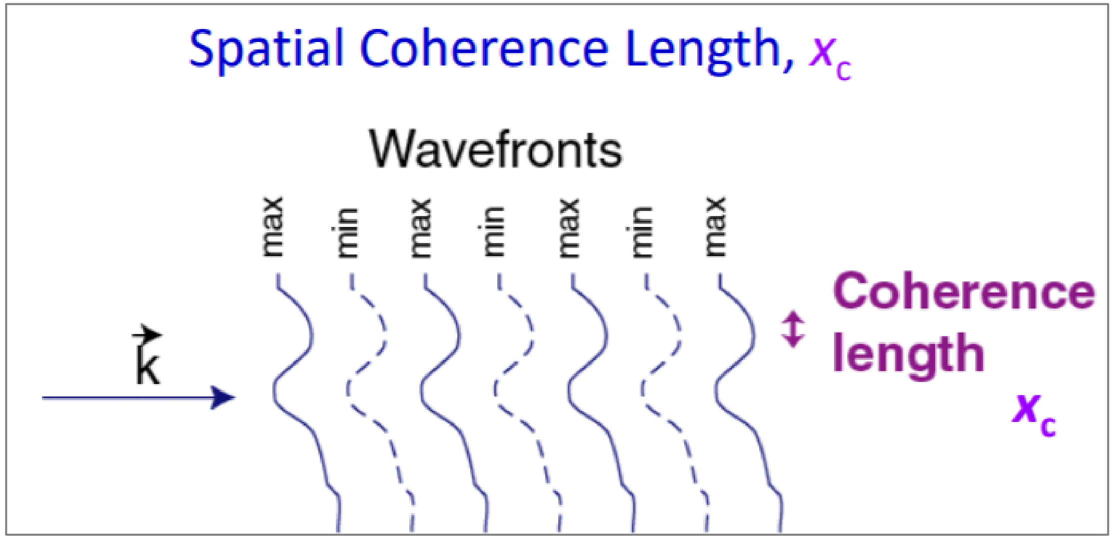
Spatial coherence

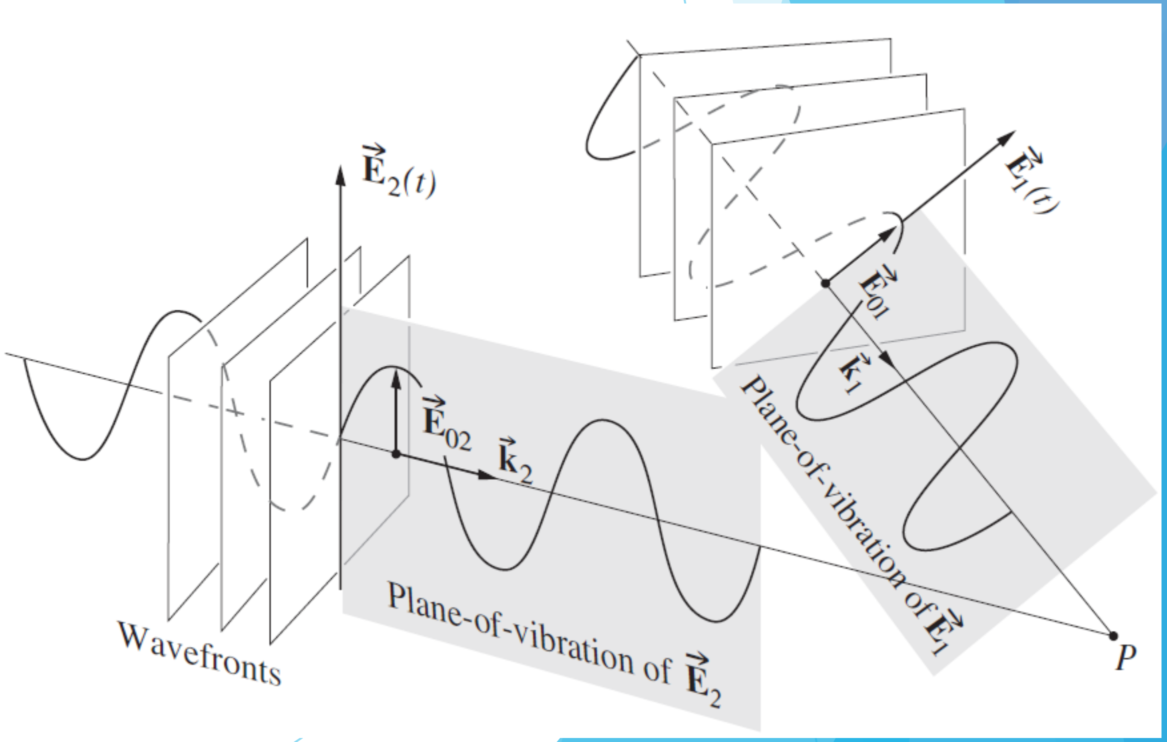
Spatial Coherence: at different points (transverse to k)→ how uniform the phase of a wavefront is



$$\mathbf{E_1} = E_1\ \mathrm{Sin}(\mathbf{k_1.r} - \omega t + \varepsilon_1)$$

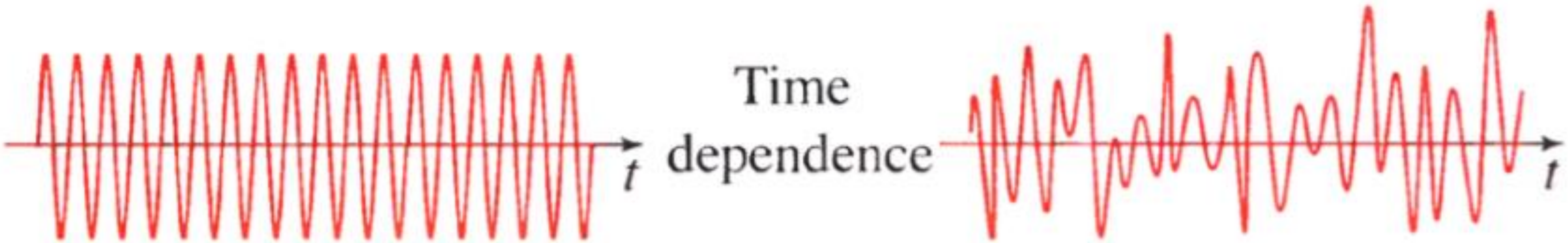
$$\mathbf{E_2} = E_2\ \mathrm{Sin}(\mathbf{k_2.r} - \omega t + \varepsilon_2)$$

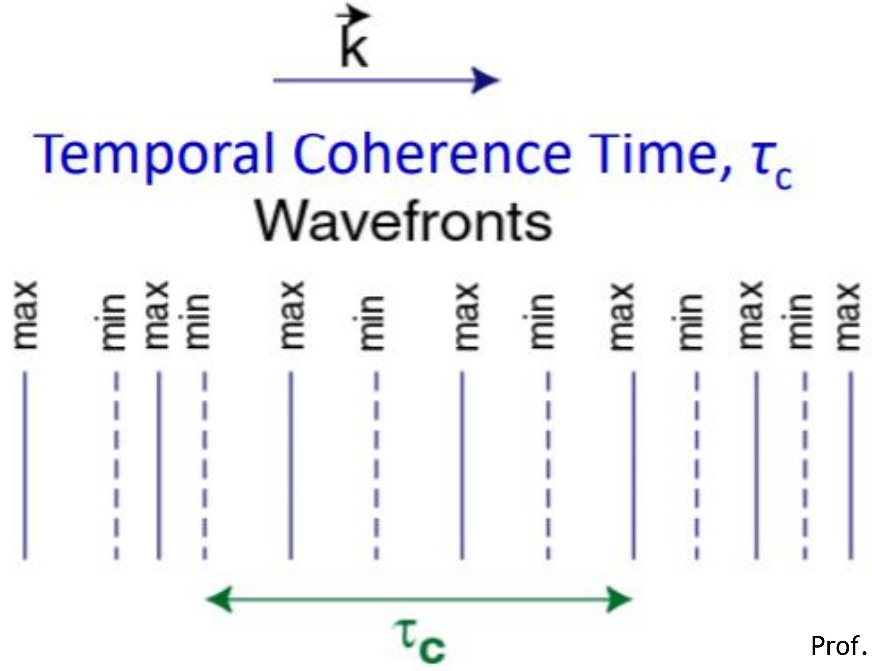




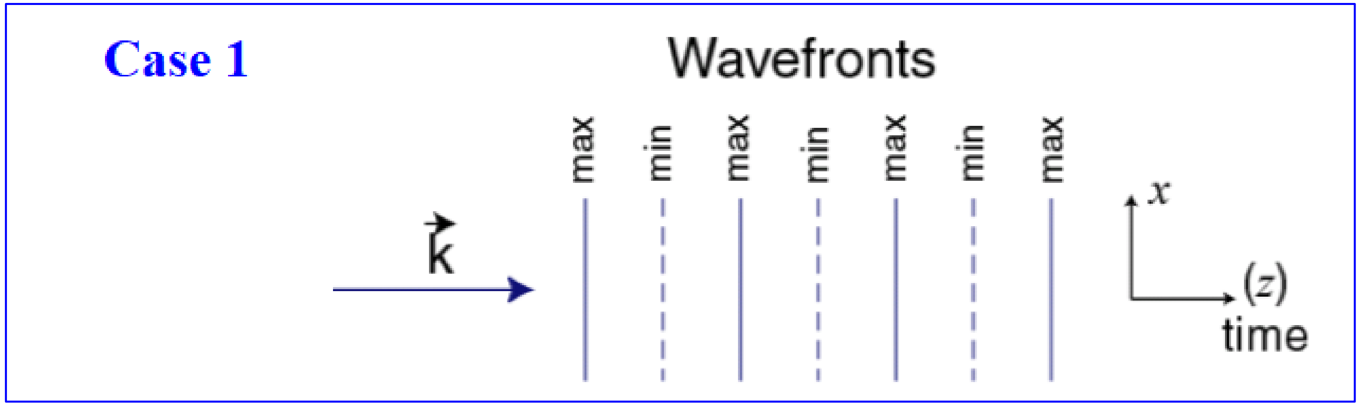
Temporal coherence

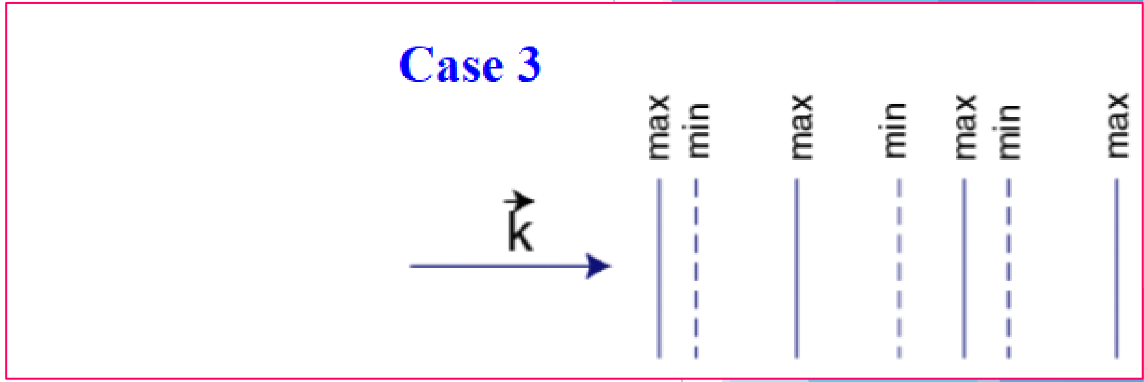
Phase correlation at different points along *k* – how monochromatic a source is

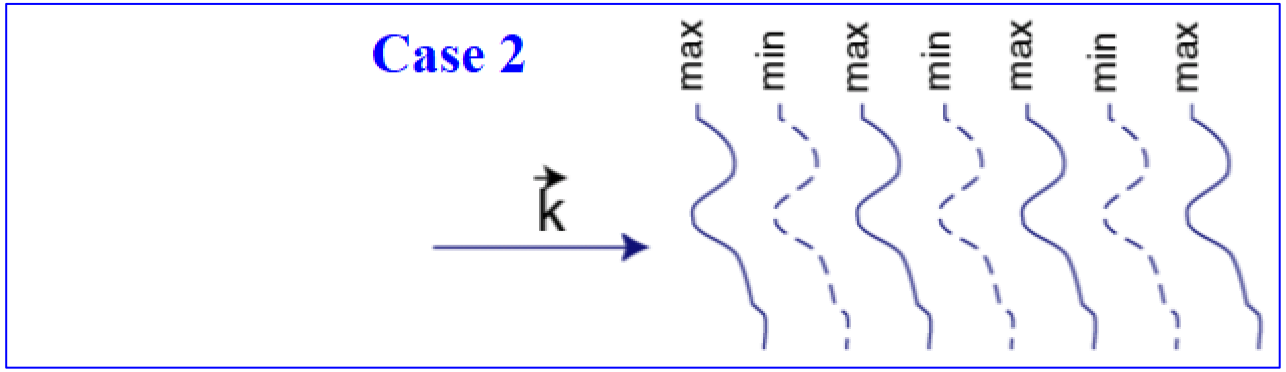


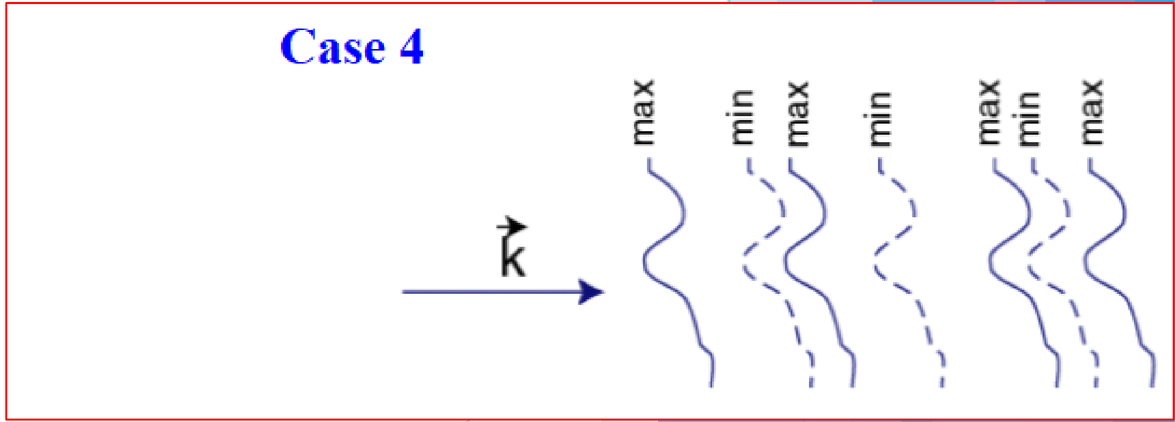


Spatial and temporal coherence









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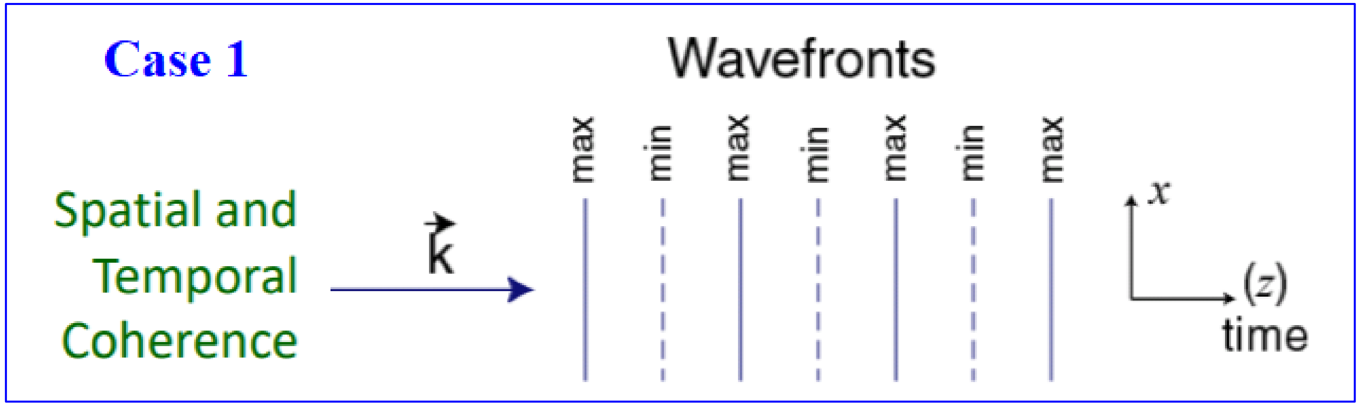

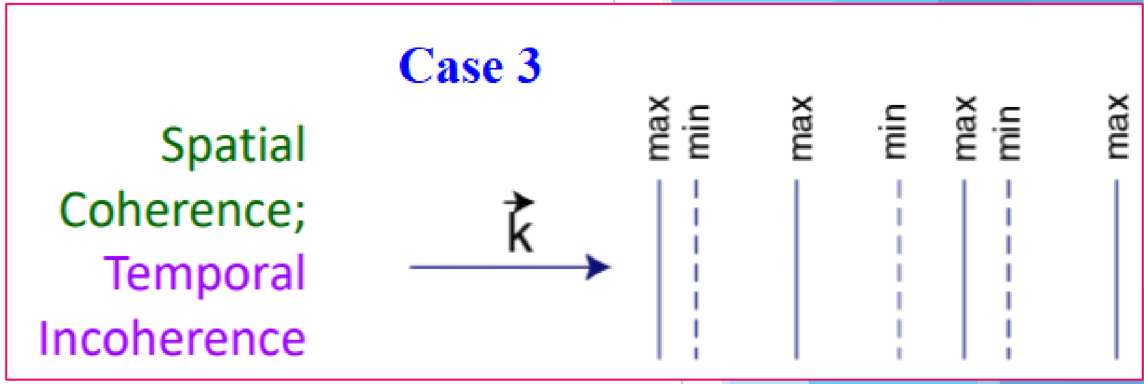

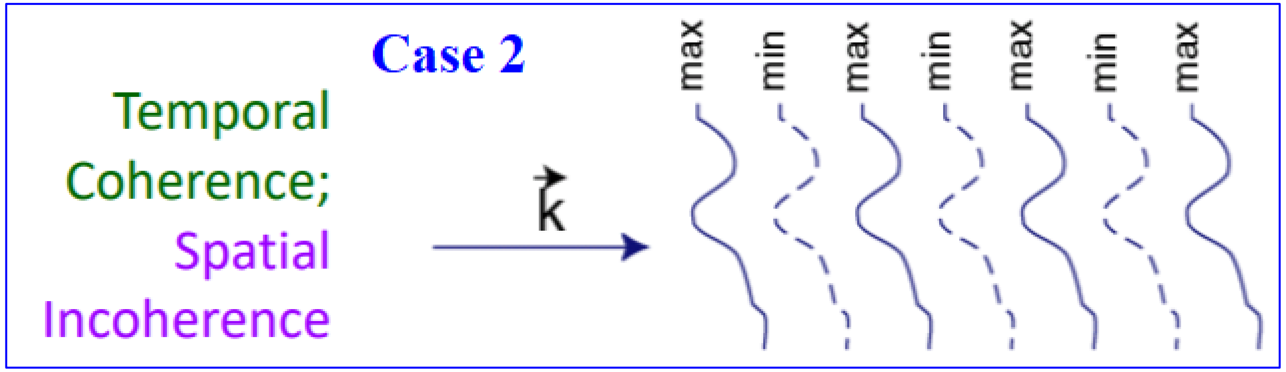

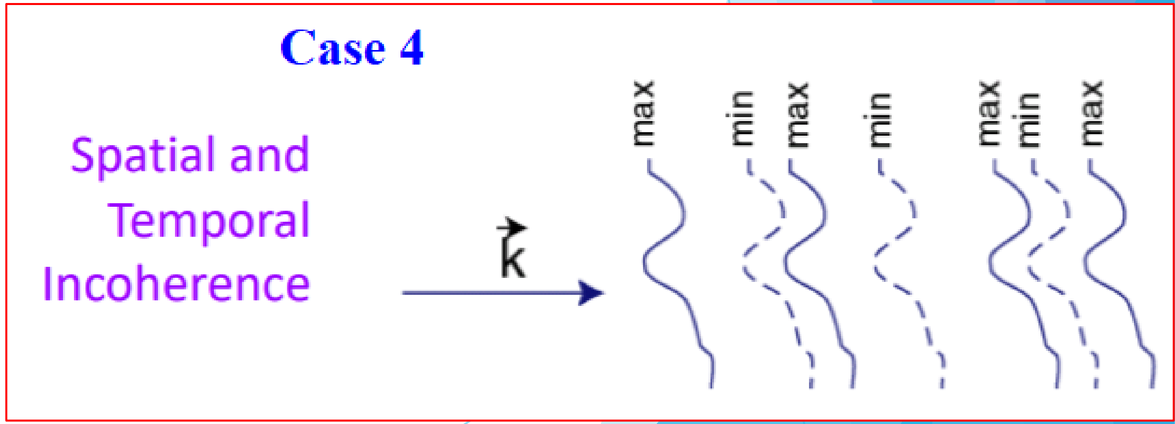

Thank You