Engineering Electromagnetics

Lecture 30

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by

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Bound currents

The first term looks just like the potential of a volume current,

$$\boxed{\mathbf{J}_b = \nabla \times \mathbf{M}}$$

while the second looks like the potential of a surface current,

$$\boxed{\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}}$$

where $\hat{\mathbf{n}}$ is the normal unit vector. With these definitions,

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}} \, d\tau' + \frac{\mu_0}{4\pi} \oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}} \, da'.$$

What this means is that the potential (and hence also the field) of a magnetized object is the same as would be produced by a volume current $\mathbf{J}_b = \nabla \times \mathbf{M}$ throughout the material, plus a surface current $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$, on the boundary.

$$\rho_b = -\nabla \cdot \mathbf{P}$$
$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$$

From Griffith book

Total current

In Sect. 6.2, we found that the effect of magnetization is to establish bound currents $\mathbf{J}_b = \nabla \times \mathbf{M}$ within the material and $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$ on the surface. The field due to magnetization of the medium is just the field produced by these bound currents. We are now ready to put everything together: the field attributable to bound currents, plus the field due to everything else—which I shall call the **free current**. The free current might flow through wires imbedded in the magnetized substance or, if the latter is a conductor, through the material itself. In any event, the total current can be written as

$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f.$$

There is no new physics in Eq. 6.17; it is simply a *convenience* to separate the current into these two parts, because they got there by quite different means:

free current is there because somebody hooked up a wire to a battery—it involves actual transport of charge; the bound current is there because of magnetization—it results from the conspiracy of many aligned atomic dipoles.

From Griffith book

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}) = \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}_f + (\nabla \times \mathbf{M})$$

$$\nabla \times \left(\frac{1}{\mu_0}\mathbf{B} - \mathbf{M}\right) = \mathbf{J}_f.$$

The quantity in parentheses is designated by the letter **H**:

$$\boxed{\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}.} \qquad \boxed{\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})}$$

In terms of **H**, then, Ampère's law reads

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}_f,}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{\text{enc}}}$$

From Griffith book

Susceptibility and permeability

$$\mathbf{M} = \chi_m \mathbf{H} \Longrightarrow \textbf{linear media}$$

The constant of proportionality χ_m is called the **magnetic susceptibility**; it is a dimensionless quantity that varies from one substance to another—positive for paramagnets and negative for diamagnets. Typical values are around 10^{-5} (see Table 6.1).

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi_m)\mathbf{H}, \text{ for linear media.}$$

Thus **B** is *also* proportional to **H**:

$$\mathbf{B} = \mu\mathbf{H},$$

where $\mu \equiv \mu_0(1 + \chi_m)$

μ is called the **permeability** of the material.[9] In a vacuum, where there is no matter to magnetize, the susceptibility χ_m vanishes, and the permeability is μ_0. That's why μ_0 is called the **permeability of free space**.

From Griffith book

Problem-1

- Calculate the magnetization of a magnetic material with length 5 cm and cross sectional area 2 cm^2 and the net magnetic moment of the material is 1Am^2.

Magnetization **M = m/V**
m=mag. moment =1 Am^2
and V=volume=Area x Length

Problem-2

- A coil of wire 0.25 m long and having 400 turns carriers a current of 15 A. Find magnitude of magnetic field strength. Compute the flux density, B, if the coil is in vacuum.
- $H = nI$
- I = 15 A
- n= N/L = 400/0.25
- $B = \mu_0 H = \mu_0 nI$

https://klus19.files.wordpress.com/2017/07/problems_co4.pdf

Problem-3

- The magnetic flux density within a bar of some material is 0.63 Tesla at an H field of 5 X 10^5 A/m. Compute the following for this material: (a) Magnetic permeability, (b) Magnetic susceptibility, (c) Type of magnetism ?

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Problem-3

- The magnetic flux density within a bar of some material is 0.63 Tesla at an H field of 5 X 10^5 A/m. Compute the following for this material: (a) Magnetic permeability, (b) Magnetic susceptibility, (c) Type of magnetism ?

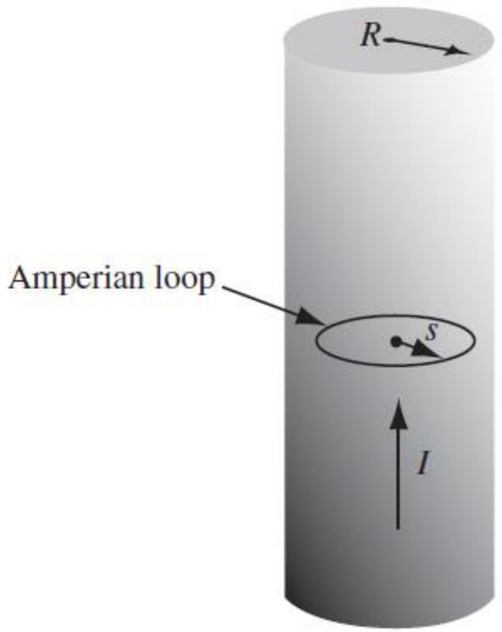
(a) Magnetic permeability, $\mu = \frac{B}{H} = \frac{0.63}{5X10^5} = 0.126\,X\,10^{-5}\,H/m$

(b) Magnetic susceptibility, $\chi = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \frac{0.126\,X\,10^{-5}}{4\,\pi X\,10^{-7}} - 1 = 1.003185 - 1 = 0.003185$

(c) Type of magnetism: Paramagnetism since the magnetic suscptibility is positive and low in magnitude.

Problem-2

Example 6.2. A long copper rod of radius R carries a uniformly distributed (free) current I (Fig. 6.19). Find $\mathbf{H}$ inside and outside the rod.



From Griffith book

Solution-2

Applying Eq. 6.20 to an Amperian loop of radius $s < R$,

$$H(2\pi s) = I_{f_{\text{enc}}} = I\frac{\pi s^2}{\pi R^2},$$

so, inside the wire,

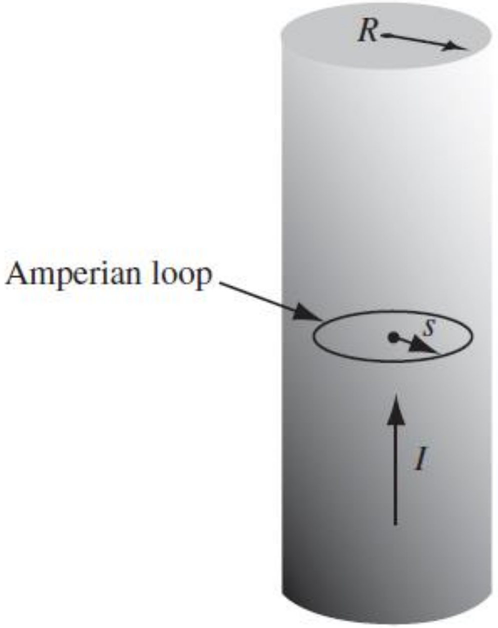
$$\boxed{\mathbf{H} = \frac{I}{2\pi R^2} s\,\hat{\boldsymbol{\phi}} \qquad (s \leq R).}$$

Outside the wire

$$\boxed{\mathbf{H} = \frac{I}{2\pi s}\,\hat{\boldsymbol{\phi}} \qquad (s \geq R).}$$

In the latter region (as always, in empty space) $\mathbf{M} = \mathbf{0}$, so

$$\mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi s}\,\hat{\boldsymbol{\phi}} \qquad (s \geq R),$$



From Griffith book

Thank You