Calculus Assignment 2

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1) Given $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

let $u(x) = \left(1+\frac{1}{x}\right)^x$

$\ln(u(x)) = x\ln\left(1+\frac{1}{x}\right)$

$\ln(u(x)) = \dfrac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$

Two possibilities to find the limit

L'Hopital rule:-

This states that for function f and g which are differentiable on an open interval I except possibly at a point c contained in I, if $\lim\limits_{x\to c} f(x) = \lim\limits_{x\to c} g(x) = 0$ (or) $\pm\infty$, $g'(x) \neq 0$ for all x in I with $x \neq c$ and $\lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$ exists, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

Here $c=\infty$, $f(x) = (1+\frac{1}{x})$, $g(x) = \frac{1}{x}$, which gives

$$\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = 1$$

(or)

using definition of the derivative :-

$$\lim_{x\to\infty}\frac{(\ln(1+\frac{1}{x}))}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\ln(1+\frac{1}{x}) - \ln(1+0)}{\frac{1}{x}-0}$$

let $\frac{1}{x} = y$

$\lim\limits_{x\to\infty}\frac{1}{x} = \lim\limits_{x\to 0}\frac{1}{x}$

$\Rightarrow \lim\limits_{y\to 0} y = 0$,

$$\Rightarrow \lim_{y\to 0} \frac{f(y)-f(0)}{y-0} \Rightarrow f'(0)=1$$

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with $f(y)=\ln(1+y)$ and $f'(y)=\frac{1}{1+y}$

we have :-

$$\lim_{x\to\infty}(\ln u(x)) = \lim_{x\to\infty}\frac{\ln(1+1/x)}{1/x} = 1$$

$$\exp(\lim_{x\to\infty}\ln(u x)) = \exp(1)$$

$$\Rightarrow e = \lim_{x\to\infty} u(x) \Rightarrow \lim_{x\to\infty}(1+1/x)^{x} = e$$

we conclude :-

$$\lim_{x\to\infty}(1+1/x)^{x} = e$$

2) Given function $f:[0,1]\to R$ is defined by

$$f(x)=\begin{cases} x & x \text{ is rational} \\ 1-x & x \text{ is irrational}\end{cases}$$

a) on $[0,1]$

for any rational $f(x)=x$ and for irrational $f(x)=1-x$

For rational number it is increasing on $[0,1]$ in case of irrational number $1-x$ will never be irrational number so their image will be unique and if $0<x_1<x_2<1$ then if x_1 & x_2 are rational then $f(x_2)>f(x_1)$ and if x_1 & x_2 are irrational $f(x)$

$f(x_1)>f(x_2)$

$\because$ The images are unique even if x is rational or irrational so

$\therefore$ f is one-one function on $[0,1]$

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b) For every rational number in {0,1] it attained as $f(x) = x$, for irrational and in con [0,1]

$f(x) = 1-x$ (for irrational number)

$f: [0,1] \longrightarrow [0,1]$

c) L.H.L

$f(x) = x$

$\lim_{x \to \frac{1}{2}^-} x = \frac{1}{2}$

RHL

$f(x) = 1-x = \frac{1}{2}$

$= \frac{1}{2}$

$\therefore$ it is continuous at $x = \frac{1}{2}$ and discontinous at every where as its has irrationals.

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3) Given $\lim_{n \to \infty} \frac{(1+\sin\pi x)^n - 1}{(1+\sin\pi x)^n + 1}$

$\Rightarrow \lim_{n \to \infty} x^n = \begin{cases} 0 & , \text{ if } x < 1 \\ 1 & , \text{ if } x = 1 \\ \infty & , \text{ if } x > 1 \end{cases}$

$\Rightarrow$ $1+\sin\pi x < 1$ $1+\sin\pi x = 1$ $1+\sin x > 1$

$L = -1$ $L = 0$ $\Rightarrow \lim_{n \to \infty} \frac{\left(1 - \frac{1}{(1+\sin\pi x)^n}\right)}{\left(1 + \frac{1}{(1+\sin\pi x)^n}\right)}$

$\Rightarrow L = 1$ $\left(\frac{1}{\infty} = 0\right)$

$\Rightarrow$ L.H.L = R.H.L = $f(x)$

at $x = 0$

L.H.L $\Rightarrow \frac{0-1}{0+1} = -1$

R.H.L $= 1$

at $x = 0$, $f(x)$ is discontinuous

Illy at $x = 1$

L.H.L $= 1$, R.H.L $= -1$ at $x = 1$, $f(x)$ is discontinuous

at $x = 2$

L.H.L $= -1$, R.H.L $= 1$ at $x = 2$, $f(x)$ is discontinuous

we have seen in $x \in (0, 2]$ as $\pi x \in [0, 2\pi]$ so, this will continous for 3, 4.... so, this is ~~limit~~ is discontinuous at $x = 0, 1, 2....$ //

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4) $f: [a,b] \rightarrow R$ and $g: [a,b] \rightarrow R$, both continuous on $[a,b]$ having same range $[0,1]$.

To prove, $f(c) = g(c)$, for $c \in [a,b]$

$\Rightarrow$ so, by intermediate value theorem, c exist, such that

$\Rightarrow f(c) = L$, [L is between $f(a)$ and $f(b)$]

similarly

$g(c) = K$, (K is between $g(a)$ and $g(b)$]

we must prove that is atleast one value in interval $[a,b]$ such that $K = L$

since $f(a) < g(a)$ so at some point

$\Rightarrow f(a) - g(a) < 0$ — ①

and at distinct point in interval, we have $\Rightarrow f(b) - g(b) > 0$ — ②

Now, let function, $h(c) = g(c) - f(c)$

as similarly to ① & ②, that $h(c) > 0$ at one point

$\Rightarrow h(c) < 0$ at another point

By intermediate value theorem, at some point on $h(c)$ must be equal to zero so,

$\Rightarrow h(c) = f(c) - g(c) = 0$

$\therefore f(c) = g(c)$

Hence $f(c) = g(c)$ for some $c \in [a,b]$

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5) Given function f such that $f(0)=0$ and f and $|f|$ both are differentiable for every $x \in R$

we can take Basic function $y=0$ as it is a constant function for every value of x, $y=0$ so, and as $f=|f|$ and The constant functions are differentiable as if $x \in [a,b]$

$$f'(x) = \frac{f(a)-f(b)}{a-b} = 0, \quad \text{as } f(a)=f(b)$$

$\therefore$ for every value of x, $f'(x)=0$//

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6) The example of a function which is not differentiable exactly at two points, is $f(x) = |x-1| + |x-2|$

$\Rightarrow$ lets see differentiability at $x = 1, 2$

Now at $x=1$ $\quad LHD = \lim_{x \to 1^-} [f(x) - f(1)/x-1$

$$LHD = \lim_{h \to 0} \frac{[f(1-h) - f(1)]}{-h} = \lim_{h \to 0} \frac{[|1-h-1| + |1-h-2| - |0| - |-1|]}{-h}$$

$$\Rightarrow \lim_{h \to 0} \left[\frac{2h}{-h}\right] = -2,$$

$$RHD = \lim_{x-1^+} \left[\frac{(f(x) - f(1)}{x-1}\right] \Rightarrow RHD = \lim_{h \to 0} \frac{(f(1+h) - f(1))}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{(h-h+1-1]}{h} \Rightarrow \lim_{h \to 0} [0/h] = 0$$

since $LHD \neq RHD$

so, Given function is not diffenetiable at $x = 1$

similarly, we can show that the given function is not differentiable at $x = 2$

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7) let a and b be the roots of the equation $e^x \cos x = 1$, then $e^a \cos a = 1$ and $e^b \cos b = 1$

let f be a function defined as $f(x) = e^{-x} - \cos x$

we observe that

i) $f(x)$ is continuous in $[a,b]$ and $\cos x$ both are continuous functions

ii) $f'(x) = -e^{-x} + \sin x$, hence function is differentiable in (a,b)

iii) $f(a) = e^{-a} - \cos a = e^{-a}(1 - e^a \cos a) = 0$ and

$f(b) = e^{-b} - \cos b = e^{-b}(1 - e^b \cos b) = 0 \quad \Rightarrow f(a) = f(b) = 0$

Thus $f(x)$ satisfies all the conditions of Rolle's theorem in $[a,b]$

hence there exists atleast one value of c in (a,b) such $f'(c) = 0$

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$f'(x) = -e^{-x} + \sin x$, $f'(c) = -e^{-c} + \sin c$

$\Rightarrow -e^{-c} + \sin c = 0 \Rightarrow \sin c = e^{-c}$

$\Rightarrow e^{c} \sin c - 1 = 0$ //

Thus, c is a root of the equation $e^{-x} - \sin x = 0$,

Hence between any two roots of equation $e^{x} \cos x = 1$, there exists atleast one root of $e^{x} \sin x - 1 = 0$ //

8) Given $\sqrt{5-2x^2} \leq f(x) \leq \sqrt{5-x^2}$ for $-1 \leq x \leq 1$,

Then $\lim_{x \to 0} f(x)$ exists if $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(x)$

Left hand limit = Right hand limit, limit is LHL (or) RHL //

$\lim_{x \to 0^-} \sqrt{5-2x^2} \leq \lim_{x \to 0^-} f(x) \leq \lim_{x \to 0^-} \sqrt{5-x^2}$, $x \to 0^-$, $x^2 \to 0$

$\Rightarrow \sqrt{5} \leq \lim_{x \to 0^-} f(x) \leq \lim_{x \to 0^-} \sqrt{5}$ By sandwitch theorum,

$\lim_{x \to 0^-} f(x) = \sqrt{5}$

For Right hand limit

$\lim_{x \to 0^+} \sqrt{5-2x^2} \leq \lim_{x \to 0^+} f(x) \leq \lim_{x \to 0^+} \sqrt{5-x^2}$ By sandwitch theorum

$\lim_{x \to 0^+} f(x) = \sqrt{5}$

$\sqrt{5-2(0)^2} \leq f(0) \leq \sqrt{5-0}$

$\therefore \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = \sqrt{5}$ As

$\Rightarrow f(0) = \sqrt{5}$

$\therefore \lim_{x \to 0} f(x)$ exists and its equal to $\lim_{x \to 0} f(x) = \sqrt{5}$ //

9) Name:- O.V.S. lokesh Reddy Roll :- EC20B1050

a) Given

$$\lim_{y\to 0} \frac{\sin(3y)\cot(5y)}{y\cot(4y)}$$

$\left[\lim_{y\to 0} \frac{\sin(3y)}{(y)} = 0/0\right]$ Thus 0/0 indefinite form

$$\Rightarrow \lim_{y\to 0}\left(\frac{\sin(3y)}{y}\right) \cdot \lim_{y\to 0} \frac{\cot(5y)}{\cot(4y)}$$

$$\Rightarrow \lim_{y\to 0}\left(\frac{\sin 3y}{y}\right) \cdot \lim_{y\to 0} \frac{\cos(5y)\sin(4y)}{\sin(5y)\cos(4y)}$$

$\left[\lim_{y\to 0}\cos(5y) = 1,\ \lim_{y\to 0}\cos(4y) = 1\right]$

$$\Rightarrow \lim_{y\to 0}\left(\frac{\sin 3y}{y}\right) \cdot \lim_{y\to 0}\left(\frac{\sin 4y}{\sin 5y}\right)$$

As, if $f(x)$ and $g(x)$ is equal to 0 or $\pm\infty$, And $h(x) = \frac{f(x)}{g(x)}$ then $f(x)/g(x)$ is indefinite form so, By L'Hospital rule $h(x) = \frac{f'(x)}{g'(x)}$ then

By L' Hospital rule

$$\Rightarrow \lim_{y\to 0}\left(\frac{3}{1}\cos(3y)\right) \cdot \lim_{y\to 0}\left(\frac{4}{5}\frac{\cos 4y}{\sin 5y}\right) \Rightarrow \frac{3\times 4}{5} \Rightarrow 2.4$$

$$\therefore \lim_{y\to 0} \frac{\sin(3y)\cot(5y)}{y\cot(4y)} = 2.4$$

b) Given $\lim_{x\to 0} 6x^2(\cot x)(\csc 2x)$.

Now $\lim_{x\to 0} 6x^2(\cot(x))\csc(2x) \Rightarrow \lim_{x\to 0} 6x^2 \frac{\cos(x)}{\sin(x)} \frac{1}{\sin(2x)}$

we know that $\sin(2x) = 2\sin x\cos x$

$$\Rightarrow \lim_{x\to 0} \cancel{6}^{3}x^2 \frac{\cancel{\cos(x)}}{\sin(x)} \frac{1}{\cancel{2}\sin(x)\cancel{\cos(x)}} \Rightarrow \lim_{x\to 0} \frac{3x^2}{\sin^2 x}$$

at $x = 0 \Rightarrow 0/0$, so we use 'L' Hospital Rule like above

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$\lim_{x\to 0} \frac{3x^2}{\sin^2 x} \Rightarrow \lim_{x\to 0} \frac{3x \cdot 2x\cdot x}{2x \sin x \cos x}$ $[\lim_{x\to 0} \cos x = 1]$

$\Rightarrow \lim_{x\to 0} \frac{3x}{\sin x} \Rightarrow \lim_{x\to 0} \frac{3}{\cos x} \Rightarrow 3$

$\therefore \lim_{x\to 0} 6x^2(\cot x)(\csc 2x) = 3$ //

c) Given $\lim_{t\to 0} \sin(\frac{\pi}{2} \cos(\tan t))$.

$\Rightarrow \sin(\lim_{t\to 0} \frac{\pi}{2} \cos(\tan t)) \Rightarrow \sin(\frac{\pi}{2} \lim_{t\to 0} \cos(\tan t))$

$\Rightarrow \sin(\frac{\pi}{2} \cos(\tan(\lim_{t\to 0} t)) \Rightarrow \sin(\frac{\pi}{2} \cos(\tan(0))) \Rightarrow \sin(\frac{\pi}{2} \cos(0))$

$\Rightarrow \sin(\frac{\pi}{2}) = 1$

$\therefore \lim_{t\to 0} \sin(\frac{\pi}{2} \cdot \cos(\tan t)) = 1$ //

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10) Given function $f: R \to R$ defined by

$$f(x) = \begin{cases} ax+2b, & x \le 0 \\ x^2+3a-b, & 0 < x \le 2 \\ 3x-5, & x > 2 \end{cases}$$

This continuous at every x

As f is continuous at $x = 0$

$\therefore \lim_{x\to 0^-} (ax+2b) = \lim_{x\to 0^+} (fx) \Rightarrow \lim_{x\to 0^-} (ax+2b) = \lim_{x\to 0^-} (x^2+3a-b)$

$\Rightarrow \therefore a(0) + 2b = 0 + 3a - b \Rightarrow a-b=0 \to (1)$

As f is continuous at $x=2$

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

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$$\lim_{x \to 2^{(-)}} x^2+3a-b = \lim_{x \to 2^+} 3x-5$$

$\Rightarrow (2)^2+3a-b = 3(2)-5 \Rightarrow 4+3a-b = 6-5$

$\Rightarrow 3a-b = -3 \rightarrow (2)$

(1) & (2) (substracting)

$a - b = 0$

$3a - b = -3$

$(-) \; (+) \quad (+)$

$-2a = 3 \Rightarrow a = -3/2$

then $-3/2 - b = 0$

$b = -3/2$

$\therefore a = -3/2, \; b = -3/2$ //

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1) let $h(x) = f(x) - g(x)$ and $h(x)$ is an continuous function as $g(x)$ & $f(x)$ is continuous

then By intermediate value theorum there may will an set of points for which let be A

$$h(x) = 0$$

$$g(x) = f(x)$$ and this set con't not be infinite

And lik if $A \in (a,b)$ $h(a) < 0$, $h(b) > 0$, then there will be some value to which $h(x) = 0$, so, The A is a closed set

then $S = \{x \in R ; f(x) = g(x)\}$ is a closed set in R.

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12) Given to prove

$$\frac{x}{1+x} < \ln(1+x) < x \quad \text{for all } x>0,$$

first part:-

let $f(x) = \ln(x+1) - \frac{x}{x+1}$, $x>0$

Differentiating $f(x)$ w.r.t x, we get

$$f'(x) = \frac{1}{1+x} - \frac{1}{(x+1)^2} \Rightarrow f'(x) = \frac{x}{(x+1)^2}$$

As $x>0$, so $f'(x)>0$ for all $x>0$ so, $f(x)$ is increasing function for all $x>0$ so,

$$f(x) > f(0)$$

$$\ln(x+1) - \frac{x}{x+1} > \ln(1) - 0$$

$$\Rightarrow \ln(x+1) > \frac{x}{x+1} \quad \text{— (1)}$$

second part:-

let $f(x) = \log(x+1)$, choose $a=0$ and $x>0$ so, that there is according to the mean value theorm an x_0 between a and x with

$$f'(x_0) = \frac{f(x)-f(a)}{x-a} \Leftrightarrow \frac{1}{1+x_0} = \frac{\log(x+1)}{x}$$

since

$$x_0>0, \Rightarrow \frac{1}{x_0+1} < 1$$

$$\Rightarrow 1 > \frac{1}{x_0+1} = \frac{\log(x+1)}{x} \Rightarrow x > \log(x+1) \quad \text{— (2)}$$

$\therefore$ from (1) & (2)

$$\frac{x}{1+x} < \ln(1+x) < x \quad \text{for all } x>0$$

Hence proved.

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13) Given that number 10 is divided into two parts such that their cubes is the least possible.

let two numbers be $x, 10-x$ then let

$$y = (x)^3 + (10-x)^3$$

As Given sum is least possible so $\frac{dy}{dt} = 0$

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 + 3(10-x)^2(-1)$$

$$0 = 3x^2 - 3(100-20x+x^2)$$

$$\Rightarrow \quad x = 5$$

then $\frac{d^2y}{dx^2}$ at $x=5 \Rightarrow \frac{d^2y}{dx^2} = 60$ (+ve)

As we know At $\frac{dy}{dx} = 0$, and $\frac{d^2y}{dx^2}$ is +ve then it gives minimum value at that x

so, The two numbers are $5, 10-5 \Rightarrow (5,5)$

Their cube sum $= 5^3 + 5^3 = 250$ //

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14) $\lim_{x \to 0} \frac{x(a+b\cos x) + c\sin x}{x^5} = \frac{1}{60}$

$\Rightarrow$ we know $\sin x = \frac{x}{\lfloor 1} - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} \cdots$

$$\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \frac{x^6}{\lfloor 6} + \cdots$$

$$\lim_{x \to 0} \frac{ax + bx\cos x + c\sin x}{x^5} = \frac{1}{60}$$

$$\Rightarrow \lim_{x \to 0} \frac{ax + bx - \frac{bx^3}{\lfloor 2} + \frac{bx^5}{\lfloor 4} \cdots + \frac{cx}{\lfloor 1} - \frac{cx^3}{\lfloor 3} + \frac{cx^5}{\lfloor 5} \cdots}{x^5}$$

$\Rightarrow$ To have the limit the numeration minimum power should be x^5

so,

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$a+b+c=0$, $\frac{b}{\lfloor 2}+\frac{c}{\lfloor 3}=0$, $\frac{b}{\lfloor 4}+\frac{c}{\lfloor 5}=\frac{1}{60}$

As

$$\lim_{x\to 0} \frac{(a+b+c)x+\left(\frac{-b}{\lfloor 2}-\frac{c}{\lfloor 3}\right)x^3+\left(\frac{b}{\lfloor 4}+\frac{c}{\lfloor 5}\right)x^5 \cdots}{x^5} = \frac{1}{60}$$

$\Rightarrow a+b+c=0$ —(1), $\frac{b}{2}+\frac{c}{6}=0$, $\frac{b}{24}+\frac{c}{120}=\frac{1}{60}$

$3b+c=0$ —(2)

$5b+c=2$ —(3)

from (2) & (3)

$b=1$, $3+c=0$, $a=-(b+c)$

$c=-3$

$a=-(1-3)$

$a=2$

$\therefore b=1, c=-3, a=2$

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15) we know Taylor series,

let f be a function with derivaties of all orders through some interval containing a as an interior point. Then Taylor series generated by f at $x=a$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 \dots$$

lets Find Tylor series of $\sqrt{1+x}$ at $x=0$

$$\Rightarrow \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} \dots$$

Given $x>0$

so, if we consider 3 terms of Tylor series, $\sqrt{1+x} < 1 + x/2$

as $-x^2/8$ is negative and $-x^2/8$ is less negative than $-x^3/8$

$$x^2/8 < x^3/8 \Rightarrow -x^2/8 > -x^3/8$$

so,
$$1 + x/2 - x^3/8 < \sqrt{1+x}$$

$$\therefore 1 + x/2 - x^3/8 < \sqrt{1+x} < 1 + x/2 \quad //$$

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16) Given function f is defined on $[0,1]$ by

$$f(x) = \begin{cases} x & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$$

let P ⇒ Partitions be $P = \{0 = x_0, x_1, \dots, x_n = 1\}$

$M_i = \sup\{f(x) : x_{i-1} \le x \le x_i\} = x_i$

$m_i = \inf\{f(x) : x_{i-1} \le x \le x_i\} = 0$

The upper Riemann sum

$$U(P,f) = \sum_{i=1}^{n} M_i \Delta x_i = \sum_{i=1}^{n} x_i \Delta x_i$$

let Assume if $x_0 = 0, x_1 = 0 + 1/n, x_2 = 2/n \dots x_n = n/n = 1$

then $U(P,f) = \lim_{n\to\infty} \sum_{i=1}^{n} i/n \cdot 1/n \Rightarrow \frac{1}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty} \frac{1}{n^2} \frac{n(n+1)}{2}$

$\Rightarrow 1/2$

$\therefore U(P,f) = 1/2$

$L(P,f) = \sum_{i=1}^{n} m_i \Delta x_i = 0$

$\overline{\int_0^1} f(x)\,dx = \inf(U(P,f)) = 1/2$, $\underline{\int_0^1} f(x)\,dx = \sup(L(P,f)) = 0$

As $\overline{\int_0^1} f(x) \ne \underline{\int_0^1} f(x)$ so, According ding to Reimann intigral theorum

there will be no Rieman integral of $f(x)$ on $[0,1]$

17) If $f:[a,b] \to R$ be Rieman integral and on $[a,b]$ and ~~$[f(x) \geq 0$, for all $x \in [a,b]]$~~ and $g:[c,d \to R]$ is a Riemann integral and if range of f is contained in $[c,d]$ then $gof(x) = g(f(x))$ is Riemann integral on $[a,b]$

because as u see $x \in [a,b]$ then $f(x) \in$ some set A then $[c,d]$ also contants A then as given $g(x)$ is Riemann-integral on $[c,d]$ then if $y=f(x)$, $g(y)$ is also Riemann-integral on $(A]$ as A is subset of $[c,d]$. //

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18) $f: [a,b] \to \mathbb{R}$ be Riemann integral on $[a,b]$ and $f(x) \geq 0$, for all $x \in [a,b]$, there exist point 'c' in $[a,b]$ such that f' is continous at c and $f(c)>0$

to prove $\int_a^b f(x)\,dx > 0$

$\Rightarrow$ f is continuous at c, there exist at a $\delta > 0$

$f(y) \geq \frac{f(x)}{2}$, for all $y \in [a,b]$, such that

$|y-x| \leq \delta$, If δ is small enough

$[x-\delta, x+\delta) \subset [a,b]$

$$\int_{x-\delta}^{x+\delta} f(t)\,dt \geq 2\delta \frac{f(x)}{2} = \delta f(x) > 0$$

$\because f(y) \geq 0$ for all $y \in [a,b]$ and $\int_a^{x-\delta} f(t)\,dt \geq 0$ and

$\int_{x+\delta}^{b} f(t)\,dt \geq 0$, Additivity theorm (shows that) $\int_a^b f(t)\,dt > 0$

(or)

$$\int_a^b f(x)\,dx > 0$$

$\therefore$ Hence proved.

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19) $f(x) = [x]$, $x \in [0,3]$

for proving 'f' is Riemann integral on $[0,3]$.

$[x]$ = the greater integral not greater than x.

$\because x \in [0,3]$, $f(x) = [x]$ is bounded and has finite number of points of discontinuity

$\therefore$ 'f' is integral on $[0,3]$

Now $\Rightarrow \int_0^3 f(x)\,dx = \int_0^3 [x]\,dx = \int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]\,dx$

(By Addive property) $\Rightarrow \int_0^1 (0)\,dx + \int_1^2 (1)\,dx + \int_2^3 2\,dx$

$\Rightarrow 0 + [x]_1^2 + [2x]_2^3$

$\Rightarrow 0 + 1 + 2 \Rightarrow 3$

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20) let $c_1 < \dots < c_k$ be all discontinuous of 'f' in $[a,b]$

let assume, $a < c_1$ and $c_k < b$ $\{c_1 = a$ or $c_k = b\}$

let $\epsilon > 0$, since 'f' is bounded, there exist $m > 0$ such that

$|f(x)| \leq m$ for all $x \in [a,b]$

let $\delta = \frac{\epsilon}{8mk}$

$\therefore$ f is continuous on $[a, c_1 - \delta]$ $[c_1 + \delta, c_2 - \delta]$ ....

f is Riemann - integrable on these intervals.

$\Rightarrow P_0 = \{a = x_0 < \dots < x_n = c_1 - \delta\}$ of $[a, c_1 - \delta]$

$P_1 = \{c_1 + \delta = y_0 < \dots < y_m = c_2 - \delta\}$ of $[c_1 + \delta, c_2 - \delta]$

$\vdots$

$P_k = \{c_k + \delta = z_0 < \dots < z_l = b\}$ of $[c_k + \delta, b]$

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Such that

$$U(P_i, f) - L(P_i, f) < \frac{\epsilon}{2(K+1)} \text{ for } i=0, \cdots$$

Now let position of $[a,b]$ by $P = P_0 \cup P_1 \cup \cdots \cup P_K$, Then

$$U(P, f) \leq U(P_0, f) + 2m\delta + U(P_1, f) + 2m\delta \cdots + L(P_K, f)$$

$$-2mK\delta + \sum_{i=0}^{K} L(P_i, f),$$

and so,

$$U(P, f) - L(P, f) \leq 4mKf + \sum_{i=0}^{K} (U(P_i, f) - L(P_i f)) < 4mK \times \frac{\epsilon}{8mK} + (K+1) \times \frac{\epsilon}{2(K+1)}$$

$$= \epsilon$$

$\therefore$ Hence, f is Reimann-integral on $[a, b]$