

IIITDM KANCHEEPURAM  
MA1000 CALCULUS - MIDTERM EXAMINATION  
JANUARY 27, 2021

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9AM – 10:30AM	ANSWER ALL QUESTIONS	MARKS: 30
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1. Calculate  $\lim_{n \rightarrow \infty} (\sqrt{n^2 + n} - n)$ . Justify your answer using the definition of convergence of a sequence. (4)
2. Prove that the sequence  $\{a_n\}$ , where  $a_1 = 10$  and  $a_{n+1} = \frac{1}{2} \left( a_n + \frac{10}{a_n} \right)$  for  $n \geq 1$ , converges. Also find its limit. (4)
3. Test the convergence of the sequence  $\{a_n\}$ , where  $a_1 = 2$  and  $a_{n+1} = 2 + \frac{a_n}{a_{n+1}}$  for  $n \geq 1$ . Also find its limit. (4)
4. Discuss the convergence of  $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^p}$ , where  $p$  is a constant. (3)
5. Let  $a_n = \begin{cases} n/2^n & \text{if } n \text{ is a prime number} \\ 1/2^n & \text{otherwise.} \end{cases}$   
Does  $\sum a_n$  converge? Give reasons for your answer. (3)
6. Show that if  $\sum a_n$  diverges, then  $\sum |a_n|$  diverges. (3)
7. Test the convergence of the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)(n+2)}{2^n}$ . Also find its sum if it converges. (4)
8. Show that if two power series  $\sum a_n x^n$  and  $\sum b_n x^n$  are convergent and equal for all values of  $x$  in an open interval  $(-c, c)$ , then  $a_n = b_n$  for every  $n$ . (2)
9. Find the Taylor series generated by  $\sin^2 x$  at  $x = 0$ . Also determine its interval of convergence. (3)