Elementary matrices

Nachiketa Mishra
IIITDM Kancheepuram, Chennai

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Find all 3×3 elementary matrices. (Assignment)

Properties of elementary matrices

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Let $$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix}, \quad (e : R_1 \longleftarrow cR_1,\ c \neq 0)$$

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$$e(A) = \begin{bmatrix} A_{21} & A_{22} \\ A_{11} & A_{12} \\ A_{31} & A_{32} \end{bmatrix}$$

$$e(I)A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} =$$

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Note: For every elementary row operation e, there exists an inverse elementary operation of the same type e_1 such that

$$e(I)e_1(I) = I = e_1(I)e(I) \quad (EE_1 = I = E_1E)$$

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Case 1: Suppose that B is row-equivalent to A.

Then B can be obtained from A by a finite sequence of elementary row operations, say

$$A = A_0 \longrightarrow A_1 \longrightarrow A_2 \longrightarrow \dots \longrightarrow A_{k-1} \longrightarrow A_k = B$$

where $e_i(A_{i-1}) = A_i$, e_i is an elementary row operation for $1 \leq i \leq k$.

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Note that $e_i(A_{i-1}) = e_i(I)A_{i-1}$, by Theorem 9 and $e_i(I)$ is an $m \times m$ elementary matrix.

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Using similar arguments,

$$B = A_k = e_k(I)e_{k-1}(I)\dots e_2(I)e_1(I)A$$

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Let $P = E_k E_{k-1} \dots E_2 E_1$ **where** E_i **is an** $m \times m$ **elementary matrix for** $1 \le i \le k$**.**

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Hence B can be obtained from A by a finite sequence of elementary row operations $e_1, e_2, \dots, e_k$.

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Hence B can be obtained from A by a finite sequence of elementary row operations $e_1, e_2, \dots, e_k$. Then B is row-equivalent to A.

Problem

Show that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \\ 8 & 10 \end{bmatrix}$ **are row-equivalent. Find a** 3×3 **matrix** P **such that** $B = PA$

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