

IIITDM KANCHEEPURAM  
MA1001 Differential Equations  
Problem Set 5

1. Solve the following differential equations:

(a)  $\frac{d^4 y}{dx^4} + m^4 y = 0$

(b)  $x \frac{d^2 y}{dx^2} - (2x - 1) \frac{dy}{dx} + (x - 1)y = 0$

(c)  $\sin^2 x \frac{d^2 y}{dx^2} = 2y$  if  $y = \cot x$  is one solution.

2. Let  $y_1$  and  $y_2$  be two solution of the differential equation  $\frac{d^2 y}{dz^2} + P(x) \frac{dy}{dz} + Q(x)y = R(x)$ . Find the condition on  $c_1$  and  $c_2$  such that  $c_1 y_1 + c_2 y_2$  is also a solution of the differential equation.

3. Let  $y = 3e^{2x} + e^{-2x} - \alpha x$  be a solution of the initial value problem

$$\frac{d^2 y}{dx^2} + \beta y = 4\alpha x, \quad y(0) = 4, \quad \text{and} \quad \frac{dy}{dx}(0) = 1, \quad \text{where } \alpha, \beta \in R.$$

Find the value of  $\alpha$  and  $\beta$ .

4. Let  $y_1(x) = x$  be a solution of the differential equation  $(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ . Find the general solution of differential equation.

5. Let  $W(y_1, y_2)$  be the Wronskian of two linearly independent solutions  $y_1$  and  $y_2$  of the equation  $y'' + P(x)y' + Q(x)y = 0$ .

(i) Calculate product  $W(y_1, y_2)P(x)$

(ii) If  $y_1 = e^{2x}$  and  $y_2 = xe^{2x}$ , then find  $P(0)$ .

6. Find the set of all linearly independent solutions of the differential equation

$$\frac{d^4 y}{dx^4} - \frac{d^2 y}{dx^2} = 0.$$

7. Let  $f(x)$  and  $xf(x)$  be solutions of the differential equation

$$y'' + P(x)y' + Q(x)y = 0.$$

Then find the solution of the differential equation  $y'' + P(x)y' + Q(x)y = f(x)$ .