Engineering Electromagnetics

Lecture 14

27/09/2023

by

Debolina Misra

Department of Physics
IIITDM Kancheepuram, Chennai, India

Coulomb's law

What is the force on a test charge Q due to a single point charge q, that is at *rest* a distance $\mathscr{r}$ away?

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}}.$$

permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$

- the force is proportional to the product of the charges
- Inversely proportional to the square of the separation distance $\mathscr{r} = \mathbf{r}_1 - \mathbf{r}_2$
- The force points along the line from q to Q
- it is repulsive if q and Q have the same sign, and attractive if their signs are opposite

Griffith book

The Electric Field

If we have *several* point charges $q_1, q_2, \ldots, q_n$, at distances $\mathscr{r}_1, \mathscr{r}_2, \ldots, \mathscr{r}_n$ from Q, the total force on Q is evidently

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{\mathscr{r}_1^2}\hat{\mathscr{r}}_1 + \frac{q_2 Q}{\mathscr{r}_2^2}\hat{\mathscr{r}}_2 + \ldots\right)$$

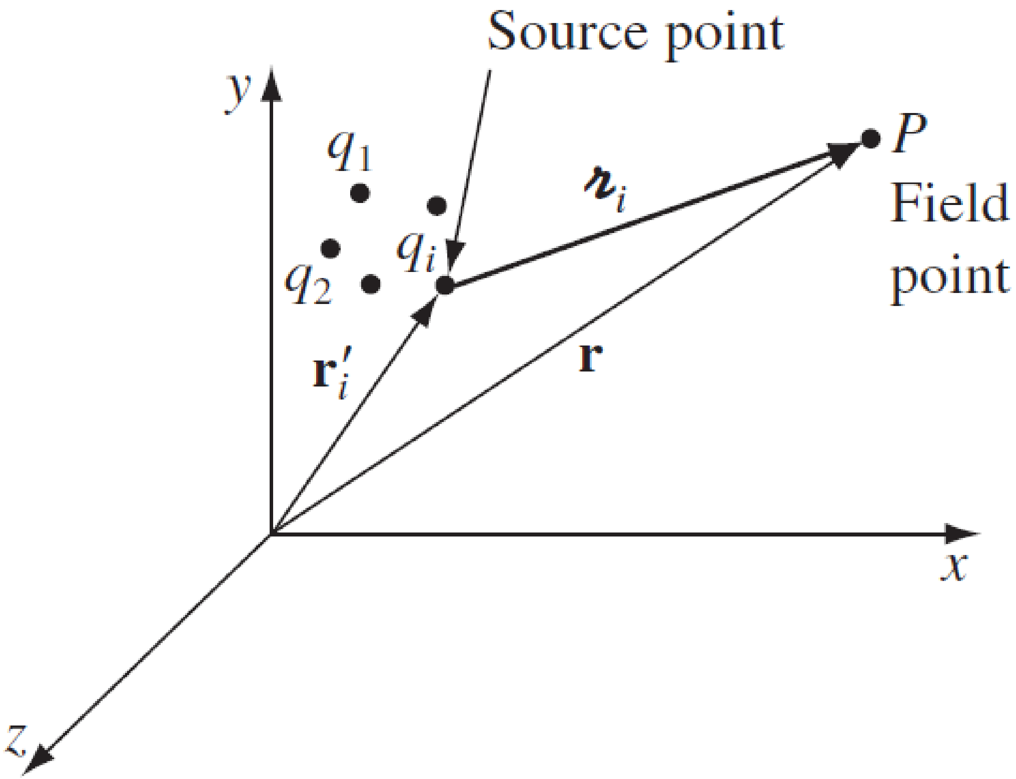
$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{\mathscr{r}_1^2}\hat{\mathscr{r}}_1 + \frac{q_2}{\mathscr{r}_2^2}\hat{\mathscr{r}}_2 + \frac{q_3}{\mathscr{r}_3^2}\hat{\mathscr{r}}_3 + \ldots\right),$$

$$\boxed{\mathbf{F} = Q\mathbf{E},} \qquad \mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i$$

electric field of the source charges.

From Griffith Book

The Electric Field



$$\mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i$$

E is called the **electric field** of the source charges.

- **E** is a function of position (**r**), because the separation vectors depend on the location of the **field point** *P*.
- no reference to the test charge *Q*.
- The electric field is a vector quantity that varies from point to point and is determined by the configuration of source charges.
- physically, **E**(**r**) is the force per unit charge that would be exerted on a test charge, if you were to place one at *P*.

From Griffith Book

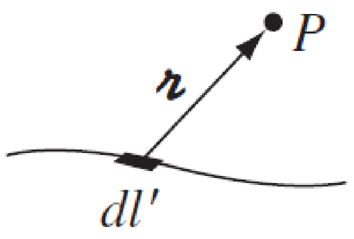
Continuous charge distributions

$$\mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i$$

If charge is distributed continuously over some region (not discrete)

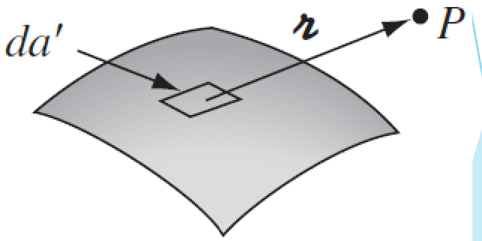
$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,dq}$$

Q: If the charge is spread out along a line with charge-per-unit-length λ, then dq=?



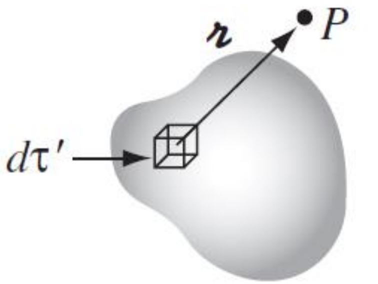
Line charge, λ

Q: over a surface, with charge-per-unit-area σ, then dq=?



Surface charge, σ

Q: a volume, with charge-per-unit-volume ρ, then dq = ?



Volume charge, ρ

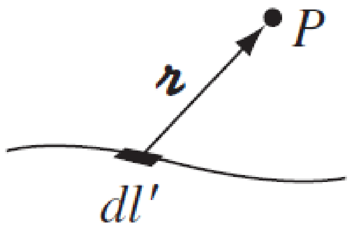
From Griffith Book

Continuous charge distributions

If charge is distributed continuously over some region (not discrete)

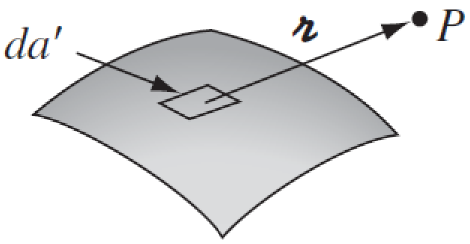
$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,dq}$$

A: If the charge is spread out along a line with charge-per-unit-length λ, then $dq=\lambda\, dl'$ (where dl' is an element of length along the line)



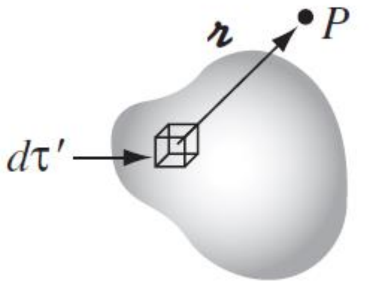
Line charge, λ

A: over a surface, with charge-per-unit-area σ,
then $dq=\sigma\, da'$ (where da' is an element of area on the surface)



Surface charge, σ

A: a volume, with charge-per-unit-volume ρ,
then $dq = \rho\, d\tau'$ (where $d\tau'$ is an element of volume)



Volume charge, ρ

From Griffith Book

E due to continuous charge distribution

$$dq \to \lambda\, dl' \sim \sigma\, da' \sim \rho\, d\tau'$$

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, dq}$$

Thus the electric field of a line charge is

$$dq = \lambda\, dl' \qquad \boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, dl'}$$

for a surface charge,

$$dq = \sigma\, da' \qquad \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, da'$$

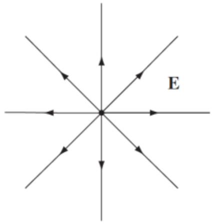
and for a volume charge,

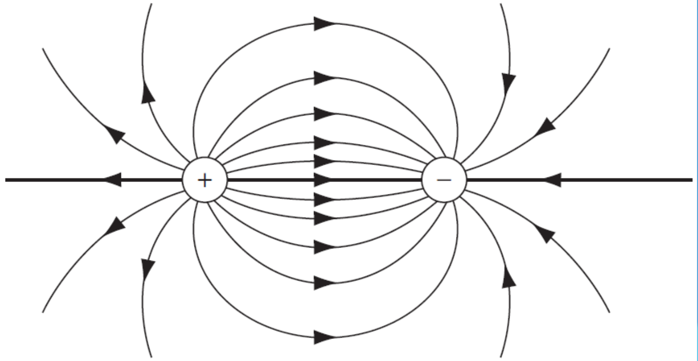
$$dq = \rho\, d\tau' \qquad \boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, d\tau'}$$

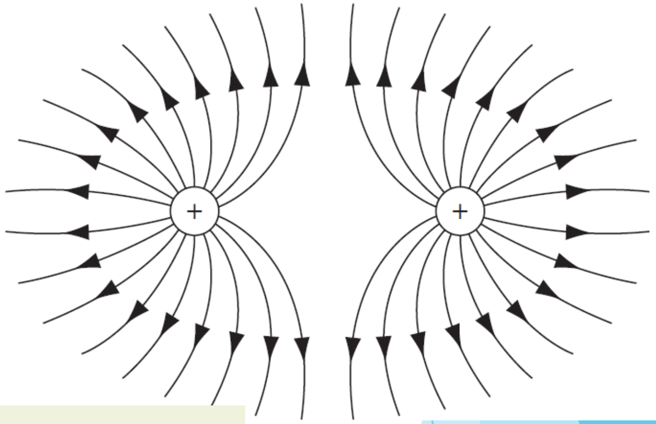
Field lines for point charges

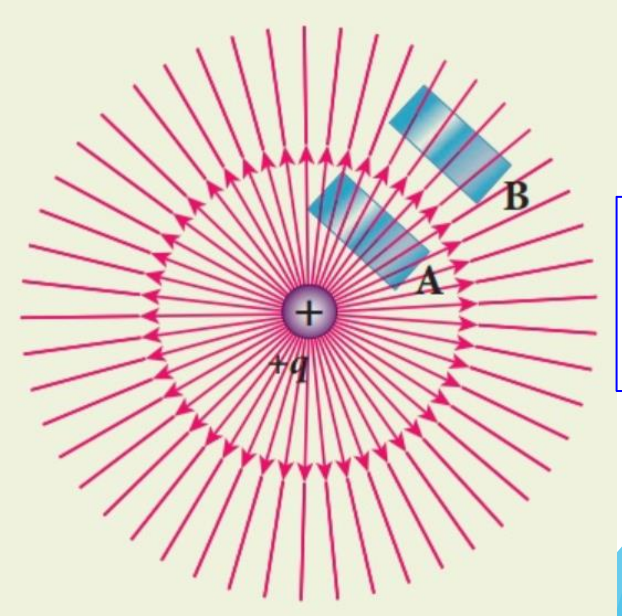
- And you must space them fairly—they emanate from a point charge symmetrically in all directions. Field lines begin on positive charges and end on negative ones
- they cannot simply terminate in midair, though they may extend out to infinity.
- **Field of any simple configuration of point charges**: Begin by drawing the lines in the neighborhood of each charge, and then connect them up or extend them to infinity
- In this model, the flux of E through a surface S is,

$$\boxed{\Phi_E \equiv \int_S \mathbf{E} \cdot d\mathbf{a}}$$









Which surface will feel **E** more strongly/denser field lines?

From Griffith Book

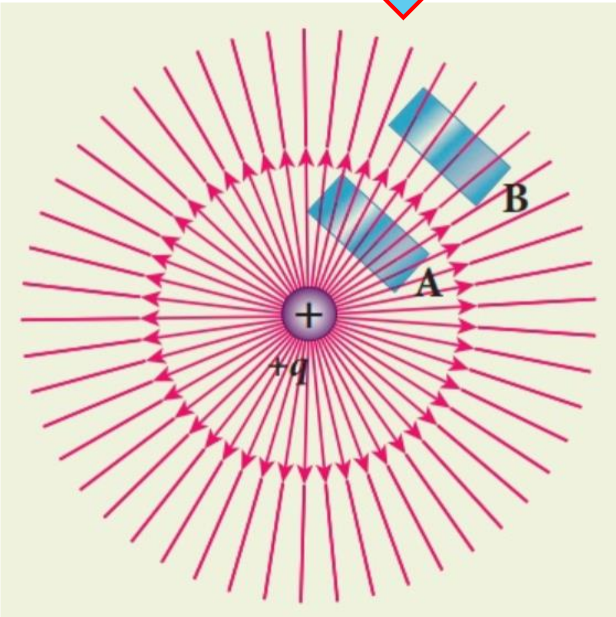
Flux

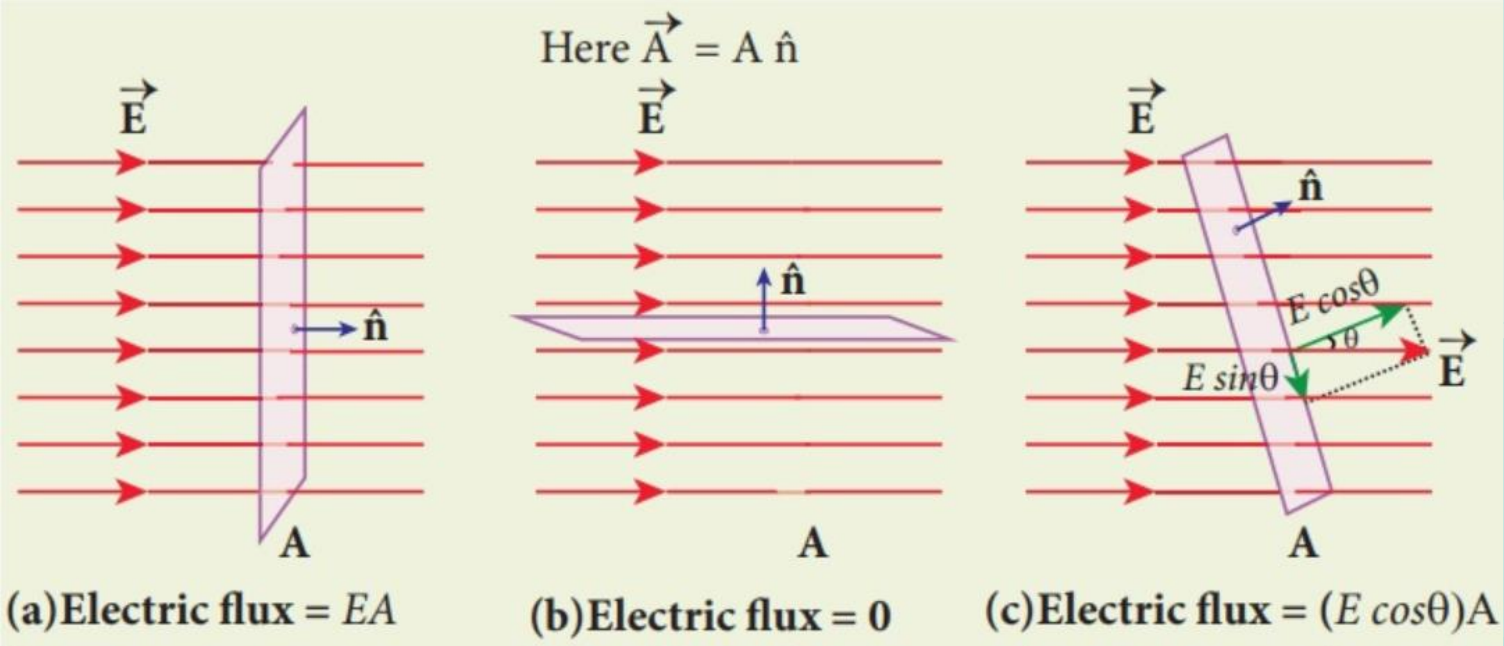
- This suggests that the flux through any closed surface is a measure of the total charge inside.
- For the field lines that originate on a positive charge must either pass out through the surface or else terminate on a negative charge inside. On the other hand, a charge outside the surface will contribute nothing to the total flux → essence of Gauss's law.

From Griffith Book

$$\Phi_E \equiv \int_S \mathbf{E} \cdot d\mathbf{a}$$

Which surface will feel **E** more strongly/denser field lines?





https://www.brainkart.com/article/Electric-Flux_38377/

- For a point charge q at the origin, calculate the flux of **E** through a spherical surface of radius r.

Flux

- In the case of a point charge q at the origin, the flux of E through a spherical surface of radius r is

$$\oint \mathbf{E} \cdot d\mathbf{a} = \int \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{\mathbf{r}}\right)\cdot(r^2 \sin\theta \, d\theta \, d\phi \, \hat{\mathbf{r}}) = \frac{1}{\epsilon_0} q$$

- the flux through any surface enclosing the charge is q/ε_0
- Now suppose that instead of a single charge at the origin, we have a bunch of charges scattered about. $\mathbf{E} = \sum_{i=1}^{n} \mathbf{E}_i$
- The flux through a surface that encloses them all is

$$\oint \mathbf{E} \cdot d\mathbf{a} = \sum_{i=1}^{n}\left(\oint \mathbf{E}_i \cdot d\mathbf{a}\right) = \sum_{i=1}^{n}\left(\frac{1}{\epsilon_0} q_i\right)$$

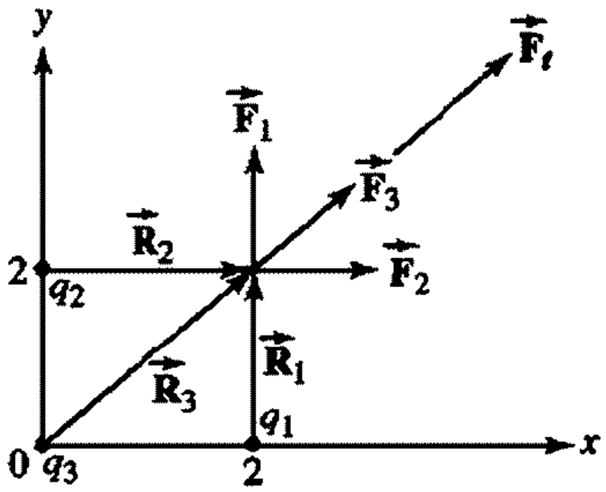
- For any closed surface

$$\boxed{\Phi_E \equiv \int_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a}} \quad \boxed{\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}}}$$

where Q_{enc} is the total charge enclosed within the surface

From Griffith Book

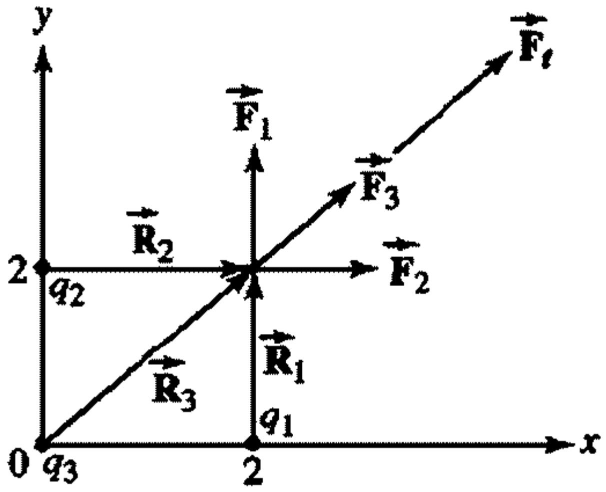
Problem-1



Three equal charges of 200 nC are placed in free space at (0, 0, 0), (2, 0, 0), and (0, 2, 0). Determine the total force acting on a charge of 500 nC at (2, 2, 0).

Solution-1

$$\vec{\mathbf{R}}_1 = \vec{\mathbf{r}} - \vec{\mathbf{r}}_1 = 2\vec{\mathbf{a}}_y \Rightarrow R_1 = 2\,\text{m}$$
$$\vec{\mathbf{R}}_2 = \vec{\mathbf{r}} - \vec{\mathbf{r}}_2 = 2\vec{\mathbf{a}}_x \Rightarrow R_2 = 2\,\text{m}$$
$$\vec{\mathbf{R}}_3 = \vec{\mathbf{r}} - \vec{\mathbf{r}}_3 = 2\vec{\mathbf{a}}_x + 2\vec{\mathbf{a}}_y \Rightarrow R_3 = 2.828\,\text{m}$$



The force on q due to q_1 is

$$\vec{\mathbf{F}}_1 = \frac{9 \times 10^9 \times 200 \times 10^{-9} \times 500 \times 10^{-9}}{2^3}[2\vec{\mathbf{a}}_y] = 225\vec{\mathbf{a}}_y\,\mu\text{N}$$

Similarly, we can compute the forces acting on q due to q_2 and q_3 as

$$\vec{\mathbf{F}}_2 = 225\vec{\mathbf{a}}_x\,\mu\text{N} \quad \text{and} \quad \vec{\mathbf{F}}_3 = 79.6[\vec{\mathbf{a}}_x + \vec{\mathbf{a}}_y]\,\mu\text{N}$$

Thus, the total force experienced by q, from (3.6), is

$$\vec{\mathbf{F}}_t = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = 304.6[\vec{\mathbf{a}}_x + \vec{\mathbf{a}}_y]\,\mu\text{N}$$

Problem-2

Two point charges of 20 nC and −20 nC are situated at (1, 0, 0) and (0, 1, 0) in free space. Determine the electric field intensity at (0, 0, 1).

Solution-2

The two distance vectors are

$$\vec{\mathbf{R}}_1 = \vec{\mathbf{r}} - \vec{\mathbf{r}}_1 = -\vec{\mathbf{a}}_x + \vec{\mathbf{a}}_z, \quad R_1 = |\vec{\mathbf{r}} - \vec{\mathbf{r}}_1| = 1.414\,\text{m}$$

and

$$\vec{\mathbf{R}}_2 = \vec{\mathbf{r}} - \vec{\mathbf{r}}_2 = -\vec{\mathbf{a}}_y + \vec{\mathbf{a}}_z, \quad R_2 = |\vec{\mathbf{r}} - \vec{\mathbf{r}}_2| = 1.414\,\text{m}$$

Substituting in equation (3.10), we obtain

$$\vec{\mathbf{E}} = 9 \times 10^9 \left[\frac{20 \times 10^{-9}}{1.414^3}(-\vec{\mathbf{a}}_x + \vec{\mathbf{a}}_z) - \frac{20 \times 10^{-9}}{1.414^3}(-\vec{\mathbf{a}}_y + \vec{\mathbf{a}}_z) \right]$$

$$= 63.67[-\vec{\mathbf{a}}_x + \vec{\mathbf{a}}_y]\,\text{V/m}$$

Thank You