Problem Set-5 Probability

January 2020

1. Establish for which constants c the following functions are densities.
   (a) $f(x) = cx$ on $(0, 1)$ and 0 otherwise.
   (b) $f(x) = cx^n$ on $(0, 1)$ and 0 otherwise, for n a non-negative integer.
   (c) $f(x) = cx^{1/2}$ on $(0, 2)$ and 0 otherwise.
   (d) $f(x) = c \sin x$ on $(0, /2)$ and 0 otherwise.
2. Suppose X has density f and Y has density g. Suppose $f(x) > g(x)$ for $1 < x < 2$. Prove that $P(1 < X < 2) > P(1 < Y < 2)$.
3. Suppose X is a standard normal variable and Y is a normal variable with mean 1 and variance 1. Prove that $P(X < 3) > P(Y < 3)$.
4. Consider rolling one fair six-sided die, so that $S = \{1, 2, 3, 4, 5, 6\}$, and $P(s) = 1/6$ for all $s \in S$. Let X be the number showing on the die, so that $X(s) = s$ for $s \in S$. Let $Y = X^2$. Compute the cumulative distribution function $F_Y(y) = P(Y \leq y)$, for all $y \in R$.
5. Let X is a normal random variable with mean -8 and mean 4. Compute each of the following in terms of the cumulative distributive function,
   (a) $P(X \geq 5)$
   (b) $P(2 \geq X \geq 7)$
   (c) $P(X \geq 3)$
6. Suppose $F_Y(y) = y^3$ for $0 \leq y < 1/2$, and $F_Y(y) = 1 - y^3$ for $1/2 \leq y \leq 1$. Compute each of the following.
   (a) $P(1/3 < Y < 3/4)$
   (b) $P(Y = 1/3)$
   (c) $P(Y = 1/2)$
7. Let X is a Exponential random variable with parameter 3. Compute the function F_X.
8. Let X be a Bernoulli random variable with parameter $1/3$, and let $Y = 4X - 2$. Compute the joint cumulative density function $F_{X,Y}$.

9. Let X be a Bernoulli random variable with parameter $1/4$, and let $Y = 7X$. Compute the joint cumulative density function $F_{X,Y}$.

10. Let X be the number of heads when flipping three fair coins. Let $Y = 1$ if $X \geq 1$, with $Y = 0$ if $X = 0$. Find the density function of Y.

11. Let X be the number showing on a fair six-sided die, so that $P(X = x) = 1/6$ for $x = 1, 2, 3, 4, 5$, and 6. Find the density function for Y.