

Solid State Electronic Devices



INDIAN INSTITUTE OF INFORMATION TECHNOLOGY,
DESIGN AND MANUFACTURING,
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Syllabus

Course Contents (with approximate breakup of hours for lecture/tutorial/practice)	<ul style="list-style-type: none">• Solid state devices – History and its relevance in the modern world. formation of energy bands in semiconductors, Density of states and Fermi level. (L3+T1)• Charge carriers in Semiconductors- Equilibrium Carrier concentration, Recombination and Generation of carriers, Carrier transport – Drift, Diffusion and their modelling in MATLAB. (L9+T2)• pn junction – derivation of dc and ac characteristics, Forward and reverse biasing, Static analysis, Breakdown processes; Transient analysis, metal semiconductor junction. Modelling of p-n junction. (L9+T3)• Bipolar junction transistors– Fundamentals and characteristics, biasing, switching, Modelling of BJT. (L4+T1)• Field Effect Transistors (JEFT, MESFET, MOSFET, HEMT), MOS capacitor, MOSFET – device physics, operation, characteristics and modelling. (L10+T3)• Optoelectronic Devices- Fundamentals of Photodiodes, Light emitting devices, Semiconductor LASERs, Solar cells, CCDs along with Nanoelectronic devices. (L6+T1)
Essential Reading	<ol style="list-style-type: none">1. Robert Pierret, Semiconductor Device Fundamentals ,1st Edition, Pearson Education, ISBN:9788177589771, 2006.2. B. G. Streetman and S. K. Banerjee, Solid State Electronic Devices, 7th Edition, Pearson, ISBN: 9780133356038, 2015.3. Neamen, Donald A., Semiconductor Physics and Devices: Basic Principles, 4th Edition, NY: McGraw-Hill, ISBN:978-0-07-352958-5, 2012.
Supplementary Reading	<ol style="list-style-type: none">1. S. M. Sze., K. K. Ng, Physics of Semiconductor Devices, 3rd Edition, United Kingdom, Wiley, ISBN: 978-0471143239, 2021.2. M. S. Tyagi, Introduction to Semiconductor Materials and Devices, 1st Edition, John Wiley, ISBN: 9788126518678, 2008.

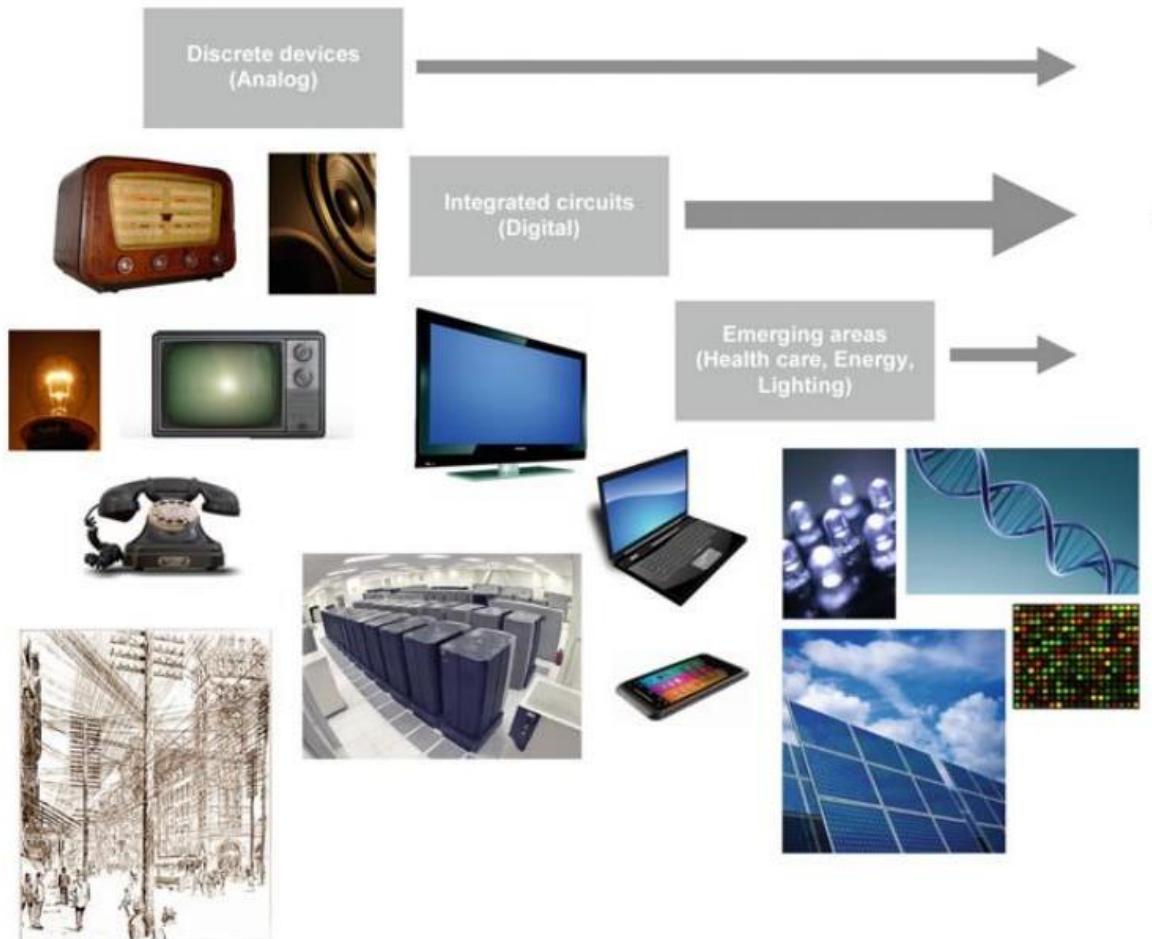


Solid state devices (L3+T1)

- History and its relevance in the modern world
- Formation of energy bands in semiconductors
- Density of states and Fermi level



History and its relevance in the modern world



- 1960: small-scale integration (SSI): < 100 transistors
- 1964: medium-scale integration (MSI): 100-1,000 transistors
- 1972: large-scale integration (LSI): 1,000-100,000 transistors
- 1980: very-large-scale integration (VLSI): > 100,000 transistors
- 1989: ultra-large-scale integration (ULSI): over million transistors
- 2022: No acronym: NVIDIA Hopper H100 GPU, 80 billion transistors, 4-nm process

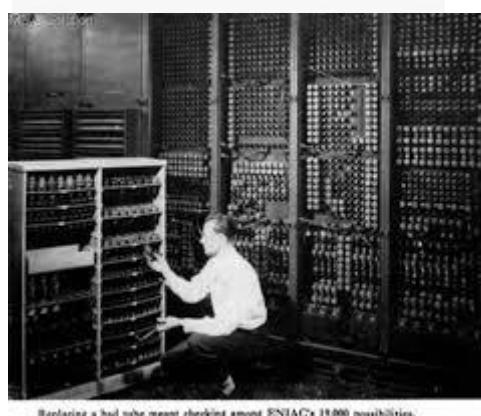
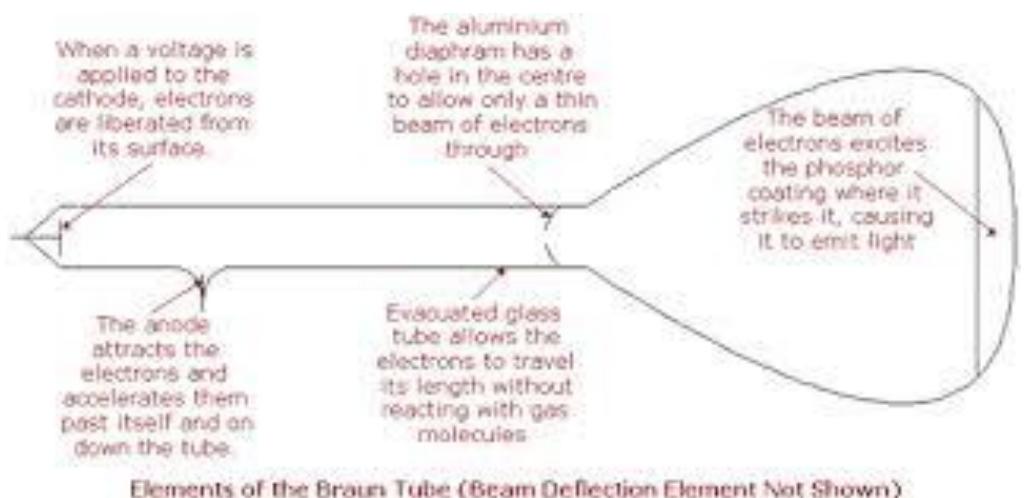
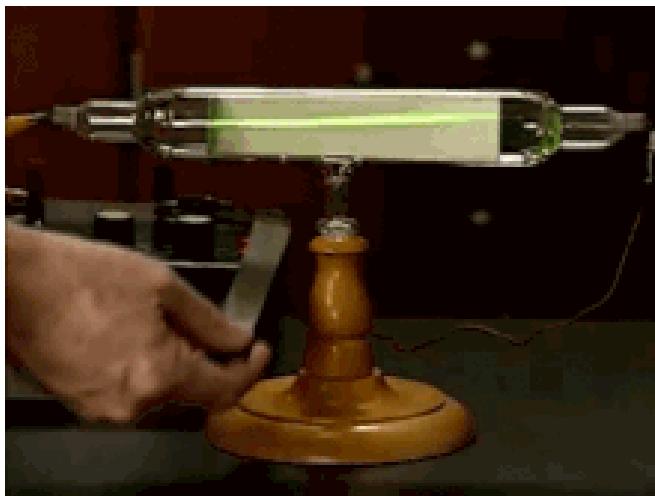
Fig. 1.1 Evolution of solid-state electronics and emerging growth areas



Year	Scientist	Semiconductor devices invention
1782	Alessandro Volta	First used semiconductor
1833	Michael Faraday	Observed electrical resistance of silver sulfide decreased with temperature
1874	Karl Braun	(Semiconductor diode effect) current flows freely in only one direction at the contact between a metal point and a galena crystal
1901	Jagadish Chandra Bose	Cat whiskers was a point-contact semiconductor rectifier used for detecting radio waves
1947	John Bardeen, Walter Brattain, and William Shockley	Transistor



CRTs and Vacuum tubes



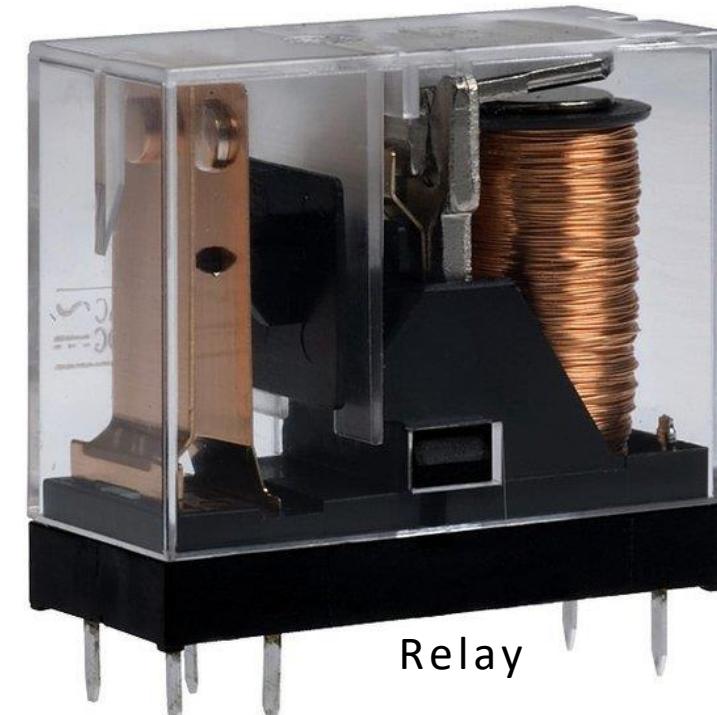
Solid state electronic components



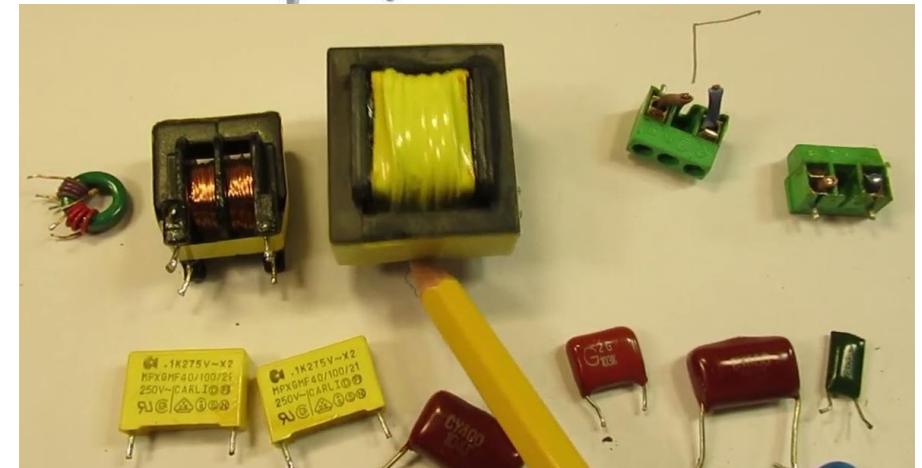
Cat Whiskers diode

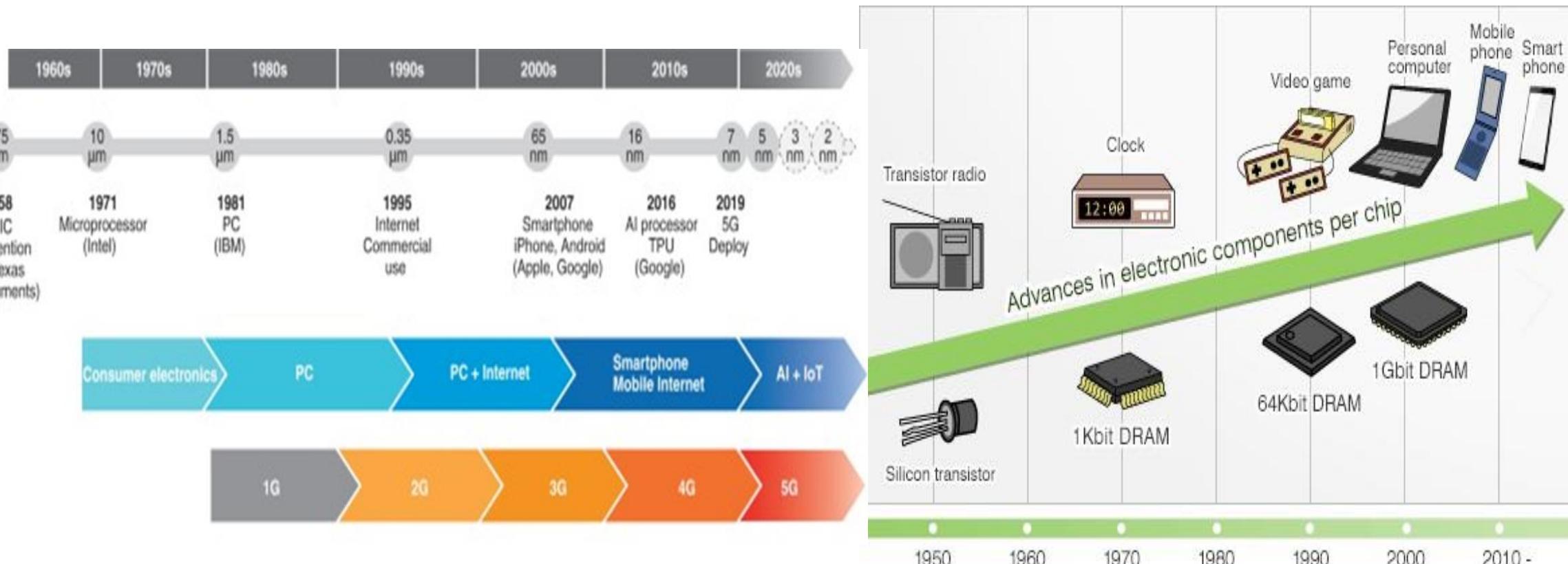


PC motherboard

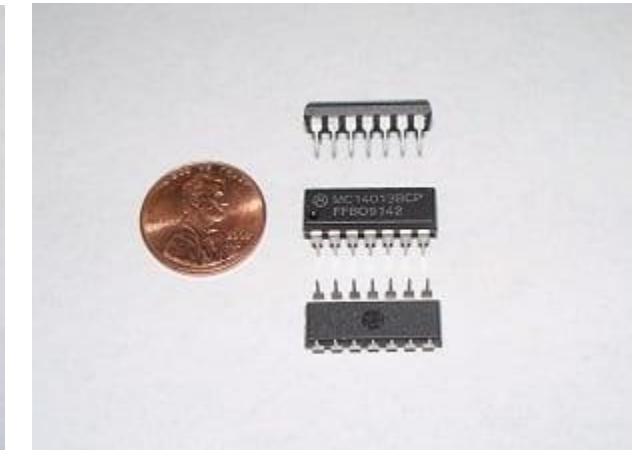
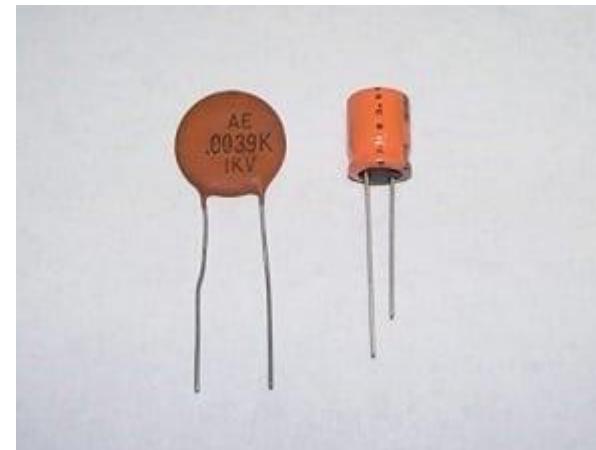


Relay





- Solid-state, as the name expresses, refers to the use of a solid material to control electrons or the movement of electrons.
- The transport of charge through a metal or a semiconductor depends not only on the **properties of the electron** but also on the **arrangement of atoms in the solid**.
- The advantages of solid-state technology:
 - Reduced equipment size
 - Increased computing power
 - Less heat and energy usage
 - Durability



Semiconductors

- A semiconductor is also a material whose electrical conductivity can be altered (called doping) through variations in temperature, applied fields, or adding impurities.
- Electronic materials fall into three categories:

Materials

- Insulators
- Semiconductors
- Conductors

Resistivity (ρ)

- $\rho > 10^5 \Omega\text{-cm}$
- $10^{-3} < \rho < 10^5 \Omega\text{-cm}$
- $\rho < 10^{-3} \Omega\text{-cm}$

Classification based on materials:

- Elemental semiconductors are formed from a single type of atom.
- Compound semiconductors are formed from combinations of column III and V elements or columns II and VI.
- Germanium was used in many early devices.
- Silicon quickly replaced germanium due to its higher bandgap energy, lower cost, and is easily oxidized to form silicon-dioxide insulating layers.

	IIIA	IVA	VA	VIA
	5 10.811 B Boron	6 12.01115 C Carbon	7 14.0067 N Nitrogen	8 15.9994 O Oxygen
	13 26.9815 Al Aluminum	14 28.086 Si Silicon	15 30.9738 P Phosphorus	16 32.064 S Sulfur
IIIB	30 65.37 Zn Zinc	31 69.72 Ga Gallium	32 72.59 Ge Germanium	33 74.922 As Arsenic
	48 112.40 Cd Cadmium	49 114.82 In Indium	50 118.69 Sn Tin	51 121.75 Sb Antimony
	80 200.59 Hg Mercury	81 204.37 Tl Thallium	82 207.19 Pb Lead	83 208.980 Bi Bismuth
				84 (210) Po Polonium



Uses of semiconductors

Elemental semiconductor

- Germanium (Ge) - development for transistors and diodes.
- Silicon (Si) – development of rectifiers, transistors, and integrated circuits (ICs).
- Si and Ge - infrared and nuclear radiation detectors.

Compound semiconductor

- Silicon carbide - making devices for specialized high-temperature applications.
- II-VI compound semiconductors such as ZnS - Fluorescent materials used in television screens
- III-V compounds - make semiconductor devices that emit light efficiently or that operate at exceptionally high frequencies.
- InSb, CdSe, or other compounds such as PbTe and HgCdTe - Light detectors
- GaN and other III-V compounds - Light-emitting diodes
- GaAs, AlGaAs, and other ternary and quaternary compounds - Semiconductor lasers

Table 1-1 Common semiconductor materials: (a) the portion of the periodic table where semiconductors occur; (b) elemental and compound semiconductors.

(a)	II	III	IV	V	VI
		B	C	N	
		Al	Si	P	S
	Zn	Ga	Ge	As	Se
	Cd	In		Sb	Te
(b)	Elemental	IV compounds	Binary III-V compounds	Binary II-VI compounds	
	Si	SiC	AlP	ZnS	
	Ge	SiGe	AlAs	ZnSe	
			AlSb	ZnTe	
			GaN	CdS	
			GaP	CdSe	
			GaAs	CdTe	
			GaSb		
			InP		
			InAs		
			InSb		



Semiconductor Classification based on conduction

Conductors:

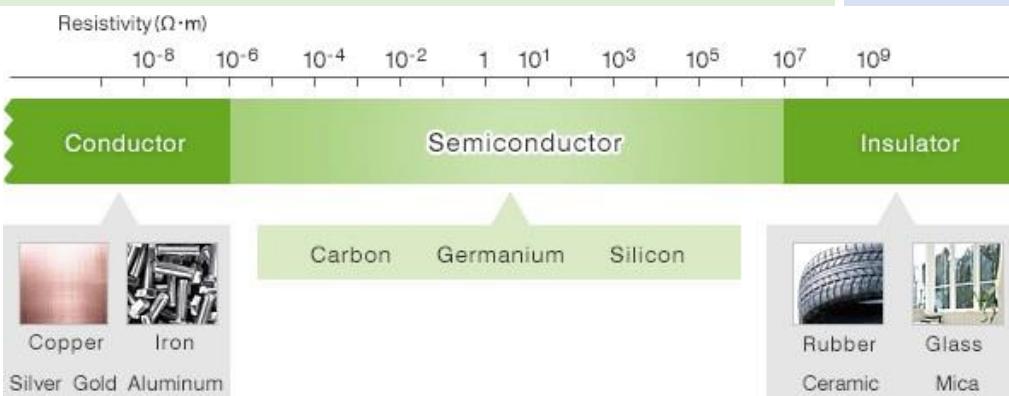
- The atomic structure of good conductors usually includes only **one electron in their outer shell**.
- It is called a valence electron.
- It is easily stripped from the atom, producing current flow.

Insulators:

- Insulators have a **high resistance** so current does not flow in them.
- Good insulators include: Glass, ceramic, plastics, & wood.
- Most insulators are compounds of several elements.
- The atoms are **tightly bound** to one another so **electrons are difficult to strip away** for current flow.

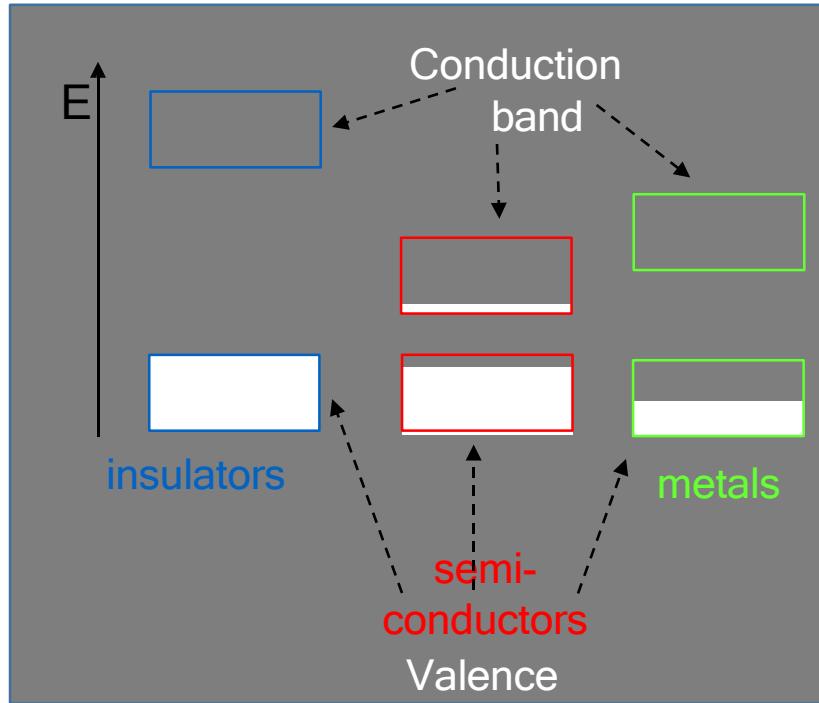
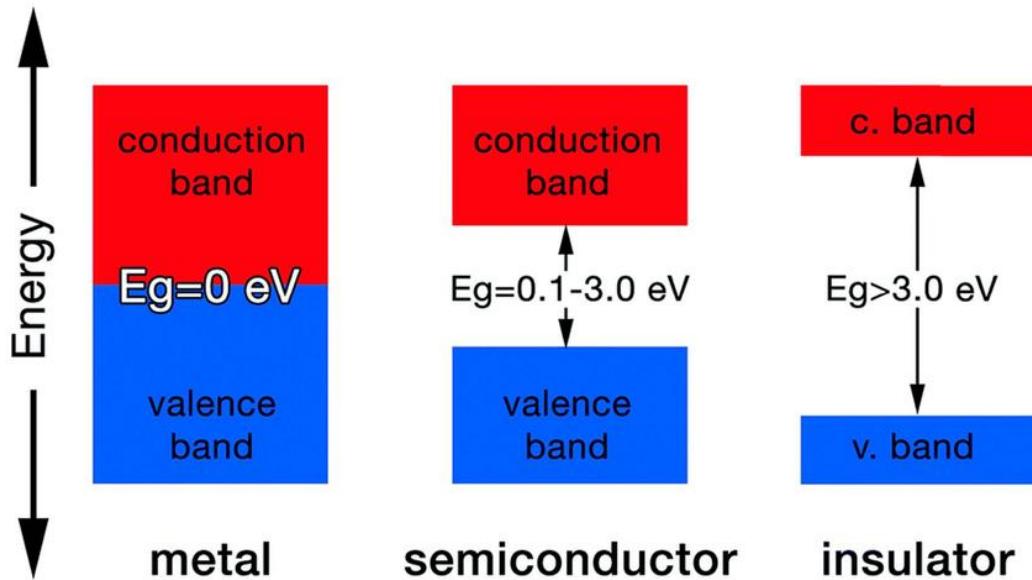
Semiconductors:

- Semiconductors are materials that essentially can be conditioned to act as good conductors, or good insulators, or **any thing in between**.
- Common elements such as carbon, silicon, and germanium are semiconductors.
- Silicon is the best and most widely used semiconductor.
- Are insulators at $T = 0$.



Energy Bandgap

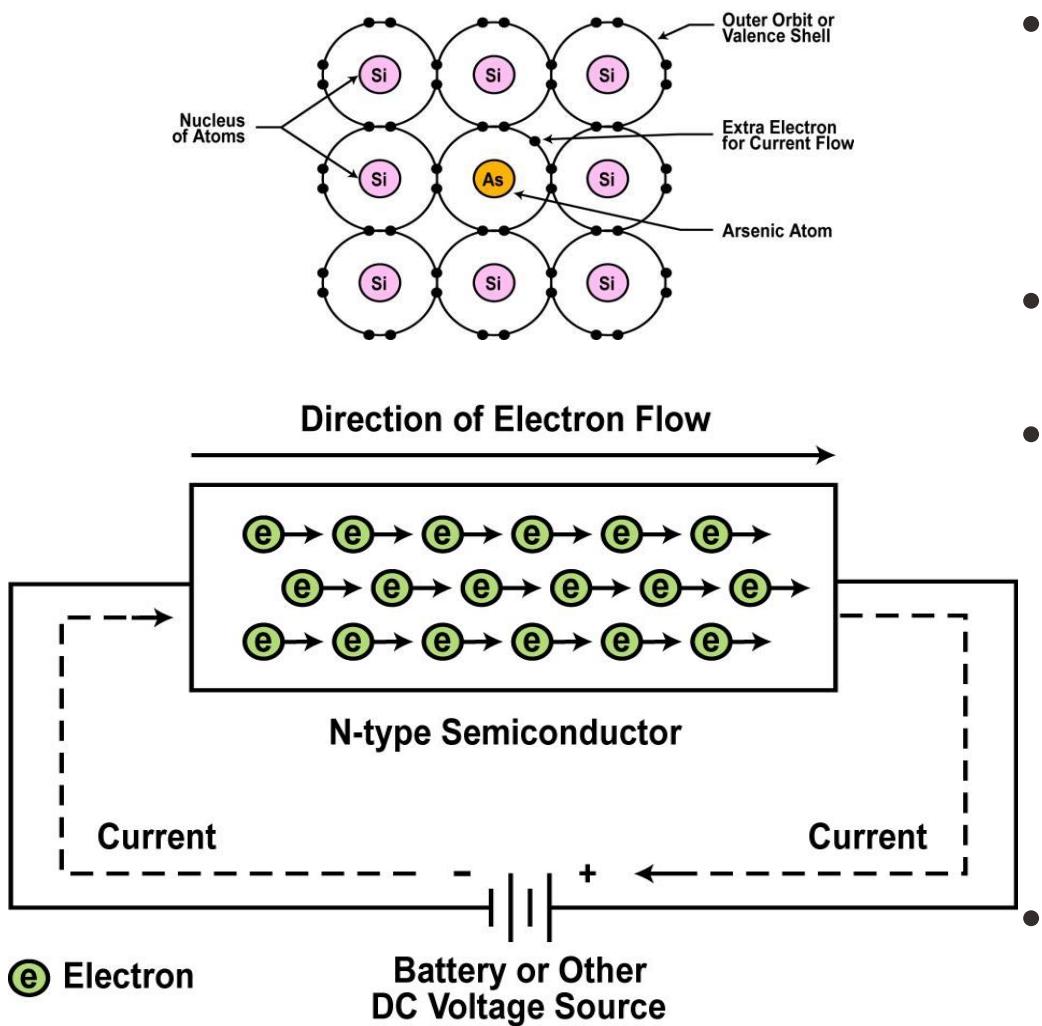
- Have a full valence band and a large energy gap (a few eV). Higher energy states are not available.



Have a partly filled band. Higher energy states are available, even at $T = 0$.

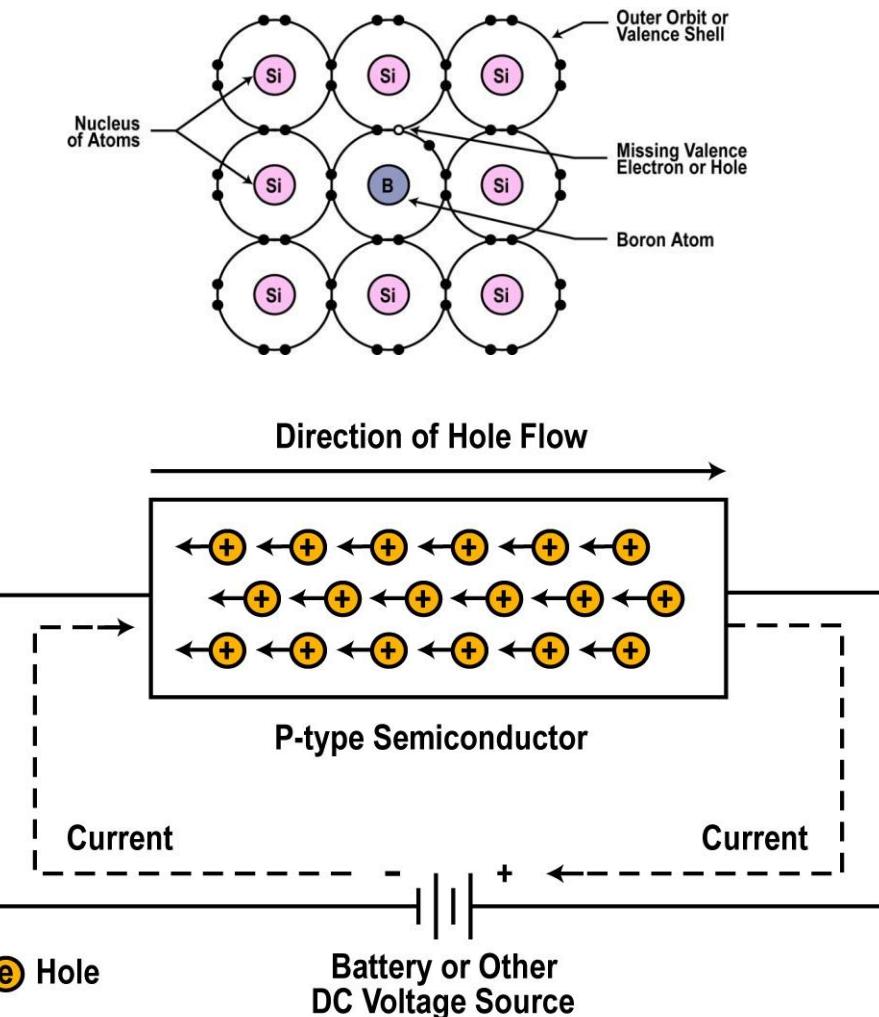
Have a small energy gap ($\sim 1 \text{ eV}$) between valence and conduction bands. Higher energy states become available (due to kT) as T increases.

Classification of semiconductors based on doping



- N-type semiconductor is formed when elements of the fifth group element of periodic table are doped (**doping of pentavalent impurity atoms**).
- **Electrons are the majority charge carrier** in n-type semiconductor while holes as minority.
- The outermost shell of arsenic has five electrons and silicon has four electrons. These electrons are called valence electrons. Thus when silicon is doped with arsenic, then the four electrons of arsenic bonds with four electrons of silicon. The bond they form is covalent bond.
One electron is still free which is responsible for the conduction.

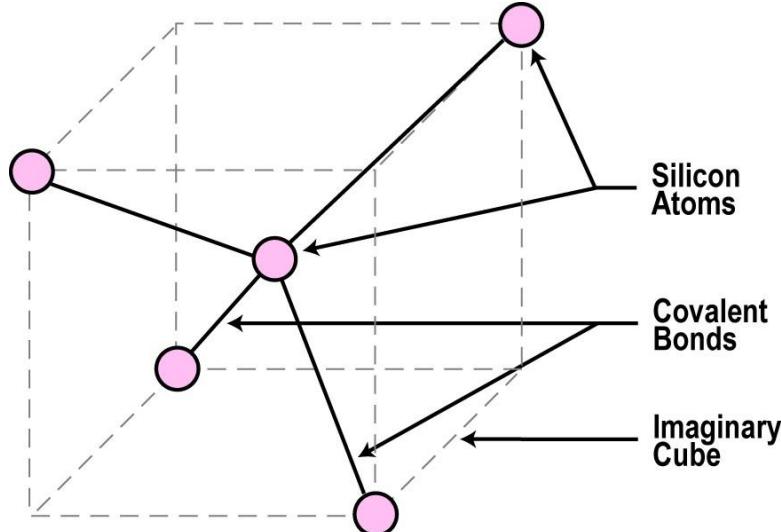
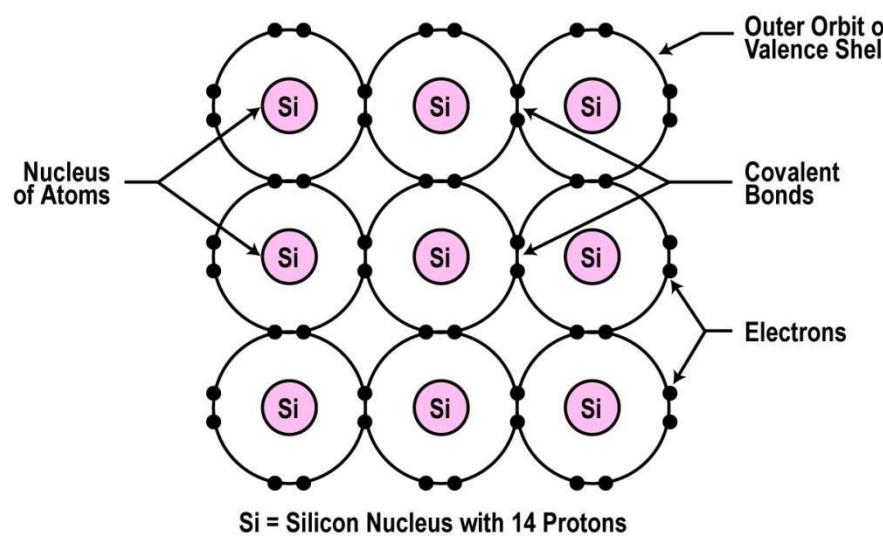
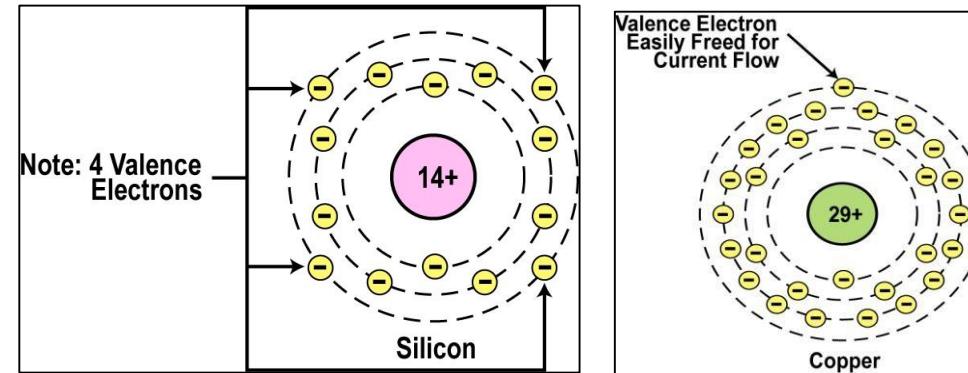
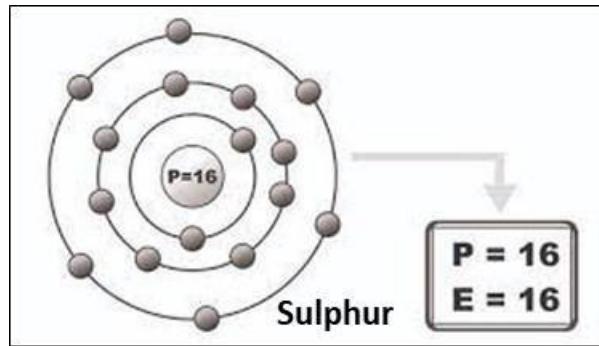
- p-type semiconductor is formed by doping trivalent impurity atoms with the semiconductor.
- In p-type semiconductor, **holes are the majority charge carriers** and electrons are the minority charge carriers.
- When silicon is doped with boron, the three electrons of boron bond with the three electrons of silicon. The bond between the electrons is the covalent bond.
- One valence electron of silicon remains free. To bond with this one free electron of silicon one electron from the neighbouring silicon atom is taken. This creates a hole or vacancy in the conductor.



S.No.	Intrinsic Semiconductor	Extrinsic Semiconductor
1.	It is a pure semiconductor without doping.	It is an impure semiconductor by doping of impurity atoms.
2.	Conductivity is low.	Conductivity is high.
3.	Conductivity depends only on temperature.	Conductivity depends both on temperature and impurity atoms.
4.	Number of free electrons is equal to the number of holes.	Number of free electrons is not equal to the number of holes.
5.	Conductivity due to electrons and holes is nearly equal.	Conductivity is mainly due to majority charge carriers.
6.	It does not conduct at zero kelvin.	It conducts at zero kelvin.
7.	Fermi level lies in between valence and conduction band.	Fermi level lies near valence in p-type and conduction band in n-type.



Atomic structure and energy levels



Crystal structure of semiconductors

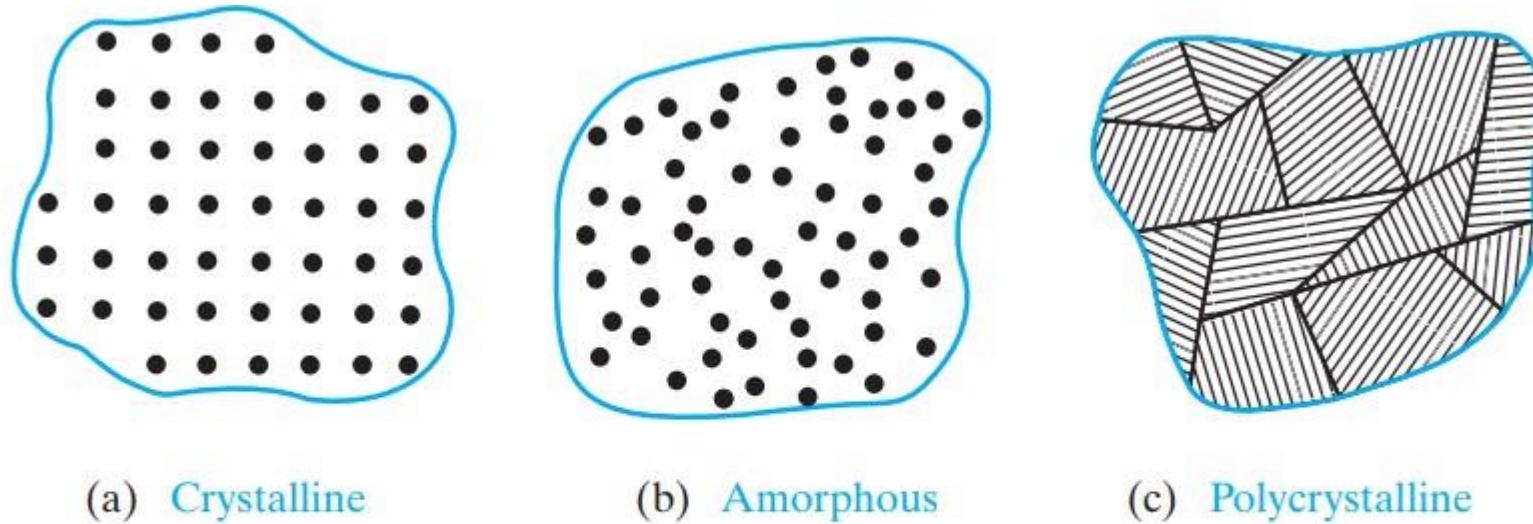
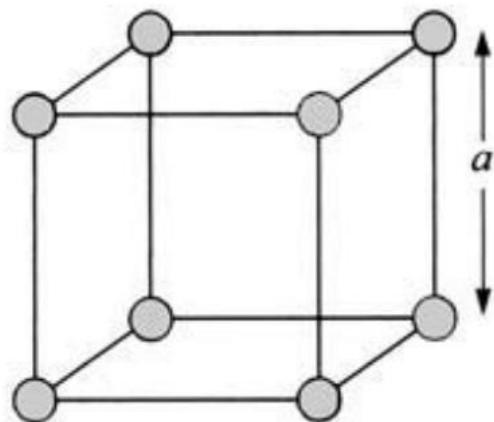


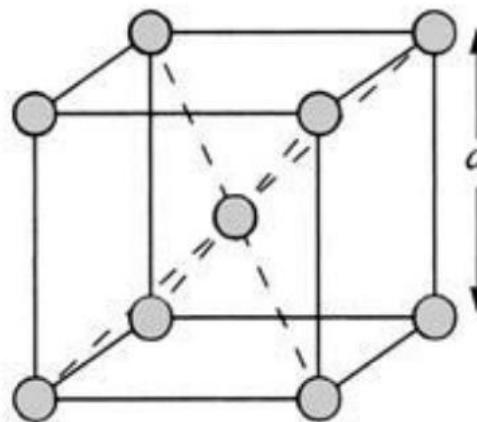
Figure 1–1

Three types of solids, classified according to atomic arrangement: (a) crystalline and (b) amorphous materials are illustrated by microscopic views of the atoms, whereas (c) polycrystalline structure is illustrated by a more macroscopic view of adjacent single-crystalline regions, such as (a).

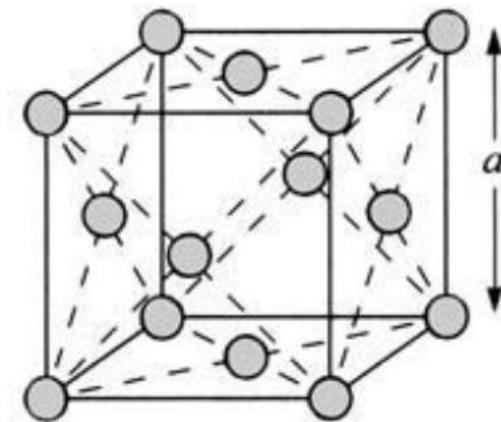
- A crystal is characterized by having a well-structured periodic placement of atoms.
- The smallest assembly of atoms that can be repeated to form the entire crystal is called a primitive cell, with a dimension of lattice constant 'a'



Simple cubic
(Po)
(a)

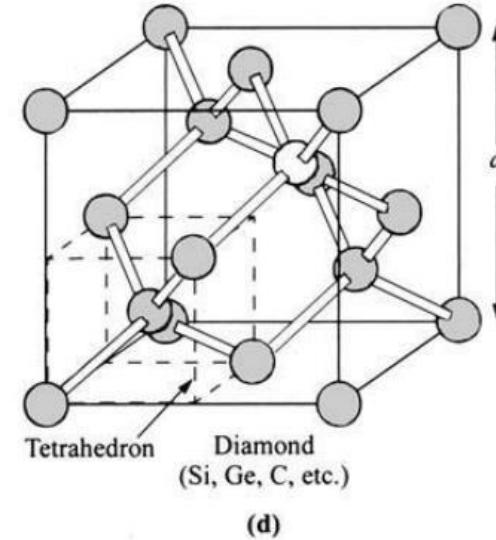


Body-centered cubic
(Na, W, etc.)
(b)

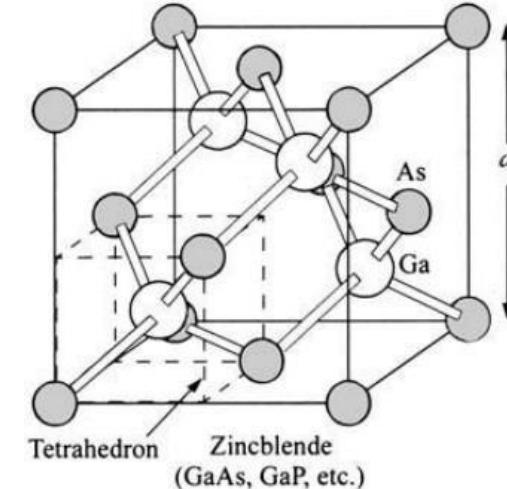


Face-centered cubic
(Al, Au, etc.)
(c)

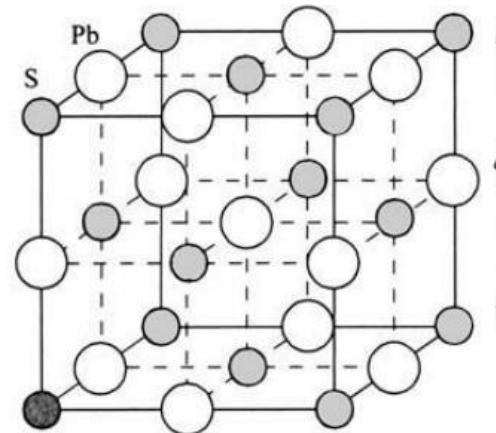
- Many important semiconductors have diamond or **zincblende** lattice structures which belong to the **tetrahedral phases**; that is, each atom is surrounded by four equidistant nearest neighbors which lie at the corners of a tetrahedron.
- The bond between two nearest neighbors is formed by two electrons with opposite spins.
- In this **rock-salt** structure, each atom has six nearest neighbors. Figure shows the wurtzite lattice, which can be considered as two interpenetrating hexagonal close-packed lattices.
- The wurtzite structure has a tetrahedral arrangement of four equidistant nearest neighbors, similar to a zincblende structure.
- A convenient method of defining the various planes in a crystal is to use Miller indices.



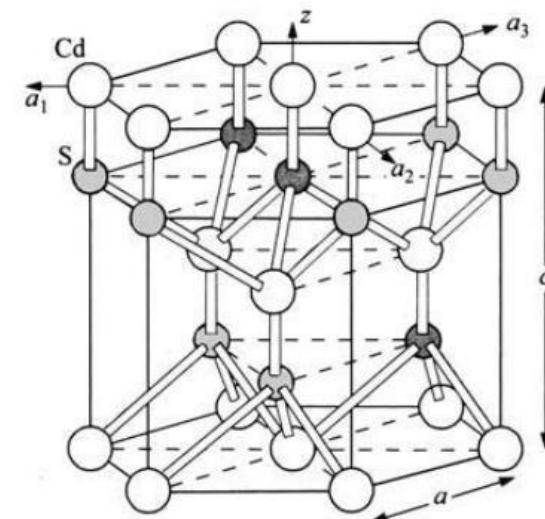
(d)



(e)



(f)



(g)



Unit cell	Number of atoms at			No. of atoms per unit cell	Volume occupied by particles (%)
	Corners	Centres	Faces		
Simple cubic	$8 \times \frac{1}{8} = 1$	0	0	1	52.4
Body centred cubic (BCC)	$8 \times \frac{1}{8} = 1$	1	0	2	68
Face centred cubic (FCC)	$8 \times \frac{1}{8} = 1$	0	$6 \times \frac{1}{2} = 3$	4	74



Problems

Objective: Find the volume density of atoms in a crystal.

Consider a single-crystal material that is a body-centered cubic, as shown in Figure 1.5b, with a lattice constant $a = 5 \text{ \AA} = 5 \times 10^{-8} \text{ cm}$. A corner atom is shared by eight unit cells that meet at each corner so that each corner atom effectively contributes one-eighth of its volume to each unit cell. The eight corner atoms then contribute an equivalent of one atom to the unit cell. If we add the body-centered atom to the corner atoms, each unit cell contains an equivalent of two atoms.

■ Solution

The number of atoms per unit cell is $\frac{1}{8} \times 8 + 1 = 2$

The volume density of atoms is then found as

$$\text{Volume Density} = \frac{\# \text{ atoms per unit cell}}{\text{volume of unit cell}}$$

So

$$\text{Volume Density} = \frac{2}{a^3} = \frac{2}{(5 \times 10^{-8})^3} = 1.6 \times 10^{22} \text{ atoms/cm}^3$$



The lattice constant of a face-centered cubic lattice is 4.25 Å. Determine the (a) effective number of atoms per unit cell and (b) volume density of atoms.

(a) Number of atoms per unit cell

$$= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$$

(b) Volume Density

$$\begin{aligned} &= \frac{4}{a^3} = \frac{4}{(4.25 \times 10^{-8})^3} \\ &= 5.21 \times 10^{22} \text{ cm}^{-3} \end{aligned}$$



Objective: Describe the plane shown in Figure 1.6. (The lattice points in Figure 1.6 are shown along the \bar{a} , \bar{b} , and \bar{c} axes only.)

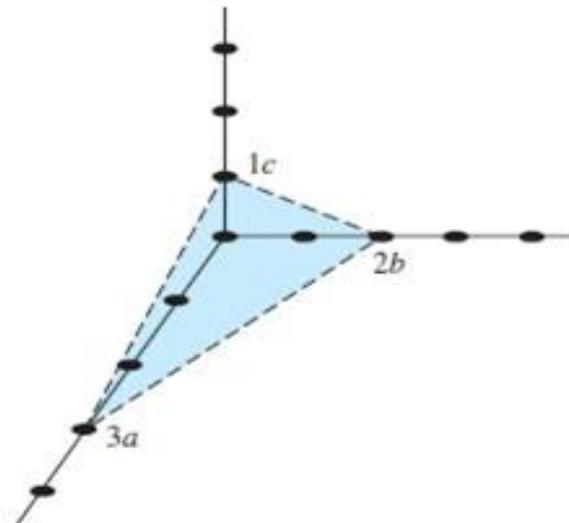


Figure 1.6 | A representative crystal-lattice plane.

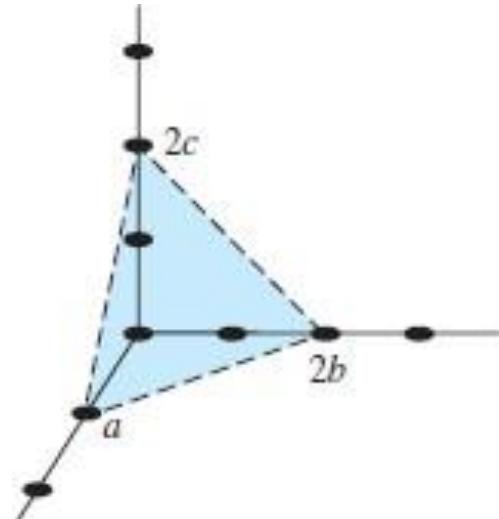
Solution

From Equation (1.1), the intercepts of the plane correspond to $p = 3$, $q = 2$, and $s = 1$. Now write the reciprocals of the intercepts, which gives

$$\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{1}\right)$$

Multiply by the lowest common denominator, which in this case is 6, to obtain $(2, 3, 6)$. The plane in Figure 1.6 is then referred to as the (236) plane. The integers are referred to as the Miller indices. We will refer to a general plane as the (hkl) plane.

Describe the lattice plane shown in Figure



Objective: Calculate the surface density of atoms on a particular plane in a crystal.

Consider the body-centered cubic structure and the (110) plane shown in Figure 1.9a. Assume the atoms can be represented as hard spheres with the closest atoms touching each other. Assume the lattice constant is $a_1 = 5 \text{ \AA}$. Figure 1.9b shows how the atoms are cut by the (110) plane.

The atom at each corner is shared by four similar equivalent lattice planes, so each corner atom effectively contributes one-fourth of its area to this lattice plane as indicated in the figure. The four corner atoms then effectively contribute one atom to this lattice plane. The atom in the center is completely enclosed in the lattice plane. There is no other equivalent plane that

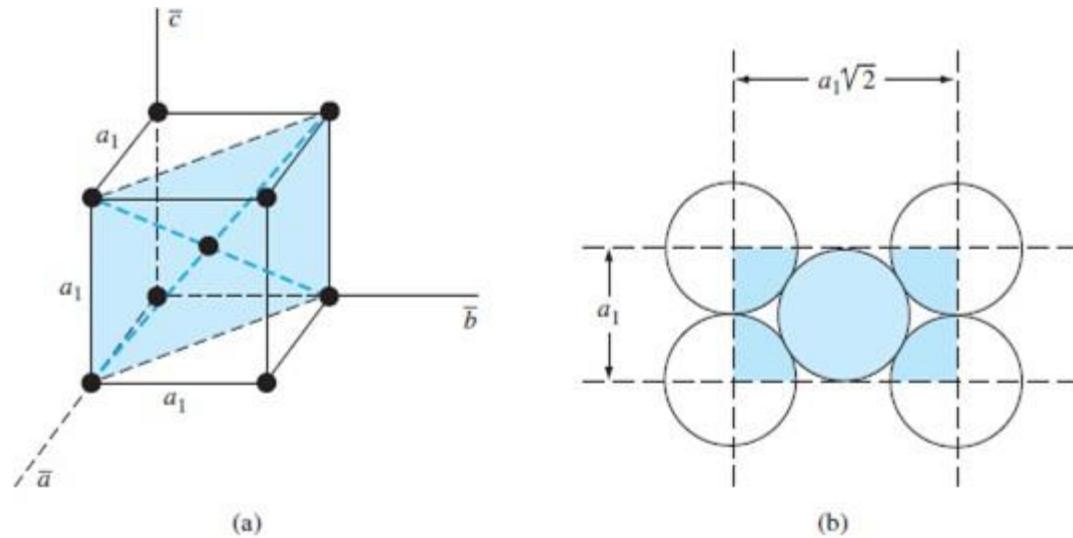


Figure 1.9 | (a) The (110) plane in a body-centered cubic and (b) the atoms cut by the (110) plane in a body-centered cubic.

Solution

The number of atoms per lattice plane is $\frac{1}{4} \times 4 + 1 = 2$

The surface density of atoms is then found as

$$\text{Surface Density} = \frac{\# \text{ of atoms per lattice plane}}{\text{area of lattice plane}}$$

So

$$\begin{aligned}\text{Surface Density} &= \frac{2}{(a_1)(a_1\sqrt{2})} = \frac{2}{(5 \times 10^{-8})^2\sqrt{2}} \\ &= 5.66 \times 10^{14} \text{ atoms/cm}^2\end{aligned}$$

Quantum Mechanics

- The behavior and characteristics of electrons in a semiconductor can be described by the formulation of quantum mechanics called wave mechanics.

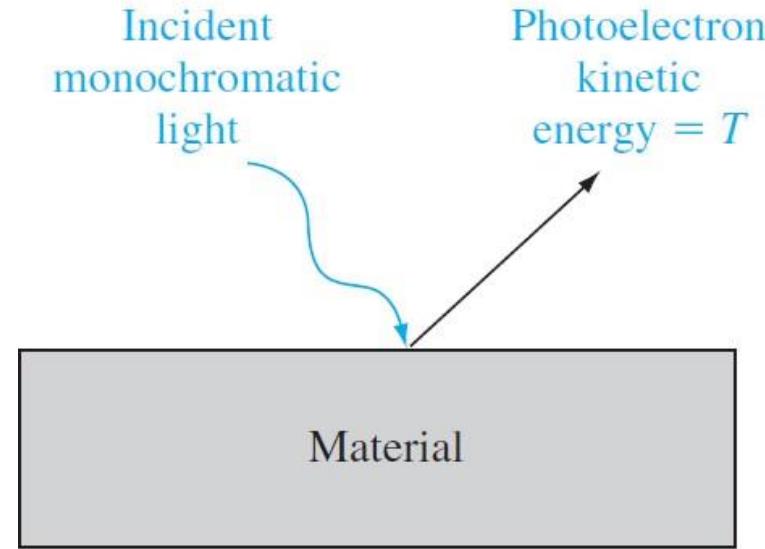
Principles:

- Principle of energy quanta
- Wave-particle duality principle
- Uncertainty principle

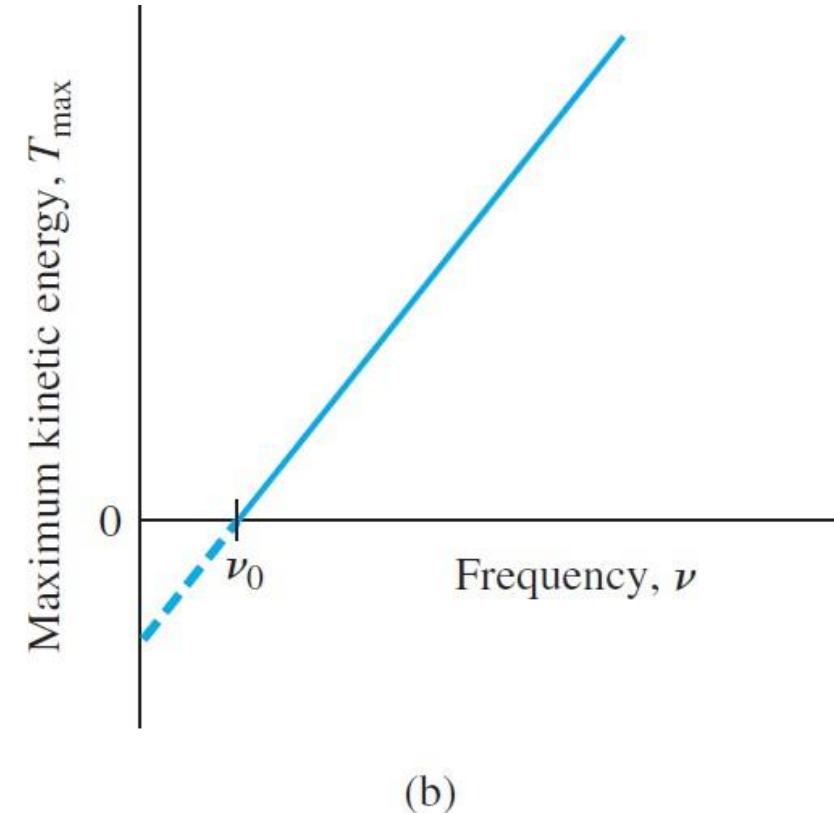


Principle of energy quanta-photoelectric effect

The photoelectric effect shows the discrete nature of the photon and demonstrates the **particle-like behavior** of the photon.



(a)



(b)

Figure 2.1 | (a) The photoelectric effect and (b) the maximum kinetic energy of the photoelectron as a function of incident frequency.

Objective: Calculate the photon energy corresponding to a particular wavelength.

Consider an x-ray with a wavelength of $\lambda = 0.708 \times 10^{-8}$ cm.

■ Solution

The energy is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34})(3 \times 10^{10})}{0.708 \times 10^{-8}} = 2.81 \times 10^{-15} \text{ J}$$

This value of energy may be given in the more common unit of electron-volt (see Appendix D). We have

$$E = \frac{2.81 \times 10^{-15}}{1.6 \times 10^{-19}} = 1.75 \times 10^4 \text{ eV}$$



Wave Particle Duality

In 1924, de Broglie postulated the existence of matter waves. He suggested that since waves exhibit particle-like behavior, particles should be expected to show wave-like properties. The hypothesis of de Broglie was the existence of a *wave-particle duality principle*. The momentum of a photon is given by

$$p = \frac{h}{\lambda} \quad (2.2)$$

where λ is the wavelength of the light wave. Then, de Broglie hypothesized that the wavelength of a particle can be expressed as

$$\lambda = \frac{h}{p} \quad (2.3)$$

where p is the momentum of the particle and λ is known as the *de Broglie wavelength* of the matter wave.



Uncertainty principle

- One way to visualize the uncertainty principle is to consider the simultaneous measurement of position and momentum, and the simultaneous measurement of energy and time.
- One consequence of the uncertainty principle is that we cannot determine the exact position of an electron, instead, determine **the probability of finding an electron at a particular position**.



- Schrodinger, in 1926, provided a formulation called **wave mechanics**, which incorporated the principles of quanta introduced by Planck, and the wave-particle duality principle introduced by de Broglie.
- On the basis of wave-particle duality principle, we will describe the **motion of electrons in a crystal by wave theory**.
- Applications of Schrodinger wave equation includes motion of an electron in free space and in a potential well.



Schrodinger wave equation

Basic Postulates

1. Each particle in a physical system is described by a wavefunction $\Psi(x, y, z, t)$. This function and its space derivative ($\partial\Psi/\partial x + \partial\Psi/\partial y + \partial\Psi/\partial z$) are continuous, finite, and single valued.
2. In dealing with classical quantities such as energy E and momentum p , we must relate these quantities with abstract quantum mechanical operators defined in the following way:

Classical variable	Quantum operator
x	x
$f(x)$	$f(x)$
$p(x)$	$\frac{\hbar}{j} \frac{\partial}{\partial x}$
E	$-\frac{\hbar}{j} \frac{\partial}{\partial t}$

and similarly for the other two directions.



3. The probability of finding a particle with wavefunction Ψ in the volume $dx dy dz$ is $\Psi^* \Psi dx dy dz$.² The product $\Psi^* \Psi$ is normalized according to Eq. (2-20) so that

$$\int_{-\infty}^{\infty} \Psi^* \Psi dx dy dz = 1$$

and the average value $\langle Q \rangle$ of any variable Q is calculated from the wavefunction by using the operator form Q_{op} defined in postulate 2:

$$\langle Q \rangle = \int_{-\infty}^{\infty} \Psi^* Q_{op} \Psi dx dy dz$$

² Ψ^* is the *complex conjugate* of Ψ , obtained by reversing the sign on each j . Thus, $(e^{ix})^* = e^{-ix}$.



The classical equation for the energy of a particle can be written:

$$\text{Kinetic energy} + \text{potential energy} = \text{total energy} \quad (2-22)$$

$$\frac{1}{2m}p^2 + V = E$$

In quantum mechanics we use the operator form for these variables (postulate 2); the operators are allowed to operate on the wavefunction Ψ . For a one-dimensional problem Eq. (2-22) becomes³

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = -\frac{\hbar}{j} \frac{\partial\Psi(x,t)}{\partial t} \quad (2-23)$$

which is the Schrödinger wave equation. In three dimensions the equation is

$$-\frac{\hbar^2}{2m} \nabla^2\Psi + V\Psi = -\frac{\hbar}{j} \frac{\partial\Psi}{\partial t} \quad (2-24)$$

where $\nabla^2\Psi$ is

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$$



Electron in free space

Recall that the time-dependent portion of the solution is

$$\phi(t) = e^{-j(E/\hbar)t} = e^{-j\omega t} \quad (2.22)$$

Then the total solution for the wave function is given by

$$\Psi(x, t) = A \exp[j(kx - \omega t)] + B \exp[-j(kx + \omega t)] \quad (2.23)$$

This wave function solution is a traveling wave, which means that a particle moving in free space is represented by a traveling wave. The first term, with the coefficient



Electron in an infinite potential well

Finally, the time-independent wave solution is given by

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad \text{where } n = 1, 2, 3, \dots \quad (2.36)$$

This solution represents the electron in the infinite potential well and is a standing wave solution. The free electron was represented by a traveling wave, and now the bound particle is represented by a standing wave.

The total energy can then be written as

$$E = E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \quad \text{where } n = 1, 2, 3, \dots \quad (2.38)$$

For the particle in the infinite potential well, the wave function is now given by

$$\psi(x) = \sqrt{\frac{2}{a}} \sin k_n x \quad (2.39)$$



Objective: Calculate the first three energy levels of an electron in an infinite potential well.

Consider an electron in an infinite potential well of width 5 Å.

■ Solution

From Equation (2.38) we have⁴

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2(1.054 \times 10^{-34})^2 \pi^2}{2(9.11 \times 10^{-31})(5 \times 10^{-10})^2} = n^2(2.41 \times 10^{-19}) \text{ J}$$

or

$$E_n = \frac{n^2(2.41 \times 10^{-19})}{1.6 \times 10^{-19}} = n^2(1.51) \text{ eV}$$

Then,

$$E_1 = 1.51 \text{ eV}, \quad E_2 = 6.04 \text{ eV}, \quad E_3 = 13.59 \text{ eV}$$



Tunneling

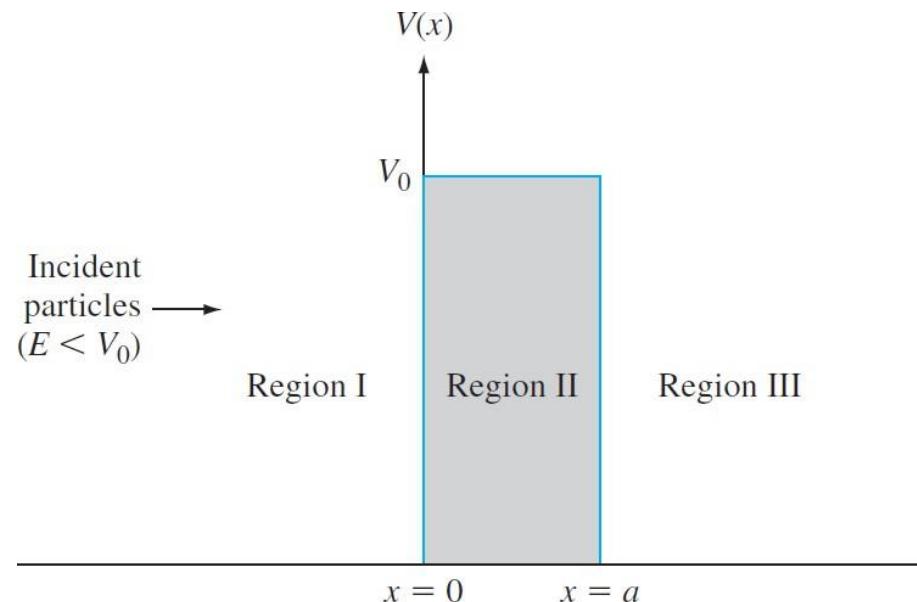


Figure 2.9 | The potential barrier function.

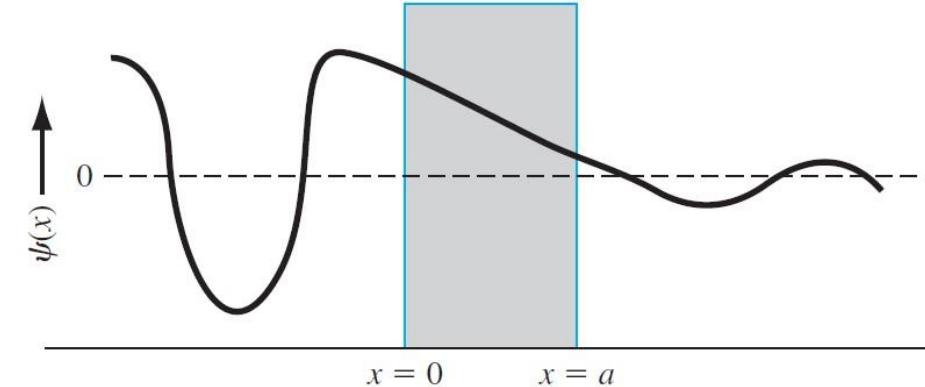


Figure 2.10 | The wave functions through the potential barrier.

$$T \approx 16 \left(\frac{E}{V_0} \right) \left(1 - \frac{E}{V_0} \right) \exp(-2k_2 a) \quad (2.63)$$

Equation (2.63) implies that there is a finite probability that a particle impinging a potential barrier will penetrate the barrier and will appear in region III. This phenomenon is called tunneling and it, too, contradicts classical mechanics. We

$$k = \frac{2\pi}{\lambda} \quad (2.27b)$$

A free particle with a well-defined energy will also have a well-defined wavelength and momentum.

- A free particle with a well-defined momentum can be found anywhere with equal probability.
- This result is in agreement with the Heisenberg uncertainty principle in which a precise momentum implies an undefined position.
- A localized free particle is defined by a wave packet, formed by a superposition of wave functions with different momentum or k values.



The wavefunction of the electron is assumed to be in the form of a plane wave moving, for example, in the x-direction with propagation constant k , also called a wavevector. The space-dependent wavefunction for the electron is

$$\psi_{\mathbf{k}}(x) = U(\mathbf{k}_x, x)e^{j\mathbf{k}_x x} \quad (3-1)$$

where the function $U(\mathbf{k}_x, x)$ modulates the wavefunction according to the periodicity of the lattice. Such wavefunctions are called Bloch functions after the physicist Felix Bloch.

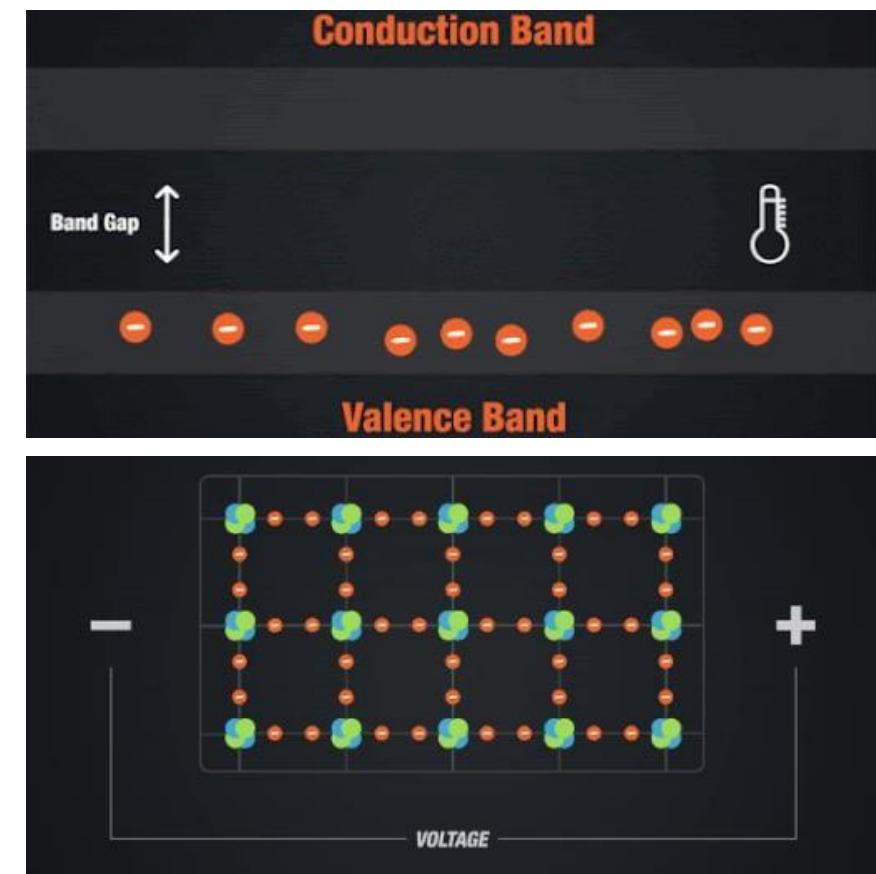
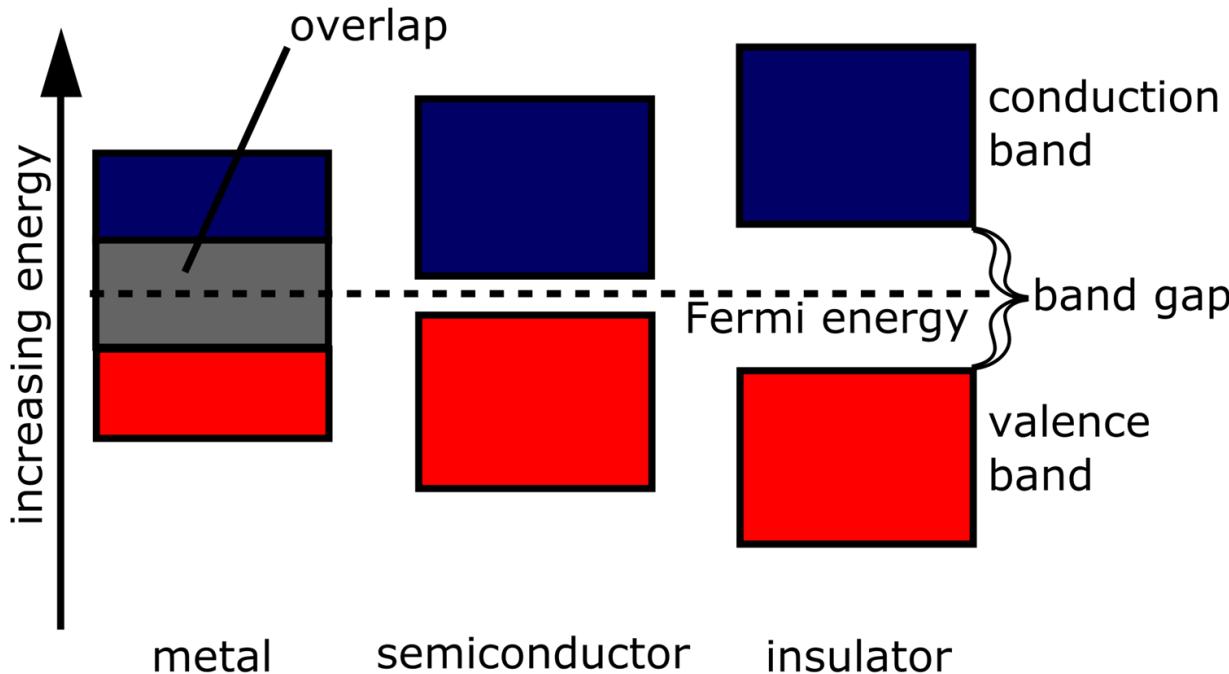


Formation of energy bands in Semiconductor

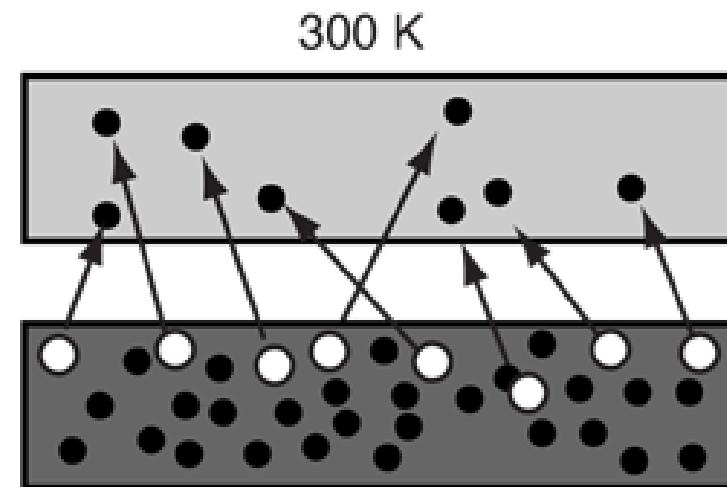
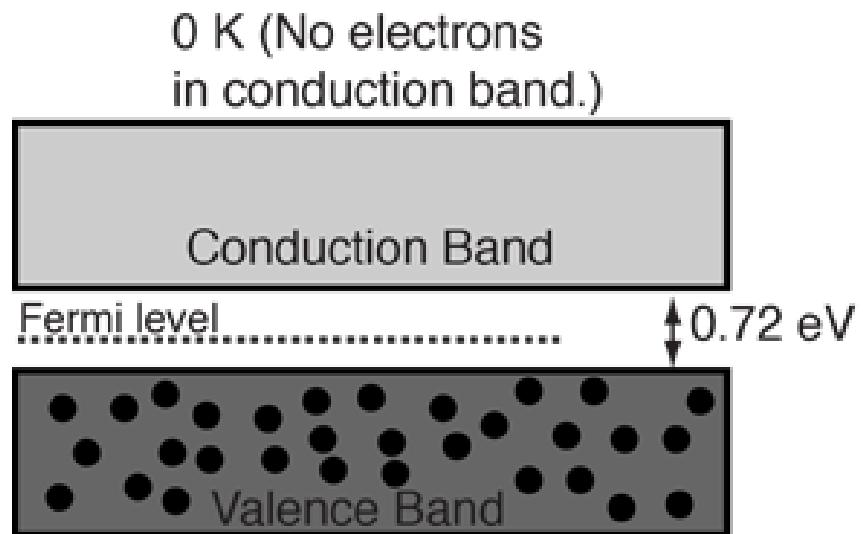
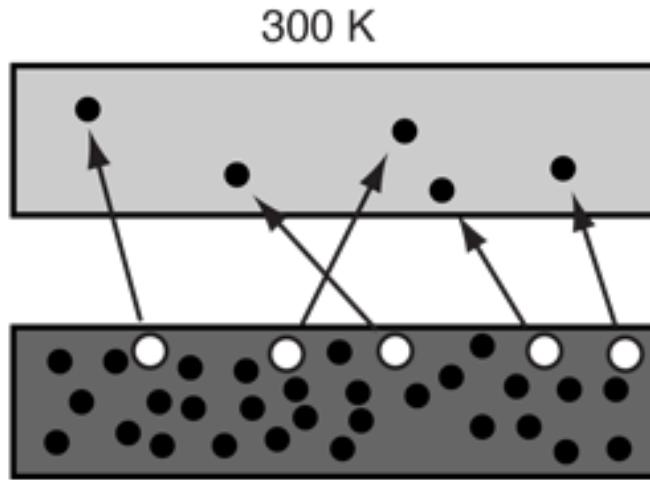
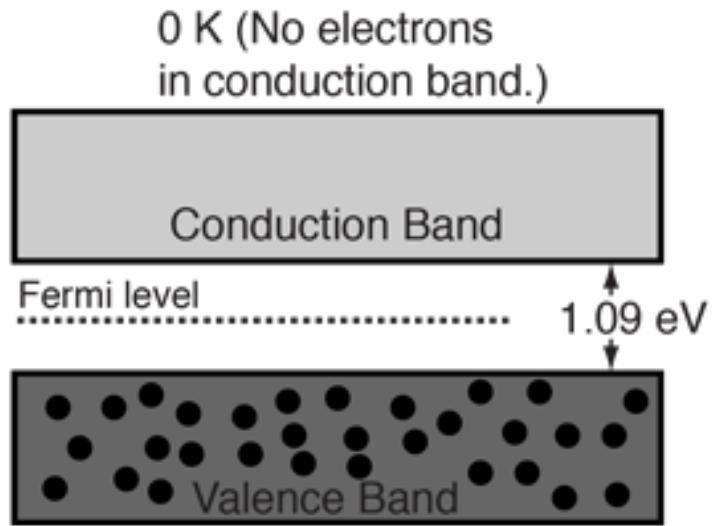
- The band in which the conduction electrons are included is called **conduction energy band**. The energy occupied by the conduction band is lower.
- These valence electrons are usually formed by a series of energy levels which in turn makes the **valence band**. This valence band is known to have the highest level of occupied energy.
- There is a space between the conduction and valence band, this space is prohibited. This prohibited space is known as the **forbidden gap**. This band lacks energy which in turn doesn't allow this band to exist at all. This results in zero electron flow in this band. This forbidden gap allows the electron to travel from the valence to the conduction state. |



- **Fermi level**, the top of the available electron energy levels at low temperatures. The position of the Fermi level with the relation to the conduction band is a crucial factor in determining electrical properties.



Energy band diagram of Silicon and Germanium



Electron Configuration for Silicon

Max electrons (s) = 2

Max electrons (p) = 6

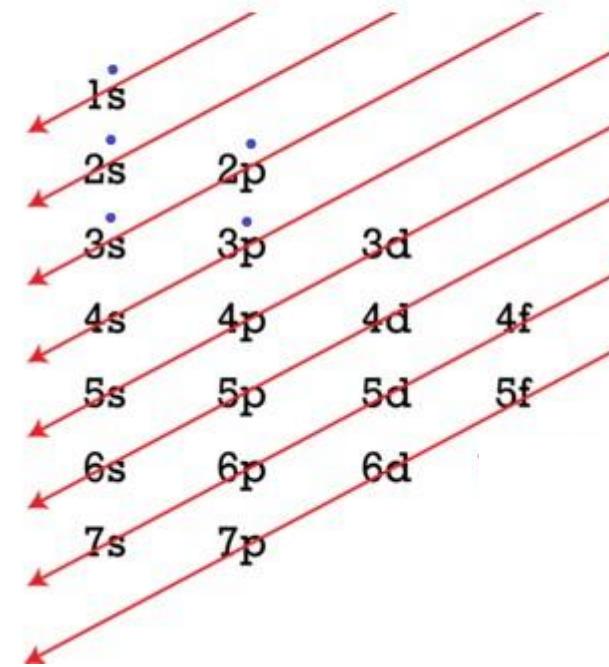
Max electrons (d) = 10

Max electrons (f) = 14

Silicon has a total of 14 electrons

Silicon Electron Configuration

$1s^2 2s^2 2p^6 3s^2 3p^2$



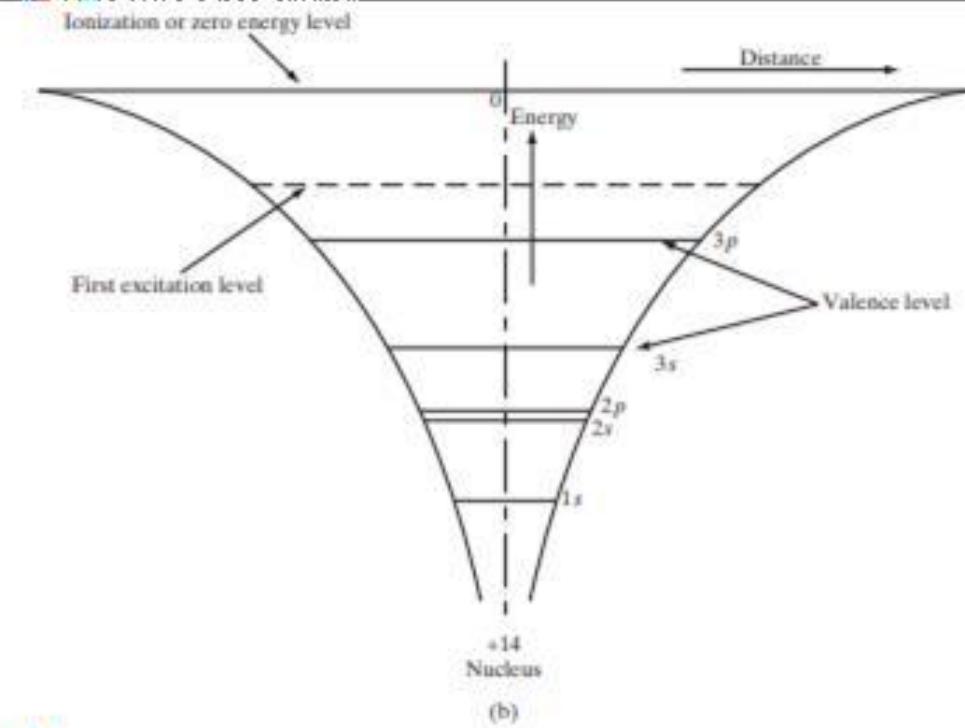
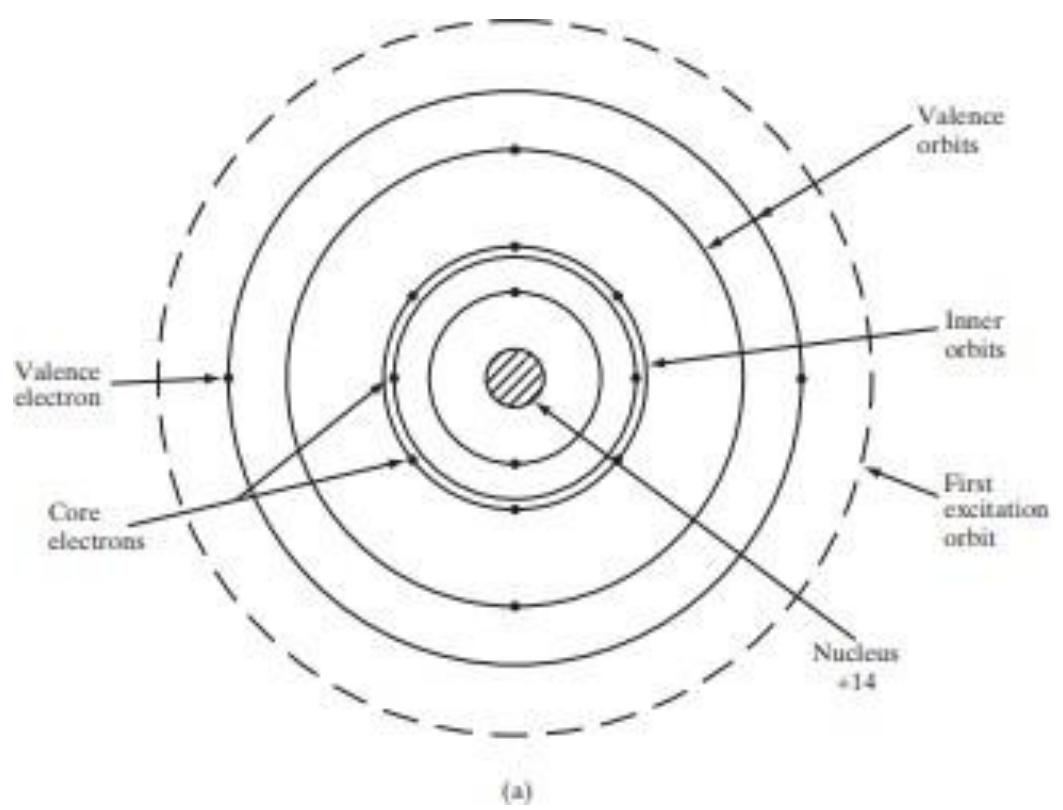
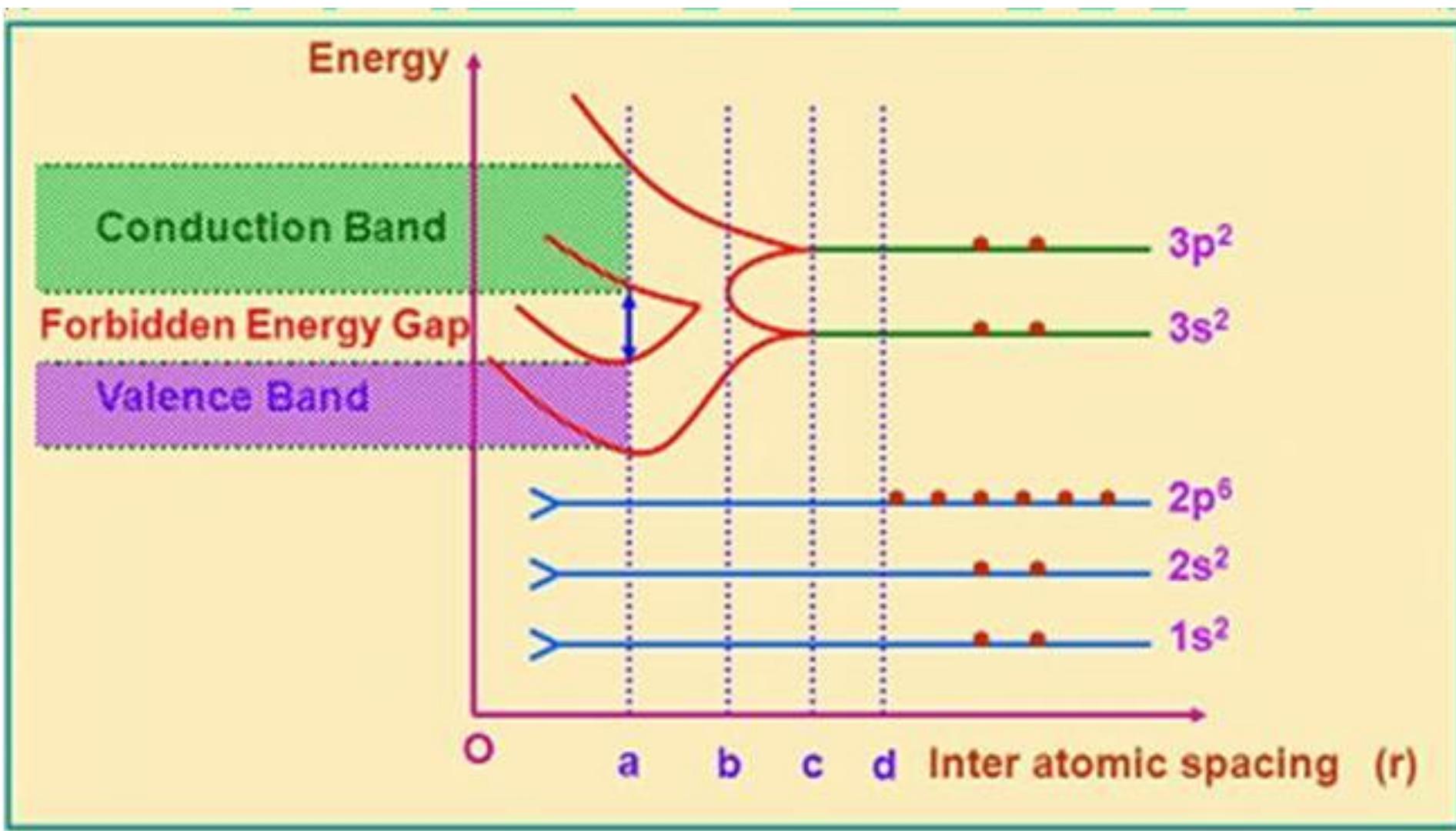


Figure 2-8

Electronic structure and energy levels in a Si atom: (a) the orbital model of a Si atom showing the 10 core electrons ($n = 1$ and 2), and the four valence electrons ($n = 3$); (b) energy levels in the Coulombic potential of the nucleus are also shown schematically.

- As isolated atoms are brought together to form a solid, various interactions occur between neighboring atoms.
- The forces of attraction and repulsion between atoms will find a balance at the **proper interatomic spacing** for the crystal.
- In the process, **important changes** occur in the **electron energy level configurations**.





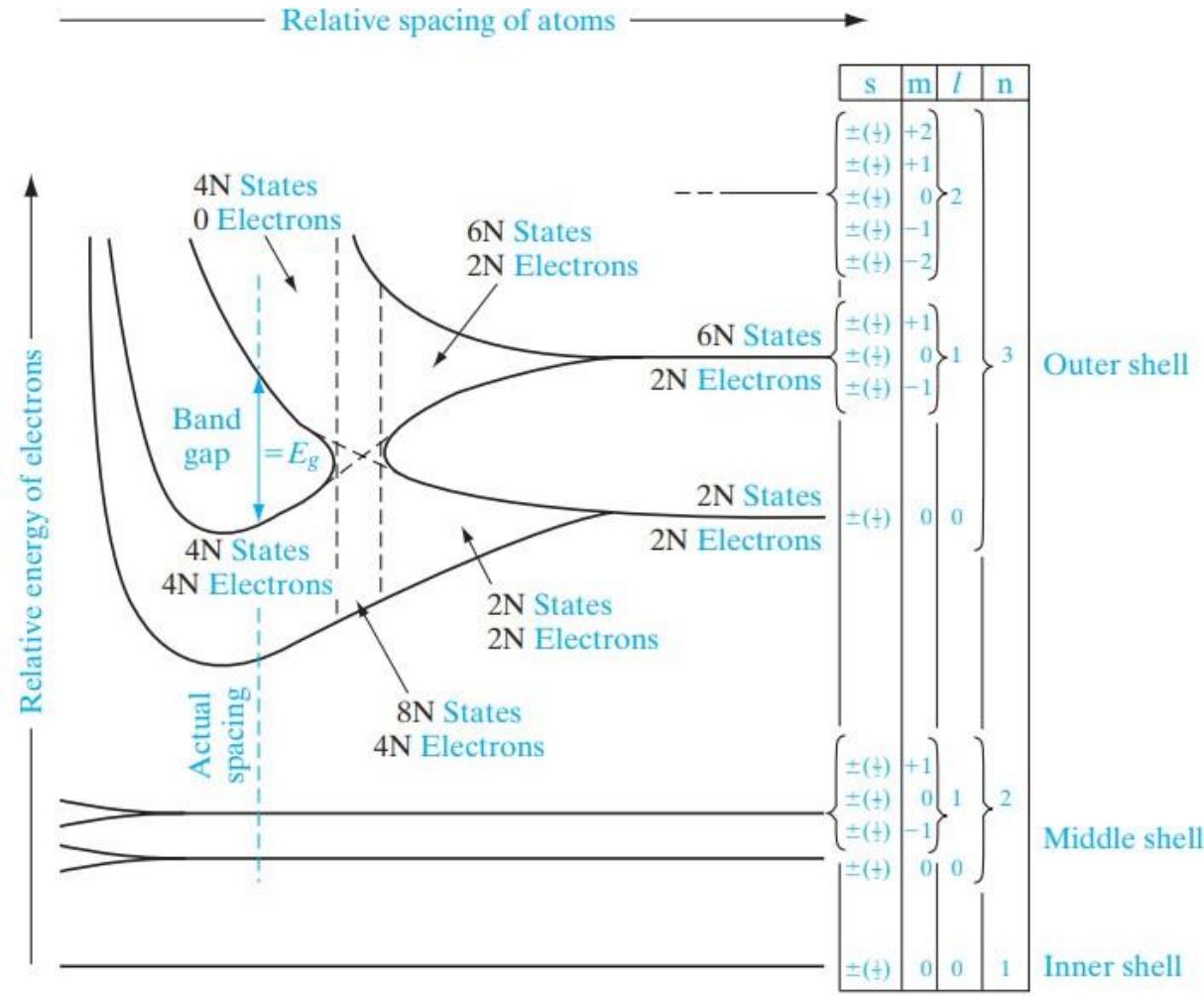


Figure 3–3

Energy levels in Si as a function of interatomic spacing. The core levels ($n = 1, 2$) in Si are completely filled with electrons. At the actual atomic spacing of the crystal, the $2N$ electrons in the $3s$ subshell and the $2N$ electrons in the $3p$ subshell undergo sp^3 hybridization, and all end up in the lower $4N$ states (valence band), while the higher-lying $4N$ states (conduction band) are empty, separated by a band gap.

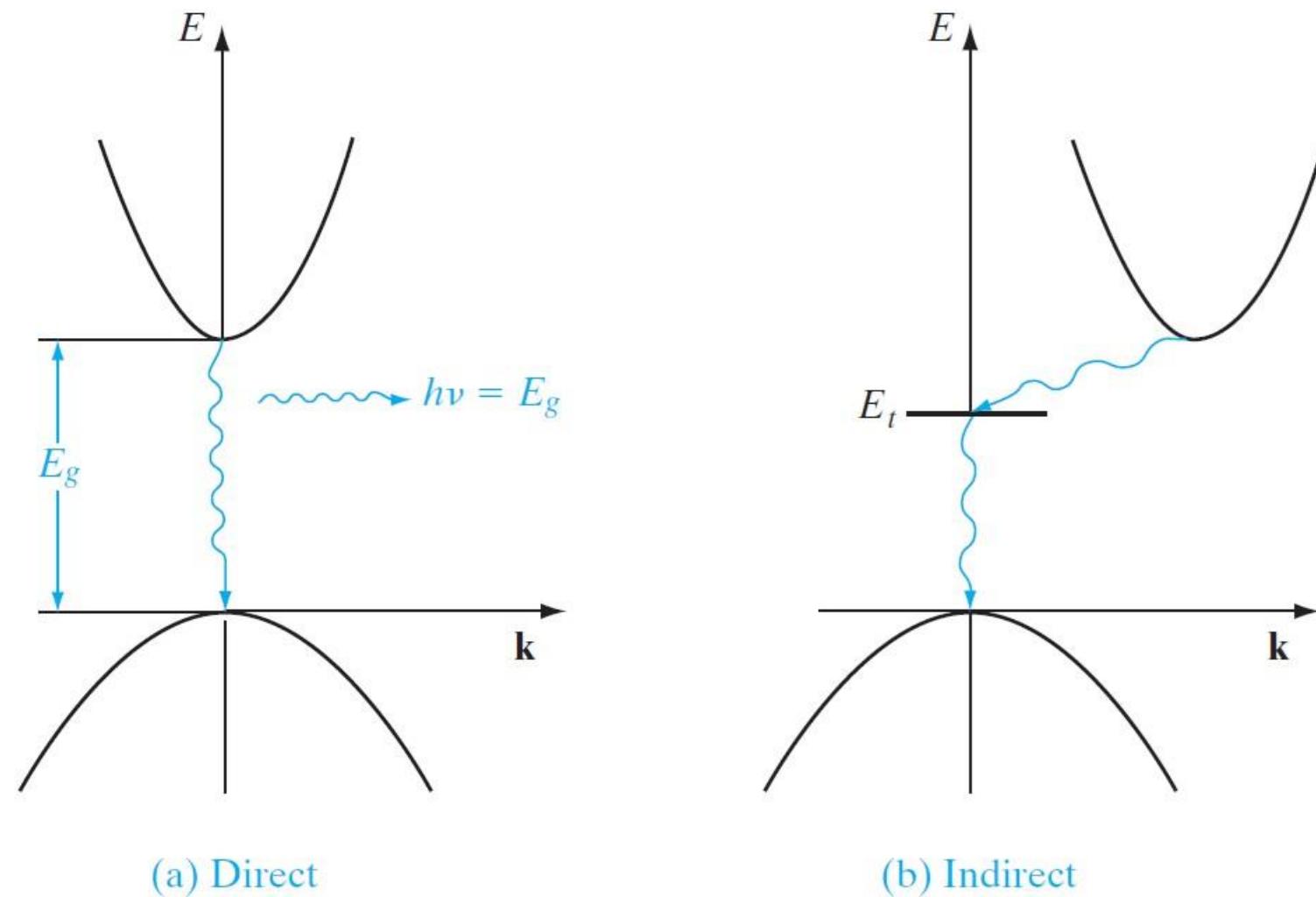
Direct and Indirect Semiconductors

- In a **direct semiconductor such as GaAs**, an electron in the conduction band can fall to an empty state in the valence band, giving off the energy difference E_g as a photon of light.
- On the other hand, an electron in the conduction band minimum of an **indirect semiconductor such as Si** cannot fall directly to the valence band maximum but must undergo a momentum change as well change in energy.



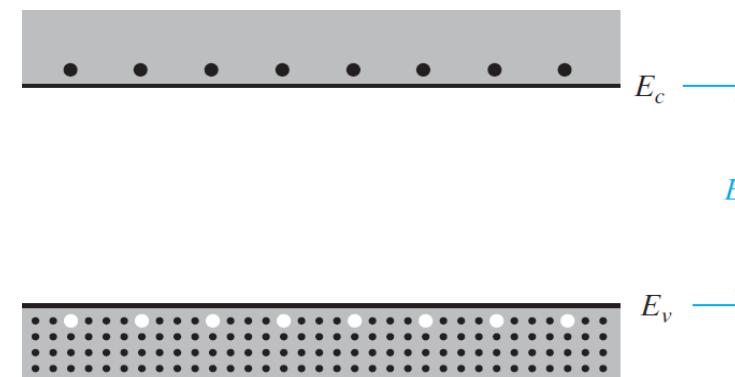
Direct and Indirect Semiconductors

Figure 3–5
Direct and
indirect electron
transitions in
semiconductors:
(a) direct
transition with
accompanying
photon emission;
(b) indirect
transition via a
defect level.

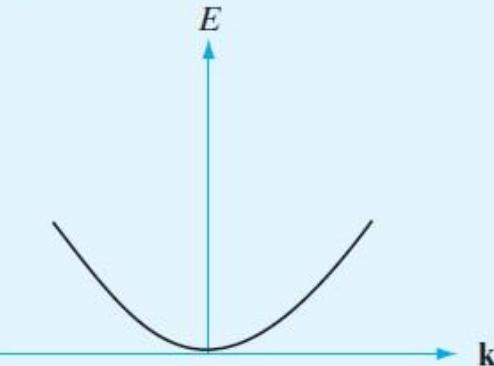


Charge carriers in semiconductors

- If the conduction band electron and the hole are created by the excitation of a valence band electron to the conduction band, they are called an electron-hole pair (abbreviated EHP).
- After excitation to the conduction band, an electron is surrounded by a large number of unoccupied energy states.
- For example, the equilibrium number of EHPs in pure Si at room temperature is only about 10^{10} EHP/cm³, compared to the Si atom density of 5×10^{22} atoms/cm³.
- Thus the few electrons in the conduction band are free to move about via the many available empty states.



Find the (E, \mathbf{k}) relationship for a free electron and relate it to the electron mass.



From Example 2–2, the electron momentum is $\mathbf{p} = m\mathbf{v} = \hbar\mathbf{k}$. Then

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{\mathbf{p}^2}{m} = \frac{\hbar^2}{2m}\mathbf{k}^2$$

Thus the electron energy is parabolic with wavevector \mathbf{k} . The electron mass is inversely related to the curvature (second derivative) of the (E, \mathbf{k}) relationship, since

$$\frac{d^2E}{d\mathbf{k}^2} = \frac{\hbar^2}{m}$$

Although electrons in solids are not free, most energy bands are close to parabolic at their minima (for conduction bands) or maxima (for valence bands). We can also approximate effective mass near those band extrema from the curvature of the band.



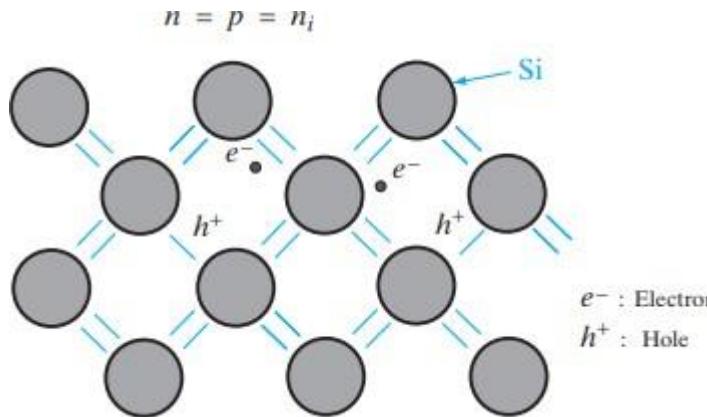
Effective mass of an electron

The effective mass of an electron in a band with a given (E, \mathbf{k}) relationship is found in Example 3–2 to be

$$m^* = \frac{\hbar^2}{d^2 E/d\mathbf{k}^2} \quad (3-3)$$



Intrinsic carrier concentration



- At higher temperatures **EHPs** are generated as valence band electrons are excited thermally across the band gap to the conduction band. These EHPs are the only charge carriers in intrinsic material

$$n = p = n_i$$

where

n = conduction band electron concentration (electrons per cm³)

p = holes in the valence band (holes per cm³)

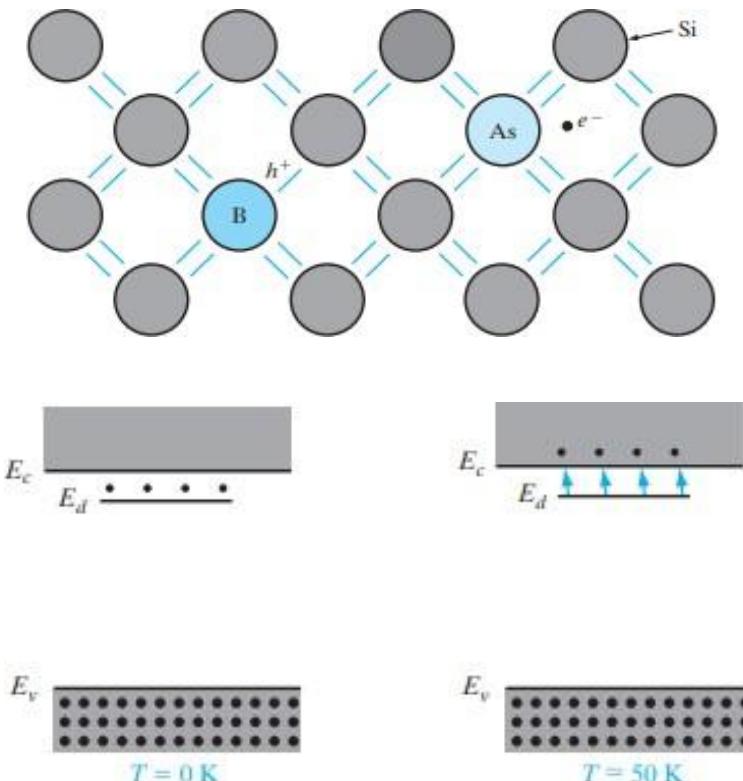
Recombination occurs when an electron in the conduction band makes a transition (direct or indirect) to an empty state (hole) in the valence band, thus annihilating the pair. If we denote the generation rate of EHPs as g_i (EHP/cm³-s) and the recombination rate as r_i , equilibrium requires that

$$r_i = g_i \quad (3-7a)$$

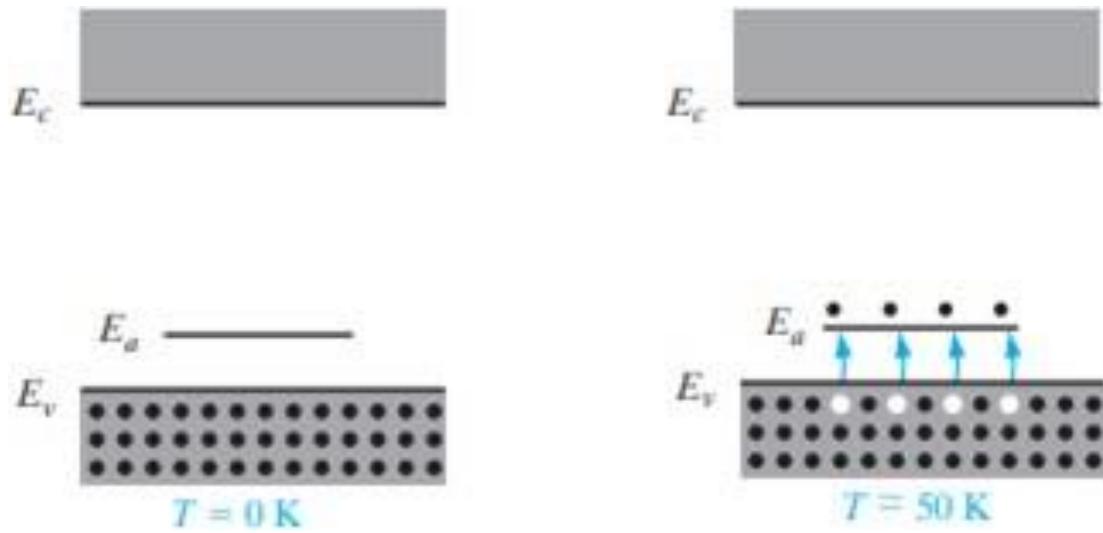


Extrinsic carrier concentration

- When a crystal is doped such that the equilibrium carrier concentrations n_0 and p_0 are different from the intrinsic carrier concentration n_i , the material is said to be extrinsic.



- An impurity from column V of the periodic table (P, As, and Sb) introduces an **energy level very near the conduction band** in Ge or Si.
- This level is filled with electrons at 0 K, and very little thermal energy is required to excite these electrons to the conduction band.
- Thus semiconductors doped with a significant number of donor atoms will **have $n_0 > (n_i, p_0)$** at room temperature. This is **n-type material**.



- Atoms from column III (B, Al, Ga, and In) introduce impurity levels in Ge or Si **near the valence band**.
- At low temperatures, enough thermal energy is available to excite electrons from the valence band into the impurity level, leaving **holes in the valence band**.
- Acceptor impurities create a **hole concentration $p_0 >$ conduction band electron concentration n_0** (this type is **p-type material**).

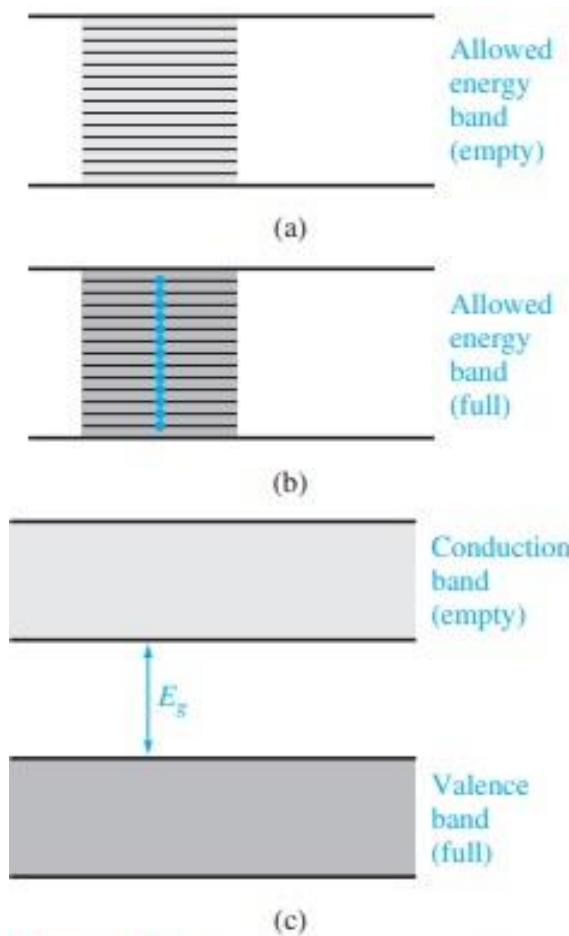


Figure 3.19 | Allowed energy bands showing (a) an empty band, (b) a completely full band, and (c) the bandgap energy between the two allowed bands.

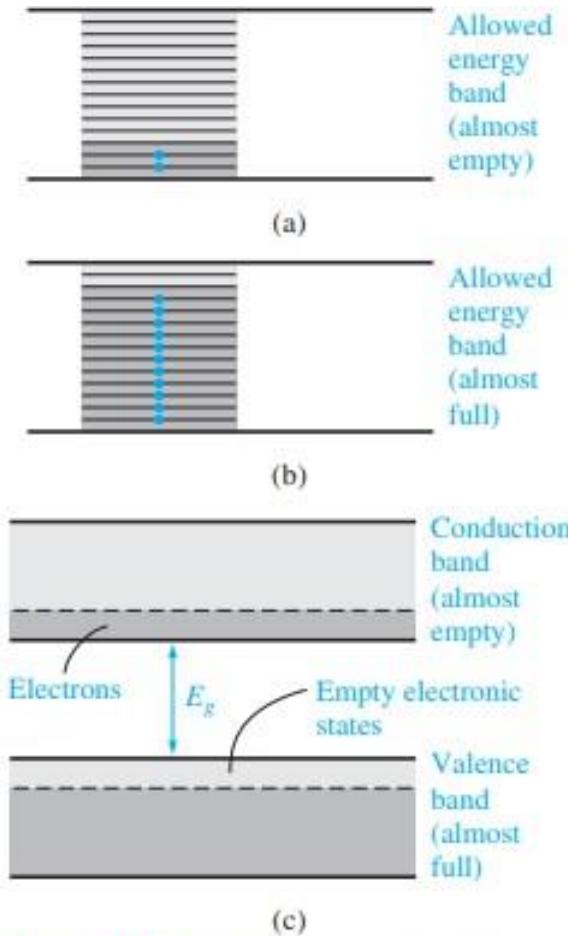


Figure 3.20 | Allowed energy bands showing (a) an almost empty band, (b) an almost full band, and (c) the bandgap energy between the two allowed bands.

Fermi levels

Electrons in solids obey *Fermi–Dirac* statistics.³ In the development of this type of statistics, one must consider the indistinguishability of the electrons, their wave nature, and the Pauli exclusion principle. The rather simple result of these statistical arguments is that the distribution of electrons over a range of allowed energy levels at thermal equilibrium is

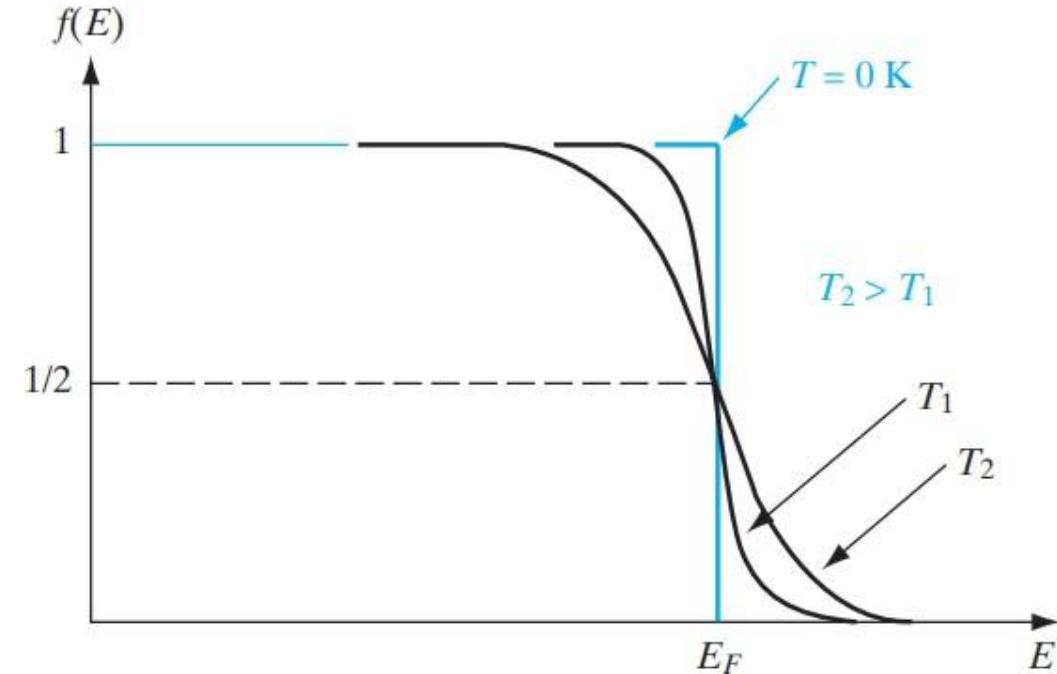
$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \quad (3-10)$$

where k is Boltzmann's constant ($k = 8.62 \times 10^{-5} \text{ eV/K} = 1.38 \times 10^{-23} \text{ J/K}$). The function $f(E)$, the *Fermi–Dirac distribution function*, gives the probability that an available energy state at E will be occupied by an electron at absolute temperature T . The quantity E_F is called the *Fermi level*, and it represents



- Calculate $f(E)$ at $E=E_f$, $T=0K$ and $E>E_f$ and $E<E_f$

- This rectangular distribution implies that at 0 K every available energy state up to E_F is filled with electrons, and all states above E_F are empty.



Objective: Calculate the probability that an energy state above E_F is occupied by an electron.

Let $T = 300$ K. Determine the probability that an energy level $3kT$ above the Fermi energy is occupied by an electron.

■ **Solution**

From Equation (3.79), we can write

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{3kT}{kT}\right)}$$

which becomes

$$f_F(E) = \frac{1}{1 + 20.09} = 0.0474 = 4.74\%$$

■ **Comment**

At energies above E_F , the probability of a state being occupied by an electron can become significantly less than unity, or the ratio of electrons to available quantum states can be quite small.

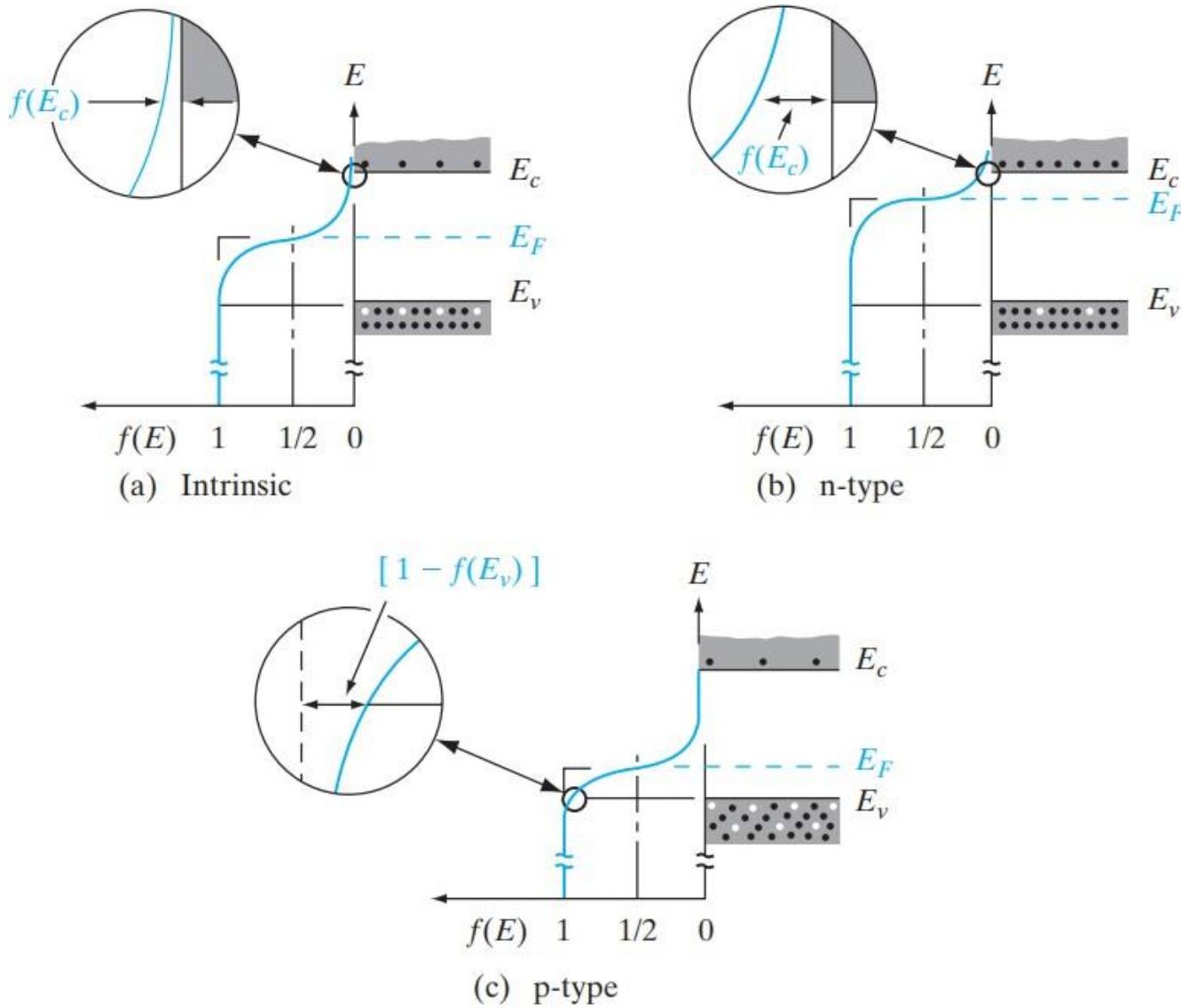


Assume the Fermi energy level is 0.30 eV below the conduction band energy E_c . Assume $T = 300$ K. (a) Determine the probability of a state being occupied by an electron at $E = E_c + kT/4$. (b) Repeat part (a) for an energy state at $E = E_c + kT$.

$$\begin{aligned}
 \text{(a)} \quad f_F(E) &\equiv \exp\left[\frac{-(E - E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(E_c + kT/4 - E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(0.30 + 0.0259/4)}{0.0259}\right] \\
 &= 7.26 \times 10^{-6} \\
 \text{(b)} \quad f_F(E) &\equiv \exp\left[\frac{-(E - E_F)}{kT}\right] \\
 &= \exp\left[\frac{-(0.30 + 0.0259)}{0.0259}\right] \\
 &= 3.43 \times 10^{-6}
 \end{aligned}$$

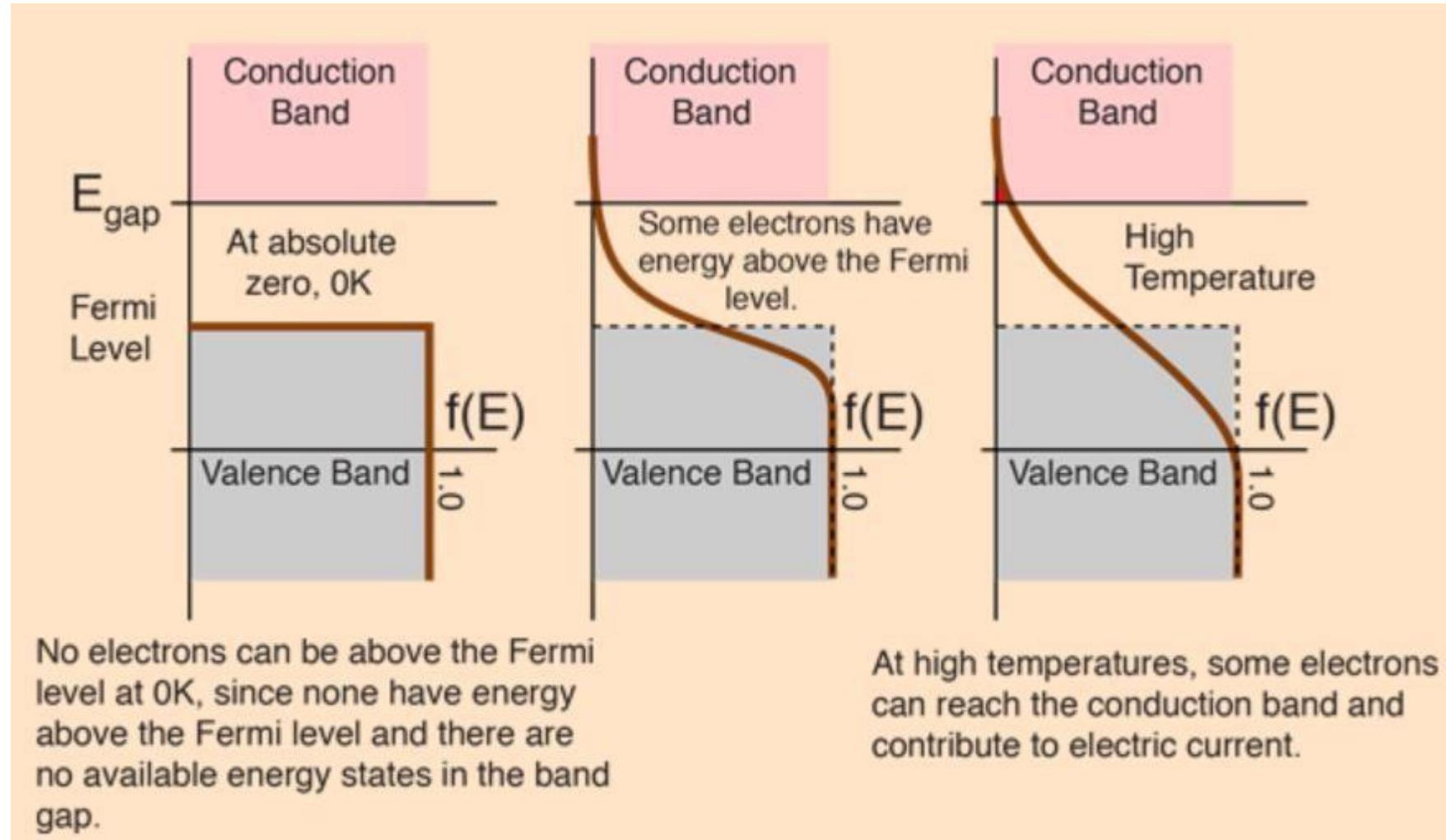


Figure 3-15
The Fermi distribution function applied to semiconductors:
(a) intrinsic material; (b) n-type material;
(c) p-type material.



Density of states and Fermi levels

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

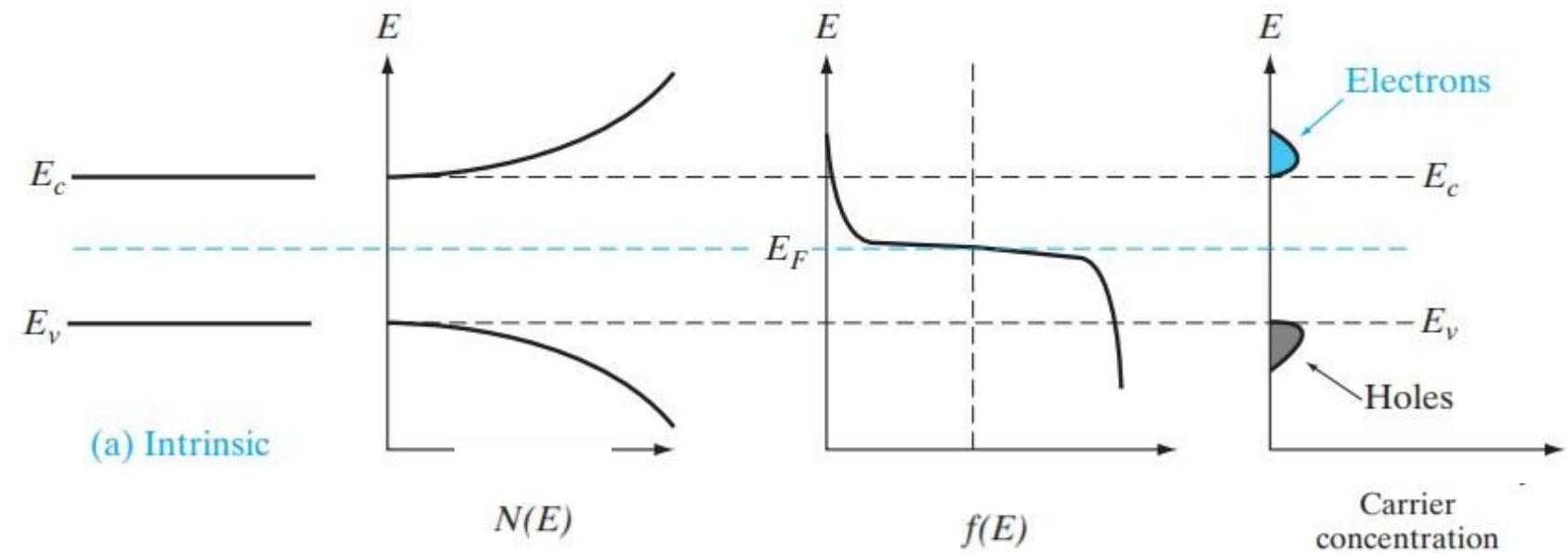


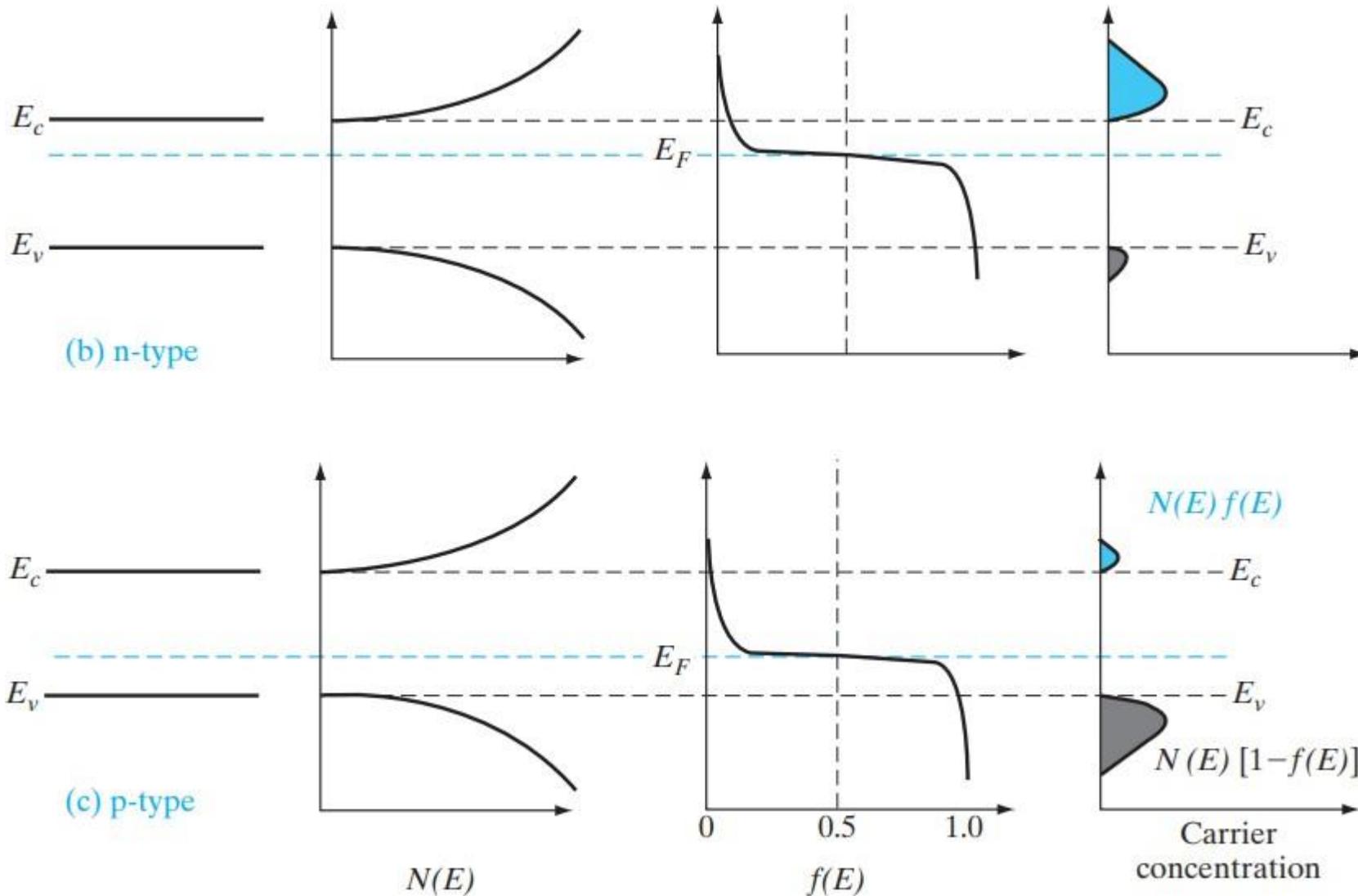
Electron and hole concentration

- The Fermi distribution function can be used to calculate the concentrations of electrons and holes in a semiconductor, if the densities of available states in the valence and conduction bands are known.

$$n_0 = \int_{E_c}^{\infty} f(E)N(E)dE$$

where $N(E)dE$ is the density of states (cm^{-3}) in the energy range dE





Concentration of electrons

$$n_0 = N_c f(E_c)$$

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \simeq e^{-(E_c - E_F)/kT}$$

$$n_0 = N_c e^{-(E_c - E_F)/kT}$$

$$N_c = 2 \left(\frac{2\pi m_n^* k T}{h^2} \right)^{3/2}$$

$$\begin{aligned} n_0 p_0 &= (N_c e^{-(E_c - E_F)/kT})(N_v e^{-(E_F - E_v)/kT}) = N_c N_v e^{-(E_c - E_v)/kT} \\ &= N_c N_v e^{-E_g/kT} \end{aligned}$$

Concentration of holes

$$p_0 = N_v [1 - f(E_v)]$$

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{(E_v - E_F)/kT}} \simeq e^{-(E_F - E_v)/kT}$$

$$p_0 = N_v e^{-(E_F - E_v)/kT}$$

$$N_v = 2 \left(\frac{2\pi m_p^* k T}{h^2} \right)^{3/2}$$

$$n_0 p_0 = n_i^2$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$



Extrinsic carrier concentration

- controlled amounts of specific dopant or impurity atoms have been added.
- the thermal-equilibrium electron and hole concentrations are different from the intrinsic carrier concentration.
- the Fermi energy will change as dopant atoms are added.
- If the Fermi energy changes from near the midgap value, the density of electrons in the conduction band and the density of holes in the valence band will change.

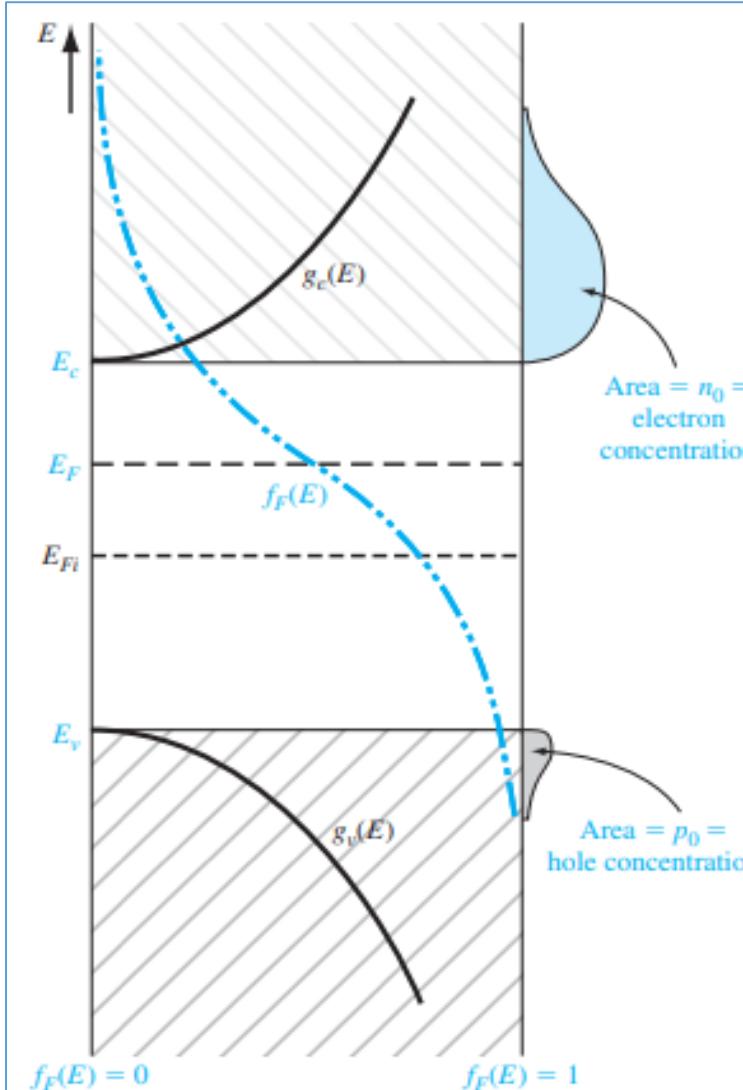


Figure 4.8 | Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is above the intrinsic Fermi energy.

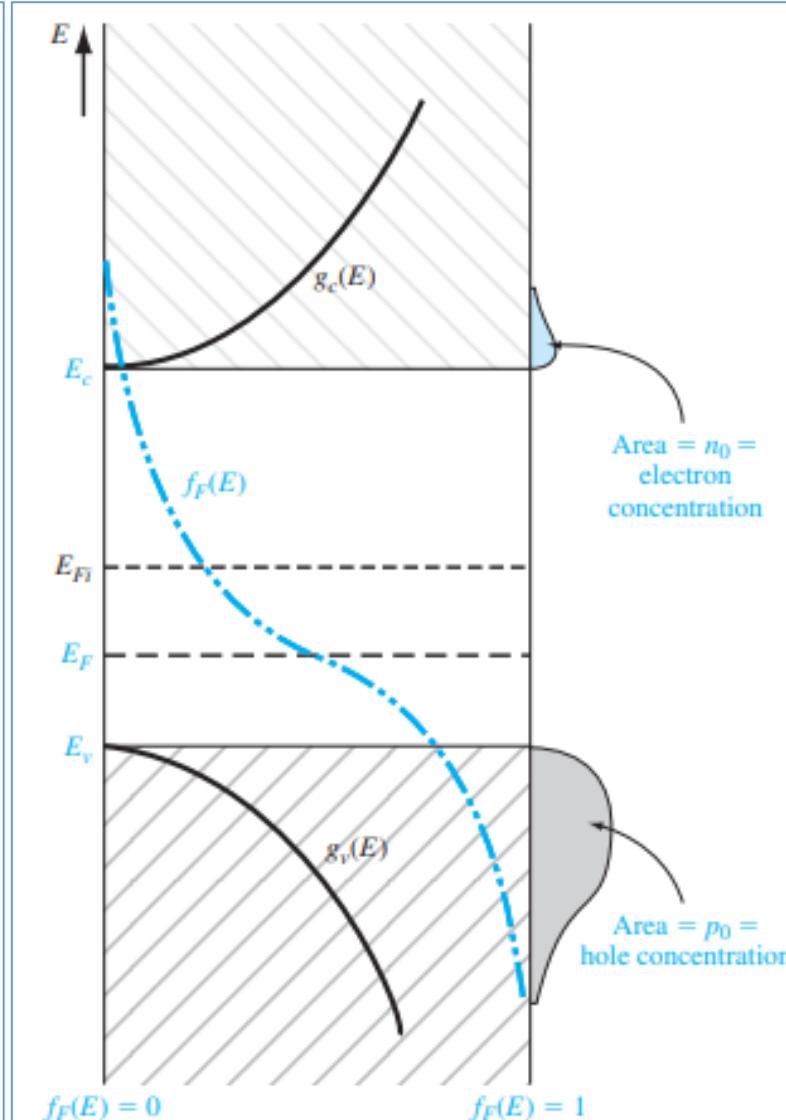


Figure 4.9 | Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is below the intrinsic Fermi energy.



- When the density of electrons is greater than the density of holes, the semiconductor is n type; donor impurity atoms have been added.
- When the density of holes is greater than the density of electrons, the semiconductor is p type; acceptor impurity atoms have been added.
- What is the equation for n_0 and p_0 ?



- If $n_0 > p_0$, the semiconductor is n-type
- $p_0 > n_0$, the semiconductor is p-type

$$n_0 = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right]$$

$$p_0 = N_v \exp \left[\frac{-(E_F - E_v)}{kT} \right]$$

$$n_0 = N_c \exp \left[\frac{-(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT} \right]$$

$$n_0 = N_c \exp \left[\frac{-(E_c - E_{Fi})}{kT} \right] \exp \left[\frac{(E_F - E_{Fi})}{kT} \right]$$



the thermal-equilibrium electron concentration can be written as

$$n_0 = n_i \exp \left[\frac{E_F - E_{Fi}}{kT} \right]$$

- Similarly calculate for p_0

$$p_0 = n_i \exp \left[\frac{-(E_F - E_{Fi})}{kT} \right]$$

intrinsic Fermi level, n_0 and p_0 change from the n_i value. If $E_F > E_{Fi}$, then we will have $n_0 > n_i$ and $p_0 < n_i$. One characteristic of an n-type semiconductor is that $E_F > E_{Fi}$ so that $n_0 > p_0$. Similarly, in a p-type semiconductor, $E_F < E_{Fi}$ so that $p_0 > n_i$ and $n_0 < n_i$; thus, $p_0 > n_0$.



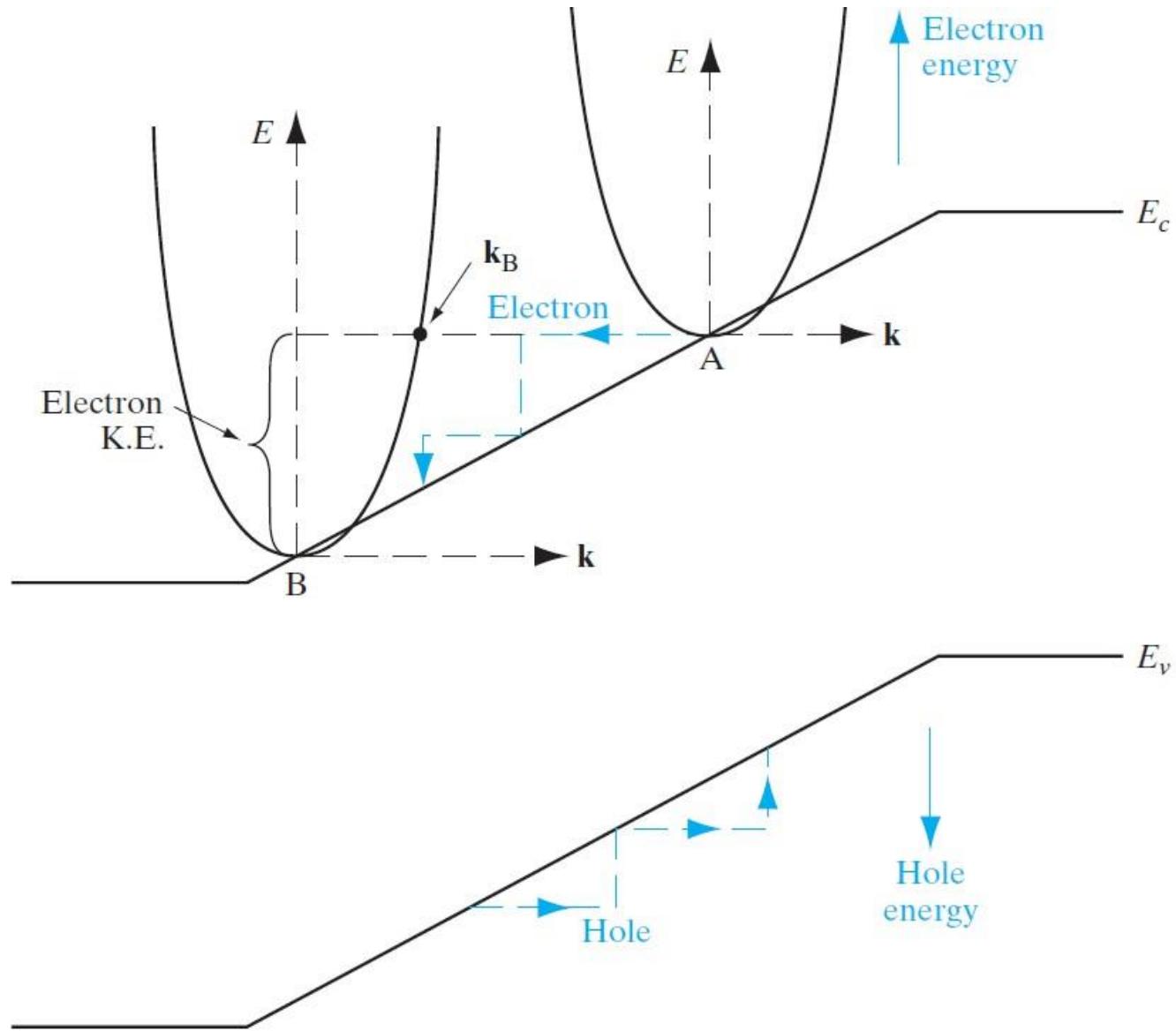
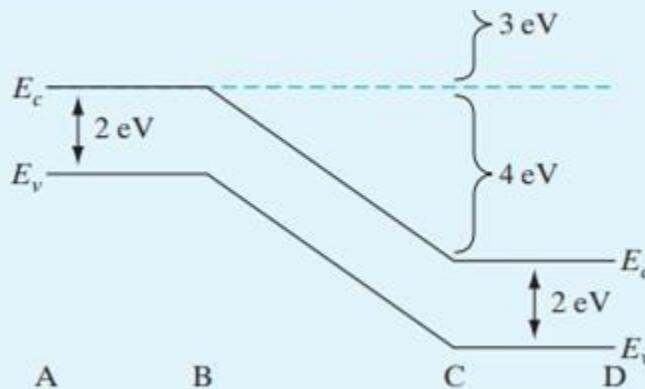


Figure 3-9
 Superimposition of the (E, \mathbf{k}) band structure on the E -versus-position simplified band diagram for a semiconductor in an electric field. Electron energies increase going up, while hole energies increase going down. Similarly, electron and hole wavevectors point in opposite directions and these charge carriers move opposite to each other, as shown.

EXAMPLE 3-1

In a long semiconductor bar ($E_G = 2 \text{ eV}$), conduction band electrons come in from the left in the positive x -direction with a kinetic energy of 3 eV. They move from location A to B to C to D. Between A and B, the electric field is zero; between locations B and C, there is a linearly varying voltage increase of 4 V; between C and D, the field is again zero. Assuming no scattering, sketch a *simplified* band diagram describing the motion of these electrons. Assuming that these electrons can be described as plane waves, with a free-electron mass, write down the wavefunction of the electrons at D. Leave your result in terms of an arbitrary normalization constant. *Draw a band diagram and give the wavefunction at D in terms of the normalization constant.*



$$\text{General wavefunction: } \Psi(x, t) = \alpha \times e^{i(kx - \omega t)}$$

$$\begin{aligned} \text{Energy at D} &= \hbar \cdot \omega = \frac{\hbar^2 \cdot k^2}{2 \cdot m_0} = 3 \text{ eV} + 4 \text{ eV} = 7 \text{ eV} \\ &= 7 \text{ eV} \cdot 1.6 \cdot 10^{-19} \frac{\text{J}}{\text{eV}} = 1.12 \cdot 10^{-18} \text{ J} \end{aligned}$$

$$\omega = \frac{1.12 \cdot 10^{-18} \text{ J}}{\hbar} = \frac{1.12 \cdot 10^{-18} \text{ J}}{1.06 \cdot 10^{-34} \text{ J} \cdot \text{s}} = 1.06 \cdot 10^{16} \text{ Hz}$$

$$\begin{aligned} k &= \sqrt{\frac{1.12 \cdot 10^{-18} \text{ J} \cdot 2 \cdot m_0}{\hbar^2}} = \sqrt{\frac{1.12 \cdot 10^{-18} \text{ J} \cdot 2 \cdot 9.11 \cdot 10^{-31} \text{ kg}}{(1.06 \cdot 10^{-34} \text{ J} \cdot \text{s})^2}} \\ &= 1.35 \cdot 10^{10} \frac{1}{\text{m}} \end{aligned}$$

Wavefunction at D:

$$\Psi(x, t) = \alpha \cdot e^{i(1.35 \cdot 10^{10} \frac{1}{\text{m}} \cdot x - 1.06 \cdot 10^{16} \text{ Hz} \cdot t)} \text{ where } \alpha \text{ is the normalization constant}$$

Problem

Objective: Calculate the probability that a quantum state in the conduction band at $E = E_c + kT/2$ is occupied by an electron, and calculate the thermal-equilibrium electron concentration in silicon at $T = 300$ K.

Assume the Fermi energy is 0.25 eV below the conduction band. The value of N_c for silicon at $T = 300$ K is $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$ (see Appendix B).

The probability that a quantum state at $E = E_c + kT/2$ is occupied by an electron is given by

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \cong \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left[\frac{-(E_c + (kT/2) - E_F)}{kT}\right]$$

$$f_F(E) = \exp\left[\frac{-(0.25 + (0.0259/2))}{0.0259}\right] = 3.90 \times 10^{-5}$$

The electron concentration is given by

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = (2.8 \times 10^{19}) \exp\left[\frac{-0.25}{0.0259}\right]$$

$$n_0 = 1.80 \times 10^{15} \text{ cm}^{-3}$$



Determine the probability that a quantum state at energy $E = E_c + kT$ is occupied by an electron, and calculate the electron concentration in GaAs at $T = 300$ K if the Fermi energy level is 0.25 eV below E_c .

$$f_F(E) = 2.36 \times 10^{-5}, n_0 = 3.02 \times 10^{13} \text{ cm}^{-3}]$$



Objective: Determine the number (#/cm³) of quantum states in silicon between E_c and $E_c + kT$ at $T = 300$ K.

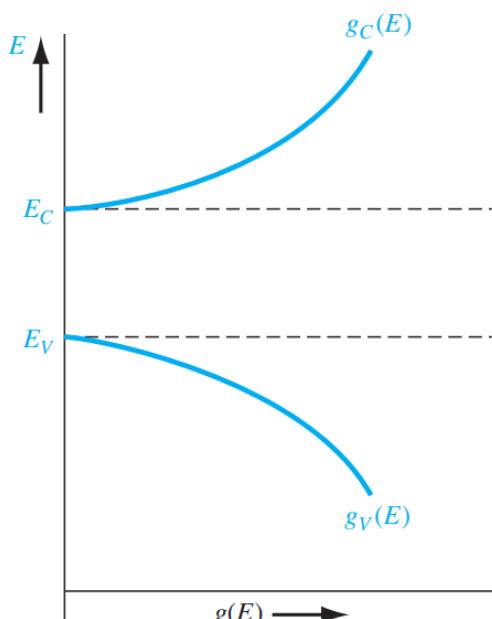
■ Solution

Using Equation (3.72), we can write

$$\begin{aligned} N &= \int_{E_c}^{E_c + kT} \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \cdot dE \\ &= \frac{4\pi(2m_n^*)^{3/2}}{h^3} \cdot \frac{2}{3} \cdot (E - E_c)^{3/2} \Big|_{E_c}^{E_c + kT} \\ &= \frac{4\pi[2(1.08)(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} \cdot \frac{2}{3} \cdot [(0.0259)(1.6 \times 10^{-19})]^{3/2} \\ &= 2.12 \times 10^{25} \text{ m}^{-3} \end{aligned}$$



- Calculate the density of states per unit volume over a particular energy range. Calculate the density of states per unit volume with energies between 0 and 1 eV.
- Consider the density of states for a free electron given by



$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c}$$

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3} \sqrt{E} \quad m = 9.11 \times 10^{-31}$$

■ Solution

The volume density of quantum states, from Equation (3.69), is

$$N = \int_0^{1 \text{ eV}} g(E) dE = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \int_0^{1 \text{ eV}} \sqrt{E} dE$$

or

$$N = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \frac{2}{3} \cdot E^{3/2}$$

The density of states is now

$$N = \frac{4\pi[2(9.11 \times 10^{-31})]^{3/2}}{(6.625 \times 10^{-34})^3} \cdot \frac{2}{3} \cdot (1.6 \times 10^{-19})^{3/2} = 4.5 \times 10^{27} \text{ m}^{-3}$$

or

$$N = 4.5 \times 10^{21} \text{ states/cm}^3$$



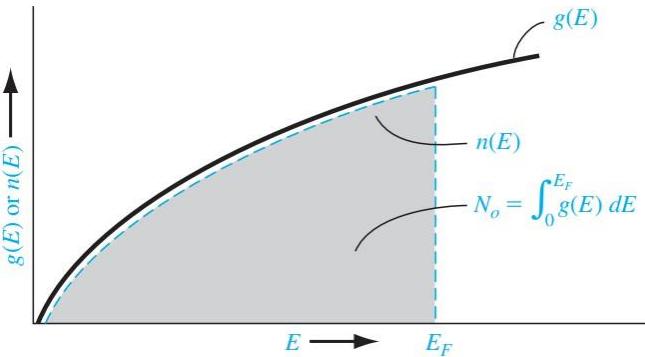


Figure 3.31 | Density of quantum states and electrons in a continuous energy system at $T = 0 \text{ K}$.

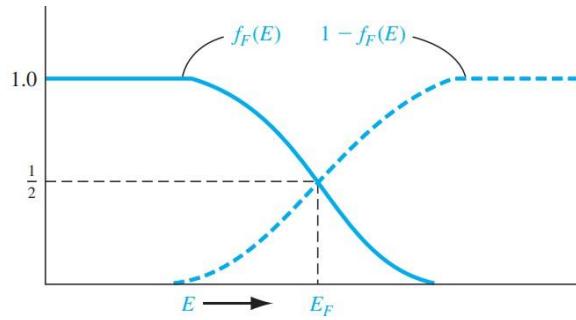


Figure 3.34 | The probability of a state being occupied, $f_F(E)$, and the probability of a state being empty, $1 - f_F(E)$.

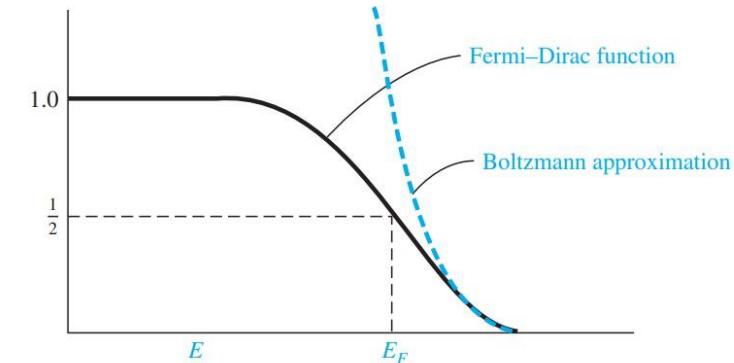


Figure 3.35 | The Fermi-Dirac probability function and the Maxwell-Boltzmann approximation.

- **N** - number of particles per unit volume
- function **g (E)** is the number of quantum states per unit volume per unit energy.
- function **f(E)** - Fermi-Dirac distribution or probability function and gives the probability that a quantum state at the energy E will be occupied by an electron.
- energy **E_F** - Fermi energy
- the distribution function **f_F(E)** - ratio of filled to total quantum states at any energy E .

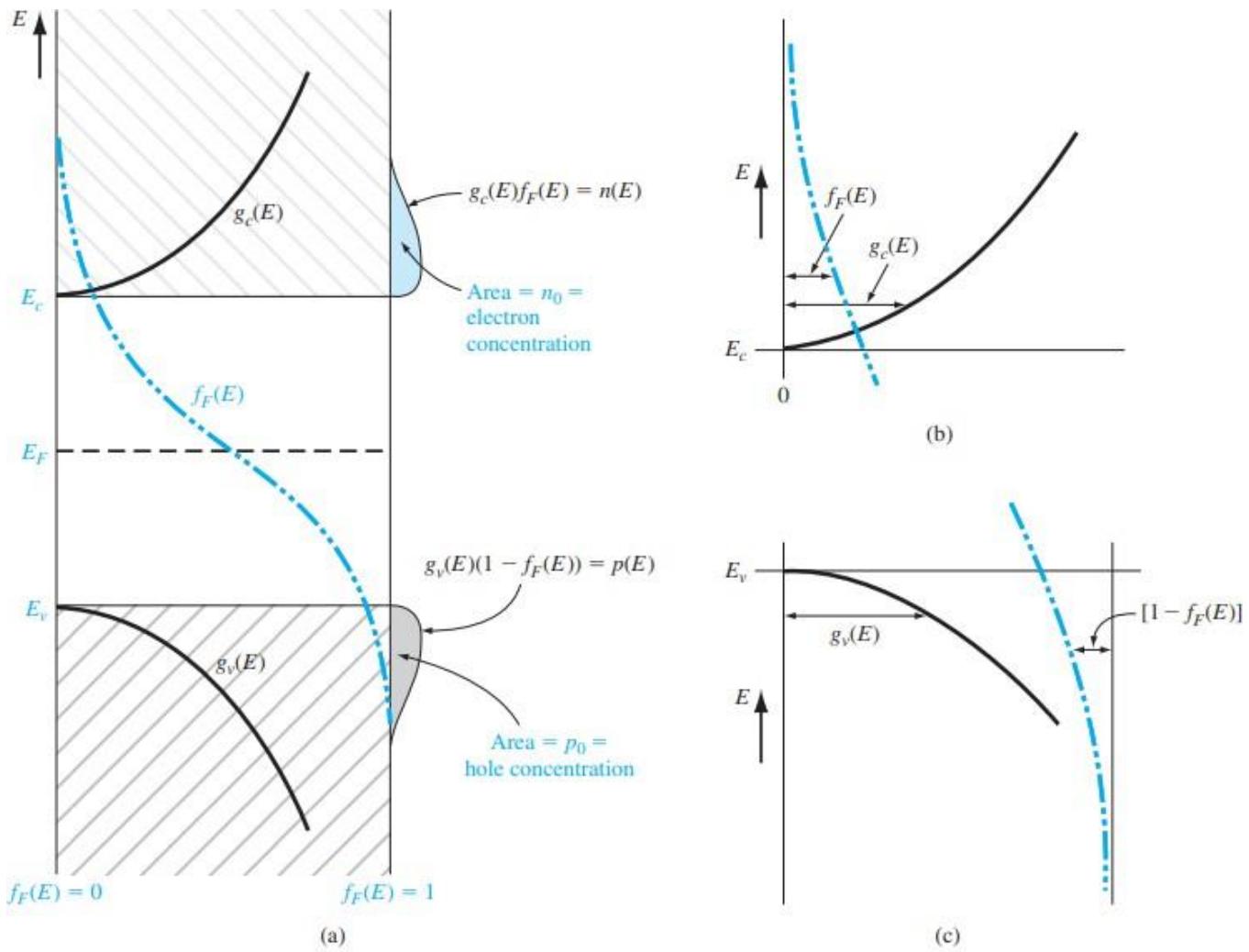


Figure 4.1 | (a) Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is near the midgap energy; (b) expanded view near the conduction-band energy; and (c) expanded view near the valence-band energy.

Objective: Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are $2.8 \times 10^{19} \text{ cm}^{-3}$ and $1.04 \times 10^{19} \text{ cm}^{-3}$, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

■ **Solution**

Using Equation (4.23), we find, at $T = 250$ K

$$\begin{aligned} n_i^2 &= (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{250}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(250/300)}\right] \\ &= 4.90 \times 10^{15} \end{aligned}$$

or

$$n_i = 7.0 \times 10^7 \text{ cm}^{-3}$$

At $T = 400$ K, we find

$$\begin{aligned} n_i^2 &= (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{400}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(400/300)}\right] \\ &= 5.67 \times 10^{24} \end{aligned}$$

or

$$n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$



- (a) Calculate the intrinsic carrier concentration in GaAs at $T = 400$ K and at $T = 250$ K. Assume that $E_g = 1.42$ eV is constant over this temperature range.
- (b) What is the ratio of n_i at $T = 400$ K to that at $T = 250$ K?

$$n_i = 3.29 \times 10^9 \text{ cm}^{-3}$$

For $T = 250$ K,

(a) For $T = 400$ K,

$$\begin{aligned} n_i^2 &= (4.7 \times 10^{17}) \left(7 \times 10^{18} \left(\frac{400}{300} \right)^3 \right. \\ &\quad \left. \times \exp \left[\frac{-1.42}{(0.0259)(400/300)} \right] \right) \\ &= 1.081 \times 10^{19} \end{aligned}$$

$$\begin{aligned} n_i^2 &= (4.7 \times 10^{17}) \left(7 \times 10^{18} \left(\frac{250}{300} \right)^3 \right. \\ &\quad \left. \times \exp \left[\frac{-1.42}{(0.0259)(250/300)} \right] \right) \\ &= 5.09 \times 10^7 \end{aligned}$$

or

$$n_i = 7.13 \times 10^3 \text{ cm}^{-3}$$

(b)

$$\frac{n_i(400)}{n_i(250)} = \frac{3.288 \times 10^9}{7.135 \times 10^3} = 4.61 \times 10^5$$



Objective: Calculate the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at $T = 300$ K.

The density of states effective carrier masses in silicon are $m_n^* = 1.08m_0$ and $m_p^* = 0.56m_0$.

■ Solution

The intrinsic Fermi level with respect to the center of the bandgap is

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4} (0.0259) \ln\left(\frac{0.56}{1.08}\right)$$

or

$$E_{Fi} - E_{\text{midgap}} = -0.0128 \text{ eV} = -12.8 \text{ meV}$$

