$\epsilon\text{-closure}(q) = \{ q' \mid q' \text{ is reachable from } q \text{ by } \underset{=}{\epsilon}, \underline{\epsilon\epsilon}, \underline{\epsilon \cdot \epsilon}, \epsilon^* \}$

$\underset{\uparrow}{0} \equiv \epsilon 0$

$\equiv \epsilon \cdots \epsilon 0$

$0 . \epsilon \epsilon . \epsilon$

$q \in \epsilon\text{-clos}(q).$

NFA — $\underline{\text{Design power}}$

— Computational power

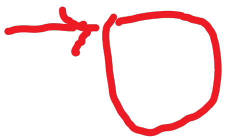
$L(\text{NFA} + \epsilon) = L(\text{NFA}) = L(\underset{=}{\text{DFA}})$

Easier

Design

Hard

States



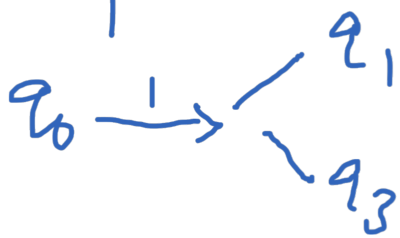
	ε-NFA / NFA+ε	NFA	DFA	Min DFA
Minimal num	10	≤10 8	8 7	6
	≥6	≥6	≥6	

Hard

$0^* 1^* 2^*$

5,4	5	≥6	6

$(11111+111)^*$



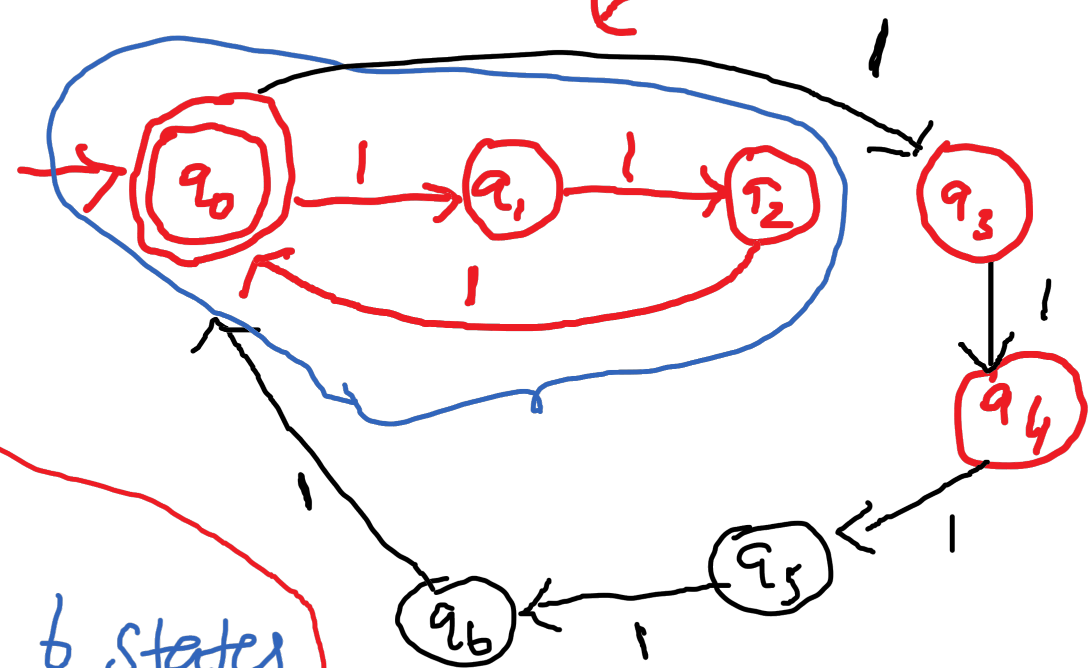
NFA ✓

DFA ✗

DFA	NFA	ϵ-NFA
6 states	<6	<6

$(11111+111)^*$

Min 8, No NFA with 7 stat

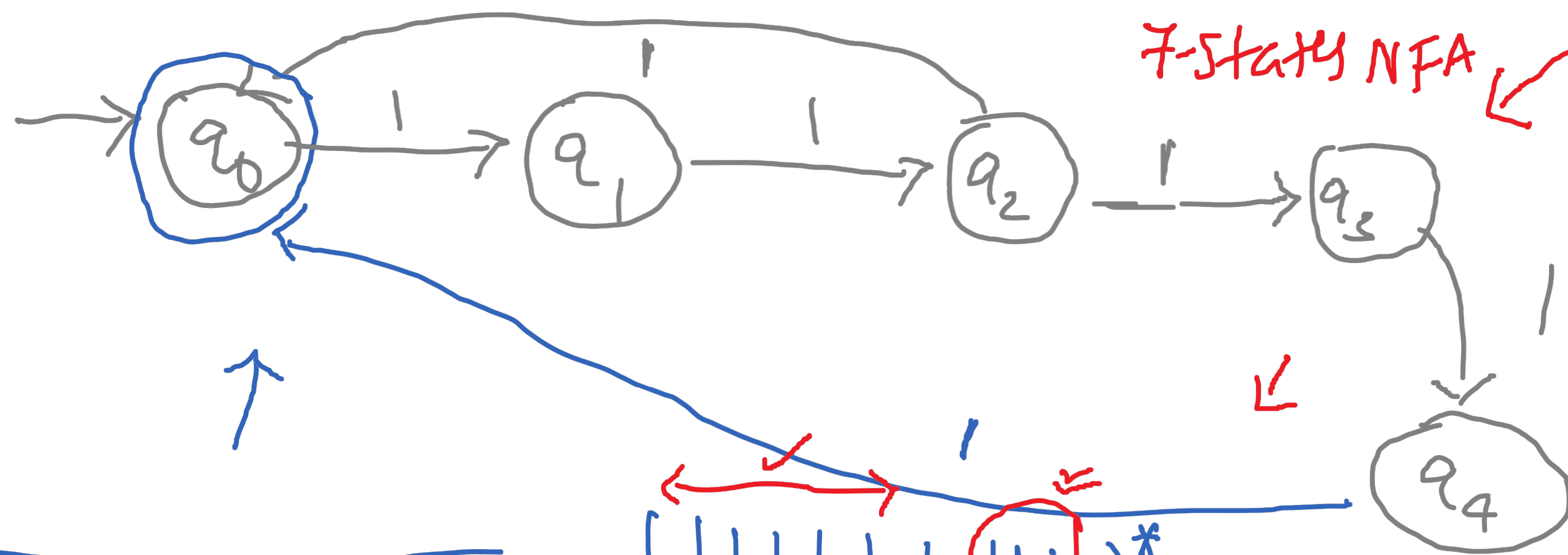


#States = 7

DFA = 7 states

? Is 7 Min

? Does $\exists$ DFA with 6 states.



$(11111 + 111)^*$

2

$(11111+111)(11111+111)$

5-NFA

7-NFA

9-NFA | 9-DFA

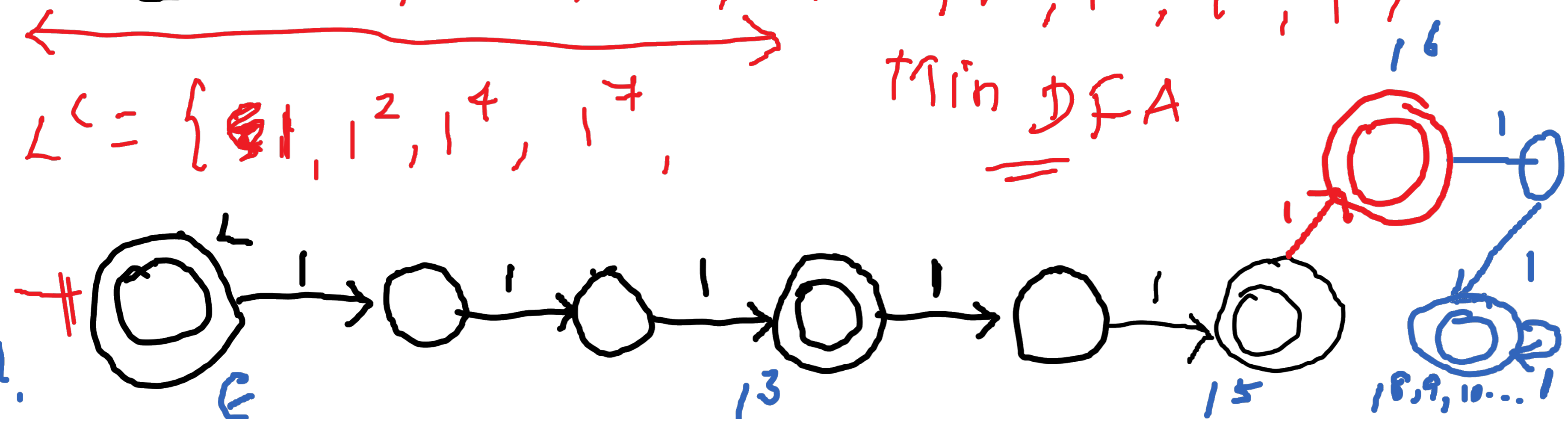
uncover

$L = \{\epsilon, 1^3, 1^5, 1^6, 1^8, 1^9, 1^{10}, 1^{11}, 1^{12}, 1^{13}, 1^{14}, \ldots\}$

$L^c = \{1, 1^2, 1^4, 1^7\}$

Min DFA

9-State DFA = MIN?



$n \geq 8$ $\qquad$ $n = f(3,5)$

↑ Lin fn

Does $\exists$ Non-neg int x, y s.t. $n = 3x + 5y$ is true.

$8 = 5 + 3$

$9 = 0 \cdot 5 + 3 \cdot 3$

$10 = 2 \cdot 5 + 0 \cdot 3$

$11 = 1 \cdot 5 + 2 \cdot 6$

$\vdots$

M. Indn

$\forall n \geq 8 \ \exists x \ \exists y$ ($x \geq 0$, $y \geq 0$) s.t. $n = 3x + 5y$

→ $\underbrace{\ldots\ldots}$

9-State DFA

9: Minimum.

ε-NFA ~ NFA | DFA ...

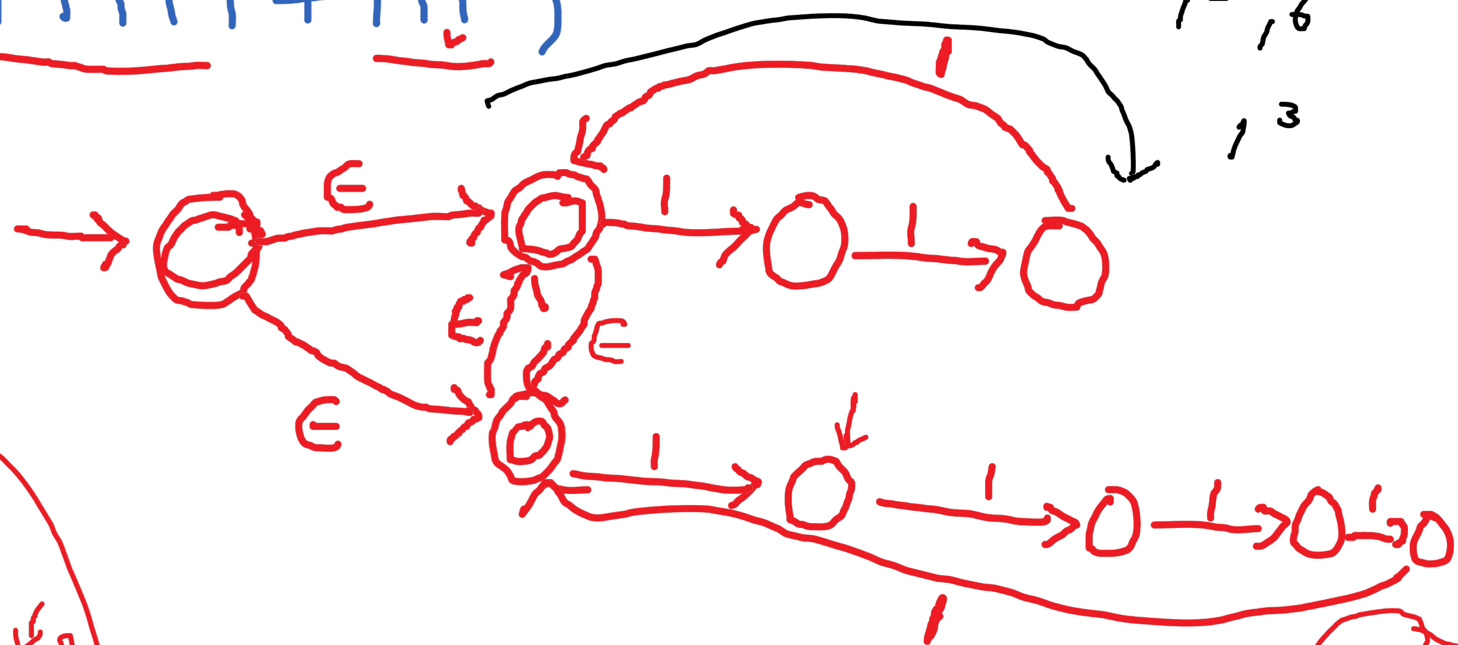
R.E $(11111 + 111)^*$

↓

ε-NFA.

$1^3, 6$

1^3



$2^Q \times \Sigma \to 2^Q$

$\delta(Q, a) \to \{q_1, q_2\}$

q_1 q_2

$\delta(\{q_1, q_2\}, a)$

E

ε-closure$(\delta(E(q_0)))$

=

δ'

F'

ε-clos$(E(q_0))$?

ϵ-NFA $\longrightarrow$ NFA

δ'

$\epsilon^* a = a = a\epsilon^* = \epsilon^* a \epsilon^*$

$\delta'(q, a)$

$=$

$\epsilon^* a \epsilon^*$

$\delta'(q, a)$

$= E(\delta(E(q), a))$

$(E(q), a)$

$E(\{q_1, q_2, q_3\})$

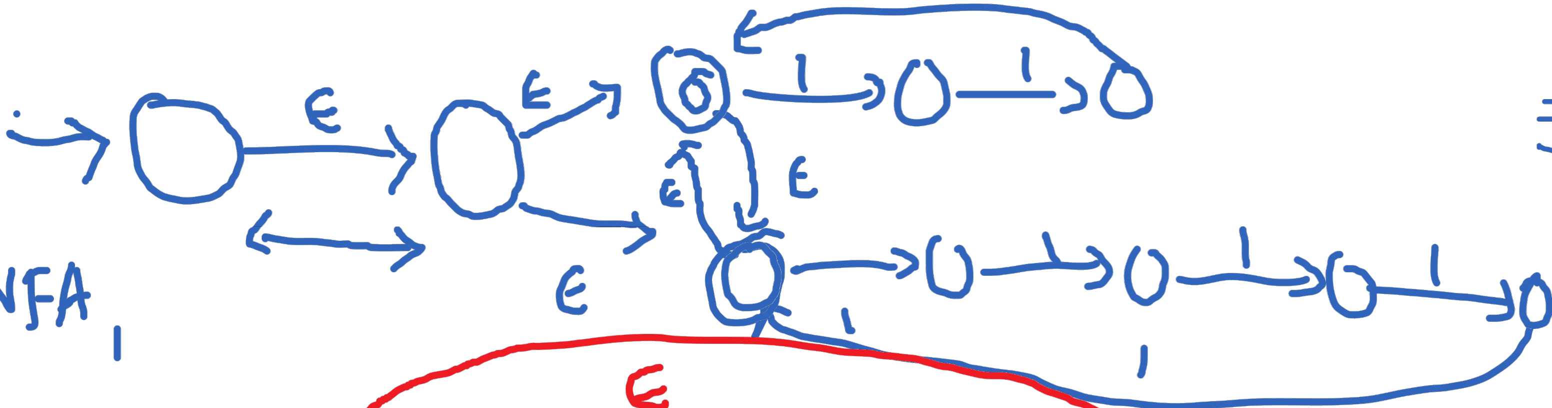
$\Downarrow$

k

$\delta(\{q_1 \ldots q_k\}, a) = \bigcup_{i=1}^{k} \delta(q_i, a)$

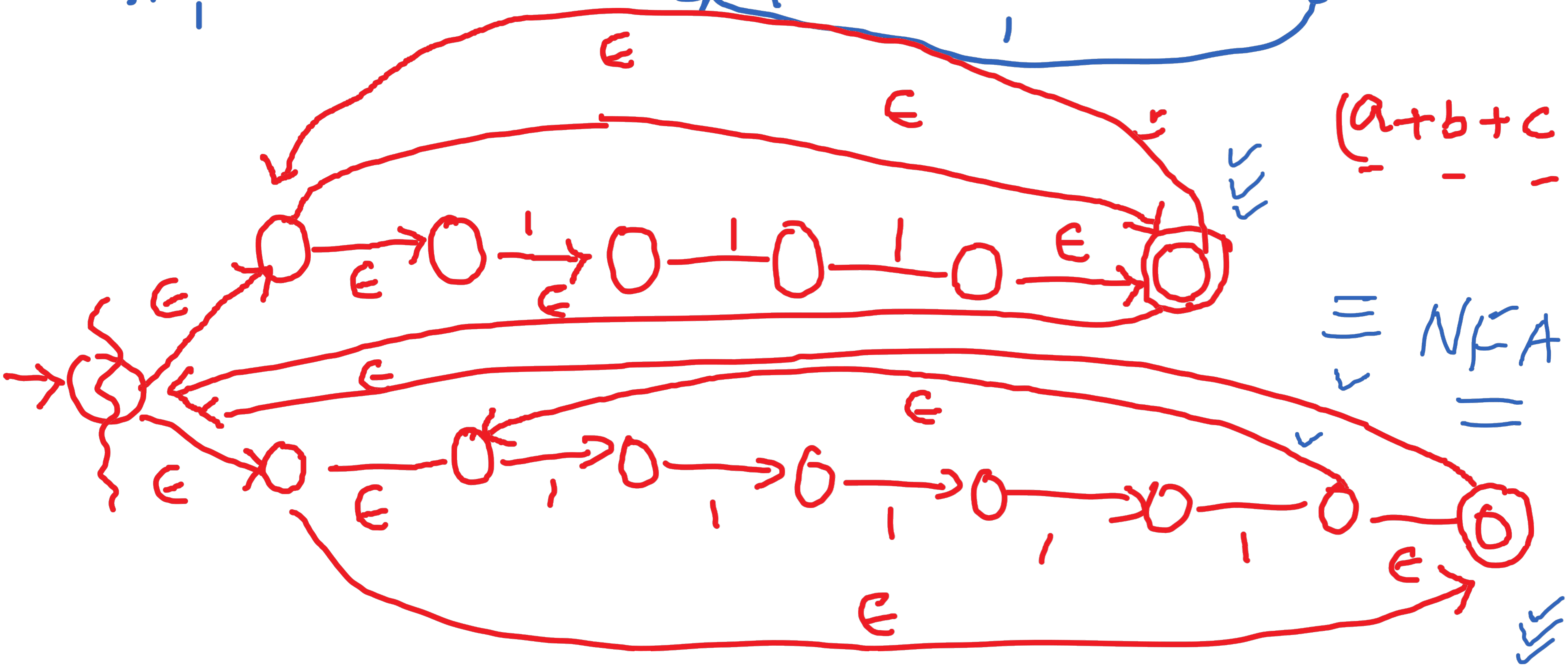
a

ε-NFA



≡ NFA.

$(a+b+c)^*$



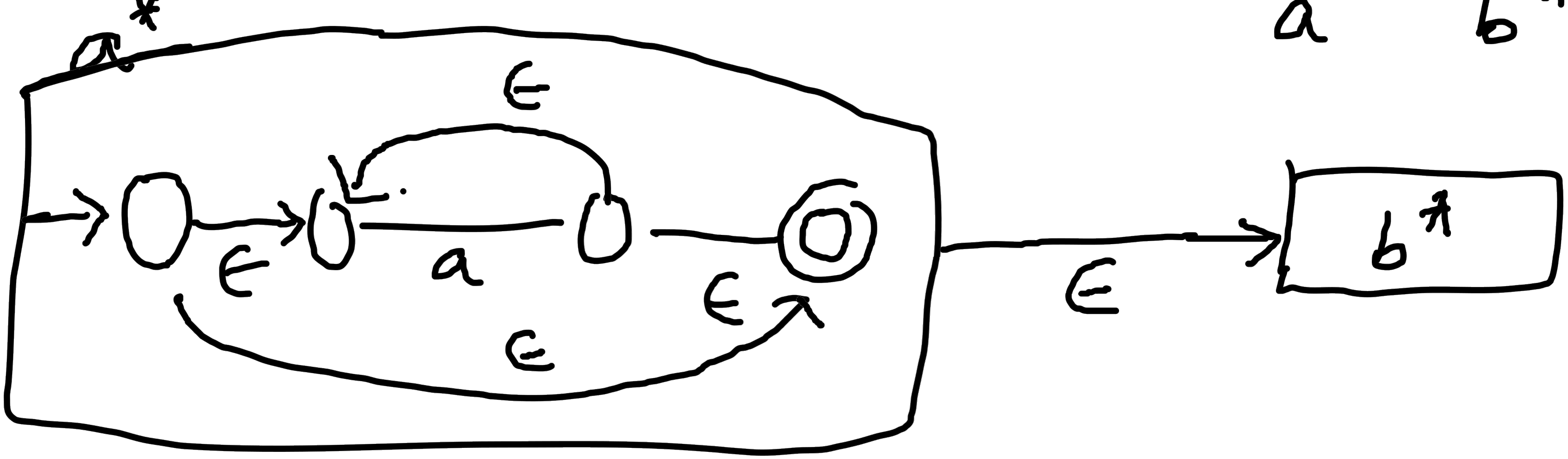
≡ NFA

≡

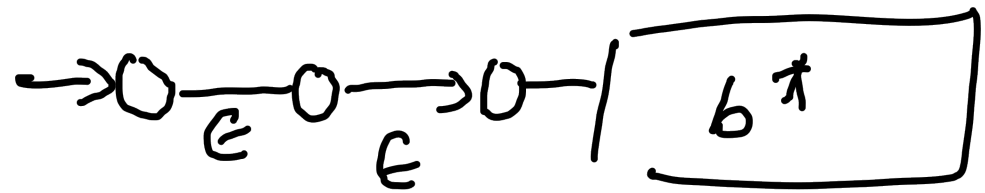
Multiple ε-NFAs ?

$a^* b^* a^* b^* b^*$

a^* b^*



a^*



$RE_1 \cdot RE_2$) 1

$RE_1 + RE_2$) 2

$(RE_1 + RE_2)^*$

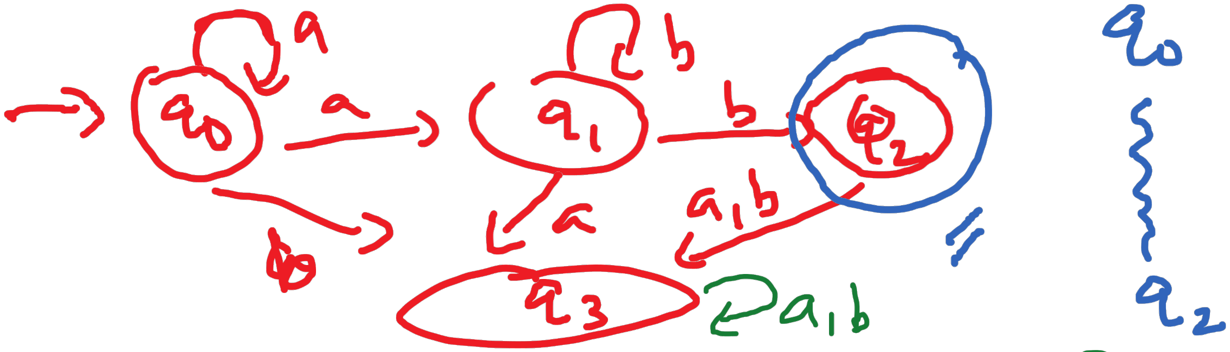
3 $(RE_1^*$

$L(RE)$

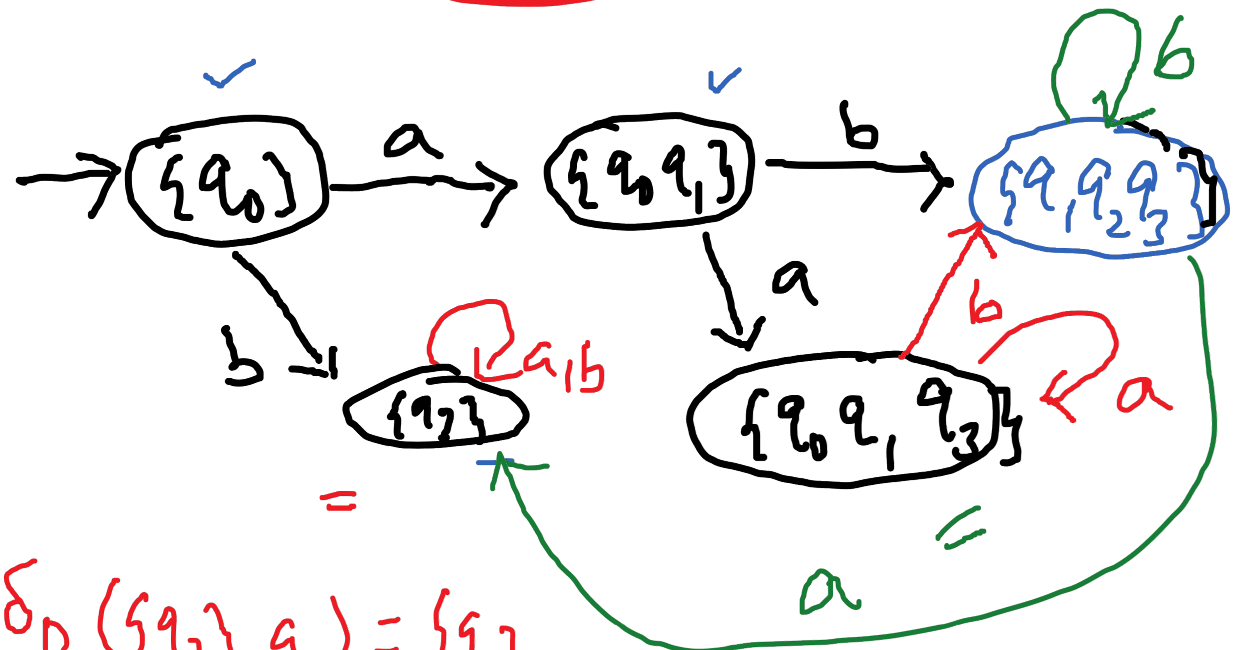
$\downarrow$

ε-NFA

$\rightarrow L(NFA)$



$$\delta_D(\{q_1 q_2 q_3\}, a) = \{q_3\}$$



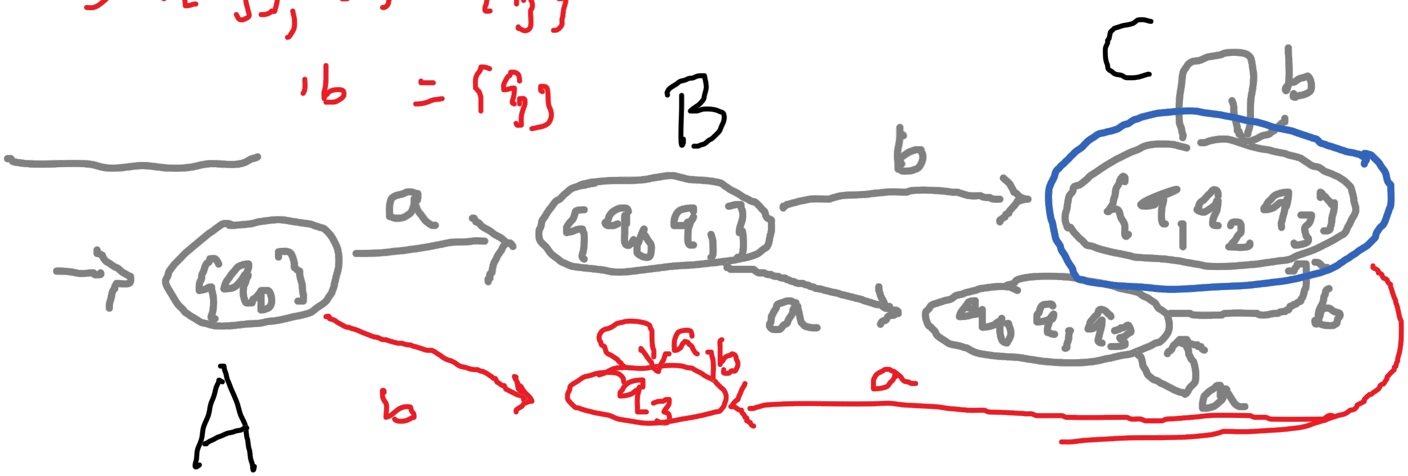
$$\delta_D(\{q_1 q_2 q_3\}, b) = \{q_1, q_2, q_3\}$$

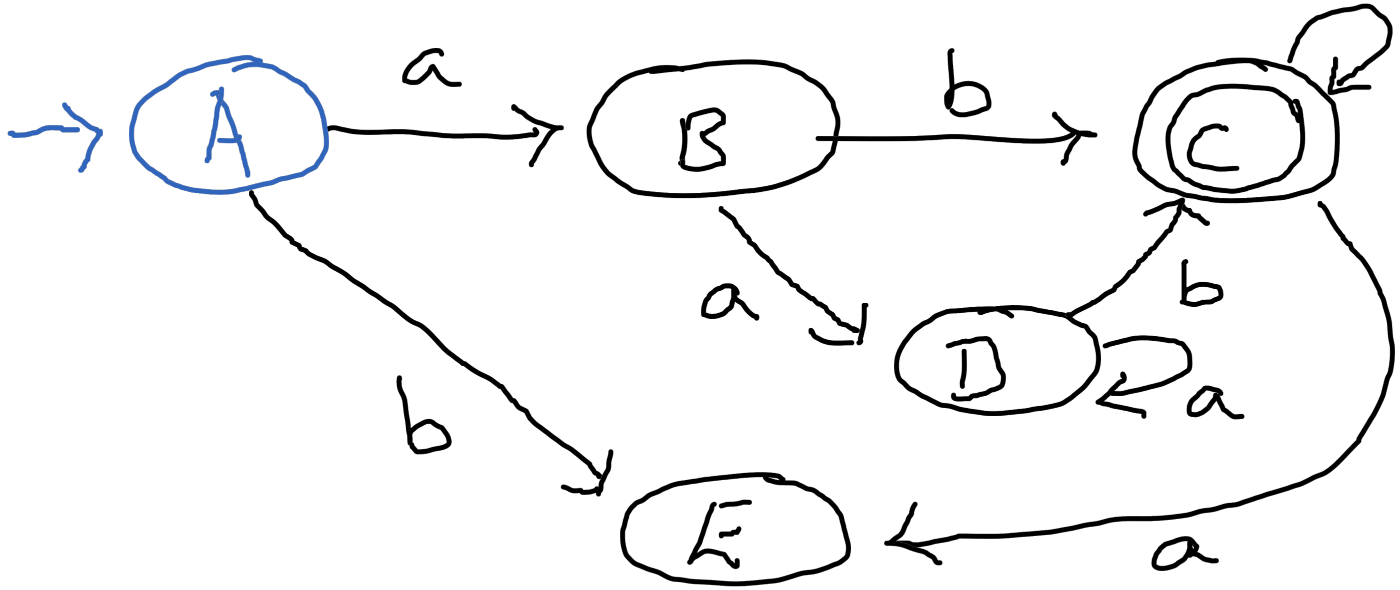
$$\delta_D(\{q_0 q_1 q_3\}, a) = \{q_0 q_1 q_3\}$$

$$\delta_D(\{q_3\}, a) = \{q_3\}$$

$$, b = \{q_3\}$$

$$\delta_D(\{q_0 q_1 q_3\}, b) = \{q_3 q_1 q_2\}$$





Guess

Tree

$\{q_0 \dots q_k\} \subseteq Q$

$\{q_0\}$

$\downarrow$

$Q' \subseteq Q$

$\downarrow$

$Q'' \subseteq Q$

$\vdots$

Path

$Q_f \cap F \neq \phi$

"Subset Construction"

NFA $\rightarrow$ $\boxed{\text{DFA}}$

Need not be MIN always!

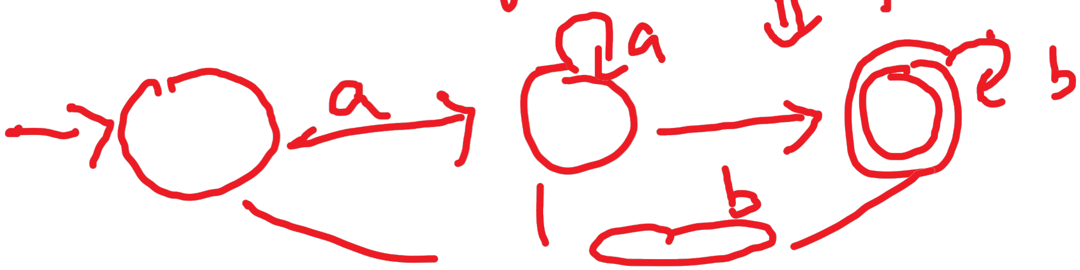
#States: 5

? Is this a MIN DFA

NO

Can we think of 4-State DFA?

$\Downarrow$



NFA

$\hat{\delta}(q_0, x)$

Comp Graph (Tree)

Non-Det $\{q_0 \ldots q_k\}$

$\{q_0 \; q_1 \; q_2\}$

$\{ \quad \}$

$x = aabbb$

q_0 q_1

q_0

q_1

q_0 .... q_3

q_2 .... q_2

Path (Det)

$\subseteq Q$

$\{ \quad \}$

$\delta(q_0, a)$ $\delta(q_1, a)$ -- $\delta(q_k, a)$

n-NFA $\rightarrow$ $(2^n - 1)$-DFA

5-NFA $\rightarrow$ 31-DFA

DFA_1 DFA_2

$\{q^1, q^2\}, a$

$q_0^1 = DFA$ $q_0^2 = DFA$

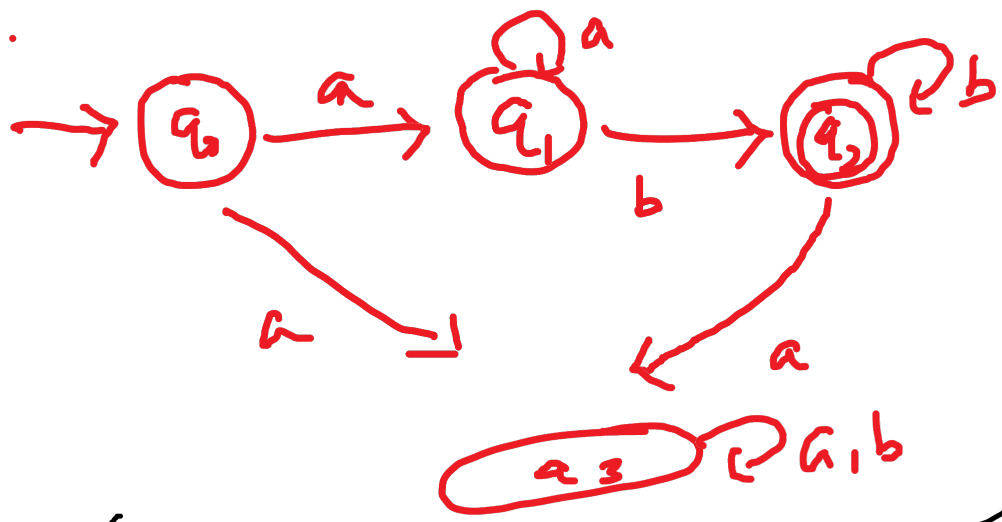
$\delta_p(\{q_0^1\ q_0^2\}, a)$

$\{q_0 \dots q_k\}, a$

Subset Const

$\{q_0 \dots q_k\}, a$

NFA (Tree)



$\underline{\underline{N}}FA \equiv \underline{\underline{D}}FA$ poly time

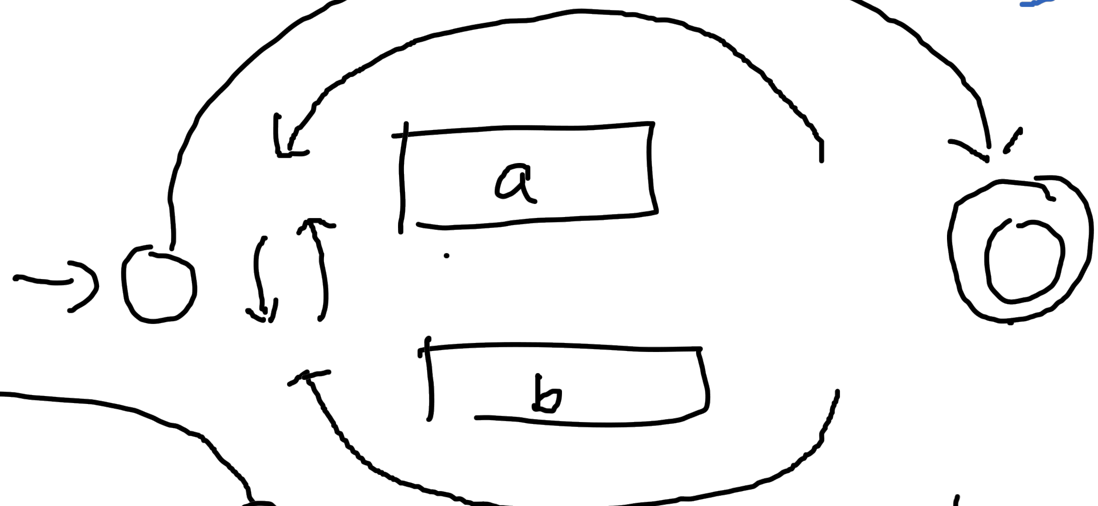
$\underline{\underline{N}}P = \underline{\underline{D}}P$

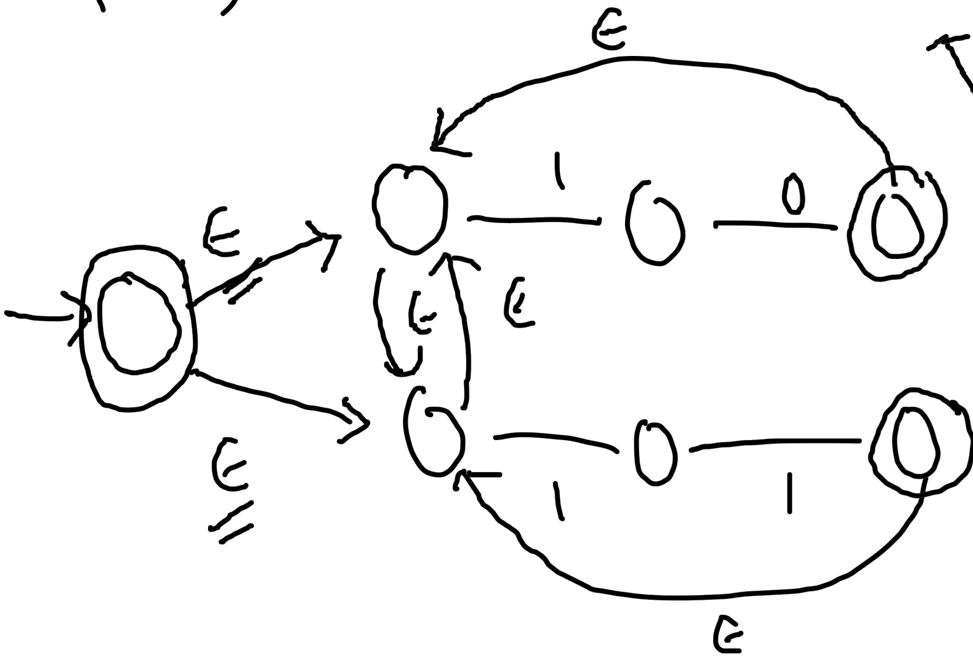
ϵ

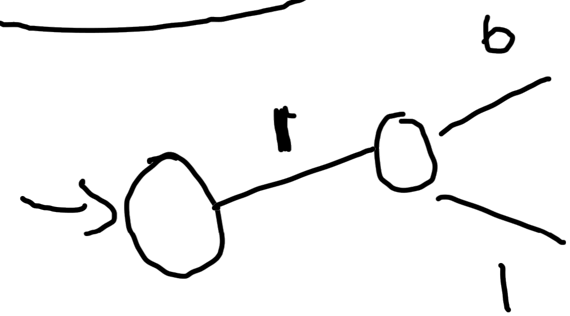
poly

$((a)+(b))^*$

$(10+11)^*$







$L_1 = \{0, 1\}$

$L_2 = \{a^n b^m \mid n, m \geq 1\}$

FSA

$= \{ab, ab^2, ab^3, ab\cdots b, \underline{a\cdots a}\,\underline{b\cdots b}\}$

∞

$\underline{\underline{\infty}}$

$\underline{\text{FSA}}$

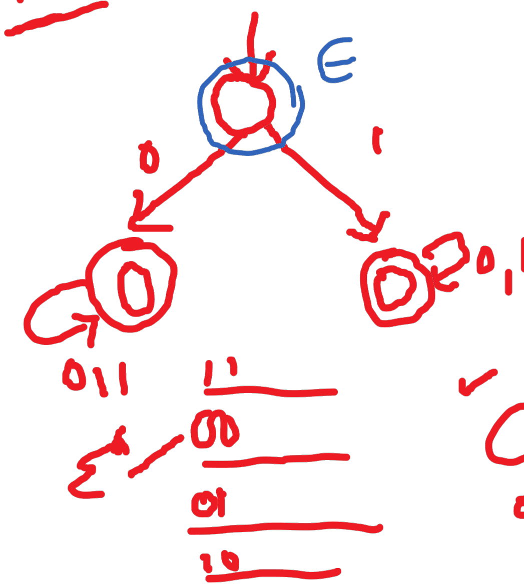
$Q_3 : L = \Sigma^*$

1)

ε_1 ε_2

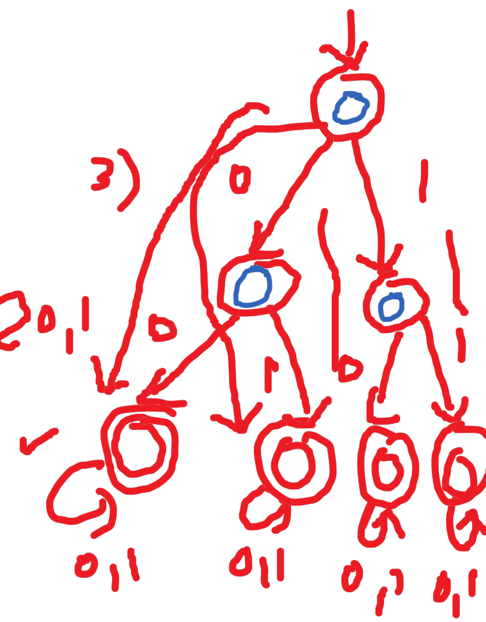
$\underline{0}$ $\underline{1}$

2)



DFA$_2$

3)

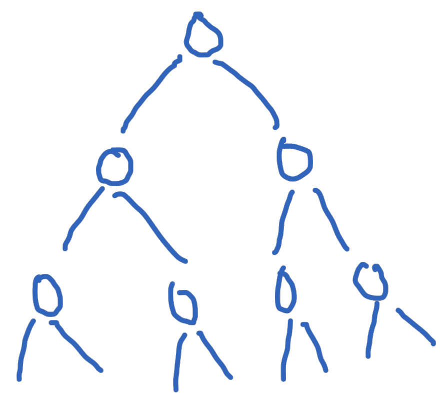


DFA$_3$

000

001

⋮



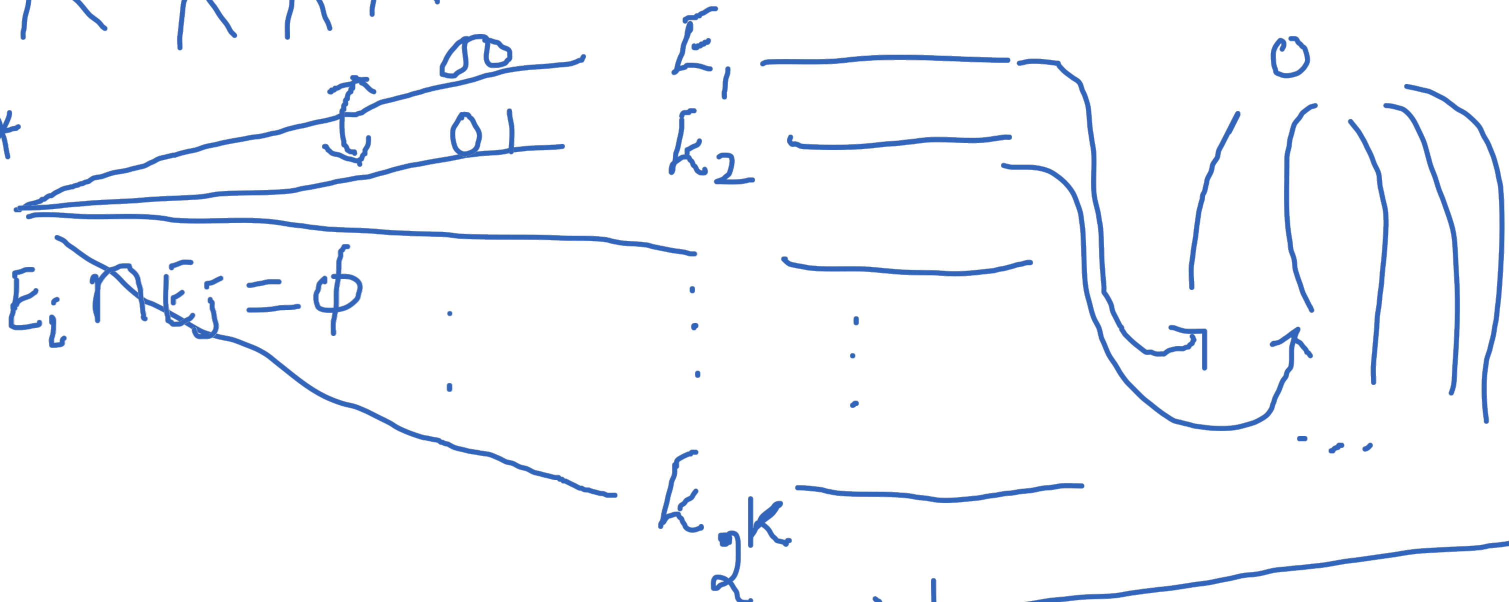
4th

5th

⋮

For every fixed integer 'k'

Draw k-level DFA



Σ n classes Σ^*

$\bigcup E_i = \Sigma^*$ $E_i \cap E_j = \phi$

B_n (#

For each fixed K ∃ FSA

∞ 'K' possible

n=5

B_n # different ways of partitioning a set

= # diff eqn classes

A = {1, 2, 3} {{1 2 3}} {1 3} 2 5

{1}{2}{3} {2 3} 1

{1 2}{3}

Σ^* — Partition Σ^* into finite no. of sets

$C\infty$

E_1 $C\infty$

$C\infty$

E_{2^k} $C\infty$

Partitioning an ∞-set ($C\infty$-set) into a finite no. of sets

B_n

We are not applying Bell no on Σ^*.

Σ^*

Σ^* : E_1 0, E_2 1

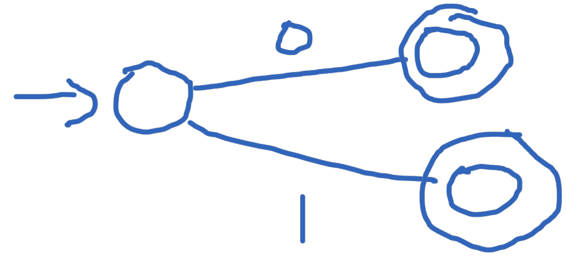
Σ^* : E_1 00, E_2 01, E_3 10, E_4 11 . . .

Σ^* : E_1, $\Sigma^* - E_1$ . . . E_8 . . . E_{2^k}

If L is finite then # DFAs is finite

∞ then # DFAs is ∞ need not be finite

DFA1



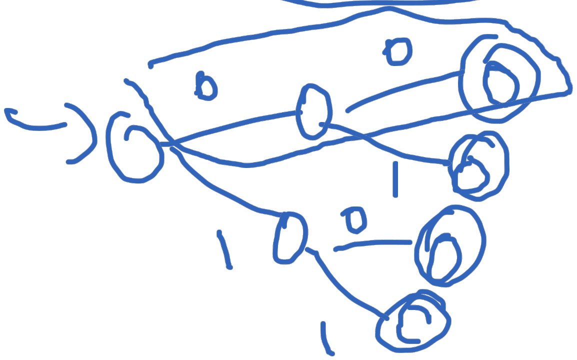
DFA2

??

L = {Finite} {0,1}

00, 01

10 11

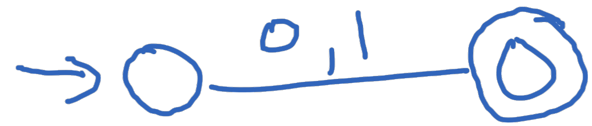


00

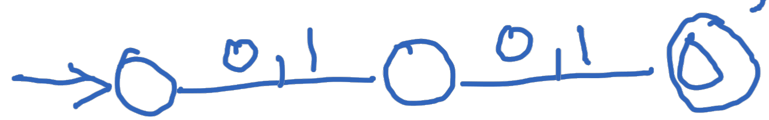
01

10

11



L = {00, 01, 10, 11}

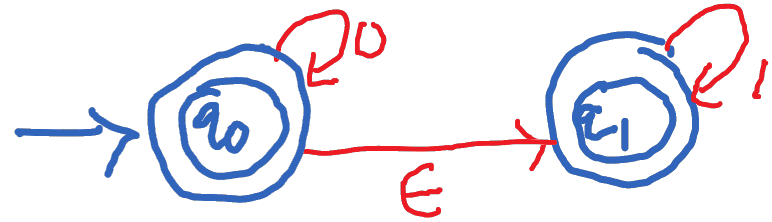


For a given L, how many diff DFAs are possible < Exact, LB, UB

FSA, in which Every st is a Final st; $0^* + 1^*$

Σ^* 

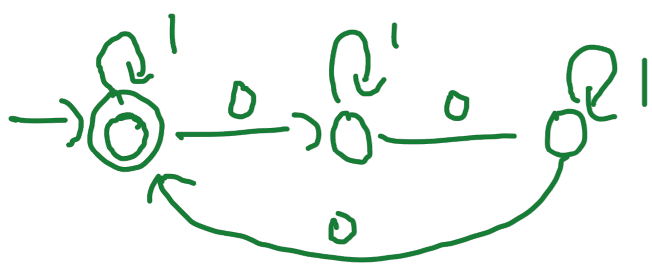
;

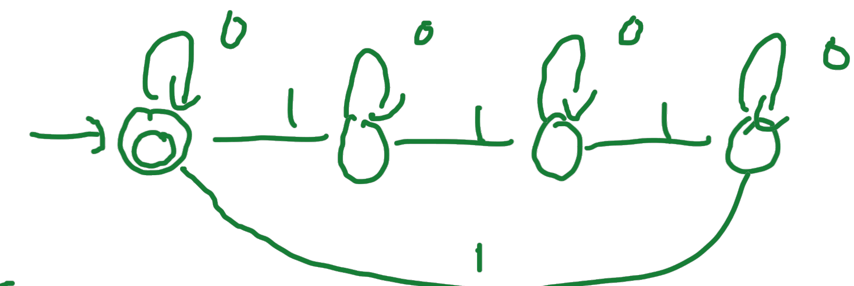


0 1

2-state DFA in which both q_0 & q_1 are final states

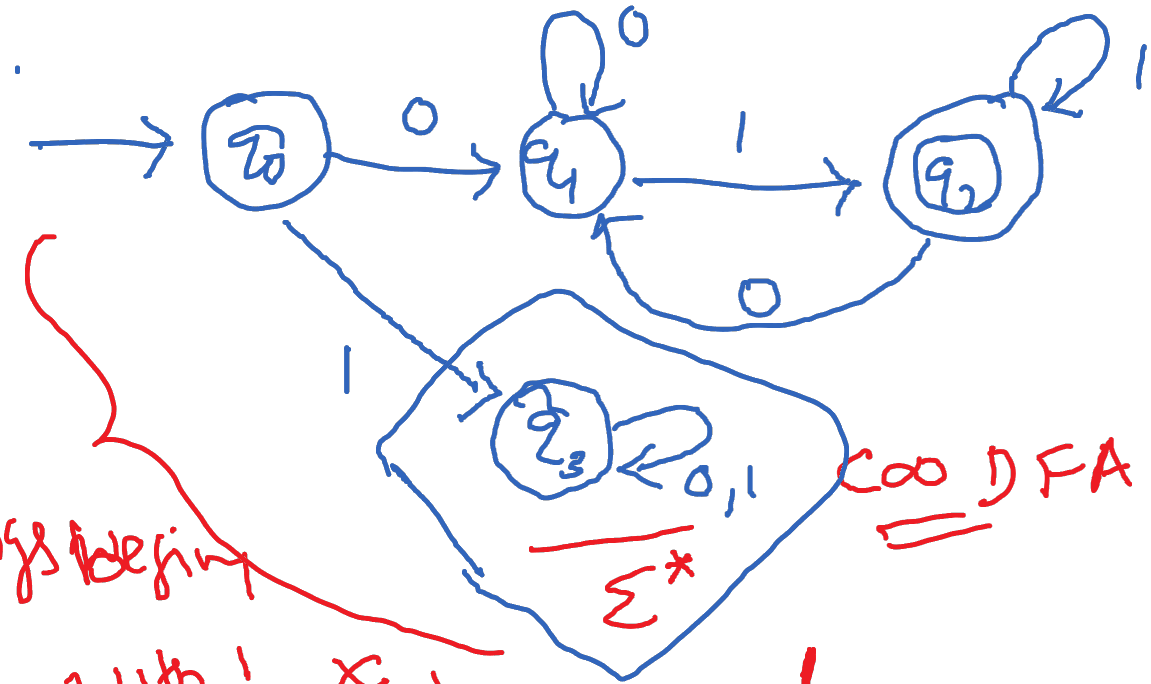
ϵ-NFA

M_1 

M_2 

$M_1 \times M_2$ = 12 states

and $F = F_1 \times F_2$



$\Rightarrow$ COO DFA,

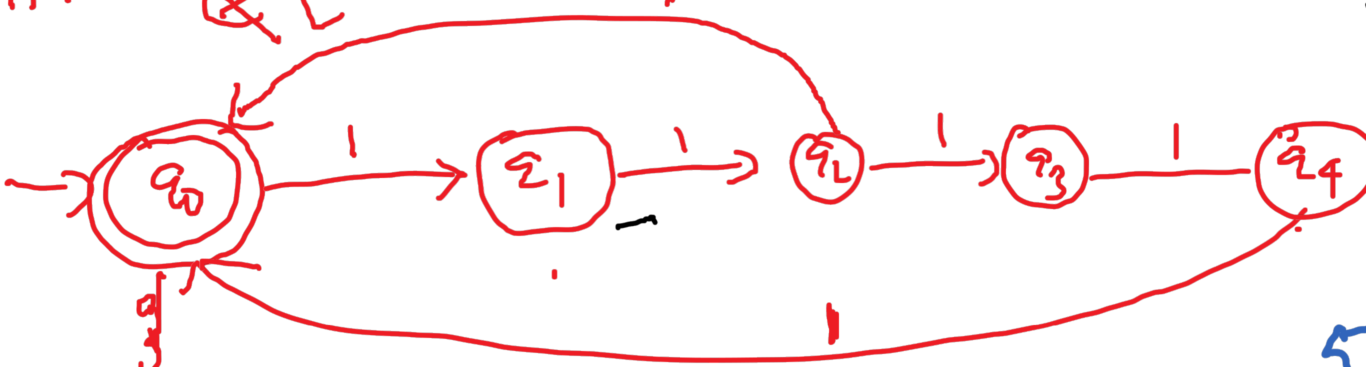
Strings begin with 1

$\notin L$

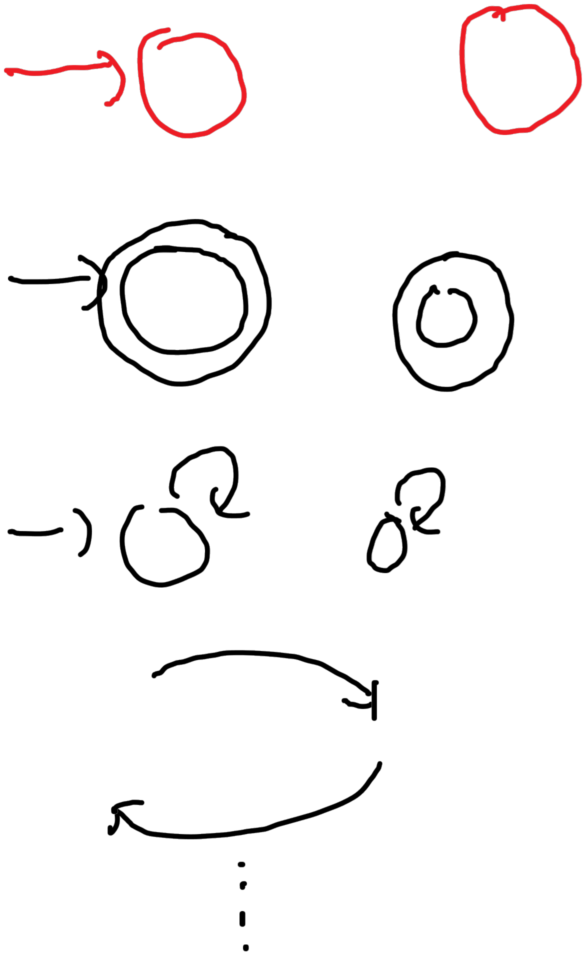
COO DFA

Σ^*

7-state NFA $\Sigma = \{1\}$

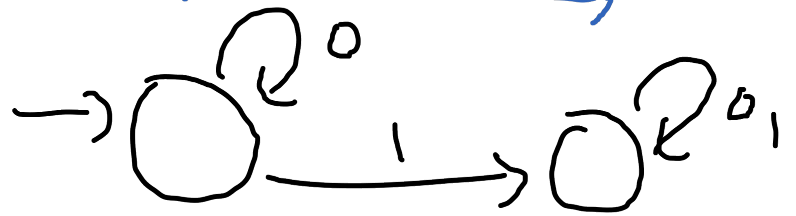


5-State NFA

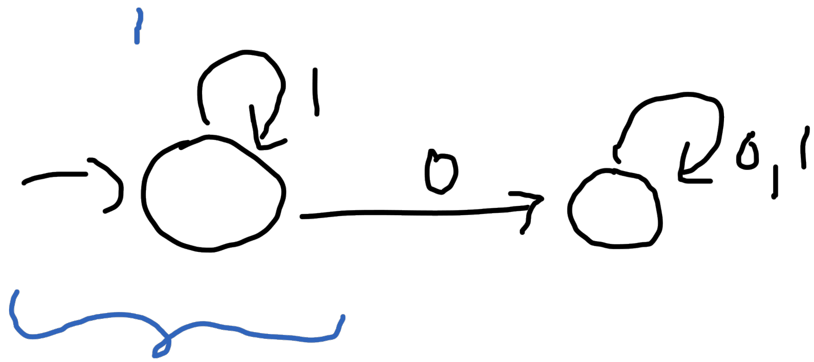


$\Sigma = \{0, 1\}$

DFA_1



DFA_2



DFA_3