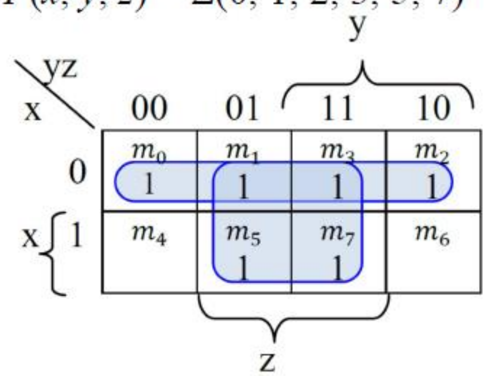
EC1001: Digital Circuits
Solutions: Assignment-3_Chapter3&4

1. Simplify the following Boolean functions, using three-variable maps:
   (a) $F(x, y, z) = \Sigma(0, 1, 2, 3, 5, 7)$
   (b) $F(x, y, z) = x'yz + xy' + yz'$

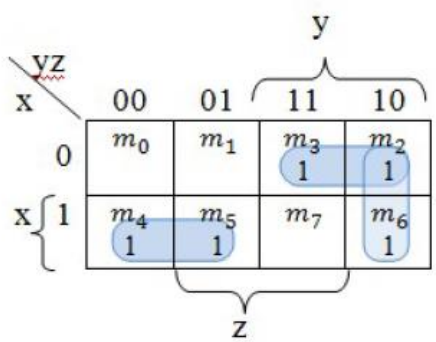
   Solution:

$$F(x, y, z) = \Sigma(0, 1, 2, 3, 5, 7)$$



$$F = x' + z$$

$$F(x, y, z) = x'yz + xy' + yz'$$



$$F = xy' + x'y + yz'$$

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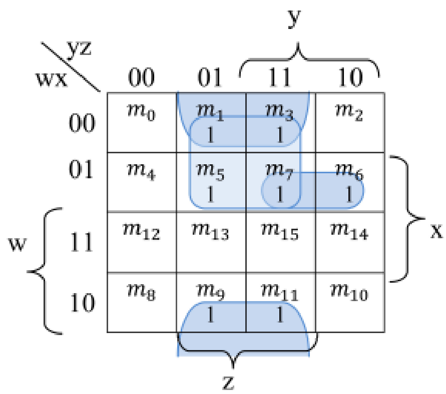
2. Simplify the following Boolean functions, using four-variable maps:
   (a) $F\,(w, x, y, z) = \Sigma(1, 3, 5, 6, 7, 9, 11)$
   (b) $F(w, x, y, z) = wxyz + wx' + wx'y + wxy + w'yz'$

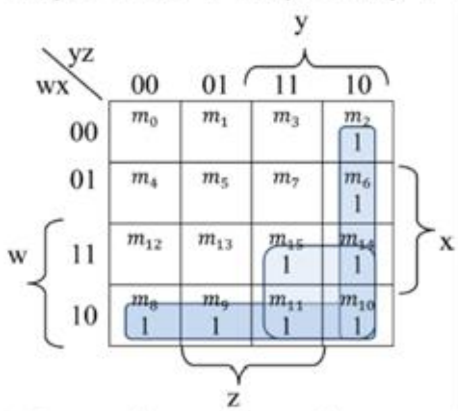
Solution:

$$F\,(w, x, y, z) = \Sigma(1, 3, 5, 6, 7, 9, 11)$$



$$F = w'z + x'z + w'xy$$

$$wxyz + wx' + wx'y + wxy + w'yz'$$



$$F = wx' + wy + yz'.$$

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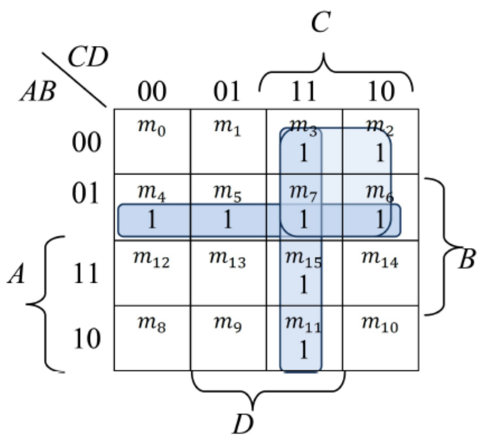
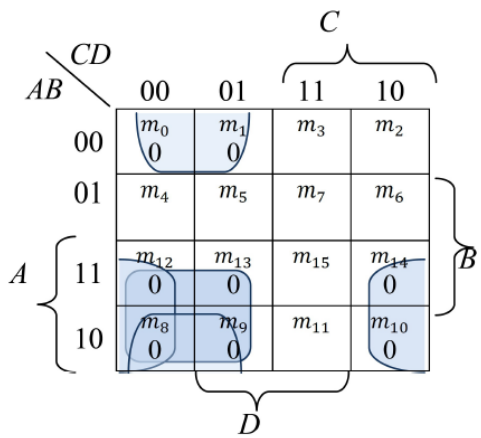
3. Simplify the following expression to (1) sum-of-products and (2) products-of-sums.
$F = A'B + A'B'C + CD$

Solution:

$A'B + A'B'C + CD$





$F = A'B + A'C + CD$

$F' = B'C' + AC' + AD'$

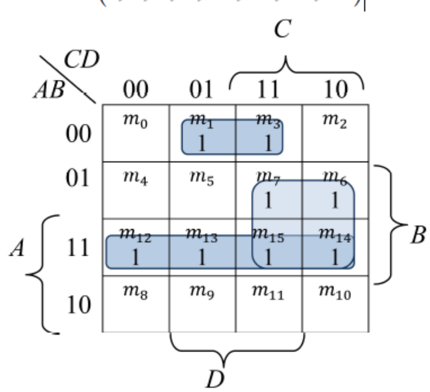
$F = (B+C)(A'+C)(A'+D)$

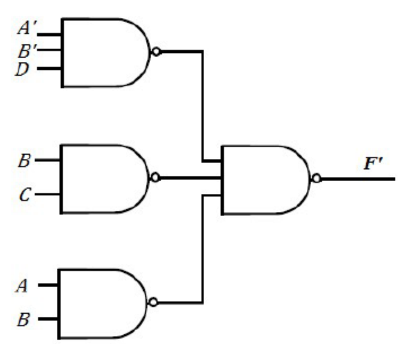
4. Draw a NAND logic diagram that implements the complement of the following function:
$F(A, B, C, D) = \Sigma(0, 2, 4, 5, 8, 9, 10, 11)$

Solution:

$F(A,B,C,D) = \Sigma(0,2,4,5,8,9,10,11)$

$F' = \Sigma(1,3,6,7,12,13,14,15)$





$F = A'B'D + BC + AB$

$= ((A'B'D + BC + AB)')' = ((A'B'D)'(BC)'(AB)')'$

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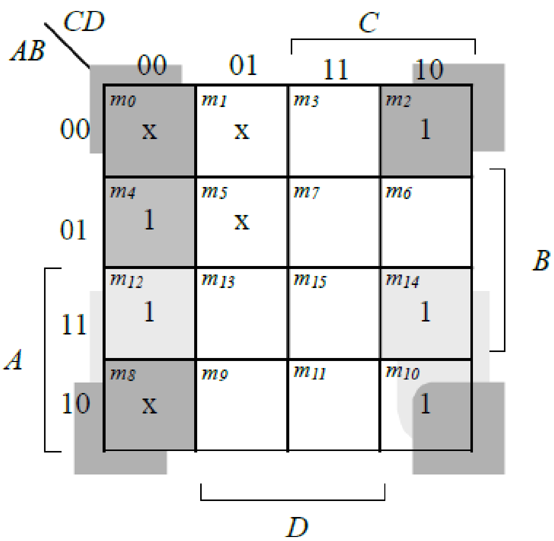
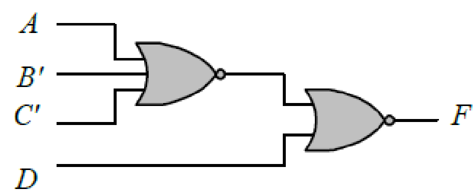
5. Implement the following Boolean function F, together with the don't-care conditions d , using no more than two NOR gates:
   $\mathrm{F}\ (\mathrm{A},\mathrm{B},\mathrm{C},\mathrm{D})\ =\ \Sigma(2,4,10,12,14)$
   $\mathrm{d}\ (\mathrm{A},\mathrm{B},\mathrm{C},\mathrm{D})\ =\ \Sigma(0,1,5,8)$
   Assume that both the normal and complement inputs are available.

   Solution:

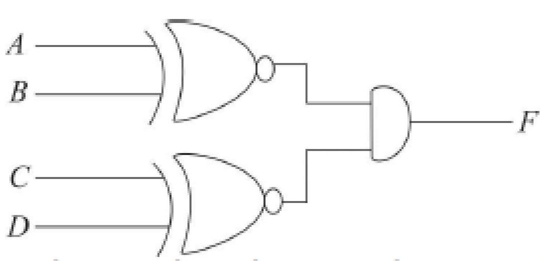




$$F = B'D' + AD' + C'D'$$
$$F' = D + A'BC$$
$$F = [D + A'BC]' = [D + (A + B' + C')']'$$

6. Implement the following Boolean expression with exclusive-NOR and AND gates.
$$F\ = A'B'C'D'\ +\ ABC'D'\ +\ A'B'CD\ +\ ABCD$$
   Solution:

$$F = A'B'C'D' + ABC'D' + A'B'CD + ABCD$$
$$= C'D'(\overline{A \oplus B}) + CD(A \oplus B)$$
$$= (\overline{A \oplus B})\,(\overline{C \oplus D})$$



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7. Design a combinational circuit with three inputs, x , y, and z, and three outputs, A, B , and C. When the binary input is 0, 1, 2, or 3, the binary output is two greater than the input. When the binary input is 4, 5, 6, or 7, the binary output is one less than the input.

   Solution:

x	y	z	A	B	C
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	0	1
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	0	1
1	1	1	1	1	0

$A = \sum(2,3,5,6,7)$

$x \backslash yz$	00	01	11	10
0	0	0	1	1
1	0	1	1	1

$A = y + xz$

$B = \sum(0,1,4,7)$

$x \backslash yz$	00	01	11	10
0	1	1	0	0
1	1	0	1	0

$B = x'y' + y'z' + xyz$

$C = \sum(1,3,4,6)$

$x \backslash yz$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$C = x'z + xz' = x \oplus z$

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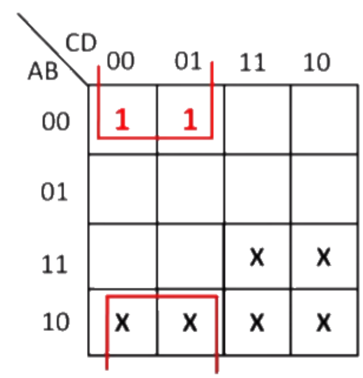
8. Design a combinational circuit that generates the 9's complement of a decimal digit represented by a gray code.

Solution:

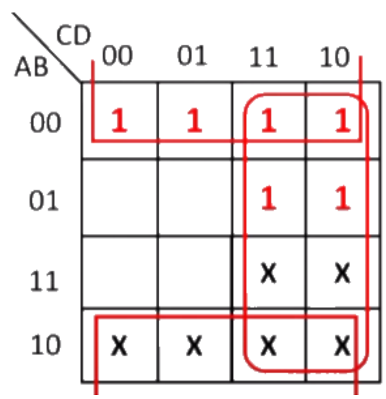
Decimal	Gray code				9's complement			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	1	1	0	1
1	0	0	0	1	1	1	0	0
2	0	0	1	1	0	1	0	0
3	0	0	1	0	0	1	0	1
4	0	1	1	0	0	1	1	1
5	0	1	1	1	0	1	1	0
6	0	1	0	1	0	0	1	0
7	0	1	0	0	0	0	1	1
8	1	1	0	0	0	0	0	1
9	1	1	0	1	0	0	0	0

Don't care terms: 8, 9, 10, 11, 14, 15

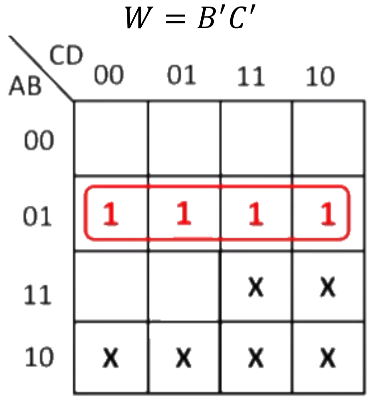
Simplified expressions (K-map):



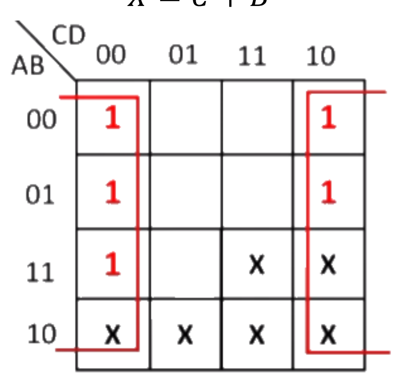
$W = B'C'$



$X = C + B'$



$Y = A'B$



$Z = D'$

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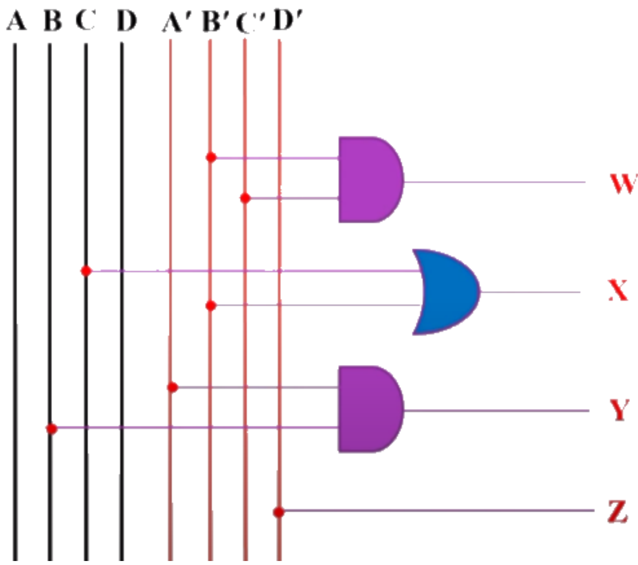


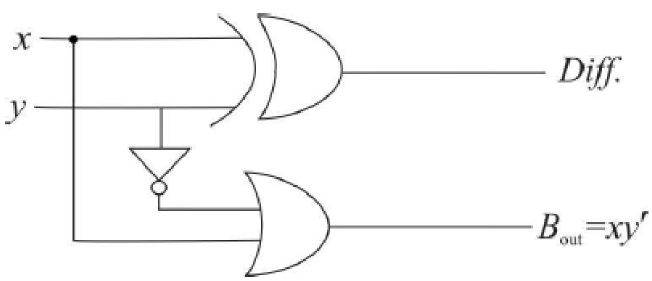
9. (a) Design a half-subtractor circuit with inputs x and y and outputs *Diff* and B_{out}. The circuit subtracts the bits y–x and places the difference in D and the borrow in B_{out}.
(b) Design a full-subtractor circuit with three inputs x, y, B_{in} and two outputs Diff and B_{out}. The circuit subtracts y–x–B_{in}, where B_{in} is the input borrow, B_{out} is the output borrow, and Diff is the difference.

Solution:

x	y	$Diff.$	$Bout$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	0

$$Diff. = x'y + xy'$$
$$= x \oplus y$$

$$Bout = xy'$$



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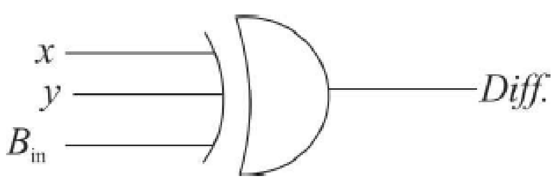
x	y	Bin	$Diff.$	$Bout$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

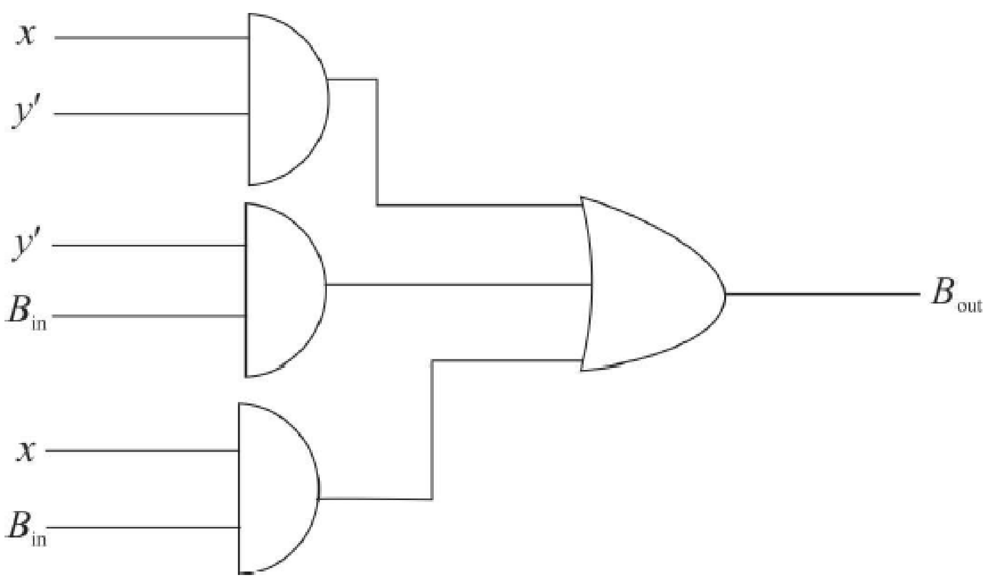
$x \backslash y$	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$$\text{Bout} = xy' + y'\,\text{Bin} + x\text{Bin}$$

$$Diff. = x'(y \oplus \text{Bin}) + x(y \oplus \text{Bin})'$$

$$= x \oplus y \oplus \text{Bin}$$





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10. Derive the two-level Boolean expression for the output carry C_4 shown in the lookahead carry generator of Fig. 4.12.

    Solution:

    $$C_4 = G_3 + P_3C_3 = G_3 + P_3(G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0)$$

    $$= G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_0$$