CS 302.1 - Automata Theory

Lecture 09

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Quick Recap

Pumping Lemma for CFL: If L is Context Free, then there exists $p > 0$ (pumping length), such that, for any $w \in L$ of length $|w| \geq p$, w can be split into five parts, i.e. $w = uvxyz$
satisfying the following conditions:

- $|vy| \geq 1$
- $|vxy| \leq p$
- $uv^ixy^iz \in L, \; \forall i \geq 0$

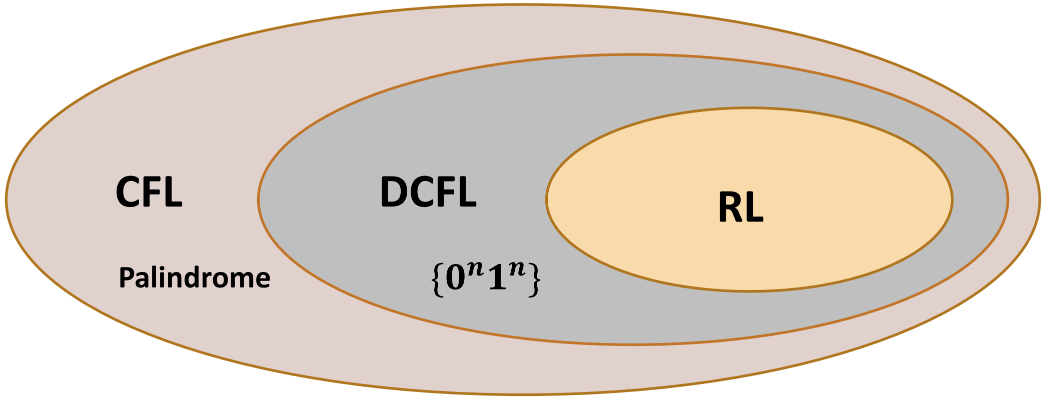
Closure properties of CFLs

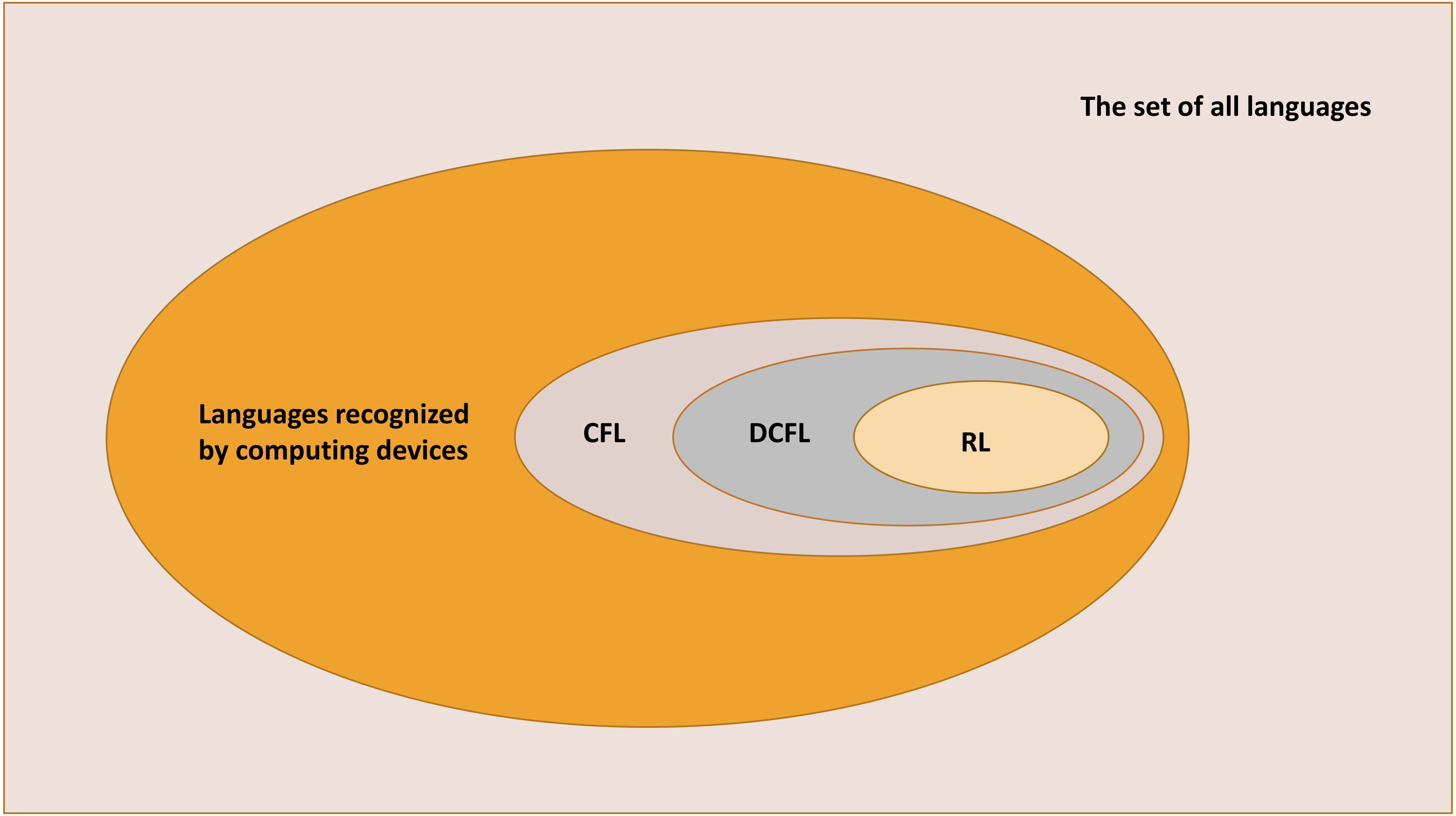
CFLs are closed under

- **Union**
- **Star**
- **Concatenation**

CFLs are NOT closed under

- **Complementation**
- **Intersection**





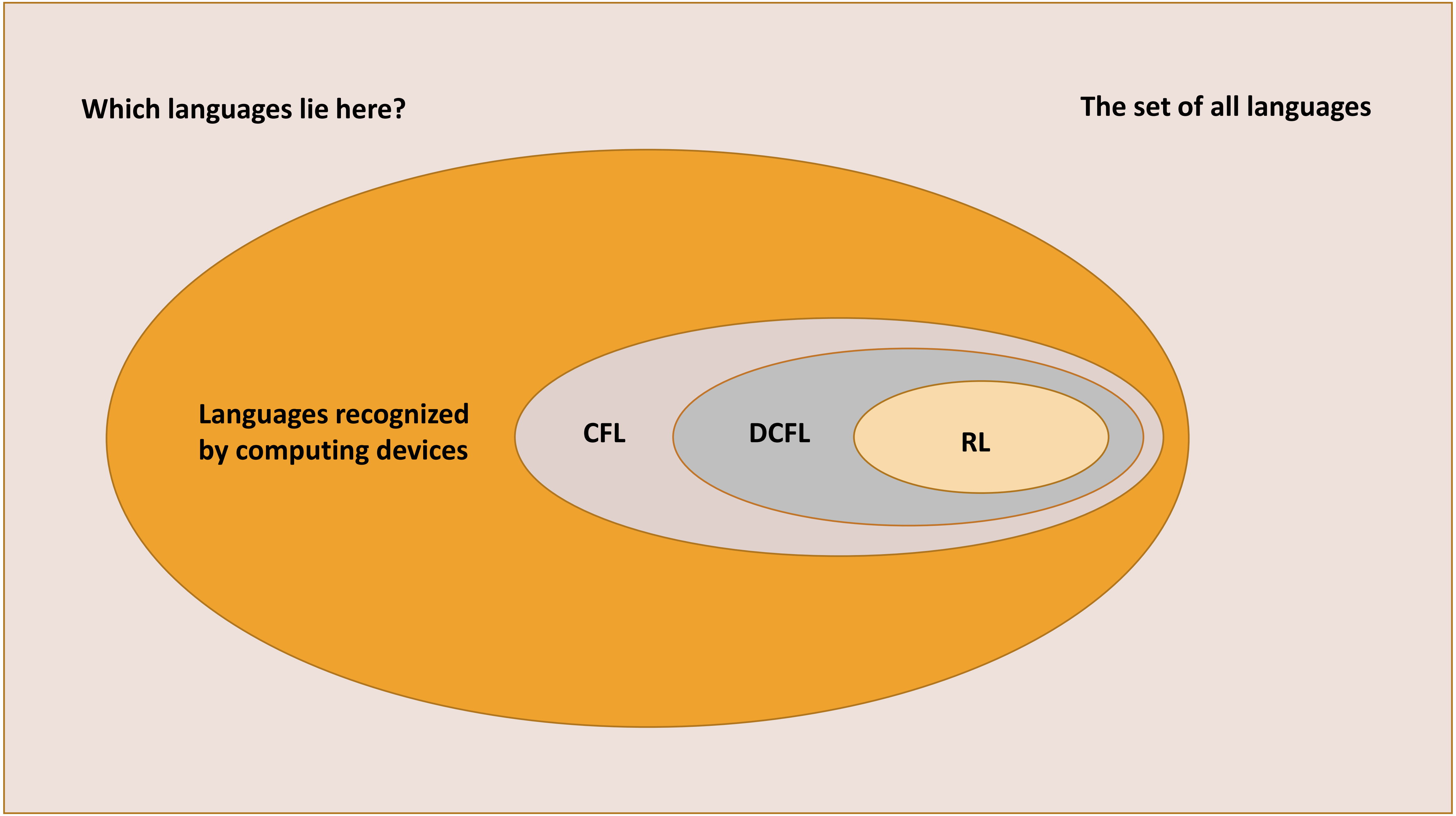
The set of all languages

Languages recognized by computing devices

CFL

DCFL

RL



Which languages lie here?

The set of all languages

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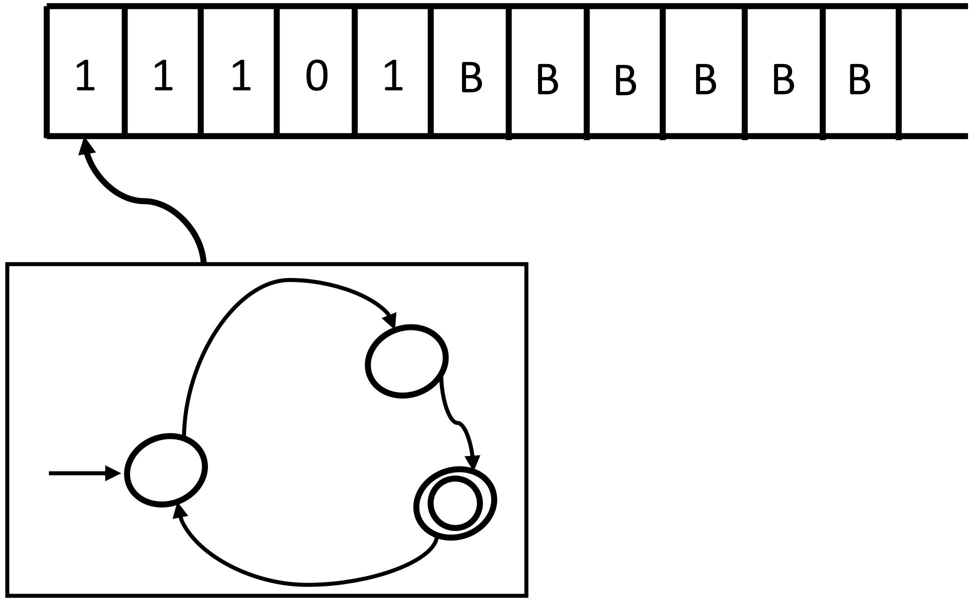
Turing Machines

- A Turing machine is a FSM that has access to a infinite tape as its memory.
- The infinite tape contains in it, the input string followed by Blanks (indicated by B)
- The Turing machine can both read from the tape and write in it – one cell at a time, using a Read/Write head.
- The Read/Write head can move to the Left or to the Right – again one cell at a time.



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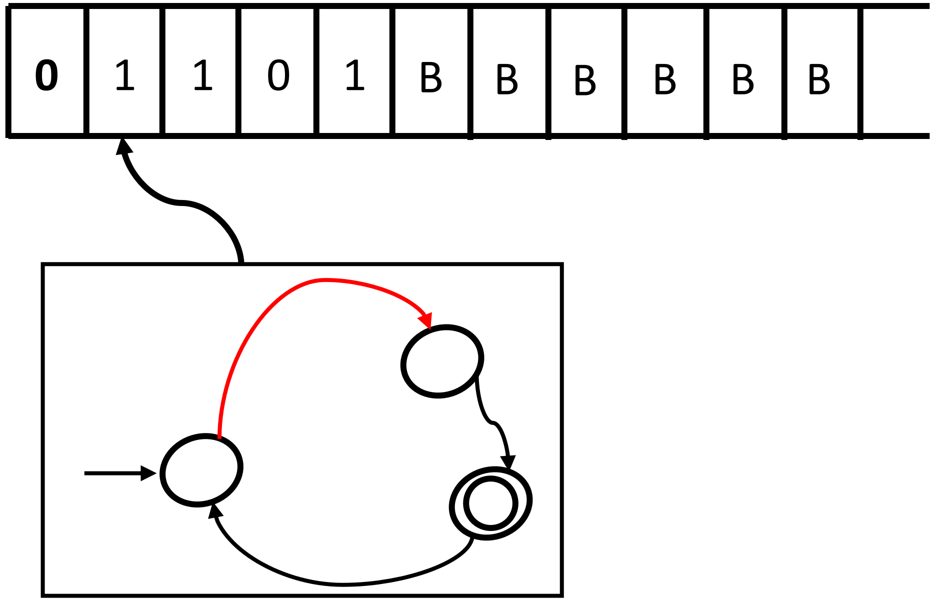

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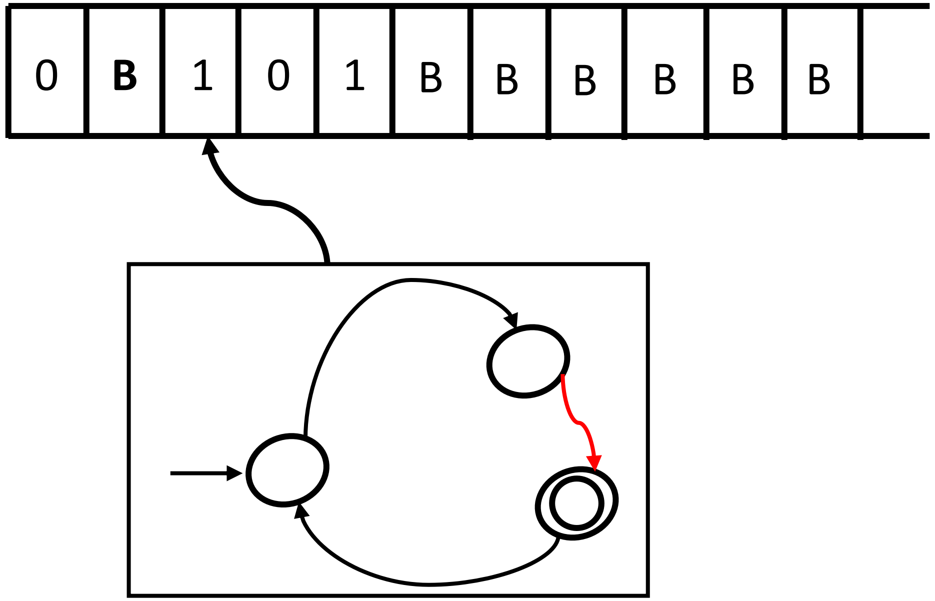

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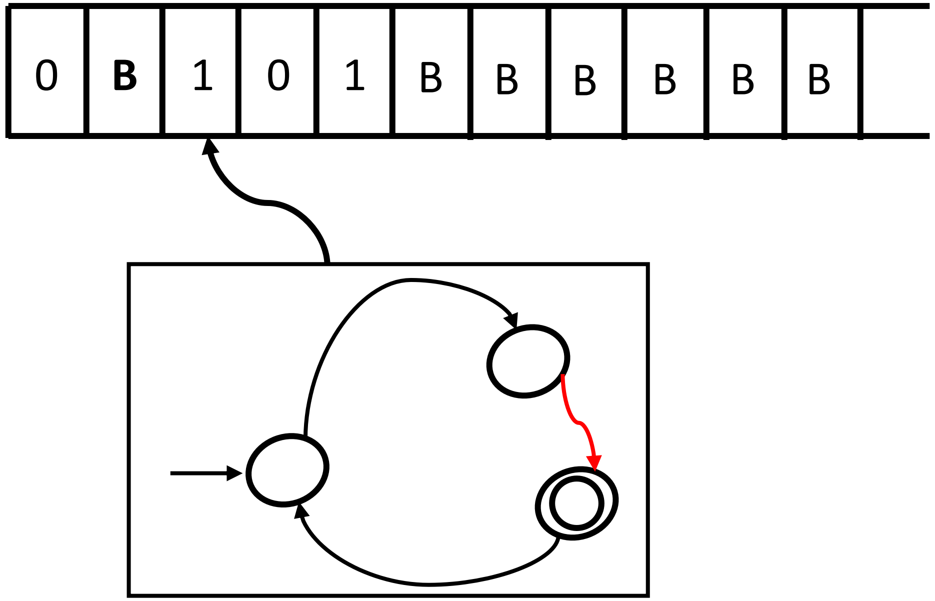

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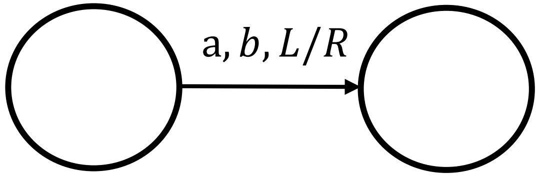
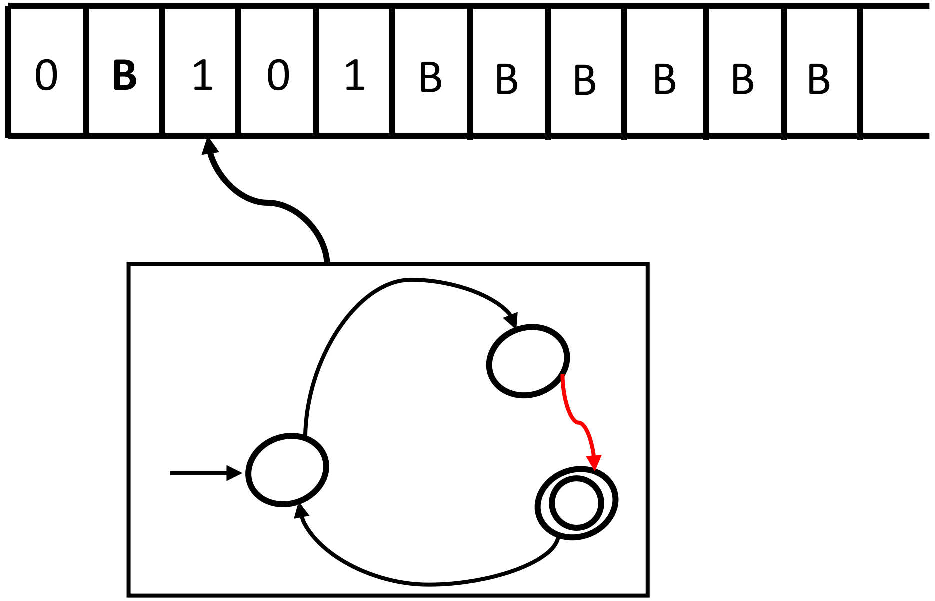
- In a way these "added features" give TMs their power. (eg: ability to write on the tape)
- Notice: acceptance/rejection of a run is not tied to the input.
- Auxiliary computation can be performed – as much as needed, even when the input string has been scanned

Turing Machines



Transition $a, b, L/R$: Read a from the tape, replace with b and move L/R



Accept state

Reject state

TM halts and **accepts/rejects** on reaching these states

TM may never halt – it may loop forever

At each step, the TM:

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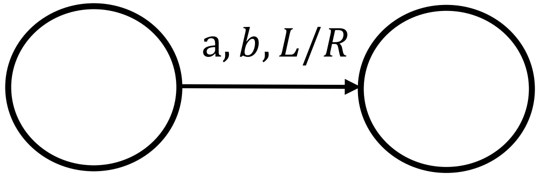

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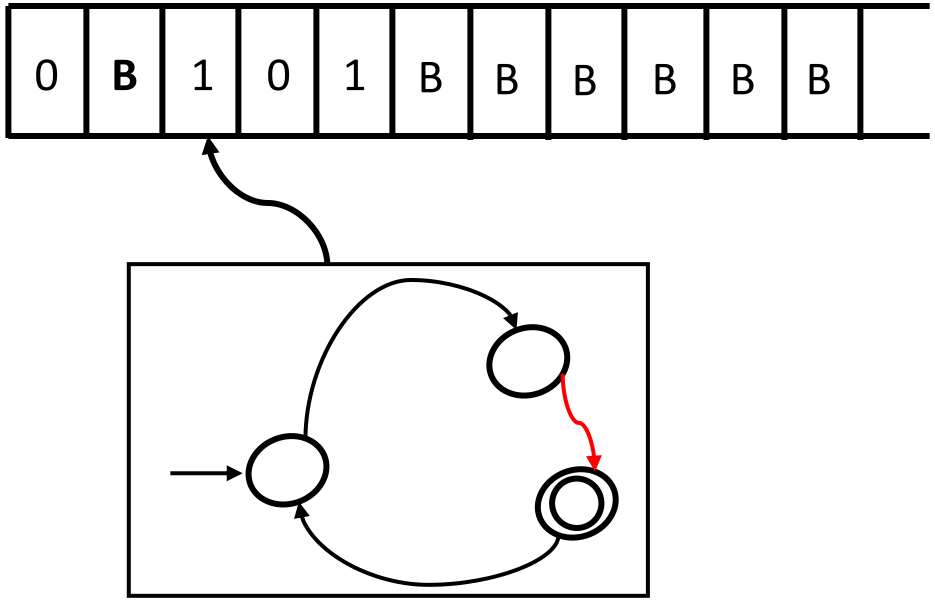

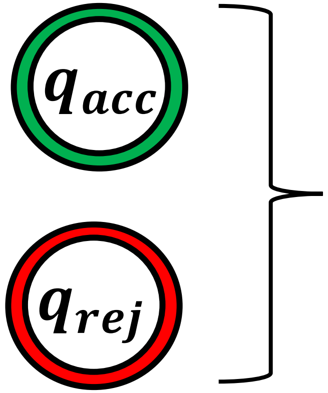

TM halts and **accepts/rejects** on reaching these states

So, given a TM M and an input ω,

$M(\omega)$ **accepts** if $\omega \in L(M)$
$M(\omega)$ **rejects** if $\omega \notin L(M)$
$M(\omega)$ **runs infinitely** if $\omega \notin L(M)$

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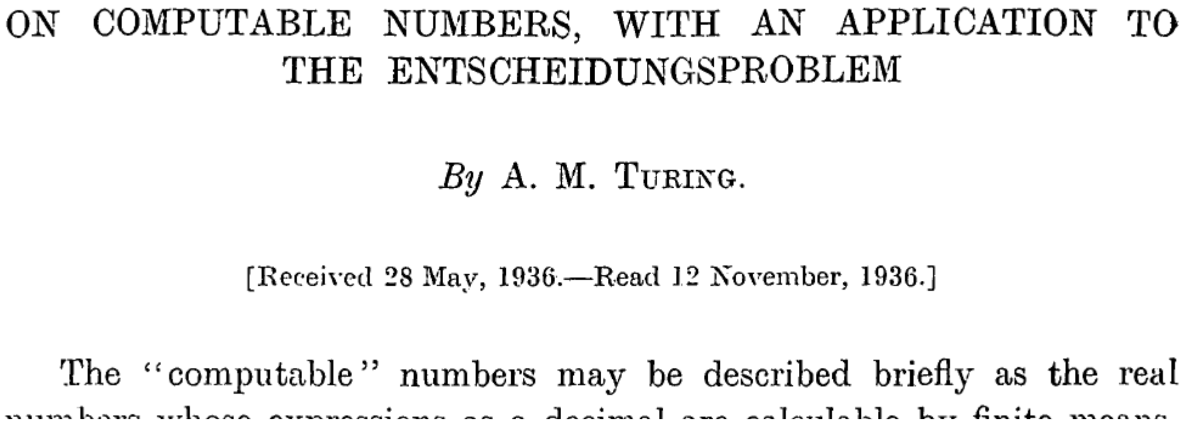

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

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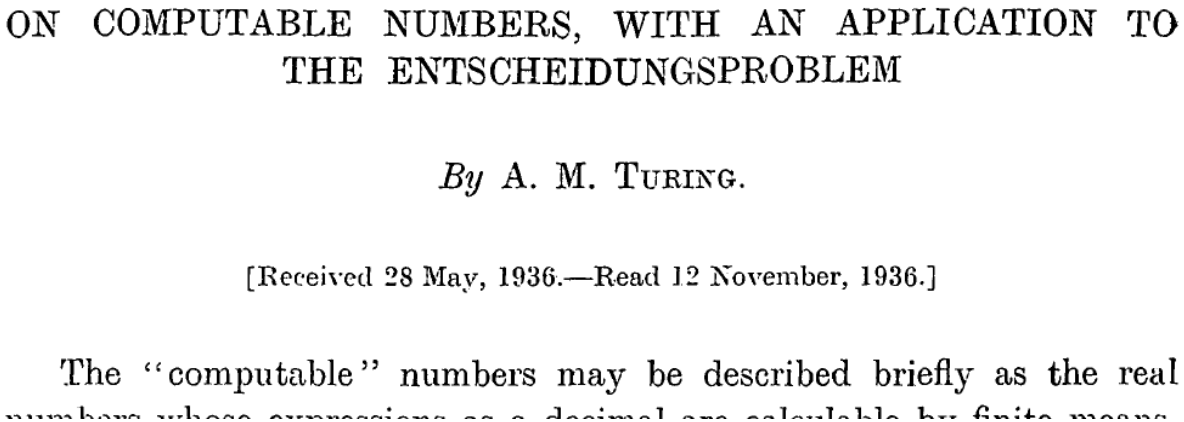

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- Turing assumed that the human brain to be a finite state machine with a finite number of states
- Consider such a human being working on a problem with a notebook, pencil and an eraser.
- The pages of the notebook are laid out on the tape – each cell consists of one page, with a finite amount of information.
- Whatever the human being does with the notebook, can be simulated on the TM: reading, writing, erasing (writing a blank), moving left or right to a new page etc.

Turing Machines

Example: Let $\boldsymbol{L = \{0^n 1^n | n \geq 1\}}$

We will try to develop the basic idea in designing the Turing Machine for this language. Note that $L = CFL$.

Idea: An accepting run of a TM for L could look something like this:

- Mark the first 0 (by replacing it with some special symbol say X)
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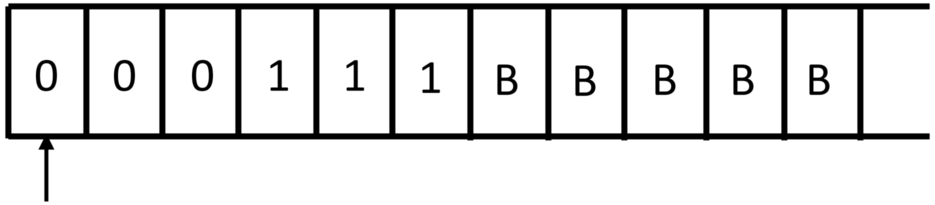

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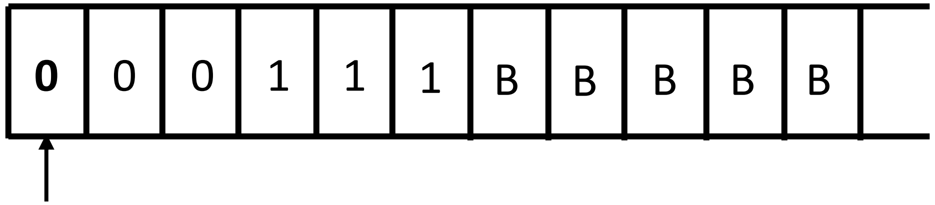

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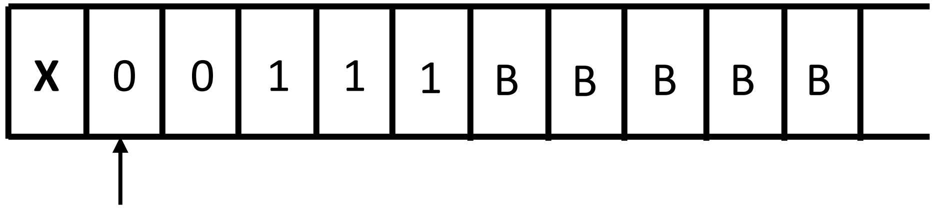

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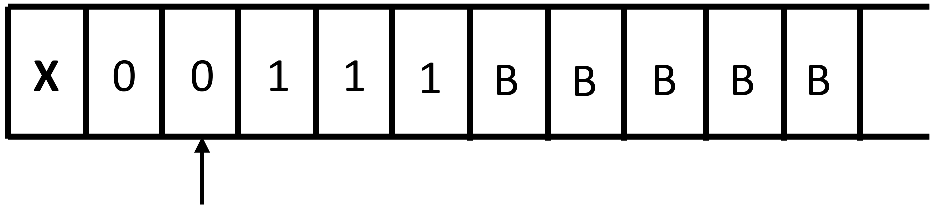

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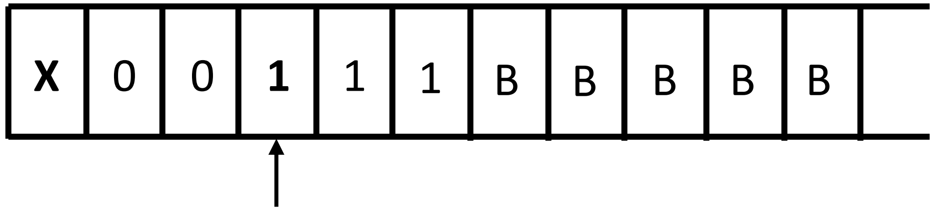

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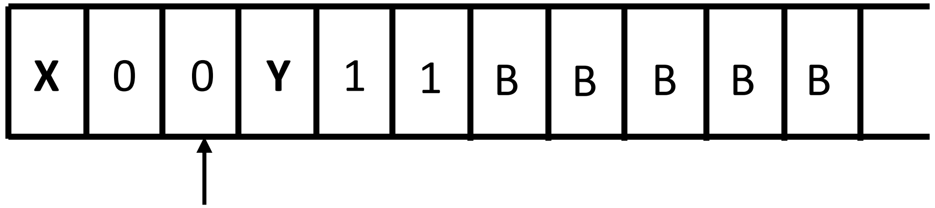

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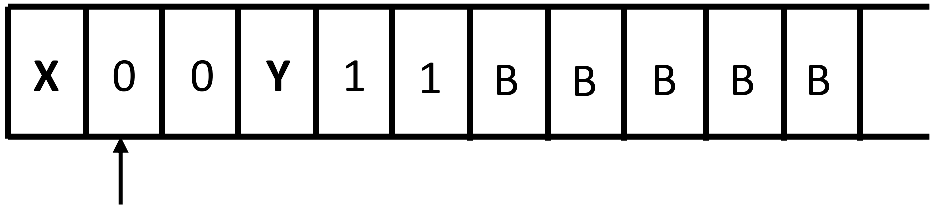

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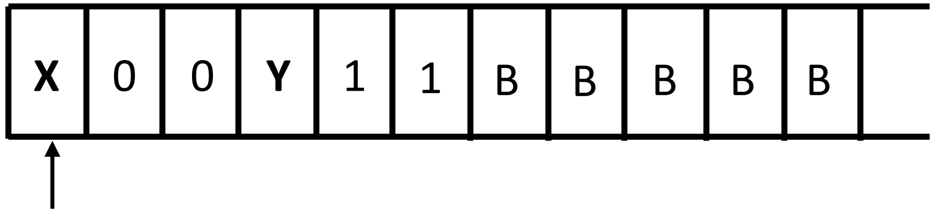

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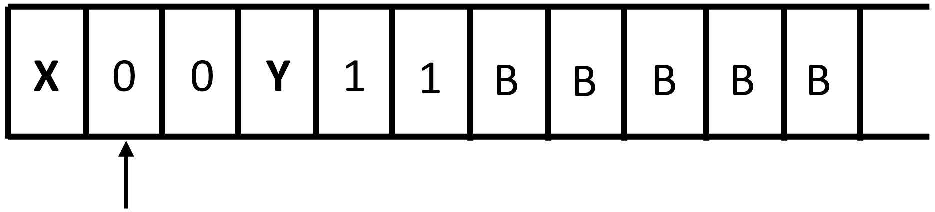

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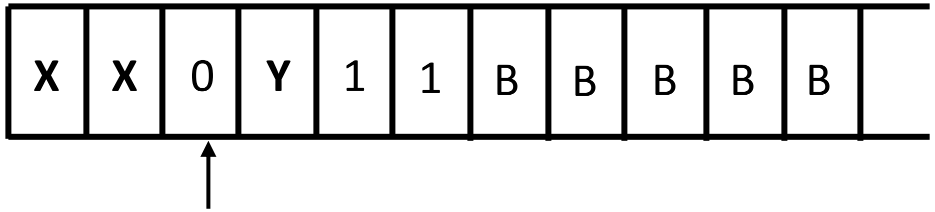

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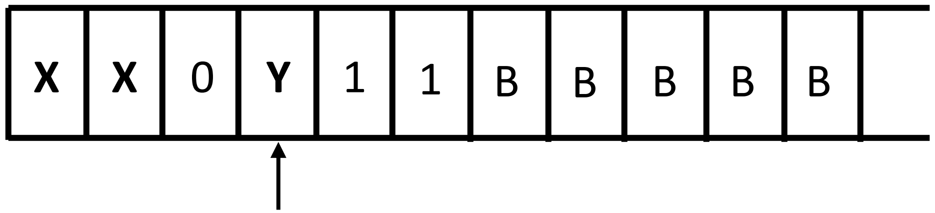

While moving right, when a Y is encountered, the head should move right as that's where the next 1 to be marked is $\Rightarrow$ **We need rules like $(\boldsymbol{Y}, \boldsymbol{Y}, \boldsymbol{R})$**

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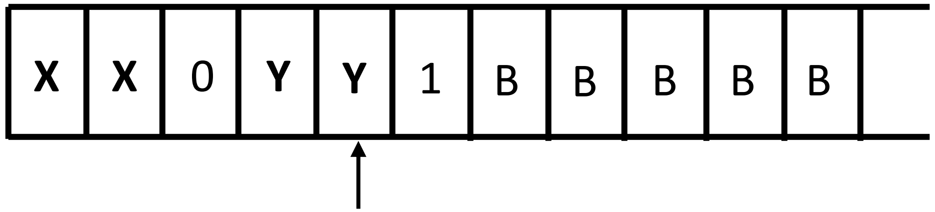

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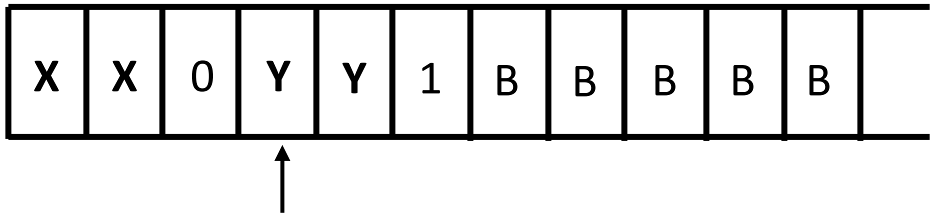

Turing Machines

Example: Let $\boldsymbol{L = \{0^n 1^n | n \geq 1\}}$

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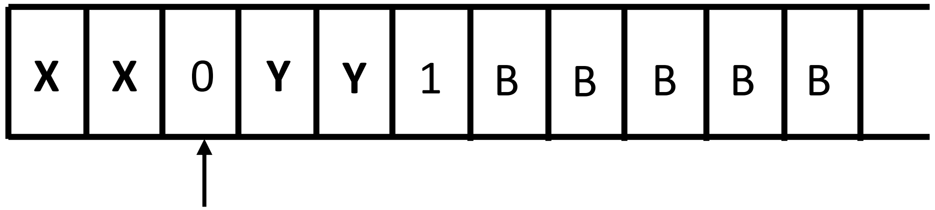

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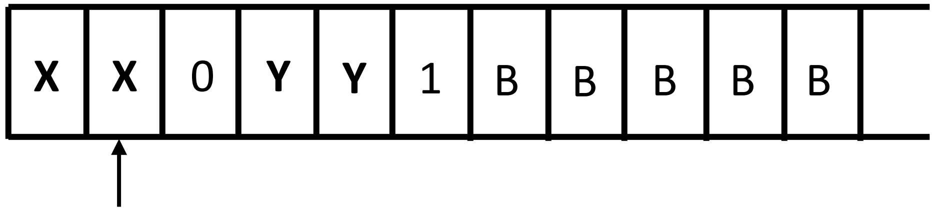

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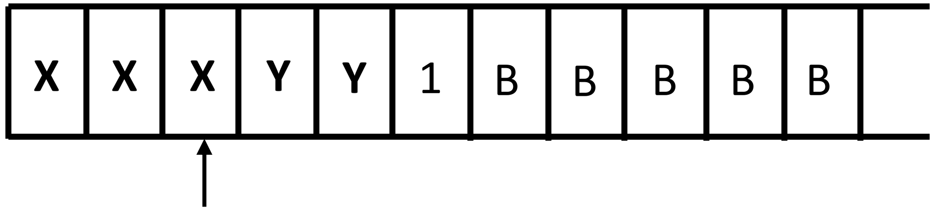

X	**X**	**X**	**Y**	**Y**	1	B	B	B	B	B	

Turing Machines

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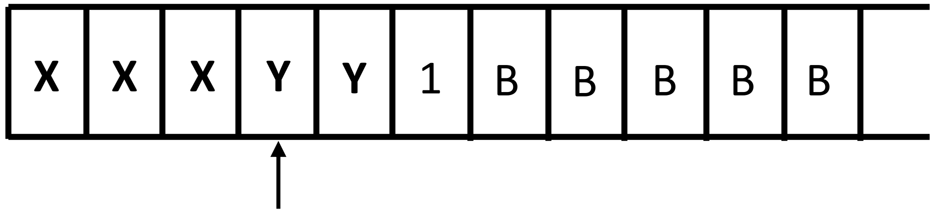

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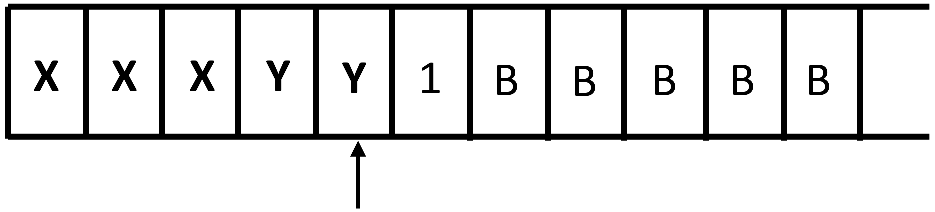

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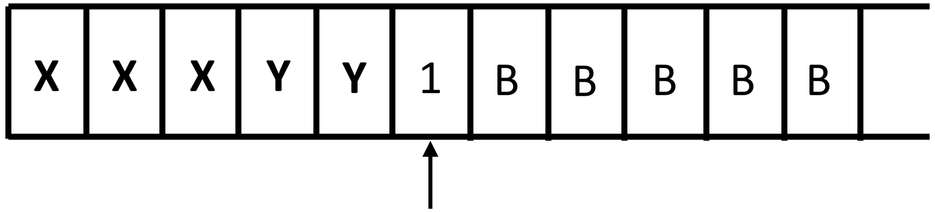

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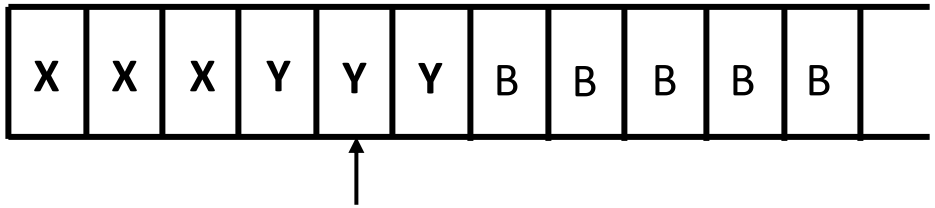

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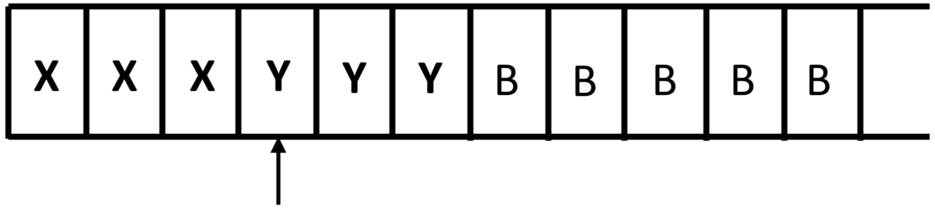

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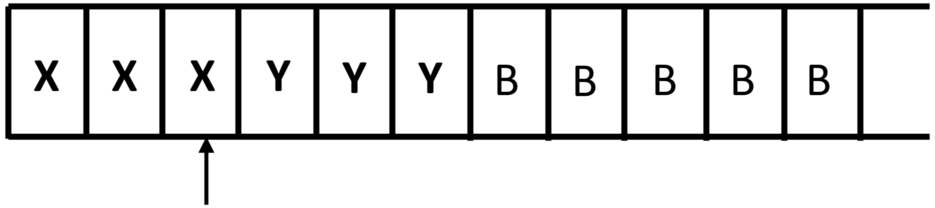

At this stage the head should move right to look for the next 0 to mark, but finds Y

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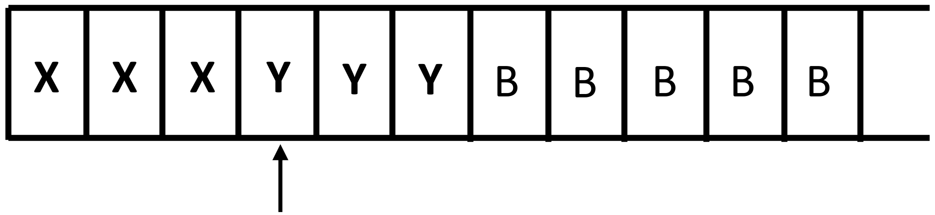

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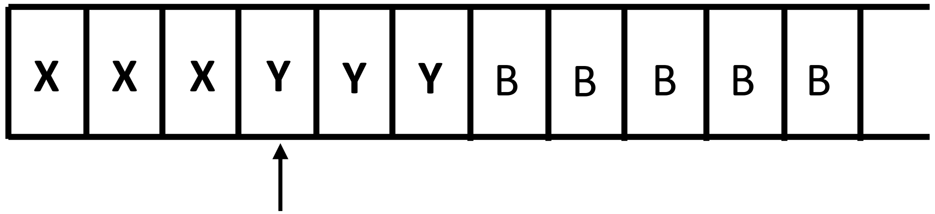

The head keeps moving right until it finds a B

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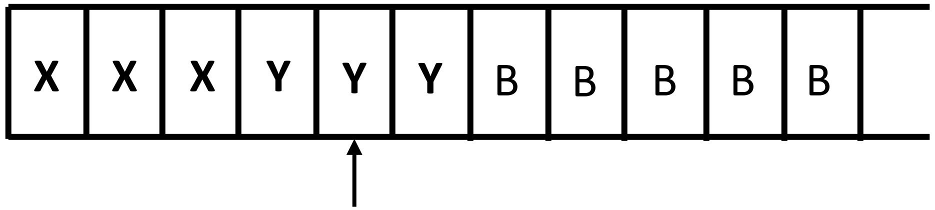

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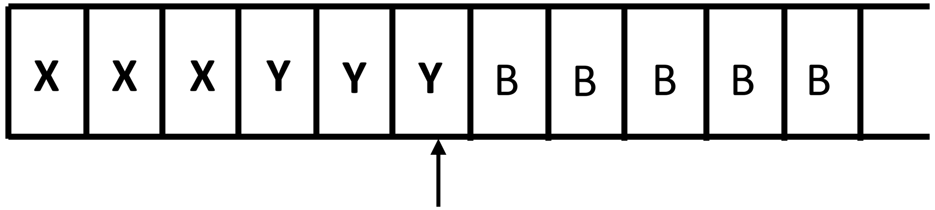

X	**X**	**X**	**Y**	**Y**	**Y**	B	B	B	B	B	

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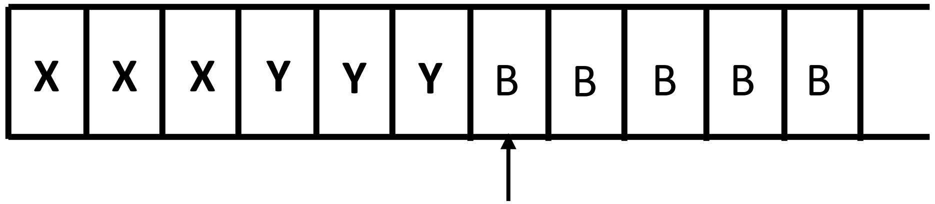

X	**X**	**X**	**Y**	**Y**	**Y**	B	B	B	B	B	

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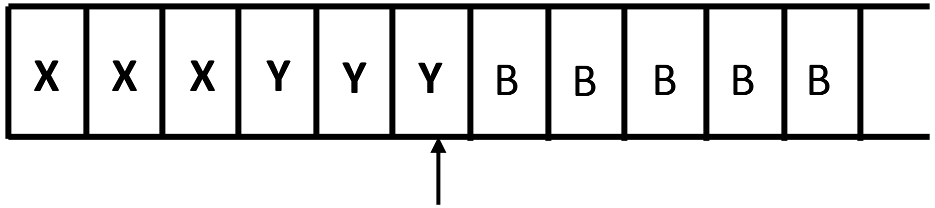
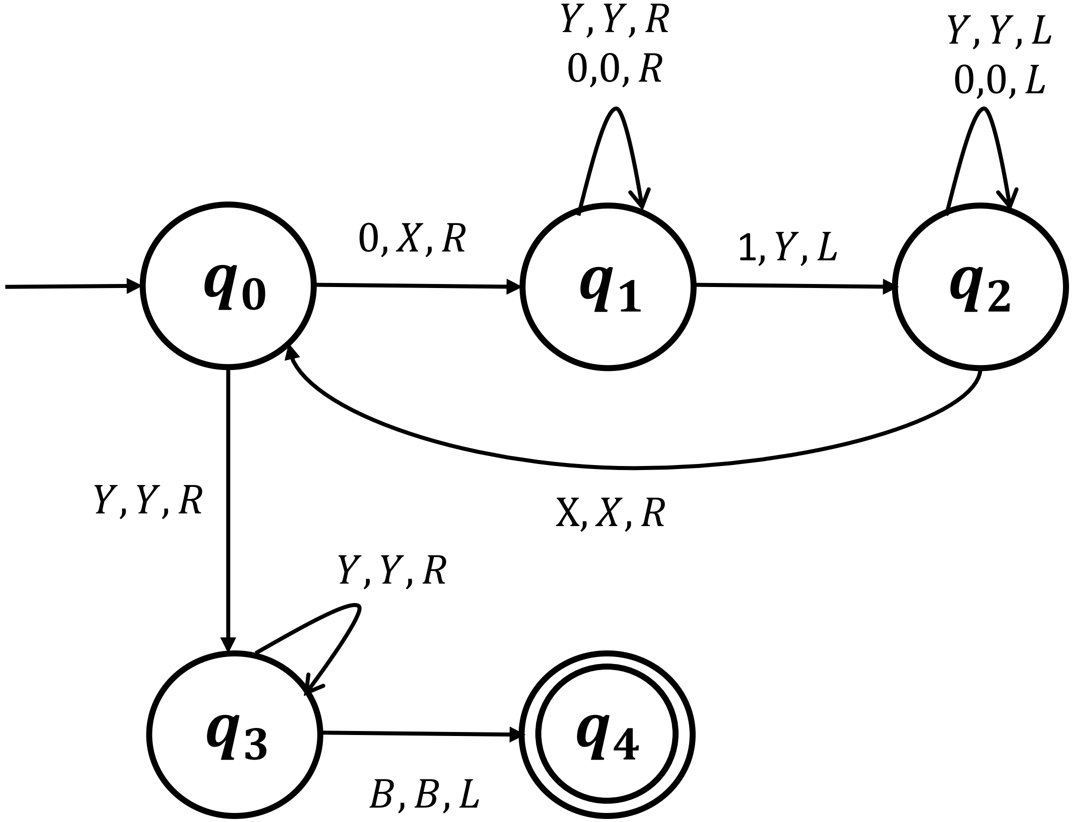


This is when the TM decides to accept the input string.

Turing Machines

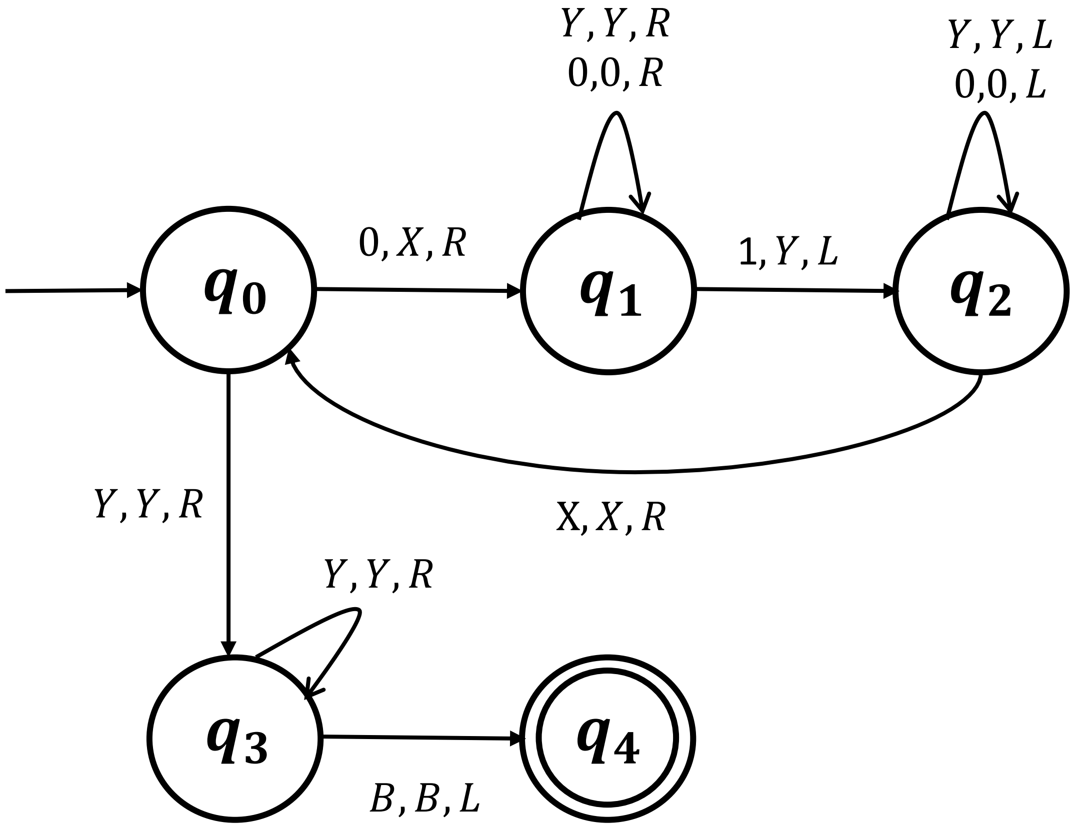
Example: Let $L = \{0^n 1^n | n \geq 1\}$

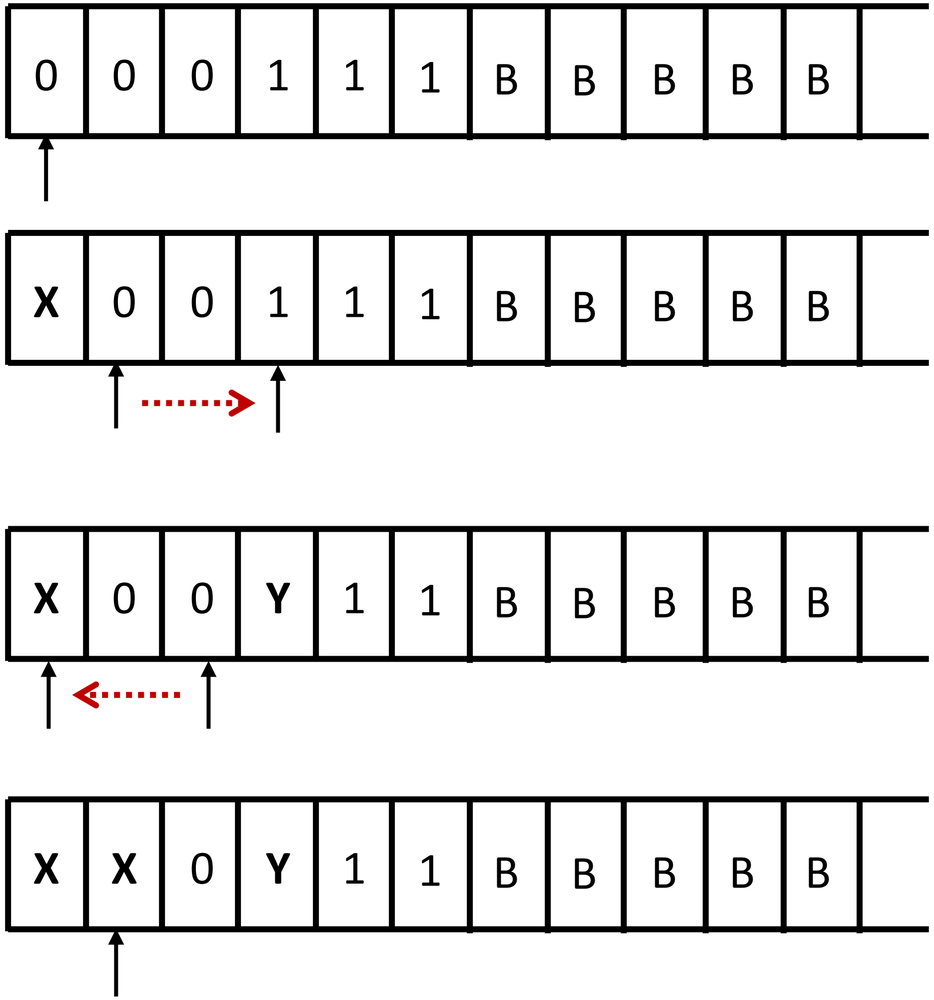


All missing transitions lead to the reject state and the input is rejected when this state is reached.

Turing Machines

Example: Let $L = \{0^n1^n | n \geq 1\}$





0	0	0	1	1	1	B	B	B	B	B

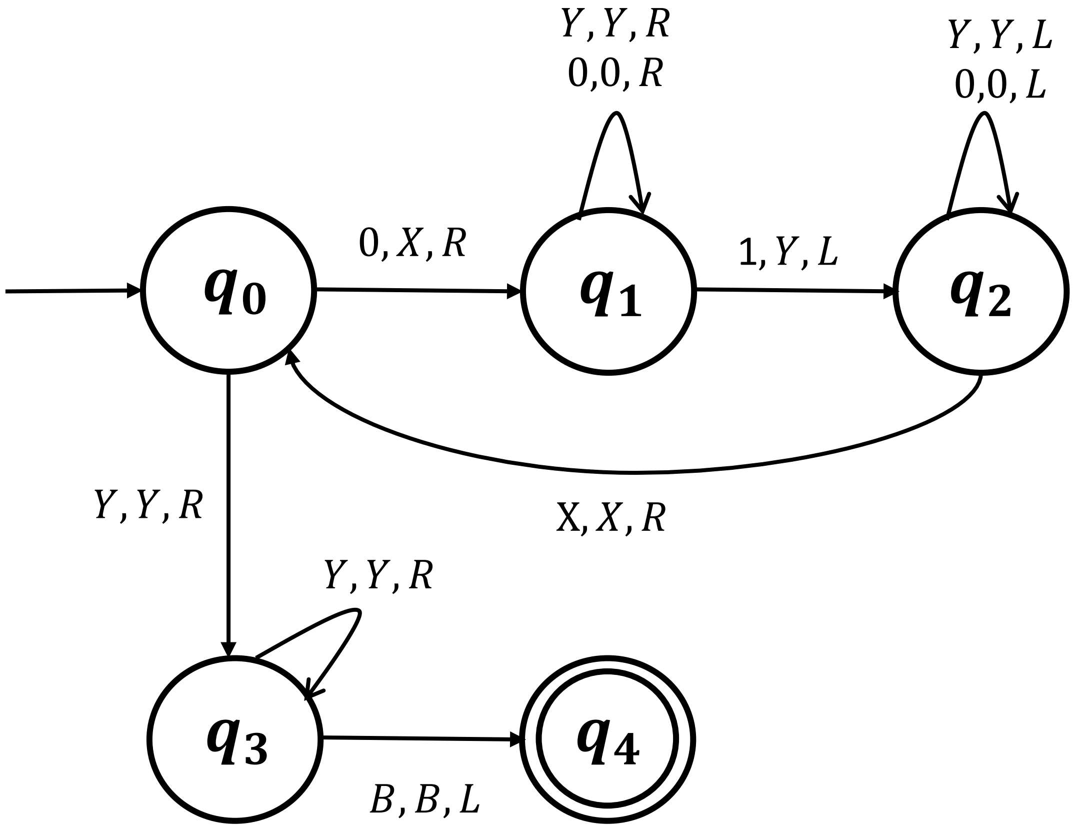
X	0	0	1	1	1	B	B	B	B	B

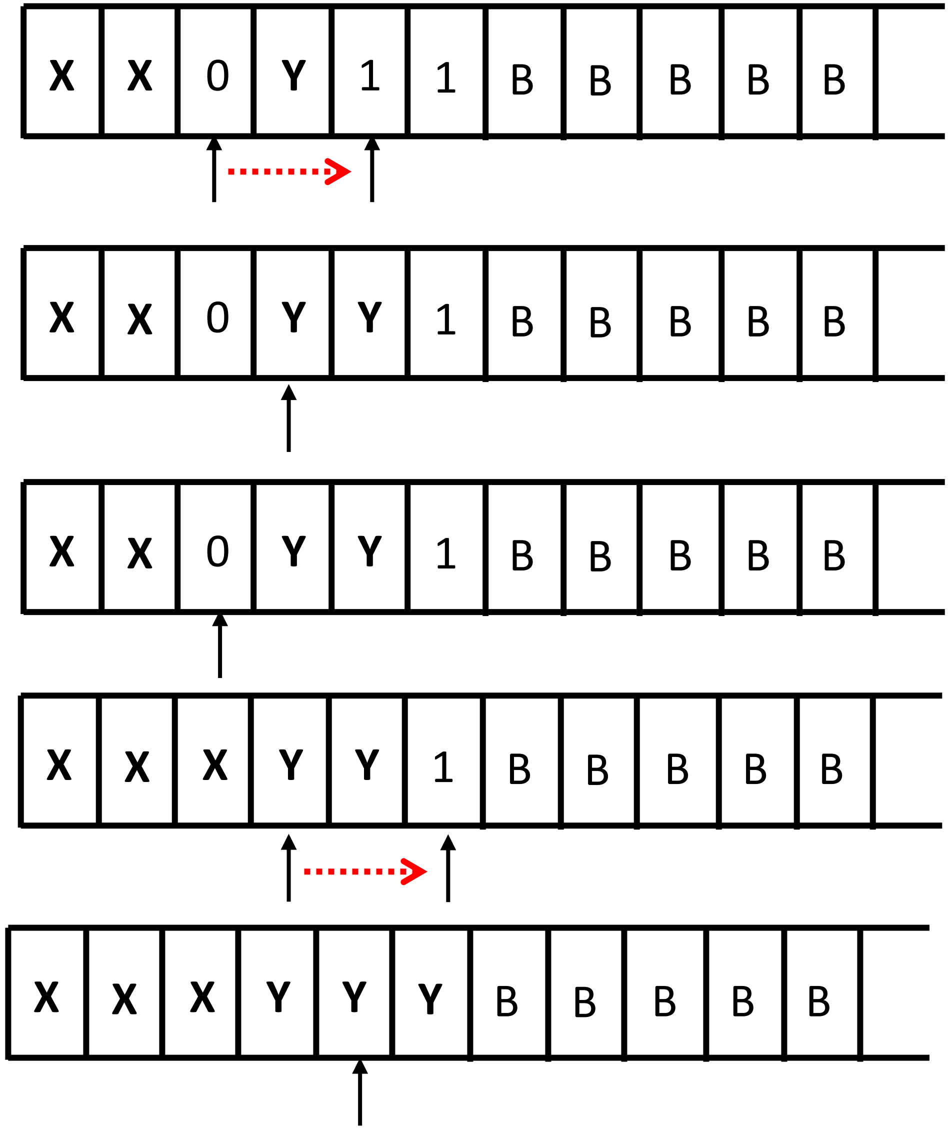
X	0	0	**Y**	1	1	B	B	B	B	B

X	**X**	0	**Y**	1	1	B	B	B	B	B

Turing Machines

Example: Let $L = \{0^n 1^n | n \geq 1\}$





X	X	0	Y	1	1	B	B	B	B	B	

X	X	0	Y	Y	1	B	B	B	B	B	

X	X	0	Y	Y	1	B	B	B	B	B	

X	X	X	Y	Y	1	B	B	B	B	B	

X	X	X	Y	Y	Y	B	B	B	B	B	

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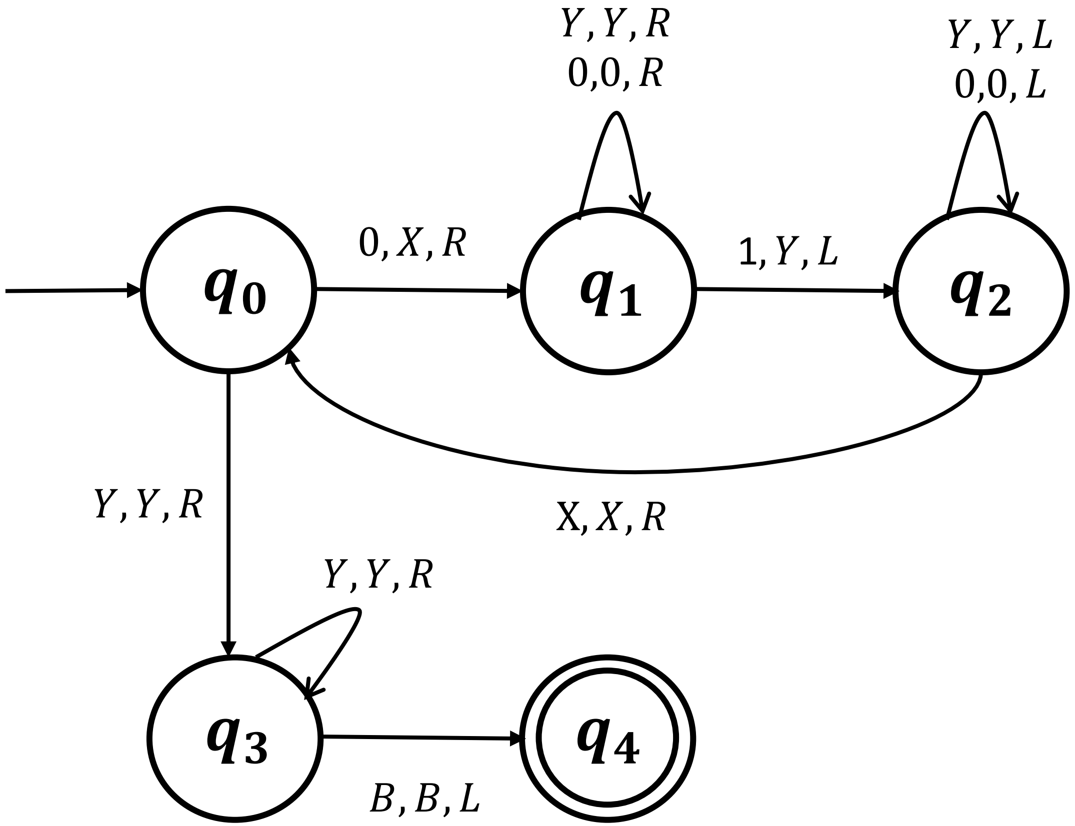
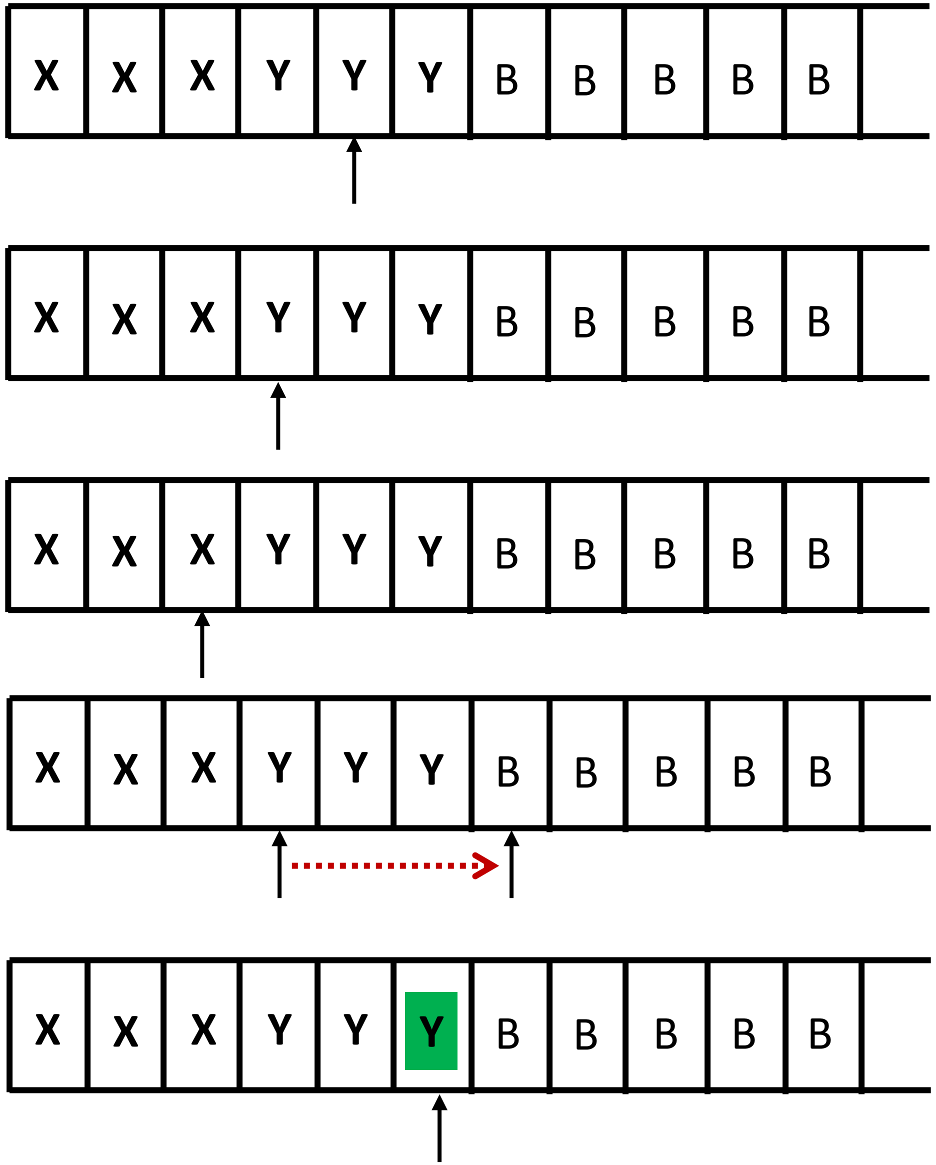
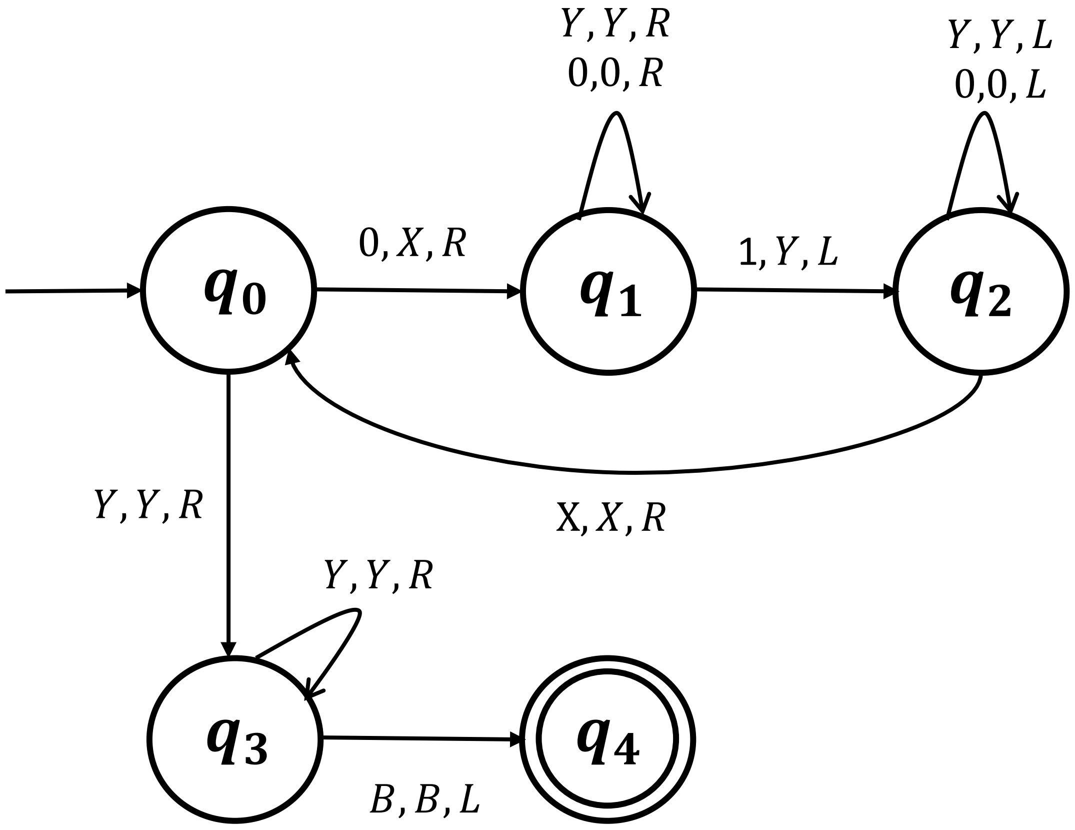


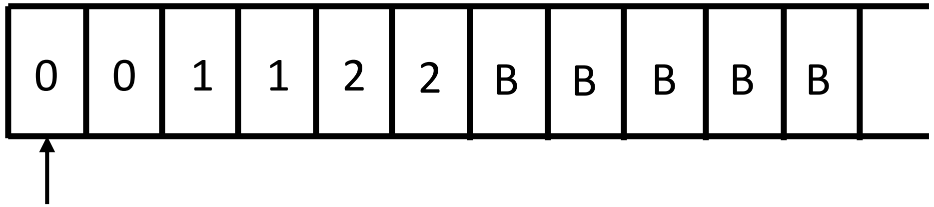


Turing Machines

Example: Let $L = \{0^n1^n2^n | n \geq 1\}$



- We will start off with the TM for $\{0^n1^n\}$ and construct the TM for $\{0^n1^n2^n\}$
- **Very similar to the TM for $\{0^n1^n\}$**, except now the FSM would count the number of $2'$s as well. So it marks the $2's$ with another symbol (say Z)



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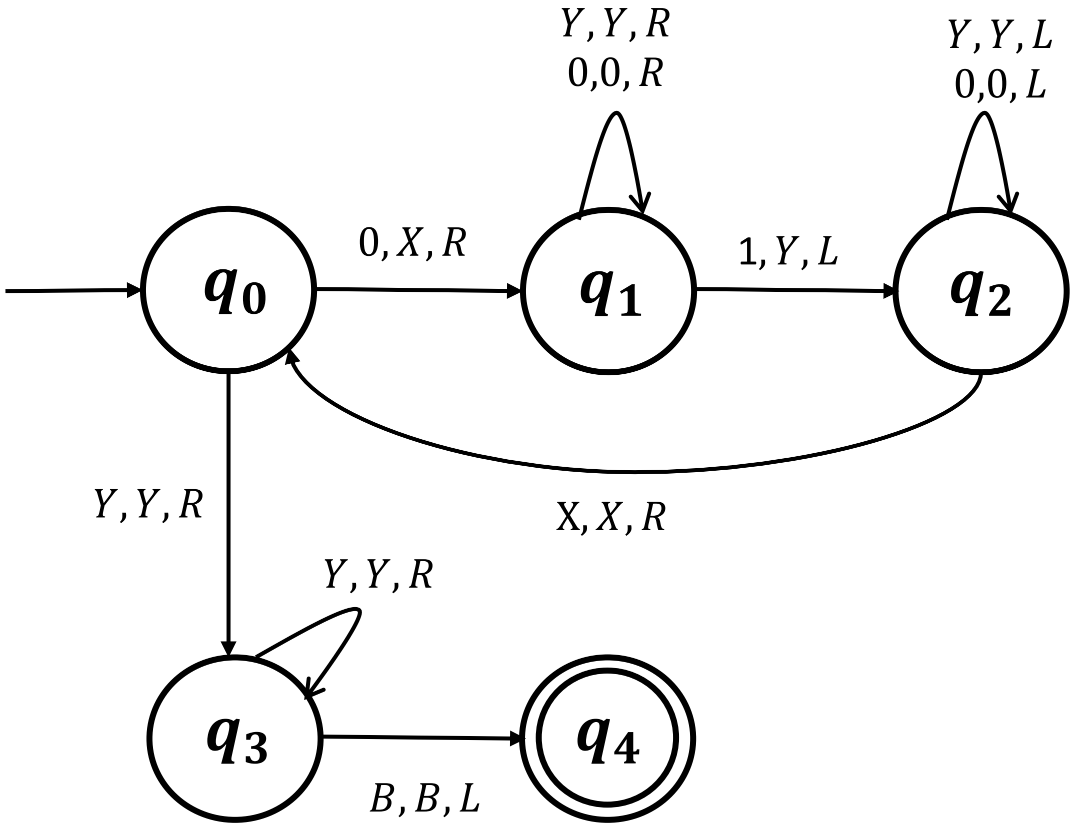

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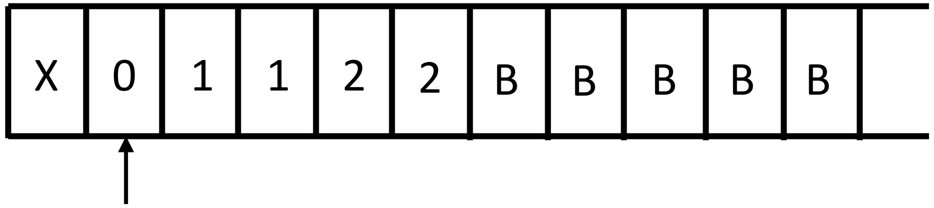

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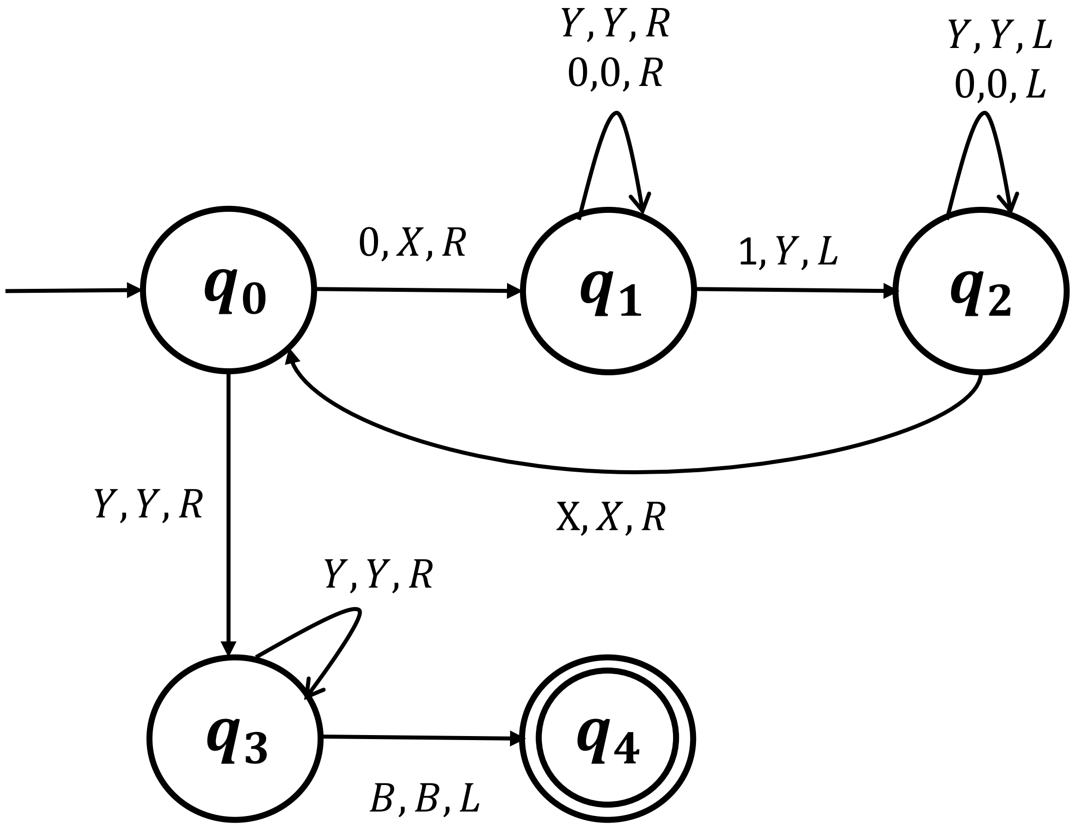

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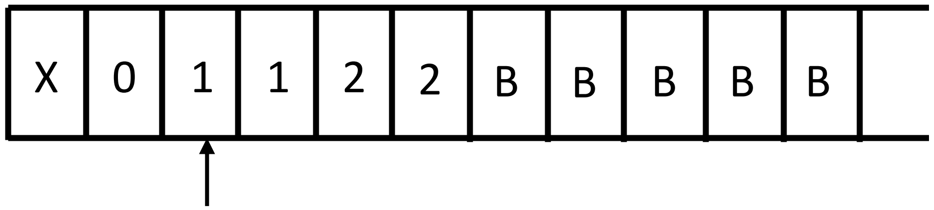

X	0	1	1	2	2	B	B	B	B	B

Turing Machines

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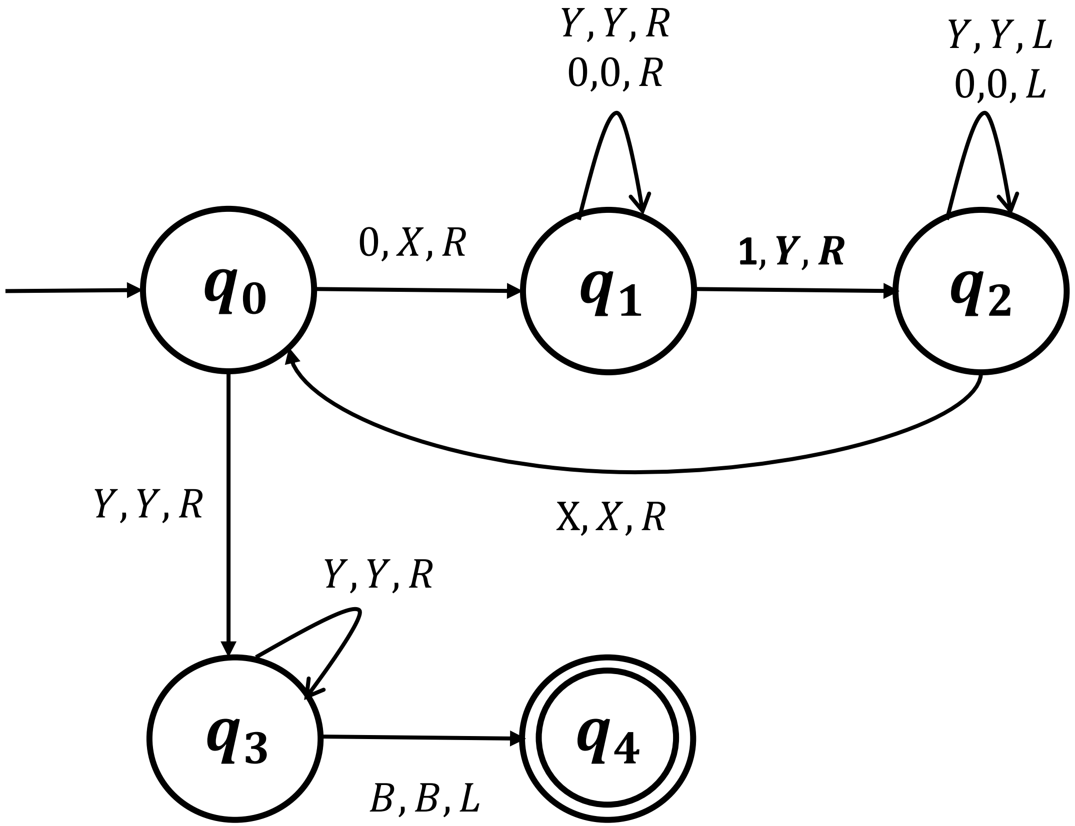
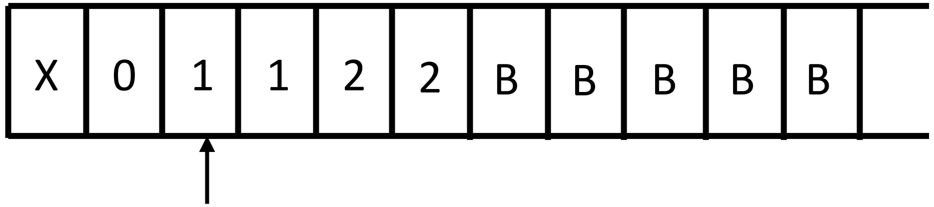
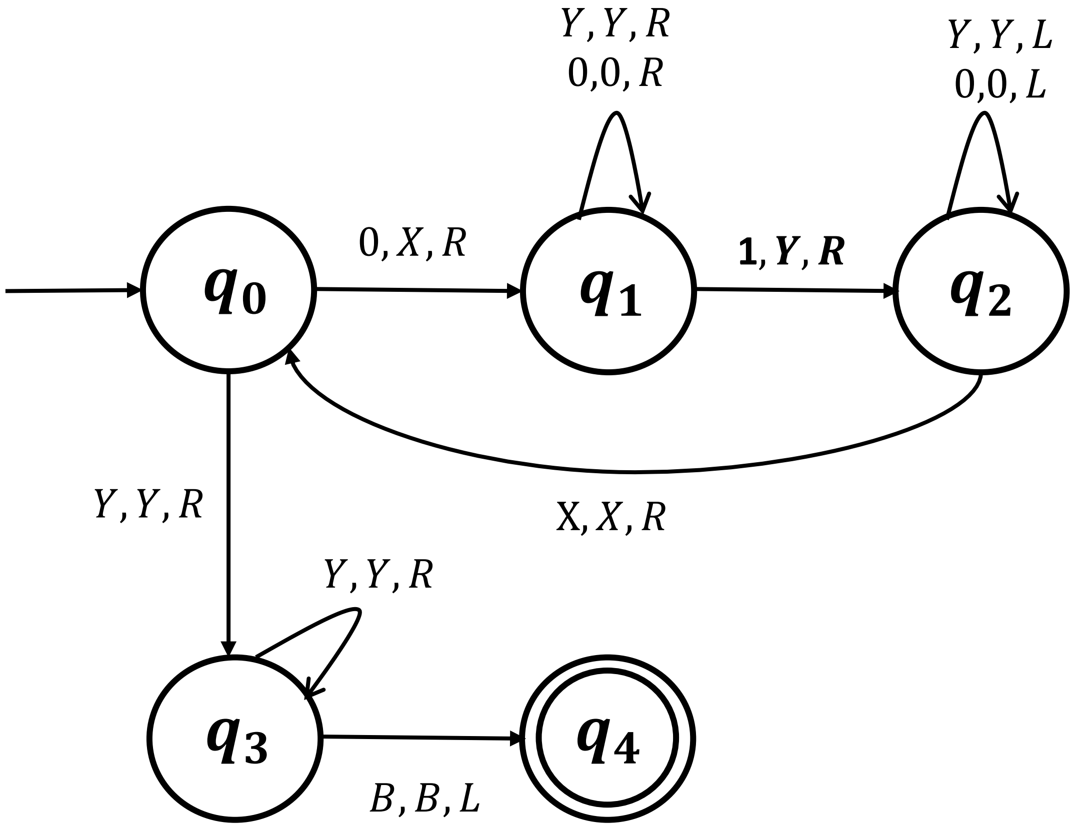


- Continue to go right to mark the next 2 with a Z.



Turing Machines

Example: Let $L = \{0^n 1^n 2^n | n \geq 1\}$



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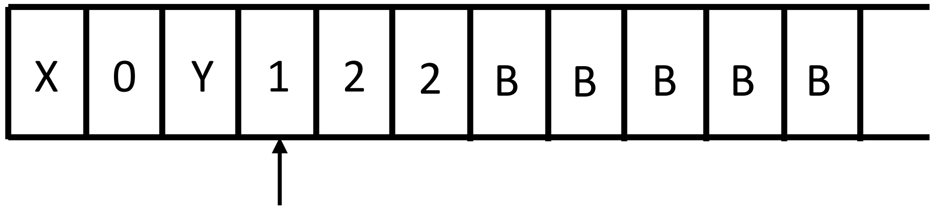
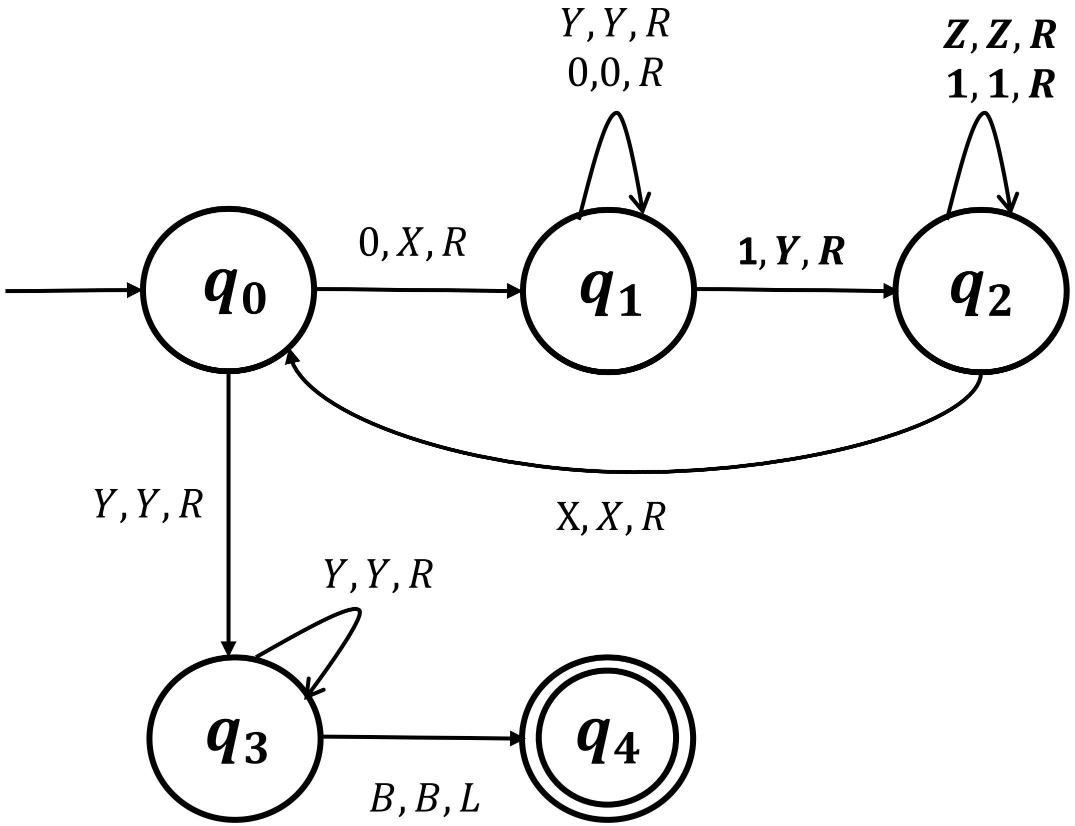
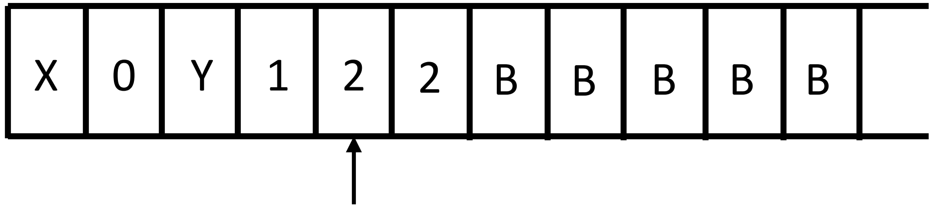


Turing Machines

Example: Let $L = \{0^n1^n2^n | n \geq 1\}$



- Continue to go right to mark the next 2 with a Z.
- Skip all the $1'$s and move right/ Also, move across (to the right) the Z's already marked.



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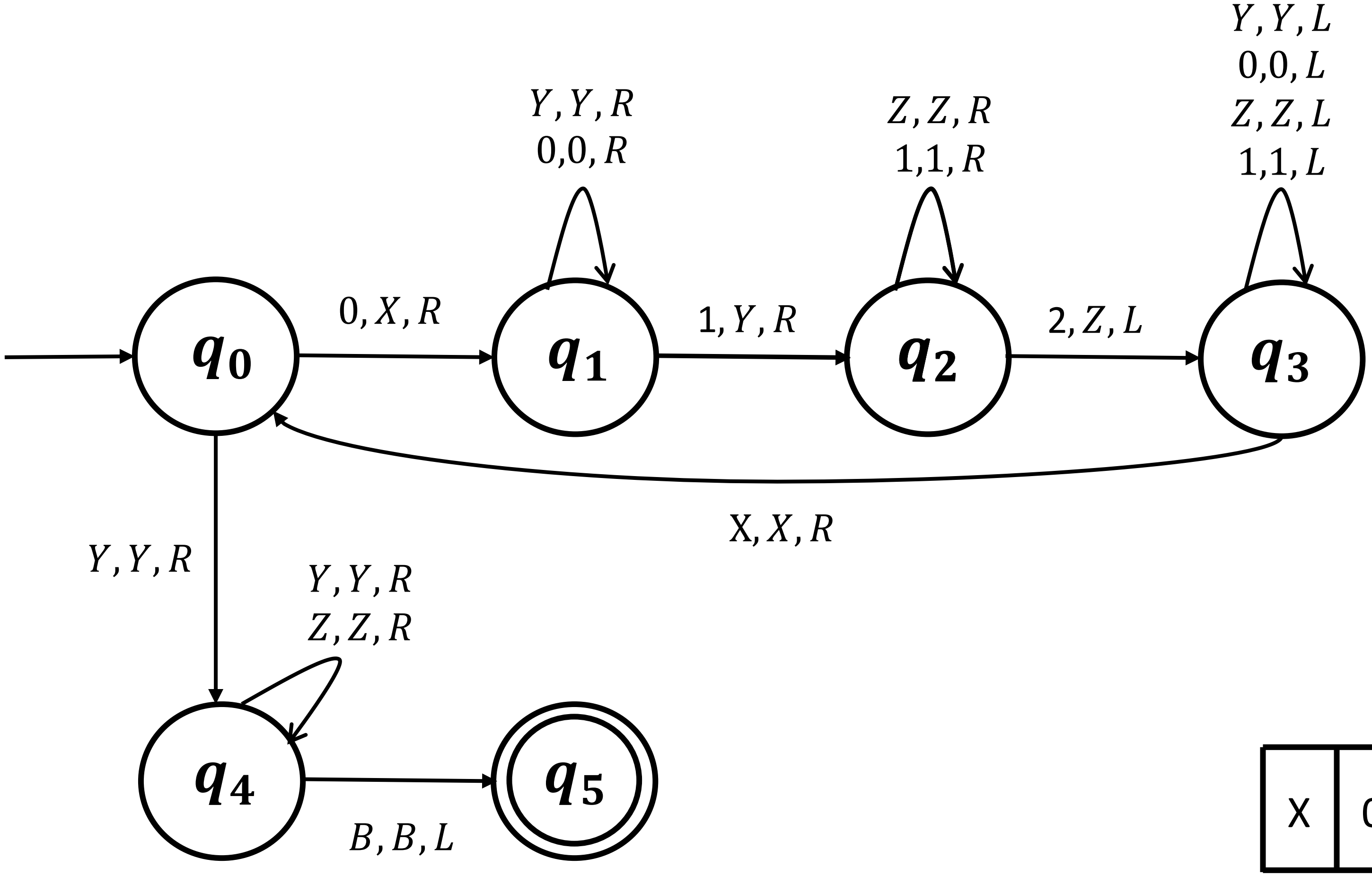

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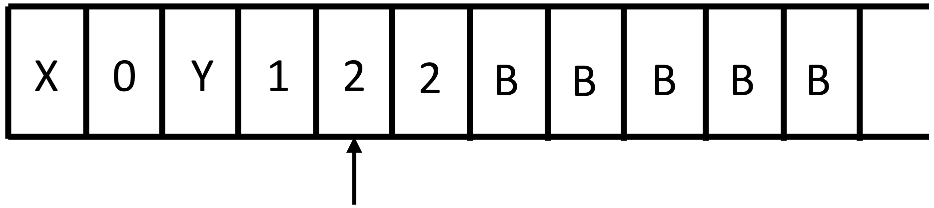

X	0	Y	1	2	2	B	B	B	B	B

Turing Machines

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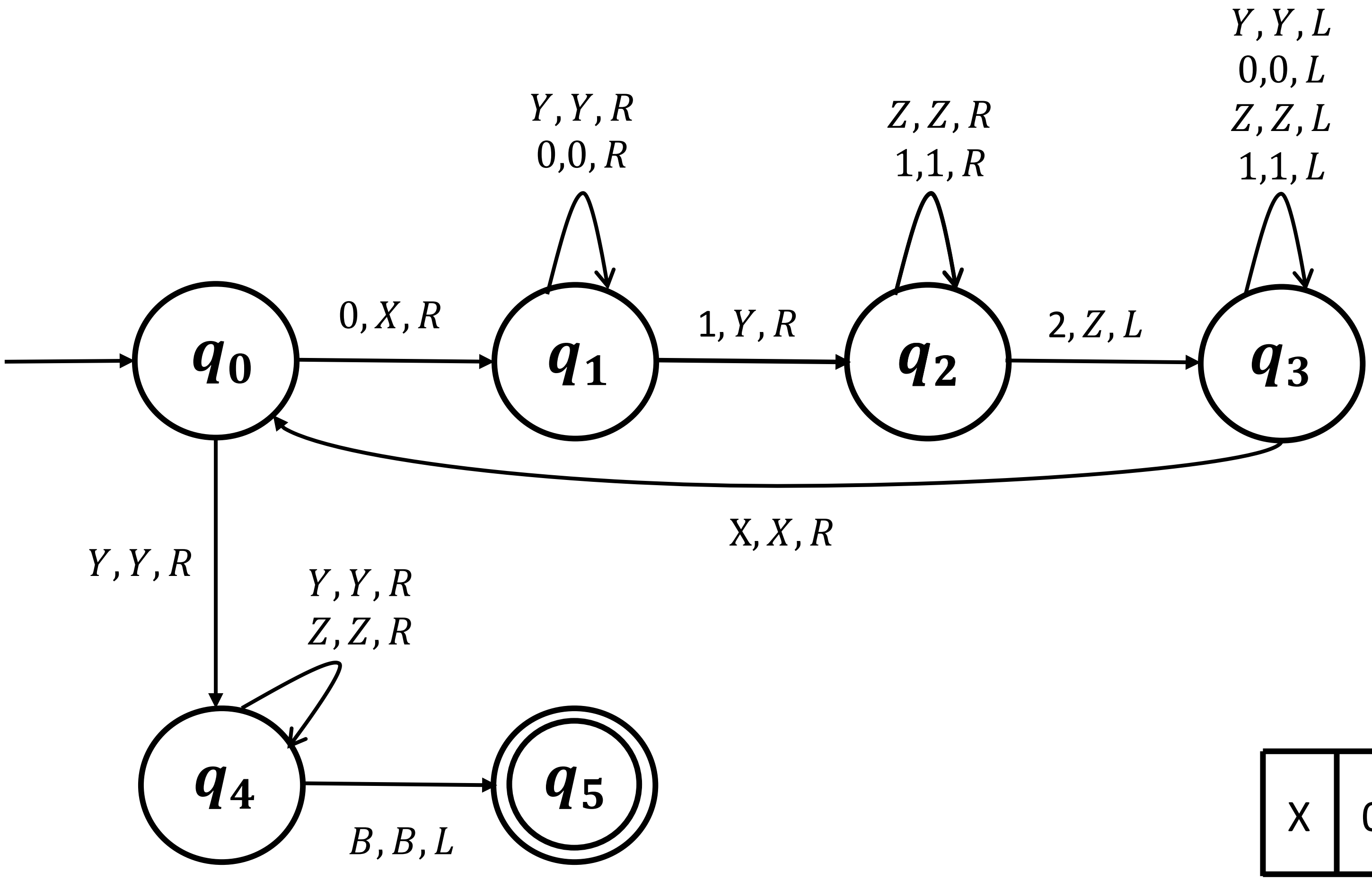

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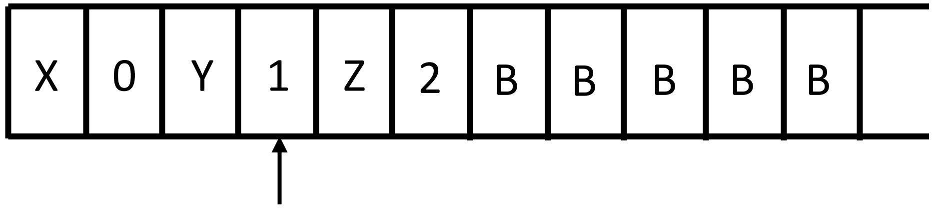
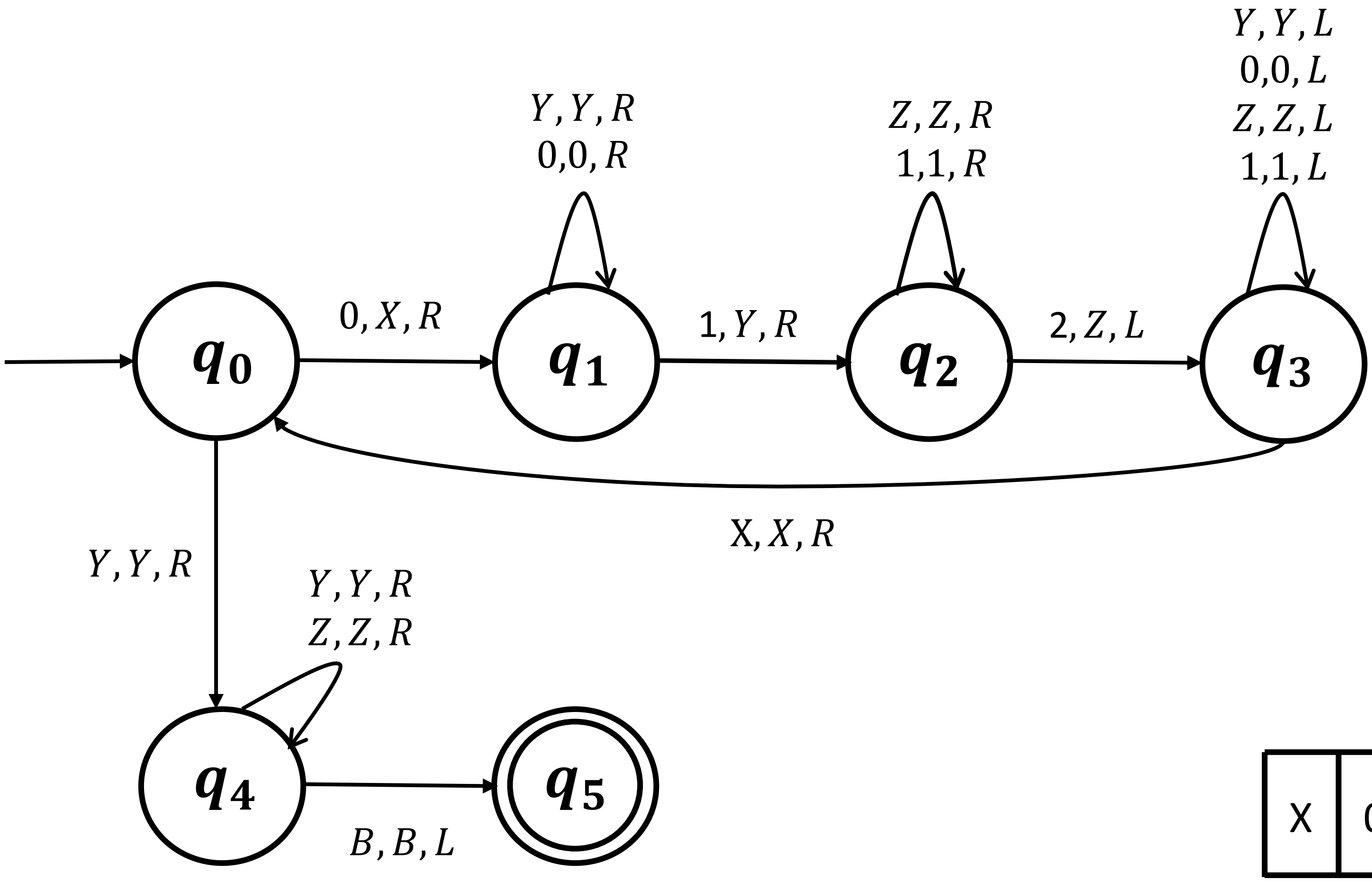


X	0	Y	1	Z	2	B	B	B	B	B

Turing Machines

Example: Let $L = \{0^n 1^n 2^n | n \geq 1\}$



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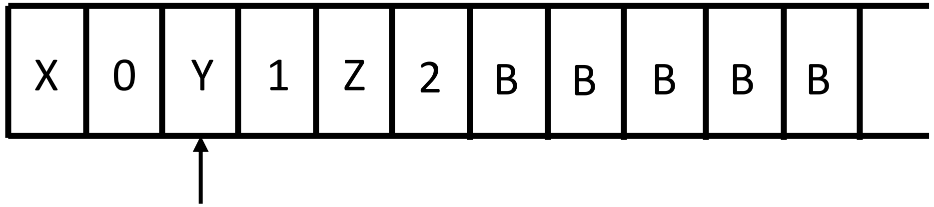

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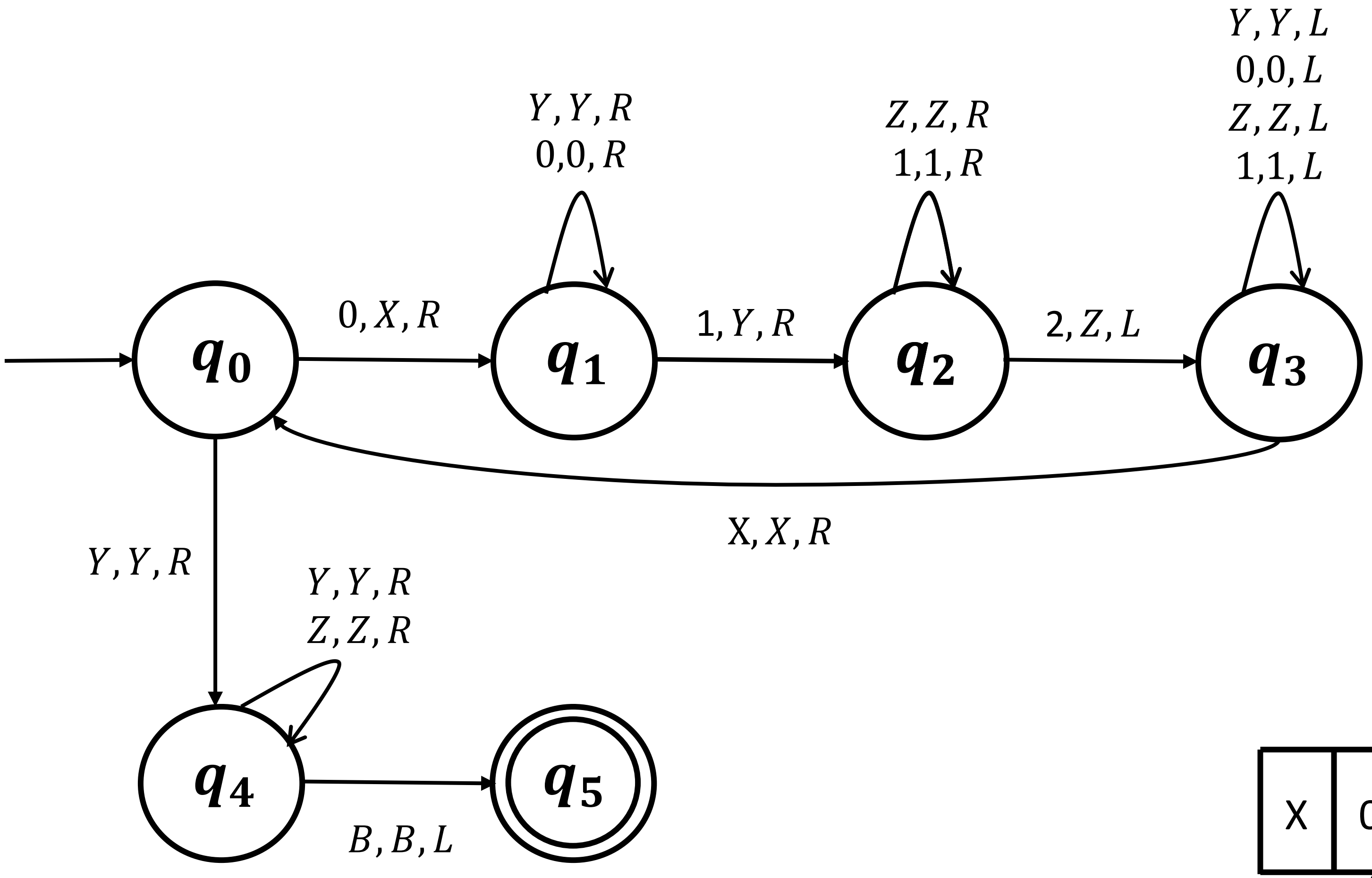

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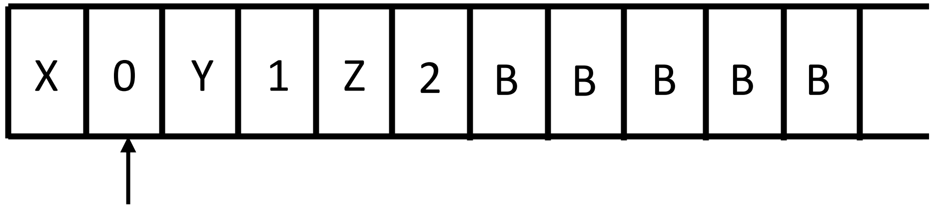

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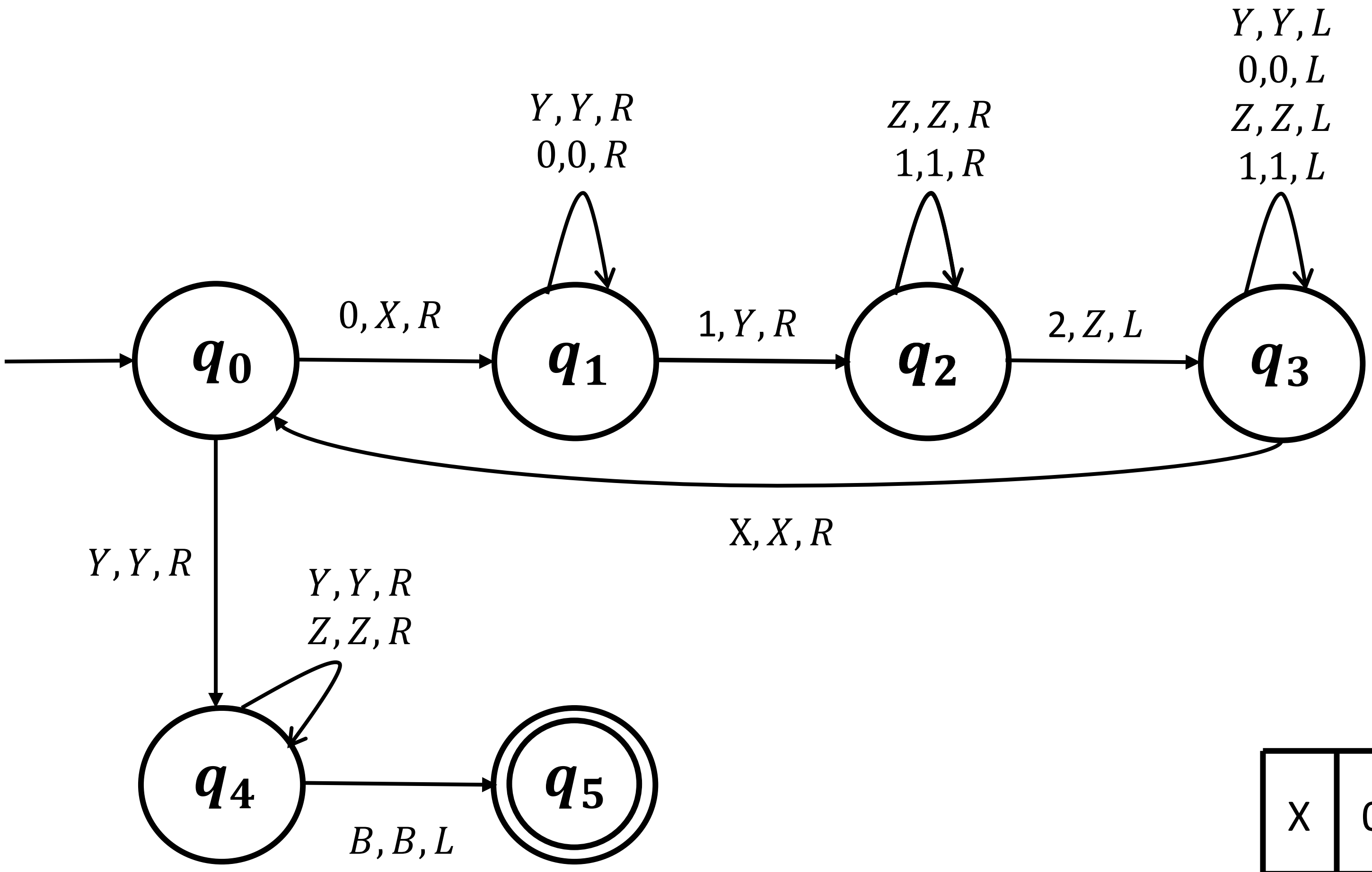

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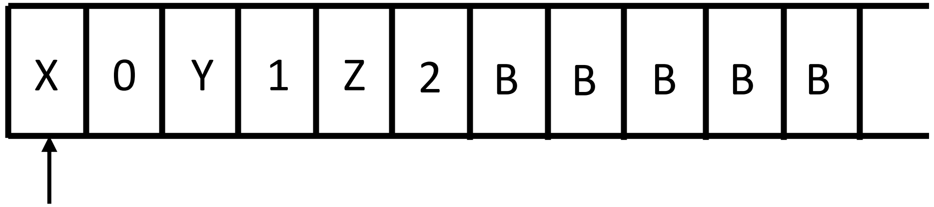
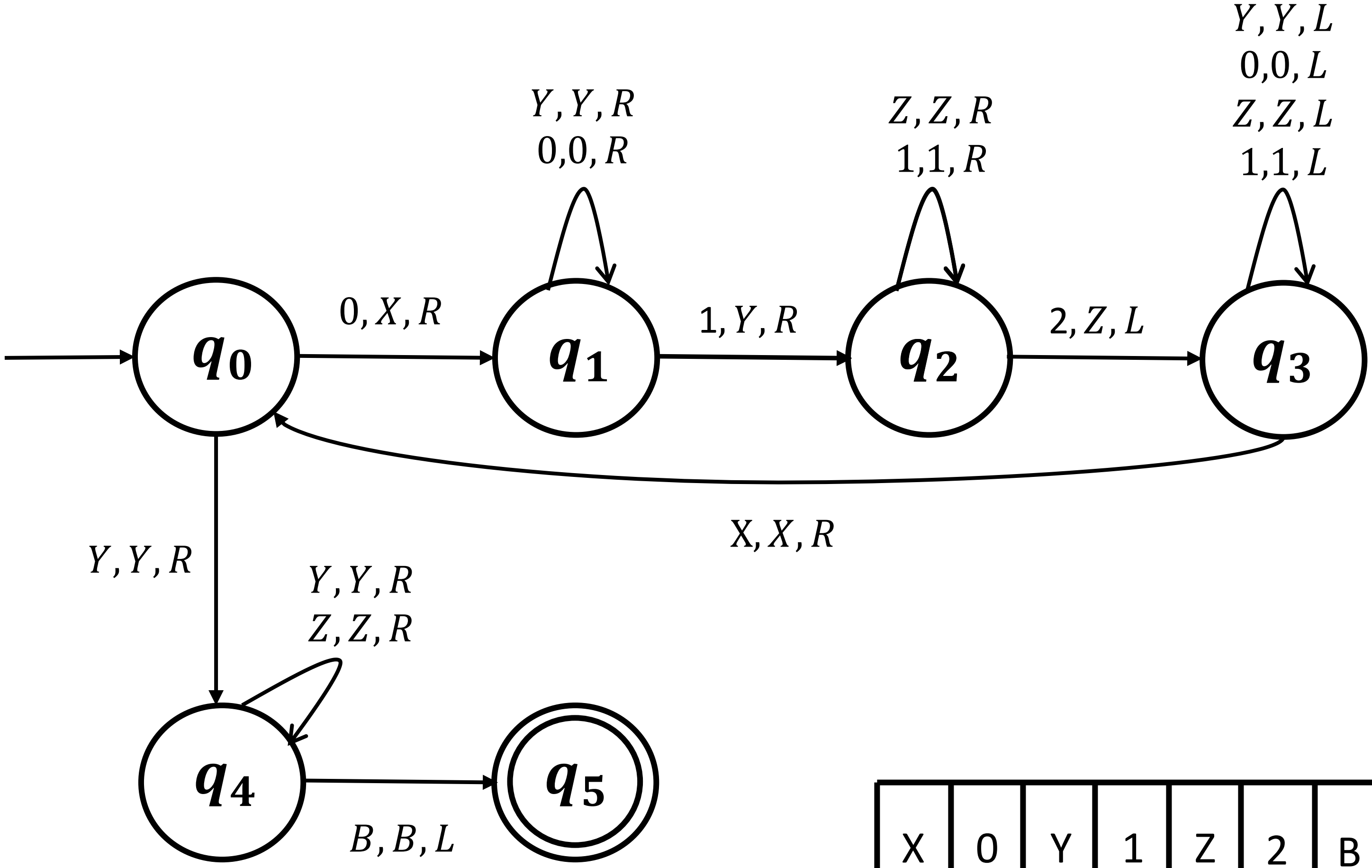


X	0	Y	1	Z	2	B	B	B	B	B

Turing Machines

Example: Let $L = \{0^n 1^n 2^n | n \geq 1\}$



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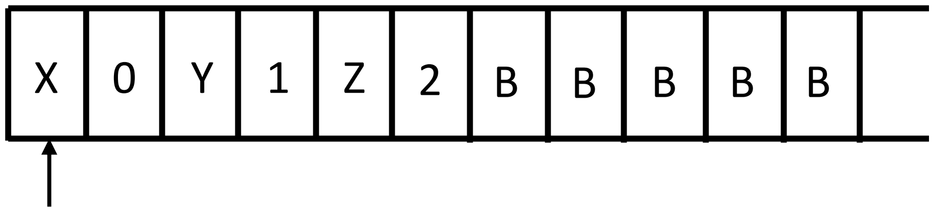

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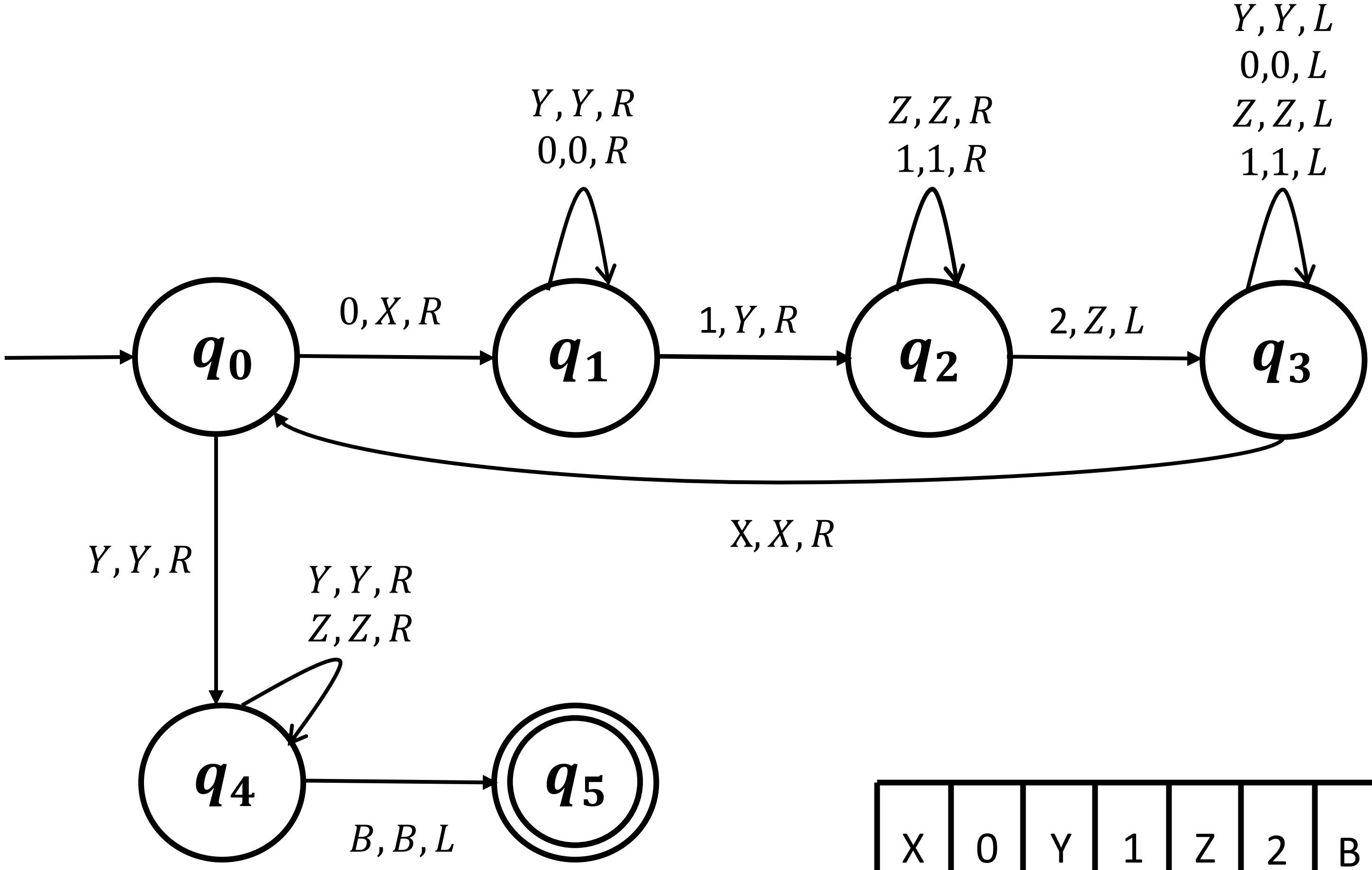

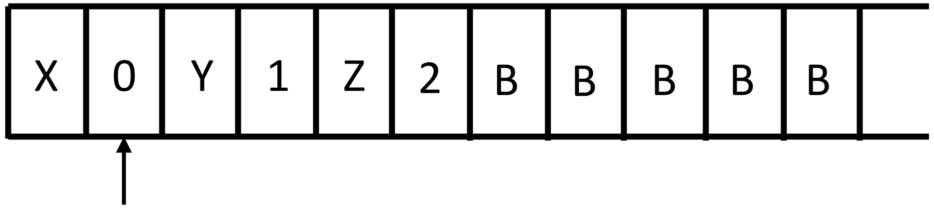

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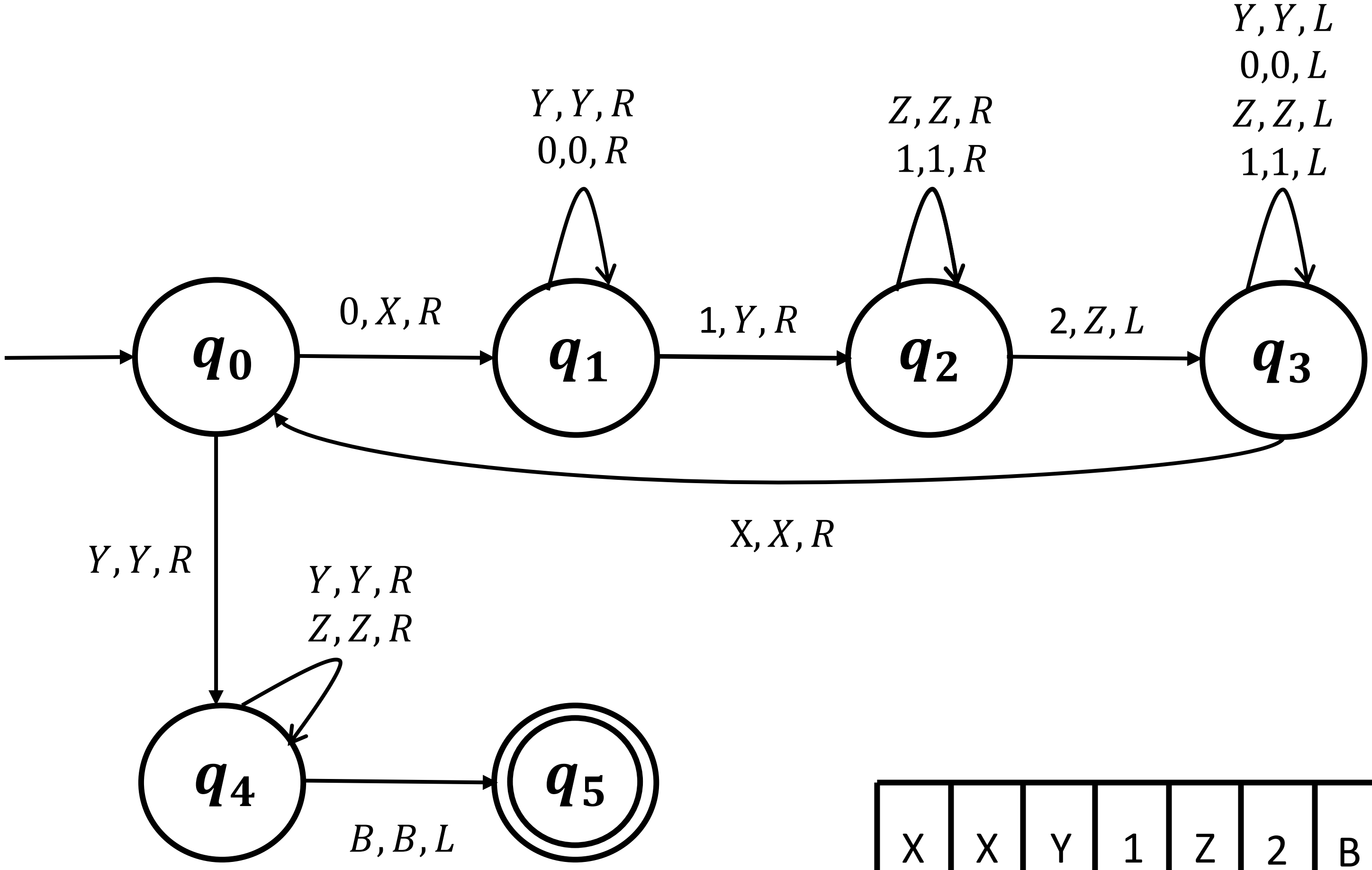

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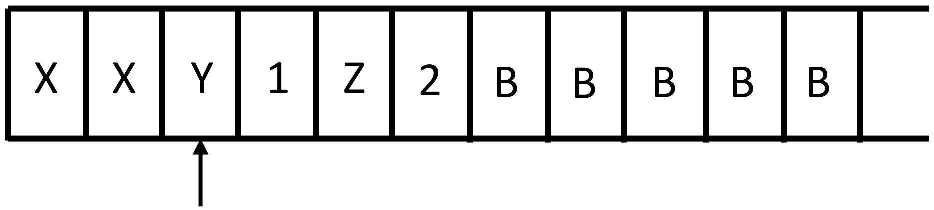

X	X	Y	1	Z	2	B	B	B	B	B

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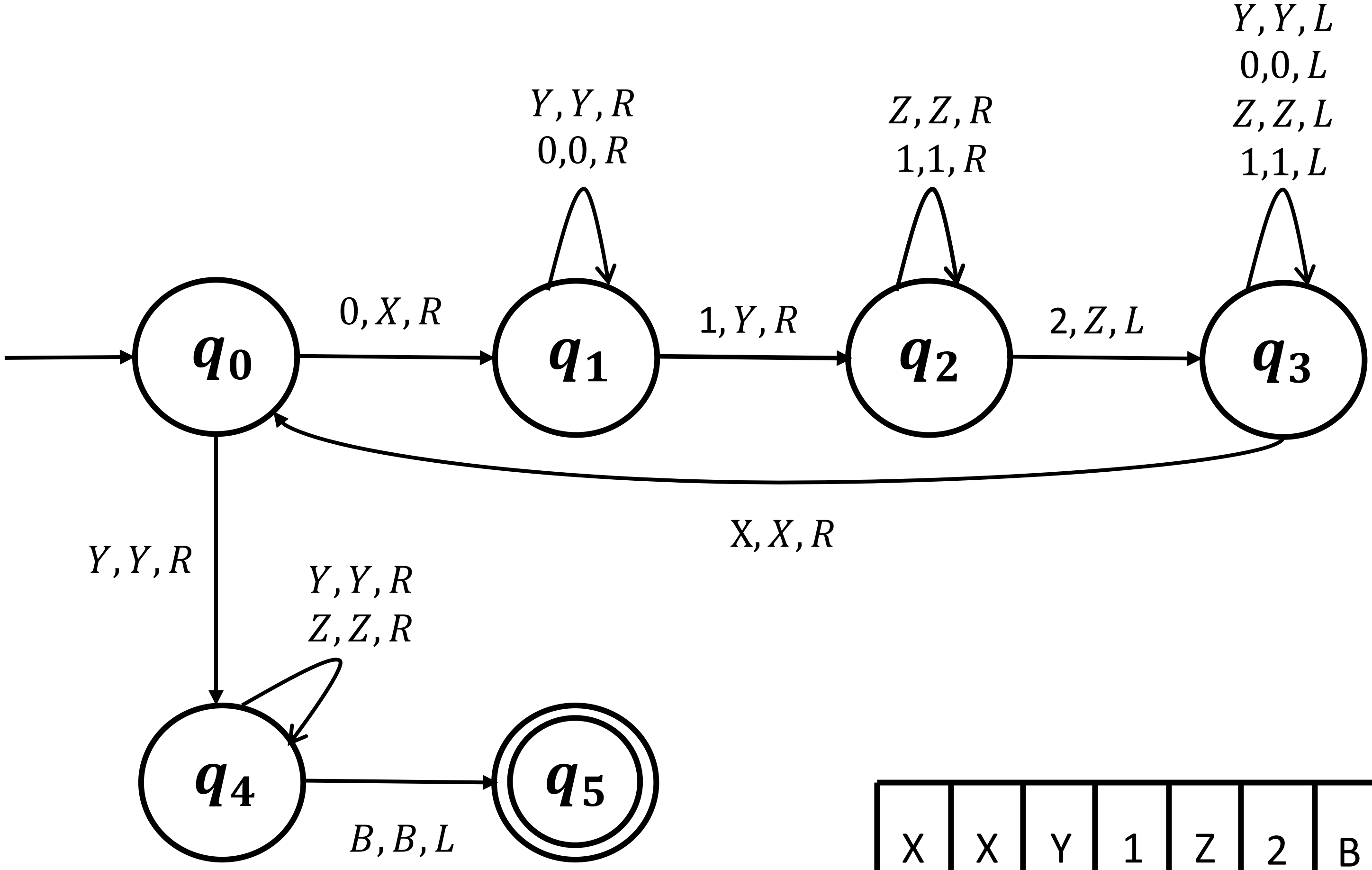

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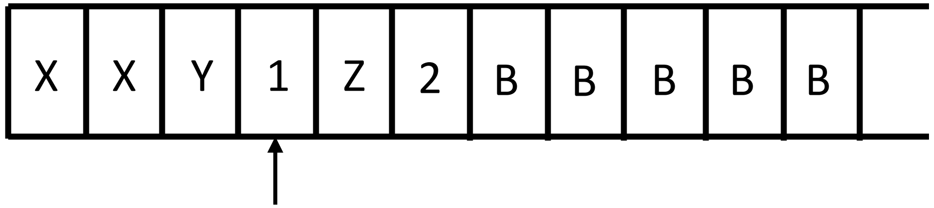

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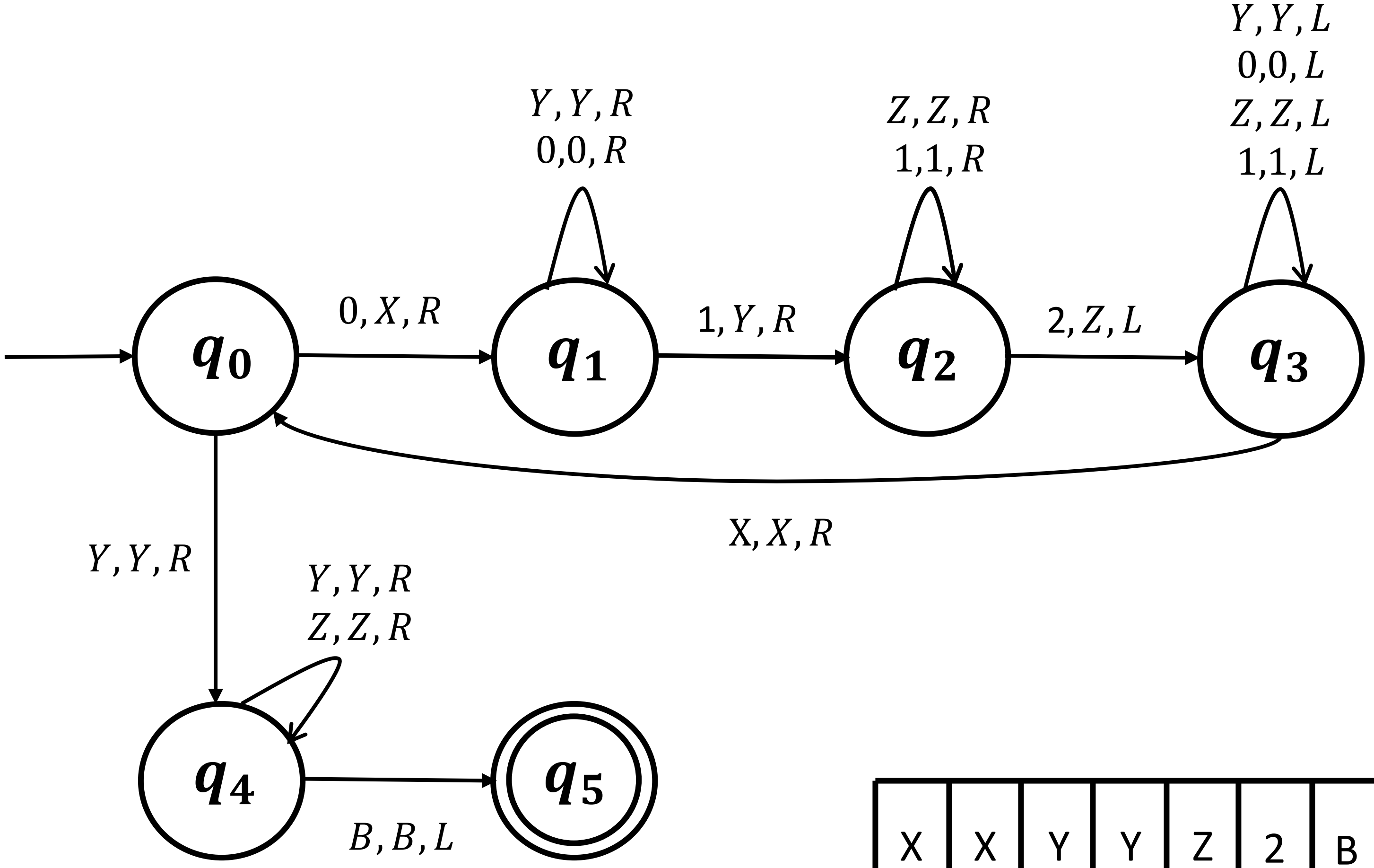

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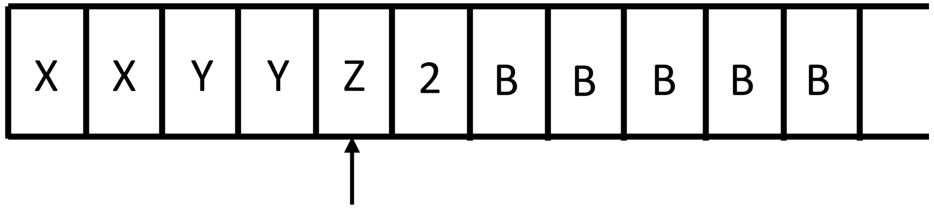

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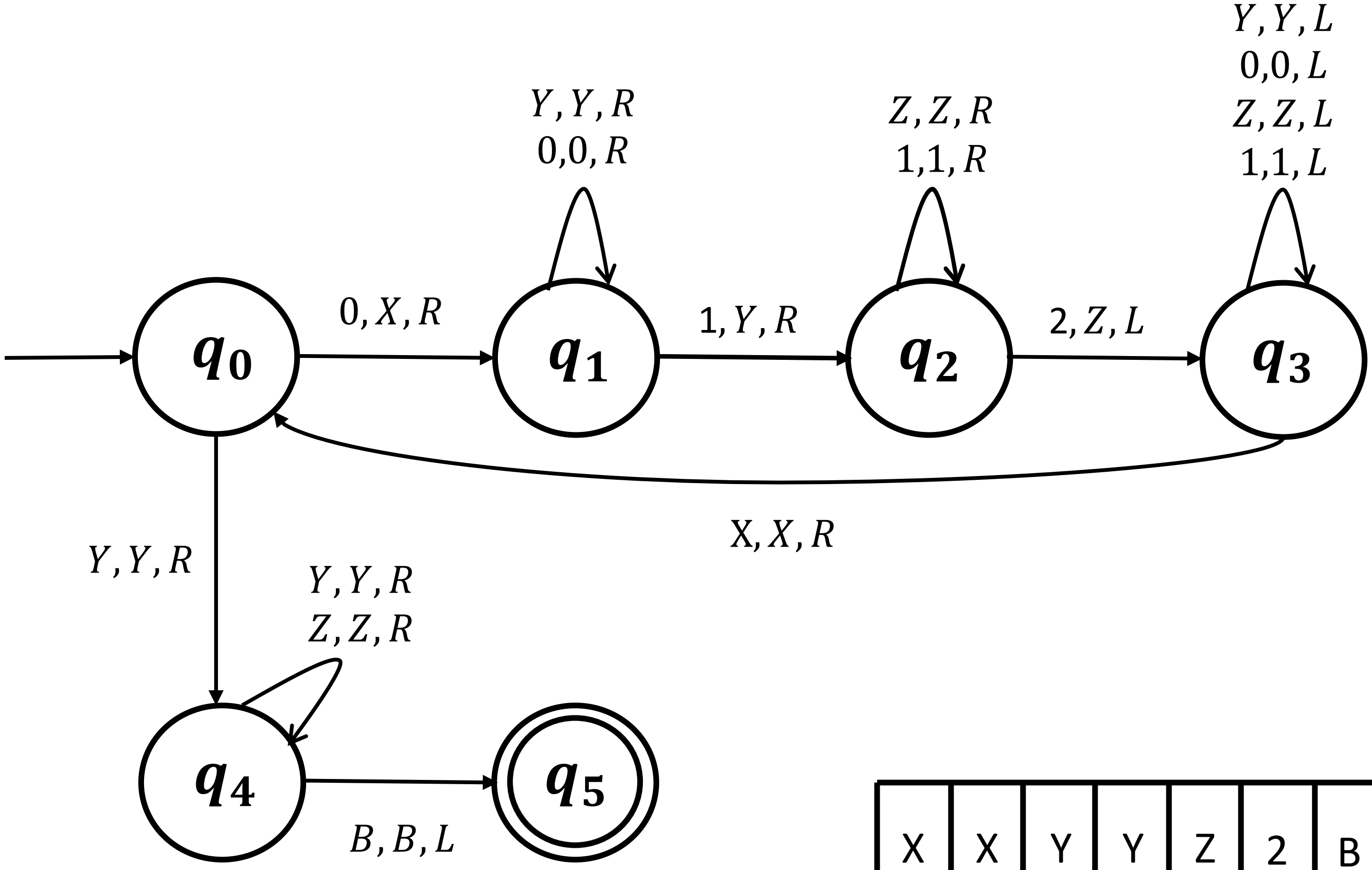

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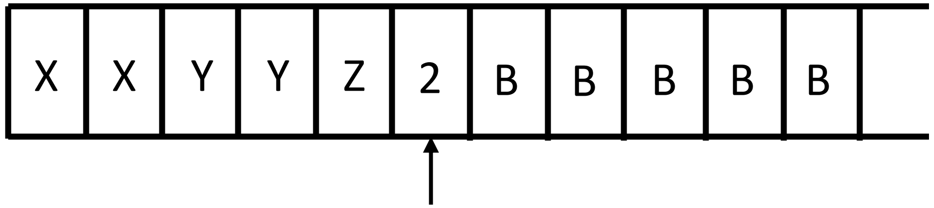

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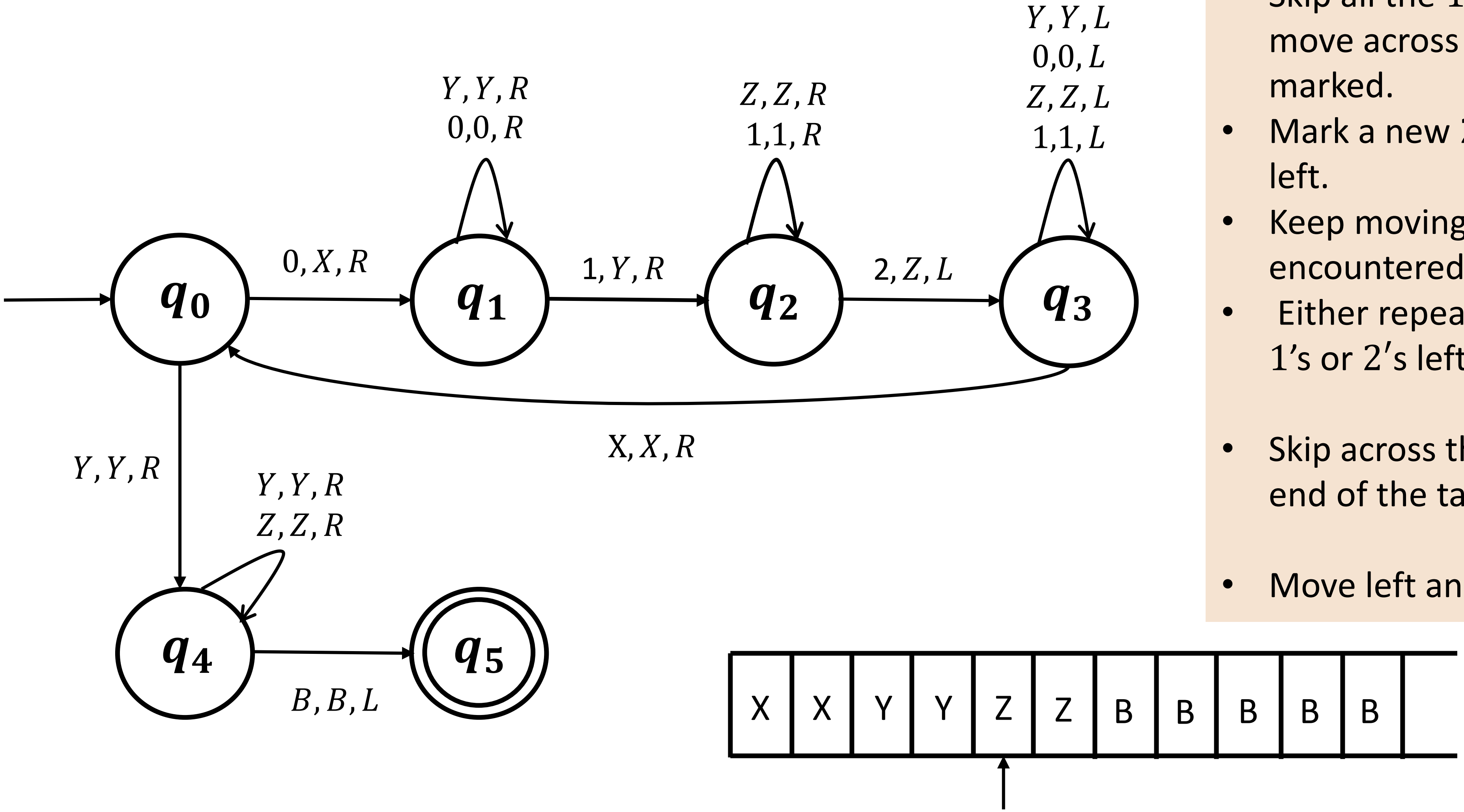

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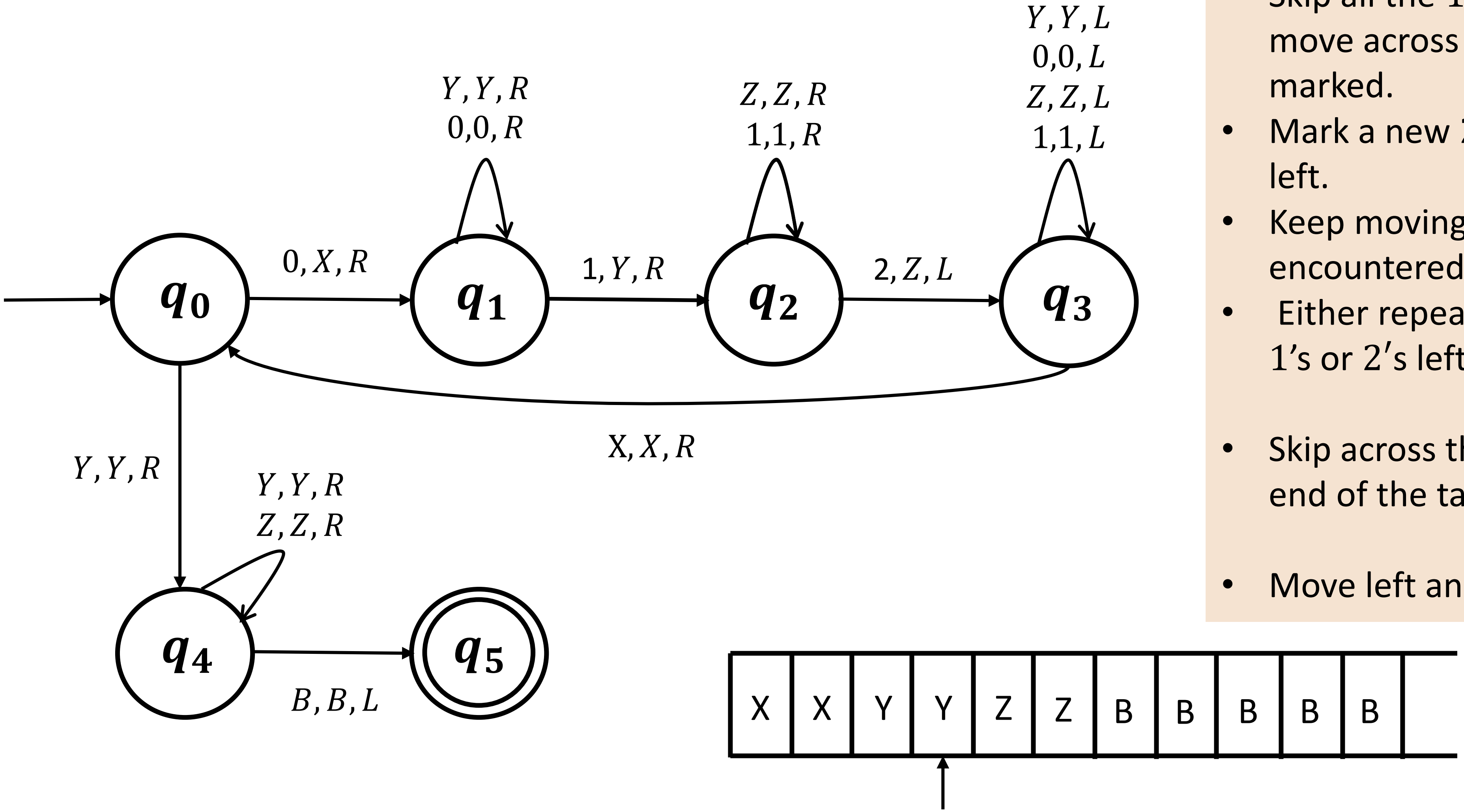

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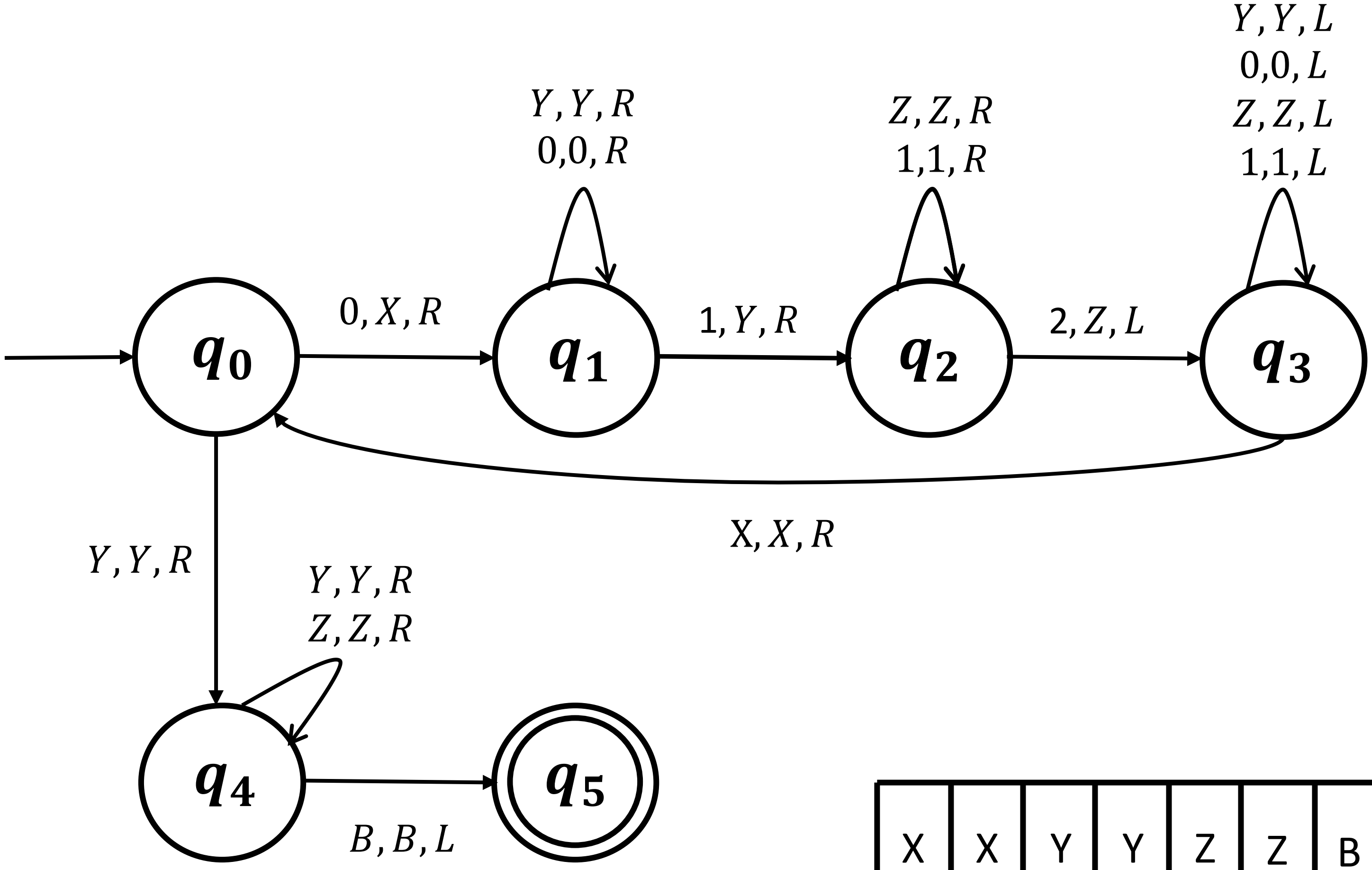

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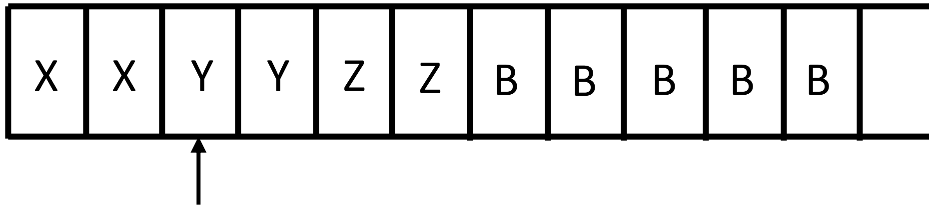

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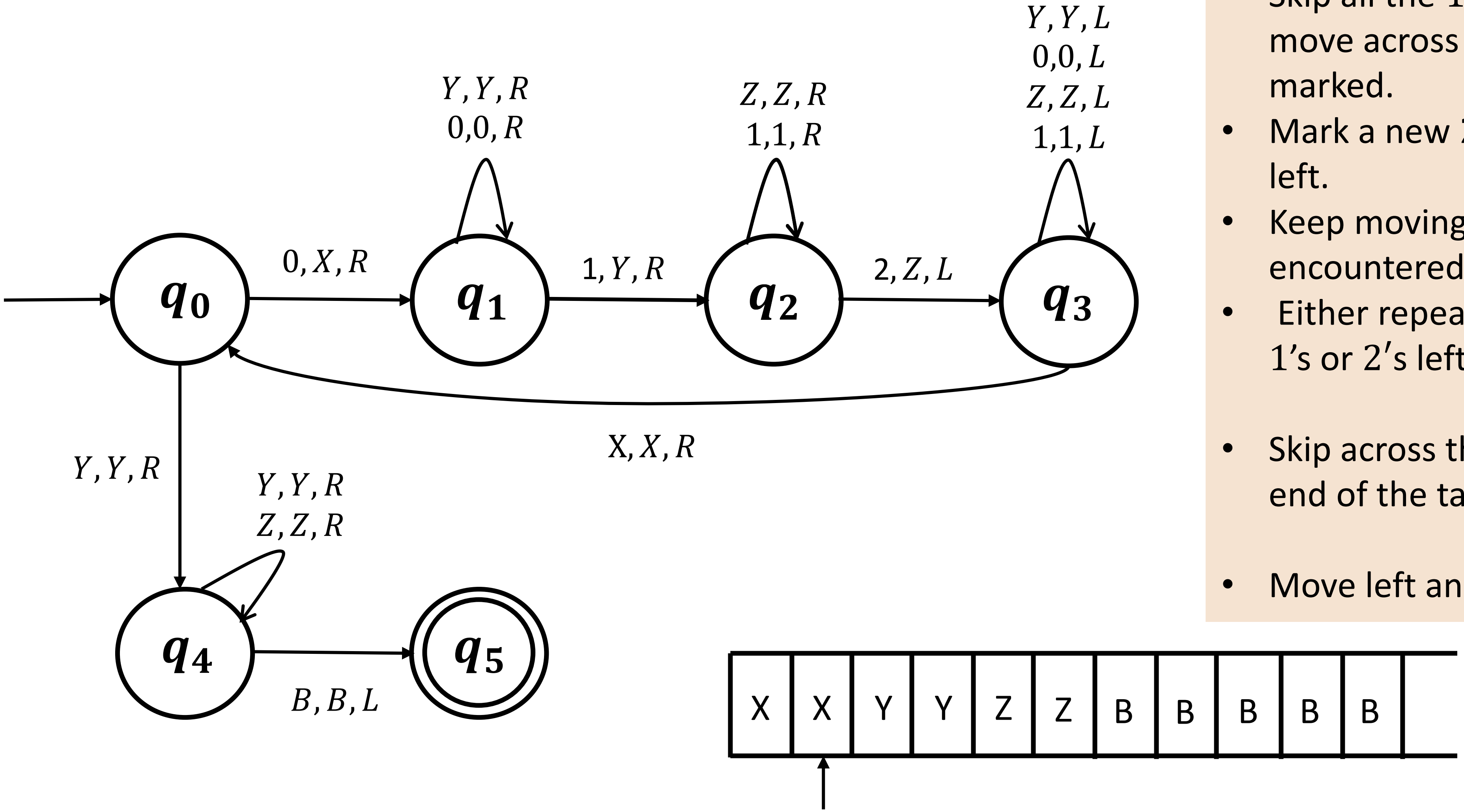

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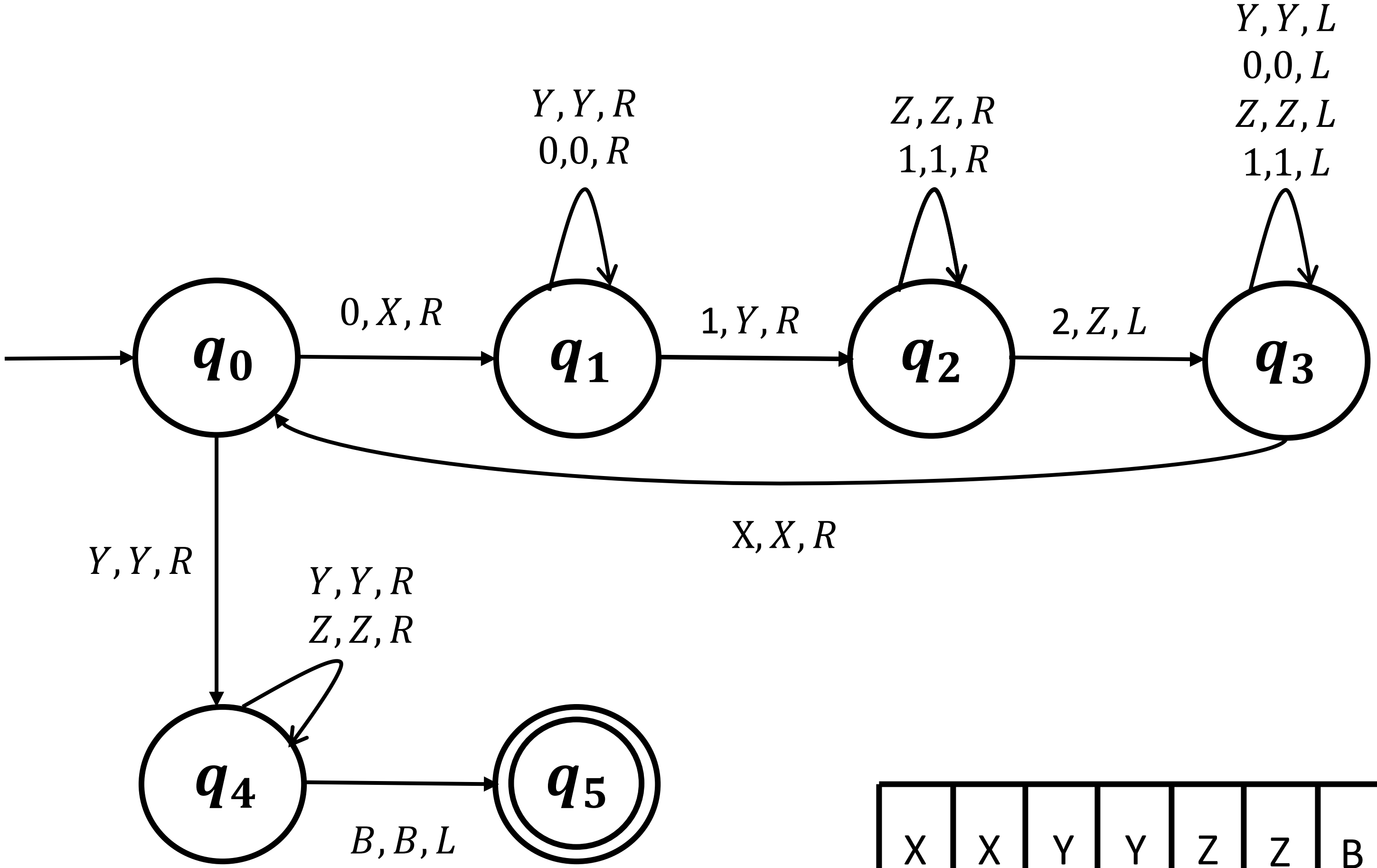

X	X	Y	Y	Z	Z	B	B	B	B	B

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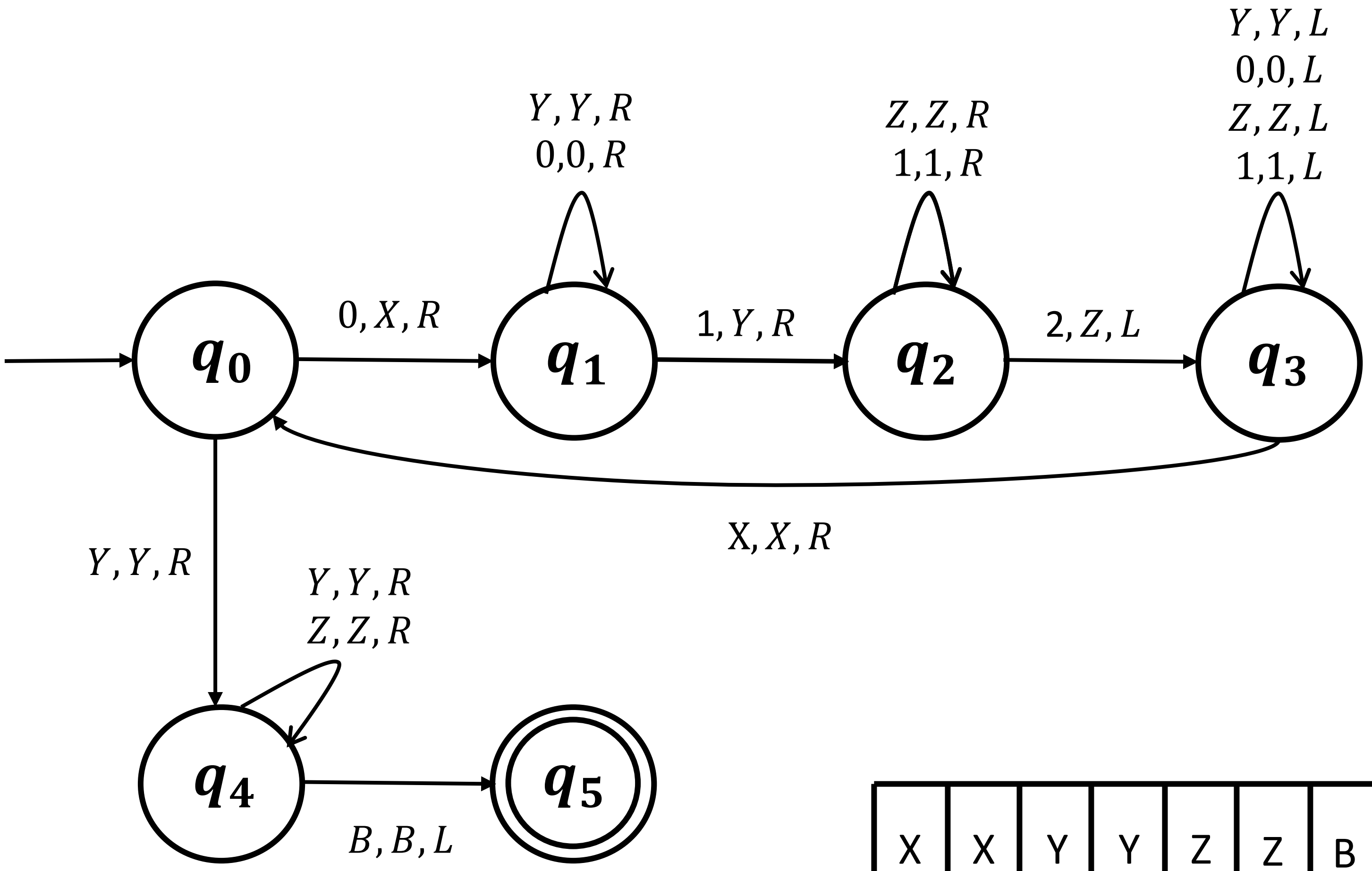

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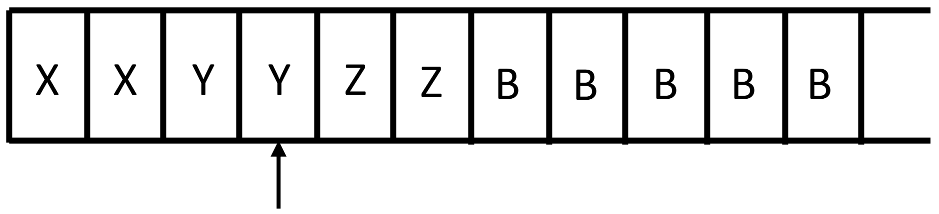

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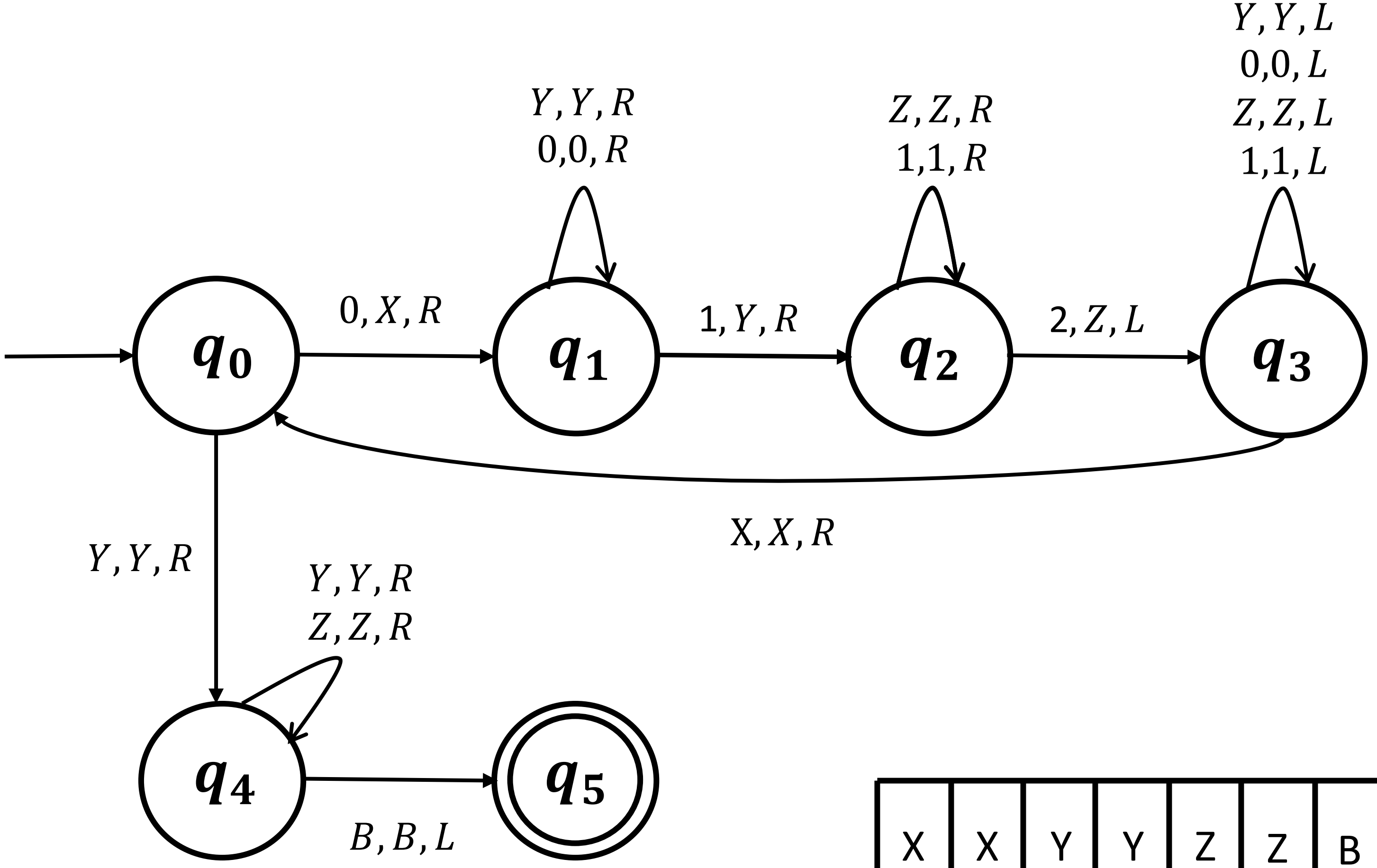

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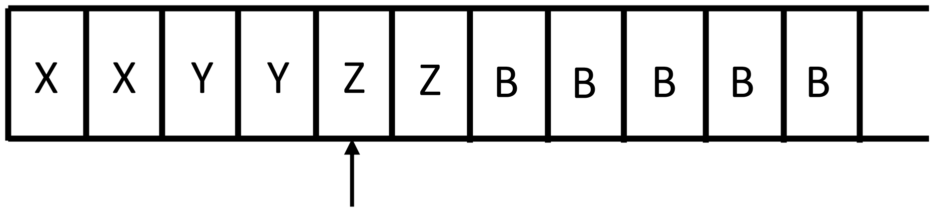

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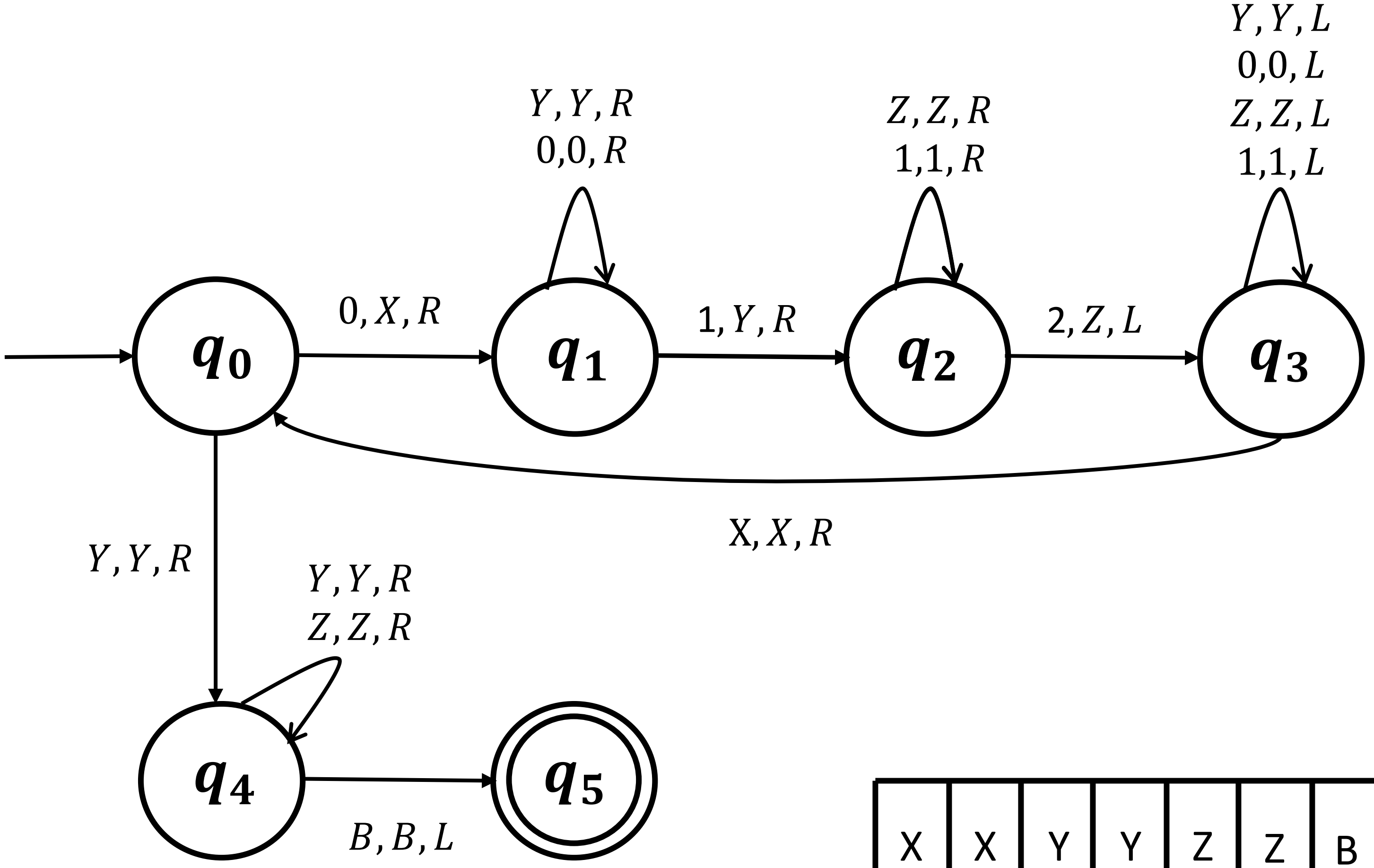

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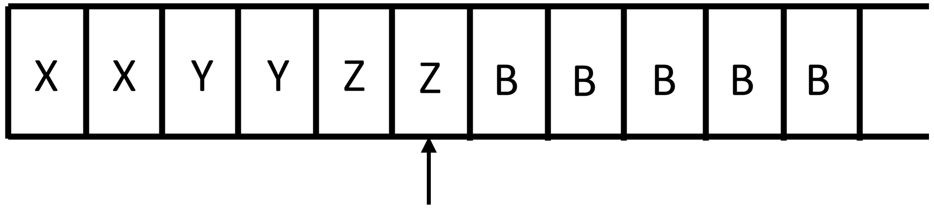

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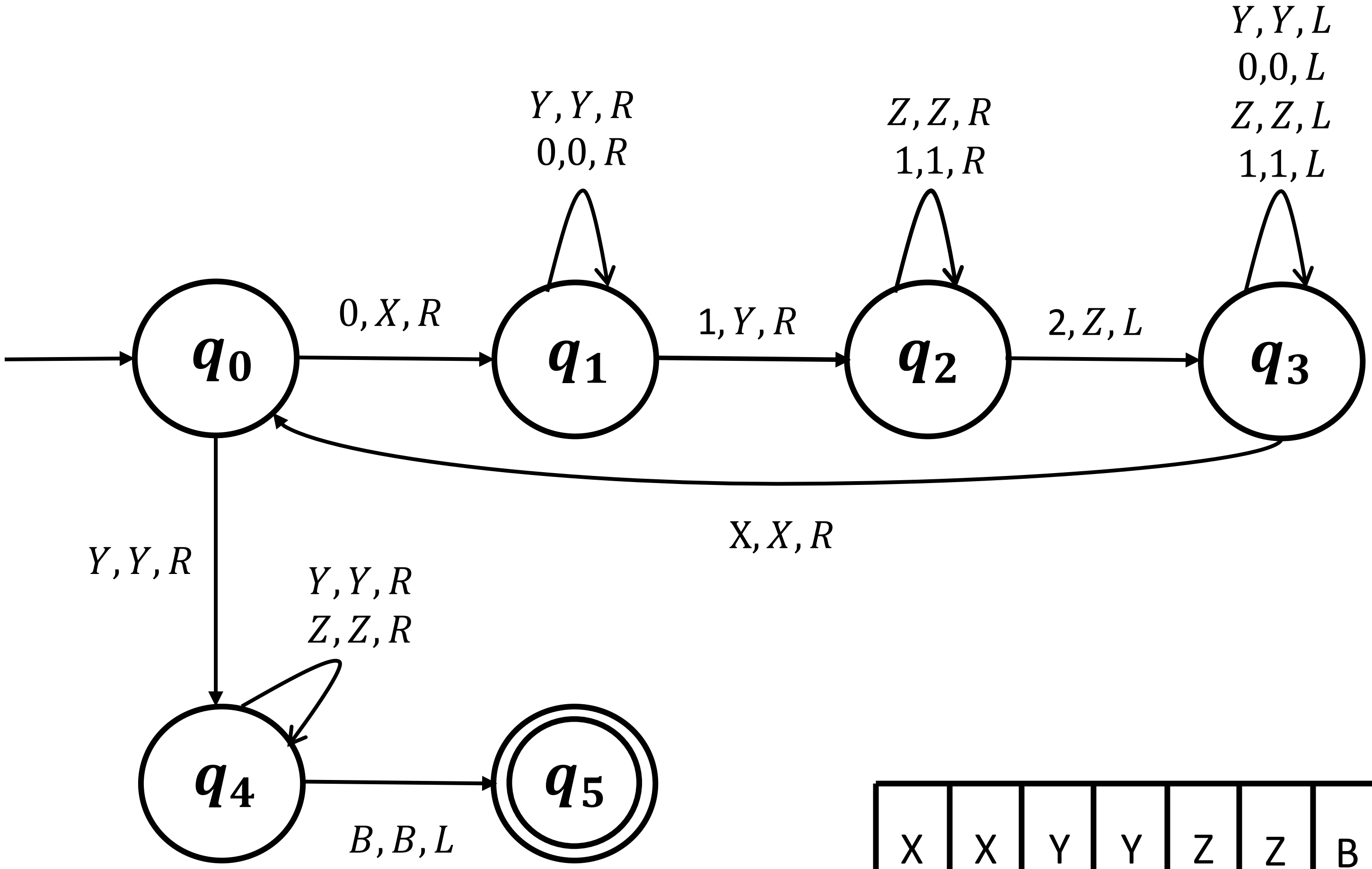

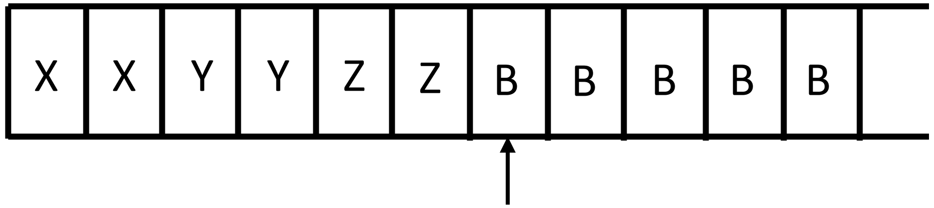

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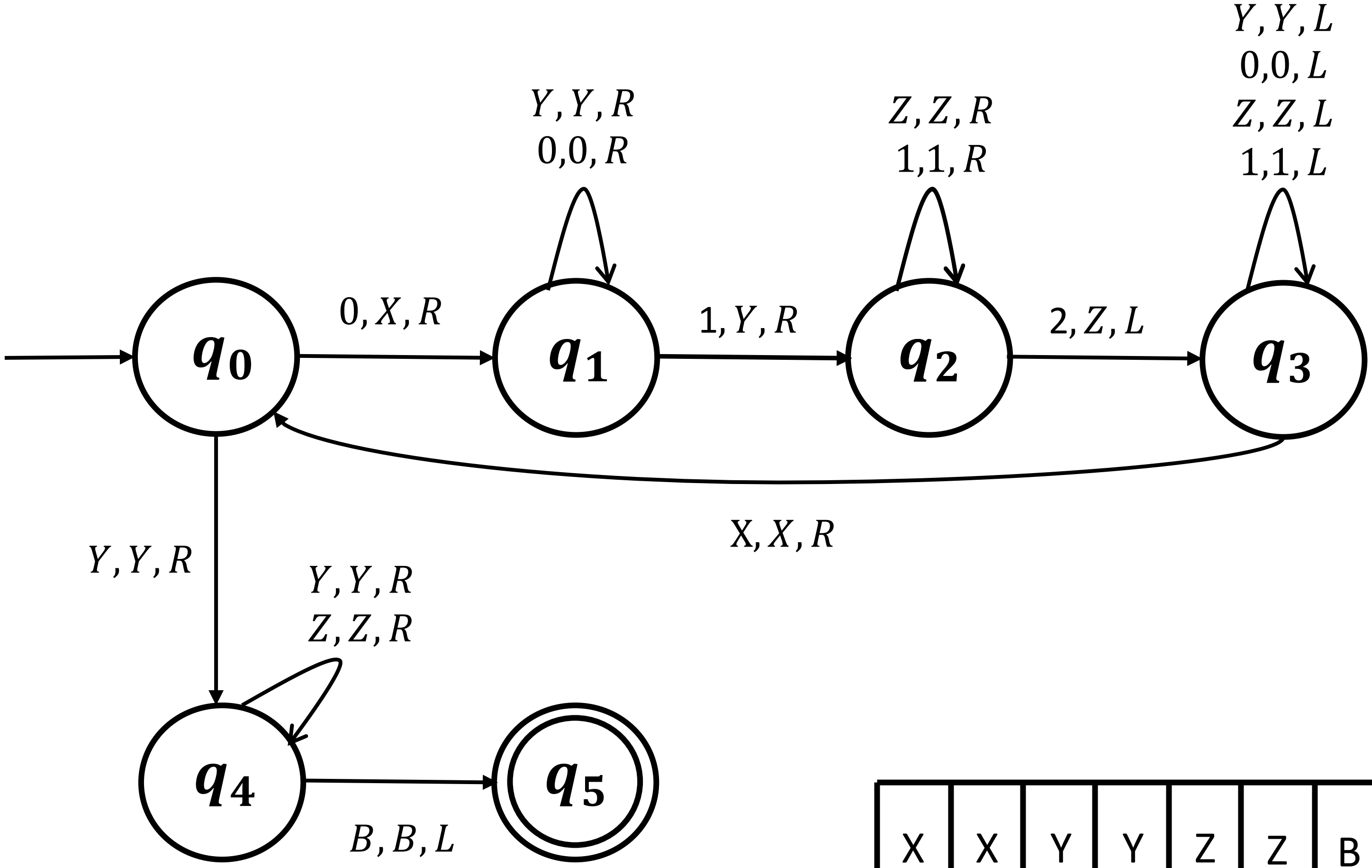

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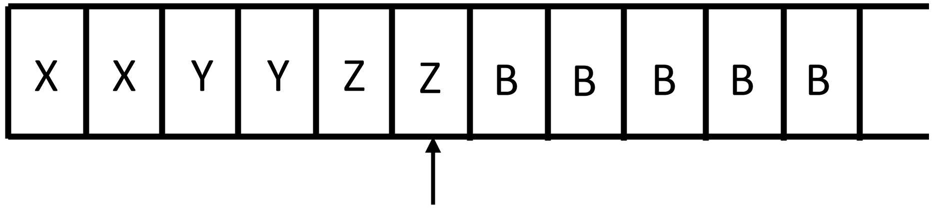

X	X	Y	Y	Z	Z	B	B	B	B	B

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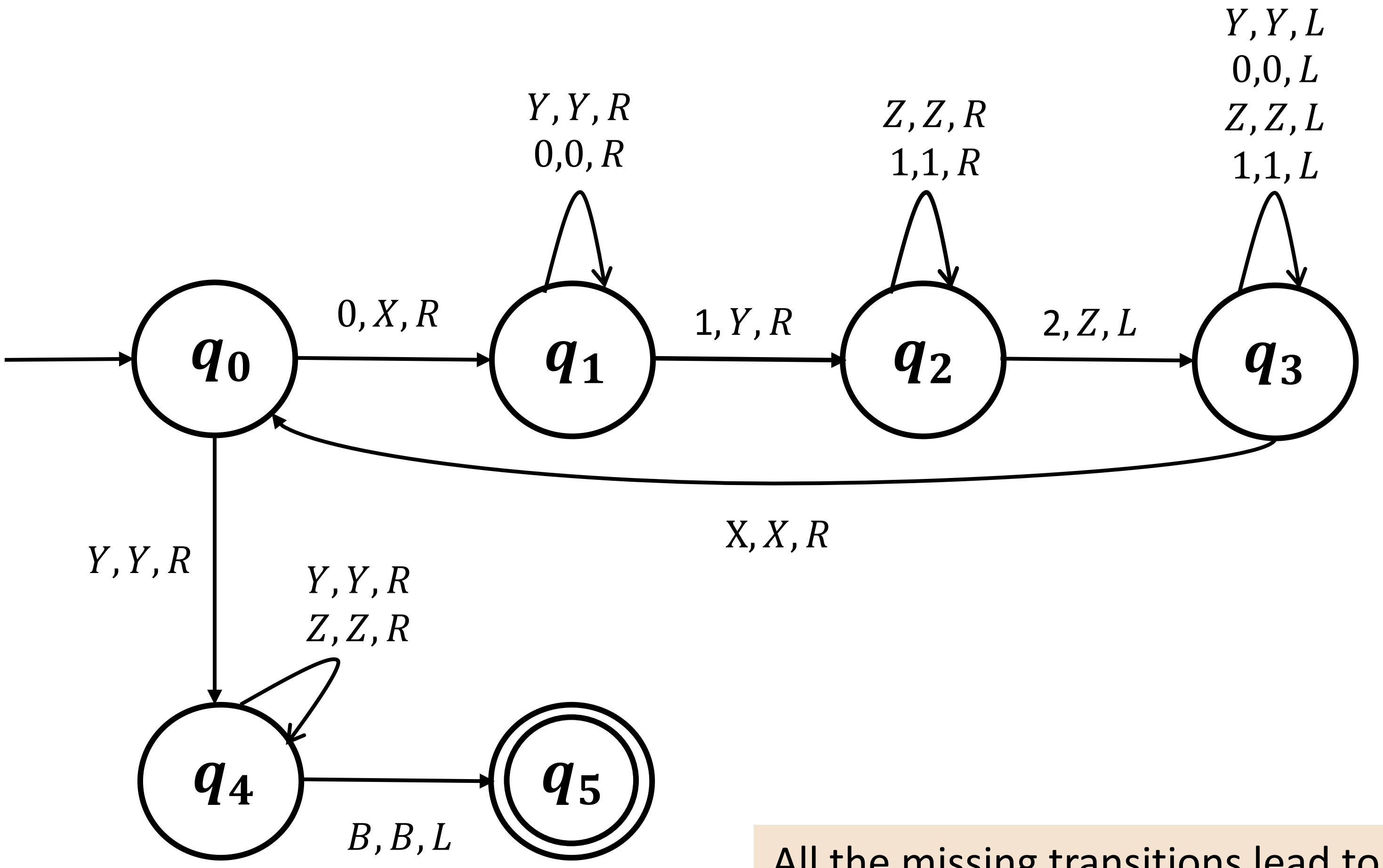

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All the missing transitions lead to a reject state and so any input not of the form $\{0^n 1^n 2^n\}$ is rejected.

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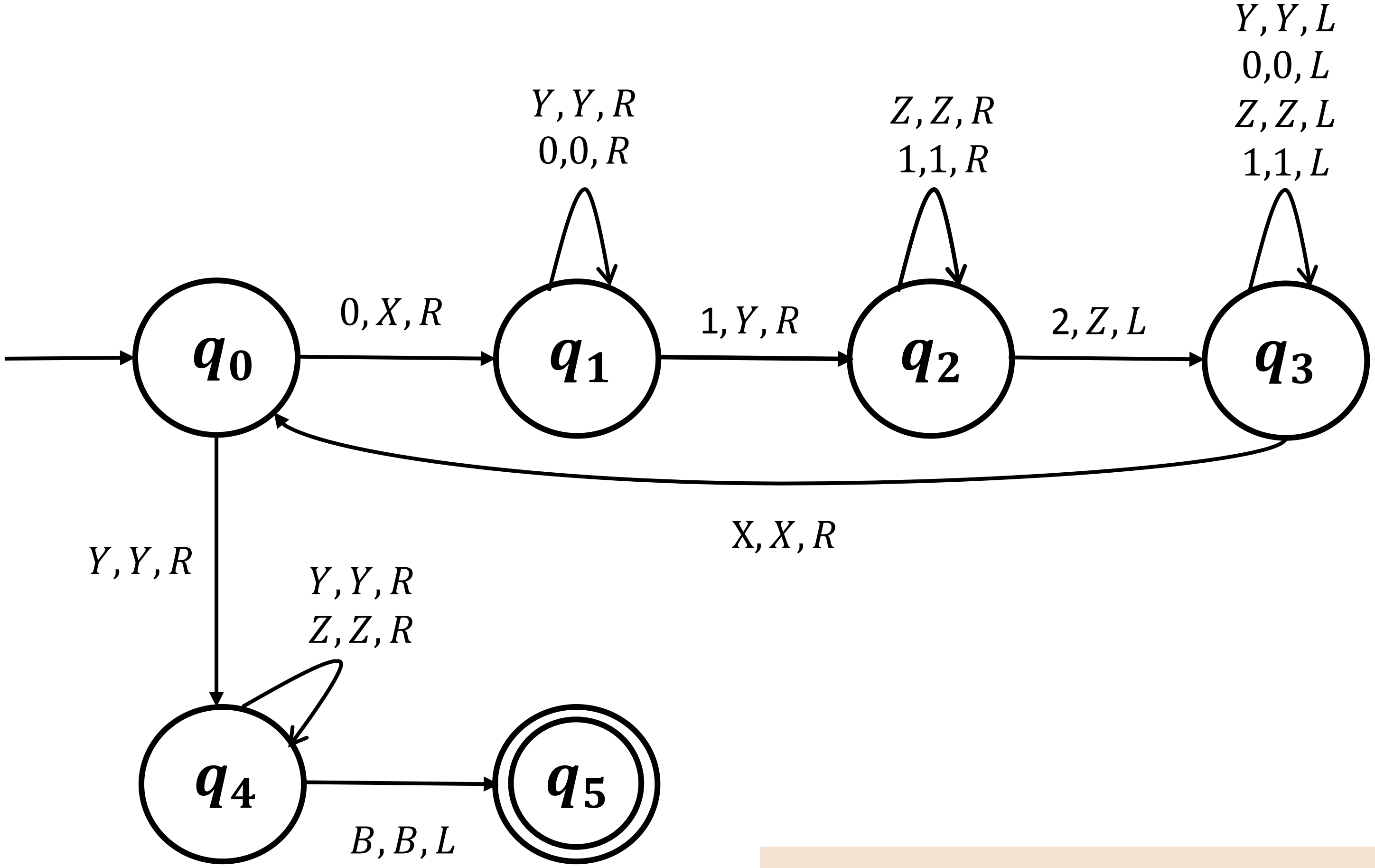

- Continue to go right to mark the next 2 with a Z.
- Skip all the $1'$s and move right/ Also, move across (to the right) the Z's already marked.
- Mark a new 2 with a Z and start moving left.
- Keep moving left until an X is encountered.
- Either repeat this process if there are $0'$s, 1's or $2'$s left to mark

or

- Skip across the Y's and Z's to the right end of the tape until a blank is found.

- Move left and accept the input string.

$CFL \subseteq Language\ recognized\ by\ TM$

Turing Machines

Formally, a Turing Machine is a 7-tuple $(\mathrm{Q}, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where

- Q is a finite set called the ***states.***
- Σ is the set of input ***alphabets*** not containing the blank symbol B**.**
- Γ is the ***tape alphabet,*** where $B \subseteq \Gamma$ and $\Sigma \subseteq \Gamma$**.**
- $\delta: Q \times \Gamma \mapsto Q \times \Gamma \times \{L, R\}$ is the ***transition function***
- $q_0 \in Q$ is the ***start state***.
- $q_{accept} \in Q$ is the ***accepting state***.
- $q_{reject} \in Q - \{q_{accept}\}$ is the ***reject state.***

Configuration of a TM: Combination of the current state, the current tape contents and the current head location.

Formally, it is a triple: $(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{x})$, where $\boldsymbol{q} \in Q, \boldsymbol{a} \in \Gamma, \boldsymbol{x} \in Z_+$

At each step, the Turing machine configuration changes. We say C_1 **yields** C_2 if the TM changes from C_1 to C_2 in one step.

Turing Machines

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A TM $\boldsymbol{M}$ **accepts** $\boldsymbol{w}$ if there exists a sequence of configurations C_1 to C_k, where

- C_1 is the start configuration M on w.
- Each C_i yields C_{i+1}.
- C_k is an accepting configuration

Language recognized a TM M:

$$\boldsymbol{L(M)} = \{\boldsymbol{w} | \boldsymbol{M} \textbf{ accepts } \boldsymbol{w}\}$$

Configuration of a TM: Combination of the current state, the current tape contents and the current head location.

$$X\ 0\ 0\ \underset{\uparrow\, q_1}{1}\ 1\ 1\ B\ B\ B\ B\ \ldots$$

At each step, the Turing machine configuration changes. We say C_1 **yields** C_2 if the TM changes from C_1 to C_2 in one step.

Start configuration:

$$\underset{\uparrow\, q_0}{0}\ 0\ 0\ 1\ 1\ 1\ B\ B\ B\ B\ \ldots$$

Accept configuration:

$$\mathrm{X}\ X\ X\ Y\ Y\ \underset{\uparrow\, q_{accept}}{Y}\ B\ B\ B\ B\ \ldots$$

Reject configuration:

$$X\ X\ X\ Y\ Y\ \underset{\uparrow\, q_{reject}}{0}\ B\ B\ B\ B\ \ldots$$

Turing Machines

Formally, a Turing Machine is a 7-tuple $(\mathrm{Q}, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where

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Configuration of a TM:

- Combination of the current state, the current tape contents and the current head location.
- At each step, the Turing machine configuration changes. We say C_1 **yields** C_2 if the TM changes from C_1 to C_2 in one step.

Next Lecture

Various TM model **variants**: Robustness of the standard TM model

Thank You!