

Automata Doubt Clarification

Thm:

If G is a Context free Grammar in CNF, then for any string w belongs $L(G)$ of length $n \geq 1$, it requires exactly $2n-1$ steps to make any derivation of w .

Proof: In CNF we only have production rules of the form

$$\text{Var} \rightarrow \text{Var Var} \quad (\text{I})$$

$$(\text{or}) \quad \text{Var} \rightarrow \text{Terminal} \quad (\text{II})$$

Suppose w is a string of length n , such that $w \in L(G)$
this means w contains n terminals.

\Rightarrow Production rule II must be used exactly n times in total.

Each time we use production rule II, we convert 1 variable to a terminal.

\therefore In order to apply II n times, we need n variables. └ ①

Initially, we start with 1 variable, S , the start variable.
Each time we apply a production rule of type I, we increment the number of variables by 1.

\therefore In order to obtain n variables, (because we need n variables ①)
We need to apply I $n-1$ times.

\therefore In total, we apply

$$\begin{array}{l} \text{type I : } n-1 \text{ times} \\ \text{type II : } n \text{ times} \end{array} = 2n-1 \text{ steps in total}$$

□