Probability and Statistics

Tutorial 5

Q1: Consider 2 Random Variables X and Y whose joint distribution is given by:

$$p_{XY}(x, y) = \frac{1}{2^{x+y}}$$

for $x, y = 1, 2, 3, \ldots$

- Find the marginal distribution of X and Y. Are X and Y independent?
- Find $\mathbb{P}(X^2 + Y \leq 10)$

Q2: Let X and Y be two independent $N(0, 1)$ random variables, and define:

$$Z = 1 + X + XY^2$$

$$W = 1 + X$$

Find $\text{Cov}(Z, W)$.

Q3: Let X and Y be jointly continuous random variables with joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 6e^{-(2x+3y)}, & x, y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Are X and Y independent?

(2) Find $\mathbb{E}[Y \mid X > 2]$.

Q4: A producer produces b items with probability p and nothing with probability $1 - p$. A consumer attempts to take $T \sim \text{Exponential}(\lambda)$ items. The actual number of items consumed is

$$C = \begin{cases} 0, & \text{if the producer produces nothing,} \\ \min(T, b), & \text{if the producer produces } b \text{ items.} \end{cases}$$

(a) Derive the distribution of C (identify its discrete and continuous parts).

(b) Compute the expected number of items consumed, $\mathbb{E}[C]$.

Q5: Let X and Y be two independent $\text{Uniform}(0, 1)$ random variables, and define:

$$Z = \frac{X}{Y}$$

(a) Find CDF of Z.

(b) Find PDF of Z.

Q6: Consider two random variables X and Y with joint PMF given below

(a) Find $\mathbb{P}(X \leq 2, Y \leq 4)$.

(b) Find the marginal PMFs of X and Y.

(c) Find $\mathbb{P}(Y = 2|X = 1)$.

(d) Are X and Y independent?

	Y		
	$Y = 2$	$Y = 4$	$Y = 5$
$X = 1$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$
$X = 2$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{8}$
$X = 3$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$

Q7: Let X and Y be two independent Uniform(0,2) random variables. Find $\mathbb{P}(\{XY < 1\})$.

Q8: Let X and Y be random variables that have the joint pdf

$$f_{X,Y}(x, y) = x + y \text{ for } 0 \leq x \leq 1, 0 \leq y \leq 1$$

What is $\mathbb{P}(X > Y|X < 1/2)$?