Towards $E[g(X, Y)]$

- What about $E[aX + bY + c]$?

$$\begin{aligned}
E[aX + bY + c] &= \sum_{x,y}(ax + by + c)p_{XY}(x, y) \\
&= a\sum_{xy} xp_{XY}(x, y) + b\sum_{xy} yp_{XY}(x, y) \\
&+ c\sum_{xy} p_{XY}(x, y) \\
&= aE[X] + bE[Y] + c.
\end{aligned}$$

- Along similar lines, one would expect:

$$\boxed{E[g(X, Y)] = \sum_x \sum_y g(x, y)p_{XY}(x, y)}$$

Finding $p_Z(\cdot)$ where $Z = g(X, Y)$.

- Suppose $Z = g(X)$. Then what is $p_Z(z)$?
- $p_Z(z) = \sum_{\{x : g(x) = z\}} p_X(x)$.
- Now suppose $Z = g(X, Y)$ then we have

$$p_Z(z) = \sum_{\{x,y : g(x,y) = z\}} p_{XY}(x, y)$$

$E[g(X,Y)]$

- How do we define $E[g(X,Y)]$?
- One way is to define $Z = g(X,Y)$ and find $E[Z] = \sum_z z p_Z(z)$
- Recall $p_Z(z) = \sum_{\{x,y:g(x,y)=z\}} p_{XY}(x,y)$
- This gives us $E[Z] = \sum_z \sum_{\{x,y:g(x,y)=z\}} z p_{XY}(x,y)$.
- This is same as $E[g(X,Y)] = \sum_{\{x,y\}} g(x,y) p_{XY}(x,y)$.

$E[g(X,Y)] = \sum_{xy} g(x,y) p_{XY}(xy)$

$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{XY}(x,y) dxdy$ (for continuous r.v)

Example of function of Random variables

- Suppose X and Y are continuous independent random variables. Let $W = max(X, Y)$ and $Z = min(X, Y)$ Find the CDF and pdf of Z.

- HW: When X and Y are exponential with parameters λ_1 and λ_2 then Z is also exponential with parameter $\lambda_1 + \lambda_2$.

Covariance of X and Y

- $Cov(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$.
- When Covariance is zero, they are said to be uncorrelated.
- (Section 4.2 Bertsekas)
- `https://en.wikipedia.org/wiki/Covariance`

Covariance of X and Y