

Midsem: Probability and Statistics (40 Marks)

Each question: 6 marks

- ✓ 1. Consider the following game with a fair die. You repeatedly roll a fair die until you get a six. The game ends, when 6 appears. The reward from each roll is the face value except that a roll of 6 yields reward 0. Find the expected total reward from the game.
- ✓ 2. Suppose X and Y are both Uniform $[0,1]$ random variables. Then prove that $P(X < Y) = 0.5$.
- ✓ 3. Let X, Y and Z be independent exponential random variables with parameters λ_1, λ_2 and λ_3 . Let $W = \min(X, Y, Z)$. Find the cdf and pdf of W .
- ✓ 4. For two random variables X and Y , prove that $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$ where Var denotes variance and Cov denotes covariance.
- ✓ 5. Consider a Gaussian random variable X with mean μ and variance σ^2 . Let $Z = aX + b$ where $a, b \in \mathbb{R}$. Derive an expression for the probability density of Z and show that Z is also a Gaussian random variable. What is the mean and variance of Z ?

Each question: 10 marks

1. Let the joint probability density function of two continuous random variables X and Y be

$$f_{X,Y}(x,y) = c(x+y), \quad 0 \leq x \leq 1, 0 \leq y \leq 1,$$

and $f_{X,Y}(x,y) = 0$ otherwise.

- ✓ (a) Find the constant c that makes $f_{X,Y}(x,y)$ a valid joint pdf. (2mks)
- ✓ (b) Find the marginal density functions $f_X(x)$ and $f_Y(y)$. (2mks)
- ✓ (c) Find the conditional density functions $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$. (2mks)
- ✓ (d) Compute the conditional expectation $E[X | Y = y]$. (2mks)
- ✓ (e) Are X and Y independent? Justify your answer. (2mks)