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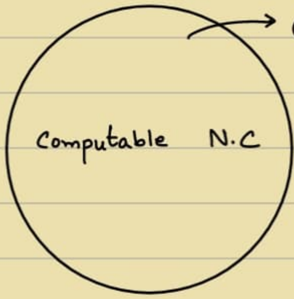
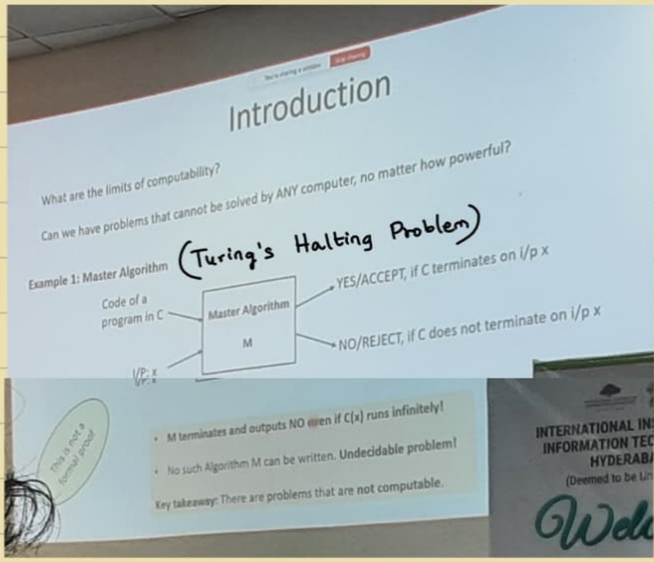
Automata Theory

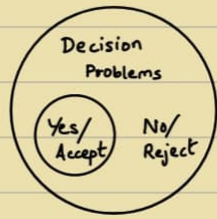
→ 

Computability

$\mathbb{N} < \mathbb{R}$

$\Rightarrow \mathbb{P}(\mathbb{N}) < \mathbb{P}(\mathbb{P}(\mathbb{N}))$

→ 



→ Properties of DFA :

(i) Single Start State

(ii) Unique Transitions

(iii) Zero or more final states

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- Saturday 9^{th}, 23^{rd} ⟶ 5 pm - 6 : 30 pm

- Q : Finite set of States

  Σ : Finite set of Alphabets

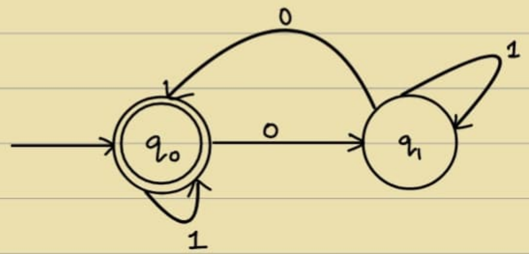
  $\delta : Q \times \Sigma \longrightarrow Q$ (Transition Function)

  q_0 : $q_0 \in Q$, Start State

  F : $F \subseteq Q$, Final/Accepting States

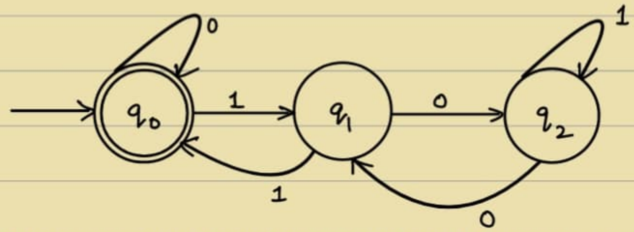
- $\Sigma = \{0, 1\}$

  $L(M) = \{ \omega \mid \omega$ has even no. of 0's $\}$



- $\Sigma = \{0, 1\}$

  $L(M) = \{ \omega \mid \omega$ is divisible by 3 $\}$



- $\Sigma = \{0, 1\}$

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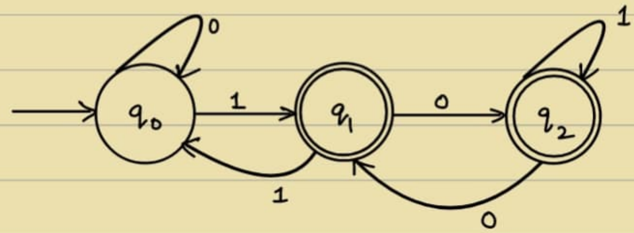


<u>Note</u>:

If L is solvable by a DFA, then L^c also has a DFA.

↳ Final ⟷ Start

→ <u>NFA</u>: [Non-deterministic Finite Automata]

Single start state + Multiple Final States

Some transitions can be missing

ε - Transitions

Multiple transactions are possible on the same input for a state

Crash → Rejecting Run

Q : Finite set of States

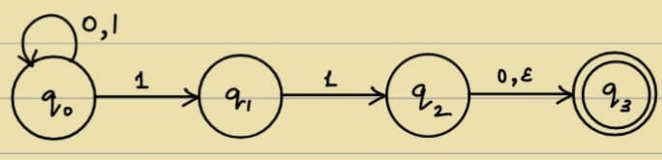
Σ : Finite set of Alphabets

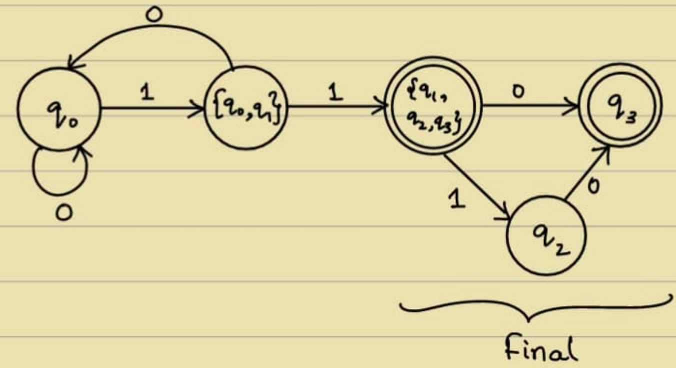
$\delta : Q \times \Sigma \longrightarrow P(Q)$ (Transition Function)

q_0 : $q_0 \in Q$, Start State

F : $F \subseteq Q$, Final/Accepting States

- NFA to DFA:





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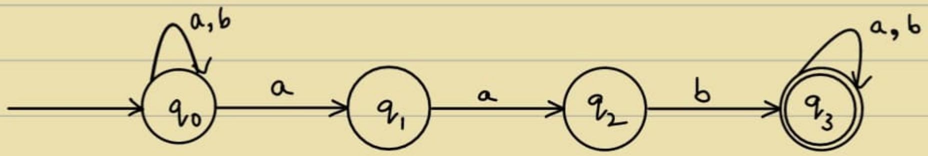
Tutorial :

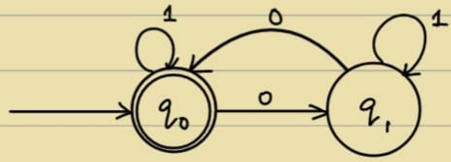
Q) $L = \{ \omega \in \{a,b\}^* \mid \omega$ contains 'aab' substring $\}$

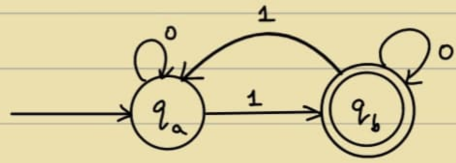


Q) $L = \{ \omega \in \{a,b\}^* \mid \omega$ contains even no. of 0s & odd no. of 1s $\}$

$L_1 = \{ \omega \in \{a,b\}^* \mid \omega$ contains even no. of 0s $\}$

$L_2 = \{ \omega \in \{a,b\}^* \mid \omega$ contains odd no. of 1s $\}$

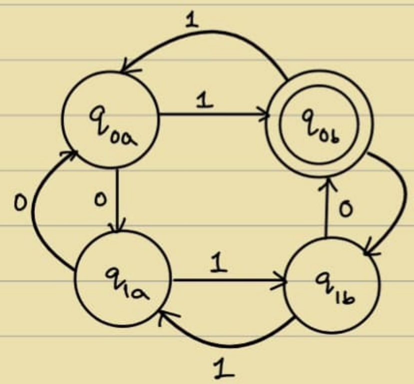




$L_1 \cap L_2 = L$

$Q = \{ q_{0a}, q_{1a}, q_{0b}, q_{1b} \}$

$F = \{ q_{0b} \}$



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$L = \{0,1\}$, $L^* = \{0,1\}^*$

$= \{ \varepsilon, 0, 1, 00, 11, \ldots \}$

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L is regular $\Rightarrow \exists p$ S.T. $\forall s \in L$ with $|s| \geq p$

$\exists x, y, z$ with $s = xyz$

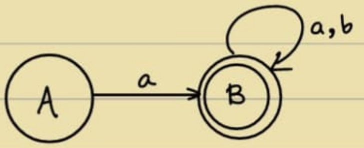
$(|xy| \leq p) \wedge (|y| \geq j) \wedge (\forall i \geq 0,\ xy^iz \in L)$

$\forall p$ S.T. $\forall s \in L$ with $|s| \geq p$

$\forall x, y, z$ with $s = xyz$

$\neg(|xy| \leq p) \wedge \neg(|y| \geq j) \wedge \neg(\forall i \geq 0,\ xy^iz \in L) \Rightarrow L$ is not regular

13/8 H.W: Convert Finite Automata to RLG & vice-versa



$A \rightarrow aB$

$B \rightarrow aB / bB / \varepsilon$

Reg $\Rightarrow$ RLG

RLG $\Rightarrow$ DFA

RLG $\Rightarrow$ LLG

LLG $\Rightarrow$ RLG

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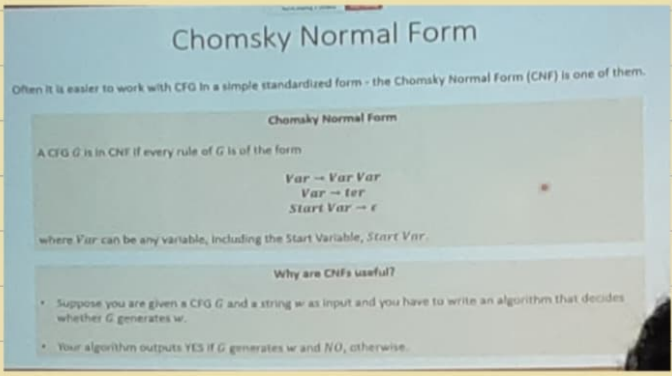

$B \overset{*}{\Rightarrow} x$, $2|x| - 1$

$C \overset{*}{\Rightarrow} y$, $2|y| - 1$

$\Big\}$ $w = xy$, $|x| \leq k$, $|y| \leq k$

$(2|x| - 1) + (2|y| - 1) + 1$

$= 2(k+1) - 1$

$A \rightarrow BC$, $B \Rightarrow x$, $C \Rightarrow y$

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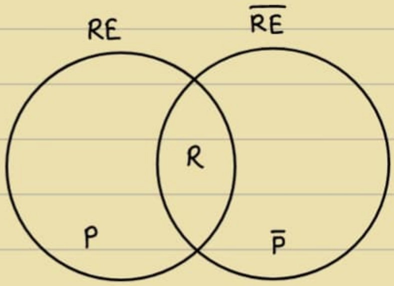
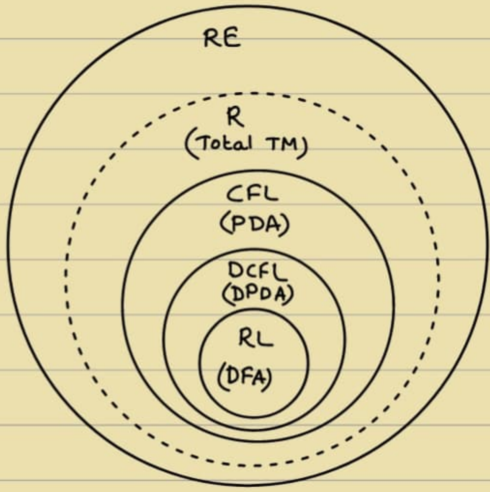
$R \subseteq RE$

$R \subseteq$ Co-RE $(\overline{RE})$



$R = RE \cap$ Co-RE



Quiz - 1 Prep :

→ M decides L if

$\forall w \in L$, $M(w)$ accepts

$\forall w \notin L$, $M(w)$ rejects

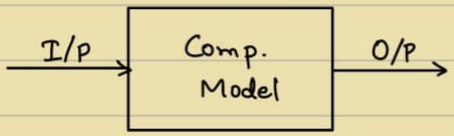
→ **DFA** : $(Q, F, \Sigma, q_0, \delta)$

$F \subseteq Q$

$\delta : Q \times \Sigma \to Q$

$q_0 \in Q$

→ Computable :



Computable :

YES instance : Device outputs YES

NO instance : Device outputs NO

→ $M \Rightarrow L$

$\overline{M} \Rightarrow \overline{L}$

$L_1 \cup L_2$, $L_1 \cdot L_2$, L_1^* , $L_1 \cap L_2$, $\overline{L_1}$ ← Regular

$L_1^* = \{x_1 x_2 \dots x_k \mid x_i \in L_1 \ \& \ k \geq 0\}$

→ **NFA** : $(Q, F, \Sigma, q_0, \delta)$

$\delta : Q \times \Sigma \to P(Q)$

$F \subseteq Q$

$q_0 \in Q$

→ GNFA ??

* Refer : Arden's Theorem $(R = RP + Q \Rightarrow R = QP^*)$

* Refer : Pumping Lemma

L is regular $\Longrightarrow$ Pumping Lemma is satisfied

Pumping lemma is NOT satisfied $\Longrightarrow$ L is NOT regular

→ DFA ≡ NFA ≡ Regular Expressions

→ Grammar : (N, T, P, S)

T → Terminals : a, b

P → Production Rules

N → Non - Terminals (Variables) : A, B, S

* Refer : Chomsky Normal form

Refer : Ambiguous Grammars (Derivations)

→ CFG : Form of $V \rightarrow (V \cup T)^*$

$L_1 \cup L_2$, $L_1 L_2$, L_1^* ← CFLs

→ PDA : DFA + $\underbrace{\text{memory}}_{\text{Stack}}$, recognizes CFGs

Transition : a, z_0, Az_0 → Operation , $\varepsilon \rightarrow$ Pop

z_0 → Top of the stack

a → I/P symbol

PDA : $(Q, F, \delta, q_0, \Sigma, T)$

δ → $Q \times \Sigma \times T \longrightarrow P(Q \times T)$

→ RL ≡ Regular Grammar ≡ RE ≡ NFA ≡ DFA ⊆ CFL ≡ CFG ≡ PDA

* Refer : Pumping Lemma for PDA

→ Turing Machine : $(Q, F, \Sigma, q_0, T, \delta, q_{rej})$

δ → $Q \times T \longrightarrow Q \times T \times \{L \times R\}$

* Refer : CYK Algorithm (Cocke - Younger - Kasami)

Myhill – Nerode Theorem

* Imp : Conversion from NFA to DFA

$L_1 \cup L_2$, $L_1 \cap L_2$, $\bar{L}$ ← Regular Languages

DFA minimization

Conversion from ε-NFA to NFA

$$L \text{ is a CFL} \Rightarrow \exists n \, \forall u \left(|u| \geq n \rightarrow \exists v \exists w \exists x \exists y \exists z \begin{pmatrix} u = vwxyz \\ |wy| \geq 1 \\ |wxy| \leq n \end{pmatrix} \wedge \forall i \left(i \geq 0 \rightarrow vw^ixy^iz \in L \right) \right)$$

$u \in L$

$$\forall n \, \exists u \left(|u| \geq n \wedge \forall v \forall w \forall x \forall y \forall z \begin{pmatrix} u = vwxyz \\ |wy| \geq 1 \\ |wxy| \leq n \end{pmatrix} \rightarrow \exists i \left(i \geq 0 \wedge vw^ixy^iz \notin L \right) \right) \rightarrow L \text{ is not CFL}$$

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$A_{TM} = \{ <M, \omega> \mid M \text{ accepts } \omega \}$

- Prove that A_{TM} is undecidable :

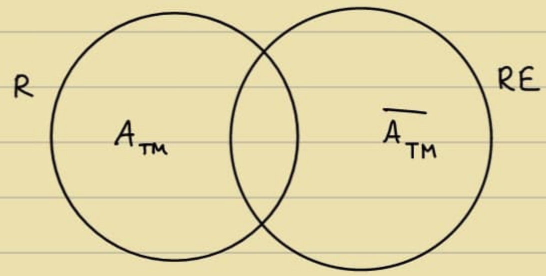
Sol: Proof by contradiction:

Suppose A be the total TM for A_{TM}

$$A(<M,\omega>) = \begin{cases} \text{Accept}, & \text{if } M(\omega) \text{ accepts} \\ \text{Reject}, & \text{if } M(\omega) \text{ doesn't accept (Reject / loop)} \end{cases}$$

Contradiction :

$$D(<D>) = \{ \text{Accepts, if } D(<D>) \text{ rejects} \}$$



φ : E_{TM} is undecidable

$\overline{A_{TM}} = \{ <M, \omega> \mid M \text{ does not accept } \omega \}$

→ Using as a subroutine (N)

$$N(<M,\omega>) = \begin{cases} \text{Accept}, & \text{if } M(\omega) \text{ does not accept (Reject / Loop)} \\ \text{Reject}, & \text{if } M(\omega) \text{ accepts} \end{cases}$$

$$E_{TM} = \{ <M> \mid M \text{ is a TM} \wedge L(M) = \emptyset \}$$

$$T_E(<T>) = \begin{cases} \text{Accept}, & L(T) = \emptyset \\ \text{Reject}, & L(T) \neq \emptyset \end{cases}$$

T does not accept any accept , if $M(\omega)$ doesn't accept

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$$N(\langle M, \omega \rangle) = \begin{cases} \text{ACCEPT}, & \text{if } M(\omega) \text{ doesn't accept (Reject/ Loop)} \\ \text{REJECT}, & \text{if } M(\omega) \text{ accepts} \end{cases}$$

$$T_E(\langle T \rangle) = \begin{cases} \text{ACCEPT}, & \text{if } L(\langle T \rangle) = \phi \\ \text{REJECT}, & \text{if } L(\langle T \rangle) \neq \phi \end{cases}$$

- $\omega \notin L \Rightarrow \omega \in \overline{L}$

$M \longrightarrow L \in RE$

$\forall \omega \in L$, $M(\omega)$ accepts

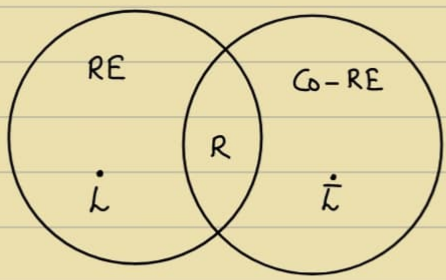
$\forall \omega \notin L$, $M(\omega)$ doesn't accept.

$$\overline{M}(\omega) = \begin{cases} \text{Run } M(\omega) \\ \text{Output ACCEPT, if } M(\omega) \text{ REJECTS} \\ \qquad\quad \text{REJECT, if } M(\omega) \text{ ACCEPTS} \end{cases}$$

For

$\forall \omega \notin L$, $\omega \in \overline{L}$, M accepts or loops

$\forall \omega \in L$, $\omega \in \overline{L}$, M rejects

- 

$L \rightarrow RE \cap Co\text{-}RE \rightsquigarrow M$

$\overline{L} \rightarrow Co\text{-}RE \rightsquigarrow \overline{M}$

Has a Recognizer & co-recognizer

$R \subseteq RE \cap Co\text{-}RE$

$RE \cap Co\text{-}RE \subseteq R$ (Dovetailing)

$\therefore R = RE \cap Co\text{-}RE$

- Closure of Co-RE:

  $L_1 \cup L_2 \equiv \overline{(\overline{L_1} \cap \overline{L_2})}$

  H.W: Closure under star

  H.W: Rice's Theorem

Quantum Computation models violates the $\underbrace{\text{Extended Church Turing Thesis}}$.

Every problem can be efficiently solvable in Polynomial Time by a Probablistic Turing Machine.

(DTP class is more powerful than the P class.)