CS 302.1 - Automata Theory

Lecture 03

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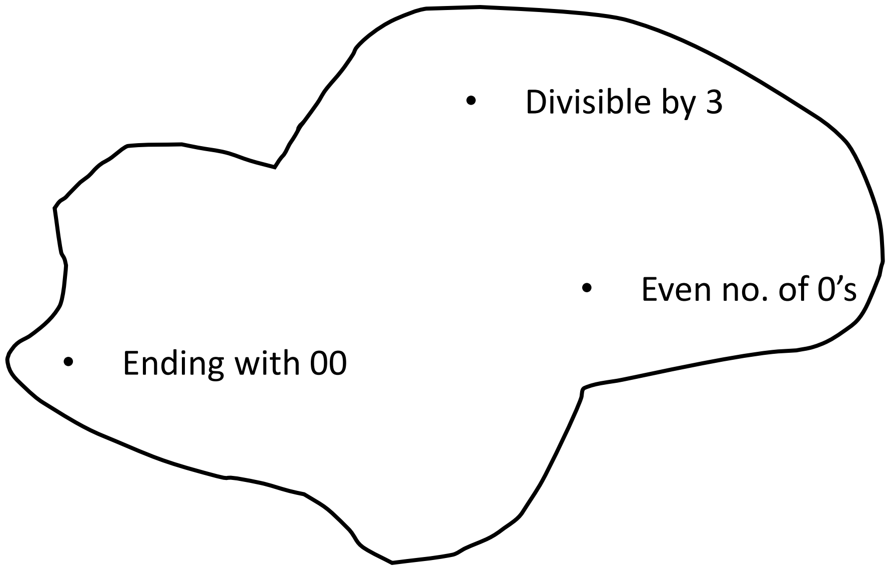
Quick Recap



- DFAs and NFAs are equivalent
- For every NFA we can obtain a “Remembering DFA” that accepts the same language.

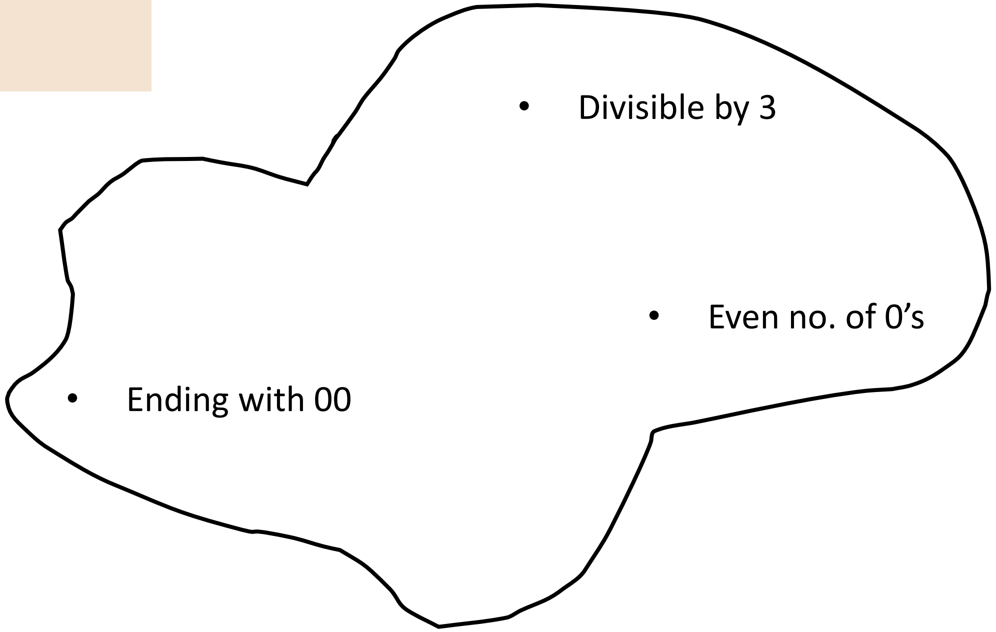
Regular Languages

A language is called a **Regular Language** if there exists some finite automata deciding it.

If M be a finite automaton (DFA/NFA) and,

$$L(M) = \{\omega | \omega \text{ is accepted by } M\}$$

$L(M)$ is regular.



Set of all regular Languages

Regular Languages

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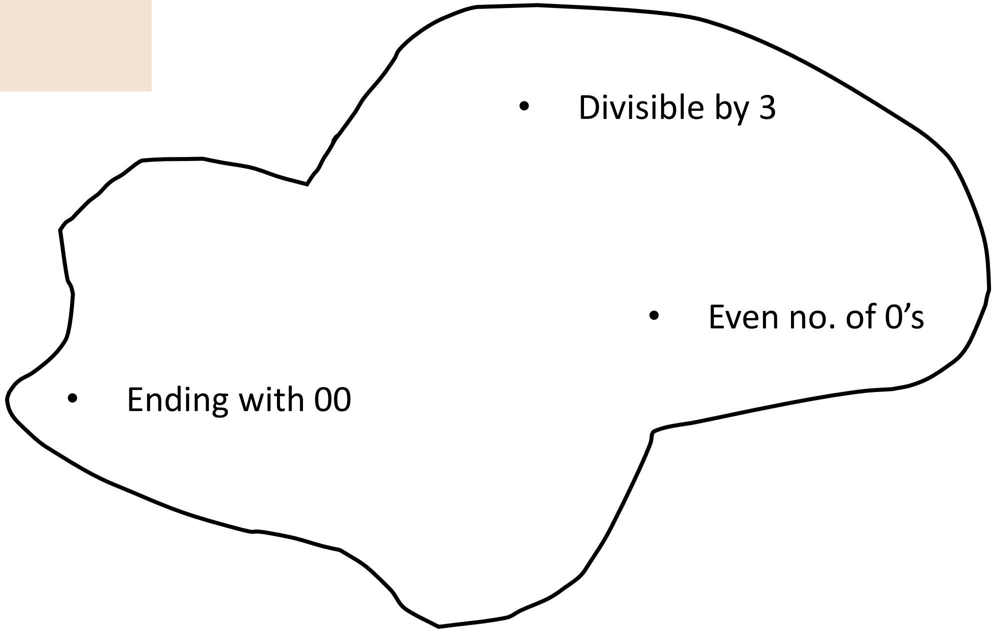
$\boldsymbol{L(M)}$ **is regular.**

- Any language has associated with it, a set of operations that can be performed on it.
- These operations help us to understand the properties of that language, e.g. closure properties
- For regular languages, this will help us prove that certain languages are non-regular and hence we cannot hope to design a finite automaton for them



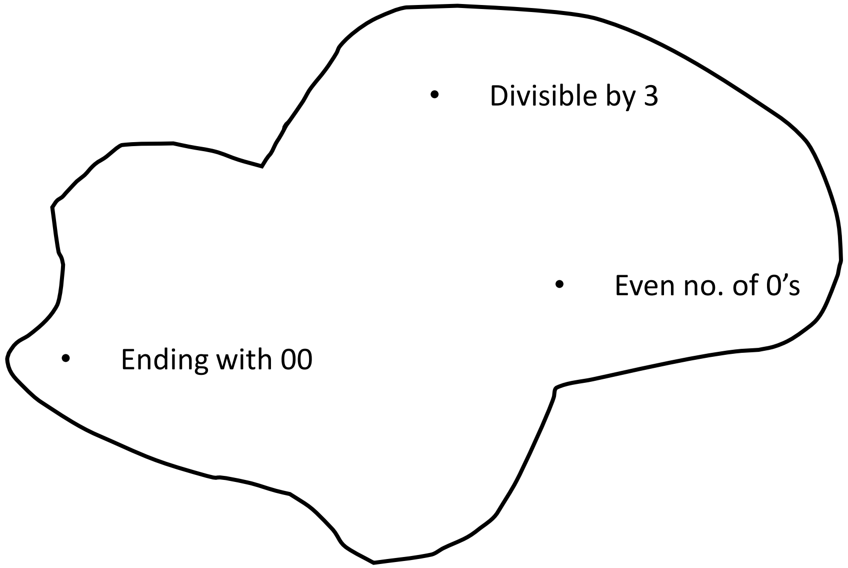
Set of all regular Languages

Regular Languages

Regular Operations:

Let L_1 and L_2 be languages. The following are the *regular operations*:

- **Union:** $L_1 \cup L_2 = \{x | x \in L_1 \text{ or } x \in L_2\}$
- **Concatenation:** $L_1 . L_2 = \{xy | x \in L_1 \text{ and } y \in L_2\}$
- **Star:** $L_1^* = \{x_1 x_2 \cdots x_k |\ k \geq 0 \text{ and each } x_i \in L_1\}$



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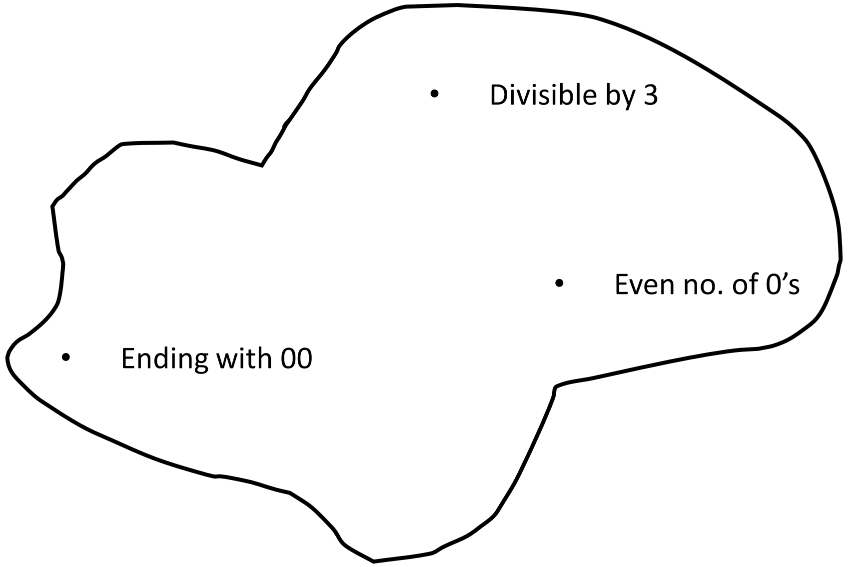

Set of all regular Languages

Star operation: It is a unary operation (unlike the other two) and involves putting together *any number of strings in L_1 together to obtain a new string.*

Note: Any number of strings includes "0" as a possibility and so the empty string ϵ is a member of L_1^*.

If $\Sigma = \{a\}$, $\Sigma^* = \{\epsilon, a, aa, aaa, \ldots \ldots\}$; If $\Sigma = \{\Phi\}, \Sigma^* = \{\epsilon\}$

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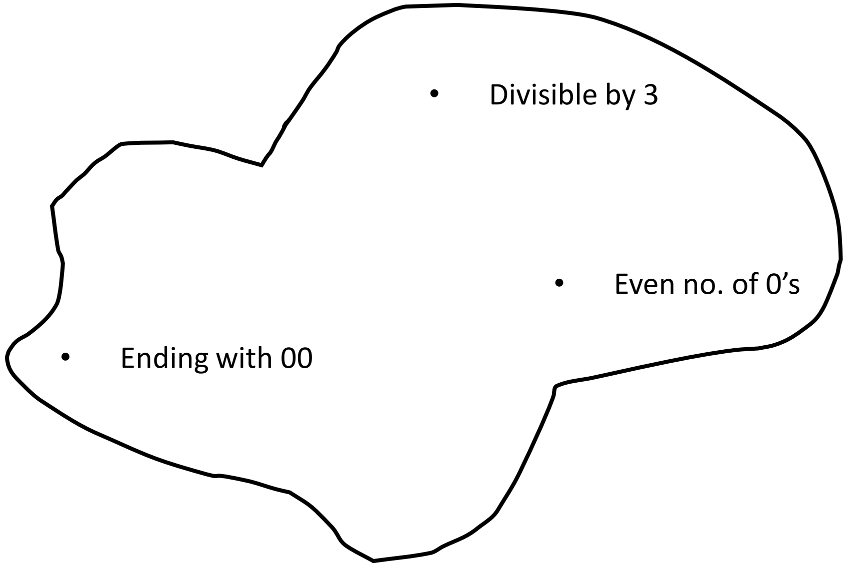

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Note: Any number of strings includes "0" as a possibility and so the empty string ϵ is a member of L_1^***.**

If $\mathrm{L} = \{0,1\}$, we have that $\mathrm{L}^* = \{0,1\}^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \dots \dots\}$

Regular Languages

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Example: Let the alphabet $\Sigma = \{a, b, \cdots, z\}$. If $L_1 = \{$*social, economic*$\}$ and $L_2 = \{$*justice, reform*$\}$, then

- $L_1 \cup L_2 = \{$*social, economic, justice, reform*$\}$

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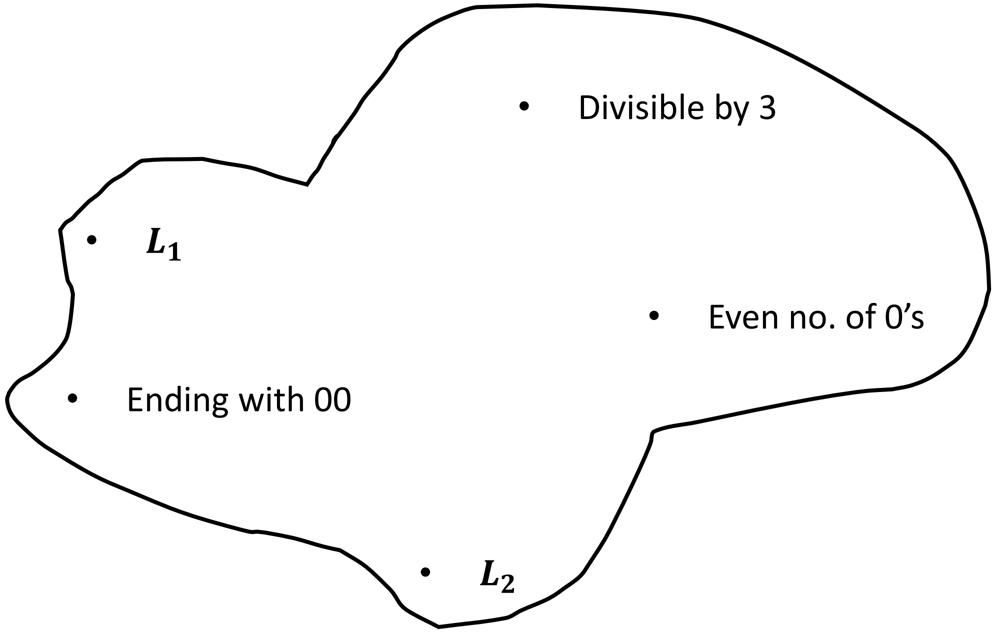
- $L_1 \cup L_2 = \{$*social, economic, justice, reform*$\}$
- $L_1 . L_2 = \{$*socialjustice, socialreform, economicjustice, economicreform*$\}$
- $L_1^* = \{\epsilon$, *social, economic, socialsocial, socialeconomic, economicsocial, economiceconomic, socialsocialsocial, socialsocialeconomic, socialeconomiceconomic,……*$\}$
- $L_2^* = \{\epsilon$, *justice, reform, justicejustice, justicereform, reformjustice, reformreform, justicejusticejustice, ……*$\}$

Closure of Regular Languages

We want to check whether the set of regular languages are **closed** under some operations.

What does this mean?

- We pick up points within the set of all regular languages (say L_1 and L_2)
- Perform *set operations* such as Union, concatenation, Star, intersection, complement etc on them.
- Observe whether the resulting language still belongs to the set of all regular languages.
- If so, we say, regular languages are **closed** under that operation.



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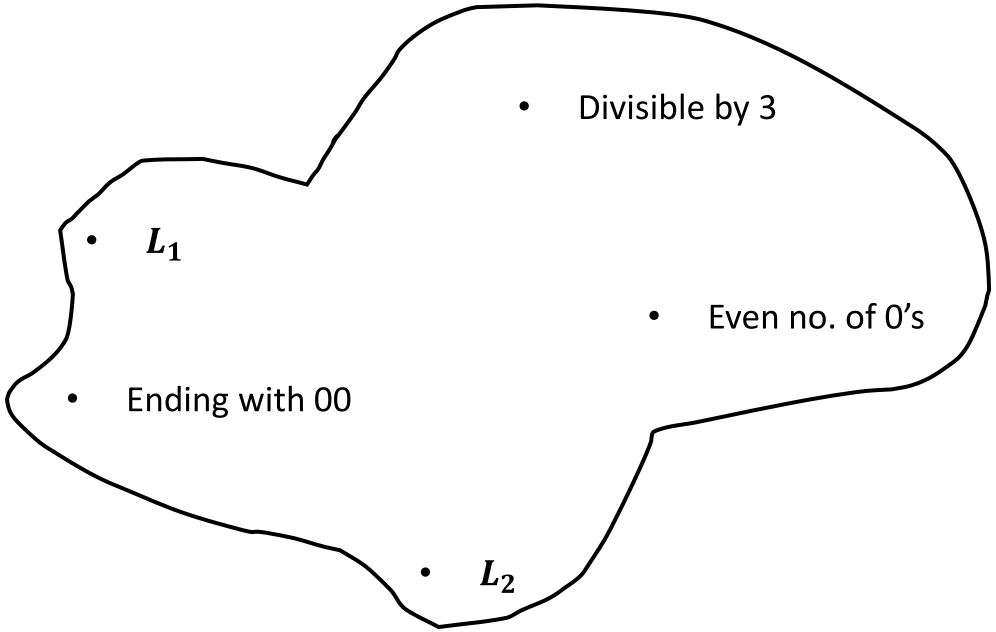


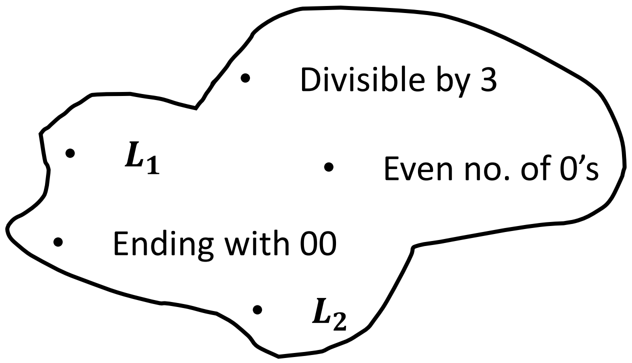
Set of all regular Languages

For example, the **natural numbers are closed under addition/multiplication** and **not under subtraction/division**.

Closure of Regular Languages

Q: Is the set of all regular languages **closed under union**?

Suppose L_1 and L_2 are regular languages. Is $L = L_1 \cup L_2$ also regular?



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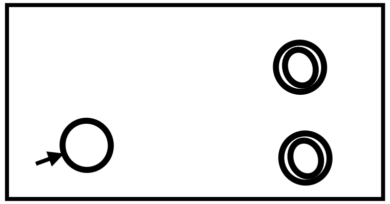
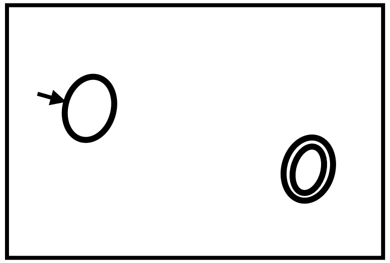
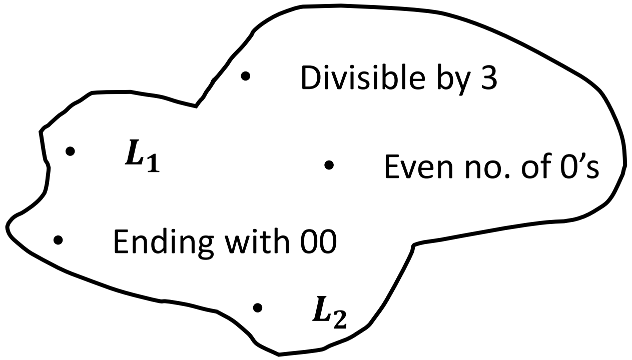
Using M_1 and M_2, we will show how to construct an NFA M that accepts $L = L_1 \cup L_2$, i.e. $L(M) = L_1 \cup L_2$.

Suppose the DFA M_1 is



And the DFA M_2 is





Set of all regular Languages

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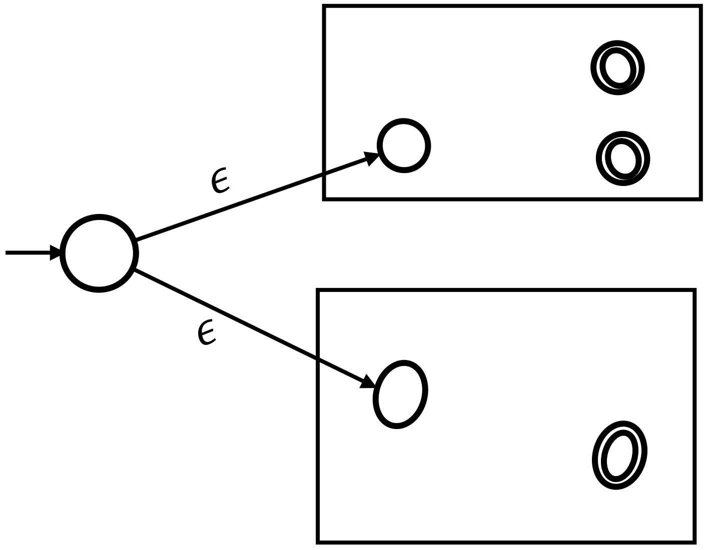
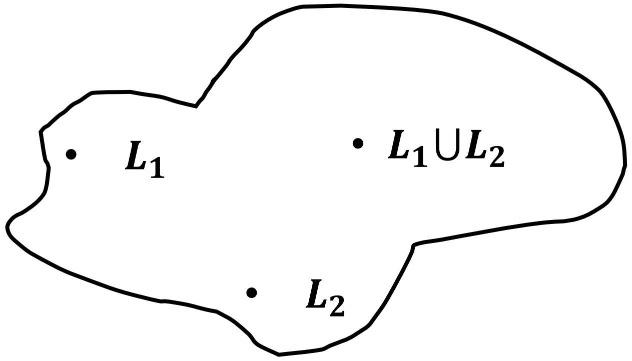
NFA M accepting $\boldsymbol{L = L_1 \cup L_2}$





Set of all regular Languages

Closure of Regular Languages

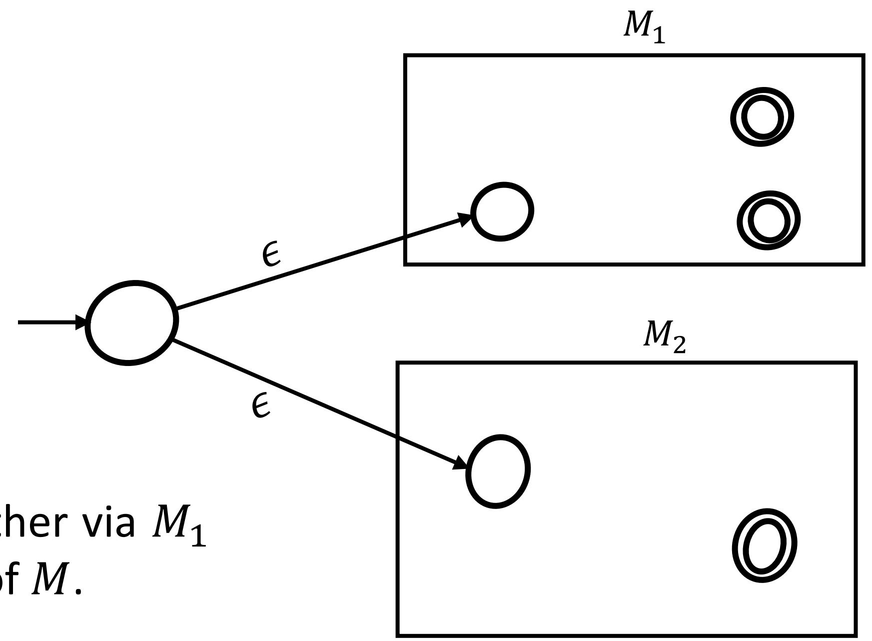
Q: Is the set of all regular languages **closed under union**?

Suppose L_1 and L_2 are regular languages. Is $L = L_1 \cup L_2$ also regular?

Proof: In order to prove that $L(M) = L_1 \cup L_2$, we show two things:

(i) $L \subseteq L_1 \cup L_2$

Let $\omega \in L$, i.e. ω is accepted by M. The final state for L can be reached either via M_1 or M_2. Thus ω must be accepted by either of them to reach the final state of M.



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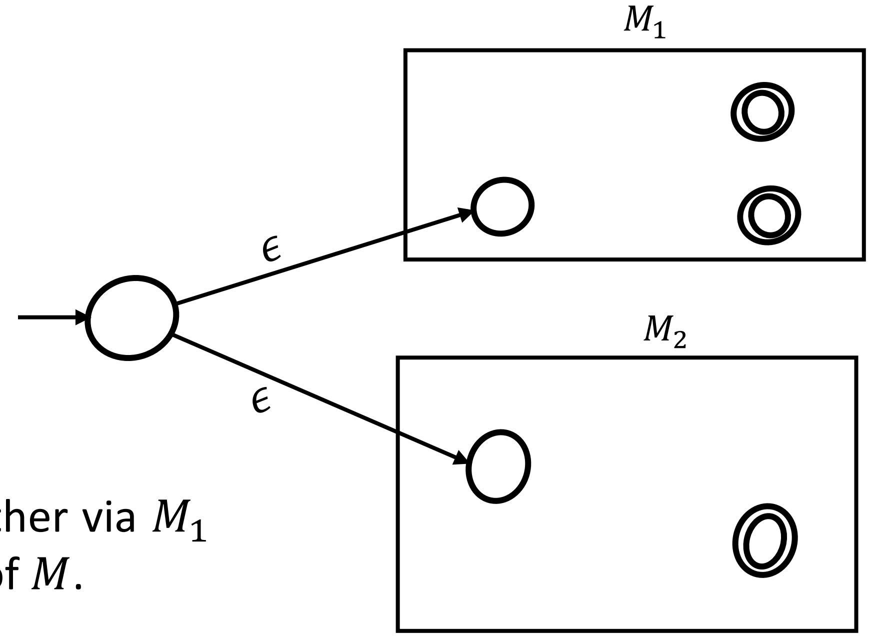
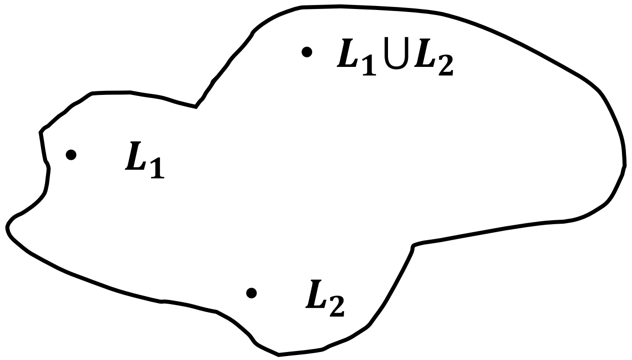
(ii) $L_1 \cup L_2 \subseteq L$

Let $\omega \in L_1 \cup L_2$. Then, $\omega \in L_1$ or $\omega \in L_2$.

Thus, ω must reach the final state of M_1 or M_2. But since the start state of M_1 or M_2 can be reached from the start state of M by taking an ϵ-transition, $\omega \in L$.





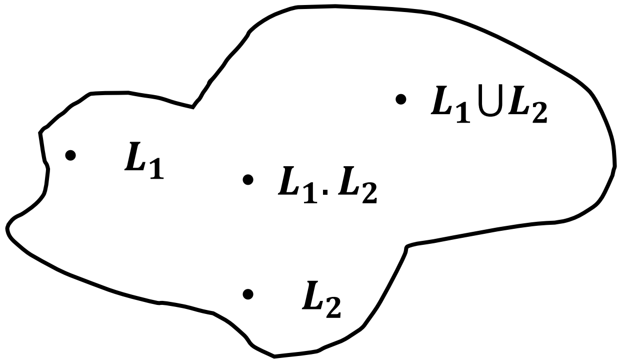
Set of all regular Languages

Closure of Regular Languages

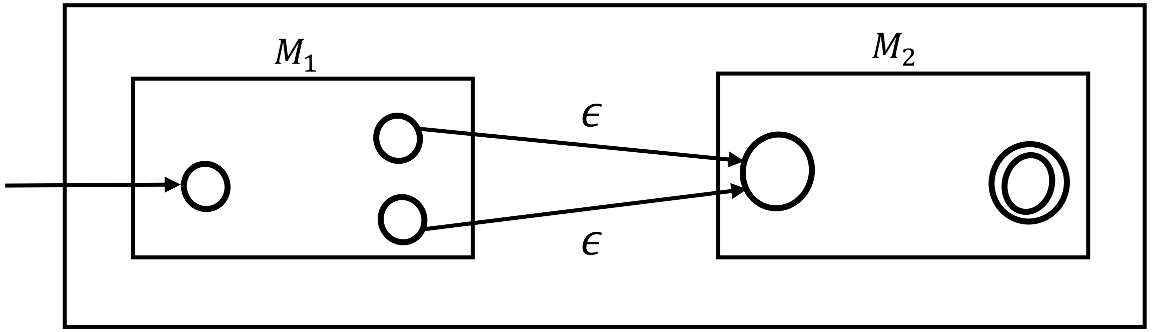
Q: Is the set of all regular languages **closed under concatenation**? Suppose L_1 and L_2 are regular languages. Is $L = L_1.L_2$ also regular?

Proof: Since L_1 and L_2 are regular, there must be a DFA M_1 that accepts L_1, i.e. $L(M_1) = L_1$ and a DFA M_2 that accepts L_2, i.e. $L(M_2) = L_2$.

Using $M_1$and M_2, we will show how to construct an NFA M that accepts $L = L_1.L_2$.



Set of all regular Languages



$L_1.L_2 = \{xy | x \in L_1 \text{ and } y \in L_2\}$

NFA M accepting $\boldsymbol{L = L_1.L_2}$

Closure of Regular Languages

Q: Is the set of all regular languages **closed under star**? Suppose L_1 is a regular language. Is L_1^* also regular?

Proof: Since L_1 is regular, there must be a DFA M_1 that accepts L_1, i.e. $L(M_1) = L_1$. Using M_1, we will show how to construct an NFA M that accepts $L = L_1^*$.

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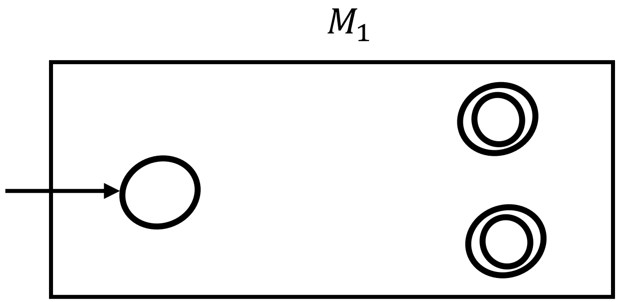

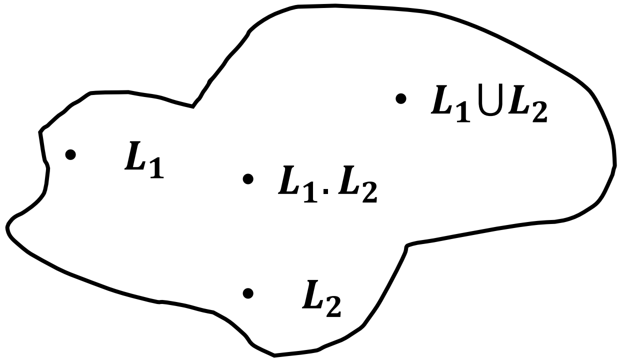

Set of all regular Languages

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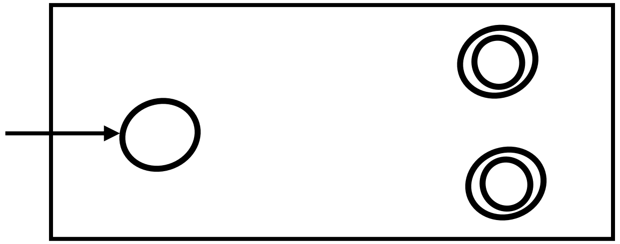
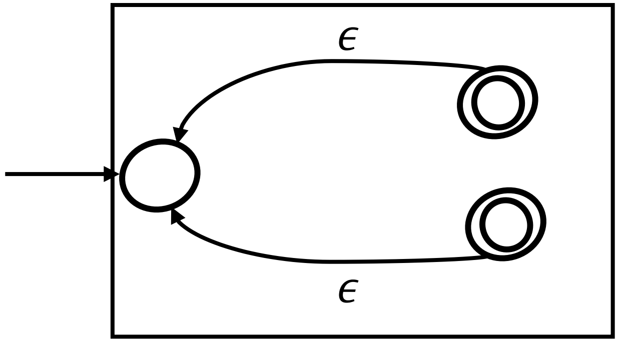
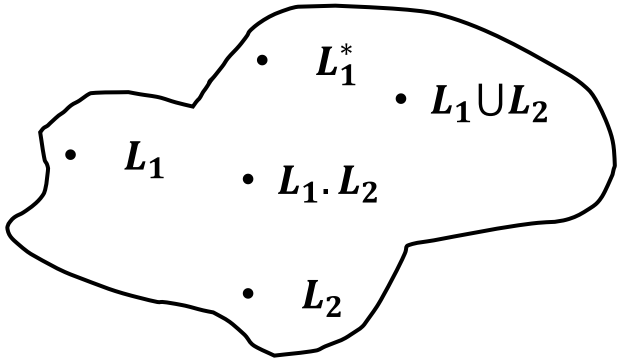




$L_1^* = \{x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in L_1\}$

Steps:

- Make ϵ-transitions from the final states of L_1 to the initial state of L_1.



Set of all regular Languages

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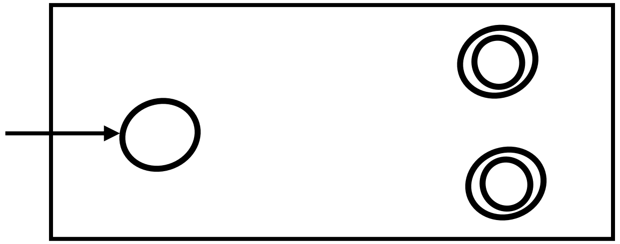

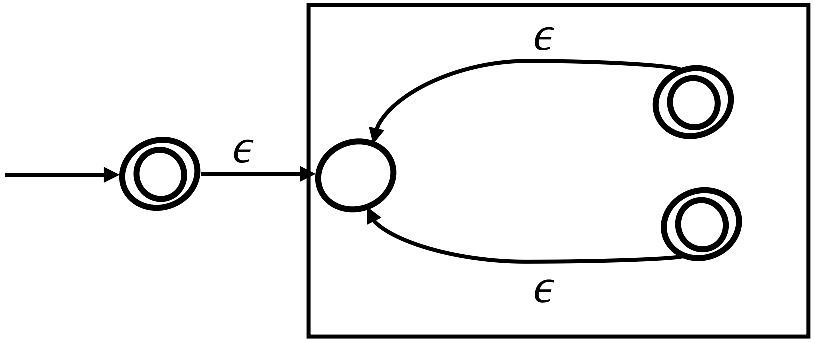

NFA accepting $\boldsymbol{L = L_1^*}$

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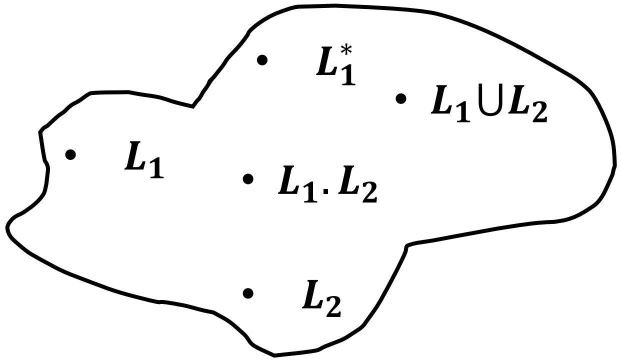


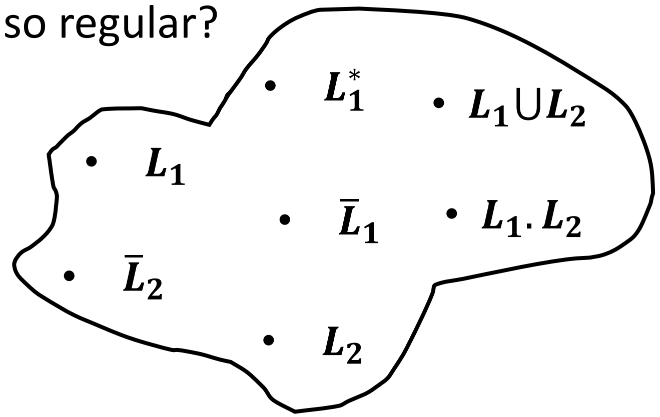
Set of all regular Languages

Steps:

- Make ϵ-transitions from the final states of L_1 to the initial state of L_1.
- Make a new final state as the start state and make an ϵ-transition from this state to the previous start state of L_1.

Closure of Regular Languages

Q: Is the set of all regular languages **closed under complement**? If L is regular, then is $\overline{L}$ also regular?



Set of all regular Languages

Proof: Given a DFA M, such that $L(M) = L$, construct the **toggled DFA** M' from M, by

(i) changing all the non-final states of M to be the final states of M' and
(ii) changing all the final states M to be the non-final states of M'.

$$\boldsymbol{L(M') = \overline{L}}$$

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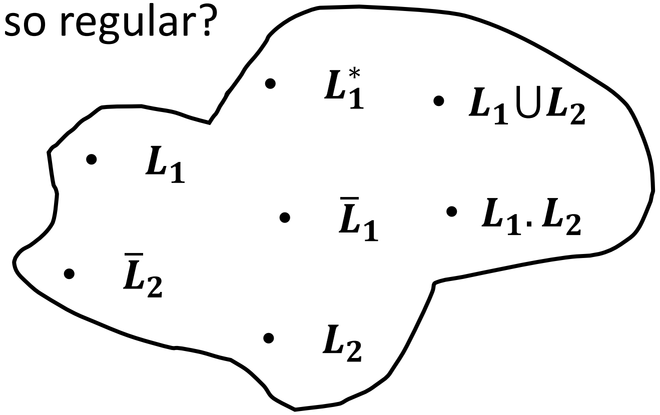

Set of all regular Languages

Q: If L is the language accepted by an NFA, does "toggling" its states result in an NFA that accepts $\overline{L}$?

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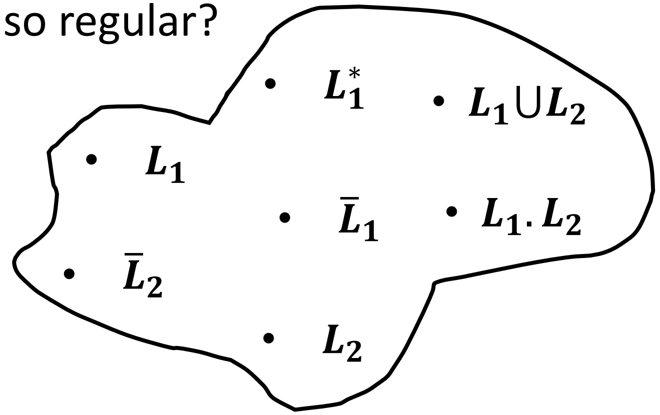

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Proof: Consider that for an input string $x \in L$**,** such that N accepts it. Suppose there is a rejecting run and an accepting run for input x. (See Table)

	NFA N	**Toggled NFA N'**
Run 1	Rejecting	
Run 2	Accepting	

Closure of Regular Languages

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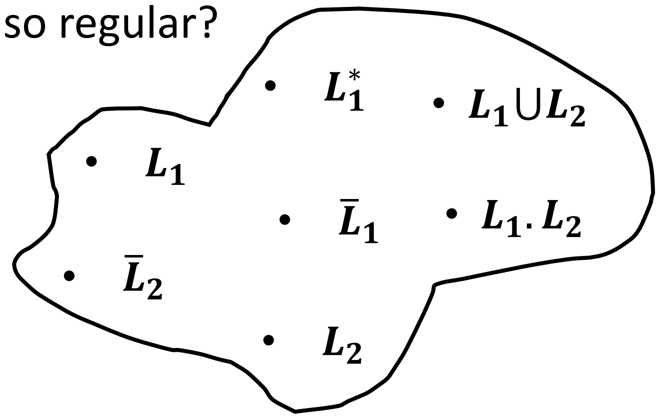

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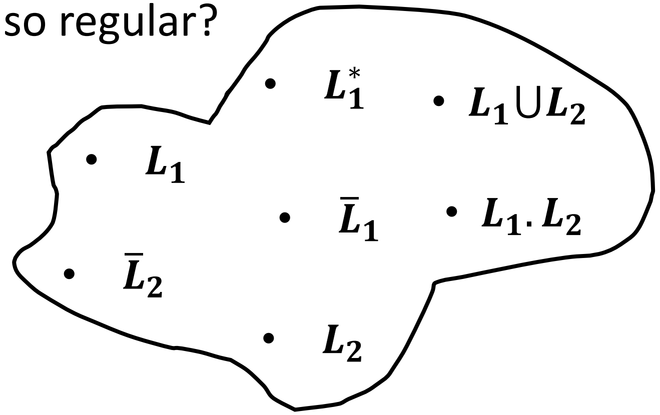
Proof: Consider that for an input string $x \in L$, such that N accepts it. Suppose there is an rejecting run and an accepting run for input x. (See Table)

For toggled NFA N' too, there are two runs for x. However, the rejecting run for N is an accepting run for N'. Thus x is accepted by both N and N'.

	NFA N	**Toggled NFA N'**
Run 1	Rejecting	Accepting
Run 2	Accepting	Rejecting

Closure of Regular Languages

Q: Is the set of all regular languages **closed under complement**? If L is regular, then is $\overline{L}$ also regular?



Set of all regular Languages

Proof: Given a DFA M, such that $L(M) = L$, construct the **toggled DFA** M' from M, by

(i) changing all the non-final states of M to be the final states of M' and
(ii) changing all the final states M to be the non-final states of M'.

$$\boldsymbol{L(M') = \overline{L}}$$

Q: If L is the language accepted by an NFA, does "toggling" its states result in an NFA that accepts $\overline{L}$?

Proof: Consider that for an input string $x \in L$, such that N accepts it. Suppose there is an rejecting run and an accepting run for input x. (See Table)

For toggled NFA N' too, there are two runs for x. However, the rejecting run for N is an accepting run for N'. Thus x is accepted by both N and N'.

	NFA N	**Toggled NFA N'**
Run 1	Rejecting	Accepting
Run 2	Accepting	Rejecting

Contradiction! So No, the **toggled NFA does not accept $\overline{L}$**.

Closure of Regular Languages

Q: Is the set of all regular languages **closed under intersection**? If L_1 and L_2 are regular, then is $L = L_1 \cap \mathrm{L}_2$ also regular?

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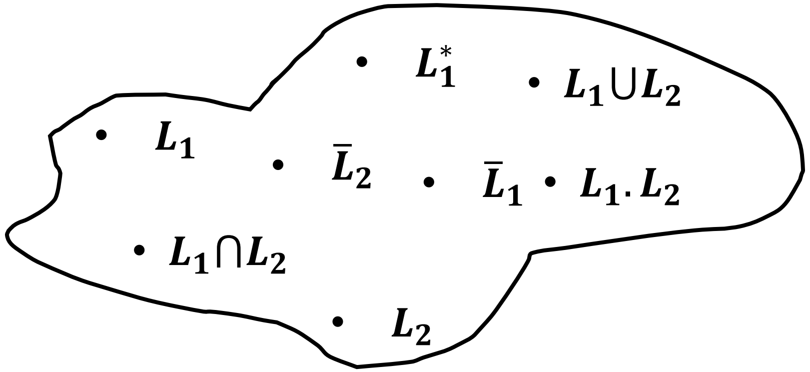
Given a DFA for L_1 and a DFA for L_2, we know how to construct an NFA for $\overline{L_1}, \overline{L_2}$ as well as for $L_1 \cup L_2$. Using these constructions and the aforementioned relationship, we can construct an NFA for $L = L_1 \cap \mathrm{L}_2$



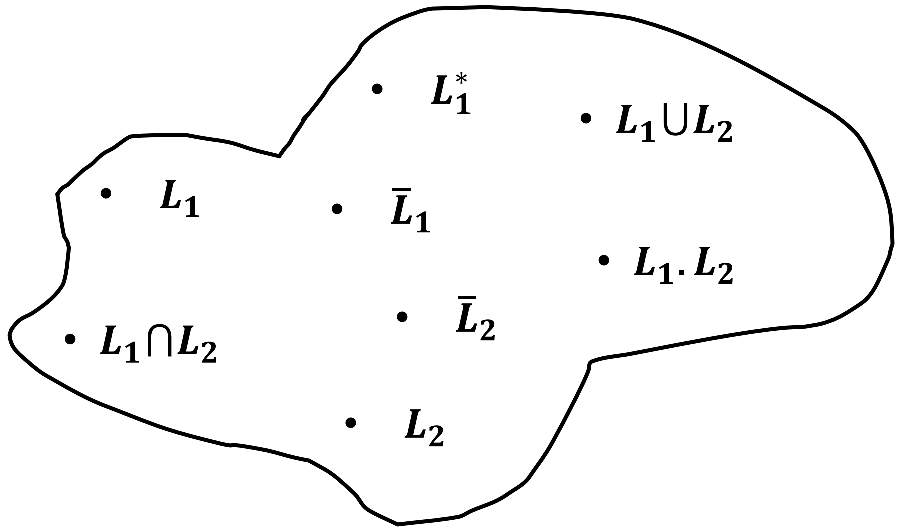
Set of all regular Languages

Closure of Regular Languages

Summary:

Regular Languages are closed under:

- **Union**
- **Intersection**
- **Star**
- **Complement**
- **Concatenation**



Set of all regular Languages

Regular Languages

If Σ is an alphabet, then

- $\Sigma^0 = \{\epsilon\}$
- $\Sigma^2 = \{a_1 a_2 | a_1 \in \Sigma,\ a_2 \in \Sigma\}$
- $\Sigma^k = \{a_1 a_2 \cdots a_k | a_i \in \Sigma\ |1 \leq i \leq k\}$
- $\Sigma^* = \{\bigcup_{i \geq 0} \Sigma^i\} = \{\Sigma^0 \cup \Sigma^1 \cup \Sigma^2\ \cdots\} = \{a_1 a_2 \cdots a_k | k \in \{0,1,\cdots\}\ \&\ a_j \in \Sigma, \forall j \in \{1,2,\cdots,k\}\}$

A Language $L \subset \Sigma^*$ and $L^* = \{\bigcup_{i \geq 0} \mathrm{L}^i\}$

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Regular Language (alternate definition): Let Σ be an alphabet. Then the following are the regular languages over Σ:

- The empty language Φ is regular
- For each $a \in \Sigma, \{a\}$ is regular.
- Let L_1, L_2 be regular languages. Then $L_1 \cup L_2$, $L_1 . L_2$, L_1^* are regular languages.

Regular Expressions

A regular expression describes regular languages algebraically. The algebraic formulation also provides a powerful set of tools which will be leveraged to prove

- languages are regular
- derive properties of regular languages

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Syntax for regular expressions (Recursive definition): R is said to be a regular expression if it has one of the following forms:

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- ϵ is a regular expression, $L(\epsilon) = \{\epsilon\}$
- Any $a \in \Sigma$ is a regular expression, $L(a) = \{a\}$

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- R_1R_2 is a regular expression if R_1 and R_2 are regular expressions, $L(R_1R_2) = L(R_1).L(R_2)$
- (R) is a regular expression if R is a regular expression, $L\left((R)\right) = R$

Regular Expressions

Syntax for regular expressions:

Regular Expression	**Regular Language**	**Comment**
Φ	$\{\}$	The empty set
ϵ	$\{\epsilon\}$	The set containing ϵ only
a	$\{a\}$	Any $a \in \Sigma$
$R_1 + R_2$	$L(R_1) \cup L(R_2)$	For regular expressions R_1 and R_2
$R_1 R_2$	$L(R_1).L(R_2)$	For regular expressions R_1 and R_2
R^*	$\left(L(R)\right)^*$	For regular expressions R
(R)	$L(R)$	For regular expressions R

Order of precedence: (), *, . , +

A language L is regular if and only if for some regular expression R, $L(R) = L$.

RE's are equivalent in power to NFAs/DFAs

Regular Expressions

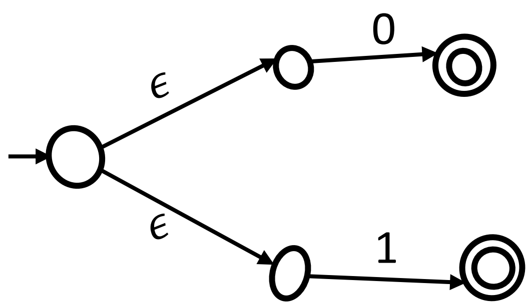
Syntax for regular expressions:

Regular Expression R	$\boldsymbol{L(R)}$
01	$\{01\}$
$01+1$	$\{01,1\}$
$(0+1)^*$	$\{\epsilon, 0, 1, 00, 01, \cdots\}$
$(01+\epsilon)1$	$\{011,1\}$
$(0+1)^*01$	$\{01,001,101,0001,\cdots\}$
$(0+10)^*(\epsilon+1)$	$\{\epsilon, 0, 10, 00, 001, 010, 0101, \cdots\}$

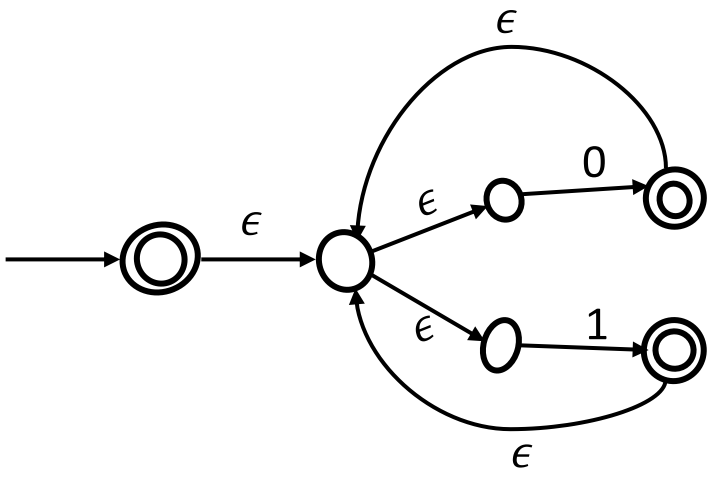
Regular Expressions

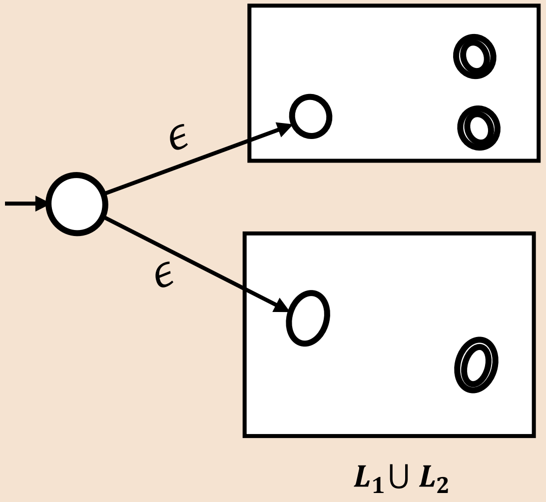
NFA for RE: $(\mathbf{0}+\mathbf{1})^*\mathbf{01}$

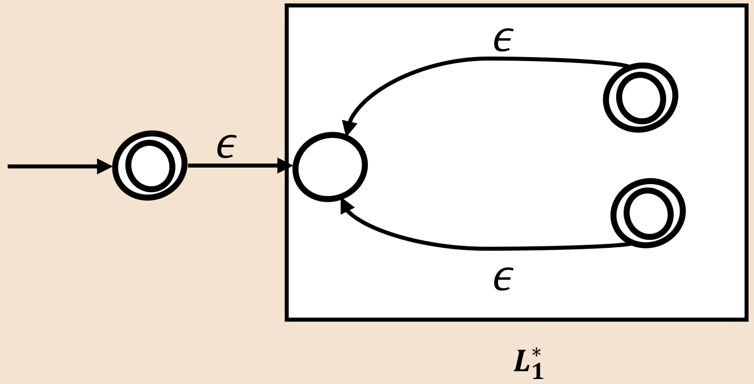
(i) NFA for $(0+1)$



(ii) NFA for $(0+1)^*$

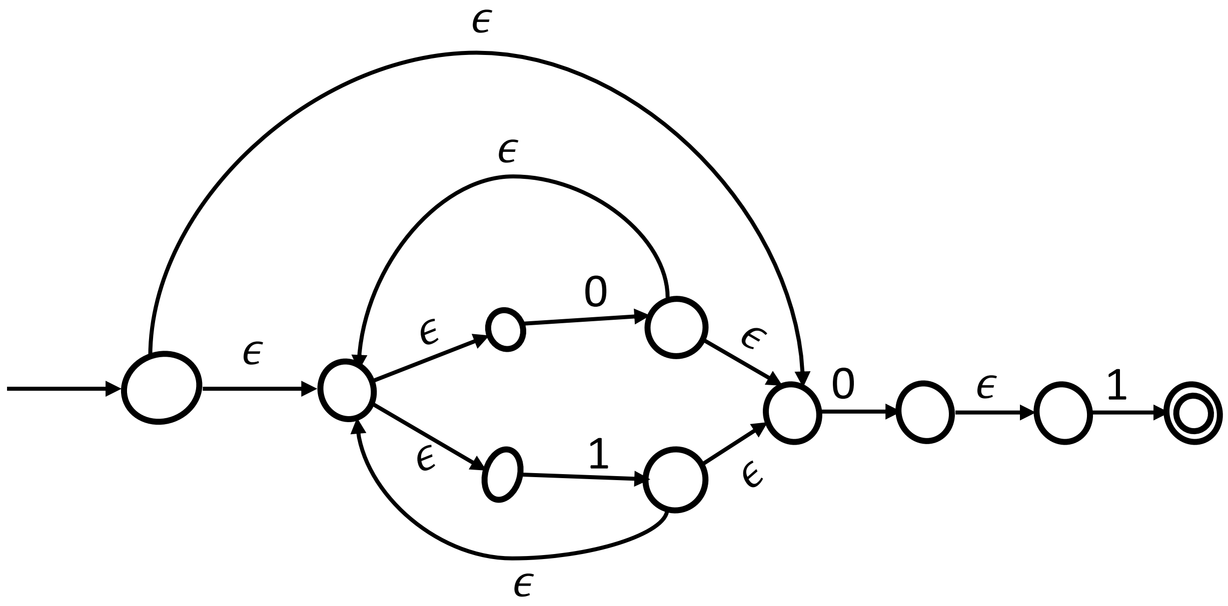


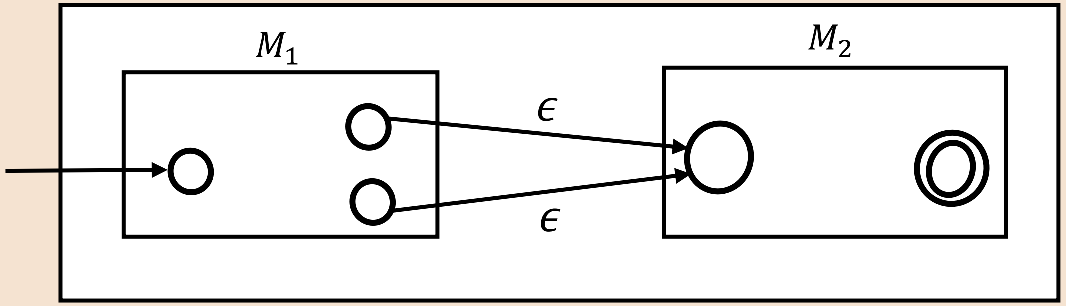




Regular Expressions

NFA for $(0+1)^*01$





$L_1 . L_2$

Regular Expressions

Let $\Sigma = \{a, b\}$.

Language	**Regular Expression**
$\{\omega \mid \omega$ ends in "ab"$\}$	$(a+b)^*ab$
$\{\omega \mid \omega$ has a single a $\}$	b^*ab^*
$\{\omega \mid \omega$ has at most one $a\}$	$b^* + b^*ab^*$
$\{\omega \mid \ \lvert\omega\rvert$ is even$\}$	$((a+b)(a+b))^* = (aa+bb+ab+ba)^*$
$\{\omega \mid \omega$ has "ab" as a substring$\}$	$(a+b)^*ab(a+b)^*$
$\{\omega \mid \lvert\omega\rvert$ is a multiple of 3$\}$	$\big((a+b)(a+b)(a+b)\big)^*$

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Some algebraic properties of Regular Expressions:

- $R_1 + (R_2 + R_3) = (R_1 + R_2) + R_3$
- $R_1(R_2R_3) = (R_1R_2)R_3$
- $R_1(R_2 + R_3) = R_1R_2 + R_1R_3$
- $(R_1 + R_2)R_3 = \mathrm{R_1R_3 + R_2R_3}$
- $R_1 + R_2 = R_2 + R_1$
- $R_1^*R_1^* = R_1^*$
- $(R_1^*)^* = R_1^*$
- $R\epsilon = \epsilon R = R$
- $R\Phi = \Phi R = \Phi$
- $R + \Phi = R$
- $\epsilon + RR^* = \epsilon + R^*R = R^*$
- $(R_1 + R_2)^* = (R_1^*R_2^*)^* = (R_1^* + R_2^*)^*$

Thank You!