CS 302.1 - Automata Theory

Lecture 01

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Introduction

In this course, we will look at:

- **Which problems are computable?**
  - Can we characterize them?
  - What about natural problems? Computers are exotic physics experiments after all.

- **Design abstract models of computation and try to understand what problems can be solved by them.**
  - Small models that are limited in power and can solve a subset of computable problems.
  - We will build increasingly powerful computational models as we go along.

- **What are the limits of computation?**
  - Are there problems that cannot be solved on the most powerful computers that will exist in the future.

Introduction

In this course, we will look at:

- Which problems are computable?
- Design abstract models of computation and try to understand what problems can be solved by them.
- What are the limits of computational models?

Consider an (extremely) simple robot which

- has a button that turns it ON and OFF
- once turned on, can either move forward or backwards
- has a sensor that recognizes an obstacle and reverses the direction of the robot.

Introduction

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States : {OFF, FORWARD, BACKWARD}

Inputs : {BUTTON, SENSOR}

Initial state: OFF

By accepting an INPUT (signal), the robot TRANSITIONS from one state to another

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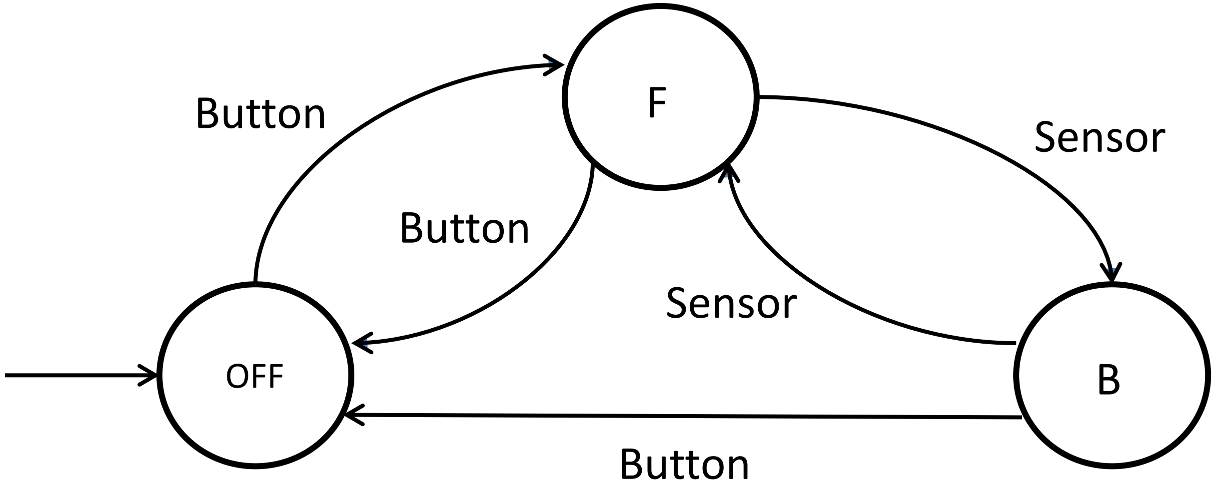
	BUTTON	SENSOR
OFF	F	X
F	OFF	B
B	OFF	F

State Transition Table



State diagram for the robot

Introduction

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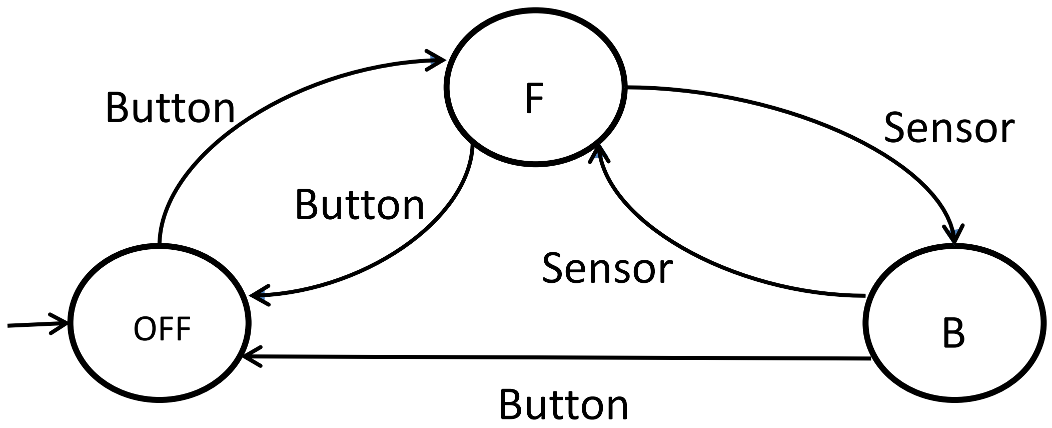


- Often computational tasks do not require an all powerful computer
- Examples: this robot, elevators, automatic doors, vending machines, ATMs etc.
- Design computational models with varying degrees of power and classify them.
- For a particular computational model, try to classify all the *problems* that can be solved by the model and those that can't be.

Introduction

In this course, we will ask questions such as :

- Can a given problem be computed by a particular computational model?

Let us explore what is meant by this.

Problem	Problem Instance
$\int f(x)dx$	$\int \sin x \, dx$
Sorting	$\frac{\pi}{3}, \frac{1}{2}, 2, \ldots\ldots\ldots\ldots\ldots$

Problem vs a specific instance of a problem

Problem vs decision problem: In order to answer these questions, we will always convert a given problem into a *decision* (YES-NO) *problem.*

Introduction

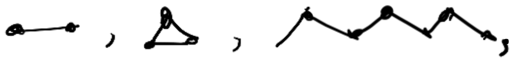
- Can a given problem be computed by a particular computational model?

Problem vs decision problem: In order to answer these questions, we will always convert a given problem into a *decision* (YES-NO) *problem.*

Problem	Decision problem
Sorting	Is the array sorted?
Graph connectivity	Is the graph connected?

By converting a problem into a decision problem is that we obtain two sets :

A YES set containing all the *instances* where the answer is YES.
A NO set containing all the *instances* where the answer is NO.

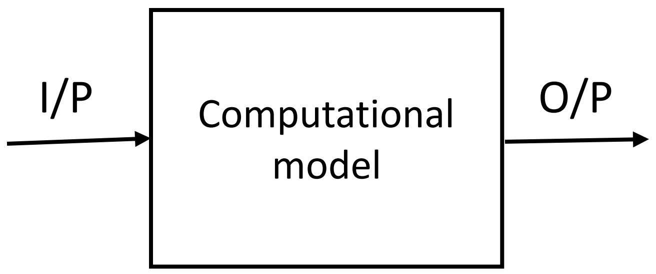
Problem: Graph Connectivity
YES set : {  ,.......}
NO set : {  ……}

Given an input instance, the computer can simply check to which set it belongs to and output accordingly.

Introduction

In this course, we will also ask questions such as :

- Can a given problem be computed by a particular computational model?



A computational model solves a problem P if,

(i) For all inputs belonging to the YES instance of P, the device outputs **YES/ACCEPT**.

AND

(ii) For all inputs belonging to the NO instance of P, the device outputs **NO/REJECT**.

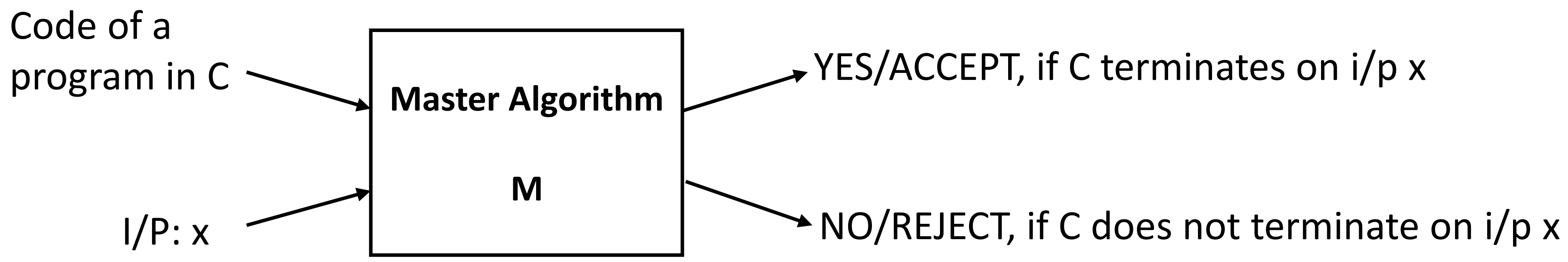
If (i) and (ii) hold, we say that the problem **P is computable** by this computational model.

Introduction

What are the limits of computability?

Can we have problems that cannot be solved by ANY computer, no matter how powerful?

Example 1: Master Algorithm



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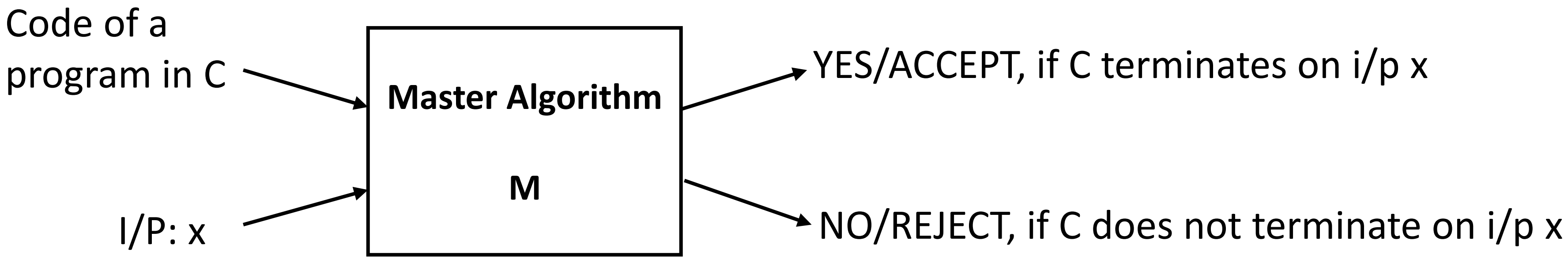
Example 1: Master Algorithm



This is not a formal proof

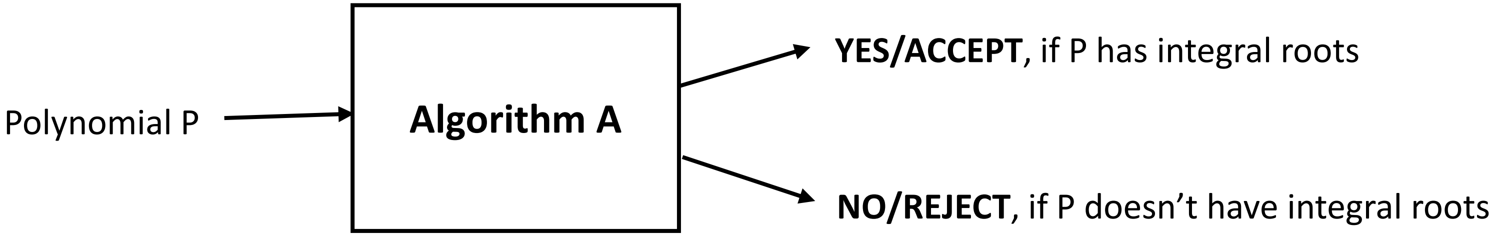
- **M terminates and outputs NO even if C(x) runs infinitely!**
- No such Algorithm M can be written. **Undecidable problem!**

Key takeaway: There are problems that are **not computable**.

Introduction

This is not a formal proof

Example 2: Does a polynomial P(x,y) with integral coefficients have integral roots?



Eg: Input Polynomial P: $x^3y^2 + xy^2 + 3x - 5 = 0$ O/P : YES/ACCEPT as (-1,1) are solutions to P

- The algorithm A proceeds by checking whether for integers $0, \pm 1, \pm 2, \cdots$. It terminates and outputs YES, whenever it finds the roots.

- What if P does not have integral roots? Algorithm A will run forever and will never terminate to output NO.

- **Undecidable problem! Key takeaway:** There are problems that are **not computable.**

Introduction

In this course we will:

- We will consider different computational models and classify them based on the problems they can solve
- Start from simple models and gradually increase their power to accommodate real computers
- Identify the problems that are not computable.

In this course we will not:

- Deal with how much time or space (memory) an algorithm would need to solve a certain problem
- Classifying the hardness of computable problems falls under the purview of Complexity Theory

Course Structure

- ❖ 13 Lectures in all

- ❖ Final Exam at the end (**35% weightage**)

- ❖ Two theory assignments (**25% weightage**)
    - Assignment 1 – will be released after Lec 3/4 (Deadline: Before Quiz 1)
    - Assignment 2 – will be released after Quiz 1 (Deadline: the week before Final exam)

- ❖ Programming assignment **(20% weightage)**
    - Released after Lec 3/4 (Deadline: before Final Exam)

- ❖ Quiz (**20% weightage**)

Tutorials and TAs

- Tutorial sessions weekly: **Wednesdays, 11:40 AM – 12:40 PM**

- Teaching Associates:
  - **Aryaman Kolhe** (aryaman.kolhe@research.iiit.ac.in)
  - **S Rajendraprasad** (rajendraprasad.s@research.iiit.ac.in)
  - **Kandi Jayanth Reddy** (kandi.reddy@students.iiit.ac.in)
  - **Niranjan Nagumalli** (niranjan.nagumalli@research.iiit.ac.in)
  - **Anurag Peddi** (anurag.peddi@students.iiit.ac.in)
  - **Sreyas Saminathan** (sreyas.saminathan@research.iiit.ac.in)

- Tutorial sessions **are not just** going to be doubt clearing/problem solving sessions.
- Several **interesting topics** will be covered.
- **My email:** shchakra@iiit.ac.in
- **Lecture slides available at my homepage:** https://sites.google.com/view/shchakra/teaching/m25-automata-theory

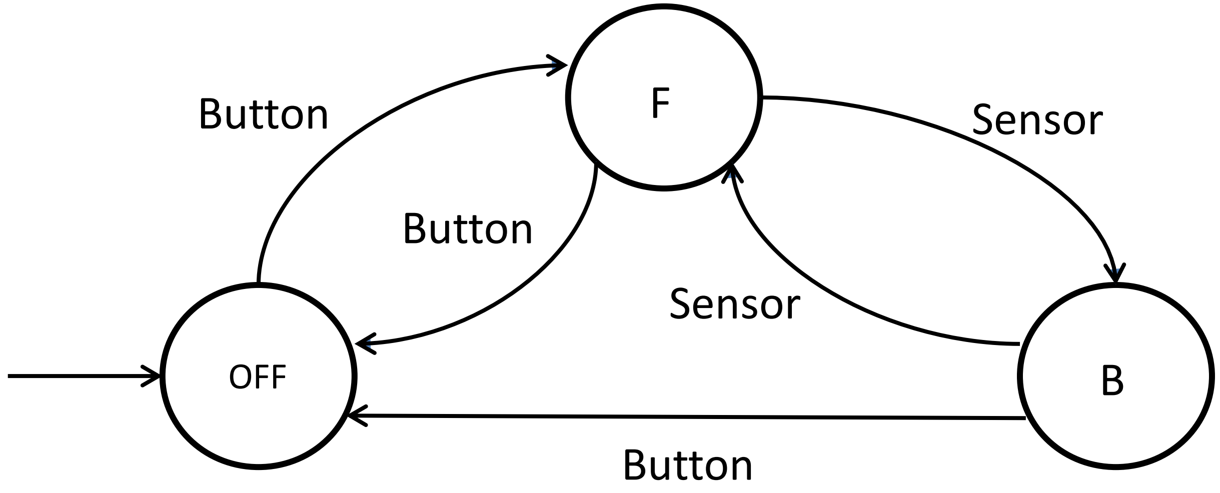
Some terminology

Alphabet	Strings/Words	Language
Any finite, non-empty set of symbols	Finite sequence of symbols from an alphabet.	Set of words/strings from the current alphabet
$\Sigma_1 = \{0,1\}$	0110, 000, 10, 10000,……	Even numbers
$\Sigma_2 = \{a, b, c, \dots, z\}$	any, word, revolution,…..	English

Generally, the empty string is denoted by ϵ

Models of computation

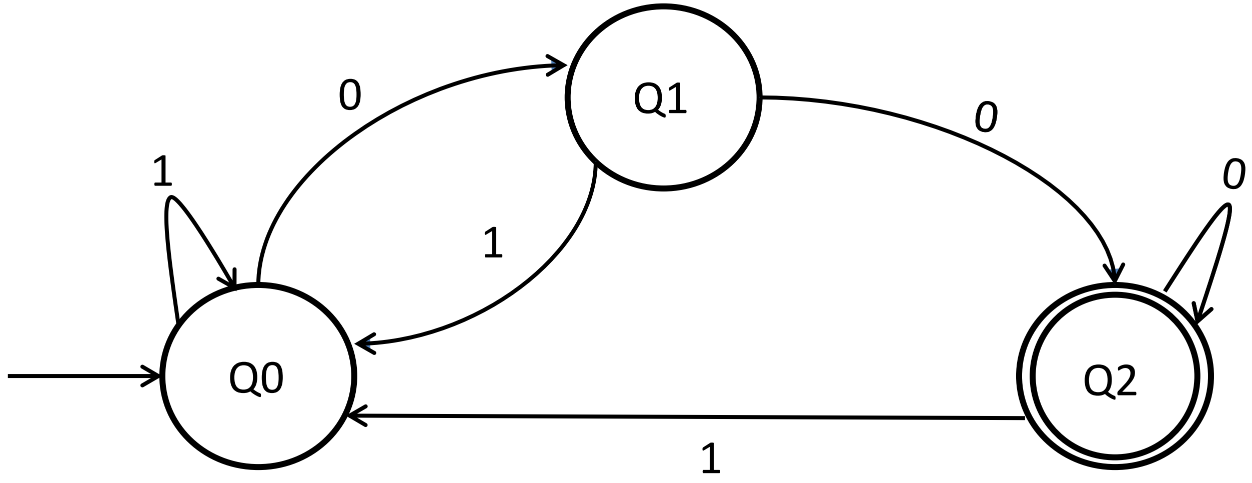
- **Deterministic Finite Automata (DFA) Model**



Models of computation

- **Deterministic Finite Automata (DFA) Model**

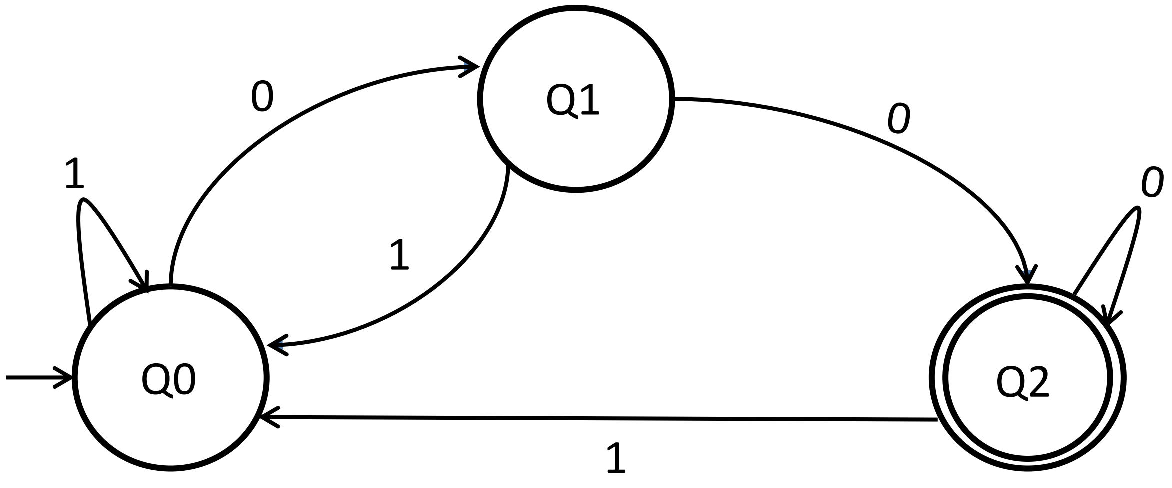
Q0: Start state, Q2: Final state



State transition diagram of the Finite State Machine

Characteristics: (i) Single start State, (ii) Unique Transitions, (iii) Zero or more final states

Deterministic Finite Automata (DFA)



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Input: Strings from alphabet $\Sigma = \{0,1\}$

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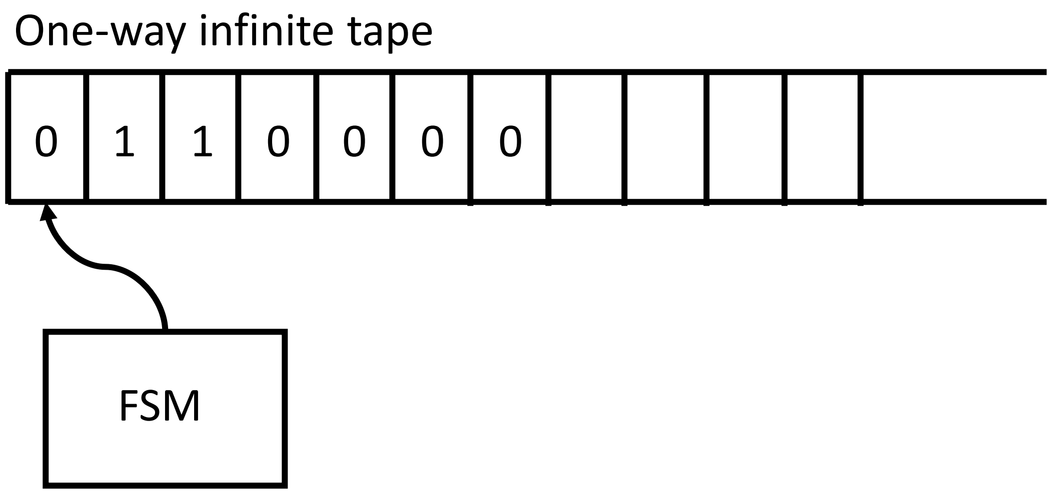

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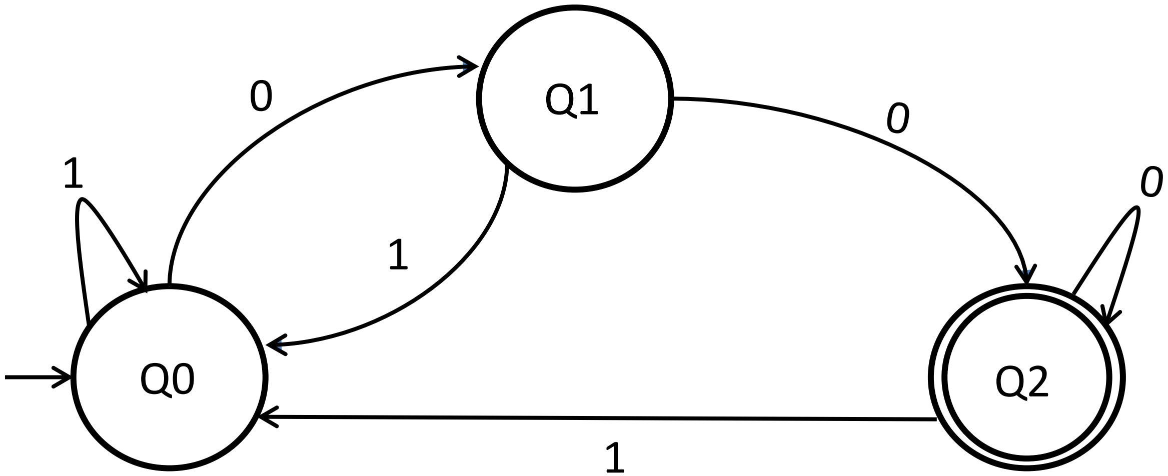

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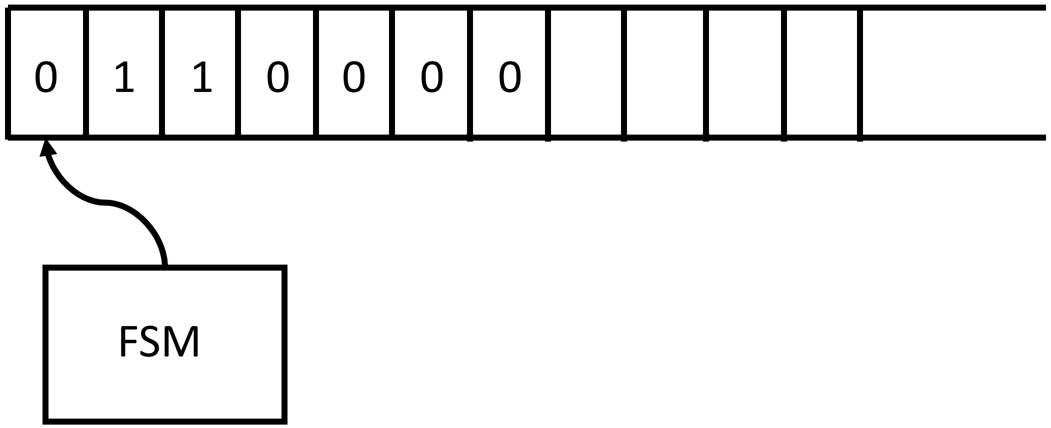
One-way infinite tape



Run:

$$\boldsymbol{Q0} \xrightarrow{0} Q1 \xrightarrow{1} Q0 \xrightarrow{1} Q0 \xrightarrow{0} Q1 \xrightarrow{0} Q2 \xrightarrow{0} Q2 \xrightarrow{0} \boldsymbol{Q2}$$

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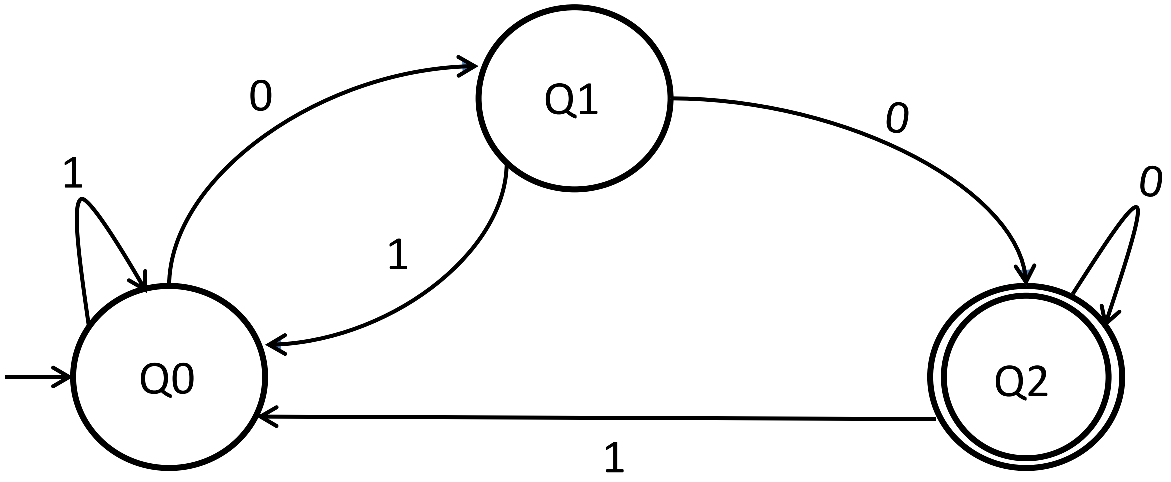

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The DFA "accepts" an input string, if it corresponds to a *run* that ends up in the final state Q2. **(Accepting Run)**

The DFA "rejects" an input string, if it corresponds to a *run* that ends up in any non-final state. **(Rejecting Run)**

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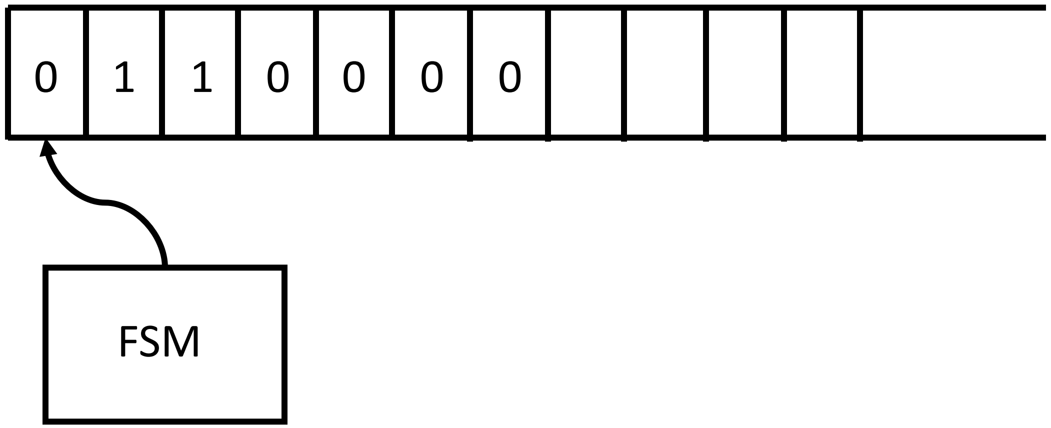

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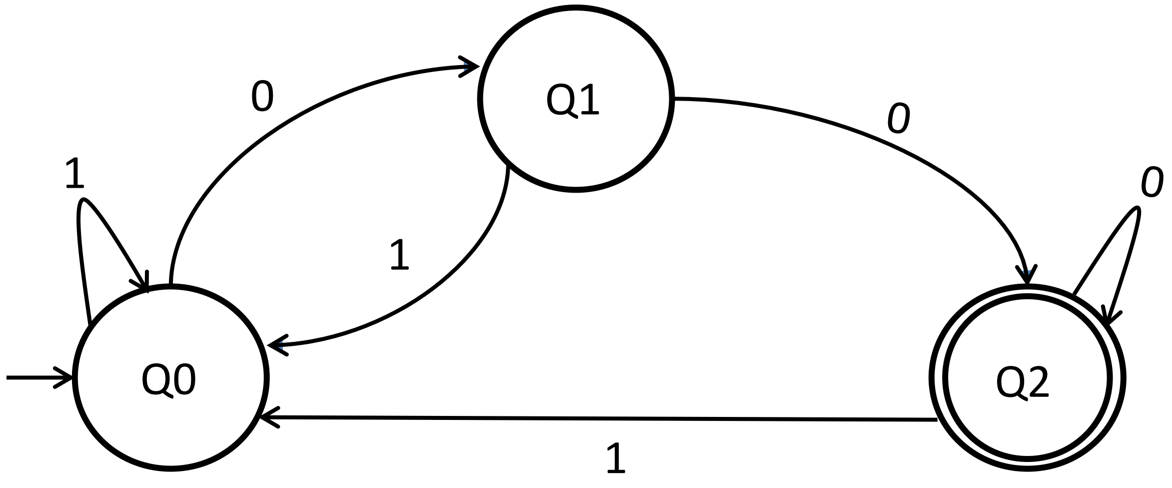

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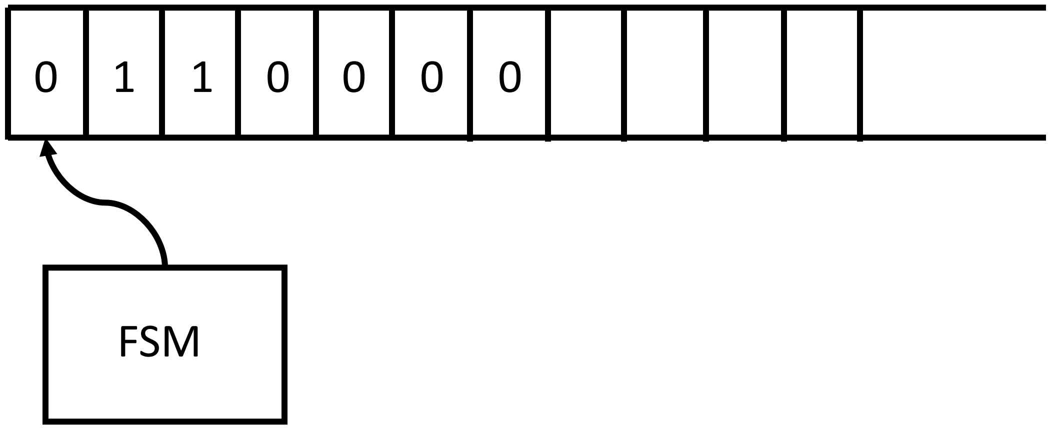

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ACCEPT = {0110000, }
REJECT = {}

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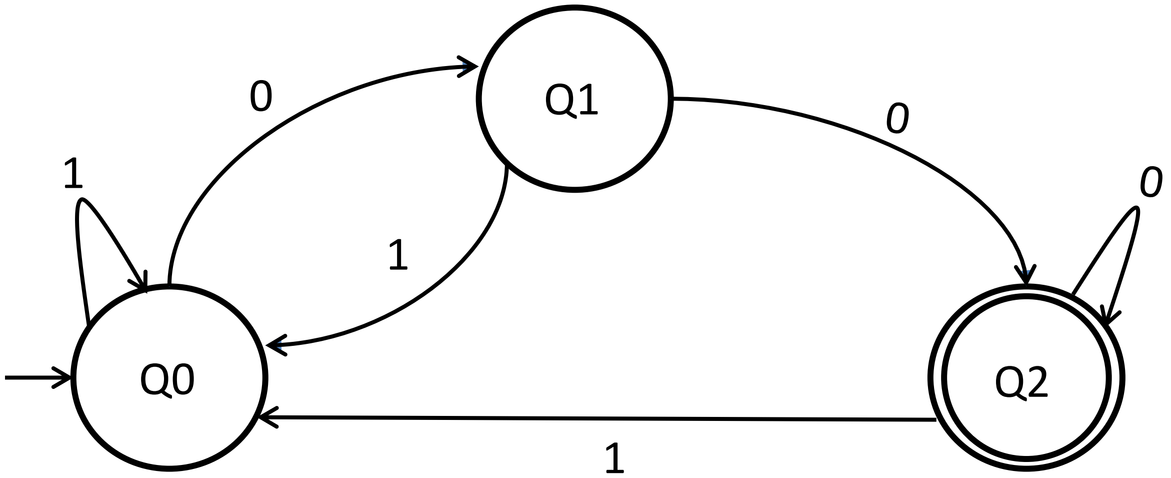

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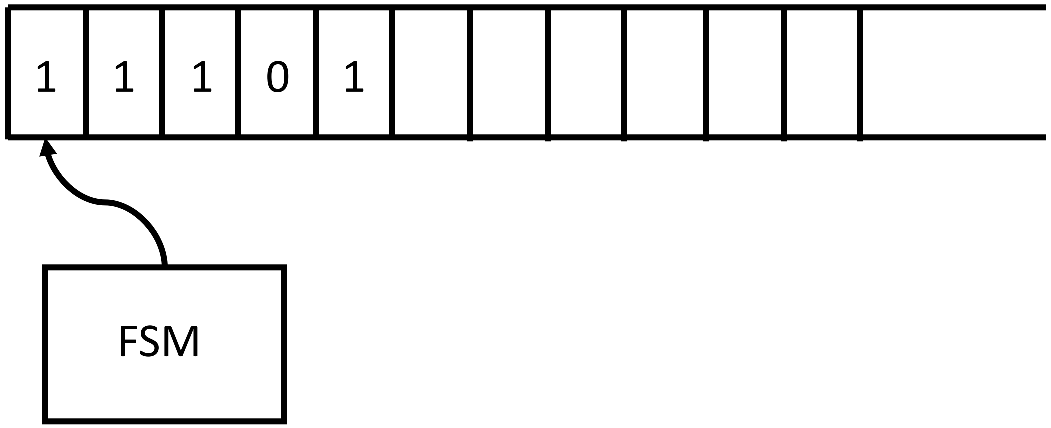

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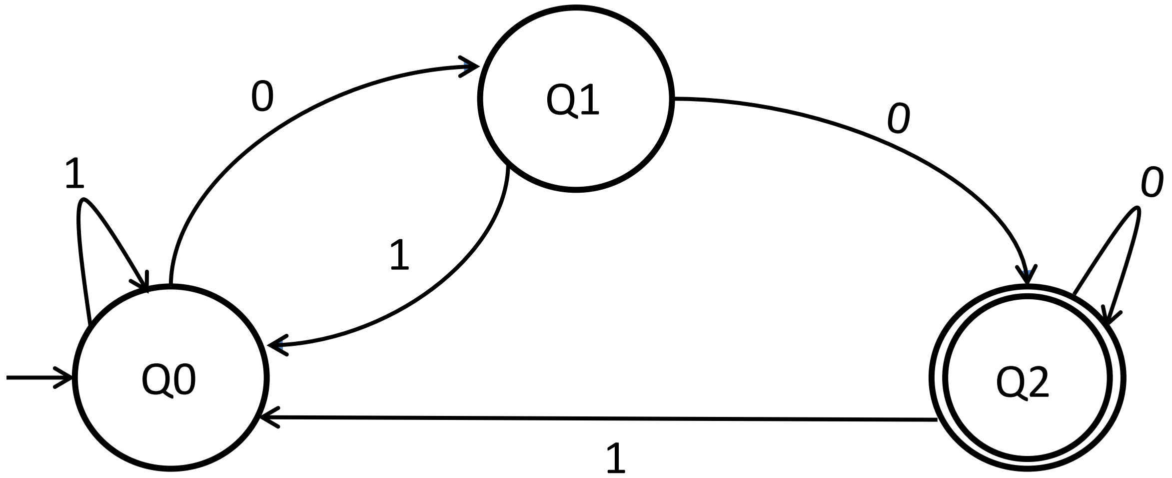

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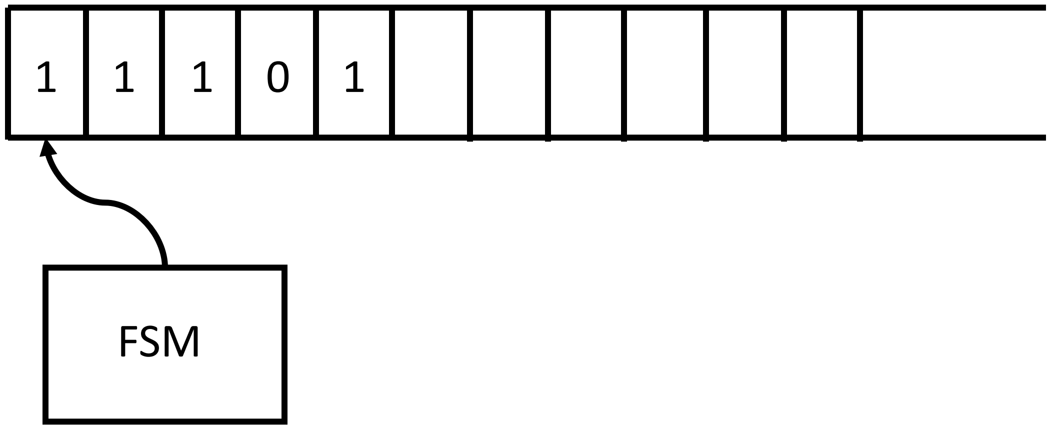

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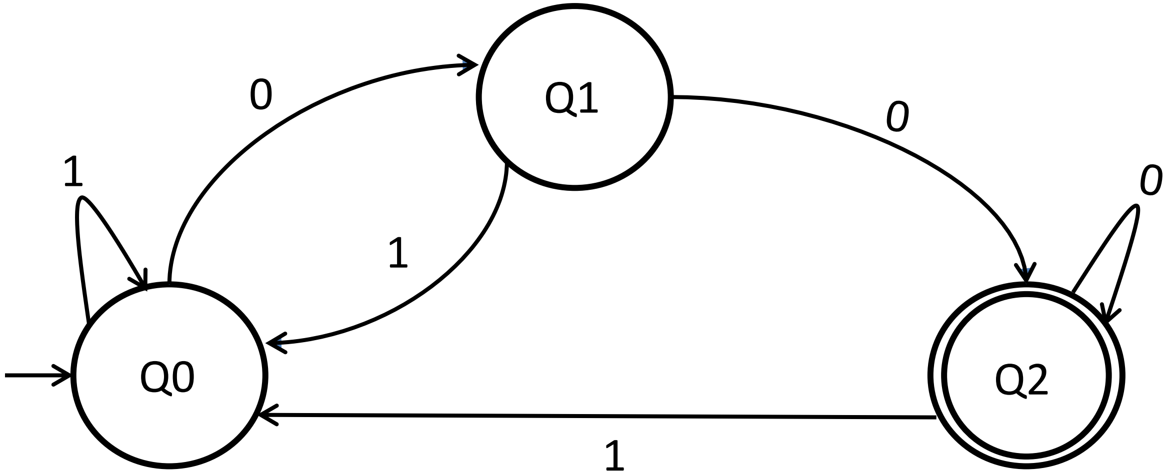

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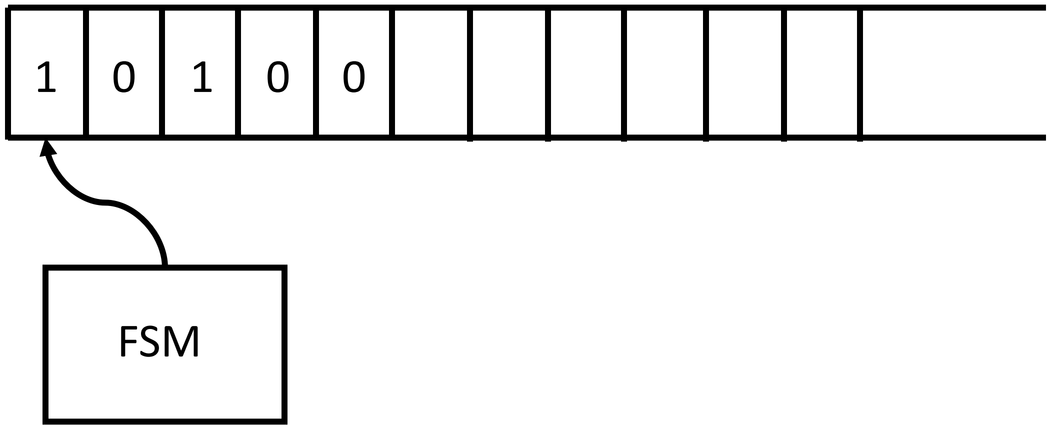

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ACCEPT = {0111000, 10100,….}
REJECT = {11101, …….}

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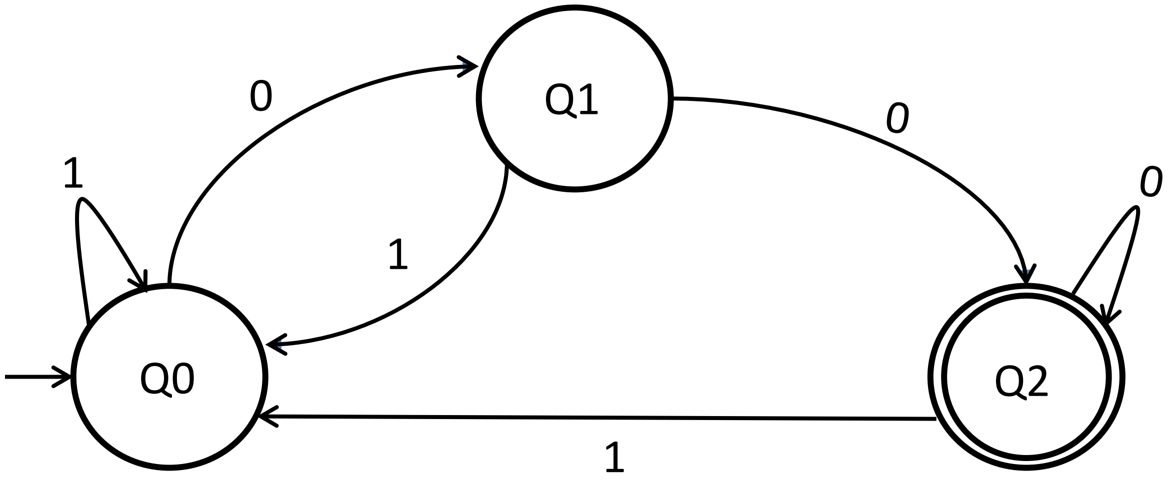

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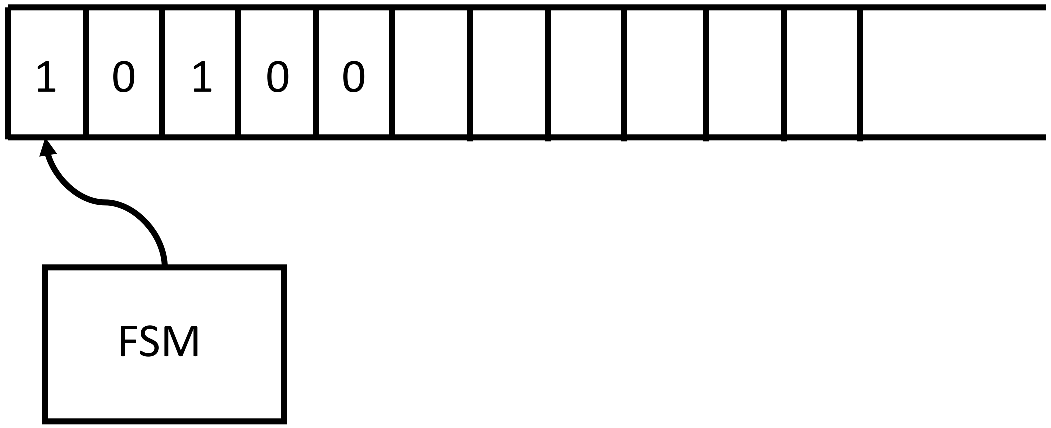

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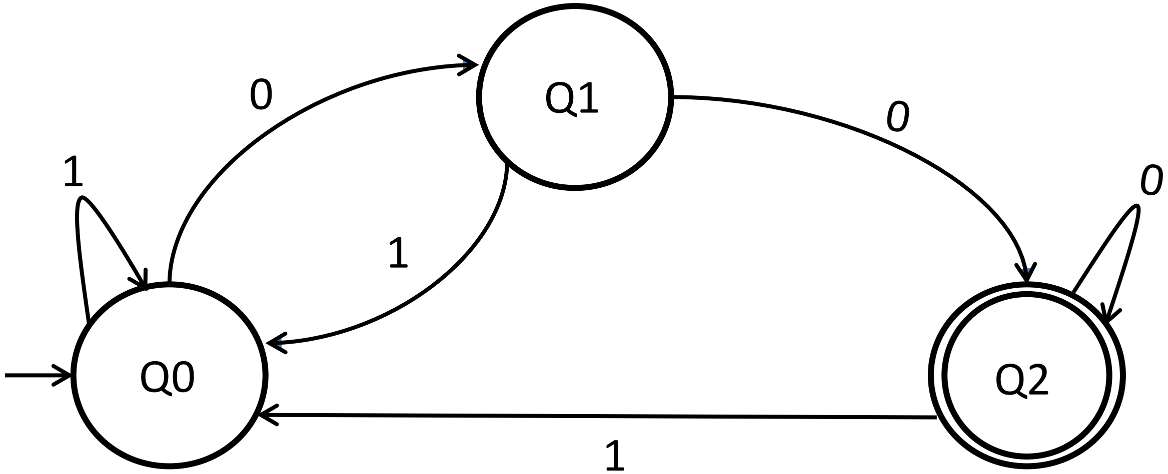

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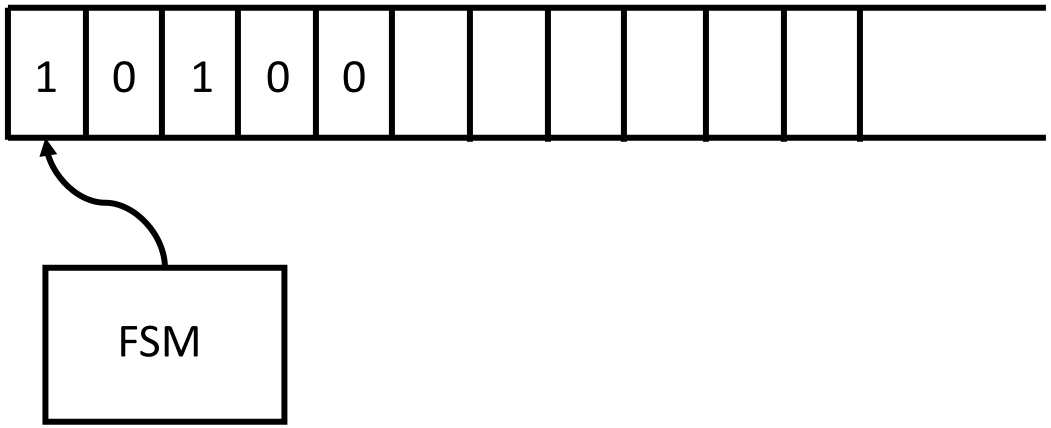

ACCEPT = {0111000, 10100, 0100, 00, 10000….}
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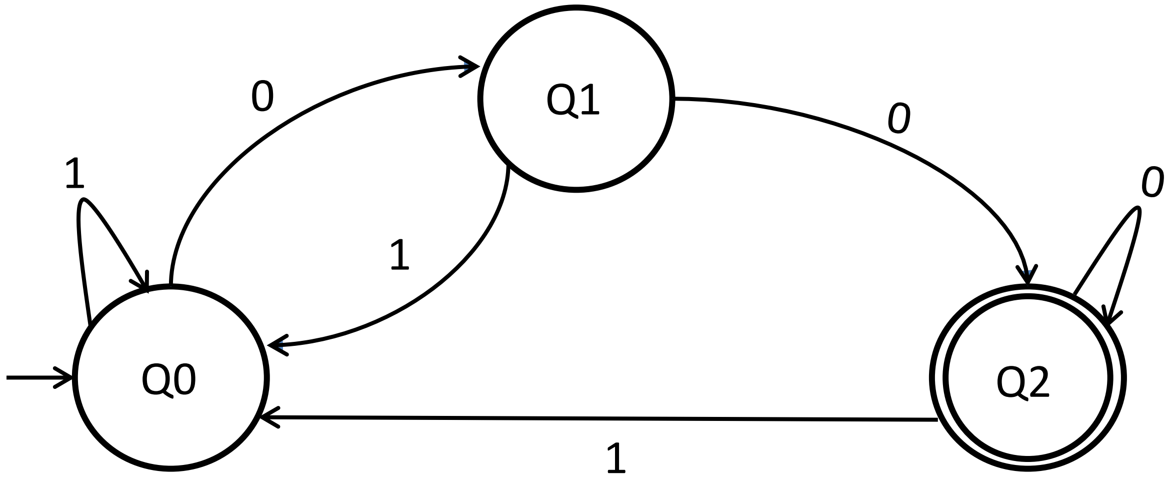
Let the DFA be M. Then, **language M accepts is**

L(M) = {$\omega|\omega$ results in an accepting run}, i.e. the set of all strings ω such that $M(\omega)$ accepts

For the example above, **L(M) = {$\omega|\omega$ ends in "00"}**

Deterministic Finite Automata (DFA)



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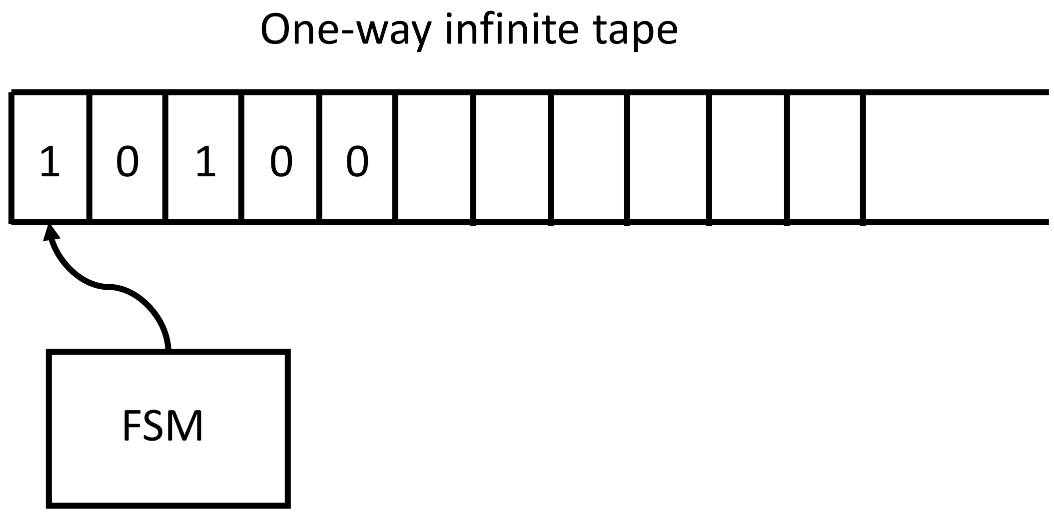


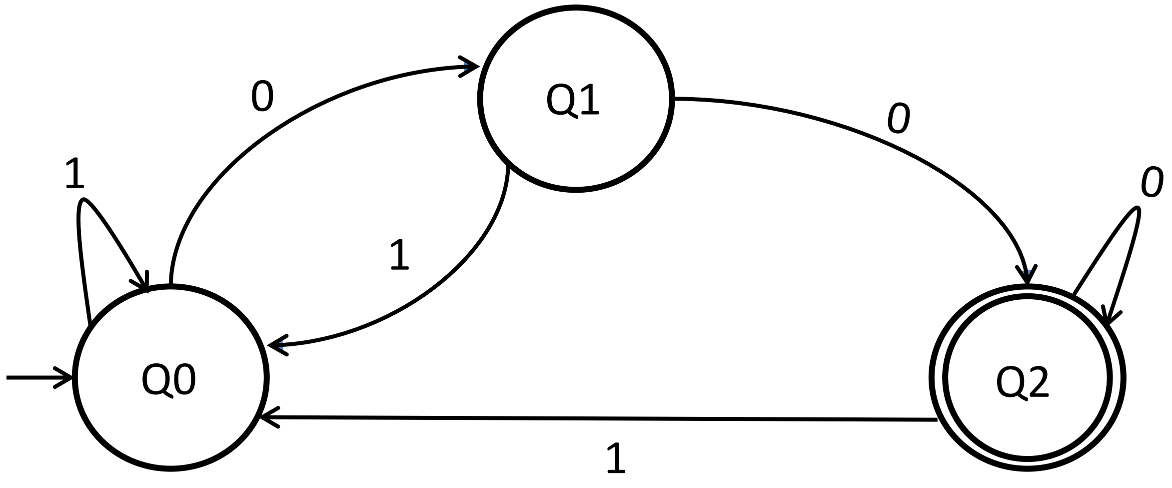
ACCEPT = {0111000, 10100, 0100, 00, 10000….}
REJECT = {11101, 0, 1, 11, 001,…….}

For any language L, we say **M solves L or M decides L if**

$$\forall \omega \in L, M(\omega) \text{ accepts}$$
$$\forall \omega \notin L, M(\omega) \text{ rejects}$$

For the example above, M **decides L= {$\omega|\omega$ ends in "00"}**

Deterministic Finite Automata (DFA)



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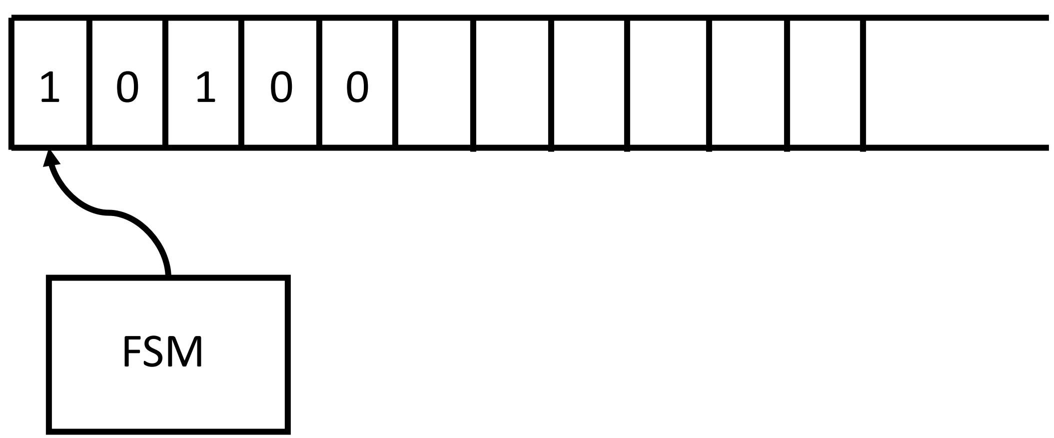

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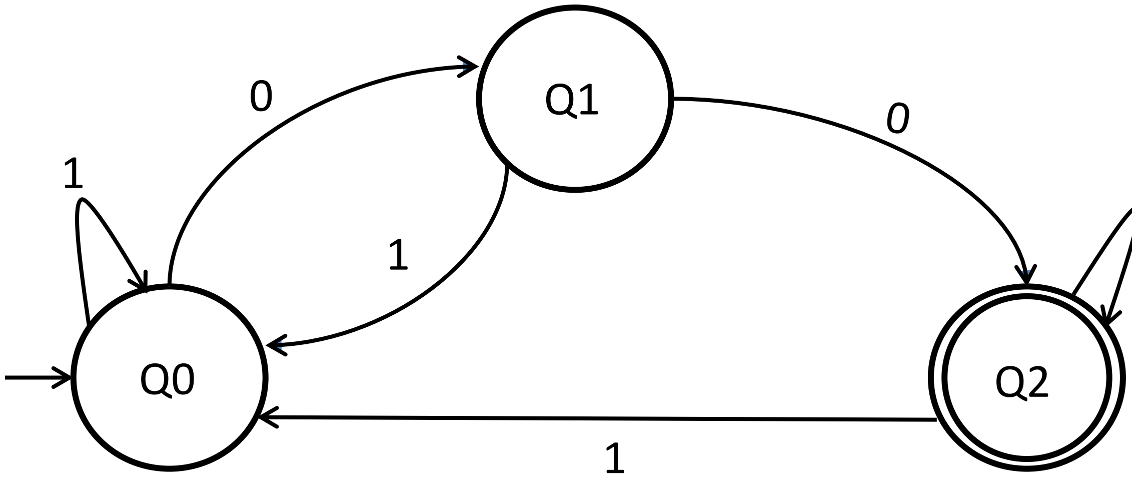

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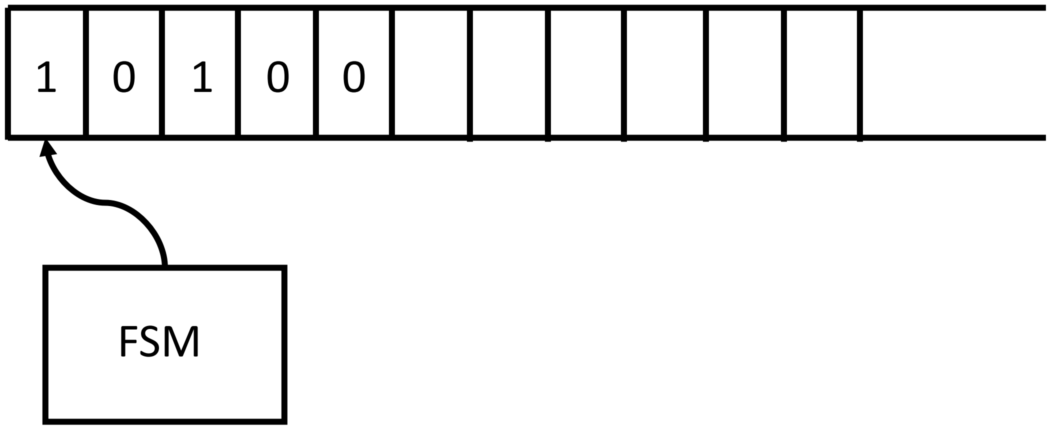
One-way infinite tape



Characteristics of DFA : (i) Single start state (ii) Unique transitions (iii) Zero or more final states

Formally, a finite automaton M is a 5-tuple $(\mathrm{Q}, \Sigma, \delta, q_0, F)$ where

- Q is a finite set called the ***states.*** — Q = {Q0, Q1, Q2}
- Σ is a finite set called the ***alphabet***. — $\Sigma = \{0,1\}$
- $\delta: Q \times \Sigma \mapsto Q$ is the ***transition function*** *(unique)*. — $(Q0,0) \mapsto Q1;\ (Q0,1) \mapsto Q0, \ldots\ldots, (Q2,1) \mapsto Q0$
- $q_0 \in Q$ is the ***start state***. — $q_0 = Q0$
- $F \subseteq Q$ are the ***final/accepting states***. — $F = Q2$

Thank You!