

Arithmetic

ADDITION OF POSITIVE NUMBERS

- Consider adding two 1-bit numbers.

The sum of 1 & 1 requires the 2-bit vector 10 to represent the value 2. We say that sum is 0 and the carry-out is 1.

0	1	0	1
+ 0	+ 0	+ 1	+ 1
-----	-----	-----	-----
0	1	1	10
			↑
			Carry-out

Figure 2.2 Addition of 1-bit numbers.

ADDITION & SUBTRACTION OF SIGNED NUMBERS

- Following are the two rules for addition and subtraction of n-bit signed numbers using the 2's complement representation system (Figure 1.6).

- Rule 1:**

- **To Add** two numbers, add their n-bits and ignore the carry-out signal from the MSB position.
- Result will be algebraically correct, if it lies in the range -2^{n-1} to $+2^{n-1}-1$.

- Rule 2:**

- **To Subtract** two numbers X and Y (that is to perform $X-Y$), take the 2's complement of Y and then add it to X as in rule 1.
- Result will be algebraically correct, if it lies in the range (-2^{n-1}) to $+(2^{n-1}-1)$.

- When the result of an arithmetic operation is outside the representable-range, an arithmetic overflow is said to occur.
- To represent a signed in 2's complement form using a larger number of bits, repeat the sign bit as many times as needed to the left. This operation is called **sign extension**.
- In 1's complement representation, the result obtained after an addition operation is not always correct. The carry-out(c_n) cannot be ignored. If $c_n=0$, the result obtained is correct. If $c_n=1$, then a 1 must be added to the result to make it correct.

OVERFLOW IN INTEGER ARITHMETIC

- When result of an arithmetic operation is outside the representable-range, an **arithmetic overflow** is said to occur.
- For example: If we add two numbers +7 and +4, then the output sum S is 1011 ($\square 0111+0100$), which is the code for -5, an incorrect result.
- An overflow occurs in following 2 cases
 1. Overflow can occur only when adding two numbers that have the same sign.
 2. The carry-out signal from the sign-bit position is not a sufficient indicator of overflow when adding signed numbers

(a)	$\begin{array}{r} 0010 \\ + 0011 \\ \hline 0101 \end{array}$	$\begin{array}{r} (+2) \\ (+3) \\ \hline (+5) \end{array}$	(b)	$\begin{array}{r} 0100 \\ + 1010 \\ \hline 1110 \end{array}$	$\begin{array}{r} (+4) \\ (-6) \\ \hline (-2) \end{array}$
(c)	$\begin{array}{r} 1011 \\ + 1110 \\ \hline 1001 \end{array}$	$\begin{array}{r} (-5) \\ (-2) \\ \hline (-7) \end{array}$	(d)	$\begin{array}{r} 0111 \\ + 1101 \\ \hline 0100 \end{array}$	$\begin{array}{r} (+7) \\ (-3) \\ \hline (+4) \end{array}$
(e)	$\begin{array}{r} 1101 \\ - 1001 \\ \hline \end{array}$	$\begin{array}{r} (-3) \\ (-7) \\ \hline \end{array}$	\Rightarrow	$\begin{array}{r} 1101 \\ + 0111 \\ \hline 0100 \end{array}$	$\begin{array}{r} \\ (+4) \\ \hline (+4) \end{array}$
(f)	$\begin{array}{r} 0010 \\ - 0100 \\ \hline \end{array}$	$\begin{array}{r} (+2) \\ (+4) \\ \hline \end{array}$	\Rightarrow	$\begin{array}{r} 0010 \\ + 1100 \\ \hline 1110 \end{array}$	$\begin{array}{r} \\ (-2) \\ \hline (-2) \end{array}$
(g)	$\begin{array}{r} 0110 \\ - 0011 \\ \hline \end{array}$	$\begin{array}{r} (+6) \\ (+3) \\ \hline \end{array}$	\Rightarrow	$\begin{array}{r} 0110 \\ + 1101 \\ \hline 0011 \end{array}$	$\begin{array}{r} \\ (+3) \\ \hline (+3) \end{array}$
(h)	$\begin{array}{r} 1001 \\ - 1011 \\ \hline \end{array}$	$\begin{array}{r} (-7) \\ (-5) \\ \hline \end{array}$	\Rightarrow	$\begin{array}{r} 1001 \\ + 0101 \\ \hline 1110 \end{array}$	$\begin{array}{r} \\ (-2) \\ \hline (-2) \end{array}$
(i)	$\begin{array}{r} 1001 \\ - 0001 \\ \hline \end{array}$	$\begin{array}{r} (-7) \\ (+1) \\ \hline \end{array}$	\Rightarrow	$\begin{array}{r} 1001 \\ + 1111 \\ \hline 1000 \end{array}$	$\begin{array}{r} \\ (-8) \\ \hline (-8) \end{array}$
(j)	$\begin{array}{r} 0010 \\ - 1101 \\ \hline \end{array}$	$\begin{array}{r} (+2) \\ (-3) \\ \hline \end{array}$	\Rightarrow	$\begin{array}{r} 0010 \\ + 0011 \\ \hline 0101 \end{array}$	$\begin{array}{r} \\ (+5) \\ \hline (+5) \end{array}$

Figure 1.6 2's-complement Add and Subtract operations.

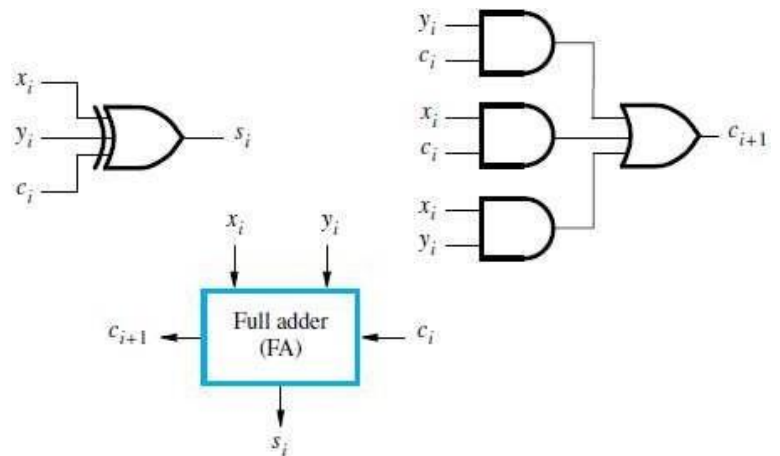
ADDITION & SUBTRACTION OF SIGNED NUMBERSn-BIT RIPPLE CARRY ADDER

- A cascaded connection of n full-adder blocks can be used to add 2-bit numbers.
- Since carries must propagate (or ripple) through cascade, the configuration is called an n-bit ripplecarry adder (Figure 9.1).

x_i	y_i	Carry-in c_i	Sum s_i	Carry-out c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s_i = \overline{x_i}\overline{y_i}c_i + \overline{x_i}y_ic_i + x_i\overline{y_i}\overline{c_i} + x_iy_ic_i = x_i \oplus y_i \oplus c_i$$

$$c_{i+1} = y_ic_i + x_ic_i + x_iy_i$$



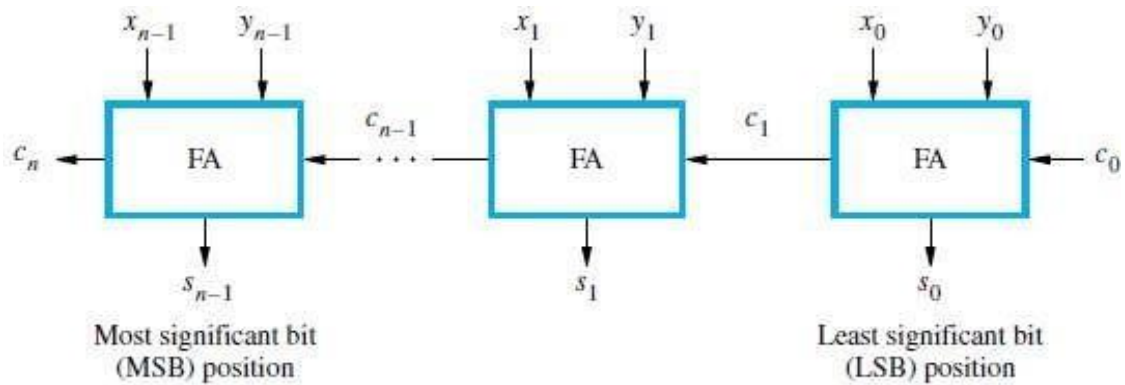
(a) Logic for a single stage

Example:

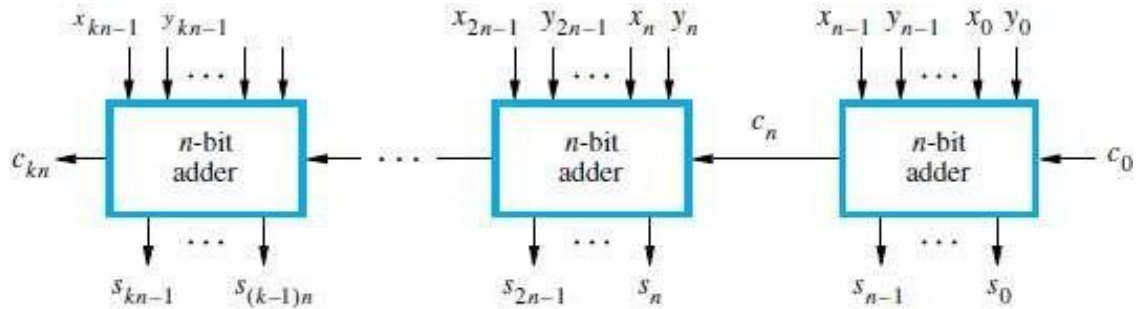
$$\begin{array}{r} X \\ + Y \\ \hline Z \end{array} = \begin{array}{r} 7 \\ + 6 \\ \hline 13 \end{array} = \begin{array}{r} 0111 \\ + 0110 \\ \hline 1101 \end{array}$$

Legend for stage i

Figure 9.1 Logic specification for a stage of binary addition.



(b) An n -bit ripple-carry adder



(c) Cascade of k n -bit adders

Figure 9.2 Logic for addition of binary numbers.

ADDITION/SUBTRACTION LOGIC UNIT

- The n -bit adder can be used to add 2's complement numbers X and Y (Figure 9.3).
- **Overflow** can only occur when the signs of the 2 operands are the same.
- In order to perform the subtraction operation $X - Y$ on 2's complement numbers X and Y ; we form the 2's complement of Y and add it to X .
- Addition or subtraction operation is done based on value applied to the Add/Sub input control-line.
- Control-line=0 for addition, applying the Y vector unchanged to one of the adder inputs. Control-line=1 for subtraction, the Y vector is 2's complemented.

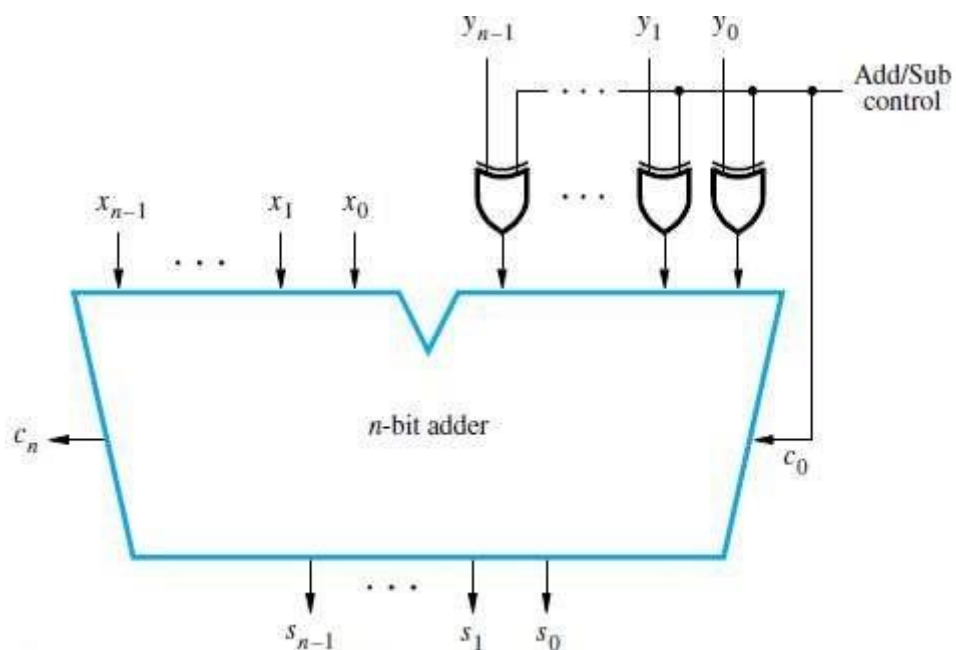


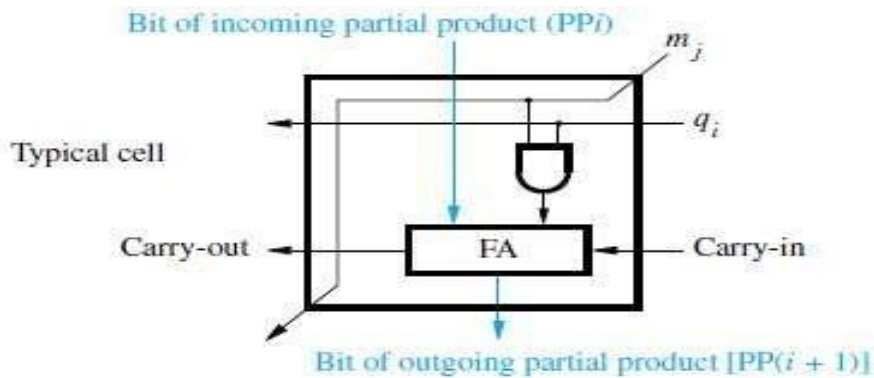
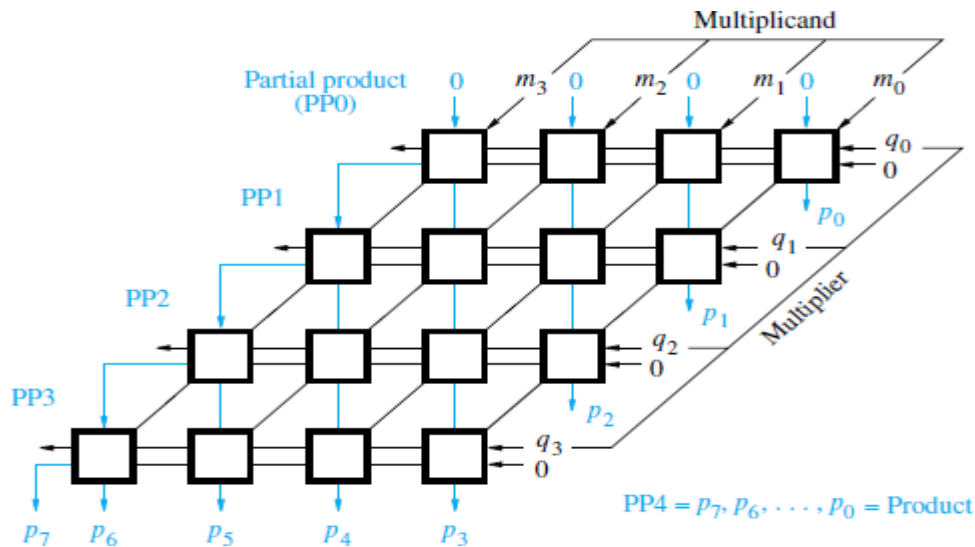
Figure 9.3 Binary addition/subtraction logic circuit.

MULTIPLICATION OF POSITIVE NUMBERS

$$\begin{array}{r}
 \begin{array}{cccc}
 & 1 & 1 & 0 & 1 \\
 \times & 1 & 0 & 1 & 1 \\
 \hline
 & 1 & 1 & 0 & 1 \\
 & 1 & 1 & 0 & 1 \\
 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 0 & 1 \\
 \hline
 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1
 \end{array}
 \end{array}$$

(13) Multiplicand M
(11) Multiplier Q
(143) Product P

(a) Manual multiplication algorithm



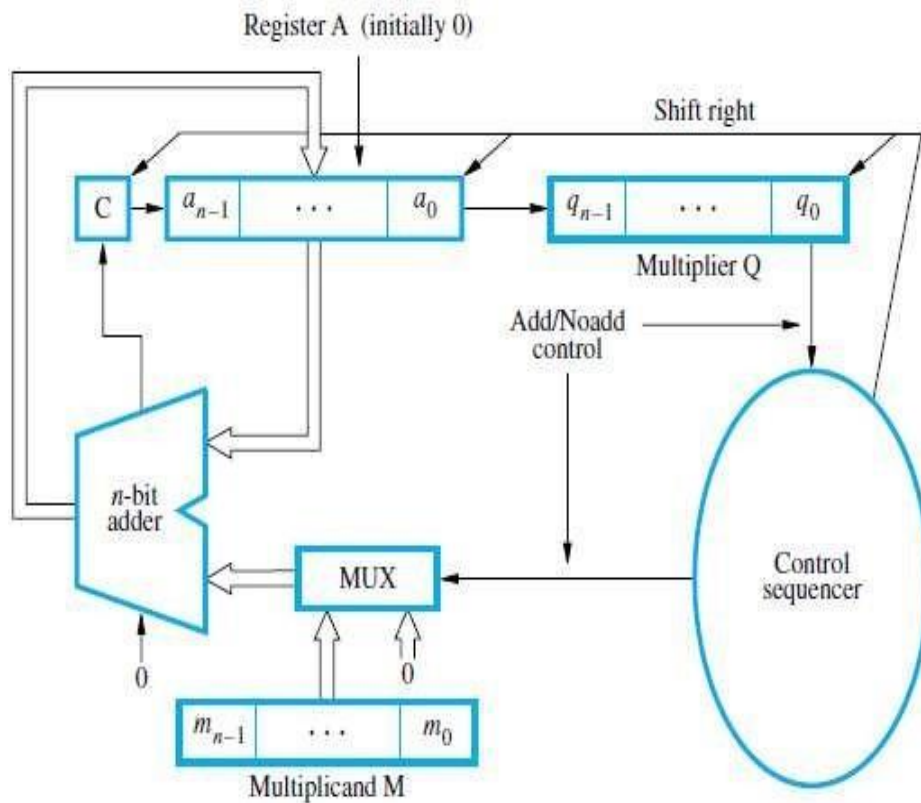
(b) Array implementation
Figure 9.6 Array multiplication of unsigned binary operands.

ARRAY MULTIPLICATION

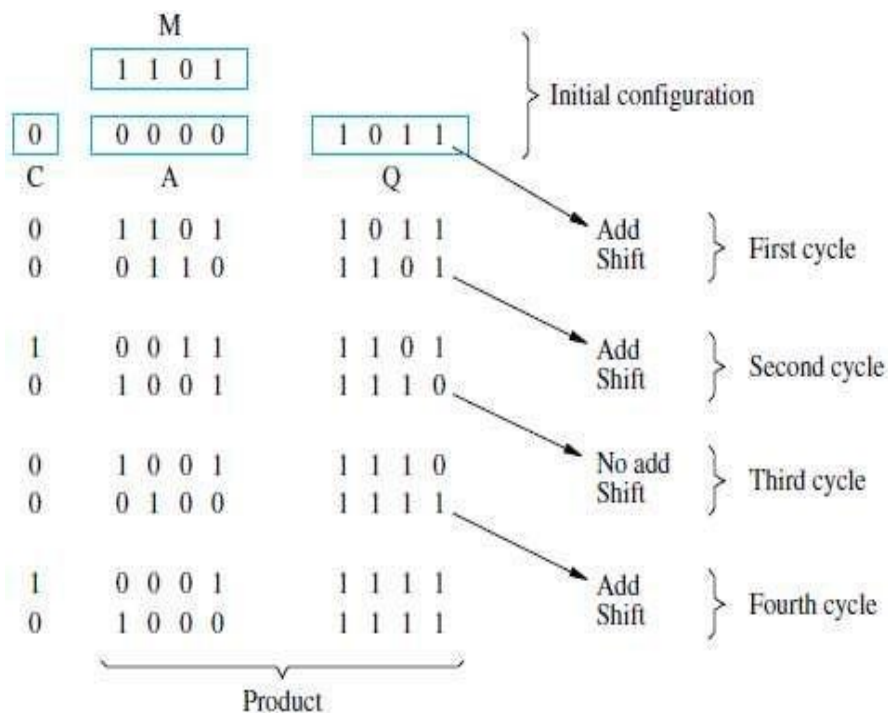
- The main component in each cell is a full adder(FA)..
The AND gate in each cell determines whether a multiplicand bit m_j , is added to the incoming partial-product bit, based on the value of the multiplier bit q_i (Figure9.6).

SEQUENTIAL CIRCUIT BINARY MULTIPLIER

- Registers A and Q combined hold PP_i (partial product) while the multiplier bit q_i generates the signal Add/Noadd.
- The carry-out from the adder is stored in flip-flop C (Figure 9.7).
- Procedure for multiplication:
 - Multiplier is loaded into register Q, Multiplicand is loaded into register M and C & A are cleared to 0.
 - If $q_0=1$, add M to A and store sum in A. Then C, A and Q are shifted right one bit-position.If $q_0=0$, no addition performed and C, A & Q are shifted right one bit-position.
 - After n cycles, the high-order half of the product is held in register A andthe low-order half is held in register Q.



(a) Register configuration



(b) Multiplication example

Figure 9.7 Sequential circuit binary multiplier.

SIGNED OPERAND MULTIPLICATION BOOTH ALGORITHM

- This algorithm
 - generates a 2n-bit product
 - treats both positive & negative 2's-complement n-bit operands uniformly (Figure 9.9-9.12).
- Attractive feature: This algorithm achieves some efficiency in the number of addition required when the multiplier has a few large blocks of 1s.
- This algorithm suggests that we can reduce the number of operations required for multiplication by representing multiplier as a difference between 2 numbers.

For e.g. multiplier(Q) 14(001110) can be represented as 010000 (16)

-000010 (2)

001110 (14)

- Therefore, product $P=M*Q$ can be computed by adding 2^4 times the M to the 2's complement of 2^1 times the M.

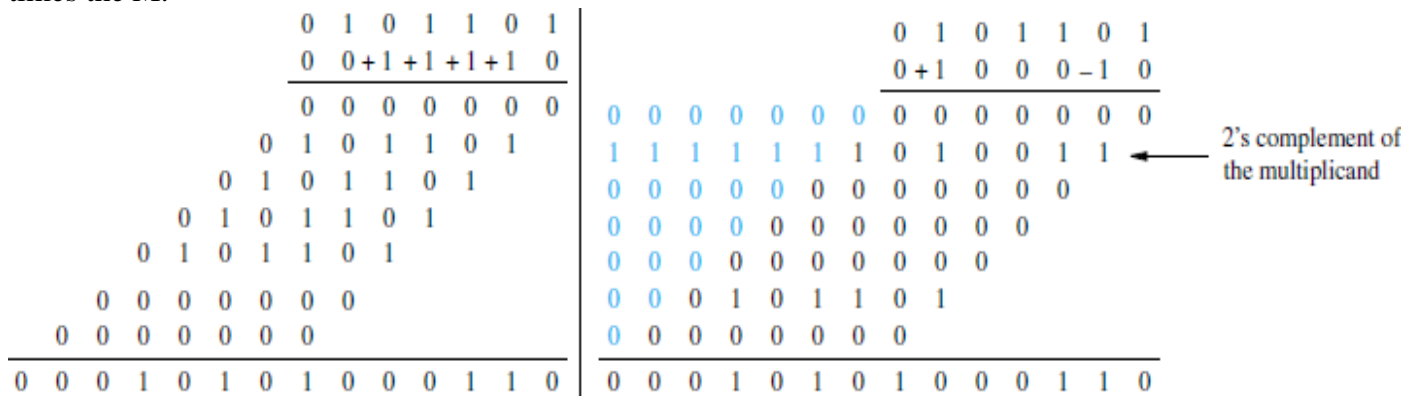


Figure 9.9 Normal and Booth multiplication schemes.

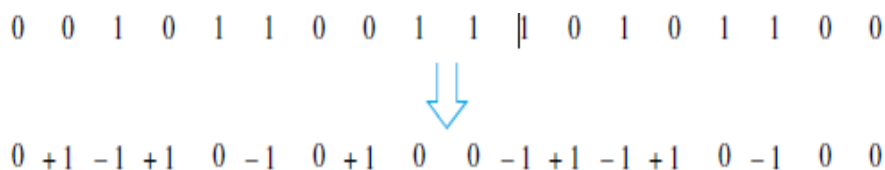


Figure 9.10 Booth recoding of a multiplier.

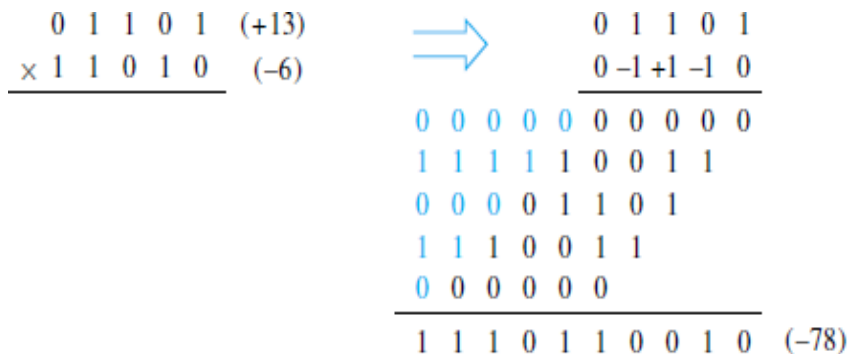


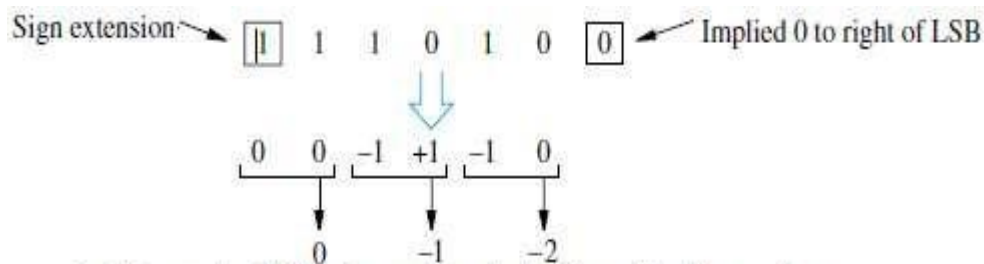
Figure 9.11 Booth multiplication with a negative multiplier.

Multiplier		Version of multiplicand selected by bit i
Bit i	Bit $i-1$	
0	0	$0 \times M$
0	1	$+1 \times M$
1	0	$-1 \times M$
1	1	$0 \times M$

Figure 9.12 Booth multiplier recoding table.

FAST MULTIPLICATION BIT-PAIR RECODING OF MULTIPLIERS

- This method
 - derived from the booth algorithm
 - reduces the number of summands by a factor of 2
- Group the Booth-recoded multiplier bits in pairs. (Figure 9.14 & 9.15).
- The pair (+1 -1) is equivalent to the pair (0 +1).



(a) Example of bit-pair recoding derived from Booth recoding

Multiplier bit-pair		Multiplier bit on the right	Multiplicand selected at position i
$i+1$	i	$i-1$	
0	0	0	$0 \times M$
0	0	1	$+1 \times M$
0	1	0	$+1 \times M$
0	1	1	$+2 \times M$
1	0	0	$-2 \times M$
1	0	1	$-1 \times M$
1	1	0	$-1 \times M$
1	1	1	$0 \times M$

(b) Table of multiplicand selection decisions

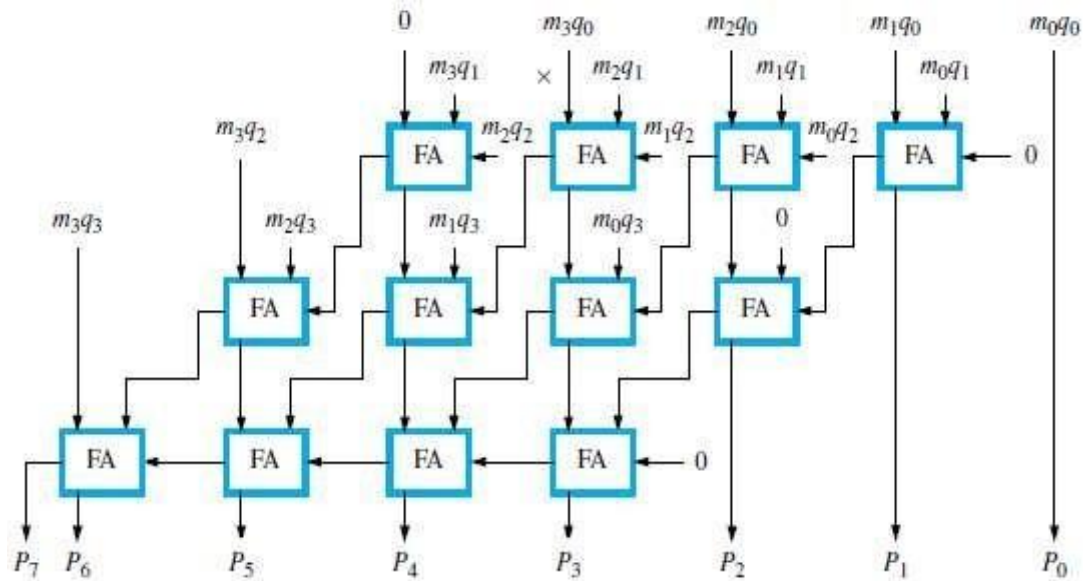
Figure 9.14 Multiplier bit-pair recoding.

$$\begin{array}{r}
 \begin{array}{cccccc} 0 & 1 & 1 & 0 & 1 & (+13) \\ \times & 1 & 1 & 0 & 1 & 0 & (-6) \\ \hline \end{array} \\
 \Downarrow \\
 \begin{array}{r}
 \begin{array}{cccccc} 0 & 1 & 1 & 0 & 1 \\ 0 & -1 & +1 & -1 & 0 \\ \hline \end{array} \\
 \begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & \\ \hline \end{array} \\
 \begin{array}{cccccccc} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & (-78) \end{array} \\
 \Downarrow \\
 \begin{array}{r}
 \begin{array}{cccccc} 0 & 1 & 1 & 0 & 1 \\ 0 & -1 & -2 & & \\ \hline \end{array} \\
 \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & & \\ \hline \end{array} \\
 \begin{array}{cccccccc} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}
 \end{array}$$

Figure 9.15 Multiplication requiring only $n/2$ summands.

CARRY-SAVE ADDITION OF SUMMANDS

- Consider the array for 4*4 multiplication. (Figure 9.16 & 9.18).
- Instead of letting the carries ripple along the rows, they can be "saved" and introduced into the next row, at the correct weighted positions.



(b) Carry-save array

Figure 9.16 carry-save arrays for a 4×4 multiplier.

				1	0	1	1	0	1	(45) M
				×	1	1	1	1	1	(63) Q
				<hr/>						
				1	0	1	1	0	1	A
			1	0	1	1	0	1		B
		1	0	1	1	0	1			C
	1	0	1	1	0	1				D
	1	0	1	1	0	1				E
1	0	1	1	0	1					F
<hr/>										
0	1	1	0	0	0	1	0	0	1	1 (2,835) Product

Figure 9.17 A multiplication example used to illustrate carry-save addition as shown in Figure 9.18.

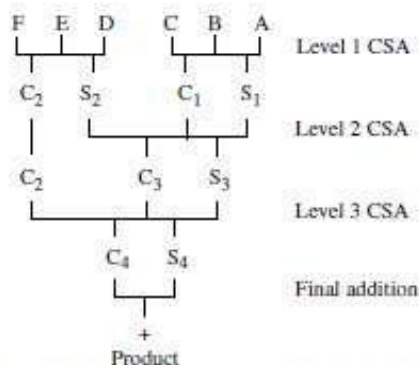


Figure 9.19 Schematic representation of the carry-save addition operations in Figure 9.18.

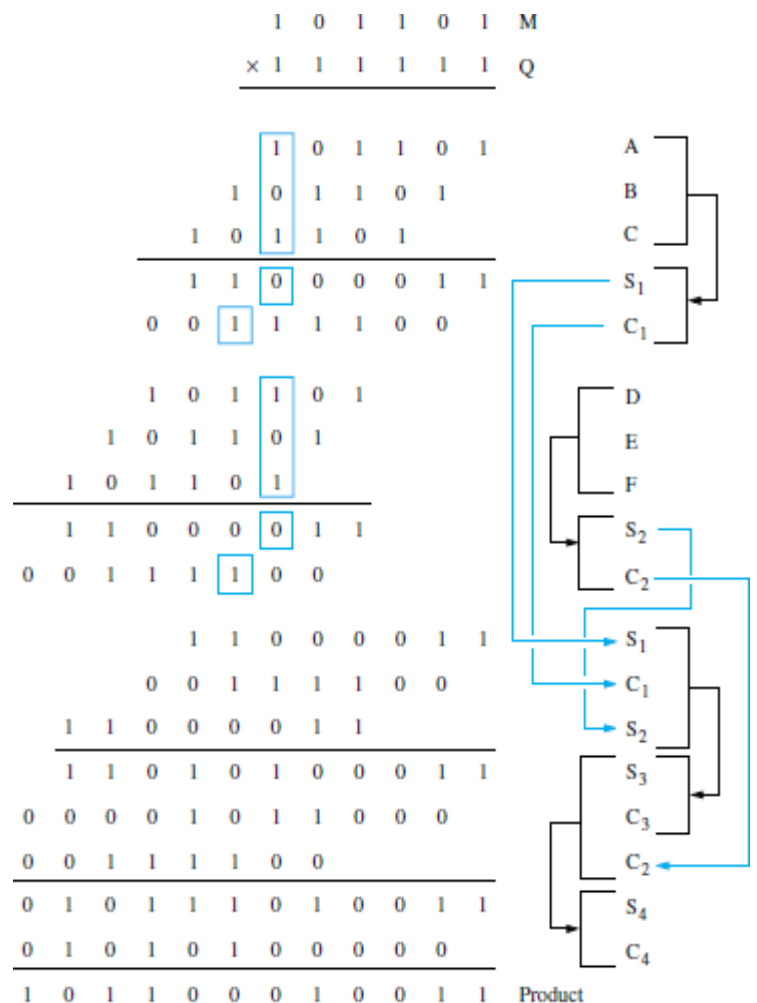


Figure 9.18 The multiplication example from Figure 9.17 performed using carry-save addition.

- The full adder is input with three partial bit products in the first row.
- Multiplication requires the addition of several summands.
- CSA speeds up the addition process.
- Consider the array for 4x4 multiplication shown in fig 9.16.
- First row consisting of just the AND gates that implement the bit products m_3q_0 , m_2q_0 , m_1q_0 and m_0q_0 .
- The delay through the carry-save array is somewhat less than delay through the ripple-carry array. This is because the S and C vector outputs from each row are produced in parallel in one full-adder delay.
- Consider the addition of many summands in fig 9.18.
- Group the summands in threes and perform carry-save addition on each of these groups in parallel to generate a set of S and C vectors in one full-adder delay
- Group all of the S and C vectors into threes, and perform carry-save addition on them, generating a further set of S and C vectors in one more full-adder delay
- Continue with this process until there are only two vectors remaining
- They can be added in a RCA or CLA to produce the desired product.
- When the number of summands is large, the time saved is proportionally much greater.
- Delay: AND gate + 2 gate/CSA level + CLA gate delay, Eg., 6 bit number require 15 gate delay, array 6x6 require $6(n-1)-1 = 29$ gate Delay.
- In general, CSA takes $1.7 \log_2 k - 1.7$ levels of CSA to reduce k summands.

INTEGER DIVISION

- An n-bit positive-divisor is loaded into register M.
An n-bit positive-dividend is loaded into register Q at the start of the operation.
Register A is set to 0 (Figure 9.21).
- After division operation, the n-bit quotient is in register Q, and the remainder is in register A.

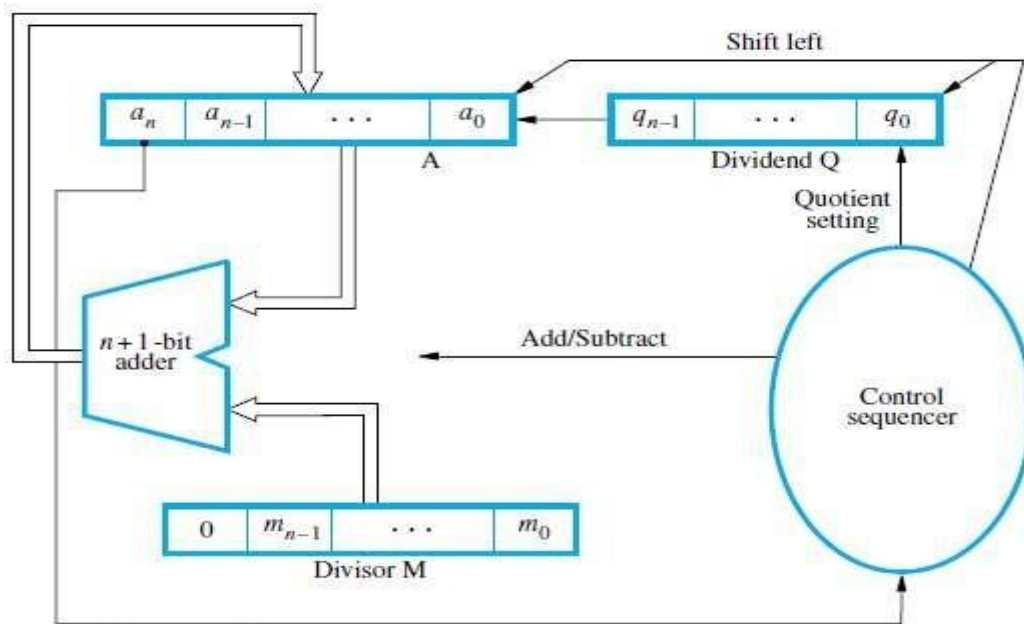


Figure 9.23 Circuit arrangement for binary division.

$$\begin{array}{r}
 21 \\
 13 \overline{) 274} \\
 \underline{26} \\
 14 \\
 \underline{13} \\
 1
 \end{array}
 \qquad
 \begin{array}{r}
 10101 \\
 1101 \overline{) 100010010} \\
 \underline{1101} \\
 10000 \\
 \underline{1101} \\
 1110 \\
 \underline{1101} \\
 1
 \end{array}$$

Figure 9.22 Longhand division examples.

NON-RESTORING DIVISION

- Procedure:

Step 1: Do the following n times

i) If the sign of A is 0, shift A and Q left one bit position and subtract M from A; otherwise, shift A and Q left and add M to A (Figure 9.23).

ii) Now, if the sign of A is 0, set q_0 to 1; otherwise set q_0 to 0. Step 2: If the sign of A is 1, add M to A (restore).

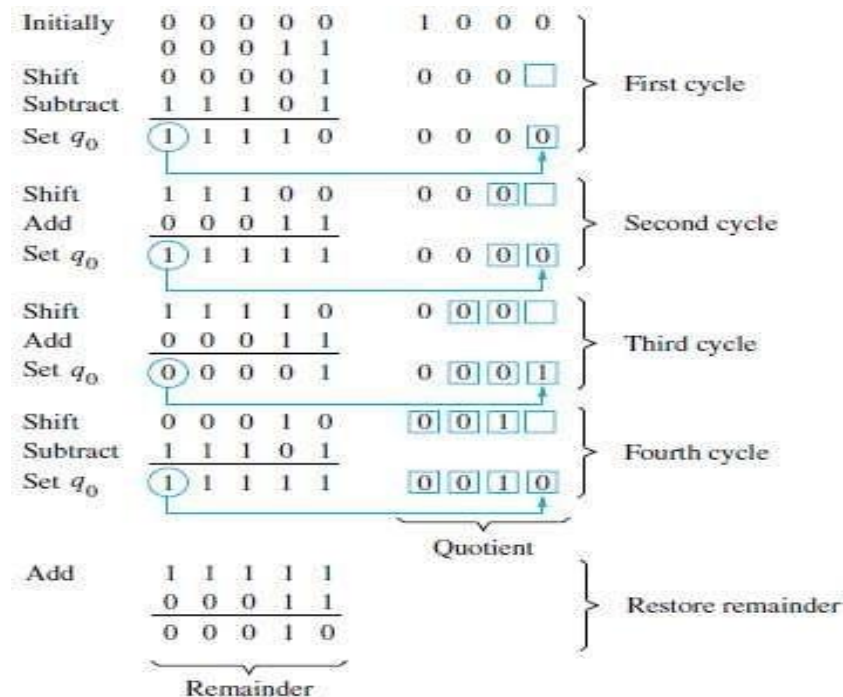


Figure 9.25 A non-restoring division example.

RESTORING DIVISION

- Procedure: Do the following n times

1) Shift A and Q left one binary position (Figure 9.22).

2) Subtract M from A, and place the answer back in A

3) If the sign of A is 1, set q_0 to 0 and add M back to A (restore A). If the sign of A is 0, set q_0 to 1 and no restoring done.

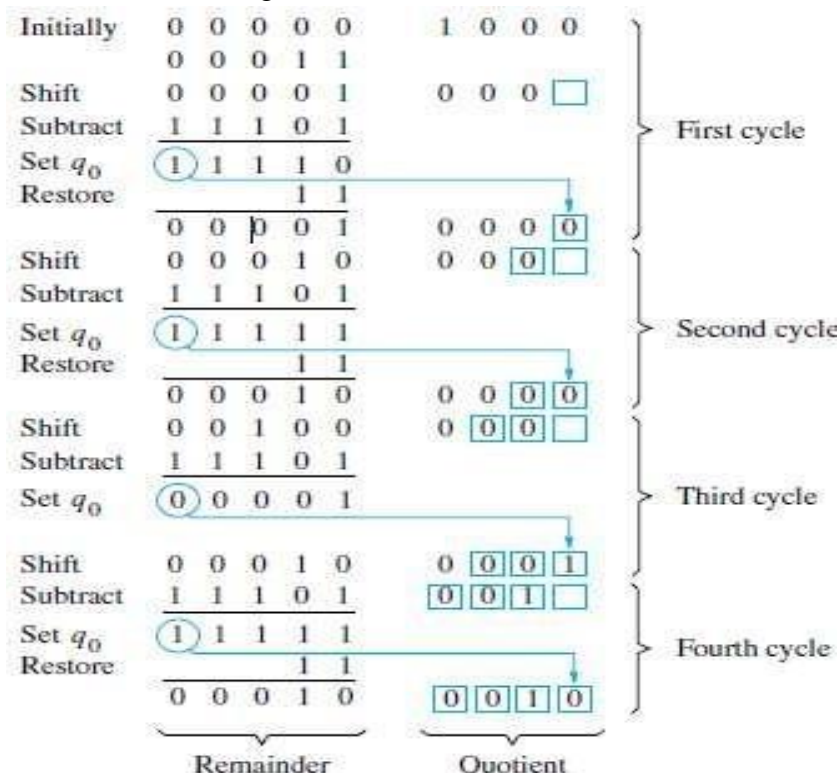


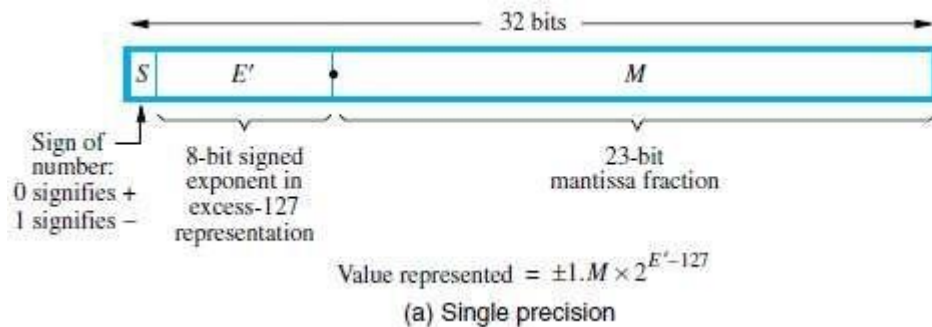
Figure 9.24 A restoring division example.

FLOATING-POINT NUMBERS & OPERATIONS IEEE STANDARD FOR FLOATING POINT NUMBERS

- Single precision representation occupies a single 32-bit word.
The scale factor has a range of 2^{-126} to 2^{+127} (which is approximately equal to 10^{+38}).
- The 32 bit word is divided into 3 fields: sign(1 bit), exponent(8 bits) and mantissa(23 bits).
- Signed exponent=E.

Unsigned exponent $E'=E+127$. Thus, E' is in the range $0 < E' < 255$.

- The last 23 bits represent the mantissa. Since binary normalization is used, the MSB of the mantissa is always equal to 1. (M represents fractional-part).
- The 24-bit mantissa provides a precision equivalent to about 7 decimal-digits (Figure 9.24).
- Double precision representation occupies a single 64-bit word. And E' is in the range $1 < E' < 2046$.



- The 53-bit mantissa provides a precision equivalent to about 16 decimal-digits.



Value represented = $1.001010 \dots 0 \times 2^{-87}$

(b) Example of a single-precision number

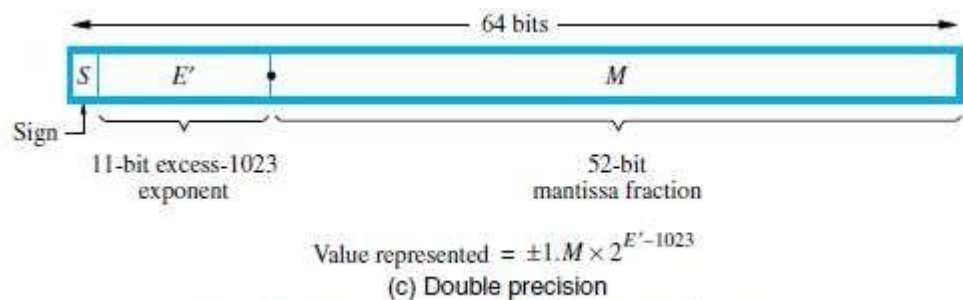


Figure 9.26 IEEE standard floating-point formats.

ARITHMETIC OPERATIONS ON FLOATING-POINT NUMBERS

• Multiply Rule

- 1) Add the exponents & subtract 127.
- 2) Multiply the mantissas & determine sign of the result.
- 3) Normalize the resulting value if necessary.

• Divide Rule

- 1) Subtract the exponents & add 127.
- 2) Divide the mantissas & determine sign of the result.
- 3) Normalize the resulting value if necessary.

• Add/Subtract Rule

- 1) Choose the number with the smaller exponent & shift its mantissa right a number of steps equal to the difference in exponents(n).
- 2) Set exponent of the result equal to larger exponent.
- 3) Perform addition/subtraction on the mantissas & determine sign of the result.
- 4) Normalize the resulting value if necessary.

IMPLEMENTING FLOATING-POINT OPERATIONS

- First compare exponents to determine how far to shift the mantissa of the number with the smaller exponent.
- The shift-count value n
 - is determined by 8 bit subtractor &
 - is sent to SHIFTER unit.
- In step 1, sign is sent to SWAP network (Figure 9.26).
 - If $\text{sign}=0$, then $E_A > E_B$ and mantissas M_A & M_B are sent straight through SWAP network. If
 - $\text{sign}=1$, then $E_A < E_B$ and the mantissas are swapped before they are sent to SHIFTER.
- In step 2, 2:1 MUX is used. The exponent of result E is tentatively determined as E_A if $E_A > E_B$ or E_B if $E_A < E_B$
- In step 3, CONTROL logic
 - determines whether mantissas are to be added or subtracted.
 - determines sign of the result.
- In step 4, result of step 3 is normalized. The number of leading zeros in M determines number of bit shifts(X) to be applied to M .

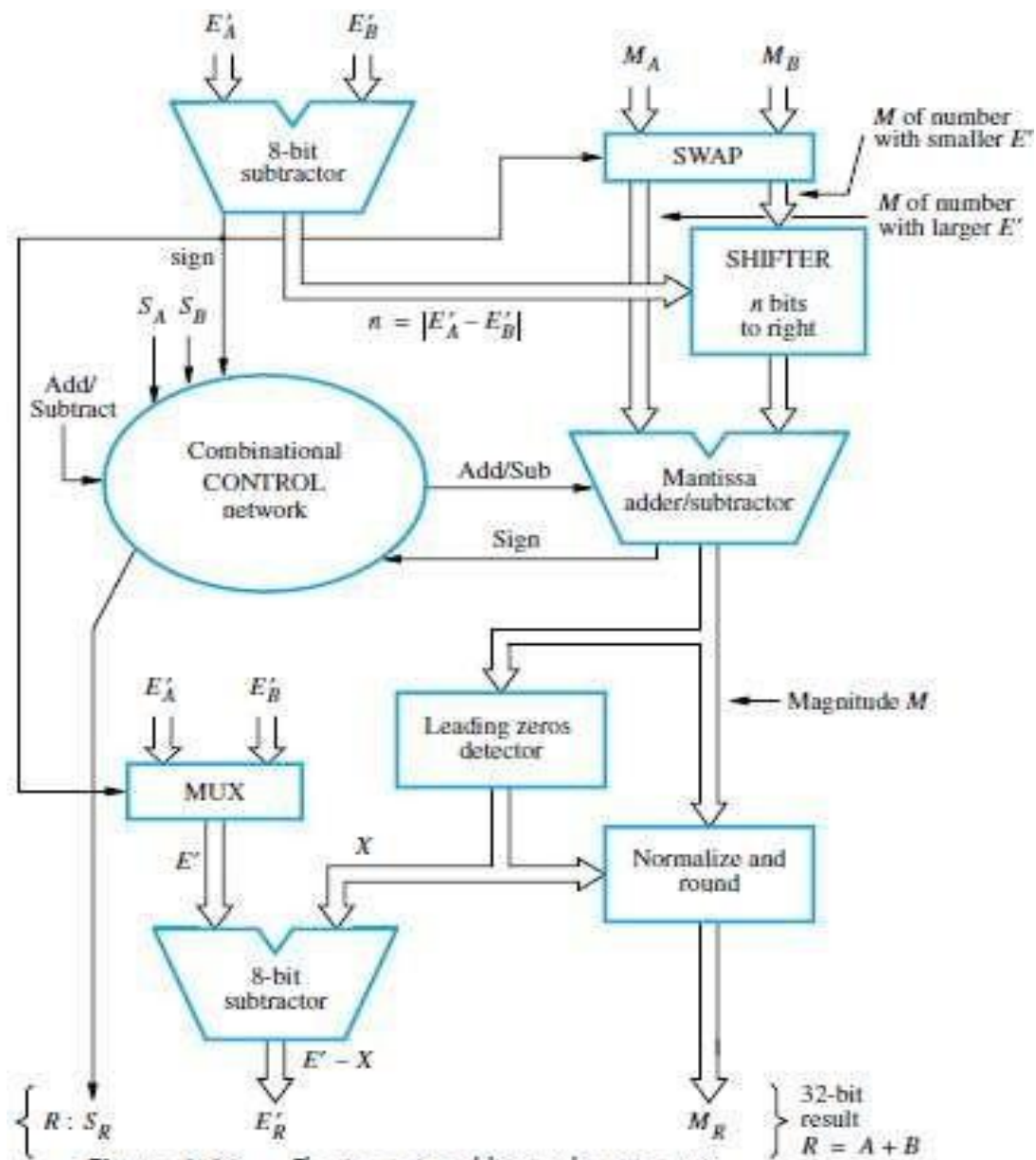


Figure 9.28 Floating-point addition-subtraction unit.

Basic Processing Unit

SOME FUNDAMENTAL CONCEPTS

- To execute an instruction, processor has to perform following 3 steps:
 - 1) Fetch contents of memory-location pointed to by PC. Content of this location is an instruction to be executed. The instructions are loaded into IR, Symbolically, this operation is written as:
 $IR \leftarrow [PC]$
 - 2) Increment PC by 4.
 $PC \leftarrow [PC] + 4$
 - 3) Carry out the actions specified by instruction (in the IR).
- The first 2 steps are referred to as **Fetch Phase**.
Step 3 is referred to as **Execution Phase**.
- The operation specified by an instruction can be carried out by performing one or more of the following actions:
 - 1) Read the contents of a given memory-location and load them into a register.
 - 2) Read data from one or more registers.
 - 3) Perform an arithmetic or logic operation and place the result into a register.
 - 4) Store data from a register into a given memory-location.
- The hardware-components needed to perform these actions are shown in Figure 5.1.

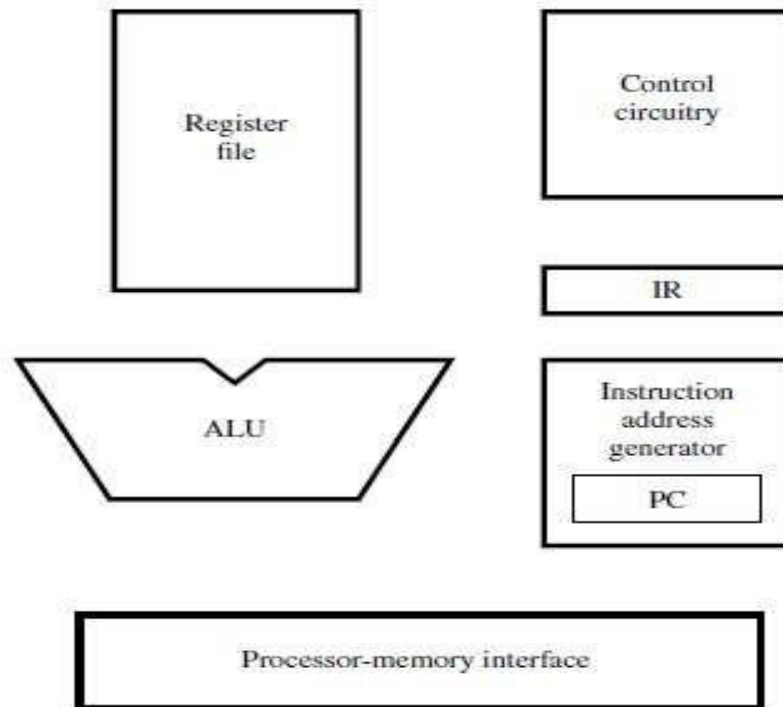


Figure 5.1 Main hardware components of a processor.

SINGLE BUS ORGANIZATION

- ALU and all the registers are interconnected via a **Single Common Bus** (Figure 7.1).
- Data & address lines of the external memory-bus is connected to the internal processor-bus via MDR & MAR respectively. (MDR \square Memory Data Register, MAR \square Memory Address Register).
- **MDR** has 2 inputs and 2 outputs. Data may be loaded
 - into MDR either from memory-bus (external) or
 - from processor-bus (internal).
- **MAR**'s input is connected to internal-bus; MAR's output is connected to external-bus.
- **Instruction Decoder & Control Unit** is responsible for
 - issuing the control-signals to all the units inside the processor.
 - implementing the actions specified by the instruction (loaded in the IR).
- Register R0 through R(n-1) are the **Processor Registers**.
The programmer can access these registers for general-purpose use.
- Only processor can access 3 registers **Y, Z & Temp** for temporary storage during program-execution.
The programmer cannot access these 3 registers.
- In **ALU**,
 - 1) „A“ input gets the operand from the output of the multiplexer(MUX).
 - 2) „B“ input gets the operand directly from the processor-bus.

- There are 2 options provided for „A“ input of the ALU.
- MUX is used to select one of the 2 inputs.
- MUX selects either
 - output of Y or
 - constant-value 4(which is used to increment PC content).

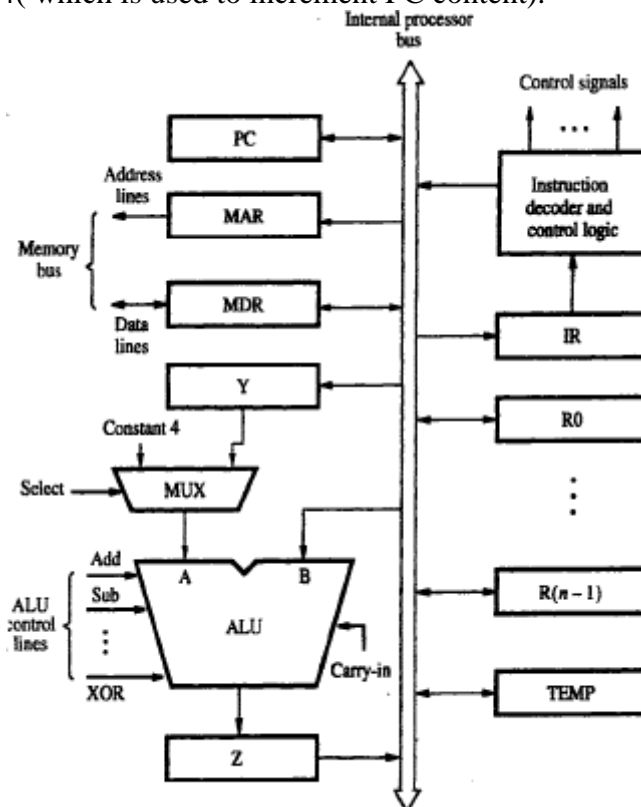


Figure 7.1 Single-bus organization of the datapath inside a processor.

- An instruction is executed by performing one or more of the following operations:
 - 1) Transfer a word of data from one register to another or to the ALU.
 - 2) Perform arithmetic or a logic operation and store the result in a register.
 - 3) Fetch the contents of a given memory-location and load them into a register.
 - 4) Store a word of data from a register into a given memory-location.

- **Disadvantage:** Only one data-word can be transferred over the bus in a clock cycle.

Solution: Provide multiple internal-paths. Multiple paths allow several data-transfers to take place in parallel.

EXECUTION OF A COMPLETE INSTRUCTION

- Consider the instruction *Add (R3),R1* which adds the contents of a memory-location pointed by R3 to register R1. Executing this instruction requires the following actions:
 - 1) Fetch the instruction.
 - 2) Fetch the first operand.
 - 3) Perform the addition &
 - 4) Load the result into R1.

Step	Action
1	PC _{out} , MAR _{in} , Read, Select4, Add, Z _{in}
2	Z _{out} , PC _{in} , Y _{in} , WMFC
3	MDR _{out} , IR _{in}
4	R3 _{out} , MAR _{in} , Read
5	R1 _{out} , Y _{in} , WMFC
6	MDR _{out} , SelectY, Add, Z _{in}
7	Z _{out} , R1 _{in} , End

Figure 7.6 Control sequence for execution of the instruction *Add (R3),R1*

- Instruction execution proceeds as follows:

Step1--> The instruction-fetch operation is initiated by
 → loading contents of PC into MAR &
 → sending a Read request to memory.

The Select signal is set to Select4, which causes the Mux to select constant 4. This value is added to operand at input B (PC's content), and the result is stored in Z.

Step2--> Updated value in Z is moved to PC. This completes the PC increment operation and PC will now point to next instruction.

Step3--> Fetched instruction is moved into MDR and then to IR. The step 1 through 3 constitutes the **Fetch Phase**.

At the beginning of step 4, the instruction decoder interprets the contents of the IR. This enables the control circuitry to activate the control-signals for steps 4 through 7.

The step 4 through 7 constitutes the **Execution Phase**.

Step4--> Contents of R3 are loaded into MAR & a memory read signal is issued. Step5--> Contents of R1 are transferred to Y to prepare for addition.

Step6--> When Read operation is completed, memory-operand is available in MDR, and the addition is performed.

Step7--> Sum is stored in Z, then transferred to R1. The End signal causes a new instruction fetch cycle to begin by returning to step1.

BRANCHING INSTRUCTIONS

- Control sequence for an **unconditional branch instruction** is as follows:

Step	Action
1	$PC_{out}, MAR_{in}, Read, Select4, Add, Z_{in}$
2	$Z_{out}, PC_{in}, Y_{in}, WMFC$
3	MDR_{out}, IR_{in}
4	$Offset-field-of-IR_{out}, Add, Z_{in}$
5	Z_{out}, PC_{in}, End

Figure 7.7 Control sequence for an unconditional Branch instruction.

- Instruction execution proceeds as follows:

Step 1-3--> The processing starts & the fetch phase ends in step3.

Step 4--> The offset-value is extracted from IR by instruction-decoding circuit.

Since the updated value of PC is already available in register Y, the offset X is gated onto the bus, and an addition operation is performed.

Step 5--> the result, which is the branch-address, is loaded into the PC.

- The branch instruction loads the branch target address in PC so that PC will fetch the next instruction from the branch target address.

- The branch target address is usually obtained by adding the offset in the contents of PC.

- The offset X is usually the difference between the branch target-address and the address immediately following the branch instruction.

- In case of **conditional branch**,

we have to check the status of the condition-codes before loading a new value into the PC. e.g.:

Offset-field-of-IR_{out}, Add, Z_{in}, If N=0 then End

If N=0, processor returns to step 1 immediately after step 4. If N=1, step 5 is performed to load a new value into PC.

MULTIPLE BUS ORGANIZATION

• **Disadvantage of Single-bus organization:** Only one data-word can be transferred over the bus in a clock cycle. This increases the steps required to complete the execution of the instruction

Solution: To reduce the number of steps, most processors provide multiple internal-paths. Multiple paths enable several transfers to take place in parallel.

• As shown in fig 7.8, three buses can be used to connect registers and the ALU of the processor.

• All general-purpose registers are grouped into a single block called the **Register File**.

• Register-file has 3 ports:

1) Two output-ports allow the contents of 2 different registers to be simultaneously placed on buses A & B.

2) Third input-port allows data on bus C to be loaded into a third register during the same clock-cycle.

• Buses A and B are used to transfer source-operands to A & B inputs of ALU.

• The result is transferred to destination over bus C.

Step	Action
1	$PC_{out}, R=B, MAR_{in}, Read, IncPC$
2	WMFC
3	$MDR_{outB}, R=B, IR_{in}$
4	$R4_{outA}, R5_{outB}, SelectA, Add, R6_{in}, End$

Figure 7.9 Control sequence for the instruction Add R4,R5,R6

• **Incrementer Unit** is used to increment PC by 4.

• Instruction execution proceeds as follows:

Step 1--> Contents of PC are

→ passed through ALU using $R=B$ control-signal &

→ loaded into MAR to start memory Read operation. At the same time, PC is incremented by 4.

Step2--> Processor waits for MFC signal from memory.

Step3--> Processor loads requested-data into MDR, and then transfers them to IR. Step4--> The instruction is decoded and add operation takes place in a single step.

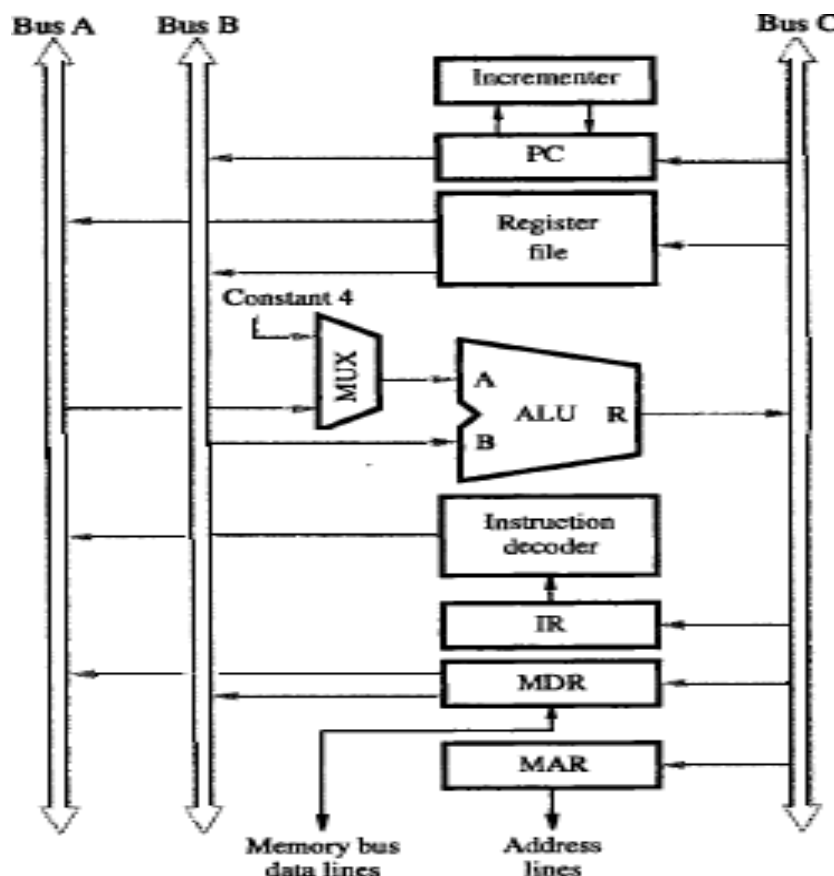


Figure 7.8 Three-bus organization of the datapath.

COMPLETE PROCESSOR

- This has separate processing-units to deal with integer data and floating-point data.

Integer Unit➔ To process integer data. (Figure 7.14).

Floating Unit➔ To process floating –point data.

- **Data-Cache** is inserted between these processing-units & main-memory. The integer and floating unit gets data from data cache.
- **Instruction-Unit** fetches instructions
→ from an instruction-cache or
→ from main-memory when desired instructions are not already in cache.
- Processor is connected to system-bus & hence to the rest of the computer by means of a **Bus Interface**.
- Using separate caches for instructions & data is common practice in many processors today.
- A processor may include several units of each type to increase the potential for concurrent operations.
- The 80486 processor has 8-kbytes single cache for both instruction and data.

Whereas the Pentium processor has two separate 8 kbytes caches for instruction and data.

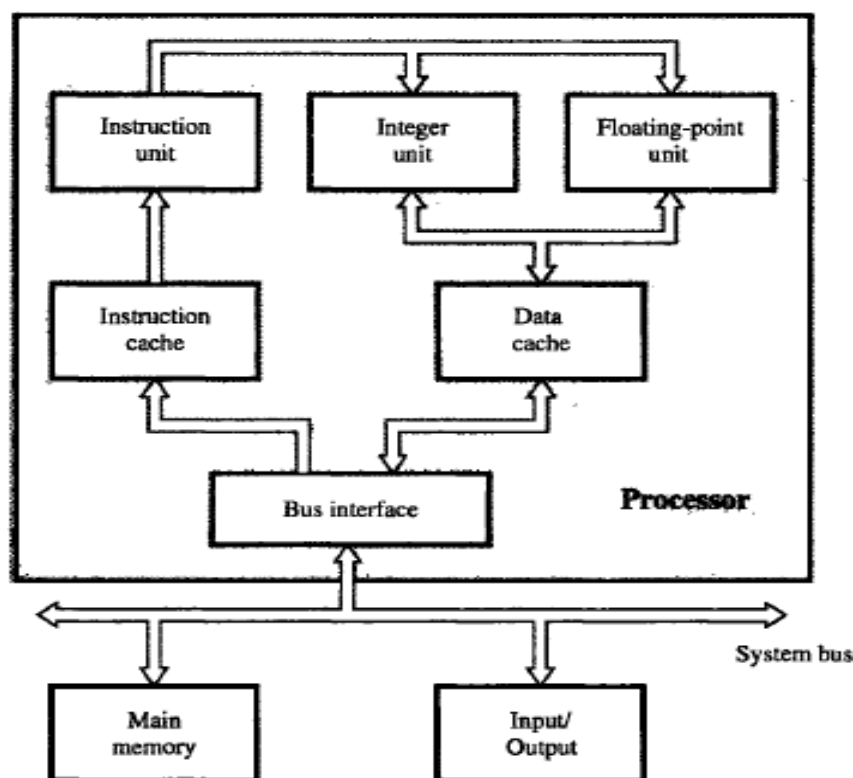


Figure 7.14 Block diagram of a complete processor.

Note:

To execute instructions, the processor must have some means of generating the control-signals. There are two approaches for this purpose:

- 1) Hardwired control and 2) Microprogrammed control.

HARDWIRED CONTROL

- Hardwired control is a method of control unit design (Figure 7.11).
- The control-signals are generated by using logic circuits such as gates, flip-flops, decoders etc.
- **Decoder/Encoder Block** is a combinational-circuit that generates required control-outputs depending on state of all its inputs.
- **Instruction Decoder**
 - It decodes the instruction loaded in the IR.
 - If IR is an 8 bit register, then instruction decoder generates 2^8 (256 lines); one for each instruction.
 - It consists of a separate output-lines INS1 through INS_m for each machine instruction.
 - According to code in the IR, one of the output-lines INS1 through INS_m is set to 1, and all

other lines are set to 0.

- **Step-Decoder** provides a separate signal line for each step in the control sequence.

- **Encoder**

- It gets the input from instruction decoder, step decoder, external inputs and condition codes.
- It uses all these inputs to generate individual control-signals: Yin, PCout, Add, End and so on.
- For example (Figure 7.12), $Z_{in} = T_1 + T_6.ADD + T_4.BR$

;This signal is asserted during time-slot T1 for all instructions.

during T6 for an Add instruction.

during T4 for unconditional branch instruction

- When **RUN**=1, counter is incremented by 1 at the end of every clock cycle. When RUN=0, counter stops counting.
- After execution of each instruction, **end** signal is generated. End signal resets step counter.
- Sequence of operations carried out by this machine is determined by wiring of logic circuits, hence the name “**hardwired**”.
- **Advantage**: Can operate at high speed.

- **Disadvantages**:

- 1) Since no. of instructions/control-lines is often in hundreds, the complexity of control unit is very high.
- 2) It is costly and difficult to design.
- 3) The control unit is inflexible because it is difficult to change the design.

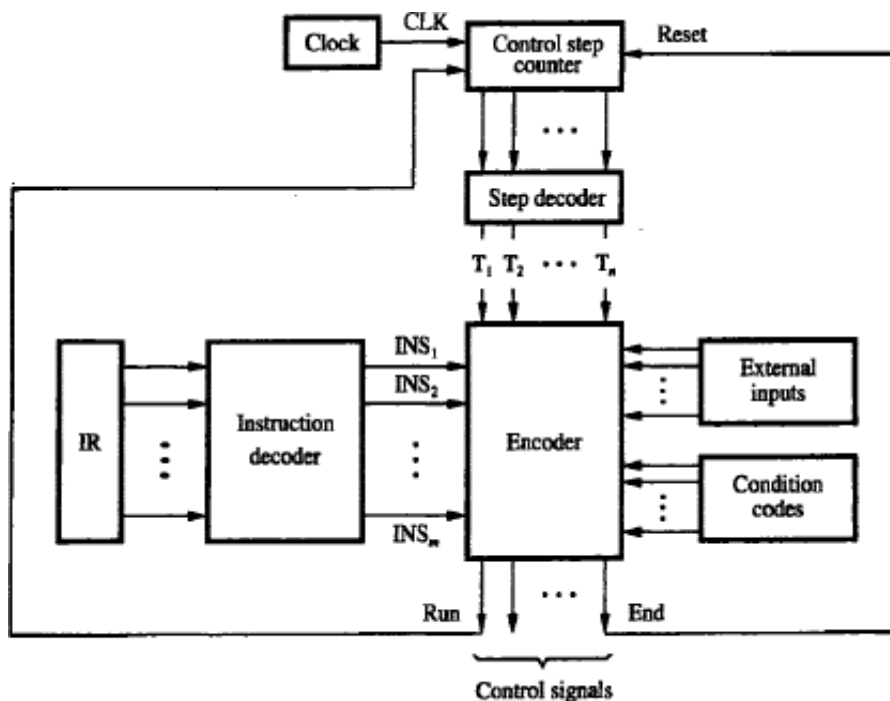


Figure 7.11 Separation of the decoding and encoding functions.

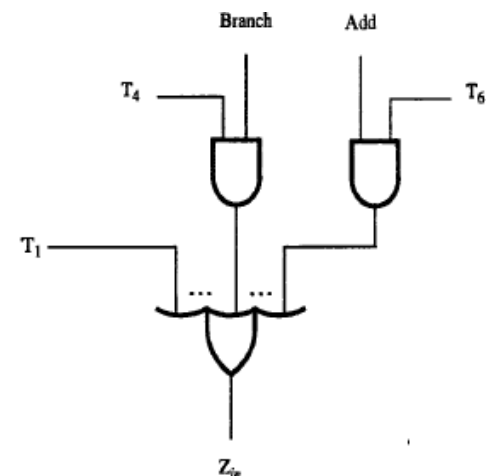


Figure 7.12 Generation of the Z_{in} control signal

MICROPROGRAMMED CONTROL

- Microprogramming is a method of control unit design (Figure 7.16).
- Control-signals are generated by a program similar to machine language programs.
- **Control Word (CW)** is a word whose individual bits represent various control-signals (like Add, PCin).
- Each of the control-steps in control sequence of an instruction defines a unique combination of 1s & 0s in CW.
- Individual control-words in microroutine are referred to as **microinstructions** (Figure 7.15).
- A sequence of CWs corresponding to control-sequence of a machine instruction constitutes the **microroutine**.
- The microroutines for all instructions in the instruction-set of a computer are stored in a special memory called the **Control Store (CS)**.
- Control-unit generates control-signals for any instruction by sequentially reading CWs of corresponding microroutine from CS.
- **μPC** is used to read CWs sequentially from CS. (μPC = Microprogram Counter).
- Every time new instruction is loaded into IR, o/p of **Starting Address Generator** is loaded into μPC.

- Then, μPC is automatically incremented by clock;
causing successive microinstructions to be read from CS.
- Hence, control-signals are delivered to various parts of processor in correct sequence.

Advantages

- It simplifies the design of control unit. Thus it is both, cheaper and less error prone implement.
- Control functions are implemented in software rather than hardware.
- The design process is orderly and systematic.
- More flexible, can be changed to accommodate new system specifications or to correct the design errors quickly and cheaply.
- Complex function such as floating point arithmetic can be realized efficiently.

Disadvantages

- A microprogrammed control unit is somewhat slower than the hardwired control unit, because time is required to access the microinstructions from CM.
- The flexibility is achieved at some extra hardware cost due to the control memory and its access circuitry.

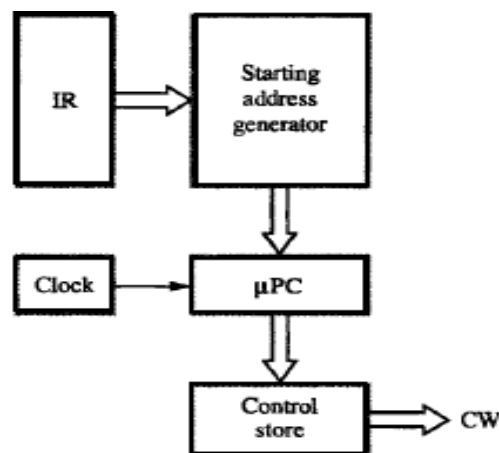


Figure 7.16 Basic organization of a microprogrammed control unit.

Micro instruction	:	PC _{in}	PC _{out}	MAR _{in}	Read	MDR _{out}	IR _{in}	Y _{in}	Select	Add	Z _{in}	Z _{out}	R1 _{out}	R1 _{in}	R2 _{out}	WMFC	End	:
1		0	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	
2		1	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	
3		0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	

Figure 7.15 An example of microinstructions for Figure 7.6.