

Unit-2

## Linear Programming Problem (LPP)

1. A carpenter makes book cases in two sizes, small and large. It takes six hours to build a large & two hours to build a small one. The profit from a large book case is ₹50 & profit from a small book case is ₹20. The carpenter can spend only 24 hours a week making book cases and must make atleast four large and three small each week. Find his maximum profit per week.

Large -  $x$

Small -  $y$

$$6x + 2y =$$

Obj function is  $= 50x + 20y \rightarrow \text{maximise}$ .

$$\text{constraints} = 6x + 2y \leq 24$$

maximise  $\epsilon$

minimise  $\geq$

$$\text{limits } x \geq 2$$

$$y \geq 3$$

A calculator Company produces two models of calculators at two different factory. factory A can produce 140 scientific calculators and 85 graphic calculators per day.

factory B can produce 60 scientific calculators & 90 graphic calculators per day. It costs 1200 ₹ per day to operate factory A & 900 rupees per day to factory B. If the company needs to produce 450 scientific calculators & 340 graphic calculators per big order. find the minimum cost to produce this order.

Let

factory A =  $x$

factory B =  $y$

objective function  $\text{min} z = 1200A + 4900B$

$$\text{constraints: } 140A + 60B \geq 460$$

$$25A + 90B \geq 340$$

$$\text{limits } A \geq 0, B \geq 0$$

A firm produces three products. The products are processed on three different machines. The time required to manufacture one unit of each 3 objects and daily capacities of the 3 machines are given in the data firm below.

Products	Time required per unit(min)			machine capacity (min/d)
	product 1	product 2	product 3	
A	2	3	2	440
B	4	-	3	470
C	2	5	-	430

It is required to determine the daily number of digits to be manufactured for each product. The profit for unit 1, 2, 3 are 4 ru, 3 ru, 6 ru respectively. It is assume that all the amounts produced are consumed in the market. Formulate the problem as L.P.P.

let product 1 =  $x_1$

product 2 =  $x_2$

product 3 =  $x_3$

obj function  $4x_1 + 3x_2 + 6x_3$

Constraints:  $2x_1 + 3x_2 + 12x_3 \leq 440$

$4x_1 + 3x_3 \leq 470$

$x_1 \geq 0$

$2x_1 + 5x_2 \leq 430$

$x_2 \geq 0$

$x_3 \geq 0$

$x_1, x_2, x_3 \in \mathbb{R}$

A person requires 10, 12 & 12 units of chemicals a, b & c respectively for his Garden. A liquid product contains 5, 2 & 1 units of a, b, c respectively for jar. A dry product contains 1, 2 & 4 units of a, b, c per bag. If the liquid product is sold for three rupees per jar, the dried product is sold per 2 ₹ per bag. How many units of each product should be purchased in order to minimize cost.

$x_1, x_2 \geq 0$

min obj function  $\rightarrow 3x_1 + 2x_2$

Constraints -  $5x_1 + x_2 \geq 10$

$2x_1 + 2x_2 \geq 12$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

A company manufactures two products  $x$  &  $y$  which requires the following resources. The resources are the capacities of machines  $m_1, m_2$  &  $m_3$ . The available capacities are 50, 25 & 15 hours respectively in the planning period. Product  $x$  requires 1 hour of machine  $m_2$  & 1 hours of machine  $m_3$ . Product  $y$  requires 2 hours of machine  $m_1$ , 2 hours of machine  $m_2$ , 1 hour of machine  $m_3$ . The profit contribution of products  $x$  &  $y$  are 5 & 4 respectively. Formulate the LPP.

$$\text{Maximize } Z = 5x + 4y$$

$$\text{Subject to:}$$

$$m_1 + m_2 + m_3 \leq 50 \rightarrow m_1 \leq 50 - m_2 - m_3$$

$$2x_1 + 2x_2 \leq 25 \rightarrow m_2 \rightarrow 4 \text{ hours}$$

$$x_1 + 2x_2 \leq 25 \rightarrow m_3 \rightarrow 1 \text{ hour}$$

$$x_1 + x_2 \leq 15 \rightarrow m_1 \rightarrow 2 \text{ hours}$$

$$x_1 + x_2 \leq 15 \rightarrow m_2 \rightarrow 2 \text{ hours}$$

$$x_1 + x_2 \leq 15 \rightarrow m_3 \rightarrow 1 \text{ hour}$$

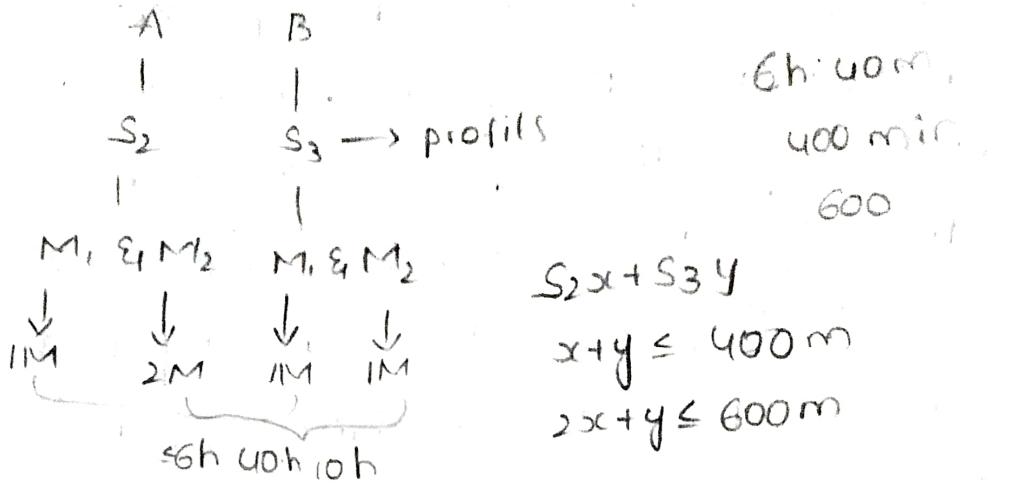
$$\text{Obj function } \text{max} = 5x + 4y.$$

$$\text{Constraints: } x + m_2 \leq 50 \quad x \geq 0$$

$$x + 2y \leq 25 \quad y \geq 0$$

$$x + y \leq 15$$

A firm manufactures two types of products A & B and sells them at a profit  $s_1$  on type A and  $s_2$  on type B. Each product on two machines  $m_1$  &  $m_2$ . Type A requires 1 minute of processing time on  $m_1$  and 2 minutes on  $m_2$ . Type B requires 1 minute on  $m_1$  and 1 minute on  $m_2$ . Machine  $m_1$  is available for not more than 6 hours and 40 minutes while machine  $m_2$  is available for 10 hours during working day. Formulate the problem as LPP so as to maximize the profit.



Graphical method.  $\rightarrow$  slack variables not considered currently.

To solve the following LPP by using graphical method.

minimize  $Z = 4x_1 + 6x_2$  is subjected to constraints

$$x_1 + x_2 \geq 8, \quad 6x_1 + x_2 \geq 12, \quad x_1, x_2 \geq 0$$

$$\text{min } Z = 4x_1 + 6x_2$$

$$\text{Given constraints } x_1 + x_2 \geq 8$$

$$6x_1 + x_2 \geq 12$$

Converting inequalities into equalities

$$x_1 + x_2 = 8 \quad \text{--- (1)}$$

$$6x_1 + x_2 = 12 \quad \text{--- (2)}$$

From eqn (1)  $x_1 + x_2 = 8$

$$\text{at } x_1 = 0$$

$$\boxed{x_2 = 8}$$

$$\text{at } x_2 = 0$$

$$\boxed{x_1 = 8}$$

From eqn (2)

$$6x_1 + x_2 = 12$$

$$\text{at } x_1 = 0$$

$$\boxed{x_2 = 12}$$

$$\text{at } x_2 = 0$$

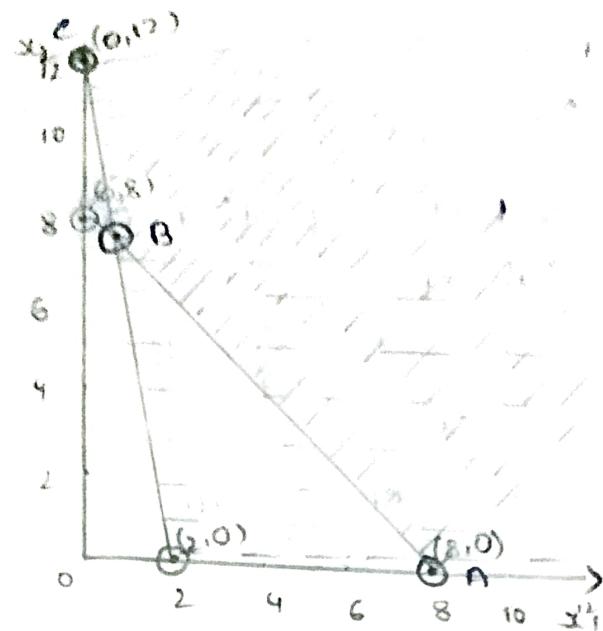
$$\boxed{x_1 = 2}$$

Q1

Q2

values of Q1, Q2

## Graph representation



From graph ABC are the feasible region.

Solving ① & ②

$$x_1 + 2x_2 = 8$$

$$\underline{\begin{array}{r} 6x_1 + x_2 = 12 \\ - \end{array}}$$

$$5x_2 = 4$$

$$x_2 = 4/5 = 0.8$$

$$x_1 = 7.2$$

$$B(0.8, 7.2)$$

$$\text{minimize } Z = 4x_1 + 6x_2$$

$$A(8,0) = 4(8) + 6(0) = 32$$

$$B(0.8, 7.2) = 4(0.8) + 6(7.2) = 46.4$$

$$C(0,12) = 4(0) + 6(12) = 72$$

At  $A(8,0)$  is minimum value  
is 32.

Solve the following LPP by using graphical method

maximizing  $Z = 12x_1 + 16x_2$  is subjected to constraints

$$10x_1 + 20x_2 \leq 120, 8x_1 + 8x_2 \leq 80, x_1, x_2 \geq 0$$

Given max  $Z = 12x_1 + 16x_2$

Given Constraints  $10x_1 + 20x_2 \leq 120$

$$8x_1 + 8x_2 \leq 80$$

# Converting inequalities into equalities

$$10x_1 + 20x_2 = 120 \quad \text{--- (1)} \rightarrow 5x_1 + 10x_2 = 60 \quad \text{--- (1')}$$

$$8x_1 + 8x_2 = 80 \quad \text{--- (2)} \rightarrow x_1 + x_2 = 10 \quad \text{--- (2')}$$

From equation (1)

at  $x_1 = 0$

$$x_2 = \frac{120}{20}$$

$$\boxed{x_2 = 6}$$

from equation (2)

at  $x_1 = 0$

$$x_2 = \frac{80}{8} = 10$$

$$x_2 = 0$$

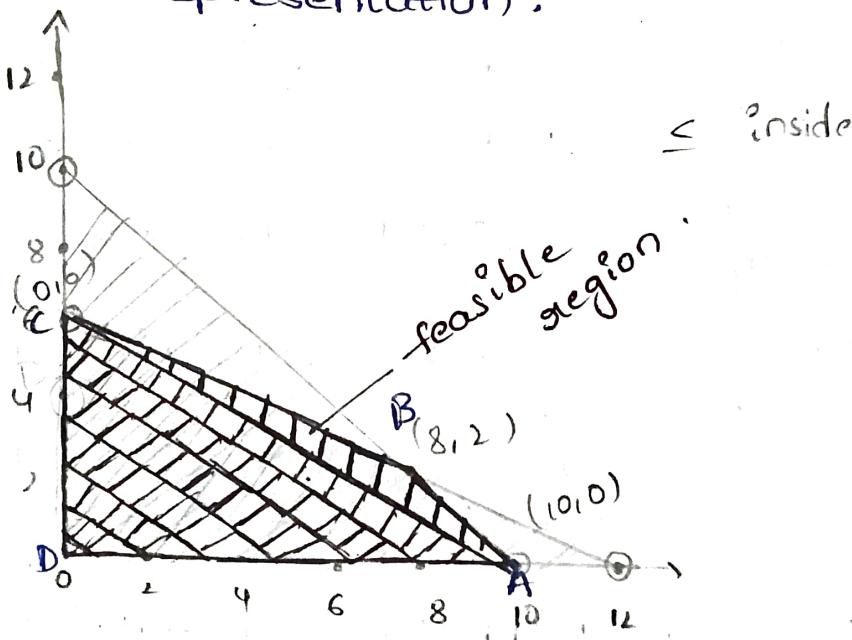
at  $x_2 = 0$

$$x_1 = \frac{80}{8} = 10$$

$$10x_1 = 120$$

$$\boxed{x_1 = 12}$$

## Graphical representation.



Solving (1) & (2)

$$80x_1 + 160x_2 = 840$$

~~$$80x_1 + 80x_2 = 800$$~~

$$80x_2 = 40$$

$$x_2 = \frac{40}{80} = 0.5$$

$$(x_1, x_2) = (9.5, 0.5)$$

Substituting  $x_2$  in

~~(2)~~

$$10x_1 + 8(0.5) = 80$$

$$8x_1 = 80 - 4$$

$$8x_1 = 76$$

$$x_1 = \frac{76}{8} = 9.5$$

$$x_1 = \frac{76}{8} = 9.5$$

iii. Solve the maximum L.L.P by using Graphical method

maximize  $Z = 12x_1 + 16x_2$  is subjected to constraints

$$10x_1 + 20x_2 \leq 120, 8x_1 + 8x_2 \leq 80$$

Solving ① & ②

$$x_1 + 2x_2 = 12$$

$$\underline{x_1 + x_2 = 10}$$

$$\underline{x_2 = 2}$$

Substituting

$$\boxed{x_1 = 8}$$

obj function:  $12x_1 + 16x_2$

$$\text{At } A(10, 0) = 120$$

$$\text{At } B(8, 2) = 96 + 32 = 128$$

$$\text{At } C(0, 6) = 6(6) = 96$$

A, B, C D is feasible region.

∴ The maximum value is at B with 128 value.

3. Solve L.L.P using Graphical method maximize  $Z = 100x_1 + 60x_2$  is subjected to constraints  $5x_1 + 10x_2 \leq 50$ ,  $8x_1 + 2x_2 \geq 16$ ,  $3x_1 - 2x_2 \geq 6$ ,  $x_1, x_2 \geq 0$ .

Given that

$$\max Z = 100x_1 + 60x_2$$

$$5x_1 + 10x_2 \leq 50, 8x_1 + 2x_2 \geq 16, 3x_1 - 2x_2 \geq 6$$

Converting inequalities in equalities

$$5x_1 + 10x_2 = 50 \quad \text{--- ①}$$

$$8x_1 + 2x_2 = 16 \quad \text{--- ②}$$

$$3x_1 - 2x_2 = 6 \quad \text{--- ③}$$

from eqn ①

$$5x_1 + 10x_2 = 50$$

$$\text{Let } x_1 = 0$$

$$\boxed{x_2 = 5.}$$

$$\text{Let } x_2 = 0$$

$$\boxed{x_1 = 10}$$

$x_1$	0	10
$x_2$	5	0

From eq<sup>n</sup> ②  $8x_1 + 2x_2 = 16$

let  $x_1 = 0$

$$5(0) + 10x_2 = 5$$

$$8(0) + 2x_2 = 16$$

$$[x_2 = 8]$$

let  $x_2 = 0$

$$[x_1 = 2]$$

$x_1$	0	2
$x_2$	8	0

From equation ③  $3x_1 - 2x_2 = 6$

let  $x_1 = 0$

$$3(0) - 2x_2 = 6$$

$$[x_2 = -3]$$

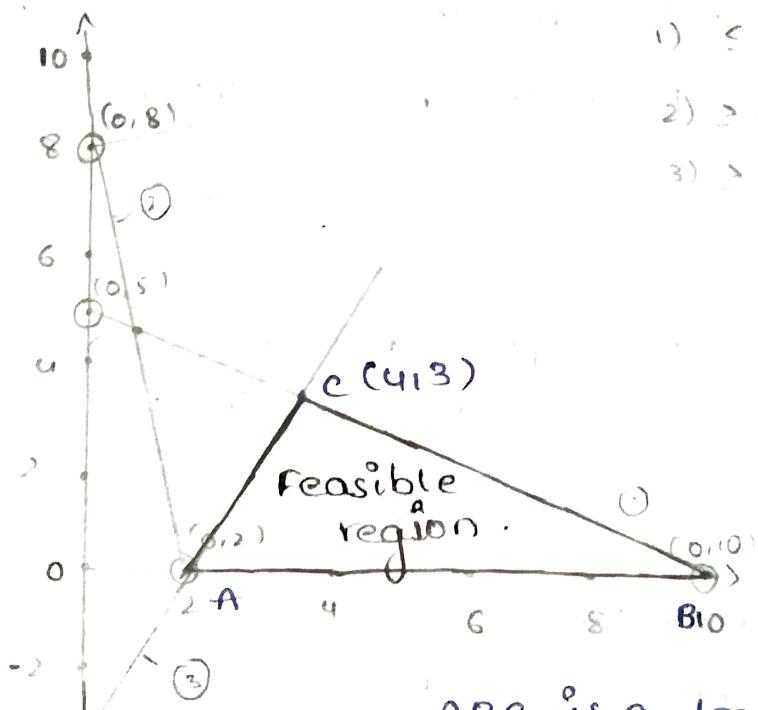
let  $x_2 = 0$

$$3x_1 = 6$$

$$[x_1 = 2]$$

$x_1$	0	2
$x_2$	-3	0

Graphical representation.



- 1)  $\leq$  - Inside
- 2)  $\geq$  - outside
- 3)  $>$  - outside

-ABC is a feasible region

$$\max z = 100x_1 + 60x_2 \geq 0$$

$$\text{at } A(0,0) = 200$$

$$\text{At } B(10,0) = 1000$$

$$\text{At } C(4,3) = 400 + 180 = 580$$

Solving ① & ③

$$5x_1 + 10x_2 = 50$$

$$8x_1 + 2x_2 = 16$$

$$20x_1 = 80 \quad [x_1 = 4] \quad [x_2 = 3]$$

$\therefore$  Then maximum is the  
B(10,0) value is of  
1000.

4. To maximize  $Z = 18x + 10y$  is subjected to constraints  
 $4x + y \leq 20$ ,  $2x + 3y \leq 30$ ,  $x, y \geq 0$ .

Given that  $Z = 18x + 10y$

Constraints  $4x + y \leq 20$

$$2x + 3y \leq 30$$

$$x, y \geq 0$$

Converting inequalities into equalities.

$$4x + y = 20 \quad \text{--- (1)}$$

$$2x + 3y = 30 \quad \text{--- (2)}$$

At  $x=0$  in (1)

$$\boxed{y = 20}$$

$$y = 0$$

$$\boxed{x = 5}$$

$x$	0	5
$y$	20	0

At  $x=0$  in (2)

$$3y = 30$$

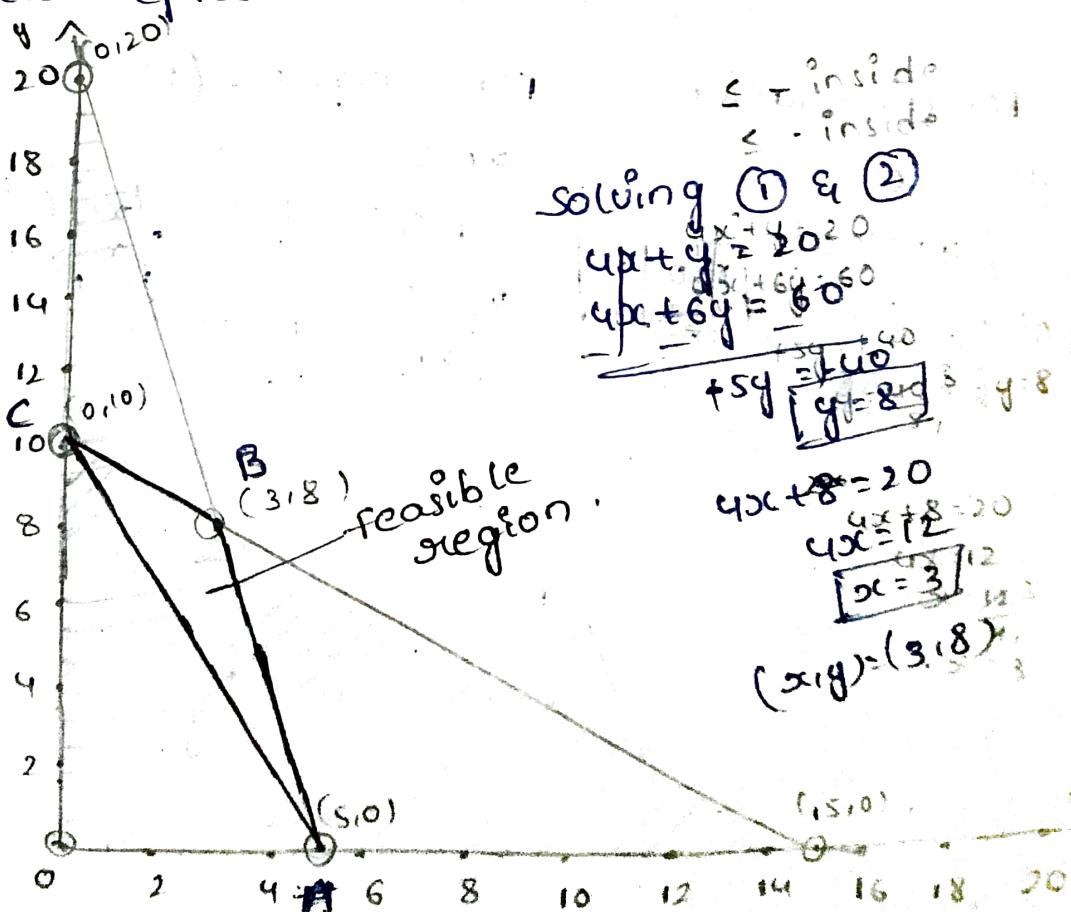
$$\boxed{y = 10}$$

$$y = 0$$

$$\begin{cases} 2x = 30 \\ x = 15 \end{cases}$$

$x$	0	15
$y$	10	0

Graphical representation.



ABC is a graphical feasible region

objective function:  $18x + 10y$

$$\text{At } A(5,0) : 18(5) + 10(0) = 90$$

$$\text{At } B(3,8) : 18(3) + 10(8) = 54 + 80 = 134$$

$$\text{At } C(0,10) : 18(0) + 10(10) = 100$$

maximize value is 134 at B(3,8)

5. To maximize  $Z = 3x_1 + 2x_2$  is subjected to const.  
 $2x_1 + x_2 \leq 100$ ,  $x_1 + x_2 \leq 80$ ,  $x_1 \leq 40$ ,  $x_1, x_2 \geq 0$

Given  $Z = 3x_1 + 2x_2$

Constraints  $2x_1 + x_2 \leq 100$

$$x_1 + x_2 \leq 80$$

Converting inequalities into equalities

$$2x_1 + x_2 = 100 \quad \text{--- (1)} \quad x_1 = 40 \quad \text{--- (3)}$$

$$x_1 + x_2 = 80 \quad \text{--- (2)}$$

for equation (1)

for equation (2)

30	40	50
20	30	40

$$x_2 = 0$$

$$x_1 = 40$$

$$x_1 = 0$$

$$x_2 = 80$$

At  $x_1 = 0$

$x_2 = 100$	$x_1$	0	50
100	$x_2$	100	0

$$x_1 = 0$$

$x_2 = 80$
------------

$$x_2 = 0$$

$x_1 = 80$
------------

$x_1$	0	80
$x_2$	80	0

At  $x_2 = 0$

$x_1 = 50$	$x_1$	0	100
50	$x_2$	100	0

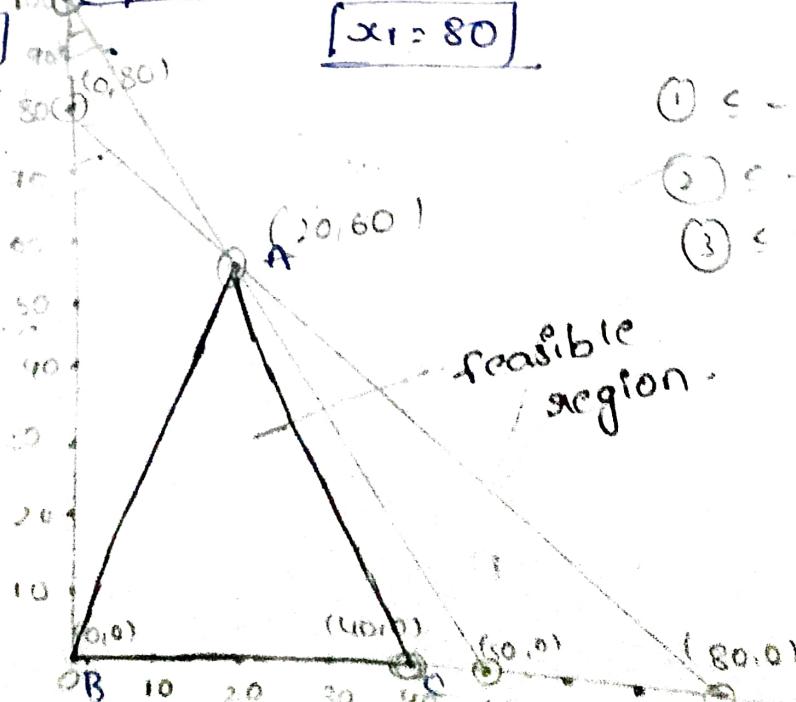
$$x_2 = 0$$

$x_1 = 80$
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(1)  $\leftarrow$  inside

(2)  $\leftarrow$  inside

(3)  $\leftarrow$  inside



Solving ① & ②

$$23,432 \div 100$$

$$2x_1 + 2y_2 \leq 100$$

$$x_1 + x_2 = 80$$

$$x_1 + x_2 = 80$$

$$x_1 = 20$$

$$x_1 = 20$$

201x1-80

$$x_1 + x_2 = 80$$

25 - 60

$$20 + x_2 = 80$$

$$x_2 = 60$$

$$(x_1, x_2) = (20, 60)$$

$\therefore A_1B_1C$  is a feasible solution region.

$$A_t = A(20, 60) = 3(20) + 2(60)$$

$$= 60 + 120 = 180$$

$$A \in B(0,0) = 0$$

$$At \quad c(40,0) = 3(40) + 2(0) = 120$$

∴ The maximum value is at A(20,60) the value is 180.

Simplex method. ( $c_j - z_j \leq 0$ ) =  $\Delta_{ij}^*$

Process → we will solve this problem in only when  
maximize opti objective equation.

If they given in minimize then we need to convert into maximize by interchanging symbols.

Step-2: Converting Inequalities into equalities with slack variables & surplus variables.

-After we take table coefficients of obj function basic variables  
Sl. no. coefficients of obj function slack variables  
Initial solution:

$C_j$	$C_B$	$b_V$	$x_1$	$x_2$	$\dots$	$S_1$	$S_2$	$\dots$
0	9	1						
$Z_j = C_B x_{j,0}$								
& add all								
$C_j - Z_j \leq 0$								

coefficients of  
 constraint  
 slack variables  
 in obj function  
 constraint in equalities

Suppose  $2x_1 + 4x_2$  is obj. function with  
 $2x_1 + 2x_2 \leq 10$ ,  $x_1 + 4x_2 \leq 5$   
obj. function  $2x_1 + 4x_2 + OS_1 + OS_2$  coefficients.

$$2x_1 + 2x_2 + S_1 = 0$$

$$x_1 + 4x_2 + S_2 = 0$$

$C_j - 2j \leq 0 \rightarrow$  Edi vache varakku cheste  
undali After this condition satisfies then  
we get  $x_1, x_2$  values. Those values will  
be substituted into obj. function. i.e. the  
final Answer.

This is the process to solve simplex method.

### Examples

1) maximize  $Z = 3x_1 + 6x_2$  is subjected to  
constraints  $4x_1 + 7x_2 \leq 8$ ,  $3x_1 + 9x_2 \leq 5$ ,  $x_1, x_2 \geq 0$

$$\text{obj. function } \max Z = 3x_1 + 6x_2$$

Converting inequalities into equalities with  
slack variables

$$4x_1 + 7x_2 + S_1 = 8$$

$$3x_1 + 9x_2 + S_2 = 5$$

2) minimize  $Z = 4x_1 + 7x_2$  is subjected to  
constraints  $x_1 + 2x_2 \geq 2$ ,  $3x_1 + 5x_2 \geq 7$ ,  $x_1, x_2 \leq 0$

$$\text{obj. fun min } Z = 4x_1 + 7x_2$$

$$\max Z = -4x_1 - 7x_2$$

Now Converting inequalities into  
equalities with slack variables

$$x_1 + 2x_2 \geq 2 \Rightarrow -x_1 - 2x_2 \leq -2$$

$$-x_1 - 2x_2 + S_1 = -2$$

$$3x_1 + 5x_2 \geq 7 \Rightarrow -3x_1 - 5x_2 \leq -7$$

$$\therefore -3x_1 - 5x_2 + s_2 = -7$$

3. maximize  $Z = 2x_1 + 6x_2$  is subjected to constraints  
 $3x_1 + 4x_2 \leq 5$ ,  $2x_1 + 9x_2 \geq 7$ ,  $4x_1 + 5x_2 \geq 9$   $x_1, x_2, x_3 \leq 0$

Obj function  $\max Z = 2x_1 + 6x_2$

Converting inequalities into equalities with slack variables.

$$3x_1 + 4x_2 \leq 5 \Rightarrow 3x_1 + 4x_2 + s_1 = 5$$

$$2x_1 + 9x_2 \geq 7 \Rightarrow -2x_1 - 9x_2 \leq -7 \quad (\text{or}) \quad 2x_1 + 9x_2 - s_2 = 7$$

$$\therefore -2x_1 - 9x_2 + s_2 = -7 \quad 0$$

$$4x_1 + 5x_2 \geq 9 \Rightarrow -4x_1 - 5x_2 \leq -9 \quad (\text{or}) \quad 4x_1 + 5x_2 - s_3 = 9$$

$$\therefore -4x_1 - 5x_2 + s_3 = -9$$

### Simplex Method.

Solve the following LPP by using the Simplex method. maximize  $Z = 12x_1 + 16x_2$  is subjected to constraints  $10x_1 + 20x_2 \leq 120$ ,  $8x_1 + 8x_2 \leq 80$   $x_1, x_2 \geq 0$

Given maximize  $Z = 12x_1 + 16x_2$

Given Constraints  $10x_1 + 20x_2 \leq 120$   
 $8x_1 + 8x_2 \leq 80$

Converting inequalities into equalities by adding slack variables.

$$10x_1 + 20x_2 + s_1 = 120$$

$$8x_1 + 8x_2 + s_2 = 80$$

Now Obj function (Z) =  $12x_1 + 16x_2 + 0s_1 + 0s_2$

		Cj	Basic Variables				Obj. Function
I	cb		x1	x2	s1	s2	
coefficients of basic variables	0	s1	10(0)	20	1	0	$\frac{120}{20} = 6 \leftarrow \min$
	0	s2	8(0)	8	0	1	$\frac{80}{8} = 10$
		$z_j^0$	0	0	0	0	
		$\Delta_{ij}^0 = C_j - z_j^0$	12	16	0	0	
		$z_j^0 = cbx_1 + cbx_2$					
							↑ max.

Eliminating  $s_1$  row in that place we generate New key equation (NKE)

NKE = Old key equation

key element

cohere key element  $\rightarrow$  intersection value of both column & row.

$$NKE = \frac{10 \quad 20 \quad 1 \quad 0 \quad 120}{20} \Rightarrow \frac{1}{2}, 1, \frac{1}{20}, 0, \frac{3}{10}, 6$$

$$\text{New } s_2 = \text{old } s_2 - \text{corresponding NKE}$$

$$\text{old } s_2 = 8 - 0 = 8$$

$$\text{Corr. NKE} = 4, 8, \frac{2}{5}, 0, 48$$

$$\text{New } s_2 = 4 - 0 - \frac{2}{5} = 3.2$$

	$C_j$	12	16	0	0	
$c_b$	$b_r$	$x_1$	$x_2$	$s_1$	$s_2$	$x_b$ Ratio
16	$x_2$	$\frac{1}{2}$	1	$\frac{1}{2}0$	0	$6 = \frac{6}{\frac{1}{2}} = 12$
0	$s_2$	4	0	$-\frac{2}{5}$	1	$32 = \frac{32}{4} = 8 \leftarrow \min$

$$Z_j = 8 \quad 16 \quad 4/5 \quad 0$$

$$\Delta_{ij}^o = C_j - Z_j = 4 \quad 0 \quad -4/5 \quad 0$$

$\uparrow$  max.

Elminating  $s_2$  row in that place we generate  
New key equation (NKE)

$$NKE = \frac{4 \quad 0 \quad -\frac{2}{5} \quad 1 \quad 32}{4} = 1 \quad 0 \quad -\frac{1}{10} \quad \frac{1}{4} \quad 8$$

New  $x_2$  = old  $x_2$  - corresponding NKE

$$\text{old } x_2 = \frac{1}{2} \quad 1 \quad \frac{1}{2}0 \quad 0 \quad 6$$

$$\text{corr NKE} = \frac{\frac{1}{2} \quad 0 \quad -\frac{1}{10} \quad \frac{1}{8} \quad 4}{4}$$

$$\text{New } x_2 = \frac{0 \quad 1 \quad \frac{1}{10} \quad -\frac{1}{8}}{4} \quad 2$$

III

	$C_j$	12	16	0	0	
$c_b$	$b_r$	$x_1$	$x_2$	$s_1$	$s_2$	
16	$x_2$	0	1	$\frac{1}{10}$	$-\frac{1}{8}$	2
12	$x_1$	1	0	$-\frac{1}{10}$	$\frac{1}{4}$	8

$$Z_j = 12 \quad 16 \quad 2/5 \quad -1$$

$$\Delta_{ij}^o = Z_j - Z_j = 0 \quad 0 \quad -2/5 \quad -1$$

$\Delta_{ij}^o = C_j - Z_j \leq 0$  so the solution

Substituting  $x_1, x_2$  values in obj function

$$\text{Obj function } Z = 12x_1 + 16x_2$$

$$= 12(8) + 16(2) \Rightarrow 96 + 32$$

$$= 128.$$

∴ the maximize optimum value is 128.

Solve the following LPP by using simplex method.  
maximize  $Z = 18x + 10y$  is subjected to  $4x + y \leq 20$ ,  
 $2x + 3y \leq 30$

$$\text{Given } Z = 18x + 10y$$

Converting inequalities into equalities with slack variables..

$$4x + y + s_1 = 20$$

$$2x + 3y + s_2 = 30$$

$$\text{Obj function } Z = 18x + 10y + 0s_1 + 0s_2$$

$c_j$	18	10	0	0		
$c_b$	bv	x	y	$s_1$	$s_2$	ratio
0	$s_1$	4	1	1	0	$20/4 = 5 \leftarrow \min$
0	$s_2$	2	3	0	1	$30/2 = 15$

$$Z_j = 0 \quad 0 \quad 0 \quad 0$$

$$\Delta_{ij}^{\text{pp}} = c_j - Z_j = 18 \quad 10 \quad 0 \quad 0$$



eliminating  $s_1$  in that place we replace  
the equation

$$NKE = \frac{4 + 1 + 0 - 20}{4} = 1 \frac{1}{4} \frac{1}{4} 0 .5$$

$$NKE = 1 \frac{1}{4} \frac{1}{4} 0 .5$$

New  $s_2$  = Old  $s_2$  - corresponding  $s_2$  NKE

$$\text{Old } s_2 = 2 \ 3 \ 0 \ 1 \ 30$$

$$C \cdot NKE = \begin{array}{r} 2 \ 1/2 \ 1/2 \ 0 \ 10 \\ \hline 0 \ 5/2 \ -1/2 \ 1 \ 20 \end{array}$$

$$\therefore \text{New } s_2 = 0 \ 5/2 \ -1/2 \ 1 \ 20$$

$$II. \quad C_j \quad 18 \quad 10 \quad 0 \quad 0$$

$C_b$	$b_r$	$x$	$y$	$s_1$	$s_2$	$x_b$	ratio
18	$x$	1	$\boxed{\frac{1}{4}}$	$\frac{1}{4}$	0	$\frac{5}{1/4}$	20
0	$s_2$	0	$\boxed{\frac{5}{2}}$	$\frac{-1}{2}$	1	$20/s_{1/2}$	8 $\leftarrow$
$z_j$		18	$\frac{9}{2}$	$\frac{9}{2}$	0		
$\Delta z_j = C_j - z_j$		0	$\frac{11}{2}$	$\frac{-9}{2}$	0		

$\uparrow$  max.

Eliminating  $s_2$  in that place we replace.

NKE

$$NKE = \frac{0 \ 5/2 \ -1/2 \ 1 \ 20}{\frac{5}{2}} = 0 \ 1 \ -1/5 \ 2/5 \ 8$$

$$NKE = 0 \ 1 \ -1/5 \ 2/5 \ 8$$

$$\text{New } \omega_c = \text{Old } \omega_c + c \cdot \text{NKE}$$

$$\text{Old } \omega_c = \begin{matrix} 1 & \frac{1}{4} & \frac{1}{4} & 0 & 5 \end{matrix}$$

$$c \cdot \text{NKE} = \begin{matrix} 0 & \frac{1}{4} & -\frac{1}{20} & \frac{1}{10} & 8 \end{matrix}$$

$$\text{New } \omega_c = \begin{matrix} 1 & 0 & \frac{3}{10} & -\frac{1}{10} & 3 \end{matrix}$$

	$C_j$	18	10	0	0		
$C_b$	$b_v$	$x$	$y$	$S_1$	$S_2$	$x_b$	ratio
18	$x$	1	0	$\frac{3}{10}$	$-\frac{1}{10}$	3	
10	$y$	0	1	$-\frac{1}{5}$	$\frac{2}{5}$	8	

$$Z_j = 18 \quad 10 \quad \frac{17}{5} \quad "15$$

$$\Delta_{ij}^{oo} = C_j - Z_j = 0 \quad 0 \quad -\frac{17}{5} \quad -1/5$$

$$\therefore C_j - Z_j \leq 0 \quad \therefore x=3, y=8$$

$$\begin{aligned} \therefore \text{obj function} &= 18(3) + 10(8) \\ &= 54 + 80 \\ &= 134 \end{aligned}$$

$\therefore$  The max value is 134

# Standard form of a linear programming problem

The standard form of a linear programming (LP) problem.

maximize or minimize:

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

subject to

$$g_1(x) = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$

$$g_2(x) = a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

- - - - -

$$g_m(x) = a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

## Components of the standard form

\* Decision Variables:  $x_1, x_2, \dots, x_n$

\* Objective function:  $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

\* Constraints:  $g_1(x), g_2(x), \dots, g_m(x)$

\* Right-hand side (RHS) values:  $b_1, b_2, \dots, b_m$

\* Non-Negativity Constraints:  $x_1, x_2, \dots, x_n \geq 0$

## Types of LP problems

\* Maximization problem: Maximize the objective function.

\* Minimization problem: Minimize the objective function.

## Importance of Standard form

- \* faster to solve: Standard form makes it easier to solve LP problems using methods like the Simplex Algorithm.
- \* Unified framework: Standard form provides a unified framework for representing and solving LP problems.
- \* Clear representation: Standard form clearly represents the decision variables, objective function, & constraints.

## Simplex Algorithm

Step-by-step to the Simplex Algorithm

Step-1: Convert to standard form

convert the linear programming problem (LPP) to standard form.

Step 2: Create initial Simplex Tableau

create initial simplex tableau.

Step 3: Choose pivot Element

choose the pivot element.

Step 4: Perform pivot operation / Step-5: Check optimality

perform the pivot operation. check the current solution is optimal

Step 6: Repeat steps 3-5

Repeat steps 3-5 until an optimal sol<sup>n</sup> is reached

Step 7: Read Solution.

Read the solution from the final simplex tableau.

## Simplex Algorithm Pseudocode

1. Initialize the simplex tableau.

2. while (not optimal) {

a. choose pivot element.

b. perform pivot operation

c. check optimality.

3. Read solution from final simplex tableau.

minimise  $Z = 2x_1 + 3x_2 + 6x_3$  is subjected to  $3x_1 - x_2 + 2x_3 \leq 7$   
 $2x_1 + 4x_2 \geq -12$ ,  $-4x_1 + 3x_2 + 8x_3 \leq 10$ ,  $x_1, x_2, x_3 \geq 0$

Given obj function  $Z = 2x_1 + 3x_2 + 6x_3$

Constraints:  $3x_1 - x_2 + 2x_3 \leq 7$

$$2x_1 + 4x_2 \geq -12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

Converting inequalities into equalities by add slack variables.

$$3x_1 - x_2 + 2x_3 + S_1 = 7$$

$$-2x_1 - 4x_2 + S_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$$

Obj function (Z) =  $-2x_1 + 3x_2 + 6x_3 + 0S_1 + 0S_2 + 0S_3$

		$C_j^o$						
$C_b$	$b_r$	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	$Z_{cb}$ ratio
0	$S_1$	-3	$\boxed{-1}$	2	1	0	0	$\frac{9}{1} = -7$
0	$S_2$	-2	$\boxed{-4}$	0	0	1	0	$\frac{12}{4} = -3$
0	$S_3$	-4	$\boxed{3}$	8	0	0	1	$\frac{10}{3} = 0/3 \leftarrow \text{max}$

$$Z_j^o = 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$Z_j^o - C_j^o = 2 \quad -3 \quad 6 \quad 0 \quad 0 \quad 0$$

$\downarrow \min$

Eliminating  $S_3$  and in that place we replace NKE equation.

$$\text{NKE} = \frac{-4 \quad 3 \quad 8 \quad 0 \quad 0 \quad 1 \quad 10}{3} = \frac{-4}{3}, \frac{8}{3}, 0, 0, \frac{1}{3}, \frac{10}{3}$$

$$\text{New } S_1 = \text{old } S_1 - \text{corres. NKE}$$

$$\text{old } S_1 = 3 - 1 \cdot 2 + 0 \cdot 0 = 7$$

$$\text{C. NKE} = \frac{4}{3}, -1 - \frac{68}{3}, 0, 0, -\frac{1}{3}, \frac{-10}{3}$$

$$\frac{5}{3}, 0, \frac{14}{3}, 1, 0, \frac{1}{3}, \frac{3}{3}$$

New  $s_2 = \text{old } s_2 - \text{corr. NKE}$

$$\text{old } s_2 = -2 \quad -4 \quad 0 \quad 0 \quad 1 \quad 0 \quad 12$$

$$\text{corr. NKE} = \frac{16}{3} \quad -4 \quad \frac{-32}{3} \quad 0 \quad 0 \quad \frac{-4}{3} \quad \frac{-40}{3}$$

$$\text{New } s_2 = -\frac{22}{3} \quad 0 \quad \frac{32}{3} \quad 0 \quad 1 \quad \frac{4}{3} \quad \frac{46}{3}$$

2<sup>nd</sup> Iteration

		$c_j^*$	-2	3	-6	0	0	0	$x_b$	ratio.
$C_B$	$b_r$	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$			
0	$s_1$	$\frac{5}{3}$	0	$\frac{14}{3}$	1	0	$\frac{1}{3}$	$\frac{3}{1}/3$	$\frac{3}{1}/3$	$= \frac{3}{1}/5 \leftarrow$
0	$s_2$	$-\frac{22}{3}$	0	$\frac{32}{3}$	0	1	$\frac{4}{3}$	$\frac{76}{3}$	$\frac{76}{3}$	$= -\frac{76}{22}$
3	$x_2$	$-\frac{4}{3}$	1	$\frac{8}{3}$	0	0	$\frac{1}{3}$	$\frac{10}{3}$	$\frac{10}{3}$	$= -\frac{10}{4}$

$$Z_j^* = -4 \quad 3 \quad 8 \quad 0 \quad 0 \quad 1$$

$$Z_j^* - c_j^* = -2 \quad 0 \quad 14 \quad 0 \quad 0 \quad 1$$

↑ min.

Eliminating  $s_1$  in that place we replace NKE  
Equation

$$\text{NKE} = \frac{\frac{5}{3} \quad 0 \quad \frac{14}{3} \quad 1 \quad 0 \quad \frac{1}{3} \quad \frac{3}{3}}{\frac{5}{3}} = 1 \quad 0 \quad \frac{14}{5} \quad \frac{3}{5} \quad 0 \quad \frac{1}{5}$$

New  $s_2 = \text{old } s_2 - \text{corr. } s_2$

$$\text{old } s_2 = -\frac{22}{3} \quad 0 \quad \frac{32}{3} \quad 0 \quad 1 \quad \frac{4}{3} \quad \frac{46}{3}$$

$$\text{corr. NKE} = \frac{-22}{3} \quad 0 \quad \frac{-308}{15} \quad \frac{-22}{5} \quad 0 \quad \frac{-22}{15} \quad \frac{682}{15}$$

$$0 \quad 0 \quad \frac{488}{15} \quad \frac{12}{5} \quad 1 \quad \frac{42}{15} \quad \frac{1062}{15}$$

New  $x_1 = \text{old } x_1 - \text{Corr. } s_1$

$$\text{old } x_1 = -4/3 + 8/3 \quad 0 \quad 0 \quad 1/2 \quad 10/5$$

$$\text{Corr. AIP} = -4/3 \quad 0 - \frac{96}{15} - 4/5 \quad 0 - 9/5 = \frac{124}{15}$$

$$\text{New } x_1 = 0 \quad 1 \quad \frac{96}{15} \quad 4/5 \quad 0 \quad 9/5 \quad \frac{124}{15}$$

	$c_j^o$	-2	3	-6	0	0	0		
$c_b$	$b_v$	$s_1$	$s_2$	$s_3$	$s_1$	$s_2$	$s_3$	$\text{ratio}$	
-2	$x_1$	0	0	$14/5$	$3/5$	0	$1/5$	$3/5$	
0	$s_2$	0	0	$\frac{468}{15}$	$\frac{12}{5}$	1	$\frac{42}{15}$	$\frac{1062}{15}$	
3	$s_2$	0	1	$\frac{96}{15}$	$4/5$	0	$9/5$		

$Z_j^o = -2 \quad 3 \quad \frac{204}{15} \quad 6/5 \quad 0 \quad 21/15$

$$Z_j^o - c_j^o = 0 \quad 0 \quad \frac{204}{15} \quad 6/5 \quad 0 \quad 21/15$$

$$Z_j^o - c_j^o \geq 0$$

These values satisfying the minimum condition.