Thứ
Ngày . . No.

BVN Đại số tuyến tính tuần 4.

GT. $A=\begin{bmatrix} 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 1 \\ -1 & 3 & 0 & -2 \end{bmatrix} \Rightarrow A^T=\begin{bmatrix} 0 & 2 & -1 \\ 1 & -5 & 3 \\ 3 & 5 & 0 \\ 5 & 1 & -2 \end{bmatrix}$

KL. Tính tích $A.A^T$ và $A^T.A$

Bgiải

⊖ $A.A^T=\begin{bmatrix} 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 1 \\ -1 & 3 & 0 & -2 \end{bmatrix}\cdot\begin{bmatrix} 0 & 2 & -1 \\ 1 & -5 & 3 \\ 3 & 5 & 0 \\ 5 & 1 & -2 \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$a_{11} = 0.0+1.1+3.3+5.5=35$
$a_{12} = 0.2+1.(-5)+3.5+5.1=15$
$a_{13} = 0.(-1)+1.3+3.0+5.(-2)=-7$
$a_{21} = 2.0-5.1+5.3+1.5=15$
$a_{22} = 2.2+(-5)^2+5^2+1^2=55$
$a_{23} = 2.(-1)-5.3+5.0+1.(-2)=-19$
$a_{31} = -1.0+3.1+0.3-2.5=-7$
$a_{32} = -2-15+0-2=-19$
$a_{33} = 1+9+0+4=14$

$\Rightarrow A.A^T=\begin{bmatrix} 35 & 15 & -7 \\ 15 & 55 & -19 \\ -7 & -19 & 14 \end{bmatrix}$

⊕ $A^T.A=\begin{bmatrix} 0 & 2 & -1 \\ 1 & -5 & 3 \\ 3 & 5 & 0 \\ 5 & 1 & -2 \end{bmatrix}\cdot\begin{bmatrix} 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 1 \\ -1 & 3 & 0 & -2 \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$

$a_{11}=0^2+2^2+(-1)^2=5$
$a_{12}=0.1+2.(-5)-1.3=-13$
$a_{13}=0+2.5+0=10$
$a_{14}=0+2.1-1.(-2)=4$

$a_{21}=0-5.2+3.(-1)=-13$
$a_{22}=1+5^2+3^2=35$
$a_{23}=1.3-5.5+3.0=-22$
$a_{24}=1.5-5.1+3.(-2)=-6$

$a_{31} = 3.0 + 5.2 + 0.(-1) = 10$

$a_{32} = 3.1 + 5.(-5) + 0.3 = -22$

$a_{33} = 3.3 + 5.5 + 0.0 = 34$

$a_{34} = 3.5 + 5.1 + 0.(-2) = 20$

$a_{41} = 5.0 + 1.2 - 2.(-1) = 4$

$a_{42} = 5.1 + 1.(-5) - 2.3 = -6$

$a_{43} = 5.3 + 1.5 - 2.0 = 20$

$a_{44} = 5.5 + 1.1 - 2.(-2) = 30$

Vậy $A^T A = \begin{bmatrix} 5 & -13 & 10 & 4 \\ -13 & 35 & -22 & -6 \\ 10 & -22 & 34 & 20 \\ 4 & -6 & 20 & 30 \end{bmatrix}$