

BTT tuần 6. BVN

①. Hệ ptinh sau có phải một hệ Cramer ko?

Nếu có hãy giải hệ = p<sup>2</sup> Cramer.

$$\begin{cases} 4x + y + z + w = 6 \\ 3x + 7y + -z + w = 1 \\ 7x + 3y - 5z + 8w = -3 \\ x + y + z + 2w = 3 \end{cases}$$

BG.

② Hệ ptinh có số ptinh = số ẩn = 4.

Mất ≠, ta có.

$$\begin{vmatrix} 4 & 1 & 1 & 1 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 4 \cdot \begin{vmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{25}{4} & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{5}{4} & \frac{-27}{4} & \frac{25}{4} \\ 0 & \frac{3}{4} & \frac{3}{4} & \frac{7}{4} \end{vmatrix} = \frac{4}{4^3} \cdot \begin{vmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 25 & -7 & 1 \\ 0 & 5 & -27 & 25 \\ 0 & 3 & 3 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-7}{25} & \frac{1}{25} \\ 0 & 0 & \frac{-128}{25} & \frac{124}{5} \\ 0 & 0 & \frac{96}{25} & \frac{172}{25} \end{vmatrix} \cdot \frac{1}{16} \cdot 25 = \frac{25}{16} \cdot \frac{1}{5} \cdot \frac{1}{25} \cdot \begin{vmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-7}{25} & \frac{1}{25} \\ 0 & 0 & -128 & 124 \\ 0 & 0 & 96 & 172 \end{vmatrix}$$

$$= \frac{-128}{16 \cdot 5} \cdot \begin{vmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-7}{25} & \frac{1}{25} \\ 0 & 0 & 1 & \frac{-31}{32} \\ 0 & 0 & 96 & 172 \end{vmatrix} = -\frac{8}{5} \cdot \begin{vmatrix} 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-7}{25} & \frac{1}{25} \\ 0 & 0 & 1 & \frac{-31}{32} \\ 0 & 0 & 0 & 205 \end{vmatrix} = -\frac{8}{5} \cdot 205 = -424 \neq 0$$

Vậy hệ ptinh có n<sub>đ</sub> duy nhất → là hệ Cramer.  
4 ẩn, 4 ptinh

$$\text{①} \quad \text{Ta có: } \begin{vmatrix} 6 & 1 & 1 & 1 \\ 1 & 7 & -1 & 1 \\ -3 & 3 & -5 & 8 \\ 3 & 1 & 1 & 2 \end{vmatrix} = X$$

$$\text{②} \quad x = \frac{-424}{-424} = 1$$

$$X = 6 \cdot \begin{vmatrix} 1 & 1/6 & 1/6 & 1/6 \\ 0 & 41/6 & -7/6 & 5/6 \\ 0 & 7/2 & -9/2 & 17/2 \\ 0 & 4 & -4 & 10 \end{vmatrix} = 6 \cdot \frac{1}{6} \cdot \frac{1}{2} \cdot \begin{vmatrix} 1 & 1/6 & 1/6 & 1/6 \\ 0 & 41 & -7 & 5 \\ 0 & 7 & -9 & 17 \\ 0 & 4 & -4 & 10 \end{vmatrix}$$

$$= \frac{1}{2} \cdot 41 \cdot \begin{vmatrix} 1 & 1/6 & 1/6 & 1/6 \\ 0 & 1 & -7/41 & 5/41 \\ 0 & 7 & -9 & 17 \\ 0 & 4 & -4 & 10 \end{vmatrix} = \frac{41}{2} \cdot \begin{vmatrix} 1 & 1/6 & 1/6 & 1/6 \\ 0 & 1 & -7/41 & 5/41 \\ 0 & 0 & -320/41 & 662/41 \\ 0 & 0 & -136/41 & 390/41 \end{vmatrix}$$

$$= \frac{41}{2} \cdot \frac{1}{41^2} \cdot \begin{vmatrix} 1 & 1/6 & 1/6 & 1/6 \\ 0 & 1 & -7/41 & 5/41 \\ 0 & 0 & -320 & 662 \\ 0 & 0 & -136 & 390 \end{vmatrix} = \frac{1}{82} \cdot (-320) \cdot \begin{vmatrix} 1 & 1/6 & 1/6 & 1/6 \\ 0 & 1 & -7/41 & 5/41 \\ 0 & 0 & 1 & -331/160 \\ 0 & 0 & 0 & 2173/20 \end{vmatrix}$$

$$= -424$$

$$\text{Vậy } X = -424 \Rightarrow x = \frac{-424}{-424} = 1$$

$$\text{③} \quad y = \frac{\begin{vmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & -1 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{vmatrix}}{-424} = \frac{y}{-424}$$

$$y = 4 \cdot 3 \cdot 7 \cdot \begin{vmatrix} 1 & 3/2 & 1/4 & 1/4 \\ 1 & 1/3 & -1/3 & 1/3 \\ 1 & -3/7 & -5/7 & 8/7 \\ 1 & 3 & 1 & 2 \end{vmatrix} = 84 \cdot \begin{vmatrix} 1 & 3/2 & 1/4 & 1/4 \\ 0 & -7/6 & -7/12 & 1/12 \\ 0 & -27/4 & -27/28 & 25/28 \\ 0 & 3/2 & 3/4 & 7/4 \end{vmatrix}$$



$$= 84 \cdot \left(-\frac{7}{6}\right) \cdot \left(-\frac{27}{14}\right) \cdot \frac{3}{2} \cdot \begin{vmatrix} 1 & 3/2 & 1/4 & 1/4 \\ 0 & 1 & -1/2 & -1/14 \\ 0 & 1 & 1/2 & -25/54 \\ 0 & 1 & 1/2 & 7/6 \end{vmatrix} = \frac{567}{2} \cdot \begin{vmatrix} 1 & 3/2 & 1/4 & 1/4 \\ 0 & 1 & 1/2 & -1/14 \\ 0 & 0 & 0 & -\frac{74}{189} \\ 0 & 0 & 0 & \frac{26}{21} \end{vmatrix} = 0$$

Uay  $y=0 \Rightarrow y=0$

$$\oplus \quad z = \begin{vmatrix} 4 & 1 & 6 & 1 \\ 3 & 7 & 1 & 1 \\ 7 & 3 & -3 & 8 \\ 1 & 1 & 3 & 2 \end{vmatrix} = \frac{z}{-424}$$

$$z = 4 \cdot 3 \cdot 7 \cdot \begin{vmatrix} 1 & 1/4 & 3/2 & 1/4 \\ 1 & 7/3 & 1/3 & 1/3 \\ 1 & 3/2 & -3/7 & 8/7 \\ 1 & 1 & 3 & 2 \end{vmatrix} = 84 \cdot \begin{vmatrix} 1 & 1/4 & 3/2 & 1/4 \\ 0 & 25/12 & -7/6 & 1/12 \\ 0 & 5/28 & -27/14 & 25/28 \\ 0 & 3/4 & 3/2 & 7/4 \end{vmatrix}$$

$$= 84 \cdot \frac{25}{12} \cdot \frac{5}{28} \cdot \frac{3}{4} \cdot \begin{vmatrix} 1 & 1/4 & 3/2 & 1/4 \\ 0 & 1 & -19/25 & 1/25 \\ 0 & 1 & -54/5 & 5 \\ 0 & 1 & 2 & 7/3 \end{vmatrix} = \frac{375}{16} \cdot \begin{vmatrix} 1 & 1/4 & 3/2 & 1/4 \\ 0 & 1 & -19/25 & 1/25 \\ 0 & 0 & -256/25 & 124/25 \\ 0 & 0 & \frac{64}{25} & \frac{172}{75} \end{vmatrix}$$

$$= \frac{375}{16} \cdot \left(-\frac{256}{25}\right) \cdot \frac{64}{25} \cdot \begin{vmatrix} 1 & 1/4 & 3/2 & 1/4 \\ 0 & 1 & -19/25 & 1/25 \\ 0 & 0 & 1 & -31/64 \\ 0 & 0 & 1 & \frac{43}{48} \end{vmatrix} = -\frac{3072}{5} \cdot \begin{vmatrix} 1 & 1/4 & 3/2 & 1/4 \\ 0 & 1 & -19/25 & 1/25 \\ 0 & 0 & 1 & -31/64 \\ 0 & 0 & 0 & \frac{265}{192} \end{vmatrix} = 848$$

Uay  $z=848 \Rightarrow z = \frac{-848}{-424} = +2$

Thứ  
Ngày  
 No

$$+ W = \frac{\begin{vmatrix} 4 & 1 & 1 & 6 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & -3 \\ 1 & 1 & 1 & 3 \end{vmatrix}}{-424} = \frac{W}{-424}$$

$$W = 4 \cdot 3 \cdot 7 \cdot \begin{vmatrix} 1 & 1/4 & 1/4 & 3/2 \\ 1 & 7/3 & -1/3 & 1/3 \\ 1 & 3/7 & -5/7 & -3/7 \\ 1 & 1 & 1 & 3 \end{vmatrix} = 84 \cdot \begin{vmatrix} 1 & 1/4 & 1/4 & 3/2 \\ 0 & 25/12 & -7/12 & -7/6 \\ 0 & 5/28 & -23/28 & -27/14 \\ 0 & 3/4 & 3/4 & 3/2 \end{vmatrix}$$

$$= 84 \cdot \frac{25}{12} \cdot \frac{5}{28} \cdot \frac{3}{4} \cdot \begin{vmatrix} 1 & 1/4 & 1/4 & 3/2 \\ 0 & 1 & -7/25 & -14/25 \\ 0 & 1 & -27/5 & -54/5 \\ 0 & 1 & 1 & 2 \end{vmatrix} = \frac{375}{16} \cdot \begin{vmatrix} 1 & 1/4 & 1/4 & 3/2 \\ 0 & 1 & -7/25 & -14/25 \\ 0 & 0 & -128/25 & -256/25 \\ 0 & 0 & 32/25 & 64/25 \end{vmatrix}$$

$$= \frac{375}{16} \cdot \left(-\frac{128}{25}\right) \cdot \left(\frac{32}{25}\right) \cdot \begin{vmatrix} 1 & 1/4 & 1/4 & 3/2 \\ 0 & 1 & -7/25 & -14/25 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{vmatrix} = 0 \quad (\text{do có 2 hàng bằng nhau})$$

Vậy  $W=0 \Rightarrow W = \frac{W}{-424} = 0$

Như vậy, hệ phương trình có nghiệm  $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ +2 \\ 0 \end{bmatrix}$