Btập 3. Hãy chéo hóa ~~(cấc)~~ trực giao các ma trận sau.

$$A=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}\qquad B=\begin{bmatrix}-3&1&2\\1&-3&2\\2&2&0\end{bmatrix}\qquad C=\begin{bmatrix}-2&0&-36\\0&-3&0\\-36&0&-23\end{bmatrix}$$

Bgiải

a)

$P_A(\lambda)=\begin{vmatrix}2-\lambda&2&2\\2&2-\lambda&2\\2&2&2-\lambda\end{vmatrix}=(2-\lambda)^3+8+8-4.(2-\lambda)-4.(2-\lambda)-4.(2-\lambda)$

$=(8-3.4.\lambda+3.2.\lambda^2-\lambda^3)+8.2+3.4.(\lambda-2)$

$=3.8-12\lambda+\cancel{12}\,6\lambda^2-\lambda^3+12\lambda-3.8$

$=-\lambda^3+\cancel{12}\,6\lambda^2=\lambda^2.(\cancel{12}\,6-\lambda)$

Vậy gtr riêng $\lambda=0$ $\lambda=\cancel{12}\,6$

⊕ Xét kgn vs $\lambda=0$

$$\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}.\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow\left[\begin{array}{ccc|c}2&2&2&0\\2&2&2&0\\2&2&2&0\end{array}\right]\xrightarrow[\substack{R_3-R_1\to R_3\\ \frac{R_1}{2}\to R_1}]{R_2-R_1\to R_2}\left[\begin{array}{ccc|c}1&1&1&0\\0&0&0&0\\0&0&0&0\end{array}\right]$$

Vậy $V_0=\left\{\begin{bmatrix}-x_2-x_3\\x_2\\x_3\end{bmatrix}\right\}=\left\{x_2.\begin{bmatrix}-1\\1\\0\end{bmatrix}+x_3.\begin{bmatrix}-1\\0\\1\end{bmatrix}\right\}=\mathcal{L}\left(\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\0\\1\end{bmatrix}\right)$

+ Đặt $u_1=(-1,1,0)$ $u_2=(-1,0,1)$

$v_1=u_1=(-1,1,0)\Rightarrow e_1=\dfrac{v_1}{\|v_1\|}=\dfrac{1}{\sqrt2}.(-1,1,0)$

$v_2=u_2-pr_{v_1}(u_2)=u_2-\dfrac{\langle u_2,v_1\rangle}{\langle v_1,v_1\rangle}.v_1=u_2-\dfrac{1+0+0}{1+1+0}.v_1$

$=(-1,0,1)-\dfrac12.(-1,1,0)=\left(-\dfrac12;-\dfrac12;1\right)=-\dfrac12.(1,1,-2)$

$\Rightarrow e_2=\dfrac{(-2v_2)}{\|(-2v_2)\|}=\dfrac{1}{\sqrt{1+1+4}}(1,1,-2)=\dfrac{1}{\sqrt6}.(1,1,-2)$

Vậy e_1,e_2 là cs trực chuẩn of V_0

① Xét kcn vs $\lambda = \cancel{12}\ 6$

$$\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} -4 & 2 & 2 & 0 \\ 2 & -4 & 2 & 0 \\ 2 & 2 & -4 & 0 \end{array}\right] \xrightarrow[\substack{\frac{R_2}{2} \to R_2 \\ \frac{R_3}{2} \to R_3}]{\substack{R_1 \leftrightarrow R_2 \\ \frac{R_1}{2} \to R_1}} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right] \xrightarrow[R_3 - R_1 \to R_3]{R_2 + 5R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & -24 & 6 & 0 \\ 0 & 6 & -6 & 0 \end{array}\right]$$

$$\xrightarrow[\frac{R_3}{6} \to R_3]{\frac{R_2}{-6} \to R_2} \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -5 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 4 & -1 & 0 \end{array}\right] \xrightarrow[R_3 - 4R_2 \to R_3]{R_1 + 5R_2 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$$\xrightarrow[\frac{R_3}{3} \to R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Vậy $V_{12} = \left\{ \cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \right\}$

Vì $\dim V_0 + \dim V_{12} = 2 + 1 = 3 \rightarrow ??$

Chọn $Q = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & \\ 1/\sqrt{2} & 1/\sqrt{6} & \\ 0 & -2/\sqrt{6} & \end{bmatrix}$ thì $Q^T . A . Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

b)

$P_B(\lambda) =$