ĐSTT BVN T13

Đề bài: Cho hệ vectơ $u_1(2,1,3,-1)$ $u_2(7,4,3,-3)$
$u_3(1,1,-6,0)$ $u_4(5,7,7,8)$

Sử dụng phép trực giao hóa, hãy tìm 1 cơ sở trực chuẩn của kg con sinh bởi 4 vectơ này

Bg giải.

⊕ $v_1 = u_1 = (2,1,3,-1) \Rightarrow w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{15}}\cdot(2,1,3,-1)$

⊕ $v_2 = u_2 - pr_{v_1}(u_2) = u_2 - \frac{\langle u_2, v_1\rangle}{\langle v_1, v_1\rangle}\cdot v_1 = u_2 - \frac{14+4+9+3}{4+1+9+1}\cdot v_1$

$v_2 = u_2 - 2v_1 = (7,4,3,-3) - 2.(2,1,3,-1) = (3,2,-3,-1) \Rightarrow w_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{23}}\cdot(3,2,-3,-1)$

⊕ $v_3 = u_3 - pr_{v_1}(u_3) - pr_{v_2}(u_3) = u_3 - \frac{\langle u_3, v_1\rangle}{\langle v_1, v_1\rangle}\cdot v_1 - \frac{\langle u_3, v_2\rangle}{\langle v_2, v_2\rangle}\cdot v_2$

$v_3 = u_3 - \frac{2+1-18+0}{4+1+9+1}\cdot v_1 - \frac{3+2+18+0}{9+4+9+1}\cdot v_2 = u_3 + v_1 - v_2$

$v_3 = (1,1,-6,0) + (2,1,3,-1) - (3,2,-3,-1) = (0,0,0,0)$

⊕ $v_4 = u_4 - pr_{v_1}(u_4) - pr_{v_2}(u_4) - pr_{v_3}(u_4)$

$= u_4 - \frac{\langle u_4, v_1\rangle}{\langle v_1, v_1\rangle}\cdot v_1 - \frac{\langle u_4, v_2\rangle}{\langle v_2, v_2\rangle}\cdot v_2 - 0$ (vì $v_3 = (0,0,0,0) \Rightarrow pr_{v_3}(u_4) = 0$)

$= u_4 - \frac{10+7+21-8}{4+1+9+1}\, v_1 - \frac{15+14-21-8}{9+4+9+1}\cdot v_2 = u_4 - 2v_1 - 0.v_2$

$= (5,7,7,8) - 2.(2,1,3,-1) = (1,5,1,10) \Rightarrow w_4 = \frac{v_4}{\|v_4\|} = \frac{1}{\sqrt{127}}\cdot(1,5,1,10)$

Vậy cơ sở trực chuẩn ~~(sinh bởi)~~ of kg con sinh bởi 4 vectơ ~~này~~ u_1, u_2, u_3, u_4 là:

$$w_1 = \begin{bmatrix} 2/\sqrt{15} \\ 1/\sqrt{15} \\ 3/\sqrt{15} \\ -1/\sqrt{15} \end{bmatrix} \quad w_2 = \begin{bmatrix} 3/\sqrt{23} \\ 2/\sqrt{23} \\ -3/\sqrt{23} \\ -1/\sqrt{23} \end{bmatrix} \quad w_4 = \begin{bmatrix} 1/\sqrt{127} \\ 5/\sqrt{127} \\ 1/\sqrt{127} \\ 10/\sqrt{127} \end{bmatrix}$$