DATA STRUCTURE AND ALGORITHMS

LECTURE 5

Trees

Reference links:

https://cs.nyu.edu/courses/fall17/CSCI-UA.0102-007/notes.php

https://www.comp.nus.edu.sg/~stevenha/cs2040.html

[M.Goodrich, chapter 8]

Lecture outline

- Tree ADT
  - Definitions
  - Applications
- Binary Tree
  - Definitions
  - Implementations
    - Linked Structure
    - Array based
  - Binary Tree Traversal
- Binary Tree Application

Tree ADT

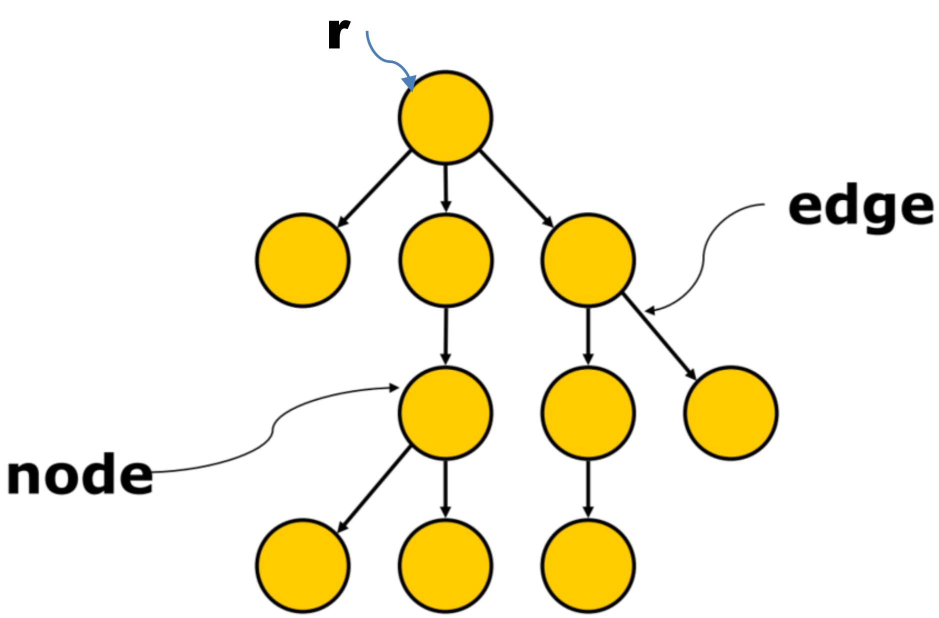
Introduction

Why trees?

- Tree structures are a breakthrough in data organization, nonlinear way. – Cách tổ chức dữ liệu phi tuyến
- Allow implementing algorithms much faster than using linear data structures, such as arrays or linked lists.
- Trees are ubiquitous in CS, covering operating systems, computer graphics, databases, etc.

Trees: definitions

- A tree is a collection of nodes. Non-empty trees have a distinguished **node** r, called root, and zero or more nonempty (sub)trees T1 , T2, . . . , Tk, each of whose roots are connected by a directed **edge** from r



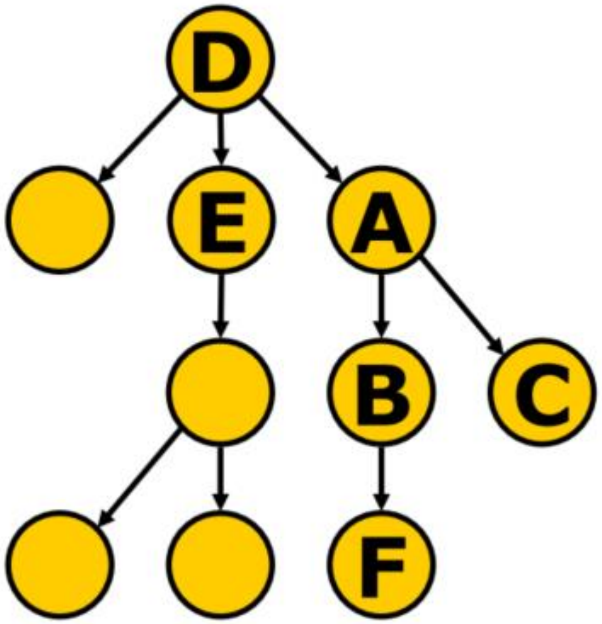
node: data object in tree

edge: links between nodes

Trees: definitions

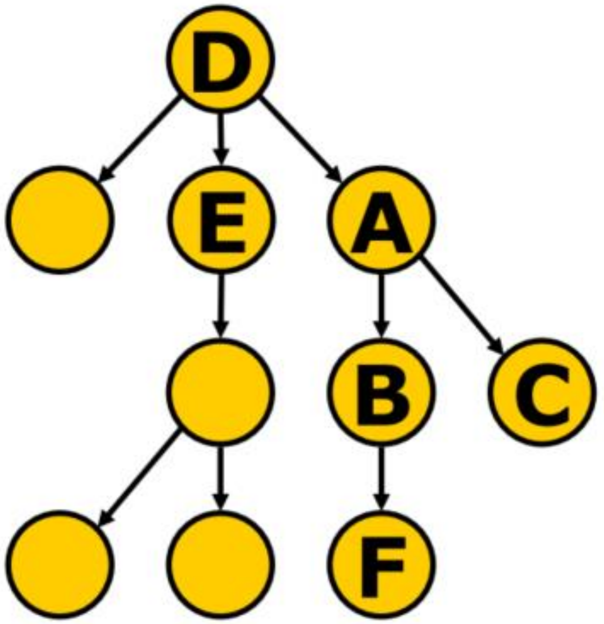
- Relationship
  - **A** is a parent of **B** and **C**
  - **B** and **C** are children of **A**
  - **B** and **C** are siblings

    (with the same parent **A**)
  - **D** is an ancestor of **B**
  - **B** is a descendant of **A** and **D**
- **D** is root (has no parent) – nút gốc
- **F** is leaf (has no children) – nút lá



Trees: definitions

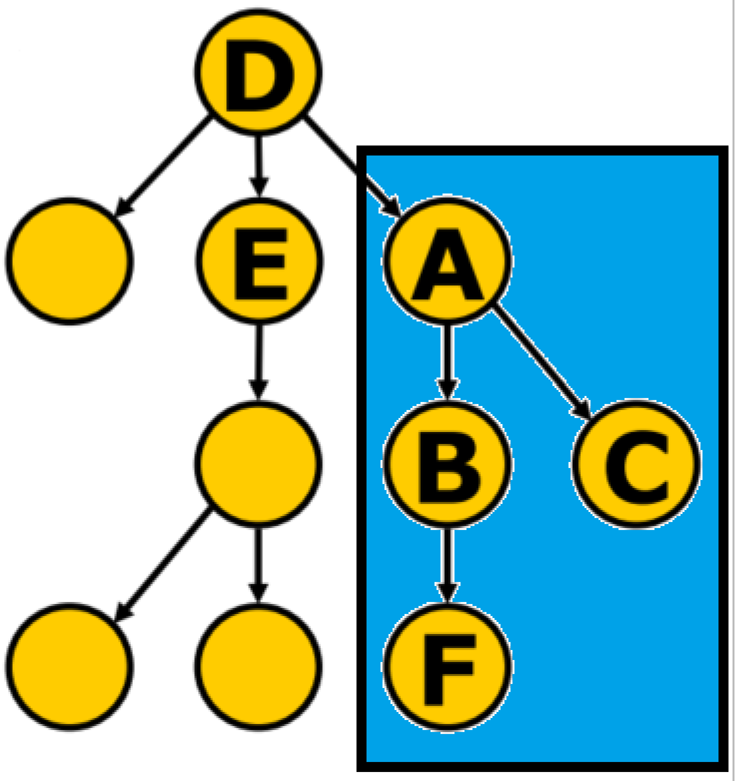
- External node – node ngoài
  - Has no children (also leaf)
    - F, C are external nodes
- Internal node – node trong
  - Has one or more children
    - E, A, B are intenal nodes



Trees: definitions

- Subtree – cây con
  - A node and all of its descendants

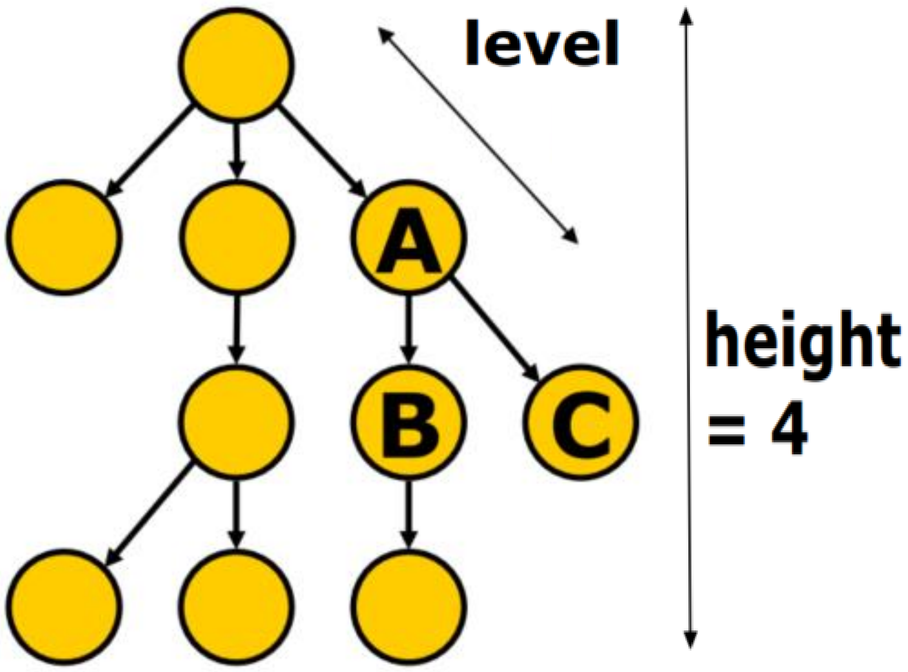
    Ex: Subtree with root **A**



  - Q: can a leaf be a subtree?

Trees: definitions

- **Level** of a node - mức của node
  - Number of nodes on the path from the root to the node
    - level of **root** is 1
    - level of **A** is 2



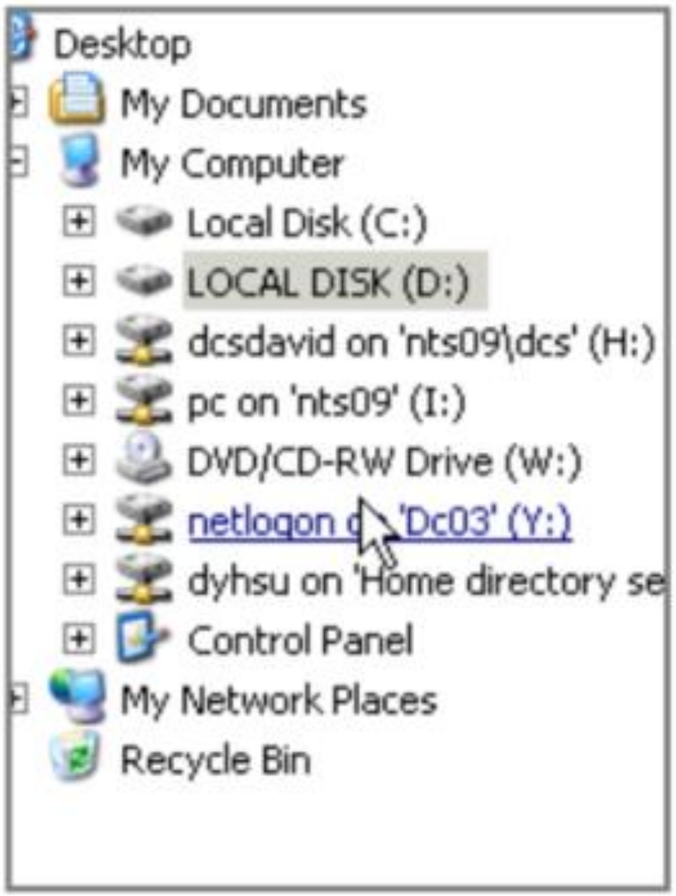
- **Height** of a tree

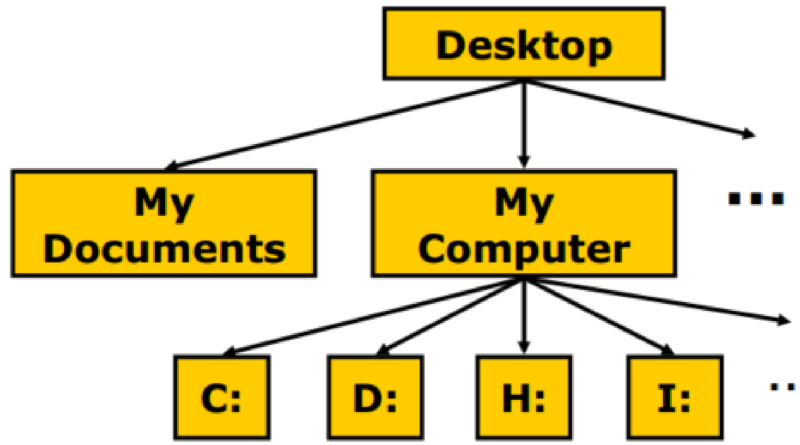
  – chiều cao của cây

  - Maximum level of the nodes in the tree

Trees: applications

- File systems

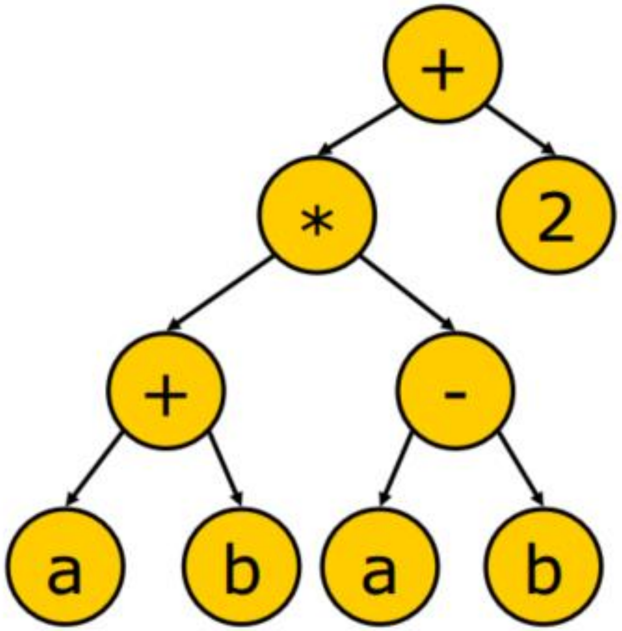




Trees: applications

- Arithmetic Expressions

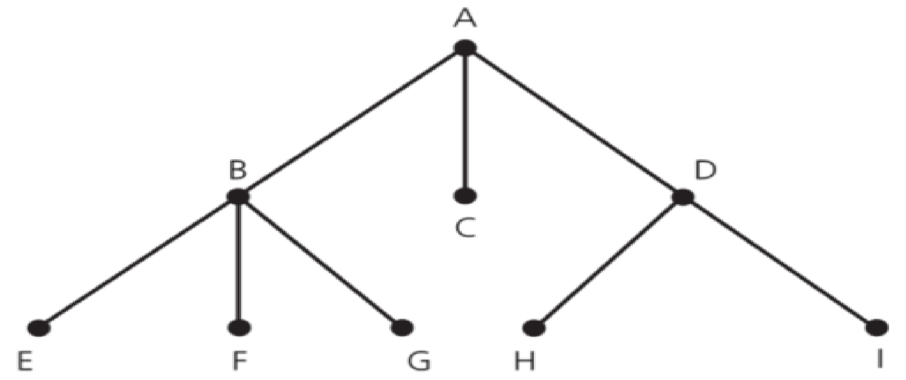
$(a+b) * (a-b) + 2$



Q: How to construct such a tree from a given arithmetic expression?

General Trees

- An *n*-ary tree (cây *n* phân)
  - A tree whose nodes each have no more than *n* children



  - The Tree ADT
    - Specification, Implementation
    - Read and handle by yourself

[M. Goodrich, subsection 8.1.2, 8.1.3]

General Trees

- Tree ADT Methods

element(v)	*// returns the content of node v*
root()	*// returns the root node of the tree.*
parent(v)	*//returns the parent of node v*
children(v)	*//returns an iterable container holding the children of node v*
isInternal(v)	*// returns true if node v is internal*
isExternal(v)	*// returns true if node v is external*
isRoot(v)	*// returns true if node v is the root*
size()	*// returns the number of nodes in the tree*
isEmpty()	*// returns true if the tree is empty*
positions()	*// returns an iterable container of all nodes in the tree*
elements()	*// returns an iterable container of the content of all node in the tree*
replace(v,e)	*// replace the contents of node v with e. return the old contents of v*

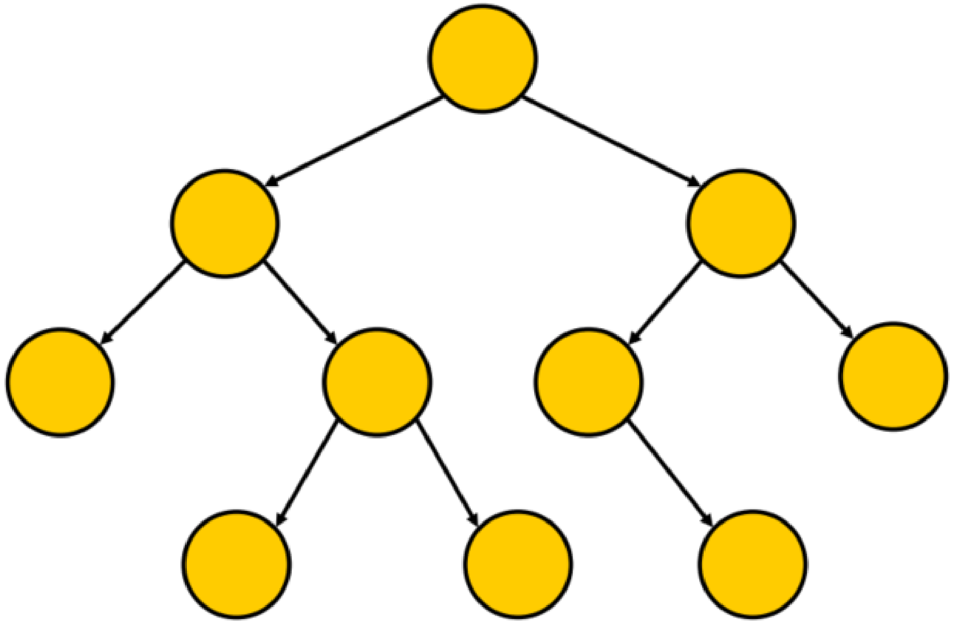
For more details on the Tree Interface [M. Goodrich, p313]

Tree ADT

Binary Tree

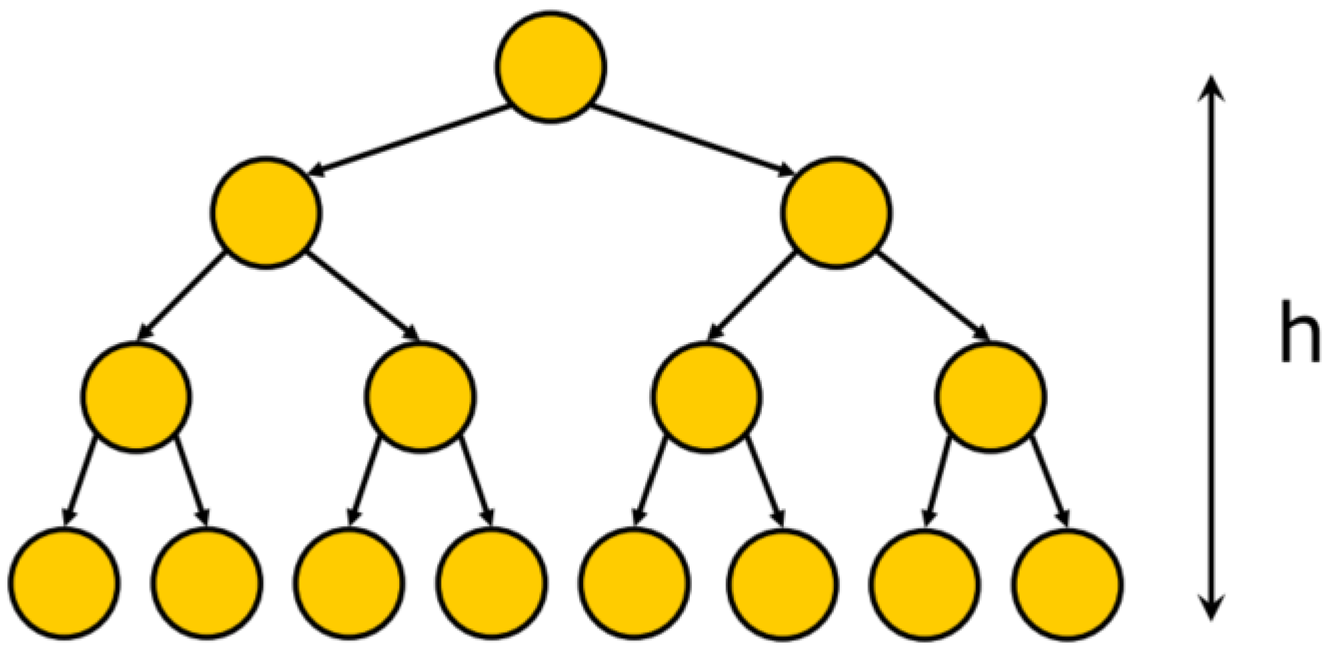
Binary Tree: Definitions

- Binary Tree = 2-ary tree (cây nhị phân)
  - Each node has at most 2 “ordered” children
  - Two children: left child, right child



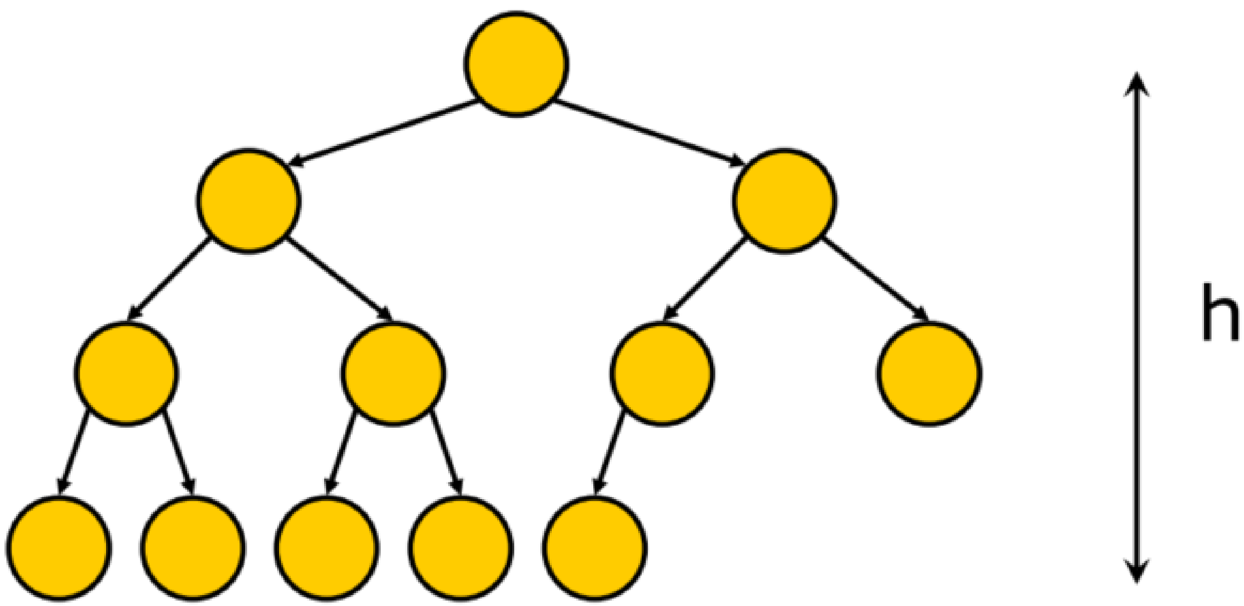
Binary Tree: Definitions

- Full Binary Tree – Cây nhị phân đầy đủ
  - All nodes at a level < h have two children (where h is the height of the tree)



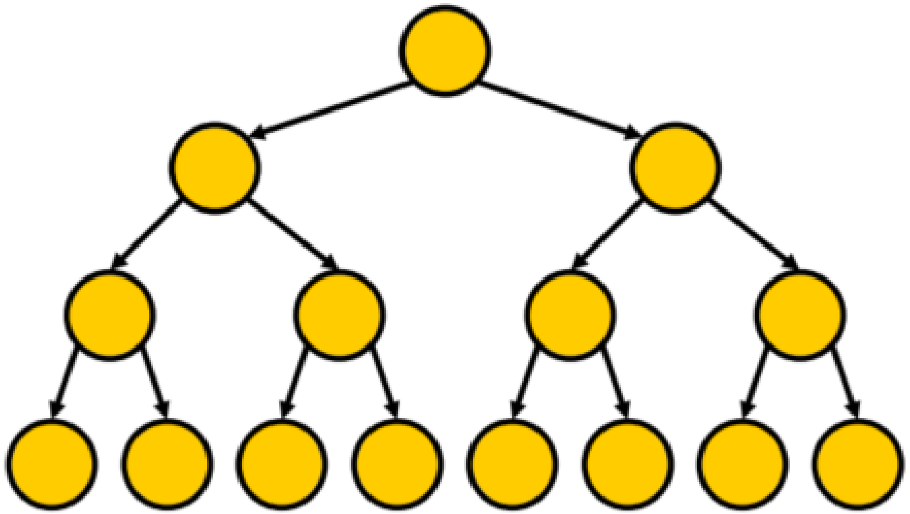
Binary Tree: Definitions

- Complete Binary Tree – Cây nhị phân hoàn chỉnh
  - Full down to level h-1
  - Level h filled in from left to right.



Binary Tree: Definitions

- Full Binary Tree Properties
  - Number of nodes in a full binary tree of height h is $N = 2^h - 1$.
  - Therefore, the height of a full binary tree is O(log N)



- h = 4
- N = 15

- Some other interesting properties [M. Goodrich, ss. 8.2.2, p321]

Binary Tree ADT: Specification

- Binary Tree Interface

```
public interface BinaryTree<E> extends Tree<E> {
    // Returns the Position of p's left child (or null if no child exists).
    Position<E> left(Position<E> p) throws IllegalArgumentException;
    // Returns the Position of p's right child (or null if no child exists)
    Position<E> right(Position<E> p) throws IllegalArgumentException;
    // Returns the Position of p's sibling (or null if no sibling exists)
    Position<E> sibling(Position<E> p) throws IllegalArgumentException;
}
```

Position meaning for node on tree

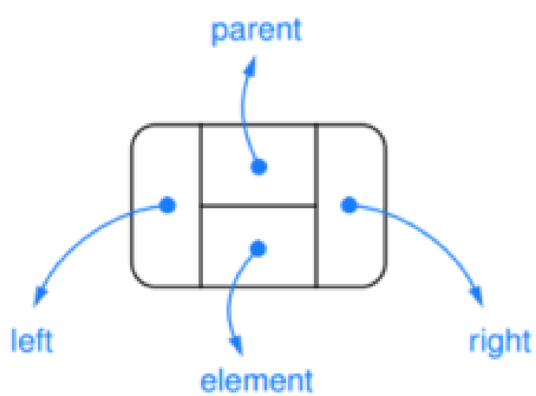
A BinaryTree interface [M.Goodrich, p.319]

Binary Tree: Implementation

- Implement Binary Tree
  - Use Linked structure
  - Array based

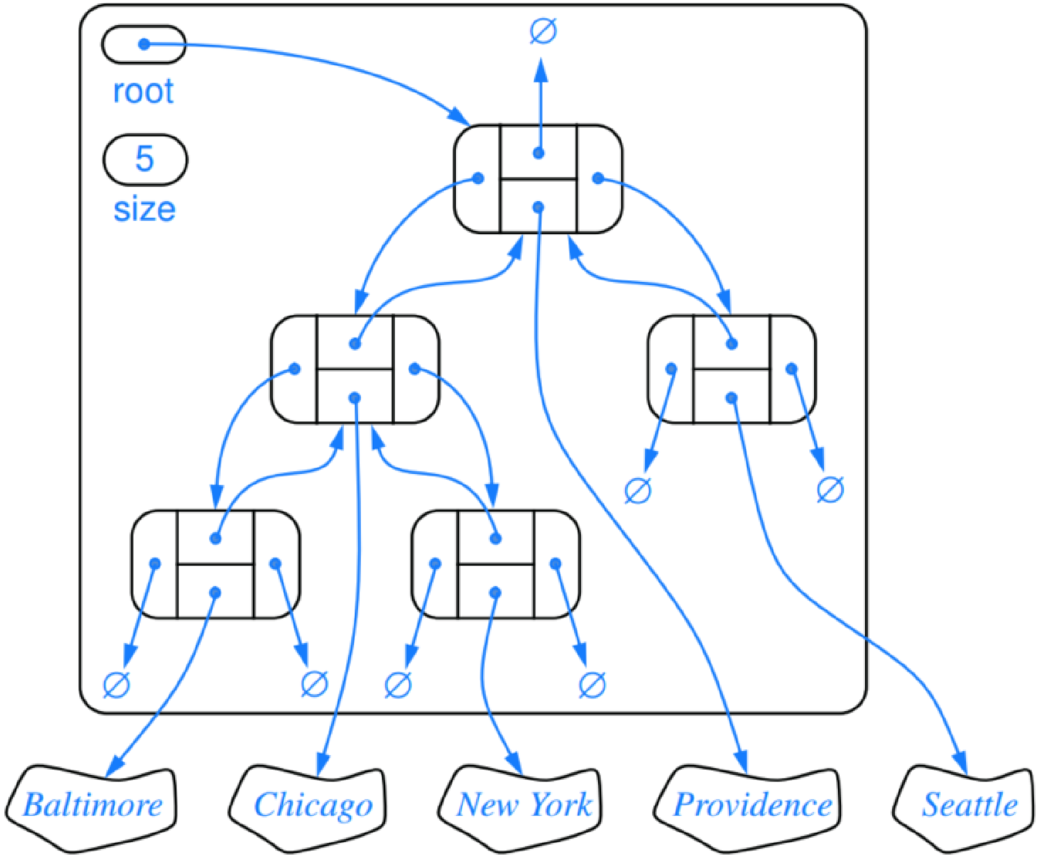
Binary Tree: Linked Structure

- Implement Binary Tree using Linked Structure



One node with:

parent, left, right: references
element: data



Binary Tree with five nodes

Source: [M.Goodrich, p323]

Binary Tree: Linked Structure

- Implement Binary Tree using Linked Structure
  - Some methods need updated

```
class LinkedBinaryTree<E>  {
        class Node<E> {
                private E element; // an element stored at this node
                private Node<E> parent; // a reference to the parent node (optional)
                private Node<E> left; // a reference to the left child (if any)
                private Node<E> right; // a reference to the right child (if any)
                //more definitions
        }
        //methods
}
```

For more detail see Code Fragment 8.9-8.11 [M.Goodrich, p326]

Binary Tree: Linked Structure

- Implement Binary Tree using Linked Structure
  - Some methods need updated

addRoot(e) : Khởi tạo nút gốc cho cây rỗng.

addLeft(p, e) : Thêm nút con trái cho nút p (chưa có con trái).

addRight(p, e) : Thêm nút con phải cho nút p (chưa có con phải).

set(p, e) : Đặt phần tử e vào nút p.

attach(p, T1,T2) : Đính 2 cây T1, T2 thành 2 cây con trái và phải của nút lá p.

remove(p) : Xóa nút p, thay thế bằng con (nếu có 1 nút con).

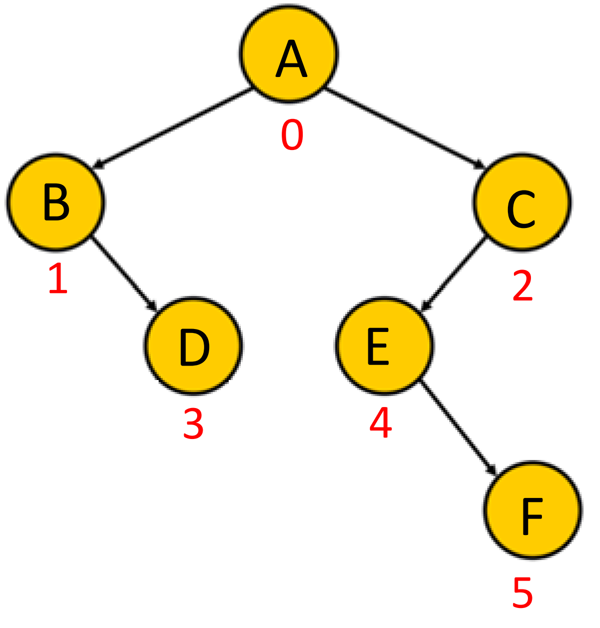
For more detail see Code Fragment 8.10, 8.11 [M.Goodrich, p328]

Binary Tree: Array-Based

- Implement Binary Tree using Array

Nodes T[]

	0	1	2	3	4	5
Data	A	B	C	D	E	F
Parent	-1	0	0	1	2	4
Left	1	-1	4	-1	-1	-1
Right	2	3	-1	-1	5	-1



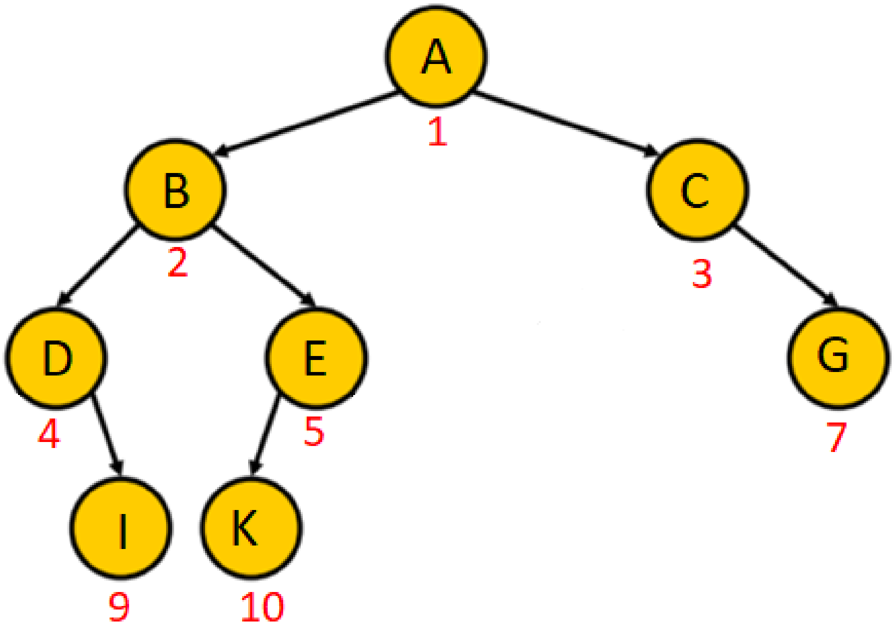
- Pros: Easy to handle each node
- Cons: Waste memory

Binary Tree: Array-Based

- Implement Binary Tree using Array
  - Effective array for Complete Binary Tree

Nodes T[]

A	B	C	D	E	Ø	G	Ø	I	K
1	2	3	4	5	6	7	8	9	10



Given that a node is stored in index position i, what is the position of its **parent**? **left child**? **right child**?

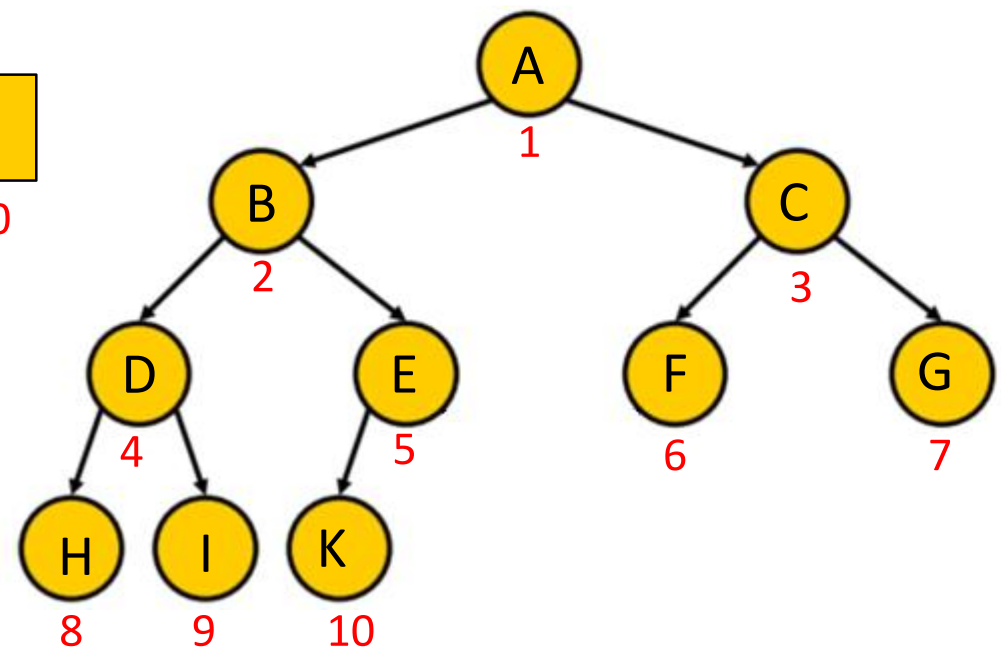
Binary Tree: Array-Based

- Implement Binary Tree using Array
  - Effective array for Complete Binary Tree

Nodes T[]

A	B	C	D	E	F	G	H	I	K
1	2	3	4	5	6	7	8	9	10

Given that a node is stored in index position i, what is the position of its **parent**? **left child**? **right child**?



Binary Tree Traversal

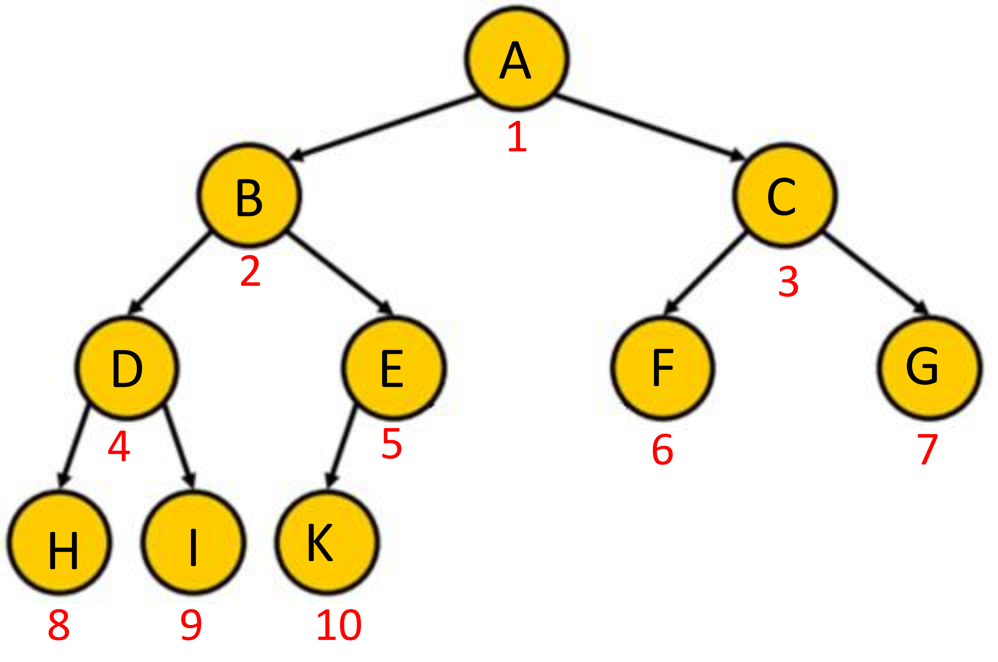
Binary Tree Traversal: Orders

- A binary tree is defined recursively: it consists of a root, a left subtree, and a right subtree
- To traverse (or walk) the binary tree is to visit each node in the binary tree exactly once
- Tree traversals are naturally recursive
- Since a binary tree has three “parts,” there are six possible ways to traverse the binary tree:
  - root, left, right
  - left, root, right
  - left, right, root
  - root, right, left
  - right, root, left
  - right, left, root

Binary Tree Traversal: Orders

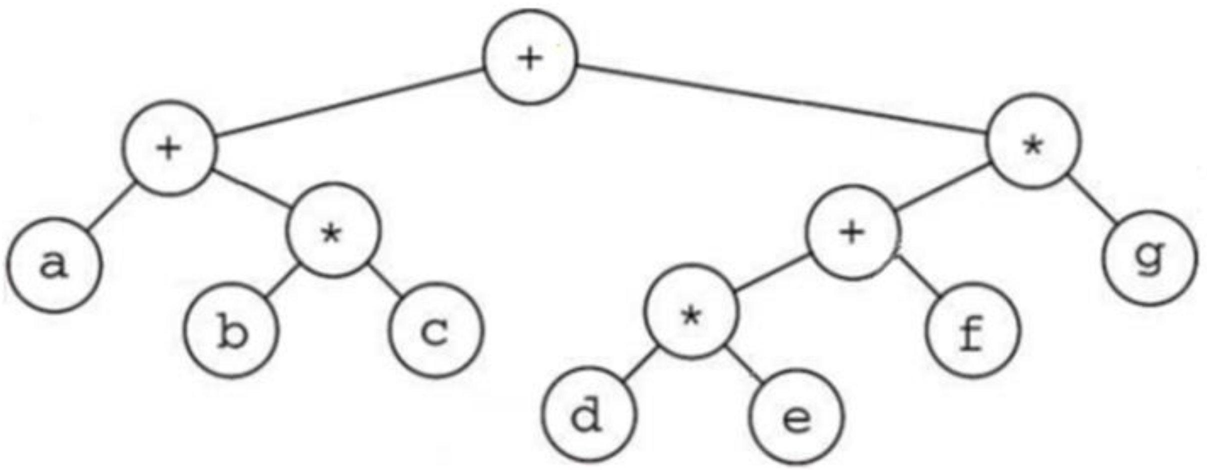
- A binary tree is defined recursively: it consists of a root, a left subtree, and a right subtree
- To traverse (or walk) the binary tree is to visit each node in the binary tree exactly once
- Tree traversals are naturally recursive
- Since a binary tree has three “parts,” there are six possible ways to traverse the binary tree:
    - root, left, right ⟹ Pre-order Travesal
    - left, root, right ⟹ In-order Travesal
    - left, right, root ⟹ Post-order Travesal

Binary Tree Traversal: Orders



- Pre-order Travesal (root, left, right) ⟹ ?
- In-order Travesal (left, root, right) ⟹ ?
- Post-order Travesal (left, right, root) ⟹ ?

Binary Tree Traversal: Orders



- Pre-order Travesal (root, left, right) ⟹ ?
- In-order Travesal (left, root, right) ⟹ ?
- Post-order Travesal (left, right, root) ⟹ ?

Pre-order Travesal

- In Pre-order Traversal, the root is visited **first**
  - Order: Root - Left - Right

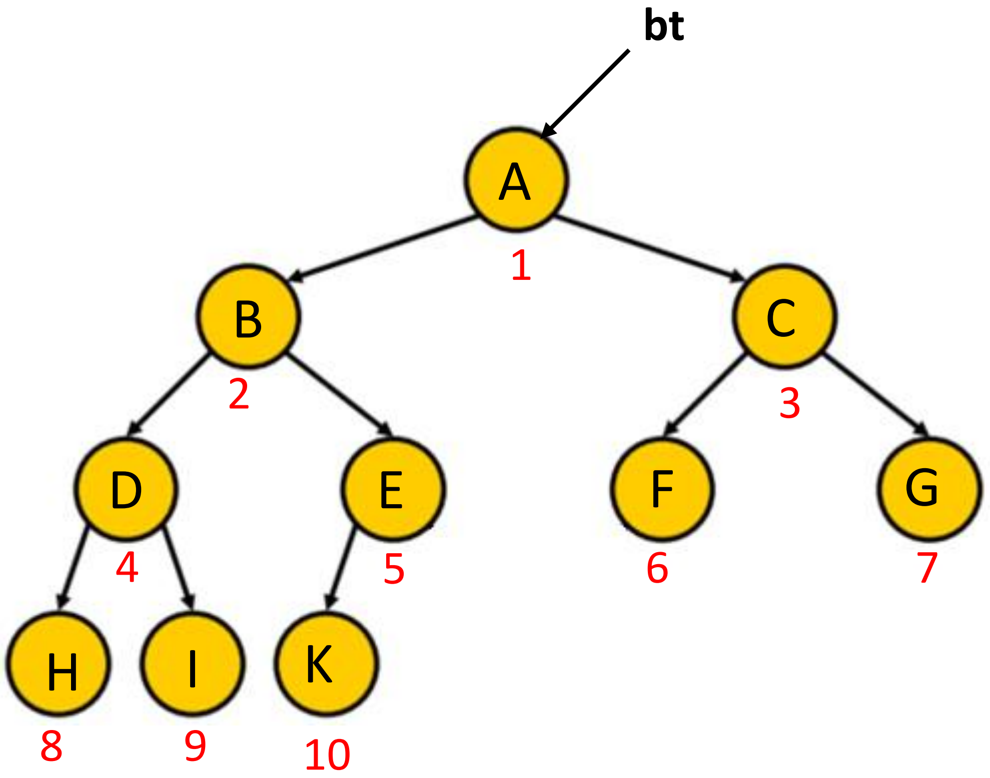
- Algorithm for preorder traversal to print out all the elements in a binary tree (bt):

```
public void preorderPrint(BinaryTree bt) {
        if (bt == null) return;
        System.out.println(bt.element);
        preorderPrint(bt.left);
        preorderPrint(bt.right);
}
```

Pre-order Travesal

- Example

```
public void preorderPrint(BinaryTree bt) {
        if (bt == null) return;
        System.out.println(bt.element);
        preorderPrint(bt.left);
        preorderPrint(bt.right);
}
```



Pre-order traversal result: **A B D H I E K C F G**

Post-order Travesal

- In Post-order Traversal, the root is visited **last**
  - Order: Left - Right - Root

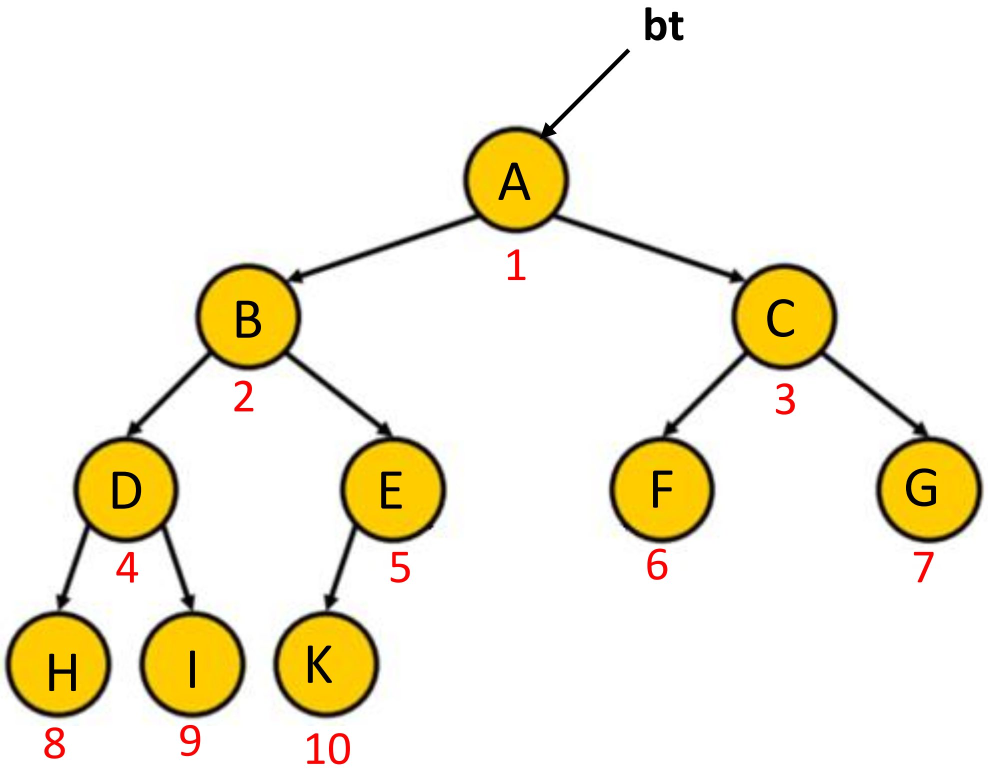
- Algorithm for postorder traversal to print out all the elements in a binary tree (bt):

```
public void postorderPrint(BinaryTree bt) {
        if (bt == null) return;
        preorderPrint(bt.left);
        preorderPrint(bt.right);
        System.out.println(bt.element);
}
```

Post-order Travesal

- Example

```
public void postorderPrint(BinaryTree bt) {
        if (bt == null) return;
        postorderPrint(bt.left);
        postorderPrint(bt.right);
        System.out.println(bt.element);
}
```



Post-order traversal result: **H I D K E B F G C A**

In-order Travesal

- In In-order Traversal, the root is visited **in the middle**
  - Order: Left - Root - Right

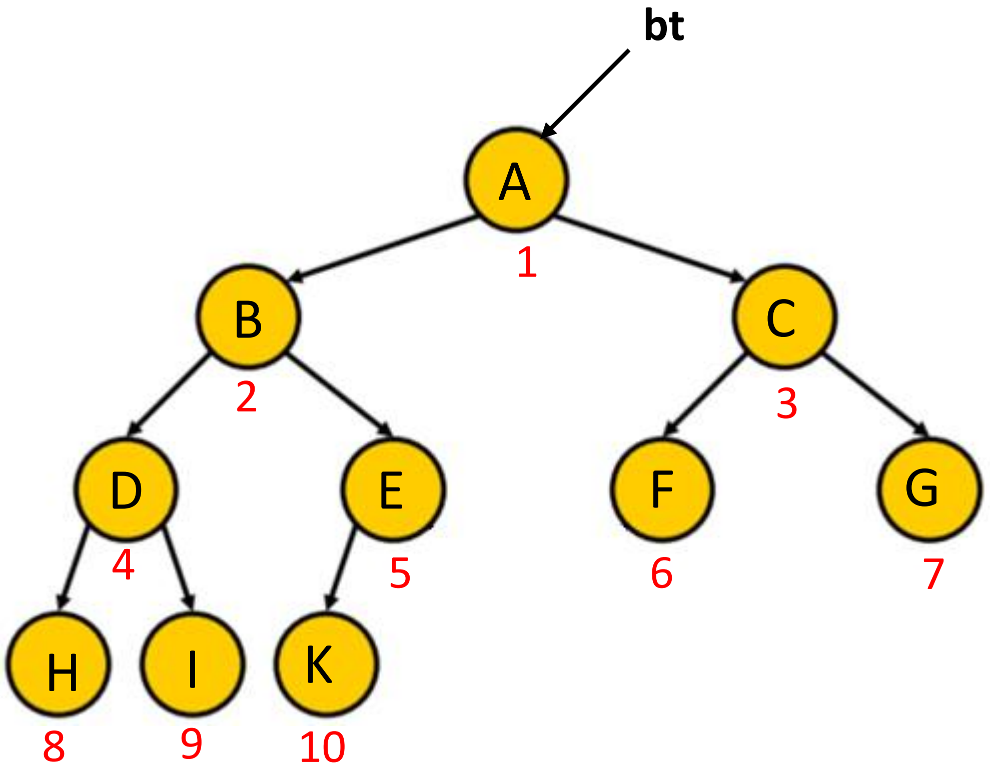
- Algorithm for In-order traversal to print out all the elements in a binary tree (bt):

```
public void inorderPrint(BinaryTree bt) {
        if (bt == null) return;
        inorderPrint(bt.left);
        System.out.println(bt.element);
        inorderPrint(bt.right);
}
```

In-order Travesal

- Example

```
public void inorderPrint(BinaryTree bt) {
        if (bt == null) return;
        inorderPrint(bt.left);
        System.out.println(bt.element);
        inorderPrint(bt.right);
}
```



Post-order traversal result: **H D I B K E A F C G**

Tree ADT

Binary Tree Application

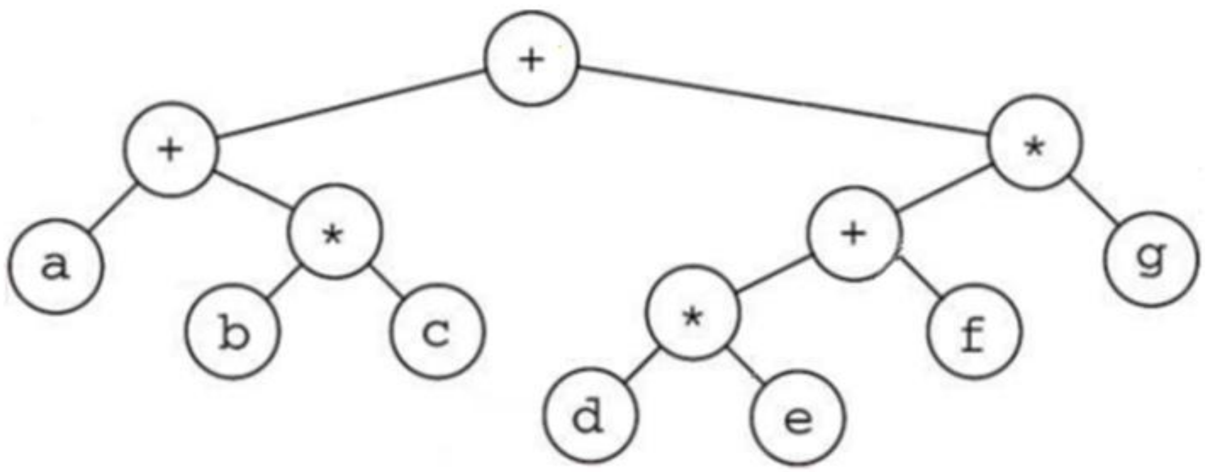
Binary Tree: Applications

- Tree ADT has a variety of applications.
- Same with binary tree. One important and well-known is **search trees** (more detail in lecture 7).
- First, we discuss about Arithmetic Expression Trees – Cây biểu thức

Binary Tree: Expression Tree

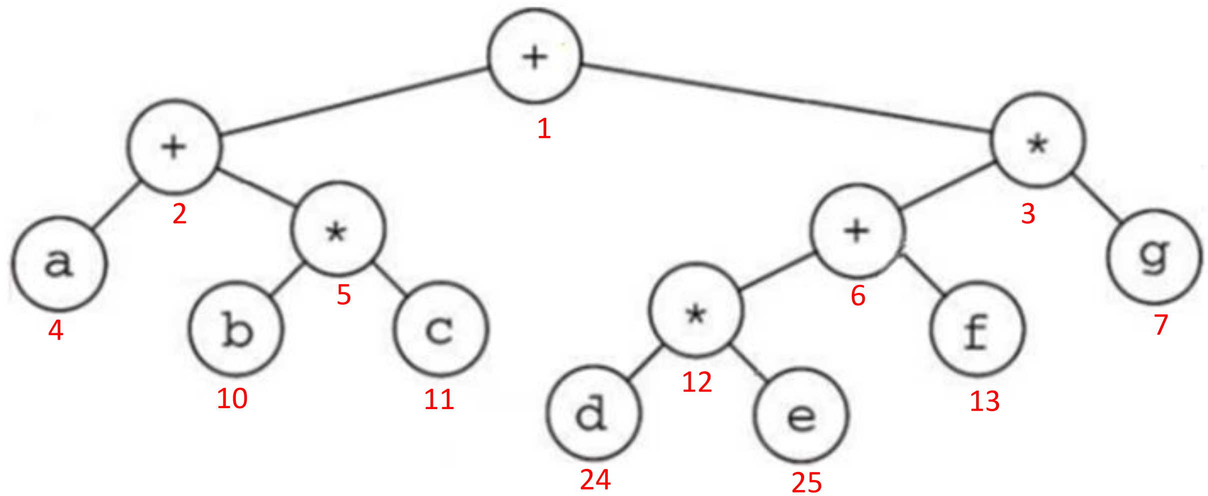
- Binary Tree for an Arithmetic Expression Tree
  - Intenal nodes: operators (toán tử ở các nút trong)
  - Leaves: operands (toán hạng ở các nút lá)
- Example:

```
(a + (b*c) + (d*e) + f)*g)
```



Binary Tree: Expression Tree

- Using array representing the tree



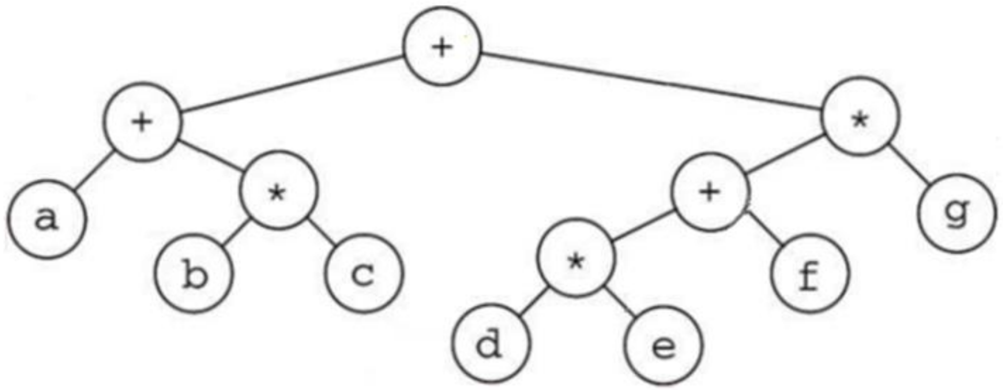
+	**+**	*****	**a**	*****	**+**	**g**			**b**	**c**	*****	**f**											**d**	**e**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

- Using linked structure is better

Binary Tree: Expression Tree

- Print Arithmetic Expressions
  - Using In-order traversal algorithm:
    - print “(“ before traversing left subtree
    - print operand or operator when visiting node
    - print “)“ after traversing right subtree

```
//Print Aithmetic Expression
void printTree(t) {
        if (t.left != null)
                print("(");
                printTree (t.left);
        print(t.element );
        if (t.right != null)
                printTree (t.right);
                print (")");
}
```

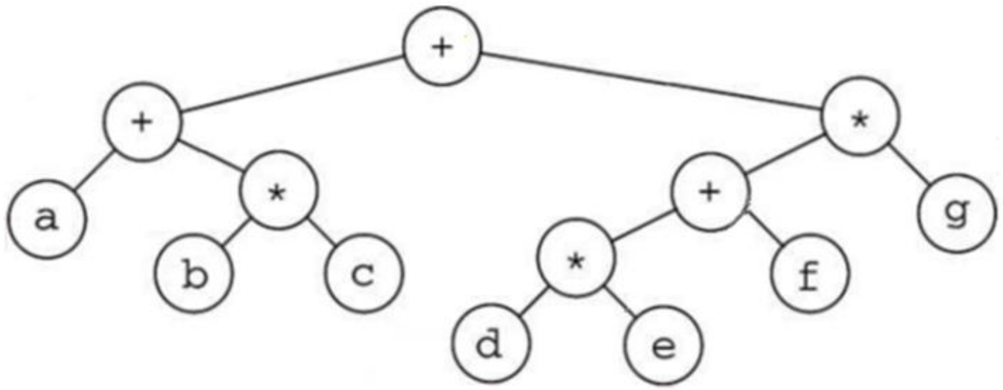


Result: ((a+(b*c))+(((d*e)+f)*g)

Binary Tree: Expression Tree

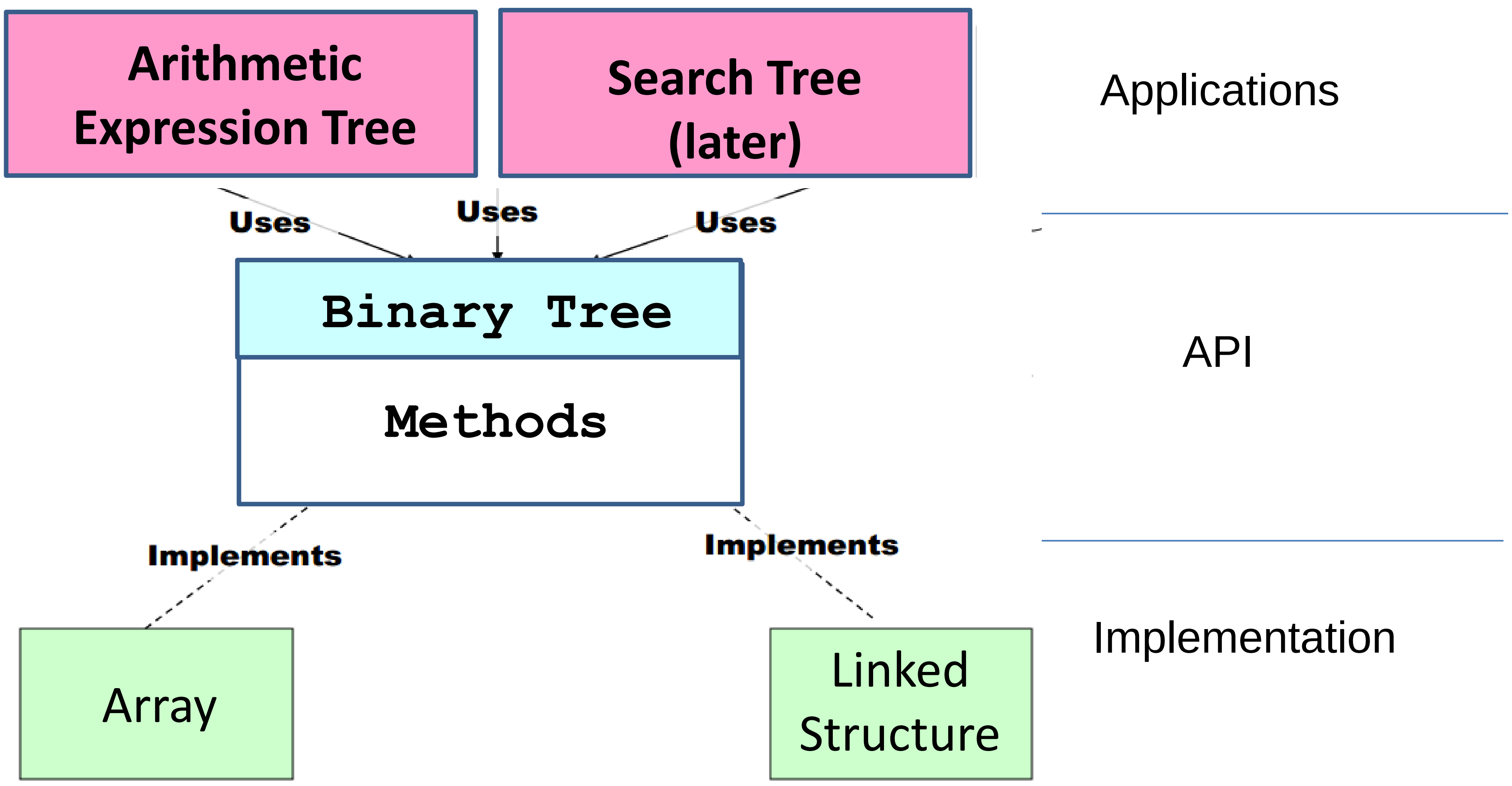
- Evaluate Arithmetic Expressions
  - Using Post-order traversal algorithm:
    - Recursively evaluate subtrees
    - Apply the operator after subtrees are evaluated

```
//Evaluate Aithmetic Expression
float evaluate(t) {
        if (t.left == null)   //external node
                return t.element;
        else  //internal node
                x = evaluate (t.left);
                y = evaluate (t.right);
                let o be the operator t.element
                 z = apply o to x and y
                return z;
}
```



Result: z=((a+(b*c))+((d*e)+f)*g)

Summary



Arithmetic Expression Tree
Search Tree (later)
Applications
Uses
Uses
Uses
Binary Tree
Methods
API
Implements
Implements
Array
Linked Structure
Implementation