Ta có

⊕ $P_A(X) = \begin{vmatrix} 1-X & 1 & 1 & 1 \\ 1 & 1-X & -1 & -1 \\ 1 & -1 & 1-X & -1 \\ 1 & -1 & -1 & 1-X \end{vmatrix} = \begin{vmatrix} 1-X & 1 & 1 & 1 \\ 0 & \frac{X-2}{1-X} & \frac{X-2}{1-X} & \frac{X^2-2X}{1-X} \\ 0 & 0 & X-2 & (X-2).(X+1) \\ 0 & 0 & 0 & X^2-4 \end{vmatrix}$ (vs $X \neq 1$) $= (X-2)^3.(X+2)$

Xét $X=1 \Rightarrow P_A(X) = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{vmatrix} = -3 \neq 0 \Rightarrow \lambda$ k' thể là $= 1$

Vậy ta có $\lambda_1 = 2$ (n. tam)

$\lambda_2 = -2$ (n. đơn)

⊕ Xét kgvtriêng ứng vs $\lambda_1 = 2$

$$\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c} -1 & 1 & 1 & 1 & 0 \\ 1 & -1 & -1 & -1 & 0 \\ 1 & -1 & -1 & -1 & 0 \\ 1 & -1 & -1 & -1 & 0 \end{array}\right] \xrightarrow[\substack{R_3 - R_1 \to R_3 \\ R_4 - R_1 \to R_4}]{\substack{-R_1 \to R_1 \\ R_2 - R_1 \to R_2}} \left[\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 = x_2 + x_3 + x_4 \\ x_2, x_3, x_4 \in \mathbb{R} \end{cases}$$

Vậy $V_2 = \left\{ x_2 . \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 . \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 . \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \middle| x_2, x_3, x_4 \in \mathbb{R} \right\} = \mathcal{L}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

⊕ Xét kg riêng ứng vs $\lambda_2 = -2$

$$\left[\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ 1 & 3 & -1 & -1 & 0 \\ 1 & -1 & 3 & -1 & 0 \\ 1 & -1 & -1 & 3 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 1 & -1 & -1 & 3 & 0 \\ 1 & 3 & -1 & -1 & 0 \\ 1 & -1 & 3 & -1 & 0 \\ 3 & 1 & 1 & 1 & 0 \end{array}\right] \xrightarrow[\substack{R_3 - R_1 \to R_3 \\ R_4 - 3R_1 \to R_4}]{R_2 - R_1 \to R_2} \left[\begin{array}{cccc|c} 1 & -1 & -1 & 3 & 0 \\ 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 4 & -4 & 0 \\ 0 & 4 & 4 & -8 & 0 \end{array}\right]$$

$$\xrightarrow[\frac{R_4}{4}\to R_4]{\frac{R_2}{4}\to R_2,\ \frac{R_3}{4}\to R_3}\left[\begin{array}{cccc|c}1 & -1 & -1 & 3 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & 0 & 1 & -1 & 0\\ 0 & 1 & 1 & -2 & 0\end{array}\right]\xrightarrow[R_4-R_2\to R_4]{R_1+R_2\to R_1}\left[\begin{array}{cccc|c}1 & 0 & -1 & 2 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & 0 & 1 & -1 & 0\\ 0 & 0 & 1 & -1 & 0\end{array}\right]\xrightarrow[R_4-R_3\to R_4]{R_1+R_3\to R_1}\left[\begin{array}{cccc|c}1 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & 0 & 1 & -1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]$$

$$\Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}-x_4\\x_4\\x_4\\x_4\end{bmatrix}=x_4.\begin{bmatrix}-1\\1\\1\\1\end{bmatrix}$$ vs x_4 bkỳ $\in \mathbb{R}$.

Vậy $V_{-2}=\mathcal{L}\left\{\begin{bmatrix}-1\\1\\1\\1\end{bmatrix}\right\}$

⊕ Nxét $\dim V_2+\dim V_{-2}=3+1=4=$ cấp of ma trận A $\Rightarrow$ Ma trận A chéo hoá đc.

Chọn $Q=\begin{bmatrix}1 & 1 & 1 & -1\\ 1 & 0 & 0 & 1\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 1\end{bmatrix}$ $\Rightarrow Q^{-1}.A.Q=\begin{bmatrix}2 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & -2\end{bmatrix}$