Questions Lecture 2

Commutative : $a + b = b + a$

$a \cdot b = b \cdot a$ for all $a, b \in \mathbb{R}$

Associativity : $(a+b)+c = a+(b+c)$ for all $a, b, c \in \mathbb{R}$

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Distributivity : $(a+b) \cdot c = ac + bc$ for all $a, b, c \in \mathbb{R}$

1. $a \boxplus b := f(a+b) = f(b+a) = b \boxplus a$

Homework 7

4.1 Lagrange interpolation

Given $n+1$ abscissae $x_0 < x_1 < \cdots < x_n$, we call

$$w(x) := \prod_{j=0}^{n} (x - x_j) \in \Pi_{n+1}$$

the node polynomial. The corresponding Lagrange polynomial are given by

$$L_i(x) := \prod_{j=0}^{n} \frac{x - x_j}{x_i - x_j} \in \Pi_n, \quad i = 0, 1, \ldots, n$$

Assignment 7

Problem 1. (Lagrange Interpolation)

$$f(x) = \frac{2}{3+2x} \quad \text{where } \{x_0, x_1, x_2, x_3\} = \{-1, -0.5, 0.5, 1\}$$

$$\therefore \quad f(x_0) = \frac{2}{3+(2 \times -1)} = \frac{2}{1} = 2$$

$$f(x_1) = \frac{2}{3-(2 \times 0.5)} = \frac{2}{2} = 1$$

$$f(x_2) = \frac{2}{3+(2 \times 0.5)} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$f(x_3) = \frac{2}{}$$

Homework 9

Question 1:

$$S(x) = \sum_{j=-1}^{3} \alpha_j B_3 (x-j) \qquad x \in [x_0, x_n]$$

we have to compute the coefficient the coeficients α_j, such that interpolation conditions:

$$S(x_i) = \sum_{j=-1}^{3} \alpha_j B_3 (x_i - j) = y_i$$

for $i = i = 0, 1, \ldots, n$ are met.

Hence, $B_3(0) = \frac{2}{3}$, $B_3(\pm 1) = \frac{1}{6}$, $B_3'(0) = 0$

$B_3'(\pm 1) = \mp \frac{1}{2}$, $B_3''(0) = -2$, $B_3''(\pm 1) = 1$

Homework 10

Question 1:

$$p(x) = ax^2 + bx + c$$
$$y = ax^2 + bx + c$$

i	0	1	2	3
x_i	-1	0	1	2
y_i	1	0	1	0

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$A^T A x = A^T b \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 4 \\ -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 4 \\ -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 18 & 8 & 6 \\ 8 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ -0.2 \\ 0.6 \end{bmatrix}$$

Question 2:

$$p(x) = a\cos(x) + b\sin(x)$$
$$y = a\cos(x) + b\sin(x)$$

$$\begin{bmatrix} \frac{3}{2} \\ 2 \\ -\frac{1}{2} \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$A^T A x = A b \Rightarrow$$

Exam Prep

1.3 Floating Point Numbers

Problem 1 (Excercise 2):

range $= -7 \le e \le 7$

bias $= 8$

$\bar{e} = e + 1000$

$\bar{e} = 0000$

cos 13 to a binary number:

13	
6	1
3	0
1	1
0	1

$\rightarrow 1101$

$13 = 1101 = 1.101 \cdot 2^3$

$\therefore e = 3_{10} = 0011$

$\bar{e} = e + 1000$

$= 0011 + 1000$

$= 1011$

Finally 01101000000001011

Exam Question:

52	
26	0
13	0
6	1
3	0
1	1
0	1

$0.375 \cdot 2 = 0.750 \rightarrow 0$

$0.750 \cdot 2 = 1.500 \rightarrow 1$

$0.5 \cdot 2 = 1 \rightarrow 1$

$\therefore 110100.011$

$\Rightarrow 1.10100011 \cdot 10^5$

Hence 01101000110001 1101

$e = 5 = 0101$

$\bar{e} = 0101 + 1000$

$= 1101$

Problem 1 (Excercise 3)

11 0110101110011101

$\bar{e} = 1101$

$= 1000 - 1101$

$= 0101$

$= 5_{10}$

$1.01101011100 \cdot 2^5$

101101.011100

$\underset{2^5\,2^4\,2^3\,2^2\,2^1\,2^0}{101101} \rightarrow 2^0 + 2^2 + 2^3 + 2^5$

$\rightarrow 1 + 4 + 8 + 32$

$\rightarrow 45$

$\underset{2^{-1}\,2^{-2}\,2^{-3}\,2^{-4}\,2^{-5}\,2^{-6}}{011100} \rightarrow \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

$\rightarrow 0.25 + 0.125 + 0.0625$

$\rightarrow 0.4375$

$\therefore -45.4375$

$\bar{e} = e + \text{bias}$

$\hat{e} = 0000$

Questions Lecture 5

1. $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 7 & 8 \\ 0 & 0 & 9 \end{bmatrix}$ $y = \begin{bmatrix} 30 \\ 30 \\ 18 \end{bmatrix}$

$b = \begin{bmatrix} 30 \\ 90 \\ 168 \end{bmatrix}$

ii $Ly = b$ using forward substitution

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 30 \\ 2 & 1 & 0 & 90 \\ 3 & 2 & 1 & 168 \end{array}\right] \Rightarrow$$

$y_1 \Rightarrow 30$

$y_2 \Rightarrow 60 + y_2 = 90$

$y_2 = 30$

$y_3 = 90 + 60 + y_3 = 168$

$150 + y_3 = 168$

$y_3 = 168 - 150$

$= \underline{\underline{18}}$

$Ux = y$ using backward substitution

$$\left[\begin{array}{ccc|c} 4 & 5 & 6 & 30 \\ 0 & 7 & 8 & 30 \\ 0 & 0 & 9 & 18 \end{array}\right]$$

$9y_3 = 18$

$y_3 = 2$

$y_2 \Rightarrow 7y_2 + 16 = 30$

$7y_2 = 14$

$y_2 = \underline{\underline{2}}$

Questions Lecture 6

1.