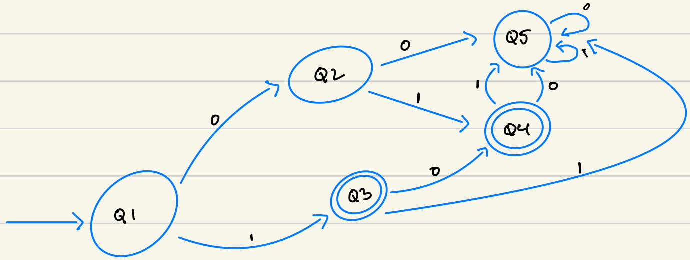
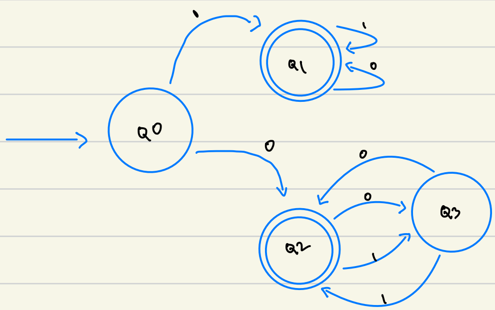
Harkeerat Singh Sawhney

Assignment 1

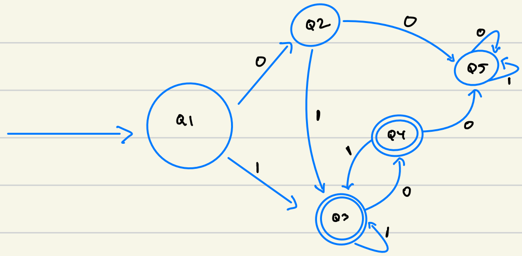
1. $L1 = \{ w \mid w$ has at most one 0 and exactly one 1 $\}$



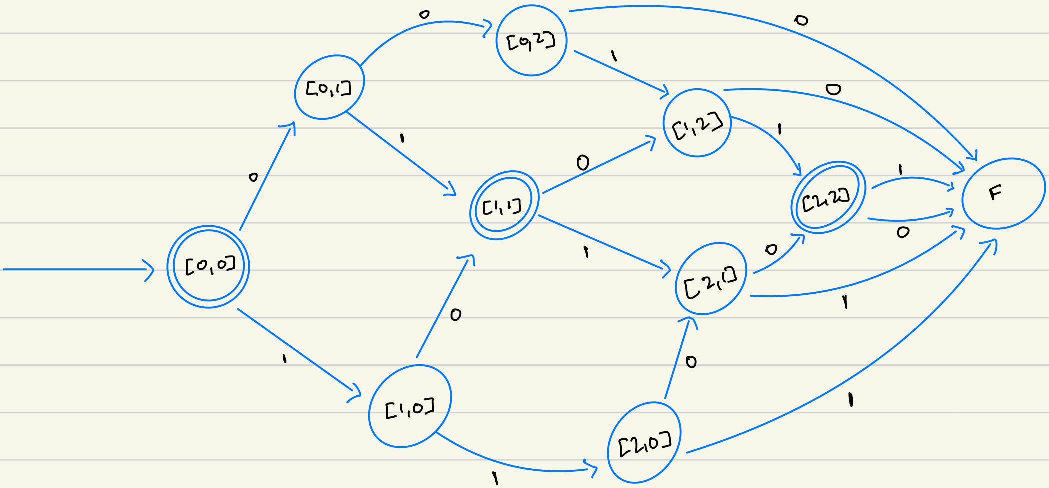
2. $L2 = \{ w \mid w$ starts with 1 or has odd length $\}$



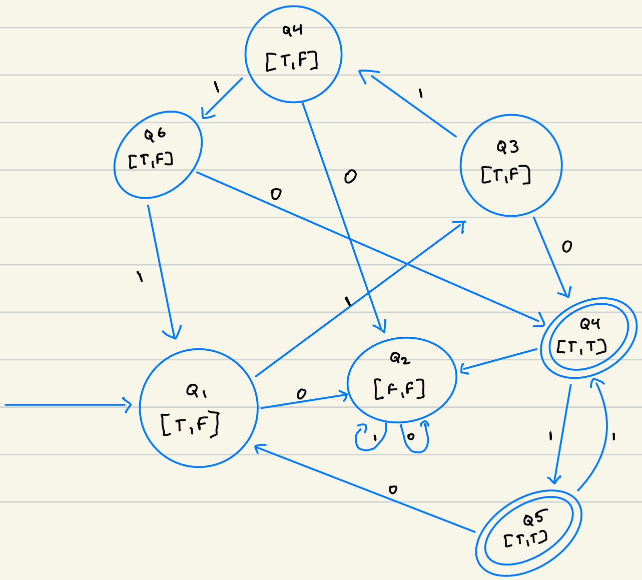
3. $L3 = \{ w \mid$ has at least one 1 and does not contain substring 00 $\}$



4. $L4 = \{ w \mid w$ has at most length 4 and has as many 0s as 1s$\}$

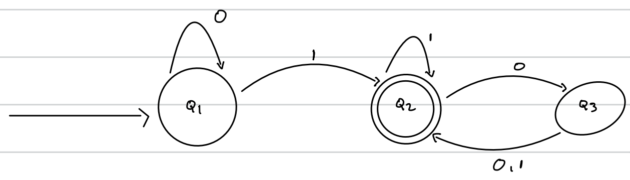


5. $L5 = \{ w \mid$ every odd position of w is 1, and w contains an odd number of 0s$\}$

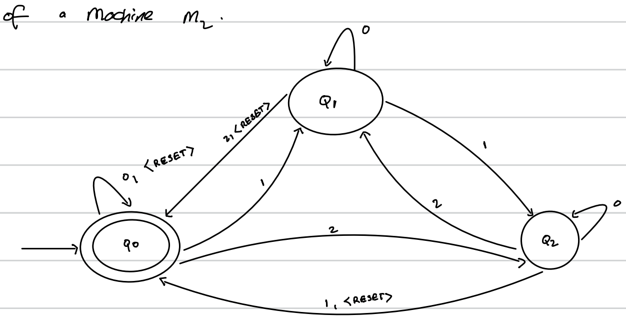


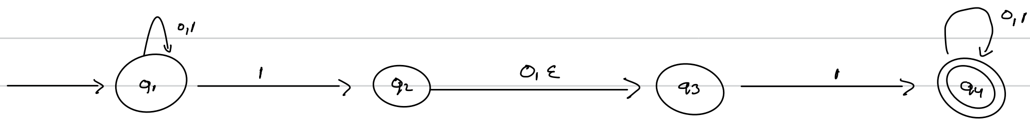
$\left\{ \begin{array}{l} \text{ODD POSITIONS} \\ \text{FILLED WITH 1s} \end{array} , \begin{array}{l} \text{ODD NO.} \\ \text{OF 0s} \end{array} \right\}$

Markov Chains



An Example of a Machine M_2.





Harkeerat Singh Sawhney

Assignment 2

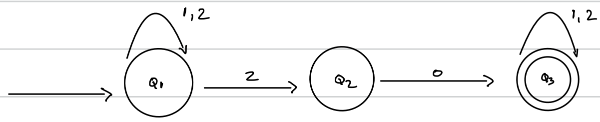
1. Proof of regular languages are closed under the Intersection Operation :

If $M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$ and

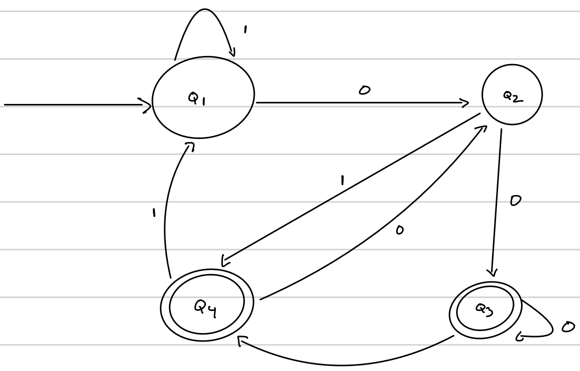
$M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$ then, $M = (Q, \Sigma, \delta, q_0, F)$ such that

- $Q = Q_1 \times Q_2 = \{(r_1, r_2) \mid r_1 \in Q_1$ and $r_2 \in Q_2\}$ * Cartesian Product
- $\Sigma = \Sigma_1, \Sigma_2$
- $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$ for $r_1, r_2 \in Q$ and $a \in \Sigma$
- $q_0 = (q_1, q_2)$
- $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$

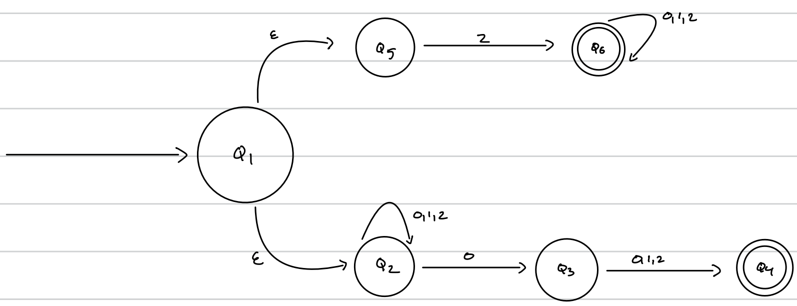
2. NFA for L2 $\{w \mid w$ contains substring 20 and w contains exactly one 0$\}$. Alphabet $\{0, 1, 2\}$



3. DFA for L3 $\{w \mid$ penultimate symbol of w is 0$\}$. Alphabet $\{0, 1\}$



4. NFA for L4 $\{w \mid$ the penultimate symbol of w is 0 or the first symbol of w is 2$\}$. Alphabet $\{0, 1, 2\}$



Mok Fazlić ①

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

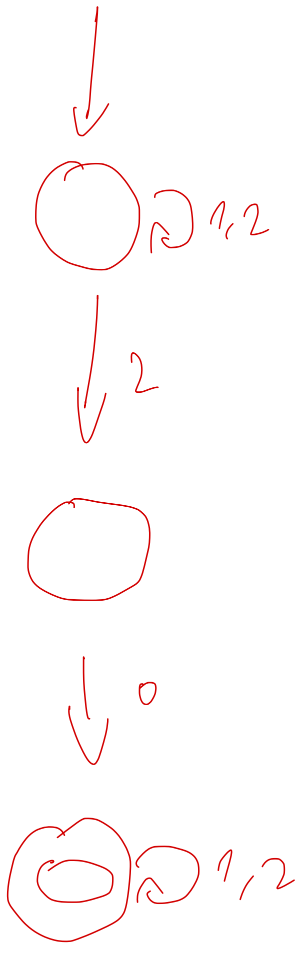
SO: $M = (Q, \Sigma, \delta, q_0, F)$ if: $Q = Q_1 \times Q_2$

$Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$ and

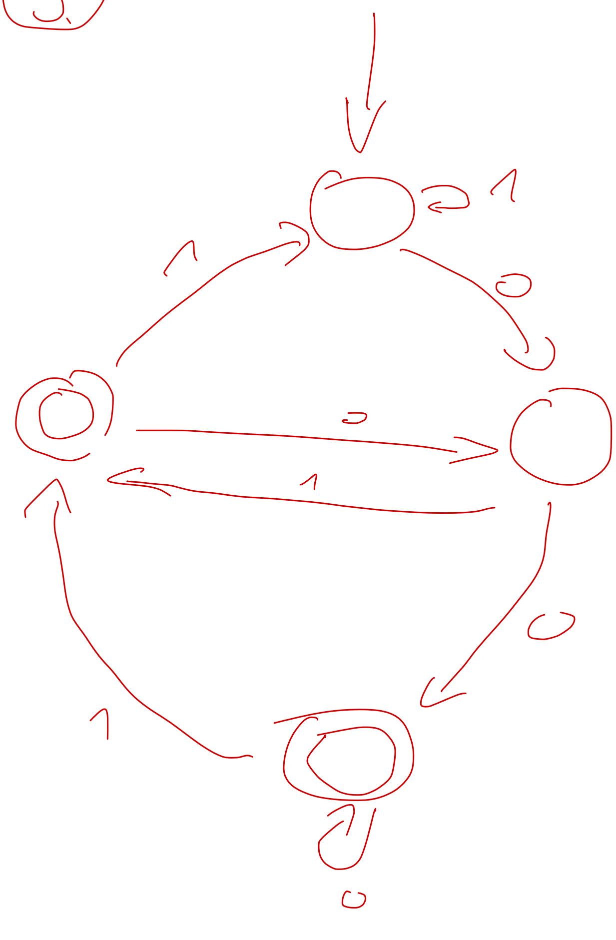
$\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$ for $r_1, r_2 \in Q$ and $a \in \Sigma$

and $q_0 = (q_1, q_2)$ and $F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\}$

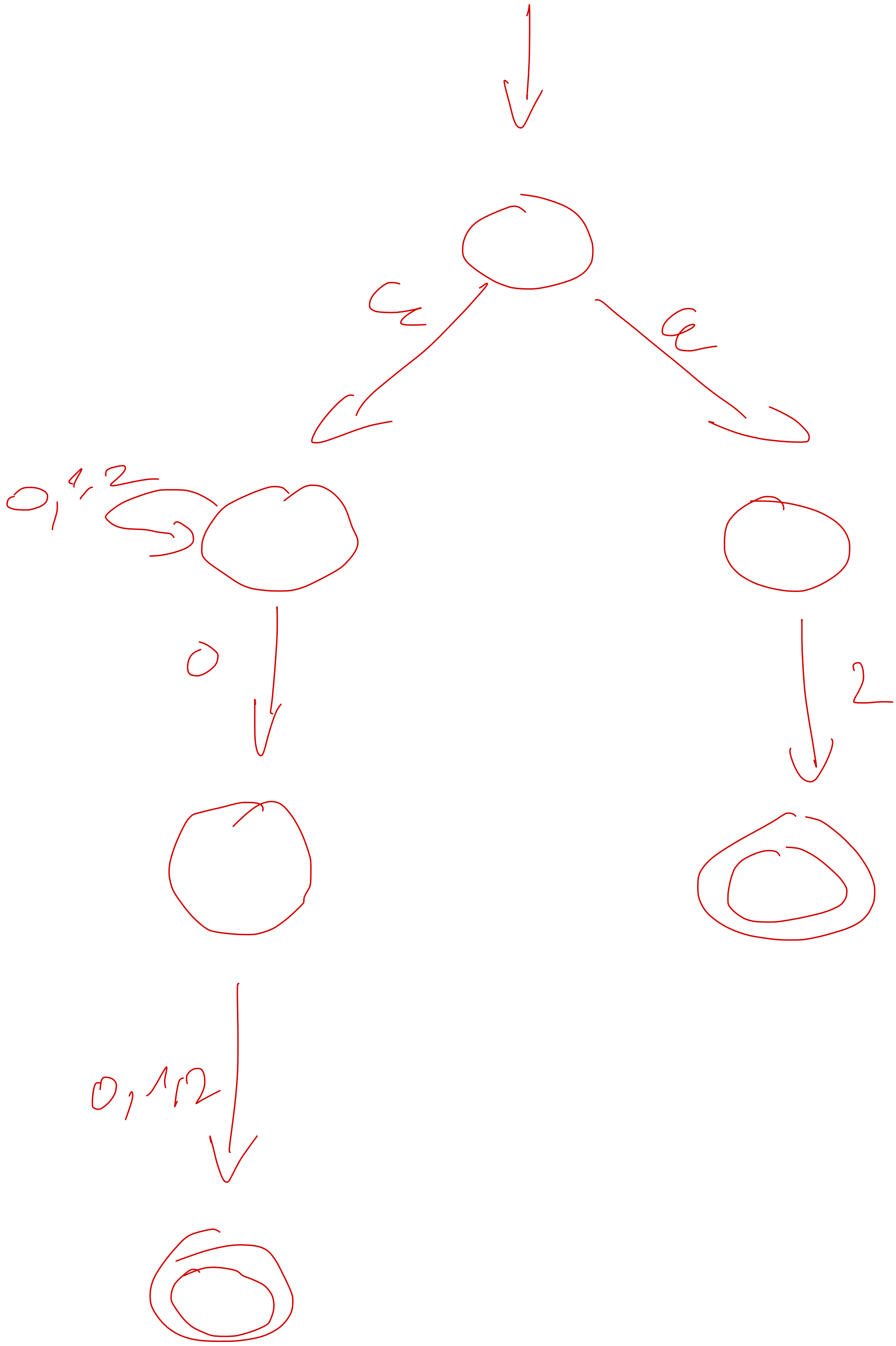
②



③



4

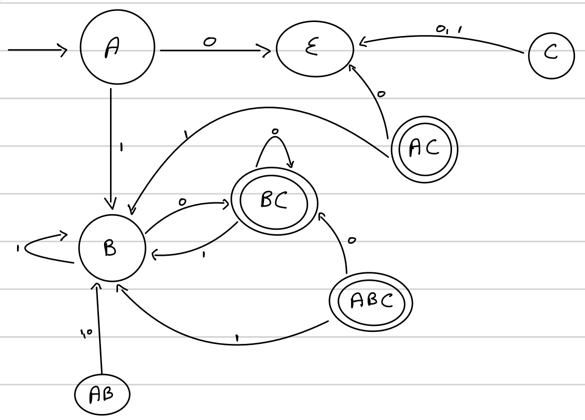


Harkeerat Singh Sawhney

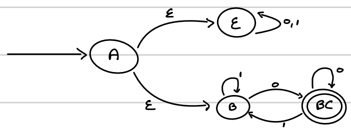
Assignment 3

1.

(a)



(b)



2.

L_1 : $(\Sigma\Sigma)^* \cup (\Sigma\Sigma\Sigma)^*$

L_2 : $(11)^* (10 \cup 01) (11)^*$

L_3 : $(\Sigma\Sigma)^* \Sigma\Sigma\Sigma$

L_4 : $1^* \cup (1^* 0 1^* 0) 1^*$

L_5 : $\epsilon \cup \Sigma \cup (\Sigma 0 \cup 0 \Sigma) \cup \Sigma\Sigma\Sigma\Sigma^*$

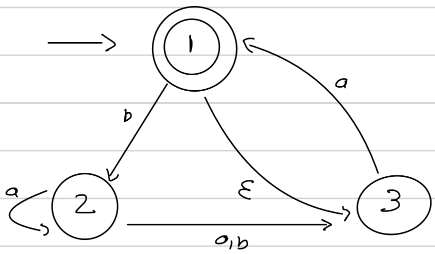
L_6 : $(1 \cup \epsilon) (00^* 1)^* 0^*$

L_7 : $((0 \cup 1) 1)^* (\epsilon \cup (0 \cup 1))$

L_8 : $((0 \cup \epsilon) 1 1^*)^*$

Conversion from ε-NFA to DFA

ε-NFA



ε-NFA	a	b	ε
1	∅	2	{1,3}
2	{2,3}	3	2
3	1	∅	3

ε-closure (1)

DFA	a	b
{1,3}	{1,3}	2
2	{2,3}	3
{2,3}	{1,2,3}	3
{1,2,3}	{1,2,3}	{2,3}
3	{1,3}	∅

$\delta_D(\{1,3\},a) = E(\delta_E(1,a) \cup \delta_E(3,a))$

$= E(\emptyset \cup \{1\})$

$= \{1,3\}$

$\delta_D(\{1,3\},b) = E(\delta_E(1,b) \cup (3,b))$

$= E(\{2\} \cup \emptyset)$

$= 2$

$\delta_D(\{2,3\},a) = E(\delta_E(2,a))$

$= E(\{2,3\})$

$= \{2,3\}$

$\delta_D(\{2,3\},b) = E(\delta_E(2,b))$

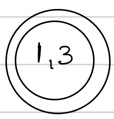
$= E(\{2,b\})$

$= 3$

$\delta_D(\{2,3\},a) = E(\delta_E(2,a) \cup \delta_E(3,a))$

$= E(\{2,3\} \cup 1)$

$= \{1,2,3\}$



Regular Expressions

$L_1 = \{ aa, ab, ba, bb \}$ $\longrightarrow$ whose length is exactly 2

✓ $aa + ab + ba + bb$

$a(a+b) + b(a+b)$

✓ $(a+b)(a+b)$

$L_1 = \{ aa, ab, ba, bb, aaa, \ldots \}$ $\longrightarrow$ whose lenght is at least 2

$(a+b)(a+b)(a+b)^*$

$*$ repersents all the possible combination.

$L_1 = \{ a, b, aa, bb, ab, ba \}$ $\longrightarrow$ whose length at most 2

$= \{ \epsilon, b, aa, bb, ab, ba \}$

$= (a+b+\epsilon)(a+b+\epsilon)$

$L_1 = \{ \ldots \}$ $\longrightarrow$ whose length is even

$L_1 = \{ \epsilon, aa, ab, ba, bb, \ldots \}$

$((a+b)(a+b))$ = length of 2 strings

$((a+b)(a+b))^*$

$L_1 = \{ \ldots \}$ $\longrightarrow$ whose length is odd

$((a+b)(a+b))^* (a+b)$

$L_1 = \{ \ldots \}$ $\longrightarrow$ Divisible by 3

$((a+b)(a+b)(a+b))^* (a+b)$

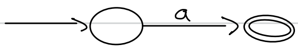
Regular Expression Practice Questions

1. $(ab)^* (a \mid \epsilon) \mid (ba)^* (b \mid \epsilon)$
2. $(aba)(a \mid b)^* (bb)$
3. $(b \mid (aaa)^*)^*$
5. $(aa^*) \mid b^*$

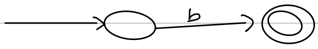
Problem 1: $a^* (b(a \cup aaa))^*$

Conversion from Regular Expression to NFA

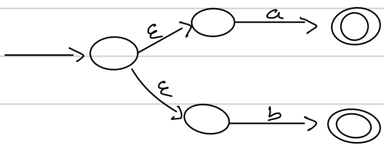
a



b

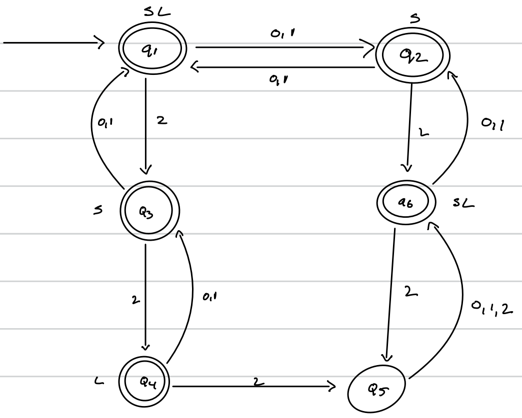


$a \cup b$



Prev - Exam

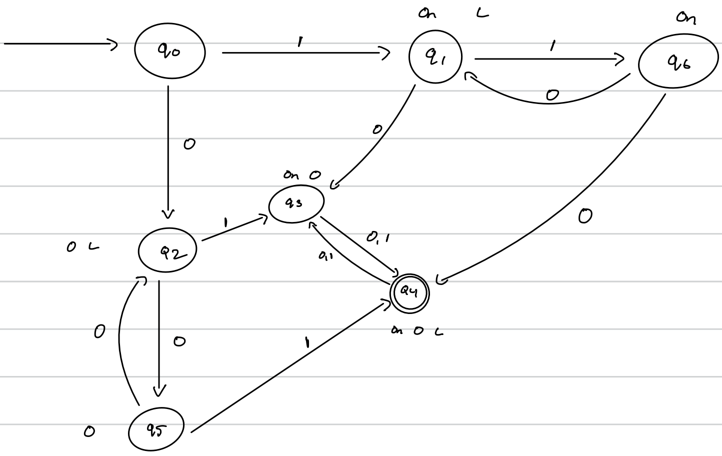
1. $(0\cup 2)^* \cup (1(0\cup 2)^* 1)^* \cup ((0\cup 2)^* 1 (0\cup 2)^* 1 (0\cup 2)^*)^* \cup (\Sigma(\Sigma\Sigma)^*)$
2. $1^* 2 1^* (0 \cup \epsilon) 1^* \cup 1^* (0\cup \epsilon) 1^* 2 1^*$
3. $\Sigma\Sigma(0\cup 2)\Sigma^*$
4.



S = sequence

L = length

5.

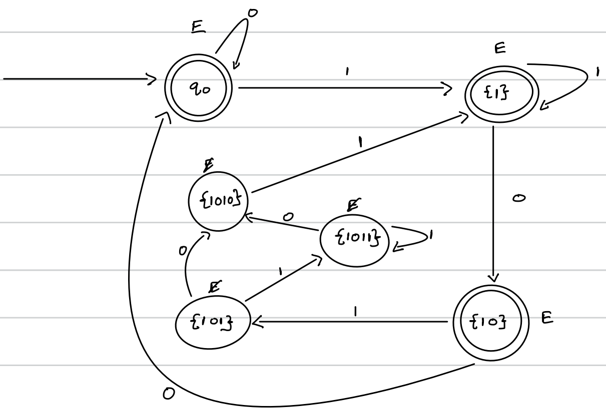


On = ones

O = zeros

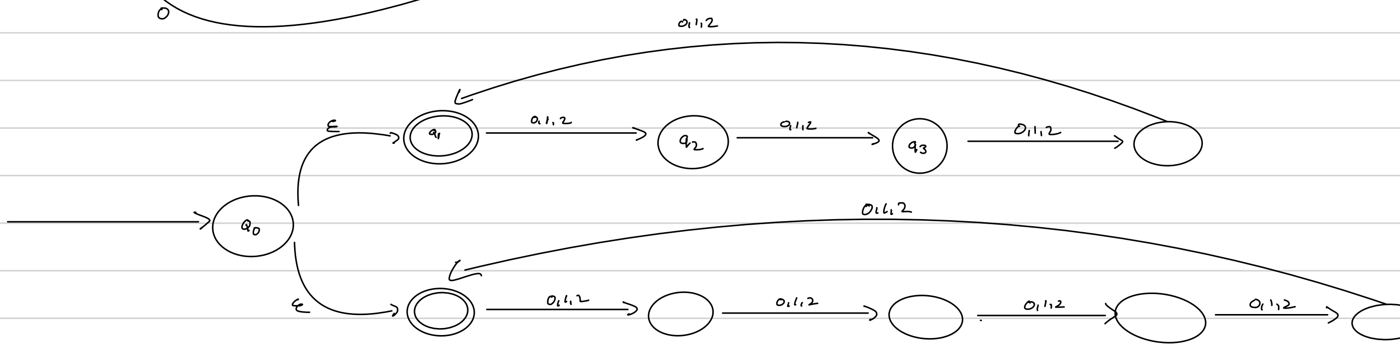
L = odd length

6.



E = Even number of occurrences

7.



Textbook Questions

1.

(a) Start state for M_1 = q_1

Start State for M_2 = q_1

(b) Set of Accept state for M_1 = $\{q_2\}$

Set of Accept state for M_2 = $\{q_1, q_4\}$

(c) Transition for M_1 :

$$q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{b} q_1 \xrightarrow{b} q_1$$

Transition for M_2 :

$$q_1 \xrightarrow{a} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} q_4$$

(d) For M_1 the machine DOESN'T ACCEPT the string

For M_2 the machine DOES ACCEPT the string

(e) When giving ε to M_1 the machine gives q_1, however q_1 is not an accept state. NO

When giving ε to M_2 the machine gives q_2, which is an accept state. YES

2. Formal Deffination for M_1 :

$M_1 = \{Q, \Sigma, \delta, q_0, F\}$

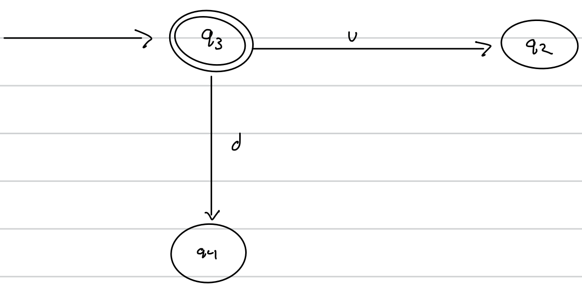
where

1. $Q = \{q_1, q_2, q_3\}$
2. $\Sigma = \{a, b\}$
3. δ is described as

	a	b
q_1	q_2	q_1
q_2	q_3	q_3
q_3	q_2	q_3

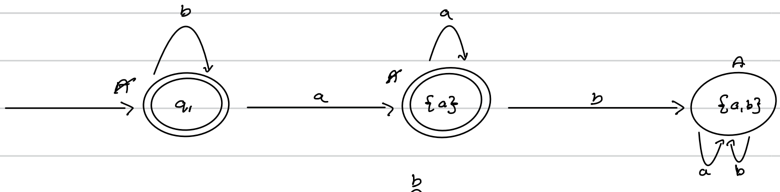
4. q_1 is the start state
5. Accept states $F = \{q_2\}$

3



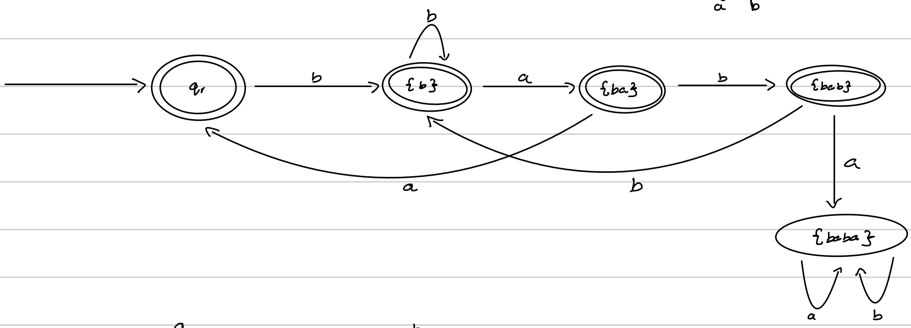
5.

(a)



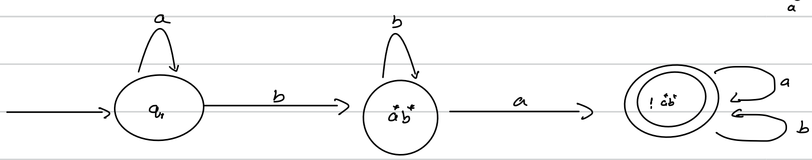
A = If substring "ab"

(b)



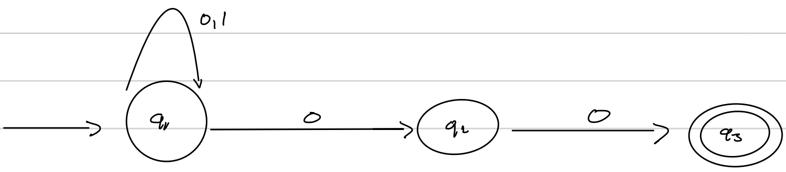
A = If substring "baba"

(c)

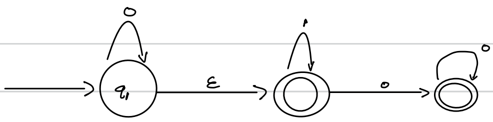


7

(a)



(e)



A

B

S

Harkeerat Singh Sawhney

Assignment 4

1. Prove that Language $L1 = \{ w \mid w$ has more 0s than 1s $\}$ is not regular

Assuming that L_1 is regular. Hence, there exists a p for L.

$w = 0^{p+1} 1^{p}$

Decompsation of w into xyz:

- $x = 1^{\alpha}$
- $y = 1^{\beta}$ $\qquad |\beta| \geq 1$
- $z = 0^{p+1} 1^{p-\alpha-\beta}$

$\therefore$ considering xy^iz for $i \geq 0$:

- $xy^iz = 0^{p+1} 1^{\alpha} 1^{i\beta} 1^{p-\alpha-\beta}$
$= 0^{p+1} 1^{p+i\beta-p}$

Here xy^iz is in L_1 iff $p + i\beta - p < p+1$
$\iff i\beta - \beta < 1$
$\iff \beta(i-1) < 1$, However $\beta \geq 1$

$\therefore$ by contradiction it leaves w outside the language, hence through proof by contradiction L_1 is a Nonregular expression.

2. Prove that Language $L_2 = \{ w \mid w$ has even length and the first half of w has more 0s than the second half of $w \}$ is not regular.

Assuming L_2 is regular. Hence there exist a p for L_2

$w = 0^{p} 1 0^{p-1}$

Decompsation of w into xyz:

- $x = 0^{\alpha}$
- $y = 0^{\beta}$ $\qquad$ where $|\beta| \geq 1$
- $z = 0^{p-\alpha-\beta} \wedge 1 0^{p-1}$

$\therefore$ considering xy^iz for $i \geq 0$:

- $xy^iz = 0^{\alpha} 0^{i\beta} 0^{p-\alpha-\beta} \wedge 1 0^{p-1}$
$= 0^{p+i\beta-\beta} 1 0^{p-1}$
$= 0^{p+\beta(i-1)} 1 0^{p-1}$, however if $p=0$ then this isn't in L_2.

Hence, by proof of contradiction w is outside the language and L_2 is a Non regular language.

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Assignment 5

1. $L1 = \{ w \mid w$ is palindrome and has at least two 0s$\}$ (and assume epsilon is a palindrome)

$S \rightarrow A$
$A \rightarrow 0B0 \mid 1A1$
$B \rightarrow 0B0 \mid 1B1 \mid 0 \mid 1 \mid \epsilon$

2. $L2 = \{ w \mid w$ starts and ends with the same symbol and has an odd number of 0s$\}$

$S \rightarrow 1A1 \mid 0A0 \mid 0$
$A \rightarrow 0B \mid 1A$
$B \rightarrow 0A \mid 1A \mid \epsilon$

3. $L3 = \{ w \mid w$ has odd length and the middle in w is a 0 and w has odd number of 1s$\}$

* Couldn't figure it out

4. $L4 = \{ w \mid w$ has even length and the first half of w does not contain 0s$\}$

$S \rightarrow 1S1 \mid 1S0 \mid \epsilon$

5. $L5 = \{ a^i b^j c^k$ with $i != j$ or $j != k$, $i, j, k >= 0 \}$

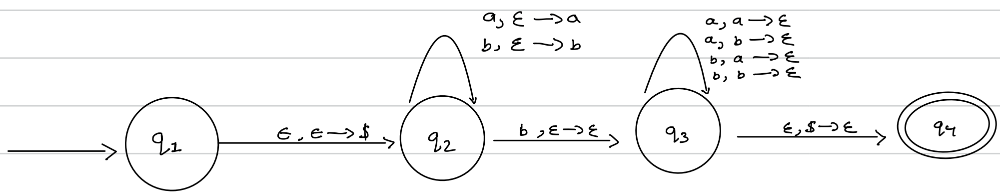
$S \rightarrow A \mid B$
$A \rightarrow A_{aB} \mid R_c$
$A_{aB} \rightarrow a A_{aB} b \mid S_a \mid S_B$
$S_a \rightarrow a S_a \mid a$
$S_B \rightarrow b S_B \mid b$
$R_c \rightarrow c R_c \mid \epsilon$

$B \rightarrow R_a \mid B_{bc}$
$B_{bc} \rightarrow b B_{bc} c \mid S_B \mid S_c$
$S_c \rightarrow c S_c \mid c$
$R_a \rightarrow a R_a \mid \epsilon$

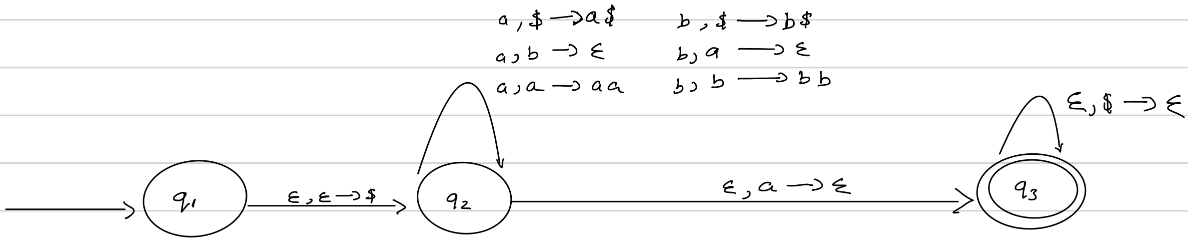
Harkeerat Singh Sawhney

Assignment 6

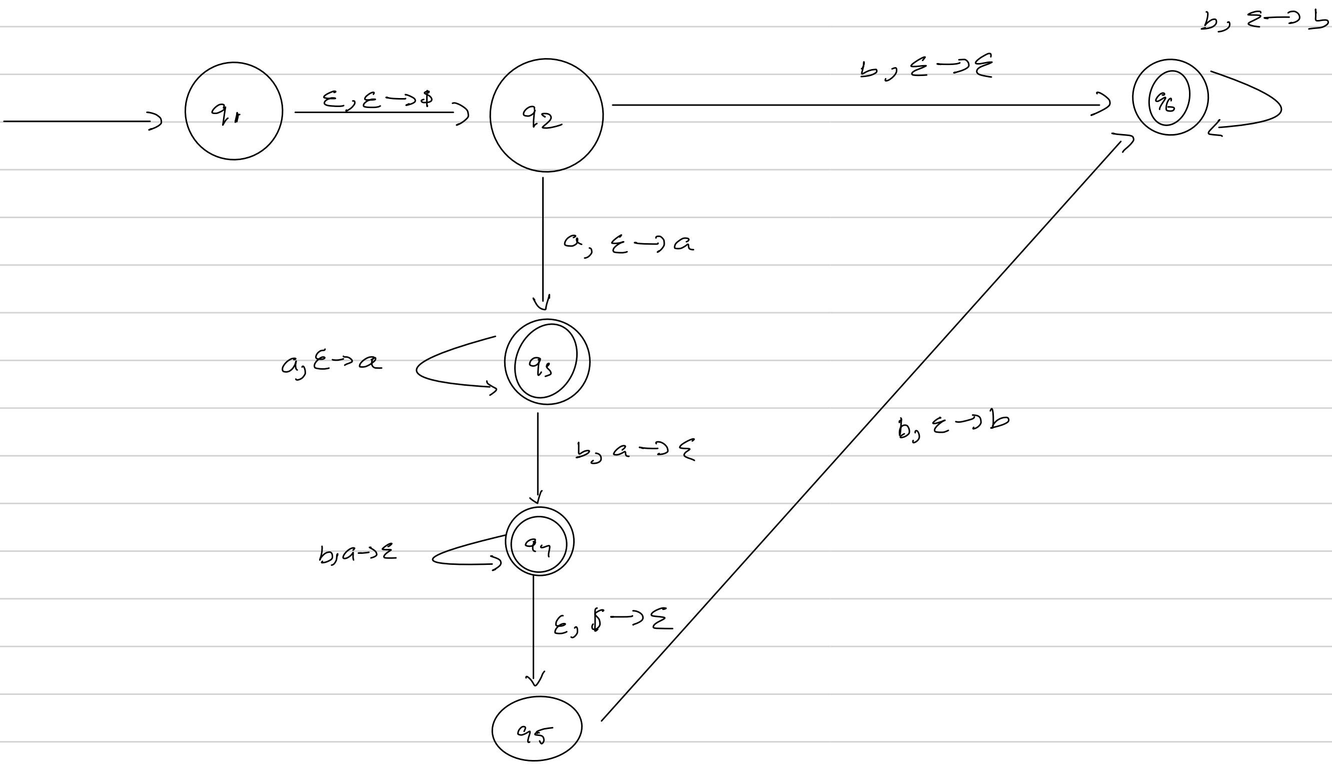
1. $L1 = \{ w \mid w$ has odd length and the middle symbol of w is a $b \}$



2. $L_2 = \{ w \mid w$ has more as than bs $\}$

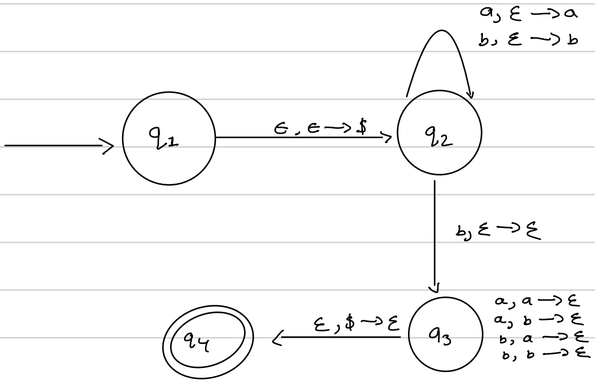


3. $L3 = \{ a^i b^j$ with $i \neq j$, $i, j >= 0 \}$

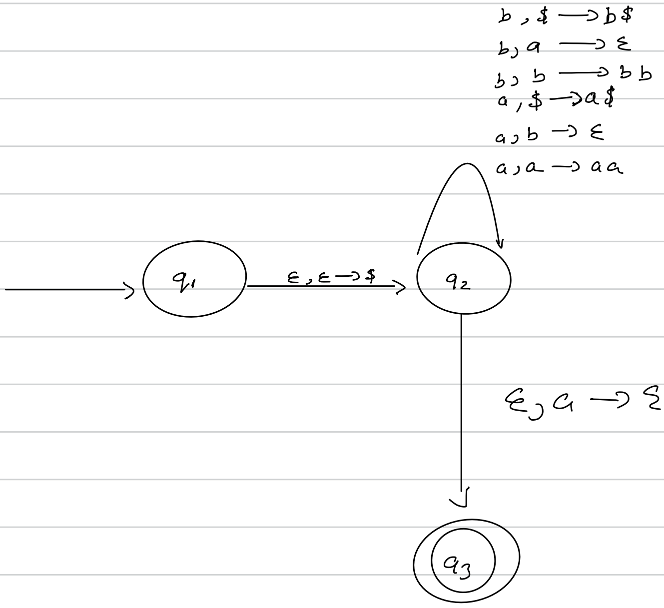


Mak Fazlik

(a)



(b)



(c)