CFL

1. $L2 = \{$ w starts and ends with same symbol and has odd number of 0s

$$S \longrightarrow 1A1 \mid 0A0 \mid 0$$
$$A \longrightarrow 1A \mid 0B$$
$$B \longrightarrow 0A \mid 1B \mid \varepsilon$$

2. $L3 = \{w \mid w$ has odd length and the middle in w is a 0 and w has odd number of 1s $\}$

$$S_E \longrightarrow 0S_E0 \mid 1S_E1 \mid 0S_o1 \mid 1S_o0$$
$$S_o \longrightarrow 1S_E0 \mid 0S_E1 \mid 0S_o0 \mid 1S_o1 \mid 0$$

3. $L4 = \{w \mid w$ has even length and the first half of w does not contain 0s $\}$

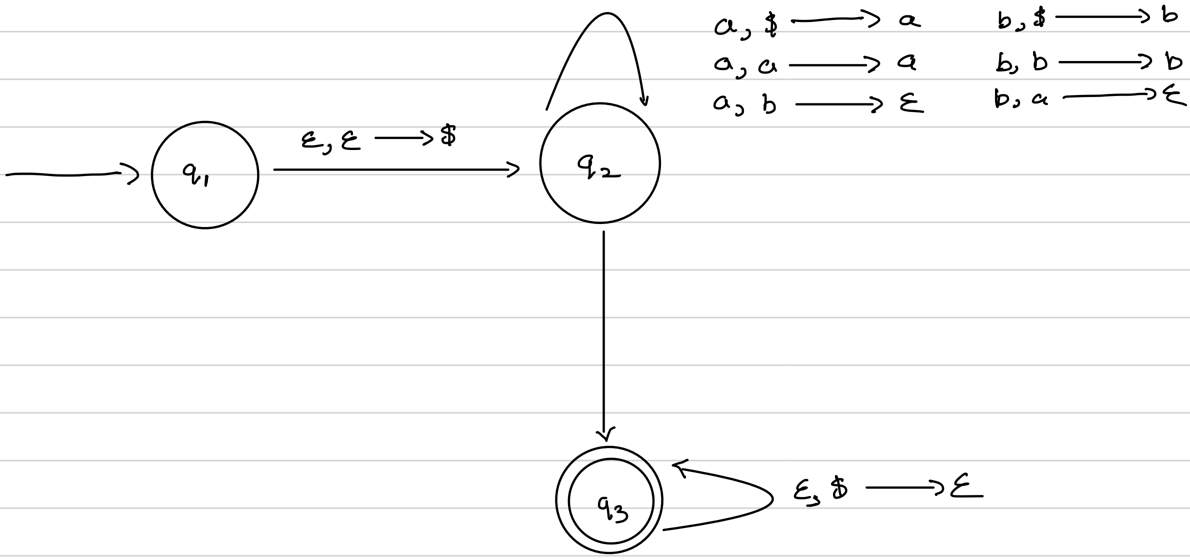
$$S \rightarrow 1S1 \mid 1S0 \mid \varepsilon$$

4. $L5 = \{a^i b^j c^k$ with $i \,!= j$ or $j \,!= k,\ i,j,k >= 0\}$

$$S \longrightarrow P_1 \mid P_2$$

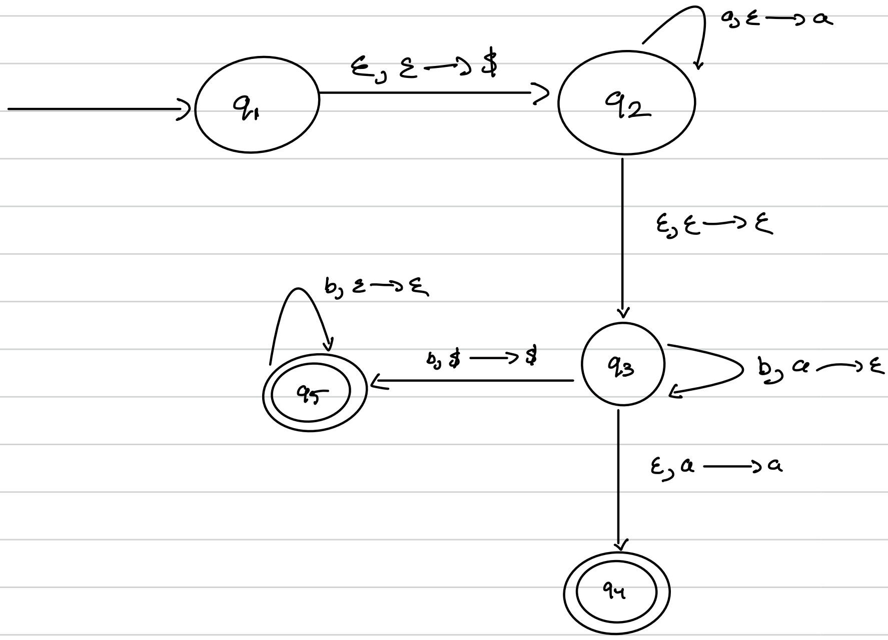
$$P_1 \longrightarrow aA_Lb \mid aA_Sb \mid cP_1 \mid \varepsilon$$
$$A_L \longrightarrow aA_L \mid a$$
$$A_S \longrightarrow bA_S \mid b$$

2.



q1
ε, ε → $
q2
a, $ → a
a, a → a
a, b → ε
b, $ → b
b, b → b
b, a → ε
q3
ε, $ → ε

3.



q1
ε, ε → $
q2
a, ε → a
ε, ε → ε
q3
b, a → ε
b, $ → $
q5
b, ε → ε
ε, a → a
q4

$L = \{ a^m b^n c^k$ with $m > n$ or $n < k;\ m, n, k > 0 \}$

$a^p \quad b^{p-1} \quad c^p$

$\boxed{xy^iz}$

$p = 1 \implies a \quad b \mid c\,c$

$xy \leq p$

$x = -$

$y = a$

$z = abcc$

pump a $= y^{10} =$ aaaaaa

Final Exam Mock

3. $L = \{ a^m b^n c^k$ with $m > n$ or $n < k;\ m, n, k > 0 \}$

① Assuming that L is a regular Language, hence there exists a p for L.

Choosing $w = a^p\, b^{p-1}\, c^p$

Looking at decomps of w into xyz:

$$x = a^{\alpha}$$
$$y = a^{\beta} \qquad \beta \geq 1$$
$$z = a^{p-\alpha-\beta}\, b^{p-1}\, c^p$$

Choosing i s.t. $xy^iz \notin L$

$$xy^iz = a^{\alpha}\, a^{i\beta}\, a^{p-\alpha-\beta}\, b^{p-1}\, c^p$$
$$= a^{p-\beta+i\beta}\, b^{p-1}\, c^p$$

$\in L \iff p - \beta + i\beta > p - 1$

$\iff -\beta + i\beta > -1$

$\iff -\beta + 2\beta > -1$ $\qquad$ choose $i = 2$

$\iff \beta > -1$

2.

Proofs

1. $N_1 = \{ Q_1, \Sigma, \delta_1, q_1, F_1 \}$
$N_2 = \{ Q_2, \Sigma, \delta_2, q_2, F_2 \}$

$N = N_1 \cup N_2$

$N = \{ Q, \Sigma, \delta, q_0, F \}$

1. $Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2$
2. $\Sigma = \Sigma_1 \cup \Sigma$
3. $\delta((r_1, r_2) a) = [\delta_1(r_2, a), \delta_2(r_2, a)]$
4. q_0 is in the pair (q_1, q_2)
5. $F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ and } r_2 \in F_2\}$

2. $N_1 = \{ Q_1, \Sigma, \delta_1, q_1, F1 \}$
$N_2 = \{ Q_2, \Sigma, \delta_2, q_2, F2 \}$
$N = N_1 \cup N_2$

$N = \{ Q, \Sigma, \delta, q_0, F \}$

1. $Q = q_0 \cup Q_1 \cup Q_2$
2. q_0 is the starting state
3. $F = F_1 \cup F_2$

3. $N_1 = \{ Q_1, \Sigma, \delta_1, q_1, F_1 \}$
$N_2 = \{ Q_2, \Sigma, \delta_2, q_2, F_2 \}$

1. $Q = Q_1 \cup Q_2$
2. same starting state as q_1

4.

1. $Q = q_0 \cup Q_1$
2. q_0 is the new start state
3. $F = q_0 \cup F_1$