Homework 1

3. Possible combinations :

Combination 1 → 2 professors , 1 associate professor , 1 assistant professor
Combination → 1 professors , 2 associate professor , 1 assistant professor
Combination → 1 professors , 1 associate professor , 2 assistant professor

Hence all the possible combination is as follows :

$$\binom{8}{2}\binom{6}{1}\binom{13}{1} + \binom{8}{1}\binom{6}{2}\binom{13}{1} + \binom{8}{1}\binom{6}{1}\binom{13}{2}$$

$$\frac{8!}{(6)!\,2!} \cdot 6 \cdot 13 + 8 \cdot \frac{6!}{(4!)\,2!} \cdot 13 + 8 \cdot 6 \cdot \frac{13!}{(11!)\,2!}$$

$$\Rightarrow (28 \cdot 6 \cdot 13) + (8 \cdot 15 \cdot 13) + (8 \cdot 6 \cdot 78) = 7488$$

Hence the possible combinations are 7488

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Homework 2

4. Fraternal set $= \frac{1}{3}$

Identical set $= \frac{2}{3}$

Hence $P(I|S)$ means probability of next set of twins who are identical.

$$\therefore \quad P(I|S) = \frac{P(I \cap S)}{P(S)}$$

$$= \frac{P(S|I)\,P(I)}{P(I \cap S) + P(F \cap S)}$$

$$= \frac{P(S|I)\,P(I)}{P(S|I)\,P(I) + P(S|F)\,P(F)}$$

$$= \frac{1 \cdot \frac{2}{3}}{1 \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3}}$$

$$= \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{6}}$$

$$= \frac{\frac{2}{3}}{\frac{5}{6}}$$

$$= \frac{2}{3} \cdot \frac{6}{5}$$

$$= \frac{12}{15} \longrightarrow \frac{4}{5}$$

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Homework 3

3.
$$f(x) = \begin{cases} (k+1)x^2 & \text{for } 0 < x < 1 \\ 0 & \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 (k+1)x^2$$

$$\Rightarrow (k+1)\int_0^1 (x^2)$$

$$\Rightarrow (k+1)\left[\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow (k+1)\frac{1}{3}$$

$$\therefore \quad (k+1)\frac{1}{3} = 1$$

$$(k+1) = 3$$

$$k = 2$$

Hence
$$\frac{1}{2} = \int_{-\infty}^{Q} 3x^2\,dx$$

$$= 3 \cdot \int_0^Q x^2\,dx$$

$$= 3 \cdot \left[\frac{x^3}{3}\right]_0^Q$$

$$= 3 \cdot \frac{Q^3}{3}$$

$$= Q^3$$

$$\therefore \quad Q^3 = \frac{1}{2}$$

$\Rightarrow$ $Q = \frac{1}{\sqrt[3]{2}}$ is the median of X

$$f(x) = \begin{cases} c(8-x) & \text{for } x = 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) $1 = \sum_{x=0}^{5} c(8-x)$

$1 = 8c + 7c + 6c\ 5c + 4c + 3c$

$1 = 33c$

$\frac{1}{33} = c$

(b) $P(x > 2) \Rightarrow \sum_{x=3}^{5} \frac{1}{33}(8-x)$

$\Rightarrow \left[\frac{1}{33}(5) + \frac{1}{33}[4) + \frac{1}{33}[3)\right]$

$\Rightarrow \frac{5}{33} + \frac{4}{33} + \frac{3}{33}$

$\Rightarrow \frac{12}{33}$

(c) $E(x^n) = \sum_{x \in \mathbb{R}} x^n f(x)$

$\Rightarrow E(x) = \sum_{x=0}^{5} x \cdot \frac{1}{33}(8-x)$

$= 0 + \frac{1}{33}(7) + \frac{2}{33}(6) + \frac{3}{33}(5) + \frac{4}{33}(4) + \frac{5}{33}(3)$

$= \frac{0+7+12+15+16+15}{33}$

$= \frac{65}{33}$

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Homework 4

8. $f(x) = \begin{cases} \frac{1}{2} & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

x by $2x \Rightarrow x \cdot 2x \Rightarrow 2x^2$ = Area of rectangle

$$E(x^n) = \int_{-\infty}^{\infty} x^n f(x)\, dx$$

$$\Rightarrow E(2x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\Rightarrow E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_0^2 x^2 \frac{1}{2}\, dx$$

$$= \frac{1}{2} \int_0^2 x^2\, dx$$

$$= \frac{1}{2} \left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{2} \left[\frac{8}{3}\right]$$

$$= \frac{8}{6}$$

$\therefore\ E(x^2) = \frac{8}{6}$, so $2E(x^2) = \frac{16}{6} \Rightarrow \frac{8}{3}$

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Homework 5

25. Probability Density Function of Poisson Distribution: $f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$ $x = 0, 1, 2, \ldots\ldots, \infty$

Hence need to find λ:

Since we know $P[\text{no-hit game}] = \frac{1}{3}$

$\Rightarrow P[X = 0] = \frac{1}{3} = e^{-\lambda}$

$\therefore -\lambda = \ln\frac{1}{3} \quad = \lambda = \ln 3$

So $f(x) = \frac{e^{-\ln(3)} \ln(3)^{x}}{x!}$

So finally: $P(x \geq 2) = 1 - [P(x=0) + P(x=1)]$

$= 1 - [\frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!}]$

$= 1 - [e^{-\lambda} + e^{-\lambda}\lambda]$

$= 1 - [e^{-\ln(3)} + e^{-\ln(3)} \ln(3)]$

$= 1 - [\frac{1}{3} + e^{-\ln(3)} . \ln(3)]$

$\approx 1 - [\frac{1}{3} + 0.3662$

$\approx 1 - [0.6995]$

≈ 0.3005

Hence $P(x \geq 2) \approx 0.3005$

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Homework 6

3 $Z = aX + Y$ and $Cov(X,Z) = \frac{1}{3}$

$Cov(X,Y) = 0$ since X and Y are independent.

Since X and Y are independent and indentically distributed:

- $Var(X) = 1$, $Var(Y) = 1$

Hence, $Corr(X,Z) := \frac{Cov(X,Z)}{\sqrt{Var(X) Var(Z)}}$

so $Var(Z) = Var(aX+Y) = E((aX+Y)^2) - E((aX+Y))^2$

$= E(a^2X^2 + 2aXY + Y^2) - (aE(X) + E(Y))^2$

* using $E(XY) = E(X)E(Y)$

$= a^2E(X^2) + 2aE(X)E(Y) + E(Y^2) - (aE(X))^2 - 2aE(X)E(Y) - (E(Y))^2$

$= a^2E(X^2) - a^2(E(X))^2 + E(Y^2) - (E(Y))^2$

$= a^2 Var(X) + Var(Y)$

$= a^2 + 1$

$Cov(X,Z) = Cov(X, aX+Y) = a\,Cov(X,X) + Cov(X,Y) = a\,Var(X) = a$

$\therefore \frac{Cov(X,Z)}{\sqrt{Var(X) Var(Z)}} = \frac{a}{\sqrt{a^2+1}}$

Finaly $\frac{a}{\sqrt{a^2+1}} = \frac{1}{3}$

$\Rightarrow a = \frac{\sqrt{a^2+1}}{3}$ $\Rightarrow a^2 = 1$

$\Rightarrow 3a = \sqrt{a^2+1}$ $\Rightarrow a^2 = \frac{1}{8}$

$\Rightarrow 9a^2 = a^2 + 1$ $=$ $a = \frac{1}{\sqrt{8}}$

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Question 7

$X_1, X_2, \ldots X_n$ random sample of size n from a Bernouli distribution.

$$\bar{X} = \frac{(x_1 + x_2 + \ldots + x_n)}{n}$$

$$u = \frac{1}{2}$$

$$Var[x_i] = \frac{1}{2}(0 - \frac{1}{2})^2 + \frac{1}{2}(1 - \frac{1}{2})^2$$

$$= 0.25$$

$\therefore$ Since sample size is Bernouli Distribution:

$$M_{x_1}(t) = q + pe^t \qquad \text{where } q = (1-p)$$

$$= \frac{1}{2} + \frac{1}{2}e^t$$

Hence $$M_{\bar{x}}(t) = M_{\frac{1}{n}} \sum_{i=1}^{n} x_1(t)$$

$$= \prod_{i=1}^{n} M_{x_1}\left(\frac{t}{n}\right)$$

$$= \prod_{i=1}^{n} \left[\frac{1}{2} + \frac{1}{2}e^{t/n}\right]^n$$

$$= \left[\frac{1}{2} + \frac{1}{2}e^{t/n}\right]^n$$

$\therefore$ t must be a value such as that $\left[\frac{1}{2} + \frac{1}{2}e^{t/n}\right]^n \neq$ undefined, and

$\bar{X}$ must follow an approximate distribution of BERNOULI (0.5)

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Question 8

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \end{cases}$$

The likehood function of the sample is given by

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$$

Therefore

$$\ln L(\theta) = \ln\left(\prod_{i=1}^{n} f(x_i;\theta)\right)$$

$$= \sum_{i=1}^{n} \ln f(x_i;\theta)$$

$$= \sum_{i=1}^{n} \ln[\theta x_i^{\theta-1}]$$

$$= n\ln(\theta) + (\theta-1)\sum_{i=1}^{n} \ln x_i$$

Maximizing $\ln L(\theta)$ with respect to θ

$$\frac{d\ln L(\theta)}{d\theta} = \frac{d}{d\theta}\left(n\ln(\theta) + (\theta-1)\sum_{i=1}^{n} \ln x_i\right)$$

$$= \frac{n}{\theta} + \sum_{i=1}^{n} \ln x_i$$

Hence, setting this derivative $\frac{d\ln L(\theta)}{d\theta}$ to 0,

$$\frac{d\ln L(\theta)}{d\theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \ln x_i = 0$$

that is $\dfrac{n}{\theta} = -\sum_{i=1}^{n} \ln x_i$

$$\frac{1}{\theta} = -\frac{1}{n}\sum_{i=1}^{n} \ln x_i$$

or $\dfrac{1}{\theta} = -\dfrac{1}{n}\sum_{i=1}^{n} \ln x_i = \overline{-\ln x}$

$$\theta = \frac{1}{\overline{-\ln x}}$$

$\therefore$ $\hat{\theta} = \dfrac{1}{\overline{-\ln x}}$ or

$$\frac{1}{\theta} = -\frac{1}{n}\sum_{i=1}^{n} \ln x_i$$

$$\hat{\theta} = -\frac{n}{\sum_{i=1}^{n} \ln x_i}$$

* to match the textbook answer

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Question 9

$$f(x;\theta) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}} \qquad -\infty < x < \infty,$$

The likehood function of the sample is given by

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$$

Thus,

$$\ln L(\theta) = \sum_{i=1}^{n} \ln f(x_i;\theta)$$

$$= \sum_{i=1}^{n} \ln \left[\frac{1}{2\theta} e^{-\frac{|x_i|}{\theta}} \right]$$

$$= n \ln\left(\frac{1}{2\theta}\right) - \sum_{i=1}^{n} \frac{|x_i|}{\theta}$$

$$= n \ln\left(\frac{1}{2\theta}\right) - \frac{1}{\theta} \sum_{i=1}^{n} |x_i|$$

Maximizing $\ln L(\theta)$ with respect to θ

$$\frac{d \ln L(\theta)}{d(\theta)} = \frac{d}{d\theta}\left(n \ln\left(\frac{1}{2\theta}\right) - \frac{1}{\theta} \sum_{i=1}^{n} |x_i| \right)$$

$$= \frac{-n}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{n} |x_i|$$

Setting this derivative $\frac{d \ln L(\theta)}{d\theta}$ to 0,

$$\frac{d \ln L(\theta)}{d(\theta)} = \frac{-n}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{n} |x_i| = 0$$

that is,

$$\frac{-n}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{n} |x_i| = 0$$

$$\Rightarrow \quad -\frac{n}{\theta} = -\frac{1}{\theta^2} \sum_{i=1}^{n} |x_i|$$

$$\Rightarrow \quad -\theta n = -\sum_{i=1}^{n} |x_i|$$

$$\Rightarrow \quad \theta n = \sum_{i=1}^{n} |x_i|$$

$$\Rightarrow \quad \theta = \frac{1}{n} \sum_{i=1}^{n} |x_i| \quad \text{or} \quad \overline{|X|}$$

$\therefore$ the estimator of θ is :

$$\hat{\theta} = \overline{|X|}$$

Now the unbiasedness of this estimator is examined

$$E(\hat{\theta}) = \theta \longrightarrow E(\overline{|X|}) = \theta$$

$$E(\overline{|X|}) = E(|X_i|)$$

* Law of Large numbers

Hence, $$E(\overline{|X|}) = \int_{-\infty}^{\infty} x f(x_i ; \theta)$$

$$= \frac{1}{2\theta} \int_{-\infty}^{\infty} x e^{\frac{-|x|}{\theta}} dx$$

$$= \frac{1}{2\theta} \left[\int_{0}^{\infty} x e^{\frac{x}{\theta}} dx - \int_{-\infty}^{0} x e^{\frac{x}{\theta}} dx \right.$$

$$= \frac{1}{2\theta} \left(\left[\theta^2 e^{-\frac{x}{\theta}} - \theta x e^{-\frac{x}{\theta}} \right]_{0}^{\infty} - \left[\theta x e^{\frac{x}{\theta}} - \theta^2 e^{\frac{x}{\theta}} \right]_{-\infty}^{0} \right)$$

$$= \frac{1}{2\theta} (\theta^2 + \theta^2)$$

$$= \frac{1}{2\theta} 2\theta^2$$

$$= \theta$$

$\therefore$ $\hat{\theta}$ is unbiased

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Question 10

$$f(x;\theta) = \begin{cases} (\theta+1)x^{-\theta-2} & \text{if } x = 0,1,2,\ldots\ldots, \infty \\ 0 & \end{cases}$$

Logarithm of the Likehood Function:

$$L(\theta) = \prod_{i=1} f(x_i;\theta)$$

thus,

$$\ln L(\theta) = \sum_{i=1}^{n} \ln f(x_i;\theta)$$

$$= \sum_{i=1}^{n} \ln (\theta+1)x_i^{-\theta-2}$$

$$= n\ln(\theta+1) - (\theta+2)\sum_{i=1}^{n} \ln(x_i)$$

Differentiating,

$$\frac{d}{d\theta} \ln L(\theta) = \frac{n}{\theta+1} - \sum_{i=1}^{n} \ln(x_i)$$

Equating this derivative to zero and solving for θ:

$$\frac{n}{\theta+1} - \sum_{i=1}^{n} \ln(x_i) = 0$$

$$\frac{n}{\theta+1} = \sum_{i=1}^{n} \ln(x_i)$$

$$\frac{\theta+1}{n} = \frac{1}{\sum_{i=1}^{n} \ln(x_i)}$$

$$\hat{\theta} = \frac{n}{\sum_{i=1}^{n} \ln(x_i)} - 1$$

$$\therefore \quad E\left[\frac{\partial^2}{\partial\theta^2} \ln[L\theta]\right] = -\frac{n}{(\theta+1)^2}$$

Hence variance :

$$\therefore \quad Var(\hat{\theta}) \simeq \frac{(\theta+1)^2}{n}$$

so,

$$\sqrt{Var(\hat{\theta})} \simeq \sqrt{\frac{(\hat{\theta}+1)^2}{n}} = \frac{\sqrt{n}}{\sum_{i=1}^{n} \ln(x_i)}$$

Hence, now the substitution could happen :

$$\therefore \left[\frac{n - z_{\frac{\alpha}{2}}\sqrt{n}}{\sum_{i=1}^{n} \ln(x_i)} - 1 \, , \, \frac{n + z_{\frac{\alpha}{2}}\sqrt{n}}{\sum_{i=1}^{n} \ln(x_i)}\right]$$

$$\left[\frac{n}{\sum_{i=1}^{n} \ln(x_i)} - 1 - z_{\frac{\alpha}{2}} \frac{\sqrt{n}}{\sum_{i=1}^{n} \ln(x_i)} \, , \, \frac{n}{\sum_{i=1}^{n} \ln(x_i)} - 1 + z_{\frac{\alpha}{2}} + \frac{\sqrt{n}}{\sum_{i=1}^{n} \ln(x_i)}\right]$$

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Homework 14

$$f(x;\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2}} \qquad -\infty < x < \infty$$

Hence, first the constant k of the likehood ratio critical region for $H_0: \mu = 3$ and $H_a: \mu \neq 3$.

$\therefore$ by using the deffination 18.5

$$w(x_1, x_2, x_3) = \frac{\max_{\theta \in \Omega_0} L(\theta, x_1, x_2, x_3)}{\max_{\theta \in \Omega} L(\theta, x_1, x_2, x_3)}$$

$$= \frac{L(\mu)}{L(\hat{\mu})}$$

$$= \frac{\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\sum_{i=1}^{3}(x_i - 3)^2}}{\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\sum_{i=1}^{3}(x_i - \bar{x})^2}}$$

$$= e^{-\frac{1}{2}\sum_{i=1}^{3}(x_i - 3)^2}$$

$$= e^{-\frac{3}{2}(\bar{x}-3)^2}$$

So with $w(x_1, x_2, x_3) = e^{-\frac{3}{2}(\bar{x}-3)^2}$,

$$C = \{(x_1, x_2, x_3) \mid w(x_1, x_2, x_3) \le k\}$$

$$= e^{\frac{3}{2}(\bar{x}-3)^2} \le k$$

$$= (\bar{x}-3)^2 \ge -\frac{2}{3}\ln(k)$$

$$= |\bar{x}-3| \ge \sqrt{-\frac{2}{3}\ln(k)}$$

Hence $C = \{(x_1, x_2, x_3) \mid |\bar{x}-3| \ge \gamma\}$ where $\gamma = \sqrt{-\frac{2}{3}\ln(k)}$

critical region : $\alpha = 0.05$

$\therefore \quad \alpha = P(|\bar{x} - 3| \geq k)$

$= P\left(\left|\frac{\bar{x}-3}{\frac{\sigma}{\sqrt{3}}}\right| \geq \frac{\sqrt{3}}{\sigma} \cdot k\right)$

$= 1 - P\left[-\frac{\sqrt{3}}{\sigma} \cdot k \leq \frac{\bar{x} - 3}{\frac{\sigma}{\sqrt{3}}} \leq \frac{\sqrt{3}}{\sigma} \cdot k\right.$

$z_{\frac{\alpha}{2}} = \frac{\sqrt{3}}{\sigma} k = \sqrt{3}\, k$

Here $k = \frac{z_{\frac{\alpha}{2}}}{\sqrt{3}}$

$= \frac{1.96}{\sqrt{3}}$ * in normal distribution

$= 1.132$

$\boxed{C = \{ (x_1, x_2, x_3) \mid |\bar{x} - 3| \geq 1.132 \}}$

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Homework 12

Since it is known that $Y_1, Y_2, Y_3, \ldots\ldots, Y_n$ are all independent random variables with $Y_i \sim N(\beta x_i, \sigma^2)$ then :

Calculating for β:

$$\ln[L(\sigma, \beta)] = \sum_{i=1} \ln f(y_i, x_i)$$

$$= -n\ln(\sigma) - \frac{n}{2} \cdot \ln(2\pi) - \frac{1}{2\sigma^2} \sum_{i=1} (y_i - \beta x_i)^2$$

$\therefore$ with this much β can be found out :

$$\frac{d}{d\beta} \ln[L(\sigma, \beta)] = \frac{1}{\sigma^2} \sum_{i=1}^{n} (y_i - \beta x_i) \cdot x_i$$

$$\Rightarrow \frac{d}{d\beta} \cdot \frac{1}{\sigma^2}\left[\sum_{i=1}^{n} (x_i \cdot y_i) - \beta \sum_{i=1}^{n} x_i^2\right] = \frac{1}{\sigma^2} \left(0 - \sum_{i=1}^{n} x^2\right) < 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - y_i) - \beta \sum_{i=1}^{n} x_i^2 = 0$$

$$\Rightarrow \beta \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} (x_i - y_i)$$

$$\therefore \quad \beta = \frac{\sum_{i=1}^{n} (x_i - y_i)}{\sum_{i=1}^{n} x_i^2}$$

Calculating σ^2 :

$$\frac{d}{d\sigma} \cdot \ln[L(\sigma, \beta)] = -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} (y_i - \beta x_i)^2 = 0$$

$$\Rightarrow \frac{d}{d\sigma}\left[\frac{-n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=3}^{n} (y_i - \beta x_i)^2\right] = \frac{n}{\sigma^2} - \frac{1}{\sigma^4} \sum_{i=1}^{n} (y_i - \beta x_i)^2$$

$$\Rightarrow \sum_{i=1}^{n} (y_i - \beta x_i)^2 = n \cdot \sigma^2$$

$$\therefore \quad n \cdot \sigma^2 = \sum_{i=1}^{n} (y_i^2 + \beta^2 x_i^2 - 2\beta$$

$$= \sum_{i=1}^{n} (y_i^2) + \beta^2 \sum_{i=1}^{n} (x_i^2) - 2\beta \sum_{i=1}^{n} (x_i y_i)$$

$$= \sum_{i=1}^{n} (y_i^2 + \beta^2 x_i^2 - 2\beta x_i y_i)$$

$$= \sum_{i=1}^{n} (y_i^2) + \left[\frac{\sum_{i=1}^{n} (x_i y_i)}{\sum_{i=1}^{n} x_i^2} \right]^2 \sum_{i=1}^{n} (x_i^2) - 2 \left[\frac{\sum_{i=1}^{n} (x_i y_i)}{\sum_{i=1}^{n} x_i^2} \right] \sum_{i=1}^{n} (x_i y_i)$$

$$\therefore n \sigma^2 = \sum_{i=1}^{n} (y_i^2) + \left[\frac{\sum_{i=1}^{n} (x_i y_i)}{\sum_{i=1}^{n} x_i^2} \right]^2 \sum_{i=1}^{n} (x_i^2) - 2 \left[\frac{\sum_{i=1}^{n} (x_i y_i)}{\sum_{i=1}^{n} x_i^2} \right] \sum_{i=1}^{n} (x_i y_i)$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} (y_i^2) + \left[\frac{\sum_{i=1}^{n} (x_i y_i)}{\sum_{i=1}^{n} x_i^2} \right]^2 \sum_{i=1}^{n} (x_i^2) - 2 \left[\frac{\sum_{i=1}^{n} (x_i y_i)}{\sum_{i=1}^{n} x_i^2} \right] \sum_{i=1}^{n} (x_i y_i)}{n}$$