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ASSIGNMENT 7 : RASTERIZATION

EXCERCISE 1

TASK 1 :

$\alpha = \frac{\pi}{2}$

$P_1 = (-1, -1, 2)$

$P_2 = (7, 1, 10)$

Mapping Points to 2-d :

$P_1 = \left(-\frac{1}{2}, -\frac{1}{2}\right)$

$P_2 = \left(\frac{7}{10}, \frac{1}{10}\right)$

$y = mx + c$

$m = \frac{dy}{dx} = \frac{[y_2 - y_1]}{[x_2 - x_1]} = \frac{\frac{6}{10}}{\frac{12}{10}} = \frac{1}{2}$

$dy = \frac{6}{10}$

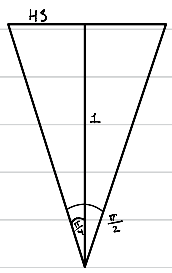
$dx = \frac{12}{10}$

$c_{P_1} = y - mx$

$= -\frac{1}{2} - \left[\frac{1}{2} \times -\frac{1}{2}\right]$

$= -\frac{1}{4}$

Calculating the pixel width :

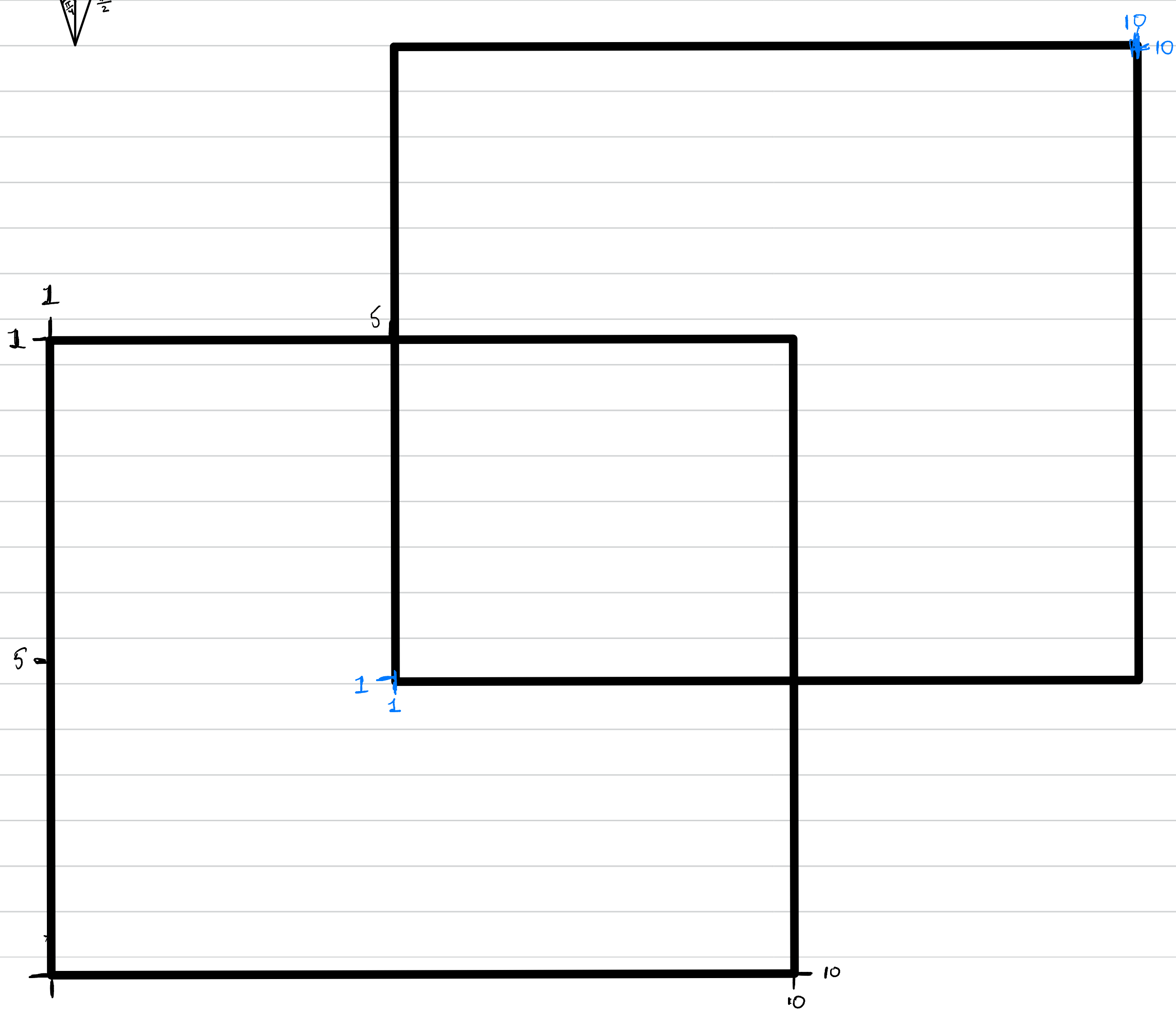


$\tan\left[\frac{\pi}{4}\right] = \frac{HS}{1} = HS = 1$

$2HS = 10$ pixel

$2 = 10$ pixel

$\frac{1}{5} = $ pixel



As we can see the mapping procedure follows the above and bellow equation:

$$x_{image} = \frac{x}{\frac{1}{5}} + 5 \quad \text{and}$$

$$y_{image} = \frac{y}{\frac{1}{5}} - 5$$

$\therefore$ the image coordinates are for P_1:

$$K_{1_x} = \frac{\frac{-1}{2}}{\frac{1}{5}} + 5$$
$$= -\frac{1}{2} \times 5 + 5$$
$$= -\frac{5}{2} + 5$$
$$= \frac{-5 + 10}{2}$$
$$= \frac{5}{2}$$
$$= 2.5$$
$$\approx 3$$
$\Rightarrow$ This evaluates to 3^{rd} pixel

$$K_{1_y} = \frac{\frac{-1}{2}}{\frac{1}{5}} - 5$$
$$= -\frac{1}{2} \times 5 - 5$$
$$= -\frac{5}{2} - 5$$
$$= \frac{-5 - 10}{2}$$
$$= \frac{-15}{2}$$
$$= -7.5$$
$$\approx -8$$
$\Rightarrow$ This evaluates to 8^{th} pixel

And for the image coordinates for P_2:

$$K_{2_x} = \frac{\frac{7}{10}}{\frac{1}{5}} + 5$$
$$= \frac{35}{10} + 5$$
$$= \frac{35 + 50}{10}$$
$$= \frac{85}{10}$$
$$= 8.5$$
$$\approx 9$$
$\Rightarrow$ This evaluates to 9^{th} pixel

$$K_{2_y} = \frac{\frac{1}{10}}{\frac{1}{5}} - 5$$
$$= \frac{5}{10} - 5$$
$$= \frac{5}{10} - \frac{50}{10}$$
$$= -\frac{45}{10}$$
$$= -4.5$$
$$\approx -5$$
$\Rightarrow$ This evaluates to 5^{th} pixel

Implicit Equation of the line through (x_1, y_1) and (x_2, y_2):

$$F(x,y) = y\,dx - x\,dy + x_1 dy - y_1 dx$$

$$y = mx + c$$

$$m = \frac{dy}{dx} = \frac{[y_2 - y_1]}{[x_2 - x_1]} = \frac{[5 - 8]}{[9 - 3]} = \frac{-3}{6} = \frac{-1}{2}$$

$$dy = -3 \xrightarrow{\text{Because y axis is flipped}} 3$$

$$dx = 6$$

Intolizing $F(m)$:

$$F(m) = -2d_y + d_x$$
$$= -6 + 6$$
$$= 0$$

$f \geq 0$ choose $P_0 \longrightarrow (4,8)$

$$F(m_0) = F(m) - 2d_y$$
$$= -6$$

$f < 0$ choose $P_1 \longrightarrow (5,7)$

$$F(m_1) = F(m) - 2d_y + 2d_x$$
$$= -6 - 6 + 12$$
$$= 0$$

$f \geq 0$ choose $P_0 \longrightarrow (6,7)$

$$F(m_0) = F(m) - 2d_y$$
$$= 0 - 6$$
$$= -6$$

$f < 0$ choose $P_1 \longrightarrow (7,6)$

$$F(m_1) = F(m) - 2d_y + 2d_x$$
$$= -6 - 6 + 12$$
$$= 0$$

$f \geq 0$ choose $P_0 \longrightarrow (8,6)$

$$F(m_0) = F(m) - 2d_y$$
$$= 0 - 6$$
$$= -6$$

$f < 0$ choose $P_1 \longrightarrow (9,5)$

The pixels set are $\left[[3,8], [4,8], [5,7], [6,7], [7,6], [8,6], [9,5] \right]$

TASK 2: We need to calculate the z-value associated with $[6,7]$ which is in the middle of the horizontal direction of the line.

$$q_1 = [3,8]$$
$$q_2 = [9,5]$$
$$q_{middle} = [6,7] \quad \text{therefore} \quad 1 - \lambda = 0.5 \Rightarrow \lambda = 0.5$$

Hence using the formula:

$$p^2 = \frac{1}{(1-\lambda)\frac{1}{p_1^2} + \lambda \frac{1}{p_2^2}}$$

$$= \frac{1}{0.5 \frac{1}{2} + 0.5 \frac{1}{10}}$$

$$= \frac{1}{0.25 + 005}$$

$$= \frac{1}{\frac{3}{10}} = \frac{10}{3}$$

TASK 3:

$A_1 = (1,0,0) \qquad A_2 = (0,1,0)$

$$A = \frac{(1-\lambda)\frac{A_1}{\rho_1^2} + \lambda\frac{A_2}{\rho_2^2}}{\frac{1}{\rho^2}}$$

$$= \frac{\frac{0.5}{2}\cdot A_1 + \frac{0.5}{10} A_2}{\frac{10}{3}}$$

$$= \frac{\left[\frac{1}{4},0,0\right] + \left[0,\frac{1}{20},0\right]}{\frac{3}{10}}$$

$$= \frac{\left[\frac{1}{4},\frac{1}{20},0\right]}{\frac{3}{10}}$$

$$= \left[\frac{5}{6},\frac{1}{6},0\right]$$