Università
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Faculty
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Computer Graphics (Fall 2022)

Assignment 4: Transformations and Barycentric Coordinates

October 13, 2022

Exercise 1 [5 points]

Let us consider a 2D coordinate system with points $p_1 = (1,1)^T$, and $p = (1.5, 2.5)^T$. Additionally, let's define a vector $\mathbf{u} = p - p_1$. Perform the following tasks:

Task 1 [1 points] Construct two matrices, $R_{90}, T \in \mathrm{R}^{3\times 3}$, such that the first one performs counter-clockwise rotation around the center of the coordinates system by angle $90°$, and the second one performs translation by vector $\mathbf{t} = (1, -2)^T$, in homogeneous coordinates.

Task 2 [1 points] Represent the points p_1 and p as well as the vector $\mathbf{u}$ using homogeneous coordinates and transform them first using the R_{90}, and then T matrices. Convert the obtained points and vector back into Cartesian coordinates and denote them by p_1', p', and $\mathbf{u}'$. Draw all the points and vectors before and after the transformation, and verify that $\mathbf{u}' = p' - p_1'$. What influence did the matrix T have on the $\mathbf{u}'$?

Task 3 [1 points] Construct a scaling matrix, $S \in \mathrm{R}^{3\times 3}$, which scales everything in x and y direction by a factor of 2. Compute new points p_1'', p'' and vector $\mathbf{u}''$ by transforming p_1', p', and $\mathbf{u}'$ using matrix S. Again perform the operation in homogeneous coordinates, and then, convert the results to Cartesian coordinates and visualize them.

Task 4 [2 points] Construct matrices $R_{90}^{-1}, T^{-1}, S^{-1} \in \mathrm{R}^{3\times 3}$, which perform inverse transformations to $R_{90}, T, S,$, i.e., clockwise rotation by $90°$, translation by vector $\mathbf{t}' = (-1, 2)^T$, and scaling by factor 0.5. Compute matrix $M = R_{90}^{-1} T^{-1} S^{-1}$, and verify that transforming points p_1'', p'', and $\mathbf{u}''$ with this matrix results in the initial values of p_1, p, and $\mathbf{u}$.

Exercise 2 [4 points]

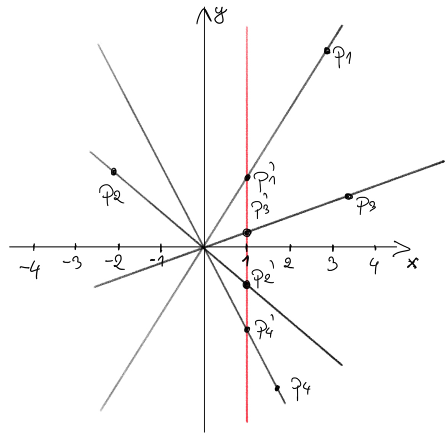
Consider a triangle made of three points $p_1 = (6,0,4)$, $p_2 = (2,0,0)$, and $p_3 = (2,4,4)$. Verify whether point $p = (4,1,3)$ belongs to the interior of the triangle.

Exercise 3 [3 points]

Using barycentric coordinates, prove that the centroid of a triangle divides its medians in ratio 2:1.

Exercise 4 [3 points]

Consider a transformation that projects all the points $p_i \in \mathrm{R}^2$ onto the line $x = 1$. For each point, the projection is performed along the line that passes through the center of the coordinate system and the point (see the image on the right). Each point p_i' is a result of applying this transformation to point p_i. Construct a matrix $M \in \mathrm{R}^{3\times3}$ which realizes this transformation using homogeneous coordinates. More specifically, after converting a point p_i into the homogeneous coordinates, multiplying it with the matrix M, and transforming it back to the Cartesian coordinates, you should obtain point p_i'. Note that this is not an affine transformation. Comment on how the matrix M transforms the points lying on the y-axis. Interpret the results both in the homogeneous and Cartesian coordinates.



Bonus exercise 5 [2 points]

Consider the same task as in Exercise 4, but the line onto which the points are projected can be now arbitrary, and it is defined by a line equation $y = ax + b$, where $a, b \in \mathrm{R}$ are constants. Derive the matrix M for this more general case.

Submission

Submit a single PDF including all the calculations you did to solve the assignments to iCorsi. You should not use any external tools, other than a simple calculator, which can provide intermediate or final results.

Solutions must be returned on October 20 2022 via iCorsi3