Harkeerat Singh Sawhney & Mak Fazlic

ASSIGNMENT 4: TRANSFORMATION AND BARYCENTRIC COORDINATES

EXCERCISE 1

TASK 1

$p_1 = (1,1)^T \qquad p = (1.5, 2.5)^T \qquad u = p - p_1$

$$R_{90} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(90) & -\sin(90) & 0 \\ \sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

TASK 2

Repersentation in Homogeneous Cordinates: $p_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad p = \begin{bmatrix} 1.5 \\ 2.5 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 0.5 \\ 1.5 \\ 0 \end{bmatrix}$

ROTATION:

We get the following results:

$$p_{1_{rotation}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$p_{rotation} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

$$u_{rotation} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix}$$

TRANSLATION

We get the following results based on the rotation:

$$p_{1_{Translate}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$p_{rotation} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2.5 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix}$$

$$u_{rotation} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix}$$

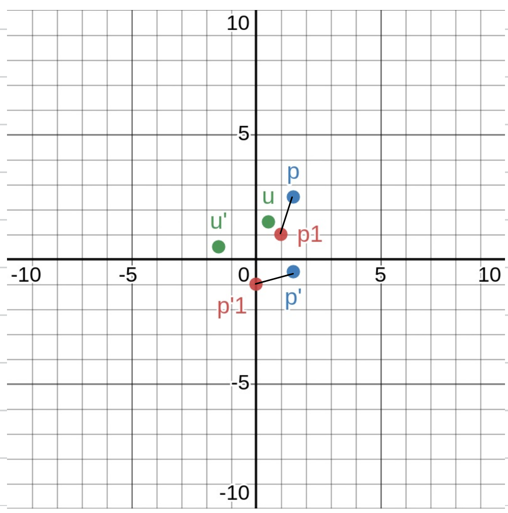
Converting Homogeneous cordinates to Cartesian coordinates:

$$p_1' = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad p' = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} \qquad u' = \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix}$$

Confirming if this is true: $u' = p' - p_1'$

$$\begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix}$$



TASK 3

$$p_1' = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad p' = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} \qquad u' = \begin{bmatrix} -1.5 \\ 0.5 \end{bmatrix}$$

Repersentation in Homogeneous Cordinates : $p_1' = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ $p' = \begin{bmatrix} 1.5 \\ -0.5 \\ 1 \end{bmatrix}$ $u' = \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix}$

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Performing the scalling :

$$p_1'' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$p'' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 \\ -0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$$

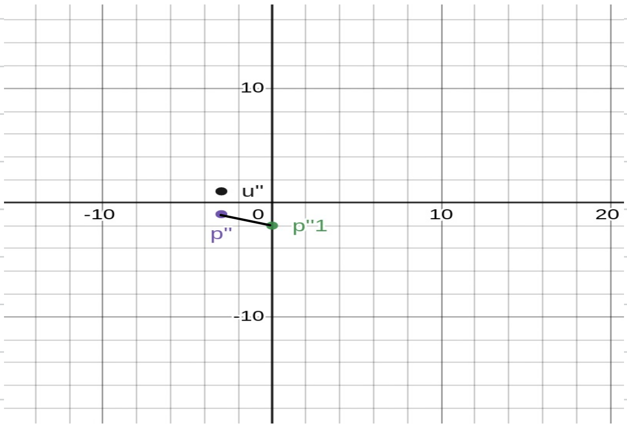
$$u'' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

Converting it back to cartestion prodct :

$p_1'' = (0, -2)$

$p'' = (-3, -1)$

$u'' = (-3, 1)$



TASK 4

The inverse transformation matrices are as follows:

$$R_{90}^{-1} = \overset{\text{Rotation (Clockwise)}}{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} \cos(90) & \sin(90) & 0 \\ -\sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

EXCERCISE 2:

First we need to find the area of the whole triangle. We have the following points for the triangle:

$$P_1 = (6,0,4) \qquad P_2 = (2,0,0) \qquad P_3 = (2,4,4)$$

Hence we do the follow:

$$\overrightarrow{P_1P_2} = \langle (2-6), (0-0), (0-4) \rangle = \langle -4, 0, -4 \rangle$$
$$\overrightarrow{P_1P_3} = \langle (2-6), (4-0), (4-4) \rangle = \langle -4, 4, 0 \rangle$$

Now doing cross product of $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$:

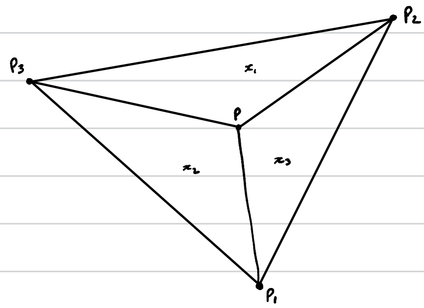
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 0 & -4 \\ -4 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -4 \\ 4 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} -4 & -4 \\ -4 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} -4 & 0 \\ -4 & 4 \end{vmatrix} \vec{k}$$
$$= [0-(-16)]\vec{i} - [0-(-16)]\vec{j} + [-16-0]\vec{k}$$
$$= 16\vec{i} - 16\vec{j} - 16\vec{k}$$

$$\therefore \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \langle 16, -16, -16 \rangle$$

$$\left\| \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} \right\| = \sqrt{(16)^2 + (-16)^2 + (-16)^2}$$
$$= \sqrt{768}$$

$$\text{Area of the triangle} = \frac{\sqrt{768}}{2} = 13.856 \text{ or } 8\sqrt{3}$$

Lets assume that P is in the triangle. Hence we should get the area for the following sub-triangles (x_1, x_2, x_3):



Finding area of x_1 we need the following points:

$$P_3 = (2,4,4) \qquad P_2 = (2,0,0) \qquad P = (4,1,3)$$

$$\overrightarrow{P_3P_2} = \langle (2-2), (0-4), (0-4) \rangle = \langle 0, -4, -4 \rangle$$
$$\overrightarrow{P_3P} = \langle (4-2), (1-4), (3-4) \rangle = \langle 2, -3, -1 \rangle$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -4 & -4 \\ 2 & -3 & -1 \end{vmatrix} = \begin{vmatrix} -4 & -4 \\ -3 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 0 & -4 \\ 2 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 0 & -4 \\ 2 & -1 \end{vmatrix} \vec{k}$$

$$= (4-12)\vec{i} - (0-(-8))\vec{j} + (0-(-8))\vec{k}$$

$$= -8\vec{i} - 8\vec{j} + 8\vec{k}$$

$$\overrightarrow{P_3P_2} \times \overrightarrow{P_3P} = \langle -8, -8, 8 \rangle$$

$$\|\overrightarrow{P_3P_2} \times \overrightarrow{P_3P}\| = \sqrt{(-8)^2 + (-8)^2 + (8)^2}$$

$$= \sqrt{192}$$

$$x_1 = \frac{\sqrt{192}}{2} = 6.928 \text{ or } 4\sqrt{3}$$

$$a_1(x_1) = \frac{4\sqrt{3}}{8\sqrt{3}} = \frac{1}{2}$$

Finding area of x_2 we need the following points:

$$P_1 = (6,0,4) \quad P_3\,(2,4,4) \quad P = (4,1,3)$$

$$\overrightarrow{P_1P_3} = \langle (2-6), (4-0), (4-4) \rangle = \langle -4, 4, 0 \rangle$$

$$\overrightarrow{P_1P} = \langle (4-6), (1-0), (3-4) \rangle = \langle -2, 1, -1 \rangle$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 4 & 0 \\ -2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 1 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} -4 & 0 \\ -2 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} -4 & 4 \\ -2 & 1 \end{vmatrix} \vec{k}$$

$$= (-4-0)\vec{i} - (4-0)\vec{j} + (-4+8)\vec{k}$$

$$= -4\vec{i} - 4\vec{j} + 4\vec{k}$$

$$\overrightarrow{P_1P_3} \times \overrightarrow{P_1P} = \langle -4, -4, 4 \rangle$$

$$\|\overrightarrow{P_1P_3} \times \overrightarrow{P_1P}\| = \sqrt{(-4)^2 + (-4)^2 + (4)^2}$$

$$= \sqrt{48}$$

$$x_2 = \frac{\sqrt{48}}{2} = 3.464 \text{ or } 2\sqrt{3}$$

$$a_2(x_2) = \frac{2\sqrt{3}}{8\sqrt{3}} = \frac{1}{4}$$

Finding area of x_2 we need the following points:

$$P_2 = (2,0,0) \quad P_1\,(6,0,4) \quad P\,(4,1,3)$$

$$\overrightarrow{P_2P_1} = \langle (6-2), (0-0), (4-0) \rangle = \langle 4, 0, 4 \rangle$$

$$\overrightarrow{P_2P} = \langle (4-2), (1-0), (3-0) \rangle = \langle 2, 1, 3 \rangle$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 4 & 0 & 4 \\ 2 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix} \bar{i} - \begin{vmatrix} 4 & 4 \\ 2 & 3 \end{vmatrix} \bar{j} \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} \bar{k}$$

$$= (0-4)\bar{i} - (12-8)\bar{j} + (4-0)\bar{k}$$

$$= -4\bar{i} - 4\bar{j} + 4\bar{k}$$

$$\overrightarrow{P_2P_1} \times \overrightarrow{P_2P} = \langle -4, -4, 4 \rangle$$

$$\|\overrightarrow{P_2P_1} \times P_2P\| = \sqrt{(-4)^2 + (-4)^2 + (4)^2}$$

$$= \sqrt{48}$$

$$x_3 = \frac{\sqrt{48}}{2} = 3.467 \text{ or } 2\sqrt{3}$$

$$\lambda_3(x_3) = \frac{2\sqrt{3}}{8\sqrt{3}} = \frac{1}{4}$$

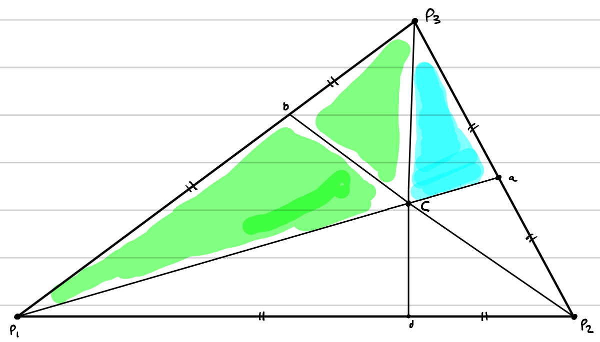
Hence, finally we have gotten the Barycentric Coordinate. If we add the Barycentric coordinates we get the following:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

Also since they are possitive we get to know that our assumption is correct. Hence p is inside the triangle

EXCERCISE 3

Lets visualize the triangle:



lets assume $2\overrightarrow{P_1C} = \overrightarrow{Ca}$

∴ that means the area of P_1P_3C is twice as big as P_3Ca. Hence let x be area for P_3Ca.

$$x = \frac{(P_3 - C) \times (C - a)}{2} \qquad \text{——— } ①$$

Then area for P_3P_1C is as:

$$2x = \frac{(P_3 - C) \times (P_1 - C)}{2} \qquad \text{——— } ②$$

Now comming back to our assumption we would substitute $2(C-a)$ in ②:

$$2x = \frac{(P_3 - C) \times 2(C - a)}{2}$$

$$2x = (P_3 - C) \times (C - a)$$

$$x = \frac{(P_3 - C) \times (C - a)}{2} \quad \text{which equals to } x.$$

EXCERCISE 4

We know that the x would allways need to be 1.
Also we know the following as well:

$$y = mx + b \qquad b = 0$$
$$y = mx$$
$$m = \frac{y}{x}$$

So y' is now equal to m. In the cartetian coordinates we want $\begin{bmatrix} 1 \\ m \end{bmatrix}$.

If convert this to the homogenous coordinates we would get the following:

$$\begin{bmatrix} 1 \\ m \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{y}{x} \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ x \end{bmatrix}$$

Hence after the trans formation we need above.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ x \end{bmatrix}$$

Therefere the tran formation matrix is as follows:

$$m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$