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Assignment 1: Ray-sphere intersection

Exercise 1:

Task 1

$$x = (\sqrt{2}, 1, 0)^T, \quad y = (1,1,1)^T$$

$$\cos\theta = \frac{x \cdot y}{\|x\|\|y\|}$$

$$\therefore \|x\| = \sqrt{(\sqrt{2})^2 + 1^2 + 0^2} = \sqrt{2 + 1 + 0} = \sqrt{3}$$

$$\|y\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Therefore $\cos\theta = \frac{\sqrt{2}+1}{3}$

$$\theta = \cos^{-1}\left[\frac{\sqrt{2}+1}{3}\right] = 36.415 \text{ deg}$$

Task 2

$$x = (\sqrt{2}, 1, 0)$$
$$y = (1, 1, 1)$$

$$x \times y = (z_1, z_2, z_3)$$

Hence $z_1 = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$, $z_2 = -\begin{vmatrix} \sqrt{2} & 1 \\ 0 & 1 \end{vmatrix}$, $z_3 = \begin{vmatrix} \sqrt{2} & 1 \\ 1 & 1 \end{vmatrix}$

$\Rightarrow z_1 = 1 - 0 \qquad z_2 = -(\sqrt{2} - 0) \qquad z_3 = \sqrt{2} - 1$

$$\therefore x \times y = (1, -\sqrt{2}, \sqrt{2} - 1)$$

$$\|x \times y\| = \sqrt{1 + 2 + 3 - 2\sqrt{2}} = \sqrt{6 - 2\sqrt{2}}$$

$$\frac{x \times y}{\|x \times y\|} = \left[\frac{1}{\sqrt{6-2\sqrt{2}}}, \frac{-\sqrt{2}}{\sqrt{6-2\sqrt{2}}}, \frac{\sqrt{2}-1}{\sqrt{6-2\sqrt{2}}}\right]$$

Task 3

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{bmatrix} \qquad z = \left[\frac{1}{\sqrt{6-2\sqrt{2}}}, \frac{-\sqrt{2}}{\sqrt{6-2\sqrt{2}}}, \frac{\sqrt{2}-1}{\sqrt{6-2\sqrt{2}}}\right]$$

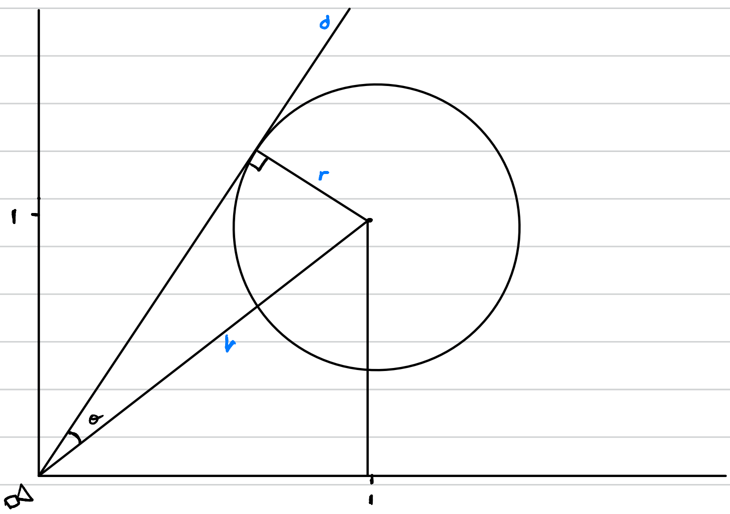
Hence $\frac{1}{\sqrt{6-2\sqrt{2}}} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ -\sqrt{2} \\ \sqrt{2}-1 \end{bmatrix}$

$$\Rightarrow \frac{1}{\sqrt{6-2\sqrt{2}}} \begin{bmatrix} 0 \\ 1-\sqrt{2} \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ \frac{1-\sqrt{2}}{\sqrt{6-2\sqrt{2}}} \\ \frac{2}{\sqrt{6-2\sqrt{2}}} \end{bmatrix}$$

$$\therefore \quad u = \begin{bmatrix} 0 \\ \frac{1-\sqrt{2}}{\sqrt{6-2\sqrt{2}}} \\ \frac{2}{\sqrt{6-2\sqrt{2}}} \end{bmatrix}$$

Excercise 2



$r = \frac{\sqrt{2}}{2}$

$c = (1,1,1)^T$

$b = \sqrt{1+1+1} = \sqrt{3}$

by using pythagoras theorem: $b^2 = r^2 + d^2$

$d = \sqrt{b^2 - r^2}$

$= \sqrt{3 - \frac{2}{4}}$

therefore $\arctan\left[\frac{\frac{\sqrt{2}}{2}}{\sqrt{3-\frac{2}{4}}}\right] = 24.09^\circ$