# Value at Risk (VaR): Understanding and Implementing Market Risk Measurement

## A Practical Guide

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- The volatility of markets reinforces the importance of prudent risk management.
- **Focus**: Market Risk - the risk of loss due to changes in the value of tradable assets.

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- Do not capture diversification effects.
- Cannot easily answer key questions: What's the probability of a certain loss? How is my overall risk?

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- VaR takes account of diversification effects

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- **Correlations ($\rho_{ij}$)**: The relationship between price changes of different assets.
  - Crucial for calculating VaR of a diversified portfolio.

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- **Monte Carlo Simulation:** Uses random price changes to simulate future price changes.

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- Simple and easy to understand.

# Historical Simulation: Implementation Steps

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4. The VAR is the portfolio value corresponding to the desired confidence level percentile.

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- May not capture rare events if the data set is not long enough.

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- Not suitable for non-linear portfolios (options).

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4. Multiply portfolio volatility by a constant relating to the required confidence level (e.g. 1.65 for 95%): $\sigma_p \cdot c$
5. Multiply this value by the portfolio value to arrive at VaR: $\text{VAR} = P \cdot \sigma_p \cdot c$

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- Can be used with spreadsheets to perform simple calculations

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- This assumption makes statistical calculations easier.
- However, real price changes often display ‘fat tails’ and skewness.
- Fat tails mean there are more outliers than expected in a normal distribution.
- Skewness means the distribution is not symmetrical around the mean.

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- VAR cannot cope with extreme price moves or 'shocks'.
- Stress testing is used to examine the effects of extreme price changes on a portfolio.
- Stress testing is often used to evaluate maximum losses on a portfolio, as this is difficult to get from VAR.

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- However, watch out for 'model risk'.
- Use of cash flow or sensitivities are not suitable for all products.

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- It is not a perfect measure; its limitations must be understood.
- Choice of method affects accuracy.
- Different VAR models have different strengths and weaknesses.
- Stress testing is essential to complement VAR and examine extreme market events.

# References


- "*Value at Risk: The New Benchmark for Managing Financial Risk, 3rd Edition*" by Philippe Jorion
- "*Implementing Value At Risk*" by Philip Best