# Introduction to Mathematical Finance A Quantitative Approach to Financial Markets

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- Note: This is an introductory session many topics will be touched on but not covered in detail.

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- **Beyond traditional finance:** It offers rigorous and quantitative methods to analyze complex financial products and strategies.

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- Competitive Advantage: Quantitative skills are highly valued in the financial industry.

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- Credit Risk: Assessing the risk of default on loans and other credit products.

# Understanding the Time Value of Money

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- Further Study: Markov models, Gaussian Processes, Stochastic processes, etc.

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- **Programming:** Python, R and other languages are fundamental tools.

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- Practical experience: Internships and competitions

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- "Methods of Mathematical Finance" by Ioannis Karatzas and Steven
   E. Shreve