# Finite-Time Consensus for Multiagent Systems With Cooperative and Antagonistic Interactions

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***Abstract*— This paper deals with finite-time consensus problems for multiagent systems that are subject to hybrid cooperative and antagonistic interactions. Two consensus protocols are constructed by employing the nearest neighbor rule. It is shown that under the presented protocols, the states of all agents can be guaranteed to reach an agreement in a finite time regarding consensus values that are the same in modulus but may not be the same in sign. In particular, the second protocol can enable all agents to reach a finite-time consensus with a settling time that is not dependent upon the initial states of agents. Simulation results are given to demonstrate the effectiveness and finite-time convergence of the proposed consensus protocols.**

***Index Terms*— Antagonistic interactions, cooperative interactions, distributed protocols, finite-time consensus, multiagent systems.**

## I. Introduction

CONSENSUS is considered as one of the hottest topics in the studies of multiagent systems that consist of a group of mobile agents operating in a cooperative manner to implement tasks for the group [1]–[6]. More detailed discussions on multiagent consensus can be found in [7] and [8]. It is stated that consensus means to drive all agents to agree on a common quantity and is a fundamental problem for coordination control of multiagent systems under certain interaction networks.

For the consensus protocol design of multiagent systems, a practically important problem is how to guarantee that all agents achieve agreement within a finite time, see, e.g., [9]–[18] for nonlinear protocols and [19] and [20] for linear iterative learning protocols. One reason is that finite-time consensus can lead to good system performances in the disturbance rejection and robustness with respect to uncertainty [13]. In fact, increasing the convergence rate plays an important role in the performance evaluation for consensus protocols, which renders finite-time consensus usually required in practical situations. It is worth noticing, however, that the existing finite-time consensus results [9]–[20] can only be applicable to multiagent networks in the presence of cooperative interactions.

Very recently, consensus problems have been studied for multiagent networks subject to antagonistic interactions [21], [22]. As claimed in [21], networks with antagonistic interactions are common especially in the social networks area, and they are represented by a class of signed graphs whose edges can be associated with either positive or negative weights. For such networks, the concept of structural balance/unbalance has been introduced, which is characterized by examining whether or not the agents can be divided into two antagonistic groups such that the weights of edges between agents in the same group are positive, whereas the weights of edges between agents in opposite groups are negative. By taking advantage of structural balance/unbalance of networks with antagonistic interactions, a unified framework has been developed for agents to reach the so-called bipartite consensus where all agents achieve agreement regarding a quantity, which is the same for all agents only in the modulus but not in the sign [21]. In [22], the bipartite consensus results have been extended to higher order multiagent systems. However, to the best of our knowledge, finite-time consensus problems on multiagent networks in the presence of antagonistic interactions have not been studied yet, and this may be of interest as concluded in [21].

In this paper, two nonlinear protocols are constructed for multiagent systems with hybrid cooperative and antagonistic interactions. By considering the Laplacian potential given in [21] for signed graphs, it is shown that all agents can be guaranteed to achieve bipartite consensus in a finite time with the proposed protocols. The multiagent systems are considered under both structurally balanced and structurally unbalanced signed graphs. In particular, the settling time determined when using the second protocol can be independent of the initial states of agents. The protocols considered in this paper can significantly extend the finite-time consensus results for multiagent systems represented as graphs in which all the edges assume nonnegative adjacency weights (see [9]–[18]) to general multiagent systems associated with signed graphs. The established theoretical results are also illustrated via simulation tests.

The remainder of this paper is organized as follows. Problem statements are given for a finite-time consensus on

Manuscript received August 10, 2014; revised January 14, 2015 and April 7, 2015; accepted April 7, 2015. Date of publication May 6, 2015; date of current version March 15, 2016. This work was supported in part by the National Basic Research Program of China (973 Program) under Grant 2012CB821200 and Grant 2012CB821201, in part by the National Natural Science Foundation of China (NSFC) under Grant 61473010, Grant 61134005, Grant 61221061, Grant 61327807, and Grant 61320106006, and in part by the Fundamental Research Funds for the Central Universities.

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Digital Object Identifier 10.1109/TNNLS.2015.2424225

networks with cooperative and antagonistic interactions in Section II. In Section III, two protocols are proposed and their corresponding finite-time bipartite consensus results are established. Simulation results and conclusions are presented in Sections IV and V, respectively.

*Notations:* $\mathscr{I}_n = \{1, 2, \ldots, n\}$, $1_n = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$, $\mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ denotes a diagonal matrix whose diagonal entries are $d_1, d_2, \ldots,$ and $d_n$ and off-diagonal entries are all zero, and $\mathrm{sign}(\cdot)$ is the sign function of any scalar, i.e.,

$$\mathrm{sign}(a) = \begin{cases} 1, & a > 0 \\ 0, & a = 0 \\ -1, & a < 0. \end{cases}$$

## II. Problem Description

Consider networks with $n$ agents, where all agents share the common state space $\mathbb{R}$. Let each agent be regarded as a vertex in a signed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$, where $\mathscr{V} = \{v_1, v_2, \ldots, v_n\}$ is the vertex set, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V} = \{(v_i, v_j) : v_i, v_j \in \mathscr{V}\}$ is the edge set, and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the adjacency matrix of the signed weights of $\mathscr{G}$ such that $a_{ij} \neq 0 \Leftrightarrow (v_j, v_i) \in \mathscr{E}$. Assume that there are no self-loops in $\mathscr{G}$, i.e., $a_{ii} = 0$, $\forall i \in \mathscr{I}_n$. If $\mathscr{G}$ is undirected, then $(v_j, v_i) \in \mathscr{E}$ implies $(v_i, v_j) \in \mathscr{E}$ and $\mathscr{A}^{\mathrm{T}} = \mathscr{A}$, i.e., $\mathscr{A}$ is symmetric. When $(v_j, v_i) \in \mathscr{E}$ is an edge, there exists information flow from $v_j$ to $v_i$, and $v_j$ is a neighbor of $v_i$. The index set of the neighbors of $v_i$ is given by $\mathscr{N}_i = \{j : (v_j, v_i) \in \mathscr{E}\}$. A path in $\mathscr{G}$ is a finite sequence $v_{i_1}, v_{i_2}, \ldots, v_{i_j}$ of distinct vertices such that $(v_{i_l}, v_{i_{l+1}}) \in \mathscr{E}$, $\forall l = 1, 2, \ldots, j-1$. The graph $\mathscr{G}$ is said to be strongly connected if any two vertices can be connected through paths. If $\mathscr{G}$ is undirected, then strong connectivity collapses into connectivity, and consequently $\mathscr{G}$ is said to be connected. By following [21], it is said that a signed graph $\mathscr{G}$ is structurally balanced if there exists a bipartition $\{\mathscr{V}_1, \mathscr{V}_2\}$ of the vertices, where $\mathscr{V}_1 \cup \mathscr{V}_2 = \mathscr{V}$ and $\mathscr{V}_1 \cap \mathscr{V}_2 = \emptyset$, such that $a_{ij} \geq 0$ for $\forall v_i, v_j \in \mathscr{V}_l$ ($l \in \{1, 2\}$) and $a_{ij} \leq 0$ for $\forall v_i \in \mathscr{V}_l$, $v_j \in \mathscr{V}_q$, $l \neq q$ ($l, q \in \{1, 2\}$), and $\mathscr{G}$ is structurally unbalanced otherwise.

This paper is interested in the finite-time consensus problem on multiagent networks described by signed undirected graphs, where $\mathscr{A}$ is symmetric. Consider the agents with the following dynamics:

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathscr{I}_n \tag{1}$$

where $x_i(t)$ is the state and $u_i(t)$ is the protocol (or input). The problem addressed in this paper is to guarantee the agents to reach bipartite finite-time consensus, i.e., there exists a settling time $T \in [0, \infty)$ such that

$$\lim_{t\to T} |x_i(t)| = c \text{ and } |x_i(t)| = c \quad \forall t \geq T, \quad i \in \mathscr{I}_n \tag{2}$$

where $c \geq 0$ is the absolute value of the state about which all agents reach consensus in a finite time.

Let $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$. In the following, the time variable $t$ will be omitted if this does not cause any confusion. Note that the consensus results to be established later can be extended to the agents' system (1) with a vector state space $\mathbb{R}^N$ ($N \geq 1$) by introducing the Kronecker product (see [24] for details). This, however, will be not detailed for simplicity.

## III. Finite-Time Consensus

### A. Laplacian Potential

To develop finite-time consensus results, the Laplacian $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ of a signed graph $\mathscr{G}$ is considered with its elements defined by (see [21, p. 936] for details)

$$l_{ij} = \begin{cases} \sum_{k\in\mathscr{N}_i} |a_{ik}|, & j = i \\ -a_{ij}, & j \neq i \end{cases}$$

which renders $L$ symmetric since $\mathscr{A}$ is symmetric. Thus, the corresponding Laplacian potential can be expressed as

$$\begin{aligned}\Phi(x) &= x^{\mathrm{T}} L x \\ &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} |a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j)^2 \\ &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} |a_{ij}|(x_j - \mathrm{sign}(a_{ij})x_i)^2. \end{aligned} \tag{3}$$

Let $\lambda_1(L) \leq \lambda_2(L) \leq \cdots \leq \lambda_n(L)$ denote all the eigenvalues of $L$. For the Laplacian potential given by (3), the following lemma can be presented.

*Lemma 1:* Consider a connected signed graph $\mathscr{G}$. If $\mathscr{G}$ is structurally balanced, then there exists a diagonal matrix $D = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ with $\sigma_i \in \{1, -1\}$ for all $i \in \mathscr{I}_n$ such that $D\mathscr{A}D$ is nonnegative, and consequently, the following results can be obtained.

1) $\Phi(x)$ is semipositive definite, and $\Phi(x) = 0$ implies $x = D1_n c'$ for some $c' \in \mathbb{R}$.
2) $\lambda_1(L) = 0$, $\lambda_2(L) > 0$, and $\Phi(x) \geq \lambda_2(L)x^{\mathrm{T}}x$ for all $x$ that satisfies $1_n^{\mathrm{T}} D x = 0$.

*Proof:* It is obvious from (3) that $\Phi(x) \geq 0$, $\forall x$ (see also the discussions of [21, eq. (2)]). From [21, Lemma 1], it follows that if $\mathscr{G}$ is structurally balanced, a diagonal matrix $D$ with diagonal elements taking from $\{1, -1\}$ exists that makes $D\mathscr{A}D = [|a_{ij}|] \in \mathbb{R}^{n\times n}$ nonnegative. We have $D^{-1} = D$, and thus

$$DLD = \mathrm{diag}\left\{\sum_{j=1}^{n}|a_{1j}|, \sum_{j=1}^{n}|a_{2j}|, \ldots, \sum_{j=1}^{n}|a_{nj}|\right\} - D\mathscr{A}D.$$

Because $(D\mathscr{A}D)1_n = [\sum_{j=1}^{n}|a_{1j}|, \sum_{j=1}^{n}|a_{2j}|, \ldots, \sum_{j=1}^{n}|a_{nj}|]^{\mathrm{T}}$, we can derive

$$\begin{aligned}(DLD)1_n = &\left[\sum_{j=1}^{n}|a_{1j}|, \sum_{j=1}^{n}|a_{2j}|, \ldots, \sum_{j=1}^{n}|a_{nj}|\right]^{\mathrm{T}} \\ &- (D\mathscr{A}D)1_n = 0.\end{aligned}$$

Due to the results of [7] for connected graphs with nonnegative edge weights, we can further obtain $\lambda_1(DLD) = 0$ corresponding to the eigenvector $1_n$ and $\lambda_n(DLD) \geq \cdots \geq \lambda_2(DLD) > 0$. This implies that the null space of $DLD$ is spanned by $1_n$. In addition, we can consider $D^2 = I$ to deduce

$$\Phi(x) = x^{\mathrm{T}}Lx = x^{\mathrm{T}}D^2LD^2x = (Dx)^{\mathrm{T}}(DLD)(Dx). \tag{4}$$

Hence, we can see that $\Phi(x) = 0$ implies $Dx = 1_n c'$ for some $c' \in \mathbb{R}$, which consequently yields result 1) of Lemma 1. Again by [21, Lemma 1], we have $\lambda_1(L) = 0$ and $\lambda_2(L) > 0$. Since $DLD$ shares the same eigenvalues with $L$, their second smallest eigenvalues are equal, i.e., $\lambda_2(DLD) = \lambda_2(L)$. Thus, it follows immediately from (4) that if $1_n^{\mathrm{T}} Dx = 0$, then [7]:

$$\Phi(x) \geq \lambda_2(DLD)(Dx)^{\mathrm{T}}(Dx) = \lambda_2(L)x^{\mathrm{T}}D^2x = \lambda_2(L)x^{\mathrm{T}}x$$

which leads to result 2) of Lemma 1. ■

In addition to Lemma 1, the following lemma gives properties of $\Phi(x)$ under structurally unbalanced signed graphs.

*Lemma 2:* Consider a connected signed graph $\mathscr{G}$. If $\mathscr{G}$ is structurally unbalanced, then the following results hold.

1) $\Phi(x)$ is positive definite, and $\Phi(x) = 0$ implies $x = 0$.
2) $\lambda_1(L) > 0$, and $\Phi(x) \geq \lambda_1(L)x^{\mathrm{T}}x$ for all $x$.

*Proof:* The proof of this lemma follows directly from [21, Corollary 2], and is omitted here. ■

It can also be deduced from the symmetry of $\mathscr{A}$ that

$$\frac{\partial \Phi(x)}{\partial x_i} = -2\sum_{j=1}^{n} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i), \quad i \in \mathscr{I}_n$$

which can be used to further derive that the derivative of $\Phi(x)$, denoted by $\dot{\Phi}(x)$, along the trajectories of (1) satisfies

$$\begin{aligned}\dot{\Phi}(x) &= \sum_{i=1}^{n} \frac{\partial \Phi(x)}{\partial x_i}\dot{x}_i \\ &= -2\sum_{i=1}^{n}\left(\sum_{j=1}^{n} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)u_i \\ &= -2\sum_{i=1}^{n}\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)u_i. \qquad (5)\end{aligned}$$

By Lemmas 1 and 2, it follows that if the protocol $u_i$ can be designed such that a settling time $T \in [0, \infty)$ can be determined to ensure the finite-time stability of $\Phi(x)$ based on (5), i.e.,

$$\lim_{t\to T} \Phi(x) = 0 \text{ and } \Phi(x) = 0 \quad \forall t \geq T \qquad (6)$$

then the finite-time consensus objective (2) can be guaranteed. Motivated by this observation, two protocols will be designed to achieve the finite-time consensus objective (2) by considering the finite-time stability (6) of the Laplacian potential in Section III-B.

### *B. Consensus Results*

For the finite-time consensus objective (2), a protocol is first proposed as

$$\begin{aligned}u_i = {}& \mathrm{sign}\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right) \\ & \times \left|\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right|^{\alpha}, \quad i \in \mathscr{I}_n \qquad (7)\end{aligned}$$

where $0 < \alpha < 1$. If $\alpha = 1$, then $\dot{x} = -Lx$ follows under the action of (7), which was studied in [21] for consensus of networks with antagonistic interactions. Now, if (7) with $0 < \alpha < 1$ is applied, then the following finite-time consensus results can be established.

*Theorem 1:* Consider the multiagent system (1) associated with a connected signed graph $\mathscr{G}$, and let the protocol (7) be applied. If $\mathscr{G}$ is structurally balanced, then the bipartite finite-time consensus objective (2) holds with $c > 0$, where the settling time $T$ is bounded by

$$T \leq \frac{(\Phi(x(0)))^{\frac{1-\alpha}{2}}}{(1-\alpha)(\lambda_2(L))^{\frac{1+\alpha}{2}}}. \qquad (8)$$

Otherwise, if $\mathscr{G}$ is structurally unbalanced, then (2) holds with $c = 0$, where

$$T \leq \frac{(\Phi(x(0)))^{\frac{1-\alpha}{2}}}{(1-\alpha)(\lambda_1(L))^{\frac{1+\alpha}{2}}}. \qquad (9)$$

*Remark 1:* From Theorem 1, it is obvious that one can solve consensus problems for multiagent networks with antagonistic interactions in finite time by modifying the protocol proposed in [21]. This can improve consensus performance of networked multiagent systems associated with signed graphs by benefiting from good properties of the finite-time consensus. One can see from (8) and (9) that the settling time $T$ depends on the initial state $x(0)$, which is the same as classical finite-time consensus results in [9]–[16]. In addition, we can choose the parameter $\alpha$ in the protocol (7) arbitrarily within the interval (0, 1). However, (8) and (9) imply that $\alpha$ affects the settling time, and thus we should take this fact into account to choose $\alpha \in (0, 1)$ such that we can achieve the specified finite-time consensus performance. In fact, to further improve the convergence speed of (7), we can add a positive feedback gain $\phi > 0$ and present

$$\begin{aligned}u_i = {}& \phi\,\mathrm{sign}\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right) \\ & \times \left|\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right|^{\alpha}, \quad i \in \mathscr{I}_n\end{aligned}$$

for which the settling time $T$ is bounded by

$$T \leq \begin{cases} \dfrac{(\Phi(x(0)))^{\frac{1-\alpha}{2}}}{\phi(1-\alpha)(\lambda_2(L))^{\frac{1+\alpha}{2}}}, & \text{if } \mathscr{G} \text{ is structurally balanced} \\ \dfrac{(\Phi(x(0)))^{\frac{1-\alpha}{2}}}{\phi(1-\alpha)(\lambda_1(L))^{\frac{1+\alpha}{2}}}, & \text{if } \mathscr{G} \text{ is structurally unbalanced.}\end{cases}$$

It is easy to see that $T$ can be determined with smaller bounds than those of (8) and (9) by choosing appropriate values of $\phi$.

To show Theorem 1, we need two useful lemmas as follows.

*Lemma 3 [9], [17]:* Given any $\xi_i \geq 0$ for $i \in \mathscr{I}_n$, if $0 < \delta \leq 1$, then

$$\sum_{i=1}^{n} \xi_i^{\delta} \geq \left(\sum_{i=1}^{n} \xi_i\right)^{\delta}.$$

*Lemma 4:* Consider a connected signed graph $\mathscr{G}$. If $\mathscr{G}$ is structurally balanced, then for all $x \in \mathbb{R}^n$

$$x^{\mathrm{T}} L^{\mathrm{T}} L x \geq \lambda_2(L) \Phi(x). \tag{10}$$

*Proof:* Since $\mathscr{G}$ is connected and structurally balanced, $L$ is semipositive definite from result 1) of Lemma 1. Hence, there exists a unique semipositive definite matrix $M$ such that $L = M^{\mathrm{T}} M = M^2$ [23]. This, together with $M = M^{\mathrm{T}}$ and $(DLD)1_n = 0$, implies

$$\begin{aligned}\left\|1_n^{\mathrm{T}} D M\right\|^2 &= \left(1_n^{\mathrm{T}} D M\right)\left(1_n^{\mathrm{T}} D M\right)^{\mathrm{T}} = 1_n^{\mathrm{T}} (D M^2 D) 1_n \\ &= 1_n^{\mathrm{T}} (DLD) 1_n = 0\end{aligned}$$

which leads to $1_n^{\mathrm{T}} D M = 0$. Further, we have $1_n^{\mathrm{T}} D(Mx) = 0$ for all $x \in \mathbb{R}^n$. It also follows from result 2) of Lemma 1 that $y^{\mathrm{T}} L y \geq \lambda_2(L) y^{\mathrm{T}} y$ for all $y \in \mathbb{R}^n$ such that $1_n^{\mathrm{T}} D y = 0$. Using these two facts, one can deduce that for all $x \in \mathbb{R}^n$

$$\begin{aligned}x^{\mathrm{T}} L^{\mathrm{T}} L x &= x^{\mathrm{T}} M^{\mathrm{T}} M M^{\mathrm{T}} M x = (Mx)^{\mathrm{T}} M^2 (Mx) \\ &= (Mx)^{\mathrm{T}} L (Mx) \\ &\geq \lambda_2(L) (Mx)^{\mathrm{T}} (Mx) = \lambda_2(L) x^{\mathrm{T}} L x = \lambda_2(L) \Phi(x)\end{aligned}$$

i.e., (10) holds. The proof is complete. ■

*Proof of Theorem 1:* In view of (5), if the derivative of $\Phi(x)$ is computed along the trajectories of (1), then one can use (7) to obtain

$$\begin{aligned}\dot{\Phi}(x) &= -2 \sum_{i=1}^{n} \left| \sum_{j=1}^{n} a_{ij} (x_j - \operatorname{sign}(a_{ij}) x_i) \right|^{1+\alpha} \\ &= -2 \sum_{i=1}^{n} \left( \left( \sum_{j=1}^{n} a_{ij} (x_j - \operatorname{sign}(a_{ij}) x_i) \right)^2 \right)^{\frac{1+\alpha}{2}}.\end{aligned} \tag{11}$$

From [21, eq. (3)], we can easily see

$$\begin{aligned}&\begin{bmatrix} \sum_{j=1}^{n} a_{1j}(x_j - \operatorname{sign}(a_{1j}) x_i) \\ \sum_{j=1}^{n} a_{2j}(x_j - \operatorname{sign}(a_{2j}) x_i) \\ \vdots \\ \sum_{j=1}^{n} a_{nj}(x_j - \operatorname{sign}(a_{nj}) x_i) \end{bmatrix} \\ &\quad = - \begin{bmatrix} \sum_{j=1}^{n} |a_{1j}|(x_i - \operatorname{sign}(a_{1j}) x_j) \\ \sum_{j=1}^{n} |a_{2j}|(x_i - \operatorname{sign}(a_{2j}) x_j) \\ \vdots \\ \sum_{j=1}^{n} |a_{nj}|(x_i - \operatorname{sign}(a_{nj}) x_j) \end{bmatrix} \\ &\quad = -Lx\end{aligned}$$

which can be inserted to deduce

$$\begin{aligned}&\sum_{i=1}^{n} \left( \sum_{j=1}^{n} a_{ij} (x_j - \operatorname{sign}(a_{ij}) x_i) \right)^2 \\ &= \begin{bmatrix} \sum_{j=1}^{n} a_{1j}(x_j - \operatorname{sign}(a_{1j}) x_i) \\ \sum_{j=1}^{n} a_{2j}(x_j - \operatorname{sign}(a_{2j}) x_i) \\ \vdots \\ \sum_{j=1}^{n} a_{nj}(x_j - \operatorname{sign}(a_{nj}) x_i) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \sum_{j=1}^{n} a_{1j}(x_j - \operatorname{sign}(a_{1j}) x_i) \\ \sum_{j=1}^{n} a_{2j}(x_j - \operatorname{sign}(a_{2j}) x_i) \\ \vdots \\ \sum_{j=1}^{n} a_{nj}(x_j - \operatorname{sign}(a_{nj}) x_i) \end{bmatrix} \\ &= (-Lx)^{\mathrm{T}} (-Lx) = x^{\mathrm{T}} L^{\mathrm{T}} L x.\end{aligned} \tag{12}$$

Due to $0 < \alpha < 1$, $0 < (1+\alpha)/2 < 1$ follows. Thus, by Lemma 3, (11) and (12) can be employed to derive

$$\begin{aligned}\dot{\Phi}(x) &\leq -2 \left( \sum_{i=1}^{n} \left( \sum_{j=1}^{n} a_{ij} (x_j - \operatorname{sign}(a_{ij}) x_i) \right)^2 \right)^{\frac{1+\alpha}{2}} \\ &= -2 (x^{\mathrm{T}} L^{\mathrm{T}} L x)^{\frac{1+\alpha}{2}}.\end{aligned} \tag{13}$$

Note that $\mathscr{G}$ is connected. If $\mathscr{G}$ is structurally balanced, then we have (10) by Lemma 4, which together with (13) leads to

$$\dot{\Phi}(x) \leq -2 (\lambda_2(L))^{\frac{1+\alpha}{2}} (\Phi(x))^{\frac{1+\alpha}{2}}. \tag{14}$$

Based on (14), one can further apply the comparison principle of differential equations [25] to obtain

$$(\Phi(x))^{\frac{1-\alpha}{2}} \leq -(1-\alpha)(\lambda_2(L))^{\frac{1+\alpha}{2}} t + (\Phi(x(0)))^{\frac{1-\alpha}{2}}, \quad t < T \tag{15}$$

which due to $\Phi(x) \geq 0$ yields (6), where $T$ satisfies (8). Again using result 1) of Lemma 1, one can deduce from (6) that (2) holds with $c = |c'| > 0$ for some $c' \in \mathbb{R}$ such that $x = D1_n c'$ due to $\Phi(x) = 0$.

If $\mathscr{G}$ is structurally unbalanced, then similarly to (10), one can obtain from results 1) and 2) of Lemma 2 that

$$\begin{aligned}x^{\mathrm{T}} L^{\mathrm{T}} L x = \left(L^{\frac{1}{2}} x\right)^{\mathrm{T}} L \left(L^{\frac{1}{2}} x\right) &\geq \lambda_1(L) \left(L^{\frac{1}{2}} x\right)^{\mathrm{T}} \left(L^{\frac{1}{2}} x\right) \\ &= \lambda_1(L) x^{\mathrm{T}} L x \\ &= \lambda_1(L) \Phi(x).\end{aligned} \tag{16}$$

As a consequence, by the comparison principle of differential equations [25] based on (13) and (16), one can derive that

$$(\Phi(x))^{\frac{1-\alpha}{2}} \leq -(1-\alpha)(\lambda_1(L))^{\frac{1+\alpha}{2}} t + (\Phi(x(0)))^{\frac{1-\alpha}{2}}, \quad t < T \tag{17}$$

and thus (6) can be also established, where the estimation of $T$ is given by (9). With result 1) of Lemma 2 and by (6), it follows that (2) holds with $c = 0$. The proof is complete. ■

*Remark 2:* If $\alpha = 1$ in (7), then one can solve (14) to obtain $\Phi(x) \leq e^{-2\lambda_2(L)t} \Phi(x(0))$ if $\mathscr{G}$ is structurally balanced. In addition, it leads to $\Phi(x) \leq e^{-2\lambda_1(L)t} \Phi(x(0))$ if $\alpha = 1$ and $\mathscr{G}$ is structurally unbalanced. By Lemmas 1 and 2, one can see that the analysis of the Laplacian potential performed in the proof of Theorem 1 is an alternative way to achieve consensus of networks with antagonistic interactions given in [21] and further can develop exponentially convergent consensus results.

To improve the performance of protocol (7) by removing the dependence of the settling time on the initial states of agents (see Remark 1), a different type of protocol is proposed as

$$u_i = \beta\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^{\frac{m}{r}} + \gamma\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^{\frac{p}{q}}, \quad i \in \mathscr{I}_n \tag{18}$$

where $\beta > 0$, $\gamma > 0$, and $m$, $r$, $p$, and $q$ are positive odd integers such that $m > r$ and $q > p$. The finite-time consensus results for the protocol (18) are presented in the following theorem.

*Theorem 2:* Consider the multiagent system (1) associated with a connected signed graph $\mathscr{G}$, and let the protocol (18) be applied. If $\mathscr{G}$ is structurally balanced, then the bipartite finite-time consensus objective (2) holds with $c > 0$, where the settling time $T$ satisfies

$$T \le \frac{1}{\lambda_2(L)}\left(\frac{n^{\frac{m-r}{2r}}}{\beta}\frac{r}{m-r} + \frac{1}{\gamma}\frac{q}{q-p}\right). \tag{19}$$

Otherwise, if $\mathscr{G}$ is structurally unbalanced, then (2) holds with $c = 0$, where

$$T \le \frac{1}{\lambda_1(L)}\left(\frac{n^{\frac{m-r}{2r}}}{\beta}\frac{r}{m-r} + \frac{1}{\gamma}\frac{q}{q-p}\right). \tag{20}$$

*Remark 3:* Theorem 2 implies that also by applying the protocol (18), one can solve the finite-time consensus problems for multiagent networks with antagonistic interactions. Moreover, when comparing (19) and (20) with (8) and (9), one can notice that the settling time given for (18) is independent of the initial states of agents. In this sense, (18) can improve the consensus performance with respect to the protocol (7). In addition, we can arbitrarily choose positive scalars $\beta > 0$ and $\gamma > 0$, and positive odd integers $m$, $r$, $p$, and $q$ satisfying $m > r$ and $q > p$ for the protocol (18). Because the settling time can be estimated in terms of these designing parameters, we should choose them to make the settling time bounded by the specified bound in order to achieve the desired finite-time consensus performance.

In addition to Lemmas 1–4, one needs two more helpful lemmas to establish Theorem 2, which are given as follows.

*Lemma 5 [17], [18]:* Given any $\xi_i \ge 0$ for $i \in \mathscr{I}_n$, if $\delta > 1$, then

$$\sum_{i=1}^n \xi_i^\delta \ge n^{1-\delta}\left(\sum_{i=1}^n \xi_i\right)^\delta.$$

*Lemma 6 [18]:* For a nonlinear scalar system

$$\dot z = -\beta z^{\frac{m}{r}} - \gamma z^{\frac{p}{q}}$$

where $\beta > 0$, $\gamma > 0$, and $m$, $r$, $p$, and $q$ are positive odd integers, if $m > r$ and $q > p$, then for any initial state $z(0)$, $\lim_{t\to T} z(t) = 0$ and $z(t) = 0$, $\forall t \ge T$, where $T$ is a settling time satisfying

$$T \le \frac{1}{\beta}\frac{r}{m-r} + \frac{1}{\gamma}\frac{q}{q-p}.$$

*Proof of Theorem 2:* If the protocol (18) is applied, then based on (5), the derivative of $\Phi(x)$ along the trajectories of (1) is given by

$$\begin{aligned}\dot\Phi(x) &= -2\sum_{i=1}^n\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)\\&\quad\times\left(\beta\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^{\frac{m}{r}} + \gamma\left(\sum_{j\in\mathscr{N}_i} a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^{\frac{p}{q}}\right)\\&= -2\beta\sum_{i=1}^n\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^{\frac{m+r}{r}}\\&\quad -2\gamma\sum_{i=1}^n\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^{\frac{p+q}{q}}\\&= -2\beta\sum_{i=1}^n\left(\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^2\right)^{\frac{m+r}{2r}}\\&\quad -2\gamma\sum_{i=1}^n\left(\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^2\right)^{\frac{p+q}{2q}}\end{aligned} \tag{21}$$

where the property that $m$, $r$, $p$, and $q$ are positive odd integers is used. Due to $m > r$ (thus, $(m+r)/(2r) > 1$), it follows from Lemma 5 that

$$\begin{aligned}&\sum_{i=1}^n\left(\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^2\right)^{\frac{m+r}{2r}}\\&\ge n^{\frac{r-m}{2r}}\left(\sum_{i=1}^n\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^2\right)^{\frac{m+r}{2r}}.\end{aligned} \tag{22}$$

Similarly, since $q > p$ implies $0 < (p+q)/(2q) < 1$, the use of Lemma 3 leads to

$$\begin{aligned}&\sum_{i=1}^n\left(\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^2\right)^{\frac{p+q}{2q}}\\&\ge \left(\sum_{i=1}^n\left(\sum_{j=1}^n a_{ij}(x_j - \mathrm{sign}(a_{ij})x_i)\right)^2\right)^{\frac{p+q}{2q}}.\end{aligned} \tag{23}$$

By inserting (12), (22), and (23) into (21), one can deduce

$$\begin{aligned}\dot{\Phi}(x) &\le -2\beta n^{\frac{r-m}{2r}}\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{n}a_{ij}(x_j-\operatorname{sign}(a_{ij})x_i)\right)^2\right)^{\frac{m+r}{2r}}\\ &\quad -2\gamma\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{n}a_{ij}(x_j-\operatorname{sign}(a_{ij})x_i)\right)^2\right)^{\frac{p+q}{2q}}\\ &= -2\beta n^{\frac{r-m}{2r}}(x^{\mathrm{T}}L^{\mathrm{T}}Lx)^{\frac{m+r}{2r}}-2\gamma(x^{\mathrm{T}}L^{\mathrm{T}}Lx)^{\frac{p+q}{2q}}. \qquad (24)\end{aligned}$$

If $\mathscr{G}$ is structurally balanced, then inserting (10) into (24) gives

$$\dot{\Phi}(x) \le -2\beta n^{\frac{r-m}{2r}}(\lambda_2(L)\Phi(x))^{\frac{m+r}{2r}}-2\gamma(\lambda_2(L)\Phi(x))^{\frac{p+q}{2q}}. \qquad (25)$$

Let $\Psi(x)=(\lambda_2(L)\Phi(x))^{1/2}$. Using (25), one has

$$\begin{aligned}\dot{\Psi}(x) &\triangleq \frac{d\Psi(x)}{dt}\\ &\le \frac{\lambda_2(L)}{2\Psi(x)}\left(-2\beta n^{\frac{r-m}{2r}}(\Psi(x))^{\frac{m+r}{r}}-2\gamma(\Psi(x))^{\frac{p+q}{q}}\right)\\ &= \lambda_2(L)\left(-\beta n^{\frac{r-m}{2r}}(\Psi(x))^{\frac{m}{r}}-\gamma(\Psi(x))^{\frac{p}{q}}\right). \qquad (26)\end{aligned}$$

Based on (26), by applying Lemma 6 to the following system:

$$\dot{\hat{z}}=\lambda_2(L)\left(-\beta n^{\frac{r-m}{2r}}\hat{z}^{\frac{m}{r}}-\gamma\hat{z}^{\frac{p}{q}}\right),\quad \hat{z}(0)\ge\Psi(x(0)) \qquad (27)$$

one can obtain $\lim_{t\to T}\hat{z}(t)=0$ and $\hat{z}(t)=0$ for $\forall t\ge T$, where $T$ is a settling time satisfying (19). By applying the comparison principle of differential equations [25] for the systems (26) and (27), $\Psi(x(t))\le\hat{z}(t)$ for all $t$ can be obtained, which leads to

$$\lim_{t\to T}\Psi(x)=0 \text{ and } \Psi(x)=0 \quad \forall t\ge T \qquad (28)$$

where $T$ is bounded by (19). Due to $\Phi(x)=(1/\lambda_2(L))(\Psi(x))^2$, it follows that (6) can be equivalently obtained based on (28). This, together with result 1) of Lemma 1, guarantees that (2) holds with $c=|c'|>0$ for some $c'\in\mathbb{R}$ such that $x=D1_nc'$ due to $\Phi(x)=0$.

Otherwise, if $\mathscr{G}$ is structurally unbalanced, then combining (16) with (24) leads to

$$\dot{\Phi}(x)\le -2\beta n^{\frac{r-m}{2r}}(\lambda_1(L)\Phi(x))^{\frac{m+r}{2r}}-2\gamma(\lambda_1(L)\Phi(x))^{\frac{p+q}{2q}}. \qquad (29)$$

With (29) and in the same way as in the above structurally balanced case, one can develop from Lemma 6 that (6) holds, where the settling time $T$ is estimated by (20). By noting this fact, result 1) of Lemma 2 can be applied to obtain that (2) holds with $c=0$. This completes the proof. ■

*Remark 4:* By Theorems 1 and 2, one can see that the finite-time consensus problems on signed networks can be solved in both cases where the associated graph for multiagent systems is structurally balanced or structurally unbalanced. This benefits from the Laplacian potential defined for signed graphs [21]. Moreover, it makes the classical finite-time consensus results on networks with all nonnegative edge weights in, e.g., [9]–[18] possible to be generalized to signed networks. In fact, if there are no negative edges in our considered graph, then it can still be viewed as the case where the graph is structurally balanced. Consequently, in this sense, the developed results of this paper are more general, which can include the existing finite-time consensus results on networks with all nonnegative edge weights in, e.g., [9]–[18] as special cases.

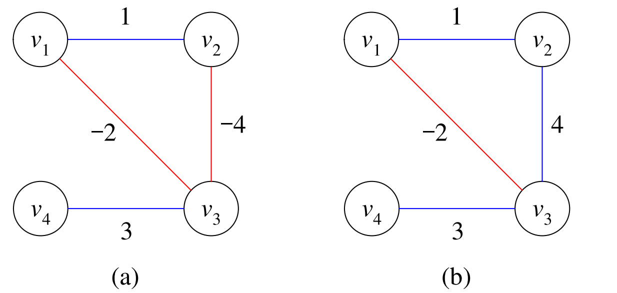


Fig. 1. Signed graphs. (a) Structurally balanced graph. (b) Structurally unbalanced graph.

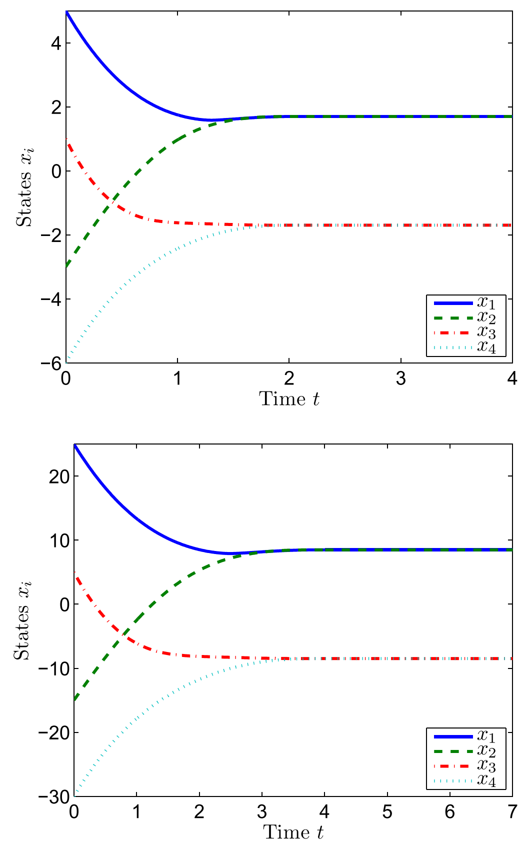


Fig. 2. Finite-time consensus of the protocol (7) under the structurally balanced graph in Fig. 1(a). Top: $x(0)=[5,-3,1,-6]^{\mathrm{T}}$. Bottom: $x(0)=[25,-15,5,-30]^{\mathrm{T}}$.

## IV. Simulation Illustrations

In this section, the simulations are given to illustrate the derived finite-time consensus results under signed networks. Consider two signed graphs in Fig. 1 to describe the interactions between agents. Clearly, the graph in Fig. 1(a) is connected and structurally balanced, whereas the graph

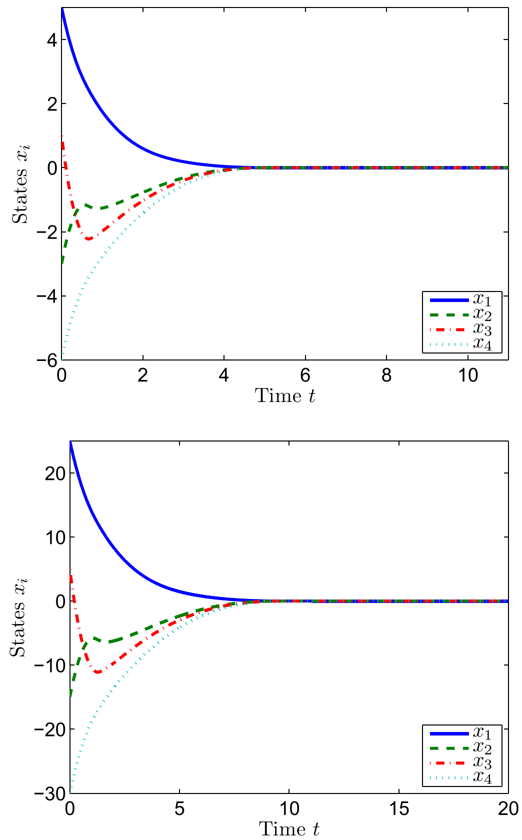


Fig. 3. Finite-time stability of the protocol (7) under the structurally unbalanced graph in Fig. 1(b). Top: $x(0) = [5, -3, 1, -6]^{\mathrm{T}}$. Bottom: $x(0) = [25, -15, 5, -30]^{\mathrm{T}}$.

TABLE I

SETTLING TIME OF PROTOCOL (7) IN DIFFERENT CASES

| $T$ $(\leq)$ | $x(0) = [5, -3, 1, -6]^{\mathrm{T}}$ | $x(0) = [25, -15, 5, -30]^{\mathrm{T}}$ |
|---|---|---|
| Graph in Fig.1(a) | 3.4078 | 6.4872 |
| Graph in Fig.1(b) | 10.3506 | 19.7040 |

in Fig. 1(b) is connected but structurally unbalanced. The initial states of agents are considered with two cases.

1) $x(0) = [5, -3, 1, -6]^{\mathrm{T}}$.
2) $x(0) = [25, -15, 5, -30]^{\mathrm{T}}$.

To perform simulations with the protocol (7), the parameter $\alpha = 0.6$ is chosen. It follows from Theorem 1 that the settling time can be estimated by (8) and (9) for the signed graphs in Figs. 1(a) and 1(b), respectively. In Table I, the estimation of the settling time is given. It is obvious that $T$ heavily depends on the initial states of the agents, regardless of whether a structurally balanced or structurally unbalanced signed graph is under consideration. In Figs. 2 and 3, the evolution of the states of all agents with respect to time is shown. It can be seen in Fig. 2 that the bipartite consensus of agents can be achieved in a finite time, and from Fig. 3 that the states of agents can be guaranteed to converge to zero in a finite time. Moreover, the finite-time consensus performance of Figs. 2 and 3 is in accordance with the computation results, which are presented in Table I.

TABLE II

SETTLING TIME OF PROTOCOL (18) IN DIFFERENT CASES

| $T$ $(\leq)$ | $x(0) = [5, -3, 1, -6]^{\mathrm{T}}$ | $x(0) = [25, -15, 5, -30]^{\mathrm{T}}$ |
|---|---|---|
| Graph in Fig.1(a) | 1.1983 | 1.1983 |
| Graph in Fig.1(b) | 4.6295 | 4.6295 |

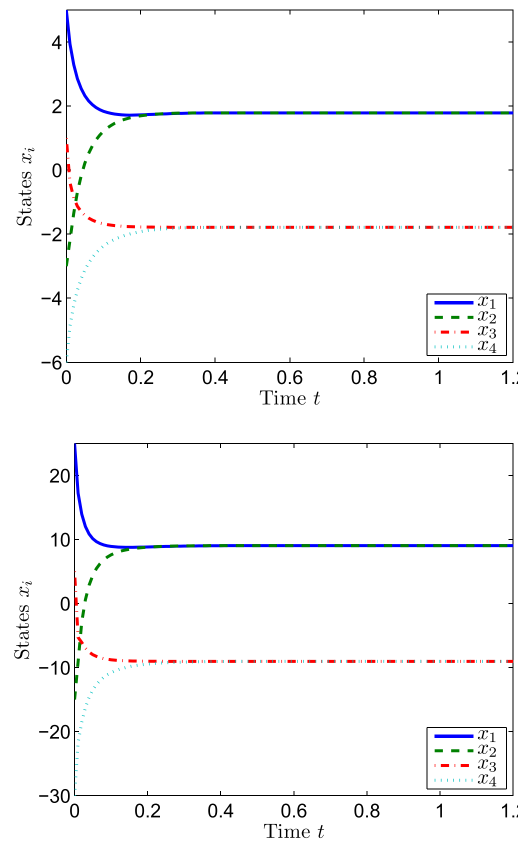


Fig. 4. Finite-time consensus of the protocol (18) under the structurally balanced graph in Fig. 1(a). Top: $x(0) = [5, -3, 1, -6]^{\mathrm{T}}$. Bottom: $x(0) = [25, -15, 5, -30]^{\mathrm{T}}$.

Next, the protocol (18) is applied using $\beta = 2$, $\gamma = 2$, $m = 9$, $r = 7$, $p = 3$, and $q = 5$. From Theorem 2, (19) and (20) can be employed to compute the bound of the settling time, where the results are given in Table II. It can be obviously observed from this table that the settling time does not depend upon the initial states of agents regardless of them operating on the structurally balanced graph in Fig. 1(a) or the structurally unbalanced graph in Fig. 1(b). To illustrate this observation, the simulations are also performed with the protocol (18) whose test results are shown in Figs. 4 and 5. The two figures obviously show that when applying the protocol (18), one can enable all agents to achieve the bipartite consensus under the structurally balanced graph in Fig. 1(a) and achieve the state convergence to zero under the structurally unbalanced graph in Fig. 1(b) with a fixed finite settling time in the presence of different initial states of agents.

Moreover, if one compares Tables I and II, then one can note that the settling time of the protocol (18) is much smaller than that of (7) especially when the magnitude of the agents' initial states is large. This discloses a better consensus performance

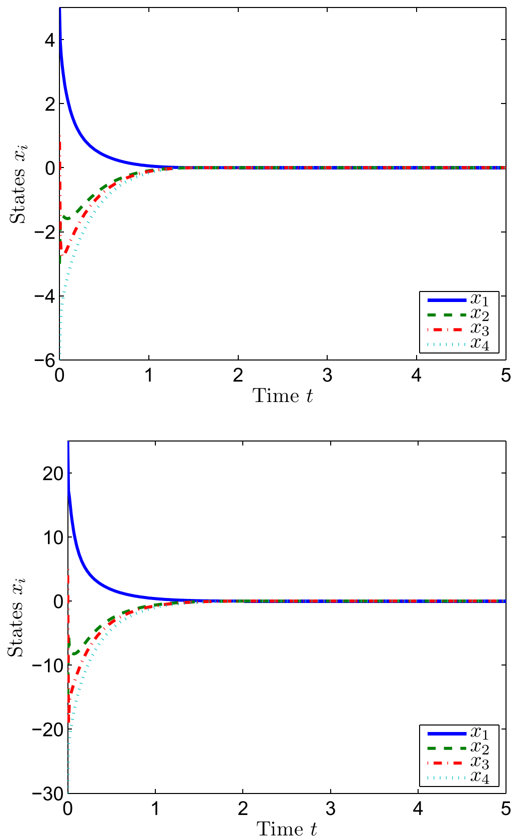


Fig. 5. Finite-time stability of the protocol (18) under the structurally unbalanced graph in Fig. 1(b). Top: $x(0) = [5, -3, 1, -6]^{\mathrm{T}}$. Bottom: $x(0) = [25, -15, 5, -30]^{\mathrm{T}}$.

of the protocol (18) relative to (7) in the sense of convergence rate, which can also be observed from comparing Figs. 4 and 5 with Figs. 2 and 3. In addition, when comparing Figs. 2–5 with [21, Fig. 2], one can observe that the signed multiagent networks under the proposed protocols (7) and (18) can indeed achieve a faster convergence rate than those under the classical feedback laws using $L$ (i.e., $\dot{x} = -Lx$ considered in [21]).

## V. Conclusion

In this paper, we have considered how to solve the proposed consensus problems for multiagent systems under signed networks of [21] within a finite time. Two classes of finite-time consensus protocols have been investigated by analyzing the Laplacian potential of signed networks, where in particular the second one makes the settling time not dependent on the initial states of agents. In addition, the finite-time consensus performances of signed networks have been illustrated with simulations.

Connected signed graphs are only considered. With slightly modifying the proposed protocols, one may extend finite-time consensus results to signed directed graphs considered in [21]. In addition, one may make extensions of the finite-time consensus protocols proposed in this paper by considering external disturbances. For example, if $\dot{x}_i = u_i + w_i$ is considered instead of (1), where $w_i$ is external disturbance such that $|w_i| \leq b_w$ for $i \in \mathscr{I}_n$ and a constant $b_w \geq 0$, then one can improve protocols (7) and (18) for $i \in \mathscr{I}_n$, respectively, as

$$\begin{aligned} u_i &= \operatorname{sign}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(x_j - \operatorname{sign}(a_{ij})x_i)\right) \\ &\quad\times \left|\sum_{j\in\mathcal{N}_i} a_{ij}(x_j - \operatorname{sign}(a_{ij})x_i)\right|^{\alpha} \\ &\quad+ \tau \operatorname{sign}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(x_j - \operatorname{sign}(a_{ij})x_i)\right) \\ u_i &= \beta\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_j - \operatorname{sign}\left(a_{ij}\right)x_i\right)\right)^{\frac{m}{r}} \\ &\quad+ \gamma\left(\sum_{j\in\mathcal{N}_i} a_{ij}(x_j - \operatorname{sign}(a_{ij})x_i)\right)^{\frac{p}{q}} \\ &\quad+ \tau \operatorname{sign}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(x_j - \operatorname{sign}(a_{ij})x_i)\right) \end{aligned}$$

where $\tau \geq b_w$. It is not difficult to validate that with the above two protocols, we can also achieve the finite-time consensus objective (2) in the same way as in the derivations of Theorems 1 and 2.

## Acknowledgment

The authors would like to thank the Associate Editor and three anonymous reviewers for their insightful comments and suggestions that improved the quality and presentation of this paper.

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