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Output Bipartite Consensus of Heterogeneous Linear Multi-Agent Systems Under Switching Topologies

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Abstract: This paper considers bipartite consensus of linear multi-agent systems over signed directed graphs, where both collaboration and competition coexist. The agent dynamics are distinct and can all have different dimensions. The communication graph can be time-varying and need not to be connected at each time. A distributed control law is proposed which enables the group to split into two disjoint subgroups, and the outputs of all agents within each subgroup achieve consensus, but the two subgroups move oppositely towards each other. Rigorous analysis and a simulation example are provided.

Key Words: Heterogeneous linear multi-agent system, Output bipartite consensus, Signed graph, Switching topology

1 Introduction

In recent years, distributed control of multi-agents systems has been intensively studied, due to its broad applications in many scenarios such as sensor networks, social networks, biological systems (see [1–5], and references therein). It is worth noting that the majority works assume interactions between agents to be collaborative. In this setup, a fundamental problem is consensus, which means all agents in a group agree to a certain quantities of interest. While there are still many interesting problems open in the topic of cooperative control of multi-agent system, another relatively new topic starts to draw attention in the control community, i.e., distributed control of multi-agent systems where both collaboration and competition coexist (see for example, [6–12], etc.). In these cases, signed graph is used to describe the interaction between agents, where a positive edge stands for collaboration, and a negative edge competition.

For multi-agent system over signed graphs, Altafini [6] first studied the bipartite consensus problem, i.e., under certain control law, the whole group of agents evolve into two subgroups, and all agents within each subgroup achieve a conventional consensus, and these two subgroups moving oppositely toward each other. The agent dynamics is extended from the single-integrator [6, 7] to linear time-invariant (LTI) system with single input [8], and further to more general multi-input multi-output LTI systems [9]. It was recognized that when the graph topology is fixed, bipartite consensus of linear multi-agent systems over signed graphs is actually dual to the conventional consensus over nonnegative graphs, and thus many control algorithms developed for conventional consensus can be adopted to solve the bipartite consensus problems [9, 10]. All the above mentioned works on bipartite consensus assumes a homogeneous multi-agent system, however, it will be more practical if all agent dynamics are allowed to be distinct, i.e., heterogeneous multi-agent systems. Compared with homogeneous multi-agent systems, heterogeneous dynamics make the bipartite consensus problem more involved. Very recently, two independent works [13, 14] studied bipartite consensus of heterogeneous linear multi-agent systems, by establishing a similar equivalence as in [10].

When the graph has a fixed topology, bipartite consensus requires the graph to be structurally balanced [6, 10]. Considering switching graph topologies, [15] finds that the DeGroot-type opinion dynamics can be polarized when the graph at each constant is structurally balanced with the same subgroups, and the union of the graphs contains a spanning tree. Noting DeGroot-type system is of single-integrator dynamics, [16] studies bipartite consensus of first-order nonlinear systems over switching topologies. However, the agent dynamics considered in both [15] and [16] are homogeneous. To the best of our knowledge, no works on bipartite consensus of heterogeneous multi-agent systems over switching graphs have been reported, which motivates our current work. In this paper, we further extend our previous results on heterogeneous linear multi-agent systems [14] to switching graphs. We require the graphs to be jointly having a spanning tree, but might be disconnected at any time interval. This work can also regarded as an extension of conventional output regulation problem [17, 18] to bipartite output consensus problem over signed graphs.

The rest of the paper is organized as follows. Basic notations and some preliminaries are given in Section 2. In Section 3, output bipartite consensus problem of heterogeneous linera multi-agent systems under switching graphs is formulated and a distributed controller is designed. In Section 4, we provide a simulation example to verify the effectiveness of the theoretical results. Finally some conclusions are drawn in Section 5.

2 Preliminaries on Signed Graphs

Notations: The n-dimensional identity matrix is written as I_n. The column vector $\mathbf{1}_n \in \mathbb{R}^n$ denotes a vector of all ones and $\bar{\mathbf{1}}_n \in \mathbb{R}^n$ denotes a vector where all elements are 1 or -1. We use $diag(\sigma_1, \ldots, \sigma_n)$ to denote the diagonal matrix with entries $\sigma_1, \ldots, \sigma_n$. The Kronecker product is denoted as $\otimes$. And $\sigma : [0, +\infty) \to \mathcal{P} = \{1, 2, \ldots, \rho\}$ is a switching signal with some positive integer ρ where $\mathcal{P}$ is called the switching index set.

Then we briefly introduce some background about graph theory. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted directed signed graph. It contains information of the nodes set $\mathcal{V} =$

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$\{v_1, v_2, \ldots, v_N\}$, edges set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ and the associated weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, where $[a_{ij}]$ is a matrix with entries a_{ij}. If node i receives information from node j we say that $(v_j, v_i) \in \mathcal{E}$, $a_{ij} \neq 0$; otherwise $a_{ij} = 0$. A direct path from node i to node j is a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$. If there is a node k such that there exists a path from k to every other vertex in the graph, then we say that the graph contains a spanning tree where k is called its root. For a set of digraphs $\mathcal{G}_i = (\mathcal{V}, \mathcal{E}_i)$, $i = 1, 2, \ldots, n$, the digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{E} = \bigcup_{i=1}^{n} \mathcal{E}_i$ is called the union of digraphs $\mathcal{G}_i$, denoted by $\mathcal{G} = \bigcup_{i=1}^{n} \mathcal{G}_i$. We denote the switching digraph as $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ and the associated weighted adjacency matrix denote as $\mathcal{A}_{\sigma(t)}$. The switching instants $t_0 = 0, t_1, t_2, \ldots$ of σ satisfy $t_{k+1} - t_k \geq \tau > 0$ for some constant τ and any $k \geq 0$, and τ is called the dwell time. In this paper, we consider signed digraphs and assume the graph is simple, which means that there are no repeated edges or self loops.

In the study of the consensus problem, the Laplacian matrix L is very important. In this paper, we use L_c to denote the Laplacian matrix of conventional nonnegative graph.

$$L_c = [l_{ij}] = diag(\sum_{j=1}^{N} a_{1j}, ..., \sum_{j=1}^{N} a_{Nj}) - \mathcal{A}. \qquad (1)$$

It is well known that the matrix L_c has rank $N-1$,i.e. $\lambda_1 = 0$ is not repeated, if and only if graph $\mathcal{G}$ has a spanning tree. But the property does not generally hold for signed digraphs. We adopt the following Laplacian matrix for signed digraphs [6] as

$$L_s = [l_{ij}] = diag(\sum_{j=1}^{N} |a_{1j}|, ..., \sum_{j=1}^{N} |a_{Nj}|) - \mathcal{A}. \qquad (2)$$

Throughout this paper, we call (1) the conventional Laplacian matrix, and (2) the signed Laplacian matrix. We adopt the following definition of structural balance in our paper, which is an import structure of signed graphs.

Definition 1. [6, 19] (Structural balance) A signed graph is structurally balanced if it has a bipartition of the nodes $\mathcal{V}_1$, $\mathcal{V}_2$, i.e., $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$, such that $a_{ij} \leq 0$, $\forall v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q$ where $p, q \in \{1, 2\}, p \neq q$, and $\varnothing$ is the empty set; otherwise, $a_{ij} \geq 0$. ∎

Lemma 1. [6] A spanning tree is always structurally balanced. ∎

Lemma 2. [6, 9] Suppose the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree. Denote the signature matrices set as

$$\mathcal{D} = \{D = diag(\sigma_1, ..., \sigma_N) \mid \sigma_i \in \{1, -1\}\}.$$

Then the following statements are equivalent.

(a) $\mathcal{G}(\mathcal{A})$ is structurally balanced;

(b) $a_{ij}a_{ji} \geq 0$, and the corresponding undirected graph $\mathcal{G}(\mathcal{A}_u)$ is structurally balanced, where $\mathcal{A}_u = \frac{\mathcal{A}+\mathcal{A}^T}{2}$;

(c) $\exists D \in \mathcal{D}$, such that $\bar{\mathcal{A}} = [\bar{a}_{ij}] = D\mathcal{A}D$ is a nonnegative matrix, i.e., $\bar{a}_{ij} = |a_{ij}|$. ∎

Lemma 3. [20] Consider the linear system

$$\dot{x} = A(t)x \qquad (3)$$

where, for every time t, $A(t) \in \mathbb{R}^{n \times n}$ is Metzler with zero row sums and every element of $A(t)$ is a bounded and piecewise continuous function of time. If there is an index $k \in \{1, \ldots, n\}$, a threshold value $\delta > 0$, and an interval length $T > 0$ such that for all $t \in \mathbb{R}$ the δ-digraph associated with $\int_t^{t+T} A(s)ds$ has the property that all nodes may be reached from the node k, then the equilibrium set of consensus states is uniformly exponentially stable. In particular, all components of any solution $x(t)$ of (3) converge to a common value as $t \to \infty$. ∎

Metzler matrix is a matrix whose off-diagonal elements are positive or zero. A δ-digraph associated with an $n \times n$ Metzler matrix A is the digraph with nodes set $\{1, 2, \ldots, n\}$ and with an edge from l to $k (k \neq l)$ if and only if the element of A on the kth row and the lth column is strictly larger than δ.

3 Problem Formulation and Controller Design

3.1 Problem Formulation

Consider the heterogeneous linear multi-agent system, distributed over a signed switching graph $\mathcal{G}_{\sigma(t)}$ of the following form:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i, \\ y_i &= C_i x_i, \qquad i = 1, \ldots, N \end{aligned} \qquad (4)$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $y_i \in \mathbb{R}^p$ are the state, control input, and output vector. Please note that all agents can have different dynamics and different system orders, but their output should share the same dimension. Our objective is to design distributed controllers for all individual agents to make sure the output of all agents to achieve bipartite consensus, a collective behavior defined as follows.

Definition 2. (Output bipartite consensus) The heterogeneous linear multi-agent system (4) achieves output bipartite consensus if $\lim_{t\to\infty}(y_i(t) - y_s(t)) = 0$, $\forall i \in p$ and $\lim_{t\to\infty}(y_j(t) + y_s(t)) = 0$, $\forall j \in q$ for some nontrivial trajectories $y_s(t)$, where $p \cup q = \{1, \ldots, N\}$ and $p \cap q = \varnothing$. ∎

Clearly, when either p or q is empty, output bipartite consensus reduces to be the conventional output consensus [17]. The trajectories $y_s(t) \not\equiv 0$ should not asymptotically vanish for nontrivial output bipartite consensus.

For the solvability of the output bipartite consensus of the system (4), we need some assumptions as follows.

Assumption 1. The triple (A_i, B_i, C_i), for all i, is controllable and observable. ∎

Assumption 2. Assume that there exists a bipartition of $\mathcal{V}$ into two nonempty subsets over $t \geq 0$, such that for each graph $\mathcal{G}$, the edges between the two subsets are negative and the edges within each subset are positive. Assume that there exists an infinite sequence of nonempty, uniformly bounded time intervals $[t_{i_k}, t_{i_{k+1}})$, where $\{i = 0, 1, \ldots\}$, $k \geq 0$, and $t_{i_{k+1}} - t_{i_k} < \nu$ for some positive ν, the union graphs $\bigcup_{j=i_k}^{i_{k+1}-1} \mathcal{G}_{\sigma(t_j)}$ contains a spanning tree. ∎

3.2 Controller Design and Analysis

The distributed output feedback control law is designed as following:

$$\begin{aligned}\dot{\zeta}_i &= S\zeta_i + \sum_{j=1}^{N}(a_{ij}\zeta_j - |a_{ij}|\,\zeta_i),\\ \dot{\hat{x}}_i &= A_i\hat{x}_i + B_i u_i + H_i(C_i\hat{x}_i - y_i),\\ u_i &= K_i(\hat{x}_i - \Pi_i\zeta_i) + \Gamma_i\zeta_i,\end{aligned} \tag{5}$$

with $\zeta_i \in \mathbb{R}^m$ and $\hat{x}_i \in \mathbb{R}^{n_i}$ for $i = 1, \ldots, N$, where S, H_i, K_i, Π_i and Γ_i are design matrices with appropriate dimensions.

According to [17], the internal reference model can be defined by a linear system

$$\begin{aligned}\dot{v} &= Sv,\\ y &= Rv,\end{aligned} \tag{6}$$

where matrices $S \in \mathbb{R}^{m\times m}$, $R \in \mathbb{R}^{p\times m}$. Assume there exist matrices $\Pi_i \in \mathbb{R}^{n_i\times m}$, $\Gamma_i \in \mathbb{R}^{p_i\times m}$ for $i = 1, ..., N$ such that

$$\begin{aligned}A_i\Pi_i + B_i\Gamma_i &= \Pi_i S,\\ C_i\Pi_i &= R,\end{aligned} \tag{7}$$

for $i = 1, ..., N$. In such a case, the models of the individual systems together with their local controller must involve the internal reference model (S, R).

Theorem 1. Consider system (4). Suppose Assumptions 1-2 hold. Then under control law (5), an output bipartite consensus is achieved if the following conditions hold,

(a) $\sigma(S) \subset j\mathbb{R}$ and (S, R) is observable, and there exist matrices $\Pi_i \in \mathbb{R}^{n_i\times m}$, $\Gamma_i \in \mathbb{R}^{p_i\times m}$ for $i = 1, ..., N$ satisfying (7);

(b) the matrices $A_i + B_iK_i$ and $A_i + H_iC_i$ are Hurwitz for $i = 1, ..., N$. ∎

Proof. Define the tracking error as $\varepsilon_i = x_i - \Pi_i\zeta_i$ and the observer errors as $e_i = x_i - \hat{x}_i$ for $i = 1, \ldots, N$. Then

$$\begin{aligned}\dot{\varepsilon}_i &= (A_i + B_iK_i)\varepsilon_i - v_i,\\ \dot{e}_i &= (A_i + H_iC_i)e_i,\\ v_i &= B_iK_ie_i + \Pi_i\sum_{j=1}^{N}(a_{ij}\zeta_j - |a_{ij}|\,\zeta_i),\\ \dot{\zeta}_i &= S\zeta_i + \sum_{j=1}^{N}(a_{ij}\zeta_j - |a_{ij}|\,\zeta_i),\end{aligned}$$

for $i = 1, \ldots, N$. We claim that $(\sigma_j\zeta_j - \sigma_i\zeta_i) \to 0$, $i, j = 1, \ldots, N$ as $t \to \infty$, where the proof of this result will be given later. By Assumption 2 and Lemma 2, for $t \geq 0$, there exists a signature matrix $D \in \mathcal{D}$ such that for each signed graph $\mathcal{G}_{\sigma(t)}(\mathcal{A}_{\sigma(t)})$, the associated graph $\mathcal{G}_{\sigma(t)}(\bar{\mathcal{A}}_{\sigma(t)})$ is a nonnegative graph, where $\bar{\mathcal{A}}_{\sigma(t)} = D\mathcal{A}_{\sigma(t)}D$, and $D = diag(\sigma_1, ..., \sigma_N)$, $\sigma_i \in \{1, -1\}$. The matrix $L_{c,\sigma(t)}$ associate with $\mathcal{G}_{\sigma(t)}(\bar{\mathcal{A}}_{\sigma(t)})$ has a property that $L_{c,\sigma(t)} = DL_{s,\sigma(t)}D$. Furthermore, H_i are chosen such that $A_i + H_iC_i$ are Hurwitz. In summary, $v_i \to 0$, $i = 1, \ldots, N$ as $t \to \infty$. Since $A_i + B_iK_i$ are Hurwitz, this implies $\varepsilon_i \to 0$, $i = 1, \ldots, N$ as $t \to \infty$ and in turn $(\sigma_j\zeta_j - \sigma_i\zeta_i) \to 0$, $i, j = 1, \ldots, N$ as $t \to \infty$. Since $C_i\Pi_i = R$ and $(\sigma_j\zeta_j - \sigma_i\zeta_i) \to 0$, $i, j = 1, \ldots, N$ as $t \to \infty$, system (4) achieves the output bipartite consensus.

So, the trick of the proof is to show that $(\sigma_j\zeta_j - \sigma_i\zeta_i) \to 0$, $i, j = 1, \ldots, N$ as $t \to \infty$.

Consider the linear switched system

$$\dot{\zeta} = ((I_N \otimes S) - (L_{s,\sigma(t)} \otimes I_m))\zeta_i \tag{8}$$

where $\zeta = [\zeta_1^T, \ldots, \zeta_N^T]^T$, $\zeta_i \in \mathbb{R}^m$, $i = 1, \ldots, N$, and $L_{s,\sigma(t)}$ is the Laplacian of the signed switching graph $\mathcal{G}_{\sigma(t)}$.

Let $\varphi = (D \otimes e^{-St})\zeta$. Then

$$\begin{aligned}\dot{\varphi} &= -(D \otimes Se^{-St})\zeta + (D \otimes e^{-St})\dot{\zeta}\\ &= -(D \otimes Se^{-St})\zeta\\ &\quad + (D \otimes e^{-St})((I_N \otimes S) - (L_{s,\sigma(t)} \otimes I_m))\zeta\\ &= -(D \otimes e^{-St})(L_{s,\sigma(t)} \otimes I_m)\zeta_i\\ &= -(DL_{s,\sigma(t)} \otimes I_m)(D \otimes I_m)(D \otimes e^{-St})\zeta\\ &= -(DL_{s,\sigma(t)}D \otimes I_m)\varphi\\ &= -(L_{c,\sigma(t)} \otimes I_m)\varphi\end{aligned} \tag{9}$$

where $\varphi = [\varphi_1^T, \ldots, \varphi_N^T]^T$, $\varphi_i \in \mathbb{R}^m$, $i = 1, \ldots, N$.

Note that (9) is equivalent to

$$\dot{\bar{\varphi}}_k = -L_{c,\sigma(t)}\bar{\varphi}_k \tag{10}$$

where $\bar{\varphi}_k = [\varphi_{1k}, \ldots, \varphi_{Nk}]^T$, $k = 1, \ldots, m$, and φ_{ik} are the kth component of φ_i, $i = 1, \ldots, N$. By the definition of the Laplacian, $-L_{c,\sigma(t)}$ is a Metzler matrix with zero row sums for any $t \geq 0$, and $-L_{c,\sigma(t)}$ is piecewise constant and bounded since $\sigma(t)$ belongs to a finite set.

Let $T = 2\nu > 0$. For any $t \geq 0$, there exists a positive k such that $[t_{i_k}, t_{i_{k+1}}) \in [t, t + T)$. Then

$$\int_t^{t+T} -l_{c,ij}(s)ds \geq \sum_{j=i_k}^{i_{k+1}-1} -l_{c,ij}(t_{j+1} - t_j) \tag{11}$$

where $l_{c,ij}$ is the (i, j) component of $L_{c,\sigma(t)}$ at time t. Since $t_{j+1} - t_j \geq \tau > 0$, the (i, j) component of

$$\sum_{j=i_k}^{i_{k+1}-1} -L_{c,\sigma(t_j)}(t_{j+1} - t_j)$$

has the same sign as the (i, j) component of

$$\sum_{j=i_k}^{i_{k+1}-1} -L_{c,\sigma(t_j)}.$$

Therefore, both of these two matrices have the same δ-digraph [18] if

$$\delta = \min_{l_{c,ij}\neq 0, i\neq j}\{-l_{c,ij} \cdot \min\{1, \tau\}\}$$

which is $\bigcup_{j=i_k}^{i_{k+1}-1}\mathcal{G}_{\sigma(t_j)}$. By (11), $\bigcup_{j=i_k}^{i_{k+1}-1}\mathcal{G}_{\sigma(t_j)}$ is a subgraph of the δ-digraph associated with $\int_t^{t+T} -L_{c,ij}(s)ds$, and the two digraphs have the same node sets. Thus, Assumption 2 implies that the union of the graphs contains a spanning tree, and hence, the δ-digraph associated with $\int_t^{t+T} -L_{c,ij}(s)ds$ has a spanning tree. By Lemma 3, all components of any solution $\varphi(t)$ of (9) converge exponentially to a common value as $t \to \infty$. Without loss of generality, let φ_0 be the common value. Then

$$\lim_{t\to\infty}\varphi_i = \varphi_0$$

and

$$\lim_{t\to\infty}\zeta_i = \lim_{t\to\infty}\sigma_i e^{St}\varphi_i = \sigma_i \lim_{t\to\infty} e^{St}\varphi_0$$

for $i = 1,\dots,N$, that is, $(\sigma_j\zeta_j - \sigma_i\zeta_i) \to 0$, $i,j = 1,\dots,N$ as $t \to \infty$.

It is clear that system (4) achieves the output bipartite consensus. □

4 Example

Consider a group of six nodes. Nodes 1 and 2 are modeled by the following third order dynamic system.

$$\begin{aligned}\dot{x}_i &= \begin{bmatrix}0 & 1 & 0\\ -2 & 0 & i\\ 0 & -2i & -0.5i\end{bmatrix} x_i + \begin{bmatrix}0\\0\\i\end{bmatrix} u_i,\\ y_i &= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix} x_i.\end{aligned} \tag{12}$$

Nodes 3 to 6 are modeled by the fourth order system (13).

$$\begin{aligned}\dot{x}_i &= \begin{bmatrix}0 & 1 & 0 & 0\\ -2 & 0 & 1 & 0\\ 0 & 0 & 0 & i\\ 0 & 0 & -2i & -0.5i\end{bmatrix} x_i + \begin{bmatrix}0\\0\\0\\i\end{bmatrix} u_i,\\ y_i &= \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} x_i.\end{aligned} \tag{13}$$

The six nodes are all observable and controllable. Choose

$$S = \begin{bmatrix}0 & 1\\ -2 & 0\end{bmatrix}, \quad R = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

and let

$$\Pi_i = \begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}, \quad \Gamma_i = \begin{bmatrix}0 & 2\end{bmatrix},$$

for $i = 1, 2$, and

$$\Pi_j = \begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix}, \quad \Gamma_j = \begin{bmatrix}0 & 0\end{bmatrix},$$

for $j = 3, ..., 6$, which satisfy (7). The feedback gains K_i and the observer gains H_i are chosen using an LQG design for each individual system independently.

Next, we assume that the switching signed digraph starts from the state $\mathcal{G}_1$ and switches every $T = \frac{1}{4}$ second to the next state according to the signed digraphs in Fig.1. The switching signal is as following:

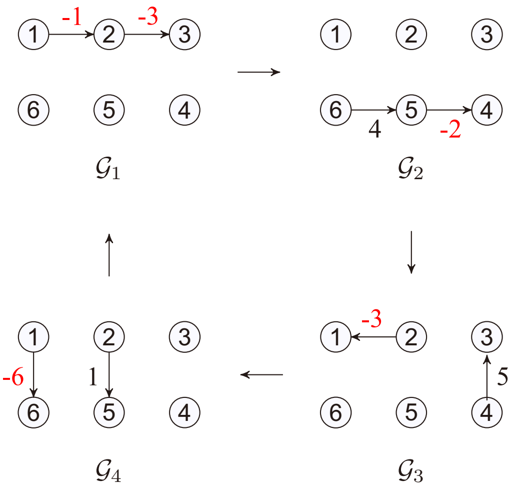

Fig. 1: Switching signed digraph $\mathcal{G}_{\sigma(t)}$ with $\mathcal{P} = \{1,2,3,4\}$.

$$\sigma(t) = \begin{cases} 1, & \text{if } sT \le t < (s+\frac{1}{4})T\\ 2, & \text{if } (s+\frac{1}{4})T \le t < (s+\frac{1}{2})T\\ 3, & \text{if } (s+\frac{1}{2})T \le t < (s+\frac{3}{4})T\\ 4, & \text{if } (s+\frac{3}{4})T \le t < (s+1)T\end{cases} \tag{14}$$

where $s = 0, 1, 2, \dots$. It can be verified that Assumption 2 is also satisfied even though, at no time instant t, the signed digraph $\mathcal{G}_{\sigma(t)}$ has a spanning tree.

For random initial states, the system output trajectories $y_{i,1}$ and $y_{i,2}$ are shown in Fig. 2. Clearly, it shows that the output bipartite consensus is achieved, with two subgroups $\mathcal{V}_1 = \{v_1, v_3, v_4\}$ and $\mathcal{V}_2 = \{v_2, v_5, v_6\}$.

5 Conclusions and Future Works

The output bipartite consensus problem for heterogeneous linear multi-agent systems over signed switching digraphs are solved. The graphs can be disconnected in each time interval, but the union of them should have a spanning tree. Moreover, the union of the graphs needs to be structurally balanced, i.e., the whole group should be able to be divided into two disjoint subgroups, and competition can only exist between these two subgroups. However, structural balance is a very stringent condition, and can be easily violated. Investigating collective behaviors over general signed graphs, which are not structurally balanced, is more interesting and challenging.

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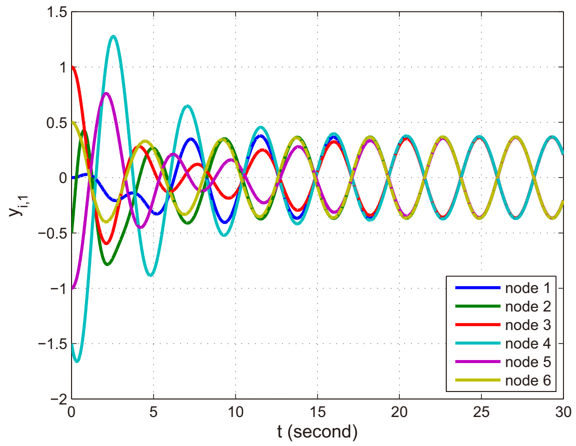
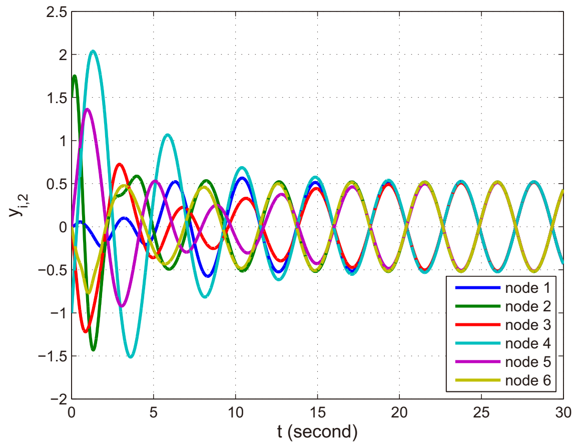


(a) Trajectories of $y_{i,1}$ for $i = 1, \ldots, 6$



(b) Trajectories of $y_{i,2}$ for $i = 1, \ldots, 6$

Fig. 2: Evolution of the system outputs $y_{i,j}$ for $i = 1, ..., 6$ and $j = 1, 2$, which show the output bipartite consensus of the six agents.

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