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Output Bipartite Consensus of Heterogeneous Linear Multi-Agent Systems

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Abstract: This paper aims to address the output bipartite consensus problem of heterogeneous linear multi-agent systems over signed directed graphs, where both collaboration and competition coexist within the group. The agents can all have different dynamics but the same output dimensions. The main idea is to relate the output bipartite consensus with the well studied conventional output consensus over nonnegative graphs, where all agents cooperate. We solve the bipartite output consensus problem by modifying two existing control protocols for conventional output consensus problems.

Key Words: Heterogeneous linear multi-agent system, Internal model principle, Output bipartite consensus, Signed graph.

1 Introduction

In the past two decades, great effort has been devoted to the study of conventional consensus of linear multi-agents systems (see [1–5] and references therein). The conventional consensus problem means that a group of agents reach an agreement, on the value of a given set of variables, typically the states or the outputs of the systems.

With the advance of study and the need of practical application, there has been tremendous interest in studying conventional output consensus for heterogeneous linear multi-agent systems, where the individual dynamics are not identical, and in particular the state dimensions may be different. In [6, 7], internal model principle of control theory plays an important role in solving leaderless output consensus problem of heterogeneous linear multi-agent systems. In [8], the reference input and disturbance are considered as leader system, and the leader-following output consensus problem is solved by designing a distributed observers of the exosystem. Nevertheless, the above mentioned results are all obtained over nonnegative graphs.

Recently, there have been a few studies of collective behaviors of networked systems over signed graphs. In signed graphs, positive edges can be associated to cooperation and negative ones antagonistic interactions, which link to trust/distrust, like/dislike, and so on. Altafini [9] stated that multiple single-integrator systems over a structurally balanced signed network could achieve bipartite consensus, that is, N agents can be divided into two subgroups, and each subgroup achieves consensus in the conventional sense, while these two subgroups moving oppositely toward each other. Bipartite consensus and bipartite flocking of integrator dynamics were further studied in [10] and [11]. Very recently, results of bipartite consensus have been extended to linear multi-agent systems, in which all nodes are assumed to have the same dynamics (see [12–15]). Motivated by the observation that individual systems are hardly exactly identical, the bipartite consensus problem will be extended to the study of output bipartite consensus in heterogeneous systems.

In this paper, we extend two controllers for heterogeneous agents over nonnegative digraphs in [6, 7], to solve the output bipartite consensus problem. The trick of the proofs about Theorem 1 and Theorem 2 in this paper is to find the equivalence relationships between output bipartite consensus over signed graphs and conventional consensus over nonnegative graphs for heterogeneous linear multi-agents systems. Through analysis of two different controllers above, we infer that some controller design technique on conventional output consensus for heterogeneous systems over nonnegative digraphs can be extended to output bipartite consensus.

The rest of the paper is organized as follows. In the next section, some definitions and results on signed graphs are presented. In Section 3, output bipartite consensus problem on heterogeneous networks for multi-agent systems is formulated, and two control approaches are proposed for output bipartite consensus problem. In Section 4, a simulation example is provided to demonstrate the feasibility of the proposed algorithms. Finally some conclusions are drawn in Section 5.

2 Preliminaries on Signed Graphs

The communication topology between the individual nodes can be shown by a graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted signed digraph (directed graph) with nodes set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, edges set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and associated adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $[a_{ij}]$ is a matrix with entries a_{ij}, i.e., the weight of edge (v_j, v_i). If node i receives information from node j, i.e., $(v_j, v_i) \in \mathcal{E}$, $a_{ij} \neq 0$; otherwise $a_{ij} = 0$. We use $\mathcal{G}(\mathcal{A})$ to explicitly denote a graph whose adjacency matrix is $\mathcal{A}$. A direct path from node i to node j is a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$. A digraph is said to have a spanning tree, if there is a node i_r (called the root), such that there is a directed path from the root to every other nodes in the graph. In this paper, we consider signed digraphs and assume the graph is simple, i.e., there are no repeated edges and self loops.

Conventionally, the Laplacian matrix L_c associated to a nonnegative graph $\mathcal{G}(\mathcal{A})$ is defined as

$$L_c = [l_{ij}] = diag(\sum_{j=1}^{N} a_{1j}, ..., \sum_{j=1}^{N} a_{Nj}) - \mathcal{A}, \tag{1}$$

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where $diag(\cdot)$ denotes a diagonal matrix by placing a vector along the diagonal direction.

The Laplacian matrix L_c plays a crucial role in studying the synchronization of multi-agent systems over nonnegative graphs. The property, that L_c has a simple eigenvalue of zero associated with a right eigenvector $\mathbf{1}_N$ when the graph has a spanning tree, is the key to study the consensus problem over nonnegative graphs, where $\mathbf{1}_N \in \mathbb{R}^N$ denotes a vector of all ones. The property does not generally hold for signed digraphs. To study collective behaviors over signed digraphs, we need to define an another Laplacian matrix. The following Laplacian matrix for signed digraphs is adopted from [9] as

$$L_s = [l_{ij}] = diag(\sum_{j=1}^{N} |a_{1j}|, ..., \sum_{j=1}^{N} |a_{Nj}|) - \mathcal{A}. \qquad (2)$$

In order to make the distinction between these two Laplacian matrices, we call (1) the conventional Laplacian matrix, and (2) the signed Laplacian matrix.

Before proceeding further, let us recall the definition of a special class of signed graphs and several lemmas about the the analysis of concensus patterns on signed graphs.

Definition 1. [9, 16] A signed graph is structurally balanced if it has a bipartition of the nodes $\mathcal{V}_1$, $\mathcal{V}_2$, i.e., $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$, such that $a_{ij} \le 0$, $\forall v_i \in \mathcal{V}_p$, $v_j \in \mathcal{V}_q$ where $p, q \in \{1, 2\}$, $p \ne q$, and $\varnothing$ is the empty set; otherwise, $a_{ij} \ge 0$. ∎

Lemma 1. [9] A spanning tree is always structurally balanced. ∎

Lemma 2. [9, 13] Suppose the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree. Denote the signature matrices set as

$$\mathcal{D} = \{D = diag(\sigma_1, ..., \sigma_N) \mid \sigma_i \in \{1, -1\}\}.$$

Then the following statements are equivalent.

(a) $\mathcal{G}(\mathcal{A})$ is structurally balanced;

(b) $a_{ij}a_{ji} \ge 0$, and the corresponding undirected graph $\mathcal{G}(\mathcal{A}_u)$ is structurally balanced, where $\mathcal{A}_u = \frac{\mathcal{A}+\mathcal{A}^T}{2}$;

(c) $\exists D \in \mathcal{D}$, such that $\bar{\mathcal{A}} = [\bar{a}_{ij}] = D\mathcal{A}D$ is a nonnegative matrix, i.e., $\bar{a}_{ij} = |a_{ij}|$. ∎

Lemma 3. [10, 13] Suppose the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree. If the graph is structurally balanced, then 0 is a simple eigenvalue of L_s and all its other eigenvalues have positive real parts; but not vice versa. ∎

The importance of Lemma 2 is that it associates a signed graph with a nonnegative graph through the diagonal matrix D.

3 Output Bipartite Consensus: Problem Formulation and Controller Design

3.1 Problem Formulation

Consider N heterogeneous agents, distributed over a signed communication graph $\mathcal{G}$, each modeled by a linear dynamical system

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k u_k, \\ y_k &= C_k x_k, \qquad\qquad k = 1, ..., N \end{aligned} \qquad (3)$$

where $x_k \in \mathbb{R}^{n_k}$ is state vector , $u_k \in \mathbb{R}^{m_k}$ is control input , and $y_k \in \mathbb{R}^p$ is output vector; the triple (A_k, B_k, C_k) is controllable and observable, for all k. The communication network is a signed digraph, which has a spanning tree and is structurally balanced. The objective is to achieve an output bipartite consensus, which is presented in Definition 2, by designing a distributed control law.

Definition 2. We say the linear multi-agent system (3) achieves output bipartite consensus if $\lim_{t\to\infty}(y_i(t) - y_s(t)) = 0, \forall i \in p$ and $\lim_{t\to\infty}(y_j(t) + y_s(t)) = 0, \forall j \in q$ for some nontrivial trajectory $y_s(t)$, where $p \cup q = \{1, ..., N\}$ and $p \cap q = \varnothing$. ∎

Clearly, when either p or q is empty, output bipartite consensus reduces to be the conventional output consensus [6, 7]. For nontrivial output bipartite consensus, the trajectories $y_s(t) \not\equiv 0$ should not asymptotically vanish.

The famous internal model principle of control theory has proven to be an efficient approach to solving conventional consensus of heterogeneous linear multi-agent systems. And the virtual exosystem that exists as part of the individual systems and their controllers, plays an important role in analyzing consensus of heterogeneous linear systems [6, 7]. In this paper, the trajectories $y_s(t)$ produced by the virtual exosystem is assumed to be described as

$$\begin{aligned} \dot{x}_s &= S x_s, \\ y_s &= R x_s, \end{aligned} \qquad (4)$$

where $x_s \in \mathbb{R}^m$, $y_s \in \mathbb{R}^p$, (S, R) is observable, and S is assumed to have at least one eigenvalue with nonnegative real part to avoid trivial output bipartite consensus, i.e., $y_k \to 0$ $(k = 1, ..., N)$.

It has been shown in [9, 13] that for identical agents, the bipartite consensus over signed digraphs is equivalent to the conventional consensus over nonnegative digraphs. Thus certain consensus control protocols are extended to solve bipartite consensus problems. In this paper, we are solving the bipartite output consensus problem of heterogeneous multi-agent systems in a similar spirit. In the next two subsections, we shall solve the bipartite output consensus problem by modifying two reported control protocols for conventional output consensus problems [6, 7]. Throughout the paper, we assume the limited information can be communicated among individual agents is restricted to the measured output information y_k, $(k = 1, ..., N)$. The communication graph $\mathcal{G}(\mathcal{A})$ is a signed digraph, which has a spanning tree and is structurally balanced.

3.2 Control Design Approach I

In this section, we show that output bipartite consensus is achieved by modifying the control law reported in [7].

Consider system (3). Design the following distributed output feedback control law

$$\begin{aligned} \dot{x}_{rk} &= A_{rk} x_{rk} + B_{rk} \sum_{j=1}^{N} (a_{kj} y_j - |a_{kj}| y_k), \\ u_k &= \Pi_{rk} x_{rk} + \Gamma_{rk} \sum_{j=1}^{N} (a_{kj} y_j - |a_{kj}| y_k), \end{aligned} \qquad (5)$$

where $x_{rk} \in \mathbb{R}^{r_k}$ for $k = 1, ..., N$, A_{rk}, B_{rk}, Π_{rk} and Γ_{rk} are design matrices with appropriate dimensions.

System (3) together with controller (5) yields

$$\begin{aligned} \dot{\bar{x}}_k &= A_{0k}\bar{x}_k + B_{0k}\sum_{j=1}^{N}(a_{kj}y_j - |a_{kj}|\, y_k), \\ y_k &= C_{0k}\bar{x}_k, \end{aligned} \tag{6}$$

where $\bar{x}_k = [x_k^T, x_{rk}^T]^T$,

$$A_{0k} = \begin{bmatrix} A_k & B_k\Pi_{rk} \\ 0 & A_{rk} \end{bmatrix}, \quad B_{0k} = \begin{bmatrix} B_k\Gamma_{rk} \\ B_{rk} \end{bmatrix},$$

$$C_{0k} = \begin{bmatrix} C_k & 0 \end{bmatrix}.$$

We assume the matrices A_{0k} are diagonalizable, and the triple (A_{0k}, B_{0k}, C_{0k}) is controllable and observable, for all k.

Consider the virtual exosystem (S, R) defined in (4). According to [7], the closed-loop system (6) can track the virtual exosystem (S, R), if there exist regular matrices T_k such that

$$\begin{aligned} T_k^{-1}A_{0k}T_k &= \begin{bmatrix} S & 0 \\ 0 & A_{pk} \end{bmatrix}, \\ C_{0k}T_k &= \begin{bmatrix} R & C_{pk} \end{bmatrix}, \end{aligned} \tag{7}$$

where A_{pk} and C_{pk} are matrices of appropriate dimensions, for $k = 1, ..., N$.

According to [7], we define an important matrix as

$$\begin{aligned} A_{LL} = &\begin{bmatrix} A_{p1} & & & \\ & A_{02} & & \\ & & \ddots & \\ & & & A_{0N} \end{bmatrix} \\ &+ \begin{bmatrix} B_{p1} & 0 & \cdots & 0 \\ -T_2\hat{B}_{q1} & B_{02} & & \\ \vdots & & \ddots & \\ -T_N\hat{B}_{q1} & & & B_{0N} \end{bmatrix} (L_c \otimes I_p) \\ &+ \begin{bmatrix} C_{p1} & & & \\ & C_{02} & & \\ & & \ddots & \\ & & & C_{0N} \end{bmatrix}, \end{aligned} \tag{8}$$

where A_{p1} and C_{p1} are from (7), $T_k^{-1}B_{0k} = \begin{bmatrix} B_{qk} \\ B_{pk} \end{bmatrix}$ for $k = 1, ..., N$, $\hat{B}_{q1} = \begin{bmatrix} B_{q1} \\ 0 \end{bmatrix}$, L_c is the conventional Laplacian matrix defined as (1) for nonnegative graph $\mathcal{G}(\bar{\mathcal{A}})$, and $\otimes$ is the symbol of the Kronecker product.

Theorem 1. Consider system (3) with control law (5). An output bipartite consensus is achieved if the following conditions hold,

(a) there exist regular matrices T_k for $k = 1, ..., N$ satisfying (7);

(b) the matrix A_{LL} defined in (8) is asymptotically stable.

∎

Proof. To prove the theorem, we need to consider linear system (9) firstly

$$\begin{aligned} \dot{z}_k &= A_k z_k + B_k\bar{u}_k, \\ w_k &= C_k z_k, \end{aligned} \tag{9}$$

where (A_k, B_k) is same as system (3), and the communication graph is $\mathcal{G}(\bar{\mathcal{A}})$. Through choosing an appropriate matrix D according to Lemma 2 (c), i.e., $\bar{\mathcal{A}} = [\bar{a}_{ij}] = D\mathcal{A}D$, we relate $\mathcal{G}(\bar{\mathcal{A}})$ to $\mathcal{G}(\mathcal{A})$. Design the following distributed output feedback control law

$$\begin{aligned} \dot{z}_{rk} &= A_{rk}z_{rk} + B_{rk}\sum_{j=1}^{N}\bar{a}_{kj}(w_j - w_k), \\ \bar{u}_k &= \Pi_{rk}z_{rk} + \Gamma_{rk}\sum_{j=1}^{N}\bar{a}_{kj}(w_j - w_k), \end{aligned} \tag{10}$$

with $x_{rk} \in \mathbb{R}^{r_k}$, A_{rk}, B_{rk}, Π_{rk} and Γ_{rk} are same as those in (5) for $k = 1, ..., N$, such that a conventional output consensus is achieved.

Next we show that system (3) with the feedback control law (5) is output bipartite consensus for system (3) is equivalent to output conventional consensus for system (9) with respect to the control laws.

Combining system (3) and the feedback control law (5), the closed-loop system is represented in a compact form as

$$\begin{aligned} \dot{x} &= [A + B\Gamma_r(L_s \otimes I_p)C]x + B\Pi_r x_r, \\ \dot{x}_r &= A_r x_r + B_r(L_s \otimes I_p)Cx, \end{aligned} \tag{11}$$

where $x = [x_1^T, ..., x_N^T]^T$, and $x_r = [x_{r1}^T, ..., x_{rN}^T]^T$. The parameters A, B, C, A_r, B_r, Γ_r and Π_r are block diagonal matrices obtained by stacking the individual dynamic together with their local controllers. The Laplacian matrix L_s is defined as (2) for signal graph $\mathcal{G}(\mathcal{A})$. Since $\mathcal{G}(\mathcal{A})$ has a spanning tree and is structurally balanced, according to Lemma 2, there is a signature matrix $D \in \mathcal{D}$ such that the associated graph $\mathcal{G}(\bar{\mathcal{A}})$ is a nonnegative graph and has a spanning tree, where $\bar{\mathcal{A}} = D\mathcal{A}D$. Then we have $L_c = DL_sD$.

The overall system (9), (10) is represented in compact form as

$$\begin{aligned} \dot{z} &= [A + B\Gamma_r(L_c \otimes I_p)C]z + B\Pi_r z_r, \\ \dot{z}_r &= A_r z_r + B_r(L_c \otimes I_p)Cz, \end{aligned} \tag{12}$$

where $z = [z_1^T, ..., z_N^T]^T$, and $z_r = [z_{r1}^T, ..., z_{rN}^T]^T$.

Define $z_k = \sigma_k x_k$ and $z_{rk} = \sigma_k x_{rk}$, where $\sigma_i \in \{1, -1\}$ and $D = diag(\sigma_1, ..., \sigma_N)$, for $k = 1, ..., N$, that is, $z = \bar{D}x$ and $z_r = \bar{D}x_r$, with

$$\bar{D} \triangleq diag(\sigma_1 \otimes I_{n_1}, ..., \sigma_N \otimes I_{n_N}), \tag{13}$$

and note that $D = D^T = D^{-1}$ and $\bar{D} = \bar{D}^T = \bar{D}^{-1}$. Then $x = \bar{D}z$ and $x_r = \bar{D}z_r$. We have

$$\begin{aligned} \dot{z} &= \bar{D}\dot{x} \\ &= \bar{D}[A + B\Gamma_r(L_s \otimes I_p)C]x + \bar{D}B\Pi_r x_r \\ &= \bar{D}[A + B\Gamma_r(L_s \otimes I_p)C]\bar{D}z + \bar{D}B\Pi_r\bar{D}z_r \\ &= Az + B\Gamma_r(D \otimes I_p)(L_s \otimes I_p)(D \otimes I_p)Cz + B\Pi_r z_r \\ &= Az + B\Gamma_r(DL_sD \otimes I_p)Cz + B\Pi_r z_r \\ &= [A + B\Gamma_r(L_c \otimes I_p)C]z + B\Pi_r z_r, \end{aligned}$$

$$
\begin{aligned}
\dot{z}_r &= \bar{D}\dot{x}_r \\
&= \bar{D}A_r x_r + \bar{D}B_r(L_s \otimes I_p)Cx \\
&= \bar{D}A_r\bar{D}z_r + \bar{D}B_r(L_s \otimes I_p)C\bar{D}z \\
&= A_r z_r + B_r(D \otimes I_p)(L_s \otimes I_p)(D \otimes I_p)Cz \\
&= A_r z_r + B_r(DL_sD \otimes I_p)Cz \\
&= A_r z_r + B_r(L_c \otimes I_p)Cz.
\end{aligned}
$$

From the above equations, they are exactly the closed-loop form (12) of system (9) with the feedback control law (10). That is to say, system (3) achieves the output bipartite consensus if and only if the system (9) achieves the conventional output consensus following from the fact that $x = \bar{D}z$.

It has been shown in [7] that, the controller (5) with the same S, R, A_{rk}, B_{rk}, Π_{rk} and Γ_{rk} as in (a), (b) of Theorem 1, achieves conventional output consensus of system (9) over nonnegative digraph $\mathcal{G}(\bar{\mathcal{A}})$. Then the system (3) achieves the output bipartite consensus. This completes the proof. □

Remark 1. *A draw back of the above mentioned control design approach is that the matrices A_{rk}, B_{rk}, Π_{rk} and Γ_{rk} are not explicitly designed, but hidden in the condition (8) in a coupled manner. In the next section, we shall explore an output feedback control law that can be explicitly designed.*

3.3 Control Design Approach II

Consider system (3). Design the following distributed state feedback control law

$$
\begin{aligned}
\dot{\zeta}_k &= S\zeta_k + \sum_{j=1}^{N}(a_{kj}\zeta_j - |a_{kj}|\,\zeta_k), \\
\dot{\hat{x}}_k &= A_k\hat{x}_k + B_k u_k + H_k(\hat{y}_k - y_k), \\
u_k &= K_k(\hat{x}_k - \Pi_k\zeta_k) + \Gamma_k\zeta_k,
\end{aligned}
\tag{14}
$$

with $\zeta_k \in \mathbb{R}^m$ and $\hat{x}_k \in \mathbb{R}^{n_k}$ for $k = 1, ..., N$, such that an output bipartite consensus is achieved, where S, H_k, K_k, Π_k and Γ_k are matrices with appropriate dimensions.

Consider the virtual exosystem defined by the pair (S, R). Assume there exist a scalar $m \in \mathbb{N}$, matrices $S \in \mathbb{R}^{m\times m}$ and $R \in \mathbb{R}^{p\times m}$, such that $\sigma(S) \subset j\mathbb{R}$ and (S, R) is observable. And at the same time, there exist matrices $\Pi_k \in \mathbb{R}^{n_k\times m}$, $\Gamma_k \in \mathbb{R}^{p_k\times m}$ for $k = 1, ..., N$ such that

$$
\begin{aligned}
A_k\Pi_k + B_k\Gamma_k &= \Pi_k S, \\
C_k\Pi_k &= R,
\end{aligned}
\tag{15}
$$

for $k = 1, ..., N$. In such case, it guarantees that the models of the individual systems together with their local controller embed the virtual exosystem (S, R) (see [6]).

Theorem 2. Consider system (3) with control law (14). An output bipartite consensus is achieved if the following conditions hold,

(a) the communication network is modeled by a signed digraph $\mathcal{G}(\mathcal{A})$ which has a spanning tree and is structurally balanced;

(b) there exist a scalar $m \in \mathbb{N}$, matrices $S \in \mathbb{R}^{m\times m}$ and $R \in \mathbb{R}^{p\times m}$, where $\sigma(S) \subset j\mathbb{R}$ and (S, R) is observable, and matrices $\Pi_k \in \mathbb{R}^{n_k\times m}$, $\Gamma_k \in \mathbb{R}^{p_k\times m}$ for $k = 1, ..., N$ satisfying (15);

(c) the matrices $A_k + B_kK_k$ and $A_k + H_kC_k$ with K_k and H_k are Hurwitz for $k = 1, ..., N$. ∎

Proof. A similar conventional output consensus problem is stated as follows.

Consider linear system (9). The communication graph is $\mathcal{G}(\bar{\mathcal{A}})$. Design the following distributed state feedback control law

$$
\begin{aligned}
\dot{\bar{\zeta}}_k &= S\bar{\zeta}_k + \sum_{j=1}^{N}\bar{a}_{kj}(\bar{\zeta}_j - \bar{\zeta}_k), \\
\dot{\hat{z}}_k &= A_k\hat{z}_k + B_k\bar{u}_k + H_k(\hat{w}_k - w_k), \\
\bar{u}_k &= K_k(\hat{z}_k - \Pi_k\bar{\zeta}_k) + \Gamma_k\bar{\zeta}_k,
\end{aligned}
\tag{16}
$$

with $\bar{\zeta}_k \in \mathbb{R}^m$ and $\hat{z}_k \in \mathbb{R}^{n_k}$ for $k = 1, ..., N$, such that a conventional output consensus is achieved. S, H_k, K_k, Π_k and Γ_k are design matrices of appropriate dimensions. It can easily be shown that we transform the output bipartite consensus problem over signed digraphs into the conventional output consensus problem over nonnegative digraphs through the fact that $\bar{\mathcal{A}} = D\mathcal{A}D$ and $x = \bar{D}z$. It has been shown in [6] that, the controller (16) with the same S, R, K_k, H_k, Γ_k, Π_k as in Theorem 2 (b) and (c), achieves conventional output consensus of system (9) over nonnegative digraph $\mathcal{G}(\bar{\mathcal{A}})$. Then the system (3) achieves the output bipartite consensus. □

4 Example

Consider a group of six nodes over a signed graph, which has a spanning tree and structurally balanced (see Fig. 1). Nodes 1 and 2 are modeled by the following third order dynamic system with $(a_1, b_1, c_1, d_1) = (2, 1, 1, 1)$ and $(a_2, b_2, c_2, d_2) = (10, 2, 1, 0)$,

$$
\begin{aligned}
\dot{x}_k &= \left[\begin{array}{cc:c} 0 & 1 & 0 \\ -2 & 0 & c_k \\ \hdashline 0 & -d_k & -a_k \end{array}\right] x_k + \begin{bmatrix} 0 \\ 0 \\ b_k \end{bmatrix} u_k, \\
y_k &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x_k.
\end{aligned}
\tag{17}
$$

Nodes 3 to 6 are modeled by the fourth order system (18), with (a_k, b_k, c_k, d_k) being $(1, 10, 1, 0)$, $(1, 1, 10, 0)$, $(5, 1, 2, 5)$ and $(2, 1, 1, 0)$, for $k = 3, ..., 6$, respectively.

$$
\begin{aligned}
\dot{x}_k &= \left[\begin{array}{ccc:c} 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & c_k \\ \hdashline 0 & 0 & -d_k & -a_k \end{array}\right] x_k + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_k \end{bmatrix} u_k, \\
y_k &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_k.
\end{aligned}
\tag{18}
$$

The six nodes are all observable and controllable. Choose

$$
S = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
$$

and let

$$
\Pi_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \Gamma_k = \begin{bmatrix} 0 & \frac{d_k}{b_k} \end{bmatrix},
$$

for $k = 1, 2$, and

$$
\Pi_j = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Gamma_j = \begin{bmatrix} 0 & 0 \end{bmatrix},
$$

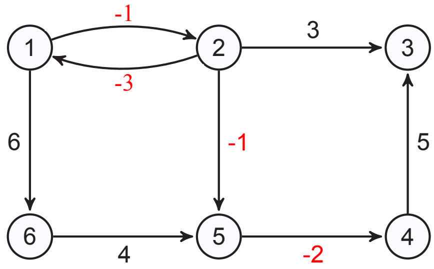
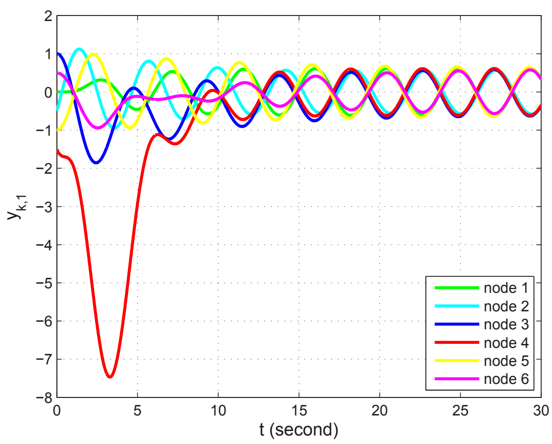


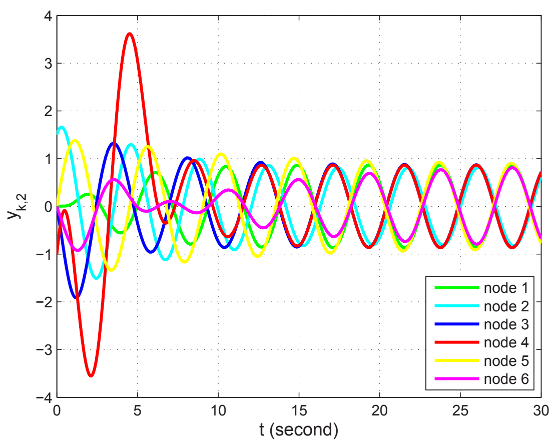
Fig. 1: A graph which has a spanning tree and is a structurally balanced.

for $j = 3, ..., 6$, which satisfy (15). The feedback gains K_k and the observer gains H_k are chosen using an LQG design for each individual system independently.

For a random initial state, the system output trajectories $y_{k,1}$ and $y_{k,2}$ are shown in Fig. 2. Clearly, it shows that the output bipartite consensus is achieved, with two subgroups $\mathcal{V}_1 = \{v_1, v_5, v_6\}$ and $\mathcal{V}_2 = \{v_2, v_3, v_4\}$.



(a) Trajectories of $y_{k,1}$ for $k = 1, \dots, 6$



(b) Trajectories of $y_{k,2}$ for $k = 1, \dots, 6$

Fig. 2: Evolution of the system outputs $y_{k,j}$, $k = 1, ..., 6$ and $j = 1, 2$, which show the output bipartite consensus of the corresponding system outputs.

5 Conclusion

This paper solved the output bipartite consensus problem for heterogeneous linear multi-agent systems over signed digraphs, which have both positive edges and negative edges. As the main result, through establishing an equivalence of output bipartite consensus and conventional output consensus for heterogeneous linear multi-agents, two output feedback controller are designed, inspired by [6, 7].

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