Adaptive Average Bipartite Consensus Control of High-order Multi-agent Systems on Coopetition Networks

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Abstract: In this paper, an average bipartite consensus control problem is investigated for a high-order multi-agent system with non-identical unknown nonlinear dynamics on a coopetition network. Firstly, the average bipartite consensus problem is formulated for a high-order multi-agent system. Then, linearly parameterized models are used to describe the unknown dynamics. Simultaneously, adaptive estimation laws are designed for the unknown dynamics and time-varying coupling weights as well. Furthermore, a bipartite consensus control is proposed for each agent with the help of the adaptive laws to guarantee the average bipartite consensus. The convergence of the close-loop multi-agent system under the proposed consensus control is analyzed. Finally, simulation results are provided to validate the theoretical results.

Key Words: Average bipartite consensus, coopetition network, high-order multi-agent system, unknown disturbances, time-varying coupling weights.

1 INTRODUCTION

During the last few years, consensus problems have been extensively studied for cooperative multi-agent systems [1] - [3]. However, the interactions among agents can not only be cooperative but also be competitive. For example, antagonistic relationship is common in some social systems [4], [5]. A signed graph is usually used to describe the coexistence of cooperation and competition [6] (i.e., coopetition), where a positive edge generally means collaboration, while a negative edge can represent an antagonistic interaction between two agents.

For cooperative multi-agent system, a widely investigated problem is the emergence of global consensus, in which all agents reach the same state in the long run. However, for cooperative-competitive (i.e., coopetitive) multi-agent system, another type of consensus phenomenon has been studied recently, i.e., bipartite consensus, which means that all agents converge to a unique value having same modulus but different signs [7]. Hu et al. investigated, in [8] - [9], the collective dynamics on coopetition network, which are described by directed signed graphs. Zhang [10] showed that a bipartite consensus of a multi-agent system with antagonistic interactions can be equivalent to a consensus problem for a cooperative multi-agent system. Valcher [11] considered a bipartite consensus for high-order coopetitive multi-agent systems. Hu [12] considered the bipartite consensus problem for a second-order multi-agent system suffering from unknown time-varying disturbances,and then designed an adaptive bipartite consensus control. By virtue of the technique of power integrator, Ma [13] addressed the bipartite output consensus problem for high-order multi-agent systems with signed digraph and input noises.

In recent years, many researchers have studied average consensus problems for cooperative multi-agent systems from various perspectives (see [14], [27] and references therein). To the best of the authors' knowledge, Altafini was the first to investigate the average bipartite consensus in [7]. Furthermore, Hu et al. showed, in [8], that if the coopetition network associated with the multi-agent system was structurally balanced, weight balanced and had a spanning tree, then an average bipartite consensus can be achieved. However, few results can be found for average bipartite consensus of high-order multi-agent systems. Motivated by the observations mentioned above, in this paper, we consider adaptive average bipartite consensus control of a high-order multi-agent system with non-identical unknown nonlinear dynamics on a coopetition network. We consider the average bipartite consensus problem by designing distributed adaptive consensus protocols with time-varying coupling weights. A projection method is used to design adaptive laws for the unknown dynamics. At the same time, the convergence of the close-loop multi-agent system is analyzed with the help of a Lyapunov function approach.

The main contributions of this paper are threefold. Firstly, the multi-agent system on a coopetition network is described by a high-order nonlinear dynamics with unknown disturbances. Secondly, the average bipartite consensus problem is investigated for the high-order multi-agent system. Finally, Adaptive bipartite consensus control design is developed together with adaptive estimation laws for unknown disturbances and time-varying coupling weights.

The remainder of this paper is organized as follows. In Sec-

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tion 2, some preliminaries are given for signed graph theory and the average bipartite consensus problem is formulated. Then some main results are presented to design the bipartite consensus control and analyze the convergence of the close-loop multi-agent system in Section 3. A simulation example is presented for illustration in Section 4. Some conclusion remarks are given in Section 5.

2 PROBLEM FORMULATION

2.1 COOPETITIVE NETWORK MODELING

An undirected signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is usually used to model an interaction network containing both cooperative and competitive interactions, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A}$ is the adjacency matrix of the signed graph. In this paper, we assume that $\mathcal{A}$ is a $(-1, 0, 1)$-matrix. If there exists an edge $(i, j) \in \mathcal{E}$, then $a_{ij} \neq 0$, otherwise, $a_{ij} = 0$. The interaction between the ith agent and the jth agent is cooperative if and only if $a_{ij} > 0$ and competitive if and only if $a_{ij} < 0$. Note that the signed graph is assumed to be a simple graph without self-loops in this paper, i.e., $a_{ii} = 0$. The degree matrix $D = \mathrm{diag}(d_1, d_2, \cdots, d_N) \in \mathbb{R}^{N\times N}$ is a diagonal matrix with its diagonal element being $d_i = \sum\limits_{j\in N_i} |a_{ij}|$, $N_i = \{j|(i, j) \in \mathcal{E}\}$. The Laplacian of an undirected signed graph $\mathcal{G}$ is defined as

$$L = D - \mathcal{A} \tag{1}$$

which is a symmetric matrix.

Definition 1 *(Structural Balance [7, 8]) A signed graph $\mathcal{G}$ is said to be structurally balanced if it contains a bipartition of the nodes $\mathcal{V}_1 = \{1, \cdots, N_0\}, \mathcal{V}_2 = \{N_0 + 1, \cdots, N\}, \mathcal{V} = \mathcal{V}_1 \bigcup \mathcal{V}_2, \mathcal{V}_1 \bigcap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \geq 0, \forall i, j \in \mathcal{V}_p$ $(p \in 1, 2)$; $a_{ij} \leq 0, \forall i \in \mathcal{V}_p, j \in \mathcal{V}_q, p \neq q$ $(p, q \in 1, 2)$. Otherwise, it is called a structurally unbalanced graph.*

In order to characterize the relationships among agents belonging to the two subnetworks $\mathcal{V}_1$ and $\mathcal{V}_2$, a gauge transformation is defined by the following diagonal matrix

$$S = \mathrm{diag}(s_1, \cdots, s_N) \in \mathbb{R}^{N\times N}, \tag{2}$$

where the diagonal entry $s_i = 1$ for $i \in \mathcal{V}_1$ and $s_i = -1$ for $i \in \mathcal{V}_2$. It is not difficult to show that $S^{-1} = S = S^T$. We know that the adjacency matrix associated with the signed graph $\mathcal{G}$ has the following block form by appropriately reordering the agents

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{pmatrix}$$

where $\mathcal{A}_{11} = \mathcal{A}_{11}^T \in \mathbb{R}^{N_0\times N_0}, \mathcal{A}_{12} = \mathcal{A}_{21}^T \in \mathbb{R}^{N_0\times(N-N_0)}, \mathcal{A}_{22} = \mathcal{A}_{22}^T \in \mathbb{R}^{(N-N_0)\times(N-N_0)}$. It is noted that $\mathcal{A}_{11}$ and $\mathcal{A}_{22}$ are nonnegative submatrices, while $\mathcal{A}_{21}$ is a nonpositive submatrix. From [8], we have known that

$$S\mathcal{A}S = \begin{pmatrix} \mathcal{A}_{11} & -\mathcal{A}_{12} \\ -\mathcal{A}_{21} & \mathcal{A}_{22} \end{pmatrix},$$

which means that, through the gauge transformation, the matrix $\mathcal{A}$ is similar with a nonnegative adjacency matrix. Let $\lambda_i(L)$ $(i = 1, \cdots, N)$ be the eigenvalues of the signed Laplacian matrix L defined by (1). Suppose that $\lambda_1(L) \leq \lambda_2(L) \leq \cdots \lambda_N(L)$. The following two lemmas are presented to show the spectral properties of a signed Laplacian matrix.

Lemma 1 *([7]) Given a signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, let L be the signed Laplacian matrix. The following facts are equivalent:*

- *$\mathcal{G}$ is structurally balanced;*
- *$\mathcal{A}$ is equivalent,through a gauge transformation, to a nonnegative matrix;*
- *$\lambda_1(L) = 0$.*

Furthermore, it is not difficult to obtain the following lemma.

Lemma 2 *Let L be the Laplacian matrix of an undirected signed graph $\mathcal{G}$ associated with a coopetition network. If $\mathcal{G}$ is structurally balanced and connected as well, then the following statements are true.*

- *L has a simple zero eigenvalue with $\mathbf{s}$ $=\mathrm{col}(s_1, s_2, \cdots, s_N) \in \mathbb{R}^N$ as its corresponding eigenvector, and all the non-zero eigenvalues are strictly positive.*
- *The smallest nonzero eigenvalue $\lambda_2(L)$ of the matrix L satisfies*

$$\lambda_2(L) = \min_{x^T\mathbf{s}=0, x\neq 0} \frac{x^T Lx}{x^T x}.$$

2.2 AVERAGE BIPARTITE CONSENSUS PROBLEM

Consider N agents with distinct dynamics. Dynamics of the ith $(i = 1, 2, \cdots, N)$ agent is described by

$$\dot{x}_i = Ax_i + Bu_i + Bf_i(x_i) \tag{3}$$

where $x_i \in \mathbb{R}^n$ is the state of agent i, $u_i \in \mathbb{R}^p$ is the control input, $f_i(x_i) \in \mathbb{R}^p$ is an unknown time-varying disturbance, and $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}$ are constant matrices. Additionally, we assume that (A, B) is a controllable pair. We say that all the agents reach an average bipartite consensus if the consensus error $x_i(t) - \frac{1}{N}\sum\limits_{j=1}^{N} s_i s_j x_j(t)$ approaches to zero as time goes to infinity, namely,

$$\lim_{t\to+\infty} x_i(t) - \frac{1}{N}\sum_{j=1}^{N} s_i s_j x_j(t) = 0. \tag{4}$$

Remark 1 *For agent $i \in \mathcal{V}_1$, one has $\lim\limits_{t\to+\infty} x_i(t) - \frac{1}{N}\sum\limits_{j=1}^{N} s_j x_j(t) = 0$. Correspondingly, for agent $i \in \mathcal{V}_2$ $\lim\limits_{t\to+\infty} x_i(t) + \frac{1}{N}\sum\limits_{j=1}^{N} s_j x_j(t) = 0$.*

3 MAIN RESULTS

Suppose that the unknown nonlinear dynamics $f_i(x_i)$ $(i = 1, 2, \cdots, N)$ are parameterized as

$$f_i(x_i) = \phi_i^T(x_i)w_i, \quad i = 1, 2, \cdots, N, \tag{5}$$

where $\phi_i \in \mathbb{R}^{m\times p}$ are the basis functions, $w_i \in \mathbb{R}^m$ are unknown constant parameters to be estimated. Each agent i estimates the parameter vector w_i by $\hat{w}_i \in \mathbb{R}^m$, thus we get the estimator $\hat{f}_i(x_i) \in \mathbb{R}^p$. So one has

$$\hat{f}_i(x_i) = \phi_i^T(x_i)\hat{w}_i, \quad i = 1, 2, \cdots, N. \tag{6}$$

Define $\Phi = \mathrm{diag}(\phi_1, \phi_2, \cdots, \phi_N) \in \mathbb{R}^{mN\times pN}$, $W = \mathrm{col}(w_1, w_2, \cdots, w_N) \in \mathbb{R}^{mN}$, $\hat{W} = \mathrm{col}(\hat{w}_1, \hat{w}_2, \cdots, \hat{w}_N) \in \mathbb{R}^{mN}$, then the global disturbance $f(x) = \mathrm{col}(f_1(x_1), f_2(x_2), \cdots, f_N(x_N)) \in \mathbb{R}^{pN}$ and its estimator $\hat{f}(x) = \mathrm{col}(\hat{f}_1(x_1), \hat{f}_2(x_2), \cdots, \hat{f}_N(x_N)) \in \mathbb{R}^{pN}$ can be expressed, respectively, by

$$f(x) = \Phi^T W, \tag{7}$$

and

$$\hat{f}(x) = \Phi^T \hat{W}. \tag{8}$$

3.1 CONTROLLER DESIGN

By using the relative states of neighboring agents, a distributed adaptive consensus protocol with a time-varying coupling weight is designed for agent i as follows:

$$\begin{aligned} u_i &= (r_i + \sigma)K \sum_{j\in N_i} |a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j) - \phi_i^T \hat{w}_i, \\ \dot{r}_i &= \varepsilon_i\{-\alpha r_i + [\sum_{j\in N_i} |a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j)]^T \Gamma \\ &\quad [\sum_{j\in N_i} |a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j)]\}, \end{aligned} \tag{9}$$

where $r_i(t)$ denotes the time-varying coupling weight associated with the i'th agent, σ , ε_i $(i = 1, 2, \cdots, N)$ and α are positive constants, $K \in \mathbb{R}^{p\times n}$ and $\Gamma \in \mathbb{R}^{n\times n}$ are the feedback gain matrices to be designed.

We define a consensus error by $e_i(t) = x_i(t) - \frac{1}{N}\sum_{j=1}^{N} s_i s_j x_j(t) \in \mathbb{R}^n$ and $M = I_N - \frac{1}{N}ss^T \in \mathbb{R}^{N\times N}$, thus we get $e = (M \otimes I_n)x$, where $e = \mathrm{col}(e_1, e_2, \cdots, e_N) \in \mathbb{R}^{nN}$. Then one has

$$\begin{aligned} \dot{e} &= \hat{A}e + (M \otimes B)u + (M \otimes B)\Phi^T W, \\ u &= (R + \sigma I)L \otimes Ke - \Phi^T \hat{W}, \end{aligned} \tag{10}$$

where $\hat{A} = I_N \otimes A \in \mathbb{R}^{nN\times nN}$, $R = \mathrm{diag}(r_1, r_2, \cdots, r_N) \in \mathbb{R}^{N\times N}, u = \mathrm{col}(u_1, u_2, \cdots, u_N) \in \mathbb{R}^{pN}$.

The error system (10) can be written in a close-loop form as follows:

$$\dot{e} = \hat{A}e + (M(R+\sigma I)L \otimes BK)e - M \otimes B\Phi^T \bar{\Omega}, \tag{11}$$

where $\tilde{w}_i = \hat{w}_i - w_i \in \mathbb{R}^m, \bar{\Omega} = \hat{W} - W = \mathrm{col}(\tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_N) \in \mathbb{R}^{mN}$.

Assumption 1 *It is assumed that the initial estimation error $\tilde{w}_i$ associated with the unknown parameter w_i satisfies*

$$\tilde{w}_i^T(0)\tilde{w}_i(0) \le W_i^{\max}. \tag{12}$$

An adaptive law is designed to estimate w_i as follows,

$$\dot{\hat{w}}_i = -\phi_i K \sum_{j\in N_i} |a_{ij}|(e_i - \mathrm{sign}(a_{ij})e_j), \tag{13}$$

if $\tilde{w}_i^T\tilde{w}_i < W_i^{\max}$ or $\tilde{w}_i^T\tilde{w}_i = W_i^{\max}$ and $-\tilde{w}_i^T\phi_i K \sum_{j\in N_i} |a_{ij}|(e_i - \mathrm{sign}(a_{ij})e_j) \le 0$; and

$$\dot{\hat{w}}_i = (\frac{\tilde{w}_i}{\tilde{w}_i^T\tilde{w}_i}\tilde{w}_i^T - I_m)\phi_i K \sum_{j\in N_i} |a_{ij}|(e_i - \mathrm{sign}(a_{ij})e_j), \tag{14}$$

if $\tilde{w}_i^T\tilde{w}_i = W_i^{\max}$ and $-\tilde{w}_i^T\phi_i K \sum_{j\in N_i} |a_{ij}|(e_i - \mathrm{sign}(a_{ij})e_j) > 0$. The adaptive law can be further written in a compact form as follows:

$$\dot{\bar{\Omega}} = \begin{cases} -\Phi L \otimes Ke, \quad if \quad \bar{\Omega}^T\bar{\Omega} < W_{\max} \quad or \\ \qquad \bar{\Omega}^T\bar{\Omega} = W_{\max} \quad and \quad -\bar{\Omega}^T\Phi L \otimes Ke \le 0, \\ (\frac{\bar{\Omega}}{\bar{\Omega}^T\bar{\Omega}}\bar{\Omega}^T - I_{mN})\Phi L \otimes Ke, \quad if \quad \bar{\Omega}^T\bar{\Omega} = W_{\max} \\ \qquad and \quad -\bar{\Omega}^T\Phi L \otimes Ke > 0, \end{cases} \tag{15}$$

where $W_i^{\max}$ and $W_{\max}$ are positive constants, $W_{\max} = \max_{i=1,\cdots,N} W_i^{\max}$.

The following lemma states the convergence of the estimation error $\tilde{w}_i$ for the unknown parameter w_i. A sketch of the proof is given due to space limitation.

Lemma 3 *For the adaptive law given by (13) - (14), if the initial value of the vector $\tilde{w}_i$ satisfies (12), then one has*

$$\bar{\Omega}^T\bar{\Omega} \le W_{\max}, \quad \forall t \ge 0. \tag{16}$$

Proof: To prove the result, the following three cases are considered:

1) When $\bar{\Omega}^T\bar{\Omega} < W_{\max}$, the conclusion follows.

2) When $\bar{\Omega}^T\bar{\Omega} = W_{\max}$ and $-\bar{\Omega}^T\Phi L \otimes Ke \le 0$

$$\bar{\Omega}^T\dot{\bar{\Omega}} = -\bar{\Omega}^T\Phi L \otimes Ke \le 0. \tag{17}$$

3) When $\bar{\Omega}^T\bar{\Omega} = W_{\max}$ and $-\bar{\Omega}^T\Phi L \otimes Ke > 0$,

$$\bar{\Omega}^T\dot{\bar{\Omega}} = -\bar{\Omega}^T\Phi L\otimes Ke + \bar{\Omega}^T\frac{\bar{\Omega}}{\bar{\Omega}^T\bar{\Omega}}\bar{\Omega}^T\Phi L\otimes Ke = 0. \tag{18}$$

Thus the proof is completed.

3.2 CONVERGENCE ANALYSIS

Now a main result is presented to show the convergence of the close-loop system (11).

Theorem 1 *Consider the multi-agent system (3). Assume that the coopetition network $\mathcal{G}$ is structurally balanced and connected. Then the average bipartite consensus can be achieved under the control law (9) and the adaptive law (13) - (14) with $\sigma > \frac{1}{2\lambda_2(L)}$, $K = -B^TP \in \mathbb{R}^{p\times n}, \Gamma = PBB^TP \in \mathbb{R}^{n\times n}$, where $P > 0$ is the unique solution to the following algebraic Riccati equation (ARE):*

$$A^TP + PA - PBB^TP + I = 0. \tag{19}$$

Proof: We define a Lyapunov candidate function

$$V = e^T(L\otimes P)e + \bar{\Omega}^T\bar{\Omega} + \sum_{i=1}^{N}\frac{r_i^2}{\varepsilon_i}, \qquad (20)$$

where $V_1 = e^T(L\otimes P)e, V_2 = \bar{\Omega}^T\bar{\Omega}, V_3 = \sum_{i=1}^{N}\frac{r_i^2}{\varepsilon_i}$.

The derivative of V_1 is given by

$$\begin{aligned}
\dot{V}_1 &= \dot{e}^T(L\otimes P)e + e^T(L\otimes P)\dot{e}\\
&= [\hat{A}e + (M(R+\sigma I)L\otimes BK)e - M\otimes B\Phi^T\bar{\Omega}]^T L\otimes Pe\\
&+ e^T L\otimes P[\hat{A}e + (M(R+\sigma I)L\otimes BK)e - M\otimes B\Phi^T\bar{\Omega}]\\
&= e^T L\otimes (A^TP+PA)e + 2e^T[L(R+\sigma I)L\otimes PBK]e\\
&- 2e^T[L\otimes PB]\Phi^T\bar{\Omega}.
\end{aligned} \qquad (21)$$

Due to $K = -B^TP$, then we know that

$$\begin{aligned}
&e^T[L(R+\sigma I)L\otimes PBK]e\\
&= -\sum_{i=1}^{N}(r_i+\sigma)[\sum_{j\in N_i}|a_{ij}|(e_i - \mathrm{sign}(a_{ij})e_j)]^T PBB^TP\\
&[\sum_{j\in N_i}|a_{ij}|(e_i - \mathrm{sign}(a_{ij})e_j)].
\end{aligned}$$

When we compute the derivative of V_2, the following two cases are considered.

1) When $\bar{\Omega}^T\Phi(L\otimes B^TP)e \le 0$, $\dot{\bar{\Omega}} = \Phi(L\otimes B^TP)e$, thus

$$\begin{aligned}
\dot{V}_2 &= 2\dot{\bar{\Omega}}^T\bar{\Omega}\\
&= 2e^T(L\otimes PB)\Phi^T\bar{\Omega}
\end{aligned}$$

and

$$\dot{V}_1+\dot{V}_2 = e^T\{L\otimes(A^TP+PA) - 2[L(R+\sigma I)L\otimes PBB^TP]\}e$$

2) When $\bar{\Omega}^T\Phi(L\otimes B^TP)e > 0$, $\dot{\bar{\Omega}} = \Phi(L\otimes B^TP)e - \frac{\bar{\Omega}}{\bar{\Omega}^T\bar{\Omega}}\bar{\Omega}^T\Phi(L\otimes B^TP)e$, therefore

$$\begin{aligned}
\dot{V}_2 &= 2\dot{\bar{\Omega}}^T\bar{\Omega}\\
&= 2e^T(L\otimes PB)\Phi^T\bar{\Omega} - 2e^T(L\otimes PB)\Phi^T\bar{\Omega} = 0
\end{aligned}$$

and

$$\begin{aligned}
&\dot{V}_1+\dot{V}_2\\
&= e^T L\otimes(A^TP+PA)e - 2e^T[L(R+\sigma I)L\otimes PBB^TP]e\\
&- 2e^T[L\otimes PB]\Phi^T\bar{\Omega}\\
&< e^T\{L\otimes(A^TP+PA) - 2[L(R+\sigma I)L\otimes PBB^TP]\}e
\end{aligned}$$

In both of these two cases, we can always have

$$\begin{aligned}
\dot{V}_1+\dot{V}_2 \le &e^T\{L\otimes(A^TP+PA)\\
&- 2[L(R+\sigma I)L\otimes PBB^TP]\}e.
\end{aligned} \qquad (22)$$

The derivative of V_3 is given by

$$\begin{aligned}
\dot{V}_3 &= \sum_{i=1}^{N}2r_i\{-\alpha r_i + [\sum_{j\in N_i}|a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j)]^T\\
&\Gamma[\sum_{j\in N_i}|a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j)]\}\\
&= -\sum_{i=1}^{N}2\alpha r_i^2 + 2e^T[LRL\otimes\Gamma]e.
\end{aligned} \qquad (23)$$

Thus, from the inequalities (22), (23), one has

$$\begin{aligned}
\dot{V} &\le e^T L\otimes(A^TP+PA)e - 2\sigma e^T[L^2\otimes PBB^TP]e\\
&- 2\sum_{i=1}^{N}\alpha r_i^2\\
&\le e^T L\otimes(A^TP+PA)e - 2\sigma e^T[L^2\otimes PBB^TP]e.
\end{aligned} \qquad (24)$$

Since $\mathcal{G}$ is structurally balanced and connected, zero is a simple eigenvalue of L and all the other eigenvalues are positive. Let $T = \begin{pmatrix}T_1 & T_2\end{pmatrix}$ be a unitary matrix with $T_1 \in \mathbb{R}^N$ and $T_2 \in \mathbb{R}^{N\times N-1}$ such that $T^TLT = \Lambda = \mathrm{diag}(\lambda_1(L), \lambda_2(L), \cdots, \lambda_N(L))$, where $0 = \lambda_1(L) < \lambda_2(L) \le \cdots \le \lambda_N(L)$ are the eigenvalues of the Laplacian matrix L. Let $\xi = \mathrm{col}(\xi_1, \xi_2, \cdots, \xi_N) = (T^T\otimes I_n)e$, thus

$$\begin{aligned}
\dot{V} &\le e^T L\otimes(A^TP+PA)e - 2\sigma e^T[L^2\otimes PBB^TP]e\\
&= \xi^T[\Lambda\otimes(A^TP+PA)]\xi - 2\sigma\xi^T[\Lambda^2\otimes PBB^TP]\xi\\
&= \sum_{i=2}^{N}\lambda_i(L)\xi_i^T[A^TP+PA-2\sigma\lambda_i(L)PBB^TP]\xi_i.
\end{aligned}$$

Define $\Lambda_1 = \mathrm{diag}(\lambda_2(L), \lambda_3(L), \cdots, \lambda_N(L))$, then we know $L = T\Lambda T^T = T_2\Lambda_1T_2^T$. According to the assumption $\sigma\lambda_2(L) > \frac{1}{2}$, and the ARE (19), $A^TP + PA - PBB^TP + I = 0$, one has

$$\begin{aligned}
\dot{V} &\le \sum_{i=2}^{N}\lambda_i(L)\xi_i^T[A^TP+PA-2\sigma\lambda_i(L)PBB^TP]\xi_i\\
&< -\sum_{i=2}^{N}\lambda_i(L)\xi_i^T\xi_i\\
&= -e^T(T_2\Lambda_1T_2^T\otimes I_n)e\\
&= -e^T(L\otimes I_n)e\\
&\le -\lambda_2(L)e^Te.
\end{aligned} \qquad (25)$$

Thus the limit $V(t)$exists, i.e.,

$$\lim_{t\to\infty}V(t) = V(\infty)$$

From the Cauchy convergence criteria, for any $a_1 > 0$, there exists a positive integer N_1 such that

$$|\ V(t_{z+1}) - V(t_z)\ | < a_1, \forall z \ge N_1.$$

Equivalently,

$$\int_{t_z}^{t_{z+1}}\dot{V}(t)dt < a_1,$$

or

$$\int_{t_z}^{t_{z+1}} \dot{V}(t)dt > -a_1,$$

which implies that

$$\lim_{t\to\infty} \int_{t}^{t+\tau} e^T(t)e(t)dt = 0$$

Therefore, from Barbalat's Lemma, one has

$$\lim_{t\to\infty} e^T(t)e(t) = 0. \qquad (26)$$

Thus, all the agents can reach average bipartite consensus on the coopetition network. The proof is complete.

4 SIMULATION RESULTS

In this section, a simulation example is provided to validate the effectiveness of the theoretic results .
Consider a network of third-order integrators, described by (3), with

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The coopetition network is illustrated in Figure 1, where nodes $1, \cdots, 7$ represent agents $1, \cdots, 7$. The blue solid edges and the red dash edges represent the cooperative interactions and the competitive interactions among agents, respectively. Agents 1, 2, 3 and agents 4, 5, 6, 7, belong two antagonistic subgroups. The basis functions are given by $\phi_i(t) = \mathrm{col}(\cos((i+1)t), \sin((i+1)t)), i = 1, 2, \cdots, 7$. Figure 2 shows that the seven agents reach average bipartite consensus. The average bipartite consensus error $e_{i,k} = x_{i,k} - \frac{1}{7}\sum_{j=1}^{7} s_i s_k x_{j,k}, i, j = 1, 2, \cdots, 7, k = 1, 2, 3$ are demonstrated in Figure 3. Furthermore, Figure 4 indicates that the parameter estimation $\bar{\Omega}(t)$ is bounded as shown in Lemma 3.

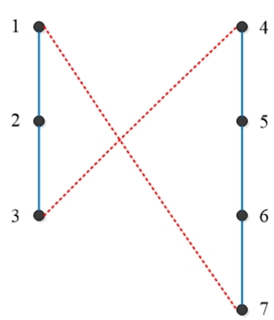
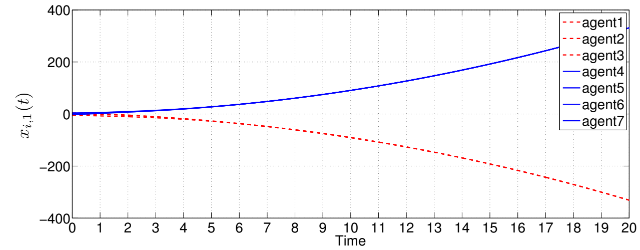


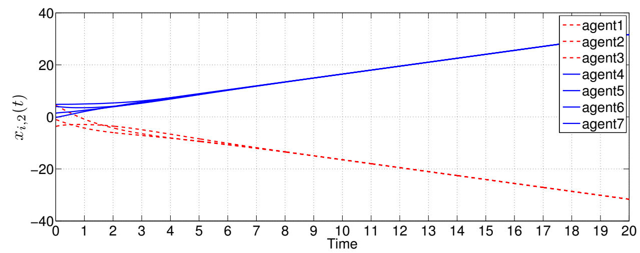
Figure 1: Coopetition network $\mathcal{G}$

5 CONCLUSION

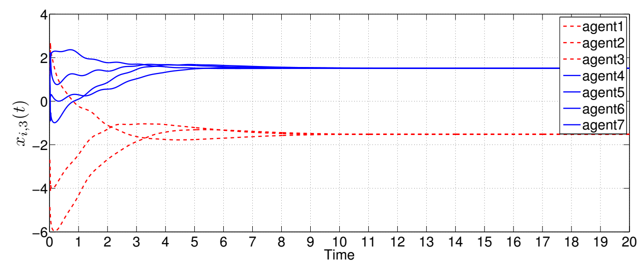
The paper has addressed an average bipartite consensus problem for high-order multi-agent systems on coopetition networks. By designing an adaptive consensus control protocol with time-varying coupling weights and adaptive laws, the convergence of the close-loop multi-agent system has been analyzed with the help of a Lyapunov function method. The simulation results have also been presented to demonstrate that all agents can reach average bipartite consensus under the proposed consensus control.



(a)



(b)



(c)

Figure 2: Evolution of the agent states $x_{i,k}(t)$ $(i = 1, \cdots, 7)$ $(k = 1, 2, 3)$

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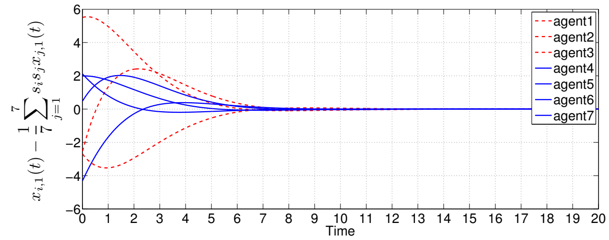
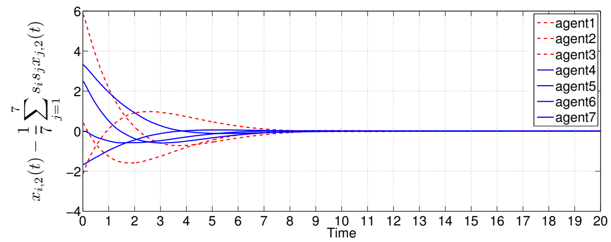
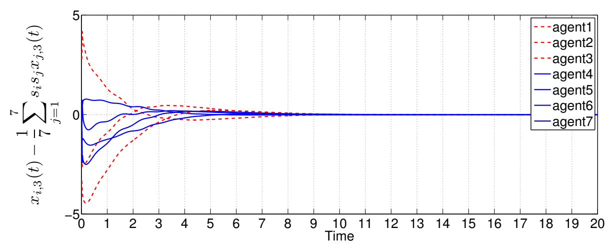


(a)



(b)



(c)

Figure 3: Evolution of the average bipartite consensus error $e_{i,k}(t)$ $(i = 1, \cdots, 7)$ $(k = 1, 2, 3)$

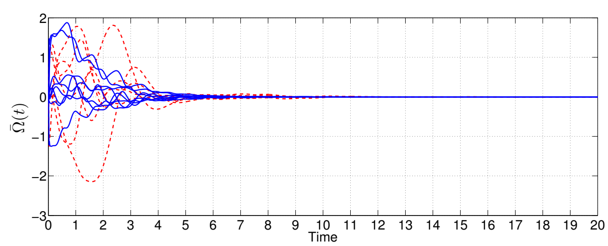


Figure 4: Evolution of the parameter estimate error $\bar{\Omega}(t)$

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