CS170–Spring 2021 — Homework nSolutions

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1. Study Group

None

2. Max Flow, Min Cut, and Duality

(a)

$$\begin{gathered} min \;\; 7x_1 + 5x_2 + 4x_3 + 4x_4 + 7x_5 \\ x_1 + 3x_3 \geq 1 \\ x_1 + x_4 + x_5 \geq 1 \\ x_2 + x_5 \geq 1 \end{gathered}$$

(b) The cut are $\{x_1, x_2\}, \{x_1, x_4, x_5\}, \{x_2, x_3, x_4\}, \{x_3, x_5\}$. For each cut, setting $x_e = 1$ for every edge crossing this cut and $x_e = 0$ for every edge not crossing this cut gives a feasible solution to the dual program.

(c) The dual problem models the min-cut problem. By LP duality, the answer for min-cut problem equals the one for max-flow problem.

3. How to Gamble With Little Regret

(a) The casino can set c_1^t for $1 \leq t \leq T$, and set any expert's cost = 0 for all days, then $max_C(\mathbb{E}(R)) = 1$.

(b) On day t, the casino can set the cost for expert i ($p_i^t = 1$) to 1, and the cost for all the other experts to 0. Then $max_C(\mathbb{E}(R)) = 1$.

(c) If $p_m = min_{1 \leq i \leq n}\, p_i$. The casino can set the cost for expert m to 1, and the cost for all the other experts to 0. Then $max_C(\mathbb{E}(R)) = 1 - p_m$. The uniform distribution minimizes the regret.

4. Global Mincut to Min s-t Cut

Choose any vertex as s, then for all the other vertices t, compute the min $s - t$ cut, the smallest min $s - t$ cut is the answer for the global mincut problem.