CS 170

1. Study Group

None

2. Counting Targets

(a) Define $f(s, i) :=$ the number of distinct length-i valid sequences with sum equal to s. Then the answer is $f(T, n)$.
Base cases: $f(s, i) = 0, s \leq 0, i > 0$, and $f(s, 1) = 1, 1 \leq s \leq m$.
Recurrence: $f(s, i) = \sum_{j=1}^{j=m} f(s - j, i - 1)$.
Runtime: $O(T * n * m) = O(m^2 n^2)$.

(b) Define $g(s, i) := \sum_{t=1}^{s} f(t, i)$. Then the answer is $g(T, n) - g(T - 1, n)$.
Base cases: $g(s, i) = 0, s \leq 0, i > 0$, and $g(s, 1) = s, 1 \leq s \leq m$.
Recurrence: $g(s, i) = g(s - 1, i) + g(s - 1, i - 1) - g(s - m - 1, i - 1)$.
Runtime: $(T * n) = O(mn^2)$.

3. Knightmare

Algorithm Description:
We use M-bit string to represent the configuration of rows of chessboard (1 means there is knight and 0 otherwise).
We solve the subproblem of the number of the valid configurations of $(n - 1) * M$ chessboard and use it to solve the $n * M$ case. Define $f(n, u, v)$ as the number of valid configurations of the first n rows with u being the $(n - 1)$-th row and v being the n-th row.

4. Geometric Knapsack

Refer to the solution.

5. GCD annihilation

Refer to the solution.