UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 1
Fall 2021

1. **Miscellaneous Review**

   (a) Show that the probability that exactly one of the events A and B occurs is $\mathbb{P}(A) + \mathbb{P}(B) - 2\mathbb{P}(A \cap B)$.

   (b) If A is independent of itself, show that $\mathbb{P}(A) = 0$ or 1.

2. **Choosing from Any Jar Makes No Difference**

   Each of k jars contains w white and b black balls. A ball is randomly chosen from jar 1 and transferred to jar 2, then a ball is randomly chosen from jar 2 and transferred to jar 3, etc. Finally, a ball is randomly chosen from jar k. Show that the probability that the last ball is white is the same as the probability that the first ball is white, i.e., it is $w/(w + b)$.

3. **Colored Sphere**

   Consider a sphere that has $\frac{1}{10}$ of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

   Hint: Carefully define some relevant events.

4. **Balls & Bins**

   Let $n \in \mathbb{Z}_{>1}$. You throw n balls, one after the other, into n bins, so that each ball lands in one of the bins uniformly at random. What is an appropriate sample space to model this scenario? What is the probability that exactly one bin is empty?