EECS 16A   Designing Information Devices and Systems I
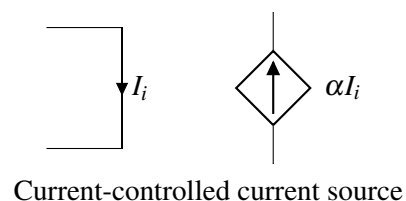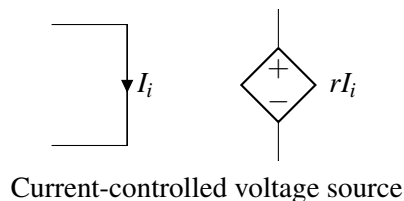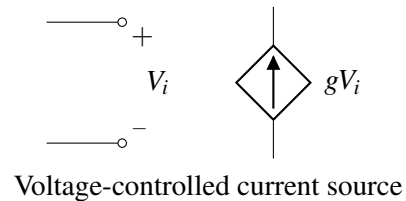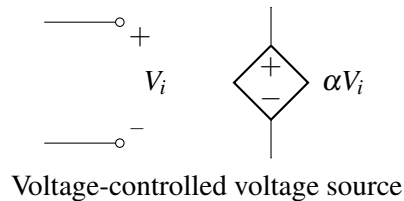## Spring 2020 Lecture Notes   Note 15A

15.1   Introduction: Superposition

Circuit analysis can be cumbersome to do by hand, and it can be difficult to understand the high-level behavior of complicated circuits given a schematic. We need tools we can use to both lessen the burden of analysis, and help us think about circuits and understand how they behave. Ultimately, our goal is to **design** interesting circuits, and in this note we will build up additional tools to help us.

15.2   Dependent Sources

At this point, we will introduce a new circuit element: dependent sources. Dependent sources act like the independent sources we've studied so far, but instead of a producing a single voltage/current, the voltage/current is *controlled* by something else in the circuit. There are four different types of dependent sources, shown below:

Voltage-controlled voltage source

Voltage-controlled current source

Current-controlled voltage source

Current-controlled current source

Here, the diamond symbol represents the source, which produces voltage or current proportional to a different voltage or current elsewhere in the circuit. Let's make this more concrete by looking at an example:
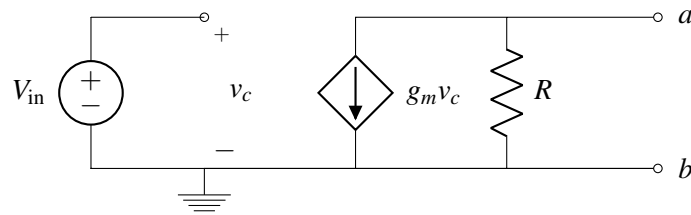
In this case, the current in the dependent source is equal a constant (g_m) times the voltage v_c, which is defined elsewhere in the circuit. Just like an independent current source, this dependent source will produce the same current, regardless of what is connected to it (unless v_c changes – then the current will change appropriately). As we'll see in future notes, dependent sources are useful tools for modeling many advanced circuit elements.

15.3 Superposition

In this section, we are going to look at circuits with multiple voltage or current sources. In particular, we would like to introduce a very useful idea in working with circuits of this type – superposition.

Let's think back to our seven-step circuit analysis procedure. To solve for the currents and node potentials in a circuit, we set up a matrix problem of the form $A\vec{x} = \vec{b}$ where $\vec{x}$ contained the unknown currents and node potentials, $\vec{b}$ contained the independent current and voltage sources, and A described the relationship between them. Since this matrix equation describes a real system, we know that there is a unique solution. Therefore, A is invertible:

$$\vec{x} = A^{-1}\vec{b}$$

This means that we can describe any current or node potential (ie. any element of $\vec{x}$) as a linear combination of the independent current and voltage sources (the elements of $\vec{b}$). For example, consider a circuit with n independent sources voltage sources $V_{s1} \ldots V_{sn}$, and m independent current sources $I_{s1} \ldots I_{sm}$. An arbitrary node potential u_i (or equivalently, an arbitrary current i_i) can be written as

$$u_i = \alpha_1 V_{s1} + \ldots + \alpha_n V_{sn} + \beta_1 I_{s1} + \ldots \beta_m I_{sm}$$

where the α's and β's are coefficients from inverting A. Since this equation is linear, we can calculate each term of this equation separately and then add them together at the end. For example, if we want to calculate the first term, $\alpha_1 V_{s1}$ we can set all of the other voltage and current sources to zero, then solve for u_i. Repeating this for every source then adding the results is equivalent to calculating u_i with all of the sources present. However, splitting up the calculations can help us see simplifications and patterns that might be less obvious with all of the sources present.

This procedure is known as **superposition** and can be summarized as follows:

**For each independent source k (either voltage source or current source)**
> **Set all other *independent* sources to 0**
>> • **Voltage source: replace with a wire**
>> • **Current source: replace with an open circuit**
> **Compute the circuit voltages and currents due to this source k**

Compute V_{out} by summing the $V_{out,k}$s for all k.

Now we ask the question: why does it make sense to replace voltage sources with wires? If we look at the I-V plot of a voltage source V_S, where I is the current going through the voltage source, then the plot would be a vertical line:
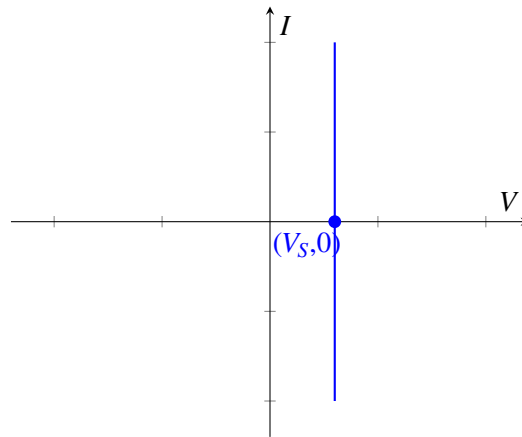

Now if we want to zero out this voltage source, we are setting $V_S = 0$. Then the I-V plot is exactly the y-axis.
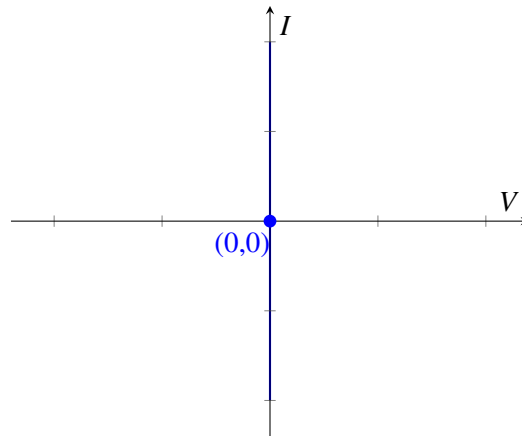

What does this mean? This means that it allows any current to go through, however the voltage drop always remains zero. This is exactly what a wire element (sometimes called a short circuit) does.

Now let's look at why we replace current sources with open circuits. If we plot the I-V graph of a current source I_S, we get the following:
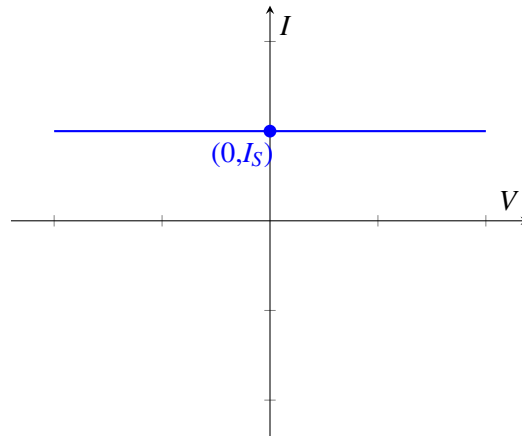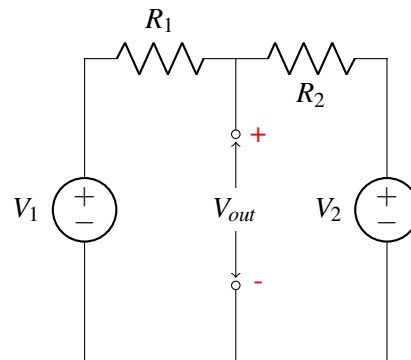
What if we turn off the current source? Then the I-V graph becomes the x-axis, i.e., the line $I = 0$.
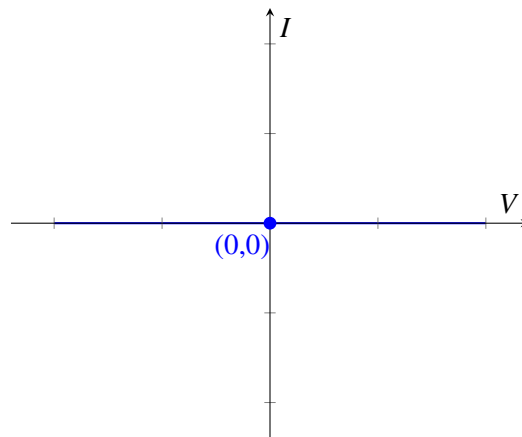
What does this mean? This means no matter what voltage you apply, there will be no current. This is equivalent to an open circuit.

Now let's illustrate this idea on the circuit below, where we would like to figure out V_{out}.

We first compute the output voltage due to V_1 and hence source V_2 will be replaced with a wire:
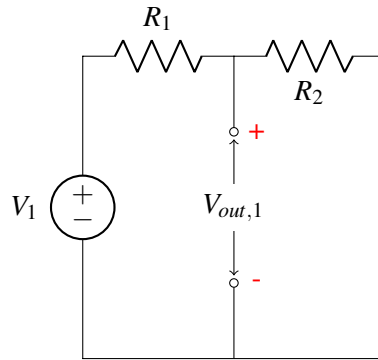
We can recognize this as a voltage divider circuit, and therefore we know that $V_{out,1} = \frac{R_2}{R_1+R_2}V_1$. Next we compute the output voltage due to V_2 and hence source V_1 will be replaced with a wire.
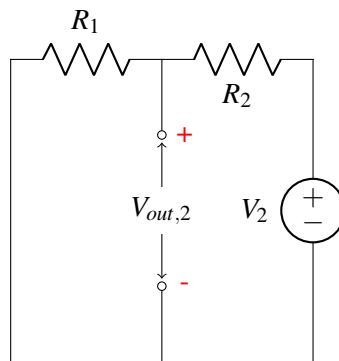
We again recognize that this is just a voltage divider circuit and therefore we can see that $V_{out,2} = \frac{R_1}{R_1+R_2}V_2$. Finally, to get the output voltage V_{out} of the original circuit, we add the contributions from each voltage source $V_{out} = V_{out,1} + V_{out,2} = \frac{R_2}{R_1+R_2}V_1 + \frac{R_1}{R_1+R_2}V_2$.

As a side note, we can apply the idea of replacing elements with equivalent elements (e.g. replacing a $V = 0$ voltage source with a wire) to resistors as well. When do resistors have an equivalent representation? We will try and demonstrate this graphically. Recall that by Ohm's law, the I-V graph across a resistor looks like
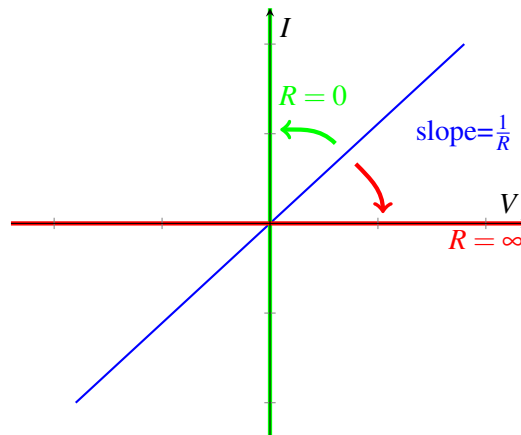
We know that the slope of the line is equal to $\frac{1}{R}$. What happens in the limit where R trends towards infinity? Then the line becomes the x-axis, which corresponds to an open circuit as we've seen earlier. Now what happens in the limit where R trends towards zero? The line becomes the y-axis, which corresponds to a wire.

To summarize, **zero voltage source and zero resistance are equivalent to wires (i.e. short circuits); zero current source and infinite resistance are equivalent to open circuits.**