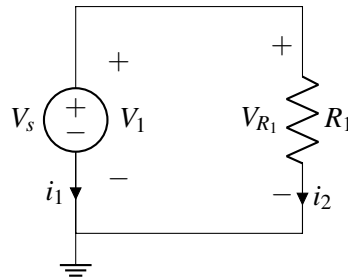EECS 16A   Designing Information Devices and Systems I
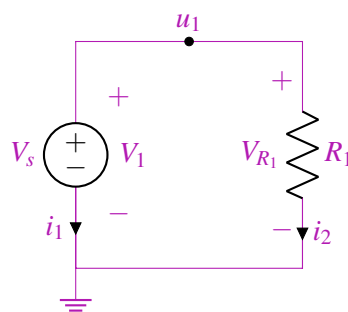Summer 2020   Discussion 3C

1. Passive Sign Convention and Power

(a) Suppose we have the following circuit and label the currents as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.

Answer: We'll start by solving the circuit for the unknown node potentials and currents.

The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = -1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

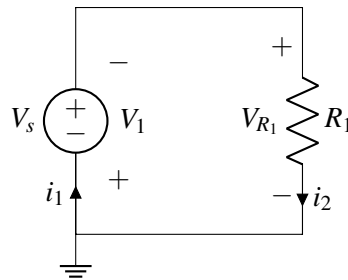From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

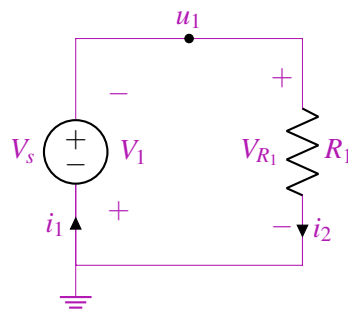Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1 V_s = -1\,\text{A} \cdot 5\,\text{V} = -5\,\text{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

(b) Suppose we change the label of the currents in the circuit to be as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.

Answer: We'll solve the circuit the same way as last time.

The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = 1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1 (-V_s) = 1\,\text{A} \cdot -5\,\text{V} = -5\,\text{W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

2. Modular Circuits

In this problem, we will explore the design of circuits that perform a set of (arbitrary) mathematical operations. (Note that the so-called analog signal processing – where these kinds of mathematical operations are performed on continuously-valued voltages by analog circuits – is extremely common in real-world applications; without this capability, essentially none of our radios or sensors would actually work.) Specifically, let's assume that we want to implement the block diagram shown below:
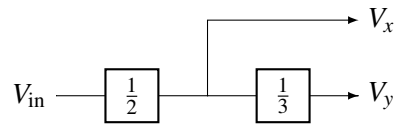
In other words, we want to implement a circuit with two outputs V_x and V_y, where $V_x = \frac{1}{2}V_{in}$ and $V_y = \frac{1}{3}V_x$.

(a) Design two voltage divider circuits that each independently would implement the two multiplications shown in the block diagram above (i.e., multiply by $\frac{1}{2}$ and multiply by $\frac{1}{3}$). Note that you do not need to include the input voltage sources in your design – you can simply define the input to each block as being at the appropriate potential (e.g., V_{in} or V_x).

Answer:
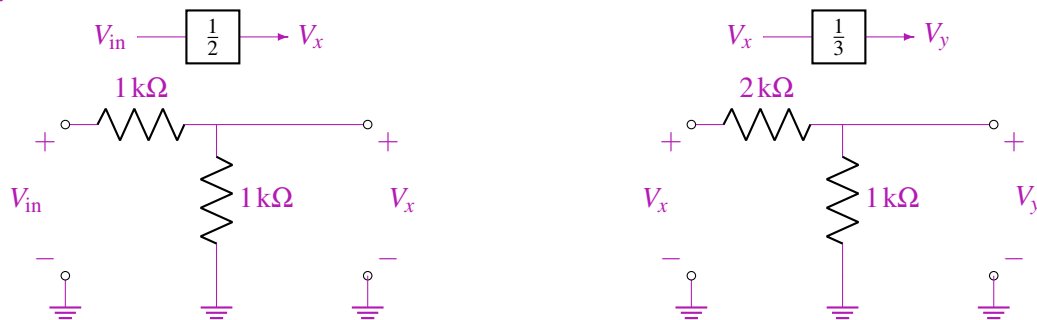
(b) Assuming that V_{in} is created by an ideal voltage source, implement the original block diagram as a circuit by directly replacing each block with the designs you came up with in part (a).

Answer:
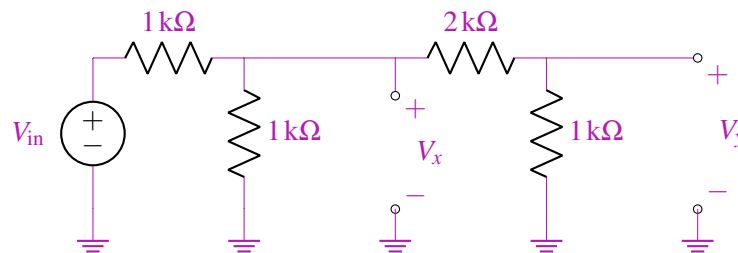

(c) For the circuit from part (b), find V_x and V_y. Do you get the desired relationship between V_y and V_x? How about between V_x and V_{in}? Be sure to explain why or why not each block retains its desired functionality.

Answer:

The relationship between V_y and V_x will be correct. We can apply the voltage divider equation to the two rightmost resistors to see this:

$$V_y = \frac{1\,\text{k}\Omega}{1\,\text{k}\Omega + 2\,\text{k}\Omega} V_x = \frac{1}{3} V_x$$

The relationship between V_x and V_{in} will not be correct. To see this, we can redraw the circuit applying resistance series and parallel rules.
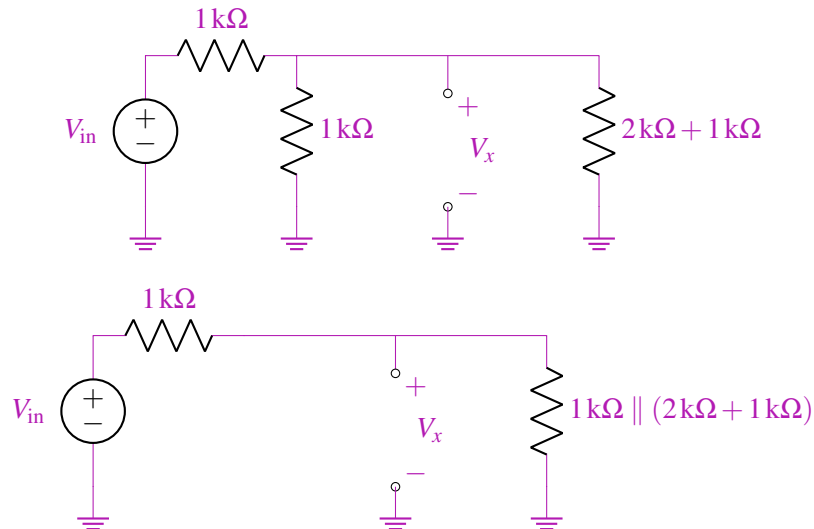
Now we can apply the voltage divider equation to see that:

$$V_x = \frac{1\,\text{k}\Omega \parallel (2\,\text{k}\Omega + 1\,\text{k}\Omega)}{1\,\text{k}\Omega + 1\,\text{k}\Omega \parallel (2\,\text{k}\Omega + 1\,\text{k}\Omega)} V_{in} = \frac{3}{7} V_{in}$$

which was not the desired relationship between V_x and V_{in}.

3. Practice: Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a Google Pixel phone, under typical usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel's battery has a battery capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will a Pixel's full battery last under typical usage conditions?

Answer:

300 mW of power at 3.8 V is about 79 mA of current. Our 2770 mA h battery can supply 79 mA for $\frac{2770\,\text{mAh}}{79\,\text{mA}} = 35\,\text{h}$, or about a day and a half.

(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has $1.602 \times 10^{-19}\,\text{C}$ of charge.)

Answer:

One hour has 3600 seconds, so the battery's capacity can be written as $2770\,\text{mAh} \times 3600\,\text{sh}^{-1} = 9.972 \times 10^6\,\text{mAs} = 9972\,\text{As} = 9972\,\text{C}$.

An electron has a charge of approximately $1.602 \times 10^{-19}\,\text{C}$, so $9972\,\text{C}$ is $\frac{9972\,\text{C}}{1.602 \times 10^{-19}\,\text{C}} \approx 6.225 \times 10^{22}$ electrons. That's a lot!

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Answer:

The battery capacity is $2770\,\text{mAh}$ at $3.8\,\text{V}$, which means the battery has a total stored energy of $2770\,\text{mAh} \cdot 3.8\,\text{V} = 10.5\,\text{Wh} = 10.5\,\text{Wh} \cdot 3600\,\text{s} = 37.9\,\text{kJ}$.

(d) Suppose PG&E charges \$0.12 per kWh. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?

Answer:

$2770\,\text{mAh}$ at $3.8\,\text{V}$ is $2770\,\text{mAh} \cdot 3.8\,\text{V} = 10.5\,\text{Wh}$, or $0.01\,\text{kWh}$. At \$0.12 per kWh, that is \$0.12·0.01 per day, or $\$0.12 \cdot 0.01 \cdot 31 = \$0.037$, or about 4 cents a month. Compare that to your cell phone data bill! Whew!

(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging into a wall plug. The wall plug can be modeled as a $5\,\text{V}$ voltage source and $200\,\text{m}\Omega$ resistor, as pictured in Figure 1. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\text{m}\Omega$, $1\,\Omega$, and $10\,\text{k}\Omega$? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of R_{bat}?
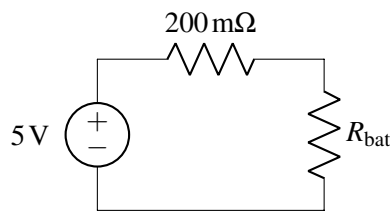

Figure 1: Model of wall plug, wire, and battery.

Answer:

The energy stored in the battery is $2770\,\text{mAh}$ at $3.8\,\text{V}$, which is $2.77\,\text{Ah} \cdot 3.8\,\text{V} = 10.5\,\text{Wh}$. We can find the time to charge by dividing this energy by power in W to get time in hours.

For $\underline{R_{\text{bat}} = 1\,\text{m}\Omega}$:
The total resistance seen by the battery is $1\,\text{m}\Omega + 200\,\text{m}\Omega = 201\,\text{m}\Omega$ (because the wire and R_{bat} are in series), so by Ohm's law, the current is $\frac{5\,\text{V}}{0.201\,\Omega} = 24.88\,\text{A}$. The voltage drop across R_{bat} is (again by Ohm's law) $24.88\,\text{A} \cdot 0.001\,\Omega = 0.024\,88\,\text{V}$. Then power is $0.024\,88\,\text{V} \cdot 24.88\,\text{A} = 0.619\,\text{W}$, and the total time to charge the battery is $\frac{10.5\,\text{Wh}}{0.619\,\text{W}} = 17\,\text{h}$.

For $\underline{R_{\text{bat}} = 1\,\Omega}$:
The total resistance seen by the battery is $1\,\Omega + 0.2\,\Omega = 1.2\,\Omega$, the current through the battery is $\frac{5\,\text{V}}{1.2\,\Omega} = 4.167\,\text{A}$, and the voltage across the battery is by Ohm's law $4.167\,\text{A} \cdot 1\,\Omega = 4.167\,\text{V}$. Then the power is $4.167\,\text{A} \cdot 4.167\,\text{V} = 17.36\,\text{W}$, and the total time to charge the battery is $\frac{10.5\,\text{Wh}}{17.36\,\text{W}} = 0.6\,\text{h}$, about $36\,\text{min}$.

For $R_{\text{bat}} = 10\,\text{k}\Omega$:
The total resistance seen by the battery is $10\,000\,\Omega + 0.2\,\Omega = 10\,000.2\,\Omega$, the current through the battery is $\frac{5\,\text{V}}{10\,000.2\,\Omega} \approx 0.5\,\text{mA}$, and the voltage across the battery is by Ohm's law $0.5\,\text{mA} \cdot 10\,\text{k}\Omega \approx 5\,\text{V}$ (up to 2 significant figures). Then the power is $5\,\text{V} \cdot 0.5\,\text{mA} = 2.5\,\text{mW}$, and the total time to charge the battery is $\frac{10.526\,\text{Wh}}{0.0025\,\text{W}} = 4210\,\text{h}$.