EECS 16A   Designing Information Devices and Systems I
Summer 2020
Discussion 5B

For Reference: Example Circuits

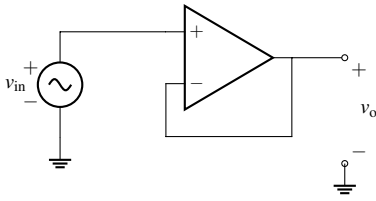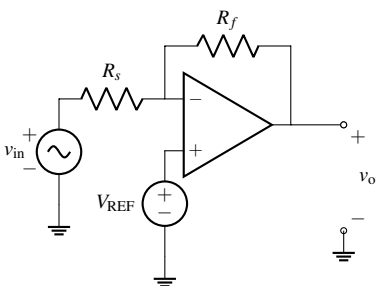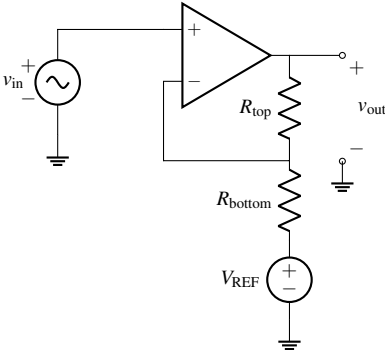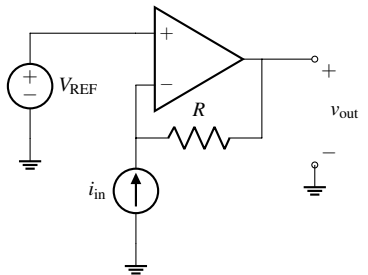| Voltage Divider	Voltage Summer	Unity Gain Buffer
$V_{R2} = V_S \left(\frac{R_2}{R_1+R_2} \right)$	$V_{\text{out}} = V_1 \left(\frac{R_2}{R_1+R_2} \right) + V_2 \left(\frac{R_1}{R_1+R_2} \right)$	$\frac{v_{\text{out}}}{v_{\text{in}}} = 1$

| Inverting Amplifier	Non-inverting Amplifier	Transresistance Amplifier
$v_{\text{out}} = v_{\text{in}} \left(-\frac{R_f}{R_s} \right) + V_{\text{REF}} \left(\frac{R_f}{R_s} + 1 \right)$	$v_{\text{out}} = v_{\text{in}} \left(1 + \frac{R_{\text{top}}}{R_{\text{bottom}}} \right) - V_{\text{REF}} \left(\frac{R_{\text{top}}}{R_{\text{bottom}}} \right)$	$v_{\text{out}} = i_{\text{in}}(-R) + V_{\text{REF}}$

1. **Noise Cancelling Headphones**

   The basic goal of a noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

   Answer:

   There are a lot of different solutions for this problem. This solution is aggressive and minimal, so be patient with your understanding. If your solution solves the same problem, you will receive credit.

   (a) Let's start by looking at the most basic part of the headphones: driving the speaker itself with the audio stream we would like to hear. In our system, the source of the audio comes from a digital-to-analog converter (DAC) that translates digital bits to analog voltages. It can be modeled as a voltage source

with min/max values of $0\,\text{V}$ and $1\,\text{V}$ and a $50\,\Omega$ source resistance. The speaker can be modeled as an $8\,\Omega$ resistor, but in order to produce loud enough sounds and not damage the speaker (driving the speaker with non-zero average voltage can damage the transducer within the speaker), it needs to be driven with a range of $-1.5\,\text{V}$ to $1.5\,\text{V}$ (relative to the ground connected to the DAC, which is the same ground used throughout the system).
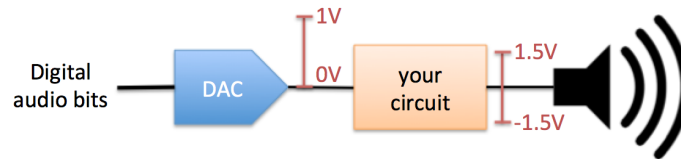

You are provided two voltage sources with values $-1.5\,\text{V}$ and $1.5\,\text{V}$, an op-amp, and any resistors you would like. Design a circuit that could drive the speaker while meeting the specifications above. Draw a block diagram for the circuit you design before you actually determine the components necessary.

Answer:

Let's go through the design method.

Step 1:

The goal of this circuit is to have transform an input that ranges from $0\,\text{V}$ to $1\,\text{V}$ into an output that ranges from $-1.5\,\text{V}$ to $1.5\,\text{V}$. Our source can be represented as shown below:
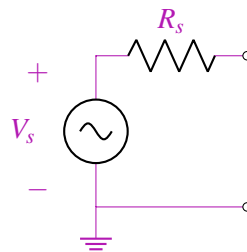
Our load is an $8\,\Omega$ resistor.

Step 2:

To achieve the goal, we need 4 things:

   i. Shift the signal to center at $0\,\text{V}$.

  ii. Provide gain to the signal to achieve a $3\,\text{V}$ range.

 iii. Maintain a low output resistance, i.e. be able to drive the $8\,\Omega$ resistor.

 iv. Maintain a high input resistance, i.e. not load the input signal.

The order of shifting and amplifying is determined by the supply voltage of the opamps. Notice if we amplify before we shift, our opamp needs to output up to $3\,\text{V}$, while we only have voltage sources of $+1.5\,\text{V}$ or $-1.5\,\text{V}$ to power the opamp with. When we shift the input signal, the shifted signal does not have to have the same amplitude. However, we can fix this by adjusting the gain of the amplifier accordingly. If we make the amplitude of the shifted input signal N, the block diagram is shown below.
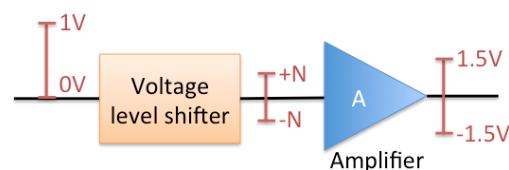
Step 3:

To solve the third and fourth points, we can simply use an op-amp to drive the output, since, in an ideal op-amp, the inputs have infinite input resistance and the output behaves like an ideal voltage source. We can use a non-inverting amplifier circuit to provide the gain of more than 1, solving the second point. Thus, we are left with the voltage shifter.

It turns out we can use a voltage summer – after all, shifting a voltage is nothing more than subtracting a fixed value. We already derived that the output of a voltage divider is $kV_1 + (1-k)V_2$, where $k = \frac{R_2}{R_1+R_2}$, so this is a matter of picking appropriate values for V_2 and k and using V_1 as the input. We also realize that there is some gain $k < 1$ applied to the input. Thus, we have to adjust the amplifier gain to $\frac{A}{k} = \frac{3}{k}$ ($A = 3$ since our output range is 3 V while our input range is 1 V).

Let's build the voltage shifter. We can only use the $-1.5\,\text{V}$ voltage source for V_2 if we want to shift the signal level down, i.e. subtract voltage. We also know that we want the shift to be half of the actual signal, since we want to center the signal around 0. Since V_1, the DAC voltage, ranges between 0 V and 1 V, the median voltage is $\frac{1}{2}\text{V}$:

$$kV_{1,med} + (1-k)V_2 = 0$$

$$(1-k)V_2 = -k \cdot (V_{1,med})$$

$$(1-k) \cdot \left(-\frac{3}{2}\right) = -k\left(\frac{1}{2}\right)$$

$$3 - 3k = k$$

$$k = \frac{3}{4}$$

Knowing our equation $k = \frac{R_2}{R_1+R_2}$, we can pick values for the resistors such that the ratio is $\frac{3}{4}$. However, remember that our DAC has a source resistance of $50\,\Omega$ which is our R_{Th}. We can use this resistance as R_1 since it is in series with the voltage source. It follows that $R_2 = 150\,\Omega$. Our voltage shifter circuit is shown below. We label the output as V_{in} to the next stage: the amplifier.
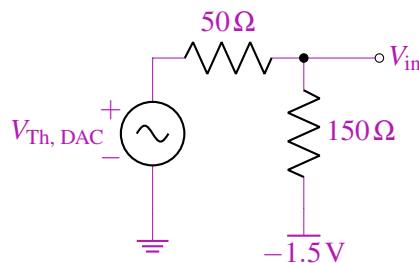
Remember that this voltage shifter actually has a gain. Specifically, $V_{in} = kV_{Th, DAC} + (1-k)(-1.5) = 0.75\,V_{Th,DAC} - 0.375$. Thus, this shifter has a gain of 0.75. Following our discussion before, we need to adjust our amplifier to have a gain of $\frac{3}{0.75} = 4$ and build the appropriate non-inverting amplifier. Since the gain of a non-inverting amplifier is $\frac{R_2+R_1}{R_1}$, we can choose $R_1 = 1\,\text{k}\Omega$ and $R_2 = 3\,\text{k}\Omega$. The final circuit is shown below.
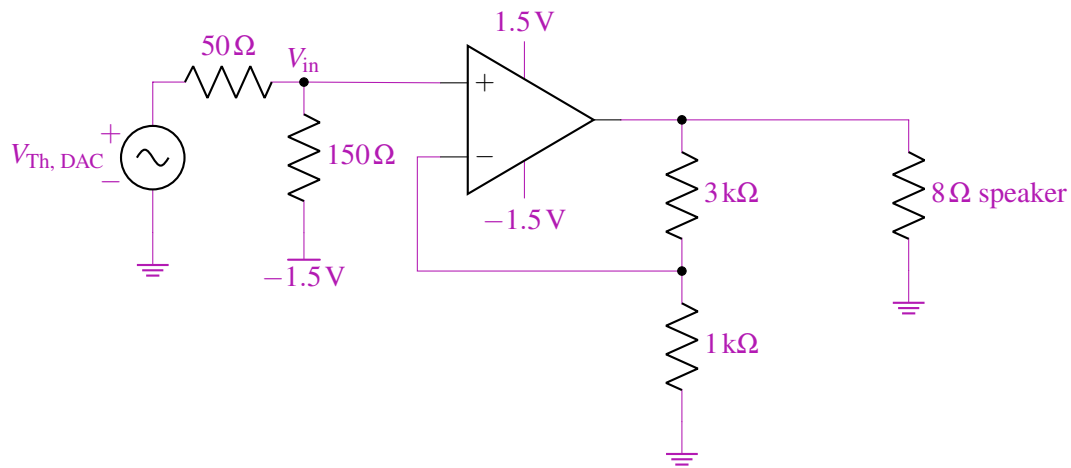
(b) Now let's look at implementing the noise cancellation. In this problem, we will assume that we do not have access to software and therefore cannot digitally remove the noise (as do most noise cancelling headphones). We will therefore focus on implementing the cancellation physically, which is to directly take the (analog) voltage produced by the microphone and subtract it out from the voltage we feed to the speaker.
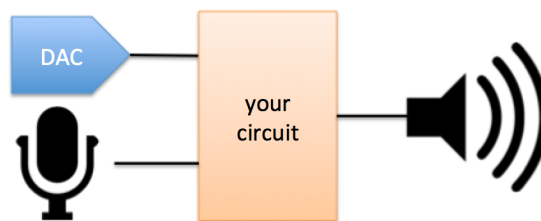
Let's assume that the microphone can be modeled as a voltage source with min/max values of $0\,V$ and $1\,V$ (relative to the DAC's ground) and a $10\,k\Omega$ source resistance. However, because the materials in the headphones attenuate some of the external sound, the loudest signals picked up by the microphone should correspond to a voltage range of only $-125\,mV$ to $+125\,mV$ driven onto the speaker.

Again draw a block diagram for your solution first.

Expand the circuit from part (a) to take the signal from the microphone and subtract it out from the signal that will be driven on to the speaker. You can use op-amps and resistors to do this, but no new voltage sources (except for the model of the microphone of course). However, note that since our speaker driver now needs to handle both the cancellation and the desired audio signal, you can assume that the supply voltages fed to the op-amp have sufficiently large magnitude to ensure that they never clip (reach the power rails). In other words, you should continue to assume that you have $\pm 1.5\,V$ voltage sources available to use in the rest of your circuit, but the op-amps are now supplied by a separate set of larger, sufficient voltage sources.

Answer:

Like in part (a), we first need to shift the range of the microphone voltage such that it centers around $0\,V$, apply gain, and then add it to our existing circuit. The most intuitive solution is to use two copies of the circuit in part (a) (though using an inverting amp instead of a non-inverting), then feed both outputs to a buffered summer so we have a small output resistance.

We keep the same circuit from part (a) for the audio. For the mic circuit, we use a similar shift, and its resistor values can be found in a similar process to (a). We are given a source resistance, so we use

that again. Centering the mic voltage to 0 again gives us a width of 0.75 V. Now however, we want our output range to be only $-125\,\text{mV}$ to $+125\,\text{mV}$, i.e. a width of 0.25 V, so we need a smaller final gain. Also, for this gain, instead of a non-inverting amp, we use an inverting amp. This takes care of the "subtraction" aspect.

The block diagram looks as follows for this proposed circuit:
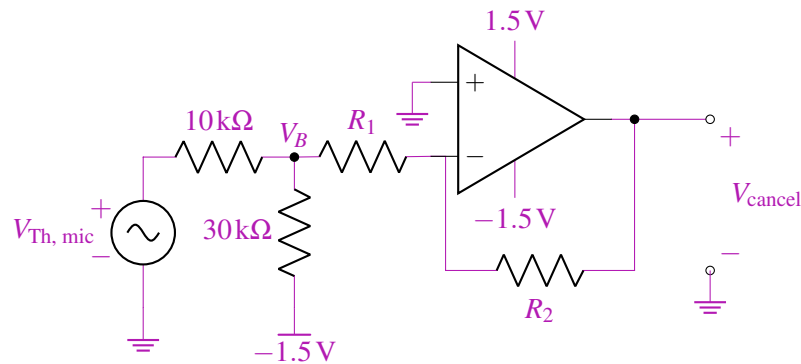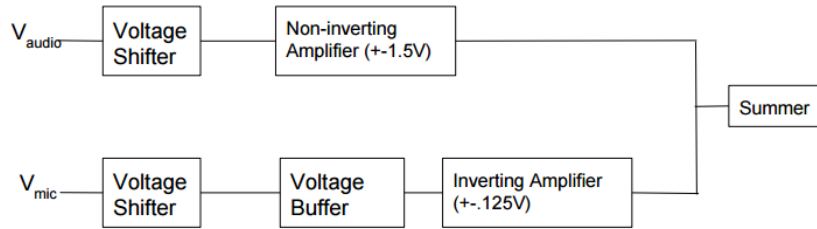
We know the gain of the inverting amp is $-\frac{R_2}{R_1}$. We want this to be $-\frac{0.25\,\text{V}}{0.75\,\text{V}} = -\frac{1}{3}$. So we set $R_1 = 3\,\text{k}\Omega, R_2 = 1\,\text{k}\Omega$.

However, there is an issue with the circuit we just build. There will be a current running through R_1. Since we connect the shifting circuit to R_1, the voltage out of voltage shifter will be affected (due to the new current). It is important then to add a voltage buffer at this node to prevent this.
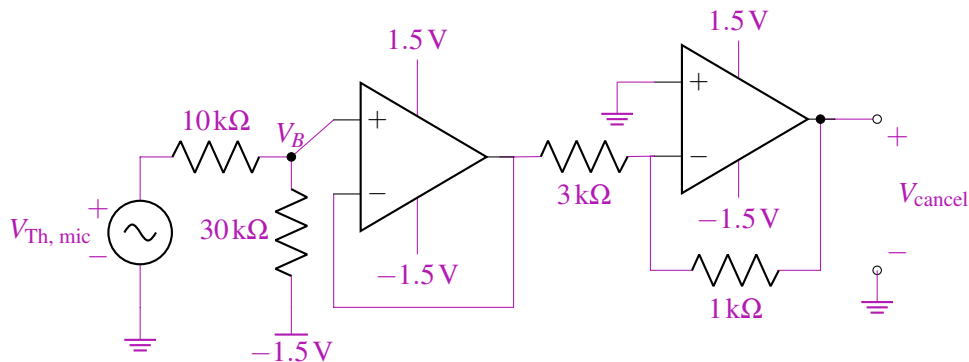
Finally, we need to add the two circuits together. Since we took care of the gains and level shift already, we can directly add these values without any further scaling, i.e. take the exact sum of them. We can use the following "summing amplifier" circuit, which combines a voltage summer and a non-inverting amplifier. We will need the non-inverting gain because a voltage summer always applies a linear combination with a gain on each term of less than 1.
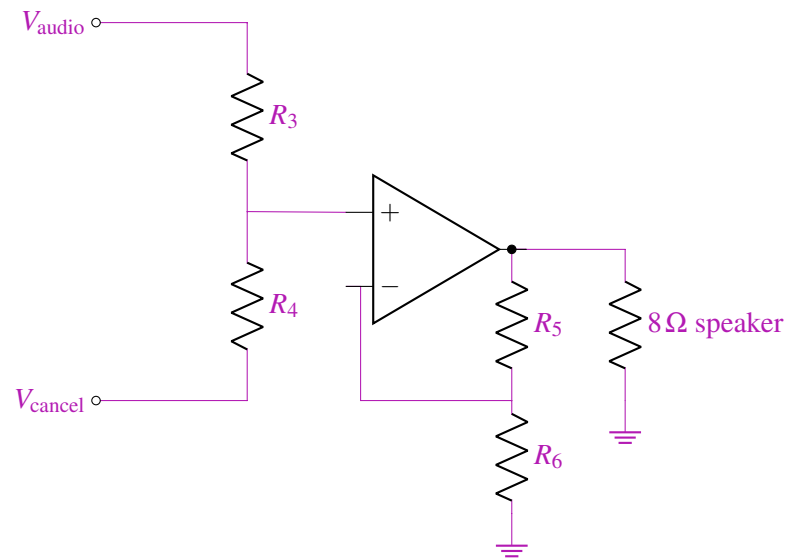
Note that since both of the inputs to this circuit will be driven with op amps, we do not have to worry about loading the previous parts, i.e. affecting their output voltages.

Setting up the equality of what we want and the equation that this circuit implements, we get:

$$V_{audio} + V_{cancel} = \left(\frac{R_4}{R_3 + R_4} V_{audio} + \frac{R_3}{R_3 + R_4} V_{cancel} \right) \left(\frac{R_5 + R_6}{R_6} \right)$$

$$= \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_5 + R_6}{R_6} \right) V_{audio} + \left(\frac{R_3}{R_3 + R_4} \right) \left(\frac{R_5 + R_6}{R_6} \right) V_{cancel}$$

The most obvious solution to this is $R_3 + R_4 = R_5 + R_6$ and $R_6 = R_3 = R_4 = R_5$. We pick all of the resistors to be $1\,\text{k}\Omega$.

We can also take this time to select the rails for this op-amp. We specified that we can use larger rails here to handle the bigger range. The maximum output voltage we can achieve is 1.5 V from the audio + 0.125 V from the mic = 1.625 V, so we need at least that large of a supply on both ends.

Putting this all together we get the following circuit: