$$r_{al} = \sqrt{\frac{r_{xc}^2 + r_{xs}^2 - 2r_{xc}r_{xs}r_{cs}}{1 - r_{cs}^2}},$$

where:

$$r_{xc} = \frac{\sum_{i=0}^{n-1} X_i \cos(a_i) - \frac{1}{n}\sum_{i=0}^{n-1} X_i \sum_{i=0}^{n-1} \cos(a_i)}{\sqrt{\left(\sum_{i=0}^{n-1} X_i^2 - \frac{1}{n}\left(\sum_{i=0}^{n-1} X_i\right)^2\right)\left(\sum_{i=0}^{n-1} \cos^2(a_i) - \frac{1}{n}\left(\sum_{i=0}^{n-1} \cos(a_i)\right)^2\right)}},$$

$$r_{xs} = \frac{\sum_{i=0}^{n-1} X_i \sin(a_i) - \frac{1}{n}\sum_{i=0}^{n-1} X_i \sum_{i=0}^{n-1} \sin(a_i)}{\sqrt{\left(\sum_{i=0}^{n-1} X_i^2 - \frac{1}{n}\left(\sum_{i=0}^{n-1} X_i\right)^2\right)\left(\sum_{i=0}^{n-1} \sin^2(a_i) - \frac{1}{n}\left(\sum_{i=0}^{n-1} \sin(a_i)\right)^2\right)}},$$

$$r_{cs} = \frac{\sum_{i=0}^{n-1} \cos(a_i)\sin(a_i) - \frac{1}{n}\sum_{i=0}^{n-1} \sin(a_i) \sum_{i=0}^{n-1} \cos(a_i)}{\sqrt{\left(\sum_{i=0}^{n-1} \sin^2(a_i) - \frac{1}{n}\left(\sum_{i=0}^{n-1} \sin(a_i)\right)^2\right)\left(\sum_{i=0}^{n-1} \cos^2(a_i) - \frac{1}{n}\left(\sum_{i=0}^{n-1} \cos(a_i)\right)^2\right)}}.$$

References

Fisher, N.I., and A.J. Lee, Nonparametric measures of angular-angular association, *Biometrica*, *69*, 315-321, 1982.

Zar, J.H., *Biostatistical Analysis*, 4th ed., 929 pp., Prentice Hall, Englewood Cliffs, New Jersey, 1999.

See Also

Chapter III-12, **Statistics** for a function and operation overview; **StatsInvChiCDF**, **StatsInvNormalCDF**, and **StatsKendallTauTest**.

StatsCircularMeans

StatsCircularMeans [***flags***] ***srcWave***

The StatsCircularMeans operation calculates the mean of a number of circular means, returning the mean angle (grand mean), the length of the mean vector, and optionally confidence interval around the mean angle. Output is to the history area and to the W_CircularMeans wave in the current data folder.

Flags

/ALPH=*val*	Sets the significance level (default 0.05).
/CI	Calculates the confidence interval (labeled CI_t1 and CI_t2) around the mean angle.
/NSOA	Performs nonparametric second order analysis according to Moore's version of Rayleigh's test where H_0 corresponds to uniform distribution around the circle. Moore's test ranks entries by the lengths of the mean radii (second column of the input) from smallest (rank 1) to largest (rank *n*) and then computes the statistic:

$$R' = \sqrt{\frac{\left(\frac{1}{n}\sum_{i=0}^{n-1}(i+1)\cos(a_i)\right)^2 + \left(\frac{1}{n}\sum_{i=0}^{n-1}(i+1)\sin(a_i)\right)^2}{n}},$$

where a_i are the mean angle entries (from column 1) corresponding to vector length rank (i+1). The critical value is obtained from Moore's distribution **StatsInvMooreCDF**.

/PSOA Perform parametric second order analysis where H_0 corresponds to no mean population direction. It assumes that the second order quantities are from a bivariate normal distribution. If this is not the case, use /NSOA above. The test statistic is:

$$F = \frac{k(k-2)}{2}\left[\frac{\bar{X}^2 S_{y^2} - 2\bar{X}\bar{Y}S_{xy} + \bar{Y}^2 S_{x^2}}{S_{x^2}S_{y^2} - S_{xy}^2}\right.$$

where

$$\bar{X} = \frac{1}{n}\sum_{i=0}^{n-1} X_i = \frac{1}{n}\sum_{i=0}^{n-1} r_i \cos(a_i),$$

$$\bar{Y} = \frac{1}{n}\sum_{i=0}^{n-1} Y_i = \frac{1}{n}\sum_{i=0}^{n-1} r_i \sin(a_i),$$

$$S_{x^2} = \sum_{i=0}^{n-1} X_i^2 - \frac{1}{n}\left(\sum_{i=0}^{n-1} X_i\right)^2,$$

$$S_{y^2} = \sum_{i=0}^{n-1} Y_i^2 - \frac{1}{n}\left(\sum_{i=0}^{n-1} Y_i\right)^2,$$

$$S_{xy} = \sum_{i=0}^{n-1} X_i Y_i - \frac{1}{n}\sum_{i=0}^{n-1} X_i \sum_{i=0}^{n-1} Y_i.$$

Here n is the number of means in *srcWave* and the critical value is computed from the F distribution, equivalent to executing:

```
Print StatsInvFCDF(1-alpha,2,n-2)
```

/Q No results printed in the history area.

/T=*k* Displays results in a table. *k* specifies the table behavior when it is closed.

k=0: Normal with dialog (default).

k=1: Kills with no dialog.

k=2: Disables killing.

/Z Ignores errors. V_flag will be set to -1 for any error and to zero otherwise.

Details

The *srcWave* input to StatsCircularMeans must be a single or double precision two column wave containing in each row a mean angle (radians) and the length of a mean radius (the first column contains mean angles and the second column contains mean vector lengths). *srcWave* must not contain any NaNs or INFs. The confidence interval calculation follows the procedure outlined by Batschelet.

V_flag will be set to -1 for any error and to zero otherwise.