FFT Amplitude Scaling

Various programs take different approaches to scaling the amplitude of an FFTed waveform. Different scaling methods are appropriate for different analyses and there is no general agreement on how this is done. Igor uses the method described in *Numerical Recipes in C* (see **References** on page III-316) which differs from many other references in this regard.

The DFT equation computed by the FFT for a complex $wave_{orig}$ with N points is:

$$wave_{FFT}[n] = \sum_{k=0}^{N-1} wave_{orig}[k] \cdot e^{2\pi i \cdot kn/N}, \quad where\ i = \sqrt{-1}$$

$wave_{orig}$ and $wave_{FFT}$ refer to the same wave before and after the FFT operation.

The IDFT equation computed by the IFFT for a complex $wave_{FFT}$ with N points is:

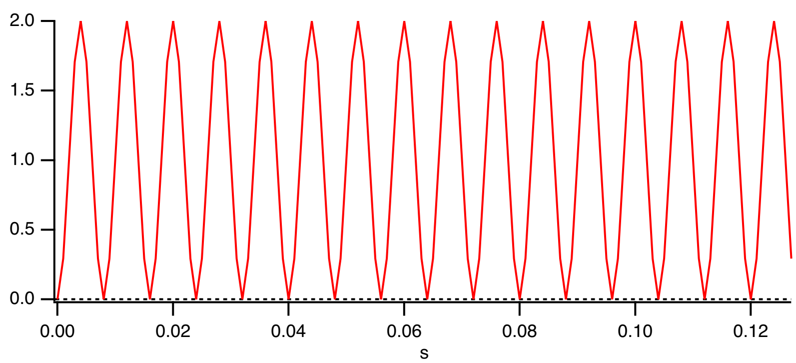
$$wave_{IFT}[n] = \frac{1}{N} \cdot \sum_{k=0}^{N-1} wave_{FFT}[k] \cdot e^{-2\pi i \cdot kn/N}, \quad where\ i = \sqrt{-1}$$

To scale $\mathrm{wave}_{\mathrm{FFT}}$ to give the same results you would expect from the continuous Fourier Transform, you must divide the spectral values by N, the number of points in $\mathrm{wave}_{\mathrm{orig}}$.

However, for the FFT of a real wave, only the positive spectrum (containing spectra for positive frequencies) is computed. This means that to compare the Fourier and FFT amplitudes, you must account for the identical negative spectra (spectra for negative frequencies) by doubling the positive spectra (but not $\mathrm{wave}_{\mathrm{FFT}}[0]$, which has no negative spectral value).

For example, here we compute the one-sided spectrum of a real wave, and compare it to the expected Fourier Transform values:

```
Make/N=128 wave0
SetScale/P x 0,1e-3,"s",wave0 // dx=1ms,Nyquist frequency is 500Hz
wave0= 1 - cos(2*Pi*125*x)    // signal frequency is 125Hz, amp. is -1
Display wave0;ModifyGraph zero(left)=3
```




```
FFT wave0
```

Igor computes the “one-sided” spectrum and updates the graph: