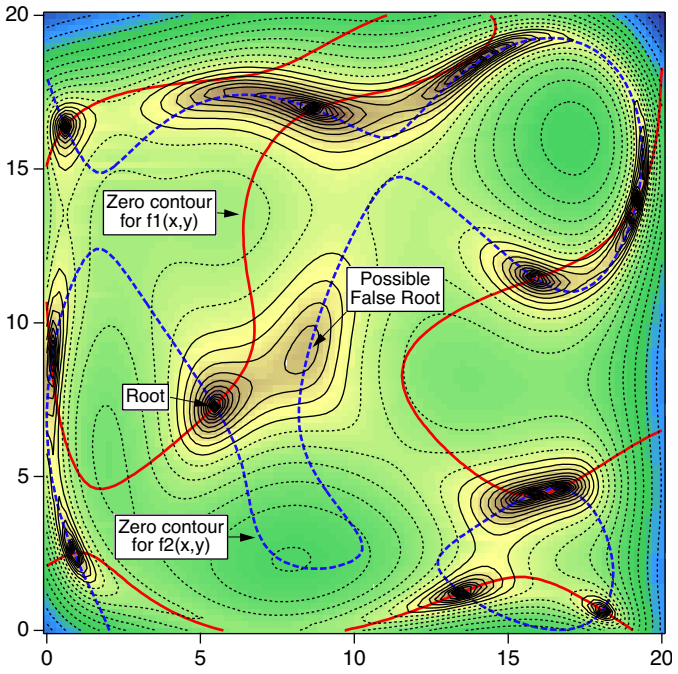
Caveats for Multidimensional Root Finding

Finding roots of multidimensional nonlinear functions is not straightforward. There is no general, foolproof way to do it. The method Igor uses is to search for minima in the sum of the squares of the functions. Since the squared values must be positive, the only places where this sum can be zero is at points where all the functions are zero at the same time. That point is a root, and it is also a minimum in the summed squares of the functions.



To find the zero points, Igor searches for local minima by travelling downhill from the starting point. Unfortunately, a local minimum doesn't have to be a root, it just has to be someplace where the sum of squares of the functions is less than surrounding points.

The adjacent graph shows how this can happen.

The two heavy lines are the zero contours for two functions (they happen to be fifth-order 2D polynomials). Where these zero contours cross are the roots for the system of the two functions.

The thin lines are contours of $f1(x,y)^2+f2(x,y)^2$, with dotted lines for high values; minima are surrounded by thin, solid contours. You can see that every intersection between the heavy zero contours is surrounded by thin contours showing that these are minima in the sum of the squared functions. One such point is labeled "Root".

There is at least one point, labelled "False Root", where there is a minimum but the zero contours don't cross. That is not a root, but FindRoots may find it anyway. For instance, a real root:

```
FindRoots /x={3,6} MyPoly2d, nn1coefs, MyPoly2d, nn2coefs
   Root found after 11 function evaluations.
   W_Root={5.4623,7.28975}
   Function values at root:
   W_YatRoot={-4.15845e-13,1.08297e-12}
```

This point is the point marked "Root". However:

```
FindRoots/x={9,10} MyPoly2d, nn1coefs, MyPoly2d, nn2coefs
   Root found after 52 function evaluations.
   W_Root={8.38701,9.10129}
   Function values at root:
   W_YatRoot={0.0686792,0.0129881}
```

You can see from the values in W_YatRoot that this is not a root. This point is marked "False root" on the figure above.

The polynomials used in this example have too many coefficients to be conveniently shown here. To see this example and others in action, try out the demo experiment. It is called "MD Root Finder Demo" and you will find it in your Igor Pro 7 folder, in the Examples:Analysis: folder.

Finding Minima and Maxima of Functions

The **Optimize** operation finds extreme values (maxima or minima) of a nonlinear function.