Hough Transform

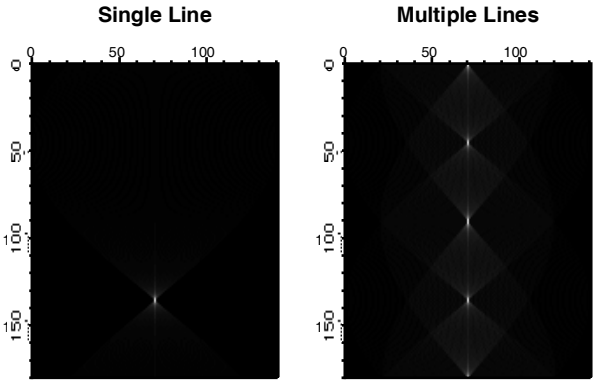
The Hough Transform is a mapping algorithm in which lines in image space map to single points in the transform space. It is most often used for line detection. Specifically, each point in the image space maps to a sinusoidal curve in the transform space. If pixels in the image lie along a line, the sinusoidal curves associated with these pixels all intersect at a single point in the transform space. By counting the number of sinusoids intersecting at each point in the transform space, lines can be detected. Here is an example of an image that consists of one line.

```
Make/O/B/U/N=(100,100) lineImage
lineImage=(p==q ? 255:0)            // single line at 45 degrees
Newimage lineimage
ImageTransform hough lineImage
Newimage M_Hough
```

The Hough transform of a family of lines:

```
lineImage=((p==100-q)|(p==q)|(p==50)|(q==50)) ? 255:0
ImageTransform Hough lineImage
```

The last image shows a series of bright pixels in the center. The first and last points correspond to lines at 0 and 180 degrees. The second point from the top corresponds to the line at 45 degrees and so on.



Fast Hartley Transform

The Hartley transform is similar to the Fourier transform except that it uses only real values. The transform is based on the *cas* kernel defined by:

$$cas(vx) = \cos(vx) + \sin(vx).$$

The discrete Hartley transform is given by

$$H(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\left\{\cos\left(\left[2\pi\left(\frac{ux}{M}-\frac{vy}{N}\right)\right]\right)+\sin\left[2\pi\left(\frac{ux}{M}-\frac{vy}{N}\right)\right]\right\}$$

The Hartley transform has two interesting mathematical properties. First, the inverse transform is identical to the forward transform, and second, the power spectrum is given by the expression:

$$P(f) = \frac{[H(f)]^2 + [H(-f)]^2}{2}$$

The implementation of the Fast Hartley Transform is part of the **ImageTransform** operation (see page V-417). It requires that the source wave is an image whose dimensions are a power of 2.

```
ImageTransform /N={18,3}/O padImage Mri      // make the image 256^2
ImageTransform fht mri
NewImage M_Hartley
```