Wigner Transform

The Wigner transform (also known as the Wigner Distribution Function or WDF) maps a 1D time signal U(*t*) into a 2D time-frequency representation. Conceptually, the WDF is analogous to a musical score where the time axis is horizontal and the frequencies (notes) are plotted on a vertical axis. The WDF is defined by the equation

$$W(t, \nu) = \int_{-\infty}^{\infty} dx U(t + x/2) U^*(t - x/2) e^{-i2\pi x\nu}$$

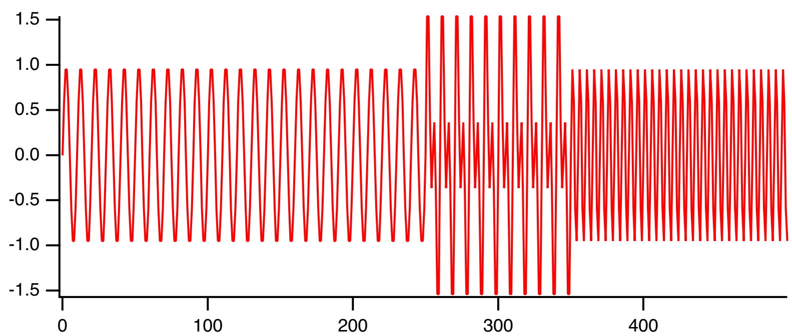
Note that the WDF W(*t*,ν) is real (this can be seen from the fact that it is a Fourier transform of an Hermitian quantity). The WDF is also a 2D Fourier transform of the Ambiguity function.

The localized spectrum can be derived from the WDF by integrating it over a finite area dtdn. Using Gaussian weight functions in both *t* and *n*, and choosing the minimum uncertainty condition dtdn=1, we obtain an estimate for the local spectrum

$$\hat{W}(t, \nu;\delta t) \propto \left| \int U(t') \exp\left[-2\pi\left(\frac{t-t'}{\delta t}\right)^2\right] \exp(-i2\pi\nu t') dt' \right|^2$$

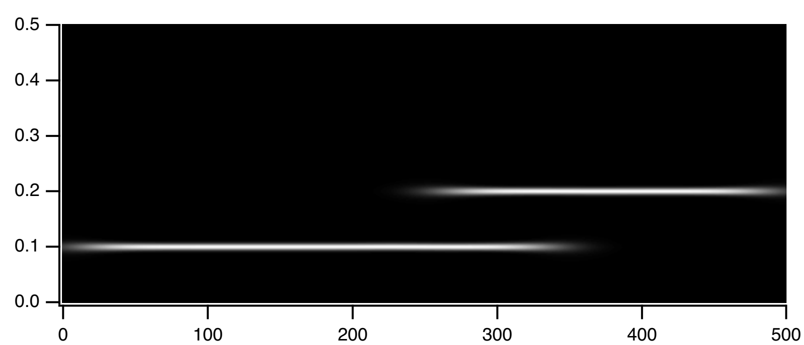
For an application of the **WignerTransform** operation (see page V-1095), consider the two-frequency signal:

```
Make/N=500 signal
signal[0,350]=sin(2*pi*x*50/500)
signal[250,]+=sin(2*pi*x*100/500)
WignerTransform /GAUS=100 signal
DSPPeriodogram signal            // Spectrum for comparison
Display signal
```



The signal used in this example consists of two “pure” frequencies that have small amount of temporal overlap:

```
Display; AppendImage M_Wigner
```



The temporal dependence is clearly seen in the Wigner transform. Note that the horizontal (time) transitions are not sharp. This is mostly due to the application of the minimum uncertainty relation dtdn=1 but it is also due to computational edge effects. By comparison, the spectrum of the signal while clearly showing