

1. a) $\text{midL}(a^*ba^*)$

$= b$, as S can be written as
 xwz thus $x=a^*$, $w=b$ and $z=a^*$.
 $\text{mid}(S)=w=b$ <

b) $\text{midL}(\{a^n b^n : n \geq 0\})$

If n is $0, 1, 2$ then the $\text{midL}(L)$ would be E . otherwise $\text{midL}(L)$ of $n \geq 3$ would be ' ab ' as that is the middle.

c) No, as the languages are finite and have a finite number of strings the string generated from $\text{midL}(L)$ may not belong in any given finite language.

d) Yes, as the languages are infinite the string found at the function $\text{midL}(f)$ will exist as a string within at least one language.

aa zccabbb

aa gbbb

aa

cc

abbccc

aabab

babbab

Z,

a) S → A | B

A → a | b | Ab | bF

S → A | B

A → a | b | Ab | bF

F → cF | E

B → aF | G

G → bG | c | Gc | cF | cW

W → E

bbababab

b)

S → bA | bsb | E

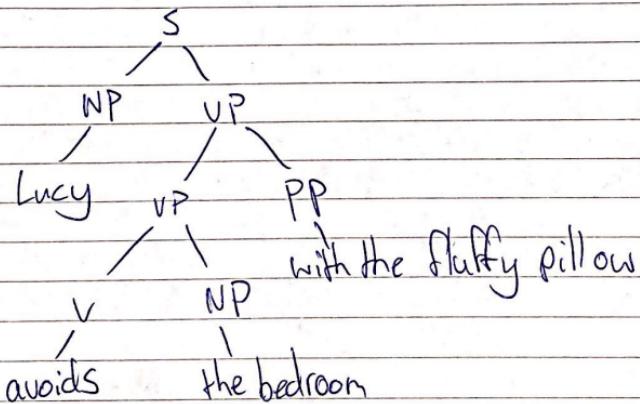
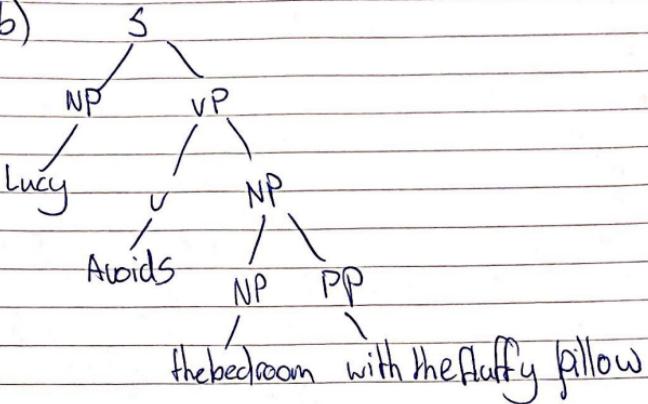
A → abA | E

c)

S → aSa | bSb | E | A

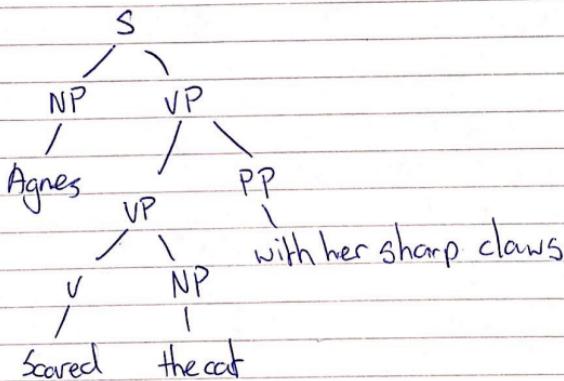
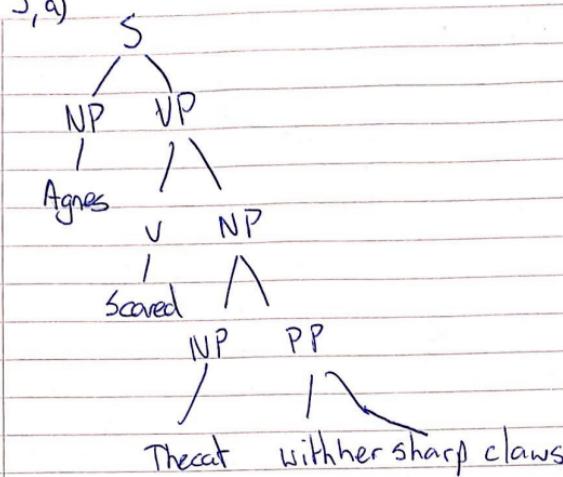
A → aaA | E

3,
b)



the above two show the ambiguity

3, a)



The above two show ambiguity

4

$$S \rightarrow T_b T$$

$$T \rightarrow A A A$$

$$T \rightarrow C$$

$$A \rightarrow a A$$

$$A \rightarrow G$$

$$C \rightarrow c C$$

$$C \rightarrow c$$

Step 1, $S \rightarrow T_b T$

$$T \rightarrow A A A$$

$$T \rightarrow C$$

$$A \rightarrow a A$$

$$A \rightarrow a$$

$$C \rightarrow c C$$

$$C \rightarrow c$$

Step 2, $S \rightarrow T_b T$

$$T \rightarrow a A a A a A$$

$$T \rightarrow c C | c$$

$$A \rightarrow a A$$

$$A \rightarrow a$$

$$C \rightarrow c C$$

$$C \rightarrow c$$

 $S \rightarrow T_b T$

$$T \rightarrow a A a A a A | c C | c$$

$$A \rightarrow a A | a$$

$$C \rightarrow c C | c$$

Step 3, $S \rightarrow T X b T$

$$T \rightarrow X_a A X_a A X_a A | X_c C | X_c$$

$$A \rightarrow X_a A | X_a$$

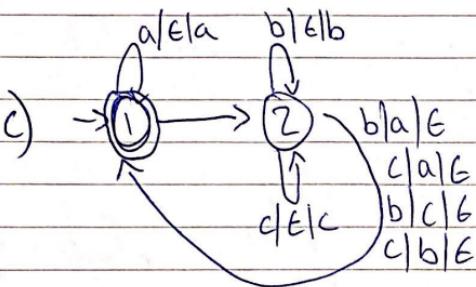
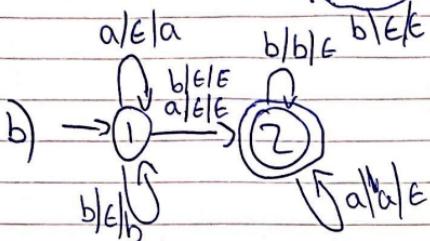
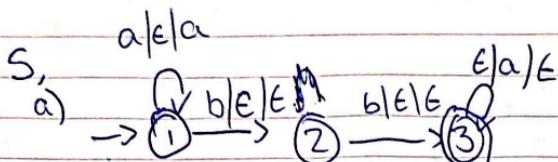
$$C \rightarrow X_c C | X_c$$

Step 4, $S \rightarrow N_x b N_x$

$$T \rightarrow X_a N_2 | X_c N_3 | X_c$$

$$A \rightarrow X_a N_2 | X_a$$

$$C \rightarrow X_c N_3 | X_c$$



6,

a) language M is a context free language with elements x and y .

there can be zero or more x 's and $y \leq x$, example, $xxxy$.

$$\begin{array}{l} b) S \rightarrow xS \mid xA \mid \epsilon \\ A \rightarrow yA \mid \epsilon \end{array}$$

c) Yes, M is deterministic as there is only one possibility for transitions from state to state. This is based on the current input.

d) Yes, $L(M)$ is deterministic context free as it can be accepted by a deterministic PDA and $L(M)$ is context free via 6(a).

7, a) $L_1 = L_2 - \text{L}_3$

f

$L_1 = \{0^n 1^n : n \geq 0\}$ context free

$L_2 = \{0^n 1^n 2^j : j \text{ is prime}, n \geq 0\}$ not context free

$L_3 = \{0^n 1^n 2^j : j \text{ is prime}\}$ not context free

b) $L_1^R = \neg L_2$

$L_1 = \{0^n 1^n 2^j : n \text{ is prime}\}$

$L_2 = \{0^n 1^n 2^j : p \geq 0\}$

e.g., $n=7$
 $p=7$

$$\begin{aligned} L_1^R &= 0000000 \ 1111111 \\ L_2 &= 1111110000000 \\ \neg L_2 &= 1111110000000 \end{aligned} \text{ equal!}$$

abbcc

8,

$$a) L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } j \leq i+k\}$$

 $i=1 \quad k=2 \quad j=2$

~~Not context free~~ is context free but not regular, the language can be generated through context free grammar but is not regular as can be shown through pumping lemma

$$b) L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } j > \max(i, k)\}$$

Not context free as there is comparison of 3 variables independently and the stack is only capable of comparing two variables at one instance.

9, a) $x = \text{00010001}$ 0101

assume L is context free

Pumping length $P = 3$

String $x = 0^p 1^p 0^p 1^p$
 divide x into $uvxyz$

divide x into $uvwxyz$

$$0^3 \ 1^3 \ 0^3 \ 1^3 = \underbrace{000}_{u} \underbrace{111}_{\text{max}} \underbrace{000111}_{z}$$

pump uv^ixy^iz let $i = 2$

0001111000111
0³, 5³, 3
0, 0, 1, , 1

thus $x^r x^r$ ~~=~~ x ; ~~under context are closed~~
~~under reverse and double (L)~~

b) Yes it is possible for $md(L)$ to be regular as it would be a finite language. For example the string sxt could be EEE and thus a finite regular language may be made.

10,

a) The Lindenmayer system also known as the "L-system", is a fractal generating system that produces these fractals in one or two dimensions. The underlying system that allows this is a string rewriting system that uses substitution with which rules are followed to perform on a string from a given alphabet. The L-system L is a triple (Σ, R, W)

b)

$$R_G = \{ F \rightarrow F+F-F-F+F \}$$

the rule is to instead of going in a square path you instead need to cut a square out of the original square path by turning in and then out.