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Q1

a)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

All outcomes are true
∴ tautology.
as $(p \wedge q) \to p$ is always True

b) ~~$p \wedge \neg q$~~

p	$-p$	$p \wedge -p$
T	F	F
T	F	F
F	T	F
F	T	F

All outcomes are False
∴ contradiction.
as $p \wedge -p$ is always False.

c)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Not all outcomes are True
∴ not a tautology.
$p \wedge q$ is not a tautology as all outcomes are ~~not~~ true.

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Q2 a) Yes it is valid.

b) $p \vee q$

$\neg p$ $\therefore$ Disjunctive syllogism

$\therefore q$

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Q3 $(p \vee q \vee \neg q) \wedge (p \vee \neg p) \wedge (\neg p \vee \neg q) \wedge (q \vee \neg q)$

p	q	-q	p v q	(p v q) v -q
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T

1

p	-p	p v -p
T	F	T
F	T	T
T	F	T
F	T	T

2

p	q	-p	-q	-p v -q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

3

q	-q	q v -q
T	T	T
T	F	T
F	T	T
F	~~T~~ F	T

4

1	2	3	4
$(p \vee q \vee \neg q)$	$\wedge (p \vee \neg p)$	$\wedge (\neg p \vee \neg q)$	$\wedge (q \vee \neg q)$
	T	F	F
	T	T	T
	T	T	T
	T	T	T

Truth value for Q3 is F
∴ satifiable. T
T
T

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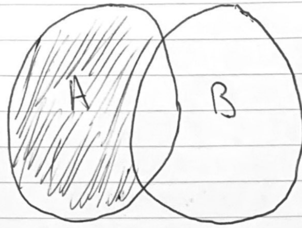
Q4 a)



b) False.
c) True.

Q5 $15a + 20b = 1$

$5(3a + 4b) = 1$
$5n = 1$
$n = 1/5$ $\therefore$ contradiction

if a and b are integers, n must be an integer, here n is not an integer therefore proof by contradiction.

$n = \frac{a}{b}$ both rational
$n' = \frac{a}{2b}$ this one is smaller.

$n' < n$ $\therefore$ proof by contradiction

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Q6

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Base case: $1^2 = \dfrac{1(1+1)(2(1)+1)}{6}$

$1^2 = 6/6$ ✓

$1 = 1$ $\therefore$ plausible

let $n = (k+1)$

$$\frac{k(k+1)(2k+1)}{6} + (k+1) = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$k(k+1)(2k+1) + 6(k+1) = (k+1)(k+2)(2k+3)$$

$$(k+1)(2k^2+k+6k+6) = (k+1)(2k^2+3k+4k+6)$$

$$(k+1)(2k^2+7k+6) = (k+1)(2k^2+7k+6)$$

$\therefore$ true for every positive integer n via induction.

Q7 $S \times S$ where $S = \{1,2,3,4\}$

a, $\left\{\begin{matrix}(1,1), (1,2), (1,3), (1,4), \\ (2,1), (2,2), (2,3), (2,4), \\ (3,1), (3,2), (3,3), (3,4), \\ (4,1), (4,2), (4,3), (4,4)\end{matrix}\right\}$

b, i, There are 8 elements in relation R

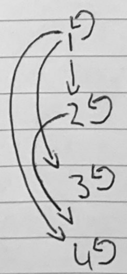
ii,

	1	2	3	4
1	1	1	1	1
2	0	1	0	1
3	0	0	1	0
4	0	0	0	1

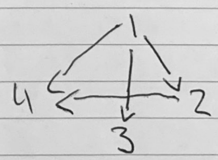
iii, Reflexive,
Antisymmetric
Transitive

iv, Yes, R is a poset of S,
as it is reflexive, antisymetric and transitive

Diagraph



Hasse Diagram



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Q8

$a-b=2k$ for some $k \in \mathbb{Z}$

a) Reflexive: $a \in \mathbb{Z}$, $a-a=2k$ ← this is an integer $\therefore aRa$

Symmetric: aRb

$$a-b=2k$$
$$b-a=-(a-b)=2(-k)$$
$$\therefore aRb,\ bRa$$

Transitive: $a,b,c \in \mathbb{Z}$

aRb and bRc

$a-b=2k_1$ and $b-c=2k_2$

$$a-b+(b-c)=2(k_1+k_2)$$
$$a-c=2(k_1+k_2)$$, this is an integer
$$\therefore aRb,\ bRc \rightarrow aRc$$

b) $2k$ has a factor of 2 $\therefore$ there must have 2 classes

$$a-b=2k$$
$$=\ldots-6,-4,-2,0,2,4,6\ldots$$

$$a-b=2k+1$$
$$=\ldots-5,-3,-1,1,3,5\ldots$$

c) $[a]:=\{x : xRa\}$

$[0]:=\{x : xR0\}$

$x-0=2k$

$x=\ldots-4,-2,0,2,4\ldots$

$[1]:=\{x : xR1\}$

$x-1=2k$

$x=\ldots-3,-1,1,3$

$[17]:=\{x : xR17\}$

$x-17=2k$

$x=2(k+8)+1$

$x=\ldots-3,-1,1,3\ldots$

These are same.

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Q8 d, The equivalence classes are either [0] or [1]

[0] = ..., -4, -2, 0, 2, 4 ...
(all even integers)

[1] = ..., -3, -1, 1, 3
(all odd integers)

$\therefore$ [0] $\cup$ [1] forms a partition of $\mathbb{Z}$

e, i) [0] + [0] = [0]
because $-4 + -4 = -8$
$-2 + -2 = -4$ $\downarrow$ follow pattern of [0]
$-0 + 0 = 0$

ii) [0] + [1] = $\mathbb{Z}$ as [0] $\cup$ [1] is a partion of $\mathbb{Z}$

iii) [1] + [1] = [0]
because [2] % 2 = [0]
or $-3 + -3 = -6$ $\downarrow$ follows pattern of [0]
$-1 + -1 = -2$

iv)

	[0]	[1]
[0]	[0]	[1]
[1]	[1]	[0]

f,

[0]	[1]	[0] $\oplus$ [1]
T	T	F
T	F	T
F	T	T
F	F	F

$\therefore$ XOR.

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Q9 $a-b=5k$ for some $k \in \mathbb{Z}$

a)

+	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

x	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]
[2]	[0]	[2]	[4]	[1]	[3]
[3]	[0]	[3]	[1]	[4]	[2]
[4]	[0]	[4]	[3]	[2]	[1]

b)

+	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[0]
[2]	[2]	[3]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]

x	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]	[5]
[2]	[0]	[2]	[4]	[0]	[2]	[4]
[3]	[0]	[3]	[0]	[3]	[0]	[3]
[4]	[0]	[4]	[2]	[0]	[4]	[2]
[5]	[0]	[5]	[4]	[3]	[2]	[1]

Similarities

c) - The addition tables both have a diagonal downwards pattern of the same class
- The multiplication tables both have the same columns and rows

Differences

- The addition table in $\mathbb{Z}_6$ has one more column and row than $\mathbb{Z}_5$
- The multiplication tables are differing in the middle.

YouTube link for explanation of Q1:
https://youtu.be/adLGSVbMaAw