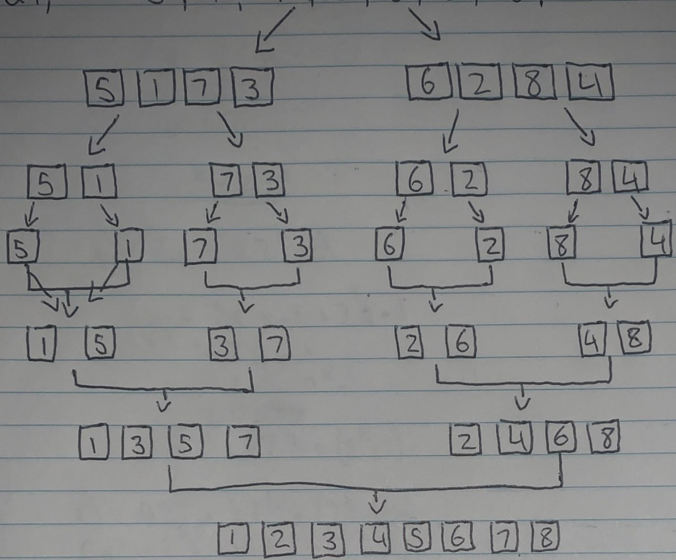
Math1510 Assignment 3

Q1, 5, 1, 7, 3, 6, 2, 8, 4



Q2,

a) $c_k = 2c_{k-1} + 2^k - 1$, $c_0 = 0$

when $k=1$
$c_1 = 2(0) + 2^1 - 1$
$= 1$

when $k=2$
$c_2 = 2(1) + 2^2 - 1$
$= 5$

when $k=3$
$c_3 = 2(5) + 2^3 - 1$
$= 17$

if we compare the outputs to the sorting number sequence: 0, 1, 3, 5, 8, 11, 14, 17, 21, 25...

1, 5 and 17 corresspond ~~those~~ to the sequence thus its the right answer. with $n = 2, 4, 8$

b) looking at $c_k = 2c_{k-1} + 2^k - 1$ you can see that it will always be less than a multiple of two, thus an odd number. With this in mind, comparing it to the worst outcome sequence and $n = 2^k$ the result is also odd, thus making it correct in general.

Q3,

a) $C_k = 2C_{k-1} + 2^k - 1$

$C_k - 2C_{k-1} = 0$

$x - 2 = 0$

$x = 2$

$\underline{c^k = A2^k}$

b) $C_k = k2^k + d$

$C_k = 2C_{k-1} + 2^k - 1$

$k2^k = 2C_{k-1}$

where $d = 2^k - 1$

$C = k2^k + (2^k - 1)$

c) $= A2^k + k2^k + (2^k - 1)$

$C_k = 2^k(A + k + 1) - 1$?

d) use $C_k = A2^k + k2^k + 2^k - 1$ to find A:

where $C_0 = 0$

$0 = A \times 2^0 + 0 \times 2^0 + 2^0 - 1$

$0 = A - 1$

$A = 1$

thus:

$C_k = 1 \times 2^k + k \times 2^k + 2^k - 1$

$= 2^k (k+2) - 1$

Q4, $a_n = 2a_{n-1} + 8a_{n-2}$ where $a_0 = 4$, $a_1 = 10$

$$a_n - 2a_{n-1} - 8a_{n-2} = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = 4, -2$$

$$a_n = ax^n + bx^n$$
$$a_n = a4^n + b(-2)^n$$
$$4 = a(4)^0 + b(-2)^0$$
$$4 = a + b$$
$$\boxed{a = 4 - b}$$
sub (b)
$$a = 4 - (1)$$
$$\boxed{a = 3}$$

$$a_n = a(4)^n + b(-2)^n$$
$$10 = a(4)^1 + b(-2)^1$$
$$10 = 4a - 2b$$
$$2b = 4a - 10$$
$$b = \frac{4a-10}{2}$$
Sub (a)
$$b = \frac{4(4-b)-10}{2}$$
$$= \frac{16-4b-10}{2}$$
$$b = 8 - 2b - 5$$
$$3b = 3$$
$$\boxed{b = 1}$$

$$a_n = 3 \times 4^n + (-2)^n$$
$$\boxed{a_n = (-2)^n + 3 \times 4^n}$$

Q5, $b_n = 8b_{n-1} - 16b_{n-2}$ where $b_0 = 1$, $b_1 = 8$

$$b_n - 8b_{n-1} + 16b_{n-2} = 0$$
$$x^2 - 8x + 16 = 0$$
$$(x-4)(x-4) = 0$$
$$x = 4$$

using: $b_n = ax^n - bnx^n$

$$1 = a(4)^0 - b(0)(4)^0$$
$$1 = a$$
$$\boxed{a = 1}$$

$$8 = a(4)^1 - b(1)(4)^1$$
$$8 = 4a - 4b$$
$$\frac{-8+4a}{4} = b$$

sub (a)

$$\frac{-8+4(1)}{4} = b$$
$$\boxed{b = -1}$$

$$b_n = 4^n - (-1)n \times (4)^n$$
$$= 4^n + n \times 4^n$$
$$= \boxed{4^n(n+1)}$$