Assigment 2 Math1510

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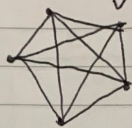
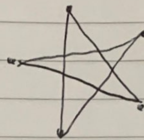
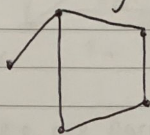
Q1, a)



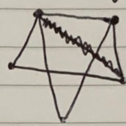
b)



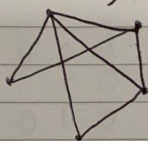
c)



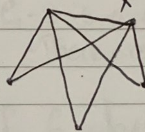
d)



e)

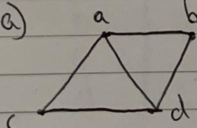


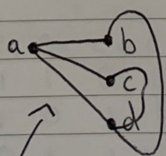
f)



- F cannot exist as the initial 4, 4 degree sequence means there will be no vertex with degree sequence 1.

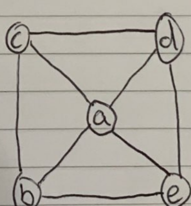
Q2, a)

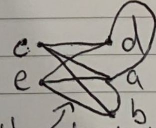




attempt at bi-partite, therefore it cant contain k_3 as a subgraph as k_3 is an odd cycle.

b)

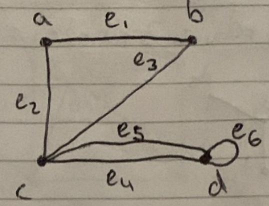




attempt at bipartite therefore it cant be bipartite as it contains an odd cycle.

Q3,

G



a, e_4 & e_5 are parallel edges
e_6 is a loop.

b,

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 1 & 0 & 2 \\ d & 0 & 0 & 2 & 1 \end{array}$$

e, - c is a cut vertice
- There are no bridges

f, 6 edges in G

c,

vertex	adjacent to
a	b, c
b	a, c
c	d, d
d	c, c, d

d,

$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline a & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 1 & 1 & 0 \\ d & 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$

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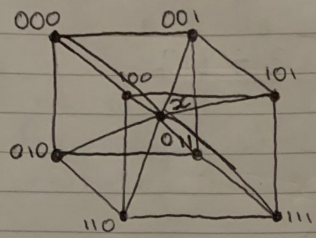
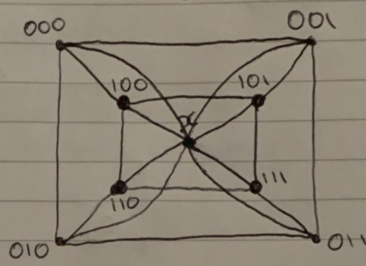
Q4,

①

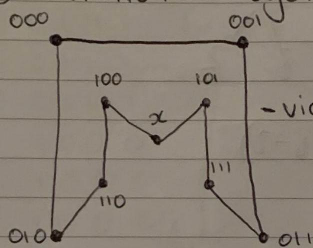


or

②

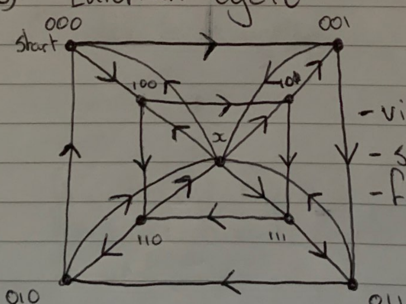


a) Hamiltonian cycle



- via graph ②

b) Eulerian cycle



- via graph ②
- starting at 000
- finishing at 000

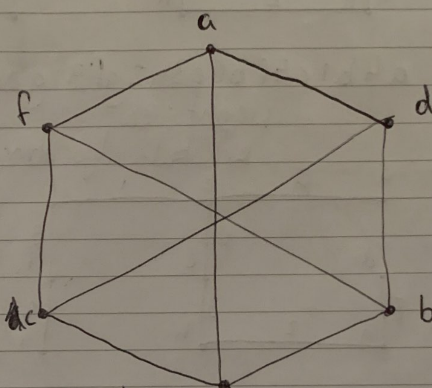
Q5,

G_1 and G_3 are isomorphic

if we arrange G_3's vertices the same as G_1 and G_2 while using the same adjacency matrix as G_3, we get:

G_3:

	a	b	c	d	e	f
a	0	0	0	1	1	1
b	0	0	0	1	1	1
c	0	0	0	1	1	1
d	1	1	1	0	0	0
e	1	1	1	0	0	0
f	1	1	1	0	0	0



∴ G_3 is isomorphic to G_1

G_2 is not isomorphic to G_1 and G_3 as there are two triangles in G_2 and none in G_1 and G_3.

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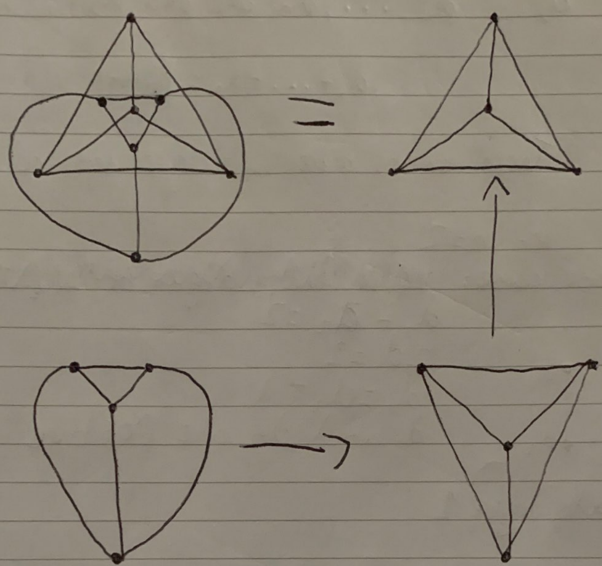
Q6,



Q7,

a) $d_1, \ldots, d_n$ are any positive integers

$$d_1 + \ldots + d_n = 2(n-1)$$

There exists a tree with degree sequence $d_1 \ldots d_n$

let $n=2$

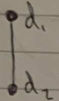
Base case:

$d_1 + d_2 = 2$

$2d = 2$

$d_1 = d_2 = 1$

d_1 and d_2 are positive integers



Both degree 1

Assume true for k

if a vertex $k+1$ is appended to the graph

This degree increases by 1 thus $d_{k+1} = 1$

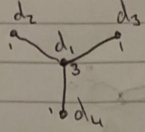
and this degree is increased by 1 therefore:

$$\sum_{a=2}^{k+1} d_a = 2(k-1)+2 \longrightarrow +2 \text{ comes from}$$

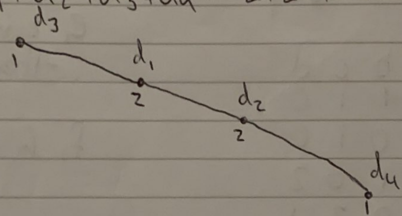
$$= 2((k+1)-1) \rightarrow \text{True for all } n \geq 2$$

Q7,

b) $d_1 + d_2 + d_3 + d_4 = 3 + 1 + 1 + 1 = 6$



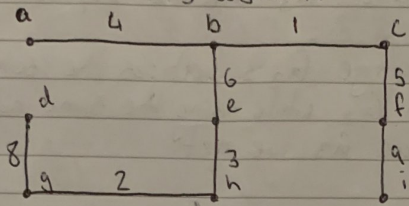
$d_1 + d_2 + d_3 + d_4 = 2 + 2 + 1 + 1 = 6$



all other trees are isomorphic to the ones above

Q8.

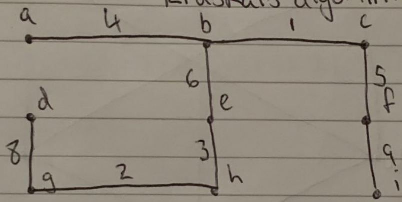
a) Prims algorithm



a4b1c5fb6e3h2gf9ig8d

weight : 38

Kruskals algorithm.



b1cg2h3ea4bc5fb6ed8gf9i

weight = 38

b) - They have the same tree
- The weight is the same.

c) Krushals algorithm is better in my opinion as in computer science it'd be easier to implement in code.

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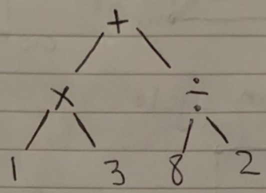
Q9,

$+ \times 1\, 3 \div 8\, 2$

a)



b) $(1 \times 3) + (8 \div 2)$

c) $= 7$