Question #4: Taylor Series

Objective: Loop

The value of the hyperbolic sine function can be defined by the Taylor series expansion as follows:

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

We can approximate $\sinh(x)$ by computing the above series with a limited number of terms:

$$\sinh(x) \approx \sum_{n=0}^{k} \frac{x^{2n+1}}{(2n+1)!}$$

Write a program to approximate $\sinh(x)$ by **summing terms until the term is less than a given threshold, ε** is reached. For instance, x = **1.0** and **ε = 0.001**, stop adding terms when a term has value smaller than ε.

	n = 0	n = 1	n = 2	n = 3
$\sinh(1.0)$	$\frac{1^0}{1!} = 1.0$	$\frac{1^3}{3!} = 0.1666666\ldots$	$\frac{1^5}{5!} = 0.0083333\ldots$	$\frac{1^7}{7!} = 0.000198$

With ε = 0.001, sinh(1) ≈ 1.0 + 0.1666666... + 0.0088333... = 1.175

Stop when the term is less than ε
(in this example, ε=0.001)

Hint: use **math.factorial(n)** to calculate the value of **n!**.

INPUT

A single line containing x and **ε** as real numbers, separated by a space.

OUTPUT

The value of **sinh(x)** obtained from the approximation method using the given threshold **ε** should be displayed with no more than 6 decimal places. Use the function **round(*value*, 6)** to round the computed **sinh(x)** value to 6 decimal places. **(Note: use round() only when displaying the result, do not use it during calculation.)**

EXAMPLES

Input (from keyboard)	Output (on-screen)
`1.0 0.001`	`1.175`
`1.0 1e-8`	`1.175201`
`5.0 0.001`	`74.203044`
`5.0 1e-8`	`74.203211`
`0.5 1e-5`	`0.521094`
`0.5 1e-6`	`0.521095`

Test Cases in Grader

Testcases quantity	Testcase characteristics
50%	10^{-2} <= ε < 10^{-8}
50%	ε <= 10^{-8}