COMS20017 – Algorithms & Data

Problem Sheet MM03 – March 2025

1 – Using *sin(2πnx)*, demonstrate the concept of superposition as follows:

(a) first plot three sine functions over the range ±3 in steps of 0.1 using *n={1/4,1,2}*. Note, plots should appear in the same graph to give a better sense of what is happening.

(b) Now plot in a different colour the sum of all the sines above.

(c) Add more sine functions over the same range and repeat step (b).

Matlab:

(a) First define the range, say x = [-3:0.1:3]

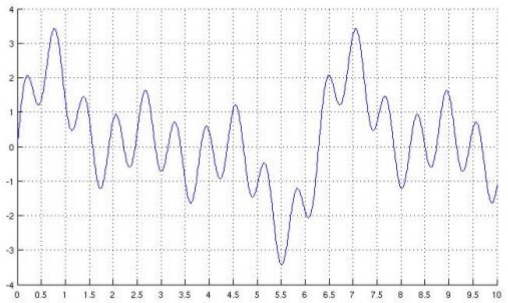
The sine function plot over the specified range with *n=1/4* is then "plot(sin(2*pi*x*1/4))"

Hold the plot. Now plot again for the other values of *n.*

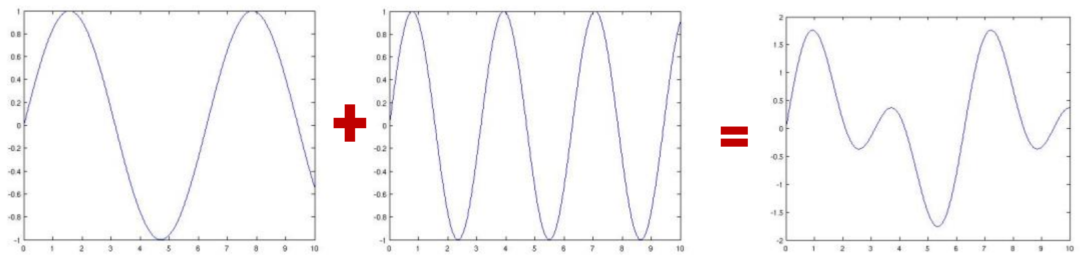
(b) Add the sines from (a) and plot the new function using 'r' as a parameter of the plot function to draw in red. See *help plot* if unsure of the syntax.

Python: see sines.py on unit github page

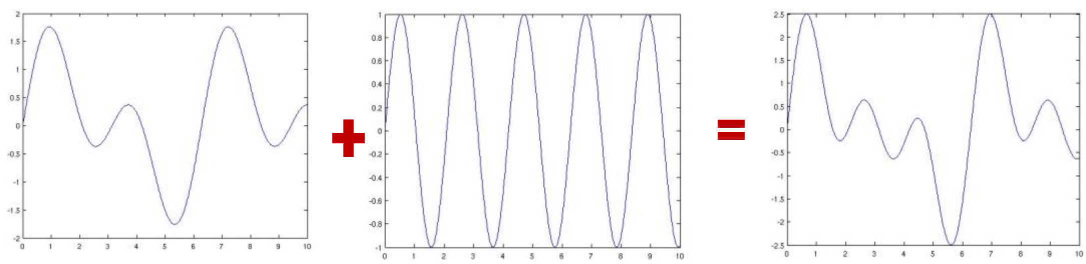
2 – Based on your understanding of the Nyquist Sampling Rate theorem, what is a sufficient sampling rate for the signal below? Hint: the signal is composed of the summation of sin(x), sin(2x), sin(3x) and sin(10x).



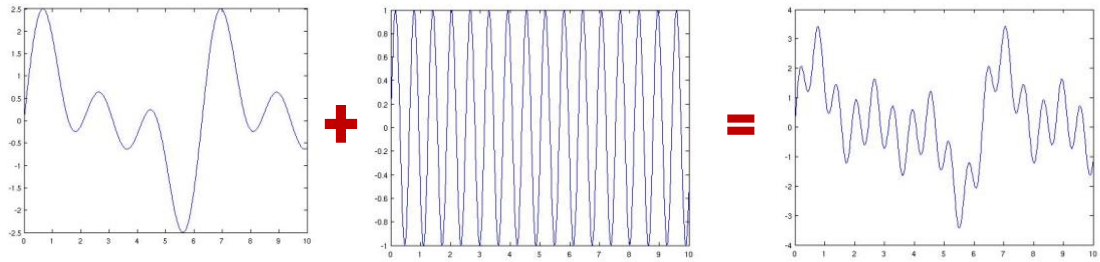
Consider the different frequency components that were used to build the signal (like in Fourier Analysis).
This shows the waves sin(x) and sin(2x) and their sum:



When adding the summed wave sin(x)+sin(2x) to the higher frequency wave sin(3x) then the wave to the right results:



Next we add sin(x)+sin(2x)+sin(3x) to sin(10x), resulting in the signal.



The highest frequency in the figure is thus that of the wave sin(10x). The frequency is thus 10/(2*pi) = 1.59 Hz. Following the Nyquist theorem, the sampling rate should be at least 3.18 Hz (2 x 1.59).

3 – Determine which is an even and which is an odd function:

(i)	$f(x) = 7x^3 - x$	odd
(ii)	$f(x) = 3x^2 + 1$	even
(iii)	$f(x) = 3x^2 \sin(x)$	odd
(iv)	$f(x) = \dfrac{3}{(-x)^4 - 4}$	even
(v)	$f(x) = \cos(x) + 5x \; - 3$	No symmetry of any kind, so it is neither even nor odd.

4 – The period of the signal $x(t) = 10 \sin 12\pi t + 4 \cos 18\pi t$ is:

a) π/4
b) 1/6
c) 1/9
d) 1/3
e) 1/30

Factor out 2π. Then, there are two waveforms of frequencies 6 and 9, respectively. Hence, the combined frequency is the highest common factor between 6 and 9 which is 3. The period is then 1/3.

5 – The following gene sequence contains significant frequencies. Design two different symbolic encodings and in each case apply your encoding to extract some of these frequencies.

ACAGAGATACAGAGATACAG......

A=1, G=C=T=0 ➔ *101010101010101010101… so period is 2, f=1/2*
A=1,G=2,C=3,T=4 ➔ *1213131412131314121 3… so period is 8, f=1/8*

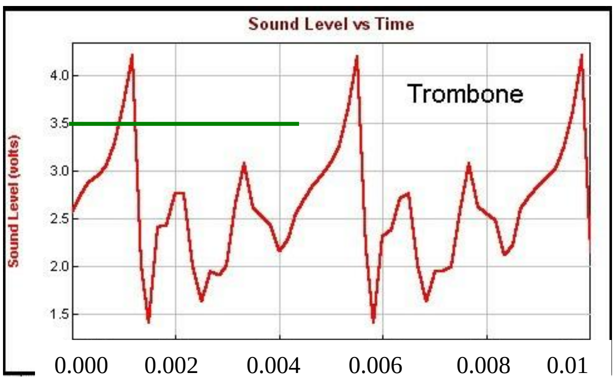
6 – If the fastest oscillations that we want to measure are at 120 Hz, which of the following is the most reasonable sampling rate?

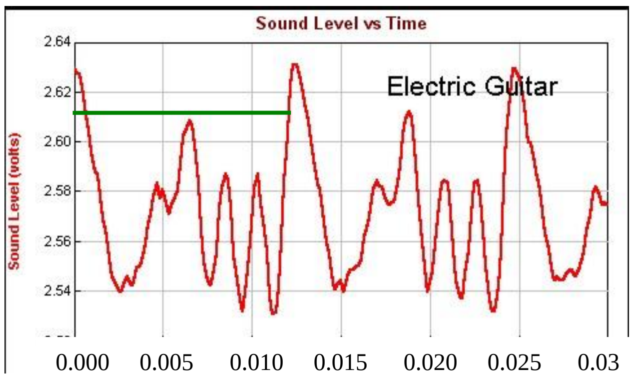
a. 60 Hz
b. 60 kHz
c. anything over 0.00833 Hz
d. 250 Hz
e. 120 Hz

Answer is d. We must sample at, or more than, twice the fastest oscillation in the measured signal.

7 – The graphs below display the amplitude of the sound wave for a Trombone and an Electric Guitar as a function of time. The y-axis is the amplitude axis and the x-axis is the time axis. Notice that each one is plotted over a different length of time.

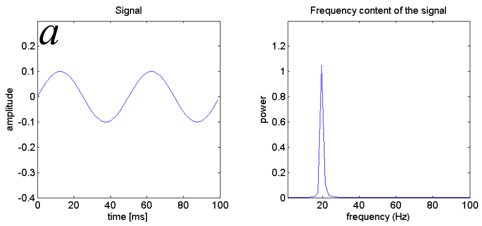
(a) Mark the period of the signal for each instrument.
(b) Approximately, how many periods are shown in these graphs for each instrument?
(c) Approximately, what is the peak amplitude in each case?
(d) Approximately, what is the frequency given the signal period in each case?
(e) Which signal contains higher frequency information? Why?





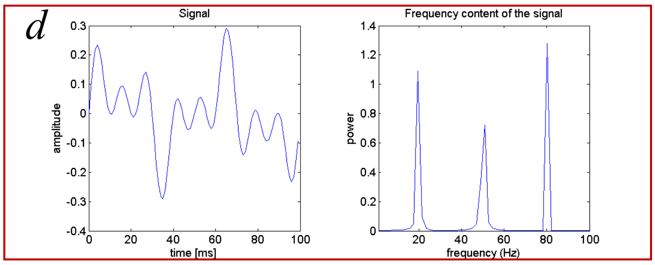
(a) Marked in Green in the diagram above, about 0.0045 and 0.012 respectively.
(b) In both cases around 2 and a bit.
(c) Trombone: about 4.2 EG: about 2.63
(d) $f = 1/T$ so $1/0.0045 = 222.2$ and $1/0.012 = 83.3$ respectively.
(e) The Trombone as it cycles more frequently than the EG over the same time period.

8 – Consider the three signals a, b, and c below, and their addition d.

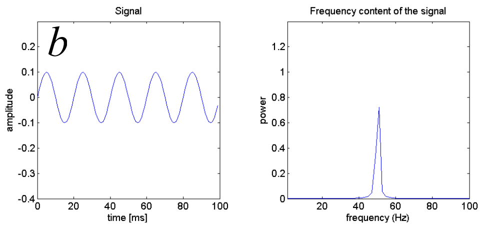


time domain frequency domain

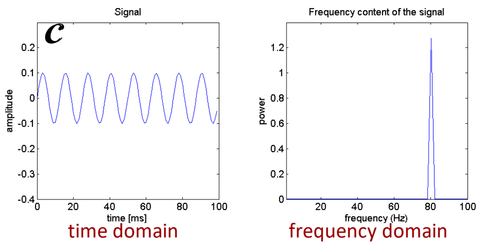
$d = a + b + c$



time domain frequency domain



time domain frequency domain



time domain frequency domain

(a) What would the frequency of the signal d look like?
(b) How many oscillations per second does signal a have?
(c) How can you determine the frequency of signal c if you did not have the frequency domain plot of that signal?

(a) The frequency of signal d would simply include the frequencies of the constituent sinusoids.
(b) Signal a has a peak frequency of 20Hz, so there are 20 oscillations per second.
(c) Looking at the time domain plot of the signal, we can count that it repeats around 8 times per 100ms, so it repeats 80 times in 1 second, and so it’s an 80Hz signal.

9 – What are the two 1D filters that can replace the 2D filter for W, if they were applied consecutively? Do the same for X.

$$W = \frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} x \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} x \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$$

10 – In the Fourier Series of function $f(x) = x$, $-2 < x < 2$, the Fourier coefficient a_2 is equal to:

A- 0
B- 2
C- 1
D- -2

It appears that $f(x)$ is odd, since $f(-x) = -x = -f(x)$. Given the function is odd, therefore all a_n terms are 0.