COMS20017 - Algorithms & Data

Problem Sheet MM02 - covariances, eigenvectors, eigenvalues, normalisation

1. Sketch a simple scatterplot to show low variance in an example data set, and then another plot to show high variance in another dataset.

2. You are given this simple 2D dataset sampled from a larger dataset:

   X = (1 2 3 4 5)
   Y = (2 4 6 8 10)

   Work out the covariance matrix of this dataset (by hand!).

3. Find the eignevalues and eigenvectors of:

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$$

4. Calculate the normalised versions of the following vectors:

   (a) $V = (2, 1, 5)$
   (b) $W = (5, -3, 8, 2)$
   (c) $Z = (2, 2, 2, 2)$

5. After a class test, the teacher computes the mean score to be 60 and the standard deviation of the test scores to be 10. If Jack's test score was 48 and Jill's was 75, what would be their individual z-score?

6. After measuring the height of all the players in its football team, a club finds that the minimum height is 155cm, while the maximum height is 197cm (probably the goalkeeper!).

   The team manager wants to rescale these heights to lie between 0 and 1.

   (a) What is the rescaled height of the central-defender in the team who is 189cm tall?

   (b) Argentinian player Messi is 169cm tall. What is his rescaled height in this range?

   (c) What would be Messi's rescaled height, if the new range was to be between 1 and 10?

7. Find the eignevalues and eigenvectors of:

$$C = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$