Cultural Transmission Models

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Abstract

Bisin and Verdier (2025) summarizes recent advances in modelling cutural transmission in the economics literature. I would like to depart from here to investigate what are possible path for future research topic and question.

1 Canonical Model of Cultural Transmission

1.1 Transmission Technology

Assume that cultural transmission is intergenerational, and there are only two cultural traits i and j. Assume that population size is normalized to 1. There are two channels for intergenerational cultural transmission: *direct vertical* or *parental* transmission, and *oblique* or *social* transmission. For the first channel, there is a probability of d^i for a child to be socialized to cultural trait i by a trait i parent. The residual probability $1 - d^i$ is for the second channel. A child is randomly matched to a member of the parental generation and is socialized to his/her trait. Conditioanl on being socialized via the social transmission channel (with probability $1 - d^i$), the probability of a child to be socialized to trait i is q^i. **Note that if a child's parent is of trait i, then the child can ONLY be socialized to trait i, while she can be socialized to $j \neq i$ only via the social channel.**

Let P^{ij} denote the transition probability that a child is socialized to trait j with trait i parent. Then the cultural transmission technology is represented by

$$P^{ii} = d^i + (1 - d^i)q^i, \qquad P^{ij} = (1 - d^i)(1 - q^i). \tag{1}$$

Then the discrete time dynamics, by Law of Large numbers, will be

$$\Delta q^i_{t+1} = (1 - q^i_t)P^{ji}_t - q^i_t P^{ij}_t.$$

In continuous time, we get the *logistic equation*:

$$\dot{q}^i = q^i(1 - q^i)(d^i - d^j), \tag{2}$$

where $d^i - d^j$ reflects the difference in parental rates. Define $f^i \equiv d^i - d^j$ to be the *relative cultural fitness*.

1.2 Choice

A parent with trait i get payoff V^{ij} if her child acquires trait j. Assume that $V^{ii} > V^{ij}$. A rational parent chooses the transmission rate d^i, facing a cost of $c(d^i)$ which increases with d^i and is convex. The expected parental payoff is $P^{ii}V^{ii} + P^{ij}V^{ij}$, where P^{ii} and P^{ij} are defined by (1). Note that this can be reduced to $V^{ij} + P^{ii}\Delta V^i$, where $\Delta V^i = V^{ii} - V^{ij}$ is defined as the *cultural intolerances* of parent i. The socialization choice problem is reduced to

$$\max_{d^i} \quad P^{ii}\Delta V^i - c(d^i) \tag{3}$$
$$\text{s.t.} \quad P^{ii} = d^i + (1-d^i)q^i.$$

The solution is $d^i = (c')^{(-1)}\left((1-q^i)\Delta V^i\right)$. It increases with ΔV^i and decreases with q^i.

Given this socialization choice setting, we can maintain a steady state / a stable equilibrium where

$$0 < q^i < 1\text{such that } f^i(q^i) = 0. \tag{4}$$

The existence of this stable equilibrium is the consequence of the property that d^i decreases with q^i, called *cultural substitution*. It implies that minority parents will socialize their children more than the majority parents.

2 Economic Models

2.1 Non-separability of Parental Effort and Consumption

Let d^i be an outcome of a production function $f(\cdot,\cdot)$. Define $d^i \equiv d\left(f(m^i, \ell^i)\right)$, where m^i is the resource input and ℓ^i is the time input. We have the following assumptions for $d(\cdot)$ and $f(\cdot,\cdot)$:

Assumption 1. *The effort function $d : \mathbb{R} \to [0,1]$ is an increasing function. It satisfies*

$$\lim_{y\to 0} d(y) = 0, \qquad \lim_{y\to +\infty} d(y) = 1. \tag{5}$$

The production function $f : \mathbb{R}^2 \to \mathbb{R}$ is twice continuously differentiable. It satisfies

$$\frac{\partial f(m^i, \ell^i)}{\partial m^i} > 0, \qquad \frac{\partial f(m^i, \ell^i)}{\partial \ell^i} > 0. \tag{6}$$
$$\frac{\partial^2 f(m^i, \ell^i)}{\partial \left(m^i\right)^2} < 0, \qquad \frac{\partial^2 f(m^i, \ell^i)}{\partial \left(\ell^i\right)^2} < 0. \tag{7}$$

It is homogeneous of degree 1, that is, for any $\alpha > 0$,

$$f(\alpha m^i, \alpha \ell^i) = \alpha f(m^i, \ell^i). \tag{8}$$

Assume that each parent is endowed with resource R and time $L = 1$ to allocate between production with wage w and socializing their child. The choice problem is

$$\begin{aligned} &\max_{x^i, m^i, \ell^i} u^i(x^i) + P^{ii}\Delta V^i \\ &\text{s.t.} x^i + m^i \leq R + w(1-\ell^i) \\ &d^i = d(f(m^i, \ell^i)) \\ &P^i i = d^i + (1-d^i)q^i. \end{aligned} \tag{9}$$

Solving this problem, we obtain the rate of transmission of the form

$$d^i \equiv d^i(q^i, \Delta V^i, R, w). \tag{10}$$

We have the following comparative statics:

(1) $\frac{\partial d^i}{\partial \Delta V^i} > 0$: An increase in ΔV^i brings an increase in the marginal utility of parental effort.

(2) $\frac{\partial d^i}{\partial q^i} < 0$: Culturally minor parents need more effort to overcome the social transmission effect.

(3) $\frac{\partial d^i}{\partial R} > 0$: Income effect from more endowment increases the effort.

(4) $\frac{\partial d^i}{\partial w}$ undetermined: On one hand, higher wage brings stronger substitution effect; on the other hand, income effect increases parental effort.

Now consider heterogeneity between cultural groups i and j. Note that the relative cultural fitness $d^i - d^j$ is an increasing function of R^i and a decreasing function of R^j. If $R^i - R^j$ is not too large, there will exist an interior steady state q^i increasing with R^i.

Harlly's note.

This logic can be applied to gender problems. Parents discriminating on female tend to get boys. Boys are born with more future value (*e.g.*, more opportunities, higher income and social status, etc.). Then $d^i - d^j$ will increase. This implies a reinforce of transmission of women discrimination.

References

Bisin, Alberto and Thierry Verdier, "Economic Models of Cultural Transmission," *NBER Working Paper*, June 2025, *33928.*