Homework #2

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ECON5345, Spring 2026
Due: Feb 22, 2026, 10:00 PM

Reminder: Empirical and simulation exercises must be submitted with "stand-alone" code and data (if any) in an appropriate format. The homework is due Sunday (data, code, and word-processed report by email to **the instructor before 10 PM Sunday**). Also, submit your word-processed report via **Gradescope**.

1. Aguiar and Gopinath (2007) highlight that shocks to trend growth are an important driver of business cycles in emerging economies. In their model, the production function is given by

$$Y_t = A_t K_t^{1-\alpha} L_t^{\alpha},$$

where the productivity A_t equals $\exp(z_t)\Gamma_t^{\alpha}$. The two productivity processes z_t and Γ_t are characterized as follows:

$$z_t = \rho_z z_{t-1} + e_t^z, \quad \text{where } |\rho_z| < 1 \text{ and } e_t^z \sim iidN(0, \sigma_z^2),$$
$$\log(\Gamma_t) = g_t + \log(\Gamma_{t-1}),$$
$$g_t = (1-\rho_g)\mu_g + \rho_g g_{t-1} + e_t^g, \quad \text{where } |\rho_g| < 1 \text{ and } e_t^g \sim iidN(0, \sigma_g^2),$$
$$\text{and } \{\epsilon_t^z\} \perp \{\epsilon_t^g\}.$$

Assume further that $\rho_z \neq \rho_g$.

a) If $a_t = \log(A_t) \sim ARIMA(p, d, q)$, what are the values of p, d, and q?
b) For this question, use the estimated parameters for the Canadian economy (Table 4). For α, see Table 3. Compute the forecast error variance for $h = 0, 1, \dots, 20$ quarters, where
$$FEV_h = E[(a_{t+h} - E[a_{t+h}|\Omega_{t-1}])^2]$$
and the information set Ω_{t-1} includes $z_{t-1}, \Gamma_{t-1}, g_{t-1}, \ldots$ (HINT: obtain the impulse responses of $\{a_t\}$ to e_t^z and e_t^g first. For e_t^g, cumsum in MATLAB can be used). Plot the results.
c) Decompose FEV_h into the mean-squared errors due to shocks to transitory components $\{e_t^z, \dots, e_{t+h}^z\}$ and permanent components $\{e_t^g, \dots, e_{t+h}^g\}$. Compute the share of the mean-squared errors due to transitory shocks out of FEV_h. Plot the results and discuss.

2. Consider $y_t = 0.8y_{t-1} + e_t + 0.4e_{t-1}$ with $var(e_t) = 3$.
   a) Compute and plot dynamic response for periods 0 through 25 for a unit shock in e_0.
   b) What are the mean, variance, and first two autocovariance?
   c) What is the autocovariance-generating function of the process? Verify your answer to b) using the autocovariance generating function.
   d) Simulate the series of sample size $T = 100$ (You may assume that $y_0 = e_0 = 0$. Do not forget to have a burn-in period: e.g., simulate series for T=200 and drop the first 100 observations. Why is the burn-in important?).
   e) Estimate ARMA(1,1) model $y_t = \alpha y_{t-1} + e_t + \theta e_{t-1}$, where $e_t \sim WN(0, \sigma^2)$ using
      i. Hannan-Rissanen procedure
      ii. MoM
   f) Replicate d) and e) 1000 times. Compute and tabulate mean bias, mean square error, and standard deviation of the parameter estimates. Plot the distribution of the estimates. You may want to use the parfor command for parallel computing in Matlab.
   g) Repeat for $T = 50$ and 500.
   h) Discuss your results.

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3. Download the U.S. series for the CPI inflation rate and real GDP growth rate (You may want to use St. Louis Fed database FRED). All series are at a quarterly frequency (aggregate [average] to quarterly frequency if necessary).

a) Estimate and plot ACF and PACF. Describe common patterns (HINT: autocorr and parcorr in Matlab).
b) Using the Box-Jenkins approach, what ARMA(p,q) model may be appropriate for these series? Discuss.
c) Using BIC, choose a parsimonious parametric AR(p) model for each series. Report the values of BIC for p=0,…,10.
d) Conduct the diagnostic checking of the estimated optimal AR model: serial correlation of residuals (e.g., Ljung-Box Q-test for autocorrelations in estimated residuals, lbqtest in Matlab; plot ACF/PACF for estimated residuals; Jarque-Bera test for normality of the error term, jbtest in Matlab).
e) Estimate the AR(p) model using data until 2014:q4. Using the estimated model, conditioning on the realized data until 2014:q4, compute point forecasts until 2019:q4 (for now, ignore sampling uncertainty in parameter estimates. HINT: iterate your AR(p) model forward). Show one and two standard deviation forecast error bands (i.e., a fan chart. HINT: simulate the model with randomly drawn shocks for periods from 2015:q1-2019:q4 many times and compute standard deviation of forecast errors across simulated paths). Compare the forecasts with the realized data from 2015:q1 to 2019:q4.

4. A time series process is deterministic if it can be predicted with certainty. The simplest example of a deterministic process is $x_t = x$ for all t. For this question, consider $x_t = \alpha * \cos(t) + \beta * \sin(t)$ with $\alpha \sim N(0,1)$ and $\beta \sim N(0,1)$, where $\alpha \perp \beta$.

a) What is the mean and variance of x_t?
b) Does $E(x_t x_{t-k})$ depend on time t? Prove.