ECON 5345 Homework 2 Report

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Question 1

a. Q1

Question 2

a. The impulse response function is shown in Figure 1.

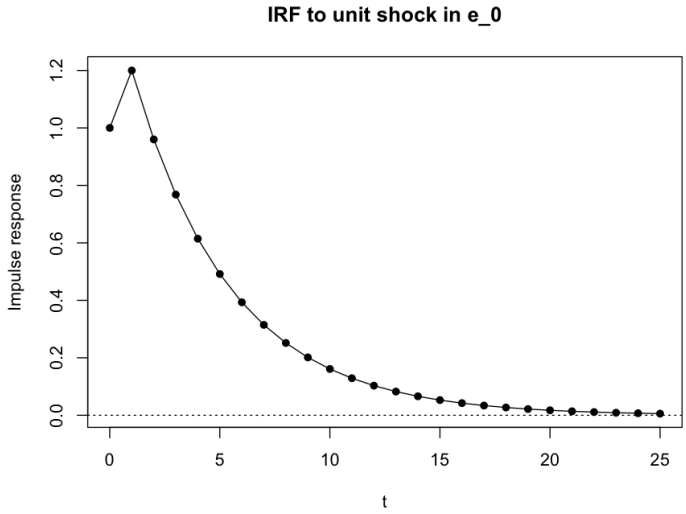

Figure 1: IRF to unit shock in e_0

Numerical results are shown in Table 1.

Table 1: Impulse response

t	irf
0	1.0000
1	1.2000
2	0.9600
3	0.7680
4	0.6144
5	0.4915
6	0.3932
7	0.3146
8	0.2517
9	0.2013
10	0.1611
11	0.1288
12	0.1031
13	0.0825
14	0.0660
15	0.0528
16	0.0422
17	0.0338
18	0.0270
19	0.0216
20	0.0173
21	0.0138
22	0.0111
23	0.0089
24	0.0071
25	0.0057

Question 3

a. Q3

Question 4

a. The expectation is

$$\begin{aligned}\mathbb{E}[x_t] &= \mathbb{E}[\alpha]\cos t + \mathbb{E}[\beta]\sin t \\ &= 0.\end{aligned}$$

The variance is

$$\begin{aligned}\mathrm{Var}[x_t] &= \mathrm{Var}[\alpha]\cos^2 t + \mathrm{Var}[\beta]\sin^2 t + 2\,\mathrm{Cov}(\alpha,\beta)\cos t \sin t \\ &= \cos^2 t + \sin^2 t \\ &= 1.\end{aligned}$$

The second line is by the independence of α and β.

b. No. Note that

$$\begin{aligned}\mathbb{E}[\alpha^2] &= \mathrm{Var}[\alpha] + (\mathbb{E}[\alpha])^2 \\ &= 1 \\ \mathbb{E}[\beta^2] &= \mathrm{Var}[\beta] + (\mathbb{E}[\beta])^2 \\ &= 1 \\ \mathbb{E}[\alpha\beta] &= \mathrm{Cov}(\alpha,\beta) + \mathbb{E}[\alpha]\mathbb{E}[\beta] \\ &= 0.\end{aligned}$$

As a result, we have

$$\begin{aligned}\mathbb{E}[x_t x_{t-k}] &= \mathbb{E}[\alpha^2]\cos t\cos(t-k) + \mathbb{E}[\beta^2]\sin t\sin(t-k) \\ &\quad + \mathbb{E}[\alpha\beta][\cos t\sin(t-k) + \sin t\cos(t-k)] \\ &= \cos t\cos(t-k) + \sin t\sin(t-k) \\ &= \cos[t-(t-k)] \\ &= \cos k.\end{aligned}$$