ECON 5345 Homework 1 Report

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Question 1

a. Note that for any t, we have

$$C_t = C_{t-3} + e_{t-2} + e_{t-1} + e_t.$$

Substituting this into the

$$\begin{aligned} \Delta C_t &\equiv \frac{C_t + C_{t+1} + C_{t+2}}{3} - \frac{C_{t-3} + C_{t-2} + C_{t-1}}{3} \\ &= \frac{e_{t-2} + 2e_{t-1} + 3e_t + 2e_{t+1} + e_{t+2}}{3}. \end{aligned}$$

b. No. They are correlated. At $t+3$, we have

$$\Delta C_{t+3} = \frac{e_{t+1} + 2e_{t+2} + 3e_{t+3} + 2e_{t+4} + e_{t+5}}{3}.$$

It is clear that

$$\text{Cov}(\Delta C_t, \Delta C_{t+3}) = \frac{2}{9}(\text{Var}[e_{t+1}] + \text{Var}[e_{t+2}]) > 0,$$

as long as $\text{Var}[e_{t+1}] + \text{Var}[e_{t+2}] > 0$.

c. No for the first part. Since e_{t-2} and e_{t-1} are known, ΔC_t is correlated with C_{t-2} and C_{t-1}.

Yes for the second part. Information known at $t-3$ only includes white noise no later then $t-3$, while ΔC_t is a linear combination of white noises after $t-3$. Given the serial uncorrelation property of white noise, they are not correlated.

d. The ACF and PACF of the change in measured consumption are shown in Figure 1. Codes in "hw1_q1d.R".

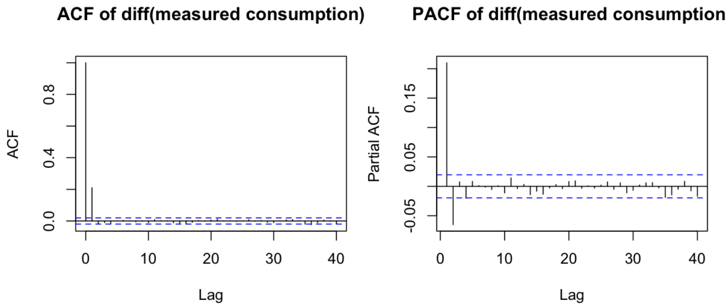
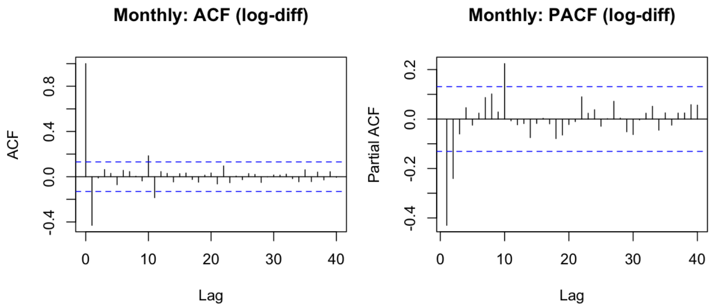
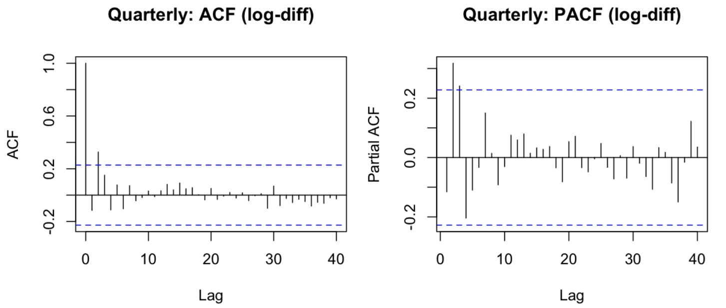


Figure 1: ACF and PACF of the change in measured consumption

e. The ACF and PACF of the change in measured consumption are shown in Figure 2a and Figure 2b. Codes in "hw1_q1e.R".



(a) Monthly data: Jan 2007–Sep 2025 (exclude months after Sep 2025 since quarterly data ends at 2025Q3).



(b) Quarterly data: 2007Q1–2025Q3.

Figure 2: ACF and PACF of the change in consumption (monthly and quarterly). Data source: FRED PCENDC96.

Monthly data shows negative autocorrelation at lag 1, while quaterly data shows positive autocorrelation at lag 1 and 2, even 3. This shows higher persistence in quarterly data, reflecting the fact that averaging over more months introduces more serial correlation, as shown in part (b).

Question 2

a. Since d is observable, the firm has the following optimization problem:

$$\max_p \Pi(p,d) = p^{-\frac{d+1}{d}}(p-1).$$

The FOC condition gives

$$-\frac{d+1}{d}p^{-\frac{d+1}{d}-1}(p-1) + p^{-\frac{d+1}{d}} = 0.$$

Multiplying both sides by $p^{\frac{d+1}{d}}$ and rearranging the terms, we get

$$p^*(d) = 1 + d.$$

b. The code is in "hw1_q2b.R". Since I used R, something may be different from the Matlab code. To find the initial point, I used the help of AI because uniform initial point produces a flat plane.The 3D plot is shown in Figure 3.

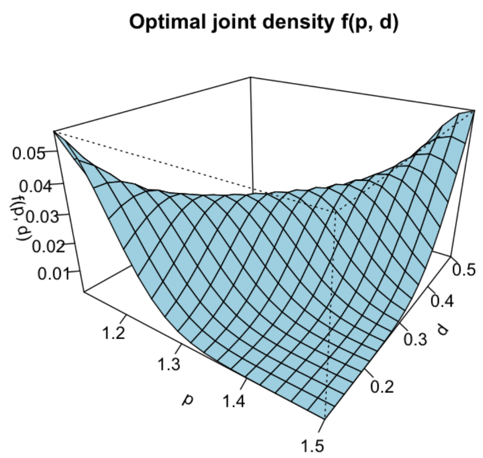


Figure 3: Optimal joint density f(p, d)

Question 3

a. No. When the observations are treated as cross-sectional data, the model becomes

$$y_{i1} = \mu_i + \rho y_{i0} + e_{i1} = \rho y_{i0} + \epsilon_{i1},$$

where $\epsilon_{i1} = \mu_i + e_{i1}$. However, note that, typically, we have

$$\begin{aligned}\mathrm{Cov}(y_{i0}, \mu_i) &= \mathrm{Cov}(\mu_i + \rho y_{i,-1} + e_{i0}, \mu_i) \\ &= \mathrm{Var}(\mu_i) + \rho \mathrm{Cov}(y_{i,-1}, \mu_i) + \mathrm{Cov}(e_{i0}, \mu_i) \\ &\neq 0.\end{aligned}$$

Then the orthogonality condition is violated since the error term also contains μ_i.

When setting $\mu_i = \mu$ for all i's, the covariance becomes zero since y_{i0} have zero covariance with constant μ.

The estimators will converge to the true value as $N \to \infty$ if the orthogonality condition holds.

b. Yes. The model now becomes

$$y_t = \mu + \rho y_{t-1} + e_t.$$

Using backward induction, we can write y_t as a function of $\{e_s | s \leq t\}$, denoted by $h(\mathbf{e_t})$, where $\mathbf{e_t}$ denotes the vector of all e_s's with $s \leq t$. Then independence between e_s's implies that

$$\mathbb{E}[y_{t-1} e_t] = \mathbb{E}[h(\mathbf{e_{t-1}}) e_t] = 0.$$

Since e_s is with zero mean, the orthogonality conditions are satisfied and OLS works.

c. Code in "hw1_q3.R". Results are shown in Table 1.

Table 1: OLS estimates

N	T	$\hat{\rho}$
100	2	-0.006711
500	2	-0.182214
1000	2	-0.038676

There is no convergence to the true value as N increases.

d. Code in "hw1_q3.R". Results are shown in Table 2.

Table 2: OLS estimates

N	T	$\hat{\rho}$
1000	5	0.450082
1000	20	0.776592

Convergence is seen by increasing T. Therefore, for $AR(1)$ model, the Nickell bias is mainly caused by the serial correlation. With T going to infinity, the bias is smaller.