# CONSUMPTION OVER THE LIFE CYCLE

Pierre-Olivier Gourinchas
Jonathan A. Parker

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## ABSTRACT

This paper employs a synthetic cohort technique and Consumer Expenditure Survey data to construct average age-profiles of consumption and income over the working lives of typical households across different education and occupation groups.  Using these profiles, we estimate a structural model of optimal life-cycle consumption expenditures in the presence of realistic labor income uncertainty.  The model fits the profiles quite well.  In addition to providing tight estimates of the discount rate and risk aversion, we find that consumer behavior changes strikingly over the life-cycle.  Young consumers behave as buffer-stock agents.  Around age 40, the typical household starts accumulating liquid assets for retirement, and its behavior mimics more closely that of a certainty equivalent consumer.  This change in behavior is mostly driven by the life-cycle profile of expected income.  Our methodology provides a natural decomposition of saving into its precautionary and retirement components.

Pierre-Olivier Gourinchas
Department of Economics
Princeton University
Princeton, NJ 08544
and NBER
pog@princeton.edu

Jonathan A. Parker
Department of Economics
University of Wisconsin
1180 Observatory Drive
Madison, WI 53706
jparker@ssc.wisc.edu

# 1. Introduction

This paper estimates a dynamic stochastic model of household expenditure. We focus on estimation of structural preference parameters and upon characterizing optimal behavior when households face exogenous, stochastic, labor income processes. We are motivated by two observations.

First, better methodology, data, and creative use of natural experiments lead to mounting evidence against consumption smoothing as an accurate description of household-level behavior. Despite generally poor-quality data on households consumption, recent tests often find that consumption responds to *expected* changes in income. At lower frequencies, there is some evidence that consumption tracks expected and unexpected income changes across groups of households.[1]

Second, recent theoretical work (Zeldes (1989), Deaton (1991), Ludvigson and Paxson (1997) and Carroll (1997b)) demonstrates that log-linearized Euler equations can provide a poor approximation to marginal utility smoothing as well as unreliable estimates of the preference parameters in uncertain environments. Income uncertainty can generate a positive covariance between expected income changes and the conditional variance of consumption growth through precautionary savings, as well as a non-zero correlation between the conditional variance of consumption growth and its higher conditional moments, typically subsumed in the error term of the approximation. The presence of uncertainty invalidates in general most tests using a linearized form of the Euler equation.

In this paper, we use household consumption data and simulation techniques to estimate a structural model of inter-temporal consumption choice with realistic levels of income uncertainty. We measure, exploit and analyze the systematic age-pattern of consumption. The estimated model is then used to re-interpret life-cycle consumption and asset accumulation behavior. We find substantial age-heterogeneity in consumption behavior that results from the interaction between, and relative strengths of, retirement and precautionary motives for saving at different ages. This, in turn is largely driven by the age-varying slope of the expected income profile. Further, we provide tight estimates of the key parameters of the household utility function.

We proceed in two steps. Using weak identifying assumptions, we construct consumption and income profiles across the working lives of "typical" men of five different educational attainments and four different occupational groupings, using quality household-level data on consumption and income from a sample of roughly 40,000 households from the Consumer Expenditure Survey (CEX) from 1980 to 1993. Consumption and income profiles are both significantly hump-shaped, despite controlling for family composition and cohort effects, and consumption tracks income reasonably well early in life.

Second, we measure labor income uncertainty from income data from the Panel

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[1]Some of the most recent papers are Lusardi (1996), Shea (1995), Souleles (forthcoming), Parker (forthcoming), Levenson (1996), Carroll and Summers (1991) and Campbell and Mankiw (1989). See Deaton (1992) or Browning and Lusardi (1996) for surveys.

Study of Income Dynamics and use our constructed profiles to estimate a canonical stochastic life-cycle model of consumer behavior. For any guess of the key parameters, we can solve numerically and recursively for the household optimal behavior and then aggregate to generate a simulated life-cycle consumption profile. By matching this simulated profile to its empirical counterpart, we estimate the parameters of the consumption problem using a Method of Simulated Moments.

To the best of our knowledge, this represents the first structural estimation of consumption functions which incorporates precautionary savings.

In our model, the optimal choice of consumption depends not only on lifetime resources and the real interest rate, but on the growth rate of the *expected profile* of income so that consumer behavior may vary systematically as households age (Hubbard, Skinner, and Zeldes (1994), Carroll (1992)). When expected income growth and the discount rate are low relative to the interest rate, consumers' behavior remains similar to that predicted by the standard life-cycle model. If, on the other hand, expected income growth or the discount rate are large relative to the interest rate, consumers behave as buffer-stock agents — wishing to consume large amounts out of future resources but instead saving enough to weather bad income draws. Since expected income profiles are hump-shaped, the model can potentially deliver average consumption profiles which are more concave than income profiles.

The paper delivers four main findings.

First, the fitted model matches the correlation between consumption and income at young ages and the general concavity of the profile that is observed in the data. By appropriately constructing the inputs to the household problem and by estimating credible preference parameters, we improve on the previous studies based mostly on simulation for a limited number of cases.

Second, we find reasonable estimates of the preference parameters. The average household has a discount rate of around $4.0 - 4.5$ percent and a marginal propensity to consume at retirement of $6 - 7$ percent. Less robust across specifications, we find a coefficient of relative risk aversion varying between 0.5 and 1. We also investigate the robustness of the model by playing a number of variations. These highlight the dimensions along which the model needs to be enriched.

Third, the paper contributes to the debate on the determinants of wealth accumulation. In our model, the relative movements of the consumption and income profiles reveal a great deal of information about the relative roles of precautionary and retirement motives for accumulating liquid assets. Defining all wealth accumulated at retirement as life-cycle assets, we can decompose saving into precautionary and life-cycle saving.[2] Our fitted model indicates that wealth is accumulated early in life for *precautionary* reasons –were it not for income uncertainty households would instead borrow against future labor income. Households older than 40 save mostly for retirement and bequests. Our fitted model hence indicates that observed saving patterns are quite consistent with forward-looking optimizing behavior in a

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[2]We do not enter the debate on the relative importance of retirement versus bequest savings. Implicitly, our retirement savings measure will include both.

life-cycle framework augmented to include income uncertainty.

Finally, and robustly, we find strikingly different consumption behavior for households at different ages: households behave like 'buffer-stock' consumers early in their working lives and more like certainty-equivalent-lifecycle-hypothesis (CEQ-LCH henceforth) households as retirement nears. We estimate that households make the transition from buffer-stock to CEQ-LCH behavior between age 40 and age 45. We conclude that a significant fraction of households consists of *target savers*, for whom the log-linearized form of the Euler equation should be expected to fail.[3] It is interesting to note that buffer-stock behavior arises early in life not from high levels of household impatience but rather from the steepness of the expected income profiles at young ages.[4] Our results are not assumed: with a sufficiently low estimated discount rate, the average consumption profile would be very similar to that of the certainty equivalent case.

Our paper is related to three main strands of literatures. First, to construct our profiles we build on the studies of life-cycle behavior in Ghez and Becker (1974), Kotlikoff and Summers (1981), and Carroll and Summers (1991), using the techniques of Deaton (1985) and Browning, Deaton, and Irish (1985). Thus we are closely related to Attanasio, Banks, Meghir, and Weber (1997) and Attanasio and Browning (1995) that argue that household level data can be explained by the CEQ-LCH once a more flexible treatment of preferences and or aggregation of commodities is allowed for. We argue instead that the data can be explained by variations in precautionary saving. To test one explanation against the other requires a model that nests preference heterogeneity and precautionary saving motives for saving. Both approaches can match certain features of averaged consumption data.[5]

Second, we build on previous studies which parameterize and simulate life-cycle consumption models with uncertainty. Hubbard, Skinner, and Zeldes (1994) and Carroll (1997a) show that the optimal consumption choices of consumers can lead to profiles which are hump-shaped and track income over the early part of life for some parameterizations. Hubbard, Skinner, and Zeldes (1995) simulate profiles so as to try to reproduce constructed profiles of assets over the life-cycle.[6] They re-interpret low-asset holding by some households as driven by means-tested gov-

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[3]This is, in part, a confirmation of Carroll and Samwick (1997) and Carroll (1993) that argue, based on asset data, that buffer-stock models apply only to households before ages 45 to 50.

[4]Carroll (1997a) argues that this is likely to be true.

[5]Attanasio, Banks, Meghir, and Weber (1997) shows that the residuals from a regression of consumption on family composition and labor supply variables are uncorrelated with age. However, if precautionary saving is part of the reason for the initial hump shape in consumption over the life cycle, this regression suffers from an omitted variable bias, which will incorrectly assign the hump to changes in demographics. Similarly, if we are fitting a model with too little preference heterogeneity to the data, then we will incorrectly assign too much of the shape to variations in precautionary saving. we try to allow, modestly, for such heterogeneity by controlling for heterogenous family size at a given age and estimating separately across occupation and education groups.

[6]These authors do not correct for family-size or cohort effects.

ernment programs.[7] Carroll and Samwick (1997) calibrate the discount rate of a structural model to replicate the sensitivity of asset holding to income uncertainty that is observed at young ages. Our approach goes beyond those studies by estimating a structural model of consumption.

Finally, a few papers estimate the optimal level of consumption as a function of a household's fully specified environment. Palumbo (1999) uses individual consumption, income and asset data to estimate individual consumption levels for retirees. We choose to rely on average profiles precisely because we do not believe that the individual-level data are of sufficient quality to support the employed technique in general. In addition, there are now several papers that use a methodology derived from our paper to address asset holding (Gakidis (1998), Cagetti (1998)) and labor supply (French (1998)) decisions.

The structure of the paper is as follows. Section 2, lays out an empirically tractable model of consumer maximization and characterizes optimal behavior. Section 3 introduces the two-step Method of Simulated Moments methodology for estimating the model. The fourth section describes the data, discusses empirical issues involved in constructing life-cycle profiles of consumption and income, and presents graphs of the profiles for various education and occupation cells. Finally, we present the results of the estimation and conclude. Appendices contain more detailed descriptions of the numerical optimization, the econometric procedure, and the CEX data.

## 2. Consumption Behavior with Stochastic Income

### 2.1. The Canonical Model with Labor Income Uncertainty

Our starting point is the basic discrete-time, life-cycle model of consumption behavior. Consumers live for $N$ periods and work for $T < N$, where both $T$ and $N$ are exogenous and fixed. In every period $t \leq T$, the consumer receives a stochastic income $Y_t$. There is one asset in the economy, totally liquid and yielding a constant gross, after-tax, real interest rate $R$. Our unit of analysis is the household. We assume that preferences take the standard additively separable expected utility form, with a discount factor $\beta$:

$$E\left[\sum_{t=1}^{N} \beta^t u\left(C_t, Z_t\right) + \beta^{N+1} V_{N+1}(W_{N+1})\right], \qquad (2.1)$$

where $C_t$ represents consumption, $W_t$ represents total financial wealth and $Z_t$ is a vector of deterministic household characteristics (e.g. family size). $V_{N+1}$ represents the value to the consumer of any assets left at the time of death, capturing any

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[7]We do not address this additional complication, but simply note that we believe that the heterogeneity in skills, abilities, and wealth across people starting their working lives makes the low-asset trap of their model very relevant for a small subset of the population and much less relevant for the typical household. See the discussion in Carroll and Samwick (1997).

bequest motive. The consumer maximizes (2.1) given an initial wealth level $W_1$, and the constraint that terminal wealth is non-negative $W_{N+1} \geq 0$.

The dynamic budget constraint is:

$$W_{t+1} = R\left(W_t + Y_t - C_t\right).$$

We further assume that the felicity function $u\left(.,.\right)$ is of the Constant Relative Risk Aversion ($CRRA$) form, with intertemporal elasticity of substitution $1/\rho$, and multiplicatively separable in $Z$:

$$u\left(C, Z\right) = v\left(Z\right)\frac{C^{1-\rho}}{1-\rho}.$$

If income were certain, the solution to this program would be standard: the consumer would choose a consumption path such that

$$\frac{C_{t+1}}{C_t} = \left(\beta R \frac{v\left(Z_{t+1}\right)}{v\left(Z_t\right)}\right)^{\frac{1}{\rho}}. \tag{2.2}$$

With constant individual characteristics, (2.2) implies a constant growth rate of consumption. Consumption increases (respectively decreases) over time when the interest rate is larger (respectively smaller) than the discount rate. The growth rate of consumption is independent of the income profile. The level of consumption is determined by the lifetime budget constraint and the terminal value function.

When individual characteristics vary over the life cycle, the growth rate of consumption may change accordingly. For instance, if the marginal utility of consumption increases with family size, consumption will grow faster as family size increases, and slower as children leave the household. These variations in individual characteristics may induce a positive correlation between consumption and income over the life cycle.

With individual income uncertainty and prudence, households hold liquid wealth to insure themselves against future contingencies. This precautionary-saving motive has potentially far-reaching and striking implications. The main consequence of income uncertainty is to increase the slope of the consumption profile. Hubbard, Skinner, and Zeldes (1994) demonstrates that this uncertainty can lead to hump-shaped consumption profiles as households save for precautionary reasons early in life and run down these assets during retirement due to lower levels of uncertainty and an increased probability of death.

Zeldes (1989), Carroll (1997a)and Deaton (1991) analyze the case in which consumers are also *impatient*: absent uncertainty, households would like to borrow in order to finance a high level of current consumption. Deaton (1991) and Zeldes (1989) impose liquidity constraints while Carroll (1997a) sets up a model in which consumers choose never to borrow. In either rendition, assets play the role of a *buffer stock* against bad income shocks. Consumers have a target level of liquid assets, above which impatience dominates and assets are decumulated,

and below which the precautionary motive dominates and assets are accumulated. Thus the theory predicts a positive correlation between expected income growth and consumption growth.

This paper explicitly incorporates uninsurable idiosyncratic income uncertainty. We adopt Zeldes' (1989) formulation, and decompose the labor income process into a permanent component, $P_t$, and a transitory component, $U_t$:

$$Y_t = P_t U_t$$
$$P_t = G_t P_{t-1} N_t$$

The transitory shocks, $U_t$, are independent and identically distributed, take the value 0 with probability $p \geq 0$, and are otherwise log-normally distributed, $\ln U_t \sim \mathcal{N}(0, \sigma_u^2)$. The log of the permanent component of income, $\ln P_t$, evolves as a random walk with drift. $G_t$ is a deterministic growth factor (specific to age $t$) while $\ln N_t$, the shock to the permanent component of income, is independently and identically normally distributed with mean zero and variance $\sigma_n^2$.[8] Thus income evolves as a *nonstationary*, serially correlated process, with both permanent and transitory shocks, and a positive probability of zero income in every period.[9]

Two points are worth noting. First, innovations to the permanent component of income are only as permanent as the remaining length of the working life: all shocks are ultimately transitory, as consumers retire and die. As a consequence, the propensity to consume out of 'permanent' shocks will vary with age, a point emphasized by Clarida (1991). This property holds true for the CEQ-LCH also. Second, in this setup consumers will choose never to borrow against future labor income, a point shown by Schechtman (1976). This follows from (a) there being a strictly positive probability that labor income will be arbitrarily close to zero for the rest of the working life and (b) the Inada condition $\lim_{c \to 0} u'(c) = \infty$. To see this, suppose the household were to borrow in the next to last working period. Then, with strictly positive probability it would be left without any wealth in the last working period. The household would then have an infinite expected marginal utility. Simple backward induction implies that it will never be optimal to borrow in any period. The precautionary motive acts as a *self-imposed* liquidity constraint. It is important to note that this holds true even when $p$, the probability of strictly zero income, is set to zero. The key here is that the income process admits a finite lower bound. This and the requirement that the consumer dies without debt *almost surely,* impose a natural borrowing limit (see Ayagari (1993)). With a strictly positive lower bound on income, the consumer could borrow only up to the present discounted value of certain future income.

Going from the model to the data, we need to make four assumptions. First, in order to solve the consumer's problem as stated, we need to specify both the

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[8]The permanent component of income is that level that would obtain without transitory shocks, as in Friedman (1957), not the present discounted value of future income streams, as in Flavin (1981).

[9]While Abowd and Card (1989) found that change in labor income was best characterized by an $MA(2)$ process, they also found little gain in moving from an $MA(1)$ to an $MA(2)$.

nature of uncertainty during retirement and a bequest function. While there have been good attempts at modelling consumer behavior after retirement,[10] we feel that we know too little about the form that uncertainty takes after retirement to use our methodology and draw inferences from post-retirement behavior. Uncertainty arises from different sources – e.g. medical expenses, the timing of death and asset returns. *Inter-vivos* bequests are important. Although these sources of uncertainty are also present to some extent in the last working years, labor income uncertainty is the dominant source of uncertainty when young. Further, high quality information on household asset holdings, together with consumption and income, is not available. Given that investment income, social security, and pensions represent the main sources of income during retirement, it is currently difficult to establish consumption patterns as a function of total wealth. Lastly, even with a proper treatment of retirement issues, one would have to make a guess about the bequest function. Instead, we make use of Bellman's optimality principle, and truncate our problem at the date of retirement.

Second, we assume that age variations in $v(Z_t)$ are common across households of the same age $t$, deterministic, and come from changes in family size, so that the evolution of the consumer problem can be captured by a single state variable.[11] This has the added advantage that we can adjust and report consumption and income profiles that maintain a constant effective family size across ages.

Third, the model imposes a single vehicle for precautionary and retirement wealth accumulation, since there is only one asset. In practice, much of net worth at retirement is accumulated in the form of illiquid wealth, only available after retirement.[12] This suggests that the relevant model of consumption behavior should incorporate an additional asset which is illiquid and accessible only after retirement. However, this would substantially complicate the problem by introducing another control variable (how much to save in liquid versus illiquid assets) and state variable (illiquid assets). In order to keep our estimation procedure tractable, we instead assume that illiquid wealth accumulates exogenously, cannot be borrowed against, and that illiquid wealth in the first year of retirement, $H_{T+1}$, is linearly related to the last permanent component of income, $P_{T+1} \equiv P_T$. These assumptions eliminate illiquid assets as a state variable of the program and contributions to illiquid assets as a control variable.[13]

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[10]See Hubbard, Skinner, and Zeldes (1994), Palumbo (1999), and Hurd (1989) .

[11]We directly control for heterogenous family size at a given age when constructing consumption and income profiles. The alternative is to model explicitly the stochastic process for changes in family size.

[12]Social security wealth is definitely illiquid and is only available as annuities after retirement. Early withdrawal of pension and savings vehicles targeted for retirement purposes, such as IRA's, 401k plans and Keogh, is often restricted and fiscally penalized, if allowed at all. One might also consider housing wealth as part of retirement wealth. Empirical evidence suggests that households run down their housing wealth only extremely late in life.

[13]We recognize that this assumption is problematic in some ways. For instance, two households with different income trajectories but the same permanent component of income at retirement are predicted to have identical accumulated illiquid wealth.

Defining the value function for the household problem at time $\tau$, $V_\tau$, our problem becomes:

$$V_\tau\left(X_\tau, P_\tau, Z_\tau\right) =$$
$$\max_{c_\tau,\ldots,c_T} E_\tau\left[\sum_{t=\tau}^T \beta^{t-\tau} v(Z_t)\frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau}\ V_{T+1}\left(X_{T+1} + H_{T+1}, P_{T+1}, Z_{T+1}\right)\right]$$
$$s.t.\ \ X_{t+1} = R\ \left(X_t - C_t\right) + Y_{t+1}, \quad X_{T+1} \geq 0,$$

where we define cash on hand $X_{t+1}$ as total liquid financial resources at time $t+1$:

$$X_{t+1} = R\left(X_t - C_t\right) + Y_{t+1} = W_{t+1} + Y_{t+1}.$$

and where income is defined as disposable income, net of saving and Social Security taxes that build illiquid assets. $H_\tau$ represents the net present value of illiquid assets accumulated at age $\tau$. The assumption that illiquid wealth cannot be borrowed against imposes the borrowing constraint that liquid wealth must be weakly positive at retirement.

Finally, we postulate a retirement value function which summarizes the consumer's problem at retirement time. The functional form we choose maintains the tractability of the problem and is flexible enough to allow robustness checks:

$$V_{T+1}\left(X_{T+1} + H_{T+1}, P_{T+1}, H_{T+1}, Z_{T+1}\right) = \kappa\, v\left(Z_{T+1}\right)\left(X_{T+1} + H_{T+1}\right)^{1-\rho},$$

for some constant $\kappa$. Under the CRRA assumption, this functional form is exactly correct if time of death or asset returns are the only sources of uncertainty after retirement (Merton (1971)). Optimal consumption is then linear in total wealth at retirement. Since almost all households have large amounts of illiquid wealth in housing, Social Security, and pensions at retirement, a linear approximation for the true consumption rule is likely to be quite close.[14]

### 2.2. Solving for Optimal Consumer Behavior

The setup of the problem combined with our particular choice of retirement value function makes the problem homogeneous of degree $1 - \rho$ in the permanent component of income $P_t$. We write the optimal consumption rule as a function of age, $t$, and the ratio of cash on hand to the permanent component of income, $x_t \equiv X_t/P_t$. The budget constraint becomes:

$$x_{t+1} = \left(x_t - c_t\right)\frac{R}{G_{t+1}N_{t+1}} + U_{t+1}, \quad 1 \leq t \leq T. \tag{2.3}$$

where lowercase letters are normalized by the permanent component of income. The following Euler equation holds in all periods prior to retirement:

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[14]To the extent that there is curvature in the relevant range of the consumption rule our assumption of linearity will bias us away from finding important effects of uncertainty on consumption.

$$u'\left(c_t\left(x_t\right)\right) = \beta R \; E\left[\frac{v(Z_{t+1})}{v(Z_t)}u'\left(c_{t+1}\left(x_{t+1}\right)G_{t+1}N_{t+1}\right)\right], \qquad (2.4)$$

where $c_t(x_t)$ represents the optimal consumption rule at time $t$ (normalized), as a function of normalized cash on hand $x_t$.

The solution to the consumer problem consists of a set of consumption rules $\{c_t\left(x_t\right)\}_{1\le t\le T}$. Given our assumptions about the value function and the proportionality of illiquid wealth and the permanent component of income at retirement, the consumption rule is linear in $x$:

$$c_{T+1} = \gamma_0 + \gamma_1 x_{T+1}, \qquad (2.5)$$

where $\gamma_1$ represents the marginal propensity to consume out of wealth, given the family size at retirement, and $\gamma_0 \equiv \gamma_1 \frac{H_{T+1}}{P_{T+1}}$.[15]

The consumption rule in the next to last period is then found as the solution to (2.4) for all values of cash on hand, where we replace $c_{T+1}$ using equation (2.5). Solving recursively generates consumption functions $c_T(x_T), .., c_1(x_1)$. A complete description of the solution method is provided in Appendix A.

## 2.3. Characterization of Individual Consumption Behavior

Figure 2.1.$a$ shows consumption rules at various ages, when the permanent component of income is flat $(G_t = G = 1)$ for a typical household working from age 25 to 65 and retiring thereafter with a consumption rule at age 66 characterized by:[16]

$$\gamma_0 = 0.001, \; \gamma_1 = 0.071.$$

When $G$ is constant, the finite horizon problem converges in the limit to the infinite horizon one, as we move further away from retirement. Consumption is always positive, increasing and concave in cash on hand. Under some additional conditions, one can also show that cash on hand can only increase if the income draw is sufficiently large.[17] Early in life, households exhibit the standard buffer stock behavior: for low level of cash on hand, typically less than the permanent component of their income $(x \le 1)$, households consume most, but never all, of their financial wealth, and move to the next period with a very low level of liquid assets. At high levels of cash on hand, the precautionary motive is small and households

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[15]In the case of full certainty after retirement and no change in the utility shifter, $\gamma_1 = \frac{1-\beta^{1/\rho}R^{1/\rho-1}}{1-\left(\beta^{1/\rho}R^{1/\rho-1}\right)^{N-T+1}}$.

[16]We discuss calibration of $\gamma$ in section 4. Other relevant parameters are $\beta = 0.960$, $\rho = 0.514$ and $R = 1.0344$. These parameters generate buffer-stock behavior. Income uncertainty is presented in Table 3.1.

[17]The key condition is that there exists an upper bound on current income. With CRRA preferences, this is enough to guarantee that cash on hand has an ergodic distribution. This condition is analyzed in more details in Deaton (1991), Ayagari (1993) and Schechtman and Escudero (1977).
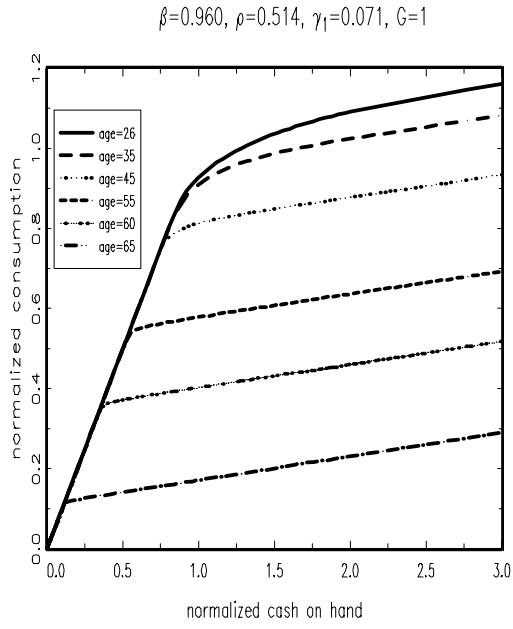
Figure 2.1.a: Consumption Rules

$\beta$=0.960, $\rho$=0.514, $\gamma_1$=0.071, G=1
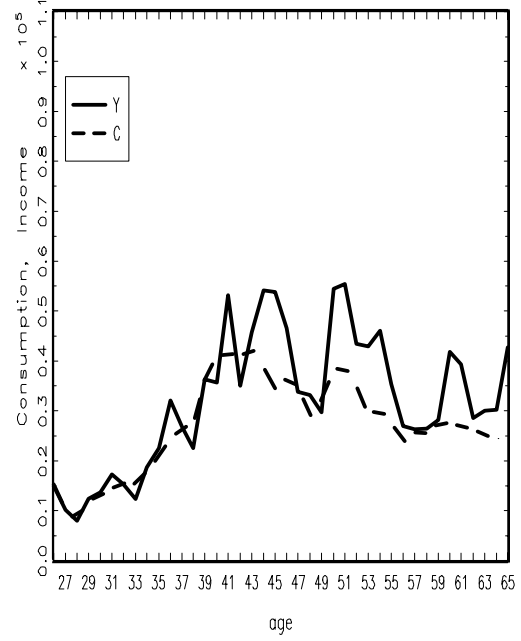
Figure 2.1.b: Individual Profile

Figure 2.1.c: Individual Profile,
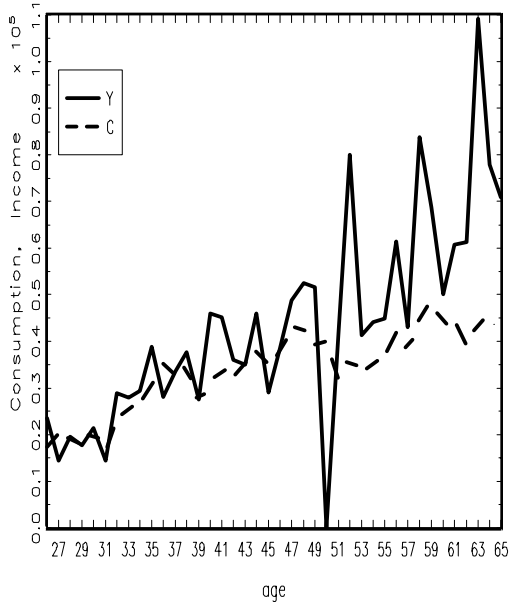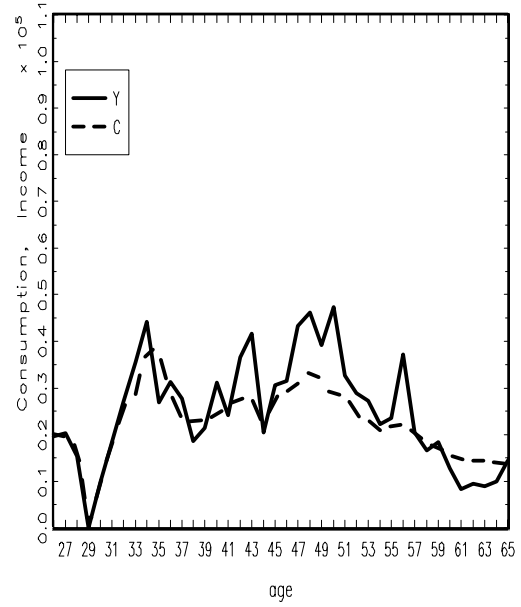0-Income Shock Age 50

Figure 2.1.d: Individual Profile,
0-Income Shock Age 29

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consume more than the income they expect to receive (which equals 1) and so run down their assets. This buffer stock behavior early in life is largely independent of the retirement part of the life cycle and is governed by the solution to the infinite horizon problem.

Figures 2.1.$b$, 2.1.$c$ and 2.1.$d$ display randomly drawn profiles of consumption for households facing typical paths of income, retirement rules, and income uncertainty.[18] In the profiles, consumption tracks income early in life, and diverges later in life. Notice further that unexpected transitory shocks are better smoothed later in life, despite the fact that they contain greater information about total resources for the remainder of the life. For most households, consumption volatility is lower later in life since the accumulation of liquid wealth intended to provide for retirement also provides self insurance against bad income shocks.

What general conditions are necessary to generate buffer-stock behavior? In a stationary infinite-horizon model, buffer stock behavior arises when agents are sufficiently impatient not to accumulate wealth without bounds. Formally, the mapping from current wealth to expected wealth in the next period must be a contraction. In the infinite horizon case, Carroll (1997a) and Deaton (1991) prove that a solution to the consumer problem exists if and only if:

$$R\beta E\left[(GN)^{-\rho}\right] < 1. \tag{2.6}$$

Thus, a higher preference for the present, steeper growth of expected income, or a lower interest rate each make buffer stock behavior more likely.

Because, in a life-cycle model income growth and behavior change with age, no such simple characterization exists. Instead, we adopt a practical approach and call a household a *buffer stock* household if it saves *more* for precautionary motives than for life-cycle (bequest or retirement) motives. A household whose saving decisions are motivated primarily by uncertainty will have the characteristics of buffer stock behavior just described.

## 3. Estimation Strategy.

This section describes the two-step method of simulated moments strategy. The general strategy is as follows. Given the parameters of the consumer problem defined in the previous section (preferences, income process,etc.) one can solve numerically for the age-dependent optimal consumption rules. We do not observe these consumption rules directly. Instead, we observe the average consumption levels over age for various groups of households. Hence we numerically simulate, for a given set of consumption rules, the associated expected consumption as a function of age only. The estimation proedure then minimizes the distance between the sample and the simulated profiles.

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[18]What is typical will be discussed in detail in Sections 3 and 4.

### 3.1. Method of Simulated Moments (MSM) Estimation.

According to section 2, consumption at age $t$ for individual $i$ depends on normalized cash on hand $x_{i,t}$, the realization of permanent component of income $P_{i,t}$, and the parameters of the consumption problem, which we now denote by $\psi \in \Psi \subset \mathbb{R}^s$. Defining the vector of state variable $s_{i,t} = (x_{i,t}, P_{i,t})$, we postulate the following data-generating process for each age, $t$:

$$\ln C_{i,t} = \ln C_t\left(s_{i,t}; \psi\right) + \epsilon_{i,t} = \ln(c_t\left(x_{i,t}; \psi\right) P_{i,t}) + \epsilon_{i,t}, \tag{3.1}$$

where $\epsilon_{i,t}$ is an idiosyncratic shock that represents measurement error in consumption levels.[19] We are interested in estimating $\psi$ and then making inference about consumption behavior. Without quality panel data on consumption, asset and income for individual households, direct estimation using (3.1) is not possible.

We do however observe the average of log-consumption at each age is defined as $\ln \bar{C}_t \equiv \frac{1}{I_t} \sum_{i=1}^{I_t} \ln C_{i,t}$ where $I_t$ represents the number of observations at age $t$ and $\ln C_{i,t}$ is defined by equation (3.1). This suggests that we can look directly at the *unconditional* expectation of log-consumption at each age:

$$\ln C_t\left(\psi\right) \equiv E\left[\ln C_t\left(s_t; \psi\right)|\psi\right] = \int \ln C_t\left(s; \psi\right)\,dF_t(s; \psi), \tag{3.2}$$

where the unconditional cumulative distribution of normalized cash on hand and permanent component of income at age $t$, $F_t\left(s; \psi\right)$ depends on age $t$ and on the parameters $\psi$. We seek to estimate the model from the following moment conditions:

$$E\left[\zeta\left(\ln C_i; \psi_0\right)\right] = 0,$$

where $\psi_0$ is the true parameter vector, $\ln C_i = \{\ln C_{i,t}\}_{t=1}^T$ and $\zeta(\ln C_i, \psi) \in \mathbb{R}^T$, with $t^{th}$ element:

$$\zeta_t\left(\ln C_i; \psi\right) = \ln C_{i,t} - \ln C_t\left(\psi\right). \tag{3.3}$$

This approach must address two further complications. First, it is difficult to estimate accurately all elements of $\psi$ in one step. This is due in part to the computational complexity of the problem and in part to the loss of information that averaging entails. Instead, we employ a two-stage estimation procedure. We partition $\psi$ into two sub-vectors $\theta \in \mathbb{R}^k$ and $\chi \in \mathbb{R}^r$. Assume that $\theta$ belongs to some compact set $\Theta \subset \mathbb{R}^k$. We use additional data and moments to estimate $\chi$ in a first stage according to:

$$E[\mu(\chi)] = 0.$$

where $\mu \in \mathbb{R}^r$.[20] We discuss this step in the next sub-section and denote the first-stage estimator by $\hat{\chi}$. For instance, rather than estimate the variance of permanent and transitory shocks to income from average consumption and income profiles,

---

[19]$\epsilon_t^i$ may also encompass missing variables such as class. Our approach remains correct as long as $\epsilon_t^i$ and the state variables are independent.

[20]The first stage is exactly identified while the second stage is overidentified.

where identification might prove difficult in practice, we use time-series moment conditions and true household-level panel data on income from the Panel Study of Income Dynamics.

The second complication arises from the analytical complexity of the unconditional expectation (3.2). The unconditional distribution for the state variables at age $t$, $F_t(s; \psi)$, is extremely cumbersome to evaluate and depends on the parameters $\psi$. To overcome this difficulty, we use the *Method of Simulated Moments*, as developed by Pakes and Pollard (1989) and Duffie and Singleton (1993). Using the budget constraint (2.3), and the process for $P_t$, we define a measurable transition function $\mathfrak{T}: \mathbb{R}^2 \mathbf{x} \mathbb{R}^2 \mathbf{x} \Theta \mathbf{x} \mathbb{R}^r \to \mathbb{R}^2$ that describes the dynamics of the state variables $s_{t+1} = (x_{t+1}, P_{t+1}) = \mathfrak{T}(s_t, \nu_{t+1}; \theta, \widehat{\chi})$, where $\nu_{t+1} = (U_{t+1}, N_{t+1})'$. This transition function can then be used to rewrite the unconditional expectation (3.2):

$$
\begin{aligned}
\ln C_t (\theta, \widehat{\chi}) &= \int \ln C_t (s, \theta, \widehat{\chi}) \, dF_t(s; \theta, \hat{\chi}) \qquad (3.4) \\
&= \iint \ln C_t (\mathfrak{T}(s, \nu; \theta, \widehat{\chi}), \theta, \widehat{\chi}) \, dF_{t-1}(s; \theta, \hat{\chi}) \, dF^\nu (\nu).
\end{aligned}
$$

From (3.4), we approximate the theoretical unconditional expectation using Monte-Carlo integration. Assume that we have an $\mathbb{R}^2 \mathbf{x} \mathbb{R}^T$-valued sequence of random variables $\{\hat{\nu}_l\}_{l=1}^{l=L}$ where $\hat{\nu}_l = (\hat{\nu}_{l,1}, ..., \hat{\nu}_{l,T})'$, that are identically independently distributed. From any initial distribution $F_1(s_1)$, candidate $\theta$ and estimated $\hat{\chi}$, we can generate the path of state variables according to:

$$
\hat{s}_{l,t+1} = \mathfrak{T}(\hat{s}_{l,t}, \hat{\nu}_{l,t+1}; \theta, \widehat{\chi}); \qquad \forall \, 1 \leq t \leq T - 1 \text{ and } 1 \leq l \leq L.
$$

The unconditional expectation $\ln C_t (\theta, \hat{\chi})$ is then simulated by:

$$
\ln \hat{C}_t (\theta, \widehat{\chi}) \equiv \frac{1}{L} \sum_{l=1}^{L} \ln C_t (\hat{s}_{l,t}, \theta, \widehat{\chi}) \rightsquigarrow \ln C_t (\theta, \widehat{\chi}),
$$

where convergence occurs as $L \to \infty$. For any parameter vector $\theta \in \Theta$, we can replace the theoretical expectation of consumption with its simulated counterpart in (3.3):

$$
\hat{\zeta}_t (\ln C_i; \theta, \widehat{\chi}) = \ln C_{i,t} - \ln \hat{C}_t (\theta; \widehat{\chi}).
$$

Estimation now simply proceeds by making the *simulated* empirical moments as close as possible to their theoretical values using sample averages:

$$
\begin{aligned}
g_t (\theta; \widehat{\chi}) &= \frac{1}{I_t} \sum_{i=1}^{I_t} \hat{\zeta}_t (\ln C_{i,t}; \theta, \widehat{\chi}) \\
&= \frac{1}{I_t} \sum_{i=1}^{I_t} \ln C_{i,t} - \ln \hat{C}_t (\theta, \widehat{\chi}) \\
&= \ln \bar{C}_t - \ln \hat{C}_t (\theta, \widehat{\chi}).
\end{aligned}
$$

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Note that we do not require repeated observations on the same households. Our second-stage estimation procedure is then a method of simulated moments estimator $(MSM)$ that minimizes over $\theta$:

$$g\left(\theta;\widehat{\chi}\right)' W g\left(\theta;\widehat{\chi}\right),$$

where $g\left(\theta;\widehat{\chi}\right) = \left(g_1, ..., g_T\right)' \in \mathbb{R}^T$ and $W$ is a `TxT` weighting matrix. In the case where $W = I_T$, the identity matrix, the estimation procedure is equivalent to minimizing the sum of square residuals:

$$S\left(\theta;\widehat{\chi}\right) = \sum_{t=1}^{T} \left(\ln \bar{C}_t - \ln \hat{C}_t(\theta;\widehat{\chi})\right)^2.$$

Note however, that we are minimizing the sum of squared residuals with a constant $T$ and that asymptotic results still apply as long as $I_t$, the numbers of observation at age $t$, goes to infinity for each age.

Under the regularity conditions stated in Pakes and Pollard (1989) and Duffie and Singleton (1993), the $MSM$ estimator $\hat{\theta}$ is both consistent and asymptotically normally distributed. However, one needs to correct the second-stage estimator of the variance-covariance matrix for the first-stage estimation.[21] Furthermore, an additional term is required to account for the fact that we have simulated moments rather than theoretical moments.

The variance-covariance estimator and its derivation is contained in Appendix $B$. We use two alternative weighting matrixes in our estimation. First, we choose a matrix which, while not fully optimal, does not depend on the fitted model. This is motivated by the observation that optimally weighting GMM estimators can worsen finite-sample bias (see for example West, Wong and Anatolyev (1998)). Second, we employ the optimal weighting matrix. Our 2-step procedure provides a test of the overidentifying restrictions in the second stage, which is derived in appendix B.

In practice, we simulate $\ln \hat{C}_t(\theta, \hat{\chi})$ by running $L = 20,000$ independents income processes for 40 years, and computing in each year the associated consumption and cash on hand. Once the optimum is found, the gradient of the moment vector is evaluated numerically and the variance-covariance matrix estimated.

### 3.2. First-Stage Estimation

The first-stage parameters $\chi$ include the real after tax interest rate $R$; the variances of the innovations to the permanent and transitory components, $p, \sigma_u^2, \sigma_n^2$; the initial distribution of liquid assets at the beginning of life $\left(w_1, \sigma_{w_1}^2\right)$ and the income and family-composition profiles. The profiles' construction will be discussed in the next section.

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[21]We thank the anonymous referees and David Card for suggesting that we move from calibration and robustness checks to formally integrating the first stage.

Table 3.1: VARIANCE OF INCOME SHOCKS

| Group | VARIANCE OF PERMANENT SHOCK | VARIANCE OF TRANSITORY SHOCK |
|---|---|---|
| TOTAL | 0.0212 | 0.0440 |
| OCCUPATION | | |
| Managerial and Prof. Speciality | 0.0175 | 0.0341 |
| Tech., Sales, and Admin. Support | 0.0235 | 0.0361 |
| Precision Prod., Craft, and Repair | 0.0175 | 0.0432 |
| Operators and Laborers | 0.0299 | 0.0458 |
| EDUCATION | | |
| Some Highschool | 0.0214 | 0.0658 |
| Highschool Degree | 0.0277 | 0.0431 |
| Some College | 0.0238 | 0.0342 |
| College Graduate | 0.0146 | 0.0385 |
| Graduate School | 0.0115 | 0.0500 |

Source: Carroll and Samwick (1997) and author's calculations. Standard errors ranges from ten percent to thirty percent of the coefficient estimate, depending largely on the sample size. These data are available from the authors.

First, we estimate the gross real interest rate, $R$, from the average real return on Moody's AAA municipal bonds and the Gross Domestic Product implicit price deflator for personal consumption expenditures used to deflate the consumption and income data. Over the sample period from January 1980 to March 1993, the average real interest rate is 3.440 percent with a standard error of 0.281.

Second, we estimate the variance of the permanent and transitory components of shocks to income, $\sigma_u^2$ and $\sigma_n^2$, using the data and methodology of Carroll and Samwick (1997).[22] Carroll and Samwick estimate these parameters from the Panel Study of Income Dynamics (PSID), which provides repeated high-quality measures of household income. The GMM procedure and data employed are designed to estimate the parameters of exactly the income process we have specified. We redo their calculations only because the definitions of occupation in the PSID and CEX do not exactly overlap. We aggregate the occupational groups in the PSID into aggregates that better match the groupings that we employ in the CEX. Table 3.1 displays the variances of the permanent and transitory shocks across education and education groups.

We calculate the probability of a truly zero income realization using the frequency of truly zero income in the PSID, as reported by Carroll (1992). Therefore, we set $\hat{p} = 0.00302$ with a standard error of 0.000764.

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[22]We thank Andrew Samwick for providing us with their raw data.

Table 3.2: LIQUID WEALTH HOLDING AT THE BEGINNING OF LIFE

RATIO OF LIQUID ASSETS TO INCOME

| GROUP | MEAN (LOG) | S.D. | MEAN (LEVEL) |
|---|---|---|---|
| TOTAL | -2.794 | 1.784 | 0.300 |
| OCCUPATION | | | |
| Managerial and Prof. Speciality | -2.261 | 1.649 | 0.405 |
| Tech., Sales, and Admin. Support | -2.574 | 1.625 | 0.285 |
| Precision Prod., Craft, and Repair | -3.312 | 2.051 | 0.299 |
| Operators and Laborers | -3.172 | 1.654 | 0.164 |
| EDUCATION | | | |
| Some Highschool | -3.772 | 1.886 | 0.136 |
| Highschool Degree | -3.296 | 1.797 | 0.186 |
| Some College | -2.756 | 1.679 | 0.260 |
| College Graduate | -2.181 | 1.554 | 0.377 |
| Graduate School | -1.887 | 1.540 | 0.495 |

Souce: Authors calculations based on the CEX measures of wealth and the constructed measure of income.

Finally, we require an initial asset level for households. We match both the typical level and the distribution in the population by estimating the ratio of liquid wealth to income for households between 24 and 28 years old. In simulating profiles, we assume that households are born with an initial draw of assets from a lognormal distribution with mean and standard deviation that we estimate from the CEX data. Table 3.2 displays our estimates of the average log normalized liquid wealth , its standard deviation in the data and the implied ratio of average normalized wealth by education and occupation groups.

## 4. CEX Consumption and Income Profiles.

This section describes the construction of the income, consumption and family composition shifter profiles. The income and family composition profiles are input to the consumer problem described in section 2. The consumption profiles need to be comparable to equation (3.2).

### 4.1. Profile Construction Methodology.

We estimate three profiles using household level data from age 26 to 65: a consumption age-profile $\left\{\bar{C}_t\right\}_{t=26}^{65}$, the average income profile $\left\{\bar{Y}_t\right\}_{t=26}^{65}$ from which we derive expected income growth $\left\{G_t\right\}_{t=27}^{65}$; and a profile for the typical shifts in

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marginal utility, $\{v(Z_t)\}_{t=26}^{65}$.

Assume that the data generating process for consumption is as defined in section 2 for each household. Define $\lambda_{i,t} \equiv v(Z_{i,t}) u'(c_{i,t})$ as the normalized marginal utility of household $i$ of age $t$. From the Euler equation (2.4) of section 2, we know that:

$$\lambda_{i,t} = \frac{1}{\beta R} \lambda_{i,t-1} (G_{t-1} N_{i,t-1})^\rho \eta_{i,t},$$

where $\eta_{i,t}$ is the (multiplicative) innovation to the marginal utility of wealth and satisfies:

$$E_{t-1}[\eta_{i,t}] = 1.$$

Iterating backward until the birth year, and substituting for $C_{i,t}$, we obtain:

$$C_{i,t} = \left(\frac{v(Z_{i,t})}{v(Z_{i,26})}\right)^{\frac{1}{\rho}} (\beta R)^{\frac{t-26}{\rho}} \left(\Pi_{l=26}^t \eta_{i,l}^{\frac{-1}{\rho}}\right) c_{25} P_{25} \quad , \quad 26 \leq t \leq 65. \qquad (4.1)$$

The first term on the right hand side represents the effect of family size variation between age 26 and age $t$. The second term reflects the drift in marginal utility that depend upon the discount factor, the real interest rate and the intertemporal elasticity of substitution $1/\rho$. The third term reflects the effect of uncertainty and precautionary saving upon consumption at age $t$, through the past and current realizations of innovations to marginal utility, $\eta_{i,t}$, due to individual income innovations. $\eta_{i,26}$ additionally captures the variation in initial cash on hand across households. The fourth term is a notional normalized consumption at age 25, which would be the same for all households. Lastly, $P_{25}$ represents the permanent component of income at age 25 and will differ only across cohorts.

The individual level data from the CEX differs from the above $C_{i,t}$ in four ways. First, individual data is mismeasured. Thus, we do not exploit the limited panel nature of the Survey, but instead rely on data from repeated cross-sections. We assume that measurement error in consumption is multiplicative so that the log of observed consumption is true consumption plus classical measurement error.

Second, year-specific events, such as the stage of the business cycle, affect the average consumption of households in each year. These effects are not included in the model. One possible justification is that aggregate fluctuations account for a small share of individual uncertainty (Pischke (1995)). We assume that time-effects enter multiplicatively.[23]

Third, the model of section 2 does not assume any variation in income across cohorts. In reality, households start life with different levels of wealth and permanent components of income and thus with different levels of consumption. Some of these differences are due to that fact that households born in later generations on average have higher values for their initial permanent component of income. This implies that there is a correlation between the effect of birth year and the effect of

---

[23]The assumption that the time effects enter multiplicatively is only a convenient short-cut. A fuller model could include aggregate shocks as state variables. This is beyond the current paper.

age on consumption. Households observed at age sixty, say, are born long before those we observe at young ages and so have on average lower lifetime resources, and lower levels of income and consumption at each age. Ignoring birth-year effects would lead to a negative bias in our estimate of the slope of income and consumption growth. We assume, as in much of the empirical literature that earnings have a time invariant age-profile so that cohort effects affect only the distance between the age-profiles of different cohorts. According to this assumption and the decomposition (4.1), the cohort effect is entirely captured by the last term, $P_{25}^b$ where $b$ indexes the cohort.

Finally, the model refers to household consumption, adjusted so that all households have the same life-cycle pattern of family size $v(Z_t)$. We therefore estimate family size adjustments and apply them to the consumption data so as to control for within-age demographic variation.[24]

Define $\tilde{C}_{i,t,\tau}$, as the observed household consumption in the CEX, where $\tau$ denotes the year. Taking logs, we can write $\ln \tilde{C}_{i,t,\tau}$ as:

$$
\begin{aligned}
\ln \tilde{C}_{i,t,\tau} &= \frac{1}{\rho} \ln \left( \frac{v(Z_{i,t})}{v(Z_{i,26})} \right) + \frac{1}{\rho} \ln \left( c_{25} \, (\beta R)^{t-26} \left( \Pi_{l=26}^t \eta_{i,l}^{-1} \right) \right) \\
&\quad + \ln \left( P_{25}^b \right) + \xi_\tau + \epsilon_i
\end{aligned}
\tag{4.2}
$$

where $\epsilon_i$ is classical measurement error and $\xi_\tau$ represents a time-effect.

Since age and cohort are correlated, we estimate directly a linear regression based on (4.2) over households with male heads aged 26 to 65. As discussed in Deaton (1985), it is not possible to separately identify the linear component of the time, age, and cohort effects in equation (4.1).[25] We make the identifying restriction that they are due to the state of the regional economy and captured by the partial correlation of consumption with the regional unemployment rate.[26] Our empirical specification is then:

$$
\ln \tilde{C}_i = f_i \pi_1 + a_i \pi_3 + b_i \pi_2 + \mathcal{U}_i \pi_4 + Ret_i \pi_5 + \varepsilon_i
\tag{4.3}
$$

where the $\pi$ parameters are to be estimated, $f_i$ is a set of family dummies, $b_i$ is a complete set of cohort dummies (less the middle one), $a_i$ is a complete set of age dummies, $\mathcal{U}_i$ is the Census region unemployment rate in year $\tau$, $Ret_i$ is a dummy for each group which is equal to 1 when the respondent is retired and $\varepsilon_i$ is a residual

---

[24]It is worth noting that the assumption that family size is exogenous might in fact mitigate the importance that we attribute to precautionary saving. If the buffer stock model is correct and having children is a form of consumption, then the decision to have children is affected by precautionary saving that is closely related to the timing of lifetime income. By normalizing by a family size adjustment over the lifecycle, one is removing that portion of changes in consumption driven by expected income changes.

[25]This follows from the annoying identity that interview year less age equals birthyear!

[26]This assumes that the time effects are not important for the linear trend in consumption and that they are observed by the household. The allocation of the trend in consumption to cohorts and age effects may be sensitive to this assumption.

that captures all the individual effects (measurement error, initial normalized cash on hand, initial income,etc.).[27]

With these estimates, we first reconstruct household-level consumption data uncontaminated by cohort and time effects:

$$\ln C_i \equiv \bar{f}_t \hat{\pi}_1 + a_i \hat{\pi}_3 + \overline{\mathcal{U}} \hat{\pi}_4 + \hat{\varepsilon}_i. \tag{4.4}$$

Thus from here on, we work with household-level consumption that represents the consumption of the observed household with the typical age-dependent family size $\bar{f}_t$, facing the average unemployment rate $\overline{\mathcal{U}}$, born in the middle cohort, and not retired.[28] It is this version of consumption to which section 3 refers.

We construct average age-profiles of consumption by averaging this data across households. So that the role of precautionary saving is highlighted, we display most of our profiles and simulations for consumption per capita and income per capita. This profile is constructed as

$$\ln C_a \equiv \bar{f} \hat{\pi}_1 + a \hat{\pi}_3 + \overline{\mathcal{U}} \hat{\pi}_4.$$

We also construct three other sets of profiles. First, and in the same manner, we construct profiles for income and the typical household family size. Second, we build smooth profiles using fifth-order polynomials in age and birthyear instead of dummies. Lastly, we estimate similar profiles separately for various occupation and education subgroups of the population. The construction of these series is similar to that of consumption, and described in Appendix $D$.

The smoothed profiles for income and family size are used as inputs to solve the model. For instance, recall that $\ln Y_{i,t} = \ln P_{i,t} + \ln U_{i,t} = \ln G_t + \ln P_{i,t-1} + \ln N_{i,t} + \ln U_{i,t}$. After removing cohort and time effects, and averaging over a large number of households, $I$, with the same characteristics:

$$\frac{1}{I}\sum_{i=1}^{I} \ln Y_{i,t} = \ln G_t + \frac{1}{I}\sum_{i=1}^{I} \ln Y_{i,t-1} + \frac{1}{I}\sum_{i=1}^{I} \ln N_{i,t} + \frac{1}{I}\sum_{i=1}^{I} \ln U_{i,t} - \frac{1}{I}\sum_{i=1}^{I} \ln U_{i,t-1}$$

Applying the Law of Large Numbers, the probability limits of the last three term are all zero. Hence, we get

$$plim \left( \frac{1}{I}\sum_{i=1}^{I} \ln Y_{i,t} - \frac{1}{I}\sum_{i=1}^{I} \ln Y_{i,t-1} \right) = \ln G_t \equiv \ln \bar{Y}_t - \ln \bar{Y}_{t-1}.$$

Thus first differencing our log-average income levels gives expected income growth rates $G_t$.

---

[27]Dummy variables are constructed for family sizes 1 through 9 and then for whether family size is 10 or greater. We also experimented with exogenous family size adjustments –assuming $f_i$ is simply family size raised to the power $-0.7$. This led to profiles which were noisier and flatter early in life.

[28]The middle cohort is 1941 and the average regional unemployment rate 7 percent.

## 4.2. The Consumer Expenditure Survey and Our Use of it

We use the Consumer Expenditure Survey ($CEX$) to construct life-cycle profiles of consumption and income. The $CEX$ contains information about consumption expenditures, demographics, income and assets, for a large sample of the US population. The Survey is conducted by the Bureau of Labor Statistics in order to construct baskets of goods for use in the bases for the Consumer Price Index, and has been run continuously since 1980. We use data from 1980 to 1993 from the family, member, and detailed expenditure files. The survey is known to have excellent coverage of consumption expenditures, to have reasonable data on liquid assets, and to have income information of moderate quality.[29] The survey interviews about 5500 households each quarter. In a household's first interview, the $CEX$ procedures are explained to them and information is collected so that they can be assigned a population weight. They are then interviewed four more times (once every three months) about detailed consumption expenditures over the previous three months. In interviews two and five, income information is collected, and in the final interview asset information is collected. Families rotate through the process, so that about 25% of households leave and are replaced in each quarter. About half of all households make it through all the interviews.

Each household contributes one data point to our sample. For each household we construct a measure of household income and consumption. Based on the characteristics of the male head of the household, we assign the household to an occupation group, an education group, a birth cohort, an interview year, and a Census region. In order to obtain a high quality sample that has the required information, we drop a significant portion of the data and make a series of adjustments. Further description of the data preparation is contained in Appendix $C$. We make note here of three major points.

First, we drop households which are classified as incomplete income reporters, which have any of the crucial variables missing, or which report changes in age over the course of the survey greater than one year or negative. We do not analyze the group of households with male heads holding less than 9 years of schooling due to very few younger households. Second, we drop all households with male heads younger then 26 or older than 70, given our focus on the working life. We are left with just under $40,000$ households. Third, we use individual as well as family level information to correct most of the top-coding in household labor income.

We construct measures of income and consumption which match the concepts in the theoretical model. First, we define consumption as total household expenditures less those on education, medical care, and mortgage payments. These categories of expenditure do not provide current utility but rather are either investments or negative income shocks.[30] So that these expenditures do not incorrectly appear as saving, they are also subtracted from income.

---

[29]See Lusardi (1996), Attanasio (1994), and Branch (1994).

[30]We are arguing that user cost of housing -repairs, maintenance, utilities, and housing services- captures the expenditures made for consumption on housing.

It should be noted that our model refers to nondurable consumption at annual frequencies. Since we are averaging expenditures across a large number of individuals and looking across one-year horizons, the distinction between durables and nondurables is not likely to matter. Further, since the buffer stock model gives strong predictions about consumption tracking income, it is important not to break the consumption-income link when studying consumption. We will also report the results when consumption is restricted to nondurables and income is defined as disposable income after durable expenditures.

Our measure of disposable labor income is comprised of after-tax family income less Social Security tax payments, mortgage payments, expenditures on medical care, spending on education, pension contributions and after-tax asset and interest income. For the first five adjustments, the related expenditures do not provide current utility but are either non-liquid investments or, in the case of medical care, simply losses of income. Further these expenditures involve a large amount of commitment and are hard to substitute intertemporally. We remove asset income since the input to our theoretical model is a profile of income net of liquid asset returns.

Finally, we put all data into real terms using the Gross Domestic Product implicit price deflator for personal consumption expenditures.[31]

### 4.3. Life Cycle Profiles

Figure 4.1 presents consumption (raw and smoothed) and income profiles for our entire sample when the family-size is held constant over the life-cycle. Even after correcting for the effects of cohort, time, and family, both profiles are still hump shaped and track each other early in life. Consumption lies above income from age 26 to 28. Given that the CEX wealth data, and better household wealth surveys, show modest increases in liquid wealth over these ranges, this feature seems likely due to misreporting of income or consumption. One possibility is underreporting the assistance which is provided by intergenerational transfers early in life. After these first few years, consumption rises with income from age 30 to age 45, when consumption drops significantly below income. This tracking is however a lot less than is observed in profiles constructed by simply averaging cross-sections. As stated above, the two main reasons are that we control for changes in family size the different wealth and incomes of different cohorts.Figures 4.2 and 4.3 give some evidence that consumption and income track each other across subgroups of the population defined by education and occupation groups.[32] These graphs are somewhat noisy. However, despite the noise, one can see that the occupation and education groups with the most pronounced humps in income present the

---

[31]It is important not to use different deflators for different items within consumption or for income and consumption. This could break the relationship between cash on hand and consumption in nominal terms which is the relationship predicted by the buffer-stock theory.

[32]We drop Service, Farming, Armed Forces and Self-Employed occupations due to small smaples.
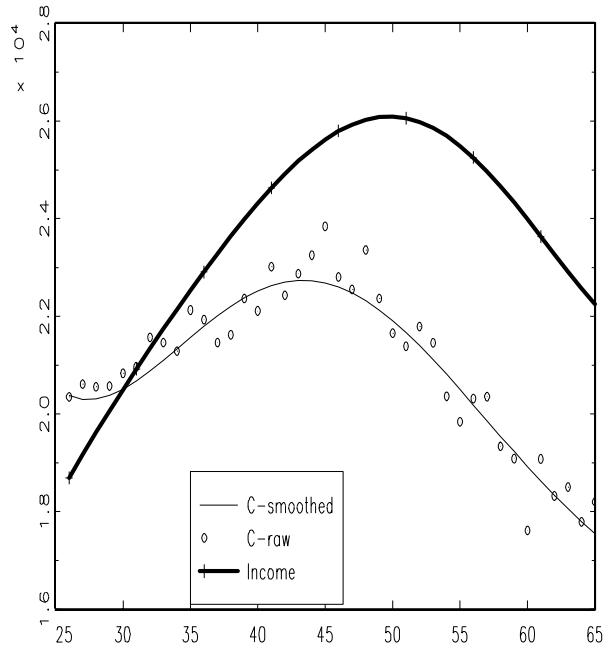
Figure 4.1: Household Consumption and Income over the LifeCycle

most pronounced humps in consumption. Further, we can formally reject the null hypothesis that the consumption profiles are flat. This is essentially a now standard test of the linearized consumption Euler equation, as studied by Attanasio and Weber (1995), Lusardi (1996).

The correlation of our consumption and income profiles over the life cycle does not reject all versions of the CEQ-LCH that lack precautionary saving. Attanasio and Weber (1995) and Attanasio and Browning (1995) argue that a richer set of preference shifters can eliminate the consumption-income correlation shown here. In Attanasio and Weber (1995) and in the linear Euler-equation approach generally used in micro data, precautionary effects are omitted so that preference shifters absorb, correctly or incorrectly, whatever variation they can in consumption. Clearly, allowing for enough preference variation can 'explain' the life-cycle correlation without reference to substantial household-level uncertainty. The approach of this paper is to adjust only for the size of the household and examine how precautionary saving explains the observed consumption behavior.

## 5. Results

We first estimate our model for the average household and discuss the implications of the fitted structural model for household behavior. We then turn to disaggregated results, by education and occupation groups.
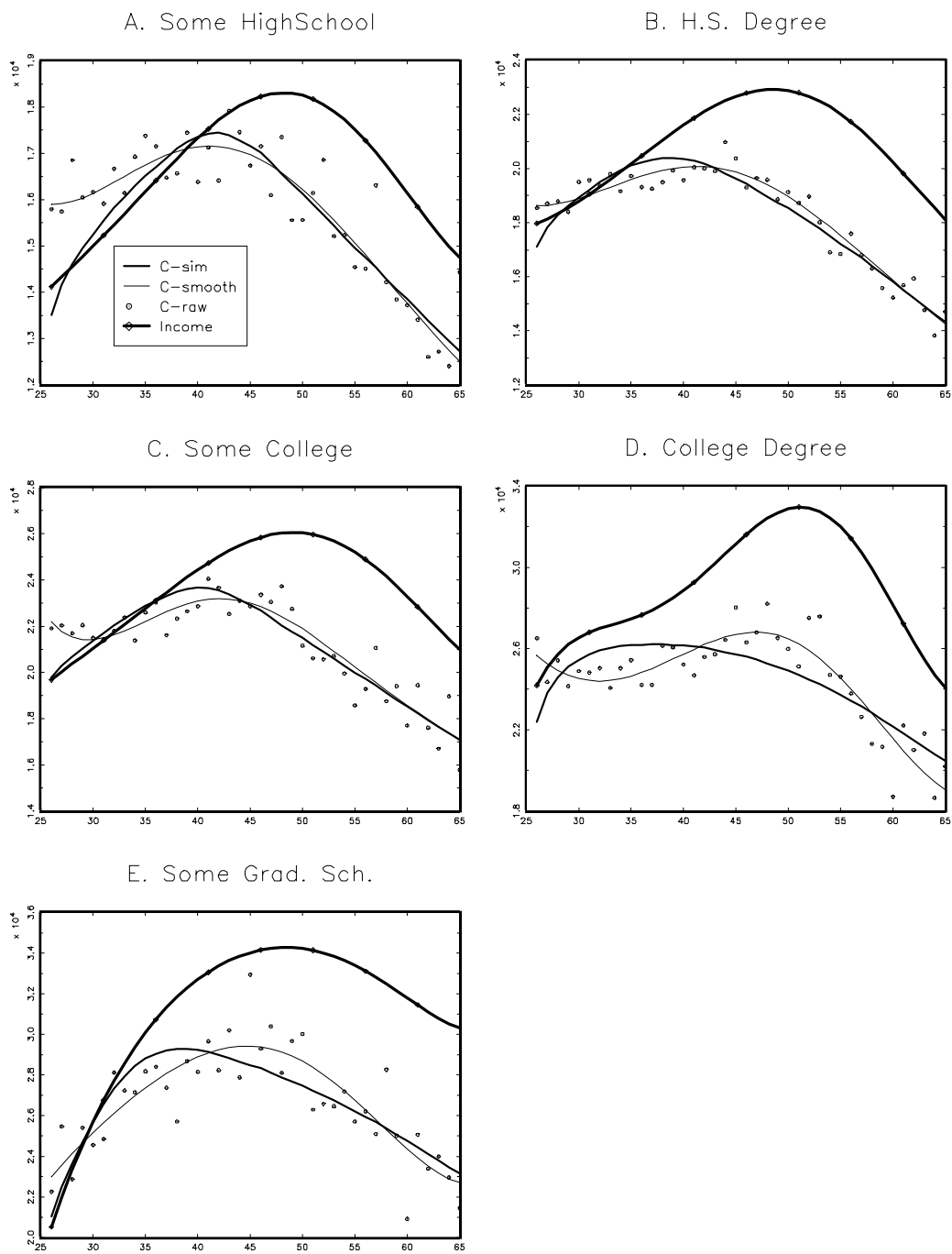
**A. Some HighSchool**

**B. H.S. Degree**

**C. Some College**

**D. College Degree**

**E. Some Grad. Sch.**
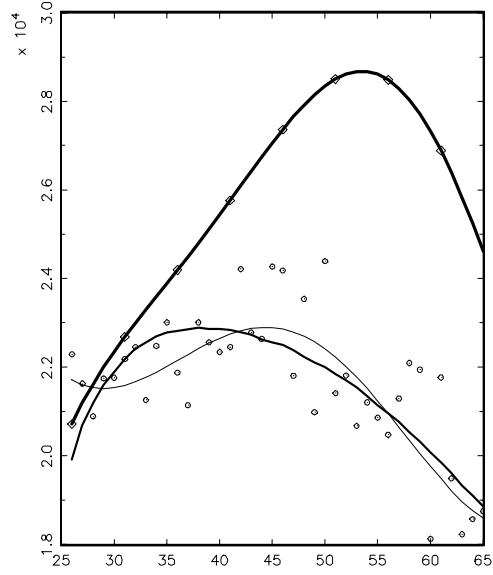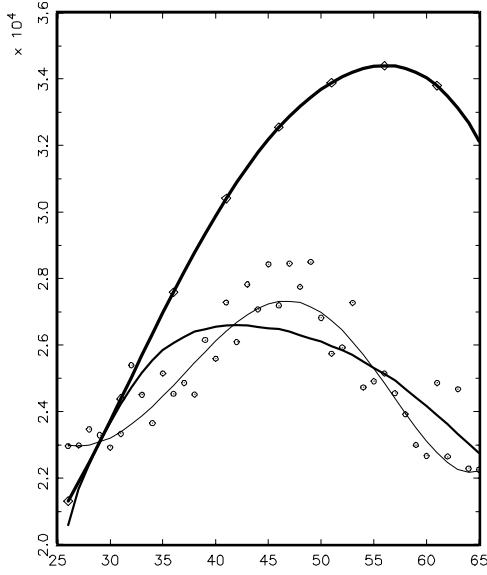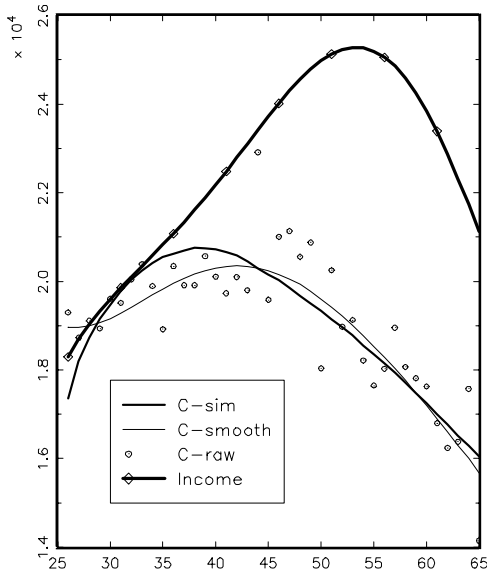
Legend:
- C-sim
- C-smooth
- C-raw
- Income

Figure 4.2: Household Consumption and Income over the LifeCycle, by Education Group.

## A. Manag., Professional



## B. Tech, Sales, Admin



## C. Precs, Prod, Craft
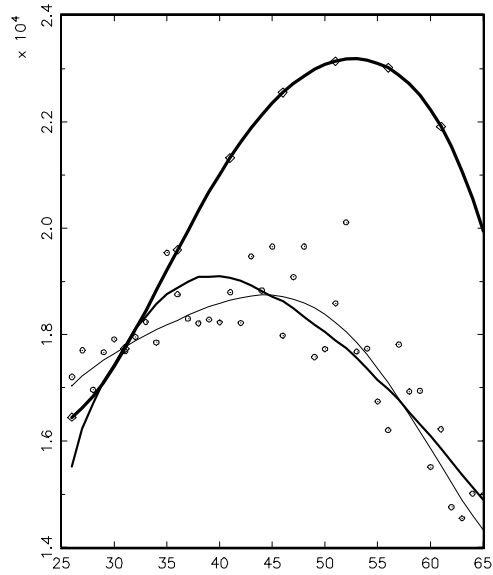


## D. Operator, Fab., Lab



Figure 4.3: Household Consumption and Income over the LifeCycle, by Occupation Group.

### 5.1. Findings from the Entire Sample

As a useful benchmark, we start by asking what the standard Life-Cycle theory would predict, assuming away all uncertainty. To give the best chance to the CEQ-LCH, we perform first difference estimation of the CEQ-LCH, not asking it to fit the mean of the consumption profile. Under certainty, equation (2.2) holds, implying, after controlling for individual characteristics, a constant growth rate of consumption over the working period:

$$\Delta \ln \bar{C}_t = \frac{1}{\rho} \ln (\beta R).$$

We estimate $\frac{1}{\rho} \ln (\beta R)$ from the coefficient on age in a least-squares regression of the profile of consumption with family size, cohort, and time effects removed, as displayed in Figure 4.1. This procedure seems trivial only because of our earlier efforts to remove changing family-size and cohort effects. It is precisely this simplicity which gives the CEQ-LCH its power. From our estimate, we use the delta method to recover the discount factor and its standard error, using the real interest rate estimated in the previous section and a choice of the coefficient of relative risk aversion of unity.[33] Since the fitted profile is essentially flat, we estimate a discount rate of 3.44% with a standard error of 0.281, after adjusting the estimate for both first order serial correlation and first-stage uncertainty. Since the CEQ-LCH consumption profile is flat and the data as shown in Figure 4.1 is hump-shaped, the certainty model performs poorly when it comes to explaining the dynamics of consumption across the life cycle.

Table 5.1 presents the results of estimating our structural model with income uncertainty. The first column displays the results when the initial, robust weighting matrix is employed; the second displays the results from the optimal weighting matrix. The standard errors are calculated (A) without and (B) with the adjustment for uncertainty in the parameters $\hat{R}$, $\hat{p}$, $\hat{\sigma}_u^2$, $\hat{\sigma}_n^2$ and $\hat{w}_1$ from the first-stage estimation.[34] The standard errors typically increase, once the uncertainty in the calibrated parameters is accounted for.

The discount rate is estimated at just over four percent, which is within conventional significance levels of the real interest rate of 3.44 percent. It is worth noting that the discount rate is within a reasonable range. Using information on the elasticity of assets with respect to uncertainty, Carroll and Samwick (1997) estimate a discount rate in the vicinity of $10 - 15\%$ and argue that even higher discount rates are needed to rationalize the findings of Hubbard, Skinner, and Zeldes

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[33]Since $\hat{\beta}R \approx 1$, the choice of $\rho$ matters little. The coefficient of relative risk aversion and the discount factor, are not separately identified because we do not have any variation in the desire to shift consumption across time.

[34]To be clear, we do not include all the first-stage parameters in the first-stage variance correction, as discussed in the previous section. In particular, we do not include the parameters that drive the family shifters and the expected income profile. We report results including uncertainty about these parameters in section 5.3 where we investigate robustness issues.

Table 5.1: STRUCTURAL ESTIMATION RESULTS

| MSM ESTIMATION | ROBUST WEIGHTING | OPTIMAL WEIGHTING |
|---|---|---|
| $\beta$ | 0.9598 | 0.9569 |
| S.E.(A) | (0.0101) | |
| S.E.(B) | (0.0179) | (0.0150) |
| $\delta$ (%) | 4.188 | 4.507 |
| S.E.(A) | (1.098) | |
| S.E.(B) | (1.949) | (1.641) |
| $\rho$ | 0.5140 | 1.3969 |
| S.E.(A) | (0.1690) | |
| S.E.(B) | (0.1707) | (0.1137) |
| $\gamma_0$ | 0.0015 | $5.68\ 10^{-6}$ |
| S.E.(A) | (3.84) | |
| S.E.(B) | (3.85) | (16.49) |
| $\gamma_1$ | 0.0710 | 0.0613 |
| S.E.(A) | (0.1215) | |
| S.E.(B) | (0.1244) | (0.0511) |
| | | |
| $\chi^2$(A) | 175.25 | |
| $\chi^2$(B) | 166.06 | 177.11 |

Note: *MSM* estimation for entire group. Standard errors calculated without (A) and with (B) correction for first stage estimation. Cell size is 36,691 households. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical values at 5% is 50.71. Efficient estimates are calculated with a weighting matrix $\hat{\Omega}$ computed from the first step.

(1994). Our lower discount rate, however, does not imply that households are not impatient enough to generate buffer stock behavior, as we will show shortly.

When we employ an optimal weighting matrix, the precision with which we estimate the discount rate declines significantly compared with the variance estimate that does not correct for uncertainty about the first stage parameters (A), but increases compared to the 2-step variance estimate (B). There is a growing literature that questions the small-sample validity of optimal weighting due to the correlation between parameter uncertainty and the weighting matrix. Optimal weighting can be more efficient; it can also be more biased. Thus we report both.

The intertemporal elasticity of substitution is estimated quite tightly, and is the sole parameter estimate that depends significantly on the weighting matrix employed. The estimated retirement rule suggests a marginal propensity to consume out of wealth at retirement ($\gamma_1$) between 6 and 7 percent, also quite reasonable. For instance, in the case of full certainty after retirement and no change in the utility shifter the marginal propensity to consume is given by

$$\left(1 - \hat{\beta}^{1/\hat{\rho}} \hat{R}^{1/\hat{\rho}-1}\right) / \left(1 - \left(\hat{\beta}^{1/\hat{\rho}} \hat{R}^{1/\hat{\rho}-1}\right)^{N-T+1}\right) = 7.58 \, \text{percent},$$

given our estimates of $\beta$, $\rho$ and $R$. Thus the estimate is very much in line with simple predictions of the model. Finally, under our assumptions, the ratio $\gamma_0/\gamma_1$ provides an estimate of the ratio of illiquid wealth to the permanent component of income at retirement. This ratio is very imprecisely estimated, but the point estimates are extremely small, around 2 percent which we regard as highly unreasonable. An alternative interpretation, which has our preference, is that households have a lower propensity to consume out of illiquid wealth, such as housing wealth.

It should be noted that both estimation methods reject the overidentifying restrictions at the 5% level. The 95% critical value for a $\chi^2(36)$ is 50.71 and the chi-square always exceeds 150. This is not entirely surprising, given the number of moments we use (40) and the sparsity of the model. The estimated model should still be taken seriously however. As we now discuss, the model does much better in an economic sense than the CEX-LCH model with which this section begins.

With our estimates in hand, we can now address how well the stochastic model fits the life-cycle consumption profile. Figure 5.1 plots the simulated (dark solid line) and actual consumption data (circles) along with the income profile (thick solide line). The stochastic life-cycle model does a much better job at fitting the consumption profile than the consumption profile with constant growth rate of $\frac{1}{\rho} \ln(\beta R)$ that would obtain under the certainty-equivalent. The consumption profile from the fitted model tracks income until around age 40 and then falls as households start increasing their liquid wealth holding in preparation for retirement.

In two places however, the model fit is not good. First, actual consumption exceeds simulated consumption early in life. This may reflect under-reported intergenerational transfers, or other sources of error in measurement. Second, the actual consumption profile is slightly flatter and peaks slightly later. Apart from

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these features however, the tight structure imposed by the model is able to deliver good predictions in terms of consumption dynamics.

Why are we able, within the context of our model to obtain such tight estimates? The first panel of Figure 5.1 plots various simulated profiles for values of $\beta$, one percent away from the point estimate. This corresponds to choices of $\beta = 0.9498$ and $\beta = 0.9698$, or equivalently, to a discount rate between 4.18 and 5.28 percent. It is immediately obvious that the profiles are very sensitive to small variations in the discount factor. With a higher discount factor, the household is willing to save more and earlier for retirement purposes. The consumption path exhibits less of a hump shape, and may even be increasing over the entire working life, as is the case in this Figure. On the other hand, for more impatient households, consumption parallels income until much later in life and then falls more precipitously to build assets for retirement. This implies a stronger concavity of the consumption profile. Thus, our method yields tight estimates of the discount factor precisely because the discount factor drives the hump shape in consumption profiles.

We now turn to the question of how household behavior changes over the life cycle. Panel b of Figure 5.1 displays the average household saving rate profile from the fitted model. Saving is defined as the discounted variation in financial wealth from one period to the next:[35]

$$S_{i,t} = \left( W_{i,t+1} - W_{i,t} \right) / R = (R - 1)/R \ W_{i,t} + Y_{i,t} - C_{i,t}.$$

Saving is equal to investment plus labor income minus consumption, i.e. is equal to disposable income minus consumption. The saving rate profile is constructed as the ratio of average saving to average disposable income at age $t$. Until about age 40, households save relatively little and consume roughly their income on average. Around age 40, retirement considerations induce an increase in saving.

To put additional structure on these two phases, define the target level of cash on hand at age $t$, $\bar{x}_t$, as the level at which cash on hand is expected to remain unchanged from one year to the next (Carroll (1997a)):

$$\bar{x}_t = E_t \left[ x_{t+1} | x_t = \bar{x}_t \right]$$

For large initial levels of cash on hand $(x_t > \bar{x}_t)$, households choose high levels of consumption and thus future levels of cash on hand are on average lower. For low levels of cash on hand $(x_t < \bar{x}_t)$, households consume less than the income they expect to receive and so on average accumulate cash on hand.

The first panel of Figure 5.2 directly computes the target level of cash on hand for consumers aged $26 - 42$, from the fitted model. One can see from the graph that the target level of cash on hand –including current income– remains small early in life, around 1.3 times the permanent component of income. Around age 40, this target increases substantially, as consumers begin to build their retirement

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[35]The discount comes from the assumption that income is received and consumption occurs at the beginning of the period.

Figure 5.1.a: Simulated Consumption Profiles for different $\beta$,
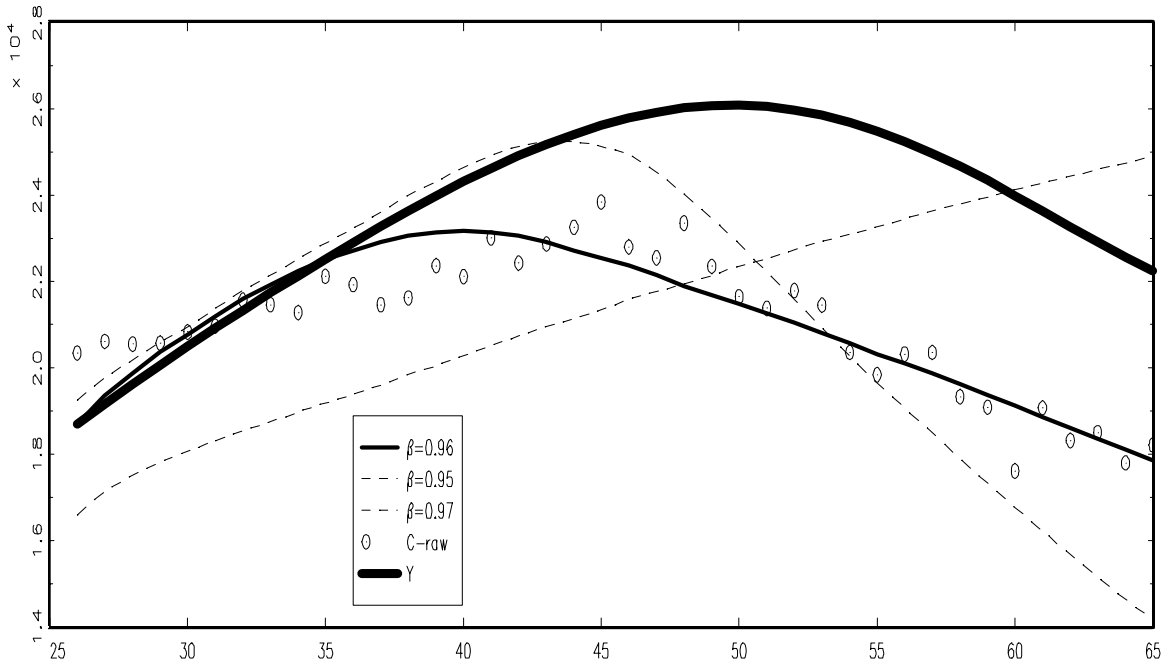$\rho=0.514$, $\gamma_0=0.001$, $\gamma_1=0.071$



Figure 5.1.b: Savings Rate Profile
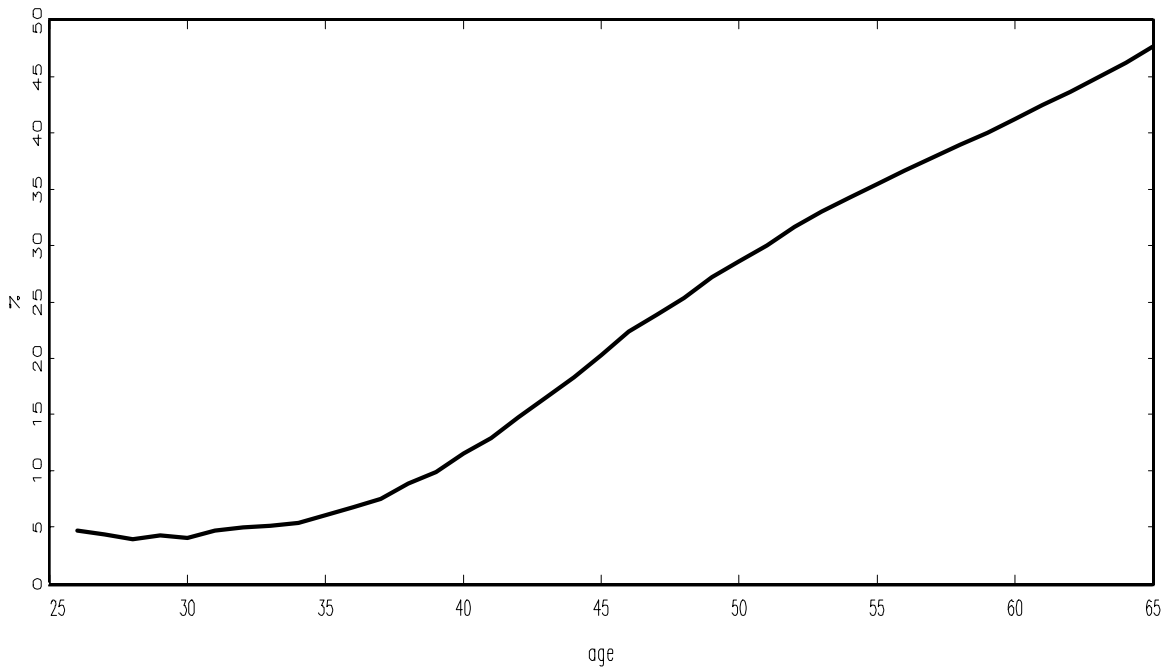R=1.0344, Normalized Mean level of Assets at 26: 0.300

Figure 5.2.a: Target Cash on Hand
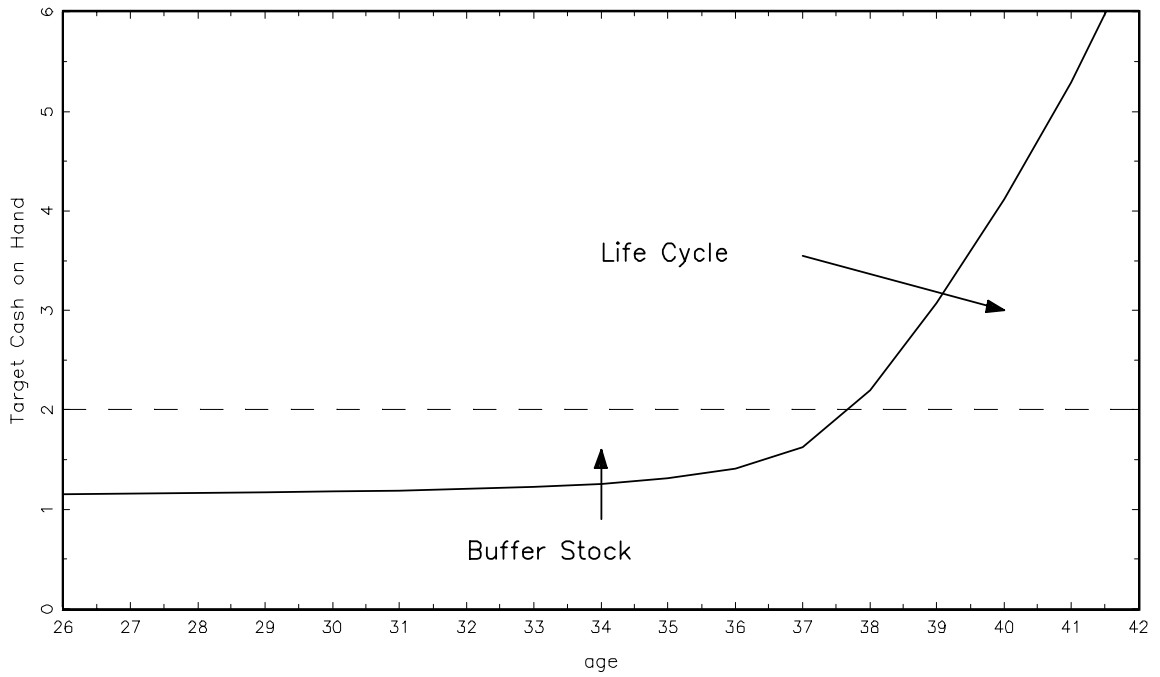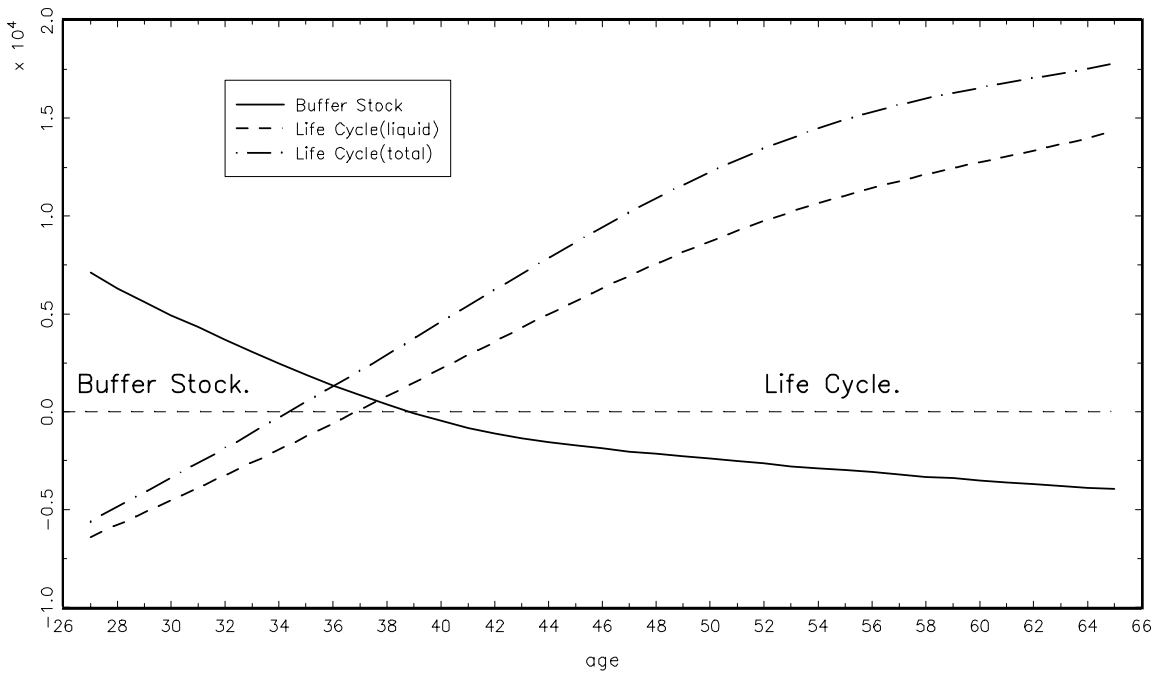
$\beta=0.960, \rho=0.514, \gamma_0=0.001, \gamma_1=0.071$



Figure 5.2.b: Life Cycle and Buffer Savings

$\beta=0.960, \rho=0.514, \gamma_0=0.001, \gamma_1=0.071$



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nest-egg. This figure shows a dramatic change in behavior. When the target level of liquid wealth is small, households behave as 'buffer stock' consumers. Good income shocks are consumed away and bad income shocks are imperfectly smoothed. Household consumption closely follows household income. Starting around age 38, agents begin to accumulate assets for retirement. Once households build large stocks of wealth, they can easily smooth high frequency movements in income and their behavior more closely mimics that of certainty-equivalent consumers.

We can also decompose total saving at each age into life-cycle and buffer-stock saving. Our previous discussion might lead the reader to think that agents have no concern for retirement when they are young and no concern for labor income uncertainty later in life. This is incorrect since consumers are rational and perfectly foresee their retirement needs. At the estimated parameters, we compute the consumption path, $\{C_t^{LC}\}$, that would occur under certainty.[36] We then define life-cycle saving as the difference between total income and life-cycle consumption

$$S_t^{LC} = \left(W_{t+1}^{LC} - W_t^{LC}\right)/R = (R-1)/R \ W_t^{LC} + Y_t - C_t^{LC}$$

Precautionary saving is defined as the complement of life-cycle saving. The second panel of Figure 5.2 plots the precautionary saving, liquid and total life-cycle saving of the typical household. The latter is defined by adding pension and social security contributions back to income. Given the estimated discount rate, CEQ life-cycle consumers would like to borrow early in life. However, precautionary saving motives cause them to hold a positive buffer stock of wealth. Around age 38, in accordance with our previous characterization, life-cycle savings becomes larger than precautionary savings. The need to build retirement savings sets in. As asset levels increase, the expected variance of consumption declines, decreasing the precautionary saving motive. This latter effect, which our previous decomposition masked, induces the agent to *run down* the buffer. As a result, the total saving rate later in life is smaller than under the certainty equivalent framework.

Figure 5.2 also clarifies why, despite a discount rate of just over 4 percent, households do not accumulate assets for retirement early in life. Since households expect their income to grow significantly from age 25 to 50, the life-cycle saving motive is for households to consume out of lifetime resources and so borrow when young to smooth consumption. However, households are smoothing marginal utility and they instead save in response to the uncertainty about their future income.

This highlights the importance of the slope of the income profile. Absent income growth, households would instead start saving for retirement early in life. This liquid wealth would double as a buffer stock. Thus, there would be no additional precautionary saving motive to accumulate this stock and household behavior would be well approximated by the CEQ LCH.

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[36]In order to do this, we calibrate the certainty case, so as to yield the same consumption rule at retirement as estimated above. In effect, this amounts to imputing some bequest motive so as to match consumption rule at retirement. Under this calibration, households exhibit strong bequest motives.

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## 5.2. Findings from Different Education and Occupation Groups

We next estimate the model separately on subgroups of the population defined by education levels and occupation. This allows us to exploit variation in the environments of different agents and allows different subgroups of the population to have different preferences. Technically, we simply follow the procedure described above on each cell, using first-stage estimates already presented by subgroup in sections 3 and 4. Each group's consumption profile is matched with different estimates using a different income profile (Figures 4.2 and 4.3), income uncertainty (Table 3.1), and initial average log liquid wealth (Table 3.2). The results are summarized in Table 5.2 and the simulated consumption profiles from the fitted model are presented in Figures 4.2 and 4.3.

The estimated discount factors are close to that obtained using the aggregate profile. The discount rate lies between 3.94 precent (Some High School) and 5.71 percent (Managerial and Professional). Interestingly, there is no clear pattern associated with the level of education. If anything, the discount rate is slightly higher for more highly educated households. By contrast, there is much wider dispersion in the estimated coefficients of relative risk aversion, from a low of 0.282 (Some High School) to a high value of 2.290 (Graduate School). The coefficients of the consumption rule at retirement are also within a reasonable range of the aggregate estimates.

Perhaps more interestingly, the overidentification tests now deliver much smaller values, indicating that the model does a better job at fitting the actual consumption profiles. Indeed, looking back at Figure 4.2 and Figure 4.3, we see that the model delivers consumption profiles very much in line with the unconstrained smoothed consumption profile estimated from the data. For all cells, the consumption profile is hump-shaped and the simulated profiles indicate that household start accumulating for retirement purposes between 35 and 40 years. The behavioral implications of this set of results are the same as those just discussed for estimation that did not allow for cross-group heterogeneity.

## 5.3. Robustness

To provide further evidence on the strengths and weaknesses of the model, we present the results of five alternative procedures: (i) estimating the initial level of liquid wealth in the second stage; (ii) estimating the retirement consumption rule in the first stage; (iii) estimating by matching age-specific growth rates rather then levels; (iv) adjusting the second-stage inference for income and family profile estimation; and (v) matching a consumption profiles for expenditures on non-durable goods only.

First, we estimate the average log cash on hand with which households begin life in the second-stage from the profile information. While the model fits the data better with this extra parameter, the second stage no longer provides precise or sensible identification of the structural parameters. The results are presented in columns 1 and 2 in Table 5.3 and in Figure 5.3.

Table 5.2: ESTIMATES FROM THE STOCHASTIC MODEL BY EDUCATION AND OCCUPATION

| GROUP: | $\beta$ | $\delta$ | $\rho$ | $\gamma_0$ | $\gamma_1$ | $\chi^2$ | $N$ |
|---|---|---|---|---|---|---|---|
| EDUCATION | | | | | | | |
| Some High School | 0.962 | 3.94 | 0.282 | 0.209 | 0.072 | 53.60 | 4,270 |
| | (0.082) | (8.92) | (1.481) | (5.04) | (2.360) | | |
| Highschool Graduate | 0.949 | 5.30 | 0.869 | $3.79\ 10^{-3}$ | 0.059 | 59.12 | 12,445 |
| | (0.015) | (1.64) | (0.220) | (20.05) | (0.049) | | |
| Some College | 0.960 | 4.15 | 0.394 | 0.351 | 0.043 | 84.21 | 9,653 |
| | (0.159) | (17.29) | (2.344) | (4.095) | (3.156) | | |
| College Graduate | 0.930 | 7.48 | 2.290 | $1.55\ 10^{-8}$ | 0.049 | 111.70 | 6,350 |
| | (0.060) | (6.97) | (0.423) | (54.60) | (0.075) | | |
| Graduate School | 0.944 | 5.93 | 1.694 | $1.06\ 10^{-7}$ | 0.057 | 87.26 | 5,973 |
| | (0.087) | (9.77) | (0.843) | (18.23) | (0.076) | | |
| OCCUPATION | | | | | | | |
| Managerial and Prof. | 0.946 | 5.71 | 1.672 | $5.20\ 10^{-8}$ | 0.050 | 115.62 | 12,693 |
| | (0.060) | (6.69) | (0.524) | (22.78) | (0.067) | | |
| Tech., Sales, Admin. | 0.953 | 4.90 | 1.059 | $2.13\ 10^{-7}$ | 0.049 | 64.02 | 6,548 |
| | (0.037) | (4.11) | (0.339) | (39.42) | (0.064) | | |
| Precision Prod., Craft | 0.953 | 4.97 | 0.990 | 0.003 | 0.054 | 52.86 | 4,469 |
| | (0.333) | (36.77) | (3.895) | (18.49) | (0.997) | | |
| Operators, Laborers | 0.953 | 4.90 | 0.867 | $3.14\ 10^{-6}$ | 0.049 | 57.58 | 6,063 |
| | (0.489) | (53.80) | (4.846) | (1365) | (2.35) | | |

Note: $MSM$ estimation in levels. Standard errors calculated with correction for first stage estimation. The next to last colum reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71.

This alternative simulated profile does a better job than the baseline model in terms of fitting household consumption during the first 5 years of life, when measured consumption lies above measured income. The procedure attains this by assigning an implausibly high value of initial cash on hand: the mean level of assets is estimated to be 12.6 times current income![37] With high initial wealth, the typical household can easily buffer labor income shocks and enjoy high consumption early in life. To fit the observed high correlation between income and consumption prior to middle age, the estimation procedure guesses that agents are very impatient (so that they run this initial wealth down) and are quite willing to substitute intertemporally. The discount rate is estimated as 14% and the coefficient of relative risk aversion at 5.2.

While it is possible that this high wealth comes from inheritances, this is at odds with all household surveys of wealth holding. In our minds, this is enough to reject this alternative scenario, however it informs us about the model. From this and other experiments, we conclude that identification hinges in part on providing the second-stage with more information than is contained in the profiles alone. In particular, from the typical consumption profile, one can accurately and sensibly estimate the preference parameters only by fixing either the retirement consumption rule or the initial value of assets. Without these parameters estimated from other sources in a first stage there is not enough information in the profiles to provide accurate estimation of the model.

In our second evaluation of the model, we estimate $\gamma_0$ and $\gamma_1$ from consumption and asset measures around the age of retirement prior to the second stage. We use the PSID to construct measures of liquid assets in 1989, income from 1989 to 1994, and total consumption from 1989 to 1994, from active saving, wealth and income measures. We then annualize the data and estimate a crude consumption function as:

$$C_i = \gamma_0 P_i + \gamma_1 X_i + \gamma_2 f_i + \varepsilon_i$$

across households. We obtain:

$$\hat{\gamma}_0 = 0.594 \quad s.e.\,(0.158)\,; \quad \hat{\gamma}_{1=}0.0774 \quad s.e.\,(0.0125)\,.$$

This estimated marginal propensity to consume at retirement (0.0710) is very close to the calibrated one (0.0774), while the ratio of illiquid wealth to the permanent component of income, at retirement, is equal to $\frac{\gamma_0}{\gamma_1} = 7.67$, a more realistic number.[38]

The third column of Table 5.3 shows that the estimated discount rate is slightly lower than in the baseline estimation and the coefficient of relative risk aversion is estimated to be quite low. While the parameters appear very tightly estimated, the overidentification restriction increases dramatically. The chi square value is

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[37]Recall that we estimate $\ln w_{25}$, and so the mean level of cash on hand is given by: $\exp\left(\ln w_{25} + \frac{1}{2}\sigma^2_{\ln w_{25}}\right)$.

[38]Gourinchas and Parker (1997) present a similar scenario based on a rule calibrated from the $CEX$ data and find that the model fits the data quite well.

1690. Finally, as shown in Figure 5.3, the simulated profiles is substantially higher than the actual profile in most years. Given retirement behavior, the model cannot deliver saving rates as high as implied by the $CEX$ profiles.

Since there may be some problems in the CEX measuring the relative levels of consumption and income, we next estimate the model in first-differences, so as to try to match the growth of consumption over the life-cycle, not its level. This eliminates one moment condition. The resulting paramter estimates (column (4) of Table 5.3 and Figure 5.3) are substantively and statistically similar to our benchmark results. More importantly, the test of overidentification does not reject the model. Since it is likely that all household consumption expenditures are not measured in the CEX, this mismeasurement may well be the reason that the data reject the baseline model.

Fourth, we correct the variance covariance in the second stage for the profiles of income and family-size preference shifts estimated in section 4. Formally, the second-step correction employs the variance-covariance matrix for the fifth-order polynomial coefficients from the smoothed income and utility shifter profiles. The result are reported in column (5) of Table 5.3 and demonstrate extremely large standard errors. That is, given the sampling uncertainty with which a fifth-order polynomial estimates the income profile, the consumption profile does not provide an accurate measure of preference parameters. Since the age-pattern of income determines the age-pattern of consumption when income is uncertain, the income profile is important in identifying the consumption profile.

Lastly, we estimate the same set of parameters using information on nondurable expenditures only. Strictly speaking, our model refers to the consumption of nondurable expenditures. However, since households do not rent all durables, a more complete model would accomodate both sorts of goods. We stop sort of doing so and investigate here what our model implies for nondurable expenditures under the assumption of full separability. That is, we impute durable expenditures as mandated expenditures and substract them from our measure of income as well as consumption.[39] The results (column (6)) are substantially different from the benchmark results. The discount rate is now very high, at 8.35 percent. This implies a very high degree of impatience from households. On the other hand, the estimate of the relative risk aversion is higher, at 1.93 with a standard error of 0.06. This value of the risk aversion implies a low degree of intertemporal substitution, while within the range of previous estimates. Further, we find a much smaller marginal propensity to consume out total wealth at the age of retirement, around 2.8 percent, with an estimated illiquid wealth equal to 0. Lastly, the overidentification test strongly rejects the model, much more than the benchmark model, indicating that the additional assumptions needed to match non durable expenditures and income may be rejected by the data.

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[39]Under this assumption, durable goods cannot be used to offset labor income uncertainty. While this is an extreme assumption, it is likely durable goods command only a fraction of their original value on second-hand markets.

Table 5.3: ROBUSTNESS CHECKS

| ESTIMATION | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Consumption Measure: | Total | Total | Total | Total | Total | NonDurable |
| OPTIMAL WEIGHTING | No | Yes | No | No | No | No |
| FIX $E[w_{25}]$ | No | No | Yes | Yes | Yes | Yes |
| METHOD | Level | Level | Level | First Diff | Level | Level |
| FIX $(\gamma_0, \gamma_1)$ | No | No | Yes | No | No | No |
| INCOME CORRECTION | No | No | No | No | Yes | No |
| | | | | | | |
| $\beta$ | 0.8741 | 0.8724 | 0.9648 | 0.9643 | 0.9598 | 0.9229 |
| S.E.(A) | (0.0984) | | | (0.0002) | (0.0202) | (0.0101) | (0.0098) |
| S.E.(B) | (0.1032) | (0.0197) | (0.0087) | (0.0206) | (1.1147) | (0.0538) |
| $\delta$ (%) | 14.406 | 14.627 | 3.6412 | 3.6992 | 4.188 | 8.350 |
| S.E.(A) | (12.881) | | | (0.0224) | (2.171) | (1.0981) | (1.1545) |
| S.E.(B) | (13.515) | (2.589) | (0.935) | (2.212) | (121.01) | (6.3189) |
| $\rho$ | 5.1817 | 5.2823 | 0.1278 | 0.1585 | 0.5140 | 1.9357 |
| S.E.(A) | (0.5640) | | | (0.0004) | (0.5855) | (0.1690) | (0.0609) |
| S.E.(B) | (0.6522) | (0.1195) | (0.0088) | (0.6669) | (16.592) | (0.3469) |
| $\gamma_0$ | $5.37\ 10^{-5}$ | $1.36\ 10^{-4}$ | 0.594 | 0.001 | 0.0015 | 0.00 |
| S.E.(A) | (20.70) | | | | (1.668) | (3.849) | (22.87) |
| S.E.(B) | (20.74) | (7.86) | | (1.675) | (1221.) | (35.93) |
| $\gamma_1$ | 0.0211 | 0.0203 | 0.0774 | 0.1455 | 0.0710 | 0.0281 |
| S.E.(A) | (0.1824) | | | | (0.3163) | (0.1215) | (0.0251) |
| S.E.(B) | (0.1847) | (0.0465) | | (0.3184) | (38.976) | (0.0361) |
| $\exp(lw_{25})$ | 2.5715 | 2.5715 | 0.0612 | 0.0612 | 0.0612 | 0.0612 |
| S.E.(A) | (0.6472) | | | | | | |
| S.E.(B) | (0.8578) | (0.4808) | | | | |
| | | | | | | |
| $\chi^2$(A) | 91.89 | | 1690 | 34.13 | 175.25 | 306.75 |
| $\chi^2$(B) | 87.59 | 87.55 | 1544 | 31.82 | 148.48 | 780.11 |

[a]

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[a]Note: $MSM$ estimation in levels and first-differences. Standard errors calculated without (A) and with (B) correction for first stage estimation. The next to last colum reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71.
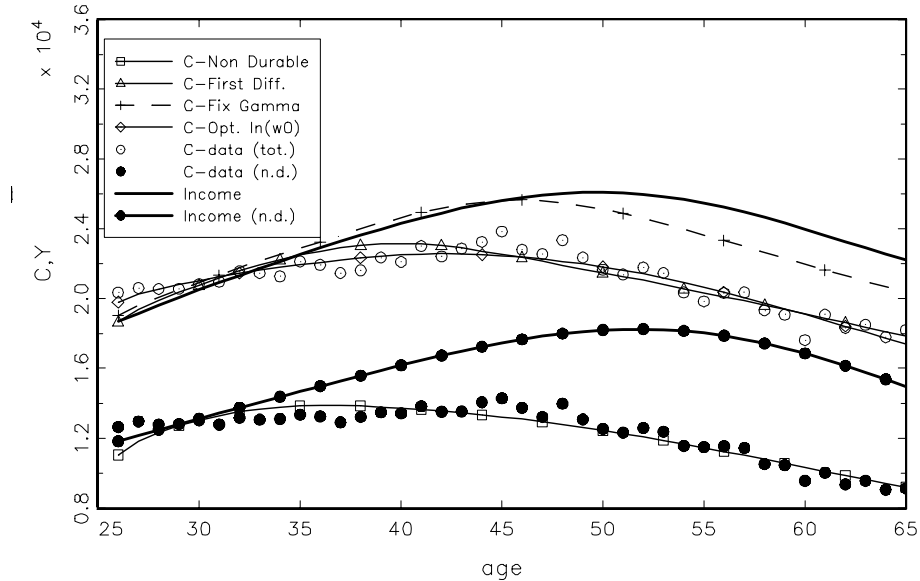
Figure 5.3: Robustness Checks

## 6. Conclusion.

Nearly all currently employed macroeconomic models include a representative agent facing representative shocks. Such a representative agent framework constitutes an extremely powerful tool, mainly because the restrictions that optimizing behavior places upon individuals carry over to the aggregate economy. Yet there is substantial evidence that the complete markets benchmark does not come close to describing household consumption behavior (Attanasio and Davis (1996), Nelson (1994)). This paper, while adopting an explicitly partial equilibrium environment, departs from the complete market assumption. We find important and substantial deviations from the canonical CEQ-LCH representative consumer model, that question the standard macroeconomic practice of representing all consumers as a single, infinitely-lived, rational agent who behaves in accordance with the Permanent Income Hypothesis.

Using individual-level data to construct average profiles of income and consumption over the working lives of households, we demonstrate that consumption remains hump-shaped, even after controlling for family and cohort effects. We then develop a model of consumption behavior embedding realistic levels of uninsurable income uncertainty and estimate *individual* consumption functions using the Method of Simulated Moments. The model fits well and yields tight estimates of the discount rate and intertemporal elasticity of substitution. To the best of our knowledge, this is the first paper which uses explicit measures of individual uncertainty and the life-cycle profile of consumption to identify structural parameters of the utility function and estimate consumption functions. This methodology

is now being used to estimate expanded models that address portfolio choice, labor supply, and retirement behavior (Gakidis (1998), Cagetti (1998) and French (1998)).

Our results indicate that consumers hold only a buffer stock of liquid assets in order to offset labor income fluctuations, until around age 40. During this period, their consumption behavior, while fully optimal, is short-sighted from the perspective of the CEQ-LCH. By contrast, older households save actively for retirement purposes and behave in a manner consistent with the CEQ-LCH. These two phases in consumer behavior are quite distinct and are at the heart of our identification procedure.

According to the estimated model, the neoclassical representative-agent model of aggregage consumption is incorrect precisely because of this changing behavior over the working life. Since households cannot insure one another against idiosyncratic risks to labor income, their behavior shanges as they age and as they experience different labor income trajectories.

An interesting avenue of research explores the aggregate implications of this micro heterogeneity. These can potentially be important. For instance, the uncovered age-variation in behavior may rationalize Campbell and Mankiw (1989) finding that roughly 40% of all agents are 'hand to mouth'. In our interpretation, this may simply reflect the consumption of young households. However, a complete assesment of the aggregate consequences of buffer stock behavior requires an investigation of the general equilibrium consequences of microeconomic heterogeneity. This is a challenging problem that has received much recent attention (Krusell and Smith (1998) and den Haan (1997)).

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# Appendices

## A. Numerical Solution to the Consumer Problem.

This appendix describes our approach to solving numerically the consumer problem.

### A.1. Gauss-Hermite quadrature.

The algorithm exploits the recursive structure of the consumer problem by solving the Euler equation for the optimal consumption rule. Assume for the time being that we know how to compute $c_{t+1}(.)$ for all possible values of cash on hand. One can rewrite the Euler equation (2.4) using the Intertemporal budget constraint (2.3) as:

$$u'(c_t(x)) = \beta R \frac{v(Z_{t+1})}{v(Z_t)} \left( p E \left[ u' \left( c_{t+1} \left( (x - c_t) \frac{R}{G_{t+1}N} \right) G_{t+1}N \right) \right] + \quad \text{(A.1)}$$
$$(1-p) E \left[ u' \left( c_{t+1} \left( (x - c_t) \frac{R}{G_{t+1}N} + U \right) G_{t+1}N \right) \Big| U > 0 \right] \right).$$

The first problem consists in evaluating the expectation in (A.1). Since $N$ and $U$ are log normally distributed, a natural way to evaluate these integrals is to perform a two-dimensional Gauss-Hermite quadrature:

$$E \left[ u' \left( c_{t+1}(x_{t+1}) G_{t+1}N \right) | U > 0 \right] = \int u' \left( c_{t+1}(x_{t+1}) G_{t+1}N \right) dF(N) \, dF(U)$$
$$= \int_{-\infty}^{\infty} f_t(n,u) \, e^{-n^2} e^{-u^2} du \, dn$$
$$\approx \sum_{i,j} f_t(n_i, u_j) \, \omega_{ij},$$

where $f_t(n,u) = \frac{1}{\pi} u' \left( c_{t+1} \left( (x - c_t) \frac{R}{G_{t+1}} e^{\sqrt{2}\sigma_n n} + e^{-\sqrt{2}\sigma_u u} \right) G_{t+1} e^{-\sqrt{2}\sigma_n n} \right)$. The weights $\omega_{ij}$ and nodes $n_i$, $u_j$ are tabulated in Judd (1998). In practice, we performed a quadrature of order 12.

One can then find the root of the Euler equation at any point $x$ using standard methods. We constrain the root to be positive and less than $x$, the current value of cash on hand. As discussed in the paper, this restriction is always satisfied when there are no illiquid assets.

### A.2. Consumption rules.

We initialize the algorithm with the consumption rule at retirement. One can show that the consumption rules for this problem are continuously differentiable as long as there are no liquidity constraints. However, in the presence of liquidity constraints, the consumption rules may exhibit a kink. See Deaton (1991)and Ayagari (1993). We effectively impose a liquidity constraint by not allowing the household to borrow against illiquid assets. This indicates that smooth approximation methods, as advocated by Judd (1992) are inappropriate. Instead, we use a standard discretization method: we specify an exogenous grid for cash-on-hand: $\left\{ x^j \right\}_{j=1}^{J} \subset [0, x^{\max}]$. In order to capture the

curvature of the consumption rule at low values of cash on hand, the grid will be finer for $x \in [0, x^{int}]$. In practice, for each value of cash on hand on the grid, $x^j$, we find the associated consumption $c^j$ that satisfies (A.1). In choosing the size and coarseness of the grid, we face the usual trade-off between precision and computing time. Adding points on the grid gives a finer approximation of the consumption rules. Since the consumption rule at age $t+1$ is the input necessary to get the consumption rule at age $t$, imprecisions could compound over time. On the other hand, the Euler equation is the innermost loop of the entire algorithm. With 100 points on the grid and 40 time period, we must solve (A.1) 4000 times. This takes approximately 1.5 minutes on a Sun Sparc Entreprise10000. We also face a decision regarding the range of cash on hand, $x^{max}$. For small values, cash on hand in sample is likely to move out of the grid. Consumption will then be evaluated using extrapolation methods, much less precise than interpolation. On the other hand, increasing the range for a fixed number of grid points implies less precise estimation of the curvature. One solution consists in endogenizing the grid so that, for instance, cash on hand remains within the grid with probability 0.95. We adopted a simpler approach consisting in checking that cash on hand, in the simulations, remains strictly inside the grid. In practice, we took $x^{max} = 40$, $x^{int} = 2$ and $J = 100$, with 50 points between 0 and $x^{int}$. We checked the quality of the approximation by solving the stationary infinite horizon problem and checking the rate of convergence to the fixed point of the functional Bellman equation. Note that even if simulated $x$ remains strictly inside the grid approximation errors can arise as from the Euler equation since evaluating expected future marginal utility may require looking outside the grid.

### A.3. Simulations

We simulate consumption profiles by generating a sequence of 20000 fictitious income processes over 40 years. For given parameters of the consumption problem, we solve for the optimal age-dependent consumption rules as explained above. Lastly, we calculate the consumption decisions of households receiving these income processes. We then construct life cycle consumption profiles by log-averaging across age.

## B. Asymptotic Variance Covariance Matrix and Test of Overidentifying Restrictions.

We follow Newey and McFadden (1994) and use an expansion method. Define the first-stage sample moments that correspond to the theoretical moments as $m(\chi) = \frac{1}{J} \sum_{j=1}^{J} \mu_j(\chi)$, where $J$ is the number of observations for the first stage. $\hat{\chi}$ is asymptotically normally distributed with a theoretical variance covariance matrix $V_\chi \equiv M_\chi^{-1} \Omega_m M_\chi^{-1'}$ where $\Omega_m = E[\mu(\chi_0)\mu(\chi_0)']$ and $M_\chi = E\left[\frac{\partial \mu(\chi_0)}{\partial \chi'}\right]$. Consistent estimation of $V_\chi$ uses the empirical counterparts to $M_\chi$ and $\Omega_m$ as is standard in $GMM$. Since the first-step is exactly identified, the choice of a weighting matrix is irrelevant here. $\hat{\chi}$ is an asymptotically linear estimator with influence function $\psi_j = -M_\chi^{-1} \mu_j(\chi_0)$ where $\sqrt{J}(\hat{\chi} - \chi_0) = \frac{1}{\sqrt{J}} \sum_{j=1}^{J} \psi_j + o_p(1)$.

The first order condition for the second stage estimator is:

$$g'_\theta\left(\widehat{\theta}, \hat{\chi}\right) W g\left(\widehat{\theta}, \hat{\chi}\right) = 0,$$

where $g_\theta\left(\widehat{\theta}, \hat{\chi}\right) = \frac{\partial g(\widehat{\theta}, \hat{\chi})}{\partial \theta'}$. Expanding $g(\hat{\theta}; \widehat{\chi})$ around $\theta_0$ and rearranging yields:

$$\sqrt{I}(\hat{\theta} - \theta_0) = -\left(g_\theta(\hat{\theta}; \widehat{\chi})' W g_\theta(\bar{\theta}; \widehat{\chi})\right)^{-1} g_\theta(\hat{\theta}; \widehat{\chi})' W \sqrt{I} g\left(\theta_0; \widehat{\chi}\right),$$

where $\bar{\theta}$ denotes a consistent mean value. Expanding the same term further, this time around $\chi_0$, we obtain:

$$\begin{aligned}
\sqrt{I}(\hat{\theta} - \theta_0) &= -\left(g_\theta(\hat{\theta}; \widehat{\chi})' W g_\theta(\bar{\theta}; \widehat{\chi})\right)^{-1} g_\theta(\hat{\theta}; \widehat{\chi})' W \times \\
&\qquad \left[\sqrt{I} g\left(\theta_0; \chi_0\right) + \sqrt{\frac{I}{J}} g_\chi\left(\theta_0; \bar{\chi}\right) \frac{1}{\sqrt{J}} \sum_{j=1}^{J} \psi_j + o_p(1)\right].
\end{aligned}$$

By the Slutsky and central limit theorems, $\sqrt{I}(\hat{\theta} - \theta_0)$ converges in distribution to a mean zero normal distribution with asymptotic covariance matrix:

$$V_\theta = (G_\theta' W G_\theta)^{-1} G_\theta' W E\left[\left(g(\theta_0; \chi_0) + \sqrt{\frac{I}{J}} G_\chi \psi\right)\left(g(\theta_0; \chi_0) + \sqrt{\frac{I}{J}} G_\chi \psi\right)'\right] W G_\theta (G_\theta' W G_\theta)^{-1},$$

where $G_\theta = E[\frac{\partial \zeta(\theta_0, \chi_0)}{\partial \theta'}]$ is $T \mathrm{x} k$ and $G_\chi = E[\frac{\partial \zeta(\theta_0, \chi_0)}{\partial \chi'}]$ is $T \mathrm{x} r$. As the mean values converge to the true ones the sample gradients converge to their theoretical counterparts.

Since our first-stage and second-stage estimator use different structural models and mostly different data, we assume that the first-stage and second-stage moments are uncorrelated. In that case, the formula simplifies to:

$$\begin{aligned}
V_\theta &= (G_\theta' W G_\theta)^{-1} G_\theta' W E\left[g(\theta_0; \chi_0) g(\theta_0; \chi_0)' + \frac{I}{J} G_\chi \psi \psi' G_\chi'\right] W G_\theta (G_\theta' W G_\theta)^{-1} \\
&= (G_\theta' W G_\theta)^{-1} G_\theta' W \left[\varsigma \Omega_g + \vartheta G_\chi V_\chi G_\chi'\right] W G_\theta (G_\theta' W G_\theta)^{-1}, \qquad \text{(B.1)}
\end{aligned}$$

where $\Omega_g = E\left[h(\theta_0, \chi_0) h(\theta_0, \chi_0)'\right]$ is $T \mathrm{x} T$, $\varsigma = \lim_{I \to \infty}\left(1 + \frac{I}{L}\right)$ and $\vartheta = \lim_{I \to \infty} \frac{I}{J}$. This makes clear that as the relative precision of the first stage increases (i.e $\vartheta \to 0$), the correction for the first stage disappears. $\varsigma$ controls for the simulation error. As the simulation becomes more and more precise, $\varsigma \to 1$. To construct an estimate of (B.1), we replace the theoretical concepts with their empirical counterparts, as is done in the usual GMM framework.

We use two different choices of the weighting matrix in our estimation. First, we choose a matrix which, while not fully optimal, does not depend on the fitted model. This is motivated by the observation that optimally weighting GMM estimators can worsen finite-sample bias (see for example West, fu Wong, and Anatolyev (1998)). We set the weighting matrix to $\Omega_g^{-1}$. This is the optimal weighting matrix when the first stage correction does not matter, i.e. when $G_\chi = 0$ or $\vartheta = 0$. This yields:

$$V_\theta = (G_\theta' \Omega_g^{-1} G_\theta)^{-1} G_\theta' \Omega_g^{-1} \left[\varsigma \Omega_g + \vartheta G_\chi V_\chi G_\chi'\right] \Omega_g^{-1} G_\theta (G_\theta' \Omega_g^{-1} G_\theta)^{-1}.$$

While $\Omega_g^{-1}$ is not the optimal weighting matrix given the first-stage correction, this case is easy to implement since an estimate of $\Omega_g$ can be obtained directly as the

sample counterpart of $E\left[(\ln C_i - E(\ln C_i))(\ln C_i - E(\ln C_i))'\right]$. Since we do not observe households across years, this is equivalent to assuming that $\Omega_g$ is diagonal with $\hat{\Omega}_{gt} = \frac{1}{I_t}\sum_{i=1}^{I_t}\left(\ln C_{i,t} - \ln \bar{C}_t\right)^2$. In the case of difference estimation, we use an identity weighting matrix since different households are observed in different years.

Second, we also employ the optimal weighting matrix $W = \left(\varsigma\Omega_g + \vartheta G_\chi V_\chi G'_\chi\right)^{-1}$. In this case, the variance formula simplifies to:

$$V_\theta = (G'_\theta\left[\varsigma\Omega_g + \vartheta G_\chi V_\chi G'_\chi\right]^{-1} G_\theta)^{-1}.$$

This requires using the estimates from our first case and constructing an estimate of $W$ using sample counterparts.

There are two other interesting special cases. First, if the second stage is exactly identified ($T = k$), the covariance matrix simplifies to $V_\theta = G_\theta^{-1}\left[\varsigma\Omega_g + \vartheta G_\chi V_\chi G'_\chi\right]G_\theta^{-1'}$. Second, if there is no cross derivative, so that $G_\chi = 0$, the first stage uncertainty does not affect the second stage. In this case the formula collapses to the familiar one in which first stage estimation does not appear: $V_\theta = \varsigma\left(G'_\theta W G_\theta\right)^{-1}G'_\theta W \Omega_g W G_\theta\left(G'_\theta W_g G_\theta\right)^{-1}$.

This methodology also provides a useful test of the overidentifying restrictions in the second stage. If the model is correct, the statistic:

$$\chi_{T-2} = \frac{I}{\varsigma}g\left(\hat{\theta};\hat{\chi}\right)'\hat{\Omega}_g^{-1}g\left(\hat{\theta};\hat{\chi}\right),$$

is distributed asymptotically as Chi-squared with $T - 2$ degrees of freedom.


## C. The CEX.

We use the CEX family, member and detailed expenditure files for years 1980 to 1993, as provided by the NBER. Most of our information about the CEX is obtained from Bureau of Labor Statistics (1993, and years 1980-1992) and conversations with BLS statisticians. Households are discarded if they are missing any of the information necessary for the regressions, if they report changes in age from the second to fifth interview of more than a year or less than zero years, if they are classified as incomplete income reporters, or if their reporting implies less than $1000 in annual income or consumption.

We use information about the reference person to assign the household to cells, unless the reference person is female. In this case we use the spouses information. If there is no spouse, or his information is missing, the household is discarded. When this cut was made it eliminated 20% of the sample. All information besides individual labor income and consumption is taken from the family files. Values are assigned to a household based on information gathered in the fifth interview, otherwise information is used from the second interview, or, if it is not available, the household is discarded. Households should not be matched across 1985 to 1986, and are not. Care is taken to assure consistency in our data despite variable classification changes through time, and across reference person and spouse. Information was provided by the Division of the CEX in the Bureau of Labor Statistics about various issues including the matching of occupation codes from 1980-81 to later years.

Pension contributions, income, Social Security contributions, and all asset income all refer to the past twelve months. Our definition of pension contributions is the sum over

the CEX subcategories and thus includes private pensions, public pensions, Railroad Retirement pensions, and self-employed, IRA, and Keogh plans. If the after-tax family income variables is topcoded, reference person and spouse labor incomes are subtracted and we add, for each, the variable created by multiplying the earnings in last paycheck by the fraction of the year the pay period covers. These labor income variables are the sole variables from the member files used. Assets and asset income refers to the sum over savings accounts, checking accounts, bonds, and stocks, as of the time of interview. Each household is assigned to a year based on the midpoint between the first and fifth interview if both data are available; otherwise simply the single interview date is used. Age is the average of both interviews if both are available, otherwise it is the single one available. Due to some extreme reports, we reset reported tax rates above 50% back to 50%, and below zero percent to zero. We perform a similar exercise for Social Security contribution rates and pension contribution rates, using 25% as the upper bound.

While topcoding is very infrequent in consumption information, the household annual income variable reflects summation over a topcoded item for roughly half a percent of our households. Since, in most years, topcoding occurs at $100,000 in income subcategories, reported individual annual labor income is the source of almost all income topcoding problems. However, households are also asked the gross amount of their paycheck and what length of time-period this paycheck covers. By multiplying these two variables together, we construct a second measure of annual labor income. Topcoding on this variable occurs only for a few cases. We correct our measure of after-tax family income by replacing the reported annual labor income in family income with our constructed measure whenever the family income variable is topcoded. We are able to correct almost all topcoding.

Consumption data is compiled from the detailed expenditure files as all expenditures by a household except for those for health care, mortgage interest, and education. The consumption level is then the average monthly expenditure times twelve. Five percent of households have consumption data for 4, 7, 10, 13, or 14 months and these households' consumption are treated as if they were over 3, 6, 9, and 12 months. That is the recall interview period extended beyond the basic three months and some expenditures are recorded in a later month. BLS statisticians recommend treating these expenditures as if they occurred in the preceding month. Those covering 1 or 2 months (one percent of the sample) were dropped.

One might be concerned about the coverage of income and consumption and thus about the relative levels of the profiles. We get reasonable relative levels of consumption and income as does Attanasio (1994), who uses relative levels in his analysis of saving in the US. Also, we have checked a total income profile and a food consumption profile against similar profiles constructed from PSID data. In the case of food, the level and shape of the two profiles were nearly identical. For total income profile, the shapes of the profiles were less similar, but still quite close. Finally, in our results section, we present evidence on estimation which instead uses only information from the changes in income and consumption. See the working paper version for further details.

The unemployment rates merged to the CEX are the regional unemployment rates for civilian population from the Household survey conducted and published by the Bureau of Labor Statistics in "Employment and Earnings." The GNP IPD PCE is from Council Of Economic Advisors (1995).

## D. Construction of Life-Cycle Profiles.

We construct profiles of consumption over the working life both smoothed and un-smoothed. Un-smoothed profiles are constructed by averaging our reconstructed measure of consumption by age. Smooth profiles are constructed by estimating an equation similar to (4.3) which fixes $\pi_2$ at the value estimated on the un-smoothed data, replaces the age and cohort dummies by fifth order polynomials, and extends the highest age to 70 to avoid some of the endpoint problems commonly encountered with polynomial smoothers.

We generate a profile for per-household-equivalent consumption for a constant family size, by replacing the typical family size, $\bar{f}_t$, for each age with its sample average, 2.8. Finally, we construct the typical profile of the shift in marginal utility caused by changes in family size for our typical household as

$$\widehat{v(Z_{i,t})^{\frac{1}{\rho}}} = \exp\left(\frac{1}{I_t} \sum_{i=1}^{I_t} f_i \hat{\pi}_2\right)$$

using $\widehat{\pi_2}$ estimated from equation (4.3), where $I_t$ is the number of households observed at age $t$. The smoothed version of this is constructed by predicting $\bar{f}_t \widehat{\pi_2}$ using a fifth-order polynomial.

We employ the exactly same procedure for income, except that the adjustment for family size heterogeneity, $\pi_2$, is set equal to its value from the consumption regression. $\bar{Y}_t = \exp[\frac{1}{I_t} \sum_{i=1}^{I_t} \ln Y_{i,t}]$ is an estimate of average income by age. To estimate separate profiles by education group or by occupation group, we add interaction terms between the categories considered and age and retirement status.