

ECON 5345 Homework 1 Report

Harlly Zhou

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Question 1

- a. Note that for any t , we have

$$C_t = C_{t-3} + e_{t-2} + e_{t-1} + e_t.$$

Substituting this into the

$$\begin{aligned}\Delta C_t &\equiv \frac{C_t + C_{t+1} + C_{t+2}}{3} - \frac{C_{t-3} + C_{t-2} + C_{t-1}}{3} \\ &= \frac{e_{t-2} + 2e_{t-1} + 3e_t + 2e_{t+1} + e_{t+2}}{3}.\end{aligned}$$

- b. No. They are correlated. At $t + 3$, we have

$$\Delta C_{t+3} = \frac{e_{t+1} + 2e_{t+2} + 3e_{t+3} + 2e_{t+4} + e_{t+5}}{3}.$$

It is clear that

$$\text{Cov}(\Delta C_t, \Delta C_{t+3}) = \frac{2}{9}(\text{Var}[e_{t+1}] + \text{Var}[e_{t+2}]) > 0,$$

as long as $\text{Var}[e_{t+1}] + \text{Var}[e_{t+2}] > 0$.

- c. No for the first part. Since e_{t-2} and e_{t-1} are known, ΔC_t is correlated with C_{t-2} and C_{t-1} .

Yes for the second part. Information known at $t - 3$ only includes white noise no later than $t - 3$, while ΔC_t is a linear combination of white noises after $t - 3$. Given the serial uncorrelation property of white noise, they are not correlated.

- d. The ACF and PACF of the change in measured consumption are shown in Figure 1. Codes in “hw1_q1d.R”.

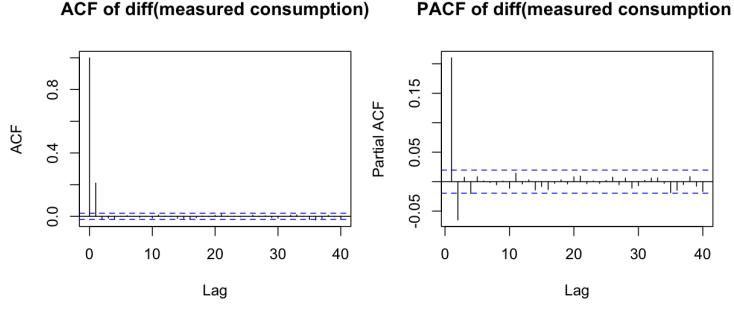
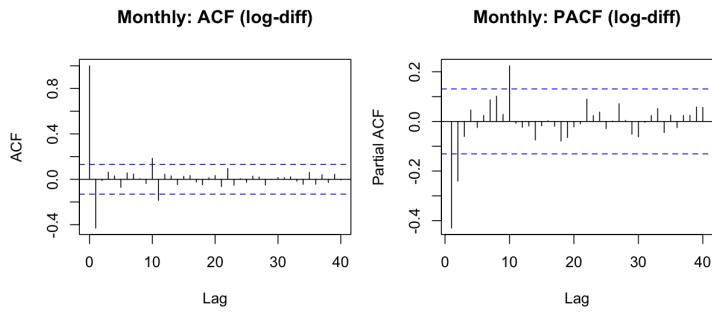
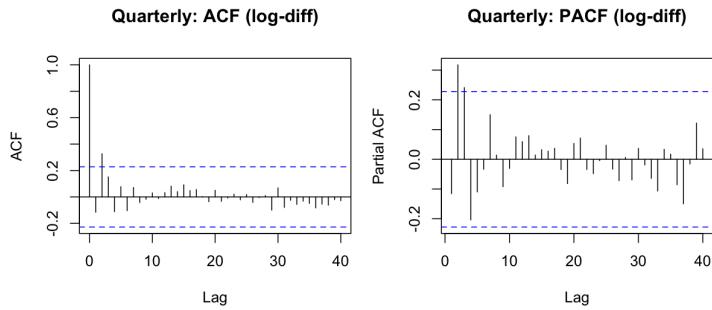


Figure 1: ACF and PACF of the change in measured consumption

- e. The ACF and PACF of the change in measured consumption are shown in Figure 2a and Figure 2b. Codes in “hw1_q1e.R”.



(a) Monthly data: Jan 2007–Sep 2025 (exclude months after Sep 2025 since quarterly data ends at 2025Q3).



(b) Quarterly data: 2007Q1–2025Q3.

Figure 2: ACF and PACF of the change in consumption (monthly and quarterly). Data source: FRED PCENDC96.

Monthly data shows negative autocorrelation at lag 1, while quarterly data shows positive autocorrelation at lag 1 and 2, even 3. This shows higher persistence in quarterly data, reflecting the fact that averaging over more months introduces more serial correlation, as shown in part (b).

Question 2

- a. Since d is observable, the firm has the following optimization problem:

$$\max_p \Pi(p, d) = p^{-\frac{d+1}{d}}(p - 1).$$

The FOC condition gives

$$-\frac{d+1}{d}p^{-\frac{d+1}{d}-1}(p - 1) + p^{-\frac{d+1}{d}} = 0.$$

Multiplying both sides by $p^{\frac{d+1}{d}}$ and rearranging the terms, we get

$$p^*(d) = 1 + d.$$

- b. The code is in “hw1_q2b.R”. Since I used R, something may be different from the Matlab code. To find the initial point, I used the help of AI because uniform initial point produces a flat plane. The 3D plot is shown in Figure 3.

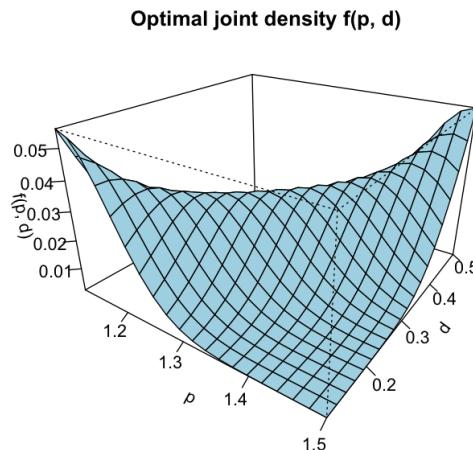


Figure 3: Optimal joint density $f(p, d)$

Question 3

- a. No. When the observations are treated as cross-sectional data, the model becomes

$$y_{i1} = \mu_i + \rho y_{i0} + e_{i1} = \rho y_{i0} + \epsilon_{i1},$$

where $\epsilon_{i1} = \mu_i + e_{i1}$. However, note that, typically, we have

$$\begin{aligned}\text{Cov}(y_{i0}, \mu_i) &= \text{Cov}(\mu_i + \rho y_{i,-1} + e_{i0}, \mu_i) \\ &= \text{Var}(\mu_i) + \rho \text{Cov}(y_{i,-1}, \mu_i) + \text{Cov}(e_{i0}, \mu_i) \\ &\neq 0.\end{aligned}$$

Then the orthogonality condition is violated since the error term also contains μ_i .

When setting $\mu_i = \mu$ for all i 's, the covariance becomes zero since y_{i0} have zero covariance with constant μ .

The estimators will converge to the true value as $N \rightarrow \infty$ if the orthogonality condition holds.

- b. Yes. The model now becomes

$$y_t = \mu + \rho y_{t-1} + e_t.$$

Using backward induction, we can write y_t as a function of $\{e_s | s \leq t\}$, denoted by $h(\mathbf{e}_t)$, where \mathbf{e}_t denotes the vector of all e_s 's with $s \leq t$. Then independence between e_s 's implies that

$$\mathbb{E}[y_{t-1} e_t] = \mathbb{E}[h(\mathbf{e}_{t-1}) e_t] = 0.$$

Since e_s is with zero mean, the orthogonality conditions are satisfied and OLS works.

Table 1: Q3(c): Fixed-effects (demeaned) OLS estimates with $\rho = 0.9$, $\sigma = 1$, $T = 2$.

N	T	$\hat{\rho}$
100	2	-0.006711
500	2	-0.182214
1000	2	-0.038676

c.

d.

Table 2: Q3(d): Fixed-effects (demeaned) OLS estimates with
 $\rho = 0.9$,
 $\sigma = 1$, $N = 1000$.

N	T	$\hat{\rho}$
1000	5	0.450082
1000	20	0.776592